Essays On Foreign Exchange Rates

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Essays On Foreign Exchange Rates

Abstract
The foreign exchange rate is one of the most important asset prices in the international financial market. My dissertation studies the determination of exchange rates from the perspective of levered financial institutions and the frictions they are facing. It consists of two chapters that shed light on the importance of levered financial institutions in exchange rate determination.

In Chapter 1, I propose an intermediary-based explanation of the risk premium of currency carry trade in a model with a cross-section of small open economies. In the model, bankers in each country lever up and hold interest-free cash as liquidity buffers against funding shocks. Countries set different nominal interest rates, while low interest rates encourage bankers to take high leverage. Consequently, bankers’ wealth drops sharply with a negative shock. This reduces foreign asset demand and leads to a domestic appreciation, which in turn makes low-interest-rate currencies good hedges. The model implies covered interest rate parity deviations when safe assets differ in liquidity. The empirical evidence is consistent with the main model implications: (i) Low-interest-rate countries have high bank leverage and low currency returns; (ii) the carry trade return is procyclical with a positive exposure to the bank stock return; and (iii) the carry trade CAPM beta increases with the stock market volatility.

In Chapter 2 (joint with Yang Liu), we study how time-varying volatility drives exchange rates through financial intermediaries’ risk management. We propose a model where currency market participants are levered intermediaries subject to value-at-risk constraints. Higher volatility translates into tighter financial constraints. Therefore, intermediaries require higher returns to hold foreign assets, and the foreign currency is expected to appreciate. Estimated by the simulated method of moments, our model quantitatively resolves the Backus-Smith puzzle, the forward premium puzzle, the exchange rate volatility puzzle, and generate deviations from covered interest rate parity. Our empirical tests verify model implications that volatility and financial constraint tightness predict exchange rates.

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ESSAYS ON FOREIGN EXCHANGE RATES

Xiang Fang

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To my parents
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career goal.
ABSTRACT

ESSAYS ON FOREIGN EXCHANGE RATES

Xiang Fang

Urban Jermann and Nikolai Roussanov

The foreign exchange rate is one of the most important asset prices in the international financial market. My dissertation studies the determination of exchange rates from the perspective of levered financial institutions and the frictions they are facing. It consists of two chapters that shed light on the importance of levered financial institutions in exchange rate determination.

In Chapter 1, I propose an intermediary-based explanation of the risk premium of currency carry trade in a model with a cross-section of small open economies. In the model, bankers in each country lever up and hold interest-free cash as liquidity buffers against funding shocks. Countries set different nominal interest rates, while low interest rates encourage bankers to take high leverage. Consequently, bankers’ wealth drops sharply with a negative shock. This reduces foreign asset demand and leads to a domestic appreciation, which in turn makes low-interest-rate currencies good hedges. The model implies covered interest rate parity deviations when safe assets differ in liquidity. The empirical evidence is consistent with the main model implications: (i) Low-interest-rate countries have high bank leverage and low currency returns; (ii) the carry trade return is procyclical with a positive exposure to the bank stock return; and (iii) the carry trade CAPM beta increases with the stock market volatility.

In Chapter 2 (joint with Yang Liu), we study how time-varying volatility drives exchange rates through financial intermediaries’ risk management. We propose a model where currency market participants are levered intermediaries subject to value-at-risk constraints. Higher volatility translates into tighter financial constraints. Therefore, intermediaries require higher returns to hold foreign assets, and the foreign currency is expected to appreciate. Estimated by the simulated method of moments, our model quantitatively resolves the Backus-Smith puzzle, the forward premium puzzle, the exchange rate volatility puzzle, and generate deviations from covered interest rate parity. Our
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Chapter 1

Intermediary Leverage and Currency Risk Premium

1.1 Introduction

Investors earn sizable excess returns on average by borrowing low-interest-rate currencies (such as Japanese yen) and investing in high-interest-rate currencies (such as Australian dollar). This investment strategy is famously known as carry trade. One widely-used approach to explain the phenomenon is based on risk (Lustig and Verdelhan, 2007; Lustig, Roussanov, and Verdelhan, 2011), which argues that low-interest-rate currencies are less risky than high-interest-rate ones. In other words, low-interest-rate currencies appreciate relative to high-interest-rate ones in bad times. However, there is no consensus yet on the sources of currency risks. In this paper, we propose a liquidity-leverage channel on how nominal interest rates affect currency risk premia through financial intermediaries. We build a dynamic model in which low-interest-rate currencies appreciate in bad times relative to high-interest-rate ones. As a result, investors require a positive risk premium to conduct carry trades.

Our model features a cross-section of small open endowment economies that differ in their nominal interest rates as exogenous policy choices. These economies are separate from each other
and face a common foreign security whose foreign currency denominated return is exogenously
given. In each economy, there are two types of agents (h and b) with heterogeneous preferences: the type-h agent has higher risk aversion and lower intertemporal elasticity of substitution (IES) than the type-b agent \(^1\). Throughout the paper, the type-b agent is called a “banker” and the type-h is a “household.” In equilibrium, the type-b agent borrows from the type-h by issuing a riskless deposit, and thus plays the role of a bank in the economy. While taking leverage, bankers face a liquidity shock (Drechsler, Savov, and Schnabl, 2018), which leads to the request of withdrawal from the bankers. Since the risky assets held by the bankers are illiquid and liquidating them incurs a large firesale loss, bankers voluntarily hold interest-free cash as liquid assets to be insulated from being affected by the funding shock. Consequently, the interest rate represents the opportunity cost of leverage, and bankers in low-interest-rate countries take high leverage.

When a negative endowment shock hits a low-interest-rate country, the wealth share of bankers sharply decreases in that country, because the bankers are highly levered. Since bankers have higher IES than households do, the wealth-weighted consumption-wealth ratio of that country increases, which induces a sharp decrease of the foreign asset demand. The domestic currency consequently appreciates, making it a good hedge against adverse endowment shocks. We call this channel the **liquidity-leverage** channel throughout this paper. On the contrary, if the same negative shock hits a high-interest-rate country with low bank leverage, the response of bankers’ wealth share is weaker, thus the exchange rate movement is milder. Therefore, the high-interest-rate currencies are not as good hedges as the low-interest-rate ones, and investors require a positive premium for the carry trade.

Our model has rich implications for the factor structures of the asset returns. Lustig et al. (2011) empirically show that the cross-section of currency portfolio returns can be priced by the carry risk factor, defined as the spread between returns of investing in high- and low-interest-rate currencies. In our model, the carry risk factor measures the magnitude of the common endowment shock faced by all countries: When the shock is positive, the carry risk factor becomes large. Consequently, low-

---

1. Empirical evidence shows that investors that hold more risky assets have higher IES (Vising-Jørgensen, 2002). Guvenen (2009) proposes a model with high-IES stock-holders, matching both asset prices and macro dynamics quantitatively.
interest-rate currencies depreciate when the carry risk factor is large, consistent with the empirical finding.

Furthermore, our model delivers a factor structure of stock valuations (Colacito, Croce, Gavazzoni, and Ready, 2018a) that stock prices of low-interest-rate countries load more on the global risk. In our model, when the same shock hits all the economies, stock valuations of low-interest-rate countries rise more because of high bank leverage.

We derive a new factor structure of the currency returns with the cross-sectional average of bankers’ return to wealth in the model. When a common positive shock hits all the countries, bankers’ wealth increases in every country while the magnitude declines with the interest rate and bank leverage. The heterogeneity in wealth redistribution across countries lead to low-interest-rate currencies’ depreciation relative to high-interest-rate ones in good times. In other words, carry trade is procyclical; and the exposure of currency carry trade return to bankers’ return to wealth increases with the interest rate. In our empirical analysis, we verify this implication by measuring the return to bankers’ wealth with the banking sector stock return in each country.

Dynamically, the factor structure of the exchange rate and the average stock return is time-varying. The time variation is first noted by Lustig, Roussanov, and Verdelhan (2008), wherein that the currency carry trade CAPM beta is particularly high during a crisis. Since our model does not have a clear definition of a crisis, we examine the correlation between currency carry trade CAPM beta and the stock market volatility. In the model, CAPM beta increases with the stock market volatility. The reason is simple: exchange rates co-move with stock returns when the amplification effect brought by leverage is strong. This is exactly when the endogenous volatility of stock return is high. We document this feature in the data in the empirical analysis. The implication of the dynamic property of CAPM beta distinguishes our model from most of the existing literature.

After exploring the cross-sectional implications on interest rates, bank leverage, exchange rates, and stock prices, we analyze the time-series relationship between interest rates and currency risk premia by introducing stochastic interest rates into the model. If a country experiences an unexpected rise in the interest rate, bankers in that country take less leverage and the required risk
premium to invest abroad decreases. This result is in line with the empirical finding of Mueller, Tahbaz-Salehi, and Vedolin (2017), wherein that the subsequent return of investing abroad is lower (higher) after the Federal Open Market Committee raises (cuts) the interest rate. Moreover, our model implies that the regression coefficient of foreign excess return on the interest rate differential between foreign and domestic currency is positive, consistent with the long-standing literature on the “forward premium puzzle”.

Our model implies that deviation from covered interest rate parity can be due to liquidity reasons. Du, Tepper, and Verdelhan (2018b) document the large and persistent deviations from covered interest rate parity (CIP) after the financial crisis, while Jiang, Krishnamurthy, and Lustig (2018) and Du, Im, and Schreger (2018a) show that dollar safe assets have privilege over foreign ones. In our model, the return to CIP arbitrage with a long dollar position is negative if the dollar safe assets are more liquid.

Despite the parsimony of the liquidity friction that only cash is held as the liquid asset, our model captures a wide range of regulation and market discipline practices concerning liquidity. The regulations include reserve requirement ratio for depository institutions, and more recent liquidity coverage ratio required by the Basel III framework. Apart from regulations, financial institutions also voluntarily hold liquid assets. In our model, we have the interest-free cash as the single type of liquid assets available. However, the model can be easily extended to incorporate other near-money safe assets that provide liquidity, such as government treasuries. The different liquidity properties of these assets lead to different liquidity premia. Nagel (2016) presents detailed empirical evidence to show that liquidity premia of near-money assets are tightly linked to the nominal interest rate.

Our liquidity-leverage mechanism is different from other models of cross-sectional currency risk premia. Existing studies (For example, Hassan, 2013; Richmond, 2018; Ready, Roussanov, and Ward, 2017) propose some source of fundamental heterogeneity (e.g, country size, trade network centrality, and composition of trade, respectively) across countries that simultaneously drive interest rates and currency risk premia. While most advanced economies use the nominal interest rate as their monetary policy tools, the literature is largely silent on how nominal interest rates are translated into exchange rate properties. In our paper, the nominal interest rates are taken as exogenous policy
choices and real exchange rate dynamics are determined by the leverage of bankers, which in turn, depends on nominal interest rates. Our *liquidity-leverage* channel thus complements the existing literature.

The empirical analysis of this paper is centered around three main model implications: (i) low-interest-rate countries have high bank leverage and low currency returns; (ii) the carry trade return is procyclical with a positive exposure to bank stock return; (iii) the carry trade CAPM beta increases with the stock market volatility.

As the first-pass validation of implication (i), figure 1.1 plots the cross-sectional relation of bank capital ratio (the inverse of bank leverage, in percent) with average forward discounts (interest rate differentials) and currency returns vis-a-vis the dollar for the ten most liquid currencies. A positive slope stands out in both graphs: Countries with lower bank capital ratio (higher leverage) tend to have lower interest rates and currency returns. The typical carry trade funding country (Japan) has substantially lower capital ratio than the typical investment country (Australia). In the empirical section, we show the same relationship for 22 advanced economies through panel regressions. The relationship is robust after controlling for each country’s inflation and GDP. Following standard asset pricing practice, we sort currencies into portfolios based on bank leverage, and find that currency returns monotonically increase from high leverage portfolios to low leverage portfolios. The spread between the lowest and highest leverage portfolios, the “Lev-factor,” is 2.0 percent per annum. Heterogeneous exposures to the “Lev-factor” accounts for the different expected returns of currency portfolios and the “Lev-factor” has a significantly positive risk price, which is consistent with our model implication.

To validate implication (ii), we construct a measure of the common endowment shock by averaging the banking sector stock returns across countries in the sample. The use of banking sector return is to highlight the role of banks in driving exchange rates, while we show the same results hold if we replace bank stock returns with country-level stock return indices in the appendix. Exposures of currency returns to the bank stock return are in line with countries’ average interest rate—low-interest-rate currencies have lower (more negative) exposures than high-interest-rate ones. In other words, carry trade is highly procyclical.
Lastly, we examine implication (iii), the time variation of carry trade CAPM beta. We construct the market risk factor by averaging the MSCI country indices and calculate the carry trade CAPM betas based on five-year rolling windows. These betas increase in the average stock return volatility, computed as the realized volatility of monthly average stock returns in the same five-year window.

This paper makes three contributions to the literature. First, we provide a new explanation of the currency risk premium featuring the liquidity-leverage channel, with nominal interest rates as exogenous monetary policy choices. Our model can accommodate many empirical findings in the literature, including the factor structure of currency and stock returns, and CIP deviation. Second, our model can simultaneously explain the time-series and cross-sectional relationship between interest rates and risk premia. Third, our empirical evidence is in line with the main model implications, especially the novel relation between interest rate, bank leverage, and currency returns in the cross-section.

**Related Literature**

Our paper is closest to three papers in the literature. Backus, Gavazzoni, Telmer, and Zin (2013) link currency risk premium to monetary policy heterogeneity across countries, but the real side of the economy is not affected by monetary policy. In our model, monetary policy affects currency risk premium in real terms. Maggiori (2017) shows that currencies of countries with a more developed financial system (such as US) appreciate in bad times, while Malamud and Schrmpf (2018) derive exchange rate dynamics from a model with monopolistic intermediaries. Neither of these two papers links currency risk premia to different countries’ interest rate levels. In both of the two papers, only intermediaries have access to the foreign asset while we relax this assumption to allow both households and bankers to freely trade the foreign asset. Moreover, our paper provides empirical evidence in support of the key mechanism of our model, while both Maggiori (2017) and Malamud and Schrmpf (2018) are theoretical studies.

Lustig and Verdelhan (2007) and Lustig et al. (2011) document the systemic variation of currency returns for cross-sectional interest rate sorted portfolios and construct pricing factors. However, these papers are silent on how economic fundamentals determine exchange rates. Economic
explanations to sources of currency risk premium highlight the fundamental heterogeneity across
countries, including monetary policy (Backus et al., 2013), country size (Hassan, 2013), trade net-
work centrality (Richmond, 2018), composition of international trade (Ready et al., 2017), long
run risk exposure (Colacito et al., 2018a), fiscal cyclicality (Jiang, 2018), and dollar external debt
(Wiriadinata, 2018). Our paper takes a different perspective by taking nominal interest rates as ex-
ogenous policy choices and proposes the liquidity-leverage channel that links nominal interest rate
and currency risk premium.

