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Essays On Inequality And Frictions In The Labor Market

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Abstract

Frictions in the labor market are important forces driving the unequal outcomes in wages, mobility, and work arrangements. I study the life cycle gender wage gap and alternative work arrangements in labor markets with information frictions and search frictions.

The first chapter proposes a novel mechanism behind the life cycle gender wage gap: female workers are offered wage contracts that suppress future job mobility because of employers' screening under asymmetric information about female job attachment. Job attachment is lower among women than men, with substantial individual-level heterogeneity. Employers value job attachment but job attachment is not directly observed. I show that the information problem of female job attachment is detrimental to female labor market outcomes. I propose a model where there is information asymmetry about female job attachment but not about male job attachment in a frictional labor market. To screen for female job attachment, employers offer separating wage contracts that distort the wage profiles of high-attachment women. The distortions suppress female job-to-job mobility, resulting in worse labor market outcomes than comparable men.

The second chapter empirically investigates the life-cycle gender differences in labor market outcomes, based on the theoretical framework developed in the previous chapter. Using data from the National Longitudinal Survey of Youth, I document substantial gender differences in the likelihood of employment-to-nonemployment transitions, confirming the central assumption of the theoretical model. I document empirical evidence on the gender differences in job mobility and wage return to tenure, consistent with the theoretical predictions. I quantitatively implement the model and assess the contributing factors to the gender wage gap. The parameterized model generates gender gaps in wage growth and distribution along the job ladder. I show that the gender differences in the job value distribution are the main contributor to the gender wage gap.

The third chapter studies alternative work arrangements in an environment with moral hazard and adverse selection. Workers are hired on a per-task basis, whose effort and marginal product may not be observed. The employer chooses between offering a bonus contract and selling off the task. Bonus contracts provide workers with insurance against output risks but subject them to organizational uncertainty. Allowing workers to buy out the task induces higher worker effort but provides no risk sharing. I characterize the tradeoff between the two arrangements. Adverse selection limits the use of the latter arrangement on low marginal product workers and hence the employer's ability to induce their effort.

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ABSTRACT

ESSAYS ON INEQUALITY AND FRICTIONS IN THE LABOR MARKET

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José-Víctor Ríos-Rull

Frictions in the labor market are important forces driving the unequal outcomes in wages, mobility, and work arrangements. I study the gender wage gap and alternative work arrangements in labor markets with information frictions and search frictions.

The first chapter proposes a novel mechanism behind the life cycle gender wage gap: female workers are offered wage contracts that suppress future job mobility because of asymmetric information about female job attachment. Job attachment is lower among women than men, with substantial individual-level heterogeneity. Employers value job attachment but job attachment is not directly observed. To screen for female job attachment, employers offer separating wage contracts that distort the wage profiles of high-attachment women. The distortions suppress female job-to-job mobility, resulting in worse labor market outcomes than comparable men.

The second chapter empirically investigates the life-cycle gender differences in labor market outcomes, based on the theoretical framework developed in the previous chapter. Using data from NLSY79, I document empirical evidence on the gender differences in labor market transitions and wage return to tenure, consistent with the theoretical assumptions and predictions. The parameterized model generates gender gaps in wage growth and distribution along the job ladder. I show that the differences in the job value distribution are the main contributor to the gender wage gap.

The third chapter studies alternative work arrangements in an environment with moral hazard and adverse selection. Workers are hired on a per-task basis, whose effort and marginal product may not be observed. The employer chooses between offering a bonus

contract and selling off the task. Bonus contracts provide workers with insurance against output risks but subject them to organizational uncertainty. Allowing workers to buy out the task induces higher worker effort but provides no risk sharing. I characterize the tradeoff between the two arrangements. Adverse selection limits the use of the latter arrangement on low marginal product workers and hence the employer's ability to induce their effort.

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CHAPTER 1

Job Attachment and Life Cycle Gender Wage Gap: A Theory

1.1. Introduction

Employment interruptions are more common among women than men, and women are more likely to take breaks from the labor force due to nonwork reasons.[1] Job attachment, the tendency to stay in the workforce, is valued by employers because of the costs of filling vacant jobs and losses of output. In this chapter, I argue that the gender difference in job attachment detriments the earning prospects of all women, including those who are less likely to experience employment interruptions. I propose a theory that characterizes a novel mechanism behind the gender wage gap: information asymmetry about female job attachment leads to wage distortions that suppress female job mobility, resulting in women's lower wage growth and slower progress on the job ladder. Wage distortions arise because wage trajectory is a useful screening device for employers to distinguish job attachment among female applicants. High-attach women value future wages more than low-attachment women because the former see a lower likelihood of future nonemployment. I show that in contrast to the direct effects of employment interruptions on people who separate from the workforce, the indirect effects of lower female job attachment are ex-ante costs borne by high-attachment women. The theoretical model investigates the effects of information asymmetry about female job attachment on women's labor market outcomes. Female workers are privately informed of their individual exogenous rate of separating from jobs into unemployment, which I refer to as “job attachment". I interpret the exogenous separation rate as stemming from issues not directly related to work, such as family responsibilities. Male workers, on the other hand, are homogeneous in job attachment. Workers accumulate human capital that increases productivity while employed. Workers and firms meet in a continuum of submarkets of different characteristics. Workers only visit the submarkets that maximize their expected payoff of job search in the trade-off between employment value and job-finding rate. Firms can enter

[1]Blau and Kahn (2017) surveyed related literature.

any submarket and post vacant jobs at a cost. Because vacancy filling is frictional and costly, firms favor workers with high job attachment. I assume that firms perfectly commit to long-term contracts in job postings. Workers with different job attachment discount future wages differently. High-attachment workers place a higher weight on future wages because they are more likely to stay in employment than low-attachment workers. Therefore, deferring wage growth by the same amount has smaller impacts on the value delivered to high-attachment workers. Firms are able to screen female workers' job attachment by taking advantage of the difference in discounting of future payoffs. In equilibrium, it is privately optimal for firms to offer separating contracts in an economy with directed search. This result arises because firms can always tailor the shape of wage contracts by a sufficiently small amount to make sure that only workers of the preferred type are attracted to the submarket. I show that wage contracts designed to attract high-attachment women are distorted due to screening, and the distortions feature additional wage backloading in order to exclude low-attachment applicants. Such distortions suppress female job-to-job mobility, leading to slower ascension along the job ladder than comparable men since moving to better jobs is a crucial factor in lifetime wage growth. On the other hand, low-attachment women experience more unemployment episodes, which sets back their career and slow down human capital growth. Taken together, women see worse outcomes in terms of landing and keeping well-paid jobs on average than comparable men, due to direct and indirect effects of low female job attachment.

1.1.1. Related Literature

This chapter is closely related to Shi (2009), who showed that firms offer increasing wage schemes to retain workers in competitive search equilibria where employed workers can search for other jobs. Similar results of wage backloading in environments with random search on the job are established in Burdett and Coles (2003) and Stevens (2004). In this chapter, there is unobserved heterogeneity about job attachment in addition to on-the-job search. Apart from reducing voluntary job-to-job mobility, there is another reason why it is privately optimal for firms to backload worker compensation: it helps to select workers with high job attachment.

The idea of using deferred benefits to screen agents with exogenous separation probabilities can be traced back to early works such as Salop and Salop (1976). The result that employers post separating contracts is in the spirit of Guerrieri et al. (2010), who studied separating equilibria with directed search and adverse selection in a static setting. In this chapter, I study screening using dynamic contracts. Apart from matching probabilities, employers use an additional screening device, the shape of wage contracts, which separates workers who differ in the valuation of future wages.

This chapter builds on the literature on on-the-job search in frictional labor markets (Burdett and Mortensen (1998), Postel-Vinay and Robin (2002), Menzio and Shi (2011)) and the directed search framework pioneered by Moen (1997). It also contributes to the growing literature of adverse selection in competitive search equilibria (e.g., Michelacci and Suarez (2006), Guerrieri (2008), Delacroix and Shi (2013)).

The rest of the chapter is organized as follows. In the next section, I present the model and analyze the equilibrium properties. The conclusion follows.

1.2. Model

Time is continuous and has an infinite horizon. There is a continuum of workers with a total measure of one. There are a large number of firms. Workers and firms discount future payoffs at the same rate $\rho > 0$. At rate $d > 0$, workers permanently leave the labor force and are replaced by an unemployed worker with no work experience. Firms live infinitely. Workers are risk-averse with instantaneous utility function $u(\cdot)$. Utility function $u(\cdot)$ satisfies $u' > 0$, $u'(0) = \infty$, and $u'' < 0$. Firms are risk-neutral and operate a constant return to scale technology.

Workers accumulate human capital while employed. When workers are without a job, the accumulation stalls and yet human capital does not depreciate. The level of human capital is the same among workers with the same cumulative work experience $x \geq 0$, and they produce the same flow output $y(x)$ while employed, where $y' \geq 0$. Work experience is publicly observed. Workers produce $b > 0$ if not employed, independent of human capital. Workers cannot borrow against or store their output.

1.2.1. Worker Type

Workers are subject to exogenous shocks that force them into unemployment while employed. The arrival rate of unemployment shocks depends on a worker's type and experience x. There are two types of workers: workers with high job attachment or type-h workers, and workers with low job attachment or type-l workers. Workers are privately informed of their own type. The rate of separation of a worker with type i and experience level x is $\delta^i(x)$, $i \in \{h, l\}$. And $\delta^l(x) > \delta^h(x)$ for any $x \geq 0$.

Type compositions differ by gender. Male workers are homogeneous with high job attachment, while female workers are heterogeneous and can be either type. Low-attachment workers are the female workers who are more likely to quit jobs to spend time in home production. Among female workers, the fraction of type-i individuals is α_i, $i \in \{h, l\}$.

1.2.2. Job Posting

Workers and firms look to match with each other in a continuum of submarkets. Firms can enter any submarket freely and post vacancies. Maintaining a posting incurs flow cost $k > 0$.

With each job posting, firms announce a contract and a worker type that the contract is intended for. Note that because a worker's type is his or her private information, there is no enforcement of the worker type requirement. Firms fully commit to the contracts. A contract specifies the firm's actions for all future contingencies and determines the value of employment of an employee at any date on the job given the employee's type. Each submarket is indexed by an intended type, the expected lifetime utility delivered to a worker of the intended type, a required experience level, and a gender. [2]

[2] It is illegal for employers to discriminate based on gender. However, regulations have never eliminated discrimination in the recruiting process or in the workplace. A recent incident has websites showing different job ads based on users' profiles. On an additional note, the expected output from a match with a female

I assume that firms do not observe and cannot verify any outside offer made to their employees, and that flow transfers must be finite. As a result, it suffices to focus on contracts that stipulate flow wage at any date on the job, or in other words, wage tenure contracts.

1.2.3. Job Search

Matching is random and anonymous. The number of matches in a submarket with applicants of measure a and vacancies of measure v is determined by a constant-return-to-scale, differentiable function $M(a, v)$. Market tightness is the vacancy-applicant ratio $\theta = v/a$. I define the job finding rate function $\mu(\theta) := M(1, \theta)$ and the job filling rate function $\nu(\theta) := M(1/\theta, 1)$.

Both unemployed and employed workers search for new jobs. Workers tradeoff between value of employment and market tightness when deciding on which submarket to search in. For workers of a given type, gender, and experience x, market tightness is a function of employment value. In a submarket that offers a higher value of employment, market tightness is also higher and hence job finding rate is lower. Because the same submarket might deliver different employment values to workers of different types, high-attachment women and low-attachment women face different market tightness functions $\Theta^h(\cdot; x)$ and $\Theta^l(\cdot; x)$, given their experience level x. On the other hand, all the male workers of experience x face the same tightness function $\Theta^m(\cdot; x)$. The gender subscript for female workers and the type subscript for male workers are omitted for exposition henceforth.

