Economics Of Buyouts

Alexander Belyakov

University of Pennsylvania
Economics Of Buyouts

Abstract
This dissertation consists of three parts. The first two, while answering important questions about investment and capital structure in their own right, develop key elements to the model that is used in the final part of the paper that addresses the main question of my work at Wharton - how private equity (PE) firms create value. Empirical analysis of a sample of companies with PE ownership in the UK shows that PE firms act as deep-pocket investors for their portfolio companies, rescuing them if they fall in financial distress. In contrast, external financing is expensive for companies without PE-ownership in financial distress. The paper builds a model that shows how companies form rational expectations about the costs of financial distress, and how these expectations affect ex-ante policies. The model explains the empirically-observed differences in how companies with and without PE-ownership invest, pay dividends, and issue debt. In particular, the model quantitatively explains the difference in leverage of companies with and without PE-ownership. The model shows that greater tax-shield benefits and superior growth of PE-backed companies can explain 6.4% of the abnormal return of PE firms. The conclusion that follows from the paper, however, is that abnormal returns PE firms cannot be replicated by other investors.

Degree Type
Dissertation

Degree Name
Doctor of Philosophy (PhD)

Graduate Group
Finance

First Advisor
Bilge Yilmaz

Subject Categories
Finance and Financial Management

This dissertation is available at ScholarlyCommons: https://repository.upenn.edu/edissertations/4046
ECONOMICS OF BUYOUTS

Alexander Belyakov

A DISSESSATION

in

Finance

For the Graduate Group in Managerial Science and Applied Economics

Presented to the Faculties of the University of Pennsylvania

in

Partial Fulfillment of the Requirements for the

Degree of Doctor of Philosophy

2021

Supervisor of Dissertation

Bilge Yilmaz, Wharton Private Equity Professor, Professor of Finance

Graduate Group Chairperson

Nancy Zhang, Ge Li and Ning Zhao Professor, Professor of Statistics

Dissertation Committee

Joao F. Gomes, Department Chair, Howard Butcher III Professor, Professor of Finance
Nikolai Roussanov, Moise Y. Safra Professor, Professor of Finance
Amir Yaron, Robert Morris Professor, Professor of Finance
ACKNOWLEDGMENT

I am very grateful to all professors at Wharton, and in particular to my supervisors Joao Gomes, Nikolai Roussanov, Amir Yaron and Bilge Yilmaz for many helpful discussions, guidance and support in writing this paper. I am very grateful to all my fellow PhD students, and in particular Alejandro Lopez-Lira, Max Miller, and Felix Nockher for their comments and feedback. I am very grateful to a big number of people with the practical knowledge of private equity industry who agreed to answer my questions, in particular Madeleine Evans, Greg Grose, Alexander Hansen, Julia Kochetygova, and Rob Seminara. Results in this paper rely on the data collected for a sample of PE-backed companies in the UK, and I am very grateful to everyone who helped with collecting the data: the whole Qynn team, and John Cushing and Anna Metcalfe in particular, Olga Kvan, and two excellent research assistants Nadezhda Belyakova and Kaan Erdognus. Rodney White Center for Financial Research, Mack Institute for Innovation Management, and Jacob Levy Equity Management Center for Quantitative Financial Research at Wharton provided financial support for this research.
This dissertation consists of three parts. The first two, while answering important questions about investment and capital structure in their own right, develop key elements to the model that is used in the final part of the paper that addresses the main question of my work at Wharton - how private equity (PE) firms create value.

Empirical analysis of a sample of companies with PE ownership in the UK shows that PE firms act as deep-pocket investors for their portfolio companies, rescuing them if they fall in financial distress. In contrast, external financing is expensive for companies without PE-ownership in financial distress. The paper builds a model that shows how companies form rational expectations about the costs of financial distress, and how these expectations affect ex-ante policies. The model explains the empirically-observed differences in how companies with and without PE-ownership invest, pay dividends, and issue debt. In particular, the model quantitatively explains the difference in leverage of companies with and without PE-ownership. The model shows that greater tax-shield benefits and superior growth of PE-backed companies can explain 6.4% of the abnormal return of PE firms. The conclusion that follows from the paper, however, is that abnormal returns of PE firms cannot be replicated by other investors.
# TABLE OF CONTENTS

ACKNOWLEDGMENT ................................................................. ii

ABSTRACT ............................................................................. iii

LIST OF TABLES ........................................................................ v

LIST OF ILLUSTRATIONS ........................................................... vii

CHAPTER 1: Leverage and Financing in Distress ........................................ 1
  1.1 Introduction ................................................................ 1
  1.2 Model .......................................................................... 11
  1.3 Model solution .............................................................. 27
  1.4 Conclusion .................................................................... 43

CHAPTER 2: Omitted Variable in Capital Structure Regressions .................... 45
  2.1 Introduction ................................................................ 45
  2.2 Model .......................................................................... 49
  2.3 Model solution .............................................................. 65

CHAPTER 3: Economics of Leveraged Buyouts: Theory and Evidence from the
  UK Private Equity Industry ....................................................... 77
  3.1 Introduction ................................................................ 77
  3.2 Discussion of the main assumption of the model ...................... 85
  3.3 How PE buyouts are structured and why it is important ............ 88
  3.4 Empirical results ......................................................... 94
  3.5 Model .......................................................................... 127
  3.6 Model Results .............................................................. 141
  3.7 Conclusion .................................................................... 156

Bibliography ........................................................................... 158
**LIST OF TABLES**

<table>
<thead>
<tr>
<th>TABLE</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>TABLE 1</td>
<td>Benchmark parameter values used to solve the model</td>
<td>28</td>
</tr>
<tr>
<td>TABLE 2</td>
<td>Results of the model solution</td>
<td>29</td>
</tr>
<tr>
<td>TABLE 3</td>
<td>Default probabilities and firm lifetime in the model</td>
<td>32</td>
</tr>
<tr>
<td>TABLE 4</td>
<td>Numbers in the data</td>
<td>35</td>
</tr>
<tr>
<td>TABLE 5</td>
<td>Empirical and simulated moments of cross-sectional distribution</td>
<td>36</td>
</tr>
<tr>
<td>TABLE 6</td>
<td>Default rates, and leverage for firms with different credit ratings</td>
<td>40</td>
</tr>
<tr>
<td>TABLE 7</td>
<td>Model with greater bankruptcy costs/costs of equity issuance</td>
<td>42</td>
</tr>
<tr>
<td>TABLE 8</td>
<td>Benchmark parameter values used to solve the model</td>
<td>67</td>
</tr>
<tr>
<td>TABLE 9</td>
<td>Leverage-predicting regressions in the model</td>
<td>70</td>
</tr>
<tr>
<td>TABLE 10</td>
<td>Cross-sectional regressions in the model</td>
<td>72</td>
</tr>
<tr>
<td>TABLE 11</td>
<td>Moments of leverage distribution in the data and in the steady-state cross-section in the model</td>
<td>74</td>
</tr>
<tr>
<td>TABLE 12</td>
<td>Sample of companies in Preqin vs. analyzed in this paper</td>
<td>96</td>
</tr>
<tr>
<td>TABLE 13</td>
<td>Leverage</td>
<td>103</td>
</tr>
<tr>
<td>TABLE 14</td>
<td>Debt management</td>
<td>106</td>
</tr>
<tr>
<td>TABLE 15</td>
<td>Debt issuance by companies with PE-ownership post-buyout</td>
<td>108</td>
</tr>
<tr>
<td>TABLE 16</td>
<td>Equity injections</td>
<td>110</td>
</tr>
<tr>
<td>TABLE 17</td>
<td>Money flow between the company and the PE</td>
<td>112</td>
</tr>
<tr>
<td>TABLE 18</td>
<td>Unconditional statistics about investments</td>
<td>114</td>
</tr>
<tr>
<td>TABLE 19</td>
<td>Conditional statistics about investments</td>
<td>116</td>
</tr>
<tr>
<td>TABLE 20</td>
<td>Operating performance</td>
<td>119</td>
</tr>
<tr>
<td>TABLE 21</td>
<td>Example of IRR calculation, Ambasador Theatre Group buyout</td>
<td>120</td>
</tr>
<tr>
<td>TABLE 22</td>
<td>IRR on the whole company, across secondary buyout deals</td>
<td>124</td>
</tr>
<tr>
<td>TABLE 23</td>
<td>Parameter values</td>
<td>143</td>
</tr>
<tr>
<td>TABLE 24</td>
<td>Performance</td>
<td>153</td>
</tr>
</tbody>
</table>
LIST OF ILLUSTRATIONS

FIGURE 1: Amount of new equity and debt (scaled by assets) issued by a median firm across firms with different leverage ............................................. 2

FIGURE 2: Cumulative distribution function (cdf) of leverage distribution in the data and in the model ......................................................... 3

FIGURE 3: Stock price discount following equity issuance announcement ......................................................... 6

FIGURE 4: Value of debt and marginal interest rates for a firm with current productivity $X = 1$ and varying coupon payments ............................................. 17

FIGURE 5: Value of debt and equity in version one of the model for a firm with $X = 1$ ......................................................... 22

FIGURE 6: Dynamic of a typical firm in the D-type environment in version two of the model ......................................................... 26

FIGURE 7: Value of debt and equity in version two of the model for a firm with $X = 1$ ......................................................... 27

FIGURE 8: Distribution of leverage and inverse coverage ratio ......................................................... 37

FIGURE 9: Leverage and 10-year default probability ......................................................... 39

FIGURE 10: Leverage and profitability in two groups of firms ......................................................... 46

FIGURE 11: Debt value and marginal interest rate for an additional unit of debt ......................................................... 57

FIGURE 12: Path of a typical firm in the model ......................................................... 59

FIGURE 13: NPV of an investment opportunity for a firm ......................................................... 63

FIGURE 14: Value of equity and debt, and the decision to issue debt to exploit benefits of tax-shield ......................................................... 64

FIGURE 15: ........................................................................................................ 68

FIGURE 16: Distributions in a simulated cross-section for firms with different values of $\gamma$ ......................................................... 73

FIGURE 17: Distribution of market-to-book ratio in the data and in the model ......................................................... 75

FIGURE 18: Total money flow between the PE and the company ......................................................... 78
CHAPTER 1: Leverage and Financing in Distress

1.1. Introduction

Interest rates to issue debt are high for firms that already have a lot of debt. This makes debt financing expensive for highly-levered firms. One would assume that firms should, therefore, substitute debt financing with equity financing when leverage becomes higher. Yet, as Figure 3 shows, highly-levered firms issue debt rather than equity.\(^1\) This implies that equity financing costs increase with firm’s leverage even faster that debt financing costs - or otherwise firms would not increase their already high leverage. This paper shows that the fact that highly levered firms are effectively cut out from equity capital markets, can explain many capital structure patterns.

First, a model that assumes that equity issuance costs are unaffected by leverage will lead to significant counterfactual implications. To show this, the paper solves a model similar to Goldstein, Ju, and Leland (2001), in which 1) firms in distress can always issue equity, and 2) costs of issuing equity do not change with leverage (either zero or proportional to the amount). In a simulated steady-state cross section, 57% of firms optimally choose to be in distress, having so much debt that their cash flow is not sufficient to pay interest expenses (and so they issue equity). This is wrong both quantitatively and qualitatively:\(^2\) firms that issue equity in the data are low-levered, and the absolute majority of firms produce enough cash to pay interest expenses. A number of other known capital structure puzzles arise: for instance, the average leverage in the model is too high (58% in the model vs. 36% in the data), or default probabilities are understated (a firm that has 20% default probability within 10 years has 50% leverage in the data and 75% in the model).\(^3\)

\(^1\)Empirical literature on equity issuance strongly supports this: Senber and Senber (1995) report complete absence of equity issuance by firms in distress, while Eckbo, Masulis, and Norli (2007) find that most equity issuances take a form of payments in mergers or employee stock compensations, and that equity issuances in which firms raise cash for business operations are rare. See Literature Review subsection A and Appendix A for a more detailed analysis of empirical research on equity issuance by firms in distress.

\(^2\)This is not the result of inadequate calibration of parameters: I took parameter values from Hennessy and Whited (2012) paper, in which they consider the same model. A similar result appears in other papers; for instance, in Bharma, Kuehn, and Strebulaev (2011) firms that optimally choose their leverage at time zero issue so much debt that they immediately put themselves in distress, with interest expenses exceeding their cash flow.

\(^3\)This again happens because firms will operate too long before defaulting by issuing equity and paying interests to debtholders.
Figure 1: Amount of new equity and debt (scaled by assets) issued by a median firm across firms with different leverage

![Graph showing issuance of new debt and equity](image)

New debt (new equity) is the amount of long-term debt (equity) issued by a firm in a given year, scaled by the book value of asset. Median rather than average values are reported because distributions of debt and equity issuances are skewed, and more so for firms with high values of leverage. Formal regression of new debt (new equity) on firm’s leverage results in the coefficient 0.35 (-0.16), both statistically significant at 1% level. Data is taken from Compustat for the period 1987-2017; see Section 3.2 for the details of data analysis. Leverage is the ratio of firm’s debt (sum of long-term and short-term) to the book value of assets.

While the above-mentioned results are based on the framework of Goldstein, Ju, and Leland (1994), it is important to emphasize that the results are not specific to this particular framework, but come from the assumption that highly-levered firms have a cheap way of external financing (equity in this case). For instance, Hennessy and Whited (2005) use a different model to study firm’s capital structure, but assume that equity issuance incurs only flotation costs. They also find that in their model “equity issuers are the most highly indebted”. In short, the assumption that highly-levered firms can easily issue equity 1) is not supported by the data or empirical literature, and 2) leads to wrong implications, both quantitatively and qualitatively.

As the next step, this paper modifies the model by assuming that firms in distress cannot issue equity, and have to issue more debt every time they do not have enough cash to pay interest expenses. Here is what happens then: when firm’s cash flow falls below the level of interest expenses, firm’s leverage is already high; by issuing more debt to cover the gap, the firm increases its leverage further, and moves closer to default. Moreover,
For each value of leverage on the horizontal axis, the bars show what is the fraction of firms in the data and in the model with leverage that does not exceed this value. Panel A shows results for the model in which firms can always issue equity, and Panel B shows results for the model in which firms whose profits are not sufficient to pay interest expenses on their debt have to issue more debt to cover the gap. Data is taken from Compustat for the period 1987-2017 for firms that have access to public bond markets (proxied by whether a firm in a given year has S&P long-term credit rating). The model is solved for the benchmark parameter values shown in Table 9. See Section 3 for details of data analysis and model simulations.

as debt is fairly priced, interest rates to issue new debt grow exponentially for firms that are deep in distress. In essence, a firm in distress risks falling in a continuous debt spiral, when interest expenses grow faster than the cash flow, and it has to continuously issue new debt to cover payments on previously issued debt. Such mechanism makes distress very costly, and affects expectations of firms that are outside of distress. Even healthy firms now choose significantly smaller leverage ex-ante, even if bankruptcy costs are low and tax-shield benefits are high.

The modified model has much better quantitative results. Firms that fall in distress either quickly recover or quickly default. As the result, only 14% of firms are in distress (have interest expenses that exceed their cash flow), and default probabilities match empirically observed numbers. Expectations about distress reduce firm’s target leverage by 20 percentage points (compared to the case when firms could issue equity in distress), and so the model correctly matches the average leverage among all firms in the data, and also the average leverage in subsets of firms grouped by their credit ratings. Moreover, the model
closely matches every quantile of leverage distribution between 35% and 100%, as shown on Figure 2 Panel B.

This last result illustrated by Figure 2 that the model correctly matches the whole right-tail distribution of leverage is particularly important for this paper. Firms in the model make capital structure choices based on what they expect will happen to them in distress. While it is impossible to directly test firms’ expectations, it is possible to study what actually happens to firms in distress vs. what the model says the expectations are. Because the match between the model-predicted and empirically-observed distributions of leverage of firms in distress is so close, it provides a strong support for the theoretical mechanism used in the model. This is the main contribution of the paper: it uncovers the mechanism that both explains the distribution of leverage of highly-leveraged firms and shows how it affects firm’s initial capital structure choice. It is also the first paper that examines this channel and shows its importance.

One of the main advantages of Leland-type models is that they are easy to solve and results are generally available in a closed form. The modification of the model that says that firms in distress cannot issue equity does not come at the cost of tractability. Tractability is preserved in the model that this paper derives, and some results become even simpler, as is the case with, for instance, the default boundary.

The paper shows that large bankruptcy costs are not necessary to match the average value of leverage in a cross-section. In the benchmark calibration, the value of bankruptcy costs is only 10%. Even with this value, the cross-sectional leverage is only 38% for the environment in which firms in distress finance the gap between the cash flow and interest expenses by issuing debt. In contrast, to match the average leverage in a cross-section within the framework in which firms in distress can also issue equity, bankruptcy costs should be at least 60%, and even higher to match other moments.

In the model, firm’s default probability depends on its current leverage, and also on how this firm will finance its interest payments in distress. Naturally, keeping firm’s leverage constant, default probability is higher if in distress the firm will have to issue debt at
exponentially increasing rates, as opposed to cheap equity. A striking result, though, is that the overall default rate is higher in the model in which firms in distress can issue equity. The reason why this happens is because firms in both models endogenously choose how much debt to issue, and in the model, in which firms in distress can only issue debt, they are much more conservative in their leverage policy ex-ante, and so default less often ex-post.

**Literature review**

A. **Equity issuance by firms in distress**

The underlying assumption of the model in this paper is that firms in distress do not issue equity and instead finance the shortfall between the cash flow and interest expenses by issuing more debt. This is a strong assumption that is imposed to achieve a closed-form solution for the function that connects firm’s leverage and external financing costs; similar results would hold in the model if distressed firms could issue equity, but equity issuance costs increased with leverage.

Empirical evidence on equity issuance by distressed firms is scant, but also mixed. Below is a review of papers that consider this question. The overall conclusion that follows is that most firms issue equity when their leverage is low; there are instances of equity issuances by financially distressed firms, but the costs are high, and such firms use equity financing because they cannot raise debt.

The first group of papers argues that most firms issue equity when their performance is good. For instance, Senber and Senber (1995) report a complete absence of equity issuance by distressed firms. Fama and French (2005) show that equity issuances are frequent, but most firms issue equity when their leverage is low. Similarly, Mikkelson and Partch (1986) and Eckbo, Masulis, and Norli (2007) find that equity issuances for cash are rare - both in absolute level and relative to public debt issuances. Some other studies provide indirect evidence that firms in distress do not issue equity. Korajczyk, Lucas, and McDonal (1990) find that firm’s leverage does not increase significantly two years before an SEO; should firms issue equity to make required debt payments when internally generated cash flow is
Data is based on SDC/Platinum database that tracks equity issuances, and shows the average stock price reaction following an equity issuance announcement. Each bar represents the average discount for a group of companies with a given leverage; for instance, among companies, whose leverage was between 30% and 40% at the time when they announced an equity issuance, the stock price fell by 3.59% on average. Negative 4.32% (red line) is the average stock price reaction following an equity issuance announcement.

insufficient, one would observe an increase in leverage prior to an SEO. DeAngelo, DeAngelo and Stulz (2008) find that the average leverage of a firm before an SEO is only 27%. Denis and McKeon (2012) show that firms, whose leverage is above the target, tend to cover financial deficit by issuing new debt and increasing leverage further.

Other authors argue in contrary that a sizable number of distressed firms issue equity, but they sell new shares at a large discount, and do so because debt financing is unavailable. Park (2017) finds that public equity offerings decrease for firms in distress, but private placements increase. Walker and Wu (2017) find that a third of all SEOs are conducted by financially distressed firms. Both of these papers, however, use the distress measure from Campbell, Hilsher, and Szilagyi (2008), which is only partially related to firm’s leverage. Indeed, the average leverage in the subsample of distressed firms in Walker and Wu is 32%, which implies that these firms are in distress for reasons other than their indebtedness, and they likely have very limited access to debt financing. This conclusion is further reinforced by Lim and Schwert (2017) who study all private placement of equity (PIPEs) by U.S. firms. They find that most firms issuing PIPEs are small distressed firms without access to debt financing.
markets: the median leverage of firms issuing PIPEs is only 7.2%, and 93% of all firms do not have credit rating. When such firms issue PIPEs, they offer shares to the market at a large discount.

Appendix A provides further empirical analysis of the correlation between the frequency of equity issuance and firm’s leverage based on Thomson Reuters data. Results show that the amount of equity issuance decreases with firm’s leverage, and the discount at which newly issued shares are offered to investors increases with leverage; this conclusion holds for all firms and also for the subsample of firms that have access to public debt markets. Similar conclusion follows from Figure 3 which is based on Compustat data.  

The model derived in this paper assumes that firms always have access to debt capital markets. Therefore, the assumption that such firms do not issue equity to pay required debt payments in distress is consistent with empirical evidence discussed above.

The question why costs to issue equity grow for firms in distress is beyond the scope of this paper. To theoretically microfound this assumption, Belyakov (2018) considers a model similar to the one used in this paper, but adds information asymmetry between firm’s manager and outside shareholders. He shows that leverage amplifies information asymmetry, and that costs to issue equity escalate with leverage as the result. Capital structure-wise, he obtains results that quantitatively similar to those in this paper, but the model itself is less tractable. Appendix B provides a simplified two-period version of that model that shows how presence of leverage amplifies information asymmetry. Such explanation is consistent with empirical findings of Hertzel and Smith (1993) and Lim and Schwer (2017) who argue that distressed firms are characterized by severe information asymmetry.

---

4The paper does not combine the two databases because they use different definitions of equity issuance. In particular, Thomson Reuters mostly considers SEOs, while Compustat partially considers private placements as well. Assuming that Compustat data is internally consistent, Figure 3 shows the relative scale of debt and equity issuance as firms leverage increases, which would not necessarily be consistent if Compustat and Thomson Reuters data for equity issuance is pooled. On the other hand, Thomson Reuters has data for the discount/premium paid for newly issued shares, which is not available in Compustat.

5The empirical sample of firms that the paper quantitatively explains also consists of firms with access to debt capital markets (firms with S&P long-term credit rating).

6In fact, an identical model
B. Underleverage puzzle

This paper is related to the line of literature that discusses the underleverage puzzle. Miller (1977) shows that present value of expected default losses seems disproportionately small compared to tax benefits of debt, implying that firms consistently issue less debt than what would be optimal to maximize the value of their shareholders. Graham (2000) estimates that tax benefits of debt add up to 5% of firm value, and also concludes that firms are on average underlevered from the point of view of a trade-off theory.

Faulkender and Petersen (2006) show that at least part of the underleverage puzzle is explained by the fact that not all firms have access to debt capital markets. They find that the difference in average leverage between firms that do and do not have S&P long-term credit rating is almost 20 percentage points. This paper assumes that all firms have access to debt capital markets; nevertheless, the underleverage puzzle remains in the model in which firms can always issue equity.

Chen (2012) provides an alternative explanation to the underleverage puzzle: he focuses on a model in which bankruptcy costs rise in bad states of the economy. He shows that firm’s target leverage at refinancing points is significantly smaller relative to a model in which bankruptcy costs do not vary with states of the economy. Nevertheless, when he simulates a cohort of firms, the average simulated leverage exceeds 40%, which is greater than the average leverage observed in the data.

Morelec, Nikolov, and Schuroff (2012) provide an alternative explanation to the underleverage puzzle that relies on the presence of a conflict of interests between shareholders and managers. They use the Goldstein-Ju-Leland framework in which firms in distress can always issue equity, but they assume that firm’s manager can divert a small fraction of the cash flow and, therefore, has incentives to keep the firm alive for longer. They show that small agency costs help resolve the underleverage puzzle. Their paper, however, only addresses the mean and median values of leverage distribution and, as authors acknowledge, the “model is statistically rejected for higher-order leverage moments and dispersion mea-

---

7See also Jensen and Meckling (1975)
sures”. Indeed, kurtosis of the simulated leverage distribution in their model is much greater than in the data, implying that the simulated leverage distribution is centered around the mean value. The higher-order moments of leverage distribution, in particular the right-tail distribution of leverage, is the main focus of this paper.

Papers that resolve the underleverage puzzle often assume high values of bankruptcy costs; this increases ex-ante costs of debt issuance and reduces firm’s incentives to issue debt. For instance, among the two papers mentioned above, Chen (2010) assumes bankruptcy costs between 40% and 80%, and Morellec, Nikolov, and Schurhoff (2012) assume bankruptcy costs of 50%\(^8\). These values are substantially greater than empirically observed among defaulted firms (between 1% and 30%).\(^9\) Glover (2016) points that there is a selection bias among firms that default; specifically, firms with low bankruptcy costs choose leverage values that are high, thus, defaulting more often and creating a bias in empirical estimates of bankruptcy costs. He uses a structural estimation approach and finds average default costs of around 40%. While Glover’s argument is valid, the underlying assumption of his structural model is that firms in distress issue equity, which, as noted earlier, is not the case in the data. This paper shows that expectations about increasing costs of external financing in distress affect firm’s initial leverage choice significantly even if actual bankruptcy costs are small.

The explanation to the underleverage puzzle provided in this paper has one advantage compared to alternative explanations: it is almost trivial. In essence, the logic follows from the backward induction: once a firm knows that tomorrow it will only be able to refinance its debt at a high rate, it wants to have less debt today. This explanation can’t be ignored simply because it relies on an assumption that does not have a solid theoretical foundation - that all forms of external financing - including equity - become more expensive as firm’s leverage increases. It is particularly important given that empirical evidence confirms that highly levered firms either only issue debt or issue equity at a high discount, as the previous subsection shows.

\(^8\)It should be noted that Morellec, Nikolov, and Schurhoff assume that default may lead to renegotiation rather than liquidation with substantially lower costs

C. Goldstein-Ju-Leland framework

This paper has an important theoretical contribution to the class of dynamic capital structure models similar to Leland (1994), Goldstein, Ju, and Leland (2001), or Strebulaev (2007): it derives in a closed form the optimal default boundary for a firm that finances the gap between the cash flow and interest expenses by issuing debt. In the seminal 1994 paper, Leland shows how to derive the optimal default boundary for the case when the firm can always issue equity. The author introduces the smooth-pasting condition for the firm’s optimal default boundary: at the time when the firm optimally defaults, both the value of equity and its derivative with respect to the value of cash flow are zero. In this paper, firms in distress issue new debt. Issuance of debt increases firm’s interest expenses going forward, and default happens when no further debt issuances are possible. This is achieved by imposing the smooth-pasting condition on the value of debt: the derivative of the value of debt with respect to the coupon payment is zero at the time when default happens. It follows from the model, however, that the smooth-pasting condition for the value of equity at default is preserved.

Tractability of the Goldstein-Ju-Leland framework is one of the main reasons why it is often used in structural estimations literature. Nevertheless, this paper shows that quantitative results of a model that assumes that firms in distress can always issue equity are unrealistic. This can be a significant concern for structural estimation papers, for which the plausibility of the underlying model is of the first-order importance. Modification of the original framework that this paper derives maintains all the tractability features of the original model, but produces much better fit to the data. This suggests that the modified framework can contribute to future structural estimation papers.

The rest of the paper is organized as follows. Section 2 presents mathematical formulation of the model. Section 3 presents model calibration and quantitative results. Section 4 concludes.

Bhamra, Kuehn, Strebulaev (2009, 2010), Chen (2010), and Glover (2016) consider more recent versions of this framework that allow for switching macroeconomic regimes. While this paper assumes that macroeconomic conditions are stable, the debt refinancing mechanism that this paper develops can be easily implemented in those models.
1.2. Model

I derive a model of firm’s optimal capital structure in two different environments, to which I refer as D-type and E-type. The ultimate goal is to compare firm’s predicted capital structure in these two environments against each other and the data. E-type and D-type environments are different by the assumption of how firms in distress finance required interest payments, with distress defined as the situation, in which firm’s time-\( t \) cash flow being lower than the required interest payment that this firm has to make at time \( t \). In the E-type environment, firms in distress issue equity to finance the difference between the cash flow and the interest payment (therefore, ”E” in the name of the model). In the D-type model, firms in distress issue new debt (therefore, ”D” in the name).

For both E-type and D-type environments, I consider two versions of the model; versions are different by the assumption of how many times firms in the economy can issue debt to exploit benefits of tax-shield. Version one maintains the set-up of Leland 1994 paper, where it is assumed that debt can only be issued at \( t_0 \). Version two has assumptions of Goldstein, Ju, and Leland (2001). In this version, firms are allowed to issue debt at \( t_0 \), and increase its amount subsequently if the financial performance is good.

I start with the description of the economy that is the same for both versions of the model, and define objective functions of agents in each version in the following subsections.

1.2.1. The economy

I consider a partial-equilibrium model of an economy with a constant risk-free rate \( r \). Firms in the economy are endowed with assets that produce an exogenous stream of cash flows \( \{X_t\} \). Evolution of \( X_t \) under the risk-neutral probability measure \( Q \) follows Geometric Brownian motion:

\[
\frac{dX_t}{X_t} = \mu dt + \sigma dW^Q_t \tag{1.1}
\]

\( ^{11} \)Note that D-type version one will have many debt issuances, but only the first will be done to exploit benefits of tax shield, and all other debt issuances will happen when the firm does not generate enough money to make required interest payments.
where $\mu$ is the risk-neutral drift, and $\sigma$ is the volatility of firm’s cash flows.

Profits in the economy are taxed at the corporate tax-rate $\tau$. I define the value of assets as the expected value of future discounted profits that these assets will produce:

$$E^U(X_t) = E \left[ \int_t^\infty e^{-r(s-t)}(1-\tau)X_s ds \right] = (1-\tau)\frac{X_t}{r-\mu}$$  \hspace{1cm} (1.2)

Firms in the economy can issue debt, and interest payments on debt are tax-deductible. Debt in the model takes the form of a perpetuity that pays a constant coupon rate $c$ per unit of time. At each point in time, a firm produces cash flow $X_t$ and has to pay $c_t$ to its debtholders. $(X_t - c_t)$ is the taxable base of the firm; for simplicity, and similar to other authors, I assume that if firm’s taxable base is negative, the firm pays negative taxes, meaning that it receives money from the government. By taking greater amount of debt, firms can reduce the amount of taxes they pay. On the other hand, high amount of debt increases the probability that the firm will not be able to service its debt obligations. When making a decision about timing and amount of debt to issue, firms trade off these benefits of debt (lower tax-payments to the government) against costs of debt (higher chances of default). Firms act in the interest of equityholders, and, therefore, I use words "firms" and "equityhodlers" interchangeably later in the text.

**Equityholders**

A firm in the economy is fully characterized by its current cash flow level $X_t$, and the coupon payment it has to make $c_t$; for simplicity, I drop subscripts $t$ in what follows.

In case $X \geq c$, the firm produces enough money to service its debt obligations, and immediate dividends to equityholders are $(1-\tau)(X-c)$. The HJB equation for this case then takes the form:\textsuperscript{12}

$$rE(X,c) = (1-\tau)(X-c) + \mu X E_x' + \frac{\sigma^2 X^2}{2} E_{xx}'' , \text{ if } X \geq c$$  \hspace{1cm} (1.3)

\textsuperscript{12}This equation is the same for both versions of the model in both E- and D-type environments
where $E = E(X, c)$ is the value of firm’s equity.

If a firm experiences a series of negative shocks, its cash flow may become insufficient to make required debt payments $c$. In this case, the firm has to finance the shortfall by raising money externally. In the E-type environment, the firm raises equity, and mathematically it is equivalent to negative dividends. Therefore, the HJB equation for the E-type environment for the case $X < c$ is the same as (3).

Equity issuances are not allowed for firms in distress in the D-type environment, and so additional debt issuance is the only source of external financing. It is assumed that there are no transaction costs of debt issuance, and newly issued debt has the same seniority level as the old debt. Denote $D(X, c)$ the value of firm’s debt, and $dD$ the value of newly issued debt. The following formula then connects changes in promised future coupon payments with the required newly debt issuances:

\[
\left( c - (X + (c - X)\tau) \right) dt = (c - X)(1 - \tau) dt = dD = dc \frac{\partial D}{\partial c}
\]

(1.4)

The very left-hand side of equation (4) is the difference between the required coupon payment $c$, and the amount of money the firm has on hands - its profits $X$, and tax return from the government $\tau(c - X)$. This difference is the shortfall that equityholders must but can not pay to debtholders; this difference should equal to change in debt value $dD$, which is in turn achieved by promising a higher coupon payment in the future. It is clear from equation (4) that it is only possible to issue new debt if $\frac{\partial D}{\partial c} > 0$, that is, if value of debt increases when the firm promises to pay more in the future. For now, consider the case when this condition is satisfied.

Equation (4) allows to derive the dynamics of $dc$ for the case when $X < c$:

\[
dc = \frac{(c - X)(1 - \tau)}{\frac{\partial D}{\partial c}} dt
\]

(1.5)

HJB equation for the equity value on the region $X < c$ in the D-type environment should
take into account that both $X$ and $c$ change:

$$rE(X, c) = \frac{(c - X)(1 - \tau)}{\partial D/\partial c} E'_c + \mu X E'_x + \frac{\sigma^2 X^2}{2} E''_{xx}, \quad \text{if} \quad X < c$$  \hspace{1cm} (1.6)

Boundary conditions for the equity value will differ between versions one and two of the model, and are discussed in the corresponding sections of the paper.

**Debtholders**

HJB equation for debtholders will differ from the HJB equation for equityholders by only the part that catches instantaneous profit. In the E-type environment, instantaneous profit to debtholders is always $cdt$. In the D-type environment, it is $cdt$ on the interval $X \geq c$, and $((X + (c - X)\tau)dt + dD)$ on the interval $X < c$. Note, however, that it follows from equation (4) that

$$(X + (c - X)\tau)dt + dD = (X + (c - X)\tau)dt + \left(c - (X + (c - X)\tau)\right)dt = cdt$$  \hspace{1cm} (1.7)

Therefore, instantaneous profit to debtholders on both intervals is $cdt$ in both E-type and D-type environments. This result should not come as a surprise: the way changes in debt were modeled in equation (4), debtholders should always get the required payment $cdt$ - either in a form of money (on the interval of $X \geq c$) or a combination of money and promises of greater future payments (on the interval of $X < c$). Hence, there is one single HJB equation for debtholders on both intervals $X \geq c$ and $X < c$ for both E- and D-type environments:

$$rD(X, c) = c + \mu XD'_x + \frac{\sigma^2 X^2}{2} D''_{xx}$$  \hspace{1cm} (1.8)

Boundary conditions will depend on the version one or two of the model.
Default

I assume that at default assets are liquidated at their price (defined in equation (2)), but a fraction $\alpha$ is lost during the liquidation process. All proceeds from assets liquidation go to debtholders, and equityholders receive nothing. Define $X_{\text{def}}(c)$ as the default boundary of a firm:

$$X_{\text{def}}(c) = \{ X : \text{firm in state } (X_t, c) \text{ defaults if and only if } X_t = X \} \quad (1.9)$$

Values of debt and equity of a firm at default then take the following form:

$$D(X_{\text{def}}(c), c) = X_{\text{def}}(c) \frac{(1 - \tau)(1 - \alpha)}{r - \mu} \quad (1.10)$$
$$E(X_{\text{def}}(c), c) = 0 \quad (1.11)$$

Equations (10) and (11) hold for both E- and D-type environments. However, conditions that determine default are going to be different between the two. In the E-type environment, timing of default is chosen by equityholders and is determined by the smooth-pasting condition for the equity value at default:

$$\frac{\partial E}{\partial X}(X_{\text{def}}(c), c) = 0 \quad (1.12)$$

Economic intuition for equation (12) is as follows. When firm’s performance is weak, the firm continuously raises equity to finance coupon payments, meaning that immediate dividends to equityholders are negative. Equityholders agree to receive negative dividends with the hope that financial performance of the firm improves in the future, and dividends become positive. The future discounted value of dividends (both positive and negative) is summarized by firm’s market capitalization $E(X, c)$, and equityholders want to choose the default boundary $X_{\text{def}}(c)$ as low as possible so that $E(X, c)$ always remains positive; therefore, equation (12)
In the D-type environment, equity issuances are not allowed, and, therefore, dividends to equityholders are always non-negative (positive on the interval $X > c$ and zero on the interval $X \leq c$). Hence, equityholders will never voluntarily choose to default the firm. However, even though equityholders do not want to default the firm, the firm may be in a situation, when internally generated cash is not sufficient to make required coupon payments, and further debt issuances are not possible because debtholders do not believe that the firm will manage to service its debt obligations. As equation (4) shows, the firm is able to issue at least some amount of new debt as long as $\frac{\partial D}{\partial c} > 0$. Hence, the following equation determines default in the D-type model:

$$\frac{\partial D}{\partial c}(X_{def}(c), c) = 0 \quad (1.13)$$

Figure 4 visualizes default in the D-type environment. The green curve on the graph shows how the value of firm’s debt changes when the firm offers a higher coupon payment to its debtholders; the blue curve shows the marginal interest rates, at which the next dollar of debt can be raised. Firm with no debt ($c = 0$) can issue the first dollar of debt at $r = 5\%$, which is used as the risk-free rate to solve the model. Marginal interest rates stay low and close to the risk-free rate for firms that have sufficiently low interest payments. However, as the coupon payment becomes very high, the green curve becomes flatter, which means that the firm has to promise to increase future interest payments by a lot to raise an additional dollar of debt. As coupon-to-cash flow ratio approaches its default value, the green curve becomes completely flat, which means that future promises of higher coupon payments do not increase debt value, or, equivalently, next unit of debt can only be issued at the infinite rate. At that point, the firm can not issue new debt, and can not pay interests on its debt out of the operating cash flow either, and so default happens.

Importantly, as Figure 4 shows, the model does not produce unrealistically high marginal interest rates for new debt. While it is true that interest rates for new debt exponentially increase up to infinity with firm’s leverage, quantitatively the mt interest rates exceed 20% only when firm’s leverage is above 80%, and interest rates exceed 40% when firm’s leverage
Figure 4: Value of debt and marginal interest rates for a firm with current productivity $X = 1$ and varying coupon payments.

The graph shows the value of firm’s debt (green curve, right axis), and marginal interest rates at which next unit of debt can be raised (blue curve, left axis). When coupon-to-cash flow ratio attains the value 2.27, the firm can’t issue new debt, and default happens. Model was solved for the benchmark set of parameters shown in Table 9 is above 96%. These numbers are empirically-plausible.

Note one very important observation: at the time of default in the D-type environment, equation (13) holds. However, the term $(\frac{\partial D}{\partial c})^{-1}$ is the term that multiplies $E'_c$ in equation (6), which is the HJB equation for the equity value in the D-type environment on the region $X < c$. Because LHS of equation (6) is finite, this implies that the term that multiplies $(\frac{\partial D}{\partial c})^{-1}$ must approach zero as $(X, c)$ approaches $(X_{def}(c), c)$, and so $E'_c(X_{def}(c), c) = 0$. Furthermore, note that

$$0 = \frac{\partial E}{\partial c}(X_{def}(c), c) = \frac{\partial E}{\partial X}(X_{def}(c), c) \frac{\partial c_{def}(X)}{\partial X}$$

(1.14)

where $c_{def}(X)$ is the inverse function of $X_{def}(c)$\textsuperscript{13}. Equation (14) implies that as long as $c_{def}(X)$ is not a constant, $E'_x(X_{def}(c), c) = 0$, which is exactly the same as equation (12). Hence, D-type model also features smooth-pasting condition for the value of equity, even

\textsuperscript{13}As will be clear from the closed-form solution derived below, $X_{def}(c)$ is indeed a function, and the inverse always exists
though in this model it is a consequence rather than the assumption.

1.2.2. Version one

In version one of the model, equityholders choose how much debt to issue at \( t_0 \); after \( t_0 \), firms in the E-type environment are not allowed to issue anymore debt, and firms in the D-type environment can only issue debt when they are in distress (\( X < c \)). After debt is issued at \( t_0 \), proceeds are distributed to equityholders in a form of immediate dividends. Equityholders choose the initial coupon payment to maximize the value of proceeds from debt issuance plus the value of equity after debt is issued.

D-type

Mathematically, firms in the D-type environment solve the following problem at \( t_0 \):

\[
\max_{c_0} \left( D(X_0, c_0) + E(X_0, c_0) \right) \tag{1.15}
\]

where \( D(X, c) \) is the function that satisfies equation (8) on the interval \( \{X, c : X \geq X_{\text{def}}(c)\} \), and has specific boundary conditions discussed below; \( E(X, c) \) is the function that satisfies equation (3) on the interval \( \{X, c : X \geq c\} \) and satisfies equation (6) on the interval \( \{X, c : c > X \geq X_{\text{def}}(c)\} \), is continuous and smooth (derivative is continuous) along the line \( X = c \), and has specific boundary conditions discussed below. Because \( D(X, c) \) and \( E(X, c) \) satisfy second-order PDEs, two conditions for each should be imposed to have the unique solution.

Note that equation (8) has a closed form solution of the following form:

\[
D(X, c) = \frac{c}{r} + B X^{\beta} c^{1-\beta} + B_2 X^{\beta_2} c^{1-\beta_2} \tag{1.16}
\]

where \( B \) and \( B_2 \) are constants to be determined, and \( \beta \) and \( \beta_2 \) are respectively the negative and the positive roots of the quadratic growth equation (17)

\[
\frac{\sigma^2}{2} \beta^2 + \left( \mu - \frac{\sigma^2}{2} \right) \beta - r = 0 \tag{1.17}
\]
The first term in equation (16) is the value of the risk-free bond with constant coupon payment \( c \). The second term converges to \(-\infty\) as \( X \) converges to 0. Economically, this term catches the effect of an increasing probability of default (and associated default losses), when firm’s performance deteriorates. The value of debt at default is known and is given by equation (10), which is one of the two boundary conditions for the value of debt. The third term in equation (16) converges to \( \infty \) as \( X \) converges to \( \infty \). Normally, value of a risky debt can not exceed the value of a riskless debt, and so \( B_2 = 0 \). It then follows from equations (10) and (13) that solutions for \( B \) and \( X_{def}(c) \) take the following form:

\[
X_{def}(c) = -c \beta \frac{r - \mu}{1 - \beta} \frac{1}{r (1 - \tau)(1 - \alpha)} \tag{1.18}
\]

\[
B = -\frac{1}{r(1 - \beta)} \left( \frac{c}{X_{def}(c)} \right)^\beta \tag{1.19}
\]

Equations (18) and (19) have two important implications. First, \( \frac{c}{X_{def}(c)} \) is a constant, and so \( B \) is also a constant, which verifies the conjecture for the debt value. Second, \( B \neq 0 \), which means that default part of equation (16) is not zero. Therefore, debt is not risk-free in the D-type environment, even though debt holders can trigger the default of a firm quite early.

