Non-Hermitian Topological Photonics: From Concepts To Applications

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Non-Hermitian Topological Photonics: From Concepts To Applications

Abstract
Recent emergence of photonic topological insulators paves a route to disorder-immune light confinement and propagation for potential applications in information processing, communication, and computing. In parallel, non-Hermitian photonics based on parity-time symmetry expands the design principles in optics to the entire complex domain of materials permittivity, providing a versatile toolbox to enable novel photonic functionality. Despite being fundamentally different, photonic topological structures integrated with optical non-Hermiticity exhibit unusual features that leverage robust light control with extraordinary degrees of freedom. This dissertation explores the synergy of topological photonics and non-Hermitian physics from the demonstrations of phenomena to the prototype of devices. We start with a complex-indexed variant of the classical Su-Schrieffer-Heeger model respecting the charge-conjugation symmetry, where non-Hermitian modulation of gain and loss enforces robust single-mode lasing with the topological zero mode selectively enhanced in a hybrid microlaser array. Beyond the selection of the topological mode, we show the creation of a topological state in the bulk of a topologically uniform photonic lattice via strategic patterning of optical non-Hermiticity, even in the absence of a topological interface. Such novel non-Hermitian control enables arbitrary topological light steering in reconfigurable non-Hermitian junctions, where chiral topological states can propagate at an interface of the gain and loss domains dynamically configured by pumping patterns. Our strategy has solved the long-standing problem of redefining the topological domain wall without altering the topological order of the structure, which would be otherwise static. The ultra-flexible and robust nature of the non-Hermitian topological light control opens the avenue to highly integrated multifunctional photonic circuitry for high-density data processing. Additionally, we exploit the highly asymmetric light transport feature associated with the unique topology in the vicinity of the non-Hermitian degeneracy, namely an exceptional point, facilitating sensitive thermal imaging and power-efficient interferometric optical modulation on-chip.

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NON-HERMITIAN TOPOLOGICAL PHOTONICS: FROM CONCEPTS TO APPLICATIONS

Han Zhao

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in

Electrical Systems and Engineering

Presented to the Faculties of the University of Pennsylvania

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ABSTRACT

NON-HERMITIAN TOPOLOGICAL PHOTONICS: FROM CONCEPTS TO APPLICATIONS

Han Zhao
Liang Feng

Recent emergence of photonic topological insulators paves a route to disorder-immune light confinement and propagation for potential applications in information processing, communication, and computing. In parallel, non-Hermitian photonics based on parity-time symmetry expands the design principles in optics to the entire complex domain of materials permittivity, providing a versatile toolbox to enable novel photonic functionality. Despite being fundamentally different, photonic topological structures integrated with optical non-Hermiticity exhibit unusual features that leverage robust light control with extraordinary degrees of freedom. This dissertation explores the synergy of topological photonics and non-Hermitian physics from the demonstrations of phenomena to the prototype of devices. We start with a complex-indexed variant of the classical Su-Schrieffer-Heeger model respecting the charge-conjugation symmetry, where non-Hermitian modulation of gain and loss enforces robust single-mode lasing with the topological zero mode selectively enhanced in a hybrid microlaser array. Beyond the selection of the topological mode, we show the creation of a topological state in the bulk of a topologically uniform photonic lattice via strategic patterning of optical non-Hermiticity, even in the absence of a topological interface. Such novel non-Hermitian control enables arbitrary topological light steering in reconfigurable non-Hermitian junctions, where chiral topological states can propagate at an interface of the gain and loss domains dynamically configured by pumping patterns. Our strategy has solved the long-standing problem of redefining the topological domain wall without altering the topological order of the structure, which would be otherwise static. The ultra-flexible and robust nature of the non-Hermitian topological light control opens the avenue to highly integrated multifunctional photonic circuitry for high-density data processing. Additionally, we exploit the highly asymmetric light
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CHAPTER 1 INTRODUCTION

The ever-expanding information explosion draws the interest in the recent transition from electronic to photonic devices for the higher power efficiency and larger data capacity. While nanophotonics is finding its pivotal role in myriad applications such as information processing, communication, and computing, the next generation of photonic engineering strives for more robust light control as well as tactical synergy of active and passive components on the same material platform [1,2]. To tackle these challenges, a considerable effort in optics and photonics research is devoted to two branches inspired by quantum physics: topological photonics where unique structural topology allows edge/interface light confinement and propagation immune to defects and disorder [3,4]; and non-Hermitian photonics with exotic effects based on a range of quantum symmetry paradigms exemplified by parity-time symmetry [5,6].

The fundamental basis behind both branches arises from the mathematical equivalence between the Schrödinger equation in quantum mechanics and the wave equation in optics [5-8]. To establish this equivalence, let us consider the basic single-particle Schrödinger equation:

\[
\frac{i\hbar}{\partial t} \psi = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x)\psi
\]  

(1-1)

where \( \psi \) is the wave function, \( \hbar \) is the reduced Planck constant, and \( m \) is the single-particle mass. The Hamiltonian \( H = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) = \hat{p}^2/2 + V(x) \) is composed of the kinetic energy operator and the static potential \( V(x) \). Meanwhile, the propagation of slowly varying envelop of electric field with negligible transverse diffraction angles obeys

\[
\frac{i}{\partial z} \frac{\partial E}{\partial z} = -\frac{1}{2k_0} \frac{\partial^2 E}{\partial x^2} + k_n(x)E,
\]  

(1-2)
where $E$ denotes the transverse electric field profile, $k_0$ is the wave number in free space, and $n(x) = n_R(x) + in_I(x)$ is the complex refractive index. It is worth noting that this paraxial equation correctly describes a broad range of electromagnetic phenomena including waveguiding and interference, while being an approximation of the full-vector Maxwell equations. The link can be built by substituting the time evolution with the $z$-dimension wave dynamics and the potential function with the spatial refractive index distribution. Therefore, the effective Hamiltonian of the paraxial wave equation can be expressed as

$$H = -\frac{1}{2k_0} \frac{\partial^2}{\partial x^2} + k_0 n(x)$$ whose eigenenergy represents the effective propagation constant along the $z$-axis.

1.1 Topological photonics

The notion of topology originates from the Gauss-Bonnet theorem which states the integral of the geometric curvature over a manifold is a quantized quantity:

$$\int_M \kappa dA = 2\pi (2 - 2g), \quad (1-3)$$

where $\kappa$ is the curvature, $g$ is named genus and generally takes integer values that counts the homeomorphic feature of the geometry. The concept was brought to physics with the milestone discovery of quantum Hall effect (QHE), where two-dimensional electron gas under out-of-plane static magnetic field exhibits robust edge conduction regardless of material impurity [9-11]. The topological nature of QHE can be characterized by the geometric Aharonov–Bohm phase in the cyclic motion of electrons in presence of quantized magnetic flux, which induces the in-plane Hall conductance

$$\sigma_{xy} = e^2 h \frac{1}{2\pi} \int \int_{BZ} \nabla_k \times \langle u(k) | \nabla_k | u(k) \rangle dk_x dk_y. \quad (1-4)$$
The gauge-invariant surface integral of the wave function in the momentum space closely relates to the genus integer that represents the topology of geometry, which is known as Thouless-Kohmoto-Nightingale-den Nijs (TKNN) number (apart from a factor of $2\pi$) [10]. The further development of topological physics revolutionizes the classification of materials according to their underlying symmetries, following the pioneering work of $\mathbb{Z}_2$ topological order in quantum spin Hall insulators under time-reversal symmetry [12-16].

The universality of topology also prevails in optics and photonics. The simplest possible photonic structures that possesses topological property are one-dimensional (1D) coupled waveguide or resonator arrays emulating the Su-Schrieffer-Heeger (SSH) dimer chain model [17, 18]. For an infinite and uniform array (Fig. 1-1a), the optical system has the effective Hamiltonian in the momentum space as

$$H = \begin{pmatrix} 0 & t_1 + t_2 e^{-2iq} \\ t_1 + t_2 e^{2iq} & 0 \end{pmatrix}, \quad (1-5)$$

where $t_1$ and $t_2$ are the couplings between the waveguides/resonators within and to the next dimer, and $q$ is the normalized momentum. The Hamiltonian has a dispersion of two symmetric bands $\omega = \pm \sqrt{t_1^2 + t_2^2 + 2t_1 t_2 \cos k}$ that are separated by a gap of size $\Delta = 2 |t' - t''|$. The symmetry of the spectrum arises from the operation $A_n, B_n \rightarrow A_n, -B_n$, whose effect is equivalent to inverting all couplings and therefore changes the sign of the effective Hamiltonian. In other words, chiral symmetry implies the existence of an operator $\chi$ such that $\chi H \chi^{-1} = -H$, thus forcing the spectrum to be symmetric around the zero line. In our case, such an operator $\chi$ corresponds to a unitary transformation

$$\sigma_z H \sigma_z = -H, \quad (1-6)$$
where $\sigma_z$ is a Pauli matrix in the space of A and B sites, and constitutes a chiral symmetry. This spectral constraint indicates the Bloch states uniformly distribute through the lattice with vanishing intra-dimer polarization. Therefore, $\frac{A}{B} = \frac{t' + t'' \exp(ik)}{\omega} = \exp(i\varphi)$ defines a complex phase $\varphi$ which depends on $k$. Changing of the phase through the Brillouin zone results in two scenarios: for $t_1 < t_2$ this phase increases by $2\pi$ (winding number 1), while for $t_1 > t_2$ the phase returns to 0 (winding number 0). The transition at $t_1 = t_2$ between both cases coincides with the closing of the gap. In a lattice with an interface across which the winding number transits from 1 to 0, there exists a bound state at the interface (Fig. 1-1b). This follows from the behaviour of the complex reflection coefficient $r = \frac{A + iB}{B + iA}$ at energies within the gap, which changes the sign across the interface. Spectral symmetry dictates that this solution sits at the middle of the band gap (zero energy) (Fig. 1-1c) [19]. Furthermore, perturbations of the couplings do not affect the chiral symmetry and do not change the winding numbers, given that the start and end points are protected by symmetry.

Fig. 1-1 Su-Schrieffer-Heeger model. (a) Schematic of an infinite and uniform dimer chain with two sublattices coupled by alternating intra-dimer coupling ($t_1$) and inter-dimer coupling ($t_2$). The black dashed lines circle a dimer unit composed of sublattices A and B. (b) Topological zero
mode locating at the transition interface across which the inter-dimer and intra-dimer interchange. The red bars represent the field amplitude distribution of the zero mode. (c) Band structure of SSH chain with a topological defect. Reproduced from [18].

