Optical Imaging Of Tissue Physiology With Exogenous Contrast Agents

Sang Hoon Chong

University of Pennsylvania

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Optical Imaging Of Tissue Physiology With Exogenous Contrast Agents

Abstract
This thesis describes experiments and analyses which push the frontier per what one can learn from optically emitting exogenous contrast agents in living tissue. The first set of experiments concurrently measured cerebral blood flow and both intravascular- and extravascular-tissue oxygen concentration in a rat brain during functional activation; the new instrumentation needed to collect this information used contrast agent phosphorescence lifetime to determine oxygen concentration and speckle contrast imaging to probe blood flow. The concurrent measurement of multiple physiological parameters with high temporal resolution (∼7 Hz) provided a unique opportunity to observe the interconnected dynamics of oxygen exchange, blood flow, and cerebral oxygen metabolism. The experiments showed that initial metabolic changes trigger a blood flow response; comprehensive theoretical modeling of the data exposed potential weaknesses of the well-known and often-used two-compartment oxygen diffusion model, and the experiments as a whole introduced a new tool for characterization of oxygen metabolism and neurovascular coupling in the brain.

The second set of experiments developed instrumentation and a simple theoretical methodology for imaging fluorescent targets in turbid media such as tissue. This approach used the ideas of spatial frequency domain fluorescence diffuse optical tomography (SFD-FDOT). The new reconstruction algorithm modified the more complex SFD-FDOT reconstruction method to rapidly acquire the depth of fluorescent target(s) and then estimate the transverse margins of the fluorescent target(s). Tissue phantom experiments demonstrated the instrumentation and algorithm, and assessed limitations. The new methodology could be useful for image guidance during tumor resection surgery, and could also provide rapid and useful constraining information for more comprehensive fluorescent tomography.

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OPTICAL IMAGING OF TISSUE PHYSIOLOGY
WITH EXOGENOUS CONTRAST AGENTS

Sang Hoon Chong

A DISSERTATION
in
Physics and Astronomy

Presented to the Faculties of the University of Pennsylvania
in
Partial Fulfillment of the Requirements for the
Degree of Doctor of Philosophy

2022

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OPTICAL IMAGING OF TISSUE PHYSIOLOGY WITH EXOGENOUS CONTRAST AGENTS

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Sang Hoon Chong

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Dedicated to my wife, Su Kyung,
and my beloved kids, Ayn and Arjun.
ACKNOWLEDGMENT

My life has been a stream of unpredictable journeys driven by my reckless dreams. It also produced many difficult challenges. The last one, a degree of Ph.D., is finally conquered and turned out to be the most demanding goal so far. It might have not come to a good end without continuous support and encouragement from many people to whom I am so grateful.

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Thank you all for granting me this incredible memory and achievement!
This thesis describes experiments and analyses which push the frontier per what one can learn from optically emitting exogenous contrast agents in living tissue. The first set of experiments concurrently measured cerebral blood flow and both intravascular- and extravascular-tissue oxygen concentration in a rat brain during functional activation; the new instrumentation needed to collect this information used contrast agent phosphorescence lifetime to determine oxygen concentration and speckle contrast imaging to probe blood flow. The concurrent measurement of multiple physiological parameters with high temporal resolution (~7 Hz) provided a unique opportunity to observe the interconnected dynamics of oxygen exchange, blood flow, and cerebral oxygen metabolism. The experiments showed that initial metabolic changes trigger a blood flow response; comprehensive theoretical modeling of the data exposed potential weaknesses of the well-known and often-used two-compartment oxygen diffusion model, and the experiments as a whole introduced a new tool for characterization of oxygen metabolism and neurovascular coupling in the brain.

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Chapter 1

Introduction

1.1 Probing Biological Tissue with Optics

Broadly, optical techniques of many kinds have been successfully employed to investigate normal and abnormal tissues [93]. Depending on desired biological goals and interrogation length scales, experimenters will choose a wavelength range, a measurement geometry (e.g., reflection/transmission, microscopic/macroscopic, etc.), and a suitable technology (e.g., absorption/fluorescence frequency-domain or time-domain spectroscopy, fluorescence/phosphorescence lifetime measurement, correlation spectroscopy, etc.).

In this thesis, all of our experiments aim to probe “near-surface” tissues in the reflection geometry. Thus, the light sources and detectors reside on the same (observing) side of the tissue sample. For this work, we predominantly (but not exclusively) employ near-infrared (NIR) light. Roughly speaking, NIR light ranges from 600 nm to 1000 nm. Compared to visible light, photons in NIR have excellent penetration depth in tissue, ranging from millimeter to centimeter scales [36]. Thus, its use enables non-invasive and minimally invasive studies of relatively deep tissues that are typically inaccessible to traditional optical microscopies [31] and methods that rely on single scattering (or no scattering). NIR spectroscopy (NIRS) provides access to the absorption of major endogenous tissue chromophores (e.g., oxy- and deoxy-hemoglobin) [111]; NIR light (pulsed or CW) can be employed to excite exogenous
contrast agents in tissue, which in turn can provide information about tissue oxygen (via fluorescence lifetime measurements) [89] or information about tumor margins (via fluorophore concentration) [144]; narrowband NIR light sources enable accurate measurements of the speckle fluctuations of reflected light, which in turn provide information about blood flow [38].

In this thesis, I will discuss optical instrumentation, algorithms, and experimental schemes we have developed to probe fluorescent/phosphorescent exogenous contrast agents in two contexts. In one study, we measure agent phosphorescence lifetime to characterize tissue oxygen gradients in the vasculature of rat-brain cortex during resting-state and functional activation. These experiments push the frontier of what is known about neurovascular coupling and oxygen metabolism. In the second, qualitatively different study, we develop fluorescence-imaging instrumentation and algorithms to locate fluorescent targets in situations relevant for image-guided tumor resection surgery. Note, while the central focus of both studies is on exogenous contrast agents, both studies also require measurement/knowledge of endogenous tissue properties.

1.2 Optical Contrast in Tissue

1.2.1 Endogenous chromophores

The tissue contains many molecules, but only a few of these chromophores absorb substantial amounts of light (chromophores) in the range of UV, VIS, and NIR parts of the optical spectrum. This large absorption is a consequence of their comparatively large concentration and oscillator strength. These chromophores are oxy- and deoxy-hemoglobin (HbO₂ and HbR respectively), water, and lipid. Importantly, as noted
above, these chromophores collectively absorb the least amount of light in the NIR part of the optical spectrum, and therefore this regime of minimum absorption is traditionally referred to as the physiological window (Fig.1)). In this window, light penetration depth in most tissues ranges from millimeters to centimeters, depending on blood concentration.

Figure 1: Absorption coefficient spectra of the major endogenous chromophores in tissue. Reprinted from Fig. 1 of Ref. 36.

When NIR light propagates in tissue, it interacts with these chromophores, and it also interacts with organelles, nuclei, cells, and their associated membranes/interfaces. Photons are lost via chromophore absorption, and photons are scattered by the organelles, etc. In most tissues, the absorption mean free path (i.e., the reciprocal of the absorption coefficient $\mu_a$) is long compared to the so-called reduced scattering mean free path (i.e., the reciprocal of the reduced scattering coefficient $\mu'_s$). In this regime, the diffusion equation (with loss) is a very good approximation for photon transport.
Diffuse optics refers broadly to techniques that utilize the diffusion equation as an underlying mathematical framework to analyze the propagation of light in turbid media, \textit{e.g.}, NIR light in most tissues. Our experiments in the reflection geometry, for example, utilize models based on the diffusion equation to predict the propagation of NIR light into and out of tissue. The model permits the determination of the medium’s absorption ($\mu_a$) and reduced scattering coefficient ($\mu_s'$) \cite{36}, and the wavelength dependence of these parameters (especially $\mu_a$) enables experimenters to deduce the concentrations of the aforementioned endogenous chromophores \cite{37}.

The NIR absorption technique, \textit{i.e.}, NIR spectroscopy (NIRS), enables many clinical and pre-clinical applications. For example, in breast cancer, tumor tissues exhibit optical properties that differ from those of normal surrounding tissues as a result of their microvasculature and metabolism; typically, for example, tumors show higher concentrations of HbO$_2$ and HbR compared to normal tissue. These differences provide spectroscopic contrast which is useful for tomographic imaging of breast cancer and for monitoring the response of cancer to therapy \cite{23,139,21,20,7,49}. In a different vein, continuous measurement of HbO$_2$ and HbR can inform experimenters about the hemodynamics and oxygen dynamics in brain tissue \cite{25}. Indeed, NIR spectroscopy and tomography of HbO$_2$ and HbR have been an important surrogate for direct molecular oxygenation measurements in tissue; moreover, if integrated with optical measurements of cerebral blood flow (CBF), it is possible to estimate cerebral oxygen consumption or the so-called cerebral metabolic rate of oxygen (CMRO$_2$).

\subsection*{1.2.2 Exogenous contrast agents}

While endogenous contrast is useful and can be probed non-invasively, there have been significant advances in the field per development of exogenous contrast
agents. Exogenous contrast agents must be introduced into tissue and are thus invasive (potentially, minimally invasive). Their utility arises from their potential to teach us about other properties in tissue, properties that cannot be rigorously determined from endogenous contrast. For example, in our experiments on functional activation in brain, we utilize exogenous porphyrin-based phosphorescence agents (Oxyphors); the lifetime of their photoluminescence is sensitive to the oxygen concentration in tissue. Oxyphors have been applied in a variety of contexts because their sensitivity to molecular oxygen concentration is relevant to many important metabolic questions. Some optical technologies used thus far with Oxyphors are two-photon microscopy [43, 51, 2, 124], fiber-based imaging [101], and camera-based imaging [44, 120]. Deployment of the phosphorescent agent in tissue enables direct spatiotemporal measurement of oxygen concentration, which is more direct than more common analyses of tissue blood oxygen dynamics based on HbO$_2$ and HbR. Typically, these exogenous contrast agents are designed to be excited by visible (green and red) light, and they emit phosphorescence light in the NIR.

Figure 2: Example spectra of Oxyphor R4 (red) and G4 (green). (a) Absorption spectra. (b) Relative emission spectra. Reprinted from Fig. 2 of Ref. 44.

Another useful exogenous contrast phenomenon is based on fluorescence for tumor detection and characterization. This general problem is a part of the motivation
for our diffuse optical imaging experiments. Specifically, an exogenous contrast agent called indocyanine-green (ICG) has been FDA-approved for clinical use. Among other things, it is the major contrast agent used for clinical angiography [126, 40]. Due to its good fluorescent efficiency and safety, it has also been increasingly deployed as a fluorescent contrast agent for image-guided surgery for tumor resection [113, 6]. In this case, the surgeon injects ICG to a patient 24 hours ahead of the tumor resection surgery; due to various factors, the ICG molecules will preferentially accumulate in and around tumor tissue via its leaky vasculature. Using this so-called ICG retention effect [108], the absorption and emission of ICG (and variants) in this context have permitted tumor detection (Fig. 3). In this case, measurement of the enhanced concentration of the agent (in space and in time) permits tumor detection and characterization. Looking forward, all-out efforts to develop cell-specific fluorescent dyes for better contrast are ongoing [72, 29]. Parallel efforts (such as ours) that aim to improve the fidelity of fluorescent contrast imaging are vital too.

Figure 3: ICG absorption and emission spectra of 6.5μM in the blood plasma. Reprinted from the package insert of ICG [3].
1.3 Central Topics of this Thesis

In the remaining chapters of this thesis, two important applications of exogenous contrast agents are utilized to probe issues at the Biophotonics frontier. Both works enable the acquisition of important information about biological tissue. In the first set of experiments, two phosphorescent contrast agents (Oxyphors) are deployed to interrogate oxygen metabolism in rat brain before, during, and after functional activation. CMRO$_2$ is an important biomarker that we gain access to as a result. In principle, information about CMRO$_2$ can teach us how to assess normal and diseased brain function [54, 129, 130]; furthermore, better information about CMRO$_2$ can help us to answer fundamental questions associated with oxygen dynamics, neurovascular coupling, and metabolic mechanisms in brain. Traditional measurement paradigms for the study of CMRO$_2$ lack the temporal, spatial, and biochemical resolution to fully elucidate dynamic neurological events, e.g., neurovascular coupling. In our investigation, we introduce a new experimental platform to estimate CMRO$_2$ with high temporal and spatial resolutions by simultaneously monitoring intra- and extra-vascular oxygen concentration before, during, and after functional activation. Additionally, we acquired images of cerebral blood flow (CBF) concurrently. Thus, assessment of CMRO$_2$ directly from Oxyphor oxygen measurements provides a novel opportunity to compare the detailed dynamics of oxygen consumption and blood flow with high temporal resolution.

The second set of experiments aims to estimate the depth and transverse margins of fluorescent targets in turbid media. This is an important task that, if carried out rapidly enough, could be utilized by a surgeon for surgical decision-making. Currently, image-guided tumor resection surgery has been increasingly popular due to real-time two-dimensional image-projection guidance, i.e., a surgeon can coarsely
distinguish near-surface tumors from normal tissue by taking two-dimensional images of tissue fluorescence. However, this technique lacks depth information which can be critical for surgeries wherein preserving healthy normal tissue is important. Our work builds on current Spatial Frequency Domain Fluorescence Diffuse Optical Tomography (SFD-FDOT) research. We developed an SFD-based algorithm and instrumentation to assess the depth and transverse margins of fluorescent targets quickly. The instrumentation and algorithm were evaluated experimentally in order to take the first step toward translation to the operating room. The work can also be useful as complimentary priori information in more comprehensive FDOT too.

1.4 Thesis Organization

In the remainder of this thesis, Chapters 2 - 4 are devoted to the first set of experiments that seek to probe CMRO$_2$ in rat brain by direct measurement of oxygen concentration. Chapters 5 & 6 report the algorithm and instrumentation and testing for a new SFD-FDOT technique for fast acquisition of depth/margin.

More specifically, Chapter 2 introduces neurovascular coupling and CMRO$_2$. Further, it explains why the platform we develop could be valuable for future studies of CMRO$_2$ (especially in comparison to existing technology). Lastly, it introduces the theoretical framework we employ to estimate CMRO$_2$ from our measurements. In Chapter 3, the primary measurement techniques are introduced along with information about detailed surgical preparation for the experiment. Chapter 4 demonstrates the full approach, reports important results from the measurements, and discusses its impact on the interpretation of neurovascular coupling and other biological phenomena in the brain.

In Chapter 5, a brief background of the general problem is given, and the theory of diffuse optics and then SFD-FDOT is introduced in detail. The aspects of the
theory of SFD-FDOT needed for our experiments with fluorescent targets is then further developed and explained. Chapter 6 reports the experimental results and system performance through a series of phantom experiments.

Lastly, Chapter 7 summarizes the results and discussion from both experiments, and provides brief suggestions for future work.
Chapter 2

Hemodynamics and Oxygen Metabolism in Rat Brain


2.1 Neurovascular Coupling and Its Significance

A continuous oxygen supply is critical for maintaining activity in the brain [160, 17, 77]. Briefly, blood carries oxygen and distributes it into brain tissue (predominantly) via a network of capillary vessels as shown in Fig. 4 (Note, the image design in the figure is closely related to Fig. 2 of Ref. 70.). Oxygen molecules diffuse from the vessels into neighboring extravascular tissues down a concentration gradient [140] which depends on the rate of oxygen consumption; in the brain, this oxygen consumption rate is called the cerebral metabolic rate of oxygen (CMRO$_2$). The oxygen is metabolized by mitochondria to generate energy via its primary pathway, oxidative phosphorylation [17, 22].

Even in their resting state, brain cells are active [95]. Thus, they require a supply of oxygen at all times. However, during functional activation more oxygen is needed, and cerebral blood flow (CBF) must rapidly respond to these changes in neuronal
activity. More oxygen must be supplied to regions where more activity occurs [71].

The cascade of events connecting variation of neuronal activity to CBF response is
known as neurovascular coupling (NVC) [28]. If NVC is compromised, then the brain
can suffer local deprivation of oxygen, which can lead to cell death, and potentially
permanent neurological disorders [77]. Therefore, comprehensive knowledge about
NVC is valuable for understanding the physiology of both healthy and abnormal
brains. One way to learn about NVC is to observe and compare hemodynamics
and oxygen dynamics in the brain during steady-state (resting-state) and during
functional activation. Information about the sequence of events following functional
activation, for example, is invaluable for ascertaining the causal connections between
various vascular and extravascular physiological parameters [60].

The pure hemodynamic parameters of importance for us (in brain) include CBF,
and the concentration of oxy- and deoxy-hemoglobin (\(\text{HbO}_2\) and \(\text{HbR}\)), or, equiva-

cently, the total hemoglobin concentration and blood oxygen saturation (THC and

Figure 4: Schematic of oxygen delivery from vasculature to tissue.
SO$_2$). The molecular oxygen parameters of importance within the vascular system are the bound and unbound oxygen concentration in arteries, arterioles, capillaries, venules, and veins. For functional activation in brain, we are specifically interested in the total (bound and unbound) oxygen concentration in the arterioles, capillaries, and venules, which we denote $C_a$, $C_c$, and $C_v$, respectively. Additionally, we are interested in unbound molecular oxygen parameters both outside the vascular system (in extravascular tissue) and within the vascular system (i.e., intravascular); here we denote unbound oxygen concentrations in the capillary intravascular space and in the extravascular space as $C_i$ and $C_e$, respectively. In our experiments, we measure these latter unbound oxygen parameters, which in turn provide access to CMRO$_2$ as described quantitatively below. Additionally, in our experiments, we measure CBF. The concurrent acquisition of CBF and CMRO$_2$ enables us to systemically compare hemodynamic and oxygen consumption responses.

2.2 Prior Measurements Sensitive to Oxidative Metabolism in Brain

Quantitative assessment of CMRO$_2$ has been of interest to the neuroscience community for many years. CMRO$_2$ can be a useful marker for early detection of tissue pathology such as in stroke [54], traumatic brain injury [129], and glioma [130]. Moreover, dynamic measurements of CMRO$_2$ during neuronal activation provide information about the brain metabolic events that underpin functional responses [13]. Positron Emission Tomography (PET) with $^{15}$O-labeled compounds (O$_2$, H$_2$O, CO) is arguably the most established method for absolute CMRO$_2$ quantification in steady-state [103, 66]. PET is compatible with human studies, but quantitative measurement of CMRO$_2$ with PET requires independent measurements of CBF and total cerebral
blood volume (CBV), and it relies on multi-parametric models for the calculation of CMRO$_2$ [102, 76]. Furthermore, PET has a low temporal resolution on the order of 2-7 minutes [45] and is, therefore, unable to reveal rapid metabolic dynamics of cerebral tissues during neuronal activation.

For dynamic tracking of brain responses, the most widely used methods are Blood-Oxygen-Level-Dependent functional Magnetic Resonance Imaging (BOLD fMRI) [117, 164] and functional Near-Infrared Spectroscopy (fNIRS) [47, 163, 115]. The central parameter in both techniques is hemoglobin oxygen saturation (S$_{tO_2}$), and, like PET, absolute quantification of CMRO$_2$ requires independent measurements of oxygen extraction fraction (OEF), CBF, and CBV, and relies on multi-parametric models [14, 64, 164, 61, 35, 39, 25].

As a result, BOLD fMRI and fNIRS are best suited to probe relative changes in CMRO$_2$, albeit with resultant computed CMRO$_2$ dynamics inherently coupled to the dynamics of CBF. However, since CMRO$_2$ and CBF need not change in the same way when responding to activation, their relative timing carries valuable information about the dynamics of NVC [65, 147], and this relative timing is inaccessible to PET, BOLD fMRI, and fNIRS which rely on CBF for calculation of CMRO$_2$.