The HJB equation for the value of equity on the interval \( X > c \) (equation (3)), also has a closed-form solution:

\[
E^{X \geq c}(X, c) = \frac{X(1 - \tau)}{r - \mu} - \frac{c(1 - \tau)}{r} + AX^\beta c^{1-\beta} + A_2 X^\beta_2 c^{1-\beta_2} \tag{1.20}
\]

Note that the third term in equation (20) converges to \( \infty \) at a very high rate as \( X \) grows to infinity\(^{15}\). Assuming that there are no speculative bubbles on the market, \( A_2 = 0 \), which

\(^{14}\)Note that for sufficiently large bankruptcy costs \( \alpha \), RHS of equation (18) may become larger than \( c \), implying that default happens when \( X > c \) (when the firm is not in distress). Of course, the firm can’t be forced to liquidate its assets as long as it is able to make required coupon payments. Essentially, \( X_{def}(c) \) should be the minimum between \( c \) and RHS of (18), but for a reasonable set of parameters (i.e. \( \alpha < 70\% \)) (18) will be the solution. Note also that (18) is always positive because \( \beta \) is negative

\(^{15}\)\( \lim_{X \to \infty} \left( \frac{X^{\beta_2} c^{1-\beta_2}}{X} \right) = \infty \)
gives one of the boundary conditions for the value of equity.

Equation (6) does not have a closed-form solution, and should be solved numerically. Denote $E^{X<c}(X,c)$ the solution to equation (6). In order for it to be the solution for the value of equity on the interval $X < c$, it must satisfy the following conditions\footnote{Equation (21) can be written as the equality of partial derivatives with respect to $X$ instead - the idea is that the derivatives from the left and from the right with respect to each variable should be continuous along the line $X = c$}:\footnote{Equation (21) can be written as the equality of partial derivatives with respect to $X$ instead - the idea is that the derivatives from the left and from the right with respect to each variable should be continuous along the line $X = c$}

\begin{equation}
X \left( \frac{1 - \tau}{r - \mu} - \frac{1 - \tau}{r} + A \right) = E^{X<c}(X,X) \tag{1.21}
\end{equation}

\begin{equation}
\frac{1 - \tau}{r} + (1 - \beta)A = -\frac{\partial E^{X<c}}{\partial c}(X,X) \tag{1.22}
\end{equation}

\begin{equation}
E^{X<c}(X_{def}(c),c) = 0 \tag{1.23}
\end{equation}

where equations (21) and (22) are the value-matching and smooth-pasting conditions along the line $X = c$, and equation (23) is the value of equity at default, which is the second boundary condition for the value of equity.

**E-type**

Firms in the E-type environment solve the same problem as firms in the D-type environment at $t_0$:

\begin{equation}
\max_{c_0} \left( D(X_0,c_0) + E(X_0,c_0) \right) \tag{1.24}
\end{equation}

where $D(X,c)$ is the function that satisfies equation (8) on the interval $\{X,c : X \geq X_{def}(c)\}$, and has specific boundary conditions discussed below; $E(X,c)$ is the function that satisfies equation (3) on the interval $\{X,c : X \geq X_{def}(c)\}$, and also has specific boundary conditions discussed below. In addition to that, firms in the E-type environment choose the timing of default, which is defined by equation (12). Conditions at the boundaries for values of both debt and equity are the same as in the D-type environment.
First, note that both $D(X, c)$ and $E(X, c)$ have closed form solutions:

$$D(X, c) = \frac{c}{r} + BX^\beta c^{1-\beta} + B_2 X^\beta_2 c^{1-\beta_2}$$  \hspace{1cm} (1.25)$$

$$E(X, c) = \frac{X(1-\tau)}{r} - \frac{(1-\tau)c}{r} + AX^\beta c^{1-\beta} + A_2 X^\beta_2 c^{1-\beta_2}$$  \hspace{1cm} (1.26)$$

From the boundary conditions at $X \gg c$, it can be concluded that $B_2 = 0$ (because value of debt should converge to the value of risk-free debt), and $A_2 = 0$ (to exclude speculative bubbles as $X \to \infty$). The remaining boundary conditions are the values of debt and equity when the firm defaults, and are given by equations (10) and (11). These equations allow to explicitly solve for values of $A$ and $B$ (see Appendix B), and $X_{def}(c)$:

$$X_{def}(c) = -c \frac{\beta}{1-\beta} \frac{r - \mu}{r}$$  \hspace{1cm} (1.27)$$

Note how default rules are different for firms in the E-type and D-type environments (equations (27) and (18) respectively). Because the timing of default is affected by decisions of debtholders in the D-type environment, bankruptcy costs $\alpha$ appear explicitly in the equation, as opposed to the solution for the default rule for firms in the E-type environment, where bankruptcy costs are only implicitly internalized by equityholders through interest rates at which debt is issued at $t_0$. However, even though for each value of $c$ the default boundary $X_{def}(c)$ in the E-type environment is lower than the default boundary in the D-type environment (by the factor $(1-\alpha)(1-\tau)$), firms in the D-type environment do not necessarily default earlier than firms in the E-type environment, and the reverse is most likely true.\footnote{This depends on the values of the parameters, but this is usually the case as Table 5 in Section 3 shows}

It is explained by the fact that firms in the D-type environment choose a more conservative debt policy: they realize that marginal interest rates to issue new debt in distress grow quickly, and so ex-ante choose a much lower leverage. In contrast, firms in the E-type environment can choose a greater leverage without a fear of going bankrupt soon.

Figure 24 shows the value of debt and equity for a firm with current productivity $X = 1$
These two graphs show the value of equity and debt in D-type and E-type environments for a firm with current productivity $X = 1$ and different coupon payments $c$. At $t_0$, the firm that starts with $X_0 = 1$ chooses $c_0$ that maximizes the value of the blue curve, which is the sum of proceeds from debt issuance (green curve on the graphs) plus the value of equity after debt is issued (red curve on the graphs). Model was solved for the benchmark set of parameters shown in Table 9.

and different values of coupon payment in the D-type and E-type environments. For the chosen calibration of parameters, firms in the D-type environment default when coupon-to-cash flow ratio exceeds the value 2.27, at which point the value of equity (red curve) falls to zero, and no further debt issuances are possible (green curve is flat). For the same set of parameters, firms in the E-type environment avoid bankruptcy for longer because they can raise external finance at a much cheaper rate (the default coupon-to-cash flow ratio is 3.23). In both environments, firms that start with $X_0 = 1$ choose the initial coupon level $c_0$ to maximize the value of the blue curve, which is the sum of the value of equity after debt is issued plus proceeds from debt issuance; the vertical line denotes the optimal initial coupon value. Note that firms in the E-type environment choose to issue debt with coupon payment that significantly exceeds their cash flow.

1.2.3. Version two

Version two of the model is different from version one of the model by the assumption that firms with strong financial performance can issue new debt after $t_0$ to further exploit benefits of tax shield. These debt issuances are modeled differently from debt issuances by firms in distress in the D-type environment, and I discuss implications of the difference later.
in the text. Debt issuances by firms in distress in the D-type environment do not change and are still characterized by equation (5) (firms in the E-type environment do not issue debt in distress at all).

Debt issuances outside of distress are modeled the same way as in Goldstein, Ju, and Leland (2001). To issue new debt to better exploit benefits of tax shield, a firm has to redeem all its debt outstanding first, and then issue new debt with a greater coupon payment. These issuances are costly, and costs are proportional to the amount of new debt issued: for each $1 of new debt raised, the firm only gets $(1 - q)$. If firm’s current cash flow level is $X$, and it pays coupon $c$ on its current debt, and the firm decides to issue new debt with the coupon level $c_{\text{new}}$, proceeds from debt issuance equal to $\left( (1 - q)D(X, c_{\text{new}}) - D(X, c) \right)$; these proceeds are distributed to equityholders as dividends. Every time the firm issues new debt, it chooses $c_{\text{new}}$ to maximize the amount of proceeds from debt issuance plus the value of equity after debt is issued. Equation below connects the value of equity before and after debt is issued:

$$E(X, c) = \max_{c_{\text{new}}} \left( (1 - q)D(X, c_{\text{new}}) - D(X, c) + E(X, c_{\text{new}}) \right)$$

The LHS of equation (28) shows the value of equity right before debt is issued, and the RHS of equation (28) shows the value of equity right after debt is issued. LHS and RHS are equal because equity value function is continuous; economically, equityholders have rational expectations about when the firm issues debt, and the share price of the firm adjusts accordingly.

The scaling property of the model allows to solve for $c_{\text{new}}$ in equation (28) easily. To understand the intuition of the scaling property, consider two firms at $t_0$ with $X^1_0 = 1$ and $X^2_0 = 2$, that is, the second firm is two times larger than the first firm. Because the model features constant return to scale, the coupon payment that the second firm chooses optimally should be two times greater than the coupon payment that the first firm chooses. As the result, values of debt and equity of the second firm should be two times greater than values of debt and equity of the first firm. Effectively, in this set-up the second firm is a
greater replica of the first firm, and so all values are proportional. Now consider just one firm but at the time when it chooses to restructure its debt upward. At the short moment when it has repurchased its outstanding debt but before it issued new debt, it has zero debt outstanding, and it is similar to a larger replica of itself at $t_0$. Hence, the new coupon payment, and the values of debt and equity of this firm after debt is issued should increase by the factor $\frac{X_t}{X_0}$ relative to values of coupon payment, debt and equity at $t_0$. Let $X_{res}(c)$ be the optimally chosen restructuring boundary of a firm:

$$X_{res}(c) = \{ X > c : \text{firm in state } (X_t, c) \text{ issues new debt if and only if } X_t = X \}$$

(1.29)

The assumption that costs to issue new debt $q$ are greater than zero guarantees that firms do not adjust their capital structure continuously. Denote $c_0$ the coupon payment that a firm chooses at $t_0$, when its cash flow is $X_0$. Equation (28) can then be rewritten in the following form:

$$E(X_{res}(c), c) = (1 - q) \frac{X_{res}(c)}{X_0} D(X_0, c_0) - D(X_{res}(c), c) + \frac{X_{res}(c)}{X_0} E(X_0, c_0)$$

(1.30)

It only remains to show how firms choose the restructuring boundary $X_{res}(c)$. Using the constant return to scale argument as before, it can be shown that $X_{res}(c)$ is proportional to $c$; denote $A_{res} = \frac{X_{res} c}{c}$. I assume that firms choose $A_{res}$ to maximize the value of equity and proceeds from debt issuance at $t_0$.

**D-type**

Note that the assumption about costly debt issuance creates some internal inconsistency in the model. Effectively, the model assumes that whenever debt is issued by a firm in distress, debt issuance is costless; however, when new debt is issued by a strongly performing firm to further exploit benefits of tax-shield, debt issuance is costly. It should be noted that the assumption of costly debt issuance was introduced by Goldstein, Ju, and Leland (2001) and is shared by most papers in this class of literature. The assumption guarantees that firms
do not adjust their capital structure continuously. By maintaining this assumption, I can better compare results of E-type and D-type models, and also preserve the feature that debt value is not adjusted continuously (debt changes in distress are not done to better exploit benefits of tax-shield, but rather to avoid bankruptcy).

Firms in the D-type environment solve the following problem at $t_0$:

$$V(X_0) = \max_{A_{res}, c_0} \left( E(X_0, c_0; A_{res}) + (1 - q)D(X_0, c_0) \right)$$

(1.31)

where $D(X, c)$ is the function that satisfies equation (8) on the interval $\{ X, c : X \geq X_{def}(c) \}$; $E(X, c; A_{res})$ is the function that satisfies equation (3) on the interval $\{ X, c : X \geq c \}$ and satisfies equation (6) on the interval $\{ X, c : c > X \geq X_{def}(c) \}$, is continuous and smooth (derivative is continuous) along the line $X = c$. As before, two boundary conditions for both $D(X, c)$ and $E(X, c)$ need to be imposed to have the unique solution.

Boundary conditions for debt value do not change from version one of the model: value of debt at default is still characterized by equation (10), and another boundary condition is equation $B_2 = 0$, which implies that the value of debt can not exceed the value of risk-free debt at any time. Because neither the HJB equation for debt value, nor boundary conditions have changed, the solution to the debt value function is still the same. Moreover, because default rule in the D-type environment is fully characterized by the debt value function, $X_{def}(c)$ is also the same:

$$D(X, c) = \frac{c}{r} + B X^\beta c^{1-\beta}$$

(1.32)

$$X_{def}(c) = -c \frac{\beta}{1-\beta} \frac{r - \mu}{r} \frac{1}{(1-\tau)(1-\alpha)}$$

(1.33)

As for the equity value function $E(X, c)$, equations (21) and (22) should hold to guarantee that the function is continuous and smooth along the line $X = c$. Furthermore, the value of equity at default is zero, as equityholders receive nothing when the firm defaults. The remaining boundary condition is the value of equity when the firm restructures its debt upward, and is given by the recursive equation (30). There is no closed-form solution for this function, and I use numerical methods to compute its value.
Figure 6: Dynamic of a typical firm in the D-type environment in version two of the model

The firm takes no action as long as its cash flow (blue curve) stays above its coupon payment (green curve) and below the upward restructuring boundary $X_{res}$ (upper red curve). When cash flow level reaches the upward restructuring boundary, the firm issues more debt, and future coupon payments increase. When firm’s cash flow is lower than its coupon payment but above the default boundary $X_{def}$ (bottom red curve), the firm slowly raises new debt to cover the shortfall between the cash flow and required coupon payments. The firm defaults when cash flow level reaches the default boundary. Values on the graph are in logs.

Figure 6 visualizes the behavior of a typical firm in the D-type environment in version two of the model. On the graph, firm’s cash flow grows steadily in the beginning, and at time $t = 167$ the firm issues more debt to further exploit benefits of tax-shield. The firm enters distress for the first time at $t = 311$, and it starts issuing debt to cover the gap between the cash flow and required interest payments. As the result, the coupon payment keeps growing all the way before firm’s performance improves sufficiently at $t = 362$; because coupon payment grows, the default boundary and the upward restructuring boundary (which are linear in $c$) also grow. The firm enters distress for the second time at $t = 375$, and, with the exception of a short moment at $t = 394$, stays in distress until it eventually defaults. Note that the rate at which coupon payment grows increases as firm’s cash flow approaches the default boundary. This is because debt issuances are endogenously more expensive for firms that are close to default.
E-type

Formulation of the problem for the E-type environment is the same as in the D-type environment with two exceptions: 1) equity value function $E(X, c)$ satisfies equation (3) on both intervals $X \geq c$ and $X < c$, and 2) default rule is determined by equation (11).

Figure 7 shows the value of debt and equity in the version two of the model for a firm with current productivity $X = 1$ and different values of coupon payment.

Figure 7: Value of debt and equity in version two of the model for a firm with $X = 1$

These two graphs show the value of equity and debt in version two of the model in D-type and E-type environments for a firm with current productivity $X = 1$ and different coupon payments $c$. At $t_0$, the firm chooses $c_0$ that maximizes the value of the blue curve, which is the sum of proceeds from debt issuance (green curve on the graphs) plus the value of equity after debt is issued (red curve on the graphs). Every time firm’s coupon level is too low relative to its cash flow level, the firm issues more debt. The grey dotted lines on each graph denote the values of $c$ at which the firm issues more debt (the left dotted line) and the value of $c$ that it chooses every time it issues more debt (the right dotted line). Model was solved for the benchmark set of parameters shown in Table 9.

1.3. Model solution

1.3.1. Parameter values

Table 9 shows parameter values that are used to solve the model. These values are taken from Strebulaev and Whited (2012) who provide the review of literature on dynamic capital structure and simulate a number of models similar to the E-type model discussed in this paper. Most parameters are hard to estimate directly in the data; moreover, there is a substantial cross-sectional heterogeneity. Therefore, some assumptions should be made.
Table 1: Benchmark parameter values used to solve the model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>risk-free rate</td>
<td>$r$</td>
</tr>
<tr>
<td>growth rate of the cash flow process</td>
<td>$\mu$</td>
</tr>
<tr>
<td>volatility of the cash flow process</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>effective corporate tax rate</td>
<td>$\tau$</td>
</tr>
<tr>
<td>bankruptcy costs</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>risk premium</td>
<td>$r_p$</td>
</tr>
<tr>
<td>debt issuance costs</td>
<td>$q$</td>
</tr>
</tbody>
</table>

Specifically mentioned, estimates of bankruptcy costs $\alpha$, which in this paper is assumed to be 10%, vary from very low to very high. For example, Gruber and Warner (1977) finds that direct bankruptcy costs are about 1% of the assets value, and Andrade and Kaplan (1998) report the value of about 20%. Some authors used a structural estimation approach to infer the bankruptcy costs from firms’ observed decisions. In particular, Davydenko, Strebulaev and Zhao (2012) find that default costs are in the range of 10% and 30%, Hennessy and Whited (2007) report values between 8.4% and 15.1%, and Glover (2016) finds the value of about 45%. Glover’s estimates are well-above estimates of other authors, but as argued by Reindl, Stoughton, and Zechner (2017), it is because Glover assumes that all firms follow optimal leverage policy, while it is not necessarily the case in the data. The authors estimate a similar model without imposing optimal capital structure and using firms stock prices instead, and find substantially lower values of bankruptcy costs (20%).

Cash flow volatility parameter $\sigma$ also does not have a precise estimate in the literature. Faulkender and Petersen (2005) report that the average implied asset volatility of firms that have access to public debt is 19%, and Schaefer and Strebulaev (2008) find 23% (also among firms that issue bonds); Reindl, Stoughton, and Zechner (2017) take a structural estimation approach with Leland-type environment and find asset volatility between 25% and 42%.

The effective corporate tax-rate $\tau$ that this paper uses implicitly aggregates the effect of corporate and personal taxes on dividends and interest payments; the resulting value $\tau = 20\%$ is based on the estimates of Graham (2000). In a model, similar to mine, Chen (2010) considers different taxes explicitly, and the resulting effective corporate tax rate in his model
Table 2: Results of the model solution

<table>
<thead>
<tr>
<th></th>
<th>Benchmark</th>
<th>$\mu = 3.5%$</th>
<th>$\sigma = 15%$</th>
<th>$\sigma = 35%$</th>
<th>$\alpha = 50%$</th>
<th>$q = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>D-type, version 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Initial coupon</strong></td>
<td>0.77</td>
<td>1.32</td>
<td>0.80</td>
<td>0.80</td>
<td>0.19</td>
<td>-</td>
</tr>
<tr>
<td><strong>Default</strong></td>
<td>0.44</td>
<td>0.24</td>
<td>0.60</td>
<td>0.32</td>
<td>0.78</td>
<td>-</td>
</tr>
<tr>
<td><strong>Leverage, $t_0$</strong></td>
<td>46.44%</td>
<td>42.60%</td>
<td>53.05%</td>
<td>42.17%</td>
<td>13.40%</td>
<td>-</td>
</tr>
<tr>
<td><strong>E-type, version 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Initial coupon</strong></td>
<td>1.36</td>
<td>2.65</td>
<td>1.28</td>
<td>1.58</td>
<td>0.84</td>
<td>-</td>
</tr>
<tr>
<td><strong>Default</strong></td>
<td>0.31</td>
<td>0.17</td>
<td>0.43</td>
<td>0.23</td>
<td>0.31</td>
<td>-</td>
</tr>
<tr>
<td><strong>Leverage, $t_0$</strong></td>
<td>69.72%</td>
<td>70.86%</td>
<td>75.29%</td>
<td>66.95%</td>
<td>48.48%</td>
<td>-</td>
</tr>
<tr>
<td><strong>D-type, version 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Initial coupon</strong></td>
<td>0.54</td>
<td>0.80</td>
<td>0.65</td>
<td>0.50</td>
<td>0.14</td>
<td>0.43</td>
</tr>
<tr>
<td><strong>Upward restructuring</strong></td>
<td>3.56</td>
<td>2.44</td>
<td>2.52</td>
<td>4.38</td>
<td>13.56</td>
<td>-</td>
</tr>
<tr>
<td><strong>Default</strong></td>
<td>0.44</td>
<td>0.24</td>
<td>0.60</td>
<td>0.32</td>
<td>0.78</td>
<td>0.44</td>
</tr>
<tr>
<td><strong>Leverage, $t_0$</strong></td>
<td>33.56%</td>
<td>25.15%</td>
<td>42.44%</td>
<td>27.89%</td>
<td>10.03%</td>
<td>26.68%</td>
</tr>
<tr>
<td><strong>E-type, version 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Initial coupon</strong></td>
<td>1.03</td>
<td>1.94</td>
<td>1.09</td>
<td>1.07</td>
<td>0.65</td>
<td>0.81</td>
</tr>
<tr>
<td><strong>Upward restructuring</strong></td>
<td>1.86</td>
<td>0.99</td>
<td>1.50</td>
<td>2.00</td>
<td>2.97</td>
<td>-</td>
</tr>
<tr>
<td><strong>Default</strong></td>
<td>0.30</td>
<td>0.14</td>
<td>0.41</td>
<td>0.22</td>
<td>0.30</td>
<td>0.29</td>
</tr>
<tr>
<td><strong>Leverage, $t_0$</strong></td>
<td>53.99%</td>
<td>48.49%</td>
<td>63.39%</td>
<td>48.26%</td>
<td>37.86%</td>
<td>43.94%</td>
</tr>
</tbody>
</table>

This table shows optimal decisions that firms make in the model. Column 2 reports model solutions for the benchmark set of parameters (Table 9); columns 3-7 report solutions of the model, in which all but one parameters are as in the benchmark set. Initial coupon refers the value $c_0$ that firms with $X_0 = 1$ optimally choose at $t_0$. Default is the ratio $X_{\text{def}}(c)/c$, and shows the cash flow level $X$ at which firms with $c = 1$ default. Upward restructuring is the ratio $X_{\text{res}}(c)/c$, and shows the cash flow level $X$ at which firms with $c = 1$ optimally choose to issue more debt to further exploit benefits of tax-shield; these additional debt issuances are not allowed in version one of the model, and so upward restructuring is not reported for version one of the model. Leverage, $t_0$ is leverage that firms in the model have right after they issue debt at $t_0$.

is around 18%.

While the model is solved under the risk neutral probability measure $\mathbb{Q}$, actual shock realizations happen under the physical probability measure $\mathbb{P}$. Therefore, in the simulation procedures discussed below, risk-premium $r_p = 5\%$ is added to the risk-neutral growth rate $\mu$.

Table 10 presents results of the model for the benchmark set of parameters, and for some variation of the parameters. As expected, firms in the E-type environment start with a greater coupon payment, operate longer before defaulting for the same value of coupon payment, and restructure their debt upward earlier, also for the same value of coupon payment. As discussed in the previous Section, even though the default boundary is lower for firms in the E-type environment, firms in the D-type environment do not necessarily default earlier because they choose lower coupon payment at $t_0$. Note also that firms in the
E-type environment often choose initial coupon payment that is greater than the cash flow level. This partially illustrates the problem that arises when equity financing is always freely available: firms optimally prefer to be in distress every time they readjust their leverage.

As Table 10 suggests, firms always issue more debt at \( t_0 \), and restructure debt upward more often when the expected growth rate of the log cash flow, which is \( \mu - \frac{\sigma^2}{2} \), is higher. Firms also operate longer before bankruptcy when \( \mu \) is high: high expected growth rate increases the value of firm’s assets. Interestingly, even though high value of \( \sigma \) lowers the expected growth rate of log cash flows, firms with greater value of \( \sigma \) postpone the default decision. High value of \( \sigma \) increases firm’s profits in good states, and losses in bad states are bounded (value of equity is always non-negative). Therefore, even though high value of \( \sigma \) reduces the value of equity when the firm is far from distress, it increases the value of the firm for firms deep in distress, and so firms wait longer before defaulting. This logic is straight for firms in the E-type environment (because firms in the E-type environment choose the timing of default), and goes through formulas implicitly in the D-type environment.\(^{18}\)

Note that consistent with equation (27), default boundary in the E-type environment is independent of bankruptcy costs \( \alpha \). This is because equityholders do not consider interests of debtholders when they choose the timing of default. Bankruptcy costs are only implicitly internalized by equityholders in the E-type environment through interest rates at which they issue debt at \( t_0 \), and that is why initial coupon payments vary with \( \alpha \). In contrast, debtholders affect the timing of default in the D-type environment, and so the default boundary is greater when \( \alpha \) is high in the D-type environment.

An important observation follows from Table 10: firm’s leverage in the model is not a perfect indicator of its indebtedness. For example, once firm’s growth rate increases from \( \mu = 2\% \) to \( \mu = 3.5\% \), firms optimally choose to issue significantly more debt (as indicated by a much higher initial coupon), but leverage falls. This happens because an increase in \( \mu \) has three effects: 1) value of firm’s equity increases, 2) the firm wants to issue debt with a higher coupon payment to better exploit benefits of tax-shield, and 3) value of firm’s debt increase because coupon payments are higher, and probability of default is lower. Due to the

\(^{18}\)Specifically, it affects the value of \( \beta \) so that the default boundary becomes lower
second effect, firm’s chosen coupon payment unambiguously increases; however, the effect on leverage is not clear, as both value of debt and equity go up. Changes in other parameters, even if they are small, may have a similar effect. This means that a model may correctly explain empirically observed leverage values because it provides a good representation of the data, or because the parameter values that it uses are estimated with an error. Therefore, in assessing the quality of a model, it is important to examine how well the model matches both market-based as well as non-market based indicators.

1.3.2. Default probabilities

As Table 10 implies, among two firms that have the same coupon payments \( c \), the firm that operates in the D-type environment will default earlier than the firm that operates in the E-type environment. This happens for two reasons: 1) default threshold is higher, and 2) interest expenses grow exponentially for firms in distress in the model with the D-type environment. On the other hand, firms in the model with the D-type environment choose lower initial leverage. Therefore, it is not straightforward which model produces a higher default probability.

To answer this question, the paper uses pairwise simulations\(^\text{19}\). For each version of the model (version one and version two), I generate two firms that are exposed to the same realization of shocks. The first firm behaves as if it lives in the E-type environment (it can issue both debt and equity), and the second firm behaves as if it lives in the D-type environment (in distress it can only issue debt); both firms make their financing decisions optimally. Each simulation is performed on the monthly basis and continues unless both firms default. Note, however, that firms in version one of the model grow on average, but they issue debt only once, and so their leverage continuously attenuates if they experience a series of positive shocks. Therefore, many simulations of version one of the model should result in no default. For this reason, simulations are additionally terminated after 60000 periods in version one of the model if at least one firm has survived this long.

\(^\text{19}\)Not formally reported here, I also estimate the annual default rate in the steady-state cross-section of firms in both types of economies in version two of the model. The default rate is higher in the E-type economy than in the D-type economy (1.25% vs. 1.19%). However, the 90% confidence intervals overlap, which does not allow to formally conclude that E-type economy has a higher default rate. Pairwise comparisons avoid this problem. Section 3.3 describes details of the simulation of a steady-state cross-section.
Table 3: Default probabilities and firm lifetime in the model

<table>
<thead>
<tr>
<th></th>
<th>Version one</th>
<th>Version two</th>
</tr>
</thead>
<tbody>
<tr>
<td>firm in E-type economy defaults earlier</td>
<td>76.8%</td>
<td>68.0%</td>
</tr>
<tr>
<td>firm in D-type economy defaults earlier</td>
<td>14.3%</td>
<td>24.3%</td>
</tr>
<tr>
<td>firms in both economies default simultaneously</td>
<td>8.9%</td>
<td>7.7%</td>
</tr>
<tr>
<td>average lifetime of a firm in E-type economy (months)</td>
<td>275.4</td>
<td>967.1</td>
</tr>
<tr>
<td>median lifetime of a firm in E-type economy (months)</td>
<td>151</td>
<td>705</td>
</tr>
<tr>
<td>average lifetime of a firm in D-type economy (months)</td>
<td>284.7</td>
<td>1030.3</td>
</tr>
<tr>
<td>median lifetime of a firm in D-type economy (months)</td>
<td>163</td>
<td>760</td>
</tr>
</tbody>
</table>

Numbers are estimated using simulations. Simulations are run on a monthly basis independently for each version of the model. Each simulation has two firms with one behaving as if it operates in the E-type environment, and one as if it operates in the D-type environment; firms in E-type model can always issue equity, and firms in distress in D-type model have to issue debt to cover the gap between interest expenses and the cash flow. Both firms in each simulation are exposed to the same realization of shocks. Simulations are continued as long as at least one firm has not defaulted. Firms in version one of the model grow on average, and their leverage attenuates, and so some simulations should result in no default; for this reason, simulations for version one are terminated after 60000 periods if at least one firm has survived. Simulations are repeated 10000 times, and numbers in columns 2 and 3 are averaged among all simulations (version two) or simulations in which both firms defaulted before $t = 60000$ (version one). Rows 1 − 3 report the fraction of simulations in which one firm defaults before the other or both firms default in the same period. Rows 5 − 7 report the average and median number of periods that the firm in each environment operated before default.

Simulations are repeated 10000 times for each version of the model, and Table 3 reports the fraction of simulations, in which the firm in the E-type environment defaults before the firm in the D-type environment or vice versa. As follows from the table, there are paths of shock realizations such that each firm can outlive the other or that both firms default in the same period. Nevertheless, for the majority of cases the firm that operates in the D-type environment operates longer than the firm that operates in the E-type environment, and the difference in average lifetime is one year for version one of the model and five years for version two of the model. Even though it is harder to pay interest expenses for firms in distress in the model with the D-type environment, they are more conservative in their initial leverage policy, and so the resulting default rate is higher in the E-type economy.

1.3.3. Leverage distribution

I first collect data on firms’ profits, interest expenses, and leverage. The sample of firms comes from Compustat for years 1981-2017.\textsuperscript{20} Firms in the financial sector (6000s SICs) and the public sector (9000s SICs) are excluded from the analysis; observations with the

\textsuperscript{20}1981 is the first year that has data on S&P long-term credit ratings
book value of assets that is less than $1 million are also excluded.

As Faulkender and Petersen (2005) find, firm’s capital structure depends a lot on whether the firm has access to public bond markets. The assumption that firms can issue debt easily is crucial in this model, and for this reason, the paper only consider firms that have S&P long-term credit rating, which is used as a proxy for whether the firm can issue public debt. Data on the S&P long-term ratings is available on monthly basis, but financial data is annual. To match the datasets, it is assumed that a firm has S&P long-term rating in a given year if it has S&P long-term rating in at least one month of that year. Data on S&P long-term ratings is available between years 1981 and 2017, and there is, on average, 1500 observations in each year. However, years 1981-1984 have only five observations combined, and year 2017 has only 146 observations.

In what follows, leverage is measured as the ratio of firm’s total debt (sum of long-term and short-term debt) to the the book value of assets. Some observations have leverage value that exceeds one, and these observations are excluded from the analysis.

This paper also considers the distribution of inverse coverage ratios (ratio of firm’s interest expenses to cash flow). There are two main reasons why the inverse coverage ratio and not the coverage ratio is chosen as a target moment. First, some firms in the data have either no debt, or very small values of debt, and, therefore, interest expenses of these firms are small compared to their cash flow. These observations significantly affect the average value of coverage ratios in the data and make it sensitive to how they are treated. For instance, unwinsorized average coverage ratio among firms with positive interest expenses is 9.8; it is 7.1 if coverage ratios are additionally winsorized at 0.1-99.9 percentiles, 5.8 if winsorized at 1-99 percentiles, and 5.1 if winsorized at 3-97 percentiles. The inverse coverage ratio avoids the problem of division by zero because all firms in the data have non-zero cash flow, and is, therefore, less sensitive to how outliers are treated: unwinsorized average inverse coverage ratio in a cross-section is 0.21; it is 0.38 if observations are winsorized at 0.1-99.9 percentiles, 0.36 if winsorized at 1-99 percentiles, and 0.36 if winsorized at 3-97 percentiles. Second, the inverse coverage ratio is a more natural parameter for the model discussed in this paper (both in E-type and D-type environments). As argued in Section 3.2, the model features
scaling property, and so firm’s equity value can be rewritten\textsuperscript{21} as $E(X, c) = X e^{\left(\frac{c}{X}\right)}$, which is correctly specified for all values of $X$ and $c$.\textsuperscript{22} At the same time, writing $E(X, c) = c e^{\left(\frac{X}{c}\right)}$ would be inconsistent (and can not be easily extended) for firms with $c = 0$.\textsuperscript{23}

The paper measures inverse coverage ratio as the ratio of firm’s total interest expenses to the value of its EBIT; values are winsorized at the 0.1% and 99.9% levels. While inverse coverage ratio has advantages over the coverage ratio, it also has one drawback: observations with negative inverse coverage ratios are somewhat misleading; these observations drive the average value of the inverse coverage ratio down, thus, creating an impression that the average inverse coverage ratio is low. Low positive inverse coverage ratio usually implies that firms in the population earn significantly more profits than they spend to pay interest expenses, which is not the case for firms with negative EBIT. There are 8.6% of observations with negative inverse coverage ratios. To account for this problem, the paper separately computes the truncated inverse coverage ratio, and the censored inverse coverage ratio. To compute the censored inverse coverage ratio, values of inverse coverage ratio are reset to zero for observations that have negative inverse coverage ratio. To compute the truncated inverse coverage ratio, observations with negative inverse coverage ratio are excluded.

Table 4 presents the summary statistics for the whole period of data, and separately for the first and second halves. The average value of leverage is 36.6%, and it does not change much before and after 2000. The average values of truncated and censored inverse coverage ratios are similar (0.69 and 0.63), implying that its value is close to zero for most firms with negative inverse coverage ratios. Figure 15 additionally shows the whole distribution of leverage and inverse coverage ratios in the data.

The next step is to study how well the model addresses the moments of distribution of leverage and inverse coverage ratios. While static results reported in Table 10 indicate that the version two of the model with the D-type environment comes close to matching empirical values (optimal leverage is 33.56%, and inverse coverage ratio is 0.56), it is misleading to

\textsuperscript{21}Note that debt value can also be rewritten this way
\textsuperscript{22}Note that because $X_t$ follows Geometric Brownian motion, it is always positive
\textsuperscript{23}It is true that all firms in the model optimally have positive values of $c$; nevertheless, to solve the model, it is necessary to correctly specify the equity value function for all values of $c$
Table 4: Numbers in the data

<table>
<thead>
<tr>
<th></th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>all years</td>
</tr>
<tr>
<td>leverage</td>
<td>36.6%</td>
</tr>
<tr>
<td># of observations</td>
<td>46 648</td>
</tr>
<tr>
<td>inverse coverage ratio</td>
<td>0.38</td>
</tr>
<tr>
<td># of observations</td>
<td>45 723</td>
</tr>
<tr>
<td>inverse coverage ratio, truncated</td>
<td>0.69</td>
</tr>
<tr>
<td># of observations</td>
<td>41 778</td>
</tr>
<tr>
<td>inverse coverage ratio, censored</td>
<td>0.63</td>
</tr>
<tr>
<td># of observations</td>
<td>45 723</td>
</tr>
<tr>
<td>fraction of firms in distress, all firms</td>
<td>18.5%</td>
</tr>
<tr>
<td>among all firms</td>
<td>18.5%</td>
</tr>
<tr>
<td>among firms with inverse coverage ratio ≥ 0</td>
<td>10.8%</td>
</tr>
</tbody>
</table>

This table reports summary statistics on firms’ leverage and inverse coverage ratios; only firms that have access to the public debt markets are considered, and the access is proxied by whether a firm has S&P long-term credit rating in a particular year. Average values are reported (except for the number of observations). Leverage is the ratio of total debt (sum of short-term and long-term debt) to the book value of assets. Inverse coverage ratio is the ratio of firm’s interest payment to the value of its EBIT. Inverse coverage ratio is winsorized at 0.1% and 99.9% values. The inverse coverage ratio is not always positive; therefore, the average values of truncated and censored inverse coverage ratios are separately considered. The average truncated inverse coverage ratio ignores firm-year observations with negative inverse coverage ratios; to compute the average censored inverse coverage ratio, value of the inverse coverage ratio is reset to zero for firm-year observations with negative inverse coverage ratio. Fraction of firms in distress refers to the fraction of firms with inverse coverage ratio negative or greater than one among all firms (second-to-last row) or the fraction of firms with inverse coverage ratio greater than one among firms with positive inverse coverage ratio (last row).

study static results that firms optimally choose. As argued by Strebulaev (2007), average values in a cross-section may differ significantly from what firms optimally choose at \( t_0 \). Therefore, before claiming success of the model, it is necessary to generate a steady-state cross-section of firms and examine its average values. The cross-sectional values are only meaningful for version two of the model: in version one, firms are not allowed to increase their leverage after \( t_0 \); because on average firms grow, the economy has a single trivial steady-state, in which all firms have zero leverage. Therefore, version one of the model (with both types of environments) has a trivial steady-state distribution, in which all firms have zero leverage and zero inverse coverage ratios.

Simulation approach is used to generate a cross-section of firms in version two of the model. For each type of the environment (E- and D-), I generate an economy populated by \( N = 3000 \) firms that operate for \( T = 3600 \) months (300 years). Firms start at \( t_0 \) with \( X_0 = 1 \) and make financing decisions optimally; if a firm defaults, it is replaced by another firm with
Table 5: Empirical and simulated moments of cross-sectional distribution

<table>
<thead>
<tr>
<th></th>
<th>E-type, version 2</th>
<th>D-type, version 2</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>fraction of firms in distress</td>
<td>57.9%</td>
<td>13.9%</td>
<td>18.5%</td>
</tr>
<tr>
<td>inverse coverage ratio</td>
<td>1.20</td>
<td>0.66</td>
<td>0.69</td>
</tr>
<tr>
<td>(truncated in the data)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>average leverage</td>
<td>58.26%</td>
<td>38.44%</td>
<td>36.64%</td>
</tr>
<tr>
<td>median leverage</td>
<td>55.89%</td>
<td>35.11%</td>
<td>34.20%</td>
</tr>
<tr>
<td>fraction of firms with</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>leverage ≥ 30%</td>
<td>99.87%</td>
<td>67.27%</td>
<td>61.04%</td>
</tr>
<tr>
<td>leverage ≥ 40%</td>
<td>89.47%</td>
<td>35.70%</td>
<td>36.00%</td>
</tr>
<tr>
<td>leverage ≥ 50%</td>
<td>64.90%</td>
<td>19.30%</td>
<td>20.68%</td>
</tr>
<tr>
<td>leverage ≥ 60%</td>
<td>39.60%</td>
<td>10.90%</td>
<td>12.00%</td>
</tr>
<tr>
<td>leverage ≥ 70%</td>
<td>22.47%</td>
<td>6.70%</td>
<td>6.45%</td>
</tr>
<tr>
<td>leverage ≥ 80%</td>
<td>11.40%</td>
<td>2.90%</td>
<td>3.01%</td>
</tr>
<tr>
<td>leverage ≥ 90%</td>
<td>4.13%</td>
<td>1.10%</td>
<td>1.09%</td>
</tr>
</tbody>
</table>

This table reports the fraction of firms in distress (firms whose interest expenses exceed the cash flow), the average inverse coverage (ratio of interest expenses to cash flow), and moments of leverage distribution in a simulated cross-section of firms and in the data. Firms in E-type model can always issue equity, and firms in distress in D-type model have to issue debt to cover the gap between interest expenses and the cash flow. Data is taken from Compustat for the period 1981-2017, and only firms with access to public debt markets are considered, which is proxied by having S&P long-term credit rating. To compute values in the model, a balanced panel of $N = 3000$ firms was simulated over $T = 3600$ months, and numbers were averaged over the last period of the simulation.

