Essays In Consumer Finance And Banking

Kian Samaee
University of Pennsylvania

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Essays In Consumer Finance And Banking

Abstract
We study two impediments to monetary policy transmission: (1) search friction in mortgage shopping (2) shadow banking. The first impediment weakens the mortgage refinancing channel of monetary policy. Many US mortgage borrowers do not refinance, despite seemingly having financial incentives to do so. We study the role of search costs in explaining this inaction, focusing on the 2009-2015 period when mortgage rates declined. We estimate a dynamic discrete choice model of refinancing and search decisions using a panel data set, which includes information on the sequence of refinancing decisions and search intensity (the number of mortgage inquiries). We find that search costs significantly inhibit refinancing through two channels. First, higher search costs directly increase the cost of refinancing. Second, they also indirectly increase lenders’ market power and thus raise the offered refinance rates. We find that the indirect market power effect dominates. In chapter two, we study an additional source of market power coming from search friction: statistical discrimination by lenders, a tool to separate borrowers who differ in search intensity. We explore how statistical discrimination affects monetary policy transmission. We build and calibrate a general equilibrium model of the mortgage market with two types of borrowers who differ in the number of lenders they meet. If lenders meet refinancers with high current interest rates, they infer it is likely they did not search in the past and is not likely to search now. So, lenders infer these refinancers as non-shoppers and more likely to offer them high refinance rates. We find statistical discrimination significantly decreases the consumption response of non-shoppers to a monetary policy shock. In chapter three, we explore the monetary policy transmission when shadow banking co-exists with a regulated banking sector. We develop a model of banking in which both banks and shadow banks do liquidity transformation. The difference is that banks have access to the discount window to manage the liquidity risk while their balance sheet is regulated. Depending on the size of the shadow banking system, the effectiveness of changing the discount window rate may weaken as shadow banks and banks can become interchangeable.

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ESSAYS IN CONSUMER FINANCE AND BANKING

Kian Samaee

A DISSERTATION

in

Economics

Presented to the Faculties of the University of Pennsylvania

in

Partial Fulfillment of the Requirements for the

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Dedicated to

my parents Afsaneh and Ali

and my wife Elmira

without their support there was no vision of success in completing my dissertation
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ABSTRACT

ESSAYS IN CONSUMER FINANCE AND BANKING

Kian Samaee

Aviv Nevo Guillermo L. Ordoñez

We study two impediments to monetary policy transmission: (1) search friction in mortgage shopping (2) shadow banking. The first impediment weakens the mortgage refinancing channel of monetary policy. Many US mortgage borrowers do not refinance, despite seemingly having financial incentives to do so. We explore the role of search costs in explaining this inaction, focusing on the 2009-2015 period when mortgage rates declined. We estimate a dynamic discrete choice model of refinancing and search decisions using a panel data set, which includes information on the sequence of refinancing decisions and search intensity (the number of mortgage inquiries). We find that search costs significantly inhibit refinancing through two channels. First, higher search costs directly increase the cost of refinancing. Second, they also indirectly increase lenders’ market power and thus raise the offered refinance rates. We find that the indirect market power effect dominates. In chapter two, we study an additional source of market power coming from search friction: statistical discrimination by lenders, a tool to separate borrowers who differ in search intensity. We explore how statistical discrimination affects monetary policy transmission. We build and calibrate a general equilibrium model of the mortgage market with two types of borrowers who differ in the number of lenders they meet. If lenders meet refinancers with high current interest rates, they infer it is likely they did not search in the past and is not likely to search now. So, lenders infer these refinancers as non-shoppers and more likely to offer them high refinance rates. We find statistical discrimination significantly decreases the consumption response of non-shoppers to a monetary policy shock. In chapter three, we explore the monetary policy transmission when shadow banking co-exists with a regulated banking sector. We develop a model of banking in which both banks and shadow
banks do liquidity transformation. The difference is that banks have access to the discount window to manage the liquidity risk while their balance sheet is regulated. Depending on the size of the shadow banking system, the effectiveness of changing the discount window rate may weaken as shadow banks and banks can become interchangeable.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ACKNOWLEDGMENT</strong></td>
<td>iii</td>
</tr>
<tr>
<td><strong>ABSTRACT</strong></td>
<td>iv</td>
</tr>
<tr>
<td><strong>LIST OF TABLES</strong></td>
<td>viii</td>
</tr>
<tr>
<td><strong>LIST OF ILLUSTRATIONS</strong></td>
<td>x</td>
</tr>
<tr>
<td><strong>CHAPTER 1</strong>: Inaction, Search Costs, and Market Power in the US Mortgage Market</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Abstract</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.3 Data</td>
<td>8</td>
</tr>
<tr>
<td>1.4 Descriptive Evidence</td>
<td>10</td>
</tr>
<tr>
<td>1.5 An Equilibrium Model of Mortgage Refinancing and Search Decisions</td>
<td>20</td>
</tr>
<tr>
<td>1.6 Estimation</td>
<td>31</td>
</tr>
<tr>
<td>1.7 Estimation Results</td>
<td>36</td>
</tr>
<tr>
<td>1.8 A Centralized Refinance Market</td>
<td>46</td>
</tr>
<tr>
<td>1.9 Conclusion</td>
<td>48</td>
</tr>
<tr>
<td>1.10 Mathematical Appendix</td>
<td>49</td>
</tr>
<tr>
<td><strong>CHAPTER 2</strong>: Mortgage Search Heterogeneity, Statistical Discrimination and Monetary Policy Transmission to Consumption</td>
<td>54</td>
</tr>
<tr>
<td>2.1 Abstract</td>
<td>54</td>
</tr>
<tr>
<td>2.2 Introduction</td>
<td>54</td>
</tr>
<tr>
<td>2.3 Data and Analysis</td>
<td>60</td>
</tr>
<tr>
<td>2.4 Model</td>
<td>67</td>
</tr>
<tr>
<td>2.5 Steady State Analysis</td>
<td>80</td>
</tr>
</tbody>
</table>
2.6 Effects of Statistical Discrimination in Steady State ........................................ 84
2.7 Effect of increasing mortgage search ................................................................. 91
2.8 Monetary Policy Transmission to Consumption .................................................. 95
2.9 Conclusions ........................................................................................................ 99
2.10 Appendix .......................................................................................................... 101

CHAPTER 3: Liquidity Management, Banks vs. Shadow Banks ............................. 108
3.1 Abstract .............................................................................................................. 108
3.2 Introduction ....................................................................................................... 108
3.3 Model .................................................................................................................. 113
3.4 Characterization ............................................................................................... 124
3.5 Steady State Equilibrium .................................................................................... 128
3.6 Numerical Results ............................................................................................. 129
3.7 Conclusion ......................................................................................................... 132
3.8 Appendix ......................................................................................................... 134
3.9 Tables and Figures ............................................................................................ 140

BIBLIOGRAPHY ................................................................................................. 140
### LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>TABLE 1</td>
<td>Unexplained mortgage rate</td>
<td>63</td>
</tr>
<tr>
<td>TABLE 2</td>
<td>Regression Results, Interest Rate</td>
<td>65</td>
</tr>
<tr>
<td>TABLE 3</td>
<td>Benchmark calibration</td>
<td>79</td>
</tr>
<tr>
<td>TABLE 4</td>
<td>Regression Results, Loan Age</td>
<td>101</td>
</tr>
<tr>
<td>TABLE 5</td>
<td>MBS and Mortgage Rate</td>
<td>103</td>
</tr>
<tr>
<td>TABLE 6</td>
<td>Regression Result, Home Equity</td>
<td>104</td>
</tr>
<tr>
<td>TABLE 7</td>
<td>Borrower Distribution, Model vs. Data</td>
<td>105</td>
</tr>
<tr>
<td>TABLE 8</td>
<td>Borrower Type Distribution</td>
<td>105</td>
</tr>
<tr>
<td>TABLE 9</td>
<td>Calibration</td>
<td>140</td>
</tr>
<tr>
<td>FIGURE 1 : Dynamic of Refinancing Decisions</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>FIGURE 2 : Incentive to Refinance</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>FIGURE 3 : Interest Rate Residual</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>FIGURE 4 : Number of Mortgage Inquiries Distribution</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>FIGURE 5 : Search and Interest Rate Across Borrowers’ Creditworthiness</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>FIGURE 6 : Search Intensity and Refinance Probability</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td>FIGURE 7 : A Borrower Refinancing and Search Decision Tree</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>FIGURE 8 : Distribution of Borrowers’ Creditworthiness</td>
<td>37</td>
<td></td>
</tr>
<tr>
<td>FIGURE 9 : Estimates of Approval Probabilities</td>
<td>38</td>
<td></td>
</tr>
<tr>
<td>FIGURE 10 : Search Cost Distribution</td>
<td>39</td>
<td></td>
</tr>
<tr>
<td>FIGURE 11 : Estimates of Search Costs in Refinancing Costs</td>
<td>41</td>
<td></td>
</tr>
<tr>
<td>FIGURE 12 : Search Costs and Refinancing</td>
<td>42</td>
<td></td>
</tr>
<tr>
<td>FIGURE 13 : Direct versus Indirect Effect of Search Costs on Refinancing</td>
<td>45</td>
<td></td>
</tr>
<tr>
<td>FIGURE 14 : A Centralized Refinance Market</td>
<td>47</td>
<td></td>
</tr>
<tr>
<td>FIGURE 15 : Effective distribution of lenders in benchmark economy</td>
<td>78</td>
<td></td>
</tr>
<tr>
<td>FIGURE 16 : Ratio of mass of borrowers in steady state of benchmark model</td>
<td>81</td>
<td></td>
</tr>
<tr>
<td>FIGURE 17 : Lifetime refinance policy in steady state of benchmark model</td>
<td>83</td>
<td></td>
</tr>
<tr>
<td>FIGURE 18 : Refinancing in steady state, with &amp; without statistical discrimination</td>
<td>87</td>
<td></td>
</tr>
<tr>
<td>FIGURE 19 : Borrowers mass with and without statistical discrimination</td>
<td>88</td>
<td></td>
</tr>
<tr>
<td>FIGURE 20 : Welfare cost of statistical discrimination</td>
<td>89</td>
<td></td>
</tr>
<tr>
<td>FIGURE 21 : Welfare difference between the two types</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>FIGURE 22 : Lenders, benchmark vs. model with more active shoppers</td>
<td>93</td>
<td></td>
</tr>
<tr>
<td>FIGURE 23 : Borrowers, benchmark vs. model with more active shoppers</td>
<td>94</td>
<td></td>
</tr>
<tr>
<td>FIGURE 24 : Welfare, Benchmark vs. Model with more active shoppers</td>
<td>95</td>
<td></td>
</tr>
<tr>
<td>FIGURE 25 : IRF to monetary policy</td>
<td>97</td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER 1 : Inaction, Search Costs, and Market Power in the US Mortgage Market

_(co-authored with Sumedh Ambokar)_

1.1. Abstract
Many US mortgage borrowers do not refinance, despite seemingly having financial incentives to do so. We explore the role of search costs in explaining this inaction, focusing on the 2009-2015 period when mortgage rates declined significantly. We estimate a (dynamic) discrete choice model of refinancing and search decisions using a proprietary panel data set, which includes information on the sequence of refinancing decisions and search intensity (the number of mortgage inquiries). We find that search costs significantly inhibit refinancing through two channels. First, higher search costs directly increase the cost of refinancing. Second, they also indirectly increase loan originators’ market power and thus raise the offered refinance rates. We find that the indirect market power effect dominates. We use our model to study an alternative market design, in which a centralized refinance market replaces the current decentralized one. We find, under specific assumptions, a centralized refinance market can significantly increase refinancing activities by eliminating market power, even if we keep the refinancing costs unchanged.

1.2. Introduction
The average refinance mortgage rates in the US declined from 6.0% in 2008 to a historical low of 3.5% in 2013. However, many mortgage borrowers failed to refinance, despite apparently having incentives to do so.\footnote{Agarwal et al. (2015b), Johnson et al. (2015) and Keys et al. (2016) document inaction in refinancing in the US.} This inaction is puzzling, since borrowers who do not refinance could lose out on substantial savings. Keys et al. (2016) argue that a household with a 30-year mortgage of $200,000 could save more than $60,000 in interest payments over the life of the loan by refinancing a 6.0% fixed-rate mortgage (FRM) at 4.5%, even after accounting for the refinance transaction costs. One explanation for this inaction could be that borrowers find it costly to search for a new mortgage. There is evidence to suggest
that search friction exists in the US mortgage market. Specifically, more than half of the mortgage borrowers contact only one lender to refinance, despite the wide dispersion of interest rates and fees for a homogeneous mortgage contract.

In this paper, we bridge the evidence on search friction and refinancing inactivity to explore the role of search costs in explaining refinancing inaction. Specifically, we ask two questions. First, what is the effect of search costs on refinancing activities? We explore two channels through which search costs inhibit refinancing. Higher search costs directly increase the cost of refinancing, and they also (indirectly) increase the loan originators’ market power and thus raise the mortgage rates offered. The second question we ask is: What is the contribution of direct versus indirect market power effect on refinancing activities? The answer to the second question is important because it would enable policy makers to evaluate which policies, mortgage designs, or market designs might be more effective in reducing search friction and, consequently, inactivity in refinancing.

To address these questions, we first use a data set, which includes information on search intensity and refinancing decisions. Next, we present evidence from the data that is indicative of refinancing inaction, search friction, and also how search intensity and refinancing decisions are related. Motivated by the evidence, we develop and estimate a dynamic equilibrium model of refinancing and search decisions. Finally, we use the estimated model to conduct counterfactual experiments.

The proprietary panel data set that we use contain detailed information on mortgage contracts and borrower characteristics. To control for the role of the borrowers’ creditworthiness in refinancing decisions, we follow the FICO® Scores and the marked-to-market loan-to-value (LTV) ratios of the borrowers over time. These data enable us to follow the sequence of borrowers’ refinancing decisions, which means that we have access to the characteristics of both old and newly refinanced mortgages. The data includes the number of mortgage inquiries per borrower. We use the inquiries as a measure of search intensity.

FICO is a registered trademark of Fair Isaac Corporation.
From our data, we first present evidence of refinancing inactivity. We argue that at least 25% of the borrowers could have reduced their interest rates by at least 1.125 percentage points if they chose to refinance between 2009 to 2013. This is equivalent to a monthly payment reduction of at least $120. Second, we provide evidence that is indicative of search friction. We document the wide dispersion of transacted refinance interest rates for homogeneous mortgage contracts. We find that the difference between the 1^{th} and the 99^{th} percentiles of this distribution is almost 1.625 percentage points. We also present a negative correlation between interest rates and the number of mortgage inquiries in the refinance market, suggesting that it pays to search more. Despite this, almost 60% of the mortgage borrowers in our dataset made only one inquiry when they refinanced their mortgages. Third, we explore the relationship between search intensity and refinancing probabilities. We document a positive correlation between these two variables. Specifically, borrowers who search more at the time of a mortgage origination are more likely to refinance their mortgages later.

To explain these facts and explore their implications, we develop an equilibrium model by incorporating search into a dynamic discrete choice model of refinancing decisions. On the demand side, borrowers first decide whether or not to refinance in each period. If they choose to refinance, they search sequentially in order to find the lowest rates. Specifically, borrowers decide whether to accept the offered rate and apply for the mortgage, or whether to reject the offer and continue searching. If they apply and the application is approved, they refinance with the offered rate. If their application is rejected, they continue searching. On the supply side, loan originators take into account that gathering many quotes is costly for borrowers. They respond accordingly by offering a distribution of rates that are higher than the marginal cost of the loan origination. This model enables us to explore how search costs, directly and indirectly through market power, inhibit refinancing.

A refinancing decision requires a cost-benefit analysis for the mortgage borrowers. Refi-
nancing costs include search costs and switching costs. The search costs of borrowers are equal to the cost of each inquiry (marginal search costs) times the total number of inquiries that they gather. The latter depends on how many times the borrowers refuse offers and the number of times their applications are declined by the loan originators. The higher the search costs, the lower the probability to refinance. This channel is what we call the direct effect of search costs on refinancing.

The benefit of refinancing comes from the flow of utilities throughout the life of a new mortgage contract. Borrowers know the distribution of the offered rates. If those who want to refinance in order to lower the interest rate of their mortgages, do not expect that they are able to reduce it significantly, they may choose not to refinance. Loan originators take into account the search friction of the borrowers and thus raise the offered refinance rates. This equilibrium effect weakens the benefit of refinancing. This channel is what we call the indirect market power effect of search costs on refinancing.

We use the model to back out the search cost distribution from the observed interest rate distribution. To do this, we need to address two complexities. First, loan originators may reject an application based on creditworthiness. Borrowers with low creditworthiness know that their chances of being approved are small; thus, they may be willing to accept a mortgage with a high interest rate in order to avoid additional search. This implies that these borrowers will behave as if their search costs are high. If we use only the interest rate distribution to estimate the search costs, we will back out search cost distribution conditional on creditworthiness, but not the unconditional one. We therefore need to separately identify the probabilities of whether loans are rejected by the loan originators or by the borrowers. We use the relationship between the number of mortgage inquiries and the interest rates to address this complexity. We document that the negative correlation between the interest rates and search intensity becomes weaker as we explore riskier borrowers, because they are more likely to accept any offer in order to avoid additional search to get approved. Finally, the relationship between the interest rate and the number
of mortgage inquiries becomes flat among the riskiest borrowers.

The second complexity comes from selection. If high search cost mortgage borrowers are less likely to refinance, then the participants in the refinance market are biased in favor of low search cost borrowers. In order to take into account this selection, we use the dynamic refinancing behavior of the borrowers. Specifically, the mentioned positive correlation between search intensity and refinancing odds helps us to infer the search costs of the borrowers based on their refinancing frequencies. The higher the search costs, the lower the refinancing probability.

We estimate the model using data from 2008 to 2015, during a period of mortgage rates’ transition of high to low. Solving a dynamic model during a transition period is computationally challenging, because we have to keep track of the distribution of offered rates and many state variables. That is why we incorporate a search model into a dynamic discrete choice framework in order to estimate the model using conditional choice probability (CCP) techniques. We build on Arcidiacono and Miller (2011) since it allows us to incorporate unobserved search costs. Specifically, we use their two-stage approach in order to estimate the model.

We find that search costs significantly inhibit refinancing. Specifically, if we completely remove search friction by assuming a zero marginal search cost for all the borrowers, the mortgage rates on outstanding mortgages decrease by 1.4 percentage points from 2009 to 2015. Eliminating the search costs directly decreases the refinancing costs. Note that we assume that switching costs are still in place. Our estimates show that, on average, the search costs are at least 30.8% of the total refinancing costs. We also find that the indirect market power effect dominates the direct effect of search costs on inhibiting refinancing. We find that almost 75% of the decrease in outstanding mortgage rates (1.4 percentage points) is attributed to the indirect market power effect.

Finally, we use our model to study an alternative market design, that under specific as-
assumptions can significantly increase refinancing activity by eliminating market power, even if we keep the refinancing costs unchanged. We specifically assume a centralized re-
finance market replaces the current decentralized one. In this market, we assume Bertrand competition among the loan originators. They compete by posting interest rates to the cen-
tralized market. Borrowers observe only one interest rate at each point in time, and they can lock in the posted rate by choosing to refinance. We assume refinancing is still costly for the borrowers. They pay for the switching costs in full, and they also pay a search cost equal to one inquiry.

**Literature Review**

We contribute to various branches of the literature. First, we contribute to the literature that studies the sources of inaction in consumer decision-making. Inaction in switching to financially more beneficial contracts is well documented in many markets (Ausubel, 1991; Handel, 2013; Honka, 2014; Heiss et al., 2016; Nelson, 2017; Fleitas, 2017). Moreover, search friction is also a well-documented feature of many markets (Brown and Goolsbee, 2002; Hortacşu and Syverson, 2004; Roussanov et al., 2018; Galenianos and Gavazza, 2019; Allen et al., 2019). Our contribution is to bridge these two evidence in order to explore the role of search costs in explaining inaction.

Second, we contribute to the studies exploring decision-making in the mortgage market. Since this market is important from both the micro and the macro perspectives, exploring the poor decision-making of borrowers in this market has received particular attention (Green and LaCour-Little, 1999; Bucks and Pence, 2008; Chang and Yavas, 2009; Woodward and Hall, 2012; Agarwal et al., 2015b; Keys et al., 2016; Agarwal et al., 2017a). We are the first that explore the role of search costs in making poor refinancing decisions. There are examples in the US that have explored the role of the creditworthiness of the borrow-
ers as a barrier to refinancing (Agarwal et al., 2015a and Lambie-Hanson and Reid, 2018). We highlight the importance of search costs in explaining inaction while we take into account the effect of creditworthiness. In this regard, our paper is similar to Andersen et al.
who study inactivity in refinancing in Denmark, where the borrowers’ creditworthiness is not a barrier to refinancing. Their paper highlights the role of inattention and the psychological costs. Similarly, Johnson et al. (2015) highlight the role of other factors than creditworthiness. They argue that suspicion about the motives of the financial institutions and time preference contribute to failures to refinance in the US.

Third, this paper explores and highlights the importance of selection in markets with search friction. In a search model, price elasticity of demand depends on the search cost distribution. Ignoring the selection results in the mis-measurement of this elasticity and, consequently, of the market power. Using price dispersion in a static search model is the standard method for estimating search costs (Hortaçsu and Syverson 2004; Hong and Shum 2006; Gavazza 2016; Salz 2017; Allen et al. 2019). Some studies use detailed information on shopping behavior to estimate search costs (De los Santos et al. 2012; Honka 2014; Honka et al. 2017), but these studies also ignore the selection. The selection problem is unlikely to be an issue in retail shopping. However, in markets, in which consumers choose long-term contracts and may be inactive in adjusting their terms to more favorable ones over time, selection can lead to considerably mis-measured search costs. We address this selection by estimating a dynamic search model.

Fourth, we contribute to an estimation method for search models. We incorporate search into a dynamic discrete model in order to use CCP techniques. Moreover, we use Arcidiacono and Miller (2011) tools to estimate a search model.

Since our paper is linked to Agarwal et al. (2017b) and our other study Ambokar and Samaee (2019) in many dimensions, we review them separately in the following.

Our paper is related to Agarwal et al. (2017b) in many aspects. We follow them to use the number of inquiries as a measure of search intensity. They have access to total number of inquiries (mortgage and nonmortgage), while we fortunately have access to the number of mortgage inquiries. We follow them to use search intensity to separately identify the
probabilities of whether loans are rejected by the loan originators or by the borrowers. Since screening in the refinance market is mostly based on the hard information of the borrowers captured by credit scores and LTV ratios, unlike them, we do not introduce an adverse selection. We instead write a model in which the borrowers’ creditworthiness is fully observable. They explore search friction in a static model in which approval process by lenders affect the search behavior. We extend their static model into a dynamic one to explore the role of search friction in refinancing inaction. This dynamic model allows us to highlight the importance of selection in the refinance market.

A lack of refinancing can weaken the transmission of monetary policy. Scharfstein and Sunderam (2016), Di Maggio et al. (2017), Beraja et al. (2018) and Auclert (2019) explored the mortgage refinancing channel of monetary policy. In our related study, Ambokar and Samaee (2019), we incorporate a mortgage market with search friction into a standard New-Keynesian general equilibrium model to explore the role of search friction in the transmission of monetary policy. In that study, we also allow for statistical discrimination by lenders based on the characteristics of the current mortgage. In both studies, we find that the loan originators’ market power induced by search friction has an important role to understand how search friction affects refinancing decisions.

1.3. Data

Our analysis relies on Equifax Credit Risk Insight Servicing and Black Knight McDash (referred to as CRISM), an anonymized panel data set that merges Equifax’s credit bureau data on consumer debt liabilities with mortgage servicing data from McDash. CRISM covers about 60% of the US mortgage market during our sample period and is well suited to studying refinancing (Lambie-Hanson and Reid, 2018).

In this data set, we have access to detailed information of the mortgage contracts (LTV ratio, Debt-to-Income ratio, location of the property, mortgage size, quarter of the origination, property type, etc.).

The Equifax data contains a borrower’s updated FICO® Score for each month. In order
to measure the borrowers’ updated LTVs, we use borrowers’ remaining principal balance in the numerator, while for the denominator (home value) we follow standard practice and assume that the value of the property (whose appraisal we observe at the time of the loan origination) evolves according to a local home price in the Zillow Home Value Index (ZHVI). We specifically follow Lambie-Hanson and Reid (2018) and Abel and Fuster (2018) approach to build marked-to-market LTV ratios for each mortgage borrower.

The formal process to refinance a mortgage is as follows. First, mortgage borrowers apply to refinance by filing an application. Depending on the loan originator, this step may be completed over the phone, online, or in person. In this application, borrowers provide information on themselves and the property. This information includes employment, income details, asset information, and details about the location and features of the property. By submitting this application, borrowers provide a consent to proceed, permitting their loan originators to move forward with the application. Second, a loan originator is required by law to provide a Loan Estimate document within three days of receiving a loan application. This document estimates the fees and closing costs for the new mortgage, such as appraisal and origination fee and title work. It also summarizes the loan terms and monthly payment. At this point, the borrower’s credit report is “pulled” by the lender in order to determine both the borrower’s eligibility for specific loans and the interest rate to be charged to the borrower. This “pull” is recorded as “an inquiry” by the credit bureau. At this stage, borrowers have already locked in the interest rates for a specific period, typically up to 45 days. By this stage, borrowers can decide whether to continue with the current loan originator or contact other originators. Third, Before approving the refinance loan, loan originators will order a home appraisal to get the property’s estimated market value. Fourth, mortgage loan officers forward the application and home appraisal to a loan processor, who will prepare and review the loan. An underwriter will then review the completed application to make a final decision based on the loan originators’ criteria. At this step, borrowers inform of the final decision on the loan application. The last step is the closing process. Once borrowers have received the final approval, they review and
sign the closing documents, and also pay the costs of processing the loan application. The borrower makes monthly payments once the mortgage is settled, which depending on the loan, are either paid directly to the loan originator or to a separate loan servicer.

A unique feature of the CRISM dataset is that we have access to the number of mortgage inquiries as a proxy for search intensity. Unlike Agarwal et al. (2017b), who use the total number of inquiries as a proxy for search intensity, including both mortgage and non-mortgage inquires, we have directly access to the number of mortgage inquiries.

We use the credit bureau data on mortgage inquiries around the “final” mortgage application (and approval) to capture the intensity of borrower search. As discussed in Agarwal et al. (2017b), it is possible that borrowers may search for mortgages informally without a credit pull, such as searching for lenders and interest rates offered on the Internet. However, the final terms offered to the borrower depend on the creditworthiness of the borrowers. Lenders can therefore only offer full contract terms after verifying the borrower’s credit score (“an inquiry”) and knowing the house characteristics. Therefore, not being able to measure such informal searches should not impact the manner in which we intend to consider borrower search.

1.4. Descriptive Evidence

In this section, we document three descriptive patterns. First, we present patterns that suggest inaction in refinancing. Second, we present evidence that is indicative of search friction. Third, we document evidence indicates that search and inaction are correlated: the higher the search intensity, the higher the refinancing odds.

The evidence provided motives a model of search and refinancing decisions. We need to take into account two complexities in developing and estimating the model. First, loan originators may reject an application. We therefore need to be able to separately identify inquiries that are rejected by the borrower or by the loan originators. Second, there is a selection in the refinance market because high search cost borrowers are less likely to refinance. In this section, we provide evidence of how we use the data on the number of
mortgage inquiries to address the first complexity and the dynamic data on refinancing decisions to address the second complexity.

1.4.1. Inaction and Incentive to Refinance

Mortgage rates declined significantly in the US in the wake of the Great Recession. Figure 1 presents the dynamic of the average FRM rate in the refinance market from 2008 to 2018. The interest rate declined from 6.0% in 2008 to the historically low rate of 3.5% in the first quarter of 2013. Moreover, the interest rate was significantly lower than that of 2008 during all periods after 2013. There were potential financial incentives for many mortgage borrowers to refinance mortgages during the periods after 2008. Figure 1 follows all the loans that were originated in 2008 until they were refinanced for the first time. From 2008 to 2010, the interest rates declined by almost 1.5 percentage points, however, almost 60% of the mortgages were still not refinanced by the end of 2010. The graph indicates that more than 20% of the mortgages remained active at the end of 2013, despite the historical low rate at the beginning of 2013.

To get a more accurate measure of the incentives for refinancing, we explore how much outstanding mortgage borrowers could save in interest rates through refinancing. More specifically, we follow each borrower with a mortgage over time by building an interest rate saving measure for them. This is the interest rate reduction that mortgage borrowers could have got by refinancing their mortgages to a new refinance rate $\hat{r}_{izt}$. The average of the refinance interest rate $\hat{r}_{izt}$ is predicted by:

$$ r_{izt} = \beta X_{it} + \mu_t + \mu_z + \epsilon_{izt} $$  \hspace{1cm} (1.4.1)

in which $r_{izt}$ is the transacted interest rate in the refinance market of borrower $i$ at time $t$ in location $z$. $X_{it}$ includes the FICO® Score groups, the loan-to-value (LTV) ratio groups, their interactions, and the remaining term of the mortgage contract. $\mu_t$ and $\mu_z$ are quarter and five-digit zip code location dummies. The following equation enables us to find the measure of interest rate saving for borrowers indexed by $i$ if they refinance at time $t$ in
Figure 1: Dynamic of Refinancing Decisions

Note: This graph follows a representative sample of fixed-rate mortgages originated in 2008 until 2018. The right axis represents the unit of the black line. The black line presents the average of the refinance rate from the transacted data. The left axis represents the columns. The gray columns present the share of active loans from the cohort of 2008 at each point in time. The blue columns demonstrate the cumulative refinance fraction of the cohort of mortgages originated 2008. The sum of blue and gray columns at each point in time is less than one. This is because loans originated in 2008 may be terminated over time for reasons other than refinance (like early pay off, default, etc.).