Topology in higher dimensions draws more interest for the defect-immune propagation states bound to the interface of two topologically distinct photonic structures, which provides an energy-efficient unidirectional channel for electromagnetic transport [20-24]. These channels are topologically protected such that any defect and disorder cannot induce backscattering or transmission loss. While the first quantum Hall electromagnetic transport was demonstrated via gyromagnetic materials in microwave frequency [20], the lack of substantial magnetic response in the optical regime forces researchers to find alternatives to create synthetic gauge field for photons. For example, one implementation using coupled ring resonators engineers opposite geometric hopping phases in a unit cell with the two degenerate circulating whispering gallery modes in each rings (Fig. 1-2) [22]. The unit cell of such structure consists square resonator arrays indirectly coupled by the anti-resonating auxiliary resonators. For the clockwise circulating power flow, the tight-binding coupled mode equations of the unit cell reads

\[ \begin{align*}
\omega a_1 &= Ja_2 + Ja_4, \\
\omega a_2 &= Ja_1 + Ja_3, \\
\omega a_3 &= Ja_2 + J \cdot e^{i\phi} a_4, \\
\omega a_4 &= Ja_1 + J \cdot e^{-i\phi} a_3,
\end{align*} \tag{1-7} \]

where \( J \) is the effective coupling amplitude between two site resonators, \( \omega \) is the eigen frequency normalized to the self-resonance of the site resonator, \((a_1, a_2, a_3, a_4)^T\) represents the field amplitudes in the four site resonators respectively, and the hopping phase \( \phi \) results from the different optical path in the auxiliary resonator.
as light hops between site 3 and site 4. For counter-clockwise circulating power flow, similar coupled mode equations apply with reversed hopping phase. The hopping phase constructs a synthetic gauge field for each of the circulation in an intimate analogy with QHE for electrons, which gives rise to two pseudo-spin dependent unidirectional edge states in the bulk band gap propagating along the perimeter of the structure.

![Fig. 1-2 Schematic of realizing one-way topological light transport in coupled ring resonators. Reproduced from [22].](image)

The ample approaches to engineer photonic band structures lead to a bloom of photonic analogies of topological insulator in many other platforms such as laser-written waveguide arrays, photonic crystals and metamaterials. More recently, the topological edge states are demonstrated to assist the generation of quantum light sources [25,26].

1.2 Non-Hermitian photonics

In parallel to the progress in topological photonics, another popular branch of research in optics is the non-Hermitian photonics. It is inspired by the seminal paper by Bender and Boettcher that states non-Hermitian systems with open boundary (thus complex potential) can possess completely real-valued eigen spectrum given that the Hamiltonian commutes with the combined parity and time reversal operators, i.e.
Such non-Hermitian systems are thus referred to as parity-time (PT) symmetric [28]. Here, the parity operator reverses the coordinate position by mirror reflection \((\hat{x} \rightarrow -\hat{x}, \hat{p} \rightarrow -\hat{p})\), while the time reversal operator flips the direction of time evolution and effectively acts as the complex conjugation \((i \rightarrow -i, \hat{p} \rightarrow -\hat{p})\). The commutation relation necessarily requires the potential be an even function for the real part and an odd function for the imaginary part. However, it is not sufficient to ensure PT symmetry as such systems can undergo spontaneous symmetry breaking crossing certain threshold value, namely exceptional point (EP) [29].

The first experimental validation of non-Hermitian quantum theory, however, was in optical settings where the non-Hermiticity can be conveniently tailored by control of gain and loss [30-32]. Inherited from the mathematical equivalence of Eq (1-1) and Eq (1-2), the refractive index of a PT symmetric optical system must sustain an even function for its real part and an odd function for its imaginary part. An example of PT symmetric Hamiltonian is a pair of coupled single-mode waveguides associated with equal gain and loss, respectively. The Hamiltonian of such a two-level system (i.e. two propagation constants) can be derived from the standard coupled mode equations:

\[
\begin{pmatrix}
\beta_0 + i\gamma & \kappa \\
\kappa^* & \beta_0 - i\gamma
\end{pmatrix}
\]

where \(\beta_0\) is the propagation constant in each individual waveguide without embedded gain or loss, \(\gamma\) takes account of the gain/loss magnitude, and \(\kappa\) denotes the reciprocal coupling between the waveguide pair. The propagation constants of the two supermodes are given by

\[
\beta_\pm = \beta_0 \pm \sqrt{g^2 - \gamma^2}.
\]

Depending on the relative relation of \(\gamma\) and \(g\), the supermodes fall into one of two contrasting phases: when \(\gamma < g\), the field amplitude
occupies the gain waveguide and the loss one equivalently, resulting in conserved light intensity and real-valued propagation constants corresponding to the unbroken PT symmetry; when $\gamma > g$, the supermodes are biased to one of the two waveguides, respectively, resulting in either light amplification or attenuation associated with a pair of conjugate propagation constants corresponding to the PT symmetry breaking (Fig. 1-3). Moreover, exactly at the transition threshold where $\gamma = g$, two supermodes coalesce to the single eigenstate $-(1,i)^T / \sqrt{2}$ with a propagation constant $\beta_0$, featuring the non-Hermitian degeneracy, i.e. an EP.

![PT symmetry and breaking](image)

Fig. 1-3 Coupled single-mode waveguides with balanced gain and loss. The supermode undergoes transition from unbroken PT symmetric phase with balanced field in the two waveguide, to broken PT phase where the supermode biased to either of the waveguide, respectively. Reproduced from [5].

Although the stringent balance of gain and loss is highly challenging from an experimental perspective due to the limited gain spectral bandwidth and inevitable fabrication errors, the majority of PT symmetric features can be observed even in
presence of a loss offset. In fact, the coupled waveguide pair with imbalanced losses can be understood as a perfect PT system superposed by a trivial background damping. Hereafter, we will refer to an optical system as PT symmetric while ignoring the imbalance of gain and loss.

Ever since the validation of such analogue from quantum mechanics to photonics, the concept of non-Hermiticity has been innovating the design paradigms of light propagation and confinement by expanding the engineering of optical materials into the whole complex permittivity domain. Notably, the emergence of non-Hermitian photonics offers revolutionary understanding on optical gain and loss: the strategic integration of detrimental absorption on optically active platform can leverage new functionalities [33-40].

1.3 The need for the synergy

The discovery of topological band theory has ushered in a new era in condensed matter physics [41]. Inspired by this groundbreaking work, topological mechanisms of optical mode formation have been proposed. However, most pioneering studies have been limited in scope, exploring only a small subset of the full design parameter space. Active optical systems involving both gain and loss provide a much wider arena. Recently, considerable effort has been made to transplant the topological notions into lasing systems [42-47], in which topological robustness collides with other physical considerations, posing diverse unexplored fundamental questions about the interplay between topological features, non-Hermitian physics and the break-down of superposition principle. The answers to these questions transform our understanding of topological robustness by revealing unique connections between topology and other types of fundamental symmetries arising from non-Hermiticity (naturally pertinent to
active systems), thus opening the door for improving robust optical device functionality, a key incentive in the research of integrated photonics over the past few decades. This new paradigm dictates a fresh look at the basic notion of topological protection in order to take into account the expanded design parameters space, and establish a connection between topological physics and various separate activities on non-Hermitian photonic systems [48-52].

From a practical perspective, while topological photonics gifts defect-free light control, redefining topological light pathways requires considerable perturbations to drive the topological phase transition inside the bulk structures that are difficult to access in the optical regime [53-57]. Such a severe limitation prevents topological photonics from being practically applied, since the topological mode only exists at the static structural boundary/interface so that most of the footprint of the photonic structure is unutilized. Optical non-Hermiticity, on the other hand, opens up another dimension of parameter space, promising possibility to overcome these drawbacks. Therefore, the synergy of non-Hermitian and topological photonics is of prominent importance in both fundamental science and innovative technology.

The dissertation is arranged as the following: in Chapter 2, we explore the complex extension of SSH model, and show that manipulation of gain and loss is capable of not only selecting the topological zero mode but also defining the position of the mode in the spatial domain. In Chapter 3, we demonstrate dynamically reconfigurable topological light propagation within the footprint of an active photonic topological insulator by only non-Hermitian control. In Chapter 4, we exploit the unique topology around exceptional point to leverage thermal mapping with enhanced sensitivity and asymmetric interferometric modulator. Finally, in Chapter 5, we conclude and present an outlook of the future work.
CHAPTER 2 TOPOLOGICAL MODES IN NON-HERMITIAN ONE-DIMENSIONAL PHOTONIC LATTICES

This chapter is adapted from the following publications:


The author of this dissertation was the primary researcher and co-author of the work.

Localized bound states with energy in the gap are commonplace in periodic systems with defects or disorder. However, bound states induced by defects are generally vulnerable to ambient randomness, impeding reliable manipulation of optical coherence. Photonic zero mode with its eigenenergy pinned at the middle of a gapped band structure is robust against local or global perturbations owing to the underlying topological protection, and is therefore intensively sought after for many device applications. Nevertheless, such static dimensionless optical mode in the Hermitian limit suffers the hybridization with the trivial collective modes due to the spatial overlap. In this chapter, we explore a complex-indexed variant of the SSH model, where non-Hermitian modulation of gain and loss enforces robust single-mode lasing with the topological zero mode selectively enhanced in a hybrid microlaser array. More than simply stabilize a Hermitian topological mode, we further show the manipulation of optical non-
Hemiticity can create a new type of topological zero mode which would disappear in the Hermitian limit.

2.1 Complex Su-Schrieffer-Heeger model

We start the exploration of non-Hermitian topology in 1D photonic lattice based on a non-Hermitian extension of the Su-Schrieffer-Heeger (SSH) model, where a tight-binding chain of coupled dimers is modulated with alternating onsite gain or loss to realize a PT symmetric lattice [58-61]. The Hamiltonian of the PT-symmetric SSH model can be formalized in the momentum space as

\[
H(k) = -i\gamma_0 I + (t_1 + t_2 \cos(ka))\sigma_x + (t_2 \sin(ka))\sigma_y + i\gamma\sigma_z, \quad (2-1)
\]

where \(k\) is the wavenumber in the momentum space, \(a\) is the lattice constant, \(t_1\) and \(t_2\) indicate the intra-dimer and inter-dimer couplings in the SSH model, \(-i\gamma_0 I\) is the background loss term in which \(I\) is a two dimensional identity matrix, \(2\gamma = \gamma_A - \gamma_B\) indicates the gain/loss contrast within one dimer (the losses of sites A and B in each dimer are represented by \(\gamma_A\) and \(\gamma_B\), respectively), and \(\sigma_{x,y,z}\) are the Pauli matrices. 

The energy dispersion can be calculated with these parameters, which reads:

\[
\epsilon = \epsilon_{\pm} + i\gamma_0 = \pm\sqrt{t_1^2 + t_2^2 + 2t_1t_2\cos(ka) - \gamma^2}, \quad (2-2)
\]

and sensitively depends on the value of \(\gamma\). Depending on the value of loss contrast, the lattice can be found in three distinct phases: unbroken PT phase for \(\gamma < |t_1 - t_2|\) (phase I), partially-broken PT phase (phase II), and fully broken PT phase for \(\gamma > |t_1 + t_2|\) (phase III).

From a topological perspective, winding numbers are often analyzed to determine the topological invariants in Hermitian systems. In a non-Hermitian system,
such parameters cannot be well defined. However, the topological nature can be equivalently probed by the global Berry phase, which, in our case, corresponds to the summation of complex Berry phase in both lower and upper bands [61]. The Berry phase in each band can be calculated:

\[ \varphi^\pm_B = \oint_k A dk \], where \( A = i \langle u_\pm | d / dk | \lambda_\pm \rangle \) is the Berry connection, and \( \langle u_\pm | \) and \( \| \lambda_\pm \rangle \) are the normalized left and right eigenvectors of the Hamiltonian matrix \( H(k) \), respectively, leading to:

\[ \varphi^\pm_B = \frac{\varphi_0}{2} \pm \frac{1}{2} \gamma_k \cos \gamma_k d\phi_k \]  

(2-3)

where, \( \gamma_k = \arctan(\rho_k / i\gamma) \), \( \rho_k = |t_A + t_B \exp(-iak)| \), and \( \phi_k = \arg(t_A + t_B \exp(-iak)) \). In Eq. (2-3), one can see both the intrinsic Berry phase in the Hermitian limit (i.e. \( \varphi_0 / 2 \)) and the non-Hermitian-induced geometric phase. Consequently, instead of being quantized, the Berry phase in each band becomes continuously varying due to the non-Hermitian-induced geometric phase. However, the global Berry phase remains quantized independent of onsite loss in the system (i.e., \( \varphi^+_B + \varphi^-_B = \varphi_0 \)), revealing the same topological nature even in different quantum phases. In other words, the non-Hermitian phase transition induced by increasing the gain/loss contrast is not expected to alter the topological phase of the system.