Formally, the job search problem of a female worker of type i, who has experience x and whose current value of employment (or unemployment) is v, is to maximize the expected gains across all submarkets:

$$\max_{v'} \mu\Big(\Theta^i(v'; x)\Big)(v' - v). \tag{1.1}$$

The policy function arising from the job search problem is denoted by $e^i(v, x)$. The value of search is denoted by $S^i(v, x)$. And the job finding rate in the optimal submarket is denoted by $g^i(v, x)$.

employee is different from that of a comparable male employee due to potentially lower job attachment.

Similarly, the job search problem of a male worker with experience x and current value v is

$$\max_{v'} \mu\Big(\Theta^m(v';x)\Big)(v'-v). \tag{1.2}$$

The corresponding policy function, value of search, and job finding rate are e^m, S^m, and g^m.

1.2.4. Value Functions

Unemployed workers consume b and keep searching for new jobs. The expected utility in unemployment of a worker with experience x and type or gender $i \in \{h,l,m\}$, $V_u^i(x)$, satisfies

$$(\rho+d)V_u^i(x) = u(b) + S^i\Big(V_u^i(x), x\Big). \tag{1.3}$$

Given a wage tenure contract $w(\cdot)$, the value of employment of a worker who has current tenure τ, work experience x_0 when starting on the job, and type or gender $i \in \{h,l,m\}$, $V^i(\tau|x_0,w)$, satisfies

$$\begin{aligned}(\rho+d)V^i(\tau|x_0,w) =& u(w(\tau)) + \delta^i(x_0+\tau)\Big(V_u^i(x_0+\tau) - V^i(\tau|x_0,w)\Big) + \\ & S^i\Big(V^i(\tau|x_0,w), x_0+\tau\Big) + \dot{V}^i(\tau|x_0,w).\end{aligned} \tag{1.4}$$

On the firm side, the value of a job is the expected present value of future profit flow. Given a wage contract $w(\cdot)$, the value of a job matched with a worker with current tenure τ, starting experience x_0, and type or gender $i \in \{h,l,m\}$, $J^i(\tau|x_0,w)$, satisfies

$$\begin{aligned}\Big(r^i(x_0+\tau) + g^i\big(V^i(\tau|x_0,w), x_0+\tau\big)\Big)J^i(\tau|x_0,w) =& \Big(y(x_0+\tau) - w(\tau)\Big) + \\ & \dot{J}^i(\tau|x_0,w),\end{aligned} \tag{1.5}$$

where $r^i(x) := \rho + d + \delta^i(x)$.

Because of unobserved heterogeneity, female workers can misreport their type to search in submarkets not intended for them. It is necessary to distinguish between the value delivered to the intended type and the unintended type by the same wage contract. For the rest of the paper, I will use V to denote the employment value of workers who truthfully report their type when entering submarkets and U to denote the employment value of workers who misreport their type. Similar to (1.4), the employment value of a contract w to a type-j worker misreporting her type follows

$$\begin{aligned}(\rho + d)U^j(\tau|x_0, w) =& u(w(\tau)) + \delta^j(x_0 + \tau)\Big(V_u^j(x_0 + \tau) - U^j(\tau|x_0, w)\Big) + \\ & S^j\Big(U^j(\tau|x_0, w), x_0 + \tau\Big) + \dot{U}^j(\tau|x_0, w). \end{aligned} \tag{1.6}$$

1.2.5. Equilibrium

Definition. A competitive search equilibrium consists of (1) market tightness functions $\Theta^i(v'; x)$ and wage tenure contracts $w(\cdot|i, v', x)$ over submarket indices (i, v', x), $i \in \{h, l, m\}$, (2) search strategies $e^i(v, x)$, $i \in \{h, l, m\}$, (3) a pair of measures of male and female workers $\{F^m, F^f\}$ over male and female submarkets (4) a pair of fractions of the two types $\{\lambda^h, \lambda^l\}$ over female submarkets such that

(i) given Θ^i, $e^i(v, x)$ is the optimal job search policy of workers with type or gender i, current value of employment or unemployment v, and current experience x,

(ii) given Θ^i and e^i, $w(\cdot|i, v', x)$ maximizes a firm's expected profits from a match that delivers v' to a type-i employee with experience x when starting on the job,

(iii) given Θ^i and w^i, a firm's expected payoff from posting a vacancy cannot exceed the flow maintenance cost k, with equality in submarkets with positive tightness and worker mass

(iv) F^i and λ^i are consistent with workers' search behavior e^i and the labor market clears:

$$\int \lambda^i(v,x)dF^f(v,x) = \alpha^i,\ i\ \in \{h,l\},$$
$$\int dF^m(v,x) = 1.$$

Part (ii) of the definition of equilibrium imposes restrictions on belief in submarkets outside the equilibrium support of worker measure. It implicates that no off-equilibrium submarket provides a higher value of search than the equilibrium value of search for any worker. The restriction is similar to the refinement of equilibrium in Guerrieri et al. (2010) and Gale (1996). As a result, it is without loss of generality to study separating equilibria.

Proposition 1. *Any equilibrium either features firms posting separating contracts or is payoff-equivalent to an equilibrium with separating contracts.*

The result of separating contracts can be proven by contradiction. Suppose that some equilibrium contract attracts both types of female workers. A firm considers a deviating contract that back-loads wage growth more and adjusts the wage level such that the more attached type is indifferent between the original contract and the deviating contract. Because the two types have different weights of future wages on the job, the less attached type finds the deviating contract worse than the original contract. Now consider the composition of workers attracted by the deviating contract. To be attractive to the less attached type, the submarket of the deviating contract must have a higher tightness and hence a higher job finding rate, which implies that the submarket offers a higher value of search to the more attached type. This constitutes a contradiction to Part (ii) of the equilibrium definition. Therefore, only high-attachment workers show up at the deviating submarket. Such a deviation can be sufficiently small because of the nature of wage contracts and induces a nontrivial change in applicant composition, allowing for a profitable deviation on the firm side. As a result, no firm offers contracts that attract both types when they strictly prefer a certain type.

There might exist submarkets in which firms are indifferent between the two types. Because of the constant return to scale matching technology, a payoff-equivalent equilibrium where every submarket features a separating contract can be constructed where the pooling submarket is proportionally divided into two separating submarkets.

1.2.6. Optimal Contracts

Because it is without loss of generality to consider separating contracts in equilibrium, a firm's profit maximization problem can be characterized as maximizing the value of a match with the intended type while excluding the other type. Specifically, in a submarket indexed by (i, v_0, x_0), the wage contract w must satisfy the other type j's incentive compatibility constraint:

$$\mu\big(\theta^i(v_0, x_0)\big)U^j(0|x_0, w) \leq \min_v \Big\{S^j(v, x_0) + \mu\big(\theta^i(v_0, x_0)\big)v\Big\}. \tag{1.7}$$

The incentive compatibility constraint implicates that no type-j female workers find it optimal to search in the said submarket.

Another constraint is the promise-keeping constraint that the contract must deliver v_0 to a matched female worker of the intended type i:

$$V^i(0|x_0, w) = v_0. \tag{1.8}$$

And the firm maximizes the expected present value of future profit flow:

$$\max_{w(\cdot)} \int_0^\infty \Big(y(x_0 + \tau) - w(\tau)\Big)\gamma^i(\tau|x_0, w)d\tau, \tag{1.9}$$

where $\gamma^i(\tau|x_0, w) := exp\{-\int_0^t r^i(x_0{+}\tau){+}g^i\Big(V^i(s|x_0, w), x_0{+}\tau\Big)ds\}$ is the effective discounting function. Suppose that the matching technology and the equilibrium market tightness are such that the composite function $\mu(\Theta^i)$ is decreasing, concave, and continuously differentiable. The following proposition characterizes the optimal contract. See Appendix for the proof.

Proposition 2. *If the composite function $\mu(\Theta^i)$ is decreasing, concave, and continuously differentiable, there exists some $\zeta \geq 0$ for any (i, v, x) such that the optimal contract $w^*(\cdot|i, v, x)$ satisfy*

$$\begin{aligned}\frac{dw^*(\tau)}{d\tau} =& \frac{u'^2\big(w^*(\tau)\big)}{u''(w^*(\tau))}\Big\{J^i(\tau|x_0, w^*)g_v^i\big(V^i(\tau|x_0, w^*), x_0+\tau\big) \\ &- \zeta\frac{\gamma^l(\tau)}{\gamma^h(\tau)}\Big[\delta^j(x_0+\tau) - \delta^i(x_0+\tau) \\ &+ g^j\big(U^j(\tau|x_0, w^*), x_0+\tau\big) - g^i\big(V^i(\tau|x_0, w^*), x_0+\tau\big)\Big]\Big\},\end{aligned} \tag{1.10}$$

where ζ and the incentive constraint (1.7) have complementary slackness.

Note that if the incentive constraint of the unintended type is slack, the optimal contract has a time derivative that is similar to Shi (2009):

$$\frac{dw^*(\tau)}{d\tau} = \frac{u'^2\big(w^*(\tau)\big)}{u''(w^*(\tau))}J^i(\tau|x_0, w^*)g_v^i\big(V^i(\tau|x_0, w^*), x_0+\tau\big). \tag{1.11}$$

In the absence of information frictions, the optimal contract is an increasing wage scheme such that the rate at which workers leave the job for a new one decreases with tenure. An increasing scheme rewards the workers with a long tenure. The optimal contract under full information achieves efficient allocation between the firm and the worker.

In the presence of information frictions, the optimal contract is distorted from the full information benchmark and the distortion results in more wage backloading as the additional term in (1.10) is positive. The magnitude of the distortion depends on the difference in valuation of future wages of the current job of the two types, as captured by $\frac{\gamma^l(\tau)}{\gamma^h(\tau)}\Big[\delta^j(x_\tau) - \delta^i(x_\tau) + g^j\big(U^j(\tau|x_0, w^*), x_\tau\big) - g^i\big(V^i(\tau|x_0, w^*), x_\tau\big)\Big]$ in (1.10).

The distortion due to adverse selection drives a wedge into the value sharing between the worker and the employer in a match, making the contracts more costly to provide for the employer than under complete information. As a result, there is less vacancy entry and hence a lower hiring rate in equilibrium in a submarket affected by adverse selection than the comparable male submarket which achieves efficient value sharing under complete information. Taken together, both the retention effect of extra wage backloading and the equilibrium result of lower hiring rates lead to lower job mobility for high-attachment women. As confirmed by the quantitative implications in the next chapter, lower female job mobility is an important driver of the gender wage gap in early career, a period with active movements across jobs.

1.3. Conclusion

In this chapter, I have proposed a novel mechanism behind the life cycle dynamics of gender wage gap. Information asymmetry about female job attachment results in costly screening on the firm side and the cost is passed onto female workers. As a result, female workers are offered distorted wage contracts that suppress job-to-job mobility. Because job-to-job transitions are an important source of lifetime wage growth, women miss out on across-job wage gains and experience a slower pace climbing the job ladder.

The theoretical result of a principle offering deferred compensation scheme to separate agents of heterogeneous attachment can have useful implications in other environments involving long-term contracts and adverse selection. This paper shows that separating contracts are generally offered in such settings because of the existence of directed search, in the spirit of Guerrieri et al. (2010). This theoretical property can prove useful for analysis in similar environments.

1.4. References

Bagger, Jesper et al. (June 2014). "Tenure, Experience, Human Capital, and Wages: A Tractable Equilibrium Search Model of Wage Dynamics." In: *American Economic Review* 104.6, pp. 1551–1596.

Blau, Francine D. and Lawrence M. Kahn (Sept. 2017). "The Gender Wage Gap: Extent, Trends, and Explanations." In: *Journal of Economic Literature* 55.3, pp. 789–865.

Bowlus, Audra J. and Louise Grogan (Apr. 2009). "Gender Wage Differentials, Job Search, and Part-time Employment in the UK." In: *Oxford Economic Papers* 61.2, pp. 275–303.

Burdett, Ken and Melvyn Coles (Sept. 2003). "Equilibrium Wage-Tenure Contracts." In: *Econometrica* 71.5, pp. 1377–1404.

Burdett, Kenneth, Carlos Carrillo-Tudela, and Melvyn Coles (July 2020). "The Cost of Job Loss." In: *The Review of Economic Studies* 87.4, pp. 1757–1798. (Visited on 09/26/2020).