Data Source: Equifax Credit Risk Insight Servicing and Black Knight McDash.

\[ \Delta r_{izt} = r_{iz} - \hat{r}_{izt} \]  

(1.4.2)

in which \( r_{iz} \) is the interest rate on the current mortgage for borrower \( i \) with a property located in location \( z \). Note that \( r_{iz} \), the interest rate on the current mortgage, is constant over time for borrower \( i \). The interest rate that the borrower can refinance their mortgage to, \( \hat{r}_{izt} \), may change over time. This occurs because the average mortgage interest rate in the market changes over time, or because the creditworthiness of the borrower, such as their
FICO® Score or LTV changes over time, or because the remaining term of the mortgage varies.

Figure 2: Incentive to Refinance

Note: We use representative sample of all outstanding fixed-rate mortgages in every quarter to generate these two graphs. Panel (a) presents the percentiles of the interest rate saving if borrowers would have chosen to refinance. The black line is the dynamic of average refinance rates, which is normalized to zero at the fourth quarter of 2008. Panel (b) replicates Panel (a) in terms of monthly payment savings.

Data Source: Equifax Credit Risk Insight Servicing and Black Knight McDash.

Figure 2 presents the distribution of this measure for all the outstanding mortgages in each quarter. To clarify, Figure 2 includes all the outstanding mortgages, not only those that were originated in 2008. The panel (a) in Figure 2 shows the 25%, 50% and 75% percentiles for the interest rate saving measure ($\Delta r_{izt}$) distribution from the fourth quarter of 2008 to 2015. We observe that the 25% percentile of the interest rate saving measure from 2009 to 2013 was typically positive. This means that at least 75% of the mortgages were potentially in the money to refinance. More interestingly, the 75% percentile shows that at least 25% of the borrowers from 2009 to 2015 could have saved more than 1.125 percentage points
through refinancing. The panel (b) in Figure 2 shows the equivalent graph in terms of monthly payment savings. This graph indicates that at least 25% of the borrowers from 2009 to 2015 could have saved almost $120 in monthly payments through refinancing.

We interpret the persistently wide positive gap between the interest rates of the outstanding mortgages and the average refinance rates as suggestive evidence of inactivity in refinancing. Inactivity in refinancing for the periods after the Great Recession has been well documented in other studies with different data sets [Agarwal et al. 2015b and Keys et al. 2016].

1.4.2. Interest Rate Dispersion and Search Intensity in the Refinance Market

![Figure 3: Interest Rate Residual](image)

Note: This graph shows the residual of transacted interest rates. We control for a long list of variables such as the FICO® Score, the loan-to-value (LTV) ratio, Debt-to-Income (DTI), income, term, demographics, five-digit zip code location dummies and quarter dummies. The difference between the 1st and the 99th distribution of the transacted interest rates is 162.5 basis points. The graph shows a wide dispersion in refinance interest rates for homogeneous mortgage contracts.

Data Source: Equifax Credit Risk Insight Servicing and Black Knight McDash.

Our data present a wide dispersion in interest rates for a homogeneous mortgage contract in the refinance market. We specifically focus on the refinance market, since this is the market that borrowers only enter for mortgage shopping. We suspect that, at the time
of purchase, most borrowers put more effort into searching for the best property than into finding the lowest mortgage rate. As a result, the dispersion in interest rates in the purchasing market may be a misleading indication of the mortgage shopping of borrowers. The following regression uses our data to document that a long list of variables cannot explain the wide dispersion in interest rates.

\[ r_{izt} = \beta X_{it} + \mu_t + \mu_z + \epsilon_{izt} \]  

(1.4.3)

in which \( r_{izt} \) is the transacted interest rate in the refinance market of borrower \( i \) at time \( t \) in location \( z \). \( X_{it} \) includes variables such as the FICO® Score, the loan-to-value (LTV) ratio, Debt-to-Income (DTI), income, term, and demographics. \( \mu_t \) and \( \mu_z \) are quarter and five-digit zip code location dummies. Figure 3 shows the interest rate residuals. The difference between the 5\(^{th} \) and the 95\(^{th} \) distribution of the transacted interest rates is 108.0 basis points. Moreover, the difference between the 1\(^{st} \) and the 99\(^{th} \) distribution of the transacted interest rates is 162.5 basis points. We also find similar dispersion in Ambokar and Samaee (2019) from a different data set in the US mortgage market. In our data, we do not have access to points and fees information. Some borrowers may pay higher upfront fees in order to lower the interest rates. These borrowers are not necessarily the low search cost ones, who could find lower rates. Ignoring the points and fees could potentially invalidate our interpretation that dispersion in interest rates is suggestive of search friction. However, Agarwal et al. (2017b) and Alexandrov and Koulayev (2018) still find that borrowers pay substantially different mortgage rates, even after adjusting for points and fees.

We use the number of mortgage inquiries as a measure of search intensity. Analyzing this variable provides more suggestive evidence of search friction in the US mortgage market. The distribution of the number of mortgage inquiries suggests a lack of mortgage shopping. Figure 4 indicates that almost 59.1% of the mortgage borrowers in our data only get one inquiry regarding refinancing a mortgage, despite the wide dispersion in the refinance
interest rates. The evidence of a lack of mortgage shopping is also documented by Alexan-
drov and Koulayev (2018) and Ambokar and Samaeel (2019), using the National Survey
Mortgage Origination (NSMO) data set.

![Figure 4: Number of Mortgage Inquiries Distribution](image)

Note: This graph shows the distribution of the number of mortgage inquiries in the refinance market between 2009-2015. The last column includes 5 and more number of inquiries. It is unlikely to observe more than 5 inquiries in our data.
Data Source: Equifax Credit Risk Insight Servicing and Black Knight McDash.

We finally explore how search intensity and refinance interest rates are correlated. The fol-
lowing regression provides the correlation between interest rate and number of mortgage
inquiries.

\[ r_{izt} = \sum_{n=1}^{5} \beta_n 1(n_{izt} = n) + \beta X_{it} + \mu_t + \mu_z + \epsilon_{izt} \]  

(1.4.4)

in which \( r_{izt} \) is the transacted interest rate in the refinance market of borrower \( i \) at time \( t \) in
location \( z \). \( X_{it} \) includes variables such as Debt-to-Income (DTI), income, and demograph-
ics. \( \mu_t \) and \( \mu_z \) are quarter and five-digit zip code location dummies. From the regression,
we can find the conditional correlation between interest rates and number of mortgage inquiries \( \{ \hat{\beta}_n \} \). We analyze this correlation for different groups of the FICO® Score and loan-to-value (LTV) ratio by running separate regressions among each group.

Figure 5 presents \( \{ \hat{\beta}_n \}_{n=1}^5 \) across the borrowers’ creditworthiness. We document that the correlation between interest rate and number of mortgage inquiries is either negative or flat across the borrowers’ creditworthiness. Figure 5 shows a negative correlation among superprime borrowers (with FICO® Scores above 740) with an LTV below 80%. This negative correlation suggests that it pays to search more.

![Figure 5: Search and Interest Rate Across Borrowers’ Creditworthiness](image)

Note: This graph shows the correlation between interest rates and number of mortgage inquiries among borrowers with different creditworthiness. As we explore riskier borrowers, the correlation becomes weak. Data Source: Equifax Credit Risk Insight Servicing and Black Knight McDash.

One complexity that we need to take into account in developing and estimating the model is that loan originators may reject an application. We therefore need to be able to separately identify inquiries that are rejected by the borrower or by the loan originators. We do
not have access to the application data for identifying these two channels. Borrowers with poor creditworthiness may accept any offer in order to avoid future searches. If we ignore such behavior, we may incorrectly classify borrowers with a low credit quality as borrowers with high search costs (Agarwal et al., 2017b). We argue that the correlation between the number of mortgage inquiries and the interest rates differ across the borrowers’ creditworthiness, and this evidence helps us to identify these two probabilities. In Figure 5, we observe that the correlation between interest rates and search intensity becomes weaker as we explore riskier borrowers. We can see this weaker correlation among prime borrowers (with FICO® Scores between 620 and 740) with an LTV above 80%. Finally, we do not observe a significant correlation between interest rate and number of mortgage inquiries among subprime borrowers (with FICO® Scores below 620) and LTV ratios above 80%.

We interpret the wide dispersion in interest rates and the lack of mortgage shopping, while it pays to search more, as suggestive evidence of search friction in the refinance market.

1.4.3. Search Intensity and Refinance

Having documented evidence suggestive of search friction and inaction in refinancing, we want to explore how these two are correlated. Specifically, this section explores the correlation between search intensity and refinancing probability.

In order to find this correlation, we specify a logit regression in equation 1.4.5. The dependent variable is the binary refinancing decision with \( \text{Refi}_{izt} \in \{0, 1\} \). Specifically, this binary variable is equal to 1 if the borrower \( i \) refinance their mortgage at time \( t \) in location \( z \). Otherwise, it is equal to 0. On the right-hand side of equation 1.4.5 we include the number of mortgage inquiries at the time of mortgage origination. Note that the number of mortgage inquiries are the fixed effects in this regression. We also control for the incentive to refinance measure that we built in section 1.4.1. The logit regression is as follows:

\[
\text{Refi}_{izt} = \sum_{n=1}^{5} \beta_n 1(n_{iz} = n) + \beta_r \Delta r_{izt} + \beta X_{it} + \mu_z + \mu_t + \epsilon_{izt}
\]  

(1.4.5)

in which \( X_{it} \) include variables such as the FICO® Score, the loan-to-value (LTV) ratio, Debt-
Figure 6: Search Intensity and Refinance Probability

Note: This graph shows the conditional correlation between refinance probabilities and the number of mortgage inquiries (search intensity) at the time of origination. More specifically, we use the number of mortgage inquiries at the time of mortgage origination as a fixed effect of a Logit regression in which the binary refinancing choice is the dependent variable. The refinancing odds for the first inquiry is normalized to one. We control for a long list of variables to find the conditional correlation such as time of origination, incentive to refinance measure, creditworthiness, etc.

Data Source: Equifax Credit Risk Insight Servicing and Black Knight McDash.

Figure 6 presents the odds ratio for the number of inquiries. It shows that borrowers who search five times to originate a mortgage are 17% more likely to refinance a mortgage every quarter, although this is conditional on them having the same incentives to refinance and the same creditworthiness. This evidence indicates that those who search more at the time of mortgage origination (shoppers) are more likely to refinance their mortgages, conditional on having the same incentives to refinance and creditworthiness. This evidence documents the heterogeneity in refinancing among borrowers, with the same incentives for refinancing and the same creditworthiness, but a different search intensity.
We need to take into account a complexity coming from selection in the refinance market in developing and estimating the model. The evidence of this section suggests that there is selection because borrowers with low search intensity, potentially high search cost borrowers, are less likely to refinance. We use the dynamic data on refinancing to address this complexity. Specifically, the documented positive correlation between search intensity and refinancing odds helps us to infer their search behavior based on their refinancing frequencies.

1.5. An Equilibrium Model of Mortgage Refinancing and Search Decisions
Motivated by the evidence from the data, we develop a model of mortgage refinancing and search decisions. We extend the static sequential random search model in Agarwal et al. (2017b) into a dynamic discrete choice framework. In a random search model, there is a distribution of offered rates for a homogeneous mortgage contract. Therefore, borrowers only know the range of interest rates they may find. This range depends on both the offered rate distribution and the borrower choice for the reservation interest rate. Given the offered rate distribution and reservation interest rate, we can find the transition probability of interest rates. Therefore, we develop a search model within a dynamic discrete choice framework by adding the transition probability of interest rates into the transition probability of a dynamic discrete choice model.

The model is an equilibrium model. On the demand side, there are borrowers who are heterogeneous in search costs and creditworthiness. They make refinancing and search decisions. On the supply side, there are homogeneous loan originators who offer rates while they take borrowers’ search friction into account. We discuss the demand in section 1.5.1 and the supply in section 1.5.2.

1.5.1. Demand
We propose a model for a borrower with a mortgage who decides whether or not to refinance. We only take into account fixed-rate mortgage (FRM) contracts, which are the dominant type of contracts in the US. The borrowers are indexed according to their type and the characteristics of their current mortgage during each period. The borrower type
includes two variables: search costs \((c)\) and FICO® Score \((\theta_t)\). The search costs are not observable to the loan originators, while the credit scores are. Search costs are persistent, meaning that borrowers’ search costs do not change over time. Credit scores \((\theta_t)\) change over time based on a Markov Process. We represent the current mortgage of a borrower by its interest rate \((r)\) and other characteristics \((x_t)\), which include current loan-to-value ratio \((LTV_t)\) and the remaining term of the mortgage \((\text{term}_t)\). \(x_t\) evolve according to a Markov Process. In Equation 1.5.1 we summarize the state variable of a borrower:

\[
z_t = (c, \theta_t, r, x_t) \tag{1.5.1}
\]

All the variables in the model are discrete. We define five groups of search costs \((c \in \{1, 2, ..., 5\})\) and eleven groups of credit scores \((\theta_t \in \{\text{below 620}, 620-639, 640-659, ... , \text{above 800}\})\). We define seven groups for LTV ratios \((LTV_t \in \{(0, 0.6], (0.6-0.7], (0.7-0.75], (0.75-0.8], (0.8-0.9], (0.9-1], (1, +\infty)\})\). We consider three groups for the remaining term of the mortgage in months \((\text{term}_t \in \{\text{below 209}, 210-329, \text{above 330}\})\). The interest rates are also discrete because we almost always observe the interest rates in increments of 12.5 basis points in the data.

Borrowers may refinance due to different incentives. Since the interest rate on a FRM contract is fixed, borrowers may want to refinance in order to get a lower rate if they can find one (a rate refinance). However, the borrowers may also refinance for other reasons, such as: cashing out a home equity (cash-out refinance), paying off a fraction of the remaining principal (pay-off refinance), or changing the term of the current mortgage (term-refinance).

If a borrower chooses to refinance, the old mortgage is terminated and they are expected to choose a new mortgage with new features (LTV, term, and interest rate). We assume a specific sequence for the borrower’s decision-making. The borrowers first choose the new LTV and term, and they then choose the interest rate of the new mortgage. We assume that
all loan originators provide all the possible combinations of mortgages with different LTV and terms. They may offer different interest rates for a homogeneous (identical LTV and term) mortgage contract, and the borrowers know that there is price dispersion. However, borrowers will not know what rate the originators are offering to them until they request a quote. Borrowers can gather quotes from various loan originators. Gathering quotes are costly for the borrowers because of search costs. These search costs may come from various sources. For some borrowers, search costs exist because their opportunity cost of time is high. For some, it exists, for instance, because they find it costly to interact with the financial institutions.

In Figure 7, we demonstrate the decision sequence of a borrower with a mortgage. Borrowers first decide whether to refinance or not. Second, those who choose to refinance, decide $j \in X$, which is the LTV and term of the new mortgage contract. $X$ includes 21 contracts that comes from all the combinations of LTV ratios and terms. We use $j \in \{0\}$ to denote borrowers who choose not to refinance. Third, those who choose to refinance a new mortgage with specific LTV and term, search sequentially to find the best rates. The borrowers know the distribution of the rates offered. Each loan originator offers a rate based on the credit score, LTV, and term at each point in time. We denote the offered rate distribution by $h_{j\theta t}(r)$ and its CDF by $H_{j\theta t}(r)$. More specifically, for contract $j$, credit score $\theta$ in period $t$, there is a distribution of interest rates. Borrowers draw i.i.d. with replacement from the offered distribution.

In the search stage, borrowers decide whether to accept the offered rate and apply for the mortgage, or whether to reject the offer and continue searching. If they apply and the application is approved, they refinance with the offered rate. However, if their application is rejected, they will search again. Note that, when borrowers choose to refinance and subsequently they start to search, they cannot go back to the original mortgage anymore. They must search until finally they refinance with a new interest rate.

Alexandrov and Koulayev (2018) explore a model in which the borrowers are unaware of price dispersion.
Figure 7: A Borrower Refinancing and Search Decision Tree

Borrower Type: Search cost \(c\) and FICO® Score \(\theta_t\),
Current Mortgage: Interest rate \(r\) and Other Characteristics \(x_t = \text{LTV}_t, \text{term}_t\)

Refinance

Choose new LTV and term
of the new mortgage
\((j \in X)\)

Search for interest rate \(r_{t+1}\)
of the new mortgage

borrower state variable
in the next period:
\((c, \theta_{t+1}, r_{t+1}, j)\)

Not Refinance

borrower state variable
in the next period:
\((c, \theta_{t+1}, r, x_{t+1})\)

Note: This graph presents the decision making on the demand side of the model. This is a Nested Logit model. Borrowers with a mortgage first decides whether to refinance or not. If they refinance, they then decide which LTV ratio and term choose for the new contract. Finally, they search for the best rates. Those who do not refinance keep the mortgage with the same rates. We also show how the state variables of those who refinance versus those who do not evolve over time.

A refinancing decision requires a cost-benefit analysis for the mortgage borrowers. The refinancing costs include search costs and switching costs. The presence of refinancing costs makes the borrower’s problem dynamic because borrowers may hold on to a mortgage for quite a while. Therefore, the benefit of refinancing comes from the flow of utilities throughout the life of the new mortgage contract plus the realization of an extreme value i.i.d. taste shock.
Equation 1.5.2 is the indirect utility of the borrower with a state variable of $z_t = (c, \theta_t, r, x_t)$ who chooses $j$ at time $t$ before receiving the taste shock.

$$u_{jt}(z_t) = \begin{cases} 
  u_{\theta x} - r & \text{if } j \in \{0\} \\
  u_{\theta x} - r - R_{j;thx,} & \text{if } j \in X
\end{cases}$$

We use $\mathcal{R}$ to denote the total refinancing costs, which is the sum of the switching costs and the search costs. The flow utility of the borrower comes from the current mortgage during every period, irrespective of whether or not they choose to refinance. If borrowers choose to refinance in period $t$, they pay for the refinancing costs. The benefit of refinancing comes from the future life of the new mortgage, which starts from period $t+1$. The utility of a borrower comes from borrower’s type and the characteristics of the mortgage. We normalize the coefficient of the interest rate to one, with the higher the interest rate of a mortgage contract, the lower the utility. Borrowers have utility over LTV ratio and term of their mortgage. These characteristics are also interacted with credit scores. $u_{\theta x}$ captures the incentives for refinancing other than lowering the interest rate.

Since the problem of a borrower is dynamic, they compare the total expected payoff for a given choice when they want to make a refinancing decision. We use $v_{jt}(z_t)$ to denote the total expected payoff (before receiving the taste shock) for choice $j$ at time $t$ when the borrower is at the state of $z_t$. After receiving the taste shock $\epsilon_{jt}$, the borrower chooses whether not to refinance ($j \in \{0\}$) or refinance, and chooses a new mortgage ($j \in X$):

$$\max_{j \in \{0, X\}} v_{jt}(z_t) + \epsilon_{jt}$$

in which $\epsilon_{jt}$ is the nested logit shock that comes from a generalized extreme value distribution. Specifically, there is no correlation between nests, and $(\text{Cov}(\epsilon_{0t}, \epsilon_{jt}) = 0, \forall j \in X)$. $1 - \sigma$ is the correlation within the nest ($\sigma \in [0, 1]$).

$v_{jt}(z_t)$ is the sum of the flow utility from Equation 1.5.2 and also a discounted expectation.
of continuation values:

\[ v_{jt}(z_t) = u_{jt}(z_t) + \beta \sum_{z_{t+1}} V_{t+1}(z_{t+1}) f_{jt}(z_{t+1}|z_t) \]  \hspace{1cm} (1.5.4)

in which \( V_{t+1}(\cdot) \) is the unconditional value function and \( f_{jt}(\cdot) \) is the transition probability.

The transition probability depends on whether or not the borrower chooses to refinance. If the borrower chooses not to refinance, they will keep the same mortgage. Since the contract is a fixed-rate mortgage, the interest rate remains unchanged. We need to follow how the credit score, the LTV ratio, and the reaming term of the contract may change over time. The LTV ratio evolves for two reasons. First, the remaining mortgage principal declines over time and, second, the value of the property may change over time. The borrower who does not choose to refinance starts the following period with a state variable of \((c, \theta_{t+1}, r_t, x_{t+1})\), as we demonstrated in the decision tree of a borrower in Figure 7. If the borrower chooses to refinance, they first choose the new LTV and the term of the new mortgage. They then search to find a rate. The set of interest rates that a borrower may find depends on the distribution of offered rates and on the reservation interest rate of the borrower. Specifically, the borrower with reservation interest rate of \( r_{jt}^* \) who chooses to refinance to contract \( j \), will get interest rate \( r_{t+1} \) with probability of \( h_{jt}(r_{t+1}|r_{jt}^*) \). In other words, \( h_{jt}(r_{t+1}|r_{jt}^*) \) is the transition probability of the interest rate for the borrower who chooses to refinance to contract \( j \). This borrower will start the following period with a state variable of \((c, \theta_{t+1}, r_{t+1}, j)\), as we demonstrated in the decision tree of a borrower in Figure 7. In fact, by incorporating the distribution of offered rates into the transition probability, we develop a search model within a standard dynamic discrete choice framework. The Equation 1.5.5 presents the transition probability:

\[
f_{jt}(z_{t+1}|z_t) = \begin{cases} 
(1 - \delta) f_{0t}(\theta_{t+1}, x_{t+1}|\theta_t, x_t) & \text{if } j \in \{0\} \\
(1 - \delta) h_{jt}(r_{t+1}|r_{jt}^*) f_{1t}(\theta_{t+1}|\theta_t), & \text{if } j \in X 
\end{cases} \]  \hspace{1cm} (1.5.5)
The transition function \( f_{0t}(\cdot) \) captures the dynamic of \((\theta, x)\) when the borrower chooses not to refinance. \( f_{1t}(\cdot) \) captures the dynamic of the credit scores when the borrower chooses to refinance and, \( h_{j\theta t}(r_{t+1}|r^*_{jc\theta t}) \) captures the transition to the new mortgage rate. As a standard dynamic discrete choice model, both \( f_{0t}(\cdot) \) and \( f_{1t}(\cdot) \) evolve according to a Markov Process.

A mortgage contract may also be terminated for reasons other than refinancing, such as default shock, moving shock, etc. We need to take this into account because it may affect the refinancing decision. For example, if the borrowers know that they want to sell the property to move to another city, they may not have the incentive to refinance the mortgage to a lower rate. \( \delta_\theta \) in Equation 1.5.5 captures the termination shock for reasons other than refinancing. We allow that the termination shock to be a function of the creditworthiness of the borrowers. For example, we expect that the higher the creditworthiness of the borrowers, the lower the default probability of the mortgage.

The transition probability of the interest rate when borrowers choose to refinance to contract \( j \), namely \( h_{j\theta t}(r_{t+1}|r^*_{jc\theta t}) \), is both a function of offered rate distribution and the choice of borrowers for reservation interest rate. We therefore need to present the borrower’s search decision in the search stage, as demonstrated in the decision tree of the borrower in Figure 7. A borrower contacts a loan originator in the search stage in order to get quotes. A borrower with an offered interest rate of \( \tilde{r} \) decides whether to accept the offer or whether to reject it and search for a lower rate. The marginal cost of a borrower of search type \( c \) for getting another quote is \( \kappa_c \). We assume that the marginal cost of the search is constant, and that it does not depend on the number of inquiries. We denote the benefit of an additional search function with \( B_{jc\theta t}(\cdot) \) in Equation 1.5.6:

\[
B_{jc\theta t}(\tilde{r}) \equiv \beta(1 - \delta_\theta)\lambda_{j\theta t} \sum_{\theta_{t+1}} \sum_{r_{t+1} \leq \tilde{r}} (V_{t+1}(c, \theta_{t+1}, r_{t+1}, j) - V_{t+1}(c, \theta_{t+1}, \tilde{r}, j)) h_{j\theta t}(r_{t+1}) f_{1t}(\theta_{t+1}|\theta_t)
\]

(1.5.6)

The marginal benefit of search captures the additional benefit of refinancing a mortgage.
with a lower interest rate. Several forces affect this marginal benefit. One is the termination shock \( \delta \). If borrowers expect that they are likely to hold on to a mortgage for quite a while, they will have low termination shock, and will have more incentive to search for better rates. The approval probability \( \lambda_{jt,t} \) also affects the marginal benefits of a search. If the borrowers know that their application is unlikely to be rejected, they will search more to find lower rates. Moreover, the marginal benefit of a search depends on the expected value of getting a lower rate. If it is possible that the borrowers will find significantly lower rates by increased searching, then they will have more incentive to search. The difference between the unconditional value function for lower rates and current rates is presented in Equation 1.5.7:

\[
V_{t+1}(c, \theta_{t+1}, r_{t+1}, j) - V_{t+1}(c, \theta_{t+1}, \tilde{r}, j) = \tilde{r} - r_{t+1} + \ln\left( \frac{P_{1,t+1}(c, \theta_{t+1}, \tilde{r}, j)}{P_{1,t+1}(c, \theta_{t+1}, r_{t+1}, j)} \right)
\]  

(1.5.7)

The first part is the interest rate saving \( \tilde{r} - r_{t+1} \), which also appears in a static model. The second part depends on the probability of refinancing in the following period. This part appears due to the dynamic nature of the problem. Specifically, we use \( P_{1,t+1}(\cdot) \) to denote the probability of refinancing in period \( t+1 \), regardless of the choice of the contract. In the appendix 1.10.1, we show the closed form solution for this probability. Finally, we can find the reservation interest rates through marginal benefit and cost of searching. The Equation 1.5.8 indicates how we find the reservation interest rate:

\[
B_{jt}(r^*) \leq \kappa_c \implies r^*_{jt}\]

(1.5.8)

in which \( r^*_{jt} \) is the maximum interest rate that satisfies the above inequality. As mentioned earlier, a refinancing decision requires a cost-benefit analysis. We described the benefit of refinancing by presenting \( v_{jt}(\cdot) \) and its components. In the following, we discuss the refinancing costs, which include search costs and switching costs. We first describe the search costs. In the search stage, borrowers search sequentially in order to find the best rates. For every draw, the borrower of type \((c)\) pays the marginal search cost \( \kappa_c \) and draws a rate \( r_{t+1} \).
from the offered rate distribution $h_{j|\theta_t}$. The draws are i.i.d. with replacement. The borrower decides whether to accept the offered rate $r_{t+1}$ and apply for the mortgage, or whether to reject the offer and continue searching. If they apply, the application is approved with the probability $\lambda_{j|\theta_t} \in [0, 1]$, and they refinance with interest rate $r_{t+1}$. However, if their application is rejected, or they choose not to apply for the loan, they will search again.

The total search costs of a borrower with search cost type $c$ who ends up getting $n$ inquiries to refinance is $\kappa_c n$. Borrowers form an expectation of the expected search costs before refinancing, and this depends on their type $(c, \theta_t)$, and on the contract they want to choose $(j)$. We use $E_{j|c|\theta_t}[n]$ to denote the expected number of inquiries. The expected search costs conditional on refinancing for a borrower of type $(c, \theta_t)$ who chooses mortgage $j$ at time $t$ is $\kappa_c E_{j|c|\theta_t}[n]$. The expected number of inquiries depends on two probabilities, namely the probability that a borrower may reject an offer and the probability that a mortgage originator declines an application. The borrowers may choose a reservation interest rate $(r^*)$ and reject any offer above that. Specifically, Equation 1.5.9 presents the expected number of mortgage inquiries:

$$E_{j|c|\theta_t}[n] = \frac{1}{\lambda_{j|\theta_t} H_{j|\theta_t}(r^*_{j|c|\theta_t})}$$  \hspace{1cm} (1.5.9)

in which $(r^*_{j|c|\theta_t})$ is the reservation interest rate that the borrowers choose.

Search costs are not the only refinancing costs that may inhibit refinancing. We also assume that there are switching costs. Such costs may include the financial costs associated with refinancing a mortgage. They may also include an unwillingness to terminate a contract and originate a new contract. We use $s_{j|c|\theta_x}$ to denote switching costs. We assume that switching costs depend on the type of the borrowers, the current and new mortgage contracts, the LTV ratios and the terms. The Equation 1.5.10 presents the refinancing cost, which is the sum of the switching cost and the search costs:

$$R_{j|c|\theta_x} = \underbrace{s_{j|c|\theta_x}}_{\text{switching cost}} + \frac{\kappa_c}{\lambda_{j|\theta_t} H_{j|\theta_t}(r^*_{j|c|\theta_t})} \underbrace{\lambda_{j|\theta_t} H_{j|\theta_t}(r^*_{j|c|\theta_t})}_{\text{search cost}}$$  \hspace{1cm} (1.5.10)
One source of inactivity in refinancing may come from the high cost of refinancing. However, since refinancing costs do not all come from the financial costs of refinancing, we use our model to estimate the total refinancing costs from refinancing frequency of the borrowers. We do that and also the model helps us to separately identify the search costs from the switching costs.

Refinancing costs have a direct effect on refinancing incentives. The direct effect of search costs on refinancing incentives go through the expected search costs \( \kappa_c E_j c \theta t[n] \). This is the most obvious channel that search costs may inhibit refinancing. In fact, borrowers may find it costly to search for a new mortgage. The second channel is that search costs indirectly increase the loan originators’ market power and thus raise the offered refinance rate. The first channel directly affects the costs of refinancing, while the second channel affects the benefit of refinancing through the equilibrium interest rates.

The Equation 1.5.5 provides the intuition of how the indirect effect of search costs may affect the benefit of refinancing. \( h_j \theta t(r_{t+1} \mid r_j c \theta t) \) governs the incentive to refinance in order to lower the interest rate of the current mortgage. If the loan originators offer high interest rates, borrowers do not have incentives to refinance.

The following section discusses the supply side of the model.

1.5.2. Supply

On the supply side, we follow Agarwal et al. (2017b) in specifying the loan originator model. Since most of the residential mortgages are originated through an originate-to-sell mechanism, we assume a static model for the supply side.

A Loan originator offers interest rate \( r \) in order to maximize its expected profit. Moreover, loan originators accept an application with exogenous probability \( \lambda_j \theta t \). Like Agarwal et al. (2017b), we do not endogenize the approval probabilities. The marginal costs of the loan originator include two parts. The first is the marginal cost of origination in the refinance market (\( \chi \)). The second is the marginal cost that depends on the type of borrower, contract
and time \((j, \theta, t)\). This part is given to the loan originator and potentially comes from how
the loan buyers price the loan using different characteristics. We find \(\hat{r}_{j\theta t}\) by running a
regression of interest rates in the refinance market on \((j, \theta, t)\) and their interactions. Note
that the changes in the funding costs, such as those due to changes in the federal funds rate, will be reflected in \(\hat{r}_{j\theta t}\).