Apart from techniques that evaluate cerebral hemodynamics, local CMRO$_2$ can also be inferred from measurements of molecular oxygen gradients around individual vessels, for example by using diffusion-based models such as the two-compartment model suggested by Krogh [75]. The partial pressure of oxygen (pO$_2$), and pO$_2$ gradients around vessels were originally measured with oxygen microelectrodes [148]. More recently, two-photon phosphorescence lifetime microscopy (2PLM) has proved useful for spatially-resolved microscopic pO$_2$ measurements [48]. 2PLM is minimally invasive and is capable of probing pO$_2$ at multiple locations near vessels in the brain [118, 82]. However, mapping oxygen by 2PLM requires long observation
times, and therefore the technique is best suited for measurements of steady-state gradients [30, 119]. On the other hand, phosphorescence lifetime oximetry has been used to observe stimulus-induced changes of brain \( \text{pO}_2 \) [4], demonstrating capability for measurements at speeds comparable with transients of neuronal activity. Lastly, microelectrodes and laser Doppler have been used concurrently to track intravascular and extravascular \( \text{pO}_2 \) in two selected locations, along with local CBF during functional stimulation; together the three signals provided information sensitive to changes in oxygen metabolism, but \( \text{CMRO}_2 \) was not computed [146].

### 2.3 Our Approach

Here we introduce and demonstrate an all-optical imaging approach for dynamic measurement of \( \text{CMRO}_2 \). The new methodology is based on the quantification of oxygen concentration gradients in the brain using two macromolecular phosphorescent probes, Oxyphors [92, 44], placed separately in the intravascular and extravascular compartments. These probes are carefully designed such that they do not diffuse across the blood-brain barrier, and the chromophores in the probes have distinguishable optical spectra. Thus, the Oxyphors can be interrogated independently and concurrently for direct measurement of oxygen gradients in both compartments. In our work, we sampled oxygen gradients in rat brain cortex at a rate of \( \sim 7 \text{ Hz} \), but this sampling frequency can potentially be increased \( > 10 \)-fold. In parallel, we measured the local CBF by laser speckle contrast imaging (LSCI). This combination enabled us to correlate measured changes in \( \text{CMRO}_2 \) with measured changes in CBF and thus ascertain the timing of the physiological events that accompany neuronal activation.

The new approach is minimally invasive, offers superior time-resolution (potentially approaching milliseconds), and creates novel opportunities for dynamic tracking and quantification of brain metabolism. Ultimately, with straightforward improve-
ments in technology, it should be possible to minimize systematic quantification errors associated with oxygen gradients, and thereby generate a robust method for in vivo dynamic quantification of absolute CMRO$_2$.

### 2.4 Oxygen Dynamics and CMRO$_2$

In our approach, CMRO$_2$ is estimated by direct measurement of oxygen concentrations in both intravascular and extravascular space. To understand how this works in practice, in this section, we present a comprehensive theoretical model.

We use the two-compartment theory [142] to model oxygen transport from the vasculature to the extravascular tissue; this theory enables us to estimate CMRO$_2$ in conjunction with the Krogh-Erlang cylinder model [75]. The variables and parameters we use are defined, along with their units, in Table 1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_i$</td>
<td>mL/100g Tissue</td>
<td>Capillary (intravascular) volume per unit mass of brain tissue.</td>
</tr>
<tr>
<td>$V_e$</td>
<td>mL/100g Tissue</td>
<td>Extravascular volume per unit mass of brain tissue.</td>
</tr>
<tr>
<td>$V_t$</td>
<td>mL/100g Tissue</td>
<td>Full tissue volume($V_i + V_e$) per unit mass of brain tissue.</td>
</tr>
<tr>
<td>CBF</td>
<td>mL/s/100g Tissue</td>
<td>Cerebral blood flow per unit mass of brain tissue.</td>
</tr>
<tr>
<td>$C_a$</td>
<td>mM</td>
<td>Arteriole O$_2$ concentration (includes both bound and unbound to Hb).</td>
</tr>
<tr>
<td>$C_c$</td>
<td>mM</td>
<td>Capillary O$_2$ concentration (includes both bound and unbound to Hb).</td>
</tr>
<tr>
<td>$C_v$</td>
<td>mM</td>
<td>Venule O$_2$ concentration (includes both bound and unbound to Hb).</td>
</tr>
<tr>
<td>$C_i$</td>
<td>µM</td>
<td>Capillary O$_2$ concentration (includes unbound O$_2$ only).</td>
</tr>
<tr>
<td>$C_e$</td>
<td>µM</td>
<td>O$_2$ concentration in the extravascular interstitial space of tissue.</td>
</tr>
<tr>
<td>P</td>
<td>cm/s</td>
<td>Permeability of O$_2$ molecule across capillary wall.</td>
</tr>
<tr>
<td>$S_c$</td>
<td>cm$^2$/100g Tissue</td>
<td>Surface area of capillary per unit mass of brain tissue.</td>
</tr>
<tr>
<td>CMRO$_2$</td>
<td>µmoles/s/100g Tissue</td>
<td>Cerebral metabolic rate of oxygen.</td>
</tr>
</tbody>
</table>
2.4.1 Intravascular-extravascular two-compartment oxygen transport model

The brain capillary network architecture is complex, and modeling capillary oxygen (O$_2$) dynamics is challenging. Therefore, we simplify the complex capillary structure to a simple cylindrical tube as has been done for many years (e.g., the Krogh-Erlang Model). Our situation and model are schematically illustrated in Fig. 4. In this model, O$_2$ is delivered by blood flowing through the penetrating arteriole, and from there it is distributed into the complex capillary network. The blood passing through the capillaries contains O$_2$ dissolved in the blood plasma; this dissolved O$_2$ is in equilibrium with O$_2$ bound to hemoglobin, which is described by the Hill equation [155]. Some of this O$_2$ diffuses into the extravascular tissue to support tissue metabolism. O$_2$ transport into and out of the capillary network is modeled by Eq. (2.1).

\[
V_i \frac{dC_c(t)}{dt} = CBF(t) \cdot [C_a(t) - C_v(t)] - PS_c \cdot [C_i(t) - C_e(t)].
\] (2.1)

The temporal rate-of-change of O$_2$ molecules, including both bound and unbound to hemoglobin, in the capillary compartment volume is shown on the left-hand side (LHS) of this equation; here $C_c$ is the O$_2$ molecule concentration (bound and unbound). $V_i$ represents the total capillary compartment volume, i.e., the total intravascular volume. The temporal rate-of-change of total O$_2$ in the capillary volume depends on the factors on the right-hand side (RHS) of the equation above. The first term on the RHS of Eq. (2.1) defines the net-flow of O$_2$ molecules into the capillary compartment; here CBF($t$) is the absolute blood flow at time $t$. $C_a$ and $C_v$ are arteriole and venule O$_2$ molecule concentrations, respectively. $C_a$ and $C_v$ include O$_2$ molecules bound to hemoglobin and O$_2$ molecules dissolved in the blood plasma; they
are in units of mM. The second term on the RHS represents the extraction rate of O₂ molecules from the capillary compartment into the extravascular tissue; here $PS_c$ is a product of the O₂ permeability, P, and the capillary surface area, $S_c$. $C_i$ and $C_e$ represent the average O₂ molecule concentration in intravascular and extravascular tissue compartments, respectively; note, $C_i$ and $C_e$ depend only on dissolved O₂ molecules (i.e., only on unbound O₂). Notice that this extraction rate of O₂ from the capillary is roughly proportional to the concentration gradient of average unbound O₂ concentration between the intravascular and extravascular compartments. The permeability contains the diffusion coefficient for O₂ and some effective lengths (slightly longer than the capillary radius. see Sec. 2.4.2).

A similar equation can be written for the extravascular unbound O₂ molecule concentration. 

$$V_e \frac{dC_e(t)}{dt} = PS_c \cdot [C_i(t) - C_e(t)] - CMRO_2(t). \quad (2.2)$$

The LHS term is the time rate of change of O₂ molecules in the extravascular compartment. It depends on the derivative of $C_e$ and the extravascular compartment volume, $V_e$, which is $V_e = V_t - V_i$, where the total tissue volume is $V_t$. In the extravascular compartment equation, Eq. (2.2), the net-flow of O₂ molecules includes the diffusive extraction of O₂ from the intravascular compartment (the first term on the RHS), and the O₂ consumed by mitochondria in tissue (the second term on the RHS) whose rate is CMRO₂ by definition. Notice that this consumption rate is independent of the amount of O₂ in the compartment.

In our measurement, both intravascular and extravascular oxygen concentrations were measured with high temporal resolution. We assume that the measurements give $C_i$ and $C_e$ respectively. With this information, Equation (2.2) allows us to estimate absolute CMRO₂ provided that $V_e$ and $PS_c$ are known. In practice, $V_e$ is well
constrained [32, 91, 121], and consequently its fractional error affects CMRO$_2$ only marginally. However, PS$_c$ is not very well constrained. There are a few publications in the literature wherein PS$_c$ is either directly estimated via measurement or theoretically computed; prior estimates of PS$_c$ range from 35 to 230 mL/s/100g Tissue [123].

### 2.4.2 Theoretical modeling of PS$_c$

![Figure 5: (a) Schematic of Krogh-Erlang cylinder model. (b) Cross-section view of a single capillary and the surrounding extravascular tissue cylindrical shell corresponding to one of the coaxial cylinders in (a).](image)

Herein, we estimate PS$_c$ based on an approach that uses Krogh’s cylinder to model the whole tissue region (both intra- and extra-vascular tissue) as described in Fig. 5. In this modeling, each cylindrical capillary is surrounded by a coaxial extravascular cylindrical shell of tissue, and the closest neighboring capillary is placed one intercapillary distance away from one another. The capillary radius is denoted $\rho_1$, and the tissue outer edge radius is denoted $\rho_2$. Oxygen escaping from the capillary at $\rho_1$ will diffuse out to the edge of the extravascular tissue at $\rho_2$. The distance from
the capillary wall to the tissue boundary is typically set to be about one-half of the average inter-capillary distance. For a fractional volume of capillary, \( V_i/V_t \), \( \rho_1 \) and \( \rho_2 \) are related via the relation below.

\[
\frac{V_i}{V_t} = \left( \frac{\rho_1}{\rho_2} \right)^2. \tag{2.3}
\]

Then, the length of the cylinder can be computed using \( \rho_2 \).

\[
L = \frac{V_i}{\pi(\rho_2)^2}. \tag{2.4}
\]

Below is the table of morphological parameters of microvasculature and tissue with a few given volume fractions of capillary.

Table 2: Radii of capillary (\( \rho_1 \)), Radii of tissue cylinder (\( \rho_2 \)) and corresponding length of capillary (L) are tabulated under the constraint of \( V_i/V_t \).

<table>
<thead>
<tr>
<th>( V_i/V_t )</th>
<th>( \rho_1 (\mu m) )</th>
<th>( \rho_2 (\mu m) )</th>
<th>L (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>2.0</td>
<td>20</td>
<td>75.8</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>25</td>
<td>48.5</td>
</tr>
<tr>
<td></td>
<td>3.0</td>
<td>30</td>
<td>33.7</td>
</tr>
<tr>
<td></td>
<td>3.5</td>
<td>35</td>
<td>24.7</td>
</tr>
<tr>
<td>2%</td>
<td>2.0</td>
<td>14</td>
<td>152</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>18</td>
<td>97.0</td>
</tr>
<tr>
<td></td>
<td>3.0</td>
<td>21</td>
<td>67.4</td>
</tr>
<tr>
<td></td>
<td>3.5</td>
<td>25</td>
<td>49.5</td>
</tr>
<tr>
<td>3%</td>
<td>2.0</td>
<td>12</td>
<td>227</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>14</td>
<td>146</td>
</tr>
<tr>
<td></td>
<td>3.0</td>
<td>17</td>
<td>101</td>
</tr>
<tr>
<td></td>
<td>3.5</td>
<td>20</td>
<td>74.2</td>
</tr>
</tbody>
</table>

The other relevant physiological parameter in the model is the oxygen diffusion coefficient, \( D \). Reported \( D \) varies from 1.4 to \( 1.9 \times 10^{-5} \) cm\(^2\)/sec at 37°C [19, 63, 52, 135]. Note, the animals used in the cited references vary from small rodent species to human brain tissue.

Having the geometrical and physiological parameters of microvasculature constrained, in steady-state, we can readily determine the concentration profile in the extravascular tissue using the diffusion equation in cylindrical coordinates. Assuming the uniform diffusion coefficient in the extravascular tissue volume, solving the
time-dependent diffusion equation below allows us to estimate oxygen concentration distribution in the extravascular tissue.

$$\frac{\partial C}{\partial t} = D \nabla^2 C - M,$$  \hspace{1cm} (2.5)

where $M$ corresponds to CMRO$_2$ density in the extravascular tissue; that is, $M = \text{CMRO}_2/V_e$, $M$ is spatially uniform in the extravascular tissue volume.

To solve this problem, we must apply boundary conditions. The diffusive $O_2$ molecule flux at the tissue outer cylinder boundary is assumed zero in the cylindrically symmetric geometry, that is, \( \frac{dC}{d\rho} \big|_{\rho=\rho_2} = 0 \). The other boundary condition is that the concentration on the capillary wall edge is assumed to be $C_i$, \textit{i.e.}, the oxygen concentration in the intravascular compartment is homogeneous. The steady-state solution to this problem was worked out by Erlang and Krogh [75] and given in by Popel [114] in more detail.

$$C(\rho) = C(\rho_1) + \frac{M}{4D}(\rho^2 - \rho_1^2) - \frac{M\rho_2^2}{2D}\ln\left(\frac{\rho}{\rho_1}\right).$$  \hspace{1cm} (2.6)

The extravascular compartment average oxygen concentration, $C_e$, is found by evaluating the integral below.

$$C_e = \frac{1}{V_e} \int_{V_e} C dV = -\frac{M}{2D} \left(\frac{\rho_2^4 \ln(\rho_2/\rho_1)}{\rho_2^4 - \rho_1^4}\right) + \frac{M}{8D} (3\rho_2^2 - \rho_1^2) + C(\rho_1).$$  \hspace{1cm} (2.7)

Equation (2.6) predicts the extravascular $O_2$ concentration distribution in the radial direction, and Eq. (2.7) provides its average. Figure 6 shows an exemplary oxygen concentration profile in the extravascular cylinder in terms of corresponding partial oxygen pressure, mmHg, as well as the volume-averaged pO$_2$. In this example, CMRO$_2$ and $D$ are set to be 3.4 $\mu$moles/s/100g Tissue and $1.7 \times 10^{-5}$ cm$^2$/s.
Let us now summarize all of the implications of Eq. (2.7), which assumes that Krogh’s cylinder model to describe the vascular system. $C_e$ is a function of the morphological factors of microvasculature such as $\rho_1$ and $\rho_2$, as well as the physiological factor $M/D$. Thus, for a given capillary volume fraction and capillary radius, the average $O_2$ concentration is a function of $M$ and $D$. Moreover, the measured $C_i$ and $C_e$ are precisely related to CMRO$_2$; Equation (2.7) implies that for specific $C_i$, $C_e$, morphological factors, and diffusion coefficient, there exists a unique value of $M$, i.e., CMRO$_2$.

By trivially rewriting Eq. (2.7), we obtain an expression for $M$ (note, $M = \text{CMRO}_2/\nu_c$). Below, we fill in intermediate steps to derive this expression.

$$C(\rho_1) - C_e = \frac{M}{2D} \left( \frac{\rho_2^4 \ln(\rho_2/\rho_1)}{\rho_2^2 - \rho_1^2} \right) - \frac{M}{8D} (3\rho_2^2 - \rho_1^2)$$

$$= \frac{M}{2D} \left[ \frac{\rho_2^4 \ln(\rho_2/\rho_1)}{\rho_2^2 - \rho_1^2} - \frac{3\rho_2^2 - \rho_1^2}{4} \right]; \quad (2.8)$$
\[ M = \frac{2D}{\rho_1^2 \ln(\rho_2/\rho_1) - \frac{3\rho_2^2 - \rho_1^2}{4}} (C(\rho_1) - C_e). \] (2.9)

Substituting \( C_i \) for \( C(\rho_1) \), we obtain an expression which relates molecular \( O_2 \) in the capillary and in the extravascular tissue to oxygen metabolism.

\[ M = \frac{2D}{\rho_1^2 \ln(\rho_2/\rho_1) - \frac{3\rho_2^2 - \rho_1^2}{4}} (C_i - C_e). \] (2.10)

This expression above, should remind the reader of the steady-state equation for \( \text{CMRO}_2 \), which we derived from the coupled differential equations for the two compartments. This equation can be manipulated to define \( \text{PS}_c \). (Note, in deriving all of the results in this section we have implicitly made the steady-state approximation.)

Since \( V_e \) is \( \pi (\rho_2^2 - \rho_1^2) L \), we have

\[
\text{CMRO}_2 = M \times V_e = \frac{2\pi D(\rho_2^2 - \rho_1^2)L}{\rho_1^2 \ln(\rho_2/\rho_1) - \frac{3\rho_2^2 - \rho_1^2}{4}} (C_i - C_e) = \frac{2\pi LD}{\rho_1^2 \ln(\rho_2/\rho_1) - \frac{3\rho_2^2 - \rho_1^2}{4}} (C_i - C_e).
\]

(2.11)

Thus, the model determines \( \text{PS}_c \) precisely.

\[ \text{PS}_c = \frac{2\pi LD}{G \left( \frac{\rho_2}{\rho_1} \right)}, \] (2.12)

\( G \left( \frac{\rho_2}{\rho_1} \right) \) is a function of a ratio between capillary radius, \( \rho_1 \), and tissue radius, \( \rho_2 \). Explicitly,

\[
G \left( \frac{\rho_2}{\rho_1} \right) = \left( \frac{\rho_2}{\rho_1} \right)^4 \ln \left( \frac{\rho_2}{\rho_1} \right) - \frac{3\left( \frac{\rho_2}{\rho_1} \right)^2 - 1}{4 \left( \frac{\rho_2}{\rho_1} \right)^2 - 1}.
\]

(2.13)

Eq. (2.13) depends only on the ratio of intercapillary distance to capillary diameter.
With caveats related to the approximations/simplifications outlined above, \( \text{PS}_c \) represents the oxygen mass transfer coefficient connecting CMRO\(_2\) to the oxygen gradient across intra- and extravascular tissue compartments. Moreover, within this model, \( P \) and \( S_c \) can be separately defined. For example, if we take \( S_c \) to be the capillary surface area, \( i.e., S_c = 2\pi \rho_1 \text{L} \), then we generate an oxygen permeability associated with tissues surrounding the capillary wall, \( i.e., P = \frac{D}{\rho_1 G \left( \frac{\rho_2}{\rho_1} \right)} \). The function, \( G \left( \frac{\rho_2}{\rho_1} \right) \), is not significantly larger than unity for typical microvascular parameters, \( e.g., \) it ranges from \( \sim 1.1 \) to \( \sim 1.6 \) for typical capillary diameter and separation when \( V_i/V_t \) is between 1 and 3\% (see Fig. 7).

![Figure 7: \( G(\rho_2/\rho_1) \) as a function of \( \rho_2/\rho_1 \).](image)

The Krogh-based model also provides an explicit relationship for the extravascular compartment relaxation time, \( T_o = V_e/\text{PS}_c \), in terms of \( \rho_1, \rho_2 \) and \( D \). This timescale arises naturally in the solution of the first order differential equation, Eq. (2.2), and can be important for the description of metabolism dynamics at early times. Interestingly, the ratio of \( T_o \) to the mean-time, \( \Delta t \), for oxygen to diffuse the distance \( \rho_2 \), is well defined in the model; the ratio \( T_o/\Delta t \) is ranges from approximately 2.1 to 3.2 for typical system parameters, and depends only weakly on \( \rho_2/\rho_1 \) in the physiologically
relevant experimental regime (*i.e.*, $V_i/V_t$ between 1 and 3%). Thus, the time for the extravascular compartment to reach a new steady-state oxygen distribution following an impulse is $2 \times$ to $3 \times$ longer than the mean time it takes for oxygen to diffuse to the extravascular tissue boundary, $\rho_2$.

In the next Chapter, we will select $\text{PS}_c$ based on the results of this theoretical model (using reasonable morphological and physiological parameters), on estimates of $\text{CMRO}_2$ from literature, and on our experimental values of $C_i$ and $C_e$.