On the other hand, the literature on time-series violation of uncovered interest rate parity has
been vast since Fama (1984). The robust positive comovement between currency returns and inter-
est rate differentials poses challenges to exchange rate modeling. Early studies with the asset pricing
approach include Bekaert (1996), Bansal (1997), Backus, Foresi, and Telmer (2001), and Brennan
and Xia (2006). Structural macro-finance models explain the forward premium puzzle under a fric-
tionless financial market (Verdelhan, 2010; Stathopoulos, 2016; Bansal and Shaliastovich, 2013;
Colacito and Croce, 2013; and Farhi and Gabaix, 2016). Alternatively, some studies introduce fi-
nancial market imperfections into standard models, such as Alvarez, Atkeson, and Kehoe (2009);
Favilukis and Garlappi (2017); Gabaix and Maggiori (2015); and Fang and Liu (2018), and so on.
Valchev (2017) explains deviation from uncovered interest rate parity by introducing a convenience
yield difference between the domestic and foreign bonds. In our paper, there is no liquidity differ-
ence between the domestic deposit and the foreign bond. Instead, nominal interest rate (liquidity
premium) affects the cyclicality of exchange rates through changing the leverage choices of the
bankers in the economy.

Recent literature highlights the role of intermediaries in exchange rate determination. For in-
stance, empirical asset pricing literature shows the pricing power of intermediary variables on cur-
currency returns (Adrian, Etula, and Shin, 2015; Adrian, Etula, and Muir, 2014; He, Kelly, and Manela,
2017; Haddad and Muir, 2017). Sandulescu, Trojani, and Vedolin (2017) derive model-free interna-
tional SDFs and show they are highly correlated with intermediary variables. Itskhoki and Mukhin
(2017) demonstrate that the financial asset demand wedge is crucial in explaining exchange rate dy-
namics. Our model highlights the liquidity frictions faced by intermediaries, while the effect of the
friction depends on nominal interest rates. Moreover, our model explains the relationship between interest rates and currency risk premia in both cross-sectional and time-series dimensions.

Our paper is related to the literature on the risk-taking channel of monetary policy. The model framework and the liquidity-leverage channel is similar to Drechsler et al. (2018), while we extend the model to the open economy. Bruno and Shin (2015) and Dell’Ariccia, Laeven, and Suarez (2017) show that banks play important roles in the risk-taking effect of monetary policy. Rey (2016) and Miranda-Agrippino and Rey (2018) demonstrate that intermediary credit and leverage are important in monetary transmission across the border. Our liquidity-leverage channel thus bridges the literatures on the risk-taking effect of monetary policy and currency risk premium.

Methodologically, our model falls into the category of international finance models with endogenous asset prices and portfolio decisions. Coeurdacier (2009), Devereux and Sutherland (2011), and Stepanchuk and Tsyrennikov (2015) are models solved in discrete time, while Baxter, Jermann, and King (1998), Jermann (2002), Pavlova and Rigobon (2007), and Pavlova and Rigobon (2012) solve continuous-time versions of such models. Our model is in continuous time, but the solution is obtained by numerically solving the fixed-point of an incomplete market economy, which is different from Pavlova and Rigobon (2012).

The rest of the paper is organized as follows. Section 1.2 lays out the structure of the model and derives equilibrium conditions. Section 1.3 shows the solutions and simulation results of the model. Section 1.4 provides empirical evidence on the main model implications. Section 1.5 concludes the paper.

### 1.2 The Model

The world consists of a cross-section of $N + 1$ small open economies. Each economy is populated with two types of agents. Both agents consume two goods: a country specific nontraded good and a common traded good. Country 1 is labeled as the provider of a global riskless asset denominated in its local nontraded good. Agents in all economies have access to the global riskless asset. We

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2. See Coeurdacier and Rey (2013) for a complete survey of the literature.
abstract away the optimization behavior of Country 1’s agents, and exogenously assume the relative price of country 1’s nontraded good in the traded good. The remaining $N$ countries are symmetric, except that they have different nominal interest rates. In the next section, we describe the economic environment of a typical small open economy.

1.2.1 Environment

Consider a representative small open endowment economy. There are two types of goods in the economy: nontraded goods ($X$) and traded goods ($Y$). Traded goods can be exported to or imported from other countries. Time is continuous and infinite. Uncertainty is characterized by a standard filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}, \{\mathcal{F}_t\})$, to which all stochastic processes are adapted. The structure of the model is close to Drechsler et al. (2018) except for its open economy features.

Endowment

The economy is endowed with a tree that pays dividends in both nontraded and traded goods. The dividend process of the nontraded good ($X$) follows a geometric Brownian motion:

$$\frac{dX}{X} = \mu_x dt + \sigma_x dB_x$$

(1.1)

$\mu_x, \sigma_x$ are positive constants. The traded good endowment ($Y$) is given by:

$$Y = \bar{\tau}X$$

(1.2)

with $\bar{\tau}$ being a constant.

Agents

The economy is populated with two types of agents ($h$ and $b$). Both agents have recursive utility, as in Duffie and Epstein (1992). We denote $\psi_j$ and $\gamma_j$ as agent $j$’s intertemporal elasticity of substitution (IES) and risk aversion. All parameters without superscript $j$ are common to both agents. The
lifetime utility of agent $j$ at time $t$, $V^j_t$ is given by:

$$V^j_t = \int_t^{\infty} f^j(C^j_\tau, V^j_\tau) d\tau$$

(1.3)

where:

$$f^j(C^j, V^j) = \frac{1 - \gamma^j}{1 - \psi^j} V^j \left\{ \left[ \frac{C^j}{((1 - \gamma^j) V^j)^{1/\psi^j}} \right]^{1 - \frac{1}{\psi^j}} - \rho \right\}$$

(1.4)

$\rho$ is the time discount factor. $C^j$ is agent $j$’s consumption basket, a constant elasticity of substitution (CES) aggregation of nontraded and traded good consumption:

$$C^j = \left[ \alpha(C^j_x)^{\theta - 1} + (1 - \alpha)(C^j_y)^{\theta - 1} \right]^{\frac{1}{\theta - 1}}$$

(1.5)

$\alpha$ measures the relative preference of the nontraded good over the traded good. $\theta$ is the elasticity of substitution between the two goods.

The two agents differ in both risk aversion and IES. Type-$b$ agents have lower risk aversion and larger IES than type-$h$ agents: $\gamma_h > \gamma_b$ and $\psi_h < \psi_b$. The assumption implies that it is more desirable for type-$h$ agents to have a smooth consumption profile across states of the world as well as across time. This assumption is consistent with the empirical evidence that investors with more holdings of risky asset have higher IES (Vissing-Jørgensen, 2002). Moreover, it is consistent with the assumption made by Guvenen (2009)—that stock-holders have higher IES. In equilibrium, type-$b$ agents borrow from type-$h$ agents in the form of riskless deposits (denominated in local nontraded good) and essentially play the role of banks in the economy. In the rest of the paper, I will call type-$h$ agents as “Households,” and type-$b$ agents as “Bankers.”

We assume that a $\kappa$ fraction of agents exit the market with the same measure of agents newly born from $t$ to $t + dt$. The wealth of exiting agents is transferred to the newly borns on an equal

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3. In Guvenen (2009), the two types of agents are homogeneous in risk aversion but one type is excluded from holding stocks. In our model, heterogeneity in risk aversion endogenously generates the heterogeneous holdings of the risky asset across the two types.

4. In our model, bankers represent broadly defined commercial banks, investment banks, hedge funds, broker dealers, and other leveraged financial institutions.
per capita basis. This assumption is standard in the two-agent dynamic models since it prevents one type of agent from dominating the other.

**Assets**

Each agent has four assets to choose from: a riskless asset denominated in the local nontraded good (deposit), a claim to the local tree (local stock), a foreign security that is risk free in Country 1’s nontraded good (foreign bond), and the country specific cash that pays zero nominal interest rate (cash). The return characteristics of these assets are as follows.

Deposit is risk free in terms of the local nontraded good, with an instantaneous real risk-free rate \( r_t \) that is endogenously determined by the intertemporal choices of the agents.

The return process of the local stock is:

\[
dR_s = \frac{X + QY}{P} dt + \frac{dP}{P} \equiv \mu_s dt + \sigma_s dB
\]

(1.6)

where \( P \) is the price of the local stock denominated in the nontraded good, while \( \frac{1}{Q} \) is the relative price of the local nontraded good in the traded good. \( \mu_s \) and \( \sigma_s \) are the endogenous drift and diffusion of the local stock return process. We group all the Brownian shocks into the vector \( B \), to be specified toward the end of this section.

The instantaneous return to foreign bond in Country 1’s nontraded good is \( r^* \). The price of Country 1’s nontraded good \( \frac{1}{Q^*} \) follows the following exogenous process:\5.

\[
\frac{dQ^*}{Q^*} = \sigma^* dB^* \equiv \sigma^* dB
\]

(1.7)

With a postulated endogenous \( Q \) process:

\[
\frac{dQ}{Q} = \mu_q dt + \sigma_q dB
\]

(1.8)

---

5. Any drift in the \( Q^* \) process can be absorbed in \( r^* \).
We can express the foreign bond return process in the local nontraded good as:

$$dR^*_f = (r^* + \mu_q + \sigma'q^*)dt + (\sigma_q + \sigma^*)dB \equiv \mu_f dt + \sigma_f dB$$  \hspace{1cm} (1.9)

$\mu_f, \sigma_f$ are the endogenous drift and diffusion of the foreign bond return in the local nontraded good.

Agents can hold cash that does not pay interest. We introduce a friction to create an incentive for agents to hold cash despite its zero interest. When an agent borrows to buy illiquid assets, she holds cash as the liquid asset to be insulated from the funding shock. The details are laid out in section 1.2.2.

**Nominal Interest Rates as Policy Choices**

We consider two versions of the model. The baseline version features constant nominal interest rates, while in the second version, we introduce nominal interest rate shocks. Central banks use nominal interest rates as policy tools. When stochastic, nominal interest rates follow Ornstein-Uhlenbeck processes:

$$di = \zeta(i_0 - i)dt + \sigma_i dB_i$$  \hspace{1cm} (1.10)

Different countries have different means of nominal interest rate, $i_0$. $\zeta > 0$ governs the persistence of nominal interest rates, and $\sigma_i$ controls their volatility.

To summarize, there are two Brownian shocks in this economy if nominal interest rates are fixed: $B_x$ and $B_q^*$. Thus, the vector $B$ is defined as a column vector:

$$B = [B_x, B_q^*]'$$  \hspace{1cm} (1.11)

If nominal interests are stochastic, we have an additional interest rate shock: $B_i$.

Our model features incomplete financial market in two aspects. First, agents in different countries are segmented: they do not have access to the trees in other countries. Second, cash holding impedes risk sharing across agents in one country.
Without loss of generality, throughout the paper, we define “real exchange rate” as the reciprocal of the country-specific nontraded good price in the traded good \( Q \).

### 1.2.2 The Liquidity Friction

This section analyzes in detail the main friction in the economy: the liquidity friction. In every instant from \( t \) to \( t + dt \), a Poisson funding shock hits the economy at rate \( \eta \). Once hit, all lenders (both domestic and foreign) will request for a withdrawal of \( \frac{\lambda}{1+\lambda} \) fraction of their assets.

The borrowers can hold four assets, while each asset differs in its liquidity. “Liquidity” is defined as how much proportion of an asset that can be liquidated to meet withdrawal needs costlessly when the funding shock hits the economy. The liquidity of each asset is as follows:

- The \( \frac{\lambda}{1+\lambda} \) fraction of deposit or foreign bond can be liquidated without additional costs. If an agent wants to sell more, she has to pay a firesale loss of \( \phi \) for each unit. Local stocks can be sold only at a fire sale price with loss \( \phi \) per unit, while all cash can be used to meet the withdrawal needs costlessly.

To summarize, the assets are ranked according to liquidity as:

- Cash > Deposit = Foreign bond > Local stock

We define \( L^D_j \) as the liquidity need of agent \( j \) when the funding shock hits, and \( L^H_j \) as the available assets owned by agent \( j \) that can be liquidated costlessly.

We denote \( w_{ph}^j, w_{pf}^j, w_c^j \) as agent \( j \)’s portfolio share in local stock, foreign bond, and cash, so that \( 1 - w_{ph}^j - w_{pf}^j - w_c^j \) is her portfolio share in the deposit. The liquidity need (in portfolio share) when the funding shock hits is:

\[
L^D_j = \frac{\lambda}{1+\lambda} \left[ \max\{w_{ph}^j + w_c^j + w_{pf}^j - 1, 0\} + \max\{-w_{pf}^j, 0\} \right]
\]

The first term is the liquidity need from domestic lenders, while the second term is the liquidity
need from foreign lenders. Meanwhile, the available share of liquid assets agent \( j \) holds is:

\[
L_H^j = \frac{\lambda}{1 + \tilde{\lambda}} \left[ \max \left\{ 1 - w_h^j - w_c^j - w_f^j, 0 \right\} + \max \{ w_f^j, 0 \} \right] + w_c^j
\]  

(1.13)

The term in the bracket is the amount of deposit and foreign bond that can be used to repay without cost. All cash \( w_c^j \) can be used to meet the liquidity need.

If an agent holds more liquid assets than needed (\( L_H^j > L_D^j \)), she can fully protect herself from the funding shock. Otherwise, she has to fire-sell \( L_D^j - L_H^j \) of illiquid assets and incur a loss.

The function of cash in the economy is to provide liquidity to insulate borrowers from the funding shock. If agent \( j \) holds one unit of cash, an additional liquidity of \( \frac{\lambda}{1 + \tilde{\lambda}} \) is provided. In fact, our model can incorporate a wide range of safe assets that differ in their liquidity properties. The yields of these safe assets are determined by how much liquidity they can provide. Consider another safe asset (e.g., treasury bill) that wherein \( \frac{\lambda}{1 + \tilde{\lambda}} \) fraction can be liquidated costlessly. The nominal yield to treasury bill \( x \) satisfies:

\[
\frac{i}{1 - \frac{\lambda}{1 + \tilde{\lambda}}} = \frac{i - x}{\tilde{\lambda} + \lambda}
\]  

(1.14)

The left-hand side is the price of liquidity for cash, while the right-hand side is the price of liquidity for treasury bills. Suppose borrowers are indifferent in holding the two assets, then the price of liquidity should be identical across the two assets. Therefore, we can solve for the following:

\[
x = (1 - \tilde{\lambda} + \lambda)i
\]  

(1.15)

If \( \tilde{\lambda} > \lambda \), the treasury bills are more liquid than the deposits. Thus, the treasury yield is smaller than the nominal interest rate.