Loan originators choose what interest rate to offer on the basis of the marginal costs and
demand function. The loan originators take the borrowers’ search friction into account and
respond accordingly. More specifically, a loan originator has a rational expectation over
the distribution of the search cost of the borrowers, who choose to refinance to mortgage
\(j\), with credit score \(\theta\) at time \(t\). In this model, the price elasticity of demand depends on
the search behavior of the borrowers. Therefore, the demand function is endogenous and
Equation 1.5.11 presents it:

\[
q_{j\theta t}(r) = \sum_{r \geq \hat{r}} \frac{\sum_{(c,x,r')} h_{j\theta t}(c,x,r') \mu_{\theta t}(c,x,r') 1\{r_{j\theta ct} = \hat{r}\}}{H_{j\theta t}(\hat{r})}
\]  

(1.5.11)
in which \(\mu\) is the mass of borrowers with a mortgage in period \(t\) with the current char-
acteristics of the borrowers and the mortgages. In appendix 1.10.2, we discuss in details
how we find the demand function. The expected profits of charging an interest rate \(r\) are therefore:

\[
\Pi_{j\theta t}(r) = (r - \hat{r}_{j\theta t} - \chi) q_{j\theta t}(r)
\]  

(1.5.12)
The loan originators choose the interest rate in order to maximize their profits. Following
Agarwal et al. (2017b), we assume a logit shock \(\epsilon_{j\theta tk}\) with Type I EV distribution. We find
the offer rate distribution as follows:

\[
\max_{r_k} \Pi_{j\theta t}(r_k) + \epsilon_{j\theta tk} \implies h_{j\theta t}(r_k) = \frac{\exp\left(\frac{\Pi_{j\theta t}(r_k)}{\epsilon_{\theta t}}\right)}{\sum_k \exp\left(\frac{\Pi_{j\theta t}(r_k)}{\epsilon_{\theta t}}\right)}
\]  

(1.5.13)
Since the loan originators take the search friction into account, their demand function is
inelastic. This means that borrowers can charge higher interest rates than the marginal
costs of originating a mortgage. We can find the average of the markups as follow:

\[
\sum \bar{r}_{i,j,t}(\bar{r}) - \bar{p}_{j,t} - \chi
\]  

(1.5.14)

The higher the search costs, the higher the markups for the loan originators. As a result, the loan originators offer higher rates than the marginal cost, which is the indirect channel by which search costs inhibit refinancing.

1.6. Estimation

In this section, we discuss how we estimate the parameters of the model. The goal is to estimate search cost distribution along with other parameters of the model. The standard approach, in industrial organization, is to back out search cost distribution from the observed price distribution (Hong and Shum, 2006). Many used this approach to estimate the search cost distribution in the mortgage market (Agarwal et al., 2017b, Alexandrov and Koulayev, 2018 and Allen et al., 2019). In section 1.4.3, we documented that: the higher the number of inquiry, the higher the probability of refinancing. If high search cost mortgage borrowers are less likely to refinance, then the participants in the refinance market are biased in favor of low search cost borrowers. By writing a dynamic model, we took into account this selection.

We estimate the model using data from 2008 to 2015, during a period of mortgage rates’ transition of high to low. Solving a dynamic model during a transition period is computationally challenging, because we have to keep track of the distribution of offered rates and many state variables. That is why we incorporate a search model into a dynamic discrete choice framework in order to estimate the model using conditional choice probability (CCP) techniques. We build on Arcidiacono and Miller (2011) since it allows us to incorporate unobserved heterogeneity. Search costs are the only unobserved heterogeneity in our model. We specifically use the two-stage Arcidiacono and Miller (2011) tools to estimate the model.
1.6.1. First Stage

We follow Arcidiacono and Miller (2011) to estimate the empirical CCPs and transition probabilities in the first stage. We incorporate a sequential search model into a dynamic discrete choice framework. As a result, the distribution of the offered rates are included in the transition function when the borrowers choose to refinance. In this stage, we should estimate this offered rate distribution. However, estimating an equilibrium search model is computationally challenging, because we need to estimate the equilibrium offered rates through solving a functional fixed point problem. We therefore complete this stage in two steps. First, we estimate the empirical refinancing probabilities. Second, given the empirical refinancing probabilities, we estimate the offered rate distribution, marginal search costs and approval probabilities.

Note that empirical refinancing probabilities cannot be directly calculated from data since search costs are unobservable. Following Arcidiacono and Miller (2011), we use EM algorithm to estimate the empirical refinancing probabilities. We assume that there are five groups of search costs \( c \in \{1, 2, 3, 4, 5\} \), higher the index number, potentially higher the (marginal) search costs. The goal of the algorithm is to classify these five groups of search costs. Mortgage borrowers within each group of search costs are similar in refinancing probability, given having the same incentive to refinance (\( \Delta r \)), credit scores, LTV ratios and terms of the mortgage contract (\( \theta, x \)). The difference across these five groups is that, conditional on other variables, they may refinance with significantly different odds. Note that, in this stage we do not use any information of search behavior of the individuals to identify the groups. Hence, the intuition for identification comes from the evidence in section 1.4.3 that, the higher the search intensity, the higher the refinancing odds.

We use \( p_{jt}(z_{it}) \) to denote the empirical refinancing probabilities. This is the probability that an individual chooses to refinance to contract \( j \) at time \( t \), given the observed state of \( z_{it} \). Denote \( L_t \) in equation 1.6.1, the likelihood of observing \( (d_{it}, z_{i,t+1}) \), conditional on state \( z_{it}, d_{it} \equiv (d_{i1t}, ..., d_{iJt}) \) is the vector of dummy variables. If individual \( i \) chooses to refinance
to contract $j$, $d_{ijt} = 1$, otherwise, it is zero.

$$L_t(d_{it}, z_{i,t+1}|z_{it}) = \prod_j [\{j = 0\} p_{0t}(z_{it}) f_{0t}(z_{i,t+1}|z_{it}) + \{j \in X\} p_{jt}(z_{it})]^{d_{ijt}} \quad (1.6.1)$$

where the transition probability conditional on not refinancing is as follow:

$$f_{0t}(z_{i,t+1}|z_{it}) = (1 - \delta_\theta) f_{0t}(\theta_{i,t+1}, x_{i,t+1}|\theta_{it}, \theta_{it}) \quad (1.6.2)$$

in which $z_{it} = (c_i, \theta_{it}, r_{it}, x_{it})$. We specify the refinancing probability of individual $i$ at time $t$ in equation $1.6.3$

$$p_{jt}(z_{it}) = \begin{cases} 
\frac{1}{1 + \exp(\beta_c x + \beta_1 \Delta r_{it})} & \text{if } j = 0 \\
\frac{\exp(\beta_c x + \beta_1 \Delta r_{it})}{1 + \exp(\beta_c x + \beta_1 \Delta r_{it})} \tilde{p}_{j|1}(c, \theta_{it}, x_{it}) & \text{if } j \in X \end{cases} \quad (1.6.3)$$

where $\Delta r_{it} = r_{it} - \hat{r}_{it}$ is the amount of interest rate saving by rate refinancing the current mortgage to a lower rate (Equation $1.4.2$). $\beta_c x$ captures interaction between search costs, creditworthiness and mortgage characteristics. $\tilde{p}_{j|1}$ denotes contract choice conditional on refinancing, $\sum_{j \in X} \tilde{p}_{j|1}(c, \theta_{it}, x_{it}) = 1$. We nonparametrically estimate $\tilde{p}_{j|1}$ and $f_{0t}$. We estimate $(\beta_c x, \beta_1)$, and $g_0(c)$ through the EM algorithm. In appendix $1.10.4$ we discuss the details of the EM algorithm. By completing this step, we find the search costs probabilities but not the marginal search costs.

In the second step of the first stage, we estimate the offered rate distributions $h_{j|\theta t}(\cdot)$, marginal search costs $\kappa_c$ and approval probabilities $\lambda_{j|\theta t}$. At the end of this step, we can fully characterize the search cost distribution. To estimate these parameters, we build a likelihood function that is quite similar to Agarwal et al. (2017b). Given the offer rate distribution $h_{j|\theta t}(r)$ and approval probability $\lambda_{j|\theta t}$, for both of which we assume a parametric form, we denote the likelihood of observing $(r_{it}, n_{it})$ conditional on reservation interest
rate $r^*$ in equation \[1.6.4\]

$$l(r_{it}, n_{it}|r^*_{jct}) = \lambda_{j\theta t} h_{j\theta t}(r_{it}) \left(1 - \lambda_{j\theta t} H_{j\theta t}(r^*_{jct})\right)^{n_{it} - 1}$$

(1.6.4)

where $n_{it}$ is the number of inquiries when a borrower refinances a mortgage. $H_{j\theta t}(.)$ is the CDF of the offered rate distribution. When we observe $(r_{it}, n_{it})$ for borrower $i$, it means that the borrower refinanced the mortgage to interest rate $r_{it}$ in the $n_{it}$th search attempt. For $n_{it} - 1$ inquiries, either the application was rejected by the loan originators with probability $1 - \lambda_{j\theta t}$ or the offered refinance rate was rejected by the borrower because the offer was above the reservation interest rate. Thus, the rejection probability of an offer is $\lambda_{j\theta t} H_{j\theta t}(r^*_{jct})$ for a borrower who chooses to refinance to a mortgage with characteristics $j$ at time $t$ with FICO® Score $\theta$. Given the offered rate distribution, we find the reservation interest rates from the Equation \[1.5.8\]. In this step, we also take into account that the offered rate distribution is an equilibrium object. We therefore find the offered rate distribution from the Equation \[1.5.13\]. From this step, we estimate the parameters from the supply side which are marginal cost of origination ($\chi$) and the standard error of the logit shock to profit ($\sigma_\pi$). Finding the offered rate distribution is a functional fixed point problem. We follow the algorithm in Agarwal et al. (2017b) to estimate the offered rate by fitting a normal distribution.

We use $\beta^\lambda$ to denote the vector of parameters that characterizes the functional form of the approval probabilities. Similarly, we use $\beta^h$ to denote the vector of parameters that characterizes the normal distribution for offered rate distribution $h^N_{j\theta t}$. Finally, we maximize the objective function in equation \[1.6.5\] to find the estimate of the parameters.

$$(\beta^\lambda, \beta^h, \sigma_\pi, \chi, \kappa_c) \in \arg\max \sum_j \sum_{\theta} \sum_t \frac{1}{N} \sum_i \sum_c \mu_{j\theta t}(c) \ln l(r_{it}, n_{it}|r^*_{jct})$$

$$- \sum_j \sum_{\theta} \sum_t \sum_r \left( h^N_{j\theta t}(r) - h_{j\theta t}(r) \right)^2$$

(1.6.5)
in which the first term is the likelihood function that we want to maximize. The second term is the distance between equilibrium offered rate \( h_{jt} \) derived in the Equation 1.5.13 and the approximated normal distribution for offered rates \( h_{jt}^N \) that we want to minimize. The reservation interest rates for any guess for offered rate distribution comes from the Equation 1.5.8.

### 1.6.2. Second Stage

Given the estimation results from section 1.6.1, we estimate the switching costs and utility parameters of the model following the second stage of Arcidiacono and Miller (2011). Intuitively, we minimize the distance between the empirical refinancing probabilities \( p_{jt}(z_t) \) estimated in 1.6.3 from the structural refinancing probabilities \( P_{jt}(z_t) \) derived in 1.10.2 to estimate the utility and switching costs. Following 1.6.1, we estimate the parameters as follow:

\[
\{u_{\theta x}, s_{jc, \theta x}\} \in \text{argmin} \left| |v_{jt}(z_t) - v_0t(z_t) + \psi_j[p_t(z_t)] - \psi_0[p_t(z_t)]|\right| \quad (1.6.6)
\]

where \( \psi_j[.] \) is the correction term for a nested logit model.

\[
\psi_k[p_t(z_t)] = \begin{cases} 
-\ln(p_{0t}) & \text{if } k = 0 \\
-\ln(p_{1t}) - \sigma \ln(p_{kt|1}) & \text{if } k \in X 
\end{cases} \quad (1.6.7)
\]

The difference \( v_{jt}(z_t) - v_0t(z_t) \) is a function of the future empirical refinancing probabilities. Calculating this can be potentially difficult if we want to simulate for many periods ahead. However, our model provides one-period finite dependence property, which makes the estimation of the parameters in the second stage fairly easy. In fact, we can characterize the \( v_{jt}(z_t) - v_0t(z_t) \) as a function of the one-period ahead empirical refinancing probabilities. The intuition behind the one period ahead finite dependence is as follows. Suppose that two borrowers with the same search costs refinance to different arbitrary contracts. If they both refinance to a same arbitrary contract in the next period, they will have the same continuation value.
1.7. Estimation Results

In this section, we discuss the estimation results. As discussed earlier, we estimate a search cost model while considering two complexities. The first complexity is that loan originators may reject an application based on creditworthiness. To address this complexity, we separately identify the probabilities of the loans being rejected by the loan originators or by the borrowers. In the Section 1.7.1, we present the estimates of the approval probabilities. The second complexity is derived from the selection. If high search cost mortgage borrowers are less likely to refinance, then the participants in the refinance market are biased in favor of low search cost borrowers. In Section 1.7.2, we argue how search cost distribution is different from the distribution of those who choose to refinance. In Section 1.7.3, we then present the estimates for search costs and their shares in total refinancing costs. In the last two Sections, we discuss the answers to the two research questions that we raised. In Section 1.7.4, we discuss the effects of search costs on refinancing activities. In Section 1.7.5, we explore the contribution of the direct and the indirect market power effect on refinancing.

1.7.1. Borrowers’ Creditworthiness and Approval Probability

Loan originators may reject an application based on the borrowers’ creditworthiness, which affects the borrowers’ search behavior. Those with low creditworthiness know that their chances of being approved are small; thus, they may be willing to accept a mortgage with a high interest rate to avoid the additional search. This implies that these borrowers will behave as if their search costs are high. If we use only the interest rate distribution to estimate the search costs, we will back out the search cost distribution conditional on creditworthiness, but not the unconditional one. In this Section, we detail the distribution of the borrowers’ creditworthiness and approval probabilities.

In Figure 8, we present the distribution of borrowers with different levels of creditworthiness in 2008. For improved data presentation, we aggregate LTV ratios into two groups: LTV ratios above 80% and those below 80%. We also aggregate the FICO® Scores into three groups: subprime (FICO® Scores below 619), prime (FICO® Scores between 620 and
Figure 8: Distribution of Borrowers’ Creditworthiness

Note: This graph presents the distribution of the borrowers’ creditworthiness in 2008. The columns from left to right present subprime (FICO® Scores below 619), prime (FICO® Scores between 620 to 739) and superprime (FICO® Scores above 740) borrowers, respectively. The first row presents the low LTV borrowers (LTV ratio below 80%). The second row presents the high LTV borrowers (LTV ratio above 80%). Data Source: Equifax Credit Risk Insight Servicing and Black Knight McDash.

739) and superprime (FICO® Scores above 740) borrowers. This distribution is directly derived from the raw data. The graph illustrates that most of the borrowers, 93%, are prime or subprime borrowers. However, almost 54% of the borrowers have LTV ratios above 80%. This result is consistent with a significant house price shock in the wake of the Great Recession. Furthermore, Figure 8 confirms that there is heterogeneity in the borrowers’ creditworthiness. If the borrowers’ approval probabilities are different, we should consider this difference in the search cost estimation.

Figure 9 displays the estimates of the approval probabilities across borrowers’ creditworthiness and time. The approval probabilities can range from 0.42 to 0.99. There is also a significant difference between the approval probabilities among borrowers with LTV ratios below 80% and those with LTV ratios above 80%.
Figure 9: Estimates of Approval Probabilities

Note: In this graph, we present the estimates of approval probabilities ($\lambda_{jt}$). The columns from left to right represent subprime (FICO® Scores below 619), prime (FICO® Scores between 620 and 739) and superprime (FICO® Scores above 740) borrowers, respectively. The first row represents the low LTV borrowers (LTV ratio below 80%). The second row represents the high LTV borrowers (LTV ratio above 80%).

Data Source: Equifax Credit Risk Insight Servicing and Black Knight McDash.

1.7.2. Search Cost Distribution and Selection

Figure 10 illustrates the distribution of the search costs. Since we estimate a dynamic model, we can verify from the estimates whether the selection exists in the refinance market. The top row presents the distribution of the search costs, and the second row presents a distribution of the search costs in a typical refinance market. This distribution is similar to the log-normal search cost distribution estimated in static search cost models in the US mortgage market (Agarwal et al. 2017b and Alexandrov and Koulayev 2018). The high search cost borrowers ($c = 5$) includes more than half of the borrowers in the mortgage market who also have a low share in the refinance market. These borrowers have low probabilities of refinancing.

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4Refinance markets are characterized by LTV ratios, terms, FICO® Scores and quarters ($j, \theta, t$). To find the typical refinance market we find the weighted average of all refinance markets.
Figure 10: Search Cost Distribution

Note: This graphs depicts the distribution of the search costs. Five groups of search costs are indexed by $c \in \{1, 2, ..., 5\}$. The higher the index, the higher the (marginal) search costs. The first row presents the distribution of search costs. The second row presents a typical search cost distribution in the refinance market. Comparing these two graphs indicates that the selection exists in the refinance market. The third row displays the reservation interest rate distribution for each search cost type. The reservation interest rates fill in the range from -0.75 to 0.875 basis points around the mean of the offered rates. For example, borrowers with search costs $c \in \{3, 4, 5\}$ choose the highest reservation interest rates whenever they refinance. Borrowers with search costs $c \in \{1, 2\}$ choose interior reservation interest rates.

Data Source: Equifax Credit Risk Insight Servicing and Black Knight McDash.

The third row in Figure 10 presents the reservation interest rate distribution for each group of search costs. Interest rates and reservation interest rates in any refinance market can fill in the range of \{-0.75, -0.625, ..., 0.875\} percentage points around the mean of the offered rates. For example, if the average offered rates in a refinance market is 4 percentage points, the offered rates can range between 3.25 and 4.875 percentage points. The third row in Figure 10 displays the distribution of the reservation interest rates of each group of...
search costs in different refinance markets (LTV ratios, terms, FICO® Scores and quarters \((j, \theta, t)\)). This graph illustrates that borrowers with search costs \(c \in \{3, 4, 5\}\) have the highest reservation interest rate (0.875 percentage points above the average offered rates) in any refinance market. Borrowers with search costs \(c \in \{1, 2\}\) are shoppers in the model. They never choose an interior value for the reservation interest rates, meanings that there are offered rates that these borrowers do not accept.

Borrowers with different search costs, in equilibrium, have different refinancing and search behavior. The search cost group \((c = 5)\) is not likely to refinance. If these borrowers refinance, they accept any offer. Borrowers with search costs \(c \in \{3, 4\}\) are more likely to refinance compared to the highest search cost group. However, like the highest search cost group, these borrowers do not search for lower rates and accept any offer. Borrowers with search costs \(c \in \{1, 2\}\) are more likely to refinance compared to other groups; they are the shoppers and search for lower rates.

### 1.7.3. Search Costs and Switching Costs

In this section, we discuss the estimation results for refinancing costs. As discussed in detail in section 1.5.1, refinancing costs include search costs and switching costs. Search costs \(\frac{\kappa_c}{\lambda j \theta t H j \theta t (r^*_{jc \theta t})}\) depend on the marginal search costs \((\kappa_c)\), approval probabilities by loan originators \((\lambda j \theta t)\) and approval probabilities by borrowers \((H j \theta t (r^*_{jc \theta t}))\). Search costs can differ across search costs groups, LTV ratios, terms, FICO® Scores and quarters \((c, j, \theta, t)\).

Based on the notation defined in the model in Section 1.5.1 we find the share of the search costs \(\frac{\kappa_c}{\lambda j \theta t H j \theta t (r^*_{jc \theta t})}\) of the refinancing costs \((R_{jc \theta t \delta t})\). The panel (a) in Figure 11 depicts the share of the search costs of the refinancing costs across different refinance markets. The share of the search costs ranges from almost 0.1 to 0.4 of the refinancing costs. The search costs include almost 30.8% of the refinancing costs on average.

The panel(b) in Figure 11 presents the search costs in monetary values. We estimate that the search costs for a mortgage of $100,000 are in the range of $400 to $2000, and are $1586.6 on average.
1.7.4. The Effect of the Search Costs on Refinancing

In this section, we address the first question of the paper: what is the effect of the search costs on refinancing activities? Since the period of 2008-2015 is a period of mortgage rates’ transition from high to low, borrowers mainly refinanced to lower their mortgage rates. Therefore, we can implicitly analyze the refinancing activities by following the dynamic of the mortgage rates on outstanding loans and its gap from the offered rates in the refinance market.

In Figure 12, we compare the benchmark economy to an alternative economy in which there is no search costs. The solid green line represents the average mortgage rates on outstanding loans in the benchmark economy. The green dashed line represents the av-
verage of the offered rate distribution. There is a gap between the mortgage rates and the offered rates, which results from inactivity in refinancing. Borrowers with search costs and switching costs choose not to refinance their mortgages to lower rates.

Figure 12: Search Costs and Refinancing

Note: The solid lines represent the average mortgage rates on outstanding loans. The dashed lines represent the average offered rates. The green lines are the estimate of the model for the benchmark economy. The red line is the alternative economy in which there are no search costs. We assume that the marginal search costs for all borrowers equal zero ($\kappa_c = 0 \forall c \in \{1, 2, ..., 5\}$).

Data Source: Equifax Credit Risk Insight Servicing and Black Knight McDash.

In the alternative economy, we assume that the marginal search costs for all borrowers equal zero ($\kappa_c = 0 \forall c \in \{1, 2, ..., 5\}$). We then use our structural model to solve for the new model. The solid red line in Figure 12 is the average of the mortgage rates in the economy without search costs. The interest rates on outstanding mortgages decline by about 1.4 percentage points on average, so the answer to the first question of the paper is that the search costs significantly inhibit refinancing.
The dotted red line in Figure 12 is the average offered rate in the alternative economy. We observe that there is a significant reduction in the offered rates. The average decline in the offered rates during this period is 1.08 percentage points. Like the benchmark model, there is a gap between the interest rates on outstanding mortgages and the offered rates in the counterfactual model because switching costs exist in the counterfactual economy. Switching costs inhibit refinancing activities in the alternative economy. The gap between the average mortgage rates and the offered rates is smaller in the model without search costs.

Search costs inhibit refinancing through two channels. The first channel is the direct effect, and the second is the indirect market power effect. The average reduction of the offered rates by 1.08 percentage points result from the elimination of the market power of the loan originators induced by search friction. This reduction in the interest rates encourages the mortgage borrowers to refinance their mortgages. In Section 1.7.5, we explain how we find the contributions of the direct effect versus the indirect effect on refinancing activities.

1.7.5. The Direct Effect versus the Indirect Effect of Search Costs on Refinancing

In this section, we address the second question of the paper: what are the contributions of the direct versus the indirect market power effect on refinancing activities?

First, we explain how we determine the direct effect. We assume that borrowers do not pay for the search costs while they still have their marginal search costs. We set \( \frac{\kappa_c}{\bar{\lambda}_{jt} H_{jt}(R_{jt})} \) equal to zero for borrowers. If borrowers choose to refinance, they do not pay for this search costs. However, the marginal search costs \( \kappa_c \) are still in place. We also assume that offered rate distributions are equal to the one in the benchmark. We assume this to keep the market power effect unchanged. Under this scenario, the borrowers will not change their search behavior when they refinance.

Second, we explore the indirect effect of the search costs on refinancing. To find the indirect effect, we assume that what borrowers pay for the search costs equals what they pay in the benchmark model. We set \( \frac{\kappa_c}{\bar{\lambda}_{jt} H_{jt}(R_{jt})} \) equal to the benchmark economy. However,
the marginal search costs for all borrowers equal zero \((\kappa_c = 0 \ \forall c \in \{1, 2, ..., 5\})\). This is as if borrowers must pay upfront search costs if they choose to refinance; however, getting an inquiry becomes free during the search process. Next, we solve for the equilibrium and find the new offered rate. This assumption eliminates the loan originators’ market power induced by search costs. All borrowers search for the lowest rates offered. In equilibrium, there is a single price equal to the minimum of the offered rate distribution in the benchmark economy.

Figure 13 compares the direct and the indirect effect. In this graph, we display the average of the mortgages rates and offered rates in the benchmark economy (solid and dashed green lines, respectively). We also present the results from Section 1.7.4, which are the average of the mortgages rates and offered rates in the counterfactual economy without search costs (solid and dashed red lines, respectively). The blue line represents the average mortgage rates on outstanding loans when we remove the search costs. Since refinancing costs decline for borrowers, it is more likely that they will choose to refinance. As a result, the average mortgage rates on outstanding loans decline. The black line represents the average mortgage rates when we remove the indirect market power effect. In this alternative economy the offered rates that borrowers choose interest rate from is exactly equal the one in the economy without search costs (dashed red line). As we can see in Figure 13 interest rates on outstanding mortgage rates are significantly lower in the economy without the indirect effect compared to the one without the direct effect. Thus, the answer to the second question of the paper is that the indirect market power effect dominates the direct effect of search costs.

To understand the direct effect, we provide an example here. We estimate that the marginal search cost for the highest search cost borrowers is at least $1712 per inquiry. In addition, applications are assumed to always be accepted. Based on our estimation results, these borrowers always receive one inquiry whenever they refinance a mortgage. One can imagine a mechanism that can evaluate the expected search costs for all borrowers
Figure 13: Direct versus Indirect Effect of Search Costs on Refinancing

Note: The graph details the direct versus indirect effect of search costs on refinancing. The solid lines are the average of the interest rates on outstanding mortgages. The dashed lines are the average of the offered rates. The green lines represent the benchmark model, and the red lines represent the alternative economy without search costs. We assume that the marginal search costs for all borrowers equal zero ($κ_c = 0 \ \forall c \in \{1, 2, ..., 5\}$). The blue line represents the mortgage rates on outstanding loans when we remove the direct effect, and the black line represents the economy when we remove the indirect market power effect.

Data Source: Equifax Credit Risk Insight Servicing and Black Knight McDash.

and subsidize them the exact amount if they choose to refinance. However, the search is still costly for borrowers. For high search cost borrowers, this means that the first inquiry is free, but they must still pay $1712 per inquiry if they want to search further. The mechanism directly encourages borrowers to refinance. We assume that the offered rate distribution remains unchanged compared to the economy without such a subsidy. This means that borrowers are not going to change their search behavior. For example, the high search cost borrowers are not going to obtain the second inquiry. To understand the indirect effect, the same environment can be assumed with a different mechanism. Borrowers must pay for their expected search costs before refinancing; however, the search cost per
inquiry becomes free when they enter the refinance market. This policy removes search friction, and loan originators lose their power to offer rates higher than the lowest possible rate. This policy encourages indirectly refinancing while the direct effect remains a barrier. Our paper finds that the second mechanism is significantly more effective in increasing refinancing. Knowing this is important because it would enable policy makers to evaluate which policies, mortgage designs or market designs might be most effective in reducing search friction and, consequently, inactivity in refinancing. In the next Section, we propose a market design that under specific assumption can eliminate the indirect effect of search costs while the direct effect remains in place.

1.8. A Centralized Refinance Market

In this section, we use our model to study a counterfactual in which borrowers can refinance their mortgages through a centralized origination market. Loan origination currently occurs in a decentralized market where borrowers contact loan originators to refinance. We impose specific assumptions to study this counterfactual. We assume that loan origination can only be done through this centralized market. In this centralized platform, markets are defined based on LTV ratios, terms and FICO Scores \((j, \theta)\). Loan originators post interest rates to markets \((j, \theta, t)\) every quarter, and we assume that a Bertrand competition exists among the loan originators when they post interest rates to the centralized market. Borrowers (with type \(\theta\)) observe only one interest rate for \((j, \theta, t)\), and they can lock in the posted rate by choosing to refinance to contract \(j\). We assume that refinancing is still costly for borrowers. They pay for the switching costs in full, and they also pay a search cost equal to number of inquiries until they get approved. We assume that the search cost is \(\frac{\kappa}{\lambda_{j,t}}\). In this alternative economy, the offered rates will be equal to the minimum range of the offered rates due to Bertrand competition.

Figure 14 presents the results for this centralized market. The red dashed line represents the offered rates for the economy without search costs. In the centralized market, the offered rates are equal to the offered rates in the economy without search costs. The black line represents the average of the mortgage rates on outstanding loans in the centralized
Note: The solid lines are the average of the interest rates on outstanding mortgages. The dashed lines are the average of the offered rates. The green lines represent the benchmark model, and the red lines represent the alternative economy without search costs. We assume that marginal search costs for all borrowers are equal to zero ($\kappa_c = 0 \ \forall c \in \{1, 2, ..., 5\}$) in the economy with no search costs. The black line represents an alternative economy in which refinance occurs in a centralized market.

Data Source: Equifax Credit Risk Insight Servicing and Black Knight McDash.

market. There is a significant reduction in the rates in this alternative economy. However, there is still a gap between the outstanding mortgage rates (solid black line) and the offered rate (dashed red line). This gap forms because the switching costs and search costs inhibit refinancing activities.

This counterfactual experiment highlights the importance of the result of the paper, which is that search friction inhibits refinancing activities mostly through the market power of the loan originators, not directly through refinancing costs. In this counterfactual economy, the market power of the loan originators is eliminated; however, we still assume that the refinancing costs are mostly in place.
1.9. Conclusion

In this paper, we highlight the evidence on search friction and inactivity in refinancing in the US mortgage market. We bridge these two pieces of evidence to explore the role of search costs in explaining refinancing inaction. We empirically demonstrate that search costs significantly inhibit refinancing. We explore two channels through which search costs affect refinancing: the direct effect and the indirect market power effect. We find that the indirect market power effect dominates the direct effect. This result indicates that the main reason that search costs inhibit refinancing is NOT that getting only one quote to refinance is a very costly action for the borrowers. The main issue is that if borrowers get only one quote, the loan originators take into account that borrowers do not get multiple quotes to find the lowest rates, thus, they respond accordingly by offering high interest rates. This indirect effect weakens the benefit of refinancing for borrowers and this is the main channel that we find search costs inhibit refinancing. This is the main result of this paper.