From the symmetry perspective, the onsite gain and loss lead to the notion of a non-Hermitian charge-conjugation symmetry

\[ \sigma_\varepsilon H \sigma_\varepsilon = -H^* \],  

(2-4)

which constraints the spectrum to be symmetric about the imaginary axis. As in the Hermitian case, all extended states still have equal amplitudes \( |A| = |B| \) on both sublattices. However, the topological interface mode now exhibits another distinct
feature: its frequency is fixed to \( \omega_0 = ig_A \), i.e. it sits on the symmetry-protected imaginary axis, while all extended states obey \( \text{Im}\{\omega\} = (g_A + g_B)/2 \).

2.2 Topological hybrid silicon microlasers

The non-Hermitian charge-conjugation symmetry of the complex SSH model hinders a new approach towards mode selection in laser systems. To demonstrate this, we design an array of coupled microring resonators emulating the complex SSH model (Fig. 2-1a). The coupling profile is precisely controlled by the separations between adjacent rings in an alternative fashion, which in turn determine the strength of the evanescent wave tunneling rate. A spacing defect in the center of the array creating a topological zero mode that decays exponentially away from the defect, and only populates every other resonator. Spectrally, the topologically-protected zero mode resides at the center of a band gap, corresponding to the resonance frequency. The distributed gain and loss respect a non-Hermitian charge-conjugation symmetry, leading to a response that robustly discriminates between the topological and non-topological states (Bloch states). Considered in the complex frequency plane, this directly translates into an enhanced gain of the topological zero mode, therefore favoring it over other states throughout the nonlinear mode competition process (Fig. 2-1b). In our experiment, a hybrid III-V-silicon semiconductor platform is chosen to deliver a robust silicon laser for maximizing its potential for photonic integrated circuits. Fig. 2-1c depicts a scanning electron microscope (SEM) picture of the fabricated topological microlaser array, consisting of nine coupled InGaAsP-silicon microring resonators on a silicon-on-insulator (SOI) substrate. As the number of resonators is odd, the zero mode is compatible with the boundary conditions, so that the main effect of the finite system size is the quantization of the extended states. In order to generate the desired gain/loss
distribution, a layer of approximately 10 nm Chromium (Cr) is deposited on top of every second resonator using overlay electron beam lithography. Finally, the gain profile is provided through uniform optical pumping applied from the top.

Fig. 2-1. Topological hybrid silicon microlaser. (a) Schematic of a topological laser array made of 9 microring resonators with alternating weak ($t_1$) and strong ($t_2$) couplings. The red halos represent the intensity profile of the oscillating zero-mode. (b) Spectral features of the topological laser array, highlighting the lasing selectivity of the topological zero-mode. (c) SEM pictures of the fabricated structure consisting of 9 rings on a hybrid III-V/silicon platform. Each ring has inner and outer radii of 3.5 µm and 4.5 µm, respectively. In the top panel of (c), we highlighted the Cr layer with artificial yellow rings for better visualization of the arrayed structure. Scale bars in (c): low magnification: 10 µm; high magnification: 2 µm.

The spectral properties of the laser action are characterized under different optical pumping levels. While the coupled microring array in principle supports multiple longitudinal modes, only the zero mode can emerge above the lasing threshold due to the introduced topology/non-Hermiticity interplay. The measured spectral evolution of the
topological hybrid silicon microlaser manifests a significant spectral narrowing from broadband photoluminescence (PL), to amplified spontaneous emission (ASE), and finally to persistent single mode lasing when well above lasing threshold (Fig. 2-2a). Due to the topological robustness associated with the zero mode, the desired single-mode operation is persistent with the resonant peak remaining well isolated around a wavelength of ~1523 nm from ASE to lasing, while the corresponding extinction ratio drastically increases to a value of approximately 20 dB. Fig. 2-2b shows the light-light curve, where the pump dependence of the total emitted intensity agrees well with the expectations for single-mode laser action, as it only displays a single threshold without further kinks.

Fig. 2-2. Topological laser action. (a) Lasing spectra of the structure of the topological hybrid silicon microlaser as a function of the pumping power much below threshold (top panel), approaching threshold (middle panel) and well above threshold (bottom panel). (b) Pump dependence of the laser emission intensity, demonstrating the fingerprint of single mode lasing.
Blue dots are experimental data and the red lines are linear fits with least-squares of the data before and after the lasing threshold, respectively.

A large overlap between the lasing mode profile and the gain material is desired in order to achieve high efficiency. In our experiment, therefore, we intentionally design a large-area single-mode laser with the transverse dimension of the hybrid ring being 1 µm wide and 720 nm thick (500 nm InGaAsP and 220 nm silicon). In this regard, while each ring supports several transverse modes, the fundamental transverse mode selected for the zero mode (TM\(_{11}\) mode in our work) occupies a much larger area of gain compared with the array of single-transverse-mode rings. In order to confirm the role of topological features in this enriched mode selection process, a control experiment was conducted using an identically-sized microlaser array without the designed distributed gain/loss profile. As expected, the hybridization through couplings of all the transverse and longitudinal modes under the uniform pumping scenario displays a broader emission spectrum with multiple peaks and a reduced peak intensity (Fig. 2-3a), with the total emission homogeneously distributed over the entire structure (Fig. 2-3b). In contrast, the zero-mode lasing in the topological array is highly reliable, despite the mode competition in each ring and across rings (Fig. 2-3c), which is a direct outcome of the interplay between the topological mode hybridization and non-Hermiticity. The lasing action of the topological zero mode is further validated by the measurement of the spatial lasing mode profile presented in Fig. 2-3d.
Fig. 2-3. Single-supermode topological laser action. (a) Multimode lasing from an identically-sized microlaser array as the topological microlaser, but without on-top Cr deposition on every second ring to introduce the distributed gain/loss profile. (b) Measured lasing mode profile of the topological microlaser without on-top Cr deposition. (c) Single-supermode lasing from the topological microlaser under the same pumping condition. (d) Measured lasing mode profile of the topological microlaser with the distributed gain/loss profile.

We have demonstrated a topologically robust single-mode hybrid silicon microlaser. Our work shows that the interplay between topology and non-Hermitian symmetries equips the emerging topological zero mode with a distinct mode profile that enables it to fully exploit the distributed gain domains, while simultaneously spoiling other states through deliberately introduced optical absorption. Realized in a hybrid III-V/silicon platform, our accomplished topological hybrid silicon microlaser supports large-area single-supermode operation, promising a highly-efficient optical source for integrated silicon photonics to robustly feed power for chip-scale communication and computing.
2.3 Creation of topological zero mode by non-Hermitian control

Beyond the selection of the topological zero mode, here, we show creation of the zero mode at the interface between two crystals with the same topological order but with distinct quantum phases, namely PT symmetry to PT symmetry breaking. Let us consider the interface shown in Fig. 2-4a, with modulated loss rates $\delta$, $\Delta > \delta$ in sublattices A and B of the left semi-array, and 0, $\sigma$ in the right semi-array. Fig. 2-4 shows a typical bifurcation diagram of energy curves (real and imaginary parts) of localized states for parameter values, $t_1/t_2 = 2.5$, $\delta = 2.5t_2$, and $\sigma = 2.5t_2$. $\Delta$ is chosen to be a controllable loss to tune the phase in the left lattice, and varies from $\delta$ to $2.5t_2$. The right semi-array is in the unbroken PT phase. When the left lattice is also in the unbroken PT phase (phase I - phase I interface), the whole system is in the same PT phase and topological order, and the states arising near the interface are the trivial Bloch states (region I in Figs. 2-4b and 2-4c). Due to this strong semi-lattice coupling, no topological defect states can be found at the interface site. For a phase II to I interface, there are two non-zero energy interface modes with eigen-energies $\varepsilon_2 = -\varepsilon_1^*$, and with the same intensity distributions and decay rate (region II). For a phase III to I interface, the two sub-lattices are fully decoupled in their real energy spectra and two zero-energy states emerge. In this limit, the dominant zero-energy interface state resembles the topologically-protected edge state of the right lattice, and reduces to it as the loss term $\Delta$ is further increased thus fully decoupling the two semi-lattices (see insets in Figs. 2-4b and 2-4c). The re-emergence of topological edge states indicates that non-Hermitian phase transition enhanced topological protection. With an increasing value of $\Delta$ the decoupling begins to split the imaginary energies of the two interface states: one interface state can arise as a dominant state, the other one becomes more dissipative.
and finally sinks into the Bloch states in the left lattice. Thus, the dominant zero-energy mode acquires an enhanced topological protection ensured by non-Hermitian phase transition. As a matter of fact, we will demonstrate that the recovered zero-energy mode is insensitive to the topological disorder at the interface.

Fig. 2-4. Bifurcation diagram of the interface modes in a SSH lattice. (a) Schematic of an interface realized by two dissipative SSH semi-lattices. Each semi-lattice is in the same topological order. (b), (c) Bifurcation diagram of the interface states. Insets show a few distributions of mode amplitudes of interface states at a few values of $\Delta / t_2$. 

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The photonic implementation of the interface structure is conducted using an array of coupled waveguides on a SOI platform where each waveguide represents a site in the complex SSH model (Fig. 2-5a). In this passive system, the PT phase transition in the lattice is realized by different loss rates introduced through selective absorption of Cr depositions on top of the waveguides (Figs. 2-5b,c). For easy excitation and test of robustness of the zero-energy interface state, we chose the strong loss regime $\Delta/t_2 = 10$, where the interface state dominates over scattered Bloch states.

To precisely characterize the zero-energy mode, it is necessary to assess a series of fundamental mode parameters including its group index and phase index, compared with the mode parameters of an unperturbed single waveguide. Here, we applied the heterodyne imaging technique to investigate the ultrafast transport dynamics to retrieve the quantitative characteristics of the interface state [62]. Since the Bloch states of the lattice are well suppressed by the introduced onsite losses, the propagation of the wave packet is robust against neighboring couplings and always remains confined in the interface waveguide (Fig. 2-5d). To reveal the zero-energy characteristics of the interface state, a control study on a single unperturbed waveguide was also conducted with the same wave packet excitation. The results demonstrate almost identical features between the interface state and the control waveguide at different time delays, regarding the pulse dispersion, shape, and duration. By sweeping the time-delay line with a constant time interval, the trace of the center of the wave packet travelling in time can be constructed for both the interface state and the single waveguide mode (Fig. 2-5e). The measured group index for the interface state is approximately $n_g = 3.97$, corresponding to an effective phase index of $n_{\text{eff}} = 2.37$, which are almost identical to the group and
phase indices of the single waveguide, evidently showing the zero-energy properties associated with the interface state.