We abstract from modeling the different liquidity properties between the foreign bond and the deposit, so that the excess return to the foreign bond only consists of a risk premium component. This assumption also simplifies analysis by ensuring all policy functions to be smooth.
1.2.3 Agents’ Optimization Problem

In this section, we characterize the optimization problem for agent \(j (j = h, b)\). The nominal price of nontraded good (numeraire) is assumed to be locally deterministic. Agent \(j\) solves the following optimization problem:

\[
V^j_0 = \max_{c^j, c'^j, w^j, w'^j} E \int_0^\infty f^j(C^j_t, V^j_t) dt
\]

s.t.: \[
dW^j/W^j = \left[ r - c^j_t - Qc^j_t + w^j_t(\mu_s - r) + w'^j_t(\mu_f - r) + w'^j_t(-i) \right] dt + \left( w^j_t\sigma_s + w'^j_t\sigma_f \right)' dB
\]

\[+ \Pi^j dt - \frac{\phi}{1 - \phi} \max(L^j_D - L^j_H, 0) dN \]

(1.16)

(1.17)

Agent \(j\) chooses optimal consumption of both the nontraded good \(c^j_t\) and the traded good \(c'^j_t\), as well as portfolio shares \(w^j_t, w'^j_t\), taking return processes as given. \(c^j_t, c'^j_t\) are consumption \(C^j_t, C'^j_t\) scaled by wealth \(W^j\). \(N\) is a non-decreasing count process with arrival rate \(\eta\), which represents the funding shock. \(\Pi^j\) is the lump-sum wealth transfer to agent \(j\).

\(L^j_D\) is the liquidity demand for withdrawal when the funding shock hits, and \(L^j_H\) is agent \(j\)'s available liquid assets, defined in equations (1.12) and (1.13). If the funding shock hits, agent \(j\) has to fire sell \(L^j_D - L^j_H\) of illiquid assets and incurs a loss of \(\phi/1 - \phi\) per unit of fire sale.

The following proposition characterizes the HJB equation for agent \(j\):

**Proposition 1.** The value function of agent \(j\) can be written as:

\[
V^j(W^j; \Omega) = \frac{(W^j)^{1 - \gamma_j}}{1 - \gamma_j} \left[ G^j(\Omega) \right]^{1 - \gamma_j} \]

(1.19)

\(G^j(\Omega)\) satisfies the following Partial Differential Equations (PDE):

\[
0 = \max_{c^j, c'^j, w^j, w'^j} \left\{ \frac{1}{1 - \psi_j} \left( \frac{c^j}{G^j} \right)^{1 - \frac{1}{\psi_j}} G^j - (\rho + \kappa) \right\} + \mu^j_w - \frac{1}{2} \psi_j(\sigma^j_w)'\sigma^j_w + \frac{1}{1 - \psi_j} \nabla G^j \mu \Omega
\]

15
\[
+ \frac{1}{2} \frac{1}{1-\gamma_j} \text{tr}(\Sigma'_{\Omega} \Sigma'_{\Omega} \mathcal{H}G^j) + \frac{1}{1-\psi_j} \psi_j - \frac{1}{1-\psi_j} \psi_j \nabla G^j \Sigma'_{\Omega} \Sigma_{\Omega} (\nabla G^j)' + \frac{1}{1-\psi_j} \psi_j \nabla G^j \Sigma'_{\Omega} \sigma'_{\omega} + \eta (V^j_+ - V^j)
\]

(1.20)

where \( c^j \) is defined in equation (1.18), and:

\[
\mu_w^j = r - c^j_s - Qc^j_s + w^j_s (\mu_s - r) + w^j_f (\mu_f - r) - w^j_i + \Pi^j
\]

(1.21)

\[
\sigma_{\omega}^j = w^j_s \sigma_s + w^j_f \sigma_f
\]

(1.22)

\( \Omega \) represents the vector of aggregate state variables. \( \Pi^j \) is the lump-sum wealth transfer. \( \nabla G^j \) is the gradient (row) vector of function \( G^j \), while \( \mathcal{H}G^j \) is the Hessian matrix of \( G^j \). \( \mu_\Omega \) is the column vector for state variable drift, while \( \Sigma_\Omega \) is the matrix for state variables’ diffusions, with each column representing one state variable’s exposures to various Brownian shocks. \( V^j_+ = V^j(W^j_+, \Omega) \), and \( W^j_+ \) is the wealth of agent \( j \) following a funding shock.

Note that agent \( j \) has \( \kappa \) probability to exit the market. It is equivalent to changing agent \( j \)’s time discount rate from \( \rho \) to \( \rho + \kappa \). A complete, term-by-term exhibition of the HJB equation can be found in Appendix .2, equation (44).

We assume that the nominal price of nontraded good is locally deterministic. This assumption serves as an equilibrium selection criterion and abstracts away the inflation risk.\(^6\)

We proceed in two steps: First, we derive agents’ optimal cash holding; second, we solve for the optimal consumption and portfolio choice decisions.

**Optimal Cash Holding**

The incentive to hold cash as liquid assets depends on the tradeoff between its benefit of avoiding a fire sale loss and nominal interest rate, the opportunity cost of holding cash. We impose the following restriction on parameters \( \eta, \lambda, \phi, i \).

---

\(^6\) Hollifield and Yaron (2003) show currency risk premium is mainly compensation for real risks instead of inflation risks.
**Assumption 1.** Parameters of \( \eta, \phi, \lambda \) satisfy\(^7\):

\[
\eta \frac{\phi}{1 - \phi} \frac{1}{1 + \lambda} > i \tag{1.23}
\]

The left-hand side is the expected fire sale loss if the borrower marginally reduces one unit of cash holding and incurs a firesale loss. The right-hand side—nominal interest rate—measures the opportunity cost of holding cash. Under Assumption 1, the benefit of holding cash overweighs the cost. Therefore, the fire sale will never happen in equilibrium, as shown in Proposition 2:

**Proposition 2.** Agent \( j \) chooses to insulate herself from the funding shock:

\[
\max(L_D^j - L_H^j, 0) = 0 \tag{1.24}
\]

A formal proof is provided in Appendix .1.1. The intuition for full insurance is that the benefit of avoiding fire sale overweighs the cost of holding cash.

The next proposition derives the optimal portfolio share on cash holding.

**Proposition 3.** Agent \( j \)'s optimal cash holding depends on her local stock holding

\[
w_c^i = \max\{\lambda (w_h^j - 1), 0\} \tag{1.25}
\]

The proof of the proposition is shown in Appendix .1.2. The intuition here is that, if agent \( j \)'s portfolio constitutes liquidity transformation, she is exposed to the funding shock and needs to hold cash for liquidity service. The only liquidity transformation occurs when the agent borrows (either from home or abroad) to buy the local stock. Therefore, only the portfolio share on the local stock matters for cash holding. If \( w_h^j < 1 \), there is no liquidity transformation, so that no cash holding is needed. If \( w_h^j > 1 \), agent \( j \) transforms \( w_h^j - 1 \) units of liquid liabilities to the illiquid local stock, and voluntarily holds cash proportionally. We assume that all the forgone interests are rebated back to the bankers.

---

\(^7\) This is a sufficient assumption (not necessary) to ensure agents to hold enough cash to protect themselves from being exposed to the funding shock.
Meanwhile, borrowing from home and investing in the foreign bond does not require additional cash holding, since the deposit and the foreign bond have the same liquidity. We make this simplifying assumption so that the excess return of foreign bond investment purely comes from the risk premium component.

**Optimal Consumption and Portfolio Choice**

The following two corollaries derive agents’ optimal consumption and portfolio decisions.

**Corollary 1.** Agent $j$’s optimality conditions for (1.20) is given by:

$$\alpha \left( \frac{c_j}{G_j} \right)^{-\frac{1}{\omega}} \left( \frac{c_j}{c_y} \right) = 1$$  \hspace{1cm} (1.26)

$$\left( 1 - \alpha \right) \left( \frac{c_j}{G_j} \right)^{-\frac{1}{\omega}} \left( \frac{c_j}{c_y} \right) = Q$$  \hspace{1cm} (1.27)

If $w_h^j \geq 1$:

$$\begin{bmatrix} w_h^j \\ w_f^j \end{bmatrix} = \frac{1}{\gamma_j} (\Sigma')^{-1} \begin{bmatrix} \mu_s - r - \lambda_i \\ \mu_f - r \end{bmatrix} + \frac{1 - \gamma_j}{1 - \psi_j} \frac{1}{\gamma_j} (\Sigma')^{-1} \Sigma_{\Omega} (\nabla G_j)^{'}$$  \hspace{1cm} (1.28)

$$w_c^j = \lambda (w_h^j - 1)$$  \hspace{1cm} (1.29)

If $w_h^j < 1$:

$$\begin{bmatrix} w_h^j \\ w_f^j \end{bmatrix} = \frac{1}{\gamma_j} (\Sigma')^{-1} \begin{bmatrix} \mu_s - r \\ \mu_f - r \end{bmatrix} + \frac{1 - \gamma_j}{1 - \psi_j} \frac{1}{\gamma_j} (\Sigma')^{-1} \Sigma_{\Omega} (\nabla G_j)^{'}$$  \hspace{1cm} (1.30)

$$w_c^j = 0$$  \hspace{1cm} (1.31)

where: $\Sigma = \begin{bmatrix} \sigma_s & \sigma_f \end{bmatrix}$, $\nabla G_j$ is the (row) vector of the gradient of $G_j$, and $\Sigma_{\Omega}$ is the matrix for state variables’ diffusions, with each column representing one state variable’s exposure to various Brownian shocks. $\Sigma_{\Omega} \Sigma$ is the covariance matrix between asset returns and state variables.

This corollary is a standard result, except for the risk premium term is augmented with $-\lambda_i$ if
We can see how the nominal interest rate affects portfolio decisions from equation (1.28). Throughout the model section, I define $w_j^h$ as leverage choice if it exceeds 1.8.

The households are more risk averse and insured by the bankers. As a result, $w_h^h > 1$, $w_h^c < 1$. Bankers borrow to buy the local stock9. As we analyze in section 1.2.2, bankers conduct liquidity transformation in their portfolios and hold cash for liquidity purposes. The nominal interest rate represents the cost of leverage taking. When the nominal interest rate is lower, bankers will demand more of the local stock.

Corollary 2 shows the Euler equation for asset $i$ and agent $j$:

**Corollary 2.** The risk premium required by agent $j$ for any asset return process $dR_i$ is given by:

$$RP_j^i = -\frac{d}{dt} \text{cov} \left( \frac{1 - \gamma_j}{1 - \psi_j} \frac{d(\tilde{P}c_j)}{Pc_j} - \gamma_j \frac{dW_j}{W_j} - (1 - \gamma_j) \frac{d\tilde{P}}{P}, dR_i \right)$$

(1.32)

where $W_j$ is the wealth of agent $j$ denominated in the nontraded good. $	ilde{P}$ is the price of the consumption basket, with:

$$\tilde{P}^{1-\theta} = \alpha^\theta + (1 - \alpha)^\theta Q^{1-\theta}$$

(1.33)

$\tilde{P}c_j \equiv \frac{\tilde{P}c_j W_j}{\tilde{P}c_j}$, represents the consumption wealth ratio of agent $j$.

The proof is provided in Appendix .1.3.

Corollary 2 is a standard result. The state price densities of agents with recursive preferences not only rely on instantaneous consumption growth, but also on the return to their wealth portfolios. Having multiple goods complicates the problem slightly by introducing $\tilde{P}$, the price of consumption basket into the state price density.

Note that for the local stock, $\mu_s - r = RP_h^h$ for households, but for bankers, the excess return of domestic stocks have two components: liquidity premium $\lambda i$ and risk premium $RP_h^b = \mu_s - r - \lambda i$.

---

8. A more precise definition of leverage is $w_j^h + w_j^c + w_j^f I(w_j^f > 0)$, where I is an indicator function. For the ease of illustration, we ignore the third term. We will choose the proper exogenous foreign asset return process so that both investors’ portfolio share on foreign asset is small relative to the portfolio share on the local stock. This is consistent with the observation that a majority of the wealth in a country is in its local assets, or the “home bias” literature (Lewis, 1999). The second term $w_j^c$ is proportional to $w_j^h - 1$ if it is positive.

9. In the numerical section, we solve the model with the conjecture that bankers take leverage with $w_h^b > 1$, and verify our conjecture after we obtain the solutions.
The risk premium component satisfies equation (1.32) for bankers. The excess return to the foreign bond satisfies equation (1.32) for both agents as it does not include a liquidity premium component. Particularly, for $j = h, b$:

$$RP_j = \mu_q + r^* - r + \sigma^* \sigma^* = -\text{cov} \left( \frac{1 - \gamma_j}{1 - \psi_j} \frac{d(Pc)}{Pe^j} - \frac{\gamma_j dW}{W^j} - (1 - \gamma_j) \frac{\tilde{P}}{P} \frac{dQ}{Q} \right)$$  \hspace{1cm} (1.34)

### 1.2.4 Market Clearing

The nontraded good market and the local stock market should clear:

$$C_h^b + C_b^b = X$$  \hspace{1cm} (1.35)

$$P = W^h w_h^h + W^b w_b^h$$  \hspace{1cm} (1.36)

Moreover, we define the total net foreign asset of the country $NFA$ as:

$$NFA = W^h w_j^h + W^b w_j^b$$  \hspace{1cm} (1.37)

The total wealth of the economy equals the value of the local stock plus the value of the net foreign asset:

$$W = W^h + W^b = P + NFA$$  \hspace{1cm} (1.38)

By Walras’ law, the market clearing condition for the deposit is automatically satisfied.