### 2.4.3 Limitations of the Model

As with many biophysical models which give exact results, the present one has potentially important limitations. For example, besides the specific numerical approximations noted above, the model ignores the heterogeneous nature of the capillary network within extravascular tissue. In practice, the capillary size, and especially the intercapillary separation, is not uniform. A slightly more complex theory should average over the distribution of regions with different morphologies. In this case, even if we continue to adopt the core cylinder model, a slightly more complex theory would quantitatively predict a distribution of $\text{PS}_c$ and $T_o$. Accounting for these distributions could change the temporal metabolism dynamics, *i.e.*, compared to replacing distributions by their average values. Another limitation derives from the fact that the extravascular tissue has both neurons and other cells, and they likely have different temporal metabolic responses to stimulation. Still another limitation is the description of the capillary wall as a uniform extension of the inner capillary; for example, the oxygen diffusion coefficient in the capillary wall could be different from the surrounding tissues. All of these factors need to be considered in data interpretation. Notably, our new methodology will make confrontation of these issues possible.
Chapter 3

Optical and Surgical Methods for Probing Rat Brain Hemodynamics and Oxygen Metabolism


3.1 Optical Probing Techniques

Three physiological parameters were optically tracked in our experiments. The molecular oxygen concentration in the intravascular and extravascular compartments, $C_i$ and $C_e$, respectively, were acquired using the phosphorescence lifetime oximetry technique. Additionally, the relative variation of cerebral blood flow (CBF) was measured using the laser speckle contrast imaging (LSCI) technique. Their principles and associated instrumentation are introduced below.

3.1.1 Two-color phosphorescence lifetime oximetry

When a molecule is excited, it returns to the ground state via multiple pathways. Some pathways are non-radiative, and other pathways are radiative. If the radiation is
emitted from a triplet state, then it is called phosphorescence (fluorescence radiation is emitted from a singlet state, see Fig.8) [143]. Herein we focus on phosphorescence phenomena.

![Figure 8: Simplified Jablonski Diagram. So: Ground State, S1: Singlet State, T1: Triplet State.](image)

When a phosphor molecule in its triplet excited state experiences a collision with an oxygen molecule, the collision provides a non-radiative decay path for energy to depart from the phosphor [156]. Therefore, the more oxygen-rich environment will lead to the rapid decay of phosphorescence emission, while the less oxygen-rich environment results in a comparatively longer phosphorescence lifetime. This relationship is described by the well-known Stern-Volmer relation [131].

\[
\frac{1}{\tau} = \frac{1}{\tau_o} + k_q [O_2],
\]

where \(\tau\), \(\tau_o\), \(k_q\) and \([O_2]\) represent the phosphorescence lifetime, the lifetime in the absence of oxygen, the quenching constant, and oxygen concentration respectively.

Our phosphorescence method utilizes this principle; it employs measurements of phosphorescence lifetime to deduce molecular oxygen concentration (pO2) in the lo-
cal microenvironment. Besides the phosphor molecule (Oxyphor), our experiments require an excitation laser pulse, as well as a means to detect and temporally resolve the phosphorescence photons emitted after the arrival of the excitation pulse. This time-resolved emission signal is fit to an exponential function to determine the lifetime, which in turn provides information about molecular oxygen concentration via a calibration curve.

The phosphorescent probes used in this study were the Oxyphors PtG4 and PtR4. PtG4 has been used previously [80, 132, 18, 15]. PtR4 was synthesized in Sergei Vinogradov’s laboratory specifically for this work (see Appendix A.2-4). PtR4 is similar to the probe PdR4 [44], except that the chromophore in PtR4 is a Pt(II) porphyrin, in place of the Pd(II) porphyrin in PdR4. The difference between Pt and Pd porphyrin-based probes has been discussed [80]. The probes were calibrated using a setup previously described [44]. The Stern-Volmer relations for both Oxyphors R4 and G4 are illustrated in Fig. 9 (Note, the figure is reprinted from Fig. 5 of Supporting Information in Ref. 44.)

![Figure 9: Stern-Volmer Relations. (a) Oxyphor R4. (b) Oxyphor G4.](image)

The instrument for two-color phosphorometry was constructed in-house (Fig. 1a).
Briefly, the excitation sources are pulsed diode lasers (Power Technology) operating at $\lambda_{max} = 635\,\text{nm}(15\,\text{mW})$ and $\lambda_{max} = 517\,\text{nm}(10\,\text{mW})$, for excitation of PtG4 and PtR4, respectively. Both diode lasers have a rise-time and fall-time of $\sim 20\,\text{ns}$, much shorter than the pulse duration. Avalanche photodiodes (APDs) are employed for light detection (C12703-01, Hamamatsu; rise-time / fall-time $\sim 3.4\,\mu\text{s}$). We will use the term channel to refer to the combination of a diode laser, optical fiber for transmitting phosphorescence light to the detector (see below), a set of optical filters, and the APD. The instrument had two channels (Ch1, Ch2). The control of data acquisition was performed by a digital board (NI USB-6351, National Instruments; 1 MHz), which communicated with the host computer via USB interface. The data-acquisition and -analysis software was written in C/C++ (Qt, Nokia).

![Figure 10](image.png)

Figure 10: (a) Scheme of the experimental setup. (b) Wide-field image of the craniotomy window over the somatosensory area. Scale bar: 1 mm.

The light beams from both lasers (Ch1 and Ch2) were focused by lenses ($f = 60\,\text{mm, } \varnothing = 12\,\text{mm}$) to the same spot (size of $\sim 0.2\,\text{mm}$) on the brain surface; the spot (marked in the white square in Fig. 10b) resided within the region of the cortex responsible for forepaw responses, and it was located $\sim 300\,\mu\text{m}$ away from the probe.
injection site (seen as a greenish area). Phosphorescence signals were collected by
single-core plastic fibers ($\varnothing = 4 \text{ mm}$, Fiberoptic Technology), whose tips were held
$\sim 2 \text{ mm}$ from the excitation spot (Fig. 10a). The phosphorescence in each channel
was passed through a series of optical filters and focused by a spherical lens ($\varnothing = 10$
$\text{ mm}$) onto the APD entrance aperture. The configuration of the optical filters in the
channels is presented in Appendix A.5.

To measure $pO_2$ in the intravascular and extravascular compartments, one phos-
phorescent probe was injected into the vasculature, and the other was injected directly
into the interstitial space of the brain tissue. A single data acquisition cycle (duration
$T = \delta t + \Delta t$) consisted of an excitation pulse ($\delta t = 20 \mu s$), during which the
laser was on, followed by the phosphorescence collection period ($\Delta t = 280 \mu s$), during
which the APD current was digitized and recorded (Note, $\delta t$ can be adjusted, typi-
cally between 10 and 30 $\mu s$, depending on the phosphorescence light level, but $T$ is
always set to 300 $\mu s$). To increase the signal-to-noise ratio (SNR), multiple cycles
(N) were executed in sequence, after which the resulting data array was transferred
to the computer. The collected phosphorescence decays were averaged and analyzed
by single-exponential fitting. The single decay curves for PtR4 and PtG4 are shown
in Fig. 11. The resulting phosphorescence decay time ($\tau$) was converted to $pO_2$
using a Stern-Volmer-like equation. Note, because of the instrument response function of
the APD, the fitting conservatively only includes phosphorescence intensity data col-
lected 20 $\mu s$ termination of excitation light pulse (see Fig. 11). Once a measurement
in one channel was completed, it was followed immediately by a measurement in the
other channel. The oxygen gradient ($\Delta [O_2]$), defined here as the difference between
volume-averaged $\alpha_i \times p_i O_2$ and $\alpha_e \times p_e O_2$ ($\Delta [O_2] = \alpha_i \times p_i O_2 - \alpha_e \times p_e O_2$), where $\alpha_i$
and $\alpha_e$ are oxygen solubility in blood plasma and in cerebrospinal fluid respectively,
was computed from the values in the two channels.
For typical acquisition settings (e.g., T=300 µs, N=30), the data collection time for each channel was ~ 10 ms, and the data transfer/analysis required another ~ 10 ms. However, due to hardware limitations, the maximum repetition rate for our two-channel acquisition was 7 Hz (~ 70 ms per channel). This significant “dead time” was caused by the internal operation of the digital board, which had to undergo a reset for each measurement cycle. In the future, with more efficient hardware and optimized collection geometry, the measurement rate can be readily increased to > 50 Hz.

### 3.1.2 Laser speckle contrast imaging

**Temporal speckle analysis.** In this section we briefly describe the theory of dynamic light scattering as it pertains to our work. (Much of this section is written informally and is not meant to be comprehensive; the interested reader is invited to read more about these ideas in the cited references.) In our experiments, we shine coherent light onto a tissue sample. The tissue contains many tiny structures that scatter light, (e.g., nuclei, red blood cells, mitochondria). Generally, some of
these structures are moving significantly and some are not. The diffuse reflection is thus composed of a superposition of the scattered fields from all of these microscopic structures in the tissue. If a single speckle (diffraction limited spot) is collected in reflection, then its time-dependent intensity is related to the local electric field at the same location in the usual way:

\[ I(t) = E(t)E^*(t). \]  \hspace{1cm} (3.2)

Here \( t \) is time. The total electric field, \( E(t) \), can be considered to be composed of a superposition of “micro” electric fields due, in the simplest case, to the tiny scattering structures in the illuminated tissue. (Note, for simplicity we have assumed it to be a scalar field.) Generally, the light can be scattered by dynamic (moving) scatterers or by static (not moving) scatterers. Each “micro” electric field has a phase, and the phases of the “micro” electric fields are randomly distributed and can vary with time if the scattering structures move. (See Ref. 56 for an in-depth discussion of these concepts.)

We are interested in the scattering structures that move significantly (the red blood cells). The red blood cells scatter the input light to produce “micro” electric fields with phases that vary in time; since the relative phases vary randomly in time, the sum of the “micro” electric field fluctuate too. Thus, the total detected field, \( E(t) \), temporally fluctuates. Of course, the corresponding total intensity will temporally fluctuate as well. One way of characterizing these fluctuations is to measure the (normalized) temporal intensity autocorrelation function of the intensity, \( g_2(t) \).

\[
g_2(t') = \frac{\langle I(t + t')I(t) \rangle_t}{\langle I(t) \rangle_t^2}. \hspace{1cm} (3.3)
\]

Here, \( t' \) is a delay time, and angled-brackets, \( \langle \rangle_t \), refers to an average over time \( t \).
In practice, it is relatively easy to measure \( g_2(t) \) by photon counting techniques. However, information about scatterer motion in the sample, such as the motions of red blood cells, is typically derived from their influence on the (normalized) electric field temporal autocorrelation function, \( g_1(t) = \frac{\langle E(t)E^*(t+t') \rangle_t}{\langle E(t)E^*(t) \rangle_t} \). Importantly, for Gaussian random fields (our case), the well-known Siegert relation [87] relates \( g_2(t) \) to \( g_1(t) \). The Siegert relation is given below.

\[
g_2(t') = 1 + \beta |g_1(t')|^2. \tag{3.4}
\]

Typically, the \( \beta \) coefficient above is a geometrical factor that depends on the number of speckle modes reaching the detector. If only one mode is detected, then \( \beta \) is one; for two modes \( \beta \) is one-half; etc.

The form of the electric field autocorrelation function, \( g_1(t') \), depends on experimental measurement geometry and on the nature of the scatterer motion. For example, in our experiment the measurement is in the backscattering (reflection) geometry, and the light is not significantly scattered, i.e., the experiment is in the single scattering regime. Moreover, the cell motion is diffusive, i.e., the particle (cell) mean-square displacement is Brownian-like, except that the diffusion coefficient differs from the traditional value computed first by Einstein, and depends instead on the shear forces (maximum velocity) in the vessels. In this case, the electric field temporal autocorrelation function has a particularly simple form: \( g_1(t') = \exp(-t'/t_c) \); \( t_c \) is the decorrelation time of the autocorrelation function which depends on the effective diffusion coefficient of the moving particle (cell). Note, if the light were multiply scattered, but the nature of the particle/cell motion was the same, then the functional form of \( g_1(t') \) would be different, but we could still extract similar information from the decay of the correlation function, since we can predict the explicit form of the
correlation function in the multiple scattering limit. A summary of various forms for $g_1(t')$ is provided in [8].

Before discussing our LSCI experiments in more detail, we point out a few more features of the single speckle temporal intensity autocorrelation function. First, notice that in the limit of long time delay wherein $t'$ goes to infinity, we expect $g_2(t')$ to approach 1. On the other hand, for $t' = 0$, $g_2(t')$ is larger than unity.

$$g_2(0) = \frac{\langle I(t)^2 \rangle_t}{\langle I(t) \rangle_t^2} = 1 + \frac{\langle I(t)^2 \rangle_t - \langle I(t) \rangle_t^2}{\langle I(t) \rangle_t^2} = 1 + (\kappa_t)^2; \quad (3.5)$$

and

$$\kappa_t = \frac{\sigma_t}{\langle I(t) \rangle_t}. \quad (3.6)$$

Here $\kappa_t$ is called temporal speckle contrast, and $\sigma_t = \sqrt{\langle I(t)^2 \rangle_t - \langle I(t) \rangle_t^2}$.

**Spatial speckle analysis.** We next focus on LSCI, which is the correlation method used in our experiment. In LSCI, either CMOS or CCD camera is used to acquire an image of the sample that is comprised of many speckles. An average pixel count, $S_i$, is recorded by the $i^{th}$ pixel of the camera via integration of the intensity, $I_i$, over an exposure time, $T$, i.e.,

$$S_{i,T} = \frac{1}{T} \int_0^T I_i(t) dt. \quad (3.7)$$

For LSCI, we are also interested in the fluctuations associated with the speckles. As the first step, we compute the mean and variance of $S_{i,T}$, i.e., $E[S_{i,T}]$ and $\text{Var}[S_{i,T}]$, respectively.

$$E[S_{i,T}] = \left\langle \frac{1}{T} \int_0^T I_i(t) dt \right\rangle_s = \frac{1}{T} \int_0^T \langle I_i(t) \rangle_s dt = \langle I \rangle_s; \quad (3.8)$$
\[
\text{Var}[S_{i,T}] = E[(S_{i,T})^2] - E[S_{i,T}]^2 = \frac{1}{T^2} \int_0^T \int_0^T \langle I_i(t_1)I_i(t_2) \rangle_s \, dt_1 dt_2 - \langle I \rangle_s^2. \tag{3.9}
\]

Here \(\langle \rangle_s\) refers to a spatial averaging operator, and \(\langle I_i(t_1)I_i(t_2) \rangle_s\) might be recognized as an ensemble averaged version of the intensity autocorrelation function. Using the Siegert relation once more, we have \(\langle I_i(t_1)I_i(t_2) \rangle_s = \langle I \rangle_s^2 [1 + \beta |g_1(t_1 - t_2)|^2]\). Combining these ideas it is straightforward to show that

\[
\text{Var}[S_{i,T}] = \frac{\beta \langle I \rangle_s^2}{T^2} \int_0^T \int_0^T |g_1(t_1 - t_2)|^2 dt_1 dt_2 = \frac{2\beta \langle I \rangle_s^2}{T^2} \int_0^T (T - t') |g_1(t')|^2 dt'. \tag{3.10}
\]

The intensity-normalized variance is then,

\[
\frac{\text{Var}[S_{i,T}]}{\langle I \rangle_s^2} = 2\beta \int_0^T \left(1 - \frac{t'}{T} \right) |g_1(t')|^2 \, dt', \tag{3.11}
\]

which we define as the square of the spatial speckle contrast, \(\kappa_s\). In LSCI, speckles are imaged onto the CCD via collection optics. Depending on the imaging, a speckle may be spread over many pixels, or a speckle might be spread over less than one pixel, or somewhere in between. For a given imaging geometry, the minimum size of a speckle is approximately the diffraction-limited spot size, \(\rho_{\text{speckle}} = 2.44\lambda(1 + M)f/\#, \) where \(\lambda, M\) and \(f/\#\) are the wavelength of the coherent light source, magnification and \(f\)-number of the collection optics [10]. Therefore, the number of speckles \((N)\) in a pixel can be computed, \textit{i.e.,} \(N = (\text{Area of a pixel}) / \pi \left(\frac{\rho_{\text{speckle}}}{2}\right)^2\).

For reflection from a homogeneous collection of microscopic scatterers undergoing diffusive motion, in the single-scattering limit (which is a good approximation for our
geometry and for brain tissues), $g_1(t')$ is well approximated as exponential, \( i.e., \)

$$g_1(t') = e^{-\frac{t'}{t_c}},$$

(3.12)

where, in diffusive dynamics, $t_c$ is

$$t_c = \frac{1}{q^2 D_{eff}}.$$  

(3.13)

$q$ is the magnitude of the scattering vector, and $D_{eff}$ is an effective diffusion coefficient of the blood cells that is in proportion to the speed of the blood flow in the imaged tissue.

Now using Eqs. (3.11) and (3.12), the spatial speckle contrast, $\kappa_s$ can be acquired.

$$\kappa_s(T) = \sqrt{\frac{\beta e^{-2\frac{t_c}{T}} - 1 + 2\left(\frac{t_c}{T}\right)}{2 \left(\frac{t_c}{T}\right)^2}},$$

(3.14)

As a consequence, for a specific camera exposure time, $T$, $\kappa_s$ can be related to the characteristic decorrelation time, $t_c$. In the limit of large exposure time, $\kappa_s$ is particularly simple. The reciprocal of $t_c$ is in proportion to the speed of the blood cells in the tissue vasculature. In practice, it is difficult to acquire an absolute speed, and therefore relative changes are sought and are sufficient in most cases. The LSCI method has been extensively validated [9, 34, 35, 69, 109, 110, 116], and the linearity of blood flow response is well established in this geometry.

In LSCI acquisition of $\kappa_s$ requires pixel sampling. In essence, we must compute the standard deviation and the mean of some small ensemble of pixel readouts, \( i.e., \), $\sigma_s$, and the mean readouts $\langle I \rangle_s$, respectively. One could compute these parameters using as few as one pixel (which would require many measurements in time); in the opposite limit, one could compute these parameters using many pixels at single point.
in time. In LSCI we are interested in obtaining images of fluctuations at high frame rates (e.g., video rates). Thus, in practice a compromise between accuracy and spatial resolution in the image must be made. While sampling many pixels over a broad area will stabilize the statistical accuracy of \( \kappa_s \), the spatial resolution of the image is lost. By contrast, sampling from smaller areas will weaken the statistical accuracy of \( \kappa_s \) but will improve the spatial resolution. The working consensus of LSCI community is to use a 7\( \times \)7 array of pixels as a sliding window (which defines a spatial resolution for the spatial contrast image) for computation of \( \kappa_s \); the 49 pixels are used to derive a mean and standard deviation of the detected pixel intensity at each location. The choice achieves both statistical significance and spatial resolution of the processed image in many practical situations [10].

In our experiment, as noted previously, a coherent laser beam illuminates the surface of the brain, and the scattered speckle from the brain is imaged onto the CMOS camera (Fig. 12a, camera pixel size is 2040 \( \times \) 2040.) in which a majority of blood vessels are not visible except for the large veins. The detected speckle will fluctuate due to motion in the brain, e.g., due to blood flow. Thus, in the spatial regions with significant blood flow, speckles will fluctuate more than in spatial regions with less blood flow. LSCI measures the speckle contrast associated with these different regions. For our analysis, the camera exposure time was set to 5 ms, and a 7\( \times \)7 moving window was used to compute \( \kappa_s \) at different spatial positions over the whole raw image. For example, in Fig. 12b, the flow information derives from the speckle contrast (the speckle variance) in each region, i.e., low and high speckle contrasts correspond to fast and slow blood flow respectively; the red arrow indicates a pial artery where the flow is fast, and the blue arrow indicates a region where there is no major vessel. Note, in our experiments, the speckle contrast did not have to be computed for the whole field of view; we only needed to investigate a smaller region.
of interest wherein functional activation occurred (marked in the white square region of $200 \mu m \times 200 \mu m$ in Fig. 10b).

Figure 12: Examples of (a) a raw speckle image, and (b) speckle contrast processed image. Scale bar indicates 1 mm.