As simulation results show, model with the D-type environment matches data moments well. Average value of leverage is only slightly higher than in the data (38.4% vs. 36.6%), but its 90% confidence interval spans between 37.1% and 39.7%, which covers 37.6% average leverage for the period before 2000 and 37.2% reported by Faulkender and Petersen (2005). More importantly, the model with the D-type environment correctly explains the fraction of firms in the right-tail of leverage distribution, independent of how the right tail is defined. In fact, 90% confidence intervals cover empirical counterparts for almost all values: 30%, 40%, 50%, 60%, 70%, 80%, and 90%). Results are reported in Table 5, and Figure 15 additionally shows the distribution of leverage and inverse coverage ratios.

\[ X = 1, \text{ thus, maintaining a balanced panel. At the end of the period } t = 3600, \text{ the following simulated moments are computed: the fraction of firms in distress (firms whose interest expenses } c \text{ are higher than the cash flow } X), \text{ average inverse coverage ratio among all firms (ratio of interest expenses to the cash flow } c/X), \text{ average and median leverage values, and the fraction of firms with high leverage (firms whose leverage exceeds certain values: 30\%, 40\%, 50\%, 60\%, 70\%, 80\%, and 90\%). Results are reported in Table 5, and Figure 15 additionally shows the distribution of leverage and inverse coverage ratios.} \]

\[ \text{As simulation results show, model with the D-type environment matches data moments well. Average value of leverage is only slightly higher than in the data (38.4\% vs. 36.6\%), but its 90\% confidence interval spans between 37.1\% and 39.7\%, which covers 37.6\% average leverage for the period before 2000 and 37.2\% reported by Faulkender and Petersen (2005). More importantly, the model with the D-type environment correctly explains the fraction of firms in the right-tail of leverage distribution, independent of how the right tail is defined. In fact, 90\% confidence intervals cover empirical counterparts for almost all values: 30\%, 40\%, 50\%, 60\%, 70\%, 80\%, and 90\%). Results are reported in Table 5, and Figure 15 additionally shows the distribution of leverage and inverse coverage ratios.} \]

\[ \text{As simulation results show, model with the D-type environment matches data moments well. Average value of leverage is only slightly higher than in the data (38.4\% vs. 36.6\%), but its 90\% confidence interval spans between 37.1\% and 39.7\%, which covers 37.6\% average leverage for the period before 2000 and 37.2\% reported by Faulkender and Petersen (2005). More importantly, the model with the D-type environment correctly explains the fraction of firms in the right-tail of leverage distribution, independent of how the right tail is defined. In fact, 90\% confidence intervals cover empirical counterparts for almost all values: 30\%, 40\%, 50\%, 60\%, 70\%, 80\%, and 90\%). Results are reported in Table 5, and Figure 15 additionally shows the distribution of leverage and inverse coverage ratios.} \]

\[ \text{As simulation results show, model with the D-type environment matches data moments well. Average value of leverage is only slightly higher than in the data (38.4\% vs. 36.6\%), but its 90\% confidence interval spans between 37.1\% and 39.7\%, which covers 37.6\% average leverage for the period before 2000 and 37.2\% reported by Faulkender and Petersen (2005). More importantly, the model with the D-type environment correctly explains the fraction of firms in the right-tail of leverage distribution, independent of how the right tail is defined. In fact, 90\% confidence intervals cover empirical counterparts for almost all values: 30\%, 40\%, 50\%, 60\%, 70\%, 80\%, and 90\%). Results are reported in Table 5, and Figure 15 additionally shows the distribution of leverage and inverse coverage ratios.} \]

\[ \text{As simulation results show, model with the D-type environment matches data moments well. Average value of leverage is only slightly higher than in the data (38.4\% vs. 36.6\%), but its 90\% confidence interval spans between 37.1\% and 39.7\%, which covers 37.6\% average leverage for the period before 2000 and 37.2\% reported by Faulkender and Petersen (2005). More importantly, the model with the D-type environment correctly explains the fraction of firms in the right-tail of leverage distribution, independent of how the right tail is defined. In fact, 90\% confidence intervals cover empirical counterparts for almost all values: 30\%, 40\%, 50\%, 60\%, 70\%, 80\%, and 90\%). Results are reported in Table 5, and Figure 15 additionally shows the distribution of leverage and inverse coverage ratios.} \]

\[ \text{As simulation results show, model with the D-type environment matches data moments well. Average value of leverage is only slightly higher than in the data (38.4\% vs. 36.6\%), but its 90\% confidence interval spans between 37.1\% and 39.7\%, which covers 37.6\% average leverage for the period before 2000 and 37.2\% reported by Faulkender and Petersen (2005). More importantly, the model with the D-type environment correctly explains the fraction of firms in the right-tail of leverage distribution, independent of how the right tail is defined. In fact, 90\% confidence intervals cover empirical counterparts for almost all values: 30\%, 40\%, 50\%, 60\%, 70\%, 80\%, and 90\%). Results are reported in Table 5, and Figure 15 additionally shows the distribution of leverage and inverse coverage ratios.} \]
The figure shows distributions of leverage and inverse coverage ratios in a simulated economy of version two of the model and in the data. Firms in E-type model can always issue equity, and firms in distress in D-type model have to issue debt to cover the gap between interest expenses and the cash flow. Data is taken from Compustat for the period 1981-2017, and only firms with access to public debt markets are considered, which is proxied by having S&P long-term credit rating. To produce a distribution in the model, a balanced panel of $N = 3000$ firms was simulated for $T = 3600$ months. The distribution is shown for the last period of the simulation. For the distribution of inverse coverage ratios in the data, only firm-year observations with positive inverse coverage ratio are considered.

threshold values of leverage considered\textsuperscript{26}. Furthermore, the fraction of firms in distress is 13.9%, and its 90% confidence intervals is between 10.8% and 16.8% - close to values in the data. The average inverse coverage ratio (0.66) falls in the range between the average

\textsuperscript{26}The only case that is not covered by 90% confidence interval is when the right tail of the distribution is defines as firms whose leverage exceeds 30%
truncated and censored inverse coverage ratio in the data, and its 90% confidence interval 0.63-0.70 covers both values. As Figure 15 shows, the model reproduces the overall shape of the distribution of inverse coverage ratios in the data, even though quantitatively, its kurtosis is greater (i.e. the distribution is narrower in the model).

On the other hand, model with the E-type environment does not explain the distribution of the right tail of firms: cross-sectional leverage is 58%, 57% of firms do not produce enough cash to service their interest expenses, and almost all firms (99.9%) have leverage value above 30%.

I further study to what extent the models can explain distribution of leverage among firms with different credit ratings in the data. Towards this end, firms in the model are matched with firms in the data according to credit ratings based on their default probabilities. Data on default probabilities for firms across credit ratings comes from Moody’s report ”Measuring Corporate Default Rates” (2006); 10-year default probabilities adjusted for issuer rating withdrawal are used. Average leverage values for firms with different credit rating are calculated using Compustat data. These numbers are shown in the first two rows of Table 6; for comparison, the table also shows leverage values across credit ratings reported by Schaefer and Strebulaev (2008) and Huang and Huang (2012). It should be noted that leverage values that I compute are smaller than those reported by both Schaefer and Strebulaev and Huang and Huang. The difference is likely caused by the fact that Huang and Huang’s sample and Schaefer and Strebulaev’s sample end before year 2004, while my sample goes up to 2017. As Table 4 suggests, leverage values are indeed lower for firm-year observations after 2003.

I then compute model-implied 10-year default probabilities for firms with different leverage values. Specifically, for each value of leverage $L$, I generate a firm that starts with this leverage at $t_0$ and operates optimally for $T = 120$ months or until it defaults. The procedure is repeated $N = 10000$ times, and default probability is measured as the fraction of simulations, in which the firm does not survive until the last period. Figure 16 shows results of the simulations. Because firms in version two of the model optimally issue more debt when their leverage falls to a very low level, there is a lower bound on the leverage
Figure 9: Leverage and 10-year default probability

This figure shows the implied 10-year default probability for firms with different values of leverage. Grey dotted lines on each graph show the 10-year default probability of bonds with different credit ratings in the data: (from bottom to top) Baa, Ba, B, Caa-C. Note that the default probability is not monotone in leverage for version two of the model: this is because firms with very low leverage issue more debt and increase their leverage, therefore, increasing their probability of default.

Note the difference in shapes of the default probability curves in D-type and E-type environments. Default probability as a function of firm’s leverage is concave around $L = 1$ in the D-type environment, and is convex in the E-type environment. This result comes from the difference in the assumption of how firms in E-type and D-type environments finance the shortfall between required interest payments and the cash flow in distress. A firm in the D-type environment has to issue more debt, and marginal interest rates are very high.
Table 6: Default rates, and leverage for firms with different credit ratings

<table>
<thead>
<tr>
<th>Credit rating</th>
<th>Data 10-year default probability</th>
<th>Baa</th>
<th>Ba</th>
<th>B</th>
<th>Caa-C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10-year default probability</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>leverage</td>
<td>4.89%</td>
<td>19.86%</td>
<td>46.12%</td>
<td>74.72%</td>
</tr>
<tr>
<td></td>
<td>leverage, reported by H&amp;H</td>
<td>31.4%</td>
<td>40.5%</td>
<td>55.5%</td>
<td>62.2%</td>
</tr>
<tr>
<td></td>
<td>leverage, reported by S&amp;S</td>
<td>43.3%</td>
<td>53.5%</td>
<td>65.7%</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>D-type, version 1</td>
<td>37%</td>
<td>50%</td>
<td>66%</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>E-type, version 1</td>
<td>36.1%</td>
<td>53.5%</td>
<td>70.6%</td>
<td>86.1%</td>
</tr>
<tr>
<td></td>
<td>D-type, version 2</td>
<td>55.0%</td>
<td>75.9%</td>
<td>91.4%</td>
<td>98.7%</td>
</tr>
<tr>
<td></td>
<td>E-type, version 2</td>
<td>33.8%</td>
<td>52.4%</td>
<td>69.8%</td>
<td>85.6%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>48.8%</td>
<td>74.8%</td>
<td>90.7%</td>
<td>98.5%</td>
</tr>
</tbody>
</table>

This table shows the average values of default probability and leverage for firms with different credit ratings in the data (rows 1-4) and in the model (rows 5-8). Leverage in the data is averaged for firm-year observations with specific S&P long-term credit rating for years 1981-2017. Leverage, reported by H&H is taken from the paper of Huang and Huang (2012), Table 1. Leverage, reported by S&S is taken from the paper of Schaefer and Strebulaev (2008), Table 7. Default probabilities in the data are adjusted for issuer rating withdrawal. To classify firms to specific credit ratings in the model, I match their implied 10-year default probability with the default rate for each credit rating in the data (row 1) using Figure 16.

For each credit rating category, I find a firm in the model that has the same 10-year default probability, and its leverage is reported in the corresponding row of Table 6. It follows from the table that version two of the model with the D-type environment matches data well, while the model with the E-type environment significantly overestimates average leverage values. This result is not surprising: keeping firm’s leverage constant, firms that...
can only issue debt in distress naturally have greater default probabilities over any finite time-horizon as compared to firms that can also issue equity. Therefore, to match any given default probability, a firm in the E-type environment should have a greater leverage value as opposed to a firm in the D-type environment.

1.3.4. Bankruptcy costs and equity issuance costs

The previous section shows that the model with the E-type environment fails to explain the right tail of the leverage distribution, and a reasonable question is if it is possible to modify the model to improve its empirical predictions. This section examines two potential approaches that may help reconcile the E-type model with the data.

The first approach is to increase initial costs of debt issuance by increasing firm’s bankruptcy costs. When bankruptcy costs are high, debtholders do not expect to recover a large fraction of their debt in default, and charge higher rates at the time when debt is issued. This should incentivize firms to issue less debt. Following this logic, I recompute the model for different values of bankruptcy costs and estimate moments from the previous Section: the average and median leverage, the average inverse coverage ratio, the fraction of firms in default, and the fraction of firms with leverage above 75%; results are reported in Table 7 Panel A. To examine the plausibility of different values of bankruptcy costs, Table 7 additionally shows debt recovery rates, which are computed as the ratio of debt value at default relative to the face value of debt.

As follows from the table, if bankruptcy costs are lower than 60%, the fraction of firms with leverage above 75% is four-to-two times greater than empirically observed (4.5%). Bankruptcy costs should go as high as $\alpha = 60\%$ to match this moment and the average leverage; however, even then the average inverse coverage ratio is 0.78 vs. 0.63-0.69 in the data. Moreover, for bankruptcy costs these large, debt recovery rates predicted by the model are too small. According to Moody’s report “Moody’s Ultimate Recovery Database” (2007), average debt recovery rates are 65% for senior secured bonds and 38% for senior unsecured bonds, and they are less than 20% in the model with bankruptcy costs $\alpha = 60\%$.

28 Other parameter values are the same as in the Table 9
Table 7: Model with greater bankruptcy costs/costs of equity issuance

<table>
<thead>
<tr>
<th>Panel A: bankruptcy costs</th>
<th>$\alpha = 5%$</th>
<th>$\alpha = 10%$</th>
<th>$\alpha = 15%$</th>
<th>$\alpha = 20%$</th>
<th>$\alpha = 30%$</th>
<th>$\alpha = 40%$</th>
<th>$\alpha = 60%$</th>
<th>$\alpha = 70%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>avg. leverage</td>
<td>60.3%</td>
<td>58.9%</td>
<td>55.5%</td>
<td>54.0%</td>
<td>50.9%</td>
<td>47.3%</td>
<td>40.6%</td>
<td>39.7%</td>
</tr>
<tr>
<td>med. leverage</td>
<td>57.9%</td>
<td>56.4%</td>
<td>52.7%</td>
<td>50.3%</td>
<td>48.2%</td>
<td>43.4%</td>
<td>37.4%</td>
<td>35.3%</td>
</tr>
<tr>
<td>avg. invc.</td>
<td>1.26</td>
<td>1.23</td>
<td>1.12</td>
<td>1.08</td>
<td>1.00</td>
<td>0.91</td>
<td>0.78</td>
<td>0.74</td>
</tr>
<tr>
<td>distress</td>
<td>65.0%</td>
<td>59.2%</td>
<td>48.9%</td>
<td>43.6%</td>
<td>37.2%</td>
<td>28.6%</td>
<td>22.3%</td>
<td>18.9%</td>
</tr>
<tr>
<td>leverage $\geq 75%$</td>
<td>18.5%</td>
<td>18.1%</td>
<td>14.1%</td>
<td>13.1%</td>
<td>9.1%</td>
<td>8.2%</td>
<td>4.6%</td>
<td>4.5%</td>
</tr>
<tr>
<td>recovery rate</td>
<td>37.5%</td>
<td>35.7%</td>
<td>33.9%</td>
<td>32.0%</td>
<td>28.2%</td>
<td>24.3%</td>
<td>16.3%</td>
<td>12.3%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: equity issuance costs</th>
<th>$\lambda = 5%$</th>
<th>$\lambda = 10%$</th>
<th>$\lambda = 15%$</th>
<th>$\lambda = 20%$</th>
<th>$\lambda = 40%$</th>
<th>$\lambda = 60%$</th>
<th>$\lambda = 80%$</th>
<th>$\lambda = 100%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>avg. leverage</td>
<td>55.7%</td>
<td>54.1%</td>
<td>53.0%</td>
<td>51.4%</td>
<td>46.7%</td>
<td>43.9%</td>
<td>41.8%</td>
<td>39.0%</td>
</tr>
<tr>
<td>med. leverage</td>
<td>52.0%</td>
<td>51.3%</td>
<td>50.0%</td>
<td>47.9%</td>
<td>42.6%</td>
<td>39.4%</td>
<td>36.6%</td>
<td>34.3%</td>
</tr>
<tr>
<td>avg. invc.</td>
<td>1.13</td>
<td>1.07</td>
<td>1.03</td>
<td>0.99</td>
<td>0.85</td>
<td>0.78</td>
<td>0.73</td>
<td>0.66</td>
</tr>
<tr>
<td>distress</td>
<td>47.2%</td>
<td>44.1%</td>
<td>40.8%</td>
<td>36.3%</td>
<td>25.6%</td>
<td>21.3%</td>
<td>18.2%</td>
<td>13.7%</td>
</tr>
<tr>
<td>leverage $\geq 75%$</td>
<td>15.1%</td>
<td>13.2%</td>
<td>11.7%</td>
<td>10.7%</td>
<td>7.4%</td>
<td>7.17%</td>
<td>6.9%</td>
<td>3.8%</td>
</tr>
</tbody>
</table>

The table compares results of the version two of the model with E-type environment for different values of bankruptcy costs $\alpha$ and equity issuance costs $\lambda$. When equity issuance costs are present, the firm has to raise $1+\lambda$ dollars of equity to get one dollar. All values are estimated using a simulated steady-state cross-section of firms. To simulate an economy, a balanced panel of $N = 1200$ firms is generated over $T = 3600$ months, and values are averaged for the final period of simulation. Avg. and med. leverage refers to the average and median value of leverage (36.6% and 34.2% in the data); avg. invc. is the average ratio of interest expenses to firm’s cash flow (0.63-0.69 in the data); distress is the fraction of firms whose interest expenses exceed the cash flow (16.5%-20.9% in the data); leverage $\geq 75\%$ is the fraction of firms whose leverage is above 75% (4.5% in the data); recovery rate is the value of firm’s debt at default divided by the face value of debt.

While not reported in Table 7, debt recovery rate is 52.3% in the model with the D-type environment and is independent of bankruptcy costs $\alpha$. The fact that debt recovery rates do not vary with bankruptcy costs level may sound counter-intuitive at first, but it happens because firm’s timing of default is endogenous in the model, and firms that have greater bankruptcy costs default earlier, when the asset value is higher. Mathematically, it follows from how the default rule is determined in the model (equations (18) and (19)). This does not mean, though, that the D-type model predicts that debt recovery rates should be constant in the data: recovery rates in the model depend on firm’s growth rate $\mu$ and volatility $\sigma$, which vary between firms.

The second approach to align the E-type model with the data is to add transaction costs of equity issuance to the model. The way the model is written, it assumes that firms that want to issue equity can do it at no cost. Empirical estimates, however, suggest that costs of equity issuance are positive and significant. For example, Clifford W. Smith Jr. (1977)
studies all equity issuances registered between 1971 and 1975, and he finds that costs vary between 2% and 15% of the proceeds amount; in a structural estimation paper, Hennessy and Whited (2007) find marginal costs of equity issuance between 5% and 10%, depending on firm’s size.

To add equity issuance costs into the model with the E-type environment, it is assumed that immediate dividends to equityholders are $(X - c)(1 - \tau)(1 + \lambda)$, when $X < c$ in equation (3). Economically, this means that firms whose cash flow is lower than required interest expenses have to raise more money than what is necessary to just pay debtholders because a fraction of proceeds is lost. I compute the model for different values of $\lambda$ between 5% and 100%, and results are shown in Table 7 Panel B.

As the table shows, equity issuance costs can potentially explain empirically observed moments of the right tail distribution of firms, but they should be very large: only when $\lambda = 100\%$ the model with the E-type environment matches every moment. However, this value of $\lambda$ implies that the firm only gets 50 cents for each value of equity raised. As discussed above, empirical estimates of equity issuance costs are smaller.

Results of this section suggest that plausible values of costs of debt or equity issuances can not address the right tail of the distribution of leverage or inverse coverage ratios, if these costs are uncorrelated with firm’s leverage. In choosing the optimal amount of debt, firms are less concerned about the absolute value of external financing costs, and are much more concerned about the relative value of external financing costs in distress, when firms need external money the most.

1.4. Conclusion

The main insight of this paper is that in order to explain why firms in the data choose seemingly low values of leverage, a model does not necessarily need to impose large costs of external financing, but these costs should be relatively higher when firms need money the most - in distress. Firms are much more conservative in their leverage policy ex-ante if they know that rates to refinance their debt will grow exponentially after several negative shocks. To have external financing costs grow endogenously with the leverage, this paper
assumes that firms in distress can only issue debt. However, any model in which the cost of equity grows with firm’s leverage will have similar results. Most importantly, the data supports this prediction - firms substitute equity financing with debt financing when their leverage is higher, implying that costs to issue equity grow faster than costs to issue debt.

The Leland-type framework is used extensively in the capital structure literature, but the assumption that firms have constant costs to issue equity is both inconsistent with the data and leads to unrealistic predictions about firm’s financing behavior. By deriving the new optimal default boundary, which establishes that firms in distress issue debt, this paper shows how both concerns can be addressed while maintaining the tractable framework of the original paper. A model with similar features can be applied more broadly to study related issues in corporate finance and asset pricing.
CHAPTER 2 : Omitted Variable in Capital Structure Regressions

2.1. Introduction

This paper shows how ex-ante differences in the quality of investment opportunities of firms can 1) translate into ex-post negative relationship between firm’s leverage and profitability, and 2) explain why cash flows have positive explanatory power in a regression with investments amount even after the market-to-book ratio is controlled for. A simple observation is at the core of this paper. If a firm has to finance some of its future investments with debt, then by issuing debt to pay dividends the firm risks not being able to raise debt at acceptable rates in the future, and will have to forgo many otherwise profitable investments. Moreover, incentives to delay debt issuance are greater for firms whose investment opportunities will be the most profitable. In extreme cases, a firm that expects its future investments to be the most profitable would not be issuing debt that is not dedicated to finance investments at all. The quality of firm’s investment opportunities is the omitted variable from the title of this paper.

To formalize the logic of the previous paragraph, this paper derives a model that captures the trade-off between issuing debt to exploit benefits of tax shield and delaying debt issuance to finance future investments. In the model, firm’s profits are taxed at the corporate level, but the firm can shield future profits from taxation by issuing debt. Occasionally, the firm finds investment opportunities, which require fixed initial investments, and allow the firm to get an extra flow of profits going forward. The NPV - or the profitability - of investment opportunities differ between firms, and firms finance these investments with debt. A firm can always issue debt and pay proceeds as dividends, but doing so increases its future funding costs and may prevent financing of otherwise profitable investment opportunities. It is important to note that the model itself contributes to the literature by simultaneously characterizing firm’s investments, debt issuances, and default decision in a very tractable manner, with many results available in a closed-form.

Think about the negative correlation between leverage and profitability. In the model, firms that eventually become the most profitable are those that ex-ante expect to find the best
This figure shows results of a model simulation. Firms in the model are different by the quality of the investment opportunities they expect, with some firms consistently getting better investment opportunities than others. As evident from the graph, firms with better investment opportunities on average end up with smaller leverage and higher profitability.

Ex-ante difference in the quality of firms’ investment opportunities also explains why cash flow is correlated with the investment amount even after controlling for the market-to-book
Which firms have the greatest cash flows? Those that invested the most in the past, and whose investments were the most profitable. Therefore, the ex-ante expected quality of firm’s investment opportunities naturally creates a spurious correlation over time between the cash flow and the investment amount. This correlation does not disappear after controlling for firm’s market-to-book ratio, as the market-to-book ratio depends non-linearly on both the quality of firm’s investment opportunities, and on firm’s leverage (as leverage determines how many of those opportunities will actually be financed).

The model has a very good quantitative fit to the data. While firms in the model target fairly high leverage value, at any point in time there are firms that already invested a lot, and firms that are still waiting for the arrival of their investments. This way, the average value of market leverage produced by the model is very close to empirically observed (27% in the data vs. 25.2% in the model), and, thus, the model explains the underleverage puzzle. Moreover, the whole distribution of market leverage in the model closely resembles distribution in the data, and matches its median (22.2% in the data vs. 22.9% in the model), standard deviation (17.1% and 16.0%), and all quantiles between 5% and 95%. The average book leverage is also matched well, with its mean being 36.6% in the data and 37.3% in the model. Furthermore, the model produces a very good fit for the whole distribution of market-to-book ratios (as Figure 8 in the main part of the paper shows), with mean and median being 2.31 and 1.78 in the model and 2.42 and 1.85 in the data.

2.1.1. Literature review

The negative relationship between

One of the significant assumptions in the paper is that firms can only finance their investment opportunities with debt. While this assumption is certainly important, especially for the quantitative part of the paper, results qualitatively would not change even if firms in the model were allowed to issue equity. Nevertheless, there is some strong evidence suggesting that indeed firms issue equity infrequently, and that cost to issue equity also grow with leverage.

Empirical evidence on equity issuance by distressed firms is scant, but also mixed. Below
is a review of papers that consider this question. The overall conclusion that follows is that most firms issue equity when their leverage is low; there are instances of equity issuances by financially distressed firms, but the costs are high, and such firms use equity financing because they cannot raise debt.

The first group of papers argues that most firms issue equity when their performance is good. For instance, Senber and Senber (1995) report a complete absence of equity issuance by distressed firms. Fama and French (2005) show that equity issuances are frequent, but most firms issue equity when their leverage is low. Similarly, Mikkelson and Partch (1986) and Eckbo, Masulis, and Norli (2007) find that equity issuances for cash are rare - both in absolute level and relative to public debt issuances. Some other studies provide indirect evidence that firms in distress do not issue equity. Korajczyk, Lucas, and McDonal (1990) find that firm’s leverage does not increase significantly two years before an SEO; should firms issue equity to make required debt payments when internally generated cash flow is insufficient, one would observe an increase in leverage prior to an SEO. DeAngelo, DeAngelo and Stulz (2008) find that the average leverage of a firm before an SEO is only 27%. Denis and McKeon (2012) show that firms, whose leverage is above the target, tend to cover financial deficit by issuing new debt and increasing leverage further.

Other authors argue in contrary that a sizable number of distressed firms issue equity, but they sell new shares at a large discount, and do so because debt financing is unavailable. Park (2017) finds that public equity offerings decrease for firms in distress, but private placements increase. Walker and Wu (2017) find that a third of all SEOs are conducted by financially distressed firms. Both of these papers, however, use the distress measure from Campbell, Hilsher, and Szilagyi (2008), which is only partially related to firm’s leverage. Indeed, the average leverage in the subsample of distressed firms in Walker and Wu is 32%, which implies that these firms are in distress for reasons other than their indebtedness, and they likely have very limited access to debt financing. This conclusion is further reinforced by Lim and Schwert (2017) who study all private placement of equity (PIPEs) by U.S. firms. They find that most firms issuing PIPEs are small distressed firms without access to debt markets: the median leverage of firms issuing PIPEs is only 7.2%, and 93% of all firms do
not have credit rating. When such firms issue PIPEs, they offer shares to the market at a large discount.

Appendix D provides further empirical analysis of the correlation between the frequency of equity issuance and firm’s leverage based on Thomson Reuters data. Results show that the amount of equity issuance decreases with firm’s leverage, and the discount at which newly issued shares are offered to investors increases with leverage; this conclusion holds for all firms and also for the subsample of firms that have access to public debt markets. Similar conclusion follows from Figure 3 which is based on Compustat data. The question why costs to issue equity grow for firms in distress is beyond the scope of this paper, but Appendix B provides a simple two-period model that shows that presence of leverage amplifies information asymmetry. Such explanation is consistent with empirical findings of Hertzel and Smith (1993) and Lim and Schwer (2017) who argue that distressed firms are characterized by severe information asymmetry.

The model derived in this paper assumes that firms always have access to debt capital markets. Therefore, the assumption that such firms do not issue equity to pay required debt payments in distress is consistent with empirical evidence discussed above.

2.2. Model

2.2.1. The economy

This paper models an infinite-horizon economy in continuous time. Markets are complete, and there is a riskless asset that pays a constant rate of interest \( r \) per unit of time\(^3\). In what follows, \( \mathbb{P} \) denotes the physical probability measure, and \( \mathbb{Q} \) denotes the risk-neutral probability measure in this economy.

---

\(^1\)The paper does not combine the two databases because they use different definitions of equity issuance. In particular, Thomson Reuters mostly considers SEOs, while Compustat partially considers private placements as well. Assuming that Compustat data is internally consistent, Figure 3 shows the relative scale of debt and equity issuance as firms leverage increases, which would not necessarily be consistent if Compustat and Thomson Reuters data for equity issuance is pooled. On the other hand, Thomson Reuters has data for the discount/premium paid for newly issued shares, which is not available in Compustat.

\(^2\)The empirical sample of firms that the paper quantitatively explains also consists of firms with access to debt capital markets (firms with S&P long-term credit rating).

\(^3\)Here and below “per unit of time” means that investors earn approximately \( r\Delta t \) within a short interval of time \( \Delta t \).
A firm in the economy is characterized by the amount of capital $K$ it has, and capital productivity $X_t$. Firm’s production technology has a constant return to scale, and instantaneous profits equal $y_t = X_tK$ per unit of time. For simplicity, capital does not depreciate, and $X_t$ evolves over time as a Geometric Brownian motion:

$$\frac{dX_t}{X_t} = \mu dt + \sigma dW^Q_t$$

(2.1)

where $\mu$ is the risk-neutral drift, $\sigma$ is volatility of capital productivity, and $W^Q_t$ is a Brownian motion under $Q$.

Government taxes firm’s profits at a constant corporate rate $\tau$, and firm’s after-tax profits are $(1 - \tau)X_tK$ per unit of time. It is assumed that firms cannot save cash, and all profits have to be paid as dividends to equityholders immediately.

Capital is traded on the outside market, and its price depends on its productivity level: price of a unit is $(1 - \tau)XH$, where $H$ is a constant. To gain intuition, consider the value of a firm that operates one unit of capital with current productivity $X_t$ that takes no actions:

$$V = \mathbb{E} \left[ \int_t^\infty e^{-r(s-t)}(1 - \tau)X_s ds \right] = \frac{(1 - \tau)X_t}{r - \mu}$$

(2.2)

If firms in the economy indeed were not taking actions, and capital markets were competitive, equation (2.2) would give the exact price of capital (from no arbitrage condition). However, as discussed later in the text, firms can invest and issue debt to exploit benefits of tax-shield, and therefore price of capital may deviate from what equation (2.2) suggests. Nevertheless, equation (2.2) shows why price of capital should grow with its productivity level.\(^4\)

\(^4\)Notice that price of capital is also proportional to the level $(1 - \tau)$: one way to look at this is to assume that after firms purchase capital, it is immediately depreciated for accounting purposes. In other words, firms pay $HX$ for a unit of capital, but the government returns them $\tau HX$ back, so the effective price that firms pay is only $(1 - \tau)HX$. Return from the government may take a form of smaller other taxes that firms pay.
2.2.2. Investments

Firms buy new capital when they find investment opportunities. Investment opportunities arrive at a rate $\lambda$ per unit of time; informally, probability to find an investment opportunity within a short period of time $\Delta t$ equals $\lambda\Delta t$. An investment opportunity allows a firm to buy pre-specified amount of capital $K_{\text{new}}$ with low-productivity $X_{\text{low}}$ and install this capital within the firm. However, once the capital is installed, it productivity grows up to $X_{\text{high}} > X_{\text{low}}$. This is what makes investment opportunities profitable for firms: they pay for low-productive capital, but install it as high-productive capital. Importantly, investment opportunities do not change the amount or productivity of firm’s existing capital: by taking an investment opportunity, the firm gets a new capital stock with its own productivity. A simple way to think about investment opportunities is that it allows a firm to open a new plant, which works independently of firm’s other plants. This would imply, however, that a pair $(K, X)$ does not fully characterize a firm, as it shows firm’s total amount of capital and productivity, while the firm may have several capital units after a number of investments. The following assumption guarantees that capital and productivity can be aggregated:

**Assumption 1** Consider a firm at time $t$ that has $K$ units of capital with productivity $X_t$ that finds an investment opportunity, which allows the firm to buy $K_{\text{new}}$ units of capital with productivity $X_{\text{low}}$ and install them within the firm with productivity $X_{\text{high}}$. Then:

1. Size of the investment opportunity is proportional to the amount of capital the firm already has:

   $$K_{\text{new}} = \delta K$$

2. Productivity of capital that the firm buys is the same as the productivity of capital that the firm already has:

   $$X_{\text{low}} = X_t$$

3. Productivity of capital once it is installed is proportional to the productivity of capital the firm buys:

   $$X_{\text{high}} = (1 + \gamma)X_t$$
4. After new capital is installed, its productivity will evolve according to the same equation (2.1) as the productivity of firm’s old capital; in other words, there are no additional idiosyncratic shocks to the productivity of new capital.

Assumption (4) guarantees that firm’s different capital units can be aggregated into one: there is only one process \( X_t \) that characterizes productivity of all capital units, and production technology is constant return to scale. Appendix 1 formally proves that capital can be aggregated.

Note how firm’s profits change when the firm takes an investment opportunity:

\[
X_tK \rightarrow X_tK + \delta K(1 + \gamma)X_t = (1 + \delta(1 + \gamma))X_tK
\]

\[
y_t \rightarrow (1 + \delta(1 + \gamma))y_t
\]

(2.3)

Note also that the price that the firm pays to take an investment opportunity \((1 - \tau)Hx_t\delta K = (1 - \tau)\delta H y_t\) - is proportional to firm’s profits. Therefore, \(y_t\) alone is a sufficient state variable to describe the firm. Evolution of \(y_t\) over time then can be computed using Ito’s lemma:

\[
\frac{dy_t}{y_t} = \mu dt + \sigma dW_t + I\{\text{firm invests}\}(1 + \delta(1 + \gamma))dN_t
\]

(2.4)

where \(dN_t\) is a Poisson process with intensity \(\lambda\), and \(I\{\text{firm invests}\}\) is the indicator function that shows whether the firm invests when it gets an investment opportunity.

By construction, investment opportunities are profitable for firms: firms buy cheap low-productive capital, but install it as high-productive capital. Nevertheless, firms that have to finance investments by issuing debt may optimally choose not take investment opportunities. This is because firms that issued a lot of debt in the past may only be able to raise new debt at very high rates, which would not justify the investment. Before the discussion of this case, however, consider the benchmark example, in which equityholders have “deep pockets”, meaning that firms can always issue equity to pay for investments.

**Benchmark example with equity financing**

Let \(v(y)\) denote the equity value of a firm whose current profits equal \(y\), and guess that the
firm always invests when it finds an investment opportunity. \( v(y) \) should then satisfy the following HJB equation:

\[
rv(y) = (1 - \tau)y + \mu yv'(y) + \frac{\sigma^2 y^2}{2} v''(y) + \lambda \left( v((1 + \delta(1 + \gamma))y) - v(y) - (1 - \tau)\delta H y \right)
\]  

(2.5)

The last term of equation (2.5) is the probability that the firm finds an investment opportunity (\( \lambda \)) multiplying the change in the value of equity after the investment is taken. Notice that because in this example investments are financed by issuing equity, there is a negative outflow of \((1 - \tau)\delta H y\) every time the firm invests.

Guess that equation (2.5) has a linear solution \( v(y) = H_0 y \). Then:

\[
rH_0 y = (1 - \tau)y + \mu H_0 y + \lambda \left( H_0 (1 + \delta(1 + \gamma))y - H_0 y - (1 - \tau)\delta H y \right)
\]

(2.6)

\[
H_0 = \frac{(1 - \tau)(1 - \lambda \delta H)}{r - \mu - \lambda \delta (1 + \gamma)}
\]

(2.7)

Now consider the case when capital markets are competitive, and price of a unit of capital exactly equals the value of a firm that operates this unit of capital, implying that \( H_0 = H \). Substitute \( H_0 = H \) into equation (2.7), and solve for \( v(y) = H_0 y \):

\[
v(y) = \frac{(1 - \tau)y}{r - \mu - \lambda \delta \gamma}
\]

(2.8)

This very simple equation is an analogue of the Gordon growth formula, which shows that the value of a firm equals to the value of its immediate dividends \((1 - \tau)y\) divided by the difference between the discounting rate \( r \) and the expected growth rate \((\mu + \lambda \delta \gamma)\). The growth rate in this case is composed of the unconditional growth \( \mu \), and the growth coming from investments \( \lambda \delta \gamma \). While \( y \) increases by \( \delta(1 + \gamma) \) every time the firm invests, the firm pays for \( \delta \) part of it, and only \( \delta \gamma \) is the additional growth.
2.2.3. Debt and debtholders

It is assumed that debt is the only source of external financing available to firms. While this assumption may seem extreme, it has been shown empirically that debt financing dominates equity financing.

Firms in the model issue debt for three purposes:

1. Interest expenses on debt are tax-deductible, and by issuing debt firms can exploit benefits of tax-shield.

2. Firms use debt to buy capital to finance investment opportunities.

3. When firm’s cash flow is lower that required interest expenses on previously issued debt, it has to issue more debt to cover the shortfall.

Note that firms whose cash flow is not sufficient to pay interest expenses face immediate default, but, as the third bullet says, they may avoid it by issuing more debt. However, firms cannot increase the amount of outstanding debt infinitely as at some point debtholders would prefer to default the firm and take its assets rather than roll over its debt and hope that firm’s financial situation recovers\(^5\).

Debt takes the form of a perpetuity that pays a constant coupon payment \(c\) per unit of time as long as there is no default. Interest expenses are tax-deductible, and, therefore, firm’s instantaneous profits equal \((1 - \tau)(y - c)dt\). For simplicity, it is assumed that when \((y - c)\) is negative, the firm pays negative taxes, which means it receives money from the government.

Debt markets are competitive, and price of debt equals the present value of future payments that debtholders expect to receive from the firm. Let \(D(y,c)\) be the value of all firm’s outstanding debt. The fact that debt can be aggregated requires that either all of firm’s debt is held by one creditor, or that different creditors have equal seniority, which is assumed to be correct. When a firm issues more debt, it increases future coupon payments, and the

\(^5\)Default happens endogenously and is discussed later. However, Belyakov (2018) provides a more detailed analysis and its implications for observed patterns of capital structure
amount of proceeds that the firm receives equals \((D(y, c_{\text{new}}) - D(y, c))\) - change in the debt value. This is a departure from the traditional assumption that debt issuances incur non-trivial transaction costs that prevent firms from continuously adjusting their leverage (i.e. Goldstein, Ju, and Leland (2001), Strebulaev (2007), Chen (2010), Bhamra, Kuehn, and Strebulaev (2009, 2010)). Instead, the paper assumes that firms are allowed to issue new debt in any quantities and as often as they want. However, even without the assumption of costly debt issuances firms in the model do not issue debt continuously: most debt is issued to finance investment opportunities, which only arrive infrequently.

Consider a firm with current cash flow \(y\) and debt that is characterized by coupon payments \(c\) that finds an investment opportunity and decides to take it. To purchase necessary capital, the firm has to pay \((1 - \tau)H\delta y\), and this amount needs to be raised by issuing more debt. The new coupon payment \(c_{\text{new}}\) that the firm has to promise to debtholders is then implicitly defined through the following equation:

\[
D((1 + \delta(1 + \gamma))y, c_{\text{new}}) - D(y, c) = (1 - \tau)H\delta y
\]  

(2.9)

The LHS of equation (2.9) shows how the value of firm’s debt changes when it takes the investment opportunity: its \(y\) increases by the factor \((1 + \delta(1 + \gamma))\) as in equation (3.2), and \(c\) increases to \(c_{\text{new}}\). As debt markets are competitive, change in debt value is the amount of proceeds that the firm receives, and this amount should equal to the price that the firm pays to buy necessary capital; therefore, RHS of equation (2.9).

For now consider the case when firms do not issue debt to exploit benefits of tax-shield (this feature is added later). Then firms with \(y > c\) only issue debt to finance investment opportunities. The value of debt \(D(y, c)\) should satisfy the following HJB equation:

\[
rD(y, c) = c + \mu y D_y' + \frac{\sigma^2 y^2}{2} D_{yy}'' + \\
\lambda I\{\text{firm invests}\} \left(D((1 + \delta(1 + \gamma))y, c_{\text{new}}) - D(y, c) - (1 - \tau)H\delta y\right)
\]  

(2.10)

\(^{6}\)The assumption of small but non-trivial costs of debt issuance proved to be a powerful tool in explaining many stylized facts - from infrequent debt issuances, to underleverage puzzle and negative relationship between leverage and profitability (i.e. Goldstein, Ju, and Leland (2001) or Strebulaev (2007)). This paper assumes that debt can be issued at no costs, but it provides a different explanation for the above-mentioned phenomena.
where the last term indicates the probability that a firm finds an investment opportunity \((\lambda)\) multiplied by whether the firm takes the investment opportunity once it finds it \((I\{\text{firm invests}\})\), multiplied by how the value of debt changes - both \(y\) and \(c\) increase, but debtholders provide funds for the firm to buy capital. Notice, however, that equation (2.9) shows that this last term of equation (2.10) equals zero. It should not be surprising: because debt capital markets are competitive, debtholders provide financing at rates which make them indifferent between whether the firm takes the investment opportunity or not. Effectively, equityholders capture the whole surplus arising from profitable investments. Therefore, equation (2.10) can be simplified:

\[
rd(y, c) = c + \mu yD' + \frac{\sigma^2 y^2}{2}D''
\]  

(2.11)

The fact that debt value does not increase when firms take profitable investment opportunities may sound unrealistic at first, as it seemingly implies that firm’s profitability does not affect debt pricing. It is not the case: as clear from equation (2.9), higher value of \(\gamma\) unambiguously increases the value of debt, and therefore, \(c_{new}\) is lower to equalize the LHS and RHS. It means that firms with more profitable investment opportunities can issue debt at lower rates.

Notice, however, that equation (2.11) holds on the region \(y > c\), where firm’s cash flow is sufficient to pay required interest expenses. On the region \(y < c\) the firm does not have enough cash to pay required interest expenses, and so it needs to issue more debt to cover the gap to avoid default. Let \(dD\) denote the additional amount of debt that needs to be issued when \(y < c\), and \(dc\) the change in coupon payments. The following formula then links together \(dc\) and \(dD\):

\[
\left(c - (y + (c - y)\tau)\right)dt = (c - y)(1 - \tau)dt = dD = dc\frac{\partial D}{\partial c}
\]  

(2.12)

The very LHS of equation (2.12) is the difference between required coupon payments \(c\) and the amount of money the firm has on hands - its profits \(y\) and tax-return from the government \(\tau(c - y)\). This difference is the amount that the firm must but cannot pay
to debtholders; the firm, therefore, raises this amount by issuing new debt, and coupon payments increase accordingly\(^7\). From equation (2.12), the dynamic of \(dc\) on the region \(y < c\) is then:
\[
dc = \frac{(1 - \tau)(c - y)}{\partial D/\partial c} dt
\] (2.13)

HJB equation for the value of debt on the region \(y < c\) should account for the fact that both \(y\) and \(c\) change\(^8\):
\[
rD(y, c) = (y + \tau(c - y)) + \left(\frac{1 - \tau}{\partial D/\partial c}\right)D'_c + \frac{\sigma^2 y^2}{2} D''_y y
\] (2.14)

which simplifies to the following:
\[
rD(y, c) = c + \mu y D'_y + \frac{\sigma^2 y^2}{2} D''_y y
\] (2.15)

which is again the same as equation (2.11). Equation (2.15) implies that pricing of debt is not affected by the fact that the firm with \(y < c\) cannot pay its interest expenses and has to issue more debt. This happens because an increase in \(c\) means greater future payments to debtholders, but also that probability of default is higher; because debt markets are competitive, an increase in \(c\) adjusts so that the two effects compensate each other, and the value of debt stays unchanged.

\(^7\)Notice that new debt can only be issued for the case when \(\partial D/\partial c > 0\)

\(^8\)Of course equation (2.14) should also account for the fact that the firm may find and take an investment opportunity, but due to the same argument as in equation (2.10), value of debt does not change in this case.
Equations (2.11) and (2.15)\(^9\) have a closed form solution of the following form:

\[ D(y, c) = \frac{c}{r} + Bc^{1-\beta}y^{\beta} + B_2c^{1-\beta_2}y^{\beta_2} \tag{2.16} \]

where \(B\) and \(B_2\) are constants to be determined from boundary conditions, and \(\beta < 0\) and \(\beta_2 > 0\) are the roots of the quadratic growth equation:

\[ r = \left( \mu - \frac{\sigma^2}{2} \right) \beta + \frac{\sigma^2}{2} \beta^2 \tag{2.17} \]

Consider the economic interpretation of the terms of equation (2.16): the first term \(\frac{c}{r}\) denotes the value of the risk-free debt that always pays \(c\). As \(\beta\) is negative, the second term is large when \(y\) is low, but it converges to zero when \(y\) increases\(^{10}\). This term captures the effect of default (and associated losses): when firm’s cash flow is high, firm’s default probability is low, and so firm’s debt is almost risk-free; however, if firm’s cash flow is low, the default probability gets bigger, and market value of debt adjusts downward. The last term, in contrast, converges to infinity as \(y\) increases. Value of risky debt can never exceed the value of the riskless debt, which implies that \(B_2 = 0\).