Burdett, Kenneth and Dale T. Mortensen (May 1998). "Wage Differentials, Employer Size, and Unemployment." In: *International Economic Review* 39.2, pp. 257–273.

Carrillo-Tudela, Carlos and Leo Kaas (Dec. 2015). "Worker Mobility in a Search Model With Adverse Selection." In: *Journal of Economic Theory* 160, pp. 340–386.

Delacroix, Alain and Shouyong Shi (May 2006). "Directed Search on the Job and the Wage Ladder." In: *International Economic Review* 47.2, pp. 651–699.

– (July 2013). "Pricing and Signaling With Frictions." In: *Journal of Economic Theory* 148.4, pp. 1301–1332.

Gale, Douglas (1996). "Equilibria and Pareto optima of markets with adverse selection." In: *Economic Theory*, p. 29.

Guerrieri, Veronica (Aug. 2008). "Inefficient Unemployment Dynamics under Asymmetric Information." In: *Journal of Political Economy* 116.4, pp. 667–708.

Guerrieri, Veronica, Robert Shimer, and Randall Wright (2010). "Adverse Selection in Competitive Search Equilibrium." In: *Econometrica* 78.6, pp. 1823–1862.

Hornstein, Andreas, Per Krusell, and Giovanni L Violante (Dec. 2011). "Frictional Wage Dispersion in Search Models: A Quantitative Assessment." en. In: *American Economic Review* 101.7, pp. 2873–2898.

Lester, Benjamin (Nov. 2010). "Directed Search With Multi-vacancy Firms." en. In: *Journal of Economic Theory* 145.6, pp. 2108–2132.

Menzio, Guido and Shouyong Shi (2011). "Efficient Search on the Job and the Business Cycle." In: *journal of Political Economy* 119.3, pp. 468–510.

Menzio, Guido, Irina A. Telyukova, and Ludo Visschers (Jan. 2016). "Directed Search Over the Life Cycle." In: *Review of Economic Dynamics* 19, pp. 38–62.

Michelacci, Claudio and Javier Suarez (2006). "Incomplete Wage Posting." In: *Journal of Political Economy* 114.6, pp. 1098–1123.

Moen, Espen R. (Apr. 1997). "Competitive Search Equilibrium." In: *Journal of Political Economy* 105.2, pp. 385–411.

Postel-Vinay, Fabien and Jean-Marc Robin (2002). "Equilibrium Wage Dispersion with Worker and Employer Heterogeneity." In: *Econometrica* 70.6, pp. 2295–2350.

Rothschild, Michael and Joseph Stiglitz (1976). "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information." en. In: *Quarterly Journal of Economics* 90.4, pp. 629–649.

Salop, Joanne and Steven Salop (Nov. 1976). "Self-Selection and Turnover in the Labor Market." In: *The Quarterly Journal of Economics* 90.4, pp. 619–627.

Shi, Shouyong (2009). "Directed Search for Equilibrium Wage-Tenure Contracts." In: *Econometrica* 77.2, pp. 561–584.

Stevens, Margaret (Apr. 2004). "Wage-Tenure Contracts in a Frictional Labour Market: Firms' Strategies for Recruitment and Retention." In: *The Review of Economic Studies* 71.2, pp. 535–551.

CHAPTER 2

Job Attachment and Life Cycle Gender Wage Gap: An Empirical Investigation

2.1. Introduction

In this chapter, I empirically investigate both the ex-ante and the ex-post impacts of the gender differences in job attachment. While employment interruptions harm one's career ex-post, uncertainty about female job attachment faced by employers during recruiting results in costs borne by women ex-ante. The empirical and quantitative analyses in this chapter are based on the theoretical framework presented in Chapter 1. I provide supporting empirical evidence and quantitatively implement the model to assess the factors affecting the life cycle gender wage gap. Using data from the National Longitudinal Survey of Youth 1979 cohort (NLSY79), I document the life cycle gender differences in labor market transitions and wage return to tenure. The theoretical model successfully generates a set of predictions that are consistent with these empirical patterns. In the quantitative implementation of the model, I show that lower female job mobility is an important source of the life cycle gender wage differences. Lower job mobility results in slower ascension along the job ladder and a smaller probability of being employed by high-value jobs among women compared with their male countrparts.

I document the gender differences in labor market turnover and tenure effects using data from NLSY79. I find persistent and increasing gender wage differentials over the life cycle, with much of the divergence taking place over the first 15 years in the labor force. I then classify labor market transitions and show that there are substantial gender differences in turnover patterns, especially during the same period of a widening gender wage gap. While women are more likely to experience employment interruptions with subsequent nonemployment spells than comparable men, women also exhibit lower job-to-job mobility. I document substantial heterogeneity on the individual level regarding job attachment and turnover, particularly among women. I show that women see higher wage growth associated with tenure, suggesting stronger selection with respect to wage tenure profile among women than comparable men.

I then quantitatively implement the model and explore its quantitative implications using simulated work history data. The parameterized model generates gender differences in job mobility and life cycle wage growth that are consistent with the empirical evidence. Women experience lower lifetime wage growth than men, due to lower wage gains from moving to better jobs. I rank jobs by the starting value of employment and find a substantial gender gap in worker distribution along the job ladder. Lower job mobility and lower job attachment result in women being retained at lower rungs of the job ladder than men, although they catch up later in the life cycle. A counterfactual analysis shows that the differences in worker distribution along the job ladder explain the majority of the life cycle gender wage gap.

This chapter contributes to the vast literature of gender differences in life cycle wage dynamics, recent examples including Card et al. (2016), Goldin et al. (2017), and Morchio and Moser (2020). Bowlus (1997) and more recently Amano-Patiño et al. (2021) studied the effects of the gender differences in labor market turnover on the wage gap with a random search framework. In this chapter, I investigate not only the ex-post, direct impacts of women's frequent job separations, but also the ex-ante, indirect costs generated by employers' response to uncertainty about women's job attachment.

The rest of the chapter is organized as follows. In the next section, I present a set of empirical patterns from NLSY79 that support the assumptions and validate the predictions of the theoretical framework of Chapter 1. Section 3 quantitatively implements and assesses the model. I conclude in the last section.

2.2. Empirical Evidence

2.2.1. Data and Sample

I use detailed survey data on individual labor market histories from a sample of workers in the National Longitudinal Survey of Youth 1979 cohort (NLSY79). The NLSY79 data consists of a national representative sample of respondents born during the years 1957 through 1964. The sample period used in this paper is 1979-2017, with certain variables collected before 1979. Respondents were 14-22 years old in 1979, and 52-60 years old in 2017. The surveyors conducted interviews annually from 1979 to 1994, and biennially thereafter. There are 27 survey rounds during the sample period.

The survey collects retrospective weekly labor market status in between survey rounds. It also collects employment and demographic information of the respondents at each survey round. The sample excludes individuals who finished schooling before 1975 due to a lack of information on prior work history. Because many respondents were still in school at the beginning of the sample period, I start tracking an individual's work history only at the time of the first non-school spell that last for more than 18 months. Observations where respondents are in military service are dropped. Because of the detailed information, I am able to track individuals over the life cycle, which means that I can construct actual work experience and track worker-employer matches over time. I call a worker-employer match "a job".[3]

I construct from the survey two datasets of labor market information, a weekly panel of labor market status and a yearly (biennial after 1994) panel of employment, and demographic information.

The labor market status panel is an unbalanced panel of 12,229 individuals over a maximum of 2,082 weeks. For each week, survey respondents report on their labor market status: employed, unemployed and looking for work, unemployed and not looking for work, and

[3] I am not able to track employers over time or across individuals in NLSY79. If an individual returns to an employer after having left the job, the new match cannot be linked to the old match in the data.

indeterminate. I aggregate the two categories of unemployment into a single category called nonemployment.

The employment panel is an unbalanced panel of 10,249 individuals over 27 survey rounds. Since respondents might hold multiple jobs in between rounds, a main job is assigned to each person-round observation by selecting the job that has the most hours worked per week during the period. Hourly real wage is trimmed by 0.5% on both tails. Individuals with less than 5 entries out of the 27 survey rounds are excluded.

2.2.2. Gender Wage Gap over the Life Cycle

Among the cohort in the sample, there exist persistent gender differences in hourly wage, as shown in Figure 2.1. The average gender wage gap widens over the life cycle, rising from 18.2% during the first year of their career to 27% with 29 years in the labor force. The gap rises steadily during the first 20 years in the labor force and levels towards the later stage of the career.

Figure 2.1: Life Cycle Gender Wage Gap

The left panel shows the average log real wage (in 2015 dollars, trimmed by 0.5%) by gender over years in the labor force. The right panel shows the scatter plot of the differences between the male and female lines in the left panel by years in the labor force, with the red line representing the local weighted average gender wage differences.

2.2.3. Labor Market Transitions

Following Royalty (1998), I define two types of labor market transitions. Employment-to-nonemployment (EN) transitions are job moves with an intervening nonemployment spell of more than 4 weeks. Job-to-job (JtJ) transitions are job moves with an intervening nonemployment spell of no more than 4 weeks. Table 2.1 presents the summary statistics of the transitions identified in the sample.

EN transitions and JtJ transitions are associated with vastly different duration and wage implications. EN transitions tend to have much longer nonemployment duration in between employment spells, and the difference between post-transition and pre-transition wages is much smaller in an average EN event than an average JtJ event. Although detailed wages right before and right after transitions are not available in the data, the pattern of wage change is consistent with a literature that documents wage losses associated with job losses and wage gains associated with job moves.[4]

Table 2.1: Summary Statistics of Labor Market Transitions

	All	Women	Men
EN Transitions			
Number of observations	21,285	11,760	9,525
Average duration of intervening nonemployment (weeks)	45.7	57.0	34.0
Average across-job wage change (nearest survey round)	5.48%	6.11%	4.88%
J2J Transitions			
Number of observations	15,074	6,589	8,485
Average duration of intervening nonemployment (weeks)	1	0.9	1
Average across-job wage change (nearest survey round)	14.07%	13.35%	14.61%

Note: Averages are trimmed at 2.5% on both sides. Pre- and post-transition wages are the hourly wage reported in the survey rounds closest to the transitions.

The frequency of labor market transitions differs substantially by gender. Figure 2.2 plots the average annual rate of EN and JtJ transitions over age by gender. At all ages, women tend to make more EN transitions and yet fewer JtJ transitions than men. The gender differences are the most pronounced among prime-age workers.

[4] For recent related papers, see Jarosch (2021) and Hahn et al. (2021).

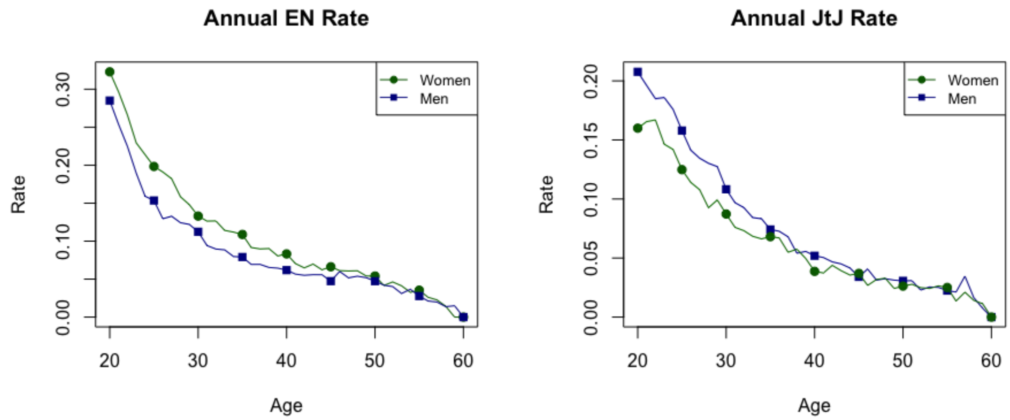


Figure 2.2: Annual Rates of Job Transitions
The left panel shows the average annual rate of employment-to-nonemployment transitions by gender over age. The right panel shows the average annual rate of job-to-job transitions by gender over age. In both panels, the green lines with circles represent the female sample and the blue lines with squares represent the male sample.