Once $\kappa$ is computed, it is then related to the decorrelation time, $t_c$ whose reciprocal is essentially proportional to local blood flow in tissue region. In the experiments, a diode laser (852nm, 600mW, LD852-SE600, Thorlabs) was positioned at a distance of $\sim 20$ cm from the brain surface, and its beam was directed at a $\sim 45^\circ$ angle relative to the surface normal. The brain was imaged using a CMOS camera (acA2040-180km, Basler AG, Germany) equipped with infinity-corrected optics. Images were acquired at the rate of 20 Hz using custom-written software (LabView, National Instruments). The LSCI field-of-view (FOV) was quite large: $6.2 \text{ mm} \times 6.2 \text{ mm}$; the FOV encompassed the functional activation region and the focal spot used for the phosphorescence oximetry. LSCI data were analyzed to quantify relative changes of CBF in the brain cortex [39].
3.2 Surgical Preparation

Twelve adult male Sprague-Dawley rats (250 – 400 g) were used in this study. A rat was anesthetized via inhalation of a nitrogen:oxygen (70:30) mixture containing isoflurane (∼ 4% for induction, and ∼ 1.5% for maintenance). A catheter was placed into the left femoral artery for continuous monitoring of the arterial blood pressure. The same catheter was used for injection of the phosphorescent probe for intravascular pO₂ measurements, as well as for injections of saline to maintain hydration. The animal’s head was fixed in a stereotaxic frame, and the intracranial pressure of the rat brain was normalized through cisterna magna. Afterward, a cranial window was made in the skull (6 mm × 4 mm) over the forepaw somatosensory area (Fig. 10b) using a saline-cooled dental drill. The dura mater was removed, after which the exposed brain surface was continuously irrigated with artificial cerebrospinal fluid heated to ∼ 37°C (see Appendix A.1 for ingredients). After the craniotomy was completed, α-chloralose was administered intraperitoneally (60 mg/kg), and the isoflurane dose was gradually decreased and then discontinued over a period of 30 min. Anesthesia was maintained by supplemental doses of α-chloralose (30 mg/kg, ∼ 1 dose/hr). Two electrodes for electrical stimulation were inserted sub-dermally into the left forepaw contralateral to the craniotomy site (Fig. 10a). The body temperature was monitored with a rectal probe and was maintained at 37.4±0.2°C using a heating pad controlled by a homeothermic monitoring system (Harvard Apparatus, MA). A detailed timeline is summarized in Fig. 13. The animals were under the care of the University of Pennsylvania Laboratory Animal Resources. All studies were approved by the University of Pennsylvania Institutional Animal Care and Use Committee.
3.3 Experimental Sequence

First, the animal’s forepaw was stimulated using an electric current from a stimulus isolator (1.5 mA, 1 ms-long pulses at 3 Hz, stimulation time 4 s), and LSCI data was collected and analyzed [39] to locate the somatosensory cortex region within the exposed craniotomy window before and after removal of the dura mater. Oxyphors PtR4 and PtG4 were administered as solutions in physiological saline. PtR4 was administered intravascularly via femoral catheter (50 µL, 200 µM), and PtG4 (0.1 µL, 300 µM) was administered by direct injection into the brain tissue using a microinjection dispenser (Picospritzer III, Parker Hannifin Precision Fluidics Division, NH) and a micropipette (tip diameter ~ 15 µm). The micropipette was inserted to a depth of 200 – 300 µm, and the Oxyphor solution was injected at a rate of ~ 0.1 µL/min. Subsequently, electrical stimulation/blood flow measurement by LSCI was repeated.
to confirm that functional responses persisted. The assemblies for phosphorescence measurements were positioned above the cranial window (Fig. 10b) and phosphorescence signals were measured. If necessary, the amount of intravascular probe was increased by injecting additional solution through the catheter.

![Diagram](image)

Figure 14: Scheme of the experiment protocol

Each stimulation cycle was 1 minute. The stimulation cycle consisted of 4 second baseline monitoring, followed by the application of 4 second stimulus (1.5 mA, 1 ms-long pulses at 3 Hz), and then a 52-second recovery. Up to 15 sequential stimulation cycles were applied, during which lifetime and LSCI data were recorded (Fig. 14 is a block diagram of the procedure).
Chapter 4

Dynamics of Oxygen Metabolism and Blood Flow in Rat Brain


4.1 Two-color, Two-Oxyphor Measurement Method

In our work, we demonstrated a novel application of the phosphorescence quenching method [145] which facilitates concurrent measurement of oxygen concentrations in both intravascular and extravascular compartments of rat brain. This approach enables us to determine vascular oxygen gradients. Critically, we identified two dendritic polyethyleneglycol (PEG)-coated phosphorescent probes, Oxyphors [80, 44] which do not cross through the blood-brain barrier [118, 82, 44, 43, 132], and which have minimally overlapping absorption and emission spectra (see Fig. 15. The laser lines (635 nm and 517 nm) of the respective channels and the LSCI laser line (852 nm) are shown by vertical bars. The absorption spectra are scaled by the respective extinction coefficients at absorption peaks ($\varepsilon_{620}(PtG4) \approx 1.0 \times 10^5 \text{ M}^{-1}\text{cm}^{-1}$, $\varepsilon_{510}(PtR4) \approx 2.0 \times 10^4$)
M$^{-1}$cm$^{-1}$). The emission spectra are scaled such that their areas are proportional to the probes’ phosphorescence quantum yields ($\eta_{\text{PtG}4} = 0.067, \eta_{\text{PtR}4} = 0.052$). The emission wavelength ranges isolated by the optical filters and seen by the detectors in Ch1 (PtG4) and Ch2 (PtR4) are shown by the shaded areas.) In our experiments, we demonstrated that the phosphorescence lifetime signals from the two probes can be retrieved with minimal cross-talk, and since one probe circulated in the blood and the other resided in the extravascular space, we showed that intravascular and extravascular pO$_2$ can be measured concurrently.

Figure 15: Optical absorption and phosphorescence spectra of Oxyphors PtG4 (Channel 1) and PtR4 (Channel 2).

A major experimental challenge for this work was to minimize optical cross-talk between two Oxyphors while preserving enough Oxyphor signal to derive the vascular oxygen gradient. Although excitation of PtR4 and PtG4 Oxyphor mixtures at 635nm produced only PtG4 phosphorescence (Fig. 15), the laser excitation at 517nm, aimed at the Q-band of PtR4, induced a weak but non-negligible emission from PtG4. Thus, multiple optical filters were needed to remove unwanted signals from each channel (see
Appendix A.5); use of these filters sacrificed ~ 40% of the PtR4 phosphorescence spectrum. This effect, in conjunction with the lower molar extinction coefficient and phosphorescence quantum yield of PtR4 (Fig. 15), caused the PtR4 signal to be 10 to 15 times weaker than that of PtG4 in vitro, even when equal amounts of the probe were dissolved in water. In vivo, this difference was further exacerbated by attenuation at 517 nm due to endogenous tissue absorption.

Thus it was necessary to increase the PtR4 concentration in brain by a factor of 10× more than PtG4. Even still, the PtR4 phosphorescence signal was ~ 10 to 20 times weaker than PtG4. Consequently, noise in Ch2 (Fig. 15) was significantly higher than in Ch1, irrespective of whether PtR4 was injected into the blood or interstitial space. Generally, the $p_eO_2$ extravascular signal had ~ 3× better signal-to-noise compared to the $p_iO_2$ intravascular signal, due to the optical advantages of PtG4 described above. Despite this challenge, oxygen ($pO_2$) readings were obtained at a rate of 7 Hz for the two channels and were sufficient to resolve transients in respiratory and vascular activity upon neuronal activation. In the experiments producing the best signal-to-noise, PtR4 was delivered intravascularly. This approach permitted us to increase PtR4 concentration via additional injections during the experiments through the catheter, if necessary.

### 4.2 Longitudinal Measurement Results

An example of raw unprocessed traces of $p_eO_2$ (Ch1/PtG4), $p_iO_2$ (Ch2/PtR4), and CBF during a series of stimulations is shown in Fig. 16a. Notably, we obtained useful single-shot data-waveforms for each electrical stimulation. All three parameters exhibited a sharp rise after stimulation, followed by a more gradual decrease to baseline levels at longer times. Between each pair of stimulations, we paused recording of $pO_2$; this minimized brain exposure to excitation light and possible associated
phototoxicity (Fig. 16b).

Figure 16: Raw unprocessed data obtained in an experiment consisting of 13 consecutive forepaw stimulations, accompanied by (a) CBF, (b) $p_iO_2$, and $p_eO_2$ measurements in the somatosensory area of the rat brain.

In Fig. 17, we exhibit superposed traces of $p_eO_2$ synchronized at the start of the electrical stimulation ($t = 0$). Each color represents a different stimulus trial, and the stimulation period is shown by a pink rectangle (this convention is consistent throughout Chapter 4.) Notice that immediately upon the start of stimulation, $p_eO_2$ exhibits a characteristic “initial dip” caused by the rapid increase in the local oxygen consumption by activated neurons. Remarkably, the dips could be seen in practically all individual stimulation events. On average, the dip magnitude was approximately $9\pm2\%$ of the average peak $p_eO_2$; the minima occurred approximately $0.8\pm0.1$ sec after the onset of stimulation. Dip minima occurrence times in previous microscopy studies are in a similar range [82, 30], but these assignments typically required averaging of hundreds of simulation trials. In the present case, the signals were strong in part because they originated from a large excitation volume that likely encompassed the entire activation region. However, this volume probably included cells whose oxygen consumption is less than that of neurons upon activation. The measured dip amplitude should therefore be expected to fluctuate, especially since illumination
areas could include greater/smaller volume fractions of neurons.

Figure 17: Overlapping traces of $p_eO_2$.

Per quantification of oxygen gradients, the average baseline intravascular oxygen partial pressure ($p_iO_2$) was $33.5 \pm 0.5$ mmHg (mean, standard deviation of the mean). This number is on the lower side of the range of intravascular $pO_2$ values reported previously [118, 82, 30, 43, 94], possibly due to the use of $\alpha$-chloralose as an anesthetic. Anesthesia is known to have pronounced effects on tissue $pO_2$ [94, 159]. The average baseline extravascular oxygen partial pressure ($p_eO_2$) clustered around $21.3 \pm 0.2$ mmHg. Therefore, the resulting intra-/extra-vascular $pO_2$ drop was $12.2 \pm 0.4$ mmHg.

In previous studies of rat brain, utilizing 2PLM and isoflurane as anesthetic, $pO_2$ drops of $\sim 15$ mmHg were seen over $25 \mu m$ distances from large single arterioles, corresponding to the gradients of $\sim 0.6$ mmHg/$\mu m$ [119]. Measurements around capillaries of a rat brain using microelectrodes (with pentobarbital anesthesia) produced somewhat lower gradients, $\sim 0.45$ mmHg [148]. In the present case, $p_iO_2$ is derived mainly from capillaries, since the total volume of blood in capillaries is larger, and
since excitation of the phosphorescent probe in thin capillaries is much more efficient than in large vessels. Our measured $p_eO_2$ represents a volume average over the entire extravascular space within the excitation volume. If we assume that the average intercapillary distance is 60 µm [85, 12], then our results give a gradient of $\sim 0.8$ mmHg/µm. Note, the measurements of $p_iO_2$ and/or $p_eO_2$ are also subject to systematic errors (see Section 4.4), which will need to be further analyzed/corrected for as the method develops.

4.3 Selection of $PS_c$ and Computation of Dynamic $CMRO_2$

4.3.1 Data-constrained selection of $PS_c$

For our calculations we choose $PS_c = 210$ [mL/sec/100g tissue]. In this subsection, we describe in detail the rationale we used to arrive at the choice, which was based on morphological and physiological considerations in the published literature. The reader who is not interested in these details is invited to skip to the next subsection. The reader who is interested in this selection should read on, in part because the full analysis exposes internal consistencies in the theory and the importance of particular assumptions.

The cylindrical model for capillary and surrounding tissue enables determination of $PS_c$ based on the full range of reported $\rho_1, \rho_2, V_i/V_t$, and $D$ and Eq. (2.12). The resulting “allowed” variation of $PS_c$ spans widely from 137 to 2470 mL/sec/100g tissue. Tables 3-5 show the values of $PS_c$ (in bold) as a function of $\rho_1, \rho_2, V_i/V_t$, and $D$; note $\rho_2$ is fixed by choice of $\rho_1$ and $V_i/V_t$. This potential variation of $PS_c$ thus deduced is larger than the range of values reported in the literature, and does not
entirely overlap with the range of values reported in the literature.

Table 3: \( P_{Sc} \) (Bold), CMRO\(_2\) vs \( D, \rho_1, \rho_2 \) for intravascular volume fraction, \( V_i/V_t \) of 0.01(1%). \( P_{Sc} \) values [mL/sec/100g Tissue] computed from (2.12) as a function of \( D \) \([10^{-5} \text{cm}^2/\text{sec}]\), \( \rho_1 \) and \( \rho_2 \) [\( \mu \text{m} \)]. Baseline CMRO\(_2\) values [\( \mu \text{moles/sec/100g Tissue} \)] are computed according to (2.11) using the \( P_{Sc} \) based on different values of \( D \) \([10^{-5} \text{cm}^2/\text{sec}]\) and \( \rho_1 \) [\( \mu \text{m} \)]. The green-shaded cells correspond to baseline CMRO\(_2\) values that are consistent with a range of prior measurements.

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Table 4: \( P_{Sc} \) (Bold), CMRO\(_2\) vs \( D, \rho_1, \rho_2 \) for intravascular volume fraction of 0.02(2%). \( P_{Sc} \) values [mL/sec/100g Tissue] computed from (2.12) as a function of \( D \) \([10^{-5} \text{cm}^2/\text{sec}]\), \( \rho_1 \) and \( \rho_2 \) [\( \mu \text{m} \)]. Baseline CMRO\(_2\) values [\( \mu \text{moles/sec/100g Tissue} \)] are computed according to (2.11) using the \( P_{Sc} \) based on different values of \( D \) \([10^{-5} \text{cm}^2/\text{sec}]\) and \( \rho_1 \) [\( \mu \text{m} \)]. The green-shaded cells correspond to baseline CMRO\(_2\) values that are consistent with a range of prior measurements.

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</table>
Table 5: \( \text{PS}_c \) (Bold), CMRO\(_2\) vs \( D, \rho_1, \rho_2 \) for intravascular volume fraction of 0.03(3%). \( \text{PS}_c \) values \([\text{mL/sec/100g Tissue}]\) computed from (2.12) as a function of \( D \) \( [10^{-5}\text{cm}^2/\text{sec}] \), \( \rho_1 \) and \( \rho_2 \) \([\mu\text{m}]\). Baseline CMRO\(_2\) values \([\mu\text{moles/sec/100g Tissue}]\) are computed according to (2.11) using the \( \text{PS}_c \) based on different values of \( D \) \( [10^{-5}\text{cm}^2/\text{sec}] \) and \( \rho_1 \) \([\mu\text{m}]\). The green-shaded cells correspond to baseline CMRO\(_2\) values that are consistent with a range of prior measurements.

<table>
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To further constrain \( \text{PS}_c \), we next utilize prior measurements of baseline (normal) CMRO\(_2\), and our absolute measurements of intravascular and extravascular compartment \( \text{pO}_2 \). Specifically, our measurements of baseline intravascular and extravascular compartment partial pressures found mean values of \( \text{p}_i \text{O}_2 = 33.5 \text{mmHg} \) and \( \text{p}_e \text{O}_2 = 21.3 \text{mmHg} \) (Fig. 16). Using these partial pressures, we can readily compute baseline volume-averaged \( C_i \) and \( C_e \). Finally, using the reported range of baseline CMRO\(_2\) and Eq. (2.11), \( i.e., \) using the result CMRO\(_2\) = \( \text{PS}_c \) \( (C_i - C_e) \), we identify a much smaller subset of \( \text{PS}_c \) which satisfy Eq. (2.11) subject to published constraints on tissue morphology and oxygen diffusion.

These results are summarized in Tables 3 - 5, which show the predicted CMRO\(_2\) as a function of \( \rho_1, \rho_2, V_i/V_t \), and \( D \) for the cases where \( 3 < \text{CMRO}_2 < 7.4 \) [41, 134, 150, 73]. The green shaded rectangles in the tables correspond to the allowed range of values of CMRO\(_2\). Note, \( \text{PS}_c \) is given in bold text, next to CMRO\(_2\) in these green shaded rectangles.

Taken together, our modeling and analysis suggest that \( \text{PS}_c \) has a value constrained approximately to the range from 180 to 430 \([\text{mL/sec/100g tissue}]\). The
choice of \( PS_c \) amongst this range of possibilities has consequences for the magnitude and dynamical response of CMRO\(_2\). For our calculations we choose \( PS_c = 210 \) [mL/sec/100g tissue]. In making this selection, we used morphological and physiological parameters towards the middle of the reported ranges; for example, \( D \approx 1.6 \times 10^{-5} \) cm\(^2\)/s and \( \rho_1 = 3.0 \) \( \mu \)m and \( \rho_2 = 30 \) \( \mu \)m. Note also, this selection of 210 mL/sec/100g tissue is similar to the value of \( PS_c \) found in the only study of rat brain [67] that had \( pO_2 \) (\( \approx 25 \) mmHg) comparable to our experiments (\( \approx 21 \) mmHg). Lastly, note that this choice is also within the range used in all prior studies; the prior studies suggest that \( PS_c \) has a value between 35 and 230 mL/sec/100g tissue [55, 59, 68, 112, 136, 142, 123, 67].

We caution the reader, however, because a small change of \( \rho_1 \) and \( \rho_2 \) to 2.5 \( \mu \)m and 25 \( \mu \)m, respectively, gives \( PS_c \approx 300 \) [mL/sec/100g tissue]. Ultimately, to ameliorate this problem, the community will need to carry out more experiments and analyses to further constrain \( PS_c \).

### 4.3.2 Computation of CMRO\(_2\)

With the value of \( PS_c \) (210 [mL/sec/100g tissue]) now determined, estimation of CMRO\(_2\) is mathematically straightforward using Eq. (2.2). An example of resultant CMRO\(_2\) is given in Fig. 18a. Recall that in contrast to prior work [61, 35, 39, 4, 86], our experiments demonstrate dynamic tracking of CMRO\(_2\) before, during, and after stimulation based on measurements of oxygen gradients; this approach enables us to dynamically measure CMRO\(_2\) without reliance on CBF. Importantly, we also independently obtained concurrent traces of CBF response. Thus, our measurements provide a unique opportunity to compare the response of all four parameters, CBF, \( p_iO_2 \), \( p_eO_2 \), and CMRO\(_2\). This capability offers physiologically important information not available to other methodologies. Especially, our ability to independently determine
$C_i, C_e$, and their time derivatives in Eq. (2.2) distinguishes the new methodology from prior analyses of CMRO$_2$ dynamics.

In previous work, because experimental determination of the derivatives, $dC_c/dt$ in Eq. (2.1) and $dC_e/dt$ in Eq. (2.2), was difficult or inaccessible, CMRO$_2$ was computed from only the instantaneous values of CBF and $C_a - C_v$ [164, 61, 35, 39, 25]. Therefore, steady-state conditions were inevitably assumed, which leads to the popular definition: $\text{CMRO}_2 = \text{CBF} \times (C_a - C_v)$. To distinguish between the older and present methods we introduce the terms *full-dynamic* and *truncated-dynamic* CMRO$_2$. Full-dynamic CMRO$_2$ is computed from the data using the full model, Eq (2.2) that includes the time derivative, $dC_e/dt$. Truncated-dynamic CMRO$_2$ is computed using Eq. (2.2) but ignoring the time derivative, which is equivalent to $\text{CMRO}_2 = \text{CBF} \times (C_a - C_v)$. Additionally, inclusion of the time-derivative introduces an exponential extravascular compartment time constant, $T_o = V_e/\text{PS}_c$, which can affect relative delays between the temporal responses of $p_{iO_2}$, $p_{eO_2}$, CBF, and that of CMRO$_2$.