### 1.2.5 State Variable Dynamics

There are two endogenous state variables: the wealth share of households ($\omega$), and scaled net foreign asset position of the economy ($\chi$). We define $\chi$ in traded goods, so that the two state variables are expressed as:

$$\omega = \frac{W^h}{W^h + W^b}, \chi = \frac{NFA}{Q}$$  \hspace{1cm} (1.39)
The dynamics of $\omega$ can easily obtained by using Ito’s lemma:

$$d\omega = \omega(1 - \omega) \left[ c_x^h - c_y^h + Q(c_y^h - c_y^b) + (w_h^h - w_h^b)(\mu_s - r) + w_i^h i + (w_f^h - w_f^b)(\mu_f - r) + \Pi^h - \Pi^b \right] dt$$

$$+ \kappa(\bar{\omega} - \omega)dt + \omega(1 - \omega) \left[ (w_h^b - w_h^h) \sigma_s + (w_f^b - w_f^h) \sigma_f \right]' dB$$

(1.40)

We can write out the dynamics of $\frac{NFA}{Q}$ from the balance of payment equation. That is:

$$d \left( \frac{NFA}{Q} \right) = (Y - C_y^h - C_y^b) dt + \frac{NFA}{Q} \sigma^r dt + \frac{NFA}{Q} (\sigma^*)' dB - \Pi_{NFA}^Q dt$$

(1.41)

Equation (1.41) describes the three components of net foreign asset change: current account adjustment, valuation change, and the lump-sum transfer $\Pi_{NFA}^Q = \eta_{NFA} \frac{NFA}{Q}$. The lump-sum transfer is from (to) the outside world to keep the scaled net foreign asset stationary, following Schmitt-Grohé and Uribe (2003). Then we can derive dynamics of $\chi$ easily using Ito’s lemma:

$$d\chi = \left( \tau - \frac{C_y^h + C_y^b}{X} + \chi \sigma^r \right) dt + \chi (\sigma^*)' dB - \eta \chi dt$$

(1.42)

When we introduce stochastic interest rates, there is another state variable, the nominal interest rate, which follows equation (1.10).

### 1.3 Numerical Solution and Model Analysis

In this section, we solve the model numerically. After discussing the parameters in section 1.3.1, we study a cross-section of six economies with different fixed nominal interest rates from 0 to 5 percent in section 1.3.2 and section 1.3.3. In section 1.3.4, we introduce stochastic interest rates and examine the time-series relation between nominal interest rate and currency risk premium. Lastly, in section 1.3.5 we discuss the deviation of covered interest rate parity (CIP, Du et al., 2018b) from the liquidity perspective.

We solve the model using Chebyshev polynomial approximation with the collocation method.
Tensor products are used to deal with multiple state variables. A detailed description of the solution procedure is provided in Appendix.2.

1.3.1 Parameter Values

Table 1.1 presents the parameters for numerical solutions. All parameters are at the annual frequency.

The parameters are grouped into five sets. Panel A contains parameters of preferences. One key ingredient of this model is the preference heterogeneity between households and bankers. Households have higher risk aversion and lower IES than bankers. Heterogeneity in risk aversion determines the risk allocation and leverage, while heterogeneity in IES drives changes in intertemporal choice and exchange rate dynamics when wealth is redistributed between the two agents. We require a substantial heterogeneity in both parameters, so that we choose $\gamma_h = 30$ and $\gamma_b = 5$. With the parameter choices, the average excess return of the local stock is 3 percent with $i = 0$. The risk aversion coefficients are high compared with most of the literature, since we do not embed additional features that enable us to have a sizable risk premium. Most asset pricing studies calibrate and estimate IES being greater than 1 (e.g., Schorfheide, Song, and Yaron (2018) estimate IES=2). In our model, we require a substantial heterogeneity in IES across agents and choose $\psi_h = 1.2$ and $\psi_b = 3$.

The effective time discount rates of both agents are $\rho + \kappa = 0.025$. The relative preference over the nontraded good $\alpha > \frac{1}{2}$ captures a bias of preference towards the local nontraded good. $\alpha = 0.975$ captures a substantial consumption home bias. The elasticity of substitution across goods $\theta$ takes a small number of 0.5. It is consistent with a number of studies in the international macroeconomics literature, such as Stockman and Tesar (1995) and Corsetti, Dedola, and Leduc (2008).

Panel B lists the parameters for endowment processes. The dividend of nontraded good follows a geometric Brownian motion with an average growth rate of 2.5 percent and a volatility of 2.5 percent. The ratio of the traded good over the nontraded good is a constant $(\frac{1-\alpha}{\alpha})^\theta = 0.160$, so that $Q = 1$ under autarky.
Panel C shows the parameters on the foreign bond return process. The instantaneous return $r^*$ is equal to 2.36 percent, while $\sigma_q^*$ is equal to 2 percent.

Panel D reports the crucial parameter that determines the cost of leverage, $\lambda$. $\lambda$ determines how much cash is needed for each unit of borrowing. We set $\lambda$ as a common parameter across all countries and calibrate it to the average ratio of liquid assets over total assets in the banking systems of the major G10 countries. With $\lambda = 0.2$, countries with different nominal interest rates have substantial differences in their costs of leverage, while bankers still have incentives to lever up.

When we introduce stochastic interest rate, we set its persistence $1 - \zeta = 0.9$. Nominal interest rate is very persistent in the data, while we choose a relatively mild number. The unconditional means of nominal interest rates change from 1 percent to 4 percent.

Lastly, in Panel E, we report the parameters that keep the economy stationary. We set $\bar{\omega} = 0.95$ so that the redistribution of wealth favors households. $\eta = 0.2$ ensures a stationary distribution of $\chi$ in the simulation.

1.3.2 Bank Leverage, Stock Return, and Real Exchange Rate

We study a cross-section of economies that differ only in nominal interest rates, ranging from $i = 0$ to $i = 0.05$. We illustrate the liquidity-leverage mechanism by analyzing the solutions of bank leverage, local stock return, exchange rate, and currency risk premium as functions of the endogenous state variables in each economy. We also show impulse responses of various variables to a positive endowment shock in each economy. For the sake of space, we only report results for the two economies with $i = 0$ and $i = 0.05$.

When solving and simulating the model, we introduce a country-specific lump-sum redistribution of wealth between households and bankers so that the ergodic means of $\omega$ are similar across countries. Details about simulation are provided in Appendix .3. In Appendix .4.2, we show the same set of results without the redistribution. The results are qualitatively similar.
Bank Leverage and Stock Return

Nominal interest rates affect bank leverage since they represent the cost of taking leverage. Equation (1.28) indicates that higher nominal rates reduce risk taking and lead to lower leverage for bankers, as the Panel A of Figure 1.2 shows. When we compare the two economies, we find that, for every given $\omega$ and $\chi$, bank leverage is higher when $i = 0$.

The higher leverage in the $i = 0$ economy makes bankers’ wealth more exposed to the endowment shock due to the following amplification mechanism. A positive endowment shock increases bankers’ wealth share, which pushes up the local stock price and further increases their wealth share. The magnitude of amplification increases with the bankers’ leverage choice. Therefore, as shown in Panel B of Figure 1.2, the low-interest-rate economy’s ($i = 0$) local stock price exposure to the endowment shock ($\sigma_{sx}$) is larger.

We note that $\sigma_{sx}$ displays a hump shape with respect to households’ wealth share $\omega$, since the amplification is strong when bankers have enough wealth to affect the local stock price while they take a high enough leverage. Panel C of Figure 1.2 shows that the risk premium of the local stock is an asymmetric hump shape along $\omega$. When the endogenous volatility ($\sigma_{sx}$) spikes, the risk premium also spikes. When endogenous volatility $\sigma_{sx}$ is close to the endowment volatility ($\sigma_x$), the risk premium is higher when wealth is accumulated in more risk-averse households, or $\omega$ being close to 1.

The impulse response functions in the two upper panels of Figure 1.4 show the different responses of local stock returns and household wealth shares to the same positive endowment shock in the two economies. The blue solid line shows the response of the local stock return and households’ wealth share in the $i = 0$ economy, while the red dashed line shows the responses in the $i = 0.05$ economy. The local stock return rises by 8 percent in the $i = 0$ economy on impact, while the 3-percent increase in the $i = 0.05$ economy is much milder. Consequently, household wealth share $\omega$ drops by 3 percent in the $i = 0$ economy, larger than the 0.5 percent drop in the $i = 0.05$ economy.
Exchange Rate and Currency Risk Premium

This section discusses the exchange rate responses to the endowment shock, and the associated currency risk premium in the two economies with \( i = 0 \) and \( i = 0.05 \). As we show in section 1.3.2, bankers’ wealth share increases in response to a positive endowment shock. Since bankers’ have higher IES than households do, the low-interest-rate economy is more desired to save with bankers’ higher wealth share in good times. Therefore, the demand for foreign asset increases sharply and the foreign currency strongly appreciates. On the contrary, the magnitude of foreign appreciation for the high-interest-rate economy is much milder.

Panel A of Figure 1.3 shows the exchange rate response to a positive endowment shock, \( \sigma_{qx} \). The exchange rate response mirrors the response of local stock returns, which is a hump-shape with \( \omega \). When we compare the \( i = 0 \) and \( i = 0.05 \) economies, we find that the foreign appreciation is larger in the low-interest-rate economy, since wealth redistribution between households and bankers is stronger. The same result can be seen from the lower panel of Figure 1.4, which shows the impulse response of foreign appreciation to a positive endowment shock.

The Euler equation (1.34) shows that the currency risk premium depends on the covariance of exchange rate movement with the state price densities for both investors, which are largely driven by \( \sigma_{sx} \). Because the low-interest-rate countries have higher stock returns and sharply depreciated exchange rates in good times, investors require a higher return to borrow low-interest-rate currencies to invest abroad. In other words, low-interest-rate currencies are good hedges to endowment shocks. Panel B in Figure 1.3 shows the currency risk premium of investing in the common foreign bond in the two economies. The currency risk premium also displays a hump shape along \( \omega \), while it is larger in the low-interest-rate country with \( i = 0 \). In Panel C of Figure 1.3, we report the ergodic distribution of the two endogenous state variables.

1.3.3 Simulation

We simulate a cross-section of six economies with fixed but different levels of nominal interest rates, ranging from 0 to 5 percent with an incremental step of 1 percent. We report the ergodic mean
of different variables for each economy in Panels A to C of Table 1.2 and confirm the implications discussed above.

The nominal interest rates affect bankers’ leverage choices as they essentially represent the cost of taking leverage. The average bankers’ leverage \((w_h^b)\) decreases with the nominal interest rate, from 1.946 to 1.373. We are not able to replicate the high bank leverage in the data quantitatively. From the market clearing condition for the local stock, we obtain:

\[
\frac{w_h^b}{1 - \omega} = 1 - \frac{\text{NFA}_W}{W} - \omega w_h^b < 1 - \frac{\text{NFA}_W}{W}
\]

(1.43)

As \(\frac{\text{NFA}_W}{W}\) is very close to 0, the largest possible bank leverage is \(\frac{1}{1 - \omega}\). Suppose \(\omega\) fluctuates between 0.4 and 0.7, the maximum possible bank leverage is 3.3. Despite the quantitative discrepancy, the model still captures the qualitative heterogeneity in bank leverage across economies with different levels of nominal interest rates.

Due to bankers’ high leverage \((w_h^b)\) and the strong amplification mechanism, low-interest-rate economies have large local stock return exposure \((\sigma_{sx})\) endogenously and thus high risk premia \((\mu_s - r)\). In the simulation, the risk premium for the local stock is as large as 3.140 percent when \(i = 0\), and declines to 1.791 percent when \(i = 0.05\). Due to higher leverage and higher risk premium for the local stock, the \(i = 0\) economy has a smaller ergodic mean of \(\omega\) than the \(i = 0.05\) economy.

The real interest rates \((r)\) are mainly determined by the wealth distribution across households and bankers in these economies. If households’ wealth share is high, the real interest rate is high because households have lower IES. Countries with lower nominal interest rates have lower households’ wealth share on average, so they tend to have low real interest rates as well. The average real interest rate ranges from 0.756 percent when \(i = 0\) to 2.392 percent when \(i = 0.05\). This is consistent with the data that low-nominal-rate economies also tend to have low real rates.

As for the exchange rates and currency risk premia, the exchange rate exposure to endowment shock \((\sigma_{qk})\) is 5.425 percent for the \(i = 0\) economy and 1.215 percent when \(i = 0.05\). A positive exposure means a foreign depreciation and domestic appreciation in bad times. A large appreciation of the low-interest-rate currency in bad times makes it a good hedge. Thus, investors require a higher
risk premium to borrow it and invest abroad. The currency risk premium is 2.061 percent in the $i = 0$
economy and 0.457 percent in the $i = 0.05$ economy. The spread is 1.604 percent on average, which is close to its empirical counterpart of 2.161 percent shown in Table 1.5.

**Factor Structures of Asset Returns**

After examining the unconditional moments of the key variables, we explore the rich factor structures of asset returns implied by our model. In the simulation, all countries experience common shocks. The asset return factor structures in our simulated data are reported in Panel D of Table 1.2.

We first look at the factor structure of currency returns. Lustig et al. (2011) find that the differences in expected returns of currency portfolios can be explained by their heterogeneous exposures to a “carry risk factor,” which is defined as the spread between returns in the highest-interest-rate currencies and the lowest-interest-rate currencies. Low-interest-rate currencies (long position) are less (more negatively) exposed to the “carry risk factor.”

We compute the analog of the “carry risk factor” in our model as $dR_f|_{i=0} - dR_f|_{i=0.05}$. It essentially measures the magnitude of the endowment shock. Low-interest-rate currencies depreciate in good times while high-interest-rate currencies appreciate. Therefore, the “carry risk factor” is large when the endowment shock is positive and large. The return exposure ($\beta_{FX}$) is computed as the regression coefficient of $dR_f$ in each economy on the risk factor. $dR_f$ is the return to borrowing the domestic currency and investing in the common foreign asset. A larger $\beta_{FX}$ for currency $i$ means that currency depreciates more in response to a positive endowment shock. The first row in Panel D shows that $\beta_{FX}$ monotonically decreases with the nominal interest rate, consistent with the empirical finding of Lustig et al. (2011).

Second, we examine the factor structure of stock returns. In a recent paper, Colacito et al. (2018a) identify a factor structure of stock valuations across countries. They find that low-interest-rate countries’ stocks have higher exposures to the cross-sectional average. We calculate the exposure of stock return ($dR_s$) to the cross-sectional average ($\bar{dR}_s$) and find that $\beta_s$ monotonically decreases with nominal interest rates. The intuition is straightforward: when the same shock hits
low-interest-rate countries, their local stock returns increase more due to high bank leverage. Therefore, our model can also replicate Colacito et al. (2018a)’s empirical finding.

Finally, we compute the exposures of currency returns ($dR_f$) to average stock return ($d\bar{R}_s$) and bank wealth portfolio return ($w_B^b dR_s$) across countries, $\beta_{FX,s}$ and $\beta_{FX,b}$ respectively. Both the average stock return $d\bar{R}_s$ and the average bank wealth portfolio return $w_B^b dR_s$ are measures of the magnitude of the common endowment shock. Therefore, both $\beta_{FX,s}$ and $\beta_{FX,b}$ decrease with the interest rate. In section 1.4.3, we empirically test this implication by measuring return to bankers’ wealth with the banking sector stock returns.