To understand the main result, we explored an alternative economy in which the current decentralized system is replaced by a centralized market for refinancing. In this centralized market, borrowers observe only one price at each point in time. Loan originators post interest rates in the centralized market and we assume that there is Bertrand competition among them. We find that a centralized market for refinancing can significantly increase refinancing activity by eliminating market power, even if the refinancing costs remain unchanged.

The results of this paper raise this question of which policies, mortgage designs or market designs might be most effective in reducing the indirect market power effect of search friction and, consequently, decreasing inactivity in refinancing.
1.10. Mathematical Appendix

1.10.1. Structural Refinance Probabilities

Given the nested logit model of a refinancing decision described in section 1.5.1, we can find the structural refinancing probabilities. The equation 1.10.1 presents the probability of refinancing of a borrower with state variable $z_t$:

$$P_{1t}(z_t) = \frac{\sum_k P_{kt}(z_t)}{1 + (\sum_k \exp(\frac{v_{kt}(z_t) - v_{0t}(z_t)}{\sigma}))^\sigma}$$  \hspace{1cm} (1.10.1)

In the language of a nested logit model, this is the probability of choosing the nest. The Equation 1.10.2 presents the choice probabilities within the nest of refinancing. Specifically, Equation 1.10.2 shows the structural probability of refinancing to contract $j$ conditional on choosing to refinance in period $t$:

$$P_{jt}(z_t) = \frac{\exp(\frac{v_{jt}(z_t) - v_{0t}(z_t)}{\sigma}) \left(\sum_k \exp(\frac{v_{kt}(z_t) - v_{0t}(z_t)}{\sigma})\right)^{\sigma-1}}{1 + \left(\sum_k \exp(\frac{v_{kt}(z_t) - v_{0t}(z_t)}{\sigma})\right)^\sigma}$$  \hspace{1cm} (1.10.2)

1.10.2. Calculating Demand

In this section we discuss how we find the demand function, $q_{j\theta t}(r)$, presented in the Equation 1.5.11. The probability that a borrower with reservation interest rate $r^*$ in market $(j, \theta, t)$ refinance with an interest rate $r$ is as follows:

$$Pr\{\tilde{r} = r | r < r^*, j, \theta, t\} = \frac{h_{j\theta t}(r)}{H_{j\theta t}(r^*)}$$  \hspace{1cm} (1.10.3)

Let $\Phi_{j\theta t}(r^*)$ and $\phi_{j\theta t}(r^*)$ be the distribution and density of the reservation interest rates, respectively, of type $\theta$ borrowers in market $j$ at time $t$. Summing over the borrower’s reservation rate yields the share of market for loan originators charging a rate less than $r$,

$$Pr\{\tilde{r} = r | j, \theta, t\} = \sum_{r^* \geq r} \frac{h_{j\theta t}(r)}{H_{j\theta t}(r^*)} \phi_{j\theta t}(r^*)$$  \hspace{1cm} (1.10.4)
Finally, since a mass $h(r)$ of loan originators charge interest rate $r$, and the borrower samples each of these lenders with equal probability, the residual demand curve for a loan originator charging rate $r$ is the above quantity divided by $h(r)$:

$$q_{jt}(r) = \sum_{r^* \geq r} \phi_{jt}(r^*) / H_{jt}(r^*)$$  \hspace{1cm} (1.10.5)

This calculation is quite similar to Agarwal et al. (2017b). The difference is that the reservation distribution depends on the search costs distribution of the borrowers with type $\theta$ who choose to refinance to contract $j$ at time $t$.

$$\sum_{(c,x,r')} \mu_{jt}(c,x,r') P_{jt}(c,x,r')$$  \hspace{1cm} (1.10.6)

in which $\mu$ is the mass of borrowers. Therefore we can find the distribution of the reservation interest rate:

$$\phi_{jt}(r^*) = \sum_{(c,x,r')} \mu_{jt}(c,x,r') P_{jt}(c,x,r') 1\{r^*_{jct} = \tilde{r}\}$$  \hspace{1cm} (1.10.7)

1.10.3. Dynamic of Borrowers Mass

We define $\mu_t(z_t)$ as the mass of borrowers with a mortgage at time $t$ in state $z_t$. Precisely, it captures the mass of borrowers at time $t$ with type $(c, \theta)$ and mortgage contract $(r, x)$. To find $\mu_{t+1}$ in every state we need to know the transition probabilities of states and refinancing choice of borrowers. Additionally, we need to know the mass of new mortgage originators.

$$\mu_{t+1}(c, \theta_{t+1}, r_{t+1}, x_{t+1}) = \sum_{\theta_t} \mu_t^0(c, \theta_t) \delta_t^0(c, \theta_t) f_{t1}(\theta_{t+1}|\theta_t)$$  \hspace{1cm} (1.10.8)

$$+ \sum_{\theta_t} \sum_{r_t} \sum_{x_t} \mu_t(c, \theta_t, r_t, x_t) (1 - \delta_t(x_t)) P_{t0}(c, \theta_t, r_t, x_t) f_{t0}(x_{t+1}, \theta_{t+1}|x_t, \theta_t)$$

$$+ \sum_{\theta_t} \sum_{r_t} \sum_{x_t} \mu_t(c, \theta_t, r_t, x_t) (1 - \delta_t(x_t)) P_{x_{t+1}0}(c, \theta_t, r_t, x_t) f_{t1}(\theta_{t+1}|\theta_t) h_{x_{t+1}0}(r_{t+1}|r^*_t(c, \theta_t, x_{t+1}))$$
The dynamic of potential borrowers are as follow,

$$
\mu_{t+1}^0(c, \theta_{t+1}) = \sum_{\theta_t} \sum_{r_t} \sum_{x_t} \mu_t(c, \theta_t, r_t, x_t) \delta_t(x_t) f_0(\theta_{t+1} | \theta_t) 
+ \sum_{\theta_t} \mu_t^0(c, \theta_t)(1 - \delta_t^0(c, \theta_t)) f_0(\theta_{t+1} | \theta_t) 
$$

(1.10.9)

We assume that there is no growth in the potential borrowers in the mortgage market:

$$
\sum_{\theta_t} \mu_t^0(c, \theta_t) + \sum_{\theta_t} \sum_{r_t} \sum_{x_t} \mu_t(c, \theta_t, r_t, x_t) = g_c \quad \forall t 
$$

(1.10.10)

$g_c$ is the mass of borrowers with search cost $c$.

Moreover, we assume the probability of becoming a homeowner is independent of search cost:

$$
\delta_t^0(c, \theta_t) = \delta_t^0(\theta_t) \quad \forall c 
$$

(1.10.11)

1.10.4. EM Algorithm

To estimate empirical CCPs, we follow the first stage EM algorithm in [Arcidiacono and Miller (2011)]. In this appendix section, we explain the expectation and maximization steps.

**Expectation Step:**

The first step of $m$th iteration is to calculate the conditional probability of being in each unobserved state given the values of the structural parameters and conditional choice probabilities from the $m$th iteration, $\{\Theta^{(m)}, \mathcal{S}^{(m)}\}$. The likelihood of the data on $i$ given the parameters at $m$th iteration is found by evaluating equation 1.10.12

$$
L(d_i, z_i; \Theta^{(m)}, \mathcal{S}^{(m)}) = \sum_{\mathcal{C}_{z}} \mathcal{S}^{(m)}(c_i | z_{i1}) \left( \prod_{t=1}^{T} L_t(d_{it}, z_{it+1} | z_{it}; \Theta^{(m)}, \mathcal{S}^{(m)}) \right) 
$$

(1.10.12)
where $\Theta \equiv (\beta_c|x, \beta_1, \beta_{j|1}(c, \theta, x))$ and $\hat{z}_1 = (\theta_1, r_1, x_1)$. To simplify, we define the following:

$$L_i^{(m)} \equiv L(d_i, z_i|\hat{z}_1; \Theta^{(m)}, g^{(m)}) \quad (1.10.13)$$

Similarly, we denote by $L_i^{(m)}(c_i = c)$ the joint likelihood of the data and unobserved state $c_i$ given the parameter evaluation at iteration $m$.

$$L_i^{(m)}(c_i = c) \equiv L(d_i, z_i, c_i = c|\hat{z}_1; \Theta^{(m)}, g^{(m)}) \quad (1.10.14)$$

where,

$$L(d_i, z_i, c_i = c|\hat{z}_1; \Theta^{(m)}, g^{(m)}) = g^{(m)}(c|\theta_1, r_1, x_1) \left( \prod_{t=1}^{T} L_t(d_{it}, z_{it+1}|z_{it}; \Theta^{(m)}, g^{(m)}) \right) \quad (1.10.15)$$

At iteration $m + 1$, the probability of $i$ being in unobserved state $c_i$, $q_i^{(m+1)}$, then follows from Bayes rule:

$$q_i^{(m+1)} = \frac{L_i^{(m)}(c_i = c)}{L_i^{(m)}} \quad (1.10.16)$$

We update the probabilities of unobserved states in equation 1.10.17.

$$g^{(m+1)}(c|\hat{z}_1) = \frac{\sum_{i=1}^{N} q_i^{(m+1)}(c_i = c)}{\sum_{i=1}^{N} 1(\theta_{i1} = \theta_1, r_{i1} = r_1, x_{i1} = x_1)} \quad (1.10.17)$$

Maximization Step

$$\Theta^{(m+1)} \equiv \arg \max \sum_{i=1}^{N} \sum_{t=1}^{T} \sum_{c} \sum_{j} q_i^{(m+1)} \ln L_t(d_{it}, z_{i,t+1}, c_i = c|z_{it}; \Theta^{(m)}, g^{(m+1)}) \quad (1.10.18)$$

To estimate the empirical CCPs, we use random sample of loans originated between 2008 to 2009 that are followed until 2015.
1.10.5. Parametric Assumptions for the First Stage

In this section, we discuss the parametric assumptions for approval probability and the offered rate distribution. In the Equation 1.10.19 we specify the approval probability:

$$\lambda_{j\theta t} = \frac{\exp(\beta_{j}^{\lambda} + \beta_{\theta}^{\lambda} + \beta_{t}^{\lambda})}{\sum_{\tilde{\theta}} \sum_{\tilde{j}} \exp(\beta_{\tilde{j}}^{\lambda} + \beta_{\tilde{\theta}}^{\lambda} + \beta_{\tilde{t}}^{\lambda})}$$ (1.10.19)

where $\{\beta_{j}^{\lambda}, \beta_{\theta}^{\lambda}, \beta_{t}^{\lambda}\}$ for all $j$ and $\theta$ are the parameters to be estimated. $\beta_{\theta}^{\lambda}$ are the dummies for FICO® Score groups, $\beta_{j}^{\lambda}$ are the dummies for LTV groups, and $\beta_{t}^{\lambda}$ are the year dummies.

Based on 1.5.3, offer rate distribution from the supply side of the model is as follows:

$$h_{j\theta t}(r) = \frac{\exp\left(\frac{(r-\hat{r}_{j\theta t}-\chi)q_{j\theta t}(r)}{\sigma_{\pi}}\right)}{\sum_{\tilde{k}} \exp\left(\frac{(r-\hat{r}_{j\theta t}-\chi)q_{j\theta t}(r)}{\sigma_{\pi}}\right)}$$ (1.10.20)

where $\{\chi, \sigma_{\pi}\}$ are parameters to be estimated given the marginal demand function $q_{j\theta t}(r)$.

In order to find the offer rate distribution from equation 1.10.20 we need to have the marginal demand function $q_{j\theta t}$, which itself is a function of $h_{j\theta t}$. This is a fixed point problem, that is time-consuming to estimate. To simplify, we guess a functional form for the offered rate distribution, $h_{j\theta t}^{N}$ and then we make sure that the guess is a good approximation of structural offered rate distribution $h_{j\theta t}$ presented in the Equation 1.10.20. We assume a normal distribution $h_{j\theta t}^{N} \sim N(\hat{r}_{j\theta t} + \beta_{j}^{h} + \beta_{\theta}^{h} + \beta_{t}^{h}, \sigma_{h})$ in which $\{\beta_{j}^{h}, \beta_{\theta}^{h}, \beta_{t}^{h}\}$ are the dummies for contract, creditworthiness and year.
CHAPTER 2: Mortgage Search Heterogeneity, Statistical Discrimination and Monetary Policy Transmission to Consumption

*(co-authored with Sumedh Ambokar)*

2.1. Abstract
In the US, half of all mortgage borrowers only consider one lender at refinancing. We investigate how statistical discrimination by lenders, a tool to separate borrowers who differ in search intensity, affects welfare and monetary policy transmission to consumption. We build and calibrate a general equilibrium model of the mortgage market with two types of borrowers who differ in the number of lenders they meet. Statistical discrimination based on the relative mass of the two types at any observable current mortgage rate and home equity results in relatively higher offer rates for non-searchers. Higher offer rates reduce the incentive to refinance. Repeated refinancing increases the separation between the two types, reinforcing the mechanism. Statistical discrimination has a big welfare cost ($3,300) for a borrower, accounting for two-thirds of the total difference in welfare between the two types. This welfare cost becomes two-thirds if non-searchers search more, but quadruples if searchers search more. Thus, the two ways to increase mortgage search, an explicit goal of the CFPB, have very different effect on welfare. Statistical discrimination halves the consumption response to a monetary policy shock of non-shoppers but does not increase it for shoppers, reducing aggregate response by a third. Hence, this indirect effect of mortgage search heterogeneity is highly relevant for policymaking.

2.2. Introduction
According to the National Survey of Mortgage Originations (NSMO), in the US, 52% homeowners only consider one lender when thinking about refinancing their mortgage. We refer to them as non-shoppers. In the data described below, we find that non-shoppers pay higher rates. This can be due to two reasons. First, non-shoppers will see only one offered rate, leading to higher accepted rates - a direct effect of reduced search. Second, lenders may use available information to statistically discriminate - a less intuitive indirect
effect. Lenders can observe the current mortgage of a refiner. If they know the number of shoppers and non-shoppers with the same mortgage, they can evaluate the probability that the refiner will not search for another quote. Higher this probability, higher the rate that lenders offer. A higher offer rate also reduces the incentive to refinance. With repeated refinancing, the difference in rates between shoppers and non-shoppers would continue to increase, making it easier to statistically discriminate.

In this environment, we ask the following questions. First, what is the welfare cost of statistical discrimination to a borrower? How does this cost change if either one-third of non-shoppers or all the shoppers search for one more quote? Both increase mortgage search, an explicit aim of the Consumer Financial Protection Bureau (CFPB), but in different ways. Second, how does variation in the ability to statistically discriminate change monetary policy transmission to consumption? Consuming the home equity extracted via mortgage refinancing has been found to be an important channel for this transmission.

To answer these questions, we build a general equilibrium model with two types of mortgage borrowers. The first type, the non-shoppers, only consider an offer from one lender. The second type, the shoppers, consider offers from two lenders. A borrower who refinances meets a random subset of identical lenders simultaneously before getting any offer. Short-lived lenders do not know the type of the borrower they meet but they observe her state (current rate and mortgage balance) and know the mass of each type in any state. This enables them to statistically discriminate. The refinance market, similar to the product market in [Burdett and Judd (1983)](https://www.jstor.org/stable/2209174), leads to rate dispersion for identical borrowers. The production side of the economy consists of standard New-Keynesian firms which allows the monetary authority’s nominal changes to have significant real changes.

We calibrate the parameters governing search cost and the fraction of borrowers who are shoppers to match the average years to refinance a mortgage according to the Freddie Mac and Fannie Mae’s Single Family Loan Level Dataset (GSE) and the fraction of refinancers who are shoppers according to the NSMO, while other parameters are chosen from the lit-
erature. To answer the first question, the following is done. Comparing borrower welfare in the benchmark economy with that in a counterfactual economy where their current state is unobservable, thus removing the ability to statistically discriminate, allows us to evaluate the welfare cost of statistical distribution. Repeating this by changing the benchmark economy to a counterfactual economy where one-third of non-shoppers consider offers from two lenders and to another counterfactual economy where shoppers consider offers from three lenders answers how the welfare cost changes with these two ways of increasing mortgage search. To answer the second question, we look at how agents in the steady states of the benchmark and the three counterfactual economies respond to an expansionary monetary policy shock: a 25 basis points unexpected reduction in the nominal risk-free rate, which is also the cost of lending in the model.

We conclude that statistical discrimination has a big welfare cost based on three steady state results. First, a borrower is willing to pay about $3,300 (30% of quarterly income) to make their current state unobservable and thus remove lenders’ ability to statistically discriminate. Non-shoppers are willing to pay much more ($5,700). This cost is small at birth, but quadruples by the age of eight years due to the increasing isolation of non-shoppers at high rates with each round of refinancing. On the other hand, shoppers are not isolated at low rates because in equilibrium, many lenders post low rates (Pareto distribution) which non-shoppers can also get; moreover, with repeated refinancing, many non-shoppers eventually end up with low rates. So, shoppers do not benefit much by statistical discrimination. Thus, the ability to statistically discriminate causes a big shift in welfare from borrowers to lenders. In the data, the rate distribution is significantly left-skewed (close to Pareto) which validates this result. Also, in steady state, the time after which a borrower refinances again is U-shaped in her state. Non-shoppers who are isolated at high rates wait and collect home equity before refinancing again, borrowers who get lower rates refinance sooner as there are more shoppers with the same rates and thus rate reduction offered is higher, and once borrowers get low enough rates, they do not refinance again. This U-shaped relation is also observed in the data, validating the mechanism.
Second, statistical discrimination accounts for two-thirds of the difference in welfare between the two types. A non-shopper is willing to pay $7,700 to become a shopper, which reduces to $2,300 if the current state is unobservable. Third, the two ways of increasing mortgage search changes the ability to statistically discriminate and thus its welfare cost in opposite directions. If one-third of non-shoppers consider two lenders, welfare cost becomes two-thirds ($2,100) of that in benchmark; but if shoppers consider three lenders, welfare cost quadruples ($13,800). If shoppers consider three lenders, the increased isolation benefits shoppers - they have a slightly negative welfare cost (-$220) and statistical discrimination now accounts for three-fourths of the difference in welfare between the two types.

We find that statistical discrimination reduces the monetary policy transmission to consumption, especially by reducing the refinancing response of non-shoppers. There is hardly any pass-through of rate reduction to non-shoppers who are isolated at high rates. Thus, few of them refinance in response to a monetary policy shock. Even when they do, they extract much smaller home equity they collect in steady state compared to shoppers. Hence, non-shoppers have a smaller consumption response.

In the two economies with more mortgage search than benchmark, the consumption response of non-shoppers changes in opposite directions. The four economies in the order of increasing ability to statistically discriminate are: unobservable current state, one-third of non-shoppers meet two lenders, benchmark, and shoppers meet three lenders. A non-shopper’s consumption increases by 1.21%, 0.93%, 0.57% and 0.31% respectively in response to the monetary shock mentioned above in these four economies. In contrast, shoppers have a bigger consumption response than non-shoppers in all economies, except the one with unobservable current state, since they collect more home equity in steady state as a result of getting lower rates sooner. In the economy with unobservable current state, both types have very similar home equity in steady state but non-shoppers have slightly higher rates due to their lack of search. Without statistical discrimination, non-
shoppers get much bigger rate reduction and thus they respond much more than benchmark. For shoppers, there is not much change in isolation across the four economies. In the economies with more mortgage search, the aggregate reduction in market power of lenders results in more home equity and thus bigger consumption response of shoppers than in benchmark. A shopper’s consumption increases by 0.88%, 1.15%, 0.84% and 0.92% respectively in response to the same shock in the four economies mentioned earlier. Thus, statistical discrimination changes monetary policy transmission to consumption at the aggregate as well as distributional level.

Consistent with the model, our empirical findings imply that otherwise identical mortgage borrowers who refinance with different unobservable search intensities get very different rates and borrow very different amounts. There is a wide range of mortgage rates (standard deviation of 26 basis points) that a refinancer gets after controlling for risk factors, lender, location, and month of origination from the GSE data. Discount points, weekly variation, and a proxy for unobserved credit risk explain only a small part of this variation. In the NSMO, non-shoppers borrow $2,750 less and pay 8 basis points higher rate than shoppers. This difference in rates falls to 3 basis points in the mortgage market for home purchases, where there is no current mortgage that lenders can use to statistically discriminate. It is difficult to identify whether a borrower will consider one or more than one lender: a probit classifier with more than 80 borrower characteristics is unable to classify 37% borrowers correctly. We use the Home Mortgage Disclosure Act Loan Application Register (HMDA LAR) to find that even before NSMO started in 2013, MSA’s with above median mortgage search activity in a year had 6 basis points lower average mortgage rates and 7.5% higher home equity extraction rates compared to MSA’s with below median search. Consistent with the relation between a borrower’s state and time after which she refinances in the model, we find in the GSE data that the number of months after origination at which borrowers refinance is U-shaped in the product of their current mortgage rate and balance. Without targeting, the distribution of borrowers in steady state matches that observed in the data at the end of 2015 when the cost of mortgage lending
was relatively stable, supporting our model’s assumptions and main results.

In our related work, Ambokar and Samaee (2019), we empirically explore the role of search costs in explaining inaction in refinancing. Hence, we estimate the search cost distribution of mortgage borrowers in the US and find that search costs, and not creditworthiness, inhibit mortgage refinancing mainly due to the resulting increase in market power of lenders. Here, the focus is on understanding the effects of the subtle mechanism of statistical discrimination. Hence, we enable lenders to statistically discriminate and find that this mechanism has a huge impact on welfare and it significantly affects monetary policy transmission. In both the papers, we find that lenders’ actions play a huge role in determining the borrowers’ actions and thus, the outcomes in the US mortgage market.

We show that considering mortgage search heterogeneity and statistical discrimination in a dynamic general equilibrium framework is important to understand the agents’ decisions in the US mortgage market and their aggregate effects. This subtle mechanism also determines the distribution of home equity as studied in Beraja et al. (2018). Different offer rates affect the potential savings as studied in Eichenbaum et al. (2018). As in Wong (2019), younger borrowers who have higher rates are more likely to refinance than older ones. Chen et al. (2013) consider labor income risk heterogeneity and Greenwald (2018) considers payment to income restrictions which are left for future work. In empirical work, Woodward and Hall (2012) first documented the substantial price dispersion in mortgage markets. Agarwal et al. (2017b) find that search costs and creditworthiness together explain mortgage search behavior whereas our focus in the other paper is on inaction in refinancing. Alexandrov and Koulayev (2018) document price dispersion in reference rates; we use contracted rates. Bhutta et al. (2019) find a wide dispersion in rates even after controlling for discount points which are not available in our data. Hurst et al. (2016) find that there is no spatial variation in GSE mortgage rates, which we confirm. Our rough estimate of the lost savings due to lack of mortgage search is in line with that in Keys et al. (2016). Allen et al. (2014) find that competition does not benefit those with high rates, similar to
non-shoppers isolated at high rates in our model.

2.3. Data and Analysis

We use multiple data sources to analyze different issues that motivate this study and become the targets for the model built in the later section. We find that there is a wide spread in the mortgage rates that borrowers get after controlling for their observable as well as unobservable risk factors, lenders, location and time using Freddie Mac and Fannie Mae’s Single Family Loan Level Dataset. Using the performance data of the loans in this dataset, we find that the refinance behavior of borrowers with respect to their mortgage rate and mortgage balance is consistent with that in the model. We find that borrowers search for their mortgage differently and their outcomes are significantly different conditional on their search behavior using the National Survey of Mortgage Originations dataset. Finally, since the survey data date backs to only 2013, we use the Home Mortgage Disclosure Act (HMDA) Loan Application Register data to see how search behavior and outcomes for the borrowers relate before 2013, but at an aggregate level, and find similar results. Below we describe our empirical results in detail.

2.3.1. Freddie Mac and Fannie Mae Loans Data

We use the single-family mortgage originations and their performance public dataset from Freddie Mac and Fannie Mae for the period 1999 to 2016. This dataset has about 65% of the mortgages originated in the US during this period. It does not record the search behavior of borrowers. It has over 60 million mortgage originations over this period. This dataset allows us to determine whether lenders offer different rates to observationally equivalent borrowers and the refinance behavior of borrowers with respect to their mortgage rate and mortgage balance.

To focus on how the mortgage rate varies within a particular homogenous type of mortgage, we first restrict the sample to the mortgages originated at fixed rate for 30 years, property is single-family owner-occupied, one-unit, without any prepayment penalty, with no insurance and not super-conforming for borrowers with FICO score at least 660. This product is highly prevalent in the sample and generates a subsample of over 19 million
mortgages over the 18 year period. The refinance behavior is also analyzed using this subsample.

**Rate dispersion**

The GSEs set fees every month for this mortgage product that vary by FICO score, LTV and loan type only. As we restrict the sample to 30 year fixed-rate fully documented mortgages, the only two dimensions of credit quality that should materially affect rates on GSE-guaranteed mortgages in any month are FICO and LTV. So, we follow the procedure used in [Hurst et al.] (2016) to obtain residual mortgage rates after controlling for borrower characteristics and time fixed effects. In particular, we run:

\[
r_{jt} = \alpha_{j0} + \alpha_{j1}D_t + \alpha_{j2}X_{it} + \alpha_{j3}D_t X_{it} + \alpha_{j4}Z_{it} + \epsilon_{jt}
\]

where \( r_{jt} \) is the loan-level mortgage rate for a loan made to borrower \( i \) during month of origination \( t \), \( D_t \) is a vector of time dummies based on the month of origination, \( X_{it} \) is a series of FICO dummies (660-679, 680-699, … 780-799, etc.) and a series of LTV dummies (50-54, 55-59, … 80-84, … 95-99, etc.) for borrower \( i \) in period \( t \) and \( Z_{it} \) is the vector (purpose of mortgage, mortgage amount, debt-to-income ratio, cumulative LTV, channel of origination, whether first home, number of borrowers on the mortgage, originator, 3-digit zip code of the property). Sample \( j \) refers to whether the mortgage was purchased by Freddie Mac or Fannie Mae. We run these regressions separately using data from each of our two GSE datasets. The results are summarized in Table 1. 92.58% and 94.48% of the variation in interest rates in the two samples is explained by this regression. Combining the errors in prediction from both the samples, the unexplained variance in rates has a standard deviation of 26.67 basis points. Some of this variation is because of weekly variation in rates and discount points bought by borrowers but [Bhutta et al.] (2019) find that discount points account for about 15 basis points variation and weekly variation in rates within a month are on average less than 10 basis points. We also check how much
of this variation is due to credit risk not observed in the data but observable to the lender via additional documents like the full credit report. Such additional information might predict payment delinquency and eventually default in the future. Hence, as a proxy for this unobserved information, we add the ex-post information about payment delinquency: the maximum number of months that the borrower has been delinquent in the first four years since origination to the vector \( Z_{it} \) in the above model. This information is found in the Performance data of these mortgages. We do not consider mortgages originated in the last four years of the sample to remove any bias. The maximum delinquency value is 0 for 90.66% of the borrowers and at least 6 for 1.23% of the borrowers in this sample. We compare the result of this model to the baseline model with the same sample size. We find that there is hardly any reduction in the standard deviation of the unexplained variance. Hence, we conclude that the unexplained variance is not due to unobserved credit risk. Thus, most of the variation found here, about four times 26.67 basis points, remains unaccounted. We repeat this exercise for only refinance mortgages which are 70% of the sample as our focus is on refinances and find that there is hardly any change in results (standard deviation reduces from 26.67 to 25.59 basis points). Hence, the annual mortgage payment of two observationally equivalent borrowers in the same area, month of origination and mortgage lender whose mortgage rate differs by one standard deviation would differ by about $360 on a mortgage of $200,000. Hence, there is a substantial amount of saving for the borrower if she shops harder for the mortgage.

**Refinance Behavior**

To verify whether the novel finding from the model in Section 2.5.1 is observed in the data, we analyze how the refinance behavior of borrowers varies with respect to their mortgage rate and mortgage balance. In particular, we find that the refinancing decision of borrowers is not a simple threshold rule but is non-monotonic in their current mortgage rate and mortgage balance. In the model, this is a result of the relative intensity of mortgage search in each submarket characterized by the current mortgage rate and mortgage balance. Hence, in the data, we build a variable which is the product of the current mort-
Table 1: Unexplained mortgage rate

<table>
<thead>
<tr>
<th>Dependent Variable: Mortgage Rate</th>
<th>Data: Freddie Mac</th>
<th>Data: Fannie Mae</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample from 1999 to:</td>
<td>2016</td>
<td>2016</td>
</tr>
<tr>
<td>Orignation Month</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>FICO, LTV dummies</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Other variables at origination (Z_i)</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Max. delinquency in first 4 years</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>Refinances only?</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
<td>Adjusted R^2</td>
<td>.9258</td>
<td>.9537</td>
</tr>
<tr>
<td>Observations</td>
<td>5800608</td>
<td>261897</td>
</tr>
<tr>
<td>RMSE</td>
<td>.2651</td>
<td>.2521</td>
</tr>
</tbody>
</table>

Note: Unexplained mortgage rate has a wide dispersion (RMSE). The dispersion does not decrease much by adding a proxy for unobserved risk (Max. delinquency in first 4 years) or by looking only at refinances.

gage rate and current mortgage balance divided by 100,000; let us label it ‘state’ and see how it affects the variable ‘loan age’: months after origination at which the loan has been refinanced. If the novel finding in the model is also observable in data, loan age should decrease and then increase in the range of the state variable, which we find is the case using the following regression:

\[ LoanAge_i = \alpha + \beta_1 State_i + \beta_2 State_i^2 + \kappa X_i + \delta D_i + \epsilon_{it} \]

where the individual borrower \( i \) refinances their mortgage in month \( D_i \) and had the characteristics \( X_i \) (FICO score, LTV, cumulative LTV, debt-to-income ratio, whether first home, number of borrowers on the mortgage) at origination. Mortgages originated before 2011 were considered for this regression so that recency does not impact the results. The results of this regression are stated in Table 4. The state variable in the data ranges between a minimum of 0 and a maximum of 59 (1 percentile: 1.277, 99 percentile: 27.435) in which loan age is decreasing then increasing in the range of the state variable based on the coefficients in the regression. Thus, the novel finding in the model is observed in the data as well, validating the main mechanism in the model.
2.3.2. National Survey of Mortgage Originations

The National Survey of Mortgage Originations (NSMO) conducted since 2013 by the Consumer Finance Protection Bureau (CFPB) and the Federal Housing Finance Agency (FHFA) allows us to look at individual-level search behavior in the mortgage market. It has about 6,000 respondents per year and a total of 24,640 observations in the sample considered. We investigate this search behavior. We check how well can borrower characteristics help identify their search behavior and what are the outcomes for borrowers with different search behavior in terms of the rates and mortgage size they get.