In practice, $V_e$ is well constrained by previous experiments, and its variation only marginally affects CMRO$_2$ response. By contrast, $\text{PS}_c$ affects both the magnitude of CMRO$_2$ and its temporal dynamics, and $\text{PS}_c$ is not as well constrained by previous experiments (*i.e.*, compared to $V_e$). We illustrate these effects in Fig. 19a. The responses shown in Fig. 19a are obtained by Gaussian low-pass filtering (Fig. 18, the raw data is marked in gray while de-noised traces are in colors, and each trace is gaussian low-pass filtered (FWHM=300 msec) for analysis of temporal dynamics) and averaging the 13 stimulation-induced $p_{iO_2}$ and $p_{eO_2}$ profiles (Fig. 16b) and, then, solving Eq. (2.2) (with and without including the time derivative) for CMRO$_2$ with the same inputs but different $\text{PS}_c$. Figure 19a exhibits the variation of full-and truncated-dynamic CMRO$_2$ at three different numerical values of $\text{PS}_c$ (80, 210, 50).
Figure 18: An example of CMRO$_2$, pO$_2$ and CBF response to a single functional stimulus. (a) CMRO$_2$ was computed from pO$_2$ data (b). (b) and (c) are single-shots of pO$_2$ and CBF responses respectively.

and 430 mL/sec/100g tissue). Notice that when PS$_c$ is large, the time constant $T_o$ gets shorter, and the traces of full- and truncated-dynamic CMRO$_2$ strongly overlap; the full-dynamic model recovers the steady-state approximation in the limit of large PS$_c$. However, for smaller PS$_c$, the full-dynamic CMRO$_2$ increasingly lags behind the truncated-dynamic trace. This lag-effect is a direct consequence of the inclusion of the time derivative ($dC_e/dt$). Ultimately, a novel feature of the new methodology is its ability to directly observe and measure these delays, thereby extracting information about the timing of physiologically-relevant events such as neurovascular coupling.

As noted above, choice of PS$_c$ is important. As described above, we used prior
experimental measurement constraints on capillary diameter [122, 152, 91, 154, 57], inter-capillary distance [85, 122, 79], intravascular volume fraction [122, 152, 91, 57, 125], oxygen diffusion coefficient [19, 63, 52, 135], and baseline CMRO$_2$ [41, 134, 150, 73], along with the predictions of the Krogh-Erlang model, Eq. (2.12), to select an optimal PS$_c$ of 210 mL/sec/100g tissue. This value of PS$_c$ gives a baseline absolute CMRO$_2$ of $\sim 3.5 \, \mu$moles/sec/100g tissue and a peak activation absolute CMRO$_2$ of $\sim 6.0 \, \mu$moles/sec/100g tissue (Fig. 19a (middle panel)) from the averaged traces obtained in the experiment shown in Fig. 16. We will use this value of PS$_c$ for the remainder of the analysis; it gives a measurable but small dynamic versus steady-state CMRO$_2$ delays.

![Figure 19](image)

Figure 19: (a) Computed traces of full-dynamic and truncated-dynamic CMRO$_2$ (see the text for definitions) for three different values of parameter PS$_c$. (b) Normalized traces of CBF, p$_e$O$_2$, p$_i$O$_2$ and full-dynamic CMRO$_2$

To put the full amplitude and temporal responses of CBF, p$_e$O$_2$, p$_i$O$_2$ and CMRO$_2$ into physiological/biochemical context, we averaged the traces of CBF, p$_e$O$_2$, p$_i$O$_2$ and CMRO$_2$ (from the 13 stimulations in Fig. 16 assuming PS$_c = 210$ mL/sec/100g tissue). In Fig. 19b, we exhibit these averaged traces normalized by their respective pre-activation baseline values. Data in this figure illustrates how the peak responses
vary in strength for the different variables and is discussed further below.

In Fig. 20, we exhibit these same traces (Fig. 19b) but normalized by their peak amplitude. (This approach is similar to that chosen in other publications, e.g., Refs. 101 and 164, for timing analyses.) Figs. 20a,c show “whole” response curves; enlargements of the responses at early times are given in Figs. 20b,d. Examination of Figs. 20b,d permits the study of relative delays in full-dynamic versus truncated-dynamic models at early times. Briefly, CMRO₂, CBF, and the intra- and extravascular pO₂ increase from their baseline levels when activation initiates a rapid increase in energy consumption by neurons and other cells. Mitochondrial oxidative phosphorylation is expected to respond nearly instantaneously by elevating the rate of the ATP (adenosine triphosphate) synthesis and consequently, CMRO₂. This rise in energy consumption is the key trigger that initiates the cascade of events for all measured parameters. Our data and analysis thus take the first steps towards the direct confrontation of the full-dynamic versus truncated-dynamic models and their underlying assumptions.

In Fig. 20, we see that the earliest event following stimulation is an initial rise in CMRO₂ due to local depletion of oxygen. This behavior is a manifestation of the “initial dip” in the trace of pₑO₂ and is apparent to varying degrees in both full-dynamic and truncated-dynamic models.

We first consider the early-time results of the full-dynamic model (Fig. 20b). The steep initial rise of CMRO₂ is a mathematical consequence of the instantaneous value of \( C_e \) and its time-derivative in the pₑO₂ dip. Subsequently, we observe a flattening of the CMRO₂ response for a brief period (0.3 – 1.3 sec), followed by a very steep rise. The initial rise of CMRO₂, at times of order 0.3 sec (or less), triggers the vascular response of upstream arterioles via a pathway known as neurovascular coupling [54, 65]. As a result, local CBF starts to increase and brings newly oxygenated blood
Figure 20: Normalized traces of CMRO\(_2\), \(p_iO_2\), \(p_eO_2\) and CBF. (a,b) compares dynamic CMRO\(_2\) with \(p_iO_2\), \(p_eO_2\) and CBF, and (c,d) compares steady-state CMRO\(_2\) with \(p_iO_2\), \(p_eO_2\) and CBF.\(^{108}\)

into the activated region. Even with our finite temporal resolution and non-negligible noise, the measurements clearly reveal the triggering event of CMRO\(_2\) in advance of the rapid increase of CBF. Once triggered, CBF grows very rapidly. When CBF reaches 20% of its peak value at \(\sim 1.4\) sec, it leads \(p_iO_2\), \(p_eO_2\), and CMRO\(_2\), though the rate-of-increase (slopes) of all variables is about the same within measurement error. The temporal trajectory of CMRO\(_2\) over this initial time period (\(\sim 1.5\) sec) is more complex than the CBF trajectory; this CMRO\(_2\) behavior is a mathematical consequence of the changing slope around the \(p_eO_2\) dip and the compartment time constant, \(T_o\). After the initial transients, the CMRO\(_2\) and \(p_iO_2\) (due to increased arterial blood) traces roughly overlap; they lead \(p_eO_2\) and lag CBF slightly.

Figures 20c,d show the results of the truncated-dynamic analysis based on the
same input data. Recall that the time derivative is set to zero in Eq. (2.2) for the truncated-dynamic model. In this case, CMRO\(_2\) and CBF overlap at early times. The early changes observed are at the limits of our resolution and noise but are consistent with CMRO\(_2\) slightly leading CBF and triggering a rapid CBF rise. The temporal trajectory of CMRO\(_2\) over the initial time period (\(~1.5\) s) is similar to CBF; CMRO\(_2\) and CBF traces overlap within experimental error, and they lead both \(p_{i}O_2\) and \(p_{e}O_2\).

We computed the important timescales for both models. A summary of these timescales, \(t_{\text{trigger}}(= t_{9\%}, t_{20\%}, t_{50\%}, t_{90\%}, t_{\text{peak}}\) and \(t_{\text{fall}}\) (the 50\% decay-time point), is given in Table 6 for each model (full- and truncated-dynamics).

Table 6: Timing of \(t_{\text{trigger}} = t_{9\%}, t_{20\%}, t_{50\%}, t_{90\%}, t_{\text{peak}}\) and \(t_{\text{fall}}\), corresponding to the 9\%, 20\%, 50\%, 90\% rise during functional activation, and the peak-time and 50\% fall-time, respectively. Estimates are given for CBF, \(p_{i}O_2\) and CMRO\(_2\) (both full- and truncated-dynamic models). The confidence interval for the timing was computed from standard error of the mean of the set of 13 traces (in parenthesis).

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<th>(t_{20%}), sec</th>
<th>(t_{50%}), sec</th>
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The differences between full- and truncated-dynamic models at early times are interesting to speculate about and offer new means to explore the underlying physiological assumptions of the models. Briefly, the results based on truncated-dynamic analysis (Fig. 20d) are largely consistent with traditional expectations wherein CMRO\(_2\) leads CBF, albeit the metabolic trigger event is only marginally resolvable due to limited signal-to-noise. The results based on full-dynamic analysis (Fig. 20b) show a clear metabolic neurovascular trigger event, but thereafter CMRO\(_2\) appears to flatten briefly, and then rises, exhibiting a slight lag with respect to CBF. While recognizing
that the finite time-resolution and measurement noise are significant, one can still argue that the observed temporal behavior for CMRO$_2$ is different from the traditional expectation, and since the full-dynamic model includes time derivatives, one might expect new features to be revealed. How can we reconcile these model-dependent CMRO$_2$ differences? One possibility is that our choice of PS$_c$ is not correct; with a larger PS$_c$ (e.g., 430 mL/sec/100g tissue) the full-dynamic model would give an early time temporal variation similar to the truncated-dynamic model; however, in this case, the CMRO$_2$ magnitudes would also be larger, and essentially all prior estimates of PS$_c$ are $\sim 2\times$ smaller than 430 mL/sec/100g Tissue, if not more [123]. A related possibility is derived from capillary network heterogeneity, which the models ignore. Network heterogeneity could lead to a distribution of PS$_c$ and thus a wider range of compartment time constants, even for the same average capillary size and separation. In a different vein, one might question the assumption of uniform metabolic response in the extravascular compartment; for example, neurons consume energy, but other cells consume energy too, and their temporal dynamics need not be the same. Such an effect could give rise to a more complex temporal variation of the CMRO$_2$ response at early (and late) times. At this stage, we do not advocate particular sets of assumptions. Rather, we emphasize that evidence from our qualitatively new methodology takes the first steps toward a more critical examination of models/model-inputs.

The temporal behaviors exhibited on longer timescales do not strongly distinguish the full-dynamic versus truncated-dynamic analyses. After reaching their peaks, all four parameters gradually return to their baselines. Interestingly, while the decay time constants of CMRO$_2$, $p_i$O$_2$ and $p_e$O$_2$ are similar, the decline of CBF is steeper than the decline of everything else (see Table 6). This observation suggests that a signal for cessation of arterial dilation is issued close to when the functional stimulus is terminated and the rate of increase of CMRO$_2$ levels off or begins to fall. It appears...
the system gives feedback to attenuate arterial blood supply when oxygen in the pool is sufficient to maintain CMRO$_2$ at an elevated level. In this regard, we note that CMRO$_2$ is expected to be independent of pO$_2$ over the entire physiological range down to very low oxygen pressures (< 1 mmHg) [17, 157, 158]. The measurements in this peak and fall-time regime continue to suggest that the vascular responses are triggered to change by variation of the energy state reflected by CMRO$_2$ [160].

Finally, we return to Fig. 19b to consider the magnitude of the various functional activation responses. We see that CMRO$_2$ at its maximum is $\sim 1.7 \times$ its baseline, while the respective increase in CBF is $\sim 1.5 \times$. In other experiments ($n = 3$, averaged over 4 – 5 stimulations) the respective increases in CBF and CMRO$_2$ were reversed, e.g., $\sim 1.4 \times$ and $1 \sim 1.2 \times$, respectively. The observation of a larger increase in CMRO$_2$ compared to CBF is somewhat unusual since many (but not all) previous studies have found that CBF response exceeds CMRO$_2$ response in magnitude [61, 35, 39, 86, 88]. It is possible that the inversion seen in our experiments was caused by some systematic measurement errors (see Section 4.4). However, we note that this inversion is not impossible, since the amount of oxygen delivered to the activation area, even upon a moderate increase of CBF, can still overwhelm the demand caused by a large increase in CMRO$_2$, due to the very high oxygen-carrying capacity of hemoglobin. Indeed, the traces of pO$_2$ showed characteristic “overshoots” similar in magnitude to overshoots reported previously [30]; the overshoots have been hypothesized to be a consequence of the anatomical structure of the brain, where arterioles triggered by neurovascular coupling supply oxygen to a much larger volume than the activated region [30, 160].
4.4 Limitations

Several limitations in the current implementation of the method could cause systematic measurement errors. In the future, it should be possible to ameliorate or account for these factors.

4.4.1 Light penetration

The optical frequencies (wavelengths) used for probing $p_iO_2$ and $p_eO_2$ in our experiment were different (by design) to eliminate the cross-talk between the two measurement channels. However, the effective depth of sampling by light is wavelength-dependent due to the endogenous absorption and scattering. Strong optical heterogeneity of tissue makes it difficult to evaluate the difference in the sampling depth, but estimates based on typical brain optical properties can deliver a rough prediction of the different penetration depths. Typically, this depth penetration can be estimated using the two-point Green’s function that describes light propagation of excitation light to the Oxyphor and then light propagation of the phosphorescence emission to the detection surface. We employed a Monte Carlo simulation algorithm, MCXLAB [46], to simulate the two-point Green’s function; this code incorporated typical brain optical properties and enabled us to deduce the comparative depth penetration of the incident excitation light and the comparative losses of escaping phosphorescent photons. Thus we could estimate the spatial origin of the measured signals due to the different Oxyphors. The optical properties for the simulation are summarized in Table 7.

As a result, the effective depth-penetration associated with the optical signals due to PtR4 and PtG4 were approximately 0.7 mm and 1.6 mm, respectively. These
Table 7: Absorption and scattering coefficients ($\mu_a$ and $\mu'_s$ respectively) for simulation. For the simulation, concentrations of HbO and HbR were set to be 60$\mu$M and 40$\mu$M respectively.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>PtR4 (Excitation/Emission)</th>
<th>PtG4 (Excitation/Emission)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength (nm)</td>
<td>517 / 670</td>
<td>634 / 770</td>
</tr>
<tr>
<td>$\mu_a$ (cm$^{-1}$)</td>
<td>2.50 / 0.13</td>
<td>0.22 / 0.10</td>
</tr>
<tr>
<td>$\mu'_s$ (cm$^{-1}$)</td>
<td>20 / 16</td>
<td>15 / 13</td>
</tr>
</tbody>
</table>

different effective sampling volumes of the two phosphors could be a source of systematic error, for example, if the oxygen concentrations or vascular architecture vary significantly over this range.

In the future, it should be possible to select two probes excitable at the same wavelength, but with both having minimally overlapping phosphorescence spectra, both in the near-infrared region. Using appropriate optical filtering, the signals of the probes could then be sampled independently, and the difference between the probed volumes will be minimized. A selection of dendritic oxygen probes with different optical parameters is available for exploration [80, 44, 43] and will be tested in the future. Another useful independent experiment could measure (for example, using electrode techniques) the differences in oxygen with depth over this range.

**4.4.2 Temperature dependence of Oxyphor**

Since the brain tissue after the craniotomy was exposed to room-temperature air, a temperature gradient from the surface down was established. Both phosphorescence decay times and oxygen quenching parameters of the probes are temperature-dependent [44]. Thus, the combination of the temperature gradient, which was unknown, with the different sampling depths for the two probes (see above) could have its own effect on the measured apparent oxygen gradients. For example, at 36.5°C,
PtG4 emission will have a lifetime of 27 $\mu$s at 40 mmHg of oxygen tension, but at 33.0°C a lifetime of 27 $\mu$s converts to 43.2 mmHg (see Fig. 21). Again, minimizing the difference between the sampling volumes should decrease the associated systematic error.

![Temperature-dependent PtG4 calibration curve. At a given phosphorescence decay time, $\tau$, a different pO$_2$ value is assigned according to the temperature.](image)

**4.4.3 Invasive injection of Oxyphor**

One of the principal strengths of all-optical methodology is its minimal invasive-ness. However, in our experiments one of the probes had to be delivered by direct injection into the brain tissue close to the measurement site, potentially damaging the brain. To circumvent this problem in the future it should be possible to introduce the extravascular pO$_2$ probe by injection into cisterna magna [43], far away from the measurement site and thus avoid tissue damage.
Chapter 5

Spatial Frequency Domain Fluorescence Diffuse Optical Tomography for Image-guided Tumor Resection Surgery (Background and Theory)


5.1 Introduction

Surgical guidance based on fluorescence imaging is rapidly becoming a method of choice for resection of tumors [113, 42, 6], especially in neuro- [16, 83, 84, 105] and thoracic-surgeries [106, 107]. The primary goal of surgical resection is to excise all cancerous tissue since leftover residual tumor cells can lead to local cancer regrowth and poor clinical outcomes [100]. Currently, surgeons rely on visual inspection, palpation, or intraoperative pathology to identify tumor boundaries and residual tumor cells; however, the specificity and spatial resolution of these methods are very limited. Fluorescence imaging addresses some of these limitations by providing spatially
resolved, real-time contrast between tumorous and healthy tissues. The technique relies on exogenous contrast agents that accumulate preferentially in tumors, such as the widely-used and FDA-approved fluorophore Indocyanine Green (ICG) and some newer cell-specific fluorophores [151]. Fluorescence light emitted by these contrast agents helps demarcate boundaries between cancerous and healthy tissue.

Currently, fluorescence image guidance for surgical tumor-resection is accomplished with commercial and research-grade imaging systems [113, 104] that utilize epi-illumination with near-infrared (NIR) light, and that image fluorescence from the surgical scene with wide-field camera-based detectors. In practice, two-dimensional (2D) fluorescence images from the tissue (including the tumor regions) are overlaid on a corresponding 2D bright-field image; this method enables surgeons to ascertain the transverse coordinate of the tumor. Since preserving normal tissue is critical for organs such as brain, accurate detection of tumor margins is important beyond a rough localization. In this regard, current fluorescence imaging still suffers from technical limitations. First, 2D fluorescence imaging is most accurate only for tumors at or near the tissue surface, and it cannot determine tumor depth. Second, diffusion of the fluorescent light from the tumor can make the image margins appear larger than the actual tumor margins [62]; this effect is more pronounced for sub-surface tumors. Improved localization of the tumorous tissues in three dimensions (3D) is therefore desirable for better surgical planning and outcomes.

To improve the localization of tumors during surgery, our work adapts a parallel set of diffuse optical fluorescence imaging advances that predate the current commercial approaches for fluorescence image guidance [23, 27, 53, 33, 1, 128, 74, 78, 50, 24, 133, 26, 137, 127]. While these early advances have rarely been deployed clinically, they hold potential for wide-field, non-contact fluorescence imaging with real-time processing. One of these techniques utilizes two different wavelengths of fluorescent
emission [81] to derive information about tumor depth [161]; however, information about tumor lateral margins is not obtained by this method. The other technique, Spatial Frequency Domain Fluorescence Diffuse Optical Tomography (SFD-FDOT), was used to produce 3D images of small point-like fluorophore targets [74] in vitro and in mouse models. SFD-FDOT illuminates tissue with the continuous wave, wide-field intensity patterns which are sinusoidally modulated at different spatial frequencies. With this scheme, the light penetration depth is controlled by the spatial frequency, and the wide-field image data collected at different spatial frequencies facilitates the 3D reconstruction of the fluorophore concentration. In principle, the SFD-FDOT method allows one to acquire 3D information about the tumor depth and lateral margins. However, prior implementations of SFD-FDOT were limited to point-like fluorophore targets (diameter of ~ 2 mm) located less than ~ 3 mm below the surface.