The impact of the gender differences in turnover on the wage gap is twofold. First, nonemployment spells are detrimental to job prospects, whose negative effects exacerbate with frequency and duration. Women experience more EN transitions in a given year, with an average duration twice that of men. Second, job-to-job transitions are an important source of wage growth, especially in early career. Workers move up the job ladder through “shopping" for betters jobs. Lower job mobility among women implicates lower gains from job shopping.

2.2.4. Individual-Level Turnover

The gender differences in transitions are robust to controlling for a number of factors that might affect worker turnover. To establish robustness, I introduce the following specification of transition probabilities for worker i in year t:

$$\mathbb{1}_{it}(\text{Transition}) = \alpha_i + \gamma_t + X_{it}\beta + \epsilon_{it}, \tag{2.1}$$

where $\mathbb{1}_{it}(\text{Transition})$ is an indicator for whether a transition occurs, α_i is the person fixed effect, γ_t is the year fixed effect, and X_{it} are the controls. The controls include job tenure, work experience, education attainment, race, marital status, full-time or part-time status, occupation, and industry. Equation (2.1) is estimated separately for EN transitions and JtJ transitions on the main jobs. The estimates of interest are the estimated person fixed effects, $\hat{\alpha}_i$, which measure the individual tendency of experiencing transitions of a certain type. In Figure 2.3, I plot the cumulative distributions of the estimated person fixed effects by gender from the two regressions. The female EN fixed effect stochastically dominates the male EN fixed effect, and the pattern reverses for the JtJ fixed effect. That is, despite the inclusion of a number of controls, women display higher tendency for EN transitions and lower tendency for JtJ transitions than comparable men. Observable factors such as occupation composition cannot fully explain the gender differences in labor market turnover.

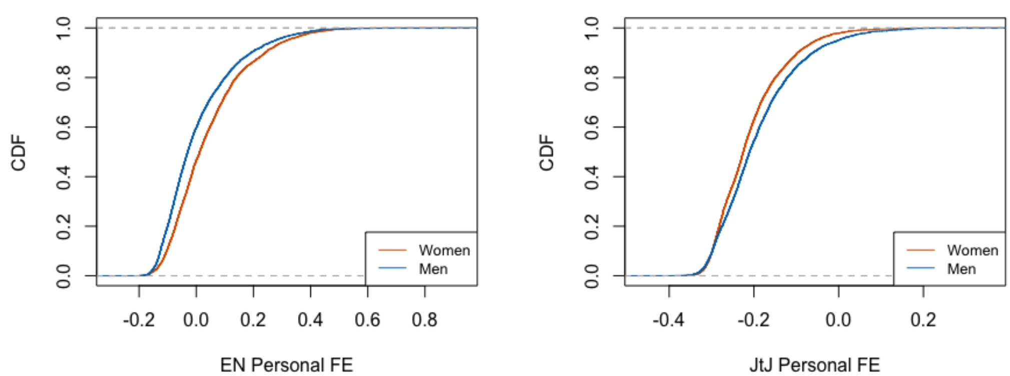


Figure 2.3: C.D.F.s of Personal Fixed Effects in Job Transition Regressions
The left panel shows the cumulative distributions of the estimated person fixed effects by gender from the EN regression. The right panel shows the cumulative distributions of the estimated person fixed effects by gender from the JtJ regression. In both panels, the orange lines represent the estimates for female respondents and the blue lines represent the estimates for male respondents.

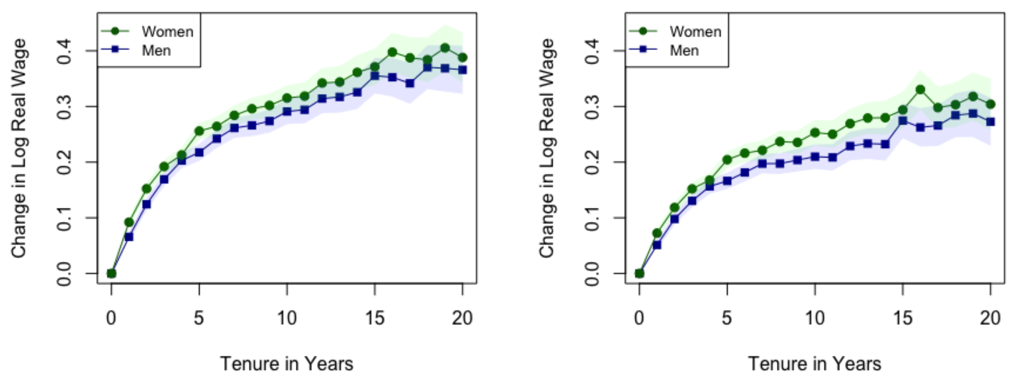
2.2.5. Return to Tenure

Next, I examine wage growth on the job, specifically the part of wage growth associated with job tenure. I estimate the following wage equation for worker i in year t:

$$log(\text{Real Wage}_{it}) = \sum_{\tau=0}^{\bar{\tau}} \mathbb{1}\{\text{Tenure}_{it} = \tau\} \cdot \eta_\tau + \alpha_i + \gamma_t + X_{it}\beta + \epsilon_{it}, \tag{2.2}$$

where $\mathbb{1}\{\text{Tenure}_{it} = \tau\}$ is an indicator for whether job tenure is τ years, α_i is the person fixed effect, γ_t is the year fixed effect, and X_{it} are the controls. I estimate the tenure coefficients both in a naive regression without controls in X_{it} and a regression with controls including experience, education, race, marital status, full-time or part-time status, occupation, and industry.

Figure 2.4 presents the estimates of tenure coefficients η_τ from (2.2), with η_0 normalized to 0. In both regressions, long job tenure is found to be associated with high wage return to tenure, which is as expected since higher reward to tenure helps retain workers from moving to other jobs. What is noteworthy is the gender difference in wage return to tenure: the return is stronger for women than men, with or without controls, suggesting stronger selection regarding wage tenure profile among women.



(a) Return to tenure, without controls (b) Return to tenure, with controls

Figure 2.4: Wage Return to Tenure

The two panels show the estimated tenure coefficients over job tenure by gender in (2.2). The left panel shows the coefficients obtained from the regression without controls. The right panel shows the coefficients obtained from the regression with controls. In both panels, the green lines with circles represent the female coefficients and the blue lines with squares represent the male coefficients.

2.3. Quantitative Implementation

In this section, I simulate the model and discuss its quantitative implications.

2.3.1. Functional Forms

I assume the following functional forms in the quantitative implementation. The output function is increasing in experience x from an initial value y_0 to a limit value y_∞:

$$y(x) = y_\infty - (y_\infty - y_0)e^{-\beta x}. \tag{2.3}$$

The measure of matches is a constant return to scale function of the measures of applicants and vacancies, (a, v):

$$M(a, v) = \bar{p}\Big((\frac{1}{a})^\psi + (\frac{1}{v})^\psi\Big)^{-\frac{1}{\psi}}. \tag{2.4}$$

Workers' flow utility is a constant relative risk aversion (CRRA) function of consumption c:

$$u(c) = \frac{c^{1-\eta}}{1-\eta}. \tag{2.5}$$

Lastly, the exogenous rate of separation among men is an eighth-order Chebyshev polynomial with respect to experience x, fitted to male EN rates in the data. By construction, high-attachment women have the same EN rate function. Given the composition of high- and low-attachment women, I am able to trace out the implied EN rate function for low-attachment women.

2.3.2. Parameters

In the quantitative model, one period is a month. Table 2.2 summarizes the parameters that are fixed without calibration. Time preference ρ is set to 0.004 to be consistent with an annual risk-free rate of 5%. The rate of permanent exit d is set to 0.002 so that the average career lasts for 40 years. The growth rate of worker productivity is 0.046 such that workers are within less than 0.0001% from maximum productivity with 25 years of work experience. Risk aversion η is set to a conventional value of 2, and the elasticity of the matching function ψ is set to 1.599 following Schaal (2017). The flow payoff of home production, b, is normalized to 0.4.

Table 2.2: Parameters Fixed before Calibration

Parameter	Description	Value
ρ	Time preference	0.004
d	Rate of exit	0.002
β	Productivity Growth	0.046
η	Risk aversion	2
ψ	Elasticity, matching function	1.599
b	Home production	0.4

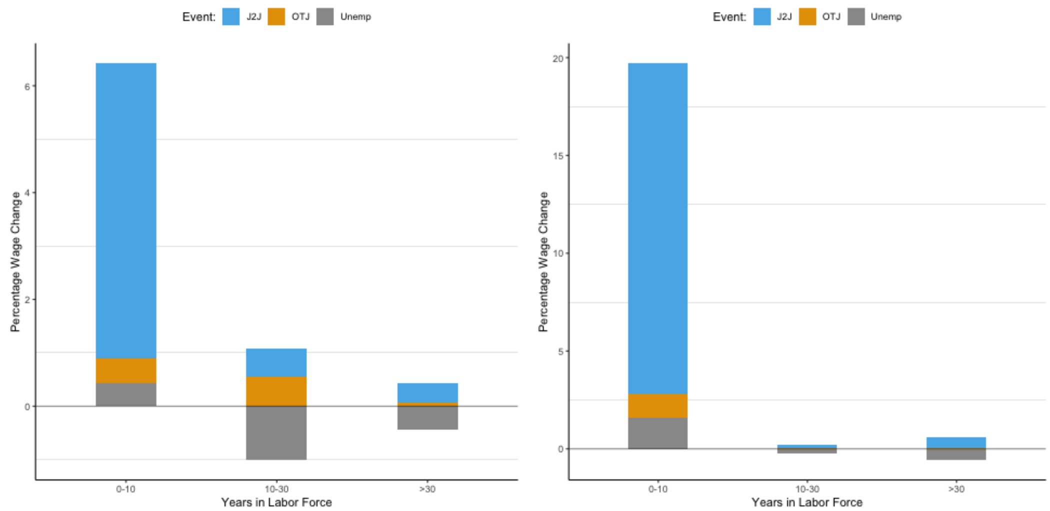
The parameters that can be calibrated are: the maximum and the minimum of output y_∞ and y_0, the fraction of high-attachment female workers α_h, the scale coefficient of matching function $\bar{p}$, and the flow cost of vacancy maintenance, k. Table 2.3 summarized the parameter values used in this implementation.

Table 2.3: Calibration Parameters

Parameter	Description	Value
$y_\infty,\ y_0$	Output function	1, 1.3
α_h	Fraction of h-type female workers	0.9
$\bar{p}$	Matching efficiency	0.5
κ	Vacancy-posting cost	2

2.3.3. Wage Growth and the Job Ladder

I simulate the monthly work histories of 2,000 individuals over 50 years with 1,000 of each gender from the parameterized model. I examine the contribution of labor market events to wage growth at different stages of the life cycle. Three types of events are considered: job-to-job transitions, employment spells, and unemployment shocks. The wage change before and after the events are aggregated by event type over three stages in the life cycle. Figure 2.5 displays the breakdown by gender. In both gender groups, most of the lifetime wage growth takes place in the early stage of the career, with job-to-job transitions being the most important factor. There is a substantial gender differential in the wage gains from job-to-job transitions. While across-job wage growth amounts to 6% of the average initial



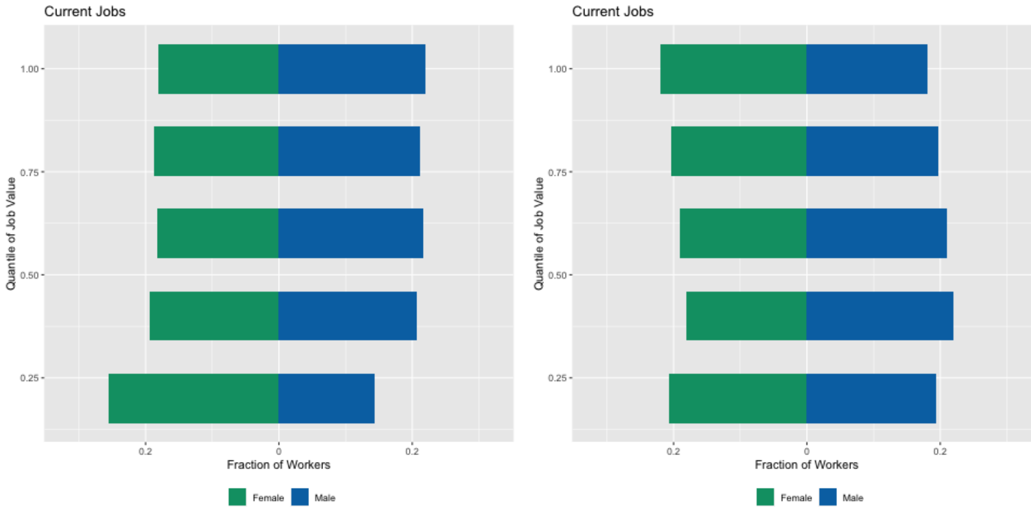
(a) Women (b) Men

Figure 2.5: Sources of Wage Growth over the Life Cycle

The two panels show the breakdown of cumulative wage growth at different stages of the life cycle. Wage growth is attributed to three sources: job-to-job (J2J) transitions, on-the-job (OTJ) spells, and reemployment with interim unemployment spells (Unemp). The left panel represents women and the right panel represents men. Wage growth is presented as a percentage of the average starting wage of the first job in the career from the simulated sample.

wage for women in the first ten years of the career, the same number for men is 15%, more than twice of the gains among women.