Fig. 2-5. Observation of the zero mode on a photonic platform. (a) Cross section of the field distribution in the waveguide array to realize the zero mode at the interface. The width and height of the ridge waveguides are $W = 400 \text{ nm}$, $H = 150 \text{ nm}$ and $h = 70 \text{ nm}$, respectively. The on-top Cr depositions has thickness of 20 nm but varied widths: $w_1 = 90 \text{ nm}$ in the right semi-lattice, while $w_2 = 80 \text{ nm}$ and $w_3 = 350 \text{ nm}$ left semi-lattice. (b) Schematic of the waveguide array, where the zero mode is excited by directly coupling quasi-TE polarized light into the interface waveguide. A set of periodic radiation holes are introduced on the top of the lossless waveguides to scatter light for far-field observations. (c) The SEM picture of the waveguide array on an SOI platform, where pseudo yellow color denotes the Cr depositions on top of waveguides. (d) Synchronized
propagation dynamics of the zero mode (upper panel) and a single waveguide control (bottom panel) at different time delays of 166 fs, 266 fs, and 366 fs. (e) Trace of the wave packet travelling in time, where dots and lines represent raw measurement data and their corresponding linear fittings. The consistence between the interface mode (red) and the single waveguide mode (blue) evidently demonstrates the zero-energy characteristics.

An important feature of the zero mode is its robustness against topological disorder. We consider a strong local perturbation of waveguide separation at the interface (Fig. 2-6a), which can even completely reverse the topological relation between intra and inter-dimer couplings, i.e. \( t_1 > t_2 \) becomes \( t_1 < t_2 \). Remarkably, the zero mode is not destroyed and maintains its zero energy and mode profile, showing large fault-tolerance. It is clearly demonstrated in the spectrum (Fig. 2-6b) that the energy of the interface state maintains at zero, regardless of the strength of the introduced local topological disorder at the interface waveguide. As a result, the field remains localized in the interface waveguide region with exponential decay tails in both semi-lattices (Figs. 2-6c and d). Field localization at the interface is weakened, with a slower exponential decay into the PT symmetric semi-lattice, if the interface waveguide shifts more right, as its coupling to the adjacent dimer (i.e. \( t'_2 \)) grows. Nevertheless, if the value of \( t'_2 \) is too high, i.e. the interface waveguide is too close to the adjacent dimer in the PT symmetric semi-lattice, they become strongly coupled to form a defect trimer. To this regard, the intensity peak slightly moves to the second B waveguide in PT symmetric semi-lattice (Fig. 2-6d), but its associated zero-energy is still protected through the phase transition. Additionally, two defect states emerge outside the continuum spectrum when the shift of the interface waveguide is larger than 150 nm (Fig. 2-6b). The energy of the two defect
states is associated with relatively large damping coefficients, thereby attenuating much faster than the zero mode.

Fig. 2-6. Robustness of the zero mode against a local perturbation. (a) Topological disorder by shifting the interface waveguide towards the PT symmetric phase semi-lattice, resulting in perturbation of local coupling parameters with $-\Delta_1$ and $+\Delta_2$. (b) Evolution of eigen spectrum with respect to the shift of the interface waveguide. Here, the two edge states on the boundary of the lattice and the flat band introduced by PT symmetry-breaking lattice have been removed to clearly show the interface state. (c)-(e) Intensity distribution of the interface state with different topological perturbations corresponding to different shifts of the interface waveguide: (c) $D = 0$ nm; (d) $D = 100$ nm; (e) $D = 200$ nm.
To test the robustness of the zero mode, the local topological perturbation is intentionally introduced to the interface waveguide, as shown in Fig. 2-7a. The interface waveguide is shifted by 100 nm towards the adjacent dimer in the PT symmetric semi-lattice, corresponding to the change of the local coupling strengths from $t_1/t_2 = 2.5$ to $t_1'/t_2' = 0.8$. In this case, therefore, the local topological order is reversed. To enable the adiabatic transition between two opposite local topological orders, the shift of the interface waveguide gradually completes over a distance of 5 µm in the z direction. The zoom-in picture of the sample clearly confirms the implementation of this topology-transition region along the interface waveguide (Fig. 2-7b). Here, we performed both numerical simulations and heterodyne measurements to characterize a wave packet propagation supported by the zero mode, showing the pulse entering, propagating, and exiting around the topology-transition region (Fig. 2-7c, Fig. 2-7d). With the same single-waveguide excitation launched at the interface, the original zero mode is well formed in the interface waveguide. While the local topology varies after the wave packet enters the transition region, the zero mode persists with strong light localization at the interface. It is clear that the shape and dispersion of the wave packet after existing the transition region remain almost unaffected, indicating the robust light transport carried by the zero mode. This is because the zero mode is protected under the PT symmetry invariants, even though the local topological order is completely reversed. The quantitative evaluation in experiments further confirms the robustness of the zero mode: the average group and phase indices during the topology transition are approximately $n_g = 3.97$ and $n_{\text{eff}} = 2.37$, which are almost identical to their counterparts of the zero mode without any disorder and perturbation.
Fig. 2-7. Experimental validation of the robustness of the zero mode. (a) The interface waveguide is adiabatically shifted 100 nm towards the PT symmetric semi-lattice over a distance of 5 µm in the $z$ direction, reversing the local topological order around the interface from $t_1/t_2 = 2.5$ to $t_1'/t_2' = 0.8$. (b) SEM picture of the topology-transition region implemented in the Si waveguide array, where pseudo yellow color denotes the Cr depositions. The dimensions of each waveguide remain the same compared with the sample in Fig. 2-5c. (c) and (d) are numerically simulated and experimentally measured ultrafast dynamics of the zero mode to probe its robustness against topological disorders, respectively. Snapshots at different time delays show the pulse entering, propagating, and exiting around the topology-transition region.
We have demonstrated a novel robust photonic zero mode in a PT symmetric optical lattice, spatially localized at the interface separating broken and unbroken PT phases with the same topological order. Through non-Hermitian engineering, the interface state can re-emerge as a dominant state in the passive system. The restoration of topologically protected defect states, in spite of a uniform topological order in the entire structure, is enabled by the spatial quantum phase transition and enhanced by non-Hermitian loss engineering in the semi-lattices. Our results suggest that non-Hermitian lattice engineering can do much more than simply stabilize a Hermitian topological mode: it can create a new type of topological state which would disappear in the Hermitian limit.

2.4 Summary

This chapter explores the topology of complex-indexed extension of SSH model respecting the non-Hermitian charge-conjugation symmetry, where the alternating onsite gain and loss facilitate robust single-mode lasing with the topological zero mode selectively enhanced in a hybrid microlaser array. Beyond the mode selection, we further demonstrate the creation of the topological zero mode in a topologically uniform lattice by PT symmetry phase transition. Our strategic quantum phase manipulation provides a genuine new route toward the creation and manipulation of topological protected states in non-Hermitian photonics.
CHAPTER 3 NON-HERMITIAN STEERING OF TOPOLOGICAL LIGHT PATHWAY

This chapter is adapted from the following manuscript recently accepted by Science, where the author of this dissertation was the primary researcher and co-author of the work:


While photonic topological insulators provide an attractive perspective to efficiently guide, switch and route light in integrated circuits, combining topological protection with reconfigurability is demanded for next generation of integrated devices. Recent efforts have been devoted to studying switching the topological phase for optical modulation [53-57] and some progresses have been achieved in the microwave regime via mechanically-controlled topological phase transition [53]. However, an effective synergy between topological guiding and ultraflexible reconfigurability remains a challenge in optics. Redefining topological light pathways requires considerable perturbations to drive the topological phase transition inside the bulk structure that are difficult to access in integrated photonic chips. Such a severe limitation prevents topological photonics from being practically applied, since the topological mode only exists at the static structural boundary/interface so that most of the footprint of the photonic structure is unutilized.

3.1 Concept of non-Hermitian topological light steering

Rather than perturbing topological robustness, here, we demonstrate the creation of a topological light transport channel via non-Hermitian control on an active photonic
platform within the bulk of an otherwise Hermitian photonic topological insulator with uniform topological property (Fig. 3-1). The topological lattice consists of coupled microring resonators supporting two topological nontrivial bandgaps on an InGaAsP multiple quantum well platform for operation in the telecom band. Non-Hermitian control is conducted by optically pumping the photonic lattice to create distributed gain (via external pumping) and loss (intrinsic material loss without pumping) domains (Fig. 3-1a). Emergence of new topological states is observed at the boundary of the gain and loss domains when the local non-Hermiticity (i.e. the gain/loss contrast) is driven across the exceptional point (EP) defined by the coalescing eigenstates. The associated phase transition induces two effectively detached topological states, of which one gets strongly attenuated in the loss domain and only the other of gain survives and enables new topological pathways for guiding light at the gain/loss domain boundary without altering the global topological properties of the photonic lattice. Therefore, non-Hermitian control can be used to actively steer topological light on-demand via projecting the designed spatial pumping patterns onto the photonic lattice (Fig. 3-1b). Consequently, guided light can be directed along any arbitrary pathway, fully utilizing the entire footprint in topologically routing optical signal to any desired output port.
Fig. 3-1. Non-Hermitian control of light propagation in a topological microring lattice. (a) Scheme of the pump-induced local non-Hermitian symmetry breaking, which creates new topological edge channels along the gain/loss interface in the bulk of the photonic lattice with uniform global topology defined by the same geometric phase $\varphi$ in the gain (red) and loss (black) plaquettes. (b) The topological edge states can be dynamically reconfigured to steer light along any boundaries defined by the arbitrarily patterned pump beam.
3.2 Theoretical approach

We consider a non-Hermitian version of the two-dimensional (2D) photonic topological microring array [22] consisting of a square lattice of site rings coupled via anti-resonant link rings. In the Hermitian limit, the topological insulating nature is engineered by the encircling phase \( \varphi = \pi/2 \) which emulates the spin-dependent magnetic flux threading a 2D electron gas. The non-trivial phase opens band gaps where the interior structure is insulating due to destructive interference, while the pseudo-spin-dependent one-way edge transport channels are protected. When each microring is with either gain or loss, the Hamiltonian in its spin subspace is

\[
H_{1,2} = -t \sum_{m,n} \left( a_{m+1,n}^+ a_{m,n} + e^{im \gamma_1} a_{m,n+1}^+ a_{m,n} + \text{h.c.} \right) - i \gamma_2 a_{m,n}^+ a_{m,n} , \tag{3-1}
\]

where \( t \) is the coupling between two site rings controlled by the ring-to-ring separation, and \( \gamma_1 (>0) \) and \( \gamma_2 (<0) \) denote the gain and loss coefficients, respectively. The topological property of the system is not altered with uniform linear gain or loss, and therefore any states in the interior are prohibited in the band gaps. However, with a non-Hermitian gain/loss junction, imbalanced field amplitude is produced between the light circulating across the two domains, leading to the breakdown of destructive interference at the interfacial site rings. With a moderate gain/loss contrast, a pair of “pseudo” interface states emerge in each band gap (Fig. 3-2a). These counter-propagating edge states strongly couple before the closure of the band gap and therefore are not topologically protected. By increasing the gain/loss contrast, the gap between the emerging states diminishes once they cross at the symmetry point in the reciprocal space, where the two eigenstates coalesce to one singularity (i.e. EP) (Fig. 3-2b). Further tuning the gain/loss contrast across the EP leads to a non-Hermitian phase transition where two newly emerged gapless interface states decouple with each other, becoming topologically
chiral and carrying two different pseudospins (Fig. 3-2c). The new topological interface state emerges via non-Hermitian control, which is biased to the gain domain and dominant over that on the loss side. With a relatively large $\Delta \gamma / t$ sufficiently above the EP (such as $>5$ in our study), the non-Hermitian chiral state possesses almost the same modal characteristics as the original topological edge state, leading to efficient coupling between them when the pathway turns from the edge into the bulk of the lattice.