Here we report on an instrument and a new rapid reconstruction algorithm for SFD-FDOT. Our analysis builds on the spatial frequency domain approach and introduces a rapid and computationally inexpensive two-step reconstruction algorithm. In the first step, we use the variation of reflected diffuse fluorescence intensity with respect to the spatial modulation frequency of incident light to estimate tumor depth. Then, using the tumor depth determined in the first step, we determine the lateral margins of the fluorophore concentration in the target plane by rapid analytical data inversion. Herein we report the principles and details of this methodology, and we exhibit the results from a series of SFD-FDOT phantom experiments; the experiments characterize the depth accuracy and lateral spatial resolution of the method. The findings reported below suggest that the methodology could be useful clinically by enabling rapid tumor localization and margin assessment. Additionally, it can be used to provide constraints for more rigorous or comprehensive fluorescence tomography.
5.2 Theory of Spatial Frequency Domain Diffuse Optical Tomography

5.2.1 Light diffusion equation in turbid media

When light propagates through an optically turbid media, such as biological tissue, it interacts with the media via absorption and scattering events. When the number of scattering events is large compared to the number of absorption events, then it is well-known that a light diffusion equation, Eq. (5.1), accurately describes the spatiotemporal transport of the fluence rate in tissue [11, 149]. We write this diffusion equation below.

\[
\nabla \cdot [D(\mathbf{r}, t) \nabla \Phi(\mathbf{r}, t)] - \mu_a(\mathbf{r}, t) \Phi(\mathbf{r}, t) + S(\mathbf{r}, t) = \frac{\partial \Phi(\mathbf{r}, t)}{\partial t},
\]

(5.1)

\[
D(\mathbf{r}, t) = \frac{1}{3[\mu_a(\mathbf{r}, t) + \mu'_s(\mathbf{r}, t)]},
\]

(5.2)

where \( \Phi, D, S, v, \mu_a \) and \( \mu'_s \) represent the light fluence rate, light diffusion coefficient, the input light source distribution, the speed of light, and the wavelength-dependent absorption and reduced scattering coefficients of light in the medium, respectively. \( \mathbf{r} \) and \( t \) represent the spatial Cartesian coordinate and time. For spatial frequency domain imaging, the source is continuous wave (CW), which makes the time-derivative of the fluence rate vanish and leads to further simplification of the diffusion equation.

\[
\nabla \cdot [D(\mathbf{r}) \nabla \Phi(\mathbf{r})] - \mu_a(\mathbf{r}) \Phi(\mathbf{r}) + S(\mathbf{r}) = 0.
\]

(5.3)

Tissue is never perfectly homogeneous. Typically, \( D(\mathbf{r}) \) and \( \mu_a(\mathbf{r}) \) deviate perturbatively from their average values \( \bar{D} \) and \( \bar{\mu}_a \). In this case, the solution of the diffusion
equation, Eq. (5.4) can be found using perturbation theory [97].

\[
\Phi(\mathbf{r}) = \int S(\mathbf{r}') G(\mathbf{r}, \mathbf{r}') d^3r' + \int G(\mathbf{r}, \mathbf{r}') [\nabla \cdot \delta \mathbf{D}(\mathbf{r}') \nabla - \delta \mu_a(\mathbf{r}')] \Phi(\mathbf{r}') d^3r'; \quad (5.4)
\]

\[
\left( \nabla^2 - \frac{\bar{\mu}_a}{\bar{D}} \right) G(\mathbf{r}, \mathbf{r}') = -\frac{\delta(\mathbf{r} - \mathbf{r}')}{\bar{D}}, \quad (5.5)
\]

Here \( D(\mathbf{r}) = \bar{D} + \delta D(\mathbf{r}) \), \( \mu_a(\mathbf{r}) = \bar{\mu}_a + \delta \mu_a(\mathbf{r}) \), and the Green’s function, \( G(\mathbf{r}, \mathbf{r}') \), satisfies the Helmholtz equation, Eq. (5.5).

The second term on the right-hand side is a convolution of the Green’s function with a function that involves the fluence rate and the spatial heterogeneity of the optical properties; this equation is intrinsically nonlinear when spatial heterogeneity exists. To solve it analytically, simplifying assumptions are needed. One of the assumptions is to assume homogeneous background optical properties. The zeroth-order approximation of the solution, in this case, is usually sufficient to describe light propagation in turbid media such as tissue (for NIR light).

For fluorescence imaging, two light sources and propagating diffusive light fields are involved. The first pertains to light that excites the fluorophore. The second pertains to fluorescence light emitted by the fluorophores (after they absorb the excitation light). To distinguish the two, we use a subscript \( \text{ex} \) for any variables/parameters relevant to the excitation light wavelength, and we use a subscript \( \text{em} \) for the emission light wavelength. Therefore, two governing light diffusion equations, Eqs. (5.4) and (5.5) are needed to fully describe the process by which fluorescence light emanates from a turbid media.

\[
\nabla \cdot \left[ D_{\text{ex}}(\mathbf{r}) \nabla \Phi_{\text{ex}}(\mathbf{r}) \right] - \mu_{a,\text{ex}}(\mathbf{r}) \Phi_{\text{ex}}(\mathbf{r}) + S(\mathbf{r}) = 0; \quad (5.6)
\]
\[ \nabla \cdot [D_{em}(r) \nabla \Phi_{em}(r)] - \mu_{a,em}(r) \Phi_{em}(r) + \varepsilon \eta C(r) \Phi_{ex}(r) = 0. \] (5.7)

Here \( \varepsilon, \eta, \) and \( C \) represent the extinction coefficient, quantum efficiency and concentration of the fluorophore at \( r \). For both Eqs. (5.6) and (5.7), solutions can be written in a similar form in Eq. (5.4). Each solution is nonlinear \emph{per se}, and solving them analytically is extremely difficult (or impossible).

As noted above, a few assumptions can simplify the problem. We assume that (1) then spatial heterogeneity of the background optical properties is negligible, and we assume that (2) the excitation light fluence rate in the medium is not affected by light absorption of the fluorescent contrast agents (the fluorophores). These approximations are not too bad, especially if the contrast agent concentration is low (as it usually is in practice). With these two assumptions, we can use the lowest-order approximations for the propagating excitation and fluorescence light fluence rates, which are:

\[ \Phi_{ex}(r) = \int_V G_{ex}(r, r')S(r')d^3r'; \] (5.8)

\[ \Phi_{em}(r) = \int_V G_{em}(r, r')\varepsilon \eta C(r')\Phi_{ex}(r')d^3r'. \] (5.9)

Here the Greens functions, \( G_{ex} \) and \( G_{em} \), satisfy Eq. (5.5) for \( \bar{D} \) and \( \bar{\mu}_a \) for excitation light and emission light wavelengths respectively. In essence, the perturbation in this problem derives from the heterogeneous contrast agent spatial distribution.

### 5.2.2 Source types

To evaluate Eqs. (5.8) and (5.9), the source term, \( S(r) \) needs modeling. There are four standard types of sources; (1) Continuous-wave (CW, the light intensity does not vary over time), (2) Frequency-domain (FD, the light intensity is modulated in time), (3) Time-resolved (TR, the input light is a pulse whose width is typically
Figure 22: Schemes of light sources. (a) CW, (b) FD, and (c) TR light source. The figure is reprinted from Ref. 36.

a few picosecond scales. The output light is broadened in time), and (4) Spatial-frequency-domain (SFD). For (1), (2), and (3), the light source is often (but not always) provided as a point source in space, either via fiber optics or by a focused laser-scanning system.

Figure 23: Two examples of spatially modulated light patterns. (left) An example of lower spatial frequency \( f = 0.02 \text{ mm}^{-1} \) light pattern. (right) An example of higher spatial frequency \( f = 0.07 \text{ mm}^{-1} \) light pattern.

The last type is particularly relevant for our work. The SFD light source uses wide-field epi-illumination on a sample. The light intensity is spatially modulated (but is CW in time, similar to (1) above); typically it is sinusoidally modulated in space. Each sinusoidal pattern is associated with a spatial frequency. The light penetration depends on the assigned spatial frequency (see Fig. 23); penetration is shallower for
larger spatial frequencies. We model the source as the sinusoidal function varying in space (and localized at the surface $z=0$).

$$\mathbf{S}(\mathbf{r}) = S_o[1 + A\cos(\mathbf{k} \cdot \mathbf{r})]\delta(z). \quad (5.10)$$

Alternatively, removing the constant background, and switching to complex notation, the source can be written:

$$\mathbf{S}(\mathbf{r}) = S_o e^{i\mathbf{k} \cdot \mathbf{r}}\delta(z), \quad (5.11)$$

where $\mathbf{k} = (k_x, k_y)$ is the wavevector of the spatial pattern.

### 5.2.3 Green’s function solutions in 2D Fourier space

Our analysis will use functions that vary in the transverse plane, *i.e.*, in the $x$- and $y$-directions. As such, we need to define forward and inverse Fourier transforms in the transverse plane. These definitions are given below for functions $f(\mathbf{q})$ and $F(\mathbf{\rho})$; we adopt the upper case variable ($F$) for the real-space functions and the lower case variable ($f$) for the Fourier-space functions.

$$f(\mathbf{q}) = \int_{S_q} F(\mathbf{\rho}) e^{-i\mathbf{q} \cdot \mathbf{\rho}} d^2 \rho; \quad (5.12)$$

$$F(\mathbf{\rho}) = \left(\frac{1}{2\pi}\right)^2 \int_{S_{\mathbf{q}}} f(\mathbf{q}) e^{i\mathbf{q} \cdot \mathbf{\rho}} d^2 q, \quad (5.13)$$

Here $\mathbf{\rho}$ is the cartesian coordinate of a transverse plane, *i.e.*, $x$ and $y$, and $\mathbf{q}$ is the corresponding Fourier variable.

Then, the Green’s function and Helmholtz equation in Eq. (5.5) can be rewritten according to the definition of the inverse Fourier transform and the Weyl expan-
\[
\left( \nabla^2 - \frac{\mu_a}{D} \right) \int_{s_q} g(q; z, z') e^{i\mathbf{q} \cdot (\rho - \mathbf{r}')} d^2 q = -\frac{\delta(z - z')}{D} \int_{s_q} e^{i\mathbf{q} \cdot (\rho - \mathbf{r}')} d^2 q, \tag{5.14}
\]

where the constant terms of \( \frac{1}{2\pi} \) were canceled on both sides. Note, it is 2D inverse Fourier transform that pertains to transverse directions so that only the transverse coordinate of the Dirac delta function in the RHS of Eq.(5.5) is written in terms of the inverse Fourier transform. Then, it is straightforward to show that Eq. (5.14) can be simplified to Eq. (5.15).

\[
\int_{s_q} \left[ \left\{ \frac{\partial^2}{\partial z^2} - (\mu_{eff})^2 - q^2 \right\} g(q; z, z') + \frac{\delta(z - z')}{D} \right] e^{i\mathbf{q} \cdot (\rho - \mathbf{r}')} d^2 q = 0, \tag{5.15}
\]

where \((\mu_{eff})^2 = 3\mu_a(\mu_a + \mu_0^s)\). The integrand has to be zero for all \( \rho \), which provides us with a differential equation of Green’s function for planar impulse at \( z' \).

\[
\left[ \frac{\partial^2}{\partial z^2} - \{Q(\mathbf{q})\}^2 \right] g(q; z, z') = -\frac{\delta(z - z')}{D}. \tag{5.16}
\]

Here, \( Q(\mathbf{q}) = \sqrt{3\mu_a(\mu_a + \mu_0^s) + |\mathbf{q}|^2} \).

Infinite medium. For our work, we seek the Green’s function solution to Eq. (5.16), wherein the input (i.e., the source) is confined to a plane \( (z') \) and is permitted to have sinusoidal spatial variation with wavevector \( \mathbf{q} \) (in the plane transverse to the \( z \)-direction). For the infinite medium, the only boundary condition is that \( g(q; z, z') \to 0 \) as \( |z - z'| \to \infty \). In this case,

\[
g(q; z, z') = \frac{e^{-Q(\mathbf{q})|z - z'|}}{2DQ(\mathbf{q})}, \tag{5.17}
\]

Here, \( Q(\mathbf{q}) = \sqrt{3\mu_a(\mu_a + \mu_0^s) + |\mathbf{q}|^2} \). Note, the Green’s function solution is isotropic.
in transverse plane in the real space. The Fourier transformed Green’s function is also symmetric in $q$.

**Semi-infinite medium.** In realistic semi-infinite medium cases, there is a boundary between the turbid media and air. Diffuse light escapes from the media to the air. This is the light we detect in our experiments. When it escapes, it does not return. Therefore, other than the light source, the only radiance traveling into the turbid medium is derived from the reflected “escaping” radiance predicted due to Fresnel reflections. This effect provides a boundary condition for the semi-infinite medium problem, the so-called partial flux boundary condition [5].

\[(G + \ell \hat{n} \cdot \nabla G)|_S = 0, \quad (5.18)\]

where $\ell$ is called the extrapolation distance [5] (which is of order but is slightly different from $1/\mu_s'$), and $S$ is the boundary surface. When the flux is normal to the surface, Eq. (5.18) reduces to Eq. (5.19) (again using Fourier transforms).

\[
\left( g + \ell \frac{\partial g}{\partial z} \right)_{z=0} = 0. \quad (5.19)
\]

The boundary conditions for the semi-infinite medium follow [98]

\[
\begin{align*}
g(q; 0, z') - \ell \frac{\partial g}{\partial z}(q; 0, z') &= 0, \\
g(q; z' + \epsilon, z') - g(q; z' - \epsilon, z') &= 0, \\
\frac{\partial g}{\partial z}(q; z' + \epsilon, z') - \frac{\partial g}{\partial z}(q; z' - \epsilon, z') &= -\frac{1}{D}, \\
g(q; z, z') &< \infty, \text{ as } z \to \infty.
\end{align*} \quad (5.20)
\]

Solving for the semi-infinite medium Green’s function is straightforward. We obtain
the solution, Eq. (5.21). Note, as was the case for the infinite-medium solution, the
semi-infinite medium solution is also symmetric in \( q \).

\[
g(q, z, z') = \frac{1}{2DQ(q)} \left[ e^{-Q(q)|z-z'|} + \frac{1 - \ell Q(q)}{1 + \ell Q(q)} e^{-Q(q)(z+z')} \right]. \tag{5.21}
\]

### 5.2.4 Fast analytic inversion algorithms for spatial frequency
domain light source

In the previous section, an expression for the SFD light source, \( S(r) \) was given. Using the expression, Eq. (5.8), the homogeneous excitation fluence rate can be written in a simple analytical form.

\[
\Phi_{ex}(\rho, z; k) = S_o \int G_{ex}(\rho', z, z') e^{i k \cdot \rho'} \delta(z) d^3 r' = S_o g_{ex}(k, z, 0) e^{i k \cdot \rho}. \tag{5.22}
\]

\( g_{ex} \) depends on sample geometry (e.g., semi-infinite, etc.).

The emission fluence rate, \( \Phi_{em} \) is also readily written. Plugging Eq. (5.22) into Eq. (5.9) gives:

\[
\Phi_{em}(r_d; k) = \varepsilon \eta \int_V G_{em}(r_d, r) C(r) \Phi_{ex}(r; k) d^3 r. \tag{5.23}
\]

\( r_d \) is the cartesian coordinate of the sample surface and equivalent to \((x_d, y_d, z_d)\). Note, typically, in our semi-infinite space we set \( z_d = 0 \).

2D Fourier transform of Eq. (5.23) renders a useful form that converts the 3D integration to a one-dimensional integration, Eq. (5.24), that in some cases can be
easily inverted.

\[
\varphi_{em}(q_d, z_d = 0; k) = \varepsilon \eta \int g_{em}(q_d, z_d = 0, z)g_{ex}(k, z, 0)c(q_d - k, z)dz,
\]

(5.24)

where \( \varphi_{em} \) and \( c \) are 2D Fourier Transform of \( \Phi_{em} \) and \( C \) respectively. \( q_d \) is the Fourier variable corresponding to \( \rho_d = (x_d, y_d) \). By substituting \( q \) for \( q_d - k \), a final convolution integral equation is acquired.

\[
\varphi_{em}(q + k, z_d = 0; k) = \varepsilon \eta \int \gamma(q, z_d = 0, z; k)c(q, z)dz;
\]

(5.25)

\[
\gamma(q, z_d = 0, z; k) = \left( \frac{\ell_{ex} \ell_{em}}{D_{ex} D_{em}} \right) \frac{e^{-[Q_{ex}(k) + Q_{em}(q+k)]z}}{[1 + \ell_{ex} Q_{ex}(k)][1 + \ell_{em} Q_{em}(q+k)]},
\]

(5.26)

Later, we will use these results to analyze the distribution of exogenous fluorophore targets in our tissue phantom experiment. For now, we will next describe how the experiments are done.

### 5.2.5 Data acquisition and analysis

In practice, the sample surface is illuminated by a sinusoidally modulated intensity pattern of the form as described above. In the experiment, we add three different phases to Eq. (5.10) to realize the complex light source, Eq. (5.11).

\[
S_i(\rho_d; k) = \frac{S_o}{2}[1 + \cos(\mathbf{k} \cdot \mathbf{\rho}_d + \theta_i)], \quad i = 1, 2, 3.
\]

(5.27)

Here \( \rho_d = (x_d, y_d) \), where \( x_d \) and \( y_d \) are Cartesian coordinates in the surface of the sample (assumed to be a plane). The quantities \( S_o \) and \( \theta_i \) represent the incident intensity and the spatial phase shift of the illumination pattern. In the experimental procedure, we use \( \theta_1 = 0 \), \( \theta_2 = 2\pi/3 \) and \( \theta_3 = 4\pi/3 \) (this choice of phases will
be explained below). In principle, it is possible to use general 2D modulation wave vectors \( k \). However, in our experiments, we modulated the incident light only along the \( x \)-axis, so that \( k = (k, 0) \), where the spatial modulation wave number is \( k = 2\pi f \), and \( f \) is the spatial frequency of the modulating sinusoid. This restriction of the modulation wave vector is sufficient for reconstructions. The use of other modulation directions can enhance the method but requires larger data acquisition times.

In the experiments, we typically employed 31 different spatial frequencies \( f \) varying from 0 to 0.15 mm\(^{-1}\). Note that, in practice, small discrepancies arise between the set of intended and actual values of \( f \) on the phantom surface due to small deviations in the estimated distance between the sample surface and the collection lens. We determine the actual values of \( f \) by analyzing the bright-field image, and we use these measured values of \( f \) in the reconstructions.

For each illumination pattern \( S_i(\rho_d; k) \), a corresponding fluorescent (intensity) emission image \( I_i(\rho_d; k) \) was recorded. The images \( I_0, I_1 \) and \( I_2 \) were then combined to obtain the complex fluorescent emission signal, \( \Phi_{em}(\rho_d; k) \) according to

\[
\Phi_{em}(\rho_d; k) = \frac{1}{3} [2I_0(\rho_d; k) - I_1(\rho_d; k) - I_2(\rho_d; k)] \\
+ \frac{i}{\sqrt{3}} [I_2(\rho_d; k) - I_1(\rho_d; k)]. \tag{5.28}
\]

This combination is well known and is needed for our analysis. Essentially, it allows us to remove the constant background in illumination and to access information about amplitude and phase (equivalent to that provided by illuminating the medium with both sine and cosine functions in the spatial modulation pattern). The resultant emission data, \( \Phi_{em} \), is Fourier transformed to estimate the depth of the fluorescent inclusion.

**Estimation of target depth.** The first step in the reconstruction algorithm is
to estimate the target depth. To this end, we utilize a simplification of Eq. (5.25). Specifically, we assume that the fluorophores are localized at a particular depth, which we refer to as $z_{\text{target}}$, so that $c(q, z) = \hat{C}(q) \delta(z - z_{\text{target}})$. In this case, Eq. (5.25) simplifies to the following form:

$$\varphi(q + k; k) = \varepsilon \eta \gamma(q + k; z_{\text{target}}; k) c(q). \quad (5.29)$$

This equation can be rearranged to relate the $z$-dependent kernel to the measured fluorescence emission and to the fluorophore concentration distribution. Note that, even if the actual fluorophore distribution is more widely spread in the $z$-direction, our method will derive a fairly good estimate of the depth. Specifically, with the assumptions outlined above, and with $q = 0$, it is straightforward to show that, for each spatial modulation wave vector $k$, we have

$$y(k) = A \exp(-z_{\text{target}} x(k)) + y_0, \quad (5.30a)$$

where

$$x(k) = Q_{\text{ex}}(k) + Q_{\text{em}}(k); \quad (5.30b)$$

$$y(k) = \varphi(k; k)[Q_{\text{ex}}(k) f_{\text{ex}} + 1][Q_{\text{em}}(k) f_{\text{em}} + 1]. \quad (5.30c)$$

The dependent variable, $y(k)$, is the normalized emission response at the modulation wave vector $k$. The independent variable, $x(k)$, can be measured or estimated; it is, essentially, twice the reciprocal of the penetration depth, which depends on the spatial modulation frequency. The parameter $A = \varepsilon \eta \left(\frac{f_{\text{ex}} f_{\text{em}}}{D_{\text{ex}} D_{\text{em}}}\right) c(0)$ depends on a number of parameters of the background medium and the fluorophores, some of which may not be directly known or measured, but it is independent of the modulation wave
vector $\mathbf{k}$. In a data set obtained by varying $\mathbf{k}$, we can view $A$ as a constant and as an adjustable parameter. Further, $y_o$ is an extra adjustable parameter added to account for the noise floor in the data since the fluorescent signal does not go to zero when $|\mathbf{k}| \to \infty$ (as predicted theoretically), perhaps as a result of background fluorescence and other noise, as well as imprecision in the theoretical model. As $A$, we assume that $y_o$ is independent of $\mathbf{k}$.