Lower job-to-job mobility among women also negatively impacts their positions on the job ladder, consistent with the predictions of the model. In Figure 2.6, I present the distribution of workers over the job ladder by gender. The job ladder is constructed by ranking jobs by the starting value of employment delivered to employees. As shown in Panel (a) of Figure 2.6, a larger mass of early-career (0-15 years) female workers work at jobs at the bottom rungs of the ladder compared with men. Panel (b) of Figure 2.6 shows that these differences diminish as women catch up subsequently in the career as the gender differences in job attachment decrease over time.



(a) 0-15 Years in Labor Force (b) 15-30 Years in Labor Force

Figure 2.6: Job Ladder

The two panels show the worker distribution over job ranking by gender. The left panel shows the distributions among workers with 0-15 years in the labor force, and the right panel shows the distributions among workers with 15-30 years in the labor force. The vertical axis in each panel shows the quantiles of jobs by the starting value of employment. In both panels, the green bars on the left represent women and the blue bars on the right represent men. The total mass by gender is one in each panel.

2.3.4. Decomposing the Life-Cycle Gender Gap

The model stipulates that the wage of a worker, given his or her gender and type, is determined by three state variables: work experience, job tenure, and the starting value of the current job. The gender differentials in the distributions of the three key variables are important in accounting for the wage gap. I measure the effect of each variable by calculating the counterfactual wage gap under the same joint distribution of the other two variables and different marginal distributions of the variable in question. Details of the decomposition can be found in the Appendix.

Figure 2.7 displays the life cycle decomposition of the female-male wage differentials. Each line depicts a counterfactual gender wage gap if the distribution of one of the wage determinants is allowed to differ by gender. The residual wage gap after accounting for the three variables represents the average gender difference in wage for men and women of the same state variables, which I call the “net wage gap".

Consistent with the gender differences on the job ladder, value of employment is the major contributor to the life cycle gender gap. Much of the gender wage gap is due to women being retained at worse jobs than men. On the contrary, tenure positively contributes to female-male wage differentials because high-attachment women tend to stay on their current jobs to claim the return to long tenure.

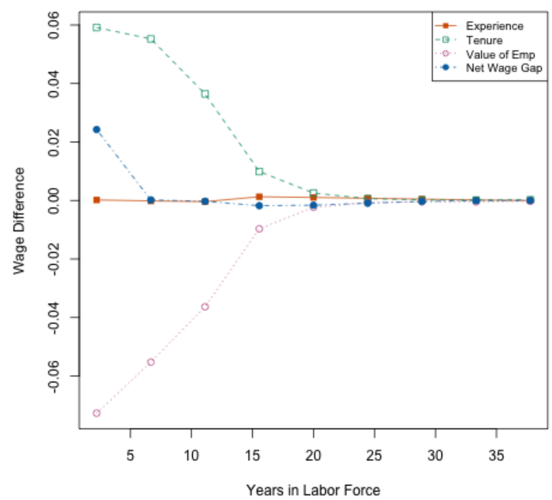


Figure 2.7: Decomposition of the Life Cycle Gender Wage Gap

The figure shows the breakdown of the average female-male wage differentials over the life cycle into four parts: the gender differentials in experience distribution, the gender differentials in tenure distribution, the gender differentials in employment value distribution, and the residualized wage difference between women and men with the same state variables.

2.4. Conclusion

In this chapter, I have provided empirical evidence supporting the assumptions and predictions of the theory in Chapter 1. The calibrated model replicates the empirical gender differentials and confirms the crucial role of job mobility in explaining the gender wage gap. A promising next is to examine the role of policy interventions. The equilibrium can be inefficient if low-attachment workers only constitute a small fraction of the labor force. Regardless of the fraction, high-attachment women bear the same cost of screening as long as there is uncertainty about their job attachment. Policy such as tax credits that subsidize newly created matches should encourage firms to offer higher hiring rates and better terms to female applicants as they are compensated for jobs with shorter durations.

2.5. References

Amano-Patiño, Noriko, Tatiana Baron, and Pengpeng Xiao (May 2021). "Human Capital Accumulation, Equilibrium Wage-Setting and the Life-Cycle Gender Pay Gap." In: *Cambridge Working Papers in Economics No. 2010*, p. 68.

Bowlus, Audra J. (Oct. 1997). "A Search Interpretation of Male-Female Wage Differentials." In: *Journal of Labor Economics* 15.4, pp. 625–657.

Bureau of Labor Statistics, U.S. Department of Labor (2019). *National Longitudinal Survey of Youth 1979 cohort, 1979-2016 (rounds 1-27).* Produced and distributed by the Center for Human Resource Research (CHRR), The Ohio State University. Columbus, OH.

Card, David, Ana Rute Cardoso, and Patrick Kline (May 2016). "Bargaining, Sorting, and the Gender Wage Gap: Quantifying the Impact of Firms on the Relative Pay of Women." In: *The Quarterly Journal of Economics* 131.2, pp. 633–686.

Fawcett, Kevin and Shouyong Shi (2018). "Learning, On-the-Job Search and Wage-Tenure Contracts." In: *SSRN Electronic Journal*.

Goldin, Claudia et al. (May 2017). "The Expanding Gender Earnings Gap: Evidence From the LEHD-2000 Census." In: *The American Economic Review* 107.5, pp. 110–114.

Hahn, Joyce K., Henry R. Hyatt, and Hubert P. Janicki (Apr. 2021). "Job Ladders and Growth in Earnings, Hours, and Wages." In: *European Economic Review* 133, p. 103654.

Jarosch, Gregor (Feb. 2021). "Searching for Job Security and the Consequences of Job Loss." In: *Working Paper*.

Keith, Kristen and Abagail McWilliams (1999). "The Returns to Mobility and Job Search by Gender." In: *Industrial and Labor Relations Review* 52.3, pp. 460–477.

Loprest, Pamela J. (2020). "Gender Differences in Wage Growth and Job Mobility." In: *The American Economic Review* 82.2, pp. 526–532.

Morchio, Iacopo and Christian Moser (Apr. 2020). "The Gender Pay Gap: Micro Sources and Macro Consequences." In: *Working Paper*.

Royalty, Anne Beeson (Apr. 1998). "Job-to-Job and Job-to-Nonemployment Turnover by Gender and Education Level." In: *Journal of Labor Economics* 16.2, pp. 392–433.

Schaal, Edouard (2017). "Uncertainty and Unemployment." In: *Econometrica* 85.6, pp. 1675–1721.

CHAPTER 3

Compensation Structure and Adverse Selection in Alternative Work Arrangements

3.1. Introduction

The rise of alternative work arrangements has transformed the landscape of work relations around the world. Alternative work arrangements are flexible, task-based, and save employers costs of retaining full-time employees (Abraham et al. (2018)). An increasing fraction of the labor force is engaging in work that requires no long-term commitment, has flexible schedules, and is done on an independent basis. The increasing adoption of new technology allows gig work platforms like Uber to connect supply and demand remotely, in the absence of centralized workplaces or close human supervision. Therefore, motivating workers through the design of compensation structure is crucial for alternative work arrangements. Gig work platforms invest heavily in data and technology to assign work and adjust compensation schemes in real-time. In this chapter, I investigate the design of compensation structure and the effects of adverse selection among workers under alternative work arrangements.

In this chapter, I explore the optimal choice between the two commonly used compensation schemes: a proportional bonus contract versus a lease contract. Many gig work platforms, such as Uber and Airbnb, have adopted a commission-based compensation scheme. Another popular compensation scheme is to sell the project to workers and allow them to keep all the proceedings, such as medallion rentals for taxi drivers. Under a bonus contract, workers share the output of the task with the firm while subject to interaction with the rest of the firm's organizational structure. Under the lease contract, a task is bought out by a worker, who carries out the task independently and keeps the entirety of the output. While the former arrangement provides risk-averse workers with valuable risk sharing, it also makes the worker subject to additional risks intrinsic to the firm's organization, which do not affect workers who buy out the tasks. For example, ride-hailing drivers work under a commission-based compensation scheme, each trip regulated by the rideshare platform. The effects of centralized ride assignment and pricing by the platform on driver revenue can be uncertain in the dynamic market of ride-hailing. In a comparison, taxi drivers rent or purchase medallions for the right to drive and retain all the earnings, working when and where they would like without the additional risks coming from a centralized organization.

The tradeoff between risk sharing and organizational uncertainty determines which compensation structure is adopted. Oftentimes, employers face two more challenges: 1) heavy reliance on compensation schemes to motivate worker effort when output is risky, and 2) asymmetric information about workers' marginal product. I propose a model to study how a firm should optimally design alternative work arrangements to tackle these concerns. In the model, a risk-neutral firm owns a task whose completion requires labor input from a risk-averse worker. The output of the task is uncertain due to a risk intrinsic to the production process, or the "production risk", and a risk that stems from the task's interaction with the rest of the firm, or the "organizational risk". The latter risk can be avoided if the firm sells the task to the worker to allow the worker to produce with full ownership of the task.

The firm considers two kinds of work arrangements. Under the first kind, the firm employs the worker and takes a predetermined share of the final output net of worker compensation. The firm compensates the worker with a base salary plus a bonus proportional to the final output, providing insurance against output uncertainty. The worker's optimal effort increases with the bonus rate. Alternatively, the firm can sell the task to the worker in exchange for an unconditional amount of transfer. The worker then carries out the task independently, the output of which is not subject to the organizational risk, but the worker is exposed to the entirety of the production risk. I show that the firm prefers lease contracts if: 1) the organizational risk is sufficiently large relative to the production risk; 2) the marginal product of worker effort is sufficiently high; 3) the worker is sufficiently risk-loving. The wage rate per unit of effort under the lease contract can be lower than under the bonus contract when the worker's outside option or the risk composition limits how much effort the firm can induce from the worker for a given amount of income uncertainty.

In the same setup, I also study the effects of asymmetric information on work arrangements when workers have private information about their productivity. High-type workers have higher marginal product of effort than low-type workers. The firm offers a menu of two contracts, one for each type. I find that adverse selection reduces the firm's ability to induce worker effort when the worker is low-type. Specifically, adverse selection limits the firm's use of lease contracts for low-type workers, because the firm tends to lower the correlation between compensation and output for low-type workers such that high-type workers would not choose the contract intended for the low type. As a result, low-type workers tend to be under-incentivized in most scenarios where both types are employable to the firm. The firm is always willing to employ high-type workers and yet may exclude low-type workers in certain parameter spaces. I characterize all the contract menus that the firm may adopt and derive the firm's payoff in each scenario.