Search Behavior

The survey reveals that half of all borrowers seriously considered only one lender and nearly 80% of them applied to only one lender. A third of the borrowers chose a lender based on past relationships, reputation and having a local branch. Figure 27 reveals the percentage of mortgage borrowers (on left) and refinancers (on right) who seriously considered 1, 2, 3, 4, 5 or more lenders in the entire sample from 2013 onwards.

Identifying who go to only one lender

To see whether borrower characteristics can help explain their search behavior, we build a dummy variable of whether the mortgage borrower seriously considered one lender or more than one lender. We use more than 80 characteristics of that mortgage like the month of origination, PMMS rate in that month, loan amount category, FICO, LTV, CLTV, DTI, PTI, first homebuyer flag, number of borrowers, whether property is in a metro, age, education, race, income, financial awareness as explanatory variables in a probit model to see whether these variables help predict the dummy variable created. We find that the model classifies 63.42% of the borrowers correctly which is a minor improvement over a random classifier which could classify about 50% correctly.

Outcomes for those who go to only one lender

We find that those who consider one lender borrow about $2850 less at about 5 basis points higher rate on average compared to those who consider more than one. This difference is
Table 2: Regression Results, Interest Rate

<table>
<thead>
<tr>
<th></th>
<th>rate</th>
<th>amount</th>
<th>rate</th>
<th>rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>More than 1 lender considered</td>
<td>-0.050***</td>
<td>0.057*</td>
<td>-0.079***</td>
<td>-0.028*</td>
</tr>
<tr>
<td>Purpose</td>
<td>Any</td>
<td>Any</td>
<td>Refinance</td>
<td>Purchase</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.355</td>
<td>0.564</td>
<td>0.396</td>
<td>0.32</td>
</tr>
<tr>
<td>Observations</td>
<td>8497</td>
<td>8497</td>
<td>3768</td>
<td>4476</td>
</tr>
</tbody>
</table>

***: p-value $\leq 0.01$, *: p-value $\leq 0.1$

Note: The difference in rate is much bigger for refinancers than for home purchasers, an indication of the effect of statistical discrimination in the refinance market that is absent in home purchasers market (NSMO)

such that the mortgage payment across these borrowers remains the same. Interestingly, for refinancers, the difference increases to 8 basis points compared to 3 basis points for home purchases. This is indicative evidence of statistical discrimination between those who look for multiple lenders versus those who look for one. It is prevalent in the refinance market but not in purchase market in which half of purchasers are first-homebuyers for whom there is no history of mortgage search which is necessary for statistical discrimination studied in this paper.

To find these results, we run regressions to explain the mortgage rate and the loan amount with the dummy variable built earlier and use the more than 80 characteristics of the mortgage mentioned above as controls. We restrict the sample to 30 year fixed rate, conforming, non-jumbo agency mortgages for these regressions. The results are summarized in Table 2. Note that the loan amount is a category variable with $50,000 interval dummies.

2.3.3. HMDA Loan Applications Register

Since the NSMO only tells us about the search behavior of borrowers from 2013 onwards, we look at the Home Mortgage Disclosure Act (HMDA) Loan Application Register public data to see how mortgage search activity affected outcomes similar to those mentioned above for the borrowers. HMDA data has loan applications since 1981 for 90% of the US mortgage market. But the public data does not allow us to identify the borrower and thus we cannot get individual search behavior as in the NSMO.

To overcome this, we build a measure of search activity in a MSA in a year which is defined
as the number of applications withdrawn by the borrower or approved by the lender but
deprecated by the borrower divided by the number of applications approved and accepted.
We compare how this search activity relates to the home equity extraction rate and the
average mortgage rate spread observed in that MSA and year. In particular, using the
Freddie Mac and Fannie Mae data, we build a MSA-year wise variable of mortgage amount
originated for home equity extraction divided by the total mortgage amount originated
and a MSA-year wise average spread between the mortgage rate and the current coupon
rate of a 30 year fixed rate agency MBS in the month of origination which is defined as
the secondary market rate in Fuster et al. (2013). This secondary market rate is the cost of
mortgage lending. We find that it is tightly connected to the effective federal funds rate
which is influenced by Federal Reserve’s open market operations. Hence, in the model, we
will set the cost of lending to be equal to the nominal risk-free interest rate in the economy.
Figure 28 shows these rates and the primary mortgage market survey (PMMS) rate since
1994 and Table 5 shows the results of regressing the primary and secondary mortgage
market rates on the effective federal funds rate.

**Areas with More Search respond more to a Rate reduction**

Even before 2013, higher search is related to lower rates and more home equity extraction.
The top panel of Figure 29 shows how search activity and home equity extraction move
together across years and that within a year, in MSA’s with above median search activity,
home equity extraction has been consistently higher. The bottom panel of Figure 29 shows
that across years, a change in search activity has coincided with an opposing change in the
average rate spread and that within a year, across MSAs, above median search MSAs have
seen a slightly lower rate spread compared to below median search MSAs.

We formally find this relation in the regression below where the dependent variable $H_{int}$
is the home equity extracted by an individual normalized by the total amount borrowed in
MSA $m$ and month $t$. Explanatory variables are $P_t$, the primary mortgage market survey
(PMMS) rate in that month, search activity in that MSA-year $S_{my}$, their interaction term
and controls $X_{int}$, FICO, LTV, CLTV, DTI, channel, number of borrowers on the mortgage, lender, month and MSA. In particular,

$$H_{int} = \alpha_0 + \alpha_1 P_t + \alpha_2 S_{my} + \alpha_3 P_t S_{my} + \alpha_4 X_{int} + \epsilon_{int}$$

The results of the regression are summarized in Table 6.

Based on this regression, we find that a one standard deviation fall in the PMMS rate results in a rise in the home equity extraction by 6.51% of the standard deviation in a MSA-year with average search activity. But in a MSA-year with 1 standard deviation higher search activity, this response is 8.95% of the standard deviation. Thus, a higher search activity area responds 37% more to a rate reduction than a lower search activity area in terms of home equity extraction.

### 2.4. Model

Our empirical findings become the basis of our main assumptions in the model and also the targets for our model\(^1\). There are two exogenous types of borrowers: those who meet 1 lender and those who meet 2 lenders at refinancing by paying the same fixed cost. Their search behavior is unobservable to short-lived banks (lenders who take deposits; I will use the terms lenders and banks interchangeably). We focus on the refinance market where statistical discrimination and monetary policy transmission to consumption is more relevant. Hence, the model does not have home purchase and mortgage origination decisions. The home size and price are also constants. The refinance mortgage is similar to the product in Burdett and Judd (1983) where a refinancer’s heterogeneous non-sequential random search for identical lenders leads to price dispersion for the product. Standard New-Keynesian

\(^1\)Apart from price dispersion and heterogeneous mortgage search detailed in Section 2.3, it typically takes 1-2 months to get an offer after application during which time another application made would be equivalent to a non-sequential search; According to NSMO, refinancers initiate contact 73% of times; Multiple applications made by refinancer within 45 days shows up as one application in a credit report, thus lenders cannot observe their search behavior in that period; Lenders profit most in the first period by selling to the GSE’s, hence short-lived. Average years to refinance a GSE loan and fraction of refinancers who seriously consider more than one lender in the NSMO survey are used to calibrate two parameters: search cost and fraction of borrowers who meet multiple lenders. Exogenous heterogeneity in search behavior, instead of search cost, also results in a tight match with untargeted distribution in the data.
firms allow changes in nominal interest rates made by a monetary authority to result in significant changes in real quantities like consumption. Below is the complete description of the model.

2.4.1. Environment

The model has discrete and infinite time indexed by \( t = 0, 1, 2, \ldots \). It has households who are born with a mortgage and make refinancing decisions and lenders who set mortgage rates to maximize their profits. Each agent is described below.

Households

**Household Types** There is a unit continuum of households who are either borrowers or savers. Each borrower is exogenously either a Type 1 or a Type 2 borrower. A Type 1 borrower meets only one lender at refinancing whereas a Type 2 borrower meets two lenders non-sequentially at refinancing.

Borrowers are more impatient than savers based on their respective rate of discounting the future, i.e., borrowers’ \( \beta_b \) is less than savers’ \( \beta_s \) which are both less than 1. Each agent survives a period with probability \( \zeta < 1 \) and is replaced by the same types. So, the effective rate of discounting of a household \( i \in \{b, s\} \) is \( \beta_{i, eff} = \zeta \beta_s \).

In any period, there is a fixed mass \( \chi_s < 1 \) of savers and \( (1 - \chi_s) \) of borrowers. Out of the borrowers, \( \alpha (1 - \chi_s) \) are Type 2 borrowers and the rest \( (1 - \alpha)(1 - \chi_s) \) are Type 1 borrowers.

**Household Commonalities** The period utility of a household \( i \in \{b, s\} \) is given by:

\[
u^i_{a,h}(c,l) = \log(c) + \xi \log(h) - \psi_i \frac{l^{1+\phi}}{1+\phi} - \hat{\eta} \{a = R\}
\]

where \( a \in \{R, N\} \) is the action taken by the household in that period: whether to refinance (\( R \)) or not to refinance (\( N \)), \( \hat{\eta} \) is the utility cost of refinancing, \( \psi_i \) is the household-type specific parameter for disutility of labor \( l \), \( \xi \) is the utility of housing \( h \) relative to the utility from consumption \( c \) and \( \phi \) is the inverse Frisch elasticity. Note that both type of borrowers

68
pay the same utility cost of refinancing but Type 1 meet only one lender whereas Type 2 meet two lenders. Each period, with probability $\lambda$, a household receives a shock by which the cost of refinancing disappears; otherwise, it is equal to a constant $\eta$. Thus,

$$\hat{\eta} = \begin{cases} 
\eta & \text{with prob. } 1 - \lambda \\
0 & \text{with prob. } \lambda 
\end{cases}$$

Moreover, each household is born with a house of price $p$ and size $h$ which are both constants. Each household pays a maintenance cost of $\delta ph$ for the house. Each household is endowed with one unit of labor per period.

**Household Differences**  Borrowers are born with a long-term mortgage of fixed size $m_0$ at a fixed rate $r_0$. They pay down $\nu < 1$ fraction of the principal and the interest on the mortgage each period. Each borrower can choose to refinance each period. On the other hand, savers own their homes mortgage-free and they own the firms and the banks in the economy. They have access to one-period nominal bonds and bank deposits.

**Banks**

Banks cannot observe the type or the refinancing cost of a refiner that meets them. There are a large number $B$ of banks. They observe the mortgage balance $m$ and the current mortgage rate $r$ of the refinancer. Each bank is short-lived across two periods. In their first period, a bank gets deposits $d$ from savers at a promised risk-free rate $i$, lends $m'$ to each refinancer by offering rate $r'$ and gets non-refinanced mortgages at cost from a bank in its second period. In their second period, a bank gets a payment $m'(1 + r')$ from each borrower, transfers the non-refinanced mortgages at cost to banks in their first period and returns deposits $d(1 + i)$ and profits $m'(r' - i)$ from each borrower to savers.

Since the banks are short-lived, their profit maximization at origination of a mortgage considers only the profit at the time of origination. This reflects the institutional behavior in the agency mortgage market in the US, where the lending banks originate to sell immedi-
ately to the agencies Fannie Mae and Freddie Mac and thus most of their profit is received at origination of the mortgage. We do not model the agencies explicitly but having risk-neutral competitive agencies would result in the short-term risk-free rate being the cost of funding for the banks, as we have in our model.

**Firms**

Firms are standard New-Keynesian to introduce rigidities in the model in order to have real aggregate impact of nominal changes. There is a competitive final good producer who purchases inputs from a unit continuum of intermediate goods producers who set the price of their intermediate good but a fraction $\Psi$ of them are unable to update their price. The intermediate good producer uses labor in a linear production function to produce their goods.

**Monetary Authority**

The monetary authority sets the nominal risk-free interest rate in the economy according to a Taylor-type interest rate rule as in [Iacoviello (2005)](lacoviello2005).

2.4.2. Decision Problems

The decision problems of the agents described above are stated below.

**Borrowers**

Individual-specific states of a borrower are its Type $j$, $j \in \{1, 2\}$, current mortgage balance $m$, current interest rate $r$ and the realization of the cost of refinancing $\hat{\eta}$. Let $\mu(j, m, r)$ be the mass of Type $j$ borrowers, $j \in \{1, 2\}$ whose current mortgage balance is $m$ and current interest rate is $r$. Note that we did not include $\hat{\eta}$ to define the mass since $\mu(j, m, r)$ is a sufficient to know that out of these, $\lambda \mu(j, m, r)$ have no cost of refinancing. Hence, let $\mu := \{\mu(j, m, r)\}_{j, m, r}$ be the distribution of borrowers over the entire state space. This distribution denotes the aggregate state of the economy and inflation and wage rate are its functions.

We define $S := \{\hat{\eta}, j, m, r; \mu\}$ as the current state of a borrower before she decides whether
to refinance. A bank cannot observe \( \eta \) and \( j \). The shock to the cost of refinancing ensures that there are refinancers of each type for every \( m, r \) in which they are present. As known from Burdett and Judd (1983), due to heterogeneity in search, there is no pure strategy solution for a bank. So, let \( F(r', K) \) be the proportion of identical banks who post no greater than \( r' \) when they meet a refinancer with mortgage balance \( m \) and rate \( r \), where \( K := \{m, r; \mu\} \) and \( r' \) cannot exceed \( r \).

Thus, the decision problem of a borrower is:

\[
V(S) = \max \{V_N(r, S), E_{r'|S}V_R(r', S)\}
\]

A borrower chooses whether to refinance (\( R \)) or not to refinance (\( N \)) based on which choice maximizes their expected lifetime value. Note that they decide to refinance first and then meet one or two banks based on their type. Type 1 meets one lender from the distribution of lenders \( F(r', K) \) whereas Type 2 meets two of them non-sequentially and chooses the minimum of the two rates that they offer, the effective distribution of lenders that they meet depends on their type as below:

\[
r'|S \sim \begin{cases} 
  dF(r', K) & j = 1 \\
  2(1 - F(r', K))dF(r', K) & j = 2
\end{cases}
\]

Conditional on their action \( a \in \{R, N\} \), households choose consumption \( c \) and labor effort \( l \) maximize their lifetime utility. Their decision problem now is:
\[ V_a(r', S) = \max_{c,l} u_{a,b}^b(c, l) + \beta^{eff} V(S') \]

where \( u_{a,b}^b(c, l) \) is the period utility mentioned above and they face the budget constraint:

\[ c + \pi(\mu)^{-1}(m(r + v) + m(1 - v)) + \delta ph = m' + w(\mu)l \]

where \( \pi(\mu) \) is the gross inflation and \( w(\mu) \) is the wage rate in the economy, \( m_r \) is the nominal interest payment and \( m_v \) is the nominal principal payment on the mortgage, \( m(1 - v) \) is the remaining nominal principal on the mortgage, \( \delta ph \) is the maintenance on the house and the new balance on the mortgage depends on the decision to refinance or not as below:

\[
m' = \begin{cases} 
\theta^{LTV} ph & a = R \\
\pi(\mu)^{-1} m(1 - v) & a = N 
\end{cases}
\]

where \( \theta^{LTV} \) is a parameter that defines the maximum ratio of the mortgage amount to the value of the home that a borrower can borrow. Thus, borrowers always refinance up to their LTV limit. This assumption simplifies the household problem and the computations in the model. It has been used commonly in the literature and has been shown to have little effect on the conclusions of the model since even without this restriction, most borrowers borrow up to their borrowing limit (e.g., [Beraja et al. (2018), Greenwald (2018)]. If they do not refinance, they continue with the remaining principal amount on their mortgage. Finally, the maximum labor effort is normalized to 1, the function \( H \) defines the law of motion of the aggregate state and the type of the borrower is persistent:
$l \leq 1, \mu' = H(\mu), j' = j$

**Banks**

For each mortgage balance $m$ and current rate $r$, a bank posts rates for the refinancers with that $(m,r)$ to maximize their profit. They do not observe the type of the borrower and hence only infer the probability of the borrower being either Type 1 or Type 2 based on the mass of borrowers in that state, and their optimal decisions in that state with and without the refinancing cost shock. In addition to that, they receive the regular mortgage payments from the non-refinancing borrowers in that submarket. Thus, a bank’s profit maximization problem is:

$$P(\mu) = \int \int \frac{1}{B} \{ \pi(\mu)^{-1} m(1 - v)(r - i)[j = 1]2 \sum q_N(j, \mathbb{K})$$

$$+ \max_{r'} (\theta^{LT} \phi(r' - i)(q_R(1, \mathbb{K}) + 2(1 - F(r', \mathbb{K}))q_R(2, \mathbb{K}))) \}dm dr \} (2.4.1)$$

where the mass of refinancers $q_R(j, \mathbb{K})$ is the fraction of mass of that borrower type who find refinancing more valuable than not refinancing. The remaining mass of that borrower type are non-refinancers $q_N(j, \mathbb{K})$. Since the highest $r'$ is $r$ and $m' \geq m$, those with $\hat{\eta} = 0$ always find it optimal to refinance. Imposing that gives:

$$q_R(j, \mathbb{K}) = \begin{cases} 
\mu(j, m, r) & \text{if } E_{r'|\{\eta, j, \mathbb{K}\}} V_R(r', \eta, j, \mathbb{K}) > V_N(r, \eta, j, \mathbb{K}) \\
\lambda \mu(j, m, r) & \text{otherwise} 
\end{cases}$$

$$q_N(j, \mathbb{K}) = \mu(j, m, r) - q_R(j, \mathbb{K})$$
The bank contracts with the refinancing Type 1 borrower that meets this bank irrespective of the rate it offers but it contracts with the Type 2 borrower only if the rate this bank offers is lower of the two rates that the Type 2 borrower gets non-sequentially. Hence the expression in the maximization problem of the bank is similar to that in [Burdett and Judd (1983)]. Hence, the distribution posted by a bank in equilibrium is also similar to that in that model. Note that the profit per borrower for the bank is the sum of the principal payment, interest payment, remaining mortgage balance payment less the gross interest payment on the savers’ deposits that financed the mortgage, i.e.,

\[ m'(r' - i) = m'(v + r' + (1 - v) - (1 + i)) \]

**Savers**

Saver’s problem is a standard problem of lifetime expected utility maximization where it earns bank profits, firm profits, risk-free return on the bonds and bank deposits and labor income and pays for the maintenance on the house, consumption, buys bonds and lends deposits to banks. Note that since savers do not have a mortgage, the cost of refinancing shock is irrelevant to them. Thus, the saver problem is:

\[
\max_{\{c_t, h_t, b_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u^{s}(c_t, l_t) \\
\text{with the budget constraint}
\]

\[ c_t + d_t + b_t + \delta ph = \pi_t^{-1}((d_{t-1} + b_{t-1})(1 + i_{t-1}) + \Pi_t^{f}) + w_t l_t + \chi_s^{-1} \Pi_t^{f} \]

with initial conditions on the bonds and deposits:
\[ b_{-1} = d_{-1} = 0 \]

and the labor effort limit normalization:

\[ l_t \leq 1 \]

In the budget constraint, \( \Pi^f_t \) are the profits from intermediate goods producers in the economy, and bank profits \( \Pi^l(\mu) \) are the profits earned by all banks:

\[ \Pi^l(\mu) = \chi^{-1}sBP(\mu) \]

and deposits demanded \( d(\mu) \) are the mortgage balances of refinancers and non-refinancers of both types of borrowers:

\[ d(\mu) = \chi^{-1}s \int \{ \theta^{LTV} ph[j = 1]2\sum qR(j, K) + \pi(\mu)^{-1}m(1 - \nu)[j = 1]2\sum qN(j, K) \} dm \]

**Firms**

The final good producer solves a static problem of choosing each intermediate good input \( y_t(k) \) to purchase at the price \( P_t(k) \) and producing the final good sold at price \( P_t \) as below:

\[
\max_{y_t(k)} P_t \left( \int_0^1 y_t(k) \frac{\xi - 1}{\xi} dk \right)^{\frac{1}{\xi}} - \int_0^1 P_t(k)y_t(k)dk
\]

The intermediate good \( k \) producer chooses price \( P_t(k) \) and produces the demanded \( y_t(k) \)
where $l_t(k)$ is labor hours and $a_t$ is the total factor productivity. In this model, I assume $a_t = a$, a constant. These intermediate good producers are subject to price-stickiness, in particular, a fraction $\Psi$ of them are unable to update their price in any period.

**Monetary Authority**

The monetary authority sets the nominal interest rate which is also the cost of lending as below:

$$1 + i_t = (1 + i_{t-1})^{\rho_i} \left( \pi_{t-1} \frac{y_{t-1}}{y_{ss}} \right)^{1 - \rho_i} u_{i,t}$$

where $i_t$ is the interest rate, $\pi_{t-1}$ is the inflation, $y_{t-1}$ is the output of the economy, $rr_{ss}$ and $y_{ss}$ are the steady-state real rate and output respectively. The white noise shock process $u_{i,t}$ has variance $\sigma^2_u$.

### 2.4.3. Equilibrium

A competitive equilibrium in the above model is defined as a sequence of prices \( \{w, i, P, \{P(k)\}_k, \pi, r\}_t \), borrower decisions \( \{c, l, a, m'\}_{j, i, t} \), saver decisions \( \{c_s, l_s, d, b\}_t \), bank decisions \( \{F\}_t \), firms’ demands \( \{y(k)\}_k, \{l(k)\}_k \), distribution of borrowers \( \{\mu\}_t \), and its law of motion \( H \) such that borrowers of both types, savers, banks and firms optimize their problems, interest rate rule holds, \( H \) is consistent with borrower and bank decisions and all the markets below clear:

- **Labor market**: \( \int l_t(k) dk = \chi_s l_{s,t} + \sum_{j=1}^{2} \int \lambda_l \theta_{s,j,t}(\mu_t) + \lambda \eta_{s,j,t}(\mu_t) \) \( d\mu_t \)

- **Bond market**: \( b_t = 0 \)
• Goods market:

\[ j = 1 \sum \{ \lambda c_{0,t} \mu_t + (1 - \lambda) c_{\eta,t} \nu_t \} d\mu_t + \chi \rho c_s \eta_t + \delta \rho h = w_t \int l_t(k) dk + \Pi_t \]

**Optimal Decision of a Bank**

The ability to observe the refinancer’s \((m, r)\), their optimal decisions and the distribution \(\mu\) allows statistical discrimination by the bank. A bank posts a rate according to its market power in the market, which is higher if a greater fraction of refinancers meet only one bank. As mentioned earlier, bank plays a mixed strategy as in Burdett and Judd (1983). At a higher rate, the profit is higher if the refinancer contracts with the bank and a lower rate, the probability of the refinancer contracting with the bank is higher. The different \(r'\) posted form a connected set and the proportion of lenders that post at most \(r'\), \(F(r', K)\), is continuous.

Equating the profit at any rate \(r'\) posted by a bank to the profit by posting \(r' = r\) in Equation 2.4.1 gives the distribution of lenders \(F(r', K)\) according to the rates \(r'\) they post once they observe \(\mathbf{K} := \{m, r, \mu\} :\)

\[ F(r', \mathbf{K}) = 1 - \frac{q_R(1, \mathbf{K})}{2q_R(2, \mathbf{K})} \frac{r - r'}{r' - i} \]

Solving for \(r'\) by setting \(F(r', \mathbf{K}) = 0\) above gives the lower bound of this distribution:

\[ r' = i + (r - i) \frac{q_R(1, \mathbf{K})}{q_R(1, \mathbf{K}) + 2q_R(2, \mathbf{K})} \]

As a numerical example, we plot this distribution in two cases with the same mortgage balance \(m\) and current mortgage rate \(r\) with the risk-free rate \(i\), but with different ratios of the two types of borrowers in the Figure 15. In either of these cases, Type 2 borrowers, who get quotes from two banks and choose the minimum, effectively get lower rates than Type 1 borrowers. In the market with a higher fraction of Type 1 borrowers, the offered rates by
banks are higher for both types of borrowers and are closer to the monopoly rate $r$. Thus, if there are more borrowers who get a quote from only one bank, even the borrowers who get two quotes end up with higher rates. Thus, the composition of types of borrowers with the same $(m, r)$ plays an important role in the optimal decision of a borrower to refinance her mortgage.

Also, for any ratio of the two types of borrowers, the distribution of lenders is Pareto. Thus, many lenders post low rates but few post high rates. Hence, Type 1 borrowers are likely to get low rates but Type 2 borrowers are very unlikely to draw two high rates. This leads to isolation of Type 1 borrowers at high rates but not of Type 2 borrowers at low rates. That is why, as we will see later, statistical discrimination costs Type 1 borrowers a lot but does not benefit Type 2 borrowers.

Figure 15: Effective distribution of lenders in benchmark economy

![Effective distribution of lenders](image)

Note: Banks post lower rates on average when there are more Type 2 borrowers with the observed $(m, r)$. Type 2 borrowers effectively get lower rates than Type 1 on average as they choose minimum of the two offers. Because of the Pareto distribution, Type 1 borrowers are likely to get low rates but Type 2 borrowers are unlikely to get high rates, leading to isolation of Type 1 borrowers at high rates.

2.4.4. Parameter Selection and Calibration

Most of the parameters are chosen from the literature. We calibrate four parameters of the model together to match four moments in the data by minimizing the maximum difference
between the moments in the data and the corresponding moments generated by the model in its steady state. The results are summarized in Table 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Target/Source</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Search Cost</td>
<td>$\eta$</td>
<td>1.116</td>
<td>Avg years to refinance (GSE)</td>
<td>3.58</td>
<td>3.57</td>
</tr>
<tr>
<td>Type 2 Borrowers</td>
<td>$\alpha$</td>
<td>.54</td>
<td>Type 2/Refinancers (NSMO)</td>
<td>.478</td>
<td>.479</td>
</tr>
<tr>
<td>Refinance Cost Shock</td>
<td>$\lambda$</td>
<td>1.25%</td>
<td>Owners who move/Q (Census)</td>
<td>1.25</td>
<td>1.25</td>
</tr>
<tr>
<td>Borrower labor disutility</td>
<td>$\psi_b$</td>
<td>11.02</td>
<td>Total borrower labor supply</td>
<td>.33</td>
<td>.33</td>
</tr>
<tr>
<td>Saver labor disutility</td>
<td>$\psi_s$</td>
<td>7.02</td>
<td>Saver labor supply</td>
<td>.33</td>
<td>.33</td>
</tr>
<tr>
<td>Inv Frisch elasticity</td>
<td>$\phi$</td>
<td>1</td>
<td>Standard</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Survival Probability</td>
<td>$\zeta$</td>
<td>99.5%</td>
<td>50 yrs owner life (Census)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fraction of savers</td>
<td>$\chi_s$</td>
<td>.681</td>
<td>SCF 1998</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Borrower discount factor</td>
<td>$\beta_b$</td>
<td>.965</td>
<td>SCF 1998</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Saver discount factor</td>
<td>$\beta_s$</td>
<td>.987</td>
<td>Avg. 10Y rate, 1993-1997</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mortgage amortization</td>
<td>$\nu$</td>
<td>.435%</td>
<td>Greenwald (2018)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max LTV ratio</td>
<td>$\theta_{LTV}$</td>
<td>.85</td>
<td>Greenwald (2018)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Housing preference</td>
<td>$\xi$</td>
<td>.25</td>
<td>Davis and Ortalo-Magne (2011)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Housing stock</td>
<td>$h$</td>
<td>8.828</td>
<td>SCF 1998</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Housing depreciation</td>
<td>$\delta$</td>
<td>5%</td>
<td>Standard</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Productivity</td>
<td>$a$</td>
<td>3.006</td>
<td>$\gamma_a = 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variety elasticity</td>
<td>$\varepsilon$</td>
<td>6</td>
<td>Standard</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price stickiness</td>
<td>$\Psi$</td>
<td>.75</td>
<td>Standard</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steady state inflation</td>
<td>$\pi_{ss}$</td>
<td>1.008</td>
<td>Greenwald (2018)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In particular, the search cost parameter $\eta$, the fraction of Type 2 borrowers $\alpha$, the borrower and saver labor disutility $\psi_b$ and $\psi_s$ respectively are calibrated together to match the average years after which a mortgage is refinanced in the Fannie Mae and Freddie Mac Loans data, the fraction of refinancers who consider more than one lender in the NSMO data, the aggregate labor supply of borrowers and the aggregate labor supply of savers. In addition to that, the refinancing cost shock is set equal to the fraction of homeowners who move per quarter in the US Census data.

I calibrate several of the standard parameters similar to the calibration in [Greenwald (2018)]. The fraction of savers $\chi_s$ and the borrower discount factor $\beta_b$ are matched using the Survey of Consumer Finances (SCF) 1998. The maximum LTV limit $\theta_{LTV}$ is set to 0.85 since there are a lot of mortgages with 80% LTV but also some with higher limits like 90% or 95%. The principal payment ratio $\nu$ is set to 0.435% as in [Greenwald (2018)]. Saver discount factor $\beta_s$ is set to match the 1993-1997 average 10-year rates (6.46%) and steady-state inflation $\pi_{ss}$.
matches the 10-year inflation expectations during the same period. Survival probability $\zeta$ matches the average life of a homeowner according to the US Census which is 50 years. Inverse Frisch elasticity $\phi$, variety elasticity in the production function $\epsilon$ and price stickiness $\Psi$ are as per the standard values in the literature. The productivity parameter $a$ is chosen so that the steady state output is 1. Housing preference parameter $\xi$ is chosen as in [Greenwald(2018)] to match housing expenditure estimated by [Davis and Ortalo-Magné(2011)]. Housing stock $h$ is chosen so that the ratio of saver house value to their income is the same as in SCF 1998 and the house price is 1 in the steady state. Housing depreciation $\delta$ value is standard in the literature.