**Time-varying Carry Trade CAPM Beta**

In this section, we discuss a distinct feature of the model on the time variation of currency carry trade exposures to the average global stock market, or its CAPM beta ($\beta_{FX,s}$). The dynamics of the carry trade CAPM beta were first noted by Lustig et al. (2008), that they are particularly high during the crisis. As there is no well-defined notion of “crisis” in our model, we explore the correlation between the carry trade CAPM beta $\beta_{FX,s}$ and the volatility of the average stock return.

In our model, $\beta_{FX,s}$ is positively related to the average stock return volatility. The intuition is simple: the exchange rate movement tracks stock return especially when the amplification is strong. This is also when the endogenous stock volatility is high. In the simulated data, we calculate the correlation between $\beta_{FX,s}$ and the average stock return volatility as 0.705. We empirically test this implication in section 1.4.4.

**1.3.4 Interest Rates and Currency Risk Premia in the Time Series**

In previous sections, we compared bank leverage and currency risk premia across countries with heterogeneous but fixed nominal interest rates. In this section, we examine our model implications on the time-series relationship between interest rates and currency risk premia. To study the

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10. We ignore the return on the foreign bond since it is very small.
time-series relationship, we introduce a stochastic nominal interest rate process of equation (1.10). Different countries have different unconditional means \((i_0)\), ranging from 1 percent to 4 percent.

A sizable body of literature has documented and explained the positive regression coefficient of currency returns on interest rate differentials, \(\beta_{FP}\). In Panel A to C of Figure 1.5, we plot the solutions of bank leverage \((\omega_b)\), exchange rate exposure \((\sigma_{q_x})\), and currency risk premium \((\mu_f - r)\) as functions of households’ wealth share \((\omega)\) and nominal interest rate \((i)\) for the \(i_0 = 0.01\) economy\(^{11}\). Since there are three state variables with stochastic interest rates, we fix \(\chi = 0\) when plotting the solutions. From the three panels, we see that the risk premium of investing abroad declines with \(i\). This is because when \(i\) increases, its exchange rate becomes less of a hedge due to the decreased leverage choice of bankers.

We run the following regressions with the simulated data in each economy:

\[
 rx_{r,t+1} = \alpha - \gamma_r i_t + \varepsilon_{r,t+1} \quad (1.44)
\]

\[
 rx_{i,t+1} = \alpha - \gamma_i i_t + \varepsilon_{i,t+1} \quad (1.45)
\]

We assume the foreign interest rate is an exogeneous constant, so that \(i^*_t\) is abstracted from the right-hand side of both regressions. The left-hand side of equation (1.44) is the return of investing abroad in excess of the real interest rate, while the left-hand side of equation (1.45) is the return of investing abroad in excess of the nominal interest rate.

Panel E of Table 1.3 show \(\gamma_r\) and \(\gamma_i\) for the four economies. \(\gamma_r(\gamma_i)\) is the regression coefficient of the excess currency returns in real (nominal) terms on the interest rate differentials. Both coefficients in the four economies are positive, consistent with the empirical findings of the literature. Quantitatively, \(\gamma_r\) is small while \(\gamma_i\) is about the same magnitude as in the data. In our model, there is no stickiness in price setting, so that a nominal rate rise is mostly reflected in an increase in inflation while the real interest rate responds only by a small magnitude induced by the decline of bankers’ wealth share.

\(^{11}\) For other values of \(i_0\), the solutions are very similar.
Panel A to D of the table are analogous to Table 1.2. In the cross-section of economies with stochastic interest rates, low-interest-rate countries have high bank leverage and their currencies appreciate the most in bad times. Therefore, investors require a positive premium to borrow these currencies and invest abroad.

Next, we study the impulse response of the currency risk premium of investing abroad to an unexpected rise in the nominal interest rate in each economy. Mueller et al. (2017) find that after the Fed unexpectedly raises the interest rate, the subsequent return of foreign currency investment declines. Panel D of Figure 1.5 displays the impulse responses of currency risk premia (in real terms) for the four economies to the same shock to the nominal interest rate. In all four economies, the currency risk premia decrease after an unexpected increase in the nominal interest rate. The currency risk premia for the four economies decline by about 3-4 basis points in response to an unexpected interest rate rise of $\sigma_i = 17.4$ basis points.

Since Mueller et al. (2017) use high-frequency data in their empirical study, it is hard to map our model implication to the data quantitatively. Therefore, we only make qualitative claims on the impulse response of currency risk premium to nominal interest rate shocks.

1.3.5 Liquidity Premium and CIP Deviation

In this subsection, we show that our model can naturally generate deviations from covered interest rate parity (CIP). CIP is a no-arbitrage condition, which states that if an investor borrows the dollar and invest in a foreign currency while hedging away the exchange rate risk, the return should be 0.

We define $i^*_t$, $i^\$_t$ as the interest rate in the foreign currency and the dollar, $q_t$ as the spot exchange rate of foreign currency in dollars, and $f_t$ as the forward rate. The return to the arbitrage, $-x_t$ ($x_t$ is called “basis”), can be expressed as:

$$-x_t = -i^\$_t + q_t + i^*_t - f_t$$  \hfill (1.46)

Under CIP, $x_t = 0$. According to Du et al. (2018b), after the recent financial crisis in 2008, the basis
(x_t) has been negative. Our model sheds light on the liquidity friction that could potentially lead to what we observe in the data\textsuperscript{12}.

Suppose only the bankers have access to the CIP arbitrage. Each unit of safe dollar liabilities is subject to a withdrawal need of $\frac{\lambda}{1+\lambda}$, while $\frac{\tilde{\lambda}}{1+\lambda}$ fraction of the foreign asset can be liquidated costlessly. The foreign asset payoff is $i_t^s - x_t$. According to equation (1.15), we obtain:

$$x_t = (\tilde{\lambda} - \lambda)i_t^s$$

Equation (1.47) shows that if $\tilde{\lambda} < \lambda$, $x_t < 0$. Intuitively, if dollar asset is more liquid than the foreign asset, investors require a positive premium to conduct the CIP arbitrage above.

Next we explore into the quantitative implication of CIP deviation on the relative liquidity of dollar and foreign safe assets. Du et al. (2018b) document an average currency basis ($x_t$) of -25 basis points, while one third of the basis is possibly caused by liquidity concerns\textsuperscript{13}. We plug in $x = -8bp, \lambda = 0.2$, and $\delta^s = 50bp$, which is equal to the time-series average of US LIBOR rate between 2010 and 2016. Then we obtain $\tilde{\lambda} = 0.04$, which implies a substantial difference in liquidity between dollar and foreign safe assets.

### 1.3.6 Model Implications

We conclude this section by summarizing the three major model implications, around which our subsequent empirical analysis is centered.

The first implication is straightforward: countries with lower nominal interest rates tend to have higher bank leverage and lower currency returns.

The second implication states the cyclicality of exchange rates: low-interest-rate currencies depreciate in good times relative to high-interest-rate ones. To highlight the role of intermediaries, we measure the “good times” with average bank stock returns in all countries.

\textsuperscript{12} In Du et al. (2018b), the authors explain CIP deviations with changes in regulations after the financial crisis, such as the leverage ratio requirement and other regulations including liquidity.

\textsuperscript{13} See Table VI of Du et al. (2018b). Another major reason that leads to CIP deviation is the change in non-risk-weighted leverage ratio requirement.
Third, our model implies that carry trade return’s comovement with the average stock return is time-varying, and it increases with the stock market volatility.

1.4 Empirical Analysis

1.4.1 Data

We mainly use three sets of data in our empirical analysis: spot and forward exchange rates, country-level banking sector balance sheet, and country-level stock return index of the banking sector. The data cover 22 advanced countries: Australia, Austria, Belgium, Canada, Denmark, Finland, Euro, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, South Korea, Spain, Switzerland, Sweden, and the UK.

The spot and one-month forward exchange rate data are from standard sources of Datastream at the monthly frequency from November 1983 to December 2016, following Lustig et al. (2011). We take the mid-price of the forward and spot quotes on the last day of each month.

We calculate each country’s banking sector capital ratio (the inverse of leverage, in percentage) using the aggregate banking data from SNL Financial, originally provided by Economist Intelligence Unit (EIU). Capital ratio is calculated as the ratio of total equity capital over assets. The bank balance sheet data are at the annual frequency from 1990 to 2015. Total assets and equity capital are simple sum of those collected from reporting banks. After 1999, all countries using the Euro are replaced with a single time series of Euro spot and forward rate; the bank capital ratio for the euro area is the average across these countries.

The country-level banking sector stock total return indices are from Datastream from January 1983 to December 2016, at the monthly frequency. The dataset includes an index for the Euro area bank stock returns, which are used after 1999 for the Euro area.

We also use each country’s GDP and inflation as control variables. Both variables are obtained from the World Bank website at the annual frequency.
1.4.2 Interest Rate, Bank Leverage, and Currency Return

Panel Regression

Using panel regressions, we show that countries with lower interest rates have lower bank capital ratio (higher leverage) and lower currency returns. Since we do not attempt to make causal statement from our regression results, it does not matter whether we use the interest rate as a regressor or a regressant. We regress interest rate\textsuperscript{14} and currency return on the bank capital ratio.

The panel regression is conducted in the two-step Fama-MacBeth procedure. First, we run a cross-sectional regression of forward discount and currency return on bank capital ratio and get a regression coefficient for each month. The Fama-MacBeth estimator is the simple average of these regression coefficients across time, while standard errors are calculated after adjusting for possible serial correlations.

The regression results are shown in Table 1.4. The left panel reports the regression coefficients of the forward discount on the bank capital ratio, while the right panel replaces the forward discount with the currency return. In column (1) of the left panel, the coefficient of forward discount on the bank capital ratio is 0.463, being significantly positive. In the cross-section, a one percent increase of the bank capital ratio is associated with a 46.3 basis point increase in the forward discount per annum. The same is true for the currency return. A one percent increase of the bank capital ratio is associated with a 27.9 basis point increase in the currency return. In the data, average bank capital ratio varies from 3 percent to 10 percent, which translates into a 3.24 percent difference in the forward discount and 1.95 percent difference in the currency return.

In column (2) of both panels, we control for the inflation (percentage change of consumer price index) in each country. The coefficient for forward discount is smaller in magnitude, but still significant. Unsurprisingly, inflation accounts for a big fraction of cross-sectional difference in the nominal interest rate across countries ($R^2$ increases from 0.134 to 0.394). However, the coefficient

\textsuperscript{14} Under the covered interest rate parity, forward discount is equal to the difference between country specific interest rate and US interest rate. Even though CIP is deviated after the crisis, the deviation is small in magnitude relative to interest rate differentials. In this section, “forward discount” and “interest rate differential” are used interchangeably.
of the currency return stays stable at 0.204 while being statistically significant, and the $R^2$ increase is mild, from 0.124 to 0.265.

In column (3), we control for each country’s log GDP (country size), which Hassan (2013) considers as an important determinant of the interest rate and currency return. In column (4) we control for both inflation and country size. Our results are robust to these controls.

The regressions in Table 1.4 include 22 advanced economies. A natural question is whether the relationship also holds for the emerging economies. We show the regression results for both advanced and emerging economies in Appendix E.2. After including the emerging economies, the relationship between the interest rate and bank capital ratio is positive, similar to the advanced economies. The regression coefficient of the currency return on bank capital ratio is also positive without controlling for inflation. However, after controlling for inflation, the regression coefficient becomes insignificant and sometimes even negative. The result suggests that inflation plays a more important role in emerging economies than in advanced economies. In the rest of the empirical analysis, we restrict our attention to the set of advanced economies.

**Portfolio Sorting**

In this section, we show the relevance of bank leverage in currency pricing by sorting currencies based on each country’s bank leverage. We sort all available currencies into three portfolios with annual rebalance, since our banking sector balance sheet variables are only available at the annual frequency.

Panel A in Table 1.5 shows the characteristics of leverage sorted currency portfolio returns. Portfolio 1 contains currencies with the highest banking sector leverage (such as Japan) and portfolio 3 contains those with the lowest banking sector leverage (such as Australia). The average average forward discount increases from 0.05 percent to 1.669 percent per annum. The bank capital ratio rises from 4.476 percent in portfolio 1 to 7.612 percent in portfolio 3. The average currency return monotonically increases from -0.687 percent in portfolio 1 to 1.465 percent in portfolio 3. The
return spread between the lowest leverage countries and the highest leverage countries is 2.152 percent on average, with a Sharpe ratio of 0.389.

In Panel B, we sort currencies based on their unconditional average forward discount, measured by the full sample average of each country. Portfolio rebalancing is also at the annual frequency. The lowest interest rate currencies have a -0.734 percent average forward discount, while the highest ones have an average of 2.540 percent. The forward discount differential between portfolio 3 and portfolio 1 is 3.274 percent, which is about twice as large as what we obtained in Panel A. The bank capital ratio increases from low-interest-rate countries to high-interest-rate countries with a spread of 1.224 percent, about one third of what we obtain when sorting on bank leverage. The return spread between portfolio 3 and 1 is 2.161 percent per annum with a Sharpe ratio of 0.343. The average return spread and Sharpe ratio is similar to those in Panel A.

In Panel C, we sort the currency portfolios based on the previous year’s average forward discount. The pattern of forward discount, bank capital ratio, and excess return are similar to Panel B while the return spread is larger. However, the capital ratio spread is smaller. The return characteristics of the leverage sorted portfolios in Panel A are closer to the unconditional forward discount sorted portfolios in Panel B. This indicates that the cross-sectional variation in bank leverage largely captures the unconditional interest rate heterogeneity, while it does not completely reflect the time-series variations.

There are two reasons that make bank leverage contain less information of currency return than interest rate. First, as our model suggests, bank leverage increases with bankers’ wealth share, while the relation between the currency risk premium and bankers’ wealth share is nonmonotonic. Second, the bank balance sheet variables are hard to measure accurately, which also adds noise to our portfolio sorting. Despite these disadvantages, we still find similarities in currency portfolios sorted on the bank leverage and the forward discount, validating our hypothesis that bank leverage is an important driver of currency returns.
Asset Pricing Test

In this subsection, we show the pricing power of the risk factor constructed from portfolios sorted on bank leverage. The risk factor (Lev-factor) is the spread between portfolio 3 and portfolio 1 in Table 1.5 Panel A. We first examine the correlation of Lev-factor with conventional “carry risk factor” in the literature, defined as the spread between portfolio 3 and portfolio 1 in Panel B and C of Table 1.5. We call these two factors “unconditional carry-factor” and “conditional carry-factor,” respectively. In Table 1.6, we regress the unconditional and conditional carry-factor on the Lev-factor. In the first column, the sensitivity of the unconditional carry-factor to the Lev-factor is 0.655, with the unexplained residual of 0.752 percent and an $R^2$ of 0.332. The two are highly correlated, and the unexplained residual is statistically insignificant. The second column reports the regression coefficients for the conditional carry-factor. The coefficient is still significantly positive but smaller in magnitude (0.444), and the unexplained residual is 2.460 percent and statistically significant. The $R^2$ is 0.119, much smaller than the regression $R^2$ with the unconditional carry-factor. The comparison further verifies that the Lev-factor better captures the variations of the unconditional forward discount sorted portfolios than the conditional forward discount sorted ones.