Fig. 3-2. Emergence of topological interface state via non-Hermitian phase transition. (a) Band structure for $\Delta \gamma= (\gamma_1 - \gamma_2)=1.4 t$ and $\varphi=\pi/2$. In addition to the edge states at the right and left physical boundaries (blue curves in the gaps), emergence of two dispersive pseudo edge states from the bulk bands are shown near the EP degeneracies at $k_y=0.25\pi/a$ (upper band) and $k_y=0.75\pi/a$ (lower band). These two states are highlighted one with red and other with black color. (b) Riemann sheets of the real and imaginary parts of the eigenspectrum, with varying gain/loss contrast and momentum, near the EP degeneracy at $(k_y=0.25\pi/a, \Delta \gamma=1.785 t, \gamma_1=-\gamma_2)$ in the upper bandgap. (C) Band structure for $\Delta \gamma=2.5 t$ showing two new anti-crossing interface states which counterpropagate at the gain/loss boundary of the lattice. The state in red curve gets amplified and the state in black curve strongly attenuates during propagation.
We confirmed this non-Hermitian transition process in 2D topological bandgaps in this array of ring resonators by calculating the topological edge states on a strip of the array that is finite in the x direction but infinite in the y direction (Fig. 3-3a). The shifts of the link rings are chosen to produce an effective magnetic field of 1/4 flux quanta through each square unit cell. Further, the gain/loss modulation in the strip is considered to be uniform along the x direction i.e. $\gamma_{mn} = \gamma_m$, but with half of the ribbon have gain and the other half have loss along the x direction (we assume an even number of site rings).

Since the system is infinite in y direction, the eigenstates of the Hamiltonian can be written as $\Psi_{m,n} = e^{i k_y y} \psi_m(k_y)$, for $-\pi \leq k_y \leq \pi$, assuming lattice constant $a = 1$. The eigenvalue equation, now reduces to one-dimensional problem of the non-Hermitian Harper-Hofstadter model depending on the parameters $k_y$ and the gain/loss contrast $\Delta \gamma = (\gamma_1 - \gamma_2)$:

$$H_m \psi_m = -i[\psi_{m+1} + \psi_{m-1} + 2 \cos(m \varphi - k_y) \psi_m] - i \gamma_m \psi_m$$ (3-2)

The corresponding dispersion relations are calculated using the transfer matrix approach for varying values of $\Delta \gamma$ and with the boundary condition $\psi_1 = \psi_m = 0$. We numerically solve for the eigen frequencies and eigenstates of the model by considering $\gamma_1 = \gamma_2$ (Fig. 3-3b). In the Hermitian limit $\Delta \gamma = 0$, the band structure exhibits four magnetic bands with a pair of counter propagating edge states that span the bandgaps, and their eigenstate wave functions are localized on opposite edges of the stripe (not shown here). For nonzero values of $\Delta \gamma$, there emerge two new states, one with $\text{Im}[\varepsilon] > 0$ and the other with $\text{Im}[\varepsilon] < 0$, at the interface of gain/loss sublattices (Figs. 3-3b and c). When $\Delta \gamma = 1.785$, at the two symmetry points $k_y = \pi/4$ and $-3\pi/4$ both the real and imaginary parts of the eigen frequency coalesce and marks the onset of the EP (Figs. 3-3d and e). The full behavior
of the non-Hermitian controlled interface state eigen frequency as a function of $\Delta \gamma$ at the symmetry point $k_y = -3\pi/4$ shows the parity-time (PT) symmetric bifurcation diagram (Fig. 3-4a). The occurrence of the PT-symmetric spectrum follows from the fact that the Hamiltonian $H_m$ satisfies PT-symmetric condition at two symmetric points $k_y = \pi/4$ and $-3\pi/4$. The field intensity distributions in the 1D ribbon site rings below, at and above the EP reveal the coupling, coalescence and decoupling between the loss and gain interface states at the gain/loss boundary. Note that the qualitative features of the non-Hermitian lattice model presented here is quite general and is verified in our experiment where the gain/loss boundaries were created in arbitrary places of the square lattice.

The qualitative features of our model can be at best understood by considering the normal form of an effective 2×2 Hamiltonian describing the two non-Hermitian induced edge states near the EP, i.e. $k_y$ near to either $\pi/4$ or $-3\pi/4$ and $\Delta \gamma/t$ near to the critical value $(\Delta \gamma/t)_c = 1.785$. Let us focus, for the sake of definiteness, to $k_y$ near to $\pi/4$, and let us set $q = k_y - \pi/4$ and $u = (\Delta \gamma/t)_c^2 - (\Delta \gamma/t)^2$. In a suitable basis, the normal form of the 2×2 non-Hermitian Hamiltonian takes the form:

$$H_{NH, edge} = \begin{pmatrix} \Omega_e & t^2 z(q, u) \\ 1 & \Omega_e \end{pmatrix}$$

(3-3)

where $\Omega_e$ is the supermode frequency corresponding to the coalescing edge states at the EP, and $z(q, u)$ is a dimensionless function of $q$ and $u$ which vanishes at $q = u = 0$. The eigenenergies are given by $\epsilon = \Omega_e \pm t \sqrt{z}$, so that at $z = 0$, i.e. at $q = u = 0$, $H_{NH, edge}$ is a Jordan normal form corresponding to the EP (i.e. PT symmetry breaking). The leading-order (linear) dependence of $z$ on $q$ and $u$ near the EP $q = u = 0$ can be obtained from the numerically-computed energy surface. In particular, in the plane $q = 0$ we require $z(q = 0, u = \beta u)$ with $\beta \approx 1$ so as to reproduce the typical EP scenario of Fig. 3-4a when $u$ crosses
the critical value $u = 0$. On the other hand, for $q \neq 0$ the EP is avoided as $u$ varies from below to above zero. For symmetry reasons one has to take a linear dependence of $z$ on $q$ with a purely imaginary coefficient. Hence we can write

$$z(q,u) = \beta u + i\alpha q ,$$

(3-4)

with $\alpha$ real. Its value can be estimated by sectioning the energy surfaces of Fig. 3-4b along the plane $u = 0$, obtaining $\alpha \approx 2$. The normal form of the effective 2×2 Hamiltonian $H_{NH,\text{edge}}$ describing the two non-Hermitian interface states near the EP finally reads

$$H_{NH,\text{edge}} = \begin{pmatrix}
\Omega_c & \beta(\Delta \gamma_c^2 - \Delta \gamma^2) + i\alpha(k_y - \pi/4)t^2 \\
1 & \Omega_e
\end{pmatrix}$$

(3-5)

Fig. 3-4 compares the energy dispersion curves of the non-Hermitian interface states versus $k_y$ for three values of the ratio $\Delta \gamma/t$ (slightly below, at and slightly above the EP), as obtained by full numerical simulations of the Harper-Hofstadter Hamiltonian and by the effective 2×2 Hamiltonian $H_{NH,\text{edge}}$ given above. The good agreement between the curves indicates that the transition to chiral non-Hermitian edge modes is well described, near the EP, by the normal form Hamiltonian $H_{NH,\text{edge}}$.
Fig. 3-3 Complex band structure of a topological non-Hermitian lattice. (a) Schematic of the non-Hermitian ribbon structure considered for band structure calculation. (b) and (c) are the imaginary parts of the band structures presented in Fig. 3-2a and Fig. 3-2c, respectively. (d) and (e) are the imaginary and real parts of the non-Hermitian band corresponding to $\gamma/t = 1.785$ for which two EPs occur at $k_y = 0.25\pi$ and $k_y = -0.75\pi$. 
Fig. 3-4 PT-symmetry breaking in the spectrum of topological non-Hermitian lattice. (a) The eigen-spectrum of the non-Hermitian interface states, at the point $k_y = -3\pi/4$ and $1.3 < \gamma/t < 2.6$, showing the PT symmetry breaking bifurcation. (b), (c), (d) show the non-Hermitian induced edge states below, at and above the EP as indicated by the gray dashed lines in (a). (e) shows the behavior of the energies $\epsilon$ versus $k_y$ near the symmetry point $\pi/4$ of the two non-Hermitian interface states for three values of $\Delta \gamma/t$ [slightly below (left panel), slightly above (right panel) and at the EP (central panel)] as obtained by full numerical simulations of the Harper-Hofstadter Hamiltonian (solid curves) and by the effective $2 \times 2$ non-Hermitian Hamiltonian $H_{NH,edge}$ (dashed curves).
3.3 Experimental demonstrations of the non-Hermitian topological states

The photonic topological lattice was fabricated on the InGaAsP multiple quantum well platform using electron beam lithography (Fig. 3-5a). We intentionally implemented shallow nanoholes on top of site rings (Fig. 3-5b) which sample the in-plane circulation of guided light in the far field. A uniform 200 nm edge-to-edge separation between the site rings and their adjacent link rings (Fig. 3-5c) opens two 70 GHz-wide band gaps. Part of the photonic topological lattice was optically pumped, which can be flexibly patterned to form any arbitrary topological pathway inside the bulk of the lattice via a spatial light modulator (SLM). The intensity of the pumping beam was precisely tuned just below the lasing threshold, offering a sufficient gain/loss contrast at the boundary of the pumping area to form the chiral non-Hermitian topological interface state while avoiding nonlinear gain saturation in each ring (see Appendix for the pump dependence of the topological routes). To validate the new topological route along the non-Hermitian heterojunction, a uniform square pattern which marginally covers a sub-area of 5×5 site rings was created (Fig. 3-5d). Due to the intrinsic amplification nature, another advantage of our InGaAsP platform is that each site ring can also act as an on-chip light source, feeding light into the topological lattice. To fully take this advantage, the light wave to probe the non-Hermitian-controlled topological edge states was launched from the periphery site ring next to the square pumping area, with a separate synchronized pumping beam above lasing threshold. Due to the time-reversal symmetry of a single ring, both clockwise and counterclockwise modes lase in the site ring. Being two pseudospins of the topological lattice, they couple along the two edges of the pumping region according to their synthetic magnetic fields, respectively, topologically routing around the pump-defined (instead of structural) turning corners without any scattering loss (Fig. 3-5e and Fig. 3-5f).
Fig. 3-5 Experimental realization of the pump-defined topological states. (a) Scanning electron microscopy (SEM) image of the photonic topological insulator on the InGaAsP platform with 8×8 site rings (before transferred to glass substrate). (b) Zoom-in SEM image showing the shallow scattering holes with 100 nm in diameter. (c) Side view of a pair of coupled link and site rings, each with the cross section of 200 nm height and 500 nm width. (d) The uniform square pump pattern formed by SLM that covers the 5×5 site rings (orange area) and the synchronized pump beam that induces the lasing incidence (red dot). (e) Simulated field amplitude distribution. White arrows show the corresponding clockwise (CW) and counter-clockwise (CCW) propagation directions. (f) Experimentally measured field amplitude at the lasing wavelength of 1486 nm. Each plotted cylinder stands for the corresponding site ring in the fabricated array.