This chapter studies the interaction of moral hazard and adverse selection in contracting problems. The literature on the dual problem goes back to early works such as McAfee and McMillan (1986) and Baron and Besanko (1987) and has been revisited with much generality in recent works such as Chade and Swinkels (2016), and Gottlieb and Moreira (2021). Chade and Swinkels (2016) show that the moral hazard problem and the adverse selection problem can be decoupled under general assumptions. Compared to their setup, my model incorporates a novel option for the principle to sell the task to the agent, adding to the trade-off between effort induction and screening. Gottlieb and Moreira (2021) show that it is optimal for the principle to offer a single contract to risk-neutral agents with limited liability under mild assumptions. I study the design of contracts for risk-averse workers without limited liability, where separating contracts can arise in much of the parameter space. Benabou and Tirole (2016) show that a monopolistic firm under-incentivizes low-skill agents under asymmetric information. A similar result emerges in my model when there is no exclusion of the low type, and a menu of separating contracts is adopted.

This chapter is also related to a growing literature on alternative work arrangements. Katz and Krueger (2019) document the increasing importance of alternative work arrangements in the US labor market from 1995 to 2015. Angrist et al. (2021) study an experiment to offer commission-based Uber drivers the option of a lease and show that drivers tend to avoid lease contracts even when it is profitable. In my model, I show that from the employer's perspective, lease contracts tend to be avoided for low marginal product workers due to adverse selection.

The rest of the chapter is organized as follows. The next section describes the setup of the model environment. Section 3 discusses the optimal bonus scheme under complete information. Section 4 discusses the optimal lease contract under complete information. Section 5 examines the effect of asymmetric information on the design of contract menus. I conclude in Section 6.

3.2. The Environment

I consider a static environment where a risk-neutral firm arranges for a risk-averse worker to conduct a task. If the worker rejects, her value in nonparticipation is $\overline{U}$. If the worker accepts, she chooses the amount of effort $d \in \mathcal{R}_+$ dedicated to the task given her marginal product $\theta \in \mathcal{R}_+$. I assume that θ is sufficiently high such that there always exists a positive surplus under the work arrangements I discuss in this chapter, a condition that I formally characterize later. Later in this chapter, I will discuss how assumptions regarding the information on productivity θ play out in different scenarios. One interpretation of the setup can be that the effort is hours worked, and the marginal product is the marginal output per hour. The output of the task, y, is determined by the worker's productivity θ, her effort d, and an output shock $u \sim \mathcal{N}(0, \sigma^2)$:

$$y = \theta d + u.$$

When the task is completed, the output is publicly observed by both parties. However, the firm observes neither the worker's effort d nor the realized output shock u. The output shock consists of two independent components. One is intrinsic to the production of the task, referred to as the production risk and denoted by u_p. And the other is associated with the organization of the firm, referred to as the organizational risk and denoted by u_f. The firm-specific component can be interpreted as the output interaction of the task with other operations within the firm. Whether the task ends up complementing or disrupting the other operations is an uncertainty that the firm needs to take into account. I assume that $u_p \sim \mathcal{N}(0, \sigma_p^2)$, and that $u_f \sim \mathcal{N}(0, \sigma_f^2)$. And hence $u \sim \mathcal{N}(0, \sigma^2)$, where $\sigma^2 = \sigma_p^2 + \sigma_f^2$.

The worker's utility consists of utility generated by consumption and disutility incurred by effort. The risk-averse worker values consumption and dislikes uncertainty in consumption. I assume that the expected utility generated by lottery x is $\mathbb{E}[x] - \rho Var(x)/2$. The disutility is increasing and convex in the effort level d. I assume that the amount of disutility associated with effort level d is $\xi d^2/2$.

Note that for normally distributed lotteries, the functional form of the expected utility is consistent with an underlying, constant-absolute-risk-aversion utility function $u(x) = -e^{-\rho x}$. The log-transformed expected utility function gives $-log(-\mathbb{E}[u(x)])/\rho = \mathbb{E}[x] - \rho Var(x)/2$, if $x \sim \mathcal{N}(\mathbb{E}[x], Var(x))$. It will become clear later that the consumption shocks under the employment arrangements that I discuss in this chapter are indeed normally distributed.

The timeline is as follows: 1. the firm and the worker agree to a contract; 2. the worker carries out the task; 3. final output is determined by the worker's productivity and action, as well as the realized output shock; 4. division of output and transfers take place between the match according to the contract.

The firm considers two types of employment contracts. The first type features a base salary and a bonus associated with the final output, motivated by the commonly used bonus contracts. The second one entails the worker making a deterministic transfer to the firm and keeping all final output, in the spirit of outsourcing arrangements for freelancers and gig workers.

3.3. Bonus Contract

The worker chooses an effort level to maximize her expected utility under contract (s, b):

$$\max_{d} s + b\theta d - \rho b^2 \sigma^2/2 - \xi d^2/2.$$

The optimal effort level is given by

$$d^*(s, b) = \theta b/\xi, \tag{3.1}$$

which increases with the worker's effective wage rate and decreases with the worker's distaste for effort. Accordingly, the worker's expected income is

$$s + \theta^2 b^2/\xi.$$

And the worker's expected utility is

$$V(s, b) = s + \frac{\theta^2 b^2}{2\xi} - \frac{\rho \sigma^2 b^2}{2}. \tag{3.2}$$

Given (3.1) and (3.2), the firm's expected profit under bonus contract (s, b), $\Pi(s, b)$, is

$$\Pi(s, b) = \frac{\theta^2}{\xi} b(1 - b) - s = \frac{\theta^2}{\xi} b - \left(\frac{\theta^2}{2\xi} + \frac{\rho \sigma^2}{2}\right) b^2 - V(s, b). \tag{3.3}$$

The result in (3.3) characterizes the value sharing between the firm and the worker, where expected total surplus $\frac{\theta^2}{\xi} b - \left(\frac{\theta^2}{2\xi} + \frac{\rho\sigma^2}{2}\right) b^2$, which is expected output net of disutility from risks and effort, is divided into the worker's expected value V and the firm's expected value Π.

Note that if $\theta^2/\xi \geq \rho\sigma^2$, there always exists positive match surplus for any bonus rate lower than one.

For the analysis of bonus contracts, I focus on the simple scheme of a base salary plus a bonus proportional to the final output.

The firm's problem is to find a proportional bonus contract that maximizes its expected profit:

$$\max_{s,b} \theta d - s - b\theta d,$$

subject to

$$d = \theta b/\xi, \tag{3.4}$$

$$s + b\theta d - \rho b^2\sigma^2/2 - \xi d^2/2 \geq \overline{U}. \tag{3.5}$$

The firm's optimal bonus rate is

$$b^f = \frac{\theta^2}{\theta^2 + \rho\xi\sigma^2}, \tag{3.6}$$

and the base salary s^f is such that the worker's participation constraint (3.5) holds.

The optimal bonus rate b^f is lower than one. Because of the worker's risk aversion, it is optimal for the firm to not load all the output risk on worker compensation. The privately optimal bonus rate b^f also maximizes the joint surplus, $\frac{\theta^2}{\xi}b - \left(\frac{\theta^2}{2\xi} + \frac{\rho\sigma^2}{2}\right)b^2$, out of all bonus contracts.

Under the firm's optimal bonus contract (s^f, b^f), the firm's expected profit is

$$\Pi(s^f, b^f) = \frac{\theta^2}{\xi}b^f - \left(\frac{\theta^2}{2\xi} + \frac{\rho\sigma^2}{2}\right){b^f}^2 - V(s^f, b^f) = \frac{\theta^4}{2\xi(\theta^2 + \rho\xi\sigma^2)} - \overline{U}.$$

3.4. Lease Contract

Under a lease contract, the firm sells the task to the worker for a deterministic price, and the worker keeps whatever output the task generates while not subject to the organizational risk. I refer to such an arrangement as a lease contract. That is, the new output function under this arrangement does not contain the organizational risk term u_f:

$$\tilde{y} = \theta d + u_p.$$

When carried on by the worker independently, the output of the task is no longer subject to interactions with the firm's operation, and therefore the worker is not exposed to the u_f shock.

A lease contract effectively has a bonus rate of 1. Given the worker's optimal effort decision setting $b = 1$ in (3.1), for the worker to receive expected utility $\overline{V}$ under the lease contract, the price of the task charged by the firm is

$$p(\overline{V}) = \frac{\theta^2}{2\xi} - \frac{\rho\sigma_p^2}{2} - \overline{V}, \tag{3.7}$$

which is also the firm's expected profit under such an arrangement. And accordingly, the worker's expected total compensation given the outside option $\overline{U}$ is

$$\frac{\theta^2}{2\xi} + \frac{\rho\sigma_p^2}{2} + \overline{U}.$$

Under the lease contract, the worker keeps the task output, bears all the production risk, and exerts the most effort. As a result, the expected task output is higher than under the bonus contract. The bonus contract stipulates a smaller share of the total risk but makes the task subject to the organizational risk. The firm chooses whichever contract that generates the higher expected profit. Proposition 3 characterizes the firm's optimal choice of work arrangement.

Proposition 3. *The firm prefers the optimal lease contract if and only if the lease contract is also the efficient contract, i.e.,*

$$\frac{\theta^2}{\xi} \geq \rho\sigma^2\frac{\sigma_p^2}{\sigma_f^2}. \tag{3.8}$$

Proof. The firm chooses the lease contract if and only if the corresponding expected profit, $p(\overline{U})$, is no less than that under the bonus contract (s^f, b^f). That is,

$$\frac{\theta^2}{2\xi} - \frac{\rho\sigma_p^2}{2} - \overline{U} \geq \frac{\theta^2}{\xi} b^f - \left(\frac{\theta^2}{2\xi} + \frac{\rho\sigma^2}{2}\right) b^{f\,2} - \overline{U}$$

After canceling $\overline{U}$ on both sides, the inequality above means that the joint surplus is larger under the optimal lease contract. Plugging in b^f from (3.6) and rearranging the terms in (3.8). □

Proposition 3 shows that the firm chooses the optimal lease contract when the organizational risk is sufficiently high relative to the production risk; marginal product is sufficiently high; marginal disutility from effort is sufficiently low; risk aversion is sufficiently low. Taken together, lease contracts are adopted for tasks where effort is crucial for output and where the organizational risk is substantial.

Next, I compare the expected compensation to effort ratio, which is a notion of the expected effective wage rate, under the two work arrangements in Proposition 4.

Proposition 4. *The worker receives a lower effective wage rate under the lease contract than under the bonus contract if and only if*

$$\overline{U} > \frac{\theta^2}{2\xi} \frac{\sigma_p^2}{\sigma_p^2 + \sigma_f^2} \tag{3.9}$$

That the outside option $\overline{U}$ appears on the (3.9) highlights how a higher $\overline{U}$ limits the firm's ability to induce more effort from the worker and hence leads to a higher expected income per unit of effort. And similarly, lower marginal product θ, higher distaste for effort ξ, or a higher organizational risk relative to the production risk limits the firm's effort induction too.

3.5. Adverse Selection

I then consider the design and choice between work arrangements when the worker's marginal product θ is her private information. The firm knows that with probability q_H, the worker has high marginal product θ_H, and that with probability q_L she has low marginal product θ_L. In practice, employers do acquire information on potential hires' productivity by learning about their education and employment history, etc., and yet these measures are not perfect. The information problem characterized here, therefore, speaks to the residual productivity that cannot be directly measured and contracted. Put differently, we focus on the firm's contracting problem with a group of observationally identical workers. The firm offers a menu that consists of two contracts, one for each worker type. The contracts are characterized by the bonus rate b, where lease contracts have a bonus rate of one, and the worker's valuation V.