To uniquely determine \(D(y, c)\), one more boundary condition needs to be imposed, which is the value of debt when the firm defaults. Absolute priority rules apply, and equityholders receive nothing; firm’s assets are transferred to debtholders, and they sell them at the market price. A fraction \(\alpha\) of assets, however, is lost in the process, and so debtholders only receive \((1 - \alpha)\) of assets value. Assume that a firm in default has \(K\) units of assets with productivity \(X\). The value of this firm’s debt then equals:

\[ D(y, c) = \frac{c}{r} + By^\beta c^{1-\beta} = (1 - \tau)(1 - \alpha)XHK = (1 - \tau)(1 - \alpha)Hy \tag{2.18} \]

It only remains to show when firms in the model default.

\(^9\)which are the same

\(^{10}\)more formally, as the ratio of \(\frac{y}{c}\) converges to infinity
This graph provides an illustration of how firm’s cash flow develops over time; notice that this graph assumes that the firm does not invest. The firm takes no action as long as its cash flow (blue curve) stays above its coupon payment (green curve). When firm’s cash flow is lower than its coupon payment but above the default boundary $y_{\text{def}}$ (bottom red curve), the firm slowly raises new debt to cover the shortfall between the cash flow and required coupon payments. The firm defaults when cash flow level reaches the default boundary. Values on the graph are in logs.

**Default**

Notice that dividends to equityholders are always non-negative: positive on the interval $y > c$ and zero on the interval $y \leq c$. Therefore, equityholders never wish to voluntarily default the firm. However, firms that have $y < c$ cannot meet interest payments and, therefore, continuously issue debt to avoid default. Debt issuances in distress cannot last infinitely, though. As equation (2.13) implies, more debt can be issued only if $\frac{\partial D}{\partial c} > 0$. Therefore, default is determined by the following condition:

$$\frac{\partial D}{\partial c} = 0$$ \hspace{1cm} (2.19)

Notice that because debt in the model is a function of two variables ($y$ and $c$), default boundary is a curve $y_{\text{def}}(c)$ rather than a single number. Together, equations (2.18) and (2.19) allow to solve for $B$ and firm’s default threshold:

$$y_{\text{def}}(c) = \frac{-\beta}{1 - \beta(1 - \tau)(1 - \alpha)r_H c}$$ \hspace{1cm} (2.20)
\[
B = -\frac{1}{r(1 - \beta)} \left( \frac{y_{def}(c)}{c} \right)^{-\beta}
\]  

(2.21)

Note that \( y_{def}(c) \) is linear in \( c \), and, therefore, the resulting \( B \) is indeed a constant. Moreover, the fact that \( \beta < 0 \) implies that \( B < 0 \), which means that value of debt gets smaller as \( y \) decreases and probability of default gets higher.

2.2.4. Equity and equityholders

The closed-form expression for the market value of debt allows to simplify equation (2.9), which shows how firm’s coupon payments increase when it takes an investment opportunity and issues debt to finance it:

\[
\frac{c_{new}}{r} + B((1 + \delta(1 + \gamma))y)^\beta c_{new}^{1-\beta} = \frac{c}{r} + By^\beta c^{1-\beta} + (1 - \tau)H\delta y
\]

(2.22)

Let \( v(y, c) \) denote the value of firm’s equity. If \( y > c \) the firm produces enough money to pay required interest expenses, and so it only issues debt infrequently to finance investment opportunities. Therefore, HJB equation for \( v(y, c) \) takes the following form on the interval \( y \geq c \):

\[
rv(y, c) = (y - c)(1 - \tau) + \mu y v'_y(y, c) + \frac{\sigma^2 y^2}{2} v''_{yy}(y, c) + \\
\lambda I\{\text{Firm invests}\} \left( v((1 + \delta(1 + \gamma))y, c_{new}) - v(y, c) \right)
\]

(2.23)

where the last term shows how the value of equity changes when the firm finds an investment opportunity, and \( c_{new} \) is implicitly defined in equation (2.22). On the interval \( y < c \) the firm does not produce enough cash to pay required interest expenses, and has to continuously issue debt to finance the shortfall, and so the HJB equation should account for that. Note that \( dc \) is defined in equation (2.13), and the firm does not pay dividends:

\[
rv(y, c) = \frac{(1 - \tau)(c - y)}{r + (1 - \beta)By^\beta c^{-\beta}} v'_c(y, c) + \mu y v'_y(y, c) + \frac{\sigma^2 y^2}{2} v''_{yy}(y, c) + \\
\lambda I\{\text{Firm invests}\} \left( v((1 + \delta(1 + \gamma))y, c_{new}) - v(y, c) \right)
\]

(2.24)
Generally speaking, equations (2.23) and (2.24) describe a second order PDE of a function of two variables, and so may be hard to solve, even numerically. However, the model satisfies the scaling feature, meaning that function \( v(y, c) \) should be homogeneous of degree one in \( y \) and \( c \). Intuitively, a firm with \((2y, 2c)\) is simply a greater replica of a firm with \((y, c)\), and, therefore, its equity value should be twice as high. Appendix 2 formally proves the homogeneity property.

Define a new variable \( z = \frac{c}{y} \) - firm’s inverse coverage ratio\(^{11}\). First note how firm’s \( z \) changes when it takes an investment opportunity. For this, divide both sides of equation (2.22) by \( y \):

\[
(1 + \delta(1 + \gamma)) \left( \frac{z_{new}}{r} + Bz_{new}^{1-\beta} \right) = z + Bz^{1-\beta} + (1 - \tau)H\delta
\]

where \( z_{new} = \frac{c_{new}}{(1 + \delta(1 + \gamma))y} \) - firm’s new inverse coverage ratio after an investment is taken. Importantly, equation (2.25) shows that firm’s inverse coverage ratio \( z_{new} \) after an investment is taken is only a function of the prior inverse coverage ratio \( z \) and not \( y \) and \( c \) separately.

Because \( v(y, c) \) is homogeneous of degree one in \( y \) and \( c \), there is a function \( f(z) \) such that \( v(y, c) = yf\left(\frac{c}{y}\right) = yf(z) \). Note that

\[
v_y'(y, c) = \left( yf(z) \right)_y' = f(z) - zf'(z)
\]

\[
v_y''(y, c) = \left( f(z) - zf'(z) \right)_y' = -f'(z)\frac{z}{y} + f'(z)\frac{z}{y} + \frac{z^2}{y}f''(z) = \frac{z^2}{y}f''(z)
\]

Furthermore, note that equation (2.13) that determines \( dc \) on the interval \( y < c \) can be partially rewritten in terms of \( z \):

\[
dc = \frac{(c - y)(1 - \tau)}{\frac{1}{r} + (1 - \beta)By^\beta c^{-\beta}} dt = y\frac{(z - 1)(1 - \tau)}{\frac{1}{r} + (1 - \beta)Bz^{-\beta}} dt
\]

\(^{11}\)Notice that the inverse coverage ratio is a more natural parameter in the model than the coverage ratio \( \frac{c}{y} \). By construction \( y \) can never be zero, while there will be firms without any debt and, therefore, with \( c = 0 \), and so the coverage ratio for these firms would be undetermined. Furthermore, if \( \frac{c}{y} \) was used as the state variable for the model, equation (2.22) would imply that \( v(y, c) = cf\left(\frac{c}{y}\right) \), which would again be undetermined for firms that have no debt.
Conditions $y > c$ and $y \leq c$ can be rewritten as $z < 1$ and $z \geq 1$. Plug equations (2.26), (2.27), and (2.28) into equations (2.22) and (2.23) and divide both sides by $y$. Then:

if $z \leq 1$

$$(r - \mu)f(z) = (1-z)(1-\tau) - \mu zf'(z) + \frac{\sigma^2 z^2}{2}f''(z) + \lambda I\{\text{Firm invests}\} \left( (1 + \delta(1 + \gamma)) f(z_{\text{new}}) - f(z) \right)$$

(2.29)

if $z > 1$

$$(r - \mu)f(z) = \left( \frac{(z - 1)(1-\tau)}{r + (1-\beta)Bz^{-\beta}} - \mu z \right) f'(z) + \frac{\sigma^2 z^2}{2}f''(z) + \lambda I\{\text{Firm invests}\} \left( (1 + \delta(1 + \gamma)) f(z_{\text{new}}) - f(z) \right)$$

(2.30)

A firm will take arriving investment opportunity only if doing so increases its equity value. Therefore, the following condition determines when a firm with an investment opportunity is indifferent between investing and not:

$$v((1 + \delta(1 + \gamma))y, c_{\text{new}}) = v(y, c)$$

(2.31)

If LHS of (2.31) is greater than the RHS, firm’s value increases if invests, and so the firm with an investment opportunity certainly takes it. On the other hand, if LHS of (2.31) is lower than the RHS, firm’s value decreases if it invests, and so the firm will forgo the investment opportunity.

Note that equation (2.31) can be rewritten in terms of $z$:

$$(1 + \delta(1 + \gamma))f(z_{\text{new}}) = f(z_{\text{inv}})$$

(2.32)

where $z_{\text{inv}}$ denotes the level such that the firm takes arriving investment opportunities if and only if $z \leq z_{\text{inv}}$.

It only remains to specify boundary conditions. Equity value should be zero at default, as
NPV of a project in the model depends on two parameters: the quality of the project (firm’s $\gamma$) and firm’s leverage. Keeping firm’s leverage constant, a higher $\gamma$ implies a greater-NPV project, which on the Figure correspond to the blue curve being above the red curve. As leverage increases, the firm can only raise new debt at a high rate (because bankruptcy risks are high). Therefore, project NPV decreases with leverage, as Panel A shows. Panel B shows a different interpretation of the same message, specifically putting the rate to raise new debt on the horizontal axis. A firm will take a project as long as its NPV is positive, which implies that high $\gamma$ firms will be investing more than low $\gamma$ firms.

firm’s assets are liquidated, and all proceeds go to debtholders. Therefore,

$$f(z_{def}) = 0$$

(2.33)

where value of $z_{def}$ follows from equation (2.20)

$$z_{def} = \frac{1 - \beta}{2\beta}(1 - \tau)(1 - \alpha)rH$$

(2.34)

The second boundary condition is more complicated, and is discussed in the Appendix B

**Tax-shield**

The discussion so far has been focused on a firm that does not issue debt to exploit benefits of tax-shield. To add this important feature, consider a firm that is characterized by a pair of $(y, c)$ that decided to issue more debt and increase its level of $c$ to $c_{res}$. In doing so, the firm gets the amount of proceeds $D(y, c_{res}) - D(y, c)$, but its equity value changes to $v(y, c_{res})$. The firm, therefore, should choose $c_{res}$ to maximize the value of the sum of
Figure 14: Value of equity and debt, and the decision to issue debt to exploit benefits of tax-shield

These two graphs show the value of equity and debt for a firm with current productivity $X = 1$ and different interest expenses $c$. At any point in time, a firm is allowed to issue new debt and increase the level of interest expenses; a firm would only do that if it increases the value of the blue curve, which is the sum of the red and the green curves. Notice that a firm whose $\gamma$ is high optimally chooses not to increase its debt level. However, a firm with a low value of $\gamma$ does issue debt. This happens because of the trade-off between the time-value of money and inability to finance future investments with debt: a firm can issue debt immediately and pay dividends, or can delay the debt issuance decision until an investment opportunity arrives and finance it later.

proceeds and equity:

$$\max_{c_{res}} \left( v(y, c_{res}) + D(y, c_{res}) - D(y, c) \right)$$  \hfill (2.35)

Take first order conditions with respect to $c_{res}$ and divide both sides by $y$:

$$f'(z_{res}) + \frac{D'(y, c_{res})}{y} = 0$$  \hfill (2.36)

At last, expand $D(y, c_{res})$ using the expression (2.16):

$$f'(z_{res}) + \frac{1}{r} + (1 - \beta)Bz_{res}^{-\beta} = 0$$  \hfill (2.37)

Note that solution $z_{res}$ to equation (2.35) does not depend on firm’s current $z$, which means firm’s policy regarding debt issuances for tax-shield purposes is independent of its current state. Firms with $z < z_{res}$ will issue enough debt so that $z = z_{res}$, and firms with $z > z_{res}$ will be waiting before their $y$ increases sufficiently.

Two important points should be mentioned about the level $z_{res}$. First of all, it is possible that there is no value $z$ that satisfies equation (2.37): such firms never issue debt to exploit
benefits of tax-shield. To understand why this could happen, recall that firms also issue debt to finance investment opportunities. If firm’s investment opportunities are very profitable or if they arrive frequently, the firm may optimally choose not to issue any debt for tax-shield purposes, and instead wait until it finds an investment opportunity. Second, it may happen that $z_{inv} < z_{res}$. This would imply that firm’s investment opportunities are not profitable enough for the firm to wait until they arrive, and the firm would rather issue debt immediately and exploit tax-shield benefits right away instead of paying high taxes to the government in waiting for investment opportunities.

2.3. Model solution

2.3.1. Simulations

To analyze cross-sectional implications of the model, this paper uses simulations approach. A virtual economy that has $N = 40000$ firms is simulated over $T = 2400$ periods with every period being one month; the next subsection explains why these numbers are chosen. In every period of the simulation, each firm observes the realization of its idiosyncratic shock and whether it gets an investment opportunity. If the firm gets an investment opportunity, it optimally decides whether to take or forgo it. Firms are also free to issue more debt to better exploit benefits of tax-shield in any period. Firms whose cash flow is below the level of their interest expenses issue debt to cover the shortfall. A firm defaults if its interest expenses in a given period exceed its cash flow, and the firm fails to issue more debt to cover the shortfall.

Firms in the simulation are different by 1) the timing of the arrival of their investment opportunities, 2) the realization of their idiosyncratic shocks, and 3) the profitability of their investment opportunities $\gamma$. By assuming that firms in a cross-section differ by the profitability of their investment opportunities, I create an environment, in which firms are ex-post heterogeneous in their profitability, and this heterogeneity is achieved through the endogenous choice to invest. All other parameters are identical between firms.

At $t = 0$, firms start with one unit of capital $K = 1$ with productivity $X = 1$ and their own profitability of future investment opportunities $\gamma$. As the economy evolves, some firms
naturally default, and these firms are replaced by newly-born firms that inherit old firm’s productivity \( X \), the number of units of capital minus bankruptcy costs \((1 - \alpha)K\), and the profitability of investment opportunities \( \gamma \). There is a concern that this way of replacing defaulted firms with new firms may affect the relationship between leverage and other variables. For instance, suppose that the true relationship between leverage and profitability is positive, and firms with the greatest leverage are also the most profitable. Resetting firm’s leverage to zero upon default will then mechanically induce the negative correlation between profitability and leverage because now most profitable firms will have no debt.\(^{12}\) To alleviate the mechanical effects, the analysis excludes firms that were in a simulation for less than \( t = 60 \) periods (5 years). This approach does not seem restrictive as the target empirical distribution of firms consists of firms with S&P long-term credit ratings, which are usually old and mature.

The economy is simulated over \( T = 2400 \) periods with every period being one month. As it takes time for the economy to achieve its steady-state, I only conduct the analysis based on the last \( t = 48 \) periods (4 years) of observations.

2.3.2. Parameters choice

Table 9 shows benchmark values of parameters that are used to solve the model. Most of the values are taken from Strebulaev and Whited (2012) who provide a review of literature on dynamic capital structure and choose neutral parameter values to simulate a number of models similar to the one discussed in this paper. Importantly, the literature does not have a consensus on the value of most of these parameters or their distribution among firms. In addressing this issue, where applicable, I discuss estimates of different authors or provide support based on the data from Compustat.

Specifically mentioned, estimates of bankruptcy costs \( \alpha \), which in this paper is assumed to be 10\%, vary from very low to very high. For example, Gruber and Warner (1977) finds that direct bankruptcy costs are about 1\% of the assets value, and Andrade and Kaplan (1998)

\(^{12}\)Note that another standard approach when defaulted firms are replaced with firms that have \( K = 1 \) and \( X = 1 \) will create the opposite problem when positive correlation between leverage and profitability is mechanically induced. This would happen because cross-sectional \( X \) on average grows
Table 8: Benchmark parameter values used to solve the model

<table>
<thead>
<tr>
<th>Panel A: Economy-wide parameters</th>
<th>This paper</th>
<th>S&amp;W (2012)</th>
</tr>
</thead>
<tbody>
<tr>
<td>risk-free rate</td>
<td>r</td>
<td>5%</td>
</tr>
<tr>
<td>effective corporate tax rate</td>
<td>τ</td>
<td>20%</td>
</tr>
<tr>
<td>risk premium</td>
<td>rp</td>
<td>5%</td>
</tr>
<tr>
<td>price of capital*</td>
<td>H</td>
<td>30.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Firm-specific parameters</th>
<th>μ</th>
<th>1.5%</th>
<th>2%</th>
</tr>
</thead>
<tbody>
<tr>
<td>volatility of the cash flow process</td>
<td>σ</td>
<td>25%</td>
<td>25%</td>
</tr>
<tr>
<td>bankruptcy costs</td>
<td>α</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>frequency of investment opportunities arrival</td>
<td>λ</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>size of investment opportunities</td>
<td>δ</td>
<td>10%</td>
<td></td>
</tr>
<tr>
<td>profitability of investment opportunities*</td>
<td>γ</td>
<td>U[2% – 6%]</td>
<td></td>
</tr>
</tbody>
</table>

This table shows parameter values used to solve the model. S&W (2012) denotes numbers used in Strebulaev and Whited (2012), where authors review the literature on dynamic capital structure and simulate a number of models similar to the one used in this paper. *Price of capital is calibrated to match the equity value of a firm with no leverage and γ = 0 (so that market-to-book ratio for this firm equals one at time zero). **Firms in the simulation differ by the profitability of their investment opportunities γ, and γ is distributed uniformly between 2% and 6%.

report the value of about 20%. Some authors used a structural estimation approach to infer the bankruptcy costs from firms’ observed decisions. In particular, Davydenko, Strebulaev and Zhao (2012) find that default costs are in the range of 10% and 30%, Hennessy and Whited (2007) report values between 8.4% and 15.1%, and Glover (2016) finds the value of about 45%. Glover’s estimates are well-above estimates of other authors, but as argued by Reindl, Stoughton, and Zechn (2017), it is because Glover assumes that all firms follow optimal leverage policy, while it is not necessarily the case in the data. The authors estimate a similar model without imposing optimal capital structure and using firms stock prices instead, and find substantially lower values of bankruptcy costs (20%).

Cash flow volatility parameter σ also does not have a precise estimate in the literature. Faulkender and Petersen (2005) report that the average implied asset volatility of firms that have access to public debt is 19%, and Schaefer and Strebulaev (2008) find 23% (also among firms that issue bonds); Reindl, Stoughton, and Zechn (2017) take a structural estimation approach with Leland-type environment and find asset volatility between 25% and 42%.
This graph shows how the distribution of $\gamma$ translates into the distribution of market-to-book ratios for firms with no leverage.

The effective corporate tax-rate $\tau$ that this paper uses implicitly aggregates the effect of corporate and personal taxes on dividends and interest payments; the resulting value $\tau = 20\%$ is based on the estimates of Graham (2000). In a model, similar to mine, Chen (2010) considers different taxes explicitly, and the resulting effective corporate tax rate in his model is around 18%.

Cash flow growth rate $\mu$ is 2% in Strebulaev and Whited (2012), but they assume that the process for cash flow is purely exogenous. In this paper, firm’s cash flow becomes higher because there is the unconditional growth rate $\mu$, but also because firms invest. Therefore, the paper assumes a smaller unconditional growth rate value of 1.5%.

While the model is solved under the risk neutral probability measure $Q$, actual shock realizations happen under the physical probability measure $P$. Therefore, in the simulation procedures discussed below, risk-premium $rp = 5\%$ is added to the risk-neutral growth rate $\mu$.

There are three parameters unique to this model (at least for the class of Leland-type models): size of firm’s investment opportunities $\delta$, frequency of investments arrival $\lambda$, and profitability of these investment opportunities $\gamma$. This paper assumes that investment opportunities arrive on average once a year ($\lambda = 1$), and size is 10% of firm’s existing capital.
Results of the model are not very sensitive to variations in $\delta$.

Instead of assuming a single value of $\gamma$ for all firms, this paper assumes that there is heterogeneity in $\gamma$ among firms. Clearly, there is no way to measure $\gamma$ for firms in the data directly. This parameter in the model, however, shows how good firm’s investment opportunities are, and so should be directly linked to the distribution of market-to-book ratios. With this in mind, the paper assumes that $\gamma$ is uniformly distributed on the interval $[2\%, 6\%]$. This region was chosen to target the distribution of market-to-book ratios in the data for firms with zero leverage; firms with zero leverage are chosen because in the model such firms correspond to newly born firms. Figure 15 shows how this distribution of $\gamma$’s translates into the distribution of market-to-book ratios for firms with zero leverage. The mean and median values in this distribution is 2.59 and 2.01 vs. 2.89 and 2.50 in the data for firms with zero leverage. Even though distribution of $\gamma$’s is chosen to target the distribution of market-to-book ratios for firms with no leverage, the following subsection shows that the simulated distribution of market-to-book ratios in the full cross-section (i.e. for firms with all values of leverage) is also very similar to the data.

To have $\gamma$ uniformly distributed between 2% and 6% for simulation purposes, I first choose 1000 numbers equally spaced over the interval $[2\%, 6\%]$. For each value in the interval, I create 40 firms that have this value as their $\gamma$. This results in $N = 40000$ firms, and I simulate these firms over $T = 2400$ periods, with the length of each period being one month.

Price of capital is a free parameter in the model, and it is normalized so that market-to-book ratio equals one for a firm with no leverage whose $\gamma = 0$.

2.3.3. Leverage and profitability

Table 9 shows results of the leverage predicting regression in the model. Specifications I, II, III, and IV use a simple model when leverage is regressed on profitability ($\log(X)$), capital ($\log(K)$), market-to-book ratio ($v(y,c)/Ay$) in univariate regressions, and then in one multivariate regression. All three coefficients are significant, and their signs coincide with the signs found in similar cross-sectional regressions based on Compustat data.
Table 9: Leverage-predicting regressions in the model

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>profitability</strong></td>
<td>−0.003∗</td>
<td>−0.001∗</td>
<td>−0.204∗</td>
<td>−0.001∗</td>
<td>−0.160∗</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td><strong>capital</strong></td>
<td>−0.023∗</td>
<td>0.006∗</td>
<td>0.003∗</td>
<td>0.003∗</td>
<td>0.278∗</td>
<td>0.006∗</td>
<td></td>
</tr>
<tr>
<td><strong>market-to-book</strong></td>
<td>−0.068∗</td>
<td>−0.071∗</td>
<td>−0.272∗</td>
<td>−0.188∗</td>
<td>−0.525∗</td>
<td></td>
<td></td>
</tr>
<tr>
<td>γ FE</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Firm FE</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

Panel A: LHS is market leverage (value of debt divided by the sum of the values of debt and equity)

<table>
<thead>
<tr>
<th></th>
<th>VIII</th>
<th>IX</th>
<th>X</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>profitability</strong></td>
<td>−0.020∗</td>
<td>−0.001∗</td>
<td>−0.204∗</td>
</tr>
<tr>
<td><strong>capital</strong></td>
<td>0.015∗</td>
<td>0.005∗</td>
<td>0.375</td>
</tr>
<tr>
<td><strong>market-to-book</strong></td>
<td>−0.026∗</td>
<td>−0.231∗</td>
<td>−0.099∗</td>
</tr>
<tr>
<td>γ FE</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Firm FE</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Panel B: LHS is book leverage

<table>
<thead>
<tr>
<th></th>
<th>XI</th>
<th>XII</th>
<th>XIII</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>profitability</strong></td>
<td>−0.050∗</td>
<td>−0.002∗</td>
<td>−0.425∗</td>
</tr>
<tr>
<td><strong>capital</strong></td>
<td>0.380∗</td>
<td>0.009∗</td>
<td>0.761</td>
</tr>
<tr>
<td><strong>market-to-book</strong></td>
<td>−0.070∗</td>
<td>−0.641∗</td>
<td>−0.380∗</td>
</tr>
<tr>
<td>γ FE</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Firm FE</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Panel C: LHS is inverse coverage ratio

This table shows results of the following regression $LHS = \beta_0 + \beta_1 \log(X) + \beta_2 \log(K) + \beta_3 + mb + \epsilon$ based on the data from a simulated cross-section of firms; see the text for details of the simulation. Specifications V, VI, IX, X, XII, and XIII also control for fixed affects. $LHS$ is: Panel A: firm’s market leverage (the ratio of the market value of firm’s debt to the sum of market values of debt and equity); Panel B: firm’s book leverage (the ratio of the market value of firm’s debt to the value of its assets); Panel C: firm’s inverse coverage ratio (in the model, the ratio of firm’s interest expenses to its cash flow $y$). Specification VII is only based on a sub-sample of firms all of which have $\gamma = 4\%$ (median value in the distribution). ∗ denote variables significant at 5% level.

What is the mechanism behind the negative correlation between leverage and profitability?

This result is mainly driven by the effect of heterogeneity in γ’s among firms. Firms with ex-post high profitability are firms that have high γ - their investment opportunities are better, and so they end up with a higher value of X after every round of investment. Firms with higher γ, on average, also have a smaller leverage. This effect is mainly driven by two factors. First, a firm with a good investment opportunity (a high-γ firm), when it raises debt to pay for an investment opportunity, is able to obtain a smaller interest rate from debtholders. Therefore, every time a high-γ firm invests, its total debt value increases by a smaller value than for a comparable low-γ firm. The second effect is purely pricing: the way leverage is measured in the model, it has firm’s equity value in denominator. Naturally, equity value of firms with high γ is much greater than the equity value of low-γ firms, and so leverage is mechanically lower for high-γ firms than for low-γ firms with the same level of debt. There is also a third factor: firms with higher γ have more incentives to delay debt issuance until investment opportunities arrive, while firms with low γ may choose to issue debt to purely exploit benefits of tax-shield even when there are no investment
opportunities. This third factor, however, does not play a role in the current calibration of the model: all firms optimally choose to never issue debt if they do not have investment opportunities.

Between the other two factors (cheaper debt and higher pricing of equity for firms with high $\gamma$) - which one has a more significant effect on the relationship between leverage and profitability? It appears that both factors have the same effect. To show that the effect of cheaper debt plays an important role, Panels B and C of Table 9 replaces leverage on the LHS of the regression by two alternative measures of firm’s indebtedness that are not affected by the pricing of equity: book value of leverage and inverse coverage ratio. Book value of leverage puts the price of capital in denominator, and price of capital is the same for all companies independent of their $\gamma$. Inverse coverage is the ratio of firm’s interest expenses and cash flow, which are not affected by firm’s future investment opportunities. As evident from the Table 9, the negative relationship between leverage and profitability remains in these regression.

One can argue that since the negative correlation between leverage and profitability in the model is purely driven by firm’s $\gamma$, the relationship should revert back to positive once the regression is controlled for $\gamma$-fixed effects (or firm fixed-effects, since $\gamma$ is fixed for every firm). Such result would be problematic for the model: in the data, the relationship between leverage and profitability remains negative even if the regression controls for firm fixed-effects. Specifications V and VI in Table 9, however, show that the model addresses this concern and produces negative relationship between leverage and profitability even after accounting for fixed-effects. To understand why the coefficient for profitability remains negative in the regressions that control for fixed-effects, remember that fixed-effects only allow for different slopes in a regression, while the relationship between the variables in the model is highly-non linear. In particular, firms with different $\gamma$’s follow different investment rules, and this interaction affects regression coefficients.

At last, to prove that the difference in $\gamma$ is indeed the factor that drives the negative relationship between leverage and profitability in the model, I additionally run a regression on a sub-sample of firms, all of which have the same $\gamma$. More specifically, Table 9 specification
Table 10: Cross-sectional regressions in the model

Panel B: Investment-predicting regressions

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
</tr>
</thead>
<tbody>
<tr>
<td>cash flow</td>
<td>0.003*</td>
<td>0.000*</td>
<td>0.045*</td>
<td>0.002</td>
<td>0.871*</td>
<td>0.083*</td>
<td>1.690*</td>
<td>0.141</td>
</tr>
<tr>
<td>market-to-book</td>
<td>0.018*</td>
<td>0.016*</td>
<td>0.016*</td>
<td>0.018*</td>
<td>0.091*</td>
<td>0.581*</td>
<td>0.559*</td>
<td>0.446*</td>
</tr>
<tr>
<td>γ FE</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Firm FE</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

This table shows results of regressions based on the data from a simulated cross-section of firms; see the text for details of the simulation. LHS variable for specifications I - IV is a dummy variable which equals one for firm-year observations with non-zero investments, and zero otherwise. LHS variable for specifications V - VIII is a log of the investment amount for firm-year observations with positive investments, and zero otherwise; results are robust if a different negative number is used instead of zero for firm-year observations with no investments. Specification I is a probit regression, and all other specifications are linear regressions. Specifications IV and VII are based on the data from a sub-sample of firms for which γ = 4 (median value of γ in the population). * denote variables significant at 5% level.

VII shows results for a regression of leverage on the three factors only based on observations for firms, for which γ = 4% (the average value of γ in the cross-section). The coefficient on profitability flips its sign to positive. This shows that heterogeneity in γ's is indeed the driving factor for the negative correlation.

2.3.4. Investments and cash flow

The next stylized fact that the model attempts to explain is why cash flow has a significant effect in a regression of investments on cash flow and market-to-book ratio. First, the model manages to replicate this stylized fact. Table 10 shows results of a regression of firm’s investments on market-to-book ratio and cash flow, and the same regressions when the LHS is the probability that a firm invests in a given period. It appears that in both cases the cash flow has significant explanatory power.

To understand the mechanism of this result, recall that in the model investment opportunities arrive randomly; the decision to invest, however, is endogenous, and depends on two factors: 1) firm’s current leverage, and 2) the profitability of the investment (firm’s γ). While firm’s market-to-book ratio is well correlated with firm’s γ, these two are not the same things. Therefore, even when one controls for firms market-to-book ratio, the effect of the γ is not fully captured. On the other hand, firms with ex-post high cash flows are firms with high γ - their cash flow is higher precisely because all their projects were good. In this
This figure shows results of a model simulation. Firms in the model are different in the quality of investment opportunities they expect, with some firms consistently getting better investment opportunities than others. As evident from the graph, firms with better investment opportunities on average have higher cash flows, higher market-to-book ratios, and invest more.

sense, γ drives spurious correlation between firm’s investment amount and the cash flow, and this correlation makes cash flow significant in the investment regression, even after one controls for firm’s market-to-book ratio. Figure 16 illustrates this result by showing the distribution of investments, cash flows, and market-to-book ratios in two sub-samples of firms.

Since the explanation for the significance of the cash flow in the investment regression comes from heterogeneity in γ, and γ is fixed for each firm at the origin, one can claim that the cash flow significance should disappear once firm-fixed effects are controlled for. Nevertheless, the model shows that the cash flow remains significant even when fixed-effects are added to the regression. This is shown in specifications II, III, XI, and XII of the Table 9. The explanation why the significance does not disappear is similar to the explanation in the previous section: firm fixed-effects only allow for different average levels of investments for
Table 11: Moments of leverage distribution in the data and in the steady-state cross-section in the model

<table>
<thead>
<tr>
<th>Panel A: Moments of leverage distribution</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>26.3%</td>
<td>25.2%</td>
</tr>
<tr>
<td>median</td>
<td>22.9%</td>
<td>22.2%</td>
</tr>
<tr>
<td>standard deviation</td>
<td>17.1%</td>
<td>16.0%</td>
</tr>
<tr>
<td>quantiles</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td>5.9%</td>
<td>5.8%</td>
</tr>
<tr>
<td>10%</td>
<td>8.33%</td>
<td>8.2%</td>
</tr>
<tr>
<td>25%</td>
<td>13.9%</td>
<td>13.6%</td>
</tr>
<tr>
<td>75%</td>
<td>34.7%</td>
<td>33.3%</td>
</tr>
<tr>
<td>90%</td>
<td>48.3%</td>
<td>45.9%</td>
</tr>
<tr>
<td>95%</td>
<td>59.47%</td>
<td>55.3%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Moments of market-to-book distribution</th>
<th>Data, winsorized</th>
<th>Data, censored</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>2.54</td>
<td>2.25</td>
<td>2.31</td>
</tr>
<tr>
<td>median</td>
<td>1.85</td>
<td>1.85</td>
<td>1.78</td>
</tr>
<tr>
<td>min</td>
<td>-12.36</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>max</td>
<td>24.93</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

This table reports summary statistics on leverage market-to-book distributions in the model and in the data. For the data part, only firms that have access to the public debt markets are considered, and the access is proxied by whether a firm has S&P long-term credit rating in a particular year. See the text in Section 3.1 for the details of simulation.

firms with low $\gamma$. In the model, however, $\gamma$ is not the only factor that determines whether and how much a firm invests; the decision to invest depends both on firm’s $\gamma$ and on its leverage, and the relationship between the variables is highly non-linear. Therefore, both variables - market-to-book ratio and cash flow - are significant even in the regressions with fixed-effects.

However, cash flow is no longer a significant factor in the regression once the regression is restricted to a sub-sample of firms that all have the same value of $\gamma$. Specifications IV and VIII of the Table 10 proves this by running the regression on a sub-sample of firms with $\gamma = 4\%$ (the mean value of $\gamma$ in the cross-section). The cash flow significance disappears, which indeed confirms that $\gamma$ drives the relationship in the model.
This Figure shows the distribution of market-to-book ratios for firms in the data (white bars) and firms in a simulated cross-section (black bars). See the text Section 3.1 for details of the simulation. Distribution in the data is based on market-to-book ratios for firms that have access to S&P long-term credit ratings in years 1978-2017.

2.3.5. Quantitative results

To study quantitative implications of the model, I first collect the data for the empirical moments that are related to the distribution of leverage and market-to-book ratios. I focus on these moments because the papers gives predictions about these moments, and they are not directly assumed in the parametrization section. The sample of firms comes from Compustat for years 1981-2017.\footnote{1981 is the first year that has data on S&P long-term credit ratings} Firms in the financial sector (6000s SICs) and the public sector (9000s SICs) are excluded from the analysis; observations with the book value of assets that is less than $1 million are also excluded.

As Faulkender and Petersen (2005) find, firm’s capital structure depends a lot on whether the firm has access to public bond markets. The assumption that firms can issue debt easily is crucial in this model, and for this reason, the paper only consider firms that have S&P long-term credit rating, which is used as a proxy for whether the firm can issue public debt. Data on the S&P long-term ratings is available on monthly basis, but financial data is annual. To match the datasets, it is assumed that a firm has S&P long-term rating in a
given year if it has S&P long-term rating in at least one month of that year. Data on S&P long-term ratings is available between years 1981 and 2017, and there is, on average, 1500 observations in each year. However, years 1981-1984 have only five observations combined, and year 2017 has only 146 observations.

As the Table 11 shows, the model manages to fully reproduce leverage distribution in the data. The model matches its mean (26.3% in the data vs. 25.2% in the model), median (22.9% vs. 22.2%), standard deviation (17.1% vs. 16.0%) and every quantile between 5% and 95%. In addition to market leverage, the model also matches the mean and median values of book leverage (37.3% and 37.1% in the model and 36.6% and 34.2% in the data, though the rest of the distribution is not matched as well as the distribution of market leverage. This is in part because the definition of book leverage that this paper uses and the definition that is used in the data are different: in the data, book leverage is measured as book value of debt divided by book value of assets. In the model, I measure book value of debt as the market value of debt divided by the value of assets. The concept of the face value of debt is not very meaningful in this model, and therefore there is no way to measure the book value of leverage in the model the same way as it is measured in the data.

The model also reproduces the distribution of the market-to-book ratios in the data well, as illustrated by the Figure 17. The model can’t account for firms with extremely high values of market-to-book ratios (above 8) or negative values, but it matches the distribution in the middle. The mean and median values of the market-to-book ratio produced by the model (2.31 and 1.78) are very close to those in the data (2.42 and 1.85).
CHAPTER 3: Economics of Leveraged Buyouts: Theory and Evidence from the UK Private Equity Industry

3.1. Introduction

Private equity (PE) industry has grown significantly since early buyouts of the 80s, reaching $4trln in 2020. Nevertheless, economics of PE firms is still poorly understood. Due to the scarcity of available data, most academic knowledge is based on small and often biased samples. Similarly, the public perception is colored by salient but not necessarily representative cases. This paper aims to address the gap in knowledge by analyzing a representative sample of PE-backed companies and explaining their behavior.

The main message of this paper is that PE-firms help their portfolio companies grow by alleviating the debt overhang problem. The debt overhang problem is less severe for PE-backed companies because PE-owners can directly inject equity in the event of a financial distress. In contrast, existing evidence shows that public and other private companies have hard time accessing equity financing in distress.\footnote{See, for instance, Senber and Senber (1995); literature review section provides an extensive discussion about equity issuance by financially distressed companies. This paper does not take a stance on why equity issuance costs are disproportionally high for companies without PE-ownership in financial distress, but a possible explanation is that public companies in financial distress are particularly affected by the information asymmetry problem. Private companies with concentrated ownership do not suffer from information asymmetry necessarily, but their owners usually have limited capital. PE firms solve both problems.} Instead, they mostly rely on debt financing, which is particularly expensive for financially distressed companies, and often exacerbates their situation. Through a dynamic quantitative model, I show how that leads to greater investment and faster growth of PE-backed companies, and use it to measure the value creation by PE firms.

In order to empirically support the premise of this paper - that PE firms provide capital to portfolio companies in situations, in which other companies have hard time raising external financing - I analyze a novel hand-collected dataset with information about a representative sample of buyouts in the UK. I first show that there are frequent money flows between sponsors and their portfolio companies and, contrary to a common stereotype, these flows go both ways. As Figure 1 illustrates, in 34% of buyouts the target company receives more
Total money flow is the difference between the amount of money received by the company from the PE and the amount of money paid by the company to the PE, during the time when the PE controls the company excluding the year when the PE acquires the company, normalized by the buyout size. Negative money flow means the company pays more money to the PE than what it receives from the PE, and positive money flow means the company receives more money from the PE than what it pays to the PE. The figure does not show 29.1% of companies that have zero money flow - those companies that neither pay nor receive money from the PE (therefore, bars on the figure add up to only 0.709).

direct equity injections from the sponsor post-buyout than what it pays in dividends.\textsuperscript{2} The fraction of companies that pay more dividends than what they receive from the sponsors is almost the same, 38%.

I further show that cases of financial distress are among factors correlated with the timing of equity injections: on average, a company receives an equity injection from the sponsor that equals 4% of the original deal amount every time its cash flow is insufficient to pay debt interests. PE owners seem to be acting as deep-pocket investors, covering the gap between needed and available cash of their portfolio companies.\textsuperscript{3} As a result, investment of PE-backed companies - both CAPEX and acquisitions of other companies - is not sensitive to the level of internal cash flow. Similarly, there is no difference in investment between

\textsuperscript{2}My sample of buyouts excludes VC and PE-growth deals, both of which presumably have even more money injections from sponsors.

\textsuperscript{3}One might argue that dividends and equity injections do not show the full picture of how money flows, as PE firms allegedly charge high fees for consulting and management services. Such fees, however, are present in less than 50% of buyouts, and combined over the whole ownership period they make less than 1% of the original deal amount.
distressed and non-distressed companies if they have PE-ownership.

I develop a quantitative framework to show how availability of external financing relates to debt overhang, and measure the value created by PE firms through their ability to relieve it. In the model, companies use debt to pay for investments that are too big to finance internally. Investment change the expected growth of cash flows, but the actual level depends on exogenous shocks. An unlucky company that is hit with a series of negative shocks may fall in financial distress, when its cash flow is not sufficient to make required debt payments. Even though such company is not profitable, it might still have economic value because the cash flow may recover in the future. Therefore, it is optimal for the company to raise external financing to avoid bankruptcy. What type of external financing is available in distress is the only difference between companies with and without PE-ownership. Companies with PE-ownership can pay negative dividends as long as it is optimal for the sponsor, while companies without PE-ownership have to rely on debt financing.

The model shows that financial distress becomes a rare event that governs investment policies of non-distressed companies. To understand the intuition, consider what happens to a company without PE-ownership that falls in distress. In order to avoid a default, the company issues debt, which increases its future interest payments. Since today’s interests cannot be covered internally, the company will likely need to issue debt again to finance even greater interests next period, forcing itself deeper into distress. Moreover, the rates on additional debt grow exponentially with every round of debt issuance as debt is fairly priced. Effectively, once a company falls in financial distress, there is a high chance of a continuous debt spiral. Expecting this, companies maintain low leverage, and sometimes forgo good investment opportunities that require external financing.

At the same time, expectations about financial distress do not affect investment policies of companies with PE-ownership as significantly. PE-owners have the ability and incentives to inject equity into their companies that are in financial distress to prevent the debt spiral. Expecting this, non-distressed companies invest more and grow faster. Smaller default probabilities also mean that debt is safer, implying that rates to issue debt are lower for

---

4In this context, ”smaller default probability” implies conditioning on the debt level. That is, among two
PE-backed companies, further increasing their investment activity.

The calibrated model shows that companies without PE-ownership stop investing in projects that require external financing once their leverage exceeds 42%, and stops investing completely if their leverage exceeds 67.4%. In contrast, companies with PE-ownership take all investment opportunities - those that can and cannot be financed internally - as long as their leverage is below 78.5%. As a result, the average growth rate of cash flows of a group of companies without PE-ownership is 8.9%, and it is 9.6% for a group of companies with PE-ownership.