To detail the experimental measurement of the non-Hermitian controlled topological light path, Fig. 3-6 shows the setup we used to configure the pumping pattern and characterize the consequent tunable topological light transport. In our experiment,
the designed gain/loss profile and the probing light were enabled by a nanosecond pulsed laser pumping beam with 8-ns pulse duration and 50 kHz repetition rate at the wavelength of 1064 nm, which provides sufficient coherent time while avoiding heat accumulation due to its corresponding low duty cycle. To separately manipulate the pump pattern and the probe light from the lasing site ring, we split the pump beam into two arms, of which one was dynamically imaged onto the microring array via a spatial light modulator and a Mitutoyo 10X near infrared (NIR) long-working distance objective (NA = 0.26) from the backside (through the glass substrate), while the other arm was reflected by a long-pass dichroic mirror (1180 nm cutoff wavelength) and focused onto the lasing site ring using another Mitutoyo 20X NIR objective (NA = 0.4) in front. To ensure the maximal temporal overlap of the probing signal with the patterned gain, the focused pump beam (on the single site ring) and the imaged pump pattern (by SLM) were synchronized via a delay line. The pump power was controlled with a variable neutral density (ND) filters, monitored by a power meter. Note that the wavelength of the induced lasing light is detuned to the topological band gap of longer wavelength when the site ring with focused pump couples to the surrounding microrings. This is because the corresponding anti-resonance supermode overlaps most with the gain distribution, while the resonance supermode (at shorter wavelength) suffers the loss from the intermediate passive link rings.
Fig. 3-6 Experimental setup to characterize the reconfigurable photonic topological insulator. BS: 50:50 beam splitter; SLM: spatial light modulator; DM: dichroic mirror; LPF: long-pass filter with cut-off wavelength at 1450 nm; FM: flip mirror.

The light intensity from the microring array was collected by the front 20× objective and was imaged to an infrared CCD camera. An exemplary recorded image, corresponding to the square pump case of Fig. 3-5d, is shown in Fig. 3-7a. Due to the mixture with the broadband spontaneous emission, it is difficult to directly visualize the propagation of the probing signal at the exact wavelength based on the captured camera image. Instead, we traced the transport trajectory of the probing signal by measuring the spectra of the light radiation at each site ring. This was achieved by moving a fully closed iris on the imaging plane, whose aperture selects only the light intensity at a single site ring for the spectral analysis (Fig. 3-7b). The light radiation passing through the iris was thus guided to the monochromator for the spectral analysis. Fig. 3-8 shows the measured spectra of the site rings under the pump region, from which the experimental results in Fig. 3-5f were reconstructed by the amplitude at the single site ring lasing wavelength (1486 nm).

It is worth emphasizing that the lasing peak at 1486 nm in the spectra of the site rings at the gain/loss boundary was absent without the induced probing signal, i.e., there is no laser action if only the patterned-pump via SLM is introduced. This is because the
intensity of the square pump was carefully controlled below the lasing threshold. Nevertheless, we observe the appearance of a second peak in the spectrum of bottom left figure between 1450 nm and 1486 nm. This peak features an amplified spontaneous emission of another longitudinal resonance mode of the site ring resonator. It is worth noting that each site ring resonator supports multiple longitudinal modes with ~10 nm free spectral range within the gain spectrum. The wavelength of the small peak locates approximately 20 nm away from the dominant peak at 1486 nm. Moreover, in this experiment, since we adjust the patterned pump power density just at the average lasing threshold of the site ring, a slight non-uniformity of a specific resonator may result in a lower threshold for a nearby longitudinal mode. This would randomly induce lasing action incoherent to the signal. However, we note that under the pump power control, such incoherent emission is always much weaker than the signal and therefore has a minimum impact on the gain distribution in the single resonator.

Fig. 3-7 Captured camera images of the lattice under square pump. (a) Image of the emission when the iris is fully open. (b) Image of the targeted single ring when the iris is fully closed.
Fig. 3-8 Imaging of the topological light transport in the square pumped region by spectral measurements. Arrows point at the corresponding site rings from which the spectra are measured.

To prove the dependence of the emerging topological states on the pump-induced non-Hermiticity, we characterized the light transport in the square pumping region (Fig. 3-9a) with increased pumping intensities before reaching the nonlinear gain saturation in each ring (confirmed in Fig. 3-9b). With a small pump power, the limited induced non-Hermiticity is not sufficient to enable the topological states, and therefore the probing light couples to the square pump region without discernible feature (Fig. 3-9c). As the pump power increases, the new topological channels are defined by the
induced non-Hermiticity. However, the propagation length of the emerging topological states is limited due to the uncompensated loss (Fig. 3-9d). With the pump intensity approaching the lasing threshold, the loss in the pumping region diminishes, leading to the desired topological light transport along the non-Hermitian-controlled interface (Fig. 3-9e).

![Pump Pattern](image)

**Fig. 3-9.** Pump dependence of the non-Hermitian topological light path in the square region. (a) Shows the pump intensity profile. Measurement of the intensity of the signal wavelength at one site ring (blue dotted circle in (a) confirmed the reach of nonlinear gain with pump power over 60 mW (b). Before the gain saturation, increasing propagation length along the boundary of the pump was observed with pump intensity (square pump) respectively at 35 mW (c), 45 mW (d) and 55 mW (e).

It is important to note, while all the simulations of band diagrams and non-Hermitian chiral modes have been performed considering a stationary (i.e. time independent) and unsaturated gain/loss coefficient in the semiconductor, we used a pulsed optical pumping in the experiment. However, for the current topological lattice
geometry and for pump parameter conditions used in the experiment (i.e. pulse duration of about 8 ns), at first instance we can safely explain the experimental results with the numerical simulations under quasi-stationary gain/loss conditions, i.e. without resort to the full time-dependent dynamical model. We can justify the above statement as follows. The gain coefficient in a semiconductor medium is given by \( g = \sigma(N-N_0) \), where \( \sigma \) is the differential gain, \( N \) is the carrier density and \( N_0 \) its transparency value. Clearly, in our experiment we have a time-varying gain coefficient \( g = g(\tau) \) owing to the pulsed optical pumping of the semiconductor. Since to realize the non-Hermitian gain/loss interface the pump level is below lasing threshold, we can neglect gain saturation effects and write a simple rate equation for the carrier density: \( \frac{dN}{d\tau} = P(\tau) - \frac{N}{\tau_c} \), where \( \tau_c \) is the recombination time of carriers (\( \tau_c = 1 \) ns in our case) and \( P(\tau) \) is proportional to the optical pump focused onto the semiconductor. If we compute the behavior of the carrier density \( N(\tau) \) versus time \( \tau \), assuming a typical pump pulse \( P(\tau) \) of duration 8 ns as in our experiment, we see that there is a time window of about 6 ns where the gain \( g(\tau) \) remains close to its peak value (it does not fall down 0.9 of the peak value) (Fig. 3-10). This is shown below by considering typical experimental values for the parameters. Note that in the experimental measurements we can limit to consider such a time window because, as the gain falls down, the light propagating along the dynamically-reconfigurable non-Hermitian path is strongly absorbed and cannot be detected by the infrared camera and monochromator. In other words, most of the light captured by the CCD camera corresponds to the highest gain window.
Fig. 3-10 Gain dynamics of by pulsed pump. (a) A typical 8 ns-pulsed pump profile for the carriers. (b) The corresponding gain calculated from rate equation, below threshold, shows almost constant distribution during the pulse duration.

The propagation speed (group velocity) of the chiral modes can be computed from the slope of the dispersion curves in the wide gaps shown in Fig. 3-2c, which is ultimately limited by the bandgap width. In our case the width of the wide gaps is 70 GHz. A rough estimate of the group velocity shows that in 1 ns light propagates over more than 80 unit cells (i.e. site rings). Hence, within the 6-ns-wide near-constant gain window, and considering that the paths of reconfigurable interfaces contains no more than 20 microrings, the quasi-stationary assumption for the gain in the semiconductor is fully justified.

3.4 Reconfigurable topological light transport

The virtue of the non-Hermitian-controlled topological light path is the convenient reconfiguration along any arbitrary shape to steer topological light within the whole footprint of the lattice. To demonstrate such versatile topological light steering, the pumping pattern was switched from the “square” to an “L” shape (Figs. 3-11a, b and c), enabling the input beam propagation along the newly formed topological domain boundaries despite the increase of turning corners in the reconfigured pumping area.
Our non-Hermitian-controlled reconfigurable light transport scheme is inherently of topological robustness against defects. Even though a defect is intentionally created along the structural edge by a notched square pumping pattern (Fig. 3-11d, e and f), the incident light detours around the defect ring without noticeable intensity drop and back reflection. Furthermore, since the pumping can locate the transport channel anywhere in the bulk, the light signal is allowed to take place at any site ring and be topologically guided. Such a novel feature was demonstrated by moving the excitation to an interior site ring (Fig. 3-11g, h and i), where the generated lasing beam was coupled with the topological states and guided along the pumping-defined perimeters. This is in stark contrast to the prior passive photonic topological insulators, where due to the insulating bulk, the topological edge states can only be accessed when probed from the edge.
3.5 Summary

We have demonstrated active topological light steering along any arbitrary route in a photonic integrated circuit via non-Hermitian control of patterned gain/loss
distribution. The non-Hermitian manipulation redefines the topological domain wall without altering the topological order of the structure, which would be otherwise static. The ultra-flexible nature of non-Hermitian topological light control is general and applies to other photonic topological insulators with the size of the unit cell at the wavelength scale [24,25]. The achievable functions can cover a large variety of photonic components and networks beyond light steering and routing, thereby promising for the development of integrated photonic circuitry for high-density data processing.
CHAPTER 4 ASYMMETRIC LIGHT TRANSPORT AROUND OPTICAL EXCEPTIONAL POINT

This chapter is adapted from the following publications:


The author of this dissertation was the primary researcher and co-author of the work.

In addition to the selection and reconfiguration of the topological states that is readily accessible in the Hermitian limit, optical non-Hermiticity constructs the unique topology featured by the non-Hermitian degeneracy, i.e. exceptional point (EP), where two or more complex eigenspectral branches intersect [63-70]. Distinct from the eigen-energy degeneracy in Hermitian systems, namely diabolic points (DPs), EP systems possess simultaneous coalescence of both eigenvalues and eigenstates in the complex eigen-spectrum. Subject to a weak parametric perturbation, the EP degeneracy can be lifted, resulting in unique eigenvalue splitting proportional to the square root of the perturbation strength. When exploited as a weak transduction signal in sensor applications, such square-root energy splitting represents pronounced enhancement of raw sensitivity superior to the linear response from the DP degeneracy [67-70]. Moreover, for PT symmetric photonic systems in particular, EP represents the transition of spontaneous symmetry breaking, where the asymmetric light transport becomes unidirectional [34,35]. In this chapter, we explore the vicinity of EP and demonstrate the
square-root topology and the skewed optical scattering matrix facilitate sensitive thermal imaging and power-efficient interferometric optical modulation on-chip.