First, note that a lease contract is preferred to any bonus contract with a bonus rate of one because it removes the organizational risk without affecting the worker's effort response. I introduce the following notation that treats the lease contract as a special case where the "effective bonus rate" is one. The firm's expected profit as a function of the effective bonus rate b and the worker's expected utility V is, for $i \in \{H, L\}$.

$$\Pi_i(b, V) = \begin{cases} \frac{\theta_i^2}{\xi} b - \left(\frac{\theta_i^2}{2\xi} + \frac{\rho\sigma^2}{2}\right) b^2 - V, \text{if } b \neq 1, \\ \frac{\theta_i^2}{2\xi} - \frac{\rho\sigma_p^2}{2} - V, \text{if } b = 1. \end{cases}$$

Recall the worker's response to the effective bonus rate is linear as in $d^*(b) = \theta b/\xi$, given her marginal product θ. Under the same work arrangement, the levels of expected utility for a high-type worker and a low-type worker differ in output consumption, $b\theta d^*(b)$, and disutility of effort, $\xi d^{*2}(b)/2$, making the total difference in value $b^2(\theta_H^2 - \theta_L^2)/(2\xi)$. Therefore, the incentive constraints for workers to choose their intended contracts from the menu are

$$V_H \geq V_L + \frac{\theta_H^2 - \theta_L^2}{2\xi} b_L^2, \tag{3.10}$$

and

$$V_L \geq V_H - \frac{\theta_H^2 - \theta_L^2}{2\xi} b_H^2. \tag{3.11}$$

I will discuss the case where the firm finds it optimal to only attract one type, but for now, I focus on the case where the firm is willing to attract both types. The firm maximizes the expected profit generated by a menu given the worker's optimal response in (3.1):

$$\max_{s_i, V_i} \sum_{i \in \{H,L\}} q_i \Pi_i(b_i, V_i),$$

subject to the incentive constraints (3.10) and (3.11) and the participation constraints for both types

$$V_i \geq \overline{U}, i \in \{H, L\}.$$

If neither of the incentive constraints binds, both types expect to receive $\overline{U}$ from the contracts offered by a profit-maximizing firm. However, under the same compensation scheme, a high-type worker receives higher utility than a low-type worker. Therefore, the incentive constraint for high-type workers must bind. And the firm's profit-maximizing implies that the participation constraint of low-type workers binds. Taken together, the two binding constraints imply that a high-type worker commands an information rent of $\frac{\theta_H^2 - \theta_L^2}{2\xi} b_L^2$ in utility.

Having set up the firm's problem, I am able to characterize the menu of work arrangements that the firm adopts in different parametric spaces. The first observation is that in the event that bonus contracts are adopted for both types, the contracts must be separating contracts, as stated in the proposition below.

Proposition 5. *The firm never offers a pooling bonus contract where* $b_H = b_L \neq 1$ *that* *attracts both types.*

Proof. Suppose that the firm offers the same bonus rate $b_H = b_L \neq 1$. The firm's optimal bonus contract lies on the continuous, differentiable segment of its expected profit function. By the reasoning above, the incentive constraint for the high type and the participation constraint for the low type bind. Solving the firm's problem yields the optimal bonus rates

$$b_H = \frac{\theta_H^2}{\theta_H^2 + \rho\xi\sigma^2},$$

the same as its full-information benchmark, and

$$b_L = \frac{\theta_L^2}{\theta_L^2 + \rho\xi\sigma^2 + \frac{q_H}{q_L}(\theta_H^2 - \theta_L^2)}. \tag{3.12}$$

The bonus rate for the low type is distorted from and lower than its full-information counterpart $b_L = \frac{\theta_L^2}{\theta_L^2+\rho\xi\sigma^2}$, so as to keep the high type from taking up the contract intended for the low type. More importantly, $b_L < b_H$, which contradicts the supposition that both types are offered the same bonus contract. □

The second observation is that the firm never adopts a menu that features a bonus contract for the high type and a lease contract for the low type, as stated in the proposition below.

Proposition 6. *The firm never uses a menu that sells the task to the low type and offers a bonus contract to the high type.*

Proof. Proof. I prove this by contradiction. If such a menu is used, the incentive constraints of the two types imply the following inequalities:

$$V_H \geq V_L + \frac{\theta_H^2 - \theta_L^2}{2\xi}, \tag{3.13}$$

$$V_L \geq V_H - \frac{\theta_H^2 - \theta_L^2}{2\xi}\tilde{b}_H^2, \tag{3.14}$$

Where denotes the bonus rate for the high type, and V_H and V_L denote the expected utility delivered to the high type and the low type respectively. By the same reasoning as the bonus menu case, if neither incentive constraint binds, both types receive $\overline{U}$ from the profit-maximizing firm, which cannot be true because then the high type would be better off taking the low-type contract. Therefore, the high type's incentive constraint (3.13) must bind.

For the two inequalities to hold at the same time, $\tilde{b}_H$ must be no less than one. However, $\tilde{b}_H \geq 1$ is a contradiction. I first discuss the case where $\tilde{b}_H > 1$. Recall that the firm's expected profit as a function of worker value V and bonus rate b in (3.3) is quadratic in b and is strictly decreasing in b at $b = 1$. Therefore, by decreasing $\tilde{b}_H$ by a small amount such that (3.14) still holds and at the same time increasing the base salary to keep the worker's value at V_H, the firm implements a profitable deviation.

Next, consider the case where $\tilde{b}_H = 1$. Again, there exists a profitable deviation for the firm. The firm finds it more profitable to switch to the lease contract from the bonus contract for high-type workers because doing so gets rid of the organizational risk while keeping the effective bonus rate at 1. Therefore, the firm can extract a higher expected profit from such a switch while promising the same level of expected utility to high-type workers. □

Another result is that the firm cannot offer both types lease contracts while also distinguishing them. This is because both types would then choose the less expensive lease given the same effective bonus rate of one.

Proposition 7. *The firm never offers separating lease contracts.*

Next, I discuss the potential contract menus that the firm may adopt. First, note that the firm may find it optimal to not attract both types. If the probability of the worker being low-type is quite small, the cost of offering separating contracts outweighs the benefits of learning about the worker's type. In this case, the firm would offer the same bonus contract or lease contract as the full-information benchmark for high-type workers as the only option on the menu, excluding low-type workers because they receive less than non-participation. The firm's expected profit generated by a menu with a bonus contract intended for the high type, $(b_H^f, \overline{U})$ and $(b_H^f, \overline{U} - \frac{\theta_H^2 - \theta_L^2}{2\xi} b_H^f)$, is

$$q_H(\frac{\theta_H^2}{2\xi} b_H^f - \overline{U}).$$

And the firm's expected profit generated by a menu with a lease contract intended for the high type, $(1, \overline{U})$, is

$$q_H(\frac{\theta_H^2}{2\xi} - \frac{\rho\sigma_p^2}{2} - \overline{U}).$$

In the event that the firm finds it optimal to attract both types, the firm adopts a menu that features separating bonus contracts, a combination of a high-type lease contract and a low-type bonus contract, or a pooling lease contract. The firm's expected profit in each case is displayed below.

The firm's expected profit generated by a menu of two bonus contracts, $(b_H^f, \overline{U} + \frac{\theta_H^2 = \theta_L^2}{2\xi} b_L)$ and $(b_L, \overline{U})$, is

$$q_H \frac{\theta_H^2}{2\xi} b_H^f + q_L \frac{\theta_L^2}{2\xi} b_L - \overline{U}.$$

The firm's expected profit generated by a menu of a lease contract for the high type and a bonus contract for the low type, $(1, \overline{U} + \frac{\theta_H^2 = \theta_L^2}{2\xi} b_L)$ and $(b_L, \overline{U})$, is

$$q_H \frac{\theta_H^2 - \rho\xi\sigma_p^2}{2\xi} + q_L \frac{\theta_L^2}{2\xi} b_L - \overline{U}.$$

Compared with the menu of two bonus contracts, the combination of a lease contract and a bonus contract is preferable under the same condition where the lease contract is preferable to the bonus contract for the high type with full information, characterized in Proposition 3.

The firm's expected profit generated using pooling lease contracts, $(1, \overline{U} + \frac{\theta_H^2 - \theta_L^2}{2\xi})$ and $(1, \overline{U})$, is

$$\frac{\theta_L^2}{2\xi} - \frac{\rho\sigma_p^2}{2} - \overline{U}.$$

In the end, the firm chooses the menu that delivers the highest expected profit out of the five menus above. Like in the full-information environment, the firm tends to adopt lease contracts when the organizational risk is disproportionally large and vice versa. And the firm tends to offer different terms to distinguish workers when the probability of the worker being high-type is sufficiently large.

3.6. Conclusion

This chapter discusses the use of bonus schemes and lease contracts in alternative work arrangements. Moral hazard problems and adverse selection problems are important concerns in these arrangements, and this chapter considers them jointly. Employers choose between the two schemes by trading off risk sharing and organizational uncertainty. The former tends to be adopted when risk sharing between employers and employees is valuable, and the latter is chosen when organizational structure adds too much uncertainty to task output. Effort motivation is crucial for both schemes, but asymmetric information about workers' marginal product restricts the use of lease contracts, which is higher-powered, among low marginal product workers, leading to efficiency loss. This result highlights the interaction between moral hazard and adverse selection in task-based, lightly supervised work.

An important issue that is left out of this chapter is workers' valuation of flexibility and autonomy. Empirical works find that gig workers dislike the stress of working under schedules set by others (Mas and Pallais (2017)) or under the sunk costs of lease contracts (Angrist et al. (2021)). Future work can be done on quantifying the cost of organization risks to contrast with the cost of inflexibility documented by the related works.

3.7. References

Abraham, Katharine et al. (2018). *Measuring the Gig Economy: Current Knowledge and Open Issues*. w24950. Cambridge, MA: National Bureau of Economic Research.

Angrist, Joshua D., Sydnee Caldwell, and Jonathan V. Hall (2021). "Uber versus Taxi: A Driver's Eye View." In: *American Economic Journal: Applied Economics* 13.3, pp. 272–308.

Baron, David P. and David Besanko (1987). "Monitoring, Moral Hazard, Asymmetric Information, and Risk Sharing in Procurement Contracting." In: *The RAND Journal of Economics* 18.4, p. 509.

Bénabou, Roland and Jean Tirole (2016). "Bonus Culture: Competitive Pay, Screening, and Multitasking." In: *journal of political economy*, p. 66.

Castillo, Juan Camilo (2018). "Who Benefits from Surge Pricing?" In: *SSRN Electronic Journal*.

Chade, Hector and Jeroen Swinkels (2021). "Disentangling Moral Hazard and Adverse Selection." In: *Working Paper*, p. 68.

Gottlieb, Daniel and Humberto Moreira (2021). "Simple Contracts with Adverse Selection and Moral Hazard." In: *Working Paper*, p. 39.

Katz, Lawrence F. and Alan B. Krueger (2019). "The Rise and Nature of Alternative Work Arrangements in the United States, 1995–2015." In: *ILR Review* 72.2, pp. 382–416.

Mas, Alexandre and Amanda Pallais (2017). "Valuing Alternative Work Arrangements." In: *American Economic Review* 107.12, pp. 3722–3759.

McAfee, R. Preston and John McMillan (1986). "Bidding for Contracts: A Principal-Agent Analysis." In: *The RAND Journal of Economics* 17.3, p. 326.

APPENDIX A

Proof of Proposition 2

Recall that g^i is the job finding rate of the optimal submarket for type-i workers to search in and that g^i is a function of the worker's current value of employment or unemployment, v, and current experience, x:

$$g^i(v,x) := p^i(e^i(v,x),x),$$

where $p^i(\cdot)$ is the job finding rate derived from the matching technology and $e^i(\cdot)$ is the policy function of workers' job search problem. Also recall that $S^i(\cdot)$ denotes the value of job search.