Empirical results strongly support the fact that companies with PE-ownership make large investment more often. The data shows that, in a sense, buyouts do not stop when a PE firm acquires a company. Instead, 59.6% of buyouts have a follow-on acquisition, meaning another company is acquired and merged with the first. Among companies that are originally bought for £100mln or more, the frequency of follow-on acquisitions is 76.7%. On average, a company with PE-ownership has one acquisition every 2.5 years. For comparison, the frequency of acquisitions by COMPSTAT companies in the UK and the US is once every five years.

I use the model to estimate what fraction of the abnormal PE return can be explained by the superior growth of their portfolio companies. Because PE-ownership increases company’s growth, the value of the company is higher when it is owned by a PE firm. Therefore, when a PE firm buys a company, there is a range of prices at which the buyout can happen. If the PE industry is small relative to the size of the public market, buyouts will happen at the lowest price that public investors accept. As a result, the PE firm pays a relatively small price for a company that produces high dividends after the buyout. Because realized dividends are disproportionately high relative to the purchase price, the average realized returns are higher than the discount rates. The difference between the average realized returns and the appropriate discount rates is the abnormal return of the private equity investments.

companies of equal sizes with equal amount of debt, the one that has PE-ownership has a smaller probability of default than the one without PE-ownership. The model predicts, however, that PE-backed companies will still default more often as they endogenously choose to have greater leverage.
I calibrate the model to produce the market excess return of 5.6%. The model then implies that the average excess return on a portfolio of PE-backed companies is 13.3%. Companies with PE-ownership are more levered, and so are more exposed to systematic risk, implying a beta of 1.2. Therefore, 6.6% of the 13.3% outperformance is compensation for the market risk. The remaining 6.7% represents the alpha that PE-firms generate, which can be further decomposed into 5.6% due to higher investments, and 1.1% due to higher benefits of tax-shield. Since this alpha is only available to specialized PE-firms, the high abnormal returns cannot be exploited by average investors.

The difference in capital structure between companies with and without PE-ownership is another results that follows from the model, and since leverage is not a moment that is used to calibrate the model, it can be used to independently validate whether the model is quantitatively plausible. For companies without PE-ownership, the average leverage in a modeled cross-section is 27%. This number is consistent with the average leverage of public companies, which is in the range between 20%-30% in the US and the UK.

The leverage of a PE-backed company in the model at the time of buyout is 49%, which is significantly larger than what companies without PE-ownership have. This number might seem small relative to the 60%-65% LBO leverage that is often quoted in the literature, but I show that prior empirical studies overestimate the average debt level in PE-buyouts due to two challenges that I address and correct for in this paper. In my sample of PE-backed companies, the average leverage at the time of buyout is only 41% (42% median). There are as many companies with leverage above 65% as companies with leverage below 5%. Large companies are more levered than small companies, but the average leverage is still only 52% among companies larger than £200mln.

The first challenge that I address in my empirical setting is data availability, which forces many studies to work with biased samples. I address it by analyzing a sample of PE-backed companies in the UK, where all private companies release their financial statement. To estimate the effect of data availability bias, I separately estimate the average leverage

---

5It is important to note that the calibration assumes that all parameters are the same for companies with and without PE-ownership, and any difference is coming from the difference in expectations about financial distress. Specifically, bankruptcy costs for both types of companies are 10% of the asset value.
The figure shows the structure of the buyout of Callcredit Information by Vitruvian Investment Partners in December 2009. The overall transaction size was £120mln, of which £23mln was a loan from a bank, and the rest was paid by the PE firm. Of the £97mln paid by the PE, only £50'000 were structured as equity, while the remaining part was structured as shareholder loans. Shareholder loans - money paid by the PE but recorded as debt on company’s balance sheet - are present in 92% of the deals in my sample, and on average they constitute 40% of the deal size.

for a subgroup of companies in my sample that also have information about their buyout structure in the LPC/Dealscan database, which is frequently used in empirical PE research. I show that the average leverage increases by 10% for this subset of companies. Perhaps surprisingly, size does not predict whether a certain deal is included into the database, and it is rather the amount of debt that is used to finance the buyout.

The second challenge that I address is that accounting rules that PE-backed companies follow make it difficult to interpret information about their cash flows and capital structure. Figure 19 illustrates this problem: PE-owners hold a big portion of the debt that is used to finance buyouts. The PE-held debt, which is called shareholder loans, is only nominally debt, but has all properties of equity, and as such, should be treated as equity. Accounting rules for the treatment of shareholder loans create a lot of paper transactions

---

6Shareholder loans do not have interest expenses, no payments on them are allowed before the actual debt is paid off, and they are written off in case company’s performance deteriorates. Most importantly, shareholder loans are only senior to company’s equity, which is also held by the PE-owner. Professional data vendors that focus on debt analysis, such as LPC/Dealscan, correctly treat them as equity. In contrast, data vendors that solely rely on companies’ reporting and do not do their own adjustments, such as AMADEUS, classify them debt.
that obscure the actual cash flow and financing data at the time of buyout and later in company’s life cycle. To get a complete picture about the financing, investment, and cash flows between companies and PE-owners, I manually analyze companies’ reports to separate paper transactions from actual transactions.\footnote{In addition to having a very high noise-to-information ratio, the mis-measurement of certain items on the balance sheet and cash flow statements of PE-backed companies is correlated with many different variables, such as what PE firm finances the deal, the complexity of the transaction, the size of the acquired company, etc.}

\textit{Related literature}

This paper is related to the line of literature that discusses the capital structure choices of companies, both with and without PE-ownership. The big-picture puzzle is the fact that companies with and without PE-ownership have very different leverage, which is hard to rationalize on the basis of traditional theories. On the side of companies without PE-ownership, Miller (1977) argues that the present value of expected default losses seems disproportionally small relative to the tax-shield benefits of debt, concluding that most companies are, on average, underlevered. Graham (2000) estimates that the value of an average company would go up by 5\% if it increased its leverage to the optimal level. This puzzle is known as the underleverage puzzle, and several papers discuss frictions that prevent companies from issuing additional debt (e.g. Faulkender and Peterson (2006)) or propose mechanisms that could rationalize the seemingly-suboptimal behavior (e.g. Chen (2012), Morellec, Nikolov, and Schuroff (2012), or Glover (2016) among others).

The flip side of the underleverage puzzle is the fact that companies with PE-ownership are more levered than most theoretical models predict, and more levered than companies without PE-ownership. For instance, Axelton, Jenkinson, Stromberg, and Weisbach (2013) study a sample of leveraged buyouts and conclude that “there appears to be no discernible relation between leverage in buyout firms and median leverage of public firms in the same industry-region-year, regardless of what leverage measure we use” (see also Gompers, Kaplan, and Mukharlyamov (2015)). Several channels have been discussed in the literature that could explain the high leverage of LBO-transactions. Ivashina and Kovner (2011) emphasize the role of the PE-banking relationship in determining the amount of debt used
in a buyout (see also Demiroglu and James (2010)). Berger, Ofek, and Yermack (1997) argue that the optimal capital structure is linked to company’s corporate governance, and it has been shown that the corporate governance changes significantly following the buyout (see, for instance, Gertner and Kaplan (1996), Acharya and Kehoe (2008), or Cornelli and Karakas (2008)).

This paper contributes to the discussion of capital structure by proposing a mechanism that quantitatively explains the leverage of companies with and without PE-ownership simultaneously. The paper focuses on the access to external financing in distress. External financing costs increase with company’s leverage for companies without PE-ownership, and so they optimally choose to follow conservative leverage policy ex-ante. Companies with PE-ownership, in contrast, can always receive an equity injection from the PE-owner, and so they optimally choose a much higher leverage. Empirically, Hotchkiss, Smith, and Stromberg (2014) show that private equity firms are indeed efficient at resolving the financial distress of their portfolio companies (see also Andrade and Kaplan (1998)).

The mechanism in this paper is most closely related to Elkamhi, Ericsson, and Parsons (2011), who show that if companies experience even modest financial distress costs prior to bankruptcy, they are much more conservative in their ex-ante leverage policy. This paper complements the Elkamhi, Ericsson, and Parsons research by explicitly showing how the financial distress costs can arise if external financing costs increase with company’s leverage (through higher default probability), and how private equity firms can mitigate these costs.

This paper is also related to the line of literature that discusses the performance of companies with PE-ownership. The majority of authors agree that PE-ownership positively affects the value of companies, and that companies with PE-ownership have higher operating growth (Harris, Siegel and Wright (2005)), better margins (Kaplan (1989)), greater investments (Chung (2009)), and increased labor productivity (Lichtenberg and Siegel (1990), or Davis, Haltiwanger, Handley, Lipsius, Lerner, and Miranda (2019)). This paper contributes to the literature by showing how better access to external financing in distress can change incentives of PE-backed companies, and how many of the empirical results that above-mentioned papers document can endogenously arise without a direct involvement from the
3.2. Discussion of the main assumption of the model

The underlying assumption of the model is that financially distressed companies without PE-ownership cannot issue equity, while companies with PE-ownership can. As a result of this assumption, external financing costs exponentially increase with company’s leverage for companies without PE-ownership, making financial distress costly, and affecting their ex-ante policies.

The assumption that companies without PE-ownership cannot issue equity in distress is strong, and is mainly made to achieve the tractability of the solution. Results of the model would hold if one assumed instead that companies without PE-ownership could issue equity, but costs of equity issuance increased with leverage. This section 1) discusses these two assumptions - that companies cannot issue equity at all or that costs of equity issuance increase with company’s leverage - from the point of view of theoretical and empirical evidence, and 2) shows what happens when the company is acquired by a private equity firm.

Start with the equity issuance costs by companies without PE-ownership. Myers and Majluf (1984) show that if there is information asymmetry between company’s insiders selling stocks and outside investors buying stocks, newly issued stocks are sold at a discount. As Appendix A shows, a simple modification of their model implies that equity issuance discount is higher if the company has some outstanding debt. The intuition behind debt and equity issuance costs is that any information asymmetry about the value of underlying assets of the company is amplified by company’s leverage because equity is a residual claim on company’s assets. While the model from Appendix A is fairly simple, it does show unequivocally that issuing equity becomes expensive for highly levered companies, and lays out the conditions that are

While Appendix A shows a very stylized model, a companion paper to this one extends the model from the appendix to a full dynamic model, and shows that the main intuition holds, and that equity issuance costs (i.e. the share price discount following the announcement of equity issuance by the company) increase with the leverage. In this model, highly-levered companies are also distressed companies, and consistent with the model predictions, distressed companies are characterized by severe information asymmetry, as shown by Hertzel and Smith (1993) and Lim and Schwert (2019).
Figure 20: Stock price discount following equity issuance announcement

Data is based on SDC/Platinum database that tracks equity issuances, and shows the average stock price reaction following an equity issuance announcement. Each bar represents the average discount for a group of companies with a given leverage; for instance, among companies, whose leverage was between 30% and 40% at the time when they announced an equity issuance, the stock price fell by 3.59% on average. Negative 4.32% (red line) is the average stock price reaction following an equity issuance announcement.

necessary to reverse this result. Specifically, only if there is an investor with 1) enough funds that 2) does not suffer from the information asymmetry problem, equity issuance costs will not be affected by company’s leverage. It is easy to see why companies with PE-ownership satisfy both of these conditions (PE-firms have money, and they are actively engaged in the management of their portfolio companies\(^9\)), while both public and private companies without PE-ownership only satisfy one. In case of public companies, there are enough deep-pocket investors that can provide financing, but managers usually know more about the company than outside investors. In case of most private companies, their managers and indeed their owners, and so there is no information asymmetry, but owners usually do not have enough resources to invest into the company.

Figure 20 shows how company’s stock price reacts to the SEO announcement, depending on company’s leverage. The figure supports the result that costs of equity issuance indeed increase with company’s leverage. Undoubtedly this result suffers from the selection bias,\(^9\)

since companies endogenously choose which type of external financing they want to raise. Nevertheless, the equity issuance discount would probably be even higher if one could control for the selection bias, since costs of debt issuance increase with company’s leverage, and so the highly levered companies that optimally issue equity should be those for which the equity issuance costs are the smallest.

Several other papers in the literature address the question of equity issuance costs by highly levered or distressed companies, but empirical evidence is mixed. Below is a review of papers that study this question. The overall conclusion that follows is that most companies issue equity when their leverage is low; there are instances of equity issuances by financially distressed companies, but the costs are high, and such companies use equity financing because they cannot raise debt.

The first group of papers argues that most companies issue equity when their performance is good. For instance, Senber and Senber (1995) report a complete absence of equity issuance by distressed companies. Fama and French (2005) show that equity issuances are frequent, but most companies issue equity when their leverage is low. Similarly, Mikkelson and Partch (1986) and Eckbo, Masulis, and Norli (2007) find that equity issuances for cash are rare - both in absolute level and relative to public debt issuances. Some other studies provide indirect evidence that companies in distress do not issue equity. Korajczyk, Lucas, and McDonal (1990) find that company’s leverage does not increase significantly two years before an SEO; should firms issue equity to make required debt payments when internally generated cash flow is insufficient, one would expect to find an increase in leverage prior to an SEO. DeAngelo, DeAngelo and Stulz (2010) find that the average leverage of a companies before an SEO is only 27%. Denis and McKeon (2012) show that companies, whose leverage is above the target, tend to cover financial deficit by issuing new debt and increasing leverage further.

Other authors argue in contrary that a sizable number of distressed companies issue equity, but they sell new shares at a large discount, and do so because debt financing is unavailable. Park (2017) finds that public equity offerings decrease for firms in distress, but private placements increase. Walker and Wu (2019) find that a third of all SEOs are conducted by
financially distressed companies. Both of these papers, however, use the distress measure from Campbell, Hilscher, and Szilagyi (2008), which is only partially related to company’s leverage. Indeed, the average leverage in the subsample of distressed companies in Walker and Wu is 32%, which implies that these companies are in distress for reasons other than their indebtedness, and they likely have very limited access to debt financing. This conclusion is further reinforced by Lim and Schwert (2019) who study all private placement of equity (PIPEs) by U.S. companies. They find that most companies issuing PIPEs are small distressed companies without access to debt markets: the median leverage of companies issuing PIPEs is only 7.2%, and 93% of all companies do not have credit rating. When such firms issue PIPEs, they offer shares to the market at a large discount.

The model derived in this paper assumes that companies without PE-ownership always have access to debt capital markets. Therefore, the assumption that such companies do not issue equity to pay required debt payments in distress is consistent with empirical evidence discussed above.

3.3. How PE buyouts are structured and why it is important

PE firms are famous for disclosing as little information as possible about their business, and even the information that is sometimes scarcely available should be properly adjusted in order to lead to meaningful conclusions. Two aspects of the structure of a PE-buyout are particularly important for the analysis in this paper: 1) the fact that the majority of money that the PE pays in the buyout is structured as shareholder loans, and is reflected as debt on company’s balance sheet, and 2) a complicated parent-subsidiary structure that PE firms create on top of the companies they acquire. This Section discusses these two issues in detail, the proper way to account for them, and why results could be misleading if the specifics of these two issues are ignored.

A. Shareholder loans

A textbook definition of a buyout says that a PE finances the deal by taking some bank debt and investing their own money. In reality, only a small portion of the PE money comes in a
form of equity, and the majority comes in a form of debt that is called shareholders loans.\textsuperscript{10} To be precise, the PE still controls the majority of company’s shares - usually above 80%; the way this is achieved is that company’s old shares are purchased using external debt and shareholder loans, and canceled, and a small number of new shares is issued, which the PE buys for a small price. In case when company’s management participates in the buyout (which is usually the case), they also mostly receive shareholders loans and a small number of shares.

To illustrate this, consider an example of a company Buckingham Bingo, an operator of bingo clubs in the UK, that was acquired by a PE firm Alchemy Partners in December 2005. The total deal value was approximately £118mln, of which £56mln was a loan from Barclays bank. The remaining £62mln was provided by the Alchemy Partners and the company’s management. Of the £62mln, only £1mln was paid for company’s shares, and the remaining £61mln were structured as shareholder loans, which were called subordinated debt and loan notes in this particular case. Figure ?? shows the balance sheet of the company in the first annual report that the company filed; both forms of loans - from Barclays bank and from the Alchemy Partners - are treated as debt.

Shareholder loans are structured as debt for accounting purposes, and have some nominal features of a debt security, such as interest expenses, maturity, and seniority over some other securities in default. Nevertheless, for all practical purposes shareholder loans should be treated as equity. The maturity of shareholder loans is usually above ten years, and it is not uncommon to see shareholder loans with maturity after the year 2050. There are interest expenses associate with shareholder loans, but they are almost always structured as payment-in-kind, which means that interest expenses are added to the face value of shareholder loans every year instead of being paid out. In fact, because large amount of bank debt is usually used to finance buyouts, companies are restricted from making any payments on shareholder loans - including interest expenses - before the bank debt

\textsuperscript{10}Shareholder loans is a collective name that I use for the rest of the paper to refer to money that PE provided to finance the buyout that was structured as debt. In reality, shareholder loans have all types of names, such as loan notes, bond notes, institutional loans, deep discounted bonds, or supermezzanine debt. Confusingly, debt from private debt providers may also be called this way, and I was carefully to make sure that debt that I identify as shareholder loans is actually the money that was given by the PE and not someone else.
Company Buckingham Bingo, an operator of bingo clubs in the UK, was acquired by a PE firm Alchemy Partners, in December 2005. Balance sheet in the 2008 annual report shows total liabilities of £131mln, but further notes show that half of the securities in the liabilities are issued to Alchemy Partners.

is repaid. Furthermore, shareholder loans are usually junior unsecured, which means that they are only senior to equity in case of bankruptcy. Since the PE itself is the holder of the shareholder loans, the company will never default on the shareholder loans. It is quite often that shareholder loans are partially or completely written off or converted into equity if company’s financial position deteriorates.

In fact, most companies controlled by PE are pretty straightforward that the majority of liabilities on their balance sheet are not real. Here is how one such company, All3Media, acquired in 2007 by a PE firm called Permira, describes the shareholder loans in its first annual reports:

“The bulk of investment from Permira and management is through unsecured Subordinated Preference Sertificates ("SPC’s") which carry a rolled up interest coupon with all interest and principal only repayable in 2016 or on sale or listing of the Group. These are treated as debt instruments and account for over £20.2 of the accrued interest costs but do not represent a cash flow strain on the company”
It is an open question why PE firms structure their stake in a form of shareholder loans rather than equity. One such reason might be that interest expenses on shareholder loans are subtracted from the pre-tax profit on the income statement, and can potentially reduce the amount of taxes the company pays. Tax-laws, however, are complicated, and not all interest expenses on shareholder loans are tax-deductible. In particular, at least one company was discussing in its annual report that it had legal disputes with HM Revenue & Custom (agency responsible for tax collection in the UK) regarding the tax relief that the company claimed for “interest payments on certain loans”. Another reason is that shareholder loans allow the PE to be formally considered a creditor, albeit a very junior, in bankruptcy proceedings, and participate in negotiations with company’s other creditors.

The preference of PE firms to structure their money as shareholder loans, and the fact that shareholder loans are formally considered as debt on the balance sheet, should significantly affect how the information that is reported by PE-backed companies is analyzed. Start with the income statement. Because shareholder loans have interests - usually very big - the amount of interest expenses that companies claim they pay on the income statement significantly overestimates what they actually pay. An average company in my sample claims that interest expenses make up 64% of its gross profit (EBITDA), while the actual interest expenses they pay are 2.5 times as small (27% of EBITDA).

Furthermore, interest expenses on shareholder loans that companies claim but do not pay create a problem with analyzing information from company’s balance sheet: liabilities constantly grow year-over-year because interest expenses on the shareholder loans are rolled up to their face value. Furthermore, in cases when company’s performance deteriorates, shareholder loans can be reevaluated, converted into equity, or written off - partially or completely. That affects the balance sheet value of both liabilities and shareholders’ equity, without any actual cash flow from investors.

Not only the original PE-investment is structured in a form of shareholder loans, but also the subsequent equity injections by the PE take this form. Moreover, the dividends that PE-backed companies pay to the PE are also structured as a repayment of shareholder loans, or payment of interest expenses on shareholder loans. In contrast, what sometimes
appears as dividends on the cash flow statement is usually dividends to join ventures or to the minority shareholders\textsuperscript{11}.

To be completely clear, shareholder loans and interest expenses on shareholder loans are by far not the only features that make the analysis of PE-backed companies complicated. For instance, buyouts usually have large transaction costs - of the order of millions of pounds. Oddly enough\textsuperscript{12}, PE-backed companies capitalize transaction costs on the balance sheet - as goodwill on the left-hand side and as liabilities or shareholders equity on the right-hand side - and amortize them later on. This paper specifically focuses on the cash flows between the company and its investors, and so I carefully examine shareholder loans for each company I consider (see the data description section below). However, any further analysis of PE-backed companies should adjust for other specifics of PE-buyouts depending on the focus of the analysis.

\textbf{B. Parent-subsidiary structure}

The second important issue of every PE-buyout is the structure of the acquisition. In order to finance an acquisition, the PE create a complicated vertically integrated parent-subsidiary chain of holding companies. The PE ultimately controls the company at the top of the structure, and the company at the bottom of the structure acquires and controls the target company, which is then called the operating company. Companies in the middle issue different types of securities to investors, and the money they raise is transferred through inter-company loan down the structure to finance the buyout. Of the companies that issue securities and receive money from investors, those that are closer to the bottom of the chain (and, therefore, closer to the operating company) issue securities with higher seniority. For instance, the company closest to the operating company issues bank debt, company slightly above issues corporate bonds (if corporate bonds are used to finance the structure), the company closer to the top issues shareholder loans, and the company at the very top issues equity.

\textsuperscript{11}Minority shareholders are shareholders of companies, in which the PE-backed company controls more than 51\% of the stake but less than 100\%  
\textsuperscript{12}Though I have to admit I do not know if companies that are not controlled by PE do a similar thing with transaction costs
There are different reasons why PE firms prefer to follow such obscure structure. Tax-optimization is certainly one of the reasons, according to industry professionals. Other than that, I saw several companies, for which some of the intermediary holding companies were established outside of the UK, and the company later expanded the business to those regions. The fact that holders of senior debt prefer to be closer to the bottom of the chain implies that bankruptcy considerations are another factor. This later point is supported by Ayotte (2019), who theoretically shows how complex levels of company structure arise when investors disagree about the value of assets that back loans.

Understanding the parent-subsidiary structure is important to properly analyze the business of PE-backed companies. Conceptually speaking, the acquired company is liable for the debt that was used to finance the buyout: its cash flow is used to repay the debt, and its assets are used as the collateral. Nevertheless, the balance sheet of the acquired company does not change much following the buyout because, formally, debt is issued by one of the parent companies. Every company in the chain files their own annual report after the buyout, with their own balance sheet and income statement.

Therefore, it is not the target company that should be analyzed, but all companies in the chain collectively. Fortunately, UK laws require that at least one company files a consolidate report for the whole group. The consolidated report shows all securities that are issued by any member of the group, and all money earned and spent. Therefore, it is the company that files consolidate reports that should be analyzed, and not the operating company, or any other individual company in the chain.

There is one big advantage, for research purposes at least, of the parent-subsidiary structure. A new company is incorporated for every buyout, and the company begins the annual reporting from scratch. This means that every dollar that is used for the buyout - whether that dollar comes from a bank, a private debt provider, the PE, or company’s managers - is reflected on the balance sheet of the new company, and nothing else is. Therefore, by adding up the values of all securities on the balance sheet of the company based on the

---

13To be precise, it does change significantly, but mostly because most things on the balance sheet are reevaluated.
first report it files, it is possible to recover the value of the company at acquisition, which otherwise would not necessarily be publicly available.

3.4. Empirical results

3.4.1. Sample selection and data sources

Preqin and UK companieshouse

This paper focuses on PE-backed companies incorporated in the UK because all companies in the UK\textsuperscript{14} - public and private - are required to file annual reports. These reports, with some rare exceptions, include balance sheet, income statement, and cash flow statement, and, thus, allow to see how companies are being managed after they are acquired by private equity firms. Scanned versions of these reports are publicly available on the UK companieshouse web-site\textsuperscript{15}.

I use Preqin database to construct an initial sample of companies. The focus of this paper is on buyouts, and, therefore, I filter the Preqin sample to exclude all deals that are not classified as buyouts or public-to-private transactions. In particular, VC (Venture Capital) and PE-growth deals are excluded. Only deals, in which the acquired company is in the UK, and in which the PE-sponsor has already exited the company, are included. Deals, in which the PE controls the company for less than a year are only included if the acquired company published at least one report while it was controlled by the PE. I take the following information from Preqin: the name of the acquired company, the name of the PE firm, the dates of investment and exit, and the type of exit.

The resulting Preqin sample has 2174 deals. Each deal represents one company, from the year it is bought by a PE, and until the year the PE exits the company. Some deals involve the same company, in case one PE sells the company to another PE, or in case the same company is bought several times in different years.

Buyouts have an obscure structure, with a lot of tax-optimization and financial engineering

\textsuperscript{14}Generally everywhere in Europe, though UK in particular is the focus of this research
\textsuperscript{15}https://www.gov.uk/government/organisations/companies-house
involved, as explained in the previous section, and as is also discussed below. Due to the level of complexity, simple aggregation of reported information is misleading, and so each deal that is studied in this paper is analyzed individually. Naturally, it required a lot of time and efforts, and instead of analyzing the whole sample of 2174 deals, I only analyzed a subsample of 410 deals. To ensure representativeness of the subsample, names of all acquired companies were sorted alphabetically and analyzed from top down. While I skipped some deals, the eventual subsample is representative of the full Preqin sample, as Table 12 shows.

The analysis of deals is based on the annual reports filed by the acquired companies, and information from the reports was corroborated by news articles in some cases. In order to obtain the reports, I matched names of acquired companies as reported in Preqin to the UK companieshouse web-site. Preqin only reports approximate names, and the legal names might be slightly (or significantly) different, i.e. All3Media vs. All3Media Holdings Limited, or Amtech vs. DeFacto 1731 Limited. Furthermore, as discussed in the previous Section, a whole parent-subsidiary structure is created to finance the acquisition, and all companies in the structure have similar names. Therefore, I manually search for each company on the UK companieshouse web-site, and then identify the ultimate parent company that published the consolidated financial reports.

Once a company is matched to the UK companieshouse web-site, I see its tax-identification number. The tax-identification numbers can be used to link companies to the AMADEUS database, which has some of the information from the reports. I rely on AMADEUS data for some of the variables that are later used in the analysis, such as sales or SIC number. Nevertheless, the majority of information is manually collected, by a company called Qynn, and by two research assistants and I.

*Acknowledging the support of Qynn*

Established in 2017, Qynn provides in-depth financial analysis, research and data on UK companies. The data within Qynn is generated from a number of public data sources including several UK Government Departments which Qynn then aggregates to create proprietary information and insight on 5.5 million UK companies stretching over 20 years. With more
Table 12: Sample of companies in Preqin vs. analyzed in this paper

<table>
<thead>
<tr>
<th>Panel A: Industries</th>
<th>Consumer &amp; Retail</th>
<th>Industrials</th>
<th>Business Services</th>
<th>Information Tech.</th>
<th>Healthcare</th>
<th>Telecoms &amp; Media</th>
<th>Food &amp; Agriculture</th>
</tr>
</thead>
<tbody>
<tr>
<td>whole sample</td>
<td>24.2% (526)</td>
<td>23.4% (508)</td>
<td>15.1% (329)</td>
<td>11.9% (259)</td>
<td>8.3% (181)</td>
<td>5.2% (114)</td>
<td>4.5% (98)</td>
</tr>
<tr>
<td>sample in this paper</td>
<td>22.2% (91)</td>
<td>20.6% (84)</td>
<td>16.2% (66)</td>
<td>8.4% (34)</td>
<td>10.1% (41)</td>
<td>6.1% (25)</td>
<td>4.7% (19)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>whole sample</th>
<th>Energy &amp; Utilities</th>
<th>Materials</th>
<th>Clean Techn.</th>
<th>Real Estate</th>
<th>Infrastructure</th>
<th>Unspecified</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.9% (63)</td>
<td>1.6% (34)</td>
<td>1.1% (24)</td>
<td>0.8% (17)</td>
<td>0.7% (16)</td>
<td>0.2% (5)</td>
<td>2174</td>
<td></td>
</tr>
<tr>
<td>sample in this paper</td>
<td>3.7% (15)</td>
<td>1.5% (6)</td>
<td>2.2% (9)</td>
<td>0.7% (3)</td>
<td>3.2% (13)</td>
<td>0.3% (1)</td>
<td>407</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Exit type</th>
<th>Trade Sale</th>
<th>SBO</th>
<th>Default/Restruct.</th>
<th>Sale to mgm.</th>
<th>IPO</th>
<th>Unspecified</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>whole sample</td>
<td>42.4% (921)</td>
<td>26.7% (581)</td>
<td>3.9% (84)</td>
<td>3.17% (69)</td>
<td>2.5% (54)</td>
<td>16.5% (358)</td>
<td>4.9% (107)</td>
</tr>
<tr>
<td>sample in this paper</td>
<td>40.9% (167)</td>
<td>35.3% (144)</td>
<td>4.2% (17)</td>
<td>2.9% (12)</td>
<td>3.2% (13)</td>
<td>6.9% (28)</td>
<td>6.6% (27)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel C: PE-sponsors (top 7 in the whole sample)</th>
<th>LDC</th>
<th>CVC</th>
<th>3i</th>
<th>Equistone</th>
<th>Livingbridge</th>
<th>Cinven</th>
<th>Graphite</th>
</tr>
</thead>
<tbody>
<tr>
<td>whole sample</td>
<td>4.0% (98)</td>
<td>3.3% (81)</td>
<td>3.0% (74)</td>
<td>2.9% (70)</td>
<td>2.1% (52)</td>
<td>1.8% (43)</td>
<td>1.7% (42)</td>
</tr>
<tr>
<td>sample in this paper</td>
<td>5.9% (27)</td>
<td>0.9% (4)</td>
<td>3.5% (16)</td>
<td>4.6% (21)</td>
<td>1.8% (8)</td>
<td>0.9% (4)</td>
<td>1.1% (5)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>whole sample</td>
<td>4.0% (88)</td>
<td>14.2% (309)</td>
<td>15.5% (336)</td>
<td>31.1% (677)</td>
<td>20.1% (437)</td>
<td>13.2% (288)</td>
<td>1.8% (39)</td>
</tr>
<tr>
<td>sample in this paper</td>
<td>0% (0)</td>
<td>9.6% (39)</td>
<td>15.7% (64)</td>
<td>32.1% (131)</td>
<td>21.8% (89)</td>
<td>18.1% (74)</td>
<td>2.7% (11)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>whole sample</td>
<td>0.6% (13)</td>
<td>4.1% (89)</td>
<td>4.7% (103)</td>
<td>18.7% (406)</td>
<td>14.6% (317)</td>
<td>25.9% (563)</td>
<td>31.5% (683)</td>
</tr>
<tr>
<td>sample in this paper</td>
<td>0% (0)</td>
<td>1.5% (6)</td>
<td>4.7% (19)</td>
<td>20.6% (84)</td>
<td>13.5% (55)</td>
<td>30.4% (124)</td>
<td>29.4% (120)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel F: Investment length</th>
<th>&lt; 1 years</th>
<th>1-2 years</th>
<th>3-4 years</th>
<th>5-6 years</th>
<th>7-8 years</th>
<th>9-10 years</th>
<th>&gt;10 years or no data</th>
</tr>
</thead>
<tbody>
<tr>
<td>whole sample</td>
<td>1.2% (27)</td>
<td>15.6% (340)</td>
<td>25.4% (553)</td>
<td>23.4% (509)</td>
<td>13.3% (290)</td>
<td>5.7% (123)</td>
<td>15.3% (332)</td>
</tr>
<tr>
<td>sample in this paper</td>
<td>0% (0)</td>
<td>14.0% (57)</td>
<td>33.3% (136)</td>
<td>28.2% (115)</td>
<td>15.7% (64)</td>
<td>3.7% (15)</td>
<td>5.1% (21)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel G: Size in USD mln (based on Preqin)</th>
<th>&lt; 1</th>
<th>1-50</th>
<th>50-100</th>
<th>100-200ln</th>
<th>200 - 1000</th>
<th>&gt; 1000</th>
<th>no data</th>
</tr>
</thead>
<tbody>
<tr>
<td>whole sample</td>
<td>0.3% (6)</td>
<td>24.5% (532)</td>
<td>10.1% (220)</td>
<td>9.3% (202)</td>
<td>11.5% (251)</td>
<td>3.5% (77)</td>
<td>40.8% (886)</td>
</tr>
<tr>
<td>sample in this paper</td>
<td>0% (0)</td>
<td>22.3% (91)</td>
<td>13.2% (54)</td>
<td>15.0% (61)</td>
<td>15.4% (63)</td>
<td>2.9% (12)</td>
<td>31.1% (127)</td>
</tr>
</tbody>
</table>

This table shows how a sample of PE-led buyouts analyzed in this paper compares to the sample of all PE-led buyouts in the UK, based on the information from Preqin database. Only 410 buyouts are analyzed because the analysis of each requires a long of data hand-collection. “whole sample” refers to all 2174 buyouts, and “sample in this paper” refers to 408 transactions analyzed in this paper. Numbers in the table show the fraction of deals in a specific category, and numbers in brackets show the number of deals in that category; for instance, 526 deals in the whole sample are in the Consumer & Retail industry, which makes 24.2% of all deals in the whole sample. In Panel B, “Trade Sale” is the exit type, in which the PE sells the company to another company; “SBO” is the exit type, in which the PE sells the company to another PE. Panel C has PE that have the most deals in the whole sample.

than 100 million data points a week being added to the system, Qynn updates in real time.

Qynn’s help was mostly used to collect the data that is reported in a standardized form.
in annual reports and is incorrectly classified by other data providers. One good example is the interest expenses paid by companies with PE-ownership. As the previous subsection discusses, shareholder loans have massive interest expenses that are recorded on the company’s income statement, but are rarely paid to the holders of shareholder loans. By analyzing the data from the statement of cash flows instead of the income statement, Qynn was able to identify the interest expenses that were actually paid, and separate those from PIK-structured interest expenses on the income statement. To show the significance of this adjustment, the accrued interest expenses that an average company in the sample reports are 2.4 times larger than the interest expenses it pays.

In total, Qynn was able to provide 30,685 data points to help with this research against a total of 407 UK companies at an average of 75 points per company. In addition, Qynn also identified a small, but significant number of instances where annual accounts had been presented, and then later restated.

*Manually-collected data*

Many important aspects of the structure and financing of buyouts are reported in a way that does not allow for automatic analysis, and, hence, required the careful reading of the reports. Therefore, I, with the help of two research assistants, examined the reports and hand-collected specific information. First, we studied the balance sheet of each company to identify which securities are actual external debt, and which securities are shareholder loans. In some cases it was easy, when the company explicitly discussed the holders of the securities it issued (as in the example of Figure 21). In other cases, companies did not explicitly say that certain loans were provided by the PE, but it was clear from other discussions in the report.\(^{16}\) In other cases, especially for larger transactions (i.e. transactions above £1bln), identifying shareholder loans was harder or even impossible, because there were too many securities on the balance sheet. For instance, company Debenhams was acquired by a group of Private Equity firms in 2003 in a £1.7bln public-to-private transaction. The first report filed by this company shows the following securities on the balance sheet: three types of

\(^{16}\)For instance, the company could discuss that it received £10mln from its shareholders, and a certain type of securities increased by £10mln in that year
senior term loan, property mortgage, deep discounted bonds, and high yield notes, with no further explanation.\textsuperscript{17} For some of such cases, including the Debenhams buyout, we found news articles that discussed the transaction and how much debt was used to finance it; using these news articles, we were able to identify the shareholder loans on the balance sheet based on which securities added up in value to the correct amount.

The second type of information we hand-collected is the flow of external financing between the company and its investors, in years following the buyout but before the PE exited the company. In particular, we collect information on how much external debt is issued and repaid every year, how much money the company receives from the PE, and how much money the company pays to the PE. There are two types of payments that companies make to the PE. The first is what would traditionally be considered as dividends. These payments are not structured as dividends, though, and instead take a form of repayment of shareholder loans and/or accrued interest expenses on shareholder loans. This type of payments we always observe and record.

The second type of payments is what’s collectively called monitoring fees. Monitoring fees can take different forms; for instance, the PE may appoint several directors to the company’s board, and charge annual payments for their service (in addition to the salary that is paid directly to the directors). Monitoring fees do not always exist, and, similar to interest expenses on shareholder loans, they sometimes accrue instead of being paid. Monitoring fees appear early on the income statement, usually as a part of COGS or SG&A expenses. Theoretically, companies are required to disclose all such payments, but there seem to be cases, in which there is no information about such fees, even though the fees were likely paid. It is, therefore, possible, that we sometimes underestimate the frequency of the monitoring fees. Nevertheless, as the following Section shows, these fees are much smaller than other payments the company makes to the PE.

At last, we manually collect information about the exit value for deals that ended with the company being sold by a PE to another PE. This type of exit is the second most common exit type in my sample, constituting 35\% of all exit types. The reason why the exit value

\textsuperscript{17}All these securities are shown in liabilities. Money raised by selling shares is less than £5mln
can be computed for these deals and not for others is because, as discussed in the previous Section, a new chain of holding companies is created for every buyout. The first consolidated report filed by the ultimate parent company of the new group, therefore, allows to back out the buyout amount. Therefore, by finding the new ultimate parent of the company after it is sold by one PE to another, we can find the value that was paid for the company. This is generally not possible for other exit types.

Comparison with companies without PE-ownership

Understanding how companies with PE-ownership operate is important because of the large number of companies that are owned by private equity firms. Equally important, however, is understanding how companies with and without PE-ownership are different from each other. The problem with comparing two group of companies is the endogenous nature of PE-ownership: private equity firms do not choose acquisition targets randomly. Therefore, any difference between companies with and without PE-ownership can be attributed to the presence of the PE-owner, as well as to unobserved factors that influenced the decision of private equity firms to buy certain companies.

One approach to partially address the selection bias would be to analyze a group of comparable companies - that is, a group of companies without PE-ownership that are similar based on observed characteristics to the sample of companies with PE-ownership. In the context of private equity, however, such approach is, at best, complicated, and can often be misleading. First, private equity firms extensively study potential target companies before the buyout, and analyze large amount of public and private information. Most of this information - think of customer concentration, supply chain management, or quality of the management team - is company-specific and rarely observable, and, hence, cannot be conditioned on in making a comparable group. Variables that are observable by a researcher, such as size, sales growth, or profitability, in practice explain less than 10% of the selection choice.

Second, numbers reported by companies with and without PE-ownership are not readily comparable before appropriate adjustments. As discussed in the previous section, some
of the accounting practices of PE-backed companies are unusual. For instance, intangible capital constitutes 60% of the total value of assets, on average, among companies with PE-ownership. Therefore, matching companies using book value of assets, or any moment that involves the book value of assets (such as profitability), is a statistical exercise before one understands what goes into intangible capital, or how intangible capital evolves over time. Intangible capital is one example; many other adjustments that companies with PE-ownership make should be taken into account before a reasonable group of comparable companies can be constructed.

In the absence of a clear group of matched companies, I compare companies with PE-ownership to two very broad group of companies: 1) all companies in COMPUSTAT Global that are incorporated in the UK (referred as “UK” companies later in the paper), and 2) all companies in COMPUSTAT North America (referred to as “US” companies later). I acknowledge that there is a limit to how much can be concluded from comparing these two group of companies, and, therefore, “US” and “UK” numbers are mostly interpreted as a benchmark in the following subsections. Nevertheless, even though the two group of companies are different, it does not mean that they are completely incomparable. Where appropriate, regressions include year fixed effects and company or industry fixed effects, to account for the fact that private equity firms target unevenly target certain companies. Furthermore, it is often clear, at least qualitatively, what other factors and to what extent can account for the observed difference between the two group of companies.

At last, all hand-collected data about companies with PE-ownership that is used in this paper is publicly available on the author’s web-site. This makes it easy for anyone with specific views on the direction of the selection bias to compare the group of companies in this paper against another group of companies.

18Both for the period between 2000 and 2019.
3.4.2. Capital structure and external financing

A. Buyout leverage

According to the UK accounting rules, securities are added to the balance sheet at the value that the company receives when it issues them. Therefore, I measure deal size by adding up the values of external debt, shareholder loans, nominal value of shares, and share premium, as shown in the first report filed by the company post-buyout. Since, as discussed in the previous Section, a new parent company is created for the purpose of buyout, all securities in its first report were issued to finance the buyout, meaning I do not occasionally overestimate the deal value\(^\text{19}\) by considering securities that could have been issued by the company before the buyout. To measure the buyout leverage, I divide the amount of external debt (i.e. all debt excluding shareholder loans) in the first report by the deal amount.

PE-backed companies are significantly less levered than traditionally thought. As Table 13 indicates, the average leverage in a PE-transaction is only 41%, and the median leverage is 43%. To put it into perspective, there are as many companies with leverage below 5% (40 companies), as companies with leverage above 65% (42 companies).

The data shows that larger deals have greater leverage. As follows from Table 13, the average and median leverage increase by roughly 10 percentage points if the deal size is restricted to be above £100mln, and even more if the deal size is restricted to be above £200mln. Nevertheless, the mean and median leverage are below 60% even among deals that are larger than £200mln. It is not infrequent to see large transaction that have very small amount of debt; for instance, PE firm Epiris acquired a footwear producer Beaconsfield Footwear in 2015 for approximately £165mln with only £40mln of debt.

There is a substantial time-variation in leverage values. Deals that were financed prior to the 2008 financial crisis used more debt than deals financed during or after the crisis, and the difference in leverage is roughly 10-15 percentage points. A part of the time-variation in leverage values, especially when deals are broken down into size groups, can likely be

\(^{19}\)To be precise, the value of these securities might be affected by what happens between the buyout and the date report is filed, which are, on average, seven month apart. I discuss later how this can affect the results.
explained by the fact that I do not correct deal size for inflation. Therefore, deals above £100mln prior to 2008 are larger than deals above £100mln after 2011; as noted above, larger deals have greater leverage. The time-variation in leverage is also consistent with the argument of Axelson, et. al (2008) that economy-wide credit conditions determine the buyout leverage. Nevertheless, leverage rarely exceeded 60% even before 2008.