4.1 Thermal imaging on a glass slide

To benefit from the sensitivity enhancement associated with EP, we conduct a multilayer design on a glass slide with a thermally deformable polymer layer of polymethyl methacrylate (PMMA) sandwiched between two Au films (Fig. 4-1a). Upon normal incidence of the He-Ne probe laser on a two-port optical system, its optical characteristics can be described using the scattering matrix [71]

\[
S = \begin{pmatrix}
    t & r_f \\
    r_b & t
\end{pmatrix},
\]

(4-1)

where \( r_f \) and \( r_b \) are the reflection coefficients in forward and backward directions, respectively, and \( t \) denotes the transmission coefficient that is the same in both directions due to reciprocity. The objective of implementing the scattering matrix is to construct a non-Hermitian EP degeneracy. The eigenvalues of the scattering matrix are

\[
u_{1,2} = t \pm \sqrt{r_f r_b}.
\]

to achieve EP, the eigenvalue splitting has to vanish, i.e., \( 2\sqrt{r_f r_b} = 0 \), corresponding to the unidirectional reflectionless condition: \( r_b \neq r_f = 0 \). We attain the EP condition at room temperature as the initial state, resulting in complete reflection darkness in the forward direction. Under temperature perturbations, the local thickness of polymer varies due to thermal expansion/suppression, which breaks the EP condition, lifting the EP degeneracy and thus causing a drastically enhanced reflection coefficient. As a consequence, spatially imaging the reflection change is equivalent to resolving and mapping the thermal response of the polymer layer, which can be further converted to a microscopic image of the temperature distribution.
It is important to note that, despite the coating of the multilayer EP structure, the glass slide is transparent under white light illumination, evidently indicated by the visible landscape behind the slide (Fig. 4-1b). Hence, in addition to the new function of temperature detection, the glass slide maintains the conventional topography-type imaging function of a microscope with transmitted light. As shown in Fig. 4-1b, the glass slide with our EP structure can be conveniently mounted on a conventional epi-fluorescent microscope system. While maintaining the transmitted-light imaging mode intact, a monochromatic laser light at the EP wavelength is applied to map the reflection variation. As the most cost-efficient and widely used laser source in many routine laboratory settings, a He-Ne laser at the wavelength of 632.8 nm, which can be seamlessly integrated with most microscopes, is chosen in our work to probe the optical-thermal signal transduction.

Fig. 4-1. Thermal sensitive microscope slide engineered at an exceptional point. (a) Schematic drawing of the multilayer EP structure. (b) Transparency of the thermal sensitive glass slide. (c) A microscope system with the devised thermal sensitive glass slide. While the white light source is for conventional microscopic imaging, the He-Ne laser (labeled by red arrow) is used as the incidence for thermal mapping. Inset: zoom-in of the glass slide in the microscope system.
The EP and its associated phase transition are both theoretically and experimentally validated through the amplitude spectrum of the generalized reflection coefficient that is half the scattering eigenvalue splitting: \( \Delta \nu / 2 = |r_L r_f| \) (Figs. 4-2a and b). Around the EP, the splitting of the square-root scattering eigenvalue sharpens the phase transition in wavelengths across the EP. Despite a slight discrepancy between the theoretical and experimental results due to fabrication imperfection, the optical response around the EP is much more drastic than that of the DP. As a result of the highly asymmetric behaviors of reflections at the EP, the abrupt phase transition across the EP can be clearly revealed if measuring only the forward reflection coefficient \(|r_f|\). This is because the backward reflection is of a large value and varies slowly around the EP, and is therefore not responsible for the sharp transition in the spectrum. This observation helps to simplify our experiments significantly. Instead of analyzing the eigenvalue splitting that requires reflection measurements from both directions, only forward reflection needs to be characterized to evaluate all the EP-related properties.

To establish a real-time optical transduction mechanism in thermal sensing, the correlation of the ambient temperature and the scattering eigenvalue splitting is characterized at the EP wavelength. Assume the transferred heat causes the thickness variation of the polymer (PMMA) layer \( L \) by a sufficiently small deformation \( \Delta L \). The scattering matrix subject to the thermal-induced perturbation then becomes

\[
S = S_0^{EP} + \Delta L \left( \begin{array}{c}
\frac{\partial t}{\partial L} & \frac{\partial r_b}{\partial L} \\
\frac{\partial r_t}{\partial L} & \frac{\partial t}{\partial L}
\end{array} \right) = \left( \begin{array}{c}
t_0^{EP} + \Delta L \frac{\partial t}{\partial L} & r_b^{EP} + \Delta L \frac{\partial r_b}{\partial L} \\
\frac{\partial r_t}{\partial L} & t_0^{EP} + \Delta L \frac{\partial t}{\partial L}
\end{array} \right),
\]

(4-2)
where \( r_f^{EP} = 0 \) and \( r_b^{EP} \neq 0 \). The square-root relation is revealed as

\[
\Delta \nu^{EP} = 2 \left( \sqrt{\frac{\partial r_f^{EP}}{\partial L}} - r_b^{EP} \right) \sqrt{\Delta L},
\]

where the sensitivity can be fully characterized by only the derivative of forward reflection and the relatively large backward reflection at the EP acts as a trivial magnifying factor. Note that for a DP degeneracy, Eq. (4-2) evolves to a linear relation with the thickness perturbation of

\[
\Delta \nu^{DP} = 2 \Delta L \frac{\partial r_f}{\partial L},
\]

since \( r_b^{DP} = r_f^{DP} = 0 \). Such a striking contrast of the eigenvalue splitting between the designed EP structure and a typical DP structure reads

\[
\frac{\Delta \nu^{EP}}{\Delta \nu^{DP}} = \sqrt{\Delta L} \left( \frac{\partial r_f^{EP}}{\partial L} \right)_b \left( \frac{\partial r_f}{\partial L} \right)_f^{EP}.
\] (4-3)

Under weak perturbations, the relation of \((\partial r_f / \partial L)_f^{EP} r_b^{EP} \gg [(\partial r_f / \partial L)_f]^{DP}\) ensures significant sensitivity improvement in terms of per unit change of the PMMA thickness (Figs. 4-2c and d).
Fig. 4-2. Characterizations of enhanced thermal sensitivity at EP. In all panels, $|\sqrt{r_1 r_2}|, |r_1|$ from the EP structure are represented by green and red curves, while the reflection $|r|$ of a DP structure (PMMA anti-reflection film) is in blue. (a) Calculated spectra of the reflection coefficients. (b) Experimentally measured spectra of the reflection coefficients. The minimum forward reflection at the EP wavelength slightly deviates from the ideal reflectionless condition. However, the sharp EP phase transition is still observed. (c) Theoretical temperature responses of the reflection coefficients at the He-Ne laser wavelength. (d) Calibration of the thermal sensitivity in terms of the reflection coefficients. Each error bar records the data of 5 separate measurements and indicates the standard deviation. The curves are the best fits of the medians of these measurements.

The purpose of the thermal mapping is to retrieve detailed temperature information of the specimen, along with conventional microscopic imaging, to enable the analysis of the correlation between multiple physical parameters. To meet this objective,
the desired technique must support thermal mapping, operated in a highly-distributed manner with high spatial resolution comparable to the resolution of the microscope. The ultimate spatial resolution of thermal mapping is only limited by the heat-transfer-induced spatial temperature overlapping of adjacent heat sources, and is evaluated to be at a 10-μm scale in our device. Here, we have validated such a highly-distributed thermal mapping function of our microscope slide and its associated microscale spatial resolution. Spatially distributed local thermal sources were effectively generated by optically casting and focusing a 3×3 square-latticed hole array onto the central layer of PMMA to locally heat up and expand the polymer through the backside of the slide. To avoid the interference with the visible measurements, we used a 10-nanosecond pulsed laser with a center wavelength of 1064 nm. Through demagnification, 9 microscale laser spots were created with a spot diameter averaged at 30 μm, as shown by the corresponding transmission image at the wavelength of 1064 nm (Fig. 4-3a). While only a small fraction of laser light can be absorbed by the polymer, the heat accumulated from absorption still induces a temperature increase around those 9 spots, which locally enlarges the thickness of the central polymer layer. Due to the heat transfer in the EP multilayer structure, the absorbed heat from laser spots spreads and raises the temperature evanescently within the vicinity of approximately 4 μm. As a result, only the local condition around the laser spots deviates from the EP condition designed for room temperature, causing the reflection variation of the probe light from the He-Ne laser by which the temperature change in the vicinity of the laser spots can be mapped. To completely eliminate the adverse effect on thermal mapping due to the transmission of the heating laser beam, a bandpass filter at 500 nm – 700 nm was placed in front of the camera to collect and image only the reflected probe light of the He-Ne laser beam to precisely read the thermal distribution on the microscope slide. It is worth noting that the
accumulated heat as well as the resulting variation of the local temperature and the deformation of the polymer layer are all linearly dependent on the power of the incident pulsed laser. Therefore, by varying the power of the pulsed laser, we observed a linear growth in forward reflection of the probe light, which was further converted to a function of temperature through the calibrated temperature-reflection correlation (Fig. 4-3b). The thermal maps retrieved from the probe light are displayed in Fig. 4-3c with different averaged incident heating laser power. Remarkably, the imaged thermal distributions spatially resolve the pattern of the heating pulsed laser spot with feature size at 30 μm.

Fig. 4-3. Thermal mapping of spatially resolved pulsed laser heating source. (a) A spatially distributed thermal source array of 3×3 spots. (b) The correlation of the power density of the pulsed laser beam and the forward reflection of the probe He-Ne laser beam measured at the center spot. Each error bar indicates the standard deviation of 3 separate measurements, and the line fits the medians of these measurements. (c) Spatially resolved thermal mappings of the heating laser spot array revealed by forward reflection of He-Ne laser beam. Panels from left to right are obtained at increasing power density of the pulsed laser beam at 100 W·cm⁻², 300 W·cm⁻² and 500 W·cm⁻², respectively. Scale bars in (a) and (c), 50 μm.
To demonstrate the capability of thermal imaging of the EP glass slide, we injected hot water into a water reservoir on the slide using a block of polydimethylsiloxane (PDMS) with a hollow octagonal cavity in the middle, which was then bonded on top of the layered EP structure, as schematically depicted in Fig. 4-4a. Following our aforementioned discussion, while the thermal mapping is performed in a reflection mode mapping the temporal evolution of the temperature distribution of injected water, the slide is still transparent to support microscopic imaging using white light in the transmitted-light mode, where the boundary of PDMS divides the entire field into two regions: an area in the reservoir where hot water injection induces the temperature change and the other area of stabilized temperature covered by PDMS (Fig. 4-4b). As a control experiment, room temperature water was first injected into the reservoir. In this case, the temperature of the glass slide surface remains at room temperature during the water injection. The detected reflection in the field of view stays uniform at its resonance minimum of the designed EP condition (Fig. 4-4c).

To observe temporal evolution of the heat transfer process, we injected hot water at approximately 43 °C and real-time monitored and mapped the temperature distribution. Figs. 4-4d-g show the measured instantaneous thermal maps at different time delays. Right after injecting a droplet of hot water, because of high thermal conductivity of the thin Au layer, the PMMA layer underneath the water reservoir region immediately responds with thermal expansion. As a consequence, the detected reflection significantly increases from the EP resonance minimum, indicating the corresponding local temperature change. As the heat dissipates in the flow direction from the water reservoir to the PDMS cladding, the temperature of water keeps dropping and so does the temperature of the PMMA layer, leading to the contraction of the PMMA layer back to the initial EP condition and its resulting decrease in reflection back to the initial
resonance EP minimum. This process has been dynamically recorded, in which the spatial temperature gradient gradually retracts in time towards the center of the reservoir. The designed thermo-sensitive microscope slide at EP successfully enables real-time monitoring of the dynamic evolution of a heat transfer process, promising in-situ control of temperature in the applications where temperature monitoring is necessary.

Fig. 4-4. Transient thermal mapping of hot water injection. (a) Schematic of the thermo-sensitive microscope slide with a bonded PDMS octagonal water reservoir for hot water injection. (b) Microscope image of the top right corner of the water reservoir by transmitted white light. (c) Thermal mapping of the reservoir edge area when the reservoir is filled with room temperature water as a control experiment. (d)-(g), Transient thermal mappings at different time using forward reflection of the He-Ne laser beam, after injection of 43 °C hot water. Dashed lines in (c)-(g) represent the boundary of water and PDMS. Scale bars in (b)-(g), 100 μm.