Let τ denote job tenure. In a submarket intended to deliver $\bar{V}$ to type-i applicants with current experience x_0, the firm maximizes the expected present value of profits by selecting a wage-tenure contract $w(\cdot)$:

$$\begin{aligned}
\max_{w(\cdot)} \quad & J^i(0|x_0,w) = \int_{\tau=0}^{\infty}[y(x_0+\tau)-w(\tau)]\gamma^i(\tau)d\tau \\
s.t. \quad & \dot{\gamma}^i(\tau) = -\gamma^i(\tau)[\rho+d+\delta^i(x_0+\tau)+g^i(V^i(\tau),x_0+\tau)] \\
& \dot{V}^i(\tau) = -u(w(\tau))+(\rho+d)V^i(\tau)-\delta^i(x_0+\tau)[V_u^i(x_0+\tau)-V^i(\tau)] \\
& \qquad\qquad - S^i(V^i(\tau)|x_0+\tau) \\
& \dot{U}^j(\tau) = -u(w(\tau))+(\rho+d)U^j(\tau)-\delta^j(x_0+\tau)[V_u^j(x_0+\tau)-U^j(\tau)] \\
& \qquad\qquad - S^j(U^j(\tau),x_0+\tau) \\
& V^i(0) = \bar{V} \\
& V^j(0) \leq \min_{v\in Y_p^i}\left\{\frac{S^j(v,x_0)}{\mu(\theta^i(\bar{v},x_0))}+v\right\}.
\end{aligned}$$

I define $\bar{U} := \min_{v\in Y_p^i}\left\{\frac{S^j(v,x_0)}{\mu(\theta^i(\bar{v},x_0))}+v\right\}$, which represents the value delivered to workers of the unintended type j if the incentive constraint binds.

The Hamiltonian of this problem can be formulated as:

$$
\begin{aligned}
\mathscr{H}_\tau :=& [y(x_0+\tau) - w(\tau)]\gamma^i(\tau) - \chi^\gamma_\tau[\rho + d + \delta^i(x_0+\tau) + g^i(V^i(\tau), x_0+\tau)] \\
& + \chi^i_\tau\Big\{(\rho+d)V^i(\tau) - u(w(\tau)) - \delta^i(x_0+\tau)[V^i_u(x_0+\tau) - V^i(\tau)] \\
& - S^i(V^i(\tau), x_0+\tau)\Big\} \\
& + \chi^j_\tau\Big\{(\rho+d)U^j(\tau) - u(w(\tau)) - \delta^j(x_0+\tau)[V^j_u(x_0+\tau) - U^j(\tau)] \\
& - S^j(U^j(\tau), x_0+\tau)\Big\},
\end{aligned}
$$

where χ^γ, χ^i, and χ^j are multipliers of the contemporaneous constraints.

The first order conditions follow:

$$
\begin{aligned}
\frac{\partial \mathscr{H}}{\partial w} =& -\gamma^i(\tau) - \chi^i(\tau)u'(w(\tau)) - \chi^j_\tau u'(w(\tau)) = 0 \\
\Rightarrow \quad \frac{\chi^i_\tau + \chi^j_\tau}{\gamma^i_\tau} =& -\frac{1}{u'(w(\tau))} \\
\frac{\partial \mathscr{H}}{\partial V^i} =& -\chi^\gamma_\tau \gamma^i(\tau) g^i_v(V^i_\tau, x_0+\tau) + \chi^i_\tau[\rho + d + \delta^i(x_0+\tau) - S^i_v(V^i_\tau, x_0+\tau)]
\end{aligned}
\tag{A.1}
$$

By the Envelope Theorem, partial derivative $S^i_v = -g^i$. Recall that I define $r^i_\tau := \rho + d + \delta^i(x_0+\tau)$ for exposition. Then I have

$$
\frac{\partial \mathscr{H}}{\partial V^i} = -\chi^\gamma_\tau \gamma^i_\tau g^i_v(V^i_\tau, x_0+\tau) + \chi^i_\tau[r^i_\tau + g^i(V^i_\tau, x_0+\tau)] = -\dot{\chi}^i_\tau. \tag{A.2}
$$

Similarly,

$$
\begin{aligned}
\frac{\partial \mathscr{H}}{\partial U^j} =& \chi^j_\tau[r^j_\tau + g^j(U^j_\tau, x_0+\tau)] = -\dot{\chi}^j_\tau \\
\Rightarrow \quad \chi^j_\tau =& \chi^j_0 exp[-\int_0^\tau (r^j_s + g^j(V^j_s, x_0+s))ds].
\end{aligned}
\tag{A.3}
$$

Lastly,

$$\frac{\partial \mathscr{H}}{\partial \gamma^i} = y(x_0+\tau) - w(\tau) - \chi^\gamma_\tau[r^j_\tau + g^i(V^i_\tau, x_0+\tau)] = -\,\dot{\chi}^\gamma_\tau$$
$$\Rightarrow \quad [y(x_0+\tau) - w(\tau)]\gamma^i_\tau + \chi^\gamma_\tau\dot{\gamma}^i_\tau = -\,\dot{\chi}^\gamma_\tau\gamma^i_\tau. \tag{A.4}$$

From (A.4), I obtain

$$\begin{aligned}\frac{d}{d\tau}(\chi^\gamma_\tau\gamma^i_\tau) &= -\,[y(x_0+\tau) - w(\tau)]\gamma^i_\tau \\ \chi^\gamma_\tau\gamma^i_\tau &= \int_\tau^\infty [y(x_0+s) - w(s)]\gamma^i_s ds + \chi^\gamma_s\gamma^i_s|_{s\to\infty} \\ &= J^i(\tau|x_0, w)\gamma^i_\tau \end{aligned} \tag{A.5}$$

Plugging (A.5) into (A.2), I get

$$-J^i(\tau|x_0,w)\gamma^i_\tau g^i_v(V^i_\tau, x_0+\tau) + \chi^i_\tau[r^i_\tau + g^i(V^i_\tau, x_0+\tau)] = -\,\dot{\chi}^i_\tau$$
$$\Rightarrow -J^i(\tau|x_0,w)g^i_v(V^i_\tau, x_0+\tau) - \frac{\chi^i_\tau\dot{\gamma}^i_\tau}{(\gamma^i_\tau)^2} = -\,\dot{\chi}^i_\tau/\gamma^i_\tau.$$

After rearranging the terms, I obtain

$$\begin{aligned}\frac{d}{d\tau}(\frac{\chi^i_\tau}{\gamma^i_\tau}) &= J^i(\tau|x_0,w)g^i_v(V^i_\tau, x_0+\tau) \\ \frac{\chi^i_\tau}{\gamma^i_\tau} &= \int_0^\tau J^i(s|x_0,w)g^i_v(V^i_s, x_0+s)ds + \chi^i_0. \end{aligned}$$

Now that properties of the multipliers are established, I take the time derivative of (A.1):

$$\frac{d}{d\tau}(\frac{\chi^i_\tau}{\gamma^i_\tau}) + \frac{d}{d\tau}(\frac{\chi^j_\tau}{\gamma^i_\tau}) = \frac{u''(w(\tau))}{u'^2(w(\tau))}\frac{dw(\tau)}{d\tau}. \tag{A.6}$$

Knowing that (A.3) stipulates

$$\begin{aligned}\frac{\chi^j_\tau}{\gamma^i_\tau} =&\chi^j_0 exp[-\int_0^\tau (r^j_s - r^i_s + g^j(U^j_s, x_0+s) - g^i(V^i_s, x_0+s))ds] \\ =&\chi^j_0\frac{\gamma^j_\tau}{\gamma^i_\tau},\end{aligned}$$

and

$$\frac{d}{d\tau}(\frac{\chi^j_\tau}{\gamma^i_\tau}) = -\chi^j_0\frac{\gamma^j_\tau}{\gamma^i_\tau}[r^j_\tau - r^i_\tau + g^j(U^j_\tau, x_0+\tau) - g^i(V^i_\tau, x_0+\tau)]. \tag{A.7}$$

Plugging (A.7) into (A.6), I obtain,

$$\begin{aligned}\frac{dw(\tau)}{d\tau} = \frac{u'^2(w(\tau))}{u''(w(\tau))}\Big\{&J^i(\tau|x_0,w)g^i_v(V^i_\tau, x_0+\tau) - \chi^j_0\frac{\gamma^j_\tau}{\gamma^i_\tau}[r^j_\tau - r^i_\tau + g^j(U^j_\tau, x_0+\tau) \\ &- g^i(V^i_\tau, x_0+\tau)]\Big\}.\end{aligned}$$

The last step is to establish that $\chi^i_0 = \zeta$. The Lagrangian of the firm's problem is

$$\mathscr{L} = \int_0^\infty (\mathscr{H}_\tau - \chi^\gamma_\tau\dot{\gamma}^i_\tau - \chi^i_\tau\dot{V}^i_\tau - \chi^j_\tau\dot{U}^j_\tau)d\tau + \zeta[\bar{U} - U^j(0)] + \xi[V^i(0) - \bar{V}],$$

where ζ and xi are the multipliers of the time-0 incentive constraint and the time-0 promise-keeping constraint.

Consider an admissible perturbation to U^j_τ, $U^j_\tau + \Delta V^j_\tau$, that obeys the constraint. The perturbation yields

$$\Delta\mathscr{L} = \int_0^\infty (\frac{\partial\mathscr{H}}{\partial U^j}\Delta U^j_\tau + \dot{\chi}^j_\tau\Delta U^j_\tau)d\tau - \chi^i_\infty\Delta U^j_\infty + \chi^j_0\Delta U^j_0 - \zeta\Delta U^j_0.$$

Since $\chi^j_\infty = 0$ and ΔU^j_∞ is bounded, $\chi^i_0 = \zeta$, the multiplier of the time-0 incentive constraint.

APPENDIX B

Decomposition of Gender Wage Gap

Let superscript F denote functions of the female population and M denote functions of the male population. Function $\bar{w}$ is the expectation of wage in a population. I use f to denote the probability density functions. For example, $f^F_{x|t,v}$ represents the conditional distribution of experience x given tenure t and starting employment value v in the female population, and $f^M_v(v)$ represents the partial distribution of starting employment value v in the male population. The full decomposition of the mean gender wage gap follows (B.1).

$$
\begin{aligned}
\bar{w}^F - \bar{w}^M
&= \int_v \Big\{ \int_t \Big[\int_x \bar{w}^F(t,v,x) \mathbf{f^F_{x|t,v}(x|t,v)} dx \Big] f^F_{t|v}(t|v) dt \Big\} f^F_v(v) dv \\
&- \int_v \Big\{ \int_t \Big[\int_x \bar{w}^F(t,v,x) \mathbf{f^M_{x|t,v}(x|t,v)} dx \Big] f^F_{t|v}(t|v) dt \Big\} f^F_v(v) dv \\
&+ \int_v \Big\{ \int_t \Big[\int_x \bar{w}^F(t,v,x) f^M_{x|t,v}(x|t,v) dx \Big] \mathbf{f^F_{t|v}(t|v)} dt \Big\} f^F_v(v) dv \\
&- \int_v \Big\{ \int_t \Big[\int_x \bar{w}^F(t,v,x) f^M_{x|t,v}(x|t,v) dx \Big] \mathbf{f^M_{t|v}(t|v)} dt \Big\} f^F_v(v) dv \\
&+ \int_v \Big\{ \int_t \Big[\int_x \bar{w}^F(t,v,x) f^M_{x|t,v}(x|t,v) dx \Big] f^M_{t|v}(t|v) dt \Big\} \mathbf{f^F_v(v)} dv \\
&- \int_v \Big\{ \int_t \Big[\int_x \bar{w}^F(t,v,x) f^M_{x|t,v}(x|t,v) dx \Big] f^M_{t|v}(t|v) dt \Big\} \mathbf{f^M_v(v)} dv \\
&+ \int_v \Big\{ \int_t \Big[\int_x \mathbf{\bar{w}^F(t,v,x)} f^M_{x|t,v}(x|t,v) dx \Big] f^M_{t|v}(t|v) dt \Big\} f^M_v(v) dv \\
&- \int_v \Big\{ \int_t \Big[\int_x \mathbf{\bar{w}^M(t,v,x)} f^M_{x|t,v}(x|t,v) dx \Big] f^M_{t|v}(t|v) dt \Big\} f^M_v(v) dv. \qquad \text{(B.1)}
\end{aligned}
$$

Using the simulated sample, I estimate the density functions assuming multivariate normal distributions. The mean wage functions are constructed with the equilibrium wage contracts of the quantitative model. Because female workers are heterogeneous by assumption, the mean wage function in the female population accounts for the fractions of high- and low-attachment women.