Next we conduct an asset pricing test by using the six portfolios sorted on bank leverage and forward discount (unconditionally) as test assets. Panel A of Table 1.7 reports the first step estimation results for the six portfolios. For the leverage sorted portfolios, the exposure to the Lev-factor monotonically increases from -0.444 for portfolio 1 to 0.556 for portfolio 3. For the forward discount sorted portfolios, the exposure to the Lev-factor also monotonically increases, from -0.228 for the portfolio with the lowest interest rate, to 0.427 for the portfolio with the highest interest rate. Heterogeneous exposures to the Lev-factor accounts for the cross-sectional variations in expected returns of these portfolios.

In Panel B of Table 1.7, we report the estimates of the risk price using both the tso-step procedure and the GMM method. When using GMM, the pricing kernel is written as a linear factor model following Lustig et al. (2011):

$$m = 1 - bf$$  \hspace{1cm} (1.48)
where $f$ is the factor, and $b$ is the price of risk. We use Hansen and Jagannathan (1997)’s scale-invariant weight matrix in the GMM estimation. We obtain similar positive estimates of the price of risk using these two methods. They are close to the unconditional mean of the Lev-factor. The risk price is statistically significant for both methods.

### 1.4.3 Procyclical Carry Trade

In this section, we test the second implication of the model—that low-interest-rate currencies are less (more negatively) exposed to average bank stock return. The cross-sectional average measures the common “good times” for all countries instead of the idiosyncratic ones. We use the bank stock return to highlight the role of intermediaries in driving exchange rate dynamics. In Appendix 5.3, we show similar results with the average country MSCI stock return indices.

We calculate the sensitivities of exchange rates to the average bank stock return ($\beta_{\text{bank}}$) for the G10 currencies and plot them against their average forward discounts in the upper panel of Figure 1.6. The two variables are highly correlated. Countries such as Japan and Switzerland have low forward discounts and their exchange rates are countercyclical. On the other hand, countries like Australia and New Zealand have high forward discounts and procyclical exchange rates.

In the lower panel of Figure 1.6, we replace the average forward discount with average bank capital ratio. The result is similar—that $\beta_{\text{bank}}$ increases with the average bank capital ratio across countries.

To see the procyclicality of carry trade returns more clearly, we regress the returns to three “carry trade” strategies on the average bank stock return. The first strategy borrows the Japanese yen and invests in the Australian dollar, “AUD-JPY.” The second strategy borrows the Swiss Franc and invests in the New Zealand dollar, “NZD-CHF.” These are the typical currency pairs involved in currency carry trades. The third strategy borrows currencies with unconditionally low forward discounts and invests in those with high forward discounts, “unconditional carry.” All the three slope coefficients are significantly positive. A one percent increase in the average bank stock return
is associated with a 30 basis point increase in the “AUD-JPY” strategy, a 25.1 basis point increase in the “NZD-CHF” strategy, and a 15.5 basis point increase in the unconditional carry strategy.

1.4.4 Time-varying Carry Trade CAPM Beta

In this section, we examine the last implication on the relationship between the carry trade CAPM beta $\beta_{FX,s}$ and the stock market volatility. We average the country-level MSCI stock return indices with equal weights and obtain a risk factor of the market return, in the similar way as in section 1.4.3. We compute $\beta_{FX,s}$ in every five-year rolling window for the long-short portfolio sorted on unconditional forward discount. The stock market volatility is measured as the realized volatility of the average stock return. Figure 1.7 plots the relationship between the stock return volatility (on the horizontal axis) and $\beta_{FX,s}$ (on the vertical axis). It is clear that $\beta_{FX,s}$ increases with the stock return volatility, with a statistically significant regression coefficient of 6.64. This implies a one percent increase in the stock market volatility is associated with an increase of 0.0664 in $\beta_{FX,s}$ for the long-short portfolio on average. The correlation between the currency carry trade CAPM beta and the stock market volatility in the data is 0.645, while the correlation in the model simulated data is 0.705.

1.5 Conclusion

This paper provides an intermediary-based explanation of the currency risk premium associated with carry trade with a liquidity-leverage channel. Countries differ in nominal interest rates. Bankers in low-interest-rate countries take high leverage for the low cost of holding liquid assets, which translates into a large drop in their wealth along with a negative shock. Due to the higher IES of bankers than of households, the total demand of foreign asset declines in bad times and the domestic currency appreciates. Therefore, investors require a positive risk premium to borrow low-interest-rate currencies. With the same liquidity-leverage channel, our model can account for the decline (rise) of foreign currency risk premium in response to an unexpected interest rate rise (cut), as well as the time series positive correlation of interest rate differential and subsequent return to invest.
abroad. Our model implies deviations from covered interest rate parity when safe assets differ in
liquidity. Empirically, our evidence is consistent with the major model implications. First, low-
interest-rate countries tend to have high bank leverage and low currency returns. This is the key
mechanism of the model. Second, carry trade is procyclical with respect to the average bank stock
return. Finally, the CAPM beta of the carry trade increases with the stock market volatility.
## 1.6 Tables and Figures

### Table 1.1: Parameter Values

<table>
<thead>
<tr>
<th>Variable</th>
<th>Notation</th>
<th>Value</th>
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</thead>
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<tr>
<td><strong>Panel A: Preference Parameters</strong></td>
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<tr>
<td>Households’ risk aversion</td>
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<td>Bankers’ risk aversion</td>
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<td><strong>Panel B: Endowment Parameters</strong></td>
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<td>Volatility of US nontraded good price</td>
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<tr>
<td>Size of funding shock</td>
<td>$\lambda$</td>
<td>0.2</td>
</tr>
<tr>
<td>Persistence of the nominal interest rate</td>
<td>$\zeta$</td>
<td>0.1</td>
</tr>
<tr>
<td>Diffusion of the nominal interest rate</td>
<td>$\sigma_i$</td>
<td>$0.004 \times \sqrt{1 - (1 - \zeta)^2}$</td>
</tr>
<tr>
<td><strong>Panel E: Other Parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wealth redistribution share</td>
<td>$\bar{\omega}$</td>
<td>0.95</td>
</tr>
<tr>
<td>Net foreign asset redistribution</td>
<td>$\eta$</td>
<td>0.2</td>
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</table>
Table 1.2: Simulation Results for A Cross-section of Economies with Fixed Interest Rates

<table>
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<tr>
<th>$i$</th>
<th>0</th>
<th>0.01</th>
<th>0.02</th>
<th>0.03</th>
<th>0.04</th>
<th>0.05</th>
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<tbody>
<tr>
<td>Panel A: Asset Prices (in percent)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_g$</td>
<td>3.896</td>
<td>3.873</td>
<td>3.872</td>
<td>3.930</td>
<td>4.032</td>
<td>4.183</td>
</tr>
<tr>
<td>$r$</td>
<td>0.756</td>
<td>1.678</td>
<td>1.950</td>
<td>1.973</td>
<td>2.093</td>
<td>2.392</td>
</tr>
<tr>
<td>$\mu_s - r$</td>
<td>3.140</td>
<td>2.195</td>
<td>1.922</td>
<td>1.957</td>
<td>1.939</td>
<td>1.791</td>
</tr>
<tr>
<td>$\sigma_{sx}$</td>
<td>8.230</td>
<td>6.533</td>
<td>5.330</td>
<td>4.174</td>
<td>3.684</td>
<td>3.161</td>
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<tr>
<td>$\sigma_{qx}$</td>
<td>5.432</td>
<td>3.725</td>
<td>2.904</td>
<td>2.276</td>
<td>1.842</td>
<td>1.225</td>
</tr>
<tr>
<td>$\mu_f - r$</td>
<td>2.061</td>
<td>1.150</td>
<td>0.881</td>
<td>0.862</td>
<td>0.752</td>
<td>0.457</td>
</tr>
<tr>
<td>Panel B: Portfolio Choices</td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$w^b_h$</td>
<td>1.899</td>
<td>1.872</td>
<td>1.779</td>
<td>1.558</td>
<td>1.445</td>
<td>1.367</td>
</tr>
<tr>
<td>$w^b_f$</td>
<td>-0.035</td>
<td>0.063</td>
<td>0.146</td>
<td>0.187</td>
<td>0.215</td>
<td>0.228</td>
</tr>
<tr>
<td>$w^h_h$</td>
<td>0.254</td>
<td>0.374</td>
<td>0.486</td>
<td>0.613</td>
<td>0.695</td>
<td>0.785</td>
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<tr>
<td>$w^h_f$</td>
<td>0.024</td>
<td>-0.051</td>
<td>-0.102</td>
<td>-0.134</td>
<td>-0.153</td>
<td>-0.139</td>
</tr>
<tr>
<td>Panel C: State Variables</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.531</td>
<td>0.569</td>
<td>0.594</td>
<td>0.586</td>
<td>0.589</td>
<td>0.626</td>
</tr>
<tr>
<td>$\chi$</td>
<td>-0.167</td>
<td>-0.186</td>
<td>-0.212</td>
<td>-0.210</td>
<td>-0.223</td>
<td>-0.221</td>
</tr>
<tr>
<td>Panel D: Return Exposures</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{FX}$</td>
<td>1.160</td>
<td>0.737</td>
<td>0.542</td>
<td>0.396</td>
<td>0.291</td>
<td>0.160</td>
</tr>
<tr>
<td>$\beta_s$</td>
<td>1.595</td>
<td>1.262</td>
<td>1.028</td>
<td>0.803</td>
<td>0.708</td>
<td>0.605</td>
</tr>
<tr>
<td>$\beta_{FX,s}$</td>
<td>1.063</td>
<td>0.732</td>
<td>0.577</td>
<td>0.459</td>
<td>0.377</td>
<td>0.259</td>
</tr>
<tr>
<td>$\beta_{FX,b}$</td>
<td>0.605</td>
<td>0.408</td>
<td>0.323</td>
<td>0.258</td>
<td>0.211</td>
<td>0.145</td>
</tr>
</tbody>
</table>
Table 1.3: Simulation Results for A Cross-section of Economies with Fixed Interest Rates

<table>
<thead>
<tr>
<th>$i_0$</th>
<th>0.01</th>
<th>0.02</th>
<th>0.03</th>
<th>0.04</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Asset Prices (in percent)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu_s$</td>
<td>3.873</td>
<td>3.867</td>
<td>3.959</td>
<td>3.972</td>
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<tr>
<td>$r$</td>
<td>1.411</td>
<td>1.534</td>
<td>1.722</td>
<td>2.034</td>
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<tr>
<td>$\mu_s - r$</td>
<td>2.462</td>
<td>2.333</td>
<td>2.237</td>
<td>1.938</td>
</tr>
<tr>
<td>$\sigma_{sx}$</td>
<td>5.955</td>
<td>5.245</td>
<td>4.517</td>
<td>4.092</td>
</tr>
<tr>
<td>$\sigma_{qx}$</td>
<td>3.774</td>
<td>3.366</td>
<td>2.815</td>
<td>2.374</td>
</tr>
<tr>
<td>$\mu_f - r$</td>
<td>1.487</td>
<td>1.352</td>
<td>1.147</td>
<td>0.779</td>
</tr>
<tr>
<td>Panel B: Portfolio Choices</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w^h_b$</td>
<td>1.803</td>
<td>1.622</td>
<td>1.553</td>
<td>1.559</td>
</tr>
<tr>
<td>$w^b_f$</td>
<td>0.076</td>
<td>0.138</td>
<td>0.196</td>
<td>0.281</td>
</tr>
<tr>
<td>$w^h_f$</td>
<td>0.402</td>
<td>0.491</td>
<td>0.603</td>
<td>0.673</td>
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<tr>
<td>$w^f_f$</td>
<td>-0.060</td>
<td>-0.116</td>
<td>-0.145</td>
<td>-0.172</td>
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<tr>
<td>Panel C: State Variables</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.561</td>
<td>0.542</td>
<td>0.574</td>
<td>0.621</td>
</tr>
<tr>
<td>$\chi$</td>
<td>-0.137</td>
<td>-0.146</td>
<td>-0.196</td>
<td>-0.312</td>
</tr>
<tr>
<td>Panel D: Return Exposures</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{FX}$</td>
<td>1.370</td>
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<td>0.649</td>
<td>0.370</td>
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<td>$\beta_s$</td>
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<td>1.058</td>
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<td>$\beta_{FX,s}$</td>
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<td>$\beta_{FX,b}$</td>
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<td>0.365</td>
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<tr>
<td>Panel E: Time Series Coefficients</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_r$</td>
<td>0.165</td>
<td>0.199</td>
<td>0.277</td>
<td>0.298</td>
</tr>
<tr>
<td>$\gamma_l$</td>
<td>1.401</td>
<td>1.446</td>
<td>1.508</td>
<td>1.482</td>
</tr>
</tbody>
</table>
Table 1.4: Bank Capital Ratio, Forward Discount, and Currency Return

<table>
<thead>
<tr>
<th></th>
<th>Forward discount</th>
<th>Currency return</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Capital ratio</td>
<td>0.463**</td>
<td>0.170**</td>
</tr>
<tr>
<td></td>
<td>(0.071)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.971**</td>
<td>0.948**</td>
</tr>
<tr>
<td></td>
<td>(0.117)</td>
<td>(0.098)</td>
</tr>
<tr>
<td>GDP</td>
<td>-0.483**</td>
<td>-0.429**</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td>(0.031)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.134</td>
<td>0.394</td>
</tr>
</tbody>
</table>

Note: Fama-Macbeth regression results of the forward discount (left panel) and the currency return (right panel) on the bank capital ratio (the inverse of leverage, in percentage). In both panels, column (1) report the univariate regression coefficients. Column (2) controls for inflation, column (3) controls for the log GDP (size) of each country, and column (4) controls both inflation and log GDP. Data are monthly including 22 countries, from Jan 1990 to Dec 2016. Annual measures of the bank capital ratio and GDP share are used repetitively for months within a year. Standard errors are Newey-West adjusted with 120 lags. ** indicates statistical significance at 5% level. * indicates statistical significance at 10% level.