The results about average leverage seem to be in contrast with findings of other authors, most of whom report that the leverage of PE transaction is usually above 60%. A number of factors could contribute to the fact that the value of leverage I find is lower than what other authors find.

First, I might underestimate the leverage value because companies file the first report some time after the buyout, on average eight month. Therefore, I do not observe the portion of debt that was used to finance the buyout and repaid during this period. This probably does not affect results much, because the majority of debt that is used to finance buyouts is structured in a way that only a small fraction of it is repaid annually. More importantly, I might also overestimate the value of shareholder loans (and, therefore, the total deal value) by the amount of interest expenses on shareholder loans that accrue to the face value over the period after the buyout and before the first report is filed.

It is unlikely, however, that debt repayments and interests on shareholder loans in the first seven months after the buyout can fully account for the difference in average leverage in this paper and in other papers. Another problem might be coming from the nature of databases that are frequently used to study the capital structure of buyouts. Several papers used AMADEUS database, but as the previous subsection discusses, AMADEUS does not distinguish between external debt and shareholder loans. In fact, as Figure 22 illustrates, the average and median leverage increase by 20 percentage points if measured incorrectly, by divining reported debt by the total value of assets, as reported in the first post-buyout year.

LPC/Dealscan is another database that is sometimes used to study the leverage of LBO

\[20\] That does not include interest payments, which are usually quarterly, but do not change the face value of debt.
Table 13: Leverage

Panel A: Size

<table>
<thead>
<tr>
<th>size</th>
<th>mean</th>
<th>q25</th>
<th>q50</th>
<th>q75</th>
<th>min</th>
<th>max</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>£133.5 mln</td>
<td>£23.0 mln</td>
<td>£50.8 mln</td>
<td>£120.3 mln</td>
<td>£3.6 mln</td>
<td>£4.4 bln</td>
<td>410</td>
<td></td>
</tr>
</tbody>
</table>

Panel B: Leverage

<table>
<thead>
<tr>
<th>years 2011 - 2019</th>
<th>whole period</th>
<th>all companies</th>
<th>mean</th>
<th>q25</th>
<th>q50</th>
<th>q75</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>size &gt; £50mln</td>
<td>46.3%</td>
<td>34.7%</td>
<td>47.2%</td>
<td>59.9%</td>
<td>206</td>
<td></td>
<td></td>
</tr>
<tr>
<td>size &gt; £100mln</td>
<td>49.7%</td>
<td>39.9%</td>
<td>52.9%</td>
<td>63.1%</td>
<td>125</td>
<td></td>
<td></td>
</tr>
<tr>
<td>size &gt; £200mln</td>
<td>51.7%</td>
<td>41.1%</td>
<td>57.3%</td>
<td>63.5%</td>
<td>66</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>years 2008-2010</th>
<th>whole period</th>
<th>all companies</th>
<th>mean</th>
<th>q25</th>
<th>q50</th>
<th>q75</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>size &gt; £50mln</td>
<td>36.6%</td>
<td>23.5%</td>
<td>38.8%</td>
<td>46.9%</td>
<td>85</td>
<td></td>
<td></td>
</tr>
<tr>
<td>size &gt; £100mln</td>
<td>44.2%</td>
<td>33.8%</td>
<td>42.9%</td>
<td>56.3%</td>
<td>26</td>
<td></td>
<td></td>
</tr>
<tr>
<td>size &gt; £200mln</td>
<td>45.7%</td>
<td>37.0%</td>
<td>42.9%</td>
<td>56.1%</td>
<td>12</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>years 2000-2007</th>
<th>whole period</th>
<th>all companies</th>
<th>mean</th>
<th>q25</th>
<th>q50</th>
<th>q75</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>size &gt; £50mln</td>
<td>55.3%</td>
<td>46.8%</td>
<td>58.2%</td>
<td>64.8%</td>
<td>91</td>
<td></td>
<td></td>
</tr>
<tr>
<td>size &gt; £100mln</td>
<td>57.9%</td>
<td>50.9%</td>
<td>60.6%</td>
<td>66.2%</td>
<td>58</td>
<td></td>
<td></td>
</tr>
<tr>
<td>size &gt; £200mln</td>
<td>58.5%</td>
<td>52.9%</td>
<td>62.5%</td>
<td>66.5%</td>
<td>25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>before 2000</th>
<th>whole period</th>
<th>all companies</th>
<th>mean</th>
<th>q25</th>
<th>q50</th>
<th>q75</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>size &gt; £50mln</td>
<td>43.9%</td>
<td>34.6%</td>
<td>49.3%</td>
<td>55.9%</td>
<td>36</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel C: Comparison with the LPC/Dealscan database

<table>
<thead>
<tr>
<th>leverage, whole period</th>
<th>in LPC</th>
<th>in LPC &amp; size &gt; £50mln</th>
<th>in LPC &amp; size &gt; £100mln</th>
<th>in LPC &amp; size &gt; £200mln</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>51.3%</td>
<td>52.7%</td>
<td>53.5%</td>
<td>53.2%</td>
</tr>
<tr>
<td>q25</td>
<td>40.6%</td>
<td>41.3%</td>
<td>41.3%</td>
<td>43.2%</td>
</tr>
<tr>
<td>q50</td>
<td>52.6%</td>
<td>54.1%</td>
<td>55.7%</td>
<td>55.8%</td>
</tr>
<tr>
<td>q75</td>
<td>62.1%</td>
<td>63.0%</td>
<td>63.7%</td>
<td>64.3%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>size, whole period</th>
<th>in LPC</th>
<th>in LPC &amp; size &gt; £50mln</th>
<th>in LPC &amp; size &gt; £100mln</th>
<th>in LPC &amp; size &gt; £200mln</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>£133.1mln</td>
<td>£22.9mln</td>
<td>£50.5mln</td>
<td>£119.8mln</td>
</tr>
<tr>
<td>q25</td>
<td>£290.0mln</td>
<td>£90.5mln</td>
<td>£147.0mln</td>
<td>£287.5mln</td>
</tr>
</tbody>
</table>

All numbers in this table are computed based on the first report that a company files after the buyout, which is usually six month after the buyout. Size is the amount of money paid to acquire the company, and is inferred from the sum of the value of all securities issued to finance the buyout. Leverage is the ratio of company’s debt to the buyout size, and company’s debt is computed as the sum of all securities that were issued to parties other than the PE and company’s management (most often to banks, more rarely to private debt providers and to public investors). LPC/Dealscan is a database that is often used to study the leverage of PE-buyouts. “in LPC” refers to deals in the sample, which also have information in the LPC/Dealscan database. q25, q50, and q75 refer to the corresponding quantiles of the distribution, and N is the number of observations.

transactions. LPC/Dealscan has reliable information about external debt (i.e. debt only includes external debt, and does not include shareholder loans), but, as most authors ac-
knowledge, it has a selection bias towards larger deals with more debt. Indeed, among 410 companies in my sample, only 76 are also in the LPC/Dealscan database and have information about debt issuance in the buyout year\textsuperscript{21}, and these companies have significantly higher average leverage than other companies.

\textbf{B. Debt management post-buyout}

In order for the PE to earn money on their investments, one of the two things should happen. Either their portfolio companies should reduce the amount of debt that was used to finance the buyout in the first place, or the value of the company should increase. This and next sections analyze the empirical evidence in favor and against both of these alternatives.

A textbook description of the life-cycle of a PE-backed company states that company’s cash flow is primarily used to repay the debt that was used to finance the buyout; by the time

\textsuperscript{21}Other 50 companies are in the LPC/Dealscan database but have no records about debt issuance in the year when the buyout happens, and other 300 companies are not in the database.
the PE exits the company, all or most of the debt is repaid, therefore, naturally increasing the value of equity and generating “healthy” returns on equity capital.

The data does not support this narrative. To show this, I compute how the value of debt on the balance sheet of PE-backed companies changes year-over-year. As Table 14 shows, PE-backed companies have more debt by the time the PE exits the company than what they had at the time of the buyout. On average, the value of debt grows by 20% between the acquisition and the exit years, from 41% relative to the deal value in the acquisition year to slightly less than 50% in the exit year. This big increase in leverage is mostly driven by smaller companies; yet, even large companies do not reduce their amount of debt.

Furthermore, in the majority of cases, the debt that is used to finance the buyout is not even structured to be repaid out of the company’s cash flow. Aside from interest expenses, only a small portion of the debt has any face value repayments before the maturity. The maturity, in turn, is usually more than five years after the buyout, implying that it exceeds the expected exit time of the PE that usually plan to exit the company within three-five years. Having said this, most of the debt that is used for acquisition has a clause that says that the debt should be immediately repaid in case of the PE sells the company to a new owner.

Even though the absolute amount of debt that PE-backed companies have increases, the overall leverage - that is the value of debt over the value of the company - seems to go down, at least for those companies, for which the exit value is known. For this subset of companies, I compute the leverage in the exit year, which is the value of debt the company has in its last year divided by the value at which it is sold to the new owner, vs. the leverage in the buyout. As Table 14 shows, the leverage at exit is only 26%, down 16 percentage points from the acquisition year. All in all, among the two alternatives that say that the PE generate money by repaying the debt vs. by increasing the value of the assets, the first has very little support in the data.

The fact that the PE-backed companies start and finish with roughly the same amount of debt implies that they either have static capital structure, or that they issue and repay
Table 14: Debt management

Panel A: Amount of debt several years after the acquisition (relative to the original deal size)

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>q25</th>
<th>q50</th>
<th>q75</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>all companies</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>acquisition year</td>
<td>41.2%</td>
<td>29.2%</td>
<td>43.1%</td>
<td>55.8%</td>
<td>388</td>
</tr>
<tr>
<td>exit year</td>
<td>49.5%</td>
<td>20.1%</td>
<td>39.4%</td>
<td>61.5%</td>
<td>388</td>
</tr>
<tr>
<td>size &gt; £50mln</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>acquisition year</td>
<td>46.6%</td>
<td>34.7%</td>
<td>47.4%</td>
<td>59.9%</td>
<td>196</td>
</tr>
<tr>
<td>exit year</td>
<td>50.1%</td>
<td>26.7%</td>
<td>45.3%</td>
<td>62.9%</td>
<td>196</td>
</tr>
<tr>
<td>size &gt; £100mln</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>acquisition year</td>
<td>49.8%</td>
<td>39.9%</td>
<td>51.9%</td>
<td>63.1%</td>
<td>120</td>
</tr>
<tr>
<td>exit year</td>
<td>49.4%</td>
<td>28.4%</td>
<td>48.7%</td>
<td>66.2%</td>
<td>120</td>
</tr>
<tr>
<td>size &gt; £200mln</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>acquisition year</td>
<td>51.5%</td>
<td>41.4%</td>
<td>56.3%</td>
<td>63.3%</td>
<td>65</td>
</tr>
<tr>
<td>exit year</td>
<td>52.3%</td>
<td>35.4%</td>
<td>51.1%</td>
<td>66.8%</td>
<td>65</td>
</tr>
<tr>
<td>companies with known exit value</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>acquisition year, $D_0/V_0$</td>
<td>41.1%</td>
<td>28.3%</td>
<td>42.2%</td>
<td>52.9%</td>
<td>107</td>
</tr>
<tr>
<td>exit year, $D_{end}/V_0$</td>
<td>60.5%</td>
<td>29.4%</td>
<td>44.9%</td>
<td>67.7%</td>
<td>107</td>
</tr>
<tr>
<td>exit year, $D_{end}/V_{new}$</td>
<td>25.9%</td>
<td>10.6%</td>
<td>18.4%</td>
<td>30.4%</td>
<td>107</td>
</tr>
</tbody>
</table>

Panel B: Frequency of follow-on debt issuance

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>≥ 4</th>
<th>0</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>all companies</td>
<td>112</td>
<td>78</td>
<td>40</td>
<td>38</td>
<td>120</td>
<td>388</td>
</tr>
<tr>
<td>size &gt; £50mln</td>
<td>52</td>
<td>44</td>
<td>20</td>
<td>20</td>
<td>60</td>
<td>196</td>
</tr>
<tr>
<td>size &gt; £100mln</td>
<td>30</td>
<td>28</td>
<td>16</td>
<td>12</td>
<td>34</td>
<td>120</td>
</tr>
<tr>
<td>size &gt; £200mln</td>
<td>19</td>
<td>16</td>
<td>6</td>
<td>9</td>
<td>15</td>
<td>65</td>
</tr>
</tbody>
</table>

Panel C: Total size of follow-on debt issuance (for companies with at least one debt issuance)

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>q25</th>
<th>q50</th>
<th>q75</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>all companies</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>rel. to original deal size</td>
<td>51.7%</td>
<td>7.4%</td>
<td>24.6%</td>
<td>65.6%</td>
<td>268</td>
</tr>
<tr>
<td>rel. to original debt amn.</td>
<td>575.1%</td>
<td>18.9%</td>
<td>60.5%</td>
<td>178.7%</td>
<td>256</td>
</tr>
<tr>
<td>size &gt; £50mln</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>rel. to original deal size</td>
<td>42.3%</td>
<td>6.3%</td>
<td>22.2%</td>
<td>65.9%</td>
<td>136</td>
</tr>
<tr>
<td>rel. to original debt amn.</td>
<td>188.7%</td>
<td>13.1%</td>
<td>49.1%</td>
<td>148.4%</td>
<td>133</td>
</tr>
<tr>
<td>size &gt; £100mln</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>rel. to original deal size</td>
<td>40.8%</td>
<td>6.3%</td>
<td>22.9%</td>
<td>65.7%</td>
<td>86</td>
</tr>
<tr>
<td>rel. to original debt amn.</td>
<td>100.8%</td>
<td>12.0%</td>
<td>49.4%</td>
<td>136.2%</td>
<td>84</td>
</tr>
<tr>
<td>size &gt; £200mln</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>rel. to original deal size</td>
<td>36.4%</td>
<td>6.2%</td>
<td>21.5%</td>
<td>62.9%</td>
<td>50</td>
</tr>
<tr>
<td>rel. to original debt amn.</td>
<td>100.9%</td>
<td>13.3%</td>
<td>49.1%</td>
<td>109.0%</td>
<td>49</td>
</tr>
</tbody>
</table>

Numbers in the table are based on acquisitions, in which the PE controls the company for at least two years. q25, q50, and q75 denote corresponding quantiles of the distribution, and N shows the number of observations. Panel A compares the amount of debt that a company has in the year when it is acquired by a PE (acquisition year), and when it is sold by the same PE (exit year). Companies with known exit value only include companies that are sold from one PE to another, as exit value is not known for most other transactions. Follow-on debt issuance is any debt issuance that happens after the buyout year. In Panel B, frequency of follow-on debt issuance is the number of years that have non-negative debt issuance (excluding the acquisition year but including the exit year if debt is issued in the exit year). "0" is the number of companies that issue no debt after the buyout year. In Panel C, only companies that have at least one follow-on debt issuance are considered. Total size of follow-on debt issuance is the amount of money received from all follow-on debt issuances, normalized by either the buyout size, or by the amount of debt issued to finance the buyout. Debt repayments are not included in the calculation, and numbers are not discounted if a company has several follow-on debt issuances. Some buyouts are financed with no debt (original debt amount is 0), which explains the mismatch in the last column of Panel C.
roughly the same amount of debt. Further analysis of the data shows that the capital structure of PE-backed companies is anything but static.

PE-backed companies frequently issue debt following the acquisition year. In fact, as Table 14 shows, almost 70% of companies issue debt at least one more time, and more than 40% of companies issue debt at least twice. This pattern is consistent among small and large companies, with 72% of companies with the value above £100mln, and 68% of companies with the value below issuing debt at least once after the buyout year.

The size of the follow-on debt issuance is also significant. For each company, I compute the total amount of debt that it issues over all years following the acquisition year (excluding the acquisition year). For an average company that issues debt at least once after the acquisition year, the total additional debt that it issues exceeds 50% of the original deal value. More strikingly, it is almost six times as large as the amount of debt issued to finance the buyout, though this is probably driven by companies which issued very little debt originally. Yet, for the median company the total additional amount of debt issued after the acquisition year is still 60% of the debt amount issued to finance the buyout.

Every time a company issues a significant amount of debt, it usually discusses the rationale in its annual report. Some of the reasons that companies often mention include one of the following. First and foremost, the loan that the PE and the bank agree on to finance the buyout usually includes the CAPEX and/or working capital facility, which are not fully drawn down at the time of the acquisition, but are available to the company down the road. If the company decides to take on that debt later, it is shown on the cash flow statement as debt issuance. Second, as the following section shows, PE-backed companies frequently acquire other companies, and debt is often issued to finance these follow-on acquisition. Third, companies sometimes refinance their original debt, by taking on new debt - often from a different bank - and fully repaying the old debt. Debt refinancings usually happen when credit conditions improve, and often come together with what is called dividends recapitalization, when a part of the proceeds from new debt issuance go to pay dividends to the PE.
Table 15: Debt issuance by companies with PE-ownership post-buyout

<table>
<thead>
<tr>
<th></th>
<th>Amount of debt added</th>
<th>Amount of debt repaid</th>
<th>Change in debt</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VI</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>distress</td>
<td>0.02***</td>
<td>−0.01</td>
<td>−0.00</td>
</tr>
<tr>
<td>t-stat</td>
<td>(1.74)</td>
<td>(−0.42)</td>
<td>(−0.50)</td>
</tr>
<tr>
<td>acquisitions</td>
<td>0.58***</td>
<td>0.47***</td>
<td>−0.33***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(26.10)</td>
<td>(4.13)</td>
<td>(−15.92)</td>
</tr>
<tr>
<td>CAPEX</td>
<td>0.40**</td>
<td>0.30</td>
<td>−0.18***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(14.04)</td>
<td>(1.56)</td>
<td>(−7.46)</td>
</tr>
<tr>
<td>debt repaid</td>
<td>0.71***</td>
<td>0.63***</td>
<td></td>
</tr>
<tr>
<td>t-stat</td>
<td>(24.83)</td>
<td>(7.23)</td>
<td></td>
</tr>
<tr>
<td>money from the PE</td>
<td>−0.41***</td>
<td>−0.43***</td>
<td>0.38***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(−10.84)</td>
<td>(−4.06)</td>
<td>(12.43)</td>
</tr>
<tr>
<td>money to the PE</td>
<td>0.62***</td>
<td>0.68***</td>
<td>−0.21***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(18.89)</td>
<td>(7.01)</td>
<td>(−6.93)</td>
</tr>
<tr>
<td>cash flow</td>
<td>−0.14***</td>
<td>−0.27***</td>
<td>0.13***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(−4.88)</td>
<td>(−3.23)</td>
<td>(5.63)</td>
</tr>
<tr>
<td>sales growth_{(t−1)→t}</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>log(DealValue)</td>
<td>0.01***</td>
<td>−0.00</td>
<td>0.01***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(2.70)</td>
<td>(−1.69)</td>
<td>(2.84)</td>
</tr>
<tr>
<td>deal leverage</td>
<td>−0.03*</td>
<td>0.09***</td>
<td>−0.06***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(−1.86)</td>
<td>(6.12)</td>
<td>(−3.13)</td>
</tr>
</tbody>
</table>

| N                      | 1270                 | 1270                  | 1270           |
| Year FE                | Yes                  | Yes                   | Yes            |
| Company FE             | No                   | Yes                   | No             |

Regressions are based on company-year observations from 351 companies for years when they were controlled by PE. Observations from the first year are always excluded; that is, if a company is acquired and controlled by a PE for three years, only observations from the second and third years are included in the regressions. LHS variable is: I and II) the amount of debt issued (positive if debt is issued and zero otherwise), III and IV) the amount debt repaid (positive if debt is repaid and zero otherwise), V and VI) the difference between issued and repaid debt. “distress” is a dummy variable that equals one if company’s cash flow is below interest expenses. “money from the PE” shows the amount of money the company receives from the PE. “money to the PE” shows the amount of money paid by the company to the PE. “Sales growth” measures sales growth between the current and the previous year. “DealValue” is the amount that was paid for the company at the time of buyout, and “deal leverage” is the fraction of the deal value that was financed by debt. All variables, except sales growth, log(DealValue), deal leverage, and dummy variables, are based on the information from the cash flow statement, and are scaled by the deal value. “debt added”, “debt repaid”, “money from the PE”, “money to the PE”, and sales growth are winsorized at 1% and 99% levels. Positive values of “acquisitions”, “CAPEX”, and “debt repaid”, and “money to the PE” indicate cash outflows; that is, debt repaid of 10 implies that debt value decreased. Positive value of “debt added” and “money from the PE” indicate a cash inflow, that is, debt added of 10 implies that debt value increased. Specifications II, IV, and VI, have errors clustered at the company-level. *, **, and *** indicate significance at the 10%, 5%, and 1% level.

C. Flows between the company and the PE

The presence of shareholders’ loans, and more generally the tendency of PE-backed companies to structure their stake as debt rather than equity, has implications for how the flow of money between the company and the PE should be analyzed. In the majority of
cases when the PE injects additional equity into the company, it is structured as new shareholders’ loans. Similarly, whenever there are some dividends paid, they are structured as repayments of shareholders’ loans or repayments of accrued interests on shareholders’ loans.

In addition to receiving the money from PE in a form of shareholders loans, PE-backed companies very often report some small proceeds from equity issuance. These proceeds usually come from the company’s management. Very often, PE-led buyouts involve the managers of the company; usually the managers pay for their shares at the time of the buyout, but sometimes they do it later, and that is reflected as proceeds from equity issuance on the cash flow statement.

Distinguishing the money coming from the PE and from the management is important: equity injections by the PE indicate that the company needs external financing as well as more generally gives information about the role of the PE in managing the company. Shares issued to the management tell very little about whether the company needs external financing. Therefore, when I describe the frequency of the transfer of money between the company and the PE below, I only count issuance and repayment of loan notes. At the same time, when I describe the total amount of money provided in a form equity, I count issuance of both new equity and loan notes. Even though issuance of shares to the management constitutes the vast majority of all instances of equity issuance in number, they are still orders of magnitude smaller than the combined amount of money that companies receive from the PE in a form of equity, in those rare cases when the PE stake is structured as equity.

PE-backed companies often receive additional financing from the PE: as Table 16 indicates, almost 45% of PE-backed companies received some additional money from the PE at least once after the acquisition year. Bigger companies receive money from the PE more often, though not by much: among companies than are smaller than £100mln, 41% receive additional PE money vs. 48% among companies larger than £100mln.

The amount of equity injections is also big. An average company, which receives money from the PE at least once, receives the amount of money that equals 28% of the original
### Table 16: Equity injections

#### Panel A: Frequency of follow-on equity injections

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>≥ 4</th>
<th>0</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>all companies</td>
<td>89</td>
<td>47</td>
<td>23</td>
<td>9</td>
<td>220</td>
<td>388</td>
</tr>
<tr>
<td>size &gt; £50mln</td>
<td>51</td>
<td>23</td>
<td>14</td>
<td>4</td>
<td>104</td>
<td>196</td>
</tr>
<tr>
<td>size &gt; £100mln</td>
<td>26</td>
<td>11</td>
<td>10</td>
<td>4</td>
<td>62</td>
<td>120</td>
</tr>
<tr>
<td>size &gt; £200mln</td>
<td>14</td>
<td>10</td>
<td>5</td>
<td>2</td>
<td>34</td>
<td>65</td>
</tr>
</tbody>
</table>

#### Panel B: Total size of follow-on equity injections (for companies with at least one equity injection)

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>q25</th>
<th>q50</th>
<th>q75</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>all companies</td>
<td>28.1%</td>
<td>4.2%</td>
<td>10.7%</td>
<td>28.6%</td>
<td>168</td>
</tr>
<tr>
<td>rel. to original deal size</td>
<td>69.9%</td>
<td>7.4%</td>
<td>20.9%</td>
<td>50.5%</td>
<td>168</td>
</tr>
<tr>
<td>size &gt; £50mln</td>
<td>18.4%</td>
<td>3.5%</td>
<td>9.3%</td>
<td>22.7%</td>
<td>92</td>
</tr>
<tr>
<td>rel. to original deal size</td>
<td>71.5%</td>
<td>6.7%</td>
<td>19.0%</td>
<td>43.7%</td>
<td>92</td>
</tr>
<tr>
<td>size &gt; £100mln</td>
<td>20.8%</td>
<td>2.8%</td>
<td>8.8%</td>
<td>22.5%</td>
<td>58</td>
</tr>
<tr>
<td>rel. to original deal size</td>
<td>96.4%</td>
<td>6.8%</td>
<td>19.8%</td>
<td>44.9%</td>
<td>58</td>
</tr>
<tr>
<td>size &gt; £200mln</td>
<td>27.2%</td>
<td>3.2%</td>
<td>6.2%</td>
<td>22.5%</td>
<td>31</td>
</tr>
<tr>
<td>rel. to original deal size</td>
<td>59.2%</td>
<td>6.8%</td>
<td>14.8%</td>
<td>74.7%</td>
<td>31</td>
</tr>
</tbody>
</table>

#### Panel C: Total money flow between the company and the PE

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>fraction</th>
<th>mean</th>
<th>q25</th>
<th>q50</th>
<th>q75</th>
</tr>
</thead>
<tbody>
<tr>
<td>all companies</td>
<td>(N = 388)</td>
<td>275</td>
<td>70.9%</td>
<td>-14.1%</td>
<td>-19.2%</td>
<td>-0.7%</td>
</tr>
<tr>
<td>total flow ≠ 0</td>
<td>127</td>
<td>32.7%</td>
<td>25.1%</td>
<td>5.6%</td>
<td>11.7%</td>
<td>27.1%</td>
</tr>
<tr>
<td>total flow &gt; 0</td>
<td>148</td>
<td>38.2%</td>
<td>-47.7%</td>
<td>-0.4%</td>
<td>-17.7%</td>
<td>-40.8%</td>
</tr>
<tr>
<td>total flow &lt; 0</td>
<td>136</td>
<td>69.4%</td>
<td>-1.8%</td>
<td>-16.2%</td>
<td>-0.1%</td>
<td>9.3%</td>
</tr>
<tr>
<td>size &gt; £50mln</td>
<td>(N = 196)</td>
<td>67</td>
<td>34.2%</td>
<td>18.2%</td>
<td>4.1%</td>
<td>9.3%</td>
</tr>
<tr>
<td>total flow ≠ 0</td>
<td>69</td>
<td>35.2%</td>
<td>-21.2%</td>
<td>-3.0%</td>
<td>-15.3%</td>
<td>-31.4%</td>
</tr>
<tr>
<td>total flow &gt; 0</td>
<td>43</td>
<td>35.8%</td>
<td>19.1%</td>
<td>3.9%</td>
<td>8.7%</td>
<td>21.9%</td>
</tr>
<tr>
<td>total flow &lt; 0</td>
<td>39</td>
<td>32.5%</td>
<td>-22.4%</td>
<td>-2.9%</td>
<td>-18.6%</td>
<td>-34.4%</td>
</tr>
<tr>
<td>size &gt; £100mln</td>
<td>(N = 120)</td>
<td>44</td>
<td>67.7%</td>
<td>1.5%</td>
<td>-14.4%</td>
<td>0.6%</td>
</tr>
<tr>
<td>total flow ≠ 0</td>
<td>23</td>
<td>35.4%</td>
<td>21.9%</td>
<td>3.9%</td>
<td>7.9%</td>
<td>21.9%</td>
</tr>
<tr>
<td>total flow &gt; 0</td>
<td>21</td>
<td>32.3%</td>
<td>-21.0%</td>
<td>-1.9%</td>
<td>-18.6%</td>
<td>-34.4%</td>
</tr>
</tbody>
</table>

Numbers in the table are based on acquisitions, in which the PE controls the company for at least two years. q25, q50, and q75 denote corresponding quantiles of the distribution, and N shows the number of observations. Follow-on equity injection is any transfer of money from the PE to the company that happens after the acquisition year, but does not include small but frequent inflows of money from stock compensations (see text for details). In Panel A, frequency of follow-on equity injections is the number of years that have non-negative equity injections (excluding the acquisition year but including the exit year if there is an equity injection in the exit year). “0” is the number of companies that receive no equity injections after the buyout year. Total size of follow-on equity injections is the total size of all follow-on equity injections, normalized by either the buyout size, or by the amount of equity issued to finance the buyout. Dividends and/or other repayments are not included in the calculation, and numbers are not discounted if a company has several follow-on equity injections. In Panel C, total money flow is the total size of follow-on equity injections minus any money paid by the company to the PE in a form of interests expenses and/or repayments of shareholder loans: negative total money flow means the company pays more money to the PE than what it receives from the PE, positive total money flow means the company receives more money from the PE than what it pays to the PE, and zero total money flow means that company neither pays, nor receives any money from the PE between the acquisition and exit years. Total money flow does not include small monitoring/service fees that companies sometimes pay to the PE, see text for details. total flow ≠ 0 only shows numbers for companies that have non-zero total money flow, and similar for total flow > 0 and total flow < 0.
buyout value, or 70% of the original equity value. The distribution of these additional equity injections is very skewed, with a few firms receiving a lot of money, and many firms receiving smaller amount. Indeed, as Penal B of the Table 16 indicates, that three quarters of companies receive the amount of money from the PE that is below the mean.

I do not systematically collect information on why the PE provides additional financing to the company. Nevertheless, based on my reading of the reports, companies usually mention two potential reasons why they receive money from the PE. First, they would sometimes receive money for follow-on acquisitions. Follows-on acquisitions are usually financed through debt issuance, but if a company has several follow-on acquisitions throughout the course of several years, the later follow-on acquisitions are usually financed by the money that the PE provides. This observation seems to indicate that the PE-backed companies have some target leverage.

The second - very frequent and important reason - explaining why the PE gives money to the company is rescue in distress, when the PE would provide money to the company because company’s cash flow is not enough to make required debt payments. This is usually - though not always - accompanied by changes to the debt structure of the company. It seems likely that in cases like this the PE renegotiates company’s debt with creditors. An example is useful here; company Amtico International was acquired by the PE firm ”AAC Capital” in an approximately £100mln transaction financed by different forms of debt. As usual, there were certain covenants associated with the debt that was used for the acquisition, and the company breached those covenants in 2009. Here is what the company reports in its 2009 annual report:

“On 13 November 2008, deep discounted bonds with an issue price of £30,024,000 were transferred by the registered holder, ABN Amro Bank MV, to AAC Capital Nebro Fund I LP, the majority investor, for a consideration of £41,637,000. The deep discounted bonds were then repaid and redeemed by AAC Capital Nebro Fund I LP through the issue of PIK loan notes with a value of £41,637,000.”

The direct equity injections of the type that I observe is not the only way for PEs to
Table 17: Money flow between the company and the PE

<table>
<thead>
<tr>
<th>Money flow:</th>
<th>PE → company: dummy</th>
<th>PE → company: amount</th>
<th>company → PE: amount</th>
<th>Net flow</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
<td>III</td>
<td>IV</td>
</tr>
<tr>
<td>distress</td>
<td>0.17***</td>
<td>0.12**</td>
<td>0.04***</td>
<td>0.04***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(4.90)</td>
<td>(2.50)</td>
<td>(5.45)</td>
<td>(2.79)</td>
</tr>
<tr>
<td>acquisitions</td>
<td>0.48***</td>
<td>0.45***</td>
<td>0.24***</td>
<td>0.25***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(6.93)</td>
<td>(2.92)</td>
<td>(14.50)</td>
<td>(2.77)</td>
</tr>
<tr>
<td>CAPEX</td>
<td>0.26</td>
<td>0.05</td>
<td>0.06***</td>
<td>0.02</td>
</tr>
<tr>
<td>t-stat</td>
<td>(2.81)</td>
<td>(0.46)</td>
<td>(2.60)</td>
<td>(0.50)</td>
</tr>
<tr>
<td>debt issued</td>
<td>-0.06</td>
<td>-0.08</td>
<td>-0.04***</td>
<td>-0.03</td>
</tr>
<tr>
<td>t-stat</td>
<td>(-1.26)</td>
<td>(-0.92)</td>
<td>(-4.01)</td>
<td>(-1.08)</td>
</tr>
<tr>
<td>debt repaid</td>
<td>0.22**</td>
<td>0.45***</td>
<td>0.18***</td>
<td>0.23***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(2.54)</td>
<td>(3.41)</td>
<td>(8.90)</td>
<td>(3.06)</td>
</tr>
<tr>
<td>cash flow</td>
<td>-0.03</td>
<td>-0.09</td>
<td>0.00</td>
<td>-0.01</td>
</tr>
<tr>
<td>t-stat</td>
<td>(-1.05)</td>
<td>(-1.06)</td>
<td>(0.67)</td>
<td>(-0.65)</td>
</tr>
<tr>
<td>sales growth(_{(t-1)}\rightarrow_t)</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00**</td>
<td>0.00</td>
</tr>
<tr>
<td>t-stat</td>
<td>(0.81)</td>
<td>(0.01)</td>
<td>(2.48)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>log(DealValue)</td>
<td>0.03**</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>t-stat</td>
<td>(3.15)</td>
<td>(1.25)</td>
<td>(1.25)</td>
<td>(0.33)</td>
</tr>
<tr>
<td>deal leverage</td>
<td>0.04</td>
<td>0.00</td>
<td>-0.07**</td>
<td>0.06***</td>
</tr>
<tr>
<td>t-stat</td>
<td>(0.71)</td>
<td>(0.05)</td>
<td>(-4.49)</td>
<td>(3.58)</td>
</tr>
</tbody>
</table>

N 1270 1270 1270 1270 1270 1270 1270 1270

Year PE Yes Yes Yes Yes Yes Yes Yes Yes
Company PE No Yes No Yes No Yes No Yes

Regressions are based on company-year observations from 351 companies for years when they were controlled by PE. Observations from the first year are always excluded; that is, if a company is acquired and controlled by a PE for three years, only observations from the second and third years are included in the regressions. LHS variable is: I and II) dummy variable showing whether there is an inflow of money from the PE to the company in a given year, III and IV) the amount of money the PE provided to the company in a given year (positive if there is equity injection and zero otherwise), V and VI) the amount of money paid by the company to the PE (positive if there is a payment and zero otherwise), VII and VIII) money given by the PE to the company minus the amount of money paid by the company to the PE (positive if the company receives more money than pays, negative if pays more than receives, and zero if neither). Amount of money paid by the company to the PE does not include monitoring fees that companies sometimes pay to the PE. LHS variables for specifications III – VIII are winsorized at 1% and 99% levels. “distress” is a dummy variable that equals one if company’s cash flow is below interest expenses. “Sales growth” measures sales growth between the current and the previous year. “DealValue” is the amount that was paid for the company at the time of buyout, and “deal leverage” is the fraction of the deal value that was financed by debt. All variables, including from the LHS, except sales growth, log(DealValue), deal leverage, and dummy variables, are based on the information from the cash flow statement, and are scaled by the deal value. Positive values of “acquisitions”, “CAPEX”, and “debt repaid” indicate cash outflows; that is, debt repaid of 10 implies that debt value decreased. Positive value of “debt issued” indicates a cash inflow, that is, debt issued of 10 implies that debt value increased. Specifications II, IV, VI, and VIII have errors clustered at the company-level. *, **, and *** indicate significance at the 10%, 5%, and 1% level.

Provide equity to the company. What also happens sometimes is the PE may attempt to repurchase the public debt of the company, if public debt was used to finance the buyout in the first place. This would usually happen if company’s financial position deteriorates after the buyout, and price of debt falls as the result. By repurchasing company’s debt, the PE effectively reduces the indebtedness of the company and helps it recover its financial health. Since such transactions are off-balance sheet, there is no way of learning about them from

112
annual reports of companies.

To further investigate the capital flows between the PE and the company, I construct the money flow variable. This variable adds up all money that is provided by the PE to the company, and subtracts the money paid by the company to the PE, every year after the acquisition year (excluding the acquisition year) and before the company exits the company. The positive money flow means that the PE invests more money into the company than what it receives from it before selling the company, and negative flow means that the company pays more money to the PE. Zero money flow means that there is no transfer of money between the company and the PE.

As it turns out, there is almost the same number of companies with negative, positive, and zero money flows: 33% of companies receive more money from the PE than what they pay to the PE, 38% pay more than what they receive, and 29% neither pay no receive any money from the PE. Surprisingly, as the size of the company increases, fewer companies have negative money flow (pay money to the PE), and more companies have positive money flow (receive money from the PE). Among companies smaller than £100mln, 41% have negative money flow and 31% have positive money flow, while among companies larger than £100mln 36% of companies have positive money flow, and 33% of companies have negative money flow.

The evidence about money flow completely refutes one of the traditional stories of the PE business. It is often argued that the PEs prioritize short-term gains over long-term benefits of their portfolio companies, and use every single opportunity to squeeze out all the resources within the company to benefit their shareholders. This is not true: not only PEs do not take away company’s resources, they are actively contributing resources to the company.
Table 18: Unconditional statistics about investments

Panel A: Size of CAPEX

<table>
<thead>
<tr>
<th></th>
<th>PE</th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>size</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>relative to sales</td>
<td>7.7%</td>
<td>10.4%</td>
<td>14.5%</td>
</tr>
<tr>
<td>relative to EBITDA</td>
<td>39.1%</td>
<td>30.5%</td>
<td>35.0%</td>
</tr>
<tr>
<td>relative to cash flow</td>
<td>35.5%</td>
<td>34.4%</td>
<td>44.7%</td>
</tr>
<tr>
<td>mean</td>
<td>7.7%</td>
<td>10.4%</td>
<td>14.5%</td>
</tr>
<tr>
<td>q25</td>
<td>1.2%</td>
<td>1.1%</td>
<td>1.7%</td>
</tr>
<tr>
<td>q50</td>
<td>2.9%</td>
<td>2.9%</td>
<td>3.9%</td>
</tr>
<tr>
<td>q75</td>
<td>7.2%</td>
<td>7.1%</td>
<td>10.7%</td>
</tr>
<tr>
<td>N</td>
<td>1709</td>
<td>23857</td>
<td>121117</td>
</tr>
</tbody>
</table>

Panel B: Size of follow-on acquisitions (for observations with non-zero acquisitions)

<table>
<thead>
<tr>
<th></th>
<th>PE</th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>size</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>relative to sales</td>
<td>13.3%</td>
<td>15.4%</td>
<td>15.2%</td>
</tr>
<tr>
<td>relative to EBITDA</td>
<td>64.0%</td>
<td>57.9%</td>
<td>57.9%</td>
</tr>
<tr>
<td>relative to cash flow</td>
<td>79.5%</td>
<td>71.4%</td>
<td>75.2%</td>
</tr>
<tr>
<td>mean</td>
<td>13.3%</td>
<td>15.4%</td>
<td>15.2%</td>
</tr>
<tr>
<td>q25</td>
<td>0.5%</td>
<td>0.5%</td>
<td>0.5%</td>
</tr>
<tr>
<td>q50</td>
<td>2.6%</td>
<td>2.6%</td>
<td>2.7%</td>
</tr>
<tr>
<td>q75</td>
<td>12.6%</td>
<td>9.9%</td>
<td>11.0%</td>
</tr>
<tr>
<td>N</td>
<td>450</td>
<td>6629</td>
<td>41048</td>
</tr>
</tbody>
</table>

Panel C: Frequency of follow-on acquisitions

<table>
<thead>
<tr>
<th></th>
<th>PE</th>
<th>UK</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>frequency</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>all companies</td>
<td>13.3%</td>
<td>15.4%</td>
<td>15.2%</td>
</tr>
<tr>
<td>size &gt; £50mln</td>
<td>64.0%</td>
<td>57.9%</td>
<td>57.9%</td>
</tr>
<tr>
<td>size &gt; £200mln</td>
<td>79.5%</td>
<td>71.4%</td>
<td>75.2%</td>
</tr>
<tr>
<td>mean</td>
<td>13.3%</td>
<td>15.4%</td>
<td>15.2%</td>
</tr>
<tr>
<td>q25</td>
<td>0.5%</td>
<td>0.5%</td>
<td>0.5%</td>
</tr>
<tr>
<td>q50</td>
<td>2.6%</td>
<td>2.6%</td>
<td>2.7%</td>
</tr>
<tr>
<td>q75</td>
<td>12.6%</td>
<td>9.9%</td>
<td>11.0%</td>
</tr>
<tr>
<td>N</td>
<td>450</td>
<td>6629</td>
<td>41048</td>
</tr>
</tbody>
</table>

“US” refers to the data from COMPUSTAT North America, and “UK” refers to the data from COMPUSTAT Global for companies that are headquartered in the UK. Observations with sic in the range 6000-6999 (financial services and institutions), with sic above 9000 (public administration), and with assets or sales below one million are excluded. “PE” refers to the main sample of companies in this paper. For all companies, CAPEX and acquisition numbers are taken from the cash flow statements. Numbers in Panel A are winsorized at 0.5% and 99.5% values. Panel B only includes company-year observations for which acquisition value is positive and not missing. Additionally, Panel B excludes all observations in the acquisition year for “PE” companies. In Panel C, frequency of acquisitions for “US” and “UK” companies is the number of years, for which the acquisition value is not zero and is not missing, divided by the total number of years that the company is in COMPUSTAT. Acquisition frequency is computed similarly for “PE” companies, except that the acquisition year is always excluded; for instance, if a company is controlled by a PE for three years, and it has an acquisition in the second year, but not the third year, acquisition frequency is 0.5. q25, q50, and q75 refer to corresponding quantiles of distribution, and N denotes the number of observations.