2.5. Steady State Analysis

Now we describe the optimal refinancing decisions during the lifetime of the borrowers in the steady state of the model and then show how the invariant distribution in this steady state matches that in the data.

2.5.1. Refinancing Policy Functions

As seen in the model, borrowers have to pay a fixed cost of refinancing in order to reduce their mortgage payments and extract home equity. The potential reduction in mortgage payment is higher if the potential rate reduction is higher but lower if more home equity is extracted. The refinancing decision thus not only depends on the current mortgage balance and the current mortgage rate but also on the composition of the two types of borrowers in that state as it determines the potential rate reduction offered by banks who statistically discriminate between the two types.

Hence, to understand these decisions, it is crucial to understand the relative distribution of the two types of borrowers in steady state in the mortgage balance - mortgage rate space. This is shown in Figure [16]. There are more Type 1 borrowers for each Type 2 borrowers at a higher mortgage rate and at a higher mortgage balance. The LTV limit mortgage balance ($m_{LTV}$) and the highest mortgage rate ($r_{max}$) is the state in which each borrower is born. Close to this state, there are much more Type 1 borrowers relative to Type 2 borrowers compared to that in the rest of the space. This is because at birth, Type 1 refinancers get
higher rate than Type 2 because of their own search behavior. Once they get a higher rate, on their next attempt to refinance, banks can infer that any refinancer with such high rate is more likely to be Type 1 refinancer and thus offer them a high rate again, whereas the opposite happens to Type 2 refinancers at lower rates. This increases the isolation of Type 1 borrowers at high rates. Type 2 borrowers do not get as much isolated at lower rates because of the Pareto distribution of lenders implies that Type 1 refinancers are also likely to end up with low rates. Thus, the relative distribution of borrowers affects the distribution of banks based on their offer rates for each \((m, r)\) state and thus affects the optimal decision of the borrowers in that state.

Figure 16: Ratio of mass of borrowers in steady state of benchmark model

Note: Ratio of mass of borrowers in steady state of benchmark model \(\mu(1, m, r) / \mu(2, m, r)\). Type 1 are isolated at high rate, high mortgage balance, thus easy to statistically discriminated.

The refinance policy of each type of borrower in each state is shown in Figure 17. Note that in each state that Type 1 refinances, Type 2 also refinances; but not vice-versa as the fixed cost of refinancing is the same for both types whereas the benefit of meeting two banks always exceeds the benefit of meeting one bank. The three regions where borrowers do not refinance are explained in the figure. Firstly, the relatively high concentration of Type 1 borrowers close to \(m_{LTV}\) and \(r_{max}\) leads to greater market power for the banks, easier inference of type leads to stronger statistical discrimination and thus the offered rate reduction is low; hence the borrowers do not refinance in these states. Secondly, once the current rate becomes low enough, borrowers do not find it optimal to spend the fixed cost of refinancing in order to reduce their mortgage payment further. Thirdly, even at higher mortgage rates, once the mortgage balance becomes low enough, the mortgage payment
is thus low enough that refinancing to get a lower rate is not worth the fixed cost.

The different arrows in Figure 17 describe the refinancing policy during the life of a typical borrower. First, at birth (at $m_{LTV}$ and $r_{max}$), both types refinance and get lower rates and stay at the same mortgage balance level $m_{LTV}$. Second, if the new mortgage rate is low enough, the borrower does not find it optimal to spend the refinancing cost to get a lower rate or to extract the home equity; thus she only repays her mortgage for the rest of her life. Third, if the new mortgage rate is not that low, the borrower refinances her mortgage after some periods. The number of periods for which she waits before refinancing depends on the potential rate improvement and the extractable home equity post refinancing. It should be noted that the borrower waits longer to refinance even though she has a higher current mortgage rate. This is because the relative mass of Type 1 borrowers to Type 2 borrowers is much higher at these high mortgage rates as seen in Figure 16 which makes it very easy for the banks to statistically discriminate in these states, thus reducing the potential rate improvement for borrowers. Eventually, when the benefit of extracting the accumulated home equity becomes high enough, the borrowers refinance at these high rates, get lower rates and cash out the home equity. Whether the borrowers refinance again in their lifetime depends on the new rate they end up with. Thus, the distribution of the two types of borrowers in the mortgage balance - mortgage rate space is crucial to determine the optimal refinance policy of the borrowers. As described in Section 2.3.1, this optimal refinance policy is also observed in the data and thus the above novel mechanism is validated.

The relevance and importance of heterogeneity in mortgage search can be seen by removing the mortgage search heterogeneity and have only Type 1 borrowers or only Type 2 borrowers in the economy with all the other parameters kept the same. In the economy with only Type 1 borrowers who meet one lender at refinancing, any lender offers only the monopoly price (current mortgage rate) to each borrower in each state if they try to refinance and hence, since there is no rate improvement, the Type 1 borrower does not find it
optimal to spend the refinancing cost and so does not refinance in any state. Her mortgage rate remains at the highest level with which she was born. On the other hand, in the economy with only Type 2 borrowers who meet two lenders at refinancing, any lender offers only the competitive price (cost of lending rate) to any borrower trying to refinance. Hence, given the calibrated cost of refinancing, the Type 2 borrower finds it optimal to refinance in each state where the rate is above the competitive rate. Her mortgage rate becomes the lowest level available as soon she refinances immediately after birth. Contrasting these vanilla optimal refinance policies with that observed in Figure 17, the optimal refinance policy of the borrowers in their lifetime crucially depends on the mortgage search heterogeneity studied in this paper. Similarly, heterogeneity in mortgage search also generates the heterogeneity in mortgage rates observed in the data.

2.5.2. Matching the Data

In Figure 30 below, we find that the model is a good representative of the data by comparing the steady state distribution of the two types of borrowers with respect to their mort-
gage balance and mortgage rate to that seen in the data. For this comparison, we choose data from the month of November 2015 as the current coupon rate of a 30 year fixed rate agency mortgage-backed securities (secondary market rate), which is the cost of financing for mortgage lenders, was relatively steady for more than 2 years before this month (See Figure 28). We also consider other months for this comparison and find similar results. To have enough data to build a distribution, we choose to work with the HMDA data. We divide the MSA’s into above-median and below-median search activity as in Section 2.3.3. First, we look at the distribution of the fraction of the initial mortgage amount that is unpaid in this month (top-left panel of Figure 30). We find that entities that search less tend to have a higher mortgage balance. That matches closely with the unpaid mortgage balance distribution in the steady state of the model (top-right panel of Figure 30). Second, we look at the distribution of the difference between the mortgage rate and the secondary market rate at the time of origination of the unpaid mortgages in this month. We find that the entities that search less tend to pay a higher mortgage rate premium. This property matches that in the model. The distribution of rates in the model is Pareto whereas the distribution in the data is highly left-skewed. One of the main results because of the Pareto distribution is that Type 1 borrowers are isolated and thus much worse off but Type 2 borrowers are not isolated and thus not much better off due to statistical discrimination. This result is valid in the data since the distribution is left-skewed. The relative difference in the rate secured by the two types of borrowers in the model matches closely with that in the data. This is shown in Table 7 which states that the difference between those who search less and those who search more in their mean of the distribution of rate or mortgage balance relative to the overall mean is comparable in the data and the model. Thus, the model is a good representative of the data and thus the steady state results and the results from the counterfactual experiments are reliable.

2.6. Effects of Statistical Discrimination in Steady State

Now we will describe the effects of statistical discrimination in the steady state of the model on borrowers at different stages of their life, on the steady state distribution of bor-
rowers, on their absolute and relative welfare. For this, we compare them in the benchmark economy which has observable mortgage to a counterfactual economy in which the current rate and balance is unobservable to the lender, they offer rates based on the aggregate ratio of the two types of borrowers and thus there is no statistical discrimination.

2.6.1. Evolution of State with Borrower’s Age

In the Figure 18 below, we show the evolution of the distribution of mortgage rate and mortgage balance of the two types of borrowers in the steady state of the model. Each borrower is born with a high rate and mortgage balance equal to the LTV limit. All of them refinance in the first period. The initial separation in the rate distribution between them thus created is because of their own search and the market power of the lender based on the aggregate ratio of the two types. Hence, it is the same in the benchmark and the counterfactual economy. But with this increased separation in types, the lenders have more information about the borrowers based on their current rate. Type 1 borrowers are likely to be at higher rates and Type 2 at lower rates. In the benchmark economy, thus, higher the rate of a refinancer, more likely she is to be of Type 1, less likely she is to meet another lender, more the market power of the lender, thus higher the offer rate. This results in increase in the separation in mean rates between the two types once they start refinancing again. After about five years of age, only those of Type 2 borrowers whose cost of refinancing becomes zero enter the market whereas a few more Type 1 borrowers enter the market to get lower rates. Hence, note that I plot values for first ten years when most of the refinancing action takes place and also that I have dropped the first period percentage refinance in the plot since it is 100. In contrast to this, in the counterfactual economy without statistical discrimination, the initial separation decreases as the lenders are no longer able to condition their offer rates based on the current rate and balance of the refinancer. So, many more Type 1 borrowers who got higher rates at the start find it optimal to refinance again and reach lower rates whereas Type 2 borrowers, who no longer gain by being at lower rates, wait longer to collect more home equity and then refinance. Thus, the difference in rates reduces sharply as they refinance again. After about five years of age, both
types refinance only if their cost of refinancing becomes zero. Thus, much of the difference in rates in the benchmark economy is explained by statistical discrimination. The initial variation in rates for Type 1 borrowers is higher as Type 2 borrowers draw twice from the same distribution and thus get a tighter distribution of rates. This variation increases as they refinance more since those at high rates get less reduction and those at low rates get more reduction because of statistical discrimination. This effect goes away when the mortgage becomes unobservable. In terms of mortgage balance, in the benchmark economy, Type 1 borrowers keep on refinancing at later ages also and hence stay at a higher mortgage balance than Type 2 borrowers. This also goes away if there is no statistical discrimination. Hence, statistical discrimination has a big impact on lender’s offer rates and thus on refinancing decisions and the resulting distribution of rates and home equity.

2.6.2. Isolation of Type 1 borrowers

We have shown in Section 2.5.1 how Type 1 borrowers become isolated at higher mortgage rate and balance in steady state. Now let us see how much of this isolation depends on statistical discrimination. The plots on the left-hand side of Figure 19 are of the relative distribution of the two types in the steady state of the benchmark economy. It can be seen how because of this relative distribution, at high mortgage rate and balance, it is much easier to statistically discriminate and infer the type of the borrower. At the same time, at low rates, there is not much isolation of Type 2 borrowers because of the Pareto distribution of lenders mentioned earlier and also because of the presence of old Type 1 borrowers who are present at these lower rates. On the other hand, on the right-hand side are the same plots for the economy without statistical discrimination. Due to repeated refinancing, the two types collect very similar home equity and they end up at higher rates mainly because of their own lack of search. There is hardly any isolation of Type 1 borrowers in any region of the state space. Thus, the isolation of Type 1 borrowers is mainly due to statistical discrimination.
2.6.3. Welfare Costs

Statistical discrimination has big welfare costs. Borrowers are willing to pay 30% of their quarterly income (about $3,300) to make mortgage unobservable and thus remove statistical discrimination. Out of this, Type 1 are willing to pay 52% and Type 2 are willing
Note: Relative mass of borrowers (Type 1/Type 2) in steady state with and without statistical discrimination. Shows isolation of Type 1 borrowers at high mortgage rates and balances in benchmark model is due to statistical discrimination.

to pay 8% of their quarterly income. Left panel of Figure 20 shows the welfare cost of statistical discrimination to borrowers at each age. Its right panel shows the average welfare cost of statistical discrimination for a borrower in economies with different fraction of Type 2 borrowers ($\alpha^*$ is the calibrated value), keeping all other parameters same. The cost is defined as the difference in values for a borrower in the economies with and without statistical discrimination expressed in consumption good equivalents. At birth, statistical discrimination costs little but as soon as they start refinancing, it costs much more to Type 1 borrowers than Type 2. Once they stop refinancing, they make smaller mortgage payments over time and so the cost of statistical discrimination diminishes. For Type 2 borrower, the
cost is positive because they do not gain much out of statistical discrimination due to the Pareto distribution and the existence of old Type 1 borrowers at low rates in steady state, as mentioned before. Even after several rounds of refinancing, about half of Type 2 borrowers have a positive cost and the other half have a smaller negative cost. Those Type 2 borrowers with low rates who decide to refinance, extract home equity, make larger mortgage payments and are worse off due to statistical discrimination (positive welfare cost) whereas those Type 2 borrowers with high rates who decide not to refinance as they will be perceived as being Type 1, collect home equity, make smaller mortgage payments over the rest of their life and thus benefit out of statistical discrimination (negative welfare cost). If there are more Type 2 borrowers in the economy, i.e., move to the right of $\alpha^*$ in the right panel of Figure 20, lender loses their average market power, fewer Type 1 are isolated at high rate and balance and thus the welfare cost of statistical discrimination decreases. Thus, this way of increasing mortgage search in the economy leads to significant reduction in welfare cost of statistical discrimination.

![Figure 20: Welfare cost of statistical discrimination](image)

Note: Welfare cost of statistical discrimination: Small at birth, increases with each round of refinancing, quadruples in eight years. Type 2's do not benefit out of it since they are not isolated at low rates due to the Pareto distribution of lenders seen earlier in Figure 15. It is the lenders who benefit by statistical discrimination. (left). It is much higher for the isolated Type 1 borrowers in the calibrated model, reduces if there are more Type 2 borrowers in economy (right).

Statistical discrimination explains more than two-thirds of the difference in welfare between the two types. Type 1 borrowers are willing to pay 70% of their quarterly income
(about $7,700) to switch to Type 2 in the benchmark economy which reduces to 21% in the counterfactual economy with unobservable mortgage. Left panel of Figure 21 shows the age-wise difference in welfare between the two types with and without statistical discrimination and the right panel shows the difference in welfare between the two types with and without statistical discrimination in economies with different fraction of Type 2 borrowers ($a^\ast$ is the calibrated value), keeping all other parameters same. The difference in value is expressed in consumption good equivalents. Similar to earlier, the difference in welfare between types is small at birth but as they start refinancing and thus getting separated, this difference increases and once they are done with refinancing, the difference in welfare diminishes as they make smaller mortgage payments. Without statistical discrimination, instead of the difference increasing, Type 1 borrowers at higher rates refinance sooner than Type 2 and close the gap in welfare and this gap keeps reducing with time. As earlier, if there are more Type 2 borrowers in the economy, i.e., move to the right of $a^\ast$ in the right panel of Figure 21, the difference in welfare between the two types decreases and statistical discrimination accounts for less of it. Thus, this way of increasing mortgage search also reduces the difference in welfare between the two types.

**Figure 21: Welfare difference between the two types**

Note: Welfare difference between the two types: At birth, it is small and not driven by statistical discrimination, but increases sharply with each round of refinancing due to increasing isolation of Type 1’s (left). Two-thirds of the welfare difference is due to statistical discrimination in the calibrated model, it would decrease if there are more Type 2’s in economy (right).
2.7. Effect of increasing mortgage search

An explicit aim of the Consumer Financial Protection Bureau (CFPB) is to increase the mortgage search in the economy. But how this mortgage search increases can be important for welfare consequences for the borrower, especially in the presence of statistical discrimination. We will consider two ways of increasing mortgage search: first in which one-third of Type 1 borrowers now become Type 2 and second in which Type 2 borrowers meet 3 lenders instead of two. We find that welfare cost becomes two-third of the benchmark level in the first case whereas it becomes four times the benchmark level in the second case. Thus, while implementing policies related to increasing mortgage search, it is important to evaluate whether non-searchers search more or searchers search more.

2.7.1. Non-searcher searches more

In this economy, we increase the fraction of Type 2 borrowers $\alpha$ from 0.54 to 0.68 based on our findings about search intensity in the HMDA data\(^2\). As seen in Section 2.6.3, this results in welfare gains for both types of borrowers and the difference in welfare between the two types also decreases. Welfare cost becomes two-third of the benchmark level and difference in welfare falls by 15%. Below we compare the steady state refinancing decisions and distribution in this economy with that in the benchmark.

Steady States Comparison

In this alternate economy with more borrowers who meet two banks, banks offer lower rates for both types of borrowers as there are fewer borrowers who meet one lender relative to those who meet two lenders. Fewer Type 1 borrowers are isolated in the high rate and mortgage balance region. Hence, refinancing is now optimal not only in each state in which refinancing is optimal in the baseline model but also in even more states for both types of borrowers. This is shown in Figure 31. Hence in the steady state distribution

\(^2\)The measure of search intensity in a MSA-year, applications withdrawn/applications accepted has mean 0.24 and standard deviation 0.07. Assuming that each borrower withdraws their application only once, about one-third (0.24/0.76) of borrowers are searchers in an MSA-year with average search intensity. In an MSA-year with one standard deviation higher search intensity, this number increases to about half (0.32/0.68). Hence, the increase in $\alpha$ by about .14 represents a one-standard deviation increase in the fraction of Type 2 borrowers in the economy.
of this alternate economy, for both types of borrowers, the mean rate is lower; since the
offered rate is lower on refinancing, the frequency of refinancing over the life of a borrower
is lower; since refinancing frequency is lower, the average home equity is also larger than
in the baseline model. This is shown in Table 8.

2.7.2. Searcher searches more

Another way of increasing mortgage search is to make Type 2 borrowers meet three lenders
instead of two when they refinance. This changes the distribution of lenders based on their
offer rates as Type 2 borrowers now choose minimum of three rates. Like earlier, many
lenders post low rates but now many also post high rates. This increases the isolation of
Type 1 borrowers as Type 2 borrowers are still unlikely to end up with those high rates.
Due to the increased isolation, Type 1 borrowers get higher rates, refinance more frequently
in the hope of getting a low rate, collect less home equity, keep making higher mortgage
payments and thus have welfare cost of statistical discrimination five times as much as
in the benchmark economy. Type 2 borrowers now benefit out of the increased statistical
discrimination and thus have a slightly negative welfare cost of statistical discrimination.
The average borrower’s welfare cost becomes four times that in benchmark case and the
difference in welfare between types becomes three times that in benchmark case. Thus, the
two ways of increasing mortgage search have opposite welfare effects.

Increased isolation of Type 1 borrower

Now, with the same mass of the two types of refinancers, the mass that a lender contracts
with as shown in Equation 2.4.1 becomes

\[
\frac{q_R(1, K) + 3(1 - F(r', K))^2q_R(2, K)}{q_R(1, K) + 3q_R(2, K)}
\]

Thus, now proportion of lenders who post no greater than \( r' \) is:

\[
F(r', K) = 1 - \frac{\sqrt{\frac{q_R(1, K)}{3q_R(2, K)}} \frac{r - r'}{r' - i}}
\]

92
with lower bound:
\[ r' = i + (r - i) \frac{q_R(1, K)}{q_R(1, K) + 3q_R(2, K)} \]

Figure 22 shows the effective distributions of lenders that the two types of borrowers meet in the benchmark economy and that where Type 2 meet 3 lenders. Now, Type 1 borrowers are much more likely to end up at higher rates and with repeated refinancing in presence of statistical discrimination, this leads to their increased isolation. They are also likely to end up at low rates and hence Type 2 borrowers are not isolated at low rates.

Figure 22: Lenders, benchmark vs. model with more active shoppers

![Graph showing effective distribution of lenders](image)

Note: Effective distribution of lenders in benchmark vs. economy where Type 2 meet 3 lenders (numerical example). If Type 2 meet 3 lenders, Type 1 borrowers are much more likely to be isolated at high rates.

Figure 23 (similar description to Figure 18) shows how Type 1 borrowers are now stuck at much higher rates. The variance of rates they get is also higher than benchmark as more lenders post extreme rates. Hence, Type 1 borrowers at high rates keep refinancing hoping to get a low rate and thus collect less home equity and make higher mortgage payments. Type 2 borrowers get much lower rates on average with a much tighter spread. This increased isolation of Type 1 borrowers increases the ability to statistically discriminate and drastically increases the welfare cost of Type 1 borrower.
Figure 23: Borrowers, benchmark vs. model with more active shoppers

Note: Borrower’s age-wise state in steady state of benchmark model and when Type 2 meet 3 lenders (thus more search). Due to increased ability to statistically discriminate in the counterfactual economy, isolation of Type 1 borrowers increases, they get much higher rates, Type 2 get lower rates than benchmark (top left); As more extreme rates are posted, variation in rates that Type 1 get increases, while Type 2 search and get low rates more often than benchmark (top right); As Type 1 are likely to get low rates, they keep on refinancing, thus their home equity stays low (bottom left); As few Type 1 get low rate soon while others keep refinancing to get low rates, the spread in mortgage balance increases (bottom right).
Increase in welfare costs

As shown in the left panel of Figure 24 (similar description to right-panel of Figure 20), welfare cost of statistical discrimination for Type 1 borrowers becomes five times (265% of quarterly income) that in benchmark economy whereas it becomes slightly negative (-2%) for Type 2 borrowers as they now benefit by statistical discrimination. For an average borrower, the welfare cost thus becomes four times (125% of quarterly income) that in benchmark. In the right panel of Figure 24 (similar description to right-panel of Figure 21), the difference in welfare between the two types now becomes three times (213% of quarterly income) that in benchmark and three-fourths of it is explained by statistical discrimination. Thus, this method of increasing search has reduced the welfare of a borrower significantly and also increased the difference in welfare between the two types.

Figure 24: Welfare, Benchmark vs. Model with more active shoppers

Note: Welfare cost of statistical discrimination - Benchmark vs. Type 2 meet 3 lenders (thus more search): Increased isolation of Type 1 borrowers cost them a lot in the counterfactual economy, Type 2 now benefit by statistical discrimination. Thus, lenders benefit much more (left). Now, statistical discrimination accounts for three-fourths of the difference in welfare between the two types, compared to two-thirds in benchmark (right).

2.8. Monetary Policy Transmission to Consumption

Mortgage refinancing is an important channel of monetary policy transmission to consumption in the US. An interest rate cut influenced by Federal Reserve’s open market operations leads to a reduction in the cost of lending which leads to a reduction in mortgage rates. This encourages households to refinance their mortgages to a lower rate and also extract their home equity at the same time, thus increasing their current consumption.
Statistical discrimination described so far in the paper can have a significant impact on this transmission mechanism since the reduction in rates offered to refinancers depends on the composition of searchers and non-searchers at any current mortgage. As seen in the steady state results, statistical discrimination hurts Type 1 borrowers much more than it benefits Type 2 borrowers. Similarly, we find in our experiments below that as statistical discrimination becomes stronger, Type 1 borrower’s consumption response becomes smaller but that of Type 2 remains almost the same. Below, we look at the response to a one-period unexpected 25 basis points expansionary monetary policy shock in four different economies with differing ability to statistically discriminate: benchmark, unobservable mortgage, one-third of Type 1 meet 2 lenders, Type 2 meet 3 lenders.

2.8.1. Benchmark Economy

Type 1 borrowers, isolated at higher rates and thus easier to statistically discriminate, respond much less to the expansionary monetary shock than Type 2 borrowers. We calculate the impulse response functions to a 25 basis points reduction in the risk-free rate in the steady state of the baseline model. As Figure 25 shows, the rate offered to Type 2 borrowers reduces more than that to Type 1 borrowers because of the isolation of Type 1 borrowers in steady state and that Type 2’s are more likely to be in states with more Type 2 borrowers. But since they already have a lower mortgage rate in steady state, the increase in the percentage of refinancers among Type 2’s is smaller. At the same time, Type 2’s have a smaller mortgage balance in steady state and thus a bigger home equity. Thus, the fewer additional Type 2 refinancers extract greater home equity than Type 1’s and thus their mortgage balance increases more. This results in Type 2 borrowers (about 0.84%) having a bigger consumption response to the monetary shock than Type 1 borrowers (about 0.57%). At the aggregate, the consumption of borrowers increases by about 0.71% in response to this monetary policy shock which is line with the consumption response found in the literature.
Note: Those who meet more lenders have a bigger consumption response to an expansionary monetary shock as in steady state, they get lower rates sooner, thus refinance less often, thus collect more home equity. Due to lower rates in steady state, fewer of them refinance to the shock (top right).

2.8.2. Economy with unobservable mortgage

This economy with no statistical discrimination results in much bigger consumption response of Type 1 borrowers than benchmark as they are no longer isolated at high rates and almost same response of Type 2 borrowers as they are not affected much by statistical discrimination. We shock the steady state of this economy with the same 25 basis points reduction in the risk-free rate. As seen in Figure 26, Type 2 get a similar rate reduction whereas Type 1 get much lower offer rates as they are not isolated at high rates. The increase in refinancers among Type 2’s is small compared to benchmark. Type 1 refinancers increase compared to benchmark because of the lower offer rates. They have bigger home equity in steady state compared to benchmark, thus their increase in mortgage balance is bigger. Home equity of Type 2 borrower is almost the same as in benchmark, hence the increase is also similar. Thus, the consumption response of Type 2 borrower (0.88%) is similar to that in benchmark but that of Type 1 (1.21%) is much bigger. Thus, without sta-
tistical discrimination, the consumption response of searchers remains almost similar but that of non-searchers increases a lot as they are no longer isolated at high rates.

Figure 26: IRF to monetary policy, model with unobservable mortgage

IRFs to -25 bps Cost of Lending Shock in Benchmark, Model with Unobservable Mortgage

Note: Without statistical discrimination, Type 1 are no longer isolated, have lower rates, more home equity, bigger consumption response; Type 2 who did not benefit much by statistical discrimination have similar response to benchmark

2.8.3. Economies with more mortgage search

Now we look at two economies described earlier with different ways of increasing mortgage search, an explicit aim of the CFPB. We find that consumption response of Type 1 borrowers moves in opposite directions compared to the benchmark whereas that of Type 2 remains almost same.

One-third of Type 1 meet 2 lenders

This economy with more search and weaker ability to statistically discriminate results in bigger consumption response than benchmark for both types of borrowers. We shock the steady state of this economy with the same 25 basis points reduction in the risk-free rate. As seen in Figure [32] the average offer rate reduces more as both types of borrowers get lower rates since there are fewer Type 1 borrowers isolated at high rates and the reduced
market power due to the presence of more Type 2 borrowers. But since both types of borrowers are at lower rates in steady state, the percentage increase in the refinancers of both types is smaller compared to that in the baseline economy. At the same time, both types of borrowers have more home equity in steady state and thus the fewer additional refinancers extract a greater amount of home equity compared to the baseline. The higher home equity extraction results in a bigger borrower consumption response in the alternate economy (1.08%, Type 1: 0.93%, Type 2: 1.15%) compared to that in the baseline economy (0.71%). Thus, having fewer borrowers who do not search for a mortgage results in a bigger borrower consumption response in the economy.

**Type 2 meet 3 lenders**

This economy with more search and stronger ability to statistically discriminate results in much smaller consumption response of Type 1 borrowers than benchmark due to their increased isolation and almost same response of Type 2 borrowers. We shock the steady state of this economy with the same 25 basis points reduction in the risk-free rate. As seen in Figure 33, the difference in offer rates for the two types increases compared to benchmark. The increased isolation of Type 1 borrower results in much higher offer rate. The increase in refinancers among Type 2’s is small as they are already at lower rate compared to benchmark. Type 1 refinancers decrease compared to benchmark because of the higher offer rates. They also have smaller home equity in steady state compared to benchmark, thus their increase in mortgage balance is lower. Home equity of Type 2 borrower is almost the same as in benchmark, hence the increase is also similar. Thus, the consumption response of Type 2 borrower (0.92%) is similar to that in benchmark but that of Type 1 (0.31%) is much smaller. Thus, if searchers search more, the consumption response of searchers remains almost similar but that of non-searchers decreases a lot due to the stronger ability to statistically discriminate.

2.9. Conclusions

In this paper, we develop a model with heterogeneity in mortgage search of borrowers who refinance leading to difference in mortgage rates not only due to their own search
but also crucially because of the lender’s ability to statistically discriminate based on their current mortgage. This ability becomes more potent with each round of refinancing as it isolates those who meet only one lender at high rates.

When calibrated to the US mortgage data, we find that this statistical discrimination has big welfare costs for borrowers, especially for those who meet only one lender to refinance their mortgage; and conversely, big welfare gains for lenders. It accounts for most of the welfare difference between those who meet one lender and others who meet multiple lenders. Increasing mortgage search is an explicit aim of CFPB. We find that the welfare consequences for borrowers of increasing mortgage search depend critically on how the mortgage search increases. In particular, if non-searchers search more, welfare increases significantly as statistical discrimination becomes less potent. But instead, if searchers search more, then welfare decreases a lot as non-searchers are increasingly isolated at high rates making it easier for lenders to statistically discriminate.

Statistical discrimination also reduces monetary policy transmission to consumption significantly, especially that of borrowers who meet one lender to refinance and are isolated at high rates. Note that our benchmark results are conservative since we restrict those who meet multiple lenders to meet exactly two lenders which is not true for one third of them. Thus, while designing any policy that affects the mortgage market, it is important to assess its impact on this less-intuitive ability to statistically discriminate in this market.

Our analysis and findings are grounded in data as we find a wide dispersion in the rates offered to observably similar refinancers, a lack of search for their mortgage among half the refinancers who thus end up with higher rates and smaller mortgages, U-shaped time-to-refinance in current mortgage rate and balance, and a larger home equity extraction rate in areas with more mortgage search activity.

Looking ahead, it would be important to understand the source of the lack of search in this market. For example, if low-income households are more likely to live in financial deserts
and thus have a low likelihood of meeting multiple lenders to get a mortgage, their resulting higher rates and lower home equity can amplify the consumption inequality borne purely out of income inequality. Incorporating other dimensions of a refinancer like her age, income, house size and price that can be used by a lender for statistical discrimination would also be important to capture this mechanism more accurately. Endogenizing search behavior with heterogeneous search costs will allow a refinancer to search differently based on her income, house size and house price; and thus allow the aggregate search and statistical discrimination to vary according to business cycles.