We can now find $z_{\text{target}}$ by nonlinear fitting of the theoretical formula, Eq. (5.30a), to the data in which $A$, $z_{\text{target}}$, and $y_o$ are viewed as $\mathbf{k}$-independent adjustable parameters. Once the depth of the target is determined, we can compute the lateral margins of the fluorophore concentration in the plane.

**Estimation of lateral margins of target.** This is the second step in our two-step algorithm. Briefly, rearranging Eq. (5.29) for $c(q)$ and taking the inverse 2D Fourier transform, we obtain a simple reconstruction formula for the transverse distribution of the fluorophore target in the plane $z = z_{\text{target}}$. The obtained relation is parameterized by $\mathbf{k}$ and, theoretically, any value of $\mathbf{k}$ can be used to obtain the transverse distribution of the fluorophore. In the reconstructions, we have used the $\mathbf{k} = 0$ for this purpose because the signal-to-noise ratio in the fluorescence images is best for unmodulated incident light. Therefore, we use the reconstruction formula

$$C(\rho, z = z_{\text{target}}) = \frac{1}{\varepsilon \eta} \int \frac{\hat{\Phi}(q; \mathbf{k} = 0)}{\gamma(q, z_{\text{target}}; \mathbf{k} = 0)} e^{-i\rho \cdot q} f_\sigma(q) \frac{d^2q}{(2\pi)^2},$$

where

$$f_\sigma(q) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{q^2}{2\sigma^2}\right)$$

is a Gaussian low-pass filtering kernel.

We apply $f_\sigma(q)$ to ameliorate the effects of noise and model imprecisions that render the integrand in Eq. (5.31) unreliable at high values of $q$. Since the denominator
of the integrand decreases exponentially with \( q \) (see Eq. (5.26)), the contribution of noise to the image is amplified when large values of \( q \) are used in a numerical reconstruction. The Gaussian filter enables us to regularize Eq. (5.31). The regularization parameter \( \sigma \) depends on the level of noise and must be determined for each data set separately. However, the reconstruction is so fast that it is possible to generate many images in real-time while tuning \( \sigma \).

5.2.6 Nonlinear fitting

Prior to fitting, the data were normalized to the maximum value. That is, both sides of Eq. (5.30a) were divided by \( y(k = 0) \). This allowed us to compare errors of the fit for different data sets quantitatively. We then obtained an estimate of the fluorophore target depth, \( z_{\text{target}} \), from the best fit of the theoretical formula, Eq. (5.30a), to the data points \( (x(k), y(k)/y(0)) \). We removed the first data point (with the smallest \( |k| \)) from the fitting procedure (it will be discussed in the next chapter). For the fitting, we used a nonlinear regression model algorithm, \texttt{fitnlm} (MATLAB 2022a, The Mathworks Inc., Natick, Massachusetts, USA), and chose the initial values for \( A, z_{\text{target}}, \) and \( y_o \) to be 5.0, 1.0, and 0.1, respectively.
Chapter 6

Spatial Frequency Domain Fluorescence Diffuse Optical Tomography for Image-guided Tumor Resection Surgery (Experiment, Results and Discussion)


6.1 Experimental Results

6.1.1 Imaging instrumentation

For testing and for eventual clinical implementation, we constructed a wide-field imaging system that consisted of a digital micro-mirror device for illuminating the tissue at different spatial frequencies, and a spectrally-separated dual camera detection system that simultaneously records bright-field reflectance images at the excitation wavelength of 808 nm and fluorescence images at the emission wavelength of 850 nm. The setup is illustrated schematically in Fig. 24. The combined illumination and imaging system was mounted on a translation stage (MN10-015-E01-13, Velmex Inc.,

Bloomfield NY, USA) and positioned above the sample with the object distance of 32 cm. Illumination was provided by a digital light projector (FC E4500 MKII, EKB Technologies Ltd. Bat-Yam Israel). Its intensity on the sample surface was approximately 0.3 mW/cm² for uniform (spatially unmodulated) illumination, well below the ANSI limit for tissue damage. A digital micro-mirror device was electronically controlled to produce spatially-modulated (at different frequencies) illumination of the sample surface with wide-field light at 808 nm. Reflected light at the excitation and fluorescent wavelengths was directed to two separate camera channels; further spectral filtering was necessary to record the fluorescence image. The fluorescence emission images recorded in the fluorescence camera are used for the reconstruction
of the depth and the transverse margin of the fluorescent targets.

6.1.2 Experimental procedure

Background tissue was mimicked using a mixture of intralipid (Intralipid 20%, Fresenius Kabi, Pune, Maharashtra, India), nigrosin (Acid Black, MP Biomedicals, Santa Ana, CA, USA), and water. The background medium had a wavelength-dependent absorption coefficient, $\mu_a$, and reduced scattering coefficient, $\mu'_s$. Respectively, these coefficients were $0.004 \text{ mm}^{-1}$ and $0.8 \text{ mm}^{-1}$ at the excitation wavelength of 808 nm, and $0.006 \text{ mm}^{-1}$ and $0.76 \text{ mm}^{-1}$ at the emission wavelength of 850 nm. The hollow cube (for containing the fluorescent target) was 3D-printed with inner side length of 10 mm and wall thickness of 1 mm (VeroWhitePlus, Stratasys Direct Manufacturing, Valencia, CA, USA), which was suspended in a tissue-simulating liquid phantom using two posts (blue). This cube was employed to simulate the fluorescent target(s); the cube was filled with a mixture of intralipid containing dissolved ICG powder (IC-GREEN, Akorn Inc., Lake Forest IL, USA). The ICG concentration was 6.5 $\mu$M.

The fluorescent contrast cubes were suspended in the liquid phantom tank using fishing lines and posts, as shown in Fig. 25. The depths of the targets were controlled by lowering or raising the surface of the background liquid phantom in the single target experiments (see Fig. 25a).

Both bright-field and fluorescence images were recorded with a 12-bit dynamic range for each phase offset (see Fig. 26); the exposure time was increased until the maximum count of a pixel reached approximately 4000 for unmodulated incident light. The adjustable fluorescence camera exposure time ranged from 800 msec for the 2 mm depth of the target to 5 sec for the 10 mm depth; these adjustments were
needed to account for the difference in detected intensity. The exposure time of the bright-field camera was about 15 msec. Accordingly, the full acquisition time for the deepest target was four times longer than that of the shallowest target. Fluorescence emission data for single-target phantoms were acquired for target depths of 2, 4, 6, 8, and 10 mm.

For two-target phantoms, two different sets of experiments were performed. In one set, two identical targets were placed at the same depth. This depth of the two targets, and the lateral separation between the targets, were varied as shown in Fig. 25b. In the second set of two-target phantom experiments, the two targets were laterally separated by either 20 or 40 mm. The depth of one target was fixed at 2
mm, while the depths of the other targets varied from 4 to 8 mm (Fig. 25c).

In all experiments, the data acquisition time for the fluorescence associated with the shallowest target (2 mm depth) was \(\sim 2\) min. The acquisition time for fluorescence associated with the deepest target (10 mm depth) was \(\sim 8\) min.

### 6.1.3 Single-target experimental results

The depth of the fluorescence contrast target varied from 2 to 10 mm, and the corresponding exponential fitting results are displayed in the left column of Fig. 27. The estimated depth is marked on top of the plot along with its standard deviation.

Note, the first data point corresponding to \(k = 0\) was not used for fittings. If used, this data point can skew the decay constant to larger values and increase the fitting errors significantly. Physically, we can justify the removal of this point by recalling that the specific intensity of the light transmitted through the top interface (tissue surface) consists of two components, sometimes referred to as diffuse and ballistic (reduced). The ballistic photons have a relatively large energy flux near the interface but then undergo a much faster exponential decay with depth compared to the diffuse photons [138]. The rate of decay of both components increases as \(k\) is increased, which results in a complex competition of the two terms. For relatively shallow targets (i.e., depths of 2 mm to 6 mm), the ballistic term can strongly perturb the \(k = 0\) measurement and, perhaps, influence the next data point as well, but it becomes negligible at larger values of \(k\). As the target moves deeper, the influence of the ballistic term is further diminished for all values of \(k\). At the depths greater than 10 mm, the ballistic term can be generally neglected. In our experiments, however, most of the targets do not satisfy this condition; the data are affected by the ballistic term, but the theory on which the reconstructions are based accounts only for the
Figure 27: Single target experiment depth estimation results. The target depth is 2 mm (a), 4 mm (b), 6 mm (c), 8 mm (d), and 10 mm (e).

Diffuse photons which obey the single-exponent decay model, Eq. (5.30a). For these reasons, the data point with \( k = 0 \) was excluded from the fitting procedure in all reconstructions. In the future, a more systematic accounting of the ballistic photons will require the use of a two-exponent decay model [138]; if successful, inclusion of the \( k = 0 \) (and other data points with relatively small values of \( k \)) will be possible and should increase the dynamic range of measurements, which is useful for more accurate depth estimation.

After the target depth and its uncertainty are estimated, we determine the trans-
verse margins of the fluorophore concentration by using Eq. (5.31). The result is shown in Fig. 28. In each panel, the figure on the left shows the reconstructed transverse images, and the yellow scale bar corresponds to 10 mm. The figure on the right shows the transverse margin (blue) in comparison to the true margin (black).

![Figure 28: Single target experiment transverse margin estimation results.](image)

The resulting reconstructions depend on the regularization parameter $\sigma$. If $\sigma$ is too large, the resulting images typically contain significant random noise, which renders the target unrecognizable (see Fig. 29a). On the other hand, if $\sigma$ is too small,
the reconstructed image is only a blob (see Fig. 29c). To determine the optimum $\sigma$, we apply a weak positivity constraint on $C(\rho, z = z_{\text{target}})$. Since $C$ is intrinsically positive, we require that the mean value of the normalized concentration over all pixels is at least one standard deviation larger than zero; this roughly corresponds to a condition that the fraction of pixels with negative values is less than 16%.

Figure 29: Transverse image of the fluorescent inclusion at 4mm depth with different values of $\sigma$. (a) A weak regularization ($\sigma = 3.77 \text{ mm}^{-1}$). (b) The optimal regularization ($\sigma = 0.46 \text{ mm}^{-1}$). (c) A strong regularization ($\sigma = 0.11 \text{ mm}^{-1}$).

The optimal $\sigma$ we choose is the maximum $\sigma$ satisfying this constraint. A resultant optimized image is shown in Fig. 29b; the transverse images in the middle column of Fig. 28b are produced using the same positivity constraint.

The lateral margin of the target was set to be the full-width-at-half-maximum
(FWHM) contour line of this distribution (the figures on the right in each panel of Fig. 28). To measure the discrepancy between the reconstructed and true margins, we computed the relative width, which we define as the square root of the ratio of the transverse target area (area confined by the blue line in the figure on the right in each panel of Fig. 28 to the true target area (100 mm²). For the single-target experiments, the reconstructed depth and the relative width are fairly accurate and are summarized in Fig. 30.

Figure 30: Reconstruction results of single-target experiments. (a) Depth-estimation results. The black dotted line has a unity slope. (b) The ratio between reconstructed image width and the true target width as a function of depth.

6.1.4 Two-target experimental results

Transverse Separation. We first placed two identical targets in the same plane but with varying lateral separation. The estimated depths and transverse margins were then computed. A few examples are shown in Fig. 31. The two targets are clearly distinguished in most cases. As expected, the ability to distinguish between the targets is reduced as the depth is increased. However, for the range of separations used in our experiments, we clearly observe two targets at every depth.
Figure 31: Reconstructed transverse margin of two-target experiment. Two targets were placed on the same depth, and it varied as well as transverse separation.

A summary of depth sensitivity and relative width is given in Figs. 32a and 32b.

Figure 32: Reconstruction results of two targets in the same vertical plane. (a) Depth-estimation results. (b) Relative width estimation results.

**Vertical Separation.** The second, more challenging problem, is to reconstruct two targets that are separated both vertically and laterally. To this end, we fixed the lateral separations to be either 20 or 40 mm. The depth of Target 2 was fixed at 2 mm below the surface, and the depth of Target 1 was varied taking the values
4, 6, and 8 mm. Since the targets were well separated transversely, even when they are located at different depths, we could safely segment the data and process each target separately (Fig. 33). We use the minimum in the peak intensity profile along the horizontal (dotted) line A to define the segmentation regions for the image, as shown by the vertical (dashed) lines B in the figure. After the depth of each target was independently estimated, we used these estimates to process the data further for each target. We obtained the transverse fluorophore distributions by utilizing the same algorithm as in the single target reconstructions.

![Image of Target 1 and Target 2 with lines A and B](image)

Figure 33: Raw data showing substantially different emission signals from Targets 1 (deep target) and 2 (superficial target). Scale bar corresponds to 10 mm.

Reconstructions with 20 and 40 mm transverse separations are presented in Fig. 34. Columns and rows represent the depth of Target 1 and the transverse separation of the targets, respectively (Target 2 is fixed at the 2 mm depth). Estimated depths are marked below each margin slice. The yellow scale bar represents a 10 mm length in the transverse plane. The vertical scale is exaggerated for better visibility.
The depth-sensitivity and depth estimates for these targets (Fig. 35a) are not as good as our results for single targets, nor for two-targets in the same vertical plane (Figs. 30b,32b). This can be understood by noting that both the experimental acquisition time and the resulting computationally processed data are different for different target depths. As described in Chapter 5, the camera exposure times needed to obtain reliable fluorescence images for deeper targets are significantly larger than those for shallow targets. In our instrument and data collection scheme, it was not possible to use large exposure times when a shallow target is present due to the possibility of overexposure. We were therefore forced to use the exposure times appropriate for the shallow target. Thus, when Target 1 is located at the depth of 10 mm, its fluorescent signal was not detectable due to the insufficient exposure time. Cross-talk of the
targets located at different depths can also cause artifacts. As expected, the errors of the transverse margins of the target are larger for smaller horizontal separations (Fig. 35b). When the transverse separation is 20 mm, then the cross-talk from Target 2 in the segmented image of Target 1 becomes significant and produces an artifact near the edge. When this effect is severe, the transverse margins of the deeper target are corrupted (see the last column of the first row of Fig. 34).

![Figure 35: Results of the two-target experiments with vertical separations. (a) Results for the reconstructed depth of Targets 1 and 2. The dashed line has the unity slope. (b) Results of the relative width of Targets 1 and 2.](image)

### 6.2 Discussion

We have introduced and demonstrated a simple method based on SFD-FDOT to estimate the depth and lateral margins of fluorescent targets in turbid media in the
reflection geometry. The present work moves beyond prior research [74] in several ways. The targets studied are extended rather than point-like and are located as deep as $\sim 1$ cm below the surface rather than at the shallow depths of $\lesssim 3$ mm. Also, we considered multiple targets located at the same and different depths rather than a single isolated point-like target. An important but subtle aspect of our work that may not be apparent is that we were able to determine the depth of each target without sensitivity to the regularization parameter (which was the case in the past); this is because we divided the reconstruction into two steps. The first step involves a simple fitting of the data to a single exponent, and the second step involves a straightforward 2D Fourier transform. Essentially, for the second step, we have utilized prior knowledge that the fluorescence signal is mostly emitted from one particular depth, i.e., at the top surface of the target. Approaches that do not rely on these simplifying assumptions generally require the inversion of a severely ill-posed operator. In the latter case, sensitivity to the regularization parameter can become strong.

### 6.2.1 Depth sensitivity

The simple algorithm proposed in this paper was successful in estimating the depth of fluorescent inclusions to within approximately 1 mm of the true depth in both single-target and two-target experiments in which the targets were at the same depth but laterally separated. Interestingly, the estimation error was minimum at the largest depth (Fig. 36); this observation can be attributed to the effects of ballistic photons which were briefly discussed in Sec. 6.1.3 and which were not accounted for in the reconstructions. We mitigated the effects of ballistic photons by removing the data point with $k = 0$ in the fitting procedure. As noted, these points are affected by the ballistic photons more strongly, especially, in the case of shallow targets. However,
the ballistic contribution still affects the data, most seriously for the target depth of 2 mm. Naturally, as the target moves away from the surface, contributions of the fast-decaying ballistic photons decrease, leading to better depth estimation. In the case of two targets at different depths, the effect of ballistic photons is stronger since at least one target is always close to the surface. This, and the experimental limitations related to the available exposure times, produced a larger error in terms of depth estimation in this set of experiments, typically, of the order of 20% to 40% of the true depth.

Clearly, technical improvements that can ameliorate some of the observed limitations are possible in future work. By increasing the number of low spatial frequencies used in the fitting, we can, potentially, use a more precise two-exponent model [138] for nonlinear fitting of depth. The theoretical model in this case will account for both ballistic and diffuse photons. This will increase the dynamic range of useful data and reduce the systematic errors of the theoretical model. In addition to reducing depth estimation errors of shallow targets, such improvements could enable imaging of targets located deeper than 10 mm below the surface.

### 6.2.2 Transverse margins

Another important advance reported in this paper is the ability to constrain the transverse margins of the fluorescent inclusions (tumors). For the most part, the transverse margins of the single targets, or two targets in the same plane, overestimated the true margins by 30% or less. For the two-target experiments with targets at different depths, these margin errors were larger; the larger error in depth-estimation propagates to the error in the transverse margin.
6.2.3 The spatial smoothing (regularization) parameter

Figure 37 shows the average value of the optimized regularization parameter $\sigma$ as a function of the target depth for single-depth reconstructions. It can be seen that the regularization parameter tends to decrease as the target depth increases. This is because the signal-to-noise ratio for the detected fluorescent intensity decreases as the pathlength of light increases and the spatial waves at high spatial frequencies in the detected intensity distribution become dominated by noise. Empirically, we note that the value of $k$ for which $y(k)$ settles into the noise plateau is roughly proportional to the optimal $\sigma$ (roughly, $k \sim \sigma$). Therefore, the value of $k$ can be considered as a guideline for choosing the optimal $\sigma$. 

Figure 36: Mean absolute errors for all depth estimation results.
6.2.4 Stability of the estimates

The proposed two-step reconstruction method utilizes a Green’s function formalism that is affected by the optical properties of the background. Therefore, errors in the background optical properties could lead to errors in the estimation of the depth and transverse margin of the fluorescent targets. We tested the stability of reconstructions by introducing a ±10% error into $\mu_a$ and $\mu'_s$. The resulting absolute error of depth estimation was less than 0.8 mm, which does not significantly impact the overall depth sensitivity. Similarly, the change in transverse margins for the targets at the depths of 2, 4, and 6 mm was less than 2%. Thus, while it is desirable to use accurate background optical properties, the method of this paper is found to be stable with respect to relative errors in the optical properties with the magnitude of 10% or less. Note, in practice these background optical properties can be readily measured using the bright-field data sets that are simultaneously taken with the fluorescence data.
6.2.5 Data acquisition and reconstruction times

Since our technique does not attempt to reconstruct full 3D shapes, the time needed to estimate the depth and transverse margins ranged from one to two seconds (Intel(R) Core(TM) i5-8600K CPU 3.60GHz, 6 Cores). Clearly, the approximate approach is different from the traditional analytic inversion and nonlinear image reconstruction techniques, which require computation times ranging from minutes to hours depending on complexity. Acquisition time is the longest temporal factor in our experiments, typically, of the order of a few minutes.

6.2.6 Limitations

The proposed SFD-FDOT estimation methodology is promising though it has the aforementioned limitations. Notably, reconstruction of absolute fluorophore concentration was not pursued; absolute concentration is not a feature currently employed for image guidance or diagnosis. Per image guidance, the analysis assumed homogeneous background optical properties, semi-infinite geometry, and a thin slab geometry of the fluorescence inclusion. In practice, heterogeneous tissue optical properties in brain or lung tissue could generate errors in the estimation of depth and transverse margin. Even though this assumption is commonly used in diffuse optics analysis, more in vivo work needs to be done to fully characterize these limitations. Additionally, the instrumentation permits the determination of optical properties of tissue layers using multiple spatial frequencies in principle [161]. This information can be used to refine data analysis and provide guidance on the interpretation of reconstruction results in the future.