3.4.3. Investments and performance

A. Investments

There are two types of investments that companies report: usual capital expenditures, which most companies have every year, and infrequent but large acquisitions. Companies report these two types of investments separately on the cash flow statement, as CAPEX and acquisitions. While CAPEX size is probably correctly reflected on the cash flow statement,
a word of caution should be mentioned about acquisitions. The cash flow statement only reflects the actual amount of money paid for acquisitions, which in some cases may underestimate their actual size for two reasons. First, if a portion of the acquisition is financed through equity (that is, the target company receives shares in the new company), the equity amount will not be reflected on the cash flow statement. Second, if a company with some amount of debt is acquired, and debt is not repaid during the acquisition, then only the part paid in cash will be shown on the cash flow statement. Both of these concerns are negligible in the case of PE-backed companies: additional acquisition are almost never paid for by giving an equity stake in the new company to the owners of the acquired company, since the PE prefers to have concentrated control in their portfolio company. Furthermore, in my experience of reading through the annual reports, companies acquired through follow-on acquisition rarely have debt, and whenever they do have debt, it is usually repaid during the acquisition.

Table 18 shows how the size of CAPEX and acquisitions is different between companies with and without PE-ownership. The results suggest that if there is any difference in the size of investments between companies with and without PE-ownership, it is probably small. Indeed, the Table implies that CAPEX is greater for US-based companies without PE-ownership when CAPEX is measured relative to sales and the cash flow, but smaller when CAPEX is measured relative to EBITDA. Acquisitions follow a similar pattern: they are slightly smaller for US- and UK-based companies without PE-ownership when measured with respect to EBITDA and cash flow, but are slightly smaller when measured relative to sales.

What is different, however, is the frequency of having an acquisition. I measure acquisition frequency for each company without PE-ownership as a fraction of years with positive acquisition amount relative to the number of years the company has data for. I measure acquisition frequency for PE-backed companies similarly, with the exception that for each company in my sample I exclude the first year (that is, if a company is in my sample for five years, I measure acquisition frequency as the number of years with non-negative acquisition between years 2-5, and divide that by 4). I do not include the first year of observation for
PE-backed companies because it always has a positive acquisition amount, which simply reflects the original buyout.

The data shows that PE-backed companies have follow-on acquisitions much more often than companies without PE-ownership. A company in the UK on average has one acquisition every five years, and a company in the US has one acquisition every three and a half years.
At the same time, a PE-backed company in the UK has one acquisition as often as once every two and a half year.

Furthermore, the follow-on acquisitions are more frequent for larger companies, and more so for companies with PE-ownership. Among PE-backed companies with size larger than £200mln, half have at least one acquisition every two years - twice as often as companies without PE-ownership in the UK and the US. One concern might be that there is some mechanical effect because PE companies exit larger companies earlier (and, therefore, the denominator would be smaller for frequency calculation). This is not the case - the median exit time is four years for all companies, and also a group of companies that are larger than £50mln, larger than £100mln, larger than £200mln, while mean exit time is 4.7 years, 4.6 years, 4.6 years, and 4.5 years respectively.

It is possible that numbers above overestimate the actual frequency of PE-backed companies to have positive acquisitions. Sometimes, when PE-backed companies acquire other companies, they do not pay the whole acquisition amount right away, but schedule deferred payments, which may sometimes depend on the performance of the acquired company. In case they later make the deferred payment, it is reflected as an acquisition on company’s cash flow statement, and I count it as a new acquisition, therefore, increasing the frequency of acquisitions. I do not think, however, that this can explain the big difference in the acquisition frequency between companies with and without PE-ownership. First of all, these deferred acquisition payments are not scheduled often. Second, it might be the case that companies without PE-ownership also extend the payment for acquisitions, which would mean that the acquisition frequency is also overestimated for companies without PE-ownership; implicit evidence suggest that it’s indeed the case, as the average size of an acquisition is similar between companies with and without PE-ownership, as indicated by the Table 18.

D. Performance

To study the performance of PE-backed companies, I analyze two measures. First, how operating performance, measured by the growth rate of sales and EBITDA, of PE-backed
companies is different from the operating performance of companies without PE-ownership. Second, I study how the market value of PE-backed companies change when they are managed by the PE.

As Table 20 shows, companies with PE-ownership grow faster than other companies: the median sales growth rate is 10.2% for PE-backed companies, and is between 6% and 7% for companies without PE-ownership. Similarly, the median growth of PE-backed companies is 8.6%, and median growth rate of EBITDA is between 4% and 6% for companies without PE-ownership. While average numbers are also greater for PE-backed companies, they are not reported because of a big number of outliers, for which the average growth rate exceed 1000%.

To partially account for the fact that private equity firms unevenly invest in companies in certain industries over the business cycle, Panel B runs a regression that controls for industry and year fixed effects. Because of the outliers, sales growth and EBITDA growth are winsorized in Panel B: sales growth is winsorized at 0.5% and 99.5% values, and EBITDA growth is winsorized at 5% and 95% values, which correspond to EBITDA growth of -179% and 236%. Regressions confirm the results in the previous paragraph: the average sales growth is 25% higher for PE-backed companies than for US-based companies, and is 20% higher than for UK-based companies. Similarly, the average EBITDA growth is 12% higher for PE-backed companies than for US-based companies, and 11% higher than for UK-based companies.

There are two known factors that correlate with operating growth: size and leverage. Large companies grow slower than small companies, and companies with high leverage grow slower than companies with small leverage. In order to account for the effect of size and leverage, Panel B also shows results for the regression, in which proxies for size and leverage are used. For PE-backed companies, size is measured as the log of the buyout value, and for companies without PE-ownership size is measured as log of assets value. Similarly, leverage for PE-backed companies is measured as buyout leverage, and leverage for companies without PE-ownership is measured as the book value of debt over the book value of assets.
Table 20: Operating performance

Panel A: Summary statistics about operational performance

<table>
<thead>
<tr>
<th></th>
<th>Sales growth</th>
<th>EBITDA growth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>q25</td>
<td>median</td>
</tr>
<tr>
<td>PE</td>
<td>-0.3%</td>
<td>10.2%</td>
</tr>
<tr>
<td>Failed Deals</td>
<td>-1.9%</td>
<td>4.3%</td>
</tr>
<tr>
<td>US</td>
<td>-5.6%</td>
<td>6.1%</td>
</tr>
<tr>
<td>UK</td>
<td>-4.6%</td>
<td>7.2%</td>
</tr>
</tbody>
</table>

Panel B: Statistics controlling for year and industry growth

<table>
<thead>
<tr>
<th></th>
<th>LHS variables: Sales growth</th>
<th>LHS variables: EBITDA growth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>q25</td>
<td>median</td>
</tr>
<tr>
<td>PE</td>
<td>0.25***</td>
<td>0.14***</td>
</tr>
<tr>
<td>Failed Deals</td>
<td>-0.06</td>
<td></td>
</tr>
<tr>
<td>UK</td>
<td>0.05***</td>
<td>-0.01</td>
</tr>
<tr>
<td>size</td>
<td>-0.05***</td>
<td>-0.05***</td>
</tr>
<tr>
<td>leverage</td>
<td>-0.03***</td>
<td></td>
</tr>
<tr>
<td>Year FE</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Industry FE</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$R^2$</td>
<td>1.2%</td>
<td>2.9%</td>
</tr>
</tbody>
</table>

In Panel A, “US” refers to the data from COMPUSTAT North America, and “UK” refers to the data from COMPUSTAT Global for companies that are headquartered in the UK. In Panel B, “UK” and “PE” refer to dummy variables that equal one for this type of companies (variables for US are included, but dummy for “US” is dropped because of col linearity). Sales growth and EBITDA growth have many outliers, and so not mean values are reported in Panel A. In Panel B, sales growth is winsorized at 0.5% and 99.5% values, and EBITDA growth is winsorized at 5% and 95% values (correspond to -179% and 236% values). *, **, and *** indicate significance at 10%, 5%, and 1% levels.

Neither the proxy for size, nor for leverage are great, mainly because they are market-based for PE-backed companies, and accounting-based for all other companies. Moreover there is no time-variation in size and leverage proxies for PE-backed companies (since for each PE-backed company there is just one buyout price, but several sales growth values), but there is time-variation in both size and leverage proxies for companies without PE-ownership. Nevertheless, it is possible, at least qualitatively, to estimate the direction of mismeasurement, which is discussed below.

In regressions that control for size and leverage, average growth rate of PE-backed companies is still higher than average growth rate of companies without PE-ownership: by 14% for sales growth and by 11% for EBITDA growth. What is the likely effect of measuring leverage and size differently for companies with and without PE-ownership? Leverage of PE-backed companies is measured with an upward bias, since the buyout leverage is used for every time-period, but leverage on average falls, as Section 3.1 shows. Therefore, the growth rates of sales and EBITDA would probably be even higher for PE-backed companies.
Table 21: Example of IRR calculation, Ambassador Theatre Group buyout

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>purchase price</td>
<td>£127,312</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>debt issued</td>
<td></td>
<td>£77,500</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>debt repaid</td>
<td>-£1,553</td>
<td>-£2,200</td>
<td>-£18,368</td>
<td></td>
<td></td>
</tr>
<tr>
<td>equity issued</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>dividends</td>
<td>-£64,505</td>
<td>-£3,412</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>selling price</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>£406,093</td>
</tr>
<tr>
<td>CF to/from investors</td>
<td>-£97,312</td>
<td>-£11,442</td>
<td>£5,612</td>
<td>£18,368</td>
<td>£406,093</td>
</tr>
<tr>
<td>IRR</td>
<td>34.2%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This table shows how IRR is measured based on the buyout of Ambassador Theatre Group in 2010 by Exponent Private Equity and subsequent sale to Providence Equity Partners in 2014. Every year, except the first and last years, the total cash flow to investors is computed as the sum of debt repaid (positive) and issued (negative), plus all other securities repaid (positive) and issued (negative), plus any dividends or interest expenses that were actually paid (positive). The first year cash flow is the total acquisition value with a negative sign; the last year cash flow is the total exit value with a positive sign. Cash flows do not include 1) small monitoring fees that companies sometimes pay to the PE-sponsor, and 2) small proceeds from equity issuance that result from stock compensations, 3) any transaction costs. All numbers in the table are in thousands.

than for other companies if leverage was measured consistently.

Mismeasurement of size has two effects on the results, which go in the opposite directions: 1) market values of assets are usually higher than book value of assets, implying that size is overestimated for PE-backed companies, 2) assets of PE-backed companies grow over time, but buyout values are used as a proxy for size, implying that size is underestimated. Since the regression coefficient on size is negative, the first effect implies that operating growth of PE-backed companies is overestimated, and second effect implies that it is underestimated.

One can try to roughly estimate the first effect quantitatively: taking average market-to-book ratio of three, the sales growth would need to be adjusted down by 0.05*\log(3) = 5.5\%,$ and EBITDA growth would need to be adjusted down by 0.01*\log(3) = 1.1\%. Both numbers - sales growth and EBITDA growth - would still remain significantly higher than the growth rates of companies without PE-ownership in the US and UK.

Ideally, one would include other controls in the regression that could potentially be correlated with operating growth, to see if the effect of the PE-ownership still remains significant in the regression. This is not feasible, however, given that the data for PE-backed companies is very limited.

The second way to analyze the performance of PE-backed companies is to study how their
market value changes while they are controlled by the PE. Since the value is only observable at the dates of buyout and exit, it is natural to use the internal rate of return (IRR). IRR is usually measured on the equity stake of the company (that is, IRR that the PE-sponsor earns), but I instead focus on the IRR of the whole company to eliminate the effect of leverage and study how the value of the whole company changes. To compute IRR, I take distributions to/from all company’s shareholders (the private equity firm, managers, external debt providers, etc.), and take the whole company value at the times of buyout and sale. The interpretation of the IRR is, therefore, the constant rate of return that an investor would earn if she held both debt and equity of the company. Under the conditions of Modigliani-Miller, one could interpret the IRRs as the annual growth rate of company’s value if the company had zero leverage.

Table 21 shows an example of how IRR is computed, which is based on the buyout of Ambassador Theatre Group by Exponent Private Equity in November 2009, and the subsequent sale of the company to Providence Equity Partners in November 2013. In 2009, the company was acquired for approximately £125mln, which was structured as £124.5mln shareholder loans and £0.5mln equity. No external debt was used to finance the acquisition, but debt was issued in the subsequent year, and shareholder loans were partially repaid. Every year, except the first (2010) and final years (2014), cash flow to investors is measured as total distributions to investors from the company (repayment of debt/shareholder loans, interest expenses or dividends) minus all external financing that the company receives from investors (new debt, equity or shareholder loans). The value in the first year is the value at which the company is acquired, and value in the last year is the value at which the company is sold. Importantly, the company had an acquisition in year 2011, which does not affect the calculations of the IRR because the acquisition itself was not a flow of money to or from investors.

There are two types of distributions that I do not include in the IRR calculation. First, I do not include monitoring fees, which companies sometimes pay to the PE-owner. These fees are usually small, are not always reported, and, when reported, it is not always clear whether they are actually paid or simply added to the balance sheet liabilities. Second, I
do not include small but frequent equity inflows from share issuance, which usually arise because of options or shares issued to the employee pension trusts.

There are several important caveats to keep in mind before interpreting the IRR numbers. Most importantly, I only compute IRRs for PE-backed companies, which were exited through a secondary buyout (the company is acquired by another PE) because the exit value is not systematically observable for other types of exit. As Section 2 explains, a new special purpose vehicle (SPV) is created for each buyout, and the SPV begins to publish annual reports from the date of buyout. It is, therefore, possible to recover the buyout price from the first report published by the SPV (for the case of secondary buyouts, the buyout price is the exit price).

What is the direction of the bias in IRR values given that they are only computed for deals that ended with a secondary buyout? In my sample, secondary buyout is the second most frequent exit type (35.3% of cases), with trade sale (exit type, in which the company is sold to another company, 40.9% of cases), default/restructuring (4.2% of cases), and IPO\textsuperscript{22} (3.2% of cases) being three other most frequent cases. In terms of profitability, at least from the perspective of returns on equity, IPOs are by far the most profitable, followed by trade sale, then secondary buyouts, and finally default/restructuring cases.\textsuperscript{23} Therefore, computed IRRs might have some downward bias.

There are also some timing problems with how IRRs are measured. Generally, IRRs are computed as if the buyout happens on the day when the first report is published, and exit happened exactly one year after the last report is published, which is not always true. Going back to the example of Ambassador Theatre Group in Table 21, periods Y2010 through

\textsuperscript{22}One could argue that it is also possible to compute the exit value - and, hence, the IRR - for companies, for which the exit type was default or IPO. For the default cases, for instance, it is tempting to say that IRR should be -100%, but that is not necessarily true, since IRR is measured on the company rather than on equity stake of the company. That is, as long as there are some payments to debtholders after the buyout date and before the default date and/or debtholders recover anything in default, the IRR will be above negative 100%. For IPO cases, it is indeed possible to recover the exit value, though not always. For 13 companies in my sample that were exited through the IPO, exit value is only available for 8 cases; because of the small number, they are not included in the analysis.

\textsuperscript{23}See, for instance, Degeorge, Martin, Phalippou (JFE, 2016). Generally, it is a well-known fact among PE professionals. One, however, should keep in mind that profitability of exit channels is known from the perspective of returns of the PE funds, not returns on the company as a whole. The ordering can change if, for instance, companies with smaller leverage are more likely to be exited through an SBO than a trade sale.
Y2014 are assumed to be exactly one year apart from each other. Nevertheless, numbers Y2010 are taken from the report published on March 27th, 2010, while the company was acquired - and the price paid - on 9th November 2009. Similarly, numbers Y2014 are taken from the report published on March 29th, 2014 (by a new SPV), but the company was sold on November 30th, 2013. To the extent the timing discrepancy affects IRR calculations, the effect is probably unsubstantial; for instance, the IRR on the Ambassador Theatre Group, would change from 34.2% to 33.6% if the timing issues were taken into account properly. Transaction costs are also ignored in measuring IRRs. It is not possible to account for them because they are not always separately reported, but whenever they are reported, they can be of the order of 7-12%.

With all these concerns in mind, Table 22 reports the computed IRRs. The average and median IRR on an investments in a PE-backed company are strikingly big 43.6% and 27.3%. Values are much higher for small companies, but average IRR is still around 15% even among companies that were acquired for more than £200 mln. There is substantial variation over time; IRRs are highest for deals that were closed before 2008 (65% on average), and relatively lower for companies that were acquired between 2008 and 2010 (22.9%).

Numbers in Table 22 are hard to rationalize by risk-exposure, as there is no indication that companies with PE-ownership are overly exposed to systematic risk. Companies that were exited during the crisis period of 2008-2011 do not have overwhelmingly negative IRRs (average is 28.9%). It is possible that private equity firms were selectively not exiting worst-performing companies in crisis, but that would imply that IRRs should be disproportionately low for companies that were exited shortly after the crisis, and there is still no sign of very low returns. One could argue that even though systematic risk is small, it is always possible to use leverage to increase risk-exposure. This would also be misleading, however, since IRRs are computed on the whole company, rather than on the equity stake of the company. In other words, company’s exposure to systematic risks should not be affected by its leverage.

Interestingly, Table 22 indicates that what Preqin classifies as secondary buyout exits also include some default cases. Specifically, there are 11 cases (10.3%), for which IRR is negative. The median IRR among deals with negative IRR is -20%, which, extrapolated over
Table 22: IRR on the whole company, across secondary buyout deals

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>q25</th>
<th>q50</th>
<th>q75</th>
<th>IRR &lt; 0</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>whole period</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>all companies</td>
<td>43.6%</td>
<td>15.7%</td>
<td>27.3%</td>
<td>45.4%</td>
<td>11</td>
<td>107</td>
</tr>
<tr>
<td>size &gt; £50mln</td>
<td>33.4%</td>
<td>13.5%</td>
<td>22.0%</td>
<td>41.2%</td>
<td>7</td>
<td>51</td>
</tr>
<tr>
<td>size &gt; £100mln</td>
<td>23.2%</td>
<td>4.5%</td>
<td>17.1%</td>
<td>33.1%</td>
<td>6</td>
<td>29</td>
</tr>
<tr>
<td>size &gt; £200mln</td>
<td>14.3%</td>
<td>-0.9%</td>
<td>15.8%</td>
<td>20.7%</td>
<td>5</td>
<td>17</td>
</tr>
<tr>
<td>deals started before 2008</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>all companies</td>
<td>48.8%</td>
<td>16.9%</td>
<td>26.6%</td>
<td>57.8%</td>
<td>4</td>
<td>51</td>
</tr>
<tr>
<td>size &gt; £50mln</td>
<td>40.4%</td>
<td>13.5%</td>
<td>24.9%</td>
<td>45.4%</td>
<td>4</td>
<td>25</td>
</tr>
<tr>
<td>size &gt; £100mln</td>
<td>19.1%</td>
<td>2.3%</td>
<td>21.4%</td>
<td>32.9%</td>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>deals closed before 2008</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>all companies</td>
<td>65.3%</td>
<td>18.7%</td>
<td>36.8%</td>
<td>84.9%</td>
<td>2</td>
<td>32</td>
</tr>
<tr>
<td>size &gt; £50mln</td>
<td>50.0%</td>
<td>13.5%</td>
<td>33.0%</td>
<td>68.2%</td>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>size &gt; £100mln</td>
<td>21.6%</td>
<td>5.3%</td>
<td>21.4%</td>
<td>45.4%</td>
<td>2</td>
<td>10</td>
</tr>
<tr>
<td>deals started between 2008 and 2010</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>all companies</td>
<td>28.8%</td>
<td>6.2%</td>
<td>18.5%</td>
<td>35.4%</td>
<td>5</td>
<td>31</td>
</tr>
<tr>
<td>size &gt; £50mln</td>
<td>16.9%</td>
<td>8.9%</td>
<td>17.1%</td>
<td>29.4%</td>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>size &gt; £100mln</td>
<td>15.3%</td>
<td>4.1%</td>
<td>14.9%</td>
<td>25.8%</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>deals closed between 2008 and 2010</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>all companies</td>
<td>22.9%</td>
<td>17.1%</td>
<td>24.9%</td>
<td>26.6%</td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>deals started after 2010</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>all companies</td>
<td>51.7%</td>
<td>24.5%</td>
<td>35.6%</td>
<td>69.5%</td>
<td>2</td>
<td>25</td>
</tr>
<tr>
<td>size &gt; £50mln</td>
<td>41.8%</td>
<td>16.7%</td>
<td>25.8%</td>
<td>69.5%</td>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>deals closed after 2010</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>all companies</td>
<td>35.6%</td>
<td>14.8%</td>
<td>26.7%</td>
<td>41.5%</td>
<td>8</td>
<td>68</td>
</tr>
<tr>
<td>size &gt; £50mln</td>
<td>25.8%</td>
<td>14.7%</td>
<td>21.3%</td>
<td>32.8%</td>
<td>4</td>
<td>29</td>
</tr>
<tr>
<td>size &gt; £100mln</td>
<td>26.9%</td>
<td>3.8%</td>
<td>15.9%</td>
<td>35.6%</td>
<td>3</td>
<td>15</td>
</tr>
</tbody>
</table>

This table shows the IRR on the company, not IRR on PE-investment. Every year, except for the first and the last years, the total cash flow to investors is computed as the sum of debt repaid (positive) and issued (negative), plus all other securities repaid (positive) and issued (negative), plus any dividends or interest expenses that were actually paid (positive). The first year cash flow is the total acquisition value with a negative sign; the last year cash flow is the total exit value with a positive sign. IRR equates the discounted stream of these cash flows to zero. IRR’s are only computed for the set of SBO transaction (the company is sold from one PE to another), since the exit value is not known for most other transactions (see text for potential selection biases). q25, q50, and q75 denote the corresponding quantiles of the distribution, IRR < 0 denotes the number of deals with negative IRR, and N denotes the total number of deals.

five years (typical investments length), means the value of the company falls by 65%. Since the equity is a levered claim on company’s assets, it is very likely that the PE’s stake is completely wiped out, even though formal bankruptcy is not triggered. What follows - the acquisition of the company by another PE - is a common way of how restructuring cases are resolved.

What do numbers in Table 22 say about returns that private equity firms earn? On one hand, private equity firms hold equity of the portfolio companies, and so their returns should be higher. On the other hand, returns of private equity firms are value-weighted among all their portfolio companies, while Table 22 reports equally-weighted returns (and IRRs of
small companies are larger than IRRs of large companies). The overall effect is ambiguous, and there is no direct way of measuring returns of private equity firms from reports of portfolio companies because it is not clear what happens to shareholder loans at the time of exit: sometimes shareholder loans are repaid as a part of the exit price, and sometimes they are written off, with all value being paid for equity. Furthermore, company’s management has a stake in company’s equity, and it is not always clear how much.

Overall, results above show two empirical facts: 1) operating performance of PE-backed companies, measured as growth in sales or EBITDA or cash flow, is higher than that of other companies, and 2) returns on PE-backed companies - or the rate at which their market values grow - exceed the values that could simply be explained by risk exposure. While each of the two facts separately can be explained in a number of equally-plausible ways, jointly they argue strongly in favor of the fact that private equity firms add value to their portfolio companies.

First, higher operating growth and higher returns of PE-backed companies are consistent with the explanation that private equity firms add value to their portfolio companies. For higher sales growth, it is almost straightforward: if private equity firms added value to their portfolio companies, one would indeed expect to see them growing faster. It is not as straightforward with higher returns, though: even if private equity firms added value to the portfolio companies, one could argue that this should be expected at the time of the buyout, and reflected in the buyout price. Returns would then adjust so that private equity firms are only compensated for the risk they take, but not for the operational improvement that they bring to the company. This type of argument, however, might be misleading in the context of private equity.

To see why, imagine that there is a kind of operational improvement that only a private equity owner can implement. The very same company can then have different values depending on whether it is owned by a private equity firm, or by other investors. At the time of buyout, there is a range of prices on which the old shareholders and the private equity can agree. As long as the private equity industry in smaller than the rest of the market,
the buyout should happen at the lowest price of the range.\textsuperscript{24} This implies that the PE buys the company at the value that does not reflect future operational improvement, helps the company implement the improvement, and later sells the company at the higher value, generating superior returns.

For the argument in the previous paragraph to hold, there should be ways to increase company’s value that are only available to the private equity owner, but not to other non-PE investors. One such possibility that is supported by empirical evidence is the fact that, thanks to the PE-sponsor, financial distress is less severe for PE-backed companies. Expectations about the severity of financial distress can, in turn, affect policies that companies choose even before they fall in the financial distress. For instance, companies may optimally forgo profitable investments that require debt issuance to be financed, if doing so unreasonably increases their leverage. The model in the following Section shows this mechanism in action; by simply guaranteeing financial support in case of distress (which is, by the way, ex-ante and ex-post optimal to both the company and the PE), the PE changes the investment policies of the company, and increases its value.

There are not that many other explanations that could rationalize the superior returns that private equity firms generate, and also account for the fact that PE-backed companies have higher operational growth relative to companies without PE-ownership. As discussed above, risk-exposure alone is unlikely to explain the whole magnitude of average returns. Similarly, even though there is a bias in the type of companies for which returns are computed (secondary buyout cases), the selection bias probably understates returns than overstates them. The only alternative explanation that could explain outperformance of PE-backed companies is that private equity firms are able to fund significantly undervalued companies.\textsuperscript{25}

\textsuperscript{24}Some clarification might be needed here. It is true that several PE funds would often try to acquire the same company, and the ultimate buyer is determined through an auction. One might argue, therefore, that each PE should bid their highest valuation for the company, and there should be no outperformance subsequently. This, however, implies that the PEs have no outside options. As long as that is not the case - and it likely is not the case if the PE industry is smaller than the market - the private equity firm will bid the price that other non-PE investor would pay for the company.

\textsuperscript{25}The notion of undervaluation is tricky for private companies, but in the context of this paragraph it implies that the price that the private equity firm pays for the company is below the expected value of properly discounted future dividends.
The hypothesis that private equity firms find undervalued companies, however, does not explain why PE-backed companies have higher operational growth. In contrast, there is an asset pricing puzzle, which shows that, at least for public companies, expected returns are lower among companies with high growth rate. One could argue that private equity firms are better at predicting the future growth of companies than other investors; they would then buy companies, for which expected future growth is higher than what company’s existing shareholders predict, and will have higher superior return. This would imply, however, that private equity firms understand companies better than their shareholders - which is unlikely.

Generally speaking, it is likely true that, at least partially, the ability of private equity firms to find undervalued companies explains the outperformance of PE-backed companies. Indeed, private equity firms mostly buy private companies, and insufficient competition among buyers could explain why prices they pay could be lower than what they would be if the company was publicly traded. However, even this mispricing is likely due to the fact that small private companies face financial constraints, which prevent them from optimal investments. In this sense, mispricing correction by private equity firms is likely very different from mispricing correction by hedge funds, which generally do nothing to companies whose shares they buy.

3.5. Model

This paper models an infinite-horizon economy in continuous time. Markets are complete, and there is a riskless asset that pays a constant rate of interest \( r \) per unit of time\(^{26} \). In what follows, \( \mathbb{P} \) denotes the physical probability measure, and \( \mathbb{Q} \) denotes the risk-neutral probability measure in this economy.

A company in the economy is characterized by three state variables: amount of capital \( K \) it has, capital productivity \( X_t \), and interest payments on debt \( c_t \). Company’s production technology has a constant return to scale, and instantaneous profits equal \( y_t = X_t K \) per unit of time. Company’s debt is perpetual, which means that the company only pays interest expenses, but not the face value of debt. At each point in time, the company produces

\(^{26}\)Here and below “per unit of time” means that investors earn approximately \( r \Delta t \) within a short interval of time \( \Delta t \)
cash flow $y_t$ and has to pay $c_t$ to its debtholders. If $y_t < c_t$, the company is in distress, meaning that it does not have enough cash to pay interest expenses, and has to raise money externally by either issuing debt or equity; if the company cannot get external financing or chooses not to, it defaults.

The difference between the cash flow and interest payments ($y_t - c_t$) is taxed at a constant corporate tax rate $\tau$. If the remainder is positive, the company may invest some of it, in a way that is described in the following section. If there is still money left after taxes and investments, it is paid to equityholders as dividends; that is, the company cannot save money.

Debt allows companies to exploit benefits of tax-shield, but it also comes with cost, both explicit and implicit. Higher leverage increases the probability of default, and makes further debt issuance costlier. Moreover, firms with greater leverage invest less, as is shown below. When choosing the optimal debt policies, companies trade off these costs and benefits.

Capital is traded on the external market, and its price depends on the productivity level: price of a unit of capital with productivity $X$ is $XH$, where $H$ is a constant.\footnote{Price of capital is proportional to $X$ because, as is shown later, the value of the company that can operate this capital is also proportional to $X$. In the data, the distribution of market-to-book ratios is mostly stable, implying that the value of the company cannot diverge from the price of capital over time. It is also natural to think that high-productive capital has a larger price than low-productive capital} Price of capital directly affects the costs of investments, and also determines how much of the debt value debtholders can recover in default.

There are two types of companies in the model, to which I refer as PE-backed, and non-PE-backed companies. The only difference between the two is the type of external financing that is available to them. PE-backed companies have access to both debt and equity, and they optimally choose which securities to issue and when. Companies without PE-ownership cannot issue equity, and only have access to debt. While the assumption that companies without PE-ownership cannot issue equity is rather extreme, the model would have similar results if the assumption was instead that all forms of external financing become more expensive when company’s leverage increases. This assumption is exogenous in the model, but the following section explicitly discusses it by showing how it can be rationalized, and
putting it in the context of available empirical evidence about equity issuance.

I start by describing the investment and financing policies of companies without PE-ownership. In order to derive the optimal policies, as well as the pricing of debt and equity, one needs to first derive the processes for the three state variables: $dX_t$, $dK_t$, and $dc_t$. It is assumed that $X_t$ follows Geometric Brownian motion under $Q$:

$$\frac{dX_t}{X_t} = \mu dt + \sigma dW_t \quad (3.1)$$

The evolution of the other two state variables is endogenous. $K_t$ will change because of the company’s investments. $c_t$ will change because the company will issue additional debt - to finance investments that cannot be financed out of the cash flow, and to cover interest expenses when the cash flow is not sufficient. Therefore, $dK_t$, and $dc_t$ should be endogenously determined in equilibrium. For instance, the process for $dc_t$ is affected by company’s current and future debt issuance, and, therefore, affects the pricing of debt, which in turn affects company’s decision to invest, which in turn affects the pricing of debt and the process for $dc_t$. Nevertheless, there is a closed-form equilibrium solution for both processes, which makes the pricing of other securities easier.

3.5.1. Investments

In the model, companies invest when they have investment opportunities. Investment opportunities allow companies to acquire more capital. There are two types of investment opportunities that companies can have: small investment opportunities available constantly, and large but infrequent investment opportunities. Loosely speaking, small investment opportunities can be thought of as CAPEX, and large investment opportunities can be thought of as acquisitions.

Large investment opportunities arrive at a rate $\lambda$ per unit of time. A large investment opportunity...

---

28Strictly speaking, the process for the productivity of company’s capital $X_t$ is also partially endogenous. The way investments are modeled, the productivity of the capital the company acquires is different from the productivity of company’s existing capital, and so the overall productivity of company’s capital will depend on the decision to invest.

29Informally, it means that the probability to find an investment opportunity is $\lambda \Delta t$ within a short interval of time $\Delta t$. 

129
opportunity allows a company to buy at most $K_{new}$ units of capital with low-productivity $X_{low}$ and install this capital inside the company. There are no costs of investment aside from the price of capital. Once the capital is installed, its productivity grows up to $X_{high} > X_{low}$. The fact that companies pay for low-productive capital, but are able to transform it into high productive capital is what makes investment opportunities profitable (or, in other words, NPV-positive).

Importantly, investment opportunities do not change the productivity of company’s existing capital: by taking a large investment opportunity, a company gets a new capital stock with its own productivity. A simple way to think about large investment opportunities is that they allow a company to establish a new plant, which works independently of company’s other plants. It would imply, however, that a pair $(K, X)$ does not fully characterize a company, as it shows company’s total amount of capital and productivity, while the company may have several capital units after a number of investments. The following assumption guarantees that capital and productivity can be aggregated:

**Assumption 2** Consider a company at time $t$ that has $K$ units of capital with productivity $X_t$ that receives a large investment opportunity, which allows the company to buy $K_{new}$ units of capital with productivity $X_{low}$ and install them within the company with productivity $X_{high}$. Then:

1. Size of the investment opportunity is proportional to the amount of capital the company already has:

$$K_{new} = \delta_2 K$$

2. Productivity of capital that the company buys is the same as the productivity of capital that the company already has:

$$X_{low} = X_t$$

3. Productivity of capital once it is installed is proportional to the productivity of capital the company buys:

$$X_{high} = (1 + \gamma)X_t$$
4. After new capital is installed, its productivity will evolve according to the same equation (3.1) as the productivity of company’s old capital; in other words, there are no additional idiosyncratic shocks to the productivity of new capital.

The assumption guarantees that company’s different capital units can be aggregated into one: there is only one process $X_t$ that characterizes productivity of all capital units, and production technology is constant return to scale. Note that company’s cash flow grows proportionally when it takes a large investment opportunity:

$$X_t K \rightarrow X_t K + (1 + \gamma)X_t \delta_2 K = (1 + \delta_2(1 + \gamma))X_t K$$

$$y_t \rightarrow (1 + \delta_2(1 + \gamma))y_t$$

(3.2)

Small investment opportunities are modeled similarly, except that the size of small investment opportunities is proportional to $dt$. At any point in time, a company can buy $\delta_1 K dt$ units of capital with productivity $X$, and install this capital within the company with productivity $(1 + \gamma)X$. To keep the aggregation intact, small investment opportunities also satisfy the assumption.

While the mathematical definition of the two types of investment opportunities is similar, they have a profoundly different effect on company’s financial policies, mainly because companies can finance small investment opportunities out of the cash flow, but they have to raise external financing to pay for large investment opportunities.

Similarly to equation (3.2), though informally, company’s cash flow grows proportionally when the company takes a small investment opportunity:

$$X_t K \rightarrow X_t K + (1 + \gamma)X_t \delta_1 K dt = (1 + \delta_1(1 + \gamma)dt)X_t K$$

$$y_t \rightarrow (1 + (1 + \gamma)\delta_1 dt)y_t$$

$$dy_t = \delta_1(1 + \gamma)y_t dt$$

(3.3)

Notice that the price that the company pays to take investment opportunities - both large

---

30In principle, profitability of large and small investment opportunities $\gamma$ might be different, but they are assumed to be the same between the two types of investments; that is, there is just one parameter $\gamma$ that characterizes the profitability of both types of investments.
and small - is also proportional to the cash flow \(HX_t\delta_1Kdt = \delta_1Hy_tdt\) for small and \(HX_t\delta_2K = \delta_2Hy_t\) for large investment opportunities). I argue that this suggests that \(y_t\) alone is a sufficient state variable to substitute both \(X_t\) and \(K_t\). It can be formally shown through the “guess and verify” method.

By construction, investment opportunities are profitable for companies: companies buy cheap low-productive capital, but install it as high-productive capital. Nevertheless, company’s leverage may prevent companies from taking some investment opportunities. First of all, leverage may cause debt overhang problem: when an overlevered company takes an investment opportunity, equityholders bear the full cost of the investment, but creditors gain some benefits (because probability of default goes down). Second, debt is the only source of external financing available to companies without PE-ownership, and price of debt (i.e. costs to issue debt) increases with company’s leverage; therefore, highly-levered companies will choose not to finance investment opportunities because costs do not justify the benefits. Therefore, highly levered companies may optimally choose to forgo some of their investment opportunities, even if all investment opportunities are ex-ante good.

Intuitively, there should be some threshold level of leverage, so that the company invests if the leverage is below this value, and the company does not invest if the leverage is above this value. In terms of the state variables \(c\) (interest expenses on company’s debt) and \(y\) (company’s cash flow), the investment threshold can be summarized as a number \(z_{inv}\), such that the company invests if \(c_t/y_t < z_{inv}\), and does not invest if \(c_t/y_t > z_{inv}\). To simplify the discussion below, I assume that \(z_{inv} < 1\).

Since company’s investments are described by the threshold rule, the evolution of company’s cash flow \(y_t\) follows the process:

\[
\begin{align*}
\frac{dy_t}{y_t} &= \left(\mu + \delta_1 (1 + \gamma)\right)dt + \sigma dW_t + \delta_2 (1 + \gamma) dN_t \quad \text{if} \quad \frac{c_t}{y_t} < z_{inv} \\
\frac{dy_t}{y_t} &= \mu dt + \sigma dW_t \quad \text{if} \quad \frac{c_t}{y_t} > z_{inv}
\end{align*}
\]

\((3.4)\)

\(^{31}\)Strictly speaking, there should be two investment thresholds - one for big investments, and another for small investments. However, separating the two does not conceptually change the math, but makes it messier, by adding an extra region to consider. Therefore, to simplify the description, I consider that the two thresholds coincide, though I do have both thresholds when I solve the model.
In order to fully characterize company’s equity and debt, one also needs to know the process for \( dc_t \). Company’s interest expenses will change every time the company increases the amount of debt. There are two main reasons for the company to issue debt: to finance investment opportunities, but also payments in distress, when company’s cash flow is not sufficient to cover interest expenses.

3.5.2. Process for \( dc \)

Before discussing the process for \( dc \), it can be useful to introduce three regions, on which \( dc \) will have a different dynamic. Region I is \( c/y \in [0, z_{inv}] \), which characterizes company’s investments. In Region I, \( dc \) will change every time a large investment opportunity arrives and requires the company to issue debt to finance the investment. Region III is \( c/y \in (1, z_{def}] \), which characterizes the distress region (with \( z_{def} \) being the ultimate default threshold). The company no longer invests in this region, and its cash flow is not sufficient to cover interest expenses \( (y_t < c) \). Therefore, the company is forced to continuously issue debt to cover interest expenses on previously issued debt. Such situation is obviously not sustainable for a long period of time, and debtholders will stop providing money to the company if interest expenses (relative to cash flow) become too high, which gives rise to the \( z_{def} \) boundary. Region II is \( c/y \in (z_{inv}, 1] \), and the company no longer invests in this region, but also has sufficient resources to pay interest expenses without raising external financing. \( dc = 0 \) in Region II. Figure 24 illustrates the three Regions.

Two things should be mentioned about the three Regions. First, it is possible that company’s cash flow will not be sufficient to both pay interest expenses and buy capital for small investments in the Region I. Second, companies with very low value of \( c/y \) (i.e. low-levered companies) may optimally prefer to issue debt to better exploit benefits of tax-shield. These issues do not affect the derivations and are added later.

Let \( D(y, c) \) be the value of company’s debt; \( D(y, c) \) has a closed-form solution, but for now we will express it as a general function. Assume \( c/y \in \) Region I, and the company receives a large investment opportunity. The company needs to issue \( H\delta y \) of additional debt to finance this investment. If newly issued debt has the same seniority as company’s old debt,
the following equation implicitly defines what is the new level of interest expenses $c_{new}$ the company will have:

$$H\delta y = D(y_{new}, c_{new}) - D(y, c)$$

(3.5)

where $y_{new} = y(1 + \delta y(1 + \gamma))$ is the new cash flow in case the investment is financed. What equation (3.5) shows is that the company promises to increase future debt payments to a new level $c_{new}$, but in return debtholders provide $H\delta y$, which the company uses to finance the investment.

Investment opportunities arrive according to a Poisson process $dN_t$, and every time an investment opportunity arrives, the company takes it. By taking an investment opportunity, the company increases its $c$ to the level $c_{new}$ explicitly defined in equation (3.5). Therefore:

$$dc = (c_{new}(y, c) - c)dN_t \quad \text{if} \quad \frac{c}{y} \in \text{Region I}$$

(3.6)

In Region II, the company does not issue debt, and so $dc = 0$. In Region III, the company continuously issues debt because its cash flow is too small to fully cover interest expenses. Let $dD$ be the value of debt the company needs to issue to avoid default at the moment when its cash flow is $y$ and current interest expenses are $c$. The following formula connects how interest expenses should change in order to issue $dD$:

$$\left(c - (y + (c - y)\tau)\right)dt = (c - y)(1 - \tau)dt = dD = dc\frac{\partial D(y, c)}{\partial c}$$

(3.7)

The very left-hand side of equation (3.7) is the difference between the required coupon payment $c$, and the amount of money the firm has on hands - its profits $y$, and tax return from the government $\tau(c - y)$. This difference is the shortfall that equityholders must but can not pay to debtholders; this difference should equal to change in debt value $dD$, which is in turn achieved by promising a higher coupon payment in the future. It is clear from equation (3.7) that it is only possible to issue new debt if $\frac{\partial D}{\partial c} > 0$, that is, if value of debt increases when the firm promises to pay more in the future. Equation (3.7) shows to derive
the dynamics of $dc$ in Region III:

$$dc = \frac{(c - y)(1 - \tau)}{\partial D/\partial c} \ dt$$ \hspace{1cm} (3.8)

Together, equations (3.4), (3.6), and (3.8) fully define the dynamic of company’s state variables:

$$\begin{cases} 
\frac{dy_t}{y_t} = (\mu + \delta_1 (1 + \gamma)) dt + \sigma dW_t + \delta_2 dN_t & \text{if } \frac{c}{y} \in [0, z_{inv}] \\
\frac{dc_t}{c_t} = (c_{new}(y_t, c_t) - c_t) dN_t \\
\frac{dy_t}{y_t} = \mu dt + \sigma dW_t & \text{if } \frac{c}{y} \in (z_{inv}, 1] \\
\frac{dc_t}{c_t} = 0 & \text{if } \frac{c}{y} \in [1, z_{def}] \\
\frac{dy_t}{y_t} = \mu dt + \sigma dW_t \\
\frac{dc_t}{c_t} = (1 - \tau)(c_t - y_t) \frac{\partial D(y_t, c_t)}{\partial c} & \text{if } \frac{c}{y} \in (1, z_{def}] 
\end{cases} \hspace{1cm} (3.9)$$

3.5.3. Debt and default

We will show that $D(y, c)$ satisfies the following system:

$$\begin{cases} 
rd(y, c) = c + (\mu + \delta_1 (1 + \gamma)) y D_y' + \frac{\sigma^2 y^2}{2} D_{yy}' & \text{if } \frac{c}{y} \in [0, z_{inv}] \\
rD(y, c) = c + \mu y D_y' + \frac{\sigma^2 y^2}{2} D_{yy}' & \text{if } \frac{c}{y} \in (z_{inv}, z_{def}] 
\end{cases} \hspace{1cm} (3.10)$$

The way to show this is to separately set up the HJB equations for $D(y, c)$ on each of the three Regions, and show that they coincide with the system (3.10). In order to set up the HJB equations, one needs to figure out the immediate payments to debtholders on each Region.