2.10. Appendix

Supplementary Figures and Tables

See main text for more figures and tables.

2.10.1. Data Analysis

Refinance behavior

Table 4: Regression Results, Loan Age

<table>
<thead>
<tr>
<th></th>
<th>Loan Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>State</td>
<td>-3.772***</td>
</tr>
<tr>
<td>$State^2$</td>
<td>0.076***</td>
</tr>
<tr>
<td>FICO</td>
<td>0.008***</td>
</tr>
<tr>
<td>LTV</td>
<td>0.012***</td>
</tr>
<tr>
<td>CLTV</td>
<td>0.055***</td>
</tr>
<tr>
<td>DTI</td>
<td>0.015***</td>
</tr>
<tr>
<td>Number of Borrowers</td>
<td>1.446***</td>
</tr>
<tr>
<td>First Home Flag=Y</td>
<td>0.685***</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.251</td>
</tr>
<tr>
<td>Observations</td>
<td>6867645</td>
</tr>
</tbody>
</table>

***: p-value < 0.01

Note: Loan Age at which it is refinanced is concave in the State (rate*balance/100,000). This is consistent with the refinance policy of borrowers in the model. (Data: GSE mortgages originated before 2011)
Figure 27: Search Behavior according to NSMO

Note: Half of all borrowers consider only one lender. (Left: All, Right: Refinancers only)

Figure 28: Cost of Mortgage Lending

Note: Cost of Mortgage Lending (MBS Rate) & Average mortgage rate (PMMS) moves with Federal Funds Rate

Search behavior (NSMO)

Cost of Lending and Federal Funds Rate

Search, Rates and Home Equity Extraction (HMDA, GSE)

2.10.2. Model Results

Steady state match with Data

Benchmark vs. More Type 2’s: Steady state comparison

Monetary Policy with More Search
Table 5: MBS and Mortgage Rate

<table>
<thead>
<tr>
<th></th>
<th>MBS Rate</th>
<th>PMMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Federal Funds Rate</td>
<td>0.81***</td>
<td>0.59***</td>
</tr>
<tr>
<td>Constant</td>
<td>1.69***</td>
<td>4.30***</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.764</td>
<td>0.75</td>
</tr>
<tr>
<td>Observations</td>
<td>302</td>
<td>302</td>
</tr>
</tbody>
</table>

***: p-value < 0.01

Note: Cost of Mortgage Lending (MBS Rate) moves much more tightly with Federal Funds Rate than the Average mortgage rate (PMMS)

Figure 29: Mortgage Search and Home Equity Extraction

Note: Mortgage Search (HMDA), Home Equity Extraction (GSE) and Rate Spread (GSE) across MSA-years. Top: Home equity extraction moves in the same direction as mortgage search, across years and MSA’s. Bottom: Average rate spread moves in opposite direction as mortgage search, across years and MSA’s.
Table 6: Regression Result, Home Equity

<table>
<thead>
<tr>
<th>Home Equity Fraction</th>
<th>PMMS Mean</th>
<th>Search Fraction</th>
<th>PMMS*Search Fraction</th>
<th>Adjusted $R^2$</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>-0.02396</td>
<td>-0.01904</td>
<td>0.005953</td>
<td>0.546</td>
<td>1624795</td>
</tr>
<tr>
<td>std. dev.</td>
<td>4.973723</td>
<td>0.2312726</td>
<td>0.0889403</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: MSA’s with more mortgage search extract more home equity in response to reduction in mortgage rates. When rates fall by 1 sd, 37% more home equity is extracted in an MSA with 1 sd more search.

Figure 30: Borrower Distribution, Model vs. Data

Note: Borrower distribution in steady state of benchmark model vs. Data (HMDA, GSE) (Untargeted). Rate spread in data is significantly left-skewed, similar to the Pareto distribution in model, validating the results.
Table 7: Borrower Distribution, Model vs. Data

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Benchmark Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Mortgage Balance Difference/Overall Mean (%)</td>
<td>1.12</td>
<td>1.27</td>
</tr>
<tr>
<td>Mean Rate Difference/Overall Mean (%)</td>
<td>2.86</td>
<td>3.13</td>
</tr>
</tbody>
</table>

Note: Relative (Type 1 - Type 2) Borrower Distribution Means in steady state of Benchmark Model vs. Data (HMDA, GSE) (Un-targeted)

Figure 31: Refinancing, Benchmark vs. Economy with more Type 2 borrowers

Note: Refinancing policy in steady states of Benchmark vs. Economy with more Type 2 borrowers. With more Type 2’s around, offer rates reduce, refinancing optimal in more states for both types.

Table 8: Borrower Type Distribution

<table>
<thead>
<tr>
<th>Weighted Means</th>
<th>$\frac{\hat{q}_1}{\hat{q}_2}$</th>
<th>$\frac{\hat{r}_1}{\hat{r}_2}$</th>
<th>$%$ who refinance</th>
<th>$1 - \frac{m}{m_{\text{TV}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type 1 Borrowers</td>
<td>2.817</td>
<td>1.26</td>
<td>3.93</td>
<td>0.383</td>
</tr>
<tr>
<td>Type 2 Borrowers</td>
<td>2.154</td>
<td>1.18</td>
<td>2.71</td>
<td>0.405</td>
</tr>
</tbody>
</table>

Note: Mean borrower type ratio, rate, refinance frequency and home equity in steady states of benchmark (Column 2) versus economy with more Type 2 (Column 3). Due to more Type 2 borrowers on average in each state, both types get lower rates, thus they refinance less often, thus collect more home equity in steady state.
Figure 32: IRF to Cost of Lending

Comparing IRFs to -25 bps Cost of Lending Shock ($\alpha = .54 \rightarrow .68$)

Note: With fewer Type 1 borrowers in economy, fewer are isolated; that and the reduced market power leads to lower rates, more home equity, bigger consumption response for both types (hence, showing only the aggregate values).
Figure 33: IRF, Benchmark Model vs Model with More Search

Note: When Type 2 meet 3 lenders, isolation of Type 1 increases, ability to statistically discriminate becomes stronger, thus consumption response of Type 1 becomes much smaller; that of Type 2 remains almost same.
CHAPTER 3: Liquidity Management, Banks vs. Shadow Banks

3.1. Abstract
How does the banking sector respond to a change in discount window rate while shadow banking coexists with a regulated banking sector? I study a dynamic equilibrium model in which banks and shadow banks provide illiquid loans by issuing short-term bonds. Banks have access to the discount window to manage the liquidity risk while regulated by the liquidity requirement. Shadow banks face the same risk, but they can only manage the liquidity shock by investing enough in the safe assets. Changing the discount window rate changes the relative advantage of banks compared to shadow banks, in terms of having access to the discount window. Depending on the size of the shadow banking system, effectiveness of monetary policy may be dampened as shadow banks and banks can become interchangeable.

3.2. Introduction
Unregulated banks, or shadow banks, coexist with regulated commercial banks. They provide services similar to banks. Shadow banks both hold risky loans and issue safe assets, or deposits. Coexistence of banks and shadow banks is an important issue, since it can change the effectiveness of a monetary policy. Since the rise of securitization in around 1990, shadow banking has increased dramatically. This paper investigates if the effectiveness of a monetary policy has been influenced by the secular growth in shadow banking in recent decades. In particular, when the federal reserve makes a specific monetary policy move, is the ultimate effect on Gross Domestic Product (GDP) different from what it would have been without shadow banking?

Empirical findings show that with the rise of securitization, the effectiveness of a monetary policy has been weakened. [Estrella et al. (2002)] demonstrates that securitization may have had a significant effect on the degree to which a given change in monetary policy can influence real GDP. In fact, the interest rate elasticity of output is close to zero in the early 2000s. [Kuttner (2000)] shows that the relative growth of asset backed securities to
the loans held by banks increases when the federal funds rate increases. Loutskina and Strahan (2009) show that when deposit costs increases, the jumbo mortgage loans issuance are more affected compared to the conforming mortgage loans. The latter are loans that can easily be securitized. Raw data also confirms these facts during the years before the financial crisis. Figure 34 shows the federal funds rate and mortgage interest rate between 2002 and 2007. The graph shows while the federal funds rate has significantly increased since 2004, the mortgage interest rate has remained almost constant.

The Federal Reserve influences the economy by affecting the liquidity cost of a bank. Banks issue deposits to finance loans. Deposits are subject to early withdrawal, or liquidity shock. Banks hold safe assets or reserves to face this shock. If banks do not have enough safe asset to meet the liquidity shock, banks borrow from either an interbank market or the discount window at the Federal Reserve. Changing the federal funds rate has a direct effect on the loan interest rate and a loan issuance decision. However, the credit market has changed significantly due to the rise of shadow banking. Shadow banks are unregulated sectors that hold risky loans. They issue safe bonds, like the deposit issued by the banks, to finance the loans. These deposit-like instruments are subject to liquidity shock as well. Shadow banks, unlike banks, do not have access to the discount window, therefore they cannot overcome the liquidity shock when needed. Suppose that the Federal Reserve increases the discount window rate. Additionally, there is an indirect effect that this policy creates. It decreases the relative advantage of banks to shadow banks in having access to the discount window. As a result, the indirect effect of the monetary policy is that shadow banks can replace banks by holding more loans and issuing bonds, or safe assets. Subsequently, shadow banks work like a buffer to smooth the effect of a monetary policy. This mechanism depends on whether the size of shadow banking is large compared to the regular banking activity. In addition, it is important to know what factors contribute to the size of a shadow banking system.

In this paper, I set up a dynamic general equilibrium model to explain the role of shadow
banking in affecting the effectiveness of a monetary policy. The agents are banks, shadow banks, entrepreneurs, and households. Banks and shadow banks are a channel between entrepreneurs and households. The entrepreneurs have access to risky technology with a single input, which is the labor of the households. The entrepreneurs are risk averse agents, which implies a demand for safe assets. Therefore, in this economy entrepreneurs are not financially constrained. They are savers in the economy by demanding safe assets. The safe asset is produced by shadow banks and banks. The safe assets are the bonds, or deposits, issued by banks and shadow banks to the entrepreneurs. Banks and shadow banks face credit and liquidity risk; they both can default. The bank’s bonds are insured by the government, so entrepreneurs do not take into account the default probability of banks when they buy the bonds issued by banks. Banks are subject to capital and liquidity requirements while shadow banks are not regulated at all. Consequently, the advantage of banks over shadow banks is that they have access to cheap deposits as well as the discount window. Both of these factors incentivize them to increase risk taking. Regulation is designed to control this moral hazard. Shadow banks are not subject to these benefits and costs. The benefit of having access to deposit insurance, as well as the discount window compared to the cost of regulation, contribute to the size of shadow banking in this economy. Any policy of the government, like changing the discount window rate, is going to change the relative advantage or disadvantage of banks to shadow banks. This implies the change in the size of the shadow banking which is an unintended consequence of a given policy. As a result, shadow banks can prevent that the federal reserve can achieve to the planned policy.

Prior to the financial crisis of 2007-2008, securitization was a very large part of the United States capital markets. It is estimated that securitization has funded approximately 64% of outstanding home mortgages and 25% of outstanding consumer credits. There are two aspects of securitization that are important in this paper. First, securitization makes it possible for shadow banks to hold loans. Traditionally, banks originated loans that they then held on their balance sheets until maturity. This is no longer the case. Other fi-
nancial institutions, called shadow banks, can hold loans issued by the banks. Second, securitization made it possible for shadow banks to access cheap borrowing. It is well documented by Gorton and Metrick (2013) that securitization has eliminated the bankruptcy cost. This is the costly bargaining over the assets of the financial institutions who have filed for bankruptcy. This reduction in bankruptcy cost is derived from the remoteness feature of the “special purpose vehicles” (SPVs) from the sponsor. This feature implies that the recovery rate of the bonds issued by the shadow banks is high.

To assess the effectiveness of the federal funds rate on GDP, I compare the economy in two different states. I will examine the economy in which the recovery rate of the bonds issued by the shadow banks is high. Then I will compare that to the economy in which the recovery rate of the bonds is low. The computational results show that as the recovery rate of the bonds issued by shadow banks increases, it results in less sensitivity of the output to a monetary policy.

Related Literature

After the financial crisis, macro finance literature has emerged to develop macroeconomic models with an explicit role for banks. Some early steps were taken by Gertler and Karadi (2011) and Cúrdia and Woodford (2009) to show the real effects of the disruption in the financial intermediation. Following these papers, a large literature has studied the role of financial intermediary equity in macroeconomic models.

This paper contributes to macro finance literature by incorporating the role of shadow banks, in addition to banks, into a general equilibrium framework. This framework emphasizes how monetary policy affects the trade off between holding assets with different liquidity while there is an unregulated banking system that simultaneously weakens the effectiveness of a given policy. The liquidity management problem presented in this paper is similar to the static reserve management problem in Poole (1968). Bianchi and Bigio (2014) incorporates the role of interbank markets to a general equilibrium model in which
banks face random transfers across each other. I borrowed a simplified version of the liquid-idity problem of Bianchi and Bigio (2014). Unlike Diamond and Dybvig (1983), Allen and Gale (1998) and most recently Gertler and Kiyotaki (2015), the liquidity shock to a bank is not an aggregate bank-run risk in my study. I align my views with Bianchi and Bigio (2014); the liquidity shock is an idiosyncratic risk that might affect only a fraction of banks.

There are few papers in the literature that consider the role of shadow banking in the econ-omy; Moreira and Savov (2017), Begenau and Landvoigt (2018), Huang (2018), Gertler et al. (2016). These papers are narrow in scope because they only examine the relation between shadow banks and a financial crisis and the role of the unconventional monetary policy. Moreira and Savov (2017) highlights the role of shadow banks in producing securities that are safe except for when the economy crashes. Begenau and Landvoigt (2018) presents a model to find optimal capital requirement when there is a risk of shifting activities to an unregulated sector. This paper is silent about the liquidity shock and access to the discount window as some of the key differences between shadow banks and banks. Gertler et al. (2016) describes a macroeconomic model with bank run. They added wholesale banks in addition to retail banks to their model. The wholesale funding arises endogenously in this model based on the relative advantage wholesale banks have in managing assets over retail banks. They also explore the relative advantage of retail banks in overcoming the agency friction that impedes lending to whole sale banks.

The general equilibrium component of the model is borrowed from Quadrini (2015), in which entrepreneurs are different from the regular households. This view is different from almost all the aforementioned papers, in which the classical view is that households are the ultimate savers to the economy and firms are the ultimate borrowers. Empirical studies show that, at least in the aggregate, this is not the case in the United States. The main savers in the economy are entrepreneurs, and the net borrowers are the households whose wealth are typically in the housing market.
3.3. Model

I present a dynamic model of banks and shadow banks that choose to hold loans and safe assets by issuing bonds and equity. The difference between banks and shadow banks is that the former is regulated by capital requirement and liquidity requirement. There are two other agents in this economy; entrepreneurs and households. The entrepreneurs hire households to produce the consumption goods. The entrepreneurs buy the bonds issued by the banks and shadow banks to buffer the risk in production. The demand for loans is coming from households.

3.3.1. Banks

There is a continuum of competitive banks. The preferences of the banks are as follows;

\[ E_0 \sum_{t \geq 0} \beta^t \text{Div}_t \]

Banks are owned by the households, therefore banks maximize the dividend payments to the households. Since the utility function of the households is linear, the discount factor of the bank is \( \beta \) which is the discount factor of the households. Bank’s assets include loans and safe assets; banks issue the loans to the households. Bank’s liabilities are the deposits they issue to the firms. They are faced with two idiosyncratic risks; credit risk and liquidity risk.

**Loans** \((I_t)\). A loan security is characterized by the pair of \((q_\ell, p)\). \(q_\ell\) is the price of the loan. Loans terminate in one period. Banks issue the loans to the households. Each unit of loan gets a performing shock from the distribution \(F(p \sim F_\alpha(p))\). \(\alpha\) is the parameter of the distribution \(F\). The support of \(p\) is in \([0, 1]\). This is what I define as credit risk in this dynamic equilibrium model. \(p = 1\) means that 100% of the loan has been performing.

**Bank Bond** \((B_t)\). A bank’s bond, or deposit, is the only liability of the banks. A bank bond security is characterized by \(q_B\) which is the price of the bond. Banks issue the bonds to the entrepreneurs and the return of the bond is guaranteed by the government. Bank
bonds might face liquidity risk. One unit of a bank bond is sold at the price of $q_B$ to the entrepreneurs. Each unit of bond is a promise to pay one unit of consumption good in the next period. Before the realization of the loan return, the entrepreneurs might ask the banks for the promised payments. This is what I define as liquidity risk, or early withdrawal shock. This shock is characterized by $\omega \in [-1,1] \sim G_{\mu_\omega, \sigma_\omega}(\omega)$. $G$ is the Cumulative Distribution Function (CDF) of the $\omega$ shock. $(\mu_\omega, \sigma_\omega)$ are the parameters of the distribution $G$. It is assumed that $\int_{-1}^{1} \omega dG(\omega) = 0$. This demonstrates that there is no net outflow or inflow of money during the liquidity shock. Suppose that a bank issues $B$ units of bonds to the entrepreneurs. If a bank receives an $\omega$ shock, banks are supposed to pay back $\omega B$ of the amount they borrowed to their creditors.

**Safe Asset** ($A_t$). Safe assets are characterized by $q_A$. Safe assets are issued by the government. The holder of one unit of safe asset pays $q_A$ to receive one in the next period.

**Regulation.** The banks are faced with two regulations; capital requirement and liquidity requirement. The capital requirement constraint is as follows;

$$q_i^B B_{t+1} \leq (1 - \theta)q_i^I I_t + q_i^A A_{t+1} \quad [CR]$$

in which $\theta \leq 1$. The higher the $\theta$ is, the more binding is the capital requirement. Liquidity requirement must hold through at the beginning of the period, at which a bank chooses its portfolio. The liquidity requirements in the beginning of the period is as follows,

$$q_i^A A_{t+1} \geq \rho q_i^B B_{t+1} \quad [LR_1]$$

in which $\rho \in [0,1]$. Before the realization of the loan return, each bank is faced with an early withdrawal shock. If a bank receives an $\omega$ shock, banks are supposed to pay back $\omega B$ to the entrepreneurs. Consequently, the liquidity requirement must hold at the end of
the period as well;

\[ A_{t+1} - \omega B_{t+1} \geq \rho (1 - \omega) B_{t+1} \quad [LR_2] \]

The above equation must satisfy for all \( \omega \). If it does not, the banks must go to the discount window to borrow the shortage. For simplicity, it is assumed that there is no interbank market for borrowing. If \( \omega = 1 \), it denotes that there is a run on a bank.

**Bank Problem, Recursive Formulation**

Each bank starts the period with net worth \( N \). Given the prices \((q_B, q_{\ell}, q_A, r_f)\), a bank chooses loan \( (I_\ell) \), Bond issuance \( (B') \), safe asset purchase \( (A') \) and dividend payment to the households \( \text{Div} \). Note that \( \text{Div} \) can be negative which means that banks can issue equity if needed. There is no cost for equity issuance. Banks face with two risks; credit risk and liquidity risk \( (p, \omega) \). Banks can choose to default after the realization of the risks.

**Problem 1.**

\[
V(N) = \max \left\{ \text{Div} + \beta \int_{-1}^{1} \int_{0}^{1} \max\{V(N'(p, \omega)), 0\} dG(\omega) dF(p) \right. \\
\left. \text{s.t.} \quad \text{Div} + q_\ell I_\ell + q_A A' \leq N + q_B B' \quad [BC] \right.
\]

\[
q_B B' \leq (1 - \theta) q_\ell I_\ell + q_A A' \quad [CR]
\]

\[
q_A A' \geq \rho q_B B' \quad [LR_1]
\]

\[
N' = p I_\ell + A' - B' - r_f x(\omega, A', B') 1 (x(\omega, A', B') \geq 0)
\]

\[
x(\omega, A', B') \equiv (\rho + (1 - \rho) \omega) B' - A'
\]

The banks are owned by the households. Banks discount the future with \( \beta \) which is the discount factor of the households. The utility function of the household is linear, so the discount factor is not stochastic. Banks choice must satisfy three conditions. \([BC]\) is the
budget constraints. $[CR]$ is the capital requirement. $[LR_1]$ is the liquidity requirement. $x(\omega, A', B')$ is the shortage of liquidity of the banks after they receive the liquidity shock. If the liquidity shock is not high, there is no liquidity cost.

3.3.2. Shadow Bank Problem

Shadow bank problem is quite similar to a bank problem. A shadow bank starts the period with net worth $(N)$. Given the prices $(q, q_a, r_f)$, a bank chooses loan ($I^l$), Bond issuance ($B'_s$), safe asset purchase ($A'$) and dividend payment to the households ($Div$). Note that $Div$ can be negative which means that the shadow banks can issue equity if needed. Note that the bond price $q_{B_s}$ is a monopoly price for a shadow bank. It will be discussed later that this price is going to be a function of the leverage choice and liquidity ratio of a shadow bank. A shadow bank can choose to default after the realization of the credit risk and liquidity risk $(p, \omega)$.

Problem 2.

$$V(N) = \max_{\{B', A', I\} \geq 0, Div} Div + \beta G\left(\frac{A'}{B'}\right) \int_{0}^{1} \max\{V(N'(p)), 0\} dF(p)$$

s.t.

$$Div + q^l I^l + q_a A' \leq N + q_{B_s} B' \quad [BC]$$

$$N' = p I^l + A' - B'$$

Shadow banks, unlike banks do not face with any regulation. A shadow bank cannot borrow from the federal reserve to overcome the liquidity shock. So banks survive if $\omega B' \leq A'$. So a shadow bank survives the liquidity shock with probability $G\left(\frac{A'}{B'}\right)$. $G(.)$ is the CDF of the liquidity shock.

There are three differences between banks and shadow banks;

- Shadow banks do not face with capital requirement
- Shadow banks do not face with liquidity requirement
• Shadow banks cannot borrow from the federal reserve to meet the liquidity shock

**Loan Market.** Loan market is a competitive market. For simplicity, it is assumed that both banks and shadow banks can issue loans directly to the households. So, there is no difference between the price in the secondary market and the price of the loan at the origination.

3.3.3. *Entrepreneur Problem*

There is a continuum measure of entrepreneurs. They have access to a technology to produce consumption goods. The only input in the production function is labor. The production function is linear in labor.

\[ Y_t = e^{z_t}H_t \quad z_t \sim N(\mu_z, \sigma_z) \]

\( H_t \) is the labor that each entrepreneur hires from the households. The production function is subject to an idiosyncratic shock \( z_t \) which is coming from a normal distribution. This shock is i.i.d across entrepreneurs and time. The entrepreneurs cannot insure this risk. They are risk averse and maximize the following utility function.

\[ E_0 \sum_{t \geq 0} \beta^t (\eta \log(C_t) + (1 - \eta) \log(B_{t+1})) \]

The entrepreneurs enjoys from consumption and the bond that is issued by the banks. This assumption gives a convenience yield to the bond of the bank comparing to the shadow banks. This guarantees that the size of banks never goes to zero. The asset of the entrepreneurs are bonds issued by banks and shadows banks.

The recursive problem of the entrepreneur is as follows;

**Problem 3.**

\[ V(N) = \max_{H \geq 0} \quad E_{z^t} \left( \max_{\{C, B', B'_t \geq 0} \eta \log(C) + (1 - \eta) \log(B') + \beta V(N'(z')) \right) \]
\[ C + q_B B' + q_{B_s} B'_{s} \leq N + (e^{z'} - w)H \]

\[ N'(z') = B' + B'_{s}(1 - F + \gamma F) \]

\[ F(p_s, \omega_s) \equiv G(\omega_s)F(p_s) + 1 - G(\omega_s) \]

The entrepreneur starts each period with net worth \( N \). Knowing the wage \((w)\), the entrepreneur chooses the labor choice \((H)\) before knowing the productivity of the production function \((z')\). After the realization of the shock, the entrepreneur allocates its net worth \((N)\) and profit \(((e^{z'} - w)H)\) to consumption \((C)\), bond issued by the banks \((B')\) and the bond issued by the shadow banks \((B'_{s})\).

The bank’s bonds are insured by the government. So, entrepreneurs get back all the promise payment of the bank’s bonds. The shadow bank’s bonds are not insured. \( F \) is the default probability of a shadow bank. In case of default the recovery rate is \( \gamma \in [0, 1] \).

The default probability of a shadow bank is a function of its leverage choice \( p_s \) and liquidity ratio choice \( \omega_s \) of a shadow bank. It is shown in Proposition 3 that there is a threshold \( \omega_s \) that shadow banks default if the liquidity shock \( \omega \) is above this threshold. Moreover, there is threshold \( p_s \) that shadow banks default if credit risk shock \( p \) is above \( p_s \). \((p_s, \omega_s)\) is given for the entrepreneurs when they choose their choice variables. These two are defined as follows:

\[ \omega_s = \frac{A'_s}{B'_s} \]

\[ p_s = \frac{B'_{s} - A'_s}{I_\ell} \]

Default happens under two scenarios; First, the liquidity shock is higher than \( \omega \). This state happens with probability \((1 - G(\omega_s))\). Second is the state in which liquidity shock is below \( \omega_s \) but a low fraction of loans are going to be performing, which means \( p \) is less than \( p_s \). This happens with probability \( G(\omega_s)F(p_s) \).
3.3.4. Household Problem

A household starts a period with net worth $N$. Given the prices $(q_k, q_S, q_s, q_\ell, w)$ and lump sum tax $T$, a household chooses consumption $C$, house purchase $K'$, share of banks $S'$, share of shadow banks $S'_s$, loan $L'$ and labor supply $H$. A household faces with the uncertainty of dividend payment by the banks and shadow banks.

**Problem 4.**

$$V(N) = \max_{\{C, L', H, K', S', S'_s\} \geq 0} \left( C - \psi \frac{H^{1+\frac{1}{\theta}}}{1 + \frac{1}{\theta}} \right) + \beta E_{e,\omega} V(N')$$

$$C + q_S S' + q_s S'_s + q_k K' \leq N + wH - T + q_\ell L'$$

$$L' \leq \kappa q_k K'$$

$$N' = S'(q_S + Div' + S'_s (q_S + Div'_s) + K'(q_k + r_k) - pL'$$

Households face with a borrowing constraint which is a fraction of the house value $\kappa q_k K'$. Household utility is linear in consumption and convex in labor choice. Households pay the lump sum tax $T$ to the government.

3.3.5. Timing

Timing of the problem is as follows;

1. All agents, i.e. banks, shadow banks, entrepreneurs and households, start the period with their networth

2. Entrepreneurs decide how many labors to hire

3. Idiosyncratic productivity shock $z$ is realized. Entrepreneurs pay the wage. They decide how much bank and shadow bank bonds to buy. Moreover they decide how much they want to consume.

4. Banks, shadow banks and households choose their portfolio.
5. **Liquidity shock** ($\omega$) to the banks and shadow banks are realized. Banks and shadow banks receive the return of the safe asset.

6. Those shadow banks whose safe assets’ return is not enough to cover the liquidity shock must default.

7. Those banks whose safe assets’ return is not enough to satisfy the liquidity requirement must borrow from the discount window.

8. **Performing loan shock** ($p$) is realized.

9. Banks and shadow banks decide whether they want to default. Those who continue pays the bond return to the entrepreneurs and dividends to the households.

10. The entrepreneurs receive the returns of the bank’s bonds. For the bankrupt banks, the government pays the full promised payments to the entrepreneurs.

11. The entrepreneurs receive the returns of shadow banks who did not default.

12. The new net worth of all agents are formed.

**Summary of the shocks** In this paper there are three idiosyncratic shocks which are i.i.d across time and agents. These three shocks are performing loan shock, liquidity shock and productivity shock. There is no correlation between shocks as well. There is no aggregate shock. Moreover, it might be expected that liquidity shock and performing loan shock must come from a strategic decisions of entrepreneurs and households respectively. This is not the case in this paper.

3.3.6. **Government**

The only loss in this economy comes from default. It is assumed that there is a partial recovery in this economy. Given the leverage and liquidity choice of a shadow bank ($p_s, \omega_s$), the maximum recovery $R_{s}^{\text{max}}$ from the default of a shadow bank is as follows:

$$R_{s}^{\text{max}} = F_s A_s + I_s \left( (1 - G)Ep + G \int_0^{p_s} pdF(p) \right)$$
\[ \mathcal{F}_s \equiv G(\omega_s)F(p_s) + 1 - G(\omega_s) \]  

(3.3.1)

\[ Ep \equiv \int_0^1 pdF(p) \]

With probability \((1 - G(\omega_s))\), the liquidity shock is higher than the liquidity ratio of the shadow banks. So the shadow bank must default. In this state, the maximum recovery is the payment of the safe asset \(A_s\) and expected payoff of loan \(l_s^tEp\). With probability \(G(\omega_s)\), a shadow bank can survive the liquidity shock. In this state, the shadow defaults if the performing shock is less than the leverage choice of a shadow bank. The maximum recovery in this state is \(\int_0^{p_s} l_s^t pdF(p) + F(p_s)A_s\).

I define the maximum recovery rate \(\bar{\gamma}_s\) in the shadow banking as follows,

\[ \bar{\gamma}_s = \frac{R_{\text{max}}}{\mathcal{F}_s B_s} \]

For later, we need a per capita version of this equation. If I divide both sides by \(I_\ell\), we have:

\[ \gamma_s = \frac{a_s}{b_s} + \frac{1}{b_s\mathcal{F}_s} \left( (1 - G)Ep + G \int_0^{p_s} pdF(p) \right) \]  

(3.3.2)

\[ a_s \equiv \frac{A_s}{I_\ell} \]

\[ b_s \equiv \frac{B_s}{I_\ell} \]

If the recovery rate of the entrepreneur from the default of a shadow bank is \(\gamma < \bar{\gamma}_s\), the total loss from shadow bank default is \((\bar{\gamma}_s - \gamma)B_s\).