4.2 Asymmetric interferometric optical modulator

Interferometric light-light switch offers a unique linear scheme to efficiently control light by light utilizing mutually coherent interaction of light beams and absorbing matters, by which coherent perfect absorption (CPA) was demonstrated [72-77]. While this strategy reduces the power requirement compared to the nonlinear approach, the
control beam still has a similar amount of power as the actual source signal in these previous works, due to the rather symmetric optical scatterings in the optical implementations. Here, we design a power-efficient light modulator by exploiting the highly asymmetric transport in the vicinity of EP under the PT symmetry paradigm, where a weak control beam can be interferometrically exploited to control an intense laser signal, resulting in two distinct modes of operation, i.e., CPA or strong scattering.

The optical transfer matrix describes the related scattering eigenstates of an optical system. For a two-port system of length $L$, a transfer matrix $M$ links the scattering eigenstates of both ports as

$$\begin{align*}
  \begin{pmatrix} A(L) \\ B(L) \end{pmatrix} &= \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} A(0) \\ B(0) \end{pmatrix},
\end{align*}$$

where $A(0)$ and $B(L)$ are inputs in left and right ports, while $B(0)$ and $A(L)$ denote outputs in left and right ports, respectively. To facilitate strong light-light interactions, the CPA condition $M_{11} = 0$ is desired, which corresponds to input light being completely absorbed. The goal we set to achieve, i.e., using a weak control beam $B(L)$ to bring a strong signal beam $A(0)$ into CPA, is satisfied when $|M_{21}| \ll 1$, which corresponds to a strongly asymmetric reflection.

Fig. 4-5a shows the schematic on a SOI platform. The modulator is designed to be 800 nm wide and 220 nm thick, embedded in a background of SiO$_2$, supporting a fundamental mode with an effective wavenumber of $k = 2.69k_0$ at the wavelength of 1550 nm, where $k_0$ is the wavenumber in free space. The non-Hermitian optical potential is enforced along the length of the waveguide with the index-absorption engineering, and reads
\[ \Delta \varepsilon = \Delta \varepsilon_0 \left( \cos(qz) + i \delta \sin(qz) \right), \quad (4-5) \]

where \( \Delta \varepsilon_0 = 0.317 \) denotes the modulation amplitude, \( \delta \) is larger than 1 (\( \delta = 1 \) corresponds to the exceptional point) to have the device operating in the symmetry breaking phase, and \( q = 2k_1 \), and the modulation regions are located at \( 4n\pi/q \leq z \leq 4n\pi/q + \pi/q \) \( (n = 1, 2, 3 \ldots) \). Due to the coupling between forward and backward propagating light by the modulated dielectric constant, the waveguide supports two degenerate Bragg modes of different absorption coefficients. The length of the device is designed to be approximately 21.9 \( \mu \text{m} \) corresponding to 38 periods, such that one degenerate mode satisfies the CPA condition, where coherent light inputs from the left and right ports are perfectly absorbed with zero output scatterings (Fig. 4-5b; upper panel). The other degenerate mode has much less absorption and thus generates strong output scatterings (Fig. 4-5b; lower panel). Assuming the incident phase of the signal remains 0, efficient switching between these two modes of operation can be achieved by tuning the incident phase of the control field from \( \pi/2 \) to \( -\pi/2 \), with an extremely remarkable extinction ratio.
Fig. 4-5. Asymmetric light-light switching. (a) Schematic of a waveguide with asymmetric reflection. The intrinsic reflection asymmetry in the vicinity of the quasi-PT exceptional point facilitates asymmetric light-light switching of a strong source signal (forward input) by a weak control field (backward input). (b) Electric field distributions of interferometrically controlled CPA and strong scattering states. The power ratio is set to 1:3.

The interferometric light switch is facilitated by the asymmetric reflection of the designed waveguide. Here a relatively small value of $\delta = 2$ was chosen to ensure that fabrication imperfections do not make the system deviate strongly from the designed CPA condition. We find that the intensity ratio $\xi$ is given by $(\delta + 1)/(\delta - 1)$, leading to $\xi$ of 3:1 between the strong signal beam and the weak control beam. To demonstrate the waveguide with the desired intrinsic scattering asymmetry, an equivalent guided-mode modulation has been designed to realize the virtual non-Hermitian function modulation with in-phase separation of real index and imaginary absorption modulations (Fig. 4-6a).
The sample was then fabricated using overlay electron beam lithography, followed by electron beam evaporation and lift-off of sinusoidal shaped Cr/Ge combos and dry etching to form the Si waveguide with cosine shaped side wall modulations, respectively (Figs. 4-6b and c).

Fig. 4-6. Asymmetric interferometric light modulator. (a) Schematic of the structure. The real index modulations are emulated using side wall modulations with cosine-varying from +71 nm to -48.5 nm; the imaginary absorption modulations are mimicked by bilayer sinusoidal shaped combo structures on top of the Si waveguide. (b) SEM picture of the device consisting of 38 periods for strong signal light switching by a weak control. (c) Zoom-in picture of the metawaveguide.

In our experiments, coherent laser beams, splitted from the same laser source, were coupled from free space to the waveguide from both ports, by means of specially designed mode converters. The experimental validation of the asymmetric light modulator required precise measurements of the ratio of outputs to inputs. To do so, We integrated two on-chip waveguide directional couplers to separate the inputs and outputs and route them to 4 respective grating couplers (Fig. 4-7a). The grating couplers
efficiently scattered input and output light to free space, which was collected by a microscope objective and further imaged onto a highly sensitive charge-coupled device (CCD) camara for final evaluations. As a result, the output scattering coefficient from the device was characterized by

\[
Q_s = 10 \log \left[ \frac{(O_1 + O_2)}{(I_1 + I_2)} \right] + C,
\]

where \(O_1\) and \(O_2\) are scattered light from two output grating couplers, \(I_1\) and \(I_2\) are scattered light from two input grating couplers, and \(C\) is a constant denoting the insertion loss to the output scatterings by the directional couplers. The incident phase of the control field was well controlled by an optical delay line constructed in free space. The intensity ratio \(\xi\) of the signal beam to the control beam was manipulated to the designed value of 3 by adjusting the coupling efficiency of the control beam, confirmed by imaging the scattered light from the corresponding input grating couplers. The spectra of the output scatterings of the fabricated non-Hermitian waveguide have been measured for both minimum and maximum \(Q_s\), corresponding to the CPA mode and the other degenerate mode of less absorption, respectively (Fig. 4-7b). At the resonant wavelength of the non-Hermitian waveguide, the CPA mode was achieved with almost no output scatterings when the incoming phase of the control was \(\phi = \pi/2\). In contrast, the mode of less absorption was excited and strong outputs were observed when the phase of the control was modulated to \(\phi = -\pi/2\) demonstrating a weak-to-intense optical switching with an extinction ratio up to approximately 60 dB.
Fig. 4-7. Characterization of the asymmetric light switching. (a) Configuration of the experiment setup. (b) Spectra of maximum (red) and minimum (blue) output scattering coefficients as a function of wavelength detuning $\Delta$. Insets: observed strong output scattering (top) and the CPA mode (bottom) at resonance.

Because the modulator was operated in the vicinity of the exceptional point, such asymmetric light-light switching in output scatterings remained as a function of wavelength detuning $\Delta$ (Fig. 4-7b). However, the phase response of output scatterings was different if moving away from the resonance. At the resonant wavelength, i.e. $\Delta = 0$, two output grating couplers manifested consistently in-phase on/off light scatterings for $O_1$ and $O_2$ in spite of interferometric control of the control (Fig. 4-8a),
wheras if $\Delta \neq 0$, output light scatterings became out-of-phase as different on/off relations were observed for $O_1$ and $O_2$ (Figs. 4-8b-c). This was because an additional phase shift was inherently associated with the Floquet-Bloch periodic boundaries due to the periodic nature of the modulation in the waveguide. Moreover, the Floquet-Bloch periodic boundaries caused the sign of the phase shift reversed if the operating wavelength crossed over the boundary of the Brillouin zone. Hence, output light scatterings showed opposite out-of-phase on/off responses with respect to interferometric control of the control at $\Delta < 0$ (Fig. 4-8b) and $\Delta > 0$ (Fig. 4-8c).
Fig. 4-8. Phase responses of outputs in light-light switching. (a) When operated at the resonance wavelength, two outputs oscillate in phase and reach their minimum simultaneously at $\varphi = \pi/2$, where almost no light is scattered from the two grating couplers of outputs. As the phase difference is flipped to $\varphi = -\pi/2$, peak output scattering from both grating couplers is obtained. (b) When operated at off-resonance wavelength $\Delta = -4.2$ nm, due to extra phase shift, the two output oscillations move forward. Since the output $O_2$ accumulates more phase than $O_1$, the responses are no more in phase and thus the two outputs reach minimum or maximum asynchronously. (c) When operated at off-resonance wavelength $\Delta = 4.6$ nm, the extra phase
shift changes sign and results in a shift of output oscillations in opposite direction. Note that neither of the output power can be completely eliminated at $\Delta \neq 0$ regardless of the phase tuning. The output power is normalized to the total incident power $I_1 + I_2$ in the plots.

4.3 Summary

Instead of engraving optical non-Hermiticity to the Hermitian topological features, this chapter showed the unique topology of the non-Hermitian exceptional point facilitates intrinsic sensitivity enhancement and highly asymmetric light transport. We exploited this non-Hermitian topology and engineered sensitive thermal imaging and power-efficient interferometric light modulator.
CHAPTER 5 CONCLUSIONS AND OUTLOOK

In this dissertation, we explored the synergy of topological photonics and non-Hermitian physics, leading to new possibilities of robust light control that have no counterparts in Hermitian systems. We offered an approach to stably enforce single-mode lasing by selectively enhancing the topological state by careful manipulation of gain and loss on an active optical platform with feedback mechanism. Beyond the selection of the topological confinement, we demonstrated the creation of topological light transport channels inside the bulk of a photonic topological insulator by dynamically configured spatial pumping patterns. The non-Hermitian reconfiguration of the topological states of light paves the avenue towards ultra-flexible and defect-immune light guiding for next-generation optical communication and information processing on integrated photonic chips. The unique non-Hermitian topology of exceptional point, i.e. the nonlinear quantitative variation in its vicinity and the highly asymmetric light scattering, can also facilitate novel photonic devices such as optical thermal sensors and asymmetric light modulators.

Our strategy of coupling non-Hermiticity with photonic topology provides new paradigms of designing photonic devices in the future generations. For example, non-blocking optical switches and routers with multiple simultaneous input-output optical data links are in urgent demand in high-density data centers. While the state-of-the-art devices based on Mach-Zehnder interferometers suffer crosstalk and instability with increasing layout complexity [78-83], our non-Hermitian reconfigurable photonic topological insulators provide a solution to these limits as the simultaneous robust topological architecture can be completely accommodated inside the footprint. Moreover,
the ultra-flexibility of the topological light transport may facilitate more efficient routes to
the emerging field of optical computation, promising a higher speed in artificial
intelligence over the electronic counterpart [84-87].


72. Y. D. Chong, L. Ge, H. Cao, A. D. Stone. Coherent perfect absorbers: time-reversed