Start with Region I. That’s the region, in which the company’s cash flow is enough to pay interest expenses $c_t dt$. Therefore, the correct HJB for the value of debt takes the following
form in Region I:

\[ rD(y, c) = c + (\mu + \delta_1(1 + \gamma)) yD'_y + \frac{\sigma^2 y^2}{2} D''_{yy} + \lambda \left( D(y_{\text{new}}, c_{\text{new}}) - D(y, c) - \delta_2 H y \right) \] (3.11)

where \( y_{\text{new}} = y(1 + \delta_2(1 + \gamma)) \) is the cash flow the company gets after taking the large investment opportunity. The last part of the equation (3.11) shows how the value of debt changes when the company issues debt to finance a large investment opportunity - future payments to debtholders increase to a new level \( c_{\text{new}} \), but debtholders give money to the company to finance the investment. Note, however, that this part equals zero, consistent with the equation (3.5). Indeed, when debtholders provide money to the company to take the investment, they do it on terms, which make them indifferent (because markets are competitive), and so the value of debt does not change when the company takes a large investment.\footnote{Notice, however, that the situation would be different, if the company had access to a alternative form of external financing, say equity. In case the investment is financed by issuing equity, \( y_t \) increases, without debtholders contributing any money. Nevertheless, the equation for debt value will still have a closed form solution, as is shown later for the case of the value of debt for companies with PE-ownership.} Therefore, \( D(y, c) \) indeed satisfies system (3.10) in the Region I.

\( D(y, c) \) trivially satisfies system (3.10) in the Region II, since the second equation of the system is literally the HJB equation for \( D(y, c) \) in that region.

Consider Region III now. In this regions, company’s cash flow is fully used to pay interest expenses, and it is not enough. The immediate payment to debtholders in this region is \( y + \tau(c - y) \), which is company’s cash flow plus the return from the government. Therefore, the correct HJB equation for \( D(y, c) \) should take the following form:

\[ rD(y, c) = y + \tau(c - y) + \frac{(1 - \tau)(c - y)}{D'_c} D'_c + \mu y D'_y + \frac{\sigma^2 y^2}{2} D''_{yy} \] (3.12)

which simplifies to

\[ rD(y, c) = c + \mu y D'_y + \frac{\sigma^2 y^2}{2} D''_{yy} \] (3.13)

which again coincides with the second equation of the system (3.10) in the Region III. Intuitively, what this shows is that debtholders are indifferent between the company paying them the full amount \( c dt \), or paying a smaller amount, but also increasing future interest
The graph shows the value of company’s debt (green curve, right axis), and marginal interest rates at which next unit of debt can be raised (blue curve, left axis). Company’s cash flow $y_t$ is fixed at one, and values are shown as the second state variable - interest expenses - changes (horizontal axis). Company’s debt value increases when interest expenses become higher, but debt also becomes riskier. Therefore, both the value of debt and interest rates to issue more debt increase with $c$. The company defaults when it can no longer issue more debt and is in distress; graphically, it means that the blue curve converges to infinity, or green curve becomes completely flat.

payments by the level $dc$. This follows from how $dc$ was modeled, which was to make debtholders even. A different way of interpreting this results is to say that debt is issued to a completely new debtholder, who gives the company $(1 - \tau)(c - y)dt$ that is used to complement company’s cash flow to pay old debtholders $cdt$ in full. As long as it happens at every point in time, every company’s debtholder receives what it was promised to at the time of debt issuance, as long as the company can raise additional debt.

The previous paragraph naturally leads to the conclusion that the company will default once it is in distress and can no longer issue additional debt. Mathematically, the company can issue additional debt as long as $\partial D/\partial c > 0$, meaning the value of debt can be increased by promising higher interest payments in the future. Therefore, the following condition determines the default threshold for a company without PE-ownership\textsuperscript{33}:

$$\frac{\partial D}{\partial c}(y_{def}(c), c) = 0 \quad (3.14)$$

\textsuperscript{33}Notice that it is consistent with the default rule being $c/y = z_{def}$
Figure 23 visualizes the default in the model. The green curve on the graph shows how the value of company’s debt changes when the company offers a higher coupon payment to its debtholders; the blue curve shows the marginal interest rates, at which the next dollar of debt can be raised. Firm with no debt \((c = 0)\) can issue the first dollar of debt at \(r = 6\%\), which is used as the risk-free rate to solve the model. Marginal interest rates stay low and close to the risk-free rate for companies that have sufficiently low interest payments. However, as the coupon payment becomes sufficiently high, the green curve becomes flatter, which means that the company has to promise to increase future interest payments by a lot to raise an additional dollar of debt. As coupon-to-cash flow ratio approaches its default value, the green curve becomes completely flat, which means that future promises of higher coupon payments do not increase debt value, or, equivalently, next unit of debt can only be issued at the infinite rate. At that point, the company cannot issue new debt, and cannot pay interests on its debt out of the cash flow either, and so default happens.

Once we know that \(D(y, c)\) satisfies system (3.10), it is easy to solve for the value of debt, since the system has a closed form solution of the following form:

\[
D(y, c) = \frac{c}{r} + B_1 y^{\beta_1} c^{1-\beta_1} + B_2 y^{\beta_2} c^{1-\beta_2} \quad \text{if} \quad \frac{c}{y} \leq z_{inv} \\
D(y, c) = \frac{c}{r} + B_3 y^{\beta_3} c^{1-\beta_3} + B_4 y^{\beta_4} c^{1-\beta_4} \quad \text{if} \quad \frac{c}{y} > z_{inv}
\]  

(3.15)

where \(\beta_1\) and \(\beta_2\) are the negative and the positive roots of the following quadratic equation:

\[
r = (\mu + \delta_1 (1 + \gamma)) \beta + \frac{\sigma^2}{2} \beta (\beta - 1) 
\]  

(3.16)

and \(\beta_3\) and \(\beta_4\) are the negative and the positive roots of the following quadratic equation:

\[
r = \mu \beta + \frac{\sigma^2}{2} \beta (\beta - 1) 
\]  

(3.17)

Coefficients \(B_1, B_2, B_3, B_4\) need to be determined through boundary conditions. It is easy to argue that \(B_2 = 0\), since the value of debt cannot exceed \(c/r\) (the value of risk-free debt), but \(y^{\beta_2} c^{1-\beta_2}\) converges to \(+\infty\) when \(y\) increases. Two boundary conditions are determined
by the value matching and smooth-pasting\textsuperscript{34} of $D(y, c)$ along the boundary $c/y = z_{inv}$.

$$\frac{c}{r} + B_1 y^{\beta_1} c^{1-\beta_1} = \frac{c}{r} + B_3 y^{\beta_3} c^{1-\beta_3} + B_4 y^{\beta_4} c^{1-\beta_4}$$  \hspace{1cm} (3.18)$$

$$\beta_1 B_1 z_{inv}^{1-\beta_1} = \beta_3 B_3 z_{inv}^{1-\beta_3} + \beta_4 B_4 z_{inv}^{1-\beta_4}$$  \hspace{1cm} (3.19)$$

The final boundary condition characterizes the value of debt in default. Absolute priority rule applies, and debtholder get the value of company’s assets. It is assumed, however, that there are some bankruptcy costs $\alpha$, which are proportional to the value of assets, so debtholders only recover $(1 - \alpha)(1 - \tau)Hy$. The value of debt in default, and the default threshold (3.14) can be summarized as follows:

$$\frac{c}{r} + B_3 y^{\beta_3} c^{1-\beta_3} + B_4 y^{\beta_4} c^{1-\beta_4} = (1 - \tau)(1 - \alpha)Hy$$  \hspace{1cm} (3.20)$$

$$\frac{1}{r} + (1 - \beta_3)B_3 z_{def}^{\beta_3} + (1 - \beta_4)B_4 z_{def}^{\beta_4} = 0$$  \hspace{1cm} (3.21)$$

Equations (3.18) - (3.21) can be simplified to the following system:

$$\begin{cases} 
B_1 z_{inv}^{\beta_1} = B_3 z_{inv}^{\beta_3} + B_4 z_{inv}^{\beta_4} \\
\beta_1 B_1 z_{inv}^{1-\beta_1} = \beta_3 B_3 z_{inv}^{1-\beta_3} + \beta_4 B_4 z_{inv}^{1-\beta_4} \\
\frac{z_{def}}{r} + B_3 z_{def}^{1-\beta_3} + B_4 z_{def}^{1-\beta_4} = (1 - \alpha)(1 - \tau)H \\
\frac{1}{r} + (1 - \beta_3)B_3 z_{def}^{\beta_3} + (1 - \beta_4)B_4 z_{def}^{\beta_4} = 0 
\end{cases}$$ \hspace{1cm} (3.22)$$

which is a system of four equations with four unknowns $\{B_1, B_3, B_4, z_{def}\}$, of which three unknowns enter the system linearly.

\textsuperscript{34}Notice that equation (3.18) is written for the derivative to be taken with respect to $y$, but the system would be equivalent to taking the derivative with respect to $c$. 

139
3.5.4. Equity value and investment policies

Denote $E(y, c)$ the value of company’s equity. In contrast to $D(y, c)$, there is no closed-form solution for $E(y, c)$. It should satisfy the following set of HJB equations:

\[
\begin{cases}
  rE(y, c) = (1 - \tau)(y - c) - H\delta_1 y + (\mu + \delta_1(1 + \gamma))yE'_y + \\
  \frac{\sigma^2 y^2}{2}E''_{yy} + \lambda(E(y_{\text{new}}, c_{\text{new}}) - E(y, c)) & \text{if } \frac{c}{y} \in [0, z_{\text{inv}}] \\
  rE(y, c) = (1 - \tau)(y - c) + \mu yE'_y + \frac{\sigma^2 y^2}{2}E''_{yy} & \text{if } \frac{c}{y} \in (z_{\text{inv}}, 1] \\
  rE(y, c) = \frac{(c - y)(1 - \tau)}{D'_c}E'_c + \mu yE'_y + \frac{\sigma^2 y^2}{2}E''_{yy} & \text{if } \frac{c}{y} \in (1, z_{\text{def}}]
\end{cases}
\]  

(3.23)

A useful simplification to solve this system is to notice that $E(y, c)$ is homogenous of degree one, and therefore can be expressed as $E(y, c) = ye(c/y) = ye(z)$, and rewrite the system (3.23) in terms of $z$. Function $E(y, c)$ should satisfy some boundary conditions, which are described in the Appendix D. The investment threshold is characterized by the following equation:

\[E(y_{\text{new}}, c_{\text{new}}) = E(y, c) \iff (1 + \delta_2(1 + \gamma))e(z_{\text{new}}) = e(z_{\text{inv}}) \]  

(3.24)

where $z_{\text{new}}$ satisfies the following equation:

\[(1 + \delta_2(1 + \gamma))\left(\frac{z_{\text{new}}}{r} + B_3z_{\text{new}}^{-1-\beta_3} + B_4z_{\text{new}}^{-1-\beta_4}\right) = \frac{z_{\text{inv}}}{r} + B_1z_{\text{inv}}^{-1-\beta_1} + H\delta_2
\]  

(3.25)

which is equivalent to $D(y_{\text{new}}, c_{\text{new}}) = D(y, c) + H\delta_2 y$

3.5.5. Issuance of debt to exploit benefits of tax-shield

Theoretically speaking, companies may choose to never let their leverage fall below a certain value, in order to optimally exploit benefits of tax-shield. That is, if their leverage is too low, companies may choose to issue additional debt even in the absence of large investment opportunities. This is easy to incorporate into the model, and the only thing that changes is that the first region becomes $[z_{\text{res}}, z_{\text{inv}}]$ instead of $[0, z_{\text{inv}}]$, where $z_{\text{res}}$ is the optimal lower-boundary for debt issuance; that is, the company will issue additional debt every time its $c/y$ falls below the level of $z_{\text{res}}$. Appendix A provides the optimality conditions for $z_{\text{res}}$. 

140
These graphs show the value of equity and debt of the same company that is not (left graph) and is (right graph) owned by a PE. Company’s cash flow is fixed at the level \( y_t \) is fixed at one, and values are shown as the second state variable - interest expenses - changes (horizontal axis).

Two things should be noted about \( z_{res} \), though. First, I depart from the traditional assumption that debt issuance incurs some small costs, and instead assume that debt can be costlessly issued by increasing \( c_t \). This means that the company will be constantly issuing some trivial amount of debt every time there is a positive \( dW_t \) shock to company’s profitability \( X_t \) if the company is at the boundary. Second, most companies will optimally choose to have \( z_{res} = 0 \), implying that companies will never issue debt to pay dividends. This could happen because the value that company’s equityholders can extract out of the investment opportunities directly depends on the cost of issuing debt, which is necessary to finance these investment opportunities. Therefore, by not issuing debt to pay dividends, companies can have lower interest rates in the future, and, therefore, finance more large investments.

3.6. Model Results

3.6.1. Parameter values

The results of the model depend on the parameter values that are used to solve the model. Most of the parameters, however, are hard to estimate directly in the data, and there is a substantial cross-heterogeneity. Another concern, particularly related to the subject of this
paper, is that PE firms do not choose buyout targets randomly. It is, therefore, possible that parameter values that describe PE-backed companies in the data are different from the parameter values that describe companies without PE-ownership. This paper does not address the selection bias, but it is important to discuss two approaches that could potentially alleviate it, and why the paper does not take them.

The first approach that could resolve the selection bias is to solve the model mathematically and find which companies PE firms prefer to acquire. Indeed, acquisitions of different companies can bring different returns to the PE: companies that need external financing the most would benefit the most from having a PE-sponsor, and, therefore, should be the preferred targets for PE-buyouts. Therefore, one could solve, within the model, for the optimal set of parameters that a company should have to maximize the return of a PE in case the PE invests in this company. The problem with this approach is that the maximization problem might be unbounded - that is, the expected return might be monotonically increasing in one or several parameters (for instance, in δ₂ - size of large investments that require external financing). The parameter space would then need to be restricted based on the type of companies that are available for buyouts in the data, and this is outside of the scope of this paper.

The second approach that could resolve the selection bias, and which is also not taken in this paper, is to structurally estimate the model, based on the data for companies that do and do not have PE-ownership. The problem with this approach is that PE-backed companies report data differently from companies without PE-ownership. Both types of companies might have similar items on their income statement and the balance sheet, but the meaning of those numbers is different. Shareholder loans is one such example: they are treated as debt on the balance sheet, but are, in fact, equity that is structured as debt. There are some other problems, unrelated to the structure of PE-buyouts, but still making the data hard to compare: for instance, the assets value of PE-backed companies likely better reflects their market value than assets of other companies, which is due to the fact that company’s assets are reevaluated to reflect the acquisition price at the time of the buyout.35

35Therefore, if one was computing the market-to-book ratio for PE-backed companies, for instance, they would find a value below but close to one, while most public companies have market-to-book ratio above
It is important to note that the previous paragraph does not imply that it is completely impossible to run a structural estimation to analyze the underlying difference between companies with and without PE-ownership. However, doing so would require a lot of efforts to adjust the data that is reported by both types of companies. This paper takes a first step in that direction by adjusting debt-related values, but data for the moments that could identify other parameters also needs to be collected.

In short, the paper does not address the selection bias, and instead explains to what extent the observed financing policies and performance of PE-backed companies can be explained by the presence of a PE-sponsor. Table 23 summarizes the parameter values used to solve the model, and all parameters are assumed to be identical between companies that do and do not have PE-ownership.

The unconditional growth rate of company’s productivity under the risk-neutral probability measure $\mu$ is $-1\%$ - that is, on average, company’s productivity falls if the company does not invest. Company’s growth rate should increase when moving from the risk-neutral probability measure to the actual probability measure, and so risk-premium$^{36}$ $4\%$ is added to the company’s growth rate when the model is simulated, therefore, increasing the unconditional growth rate up to $3\%$. By adding small investments, the company can increase the

---

36Risk-premium is probably a confusing name for a variable, since I later speak about risk-premium on the market portfolio and the PE-portfolio. In the context of the current paragraph, risk-premium is the difference in the growth rate of company’s productivity under the actual probability measure and the risk-neutral probability measure. Since market portfolio is a levered claim on company’s asset, $4\%$ difference in the growth rate becomes the 5.6% market excess return, which is roughly consistent with the data. This paper does not take a stance on where the $4\%$ risk-premium comes from, which might be from the risk-aversion of the representative agent, or some other unrelated market factors.

---

<table>
<thead>
<tr>
<th>Company-specific parameters</th>
<th>Economy-wide parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>unconditional growth rate $\mu$</td>
<td>risk-free rate $r_f$ $6%$</td>
</tr>
<tr>
<td>size of small inv. opportunities $\delta_1$</td>
<td>risk premium $r_p$ $4%$</td>
</tr>
<tr>
<td>size of large inv. opportunities $\delta_2$</td>
<td>corporate tax rate $\tau$ $20%$</td>
</tr>
<tr>
<td>intensity of arrivals of large inv. opportunities $\lambda$</td>
<td>price of capital $H$ $13.5$</td>
</tr>
<tr>
<td>intensity of PE-exits $\lambda_{PE}$</td>
<td>0.15</td>
</tr>
<tr>
<td>bankruptcy costs $\alpha$</td>
<td>10%</td>
</tr>
<tr>
<td>idiosyncratic volatility $\sigma_{ID}$</td>
<td>22%</td>
</tr>
<tr>
<td>systematic volatility $\sigma_s$</td>
<td>12%</td>
</tr>
<tr>
<td>total volatility $\sigma$</td>
<td>25%</td>
</tr>
</tbody>
</table>
growth rate up to \( \mu + \delta_1(1 + \gamma) \), which becomes 5.2% annually (under the actual probability measure). The growth rate can further be increased by taking large investment opportunities: the growth rate of a company that takes all large investment opportunities is, on average, \( \mu + \delta_1(1 + \gamma) + \lambda \delta_2(1 + \gamma) \), which is 9.6%.

Price of capital \( H \) has a significant influence on the results of the model, since it controls the profitability of company’s investments (to the same extent that \( \gamma \) does), and also the value of debt in default, since the model assumes that debtholders in default recover company’s assets at their market price after bankruptcy costs are accounted for. The value of \( H \) in the model is 13.5, and two moments discipline it. First, a company without PE-ownership and zero leverage has market-to-book ratio of one, this way linking the price of capital to the value of a company that operates this capital. Second, the recovery rate of debt in default (relative to the face value) is 47%, which is roughly consistent with the recovery rate on rated debt.\(^{37}\)

Estimates of bankruptcy costs \( \alpha \), which in this paper is 10%, vary from very low to very high. For example, Gruber and Warner (1977) finds that direct bankruptcy costs are about 1% of the assets value, and Andrade and Kaplan (1998) report the value of about 20%. Some authors used a structural estimation approach to infer the bankruptcy costs from companies observed decisions. In particular, Davydenko, Strebulaev and Zhao (2012) find that default costs are in the range of 10% and 30%, Hennessy and Whited (2007) report values between 8.4% and 15.1%, and Glover (2016) finds the value of about 45%. Glovers estimates are well-above estimates of other authors, but as argued by Reindl, Stoughton, and Zechners (2017), it is because Glover assumes that all companies follow optimal leverage policy, while it is not necessarily the case in the data. The authors estimate a similar model without imposing optimal capital structure and using stock prices instead, and find substantially lower values of bankruptcy costs (20%).

Intensity of PE-exit is assumed to be \( \lambda_{PE} = 0.15 \), implying that the PE usually controls a

\(^{37}\)For full disclosure, the range of recovery rates in the data is wide: 14% on junior subordinated bonds, to 75% on first lien bank loan. The recovery rate in the model should roughly aggregate the recovery rate on different types of debt that companies in the data have. In this sense, 47% recovery rate in the model is similar to 54% average recovery rate of the first lien bonds and 46% average recovery rate of the unsecured bank loans https://www.researchpool.com/download/?report_id=1751185&show_pdf_data=true.
company for six and a half years. It is slightly higher than the average number of annual reports that I observe for companies in my sample (5.6), but the number of annual reports underestimates the length of a PE-investments.

3.6.2. Capital structure and financial policies

The solution of the model shows that companies without PE-ownership only issue debt when they have large investment opportunities, and do not issue debt to pay dividends. In other words, companies optimally choose to maintain low leverage even though some debt could be issued at almost the risk-free rate while giving the company tax-shield advantage. Companies avoid leverage because greater leverage increases the rates at which additional debt can be raised in the future and, therefore, makes future investment opportunities less profitable. Moreover, companies with high leverage suffer from the debt overhang problem: equityholders control company’s investments, but parts of the benefits of the investments go to debtholders. If a big enough share of benefits of an investment goes to debtholders, but equityholders pay the full price, they will optimally choose to forgo the investment.

The debt overhang problem that affect companies without PE-ownership deserves some special discussion. The way large investments are modeled, they are always financed by issuing debt. Therefore, independent of company’s leverage, debtholders do not get benefits from large investments. Equation (3.5) shows this mathematically. Intuitively, it follows from the assumption that debt markets are competitive: if a company does not raise debt to finance a large investment opportunity, company’s debt value will not change. Therefore, the current value of debt becomes the outside option for company’s debtholders, and they will provide debt on terms that make them indifferent to the outside option. Therefore, whenever a company takes a large investment opportunity, the benefits fully go to equityholders, eliminating the debt overhang problem. Note, however, that companies with high leverage will still forgo large investments because debt issuance costs are high.\(^{38}\) In contrast, small

\(^{38}\)In this sense, otherwise profitable investment opportunities become ex-post not profitable to both equityholders and debtholders. This happens because an investment opportunity, if is taken and financed by issuing debt, increases company’s leverage, therefore, increasing the default probability. Note that this problem could not be solved by debt renegotiation if the money still had to come from debtholders - issuing debt at terms that leave debtholders indifferent is already the best outcome that debt renegotiation could achieve. The only solution, in which the investment could be taken, while still increasing the value of at least one party and not hurting the other, would be to let the company issue equity. However, if equity
investment opportunities that the company can finance out of the operating cash flow can become a subject to debt overhang problem, and therefore companies have incentives to keep the leverage low.

Companies without PE-ownership take large investment opportunities as long as their leverage is below 41.5%, which corresponds to the inverse coverage ratio of $z = 0.4$. At that point, the marginal interest rate to issue more debt is 9.1%. The threshold is different for small investments: a company takes them as long as the cash flow is sufficient to both pay interest expenses and invest. The cutoff leverage for small investment opportunities is 67.4%, which corresponds to the inverse coverage ratio of $z = 0.73$. If the company was to take small investment opportunities after that point, it would have to issue at least some amount of debt, since the cash flow alone would not be sufficient to cover both interest expenses and investments, but the marginal interest rate to issue more debt is 17.8% at that point. As long as company’s inverse coverage ratio is between 0.73 and 1, the company does not take any investment opportunities - small or large - but also does not need to raise external financing to cover debt payments since the cash flow is still above the level of interest expenses. Once the inverse coverage ratio exceeds one, the company constantly issues debt to pay interest expenses, at the marginal interest rate that constantly grows. The default happens if the inverse coverage ratio exceeds 1.23.

The solution looks very different for companies with PE-ownership. To start with, these companies issue significant amount of debt even when they do not need debt to finance investment opportunities. Therefore, the buyout leverage is 48.9%, which is very close to the empirical estimates. This leverage corresponds to the inverse coverage ratio of $z = 0.45$. Two factors explain the high leverage of PE-backed companies in the model. First, and most important, high leverage does not prevent PE-backed companies from investments. In case debt financing is too expensive, the PE-backed company can always get an equity injection from the PE-sponsor to pay for the investment. It does not mean that PE-ownership completely eliminates the debt overhang problem - companies with very high value of leverage still do not invest - but PE-ownership does significantly alleviate it.

issuance costs increase with leverage, as is the case in the data, there is no solution.
Second, PE-ownership makes company’s debt less risky: in distress, when company’s cash flow is not sufficient to cover interest expenses, a PE-backed company does not need to continuously issue more debt at exponentially increasing rates to cover interest expenses on previously issued debt, and instead can get money from the PE-sponsor. This effectively cuts the debt spiral that companies without PE-ownership are prone to in distress, and, therefore, reduces the probability of default for PE-backed companies.

Figure 25 analyses to what extent the capital structure in a buyout is affected by direct and indirect costs of debt issuance. Bankruptcy costs $\alpha$ control the direct costs of debt issuance, as they change the recovery rates for debtholders in default. Profitability of company’s future investment opportunities $\gamma$ controls the indirect costs of debt issuance, as highly levered companies will have to forgo some of the future investments. As follows from the figure, the buyout leverage is much more sensitive to the indirect costs of debt issuance.

Furthermore, PE-backed companies issue additional debt to pay dividends when their leverage falls below 17.3%, which corresponds to the inverse coverage ratio of $z = 0.16$; every time this happens, PE-backed companies bring the leverage back to its optimal level of 48.9% ($z = 0.45$). PE-backed companies take all investment opportunities - large and small - as long as the leverage stays below 78.5% (or $z = 0.94$). Investment opportunities are paid for by issuing debt if company’s leverage is below the optimum level of 48.9%, and
by receiving money from the PE-sponsor if the leverage is above that value. As long as company’s inverse coverage ratio stays between 1 and 1.8, the company continuously issues equity to pay interest expenses, and default happens when the inverse coverage ratio exceeds the 1.8 threshold.

3.6.3. Cross-sectional distributions

The previous subsection describes optimal financial and capital structure choices. However, the actual distribution of leverage and/or investments can look very different, as the result of idiosyncratic shocks and companies’ responses to them. Therefore, I simulate a cross-section of companies to study how it compares to the data.

In order to run a simulation, I generate a group of 500 companies with PE-ownership, and another group of 500 companies without PE-ownership. All 1000 companies start with the same cash flow $y_0 = 1$, and are optimally levered (i.e. companies without PE-ownership have no debt, and companies with PE-ownership have 48.9% leverage). Throughout the simulation, companies make decision about investments and debt issuance consistent with the solution to the model. Every period, each company receives a Brownian shock that affects its cash flow and inverse coverage ratio. Furthermore, all companies have a small investment opportunity, and some companies, in addition, receive a large investment opportunity. Companies optimally choose if they want to invest, and how to finance the investment. Companies without PE-ownership that are in distress issue debt to pay interest expenses that are not covered by the cash flow. At the end of each period, some companies may default, in which case they are replaced by newly born optimally levered companies of the same type, with the productivity of the defaulted companies, but only $(1 - \alpha)$ fraction of the assets. In addition, companies with PE-ownership sometimes are sold to the market, in which case they are also replaced by another optimally levered company with the same productivity and same value of assets. Companies with PE-ownership may issue additional debt if their leverage falls to a very low value as the result of a sequence of several positive shocks, in which case their leverage is reset to the 48.9% level. Each period of a simulation is $1/36$ of a year, and the simulation is run for 3600 periods (600 years).
Figure 26: Steady-state distribution

Distribution of leverage

Graphs are based on the result of model simulation. The simulation has 500 companies with PE-ownership, and 500 companies without PE-ownership, and runs for 600*36 periods (600 years), with each period being 1/36th of a year. When a company defaults or is sold (in case of PE-backed companies) during a simulation, it is replaced by another company with the same productivity, thus, maintaining a balanced sample. Distributions are based on the company-period observations from the final 300 years of the simulation. Inverse coverage ratio is the ratio of interest expenses to company’s cash flow.

Figure 26 shows the cross-sectional distribution of leverage and inverse coverage ratios of companies with and without PE-ownership. These cross-sections distributions are based on the observations from the last 1800 periods of the simulation. The average and median leverage of companies without PE-ownership is 26.6% and 24.4% correspondingly. These numbers seems to be consistent with leverage values in the data, though there are different opinions on how leverage should be measured, or what is the appropriate comparable group of companies. For instance, Rajan and Zingales (1995) report that the average and median values of book leverage in the US are 31% and 27%, but when assets are adjusted for the
Graphs are based on the results of model simulations. Each simulation has 500 companies with PE-ownership, and 500 companies without PE-ownership, and runs for 100*36 periods (100 years), with each period being 1/36th of a year. All 100 companies start simulations with the same $\log(y_t) = 0$, but $y_t$ changes during the simulation as the result of company's investments, and also idiosyncratic shocks. If a company defaults or is sold (in case of PE-backed companies) during the simulation, it is replaced by another company with the same productivity, thus, maintaining a balanced sample. Average $\log(y_t)$ is measured for both groups of companies in the final period of 24th, 49th, and 99th year of the simulation. Simulation is repeated 400 times, and distributions of average productivity is shown on the graph.

market value of equity, the values change to 24% and 20%. Numbers for the UK in their paper are smaller: they find that the average and median book leverage in the UK are 21% and 18%, but average and median market leverage are 14% and 16%. Faulkender and Petersen (2005) argue that many companies have low leverage because they do not have access to debt capital markets, and so they restrict the sample of companies to those with S&P credit rating; they find that the average and median book leverage values in the US are 37% and 35%, and average and median values of the market leverage are 28% and 26%.

The average and median inverse coverage ratios of companies without PE-ownership in the model is 0.25 and 0.22. The inverse of that - the coverage ratio\(^{39}\) - has mean 6.27 and median 4.57.\(^{40}\) Rajan and Zingales (1995) report that the median interest coverage ratio in the US is between 2.41 and 4.05, and median interest coverage ratio in the UK is between 4.79 and 6.44.

The average and median values of leverage are both 0.4 for companies with PE-ownership in the model. The leverage falls from 48.9% in the acquisition year because companies,\(^{39}\)Note that this paper uses the inverse coverage ratio rather than the coverage ratio variable because coverage ratio is not defined for companies with no debt. Also, all function in the model can be rewritten in terms of the inverse coverage ratio, but not the coverage ratio - again, because the value for companies with no debt would not be defined: $E(y, c) = ye(c/y)$, which could not be rewritten to $E(y, c) = ce(y/x)$

\(^{40}\)Difference between 6.27 and 1/0.25 is due to Jensen’s inequality.
on average, increase the value of their assets. It is hard to find the data counterpart for the average value of leverage in a cross-section of PE-backed companies, mainly because there is no good way of measuring company’s leverage after the buyout in the data. The market value of assets is not observed, and the book value of assets is misleading. One way to compare would be to look at the value of the leverage of the company when the PE sells it, in the data and in the model. As Table 14 shows, the average and median values of leverage of a company at exit are 26% and 18%. Comparable numbers in the model are 33% and 34%, implying that the leverage distribution for PE-backed companies is skewed to the right. There might, however, be another reason why the model overestimates company’s leverage at exit. As the next subsection shows, the model does not fully explain the outperformance of PE-backed companies, likely because there are other reasons how PE firms increase the value of their portfolio companies. Market timing might be one of such factor - PE firms might be able to find undervalued private companies and sell them later, therefore, increasing the value of the company by more than what the model explains (as all companies are fairly valued in the model). This could explain the discrepancy of the exit leverage in the model and in the data.

Turning to investments, almost all companies take small investment opportunities. Among companies without PE-ownership, 99.5% take a small investment opportunity every period, and the number is 99.8% for companies with PE-ownership. The same fraction of PE-backed companies take large investment opportunities when they arrive, but only 84.6% of companies without PE-ownership do so. Companies without PE-ownership finance all large investments by issuing debt, which is simply the results of the model assumption. PE-backed companies, in contrast, can choose whether they want to finance large investment opportunities by issuing debt or equity. The simulation shows, that the debt is issued in 72.8% of cases, and only 27.2% of large investments are financed by money from the PE-sponsor.

As PE-backed companies take large investment opportunities more frequently, they also grow faster. To show this, I simulate the model 400 times, and measure the average productivity of companies with and without PE-ownership after 25, 50, and 100 years of simu-
lation. Figure 27 shows that indeed average productivity diverges. This is consistent with the empirical results. PE-backed companies on average have CAPEX and acquisitions of the same size as companies without PE-ownership, but the acquisition frequency is higher. As the result, PE-backed companies grow faster than the rest of the economy.

3.6.4. Performance

I next turn to the question to what extent the model can explain the outperformance of PE-backed companies vs. the rest of the market. Note, however, that companies with PE-ownership should grow faster than the market, since they are more levered and, therefore, more exposed to the aggregate shocks. A more interesting question is whether companies with PE-ownership outperform the market after accounting for risk.

To answer this question, I consider an index of companies with and without PE-ownership. The indices show how the wealth of an investor would change over time if she invested her wealth into a value-weighted portfolio of companies with and without PE-ownership that have the same productivity, and whose leverage is distributed according to the steady state distribution from Figure 26. Indices account for default, difference in average growth rate between companies with and without PE-ownership, and the fact that PE firms sometimes sell and buy their portfolio companies. Appendix D explains the construction in details. I repeat simulation 400 times.

Table 24 shows the relative performance of PE and non-PE indices across 400 simulations. “PE-equity” shows the performance of the PE firms itself (or investors in the PE), and “PE: equity+debt” shows the evolution of wealth of an investor who invests in both equity and debt of companies with PE-ownership. Hypothetical counterparts to the “market” would be S&P 500 index, to the “PE: equity” would S&P if all companies in S&P 500 were fully owned by PE firms, and to “PE: equity + debt” would be a portfolio that combined shares and debt of companies in S&P 500 if they were all owned by the PE.

One issue with comparing returns on the market and PE industry in the data is that returns are measured differently. Asset pricing literature usually measures average returns that the market (or portfolios) produce. In contrast, there is no way of studying immediate
returns of PE funds since the value of their investments is not known before their investments are realized. Therefore, usually IRRs of PE industry is computed. Nevertheless, comparing IRRs of PE-industry with average returns on the market is misleading because of Jensen’s inequality. The simplest way to see that is to notice that while the average return on the S&P 500 between 1960 and 2015 was 11.24% a year, the IRR on the market over the same time period was only 9.83%.

Fortunately, these issues can be addresses within the model, as the model shows both IRR and average returns on any type of investment. As Table 24 shows, the model explain the market performance well: the average annual return is 11.6%, which corresponds to the market IRR of 10.3%. The average risk-premia is 5.6% that arises because of the levered difference in the growth rate under the actual and risk-neutral probability measures. Average returns on the PE:equity index are greater than average market returns, which is partially explained by a greater exposure to the systematic risks of PE-backed companies (they are more levered, and so volatility and beta with respect to the market are greater for PE-backed companies). Nevertheless, even accounting for risk-exposure, PE:equity index produces 6.7% alpha.

This is how the model explains outperformance. Company’s future cash flows increase when it is acquired by a PE, because the company will invest more, and because it will have greater tax-shield benefits. Nevertheless, the PE pays the price that all other investors on the market assign to the company. The price that is paid by the PE - and, therefore, observed by an econometrician - is smaller than the value of future cash flows discounted

<table>
<thead>
<tr>
<th></th>
<th>market</th>
<th>PE: equity</th>
<th>PE: equity + debt</th>
</tr>
</thead>
<tbody>
<tr>
<td>total return</td>
<td>11.55%</td>
<td>19.28%</td>
<td>13.36%</td>
</tr>
<tr>
<td>risk-free rate</td>
<td>6%</td>
<td>6%</td>
<td>6%</td>
</tr>
<tr>
<td>risk-premium</td>
<td>5.55%</td>
<td>6.57%</td>
<td>4.13%</td>
</tr>
<tr>
<td>alpha:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>alpha due to tax-shield benefits</td>
<td>6.71%</td>
<td>1.04%</td>
<td>3.23%</td>
</tr>
<tr>
<td>alpha due to greater investments</td>
<td>5.67%</td>
<td>0.57%</td>
<td>2.66%</td>
</tr>
<tr>
<td>IRR</td>
<td>10.3%</td>
<td>16.9%</td>
<td></td>
</tr>
<tr>
<td>beta</td>
<td>1</td>
<td>1.19</td>
<td>0.76</td>
</tr>
<tr>
<td>volatility</td>
<td>16.6%</td>
<td>19.7%</td>
<td>12.4%</td>
</tr>
</tbody>
</table>

Table 24: Performance
Indices show the log of the value of a dollar over time if this dollar is invested in a group of companies without PE-ownership (left graph) or with PE-ownership (right graph), according to weights that correspond to the steady-state distribution of these companies (refer to Figure 26 for the cross-sectional distribution). Indexes were computed by simulating the model 400 times, and the grey area shows the area within which 95% of the indices fall. Red line on the left graph and blue line on the right graph show indices from one of the simulations.

at the appropriate rate. As the result, future realized returns will, on average, be higher than expected returns.

Going deeper, what happens is that company’s ownership determines its future cash flows, and, therefore, its value. An important assumption is that the buyout happens at the lower price that the market assigns to the company and not the price that the PE assigns, which is rationalized by the fact that the PE industry is small relative to the rest of the market. That is, there is a large number of public investors who are indifferent between buying and selling a company at a low price, and a small number of big investors (PE), for whom the value of the company is bigger.

At first sight, the arguments in the previous is at odds with theoretical models, such as Grossman and Hart (1980), that say that corporate takeovers should not happen in equilibrium even if the value of the company could be greater under a different ownership. In the model of Grossman and Hart, a company is owned by a group of atomistic investors, and an outsider can increase company’s value if she acquires the majority control of the company. Nevertheless, takeover does not happen in equilibrium - and the value of the
company remains low - because there is no price at which the majority of current investors would agree to sell their shares; investors do not internalize their actions, and, therefore, no one wants to sell their share at a price below the post-takeover price. This argument would not work for PE-buyouts because they are structured differently: PE leaves no shares to outside investors (other than company’s managements). Therefore, company’s old investors do not have an option to hold their shares hoping that the buyout would go forward anyway, and their share price would go up.

As discussed above, the model explains alpha of PE:equity index through greater tax-shield benefits and greater investments (or faster growth that comes as the result of bigger investments). To what extent is alpha attributed to each of these factors? There is no definitive answer to this question, since the two channels interact with each other and with other factors; for instance, because companies with PE-ownership invest more, they grow faster, and, therefore, issue debt more often, which in turn means they exploit greater benefits of tax-shield, and pay less taxes. Appendix D shows how the two effects are disentangled. I find that of the total alpha of 6.7%, greater investments explain 5.7%, and tax-shield benefits explain 1%. In other words, greater tax-shield benefits and greater investments explain 15% and 85% of the outperformance that this paper can explain.

Average return of 19.3% of the PE:equity index corresponds to 16.9% IRR. This value seems similar to the IRR that other researchers computed based on the LP data: 14-17% in Ang, et.al (2018), 15% Driessen, et.al (2012), and Franzoni, et.al (2012). One should remember, however, that returns based on the LP data are computed after accounting for the PE fees, and, therefore, the actual returns that the PE earns should be higher. Axelson, et.al (2014) claim that gross excess return over the market is 8.3% - 8.6%, implying gross total IRR of roughly 20%.

Another way to study whether the model fully exploits the outperformance of PE is to study within the model the IRR on the whole company, that is company’s debt and equity, and compare it to the results in Section 3. The third column of Table 24 shows that the IRR of

41 The main additional factor, whose effect multiples the effect of tax-shield benefits and greater investments, is the difference in growth rate under the actual and risk-neutral probability measure.
the PE: equity+debt index is 13.36%, which is significantly lower than 32% in the Section 2. A number of factors can justify that. First, as already discussed, empirical results do not account for any transaction costs, which are high in the case of buyouts. Second, value-weighted returns rather than equal-weighted returns should be studied, and average returns on big companies are smaller than smaller companies.

Nevertheless, it seems that the model does not fully account for the alpha generated by PE-firms in the data, whether one uses returns based on the data from LPs, or IRRs inferred from the values of the company at the time of the buyout and sale. This difference in return is, therefore, accounted by factors outside of the model. The main such factor is likely the market timing. The model assumes that PE randomly choose companies that they acquire, while PE-firms likely carefully choose in which companies to invest. This implies that PE can find companies that are undervalued relative to their fundamentals, which can explain the difference between the observed outperformance, and outperformance predicted by the model.

3.7. Conclusion

This paper empirically analyzes a sample of companies with PE-ownership, specifically studying their capital structure, investment policies, and performance. This analysis contributes to the literature for three main reasons. First, the paper analyzes a representative sample of PE-backed companies, which mitigates the selection bias concerns that are frequent in other studies of PE-backed companies. Second, the paper identifies the owner of every single security that is issued to finance the buyout, and separates external debt from shareholder loans - securities that are recorded as debt but are paid for and held by the private equity owner. Third, the paper separates paper transactions that happen between the company and the company’s investors (the PE and external debtholders) from transactions that involve the actual exchange of money.

The results of the analysis defy many of the common stereotypes about private equity business. It turns out that the leverage of companies with private equity ownership is significantly lower than traditionally considered, with many buyouts financed with either
no or just a small amount of debt. Neither private equity owners show any evidence of over-extraction. An almost equal number of PE-backed companies pay dividends and receive money in follow-on equity injections from the PE-owner. If anything, private equity owners relax the financial constraints of their portfolio companies and allow them to invest when internal cash flow is low. The ability to receive equity injection from the private equity owner is particularly crucial for financially-distressed companies: as the analysis in this paper shows, such companies do not reduce their investments.

The theoretical model then explores the role of financial distress in explaining the behavior of companies with and without PE-ownership. Financial distress happens rarely, but it represents an event in which the company desperately needs external financing. For companies without PE-ownership, however, external financing is very expensive in distress, and so they risk falling in a continuous debt spiral every time financial distress happens. In contrast, PE-owners act as deep-pocket investors, and can rescue their portfolio companies from the financial distress.

Quantitative results of the model show the importance of the ability of PE-firms to relax the financial constraints of their portfolio companies in distress. The model abstracts away from all other potential benefits that private equity firms provide to their portfolio companies and/or their ability to select specific companies. Nevertheless, the model quantitatively reconciles the empirically-observed difference in the capital structure of companies with and without PE-ownership, and explains a large portion of the abnormal returns that private equity firms demonstrate in the data.
: Bibliography