Following the shadow bank argument, we can find the maximum recovery rate of banks
as follows. $\gamma_B$ solves the following equation:

$$
\int_{\omega_B} \int_{p_B(\omega)} \{ pI_\ell + A - \gamma_B B \} dG(\omega) dF(p) + G(\omega_B)(\int_0^{p_B} I_\ell p dF(p) + F(p_B)(A_B - \gamma_B B)) = 0
$$

$$
\tilde{\gamma}_B = \frac{a}{b} + \frac{1}{b F_B} \left( G(\omega_B) \int_0^{p_B} p dF(p) + \int_{\omega_B} \int_{p_B(\omega)} p dF(p) dG(\omega) \right) \tag{3.3.3}
$$

$$
F_B \equiv G(\omega_B) F(p_B) + \int_{\omega_B} \int_{p_B(\omega)} dF(p) dG(\omega) \tag{3.3.4}
$$

$F_B$ is the default probability of a bank.

Budget Balance of the government is as follows,

$$
T + r_f \int_{\omega_B} \int_{p_B(\omega)} \{(\rho + (1 - \rho)\omega)B' - A'\} dF(p) dG(\omega) + q_A A' S
$$

$$
= A_S + (1 - \gamma_B)B' F_B \tag{3.3.5}
$$

Here for simplicity, I do not differentiate between government and the federal reserve. Above equation is the combined budget balance of the government and the federal reserve. Left hand side of the above equation is the resources of the government. $T$ is the lump sum tax from the households. second term is the income of the fed from lending through the discount window to the banks. the federal reserve lends with interest rate ($r_f$) to the banks if they are short in liquidity. The third term is the borrowing of the government from issuing safe assets ($A'_S$) to both banks and shadow banks. The right hand side of the above equation is the expenses of the government. The first term is the return of the safe asset. The second term is the payment to the entrepreneur if the banks default. $\gamma_B \leq \tilde{\gamma}_B$ is the recovery rate of the assets of the banks by government.
3.3.7. Market Clearing Conditions

1. Labor Market:
   \[ H_e = H_h \]

2. Housing Market:
   \[ K' = 1 \]

3. Bank Bond Market
   \[ B'_c = B' \]

4. Shadow Bank Bond Market
   \[ B'_{c,s} = B'_s \]

5. Safe Asset
   \[ A'_s = A' + A'_s \]

6. Loan Market
   \[ L' = I' + I'_s \]

7. Goods Market
   \[ C_h + C_e + (\gamma_s - \gamma_e)B'_s + (\gamma_B - \gamma_B)B' = \mu H + r_k \quad , \mu = \mu \epsilon_z + 0.5 \sigma^2 \]

**Definition of Equilibrium.** A competitive equilibrium is a sequence of bank choices \( \{\text{Div}, B', A', I'\} \), shadow bank choices \( \{\text{Div}_s, B'_s, A'_s, I'_{s}\} \), entrepreneur choices \( \{H, C, B', B'_s\} \) and household choices \( \{C, L', H, K', S', S'_s\} \), government policies \( \{A'_s, T, r_f\} \) and prices \( \{q_A, q_B, q_{B_s}, q_{L}, q_{S}, q_{S_s}\} \) such that

1. \( \{\text{Div}, B', A', I'\} \) solves Problem 1
2. \{Div_s, B'_s, A'_s, I'_s\} solves Problem 2

3. \{H, C, B', B'_s\} solves Problem 3

4. \{C, L', H, K', S', S'_s\} solves Problem 4

5. All seven markets are clear

6. Government is budget balanced. Indeed, equation 5 holds.

3.4. Characterization

3.4.1. Characterization of the Bank Problem

Define the per loan variables as follow;

\[ a' = \frac{A'}{I'} \]
\[ b' = \frac{B'}{I'} \]

**Proposition 1.** Bank problem can be simplified into two sub problems,

\[ V(N) = \max_{\{I_l\} \geq 0} N + I_l \Pi \]

\[ \Pi = \max_{b'a'} \left[ q_B b' - q_A a' - q_\ell + \beta \int_{\omega_B} \int_{p_B(\omega)} (p - p_B(\omega)) dF(p) dG(\omega) + \beta G(\omega_B) \int_{p_B} (p - p_B) dF(p) \right] \]

s.t.

\[ q_B b' \leq (1 - \theta) q_\ell + q_A a' \] [CR]

\[ q_A a' \geq \rho q_B b' \] [LR1]

\[ p_B \equiv b' - a' \]

\[ \omega_B \equiv \frac{1}{1 - \rho} \left( \frac{a'}{b'} - \rho \right) \]
\[ p_B(\omega) = \begin{cases} 
  p_B + r_f ((\rho + \omega(1 - \rho))b' - a') & \omega \geq \omega_B \\
  p_B & \omega < \omega_B 
\end{cases} \]

Moreover, as along as \( I_\ell > 0 \), \( \Pi = 0 \) must hold.

**Proof.** It is in the appendix.

Proposition 1 shows that the problem of the bank can be simplified as two sub problems. The first is a linear problem in loan choice of the banks. It is shown in the appendix that the value function is linear in the net worth. Moreover, as long as the bank issue positive loans the profit per unit of loan must be zero. The second sub problem is a profit maximization problem. In this problem, banks choose the optimal bond to loan and safe asset to loan ratios. \( p_B \) can be think of as the net leverage of a bank. If performing loan shock is bellow \( p_B \), the bank defaults. \( \omega_B \) is the liquidity ratio. If the liquidity shock (\( \omega \)) is above \( \omega_B \), the bank must pay \( r_f \) for every unit of borrowing from the federal reserve. In this sub problem, the capital requirement (\( \theta \)) can be think of as a constraint on \( p_B \). The liquidity requirement \( \rho \) can be think of as a constraint on \( \omega_B \).

In case either the capital requirement or liquidity requirement is binding, the characterization of the bank problem is a solution to the followings;

\[ b' \leq \min\left\{ \frac{(1 - \theta)q_\ell + q_Aa'}{q_B}, \frac{q_Aa'}{\rho} \right\} \]

\[ \Pi = 0 \]

### 3.4.2. Characterization of the Shadow Bank Problem

**Proposition 2.** Shadow bank problem can be simplified in two sub problems,

\[ V(N) = \max_{(I_\ell) \geq 0} N + I_\ell \Pi_S \]
\( \Pi_S = \max_{b', a'} q_B b' - q_A a' - q_\ell + \beta G(\omega_s) \int_{p_\ell} (p - p_s) dF(p) \)

\[ p_s \equiv b' - a' \]
\[ \omega_s \equiv \frac{a'}{b'} \]

As long as \( I_\ell > 0 \), \( \Pi_S = 0 \).

**Proof.** It is in the appendix

For shadow banks, \( q_{B_s} \) is going to be a monopoly price for every shadow bank. It means that the choice of \((a_s, b_s)\) of the shadow banks is going to change the interest rate that they are expected to pay. The price function \( q_{B_s} \) is going to come from the problem of the entrepreneurs.

3.4.3. Characterization of the Entrepreneur Problem

**Proposition 3.** The solution to the entrepreneur problem is as follows;

1. Given wage and net worth \((w, N)\), the entrepreneur chooses labor,

\[ H = \phi(w)N \]

in which \( \phi(w) \) solves the following:

\[ \mathbb{E}_z \left\{ \frac{z' - w}{1 + (z' - w)\phi(w)} \right\} = 0 \quad \forall w \geq 0 \]

2. Given the labor choice and productivity shock, the entrepreneur chooses \((c, b', b'_s)\),

\[ c = \eta(1 - \beta) \]
\[
\begin{align*}
\dot{b}' &= \begin{cases} 
(1 - \eta)(1 - \beta) \left( q_B - \frac{q_{b_k}}{1 - \lambda_s + \gamma} \right)^{-1}, & \frac{1 - \eta(1 - \beta)}{(1 - \eta)(1 - \beta) + \hat{\beta}} \geq \left( q_B - \frac{q_{b_k}}{1 - \lambda_s + \gamma} \right)^{-1} \geq 0 \\
(1 - \eta)(1 - \beta) + \hat{\beta} & \text{otherwise}
\end{cases}
\end{align*}
\]

\[
q_{b_s} b'_s = \begin{cases} 
\beta + (1 - \eta)(1 - \beta) \left( 1 - \frac{q_{b_k}}{1 - \lambda_s + \gamma} \right)^{-1}, & \frac{1 - \eta(1 - \beta)}{(1 - \eta)(1 - \beta) + \hat{\beta}} \geq \left( q_B - \frac{q_{b_k}}{1 - \lambda_s + \gamma} \right)^{-1} \geq 0 \\
0 & \text{otherwise}
\end{cases}
\]

in which,

\[
\begin{align*}
C(z') &= c(1 + g(z')) N \\
B'(z') &= b'(1 + g(z')) N \\
B'_s(z') &= b'_s(1 + g(z')) N \\
g(z') &\equiv (e^{z'} - w)\phi(w)
\end{align*}
\]

**Proof.** It is in the appendix.

Proposition 3 shows the labor choice of the entrepreneur is a linear function of its net worth. The net worth of an entrepreneur is composed of the banks and shadow banks’s bonds. So by growing the total size of bonds in this economy, entrepreneurs demand more labor. The coefficient \(\phi\) is just a function of \(w\). It can be shown that it is decreasing in \(w\).

After the realization of the productivity shock, an entrepreneur pays the wage to the house-holds and choose how much to save and consume. As we see in the characterization of the entrepreneur’s problem, the consumption and saving decisions are linear in the asset of the entrepreneurs. She always consumes a constant fraction \((\eta(1 - \beta))\) of its asset. The rest is divided to buy banks and shadow banks bonds. Because of the convenience yield
of the banks’s bonds it is never optimal to buy zero of the bank bonds. If the interest rate on the shadow bank’s bonds is low, the entrepreneurs choose to hold only banks’ bonds.

3.4.4. Characterization of the Household Problem

**Proposition 4.** If \( q_k < \beta E p \), household solution is as follows,

\[
H = \left( \frac{w}{\psi} \right)^\nu
\]

\[
L' = \kappa q_k K'
\]

**Proof.** It is in the appendix \( \square \)

Labor choice of a household is an increasing function in wages. The loan choice of a household is constrained by the house value.

3.5. Steady State Equilibrium

The experiment of interest in this paper is to compare the reaction of all endogenous variables to change in the discount window rate in two economies. One in which the recovery rate of the shadow banking \( \gamma_B \) is high with the one with low \( \gamma_B \). I compare the steady state of these two economies.

In the steady state, the expected net worth growth of the entrepreneur must be zero:

\[
E\{N_e'\} = N_e
\]

\[
\implies E\{B' + (1 - F_s)B'_s + \gamma F_s B'_s\} = N
\]

\[
\implies b + (1 - F_s)b_s + \gamma F_s b_s = \frac{1}{1 + g}, \quad g = (\mu - w)\phi(w)
\]

Given this and the policy function of the entrepreneurs, if entrepreneurs choose non zero holding of the shadow bank’s bonds, we have
\[ q_{B_s} = \beta (1 - F_s + \gamma F_s)(1 + g) \]

1 + g is the MRS of the entrepreneur in the steady state. \( F_s \) is a function of liquidity ratio and leverage choice of the entrepreneurs. This price rule is the one that the shadow banks face. When the size of shadow banks is non-zero, the loan price is coming from the profit condition of the shadow banks. Form the problem of the shadow bank, we know that changes in loan price \( q_{\ell} \) do not change the optimal safe asset and bond choice of the shadow banks \((a_s, b_s)\). Given the price rule \( q_{B_s}(a_s, b_s) \), shadow banks find optimal \((a_s, b_s)\). Zero profit condition of the shadow banks \((\Pi_s = 0)\) pins down the \( q_{\ell} \).

Suppose that the discount window rate increases. The immediate effect is that, at the previous prices, profit of the banks is going to be negative. So loan price must be lower to satisfy the zero profit condition of the banks. In the economy in which the recovery rate is low, and consequently the size of the shadow banking is very small, this effect is large. In contrast in the economy in which shadow banking size is high, shadow banks prevent that the price of loans drop too much. This is because, by decrease in the price shadow banks can increase the leverage and replace the banks by issuing more loans and bonds.

3.6. Numerical Results

The experiment of interest is to compare the reaction to the federal funds rate of an economy with high recovery rate of the shadow banking \( \gamma_B \) with the one with low recovery rate. The results for the numerical example is shown in the figures 4 to 8. This part is just a numerical analysis and not a serious calibration. The parameters of the model are reported in Table 9. It is assumed that the CDF of the performing loan shock has the following functional form,

\[ F(p) = p^{1 + \alpha}, \quad p \in [0, 1] \]

The CDF of productivity shock and liquidity shock is Gaussian with parameters defined in Table 9. Through out of all this section it is assumed that the government keep the price
of the safe bond at one \((q_A = 1)\). So lump sum tax is found such that the government is budget balanced. Before the final results are discussed, it is worth discussing the policy functions.

3.6.1. Policy Functions, Banks

Figure 35 shows the policy functions of the banks when discount window rate changes. To draw this graph, first all the steady state prices have been found for the economy in which the discount window rate is \(r_f = 0.025\) and the recovery rate in shadow banking is \(\gamma_s = 0.95\). Keeping all the prices constant at the given steady state, I found the policy functions of the banks while the discount window rate changes. As the discount window interest rate increases, profit of the banks decrease because banks are supposed to pay more when they face with a high liquidity shock. The safe asset to loan \((a = \frac{A}{ℓ})\) is constant and then increases as the discount window rate increases. The reason that \(a\) is constant up to a threshold is that the liquidity requirement of the bank is binding. So in this region, banks are not going to increase the safe asset ratio although the liquidity cost is increasing. When the discount window rate is high enough, liquidity cost is high, so banks start to hold more safe assets than they are forced to hold by the regulation.

3.6.2. Policy Functions, Shadow Banks

Figure 36 shows the policy functions of the shadow banks for different recovery rate \(\gamma_B\) of its assets. The assets of a shadow banks is seized by the entrepreneurs when a shadow bank defaults. The prices in this figure are the steady state prices when \(\gamma = 0.95\) and the discount window rate \(r_f = 0.025\). Policy functions are the response of the shadow bank choices to the changes in the recovery rate. This shows that as the recovery rate increases the profit of a shadow bank increases. The reason is that as the recovery rate increases the interest rate that the shadow banks pay to the entrepreneurs is going to decline.

Safe asset to loan ratio \((a_s)\) decreases. The reason is that by the increase in the recovery rate, shadow banks can hold less safe assets since the bankruptcy cost is lower. This is what we can see also in the liquidity ratio. This means \(ω = \frac{a_s}{b_s}\) increases which means that the probability of default due to a liquidity shock increases.
Shadow bank bond holding to loan ratio increases. The graph shows that as the recovery rate starts to increase shadow banks net leverage increases as well \((p_s \equiv b_s - a_s)\).

Since the liquidity ratio and leverage increase as the recovery rate increases, the default probability of a shadow bank increases. Because of this higher risk taking, the maximum recovery rate of a shadow bank decreases.

3.6.3. Steady State Comparison

This section is an explanation for figures 4-6. The experiment to find these graphs is as follows. The solid line is the steady state results for the economy with low shadow banking recovery rate \(\gamma_B = 0.1\). The dash-line is the steady state results for the one with high recovery rate \(\gamma_B = 0.95\). The x-axis of both graphs is the discount window rate changing from 0 to 0.1. So the experiment is to assess the reaction of all endogenous variables to changes to the discount window rate in the economy with high and low recovery rate. To visually see the contrast, I compare two economies with large difference in the recovery rate (i.e. 0.1 to 0.95). The results hold if two economies with close recovery rates are compared.

The choice of all parameters are reported in Table 9. The general theme in all graphs is that the changes of all endogenous variables to the changes of the discount window rate is less sensitive in the economy with high recovery rate. In this economy the size of shadow banking is larger. Moreover, in almost all graphs that are increasing in the discount window rate, we see convexity. And in almost all graphs that are decreasing in the discount window rate, we observe a concavity. This happens because, as the discount window rate increases shadow bank starts to grow. So the marginal effect of the increase in discount window decreases. This explains the convexity and concavity which are observed in the figures 4-6.

Figure 37 illustrates the five endogenous prices in this economy. As the discount window rate increases, the wages increases. The changes in wages in the economy with higher recovery rate is lower. Moreover, loan price is not responding to the changes in the discount
window rate. This is what we see also in the raw data reported in Figure 34. When the recovery rate is high, interest rate on shadow bank bonds increases with the federal funds rate, but the banks bond rate interest rate decline. We will see in the following graphs that this incentivizes the entrepreneurs to replace shadow bank bonds with bank bonds. House price is also less responding to the changes in the discount window rate in the economy with high recovery rate.

Figure 38 demonstrates the results for aggregate bond and loan in the economy. Again, it is apparent that the reaction of total loans and total bonds to changes in the discount window rate in the economy with high recovery rate is lower. In the economy with low recovery rate, when discount window rate is very low, loans and bonds issued by the shadow banks are zero. This means that there is no shadow banking in these states. As the discount window rate increases more, shadow banks start to hold loans and issue bonds. Simultaneously, banks decrease loan and bond holdings.

Figure 39 shows the steady state results for banks and shadow banks. As the discount window rate increases, the safe asset to loan ration of banks start to increase after a threshold. Interestingly, shadow banks do not change the leverage and safe asset ratios by changing the discount window. So increase in the discount window rate just shifts the activities to the shadow banking, without increasing the default probability of shadow banks. Note that, in the economy with higher recovery rate, the default probability of the shadow banks are higher.

3.7. Conclusion
This paper proposes a novel view on monetary policy. The unintended consequences of a given monetary policy move can be quite large; this makes the initial goal of the monetary policy hardly feasible. This paper argues when the shadow banking is large in an economy, a policy move simply shifts the activities from banks to shadow banks or vice versa without having significant effects on GDP, loan issuance, and loan interest rates.

This paper closely aligns with the economy of the United States in the years immediately
before the great recession. As is illustrated in Figure 34, despite the sharp increase in the federal funds rate, mortgage interest rate remained almost unchanged. DSGE models, focusing on the transmission of a monetary policy, cannot capture this fact. Greenwald (2018) specifically highlights the mortgage credit channel of a monetary policy. This paper fails to capture the relation between mortgage interest rate and the federal funds rate before the financial crisis.

To conclude, this paper proposes the importance of considering the role of shadow banking activities, in addition to the regulated banking activities. A monetary policy can be quite ineffective in the real economy if the shadow banks have a large share in the loan and bond markets.
3.8. Appendix

Proof of Proposition 1. \(\text{Div}\) can be negative. Replacing for \(\text{Div}\) from the budget constraint, we have:

\[
V(N) = \max_{\{B', A', I\} \geq 0, \text{Div}} N + q_B B' - q_\ell I_\ell - q_A A' + \beta E_{p, \omega} \max\{V(N'(p, \omega)), 0\}
\]

Conditional on survival of the bank, we can guess that

\[
V(N(p, \omega)) = N(p, \omega) \quad \forall p \in [0, 1], \quad \forall \omega \in [-1, 1]
\]

If the guess is true,

\[
V(N) = \max_{\{B', A', I\} \geq 0, \text{Div}} N + q_B B' - q_\ell I_\ell - q_A A' + \beta E_{p, \omega} \max\{N'(p, \omega), 0\}
\]

\[
= \max_{\{B', A', I\} \geq 0, \text{Div}} N + q_B B' - q_\ell I_\ell - q_A A' + \beta E_{p, \omega} \max\{p I_\ell + A' - B' - r_f x(\omega, A', B') \mathbb{1}(x(\omega, A', B') \geq 0), 0\}
\]

\[
= \max_{\{I, B', A'\} \geq 0} N + I \Pi
\]

\[
\Pi = \max_{B', a'} [q_B b' - q_A a' - q_\ell + \beta \int_{\omega_B} \int_{p_B(\omega)} (p - p_B(\omega)) dF(p) dG(\omega) + \beta G(\omega_B) \int_{p_B} (p - p_B) dF(p)]
\]

\[q_B b' \leq (1 - \theta)q_\ell + q_A a' \quad [CR]\]

\[q_A a' \geq \rho q_B b' \quad [LR_1]\]

\[p_B \equiv b' - a']

134
\[ \omega_B \equiv \frac{1}{1 - \rho} \left( \frac{d'}{b'} - \rho \right) \]

\[ p_B(\omega) = \begin{cases} 
  b' - a' + r_f ((\rho + \omega(1 - \rho))b' - a') & \omega \geq \omega_B \\
  b' - a' & \omega < \omega_B 
\end{cases} \]

Since the return is linear in \( I_e \), we must have,

\[ \Pi_B = 0 \]

\( \square \)

Proof of Proposition 3.

\[ V(N) = \max_{\{H\} \geq 0} E_{z'} \left( \max_{C, B', B'_s} \eta \log(C) + (1 - \eta) \log(B') + \beta V(N'(z')) \right) \]

\[ C + q_B B' + q_B B'_s \leq N + (z' - w) H \quad [\lambda(z')] \]

\[ N'(z') = B' + B'_s (1 - F(p_s, \omega_s) + \gamma \mathcal{F}(p_s, \omega_s)) \]

\[ \mathcal{F}(p_s, \omega_s) \equiv G(\omega_s)(1 - F(p_s)) + (1 - G(\omega_s)) \]

FOC,

\[ [C(z')] : \quad \frac{\eta}{C(z')} = \lambda(z') \]

\[ [B'(z')] : \quad (1 - \eta) \frac{1}{B'(z')} + \beta V'(N'(z')) = \lambda(z') q_B \]

\[ [B'_s(z')] : \quad \beta V'(N'(z'))(1 - F(p_s, \omega_s) + \gamma \mathcal{F}(p_s, \omega_s)) \leq \lambda(z') q_{B_s} \]

\[ [H] : \quad E_{z'} \{(z' - w) \lambda(z')\} = 0 \]

135
\[ V'(N) = E_{z'} \{ \lambda(z') \} \]

Guess that,

\[ H = \phi N \]

\[ C(z') = c(1 + g(z'))N \]
\[ B'(z') = b'(1 + g(z'))N \]
\[ B'_s(z') = b'_s(1 + g(z'))N \]
\[ g(z') \equiv (e^{z'} - w)\phi(w) \]

Plug in the FOC of \([H]\):

\[ E_{z'} \{ (z' - w)\lambda(z') \} = 0 \]
\[ E_{z'} \{ (z' - w) \frac{C(z)}{C(z')} \} = 0 \]
\[ E_{z'} \{ \frac{z' - w}{1 + (z' - w)\phi} \} = 0 \]

Above equation characterizes \( \phi(w) \).

From FOC of \([B'(z')]\):

\[
\frac{(1 - \eta)}{\eta} \frac{c}{b'} + \beta E_{z'} \frac{C(z)}{C(z')} = q_B
\]

\[
E_{z'} \frac{C(z)}{C(z')} = E_{z'} \frac{c(1 + (z' - w)\phi)N}{c(1 + (z' - w)\phi)(b' + b'_s(1 - F(p, w_s) + \gamma F(p, w_s)))}
\]

\[
= E_{z'} \frac{c(1 + (z' - w)\phi)N}{c(1 + (z' - w)\phi)(b' + b'_s(1 - F + \gamma F)(1 + (z - w)\phi)N)}
\]

\[
= \frac{1}{b' + b'_s(1 - F + \gamma F)} E_{z'} \{ \frac{1}{1 + (z' - w)\phi} \}
\]
Since \( E_{\zeta'} \{ \frac{1}{1+(z-w)\phi} \} = 1 \),

\[
E_{\zeta'} \frac{C(\tilde{z})}{C(z')} = \frac{1}{b' + b'_s(1 - F + \gamma F)}
\]

\( q_B b' = \frac{(1 - \eta)}{\eta} c + \frac{\beta b'}{b' + b'_s(1 - F + \gamma F)} \)

From FOC of \( B'_s \),

\[
\frac{\beta(1 - F + \gamma F)}{b' + b'_s(1 - F + \gamma F)} \leq q_B
\]

The budget constraints is

\[
c + q_B b' + q_B b'_s = 1
\]

Either \( B'_s \) is strictly positive (FOC of \( B'_s \) holds with equality) or \( B'_s \) is zero, we can show that,

\[
q_B b' + q_B b'_s = \frac{1 - \eta}{\eta} c + \beta
\]

Combining with budget constraint, we have:

\[
c = \eta(1 - \beta)
\]

- If \( b_s > 0 \), From FOC of \( [B'] \),

\[
q_B b' = (1 - \eta)(1 - \beta) + b' \frac{q_B}{1 - F + \gamma F} \implies
\]

\[
b' = (1 - \eta)(1 - \beta) \left( q_B - \frac{q_B}{1 - F + \gamma F} \right)^{-1}
\]

\[
q_B b'_s = \beta + (1 - \eta)(1 - \beta) \left( 1 - (q_B - \frac{q_B}{1 - F + \gamma F})^{-1} \right)
\]

- If \( b_s = 0 \),

\[
q_B b' = 1 - \eta(1 - \beta)
\]
Proof of Proposition 4.

\[ V'(N') = 1 \]

\[ [H] : \quad w - \psi H^{1/2} = 0 \implies H = (\frac{w}{\psi})^v \]

\[ [K'] : \quad -q_k + \beta V'(N') (q_k + r_k) + \lambda k q_k = 0 \]

\[ [L'] : \quad q_\ell - \lambda - \beta E p = 0 \]

If \( q_\ell > \beta E p \), we have the following,

\[ \lambda = q_\ell - \beta E p \]

Plug \( \lambda \) in \([K']\),

\[ q_k (1 - \beta - \kappa (q_\ell - \beta E p)) = \beta r_k \]

\[ q_k = \frac{\beta r_k}{1 - \beta - \kappa (q_\ell - \beta E p)} \]

\[ L' = \frac{\kappa \beta r_k}{1 - \beta - \kappa (q_\ell - \beta E p)} \]

Algorithm to Characterize the Steady State.

1. Guess \( w \)

2. Given \( w \), find \( \phi(\omega) \) and \( g \)

3. Find \( q_{B_s} \) and \( (\omega_s, p_s) \)

\[ q_{B_s} = \beta (1 - F + \gamma F) (1 + g) \]

4. imposing \( \Pi_s = 0 \), find \( q_\ell \)
5. Find $q_B$ such that $\Pi_B = 0$

6. Check if the condition for $b_s > 0$ is satisfied.

7. If $b_s > 0$, check the the excess demand for loan

$$\frac{1 - \eta(1 - \beta)}{(1 - \eta)(1 - \beta)} \geq \left( q_B - \frac{q_B}{1 - F' + \gamma F} \right)^{-1} \geq 0$$

$$q_B - \beta(1 + g) \geq \frac{(1 - \beta)(1 - \eta)}{1 - \eta(1 - \beta)}$$

$$q_B - \beta g \geq \beta + \frac{(1 - \beta)(1 - \eta)}{1 - \eta(1 - \beta)}$$

8. If $b_s = 0$.

- Find $q_B$

  $$b = \frac{1}{1 + g} \quad q_B = (\beta + (1 - \eta)(1 - \beta))(1 + g)$$

- Find $q_\ell$ from $\Pi_B = 0$

- Check excess demand for loan
3.9. Tables and Figures

Table 9: Calibration

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount Rate</td>
<td>$\bar{\beta}$</td>
<td>0.97</td>
</tr>
<tr>
<td>Productivity, Mean</td>
<td>$\mu_z$</td>
<td>0</td>
</tr>
<tr>
<td>Productivity, Variance</td>
<td>$\sigma_z$</td>
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</tr>
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<td>Leisure, HH utility</td>
<td>$\nu$</td>
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<tr>
<td>Leisure, HH utility</td>
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<td>Performing Loan Shock, Dist</td>
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<tr>
<td>Loan to Value Ratio</td>
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</tr>
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<td>Liquidity Requirement</td>
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<tr>
<td>Capital Requirement</td>
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<td>Liquidity Shock, Variance</td>
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</tr>
<tr>
<td>Housing Return</td>
<td>$r_k$</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Note: These are the parameters of the numerical analysis. The parameters related to households and entrepreneurs come from Quadrini (2015). The parameters related to the distribution of the liquidity come from Bianchi and Bigio (2014).

Figure 34: Average Mortgage Interest Rate and Federal Funds Rate

Note: The flat line is the average mortgage interest rate. The other one is the federal funds rate.
Figure 35: Bank Policy Function

Note: To get this graph, the steady state with parameters given in the Table 1 is found. In addition, it is assumed that the government keep the price of the safe asset at one ($q_A = 1$). Moreover the recovery rate of the shadow banks is $\gamma_B = 0.95$, banks $\gamma = 0$ and the discount window rate is $r_f = 0.025$. The above graph comes from changing the discount window rate while keeping all the prices constant at the mentioned steady state.
Figure 36: Shadow Bank Policy Function

Note: To get this graph, the steady state with parameters given in the Table 1 is found. In addition, it is assumed that the government keep the price of the safe asset at one ($q_A = 1$). Moreover the recovery rate of the shadow banks is $\gamma_B = 0.95$, banks $\gamma = 0$ and the discount window rate is $r_f = 0.025$. The above graph comes from changing the discount window rate while keeping all the prices constant at the mentioned steady state.
Figure 37: Steady State, Prices

Note: This figure shows all five endogenous prices in the model. Note that, safe asset price is fixed at one in all the cases ($q_A = 1$). x-axis of all subgraphs is the discount window rate ranging from 0 to 0.1. $\gamma_B$ is the recovery rate of the shadow banking bonds in case of default. $\gamma_B^{\text{high}} = 0.9$ and $\gamma_B^{\text{low}} = 0.1$
Figure 38: Steady State, Bond and Loans

Note: Left column shows the aggregate bond issued by banks, shadow banks and total bond holdings. The right column shows the loan issued by banks, shadow banks and total loan in the economy. x-axis of all subgraphs is the discount window rate ranging from 0 to 0.1. $\gamma_B$ is the recovery rate of the shadow banking bonds in case of default. $\gamma_B^{high} = 0.9$ and $\gamma_B^{low} = 0.1$
Figure 39: Steady State, Bank and Shadow Bank Choices

Note: the top subplots are the consumption by households and entrepreneurs. The below subplots are the total dividend payments of the banks and shadow banks to the households. The dividend is already included in the consumption of the households. x-axis of all subgraphs is the discount window rate ranging from 0 to 0.1. $\gamma_B$ is the recovery rate of the shadow banking bonds in case of default. $\gamma_B^{high} = 0.9$ and $\gamma_B^{low} = 0.1$
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