Per the semi-infinite geometry, this approximation is fairly good for most situa-
tions in the surgical theatre. In practice, this instrumentation can be readily modified to concurrently measure surface profiles and thereby deduce the magnitude/character of the assumption error; this will also provide concrete data which could be employed to modify the current approach to perturbatively include corrections due to surface curvature.

Per the thin slab approximation for the fluorescence inclusion, in practice, the high absorption coefficient of ICG will cause the top surface of the target to absorb most of the excitation light propagating downward through the target, and consequently the fluorescence emission will be dominated by the signal from the top surface. Thus, the thin slab approximation is useful, as long as the user appreciates that the target depth corresponds to the top surface of the fluorescent region. Note also, even if tumor tissue is not flat on the top surface, the proposed technique will provide a useful estimate of depth that is skewed toward the shallowest part of the tumor.

Another challenge is that our theory only concerns diffuse photons, not ballistic photons. For accurate estimation near surface, it is desirable to develop a comprehensive theory to account for the ballistic photon contribution. Subsequent experiments should characterize the cost versus benefit for practical implementation of the more complex model.

Finally, it is desirable to make measurements quickly in the operating room. In our current setup, the longest part of the procedure is data acquisition (up to eight minutes for the deepest occlusions). In the future, this can be improved by eliminating unnecessary (redundant) spatial modulation frequencies (at high-\(q\)) and increasing the excitation light intensity. Note, the current light intensity on the sample surface is smaller than the ANSI limit by the order of 3 (see Appendix A of Ref. 96).
6.3 Summary and Outlook

We modified the SFD-FDOT technique for rapid estimation of the fluorophore target depth and lateral margin. The new methodology was demonstrated to provide depth sensitivity in a variety of experimental situations. The width of the target was estimated with a reasonable negative margin, although this information became less reliable for multiple targets with large vertical separations. These advances build upon the prior work where the target size and depth were limited to point-like occlusions located less than 3 mm below the surface. Moreover, the prior image reconstructions were sensitive to the regularization parameter and the depth resolution was low. The simple and straightforward analytic estimates proposed in the present work are potentially attractive for neuro- and thoracic-surgeries because the technique can rapidly and accurately deliver information about the depth and transverse margins of a fluorescing target.

More broadly, the fluorophore target information obtained in this simple way can provide priors to constrain more complex fluorescence tomography. Applications, in this case, could extend beyond image guidance during tumor resection surgery. Looking forward, the technique can be improved with more rapid and complete data acquisition and type. Additionally, the use of improved theoretical models accounting for the presence of ballistic light can also lead to improvements.
Chapter 7

Summary

Optical imaging of photoluminescence from exogenous contrast agents provides unique opportunities to measure observables that are not easily accessed. The lifetimes of phosphorescent dyes enabled us to measure of oxygen concentration in the vascular and extravascular compartments of the brain, and thereby test traditional models for cerebral metabolism and neurovascular coupling. Moreover, fluorescence imaging is increasingly utilized for image-guided tumor resection surgery, which has largely relied on 2D images in reflection; our technical developments in fluorescence imaging theory and instrumentation, and corresponding experimental demonstrations, have provided advances which can improve upon current techniques for fast characterization of tumor depth and transverse margins.

More specifically, Cerebral Metabolic Rate of Oxygen consumption (CMRO\textsubscript{2}) is a key physiological variable that characterizes brain oxygen metabolism in steady-state and during functional brain activation. We measured the spatiotemporal variation of CMRO\textsubscript{2} in steady-state and during functional activation using with two Oxyphors (one in the vasculature and one in the extravascular tissue) for the first time. To this end, we introduced a new optical technique for minimally-invasive real-time measurement of the oxygen gradients between intravascular and extravascular tissue compartments of the brain using phosphorescence lifetime oximetry. The method enabled computation and tracking of CMRO\textsubscript{2} during functional activation with high temporal resolution (~ 7 Hz), which, in principle, can be improved much more. In parallel, we measured cerebral blood flow (CBF) using laser speckle fluctuation imaging. No-
tably, in contrast to other approaches, our assessment of CMRO$_2$ does not require measurements of CBF or hemoglobin oxygen saturation thus making it possible to compare CBF and CMRO$_2$ as two independent variables.

Simultaneous measurement of p$_i$O$_2$ and p$_e$O$_2$, from which CMRO$_2$ is deduced, enabled us to explore traditional compartmental models of oxygen dynamics and metabolism in a more rigorous way. In particular, we were able to analyze CMRO$_2$ response using the full dynamic equations (with time-derivatives included) for the first time. Thus, we could compare the predictions of the full-dynamic and the truncated-dynamic or steady-state models using the same input data. The temporal differences observed at early times took first steps towards more rigorous confrontation of the common assumptions made by the community. The truncated-dynamic CMRO$_2$ closely followed conventional expectation with the dynamics of CBF rising nearly simultaneously but slightly ahead of CMRO$_2$. However, in case of the full-dynamic CMRO$_2$, CMRO$_2$ rapidly rises ahead CBF, p$_i$O$_2$ and p$_e$O$_2$, triggering CBF response, and then plateaus (briefly) and rises again (following CBF with a slight time-lag).

Importantly, these results depend critically on the molecular oxygen mass transfer coefficient, PS$_c$ for transport from inside to outside the capillaries. Our work suggests that the community needs to measure this parameter better. Taken together, the independent records of intravascular and extravascular partial pressures of oxygen (p$_i$O$_2$ and p$_e$O$_2$), CBF and CMRO$_2$ provided new information about the physiological events that accompany neuronal activation.

In a different vein, our work with Spatial Frequency Domain Fluorescence Diffuse Optical Tomography (SFD-FDOT), which was motivated by fluorescence image-guided surgery for tumor resection, advanced the SFD field and showed how current image-guided surgery techniques based on fluorescence can be improved per depth-specificity and determination of lateral margins. Specifically, we demonstrated rapid
reconstruction algorithms and associated instrumentation using the SFD-FDOT technique, but with compromises that save time and reduce computational complexity. The algorithms utilized the variation of fluorescence intensity with respect to spatial-modulation frequency to compute target depth. Then, using the aforementioned depth information, the lateral margins of fluorescent targets are determined via analytical inversion of the data. Both steps are computationally inexpensive and fast.

We characterized system performance in experiments with fluorescent contrast targets embedded in tissue-simulating phantoms. Single targets (not point-like) were imaged at varying depths as large as 1 cm, and multiple (two) targets at similar and different depths were also imaged. The experiments suggest that the rapid SFD-FDOT approach could be useful in resection surgery and as a first step for more rigorous SFD-DOT. The experiments also permitted evaluation of current limitations.

Looking forward, we can make clear progress in both arenas by addressing the limitations discussed in Chapter 4 & 6. For simultaneous oxygen measurements, further elucidation and accounting of systematic errors due to the different optical depth penetration (for light excitation of the two Oxyphors) will generate more robust and reliable information about oxygen gradients between intra-/extravascular compartments. Improvements in time-resolution by as much as 50x should also be readily possible with better electronics. With these enhancements, further experiments could shed light on oxygen metabolism study in a variety of situations, e.g., hypoxia, hyperoxia, hypercapnia, etc. Per the SFD-FDOT work, we next need to apply it in clinical settings, i.e., neuro- and thoracic-surgery. This step is important to confirm performance in vivo and to understand practical issues that could improve instrumentation. Additionally, development of a theory to account for ballistic photons will improve depth sensitivity and multiple target experiment results.
Appendix


A.1 General Information

All solvents and reagents were purchased from standard commercial sources and used as received. Polyethyleneglycol-amine monomethyl ether (MeO-PEG-NH₂), Av. MW 1000, was obtained from Laysan Bio. CBzNH-AG¹OH and PtTPP-OH were synthesized as described previously [80, 44]. Size-exclusion chromatography (SEC) was performed using Bio-Beads S-X1 (40 – 80 µm bead size, Biorad) using THF as a mobile phase. Column chromatography was performed using Selecto™ silica gel (Fisher Scientific). Analytical thin-layer chromatography (TLC) was carried out using silica gel matrix with a fluorescent indicator (layer thickness 200 µm) on aluminum support (SiliCycle). ¹H and ¹³C NMR spectra were recorded on Bruker Neo400 (400.17 MHz for ¹H, 100.6 MHz for ¹³C), UNI600 (600.1 MHz for ¹H, 150.9 MHz for ¹³C) and UNI500 (500.4 MHz for ¹H, 125.8 MHz for ¹³C) spectrometers. Deuterated solvents (CDCl₃, CD₂Cl₂, DMSO-d₆) were purchased from Cambridge Isotope Laboratories, Inc. The NMR data were analyzed using MestReNova software (Mestrelab Research). Mass spectra were recorded on a MALDI-TOF Bruker Daltonics Microflex LRF instrument using α-cyano-4-hydroxycinnamic acid (CCA) as a matrix (positive-ion mode). Samples were prepared by mixing a solution of the analyte in THF or
MeOH (10 µL, ~1 mM) with a solution of the matrix (100 µL, 10 mg/ml, 0.053 M) in CH₂Cl₂/2-propanol (9:1). The sample, approximately 1 µL, was deposited on the probe tip, dried and analyzed.

Artificial cerebrospinal fluid (aCSF) was supplied by the Cerebrovascular Group in the Department of Anesthesiology and Critical Care at the University of Pennsylvania. The ingredients for preparation of 1 L of aCSF are listed below: diH₂O (1 L), KCl (0.220 g), MgCl₂ (0.132 g), CaCl₂ (0.221 g), urea (0.402 g), dextrose (0.665 g), NaCl (7.710 g), NaHCO₃ (0.207 g / 100 mL; added on the day of use).

**A.2 Synthesis of Oxyphor PtR4**

**CBzNH-AG¹(GLuO'Bu)₂.** To a solution of N-(benzyloxycarbonyl)-3,5-dicarboxyphenyl glycineamide, CBzNH-AG¹OH (0.744 g, 2 mmol), in DMF (20 ml), a solution of CDMT (0.878 g, 5.0 mmol) in DMF (4 ml) was added at 0°C, followed by immediate addition of N-methylmorpholine (1.1 mL, 10 mmol). The reaction mixture was stirred at 0°C for 1 h, then a solution of glutamic acid di-tert-butyl ester hydrochloride (1.302 g, 4.4 mmol) in DMF (20 ml) was added, and the reaction mixture was stirred at room temperature for 48 hours. The solvent was removed in vacuum, the resulting oil was dissolved in CH₂Cl₂ (300 ml), washed with HCl aq. (1N, 100 ml), then with NaHCO₃ aq. (10%, 2 × 100 ml) and dried over Na₂SO₄. The product was purified by column chromatography (silica gel, CH₂Cl₂/MeOH, 25 : 1) to give the title compound as a yellowish solid. Yield 1.22 g (78%). 1H NMR (DMSO-d₆), δ (ppm): 1.40 (s, 18H), 1.43 (s, 18H), 1.89 – 2.10 (m, 4H), 2.35 (t, J = 7.6 Hz, 4H), 3.85 (d, J = 6.0 Hz, 2H), 4.33 – 4.39 (m, 2H), 5.07 (s, 2H), 7.23 – 7.39 (m, 5H), 7.61 (t, J = 6.4 Hz, 1H), 8.00 (s, 1H), 8.19 (s, 2H), 8.75(d, J = 7.2 Hz, 2H ), 10.28 (s, 1H); ¹³C NMR (DMSO-d₆), δ (ppm): 26.4, 28.1, 28.2, 31.8, 53.1, 66.0, 80.2, 81.1, 121.7, 128.20, 128.25, 128.8,
135.5, 137.5, 139.3, 157.1, 166.9, 171.4, 171.9; MALDI-TOF (m/z): calcd. for C_{44}H_{62}N_{4}O_{13}: 854.4, found: 893.5 [M+K]^+.

**Scheme S1.** Synthesis of the Gen2 dendron NH$_2$-AG$^1$(GluO$'$Bu)$_2$.

Reagents and conditions: (a) CDMT, N-methylmorpholine, DMF, r.t., 48 h (78%); (b) 10 wt. % palladium on carbon, THF, r.t., 14 – 15 h (85%).

NH$_2$-AG$^1$(GluO$'$Bu)$_2$. CBzNH-AG$^1$(GluO$'$Bu)$_2$ (1.2 g, 1.40 mmol) was dissolved in THF:MeOH mixture (100 mL, 1:1), and the solution was deoxygenated by Ar bubbling during 10 – 15 min. Pd/C (10 wt%, 100 mg) was added to the solution, and the resulting mixture was flushed with hydrogen gas (H$_2$) and left under stirring.
in H₂ atmosphere (1 atm) for 14 – 15 h. The resulting mixture was passed through a Celite 545 column and the product was washed off the column with THF:MeOH mixture (200 mL, 1 : 1). Concentrating the washings afforded the title compound as a light-yellow solid. Yield: 0.86 g (85%). ¹H NMR (DMSO-d₆), δ (ppm): 1.38 (s, 18H), 1.41 (s, 18H), 1.88-1.96 (m, 2H), 1.99-2.07 (m, 2H), 2.33 (t, J = 5.6 Hz, 4H), 3.29 (s, 2H), 4.31 – 4.35 (m, 2H), 7.99 (s, 1H), 8.23 (s, 2H), 8.72 (d, J = 6.0 Hz, 2H); ¹³C NMR (DMSO-d₆), δ (ppm): 26.4, 28.1, 28.2, 31.8, 45.9, 53.1, 80.3, 81.2, 121.5, 121.6, 135.4, 139.2, 166.9, 171.4, 172.0, 172.8; MALDI-TOF (m/z): calcd. for C₃₆H₅₆N₄O₁₁: 720.4, found: 759.5 [M+K]⁺.

**Scheme S2.** Synthesis of Oxyphor PtR4.

Reagents and conditions: (a) HBTU, DIPEA, DMF, r.t., 96 h (70%); (b) TFA, DCM,
r.t., 2 h (90%); (c) NH$_2$-PEG (Av. MW~1000), HBTU, DIPEA, DMF, r.t., 10 – 12 days (63%).

**PtTPP-(AG$^1$GluO'Bu)$_8$**. To a solution of PtTPP-OH (0.040 g, 0.034 mmol) in DMF (5 ml), HBTU (0.210 g, 0.554 mmol) was added, and the reaction mixture was stirred for 10 min at room temperature. N,N-diisopropylethylamine (0.19 mL, 0.554 mmol) was added to the mixture, immediately followed by addition of NH$_2$-AG$^1$(GluO'Bu)$_2$ (0.4 g, 0.544 mmol). The resulting mixture was stirred at r.t. for 96 h. The reaction mixture was poured into aqueous HCl (5%, 30 ml). The resulting red precipitate was collected by centrifugation, washed with water (2 × 10 ml) and dried in vacuum. After purification by SEC (S-X1, THF), the title porphyrin-dendrimer was isolated as a red solid. Yield: 0.162 g (70%). $^1$H NMR (DMSO-$d_6$), δ (ppm): 1.34, 1.35 and 1.38 (s, 288H), 1.87 – 2.05 (m, 32H), 2.30 – 2.33 (m, 32H), 4.18 (br s, 16H), 4.31 – 4.35 (m, 16H), 6.62 (s, 4H), 6.86 (s, 8H), 7.99 (s, 8H), 8.21 (s, 16H), 8.71 (d, $J = 6.0$ Hz, 16H), 8.88 – 8.92 (m, 16H), 9.01 (s, 4H), 9.34 (s, 8H), 10.4 (s, 8H).

**PtTPP-(AG$^1$GluOH)$_8$**. To a solution of PtTPP-(AG$^1$GluO'Bu)$_8$ (0.115 g, 0.017 mmol) in dichloromethane (10 ml), trifluoroacetic acid (5 mL) was added dropwise, and the mixture was stirred at r.t. for 2 h. The solvents were removed in a vacuum, after which DCM (10 mL) was added and evaporated again. This procedure was repeated twice to remove the remaining traces of TFA, after which the resulting precipitate was dried. The product was used in the following step without further purification. Yield: 0.076 g (90%).

**Oxyphor PtR4**. To a solution of PtTPP-(AG$^1$GluOH)$_8$ (0.075 g, 0.015 mmol) in DMF (10 ml), HBTU (0.364 g, 0.96 mmol) was added, and the reaction mixture was stirred for 10 min at room temperature. DIPEA (0.67 mL, 3.84 mmol) was added to
the mixture, followed by the addition of solid mPEG-Amine (MW 1000, 0.96 g, 0.96 mmol). The reaction mixture was stirred at room temperature for 7 days. Additional portions of HBTU (0.182 g, 0.48 mmol) and mPEG-Amine (MW 1000, 0.48 g, 0.48 mmol) were added to the mixture, and stirring continued for additional 5 days. The reaction mixture was poured into diethyl ether (100 mL) and sonicated. The obtained mixture was centrifuged, the supernatant was discarded, and the remaining dark red viscous material was dissolved in THF (10 mL) and subjected to SEC (Bioard S-X1, THF). The red-orange fraction was collected, the solvent was removed in a vacuum and the product was dried. PtR4 was isolated as a dark red solid. Yield: 0.345 g (63%).

A.3 Photophysical Properties of Oxyphor PtR4

Figure 38: Absorption (a) and phosphorescence (b) spectra of Oxyphor PtR4 in 50 mM phosphate buffer solution (pH 7.1, \( \lambda_{ex} = 512 \) nm).

The absorption spectrum of PtR4 recorded in 50 mM phosphate buffer solution (pH 7.2) shows no traces of aggregation (Fig. 38). The photophysical properties of Oxyphor PtR4 are summarized in Table A1. The phosphorescence emission maximum is 727 nm. The phosphorescence quantum yield was measured in an aqueous
solution (phosphate buffer, pH 7.2, 23°C) relative to the fluorescence of rhodamine 6G (Rh6G) in EtOH ($\phi_{fl} = 0.94$) and found to be 0.053.

Table 8: Photophysical constants of Oxyphor PtR4

<table>
<thead>
<tr>
<th>Probe</th>
<th>Absorption, nm ($\varepsilon \times 10^{-4}$, M$^{-1}$cm$^{-1}$)$^c$</th>
<th>Emission$^a$</th>
<th>$\phi / \tau_0$ (µs)$^b$</th>
<th>$\lambda_{max}$ nm</th>
</tr>
</thead>
<tbody>
<tr>
<td>PtR4</td>
<td>403 (30.9)</td>
<td>660, 727</td>
<td>0.053 / 44</td>
<td></td>
</tr>
<tr>
<td></td>
<td>512 (3.2)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^a$The emission spectra were recorded in 50 mM phosphate buffer solution (pH 7.2, 23°C), deoxygenated by Ar. $^b$The phosphorescence quantum yield ($\phi$) was determined relative to the fluorescence of Rh6G ($\phi_{fl}=0.94$) in EtOH; the phosphorescence lifetime at zero oxygen ($\tau_0$) was measured using a solution deoxygenated by Ar. $^c$The extinction coefficient was determined assuming the molecular weight of 36,410 Da.
A.4 Oxygen Quenching Properties of Oxyphor PtR4

Figure 39: Phosphorescence quenching plot of Oxyphor PtR4.

The oxygen quenching plots of Oxyphor PtR4 were measured in buffered aqueous solutions (20 mM phosphate, pH 7.2) in the absence and presence of bovine serum albumin (BSA, 5%) and found to be nearly identical (Fig. 39). The data were fitted to an empirical bi-exponential form (shown in Fig. 39), and its coefficients were used to convert the phosphorescence lifetime measured in vivo to pO₂. The phosphorescence lifetime of Oxyphor PtR4 changes from $\sim 8 \mu$s at air saturation to $\sim 43 \mu$s at zero-oxygen at (36.8°C).

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A.5 Optical Filter Configuration of Two-channel Phosphorometer

Figure 40: Optical absorption and emission spectra of the probes and associated laser lines and optical filters in Channels 1 and 2 of the phosphorometer. The spectral ranges seen by the detectors (APDs) are shown by shaded areas. \textit{lp} is the abbreviation for \textit{long-pass filter}, and \textit{sp} is for \textit{short-pass filter}. 
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