Table 1.5: Currency Portfolios

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(3)-(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Leverage sorted portfolios</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Forward discount</td>
<td>0.050</td>
<td>1.068</td>
<td>1.669</td>
<td>1.619</td>
</tr>
<tr>
<td>Bank capital ratio</td>
<td>4.476</td>
<td>5.738</td>
<td>7.612</td>
<td>3.135</td>
</tr>
<tr>
<td>Excess return</td>
<td>-0.687</td>
<td>0.182</td>
<td>1.465</td>
<td>2.152</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>8.513</td>
<td>9.165</td>
<td>8.711</td>
<td>5.550</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>-0.081</td>
<td>0.020</td>
<td>0.168</td>
<td>0.388</td>
</tr>
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</table>

Panel B: Forward discount sorted portfolios (unconditional)

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(3)-(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forward discount</td>
<td>-0.734</td>
<td>1.128</td>
<td>2.540</td>
<td>3.274</td>
</tr>
<tr>
<td>Bank capital ratio</td>
<td>5.324</td>
<td>5.738</td>
<td>6.548</td>
<td>1.224</td>
</tr>
<tr>
<td>Excess return</td>
<td>-0.526</td>
<td>-0.226</td>
<td>1.635</td>
<td>2.161</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>-0.069</td>
<td>-0.023</td>
<td>0.177</td>
<td>0.343</td>
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</tbody>
</table>

Panel C: Forward discount sorted portfolios (conditional)

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(3)-(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forward discount</td>
<td>-1.054</td>
<td>0.868</td>
<td>3.250</td>
<td>4.303</td>
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<tr>
<td>Bank capital ratio</td>
<td>5.348</td>
<td>6.035</td>
<td>6.199</td>
<td>0.851</td>
</tr>
<tr>
<td>Excess return</td>
<td>-0.952</td>
<td>-0.221</td>
<td>2.462</td>
<td>3.414</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>8.271</td>
<td>8.575</td>
<td>10.107</td>
<td>7.141</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>-0.115</td>
<td>-0.026</td>
<td>0.244</td>
<td>0.478</td>
</tr>
</tbody>
</table>

Note: Panel A reports statistics to the three portfolios sorted on bank leverage. Portfolio 1 includes countries with the highest bank leverage while portfolio 3 includes countries with the lowest bank leverage. Rebalancing is annual. Panel B reports the statistics to the three portfolios sorted on full sample average forward discount. Panel C reports the statistics to the three portfolios sorted on average forward discount in the previous year with annual rebalancing. All numbers are annualized.
### Table 1.6: Correlation of Lev-factor with (Un)conditional Carry-factor

<table>
<thead>
<tr>
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<th>Conditional</th>
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</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.752</td>
<td>2.460**</td>
</tr>
<tr>
<td></td>
<td>(1.019)</td>
<td>(1.325)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.655**</td>
<td>0.444**</td>
</tr>
<tr>
<td></td>
<td>(0.053)</td>
<td>(0.069)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.332</td>
<td>0.119</td>
</tr>
</tbody>
</table>

Note: This table shows the regression coefficients of the conditional and the unconditional carry risk factors on the Lev-factor. Lev-factor is constructed from portfolios in Panel A of Table 1.5 while conditional and unconditional carry risk factors are constructed from portfolios in Panel B and C of Table 1.5, respectively. All numbers are annualized. ** indicates statistical significance at 5% level. * indicates statistically significance at 10% level.

### Table 1.7: Estimation of the Factor Model: Lev-factor

#### Panel A: Risk Factor Exposure

<table>
<thead>
<tr>
<th></th>
<th>Lev-1</th>
<th>Lev-2</th>
<th>Lev-3</th>
<th>Fd-1</th>
<th>Fd-2</th>
<th>Fd-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.269</td>
<td>0.060</td>
<td>0.269</td>
<td>-0.036</td>
<td>-0.127</td>
<td>0.716</td>
</tr>
<tr>
<td></td>
<td>(1.613)</td>
<td>(1.813)</td>
<td>(1.613)</td>
<td>(1.484)</td>
<td>(1.927)</td>
<td>(1.769)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-0.444**</td>
<td>0.056</td>
<td>0.556**</td>
<td>-0.228**</td>
<td>-0.046</td>
<td>0.427**</td>
</tr>
<tr>
<td></td>
<td>(0.084)</td>
<td>(0.094)</td>
<td>(0.084)</td>
<td>(0.077)</td>
<td>(0.100)</td>
<td>(0.092)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.084</td>
<td>0.001</td>
<td>0.125</td>
<td>0.028</td>
<td>0.001</td>
<td>0.066</td>
</tr>
<tr>
<td>Obs</td>
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<td>312</td>
<td>312</td>
<td>312</td>
<td>312</td>
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</tbody>
</table>

#### Panel B: Price of Risk

<table>
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<tr>
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<th>Two-step</th>
<th>GMM</th>
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</thead>
<tbody>
<tr>
<td>2.552*</td>
<td>2.134**</td>
<td></td>
</tr>
<tr>
<td>(1.411)</td>
<td>(1.073)</td>
<td></td>
</tr>
</tbody>
</table>

Note: Panel A in this table shows the exposure to the Lev-factor across the three currency portfolios sorted on unconditional average forward discount. Panel B reports the estimated price of the Lev-factor risk using the two-step and GMM methods. Standard errors in two-step estimates are adjusted according to Shanken (1992). Hansen and Jagannathan (1997)’s scale-invariant weight matrix is used in the GMM estimation. Andrews (1991)’s optimal choice of lags is used when computing standard errors. All estimates of price of risk are annualized. ** indicates statistical significance at 5% level. * indicates statistically significance at 10% level.
Table 1.8: Procyclical Carry Trade

<table>
<thead>
<tr>
<th></th>
<th>“AUD-JPY”</th>
<th>“NZD-CHF”</th>
<th>unconditional carry</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>-0.092</td>
<td>1.709</td>
<td>0.607</td>
</tr>
<tr>
<td></td>
<td>(2.580)</td>
<td>(2.161)</td>
<td>(1.163)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.300**</td>
<td>0.251**</td>
<td>0.155**</td>
</tr>
<tr>
<td></td>
<td>(0.041)</td>
<td>(0.035)</td>
<td>(0.019)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.121</td>
<td>0.121</td>
<td>0.148</td>
</tr>
</tbody>
</table>

Note: This table shows the regression coefficients of three carry trade strategies’ returns on the average bank stock return. The first column shows the result for the “AUD-JPY” strategy, the second column for the “NZD-CHF” strategy, and the third column for the unconditional carry strategy. All numbers are annualized. ** indicates statistically significance at 5% level. * indicates statistically significance at 10% level.
Figure 1.1: Bank Capital Ratio, Interest Rate, and Currency Returns

Note: Observations span from 1990 to 2016, with the currency data at monthly frequency and the bank balance sheet data at annual frequency. We select the most liquid G10 countries with relatively long sample for comparability. We use German currency data before 1999 and Euro currency data after 1999.
Figure 1.2: Leverage, Local Stock Return Exposure, and Excess Stock Return

Panel A: Bank Leverage

Panel B: Local Stock Return Exposure

Panel C: Excess Local Stock Return

Note: This figure shows the solutions of bank leverage, excess local stock return, and local stock return exposure for economies with fixed nominal interest rate $i = 0$ and $i = 0.05$ in each panel, respectively. The graphs are on the same scale. $\omega$ represents the households’ wealth share, and $\chi$ represents the scaled net foreign asset position.
Figure 1.3: Exchange Rate Exposure, Currency Risk Premium, and Ergodic Distribution

Panel A: Exchange Rate Exposure

Panel B: Currency Risk Premium

Panel C: Ergodic Distribution

Note: This figure shows the solutions of exchange rate exposure and currency risk premium for economies with fixed nominal interest rate $i = 0$ and $i = 0.05$ in Panel A and B. Panel C shows the ergodic distribution of the two state variables. The graphs are on the same scale. $\omega$ represents the households’ wealth share, and $\chi$ represents the scaled net foreign asset position.
Figure 1.4: Impulse Responses to a Positive Endowment Shock

Note: This figure shows the impulse responses of various variables to a one standard deviation positive endowment shock in the two economies with \( i = 0 \) and \( i = 0.05 \). The solid blue line represents the \( i = 0 \) (low-interest-rate) economy and the dashed red line represents the \( i = 0.05 \) (high-interest-rate) economy. Impulse responses are obtained by simulation of \( N = 2,000 \) parallel economies and taking their average.
Figure 1.5: Solutions for the Stochastic Interest Rates

Panel A: Bank Leverage

Panel B: Exchange Rate Exposure

Panel C: Currency Risk Premium

Panel D: Currency risk premium responses

Note: Panel A to C show the solutions of bank leverage $w_h$, exchange rate exposure $\sigma_q$, and currency risk premium $\mu_f - r$ as functions of households' wealth share $\omega$ and nominal interest rate $i$, while fixing $\chi = 0$. Panel A to C are solutions to the economy with $i_0 = 0.01$. Panel D shows the impulse responses of currency risk premium (in percentage) to a positive nominal interest rate shock for the four economies. Impulse responses are obtained by simulation of $N = 1,000$ parallel economies and taking their average.
Figure 1.6: Procyclical Carry Trade

Note: This figure plots the relationship between a country’s average forward discount (the upper panel) and average bank capital ratio (the lower panel) and the exchange rate beta with respect to the average bank stock return for the G10 currencies (vis-a-vis dollar). Data range from November 1983 to December 2016. Euro exchange rate is used for “DEM” after 1999.
Figure 1.7: Currency Beta and Stock Market Volatility

Note: This figure plots the relationship of time-varying currency beta of the carry trade portfolio to average stock return (using five-year rolling window) and the stock market volatility (monthly realized volatility of the average stock return across countries). Each dot represents an observation, and the solid blue line is the empirical fitted line.
Chapter 2

Volatility, Intermediaries, and Exchange Rates

2.1 Introduction

Exchange rates are puzzling in many aspects. First, exchange rates are disconnected from economic fundamentals, especially relative consumption growth rate, which is in sharp contrast to implications of most international macro-finance models (Backus and Smith, 1993). Second, high-interest-rate currencies do not depreciate as the uncovered interest parity suggests. On the contrary, they often appreciate in subsequent periods (Hansen and Hodrick, 1980; Fama, 1984), known as the “forward premium puzzle”. As a result, excess returns of currency investment can be predicted by interest rate differentials. Third, it is hard to obtain exchange rate volatility close to data in standard international macro-finance models (Chari, Kehoe, and McGrattan, 2002; Brandt, Cochrane, and Santa-Clara, 2006). Lastly, covered interest rate parity, a classic no-arbitrage condition in the currency market, is violated for a decade after the global financial crisis (Du et al., 2018b). In this paper, we attempt to resolve these puzzles by focusing on the role of leveraged financial intermediaries in exchange rate determination.

15. This chapter is coauthored with Yang Liu.
Financial intermediaries are major participants in the foreign exchange (FX) market. More than 85% of turnovers in the FX market have financial institutions involved, according to the recent BIS triennial surveys. Moreover, non-dealer financial institutions account for more than half of turnovers. With respect to aggregate portfolio holding, the BIS reporting banks hold about half of the countries’ total external claims and more than 40% of total external liabilities in 21 OECD countries.¹⁶

The recent intermediary asset pricing literature has shown the importance of intermediaries on a broad class of asset returns (for example, Brunnermeier and Pedersen, 2009; Adrian et al., 2014; He and Krishnamurthy, 2013; He et al., 2017). It is natural to study exchange rates through the lens of an intermediary-based model. An essential feature of financial intermediaries is the constraint on taking leverage. The financial constraint is tightly linked to the volatility in the economy because of the value-at-risk (VaR) rule adopted by major financial institutions (Adrian and Shin, 2014). The VaR rule states that the size of the balance sheet shrinks with the rise of volatility in the economy.

In light of the dominance of intermediaries in the FX market and the constraints they are facing, we introduce these features into an otherwise standard international asset pricing model. The model has two ex-ante identical countries, home and foreign. Both countries have a continuum of homogeneous households and intermediaries. Households only have access to a risk-free money market account in local intermediaries. Intermediaries take deposits and invest in the local risky asset and the international bond. Both intermediaries face value-at-risk induced financial constraints, such that the size of the balance sheet cannot exceed a fraction of their market values (Gertler and Kiyotaki, 2010). The fraction increases with the volatility in the economy. In equilibrium, constrained from taking leverage, intermediaries’ marginal value of assets is higher than that of their liabilities. Since intermediaries are the only traders on intermediated assets including the local risky asset and the international bond, they require an excess return on those assets. The exchange rate change is a large component in international bond returns, so exchange rate dynamics are driven by the financial constraint. A higher volatility in the home country tightens its intermediaries’ constraint and increases the difference between the two marginal values, and thus leads to an expected foreign

¹⁶. Details will be shown in section 2.
We estimate the model using the simulated method of moments (SMM), and show that the model can resolve the four exchange rate puzzles quantitatively. We resolve the Backus-Smith puzzle by replacing the standard consumption Euler equation with an intermediary Euler equation, so that consumption and exchange rates are disconnected. As for the forward premium puzzle, when volatility increases in the home country, its interest rate declines. Meanwhile, because of a higher excess return required by home intermediaries, there is an expected foreign appreciation. The exchange rate volatility is closer to data, as the financial constraint amplifies the shocks in the economy. Finally, the tightened banking regulations after the global financial crises constrain the intermediaries from making arbitrage in the currency forward market and generate deviations from covered interest rate parity. Moreover, the model generates the cyclicity of CIP deviations consistent with empirical evidence documented by Avdjiev, Du, Koch, and Shin (2018). The deviations are large when home currency is strong, and when volatility is large.

We examine additional empirical implications of our model. First, we link exchange rates to the most direct measure of value-at-risk for currency traders, the exchange rate volatility. We find that a higher dollar exchange rate volatility predicts an appreciation of foreign currencies, and a higher currency return borrowing dollar and investing in foreign currencies. Second, as a more direct test on the channel of intermediaries and financial constraint, we measure the tightness of financial constraint in the US using the annual growth rate of US financial commercial paper outstanding. An increase in commercial paper is associated with looser financial conditions. We show that a higher amount of US financial commercial paper outstanding predicts a foreign depreciation and a lower foreign currency return. The predictability is preserved after controlling for other predictors in the literature. Third, when we include the commercial paper and exchange rate volatility in the standard regression of currency returns on interest rate differentials, the coefficient on interest rate becomes smaller, and less significant. It indicates that our mechanism is supported by data in resolving the forward premium puzzle.

Related Literature
A vast literature resolves the exchange rate puzzles in complete market settings. The leading models include habit formation (Verdelhan, 2010; Stathopoulos, 2016), long run risks (Colacito and Croce, 2011, 2013; Bansal and Shaliastovich, 2013), disaster risks (Farhi and Gabaix, 2016), etc. There are also various attempts to explain these puzzles in incomplete market models, including Corsetti et al. (2008), Maurer and Tran (2016) and Favilukis, Garlappi, and Neamati (2015). Lustig and Verdelhan (2016) shows that standard models with only financial market incompleteness cannot resolve multiple exchange rate puzzles simultaneously. Additional frictions are added into standard models to account for these puzzles: market segmentation (Alvarez, Atkeson, and Kehoe, 2002; Alvarez et al., 2009 and Chien, Lustig, Naknoi et al., 2015), nominal rigidity (Chari et al., 2002), search frictions (Bai and Ríos-Rull, 2015), infrequent portfolio decisions (Bacchetta and Van Wincoop, 2010). Itskhoki and Mukhin (2017) show the key friction to explaining exchange rate puzzles is the financial shock.

The literature of financial frictions emphasizes several types of constraints faced by capital market participants. Bernanke, Gertler, and Gilchrist (1999), Kiyotaki and Moore (1997), Gertler and Kiyotaki (2010), and Brunnermeier and Sannikov (2014) study the amplification effect of financial frictions on the macroeconomy. Jermann and Quadrini (2012) uncover that the time variation of financial constraint is an important source of aggregate fluctuations and financial flows. In the asset pricing literature, theoretical models show the importance of intermediaries in asset returns (Brunnermeier and Pedersen, 2009; He and Krishnamurthy, 2013; Li, 2013). Adrian et al. (2014), He et al. (2017), and Haddad and Muir (2017) find strong supportive evidence for a broad class of asset returns including stocks, bonds, and more complex securities such as mortgage-backed securities and derivatives. In international finance, Mendoza (2010) and Perri and Quadrini (2018) show that real shocks are amplified by financial frictions, leading to financial crisis. Dedola, Karadi, and Lombardo (2013) studies the transmission of shocks to financial constraints across countries.

Recently, the role of financial intermediation in exchange rate determination is emphasized by Gabaix and Maggiori (2015). They propose a theory with imperfect intermediation in the international financial market. Exchange rates are determined jointly by capital flows and intermediary balance sheet. Malamud and Schrmpf (2018) develop a theoretical model in which intermediaries
exploit their market power to seek rent, and explain the safe haven properties of exchange rates and CIP deviation. Our paper is different from them in several aspects. First of all, they provide theoretical frameworks while we bring the model to data and resolve the four puzzles in a quantitative manner. Second, we highlight the link between stochastic volatility and financial constraint fluctuations through VaR. Third, their model has a single global intermediary that intermediates capital flows, while our model studies risk sharing across countries of intermediaries with different financial constraints. Lastly, we provide supportive empirical evidence on our mechanism. Sandulescu et al. (2017) use a model-free approach to estimate international SDFs, and show strong links between model-free international SDFs and intermediary balance sheets as well as volatility.

The rest of the paper is organized as follows. Section 2 lays out some institutional features of the foreign exchange market and shows the preeminent role of leveraged financial institutions in exchange rate determination. Section 3 presents the model and section 4 illustrates how this model can qualitatively resolve the four exchange rate puzzles. In section 5 we estimate our model and show that the model can resolve the four exchange rate puzzles quantitatively. Empirical implications of the model are tested in section 6. Section 7 concludes the paper.

2.2 The Relevance of Financial Institutions

This section sketches the basic structure of the foreign exchange market. We show that financial institutions play a major role in the foreign exchange market, and thus exchange rate determination.

The foreign exchange market is the largest financial market in the world, with daily trading volume exceeding five trillion dollars in 2016, according to the BIS triennial survey. The structure of the foreign exchange market is two-tier: the inter-dealer market and dealer-customer market. Most inter-dealer transactions are high-frequency market-making transactions. These high-frequency transactions are not our considerations, as the half-life of inventory for dealers is only between

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17. Though there have been tremendous changes in the foreign exchange market in the recent decades, we describe the common features of the market across time. The new changes include the use of electronic trading systems, the increase of foreign exchange transactions between financial institutions, etc. For more institutional details of the foreign exchange market, see Osler (2008) and King, Osler, and Rime (2011).
1 to 30 minutes, and dealers usually end the day with a small amount of inventories (Bjønnes and Rime, 2005). There are several exceptions, according to Sager and Taylor (2006), that dealers take speculative positions in propriety trading with horizons from one day to three months. These longer horizon speculations are within our consideration in the paper.

Behaviors in the dealer-customer market are important determinants of exchange rates at monthly, quarterly, or annual frequencies. Main categories of customers include financial customers, corporate customers\textsuperscript{18}, and retail customers\textsuperscript{19}. Financial customers can be divided into two groups: real money investors and levered investors. Real money investors include mutual funds, pensions funds, endowments, and so on, which do not take leverage and infrequently adjust their portfolios. Levered investors include non-dealer commercial banks, hedge funds, and commodity trading advisors, and so on. They take high leverages and actively manage their portfolios.

We show that levered investors account for a substantial portion of turnovers in the FX market in Table 2.1 and Figure 2.1. Table 2.1 shows the fraction of FX turnovers by different entities from 1998 to 2016. Turnovers associated with nondealer financial institutions keep increasing and rise to 51% in 2016. Starting from 2013, the BIS triennial survey makes a detailed split of nondealer financial institutions into nonreporting banks (24%, 22\textsuperscript{20}), institutional investors (11%, 16%), hedge funds and PTFs (11%, 8%), official sector (1%, 1%), and other institutions (6%, 4%). Nonreporting banks, hedge funds and PTFs, and part of institutional investors are considered as levered investors. Meanwhile, nonfinancial transactions account for no more than 20% of all turnovers, and it has been declining in the recent decades. These facts motivate us to focus on the behavior of levered institutions to study exchange rates.

Besides looking at turnovers in the FX market, we also show the important role of banks in holding cross-border claims and liabilities using aggregate banking data. Figure 2.1 plots the time series weighted average of the ratio of banking claims (liabilities) over total claims (liabilities) from

\textsuperscript{18} Corporate customers trade for real purposes, such as production, investment, and dividend payout. The size of corporate transactions is small relative to financial transactions.

\textsuperscript{19} Retail customers, accounting for a very small fraction, are not studied in this paper.

\textsuperscript{20} Numbers in 2013 and 2016, respectively.
1977 to 2014\textsuperscript{21}. In the late 1970s and early 1980s, banks account for about half of external claims and 40 percent of external liabilities. This number declined substantially in the late 1990s, to 40 percent (claims) and 30 percent (liabilities) at the trough, possibly due to the global stock market boom. It rebounded back quickly in the 2000s until the global financial crisis in 2007.

Generally, levered financial intermediaries are constrained in taking leverage, so do the FX market participants. Speculative positions are constrained for various reasons, such as regulation, risk management and avoidance of excess risk taking for each trader. Banks in different countries are subject to the Basel regulatory capital adequacy framework with a minimal risk-weighted capital ratio of 8 percent and non-risk-weighted leverage ratio of 3 percent. Beyond regulation, FX market participants face market disciplines in balance sheet management, usually in the form of value-at-risk (VaR) constraint (Sager and Taylor, 2006). In practice, most intermediaries adopt VaR as their portfolio risk management model. It calculates the worst possible loss that will not exceed a given probability over a period. Intermediaries collect data on portfolio positions and market conditions to calculate their value-at-risk. They use different models to derive time variation in risk, such as ARCH, GARCH, and exponentially-weighted moving average models. When the Basel Committee on Banking Supervision allowed commercial banks to use their internal VaR model as the basis for market risk charge in 1998, the VaR model is widely accepted by the industry (Jorion, 2010). Usually, position limits are imposed on traders to avoid individual excess risk taking (Osler, 2008). Therefore, we argue that the variation in intermediaries’ financial constraint is the distinct feature of levered institutions that bring us new insights into exchange rate studies.

To sum up, we provide descriptive evidence on the preeminent role financial institutions (banks) play in the international financial market and their distinct feature of facing leverage constraints. In the next sections, we incorporate these features into an otherwise standard international asset pricing model and show these features help us resolve the exchange rate puzzles documented in the literature.

\textsuperscript{21} Countries include Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, South Korea, Spain, Sweden, Switzerland, UK, and US.
2.3 The Model

There are two ex-ante identical countries in the economy, home and foreign, each populated with a unit measure of households and endowed with a Lucas tree. The home tree delivers good $X$, and the foreign tree delivers good $Y$, both of which are tradable. In both countries, each household owns an intermediary and sends out a manager to operate it. Households make deposits in local intermediaries. Intermediaries combine deposits and their own net worth to invest in risky assets. There are two available risky assets, a claim to the local Lucas tree and an international bond. Intermediation is imperfect, in the form that the intermediaries in each country face a financial constraint, whose tightness is determined by the volatility in the local economy. Every period, a fixed fraction of intermediaries exit the market and rebate back their net worth to their owners, while the same measure of new intermediaries is set up with some initial funds to keep the measure of intermediaries stationary. The structure of the economy in each country is similar to Gertler and Kiyotaki (2010).

We describe the behavior of households and intermediaries in detail in the following subsections.

2.3.1 Households

Households in the home and foreign countries are endowed with a Lucas tree with different goods, $X$ for home and $Y$ for foreign. They follow cointegrated processes:

$$\log X_{t+1} - \log X_t = \mu + \tau (\log Y_t - \log X_t) + \sigma_{X,t} \epsilon_{X,t+1}$$

$$\log Y_{t+1} - \log Y = \mu - \tau (\log Y_t - \log X_t) + \sigma_{Y,t} \epsilon_{Y,t+1}$$

Volatilities are stochastic, following:

$$\log X_{t+1} - \log X_t = \mu + \tau (\log Y_t - \log X_t) + \sigma_{X,t} \epsilon_{X,t+1}$$

$$\log Y_{t+1} - \log Y = \mu - \tau (\log Y_t - \log X_t) + \sigma_{Y,t} \epsilon_{Y,t+1}$$

(2.1)
\[
\begin{align*}
\log(\sigma_{X,t+1}) &= (1 - \rho_\sigma) \log \bar{\sigma} + \rho_\sigma \log(\sigma_{X,t}) + \sigma_\sigma \eta_{X,t+1} \\
\log(\sigma_{Y,t+1}) &= (1 - \rho_\sigma) \log \bar{\sigma} + \rho_\sigma \log(\sigma_{Y,t}) + \sigma_\sigma \eta_{Y,t+1}
\end{align*}
\] (2.2)

The four shocks follow the standard normal distribution. The two goods aggregate into a consumption basket. The aggregator takes the form of constant elasticity of substitution:

\[
C = \left( (1 - \alpha) C_X^{\sigma-1} + \alpha C_Y^{\sigma-1} \right)^{\frac{1}{\sigma}}, C^* = \left( (1 - \alpha) C_Y^{* \sigma-1} + \alpha C_X^{* \sigma-1} \right)^{\frac{1}{\sigma}}
\]

\(C_X, C_Y\) are home households’ consumption of \(X\) and \(Y\), while variables with an asterisk refer to the foreign counterpart. Households in the home and foreign countries put different weights on \(X\) and \(Y\) with consumption home bias, i.e., \(\alpha < \frac{1}{2}\). \(\sigma\) is the price elasticity of substitution between \(X\) and \(Y\). We choose the home composite good as numeraire and define real exchange rate as the price of foreign composite good \(Q_t\). An increase in \(Q_t\) means a real appreciation of the foreign currency.

In every period, given composite consumption of \(C\), and prices \(P_X, P_Y\), home households choose how much \(X\) and \(Y\) to consume. Home households solve the intratemporal optimization problem:

\[
\min_{C_X, C_Y} P_X C_X + P_Y C_Y
\]

s.t. : \(C = \left( (1 - \alpha) C_X^{\sigma-1} + \alpha C_Y^{\sigma-1} \right)^{\frac{1}{\sigma}}\)

The allocation between \(X\) and \(Y\) are solved as:

\[
C_X = \frac{C(P_Y(P_X\frac{\alpha}{1-\alpha})^{-\sigma} + P_X(P_Y\frac{\alpha}{1-\alpha})^{-\sigma})}{P_Y + P_X(P_Y\frac{\alpha}{1-\alpha})^{-\sigma}}, C_Y = \frac{C}{P_Y + P_X(P_Y\frac{\alpha}{1-\alpha})^{-\sigma}}
\] (2.3)

For foreign households, the price of \(X\) and \(Y\) in foreign consumption basket are \(\frac{P_X}{Q}, \frac{P_Y}{Q}\), thus the
solution to foreign intratemporal optimization problem is:

\[ C_X^* = \frac{C^* (\frac{P_X}{P_Y})^{\frac{\alpha}{1-\alpha}}}{P_Y + P_X (\frac{P_X}{P_Y})^{\frac{\alpha}{1-\alpha}}} - \sigma Q \]

\[ C_Y^* = \frac{C^* Q}{P_Y + P_X (\frac{P_X}{P_Y})^{\frac{\alpha}{1-\alpha}}} - \sigma \] (2.4)

All households have identical Constant Relative Risk Aversion (CRRA) preferences over their country-specific consumption basket with risk aversion \( \gamma \). A fraction \( \alpha_1 \) of the endowment goes to the households as labor income, while the remaining are capitalized as a risky financial asset. These financial assets are interpreted broadly as bank loans and other fixed income securities that are generally intermediated by the financial sector. Households do not hold the risky financial assets directly. The only financial asset they have access to is a money market account offered by the local intermediaries, paying one unit of consumption basket risklessly in the subsequent period.

This assumption is consistent with the empirical evidence on households’ limited participation in the stock market (Vissing-Jørgensen, 2002) and passive portfolio behaviors without rebalancing (Chien, Cole, and Lustig, 2012). We interpret the labor income component of the endowment as cash flows received by passive investors. Moreover, this assumption is an extreme case of large efficiency loss for households to trade risky assets, while intermediaries have a comparative advantage in investment expertise, as in Brunnermeier and Sannikov (2014).

Households solve a standard intertemporal optimization problem:

\[ \max E \sum_{t=0}^{\infty} \frac{c_t^{1-\gamma} - 1}{1-\gamma} \]

s.t. \( C_t + D_t = \alpha_0 P_{X,t} X_t + R_{f,t-1} D_{t-1} + \Pi_t \)

\( D_t \) is the deposit by households into intermediaries at time \( t \), while \( R_{f,t-1} D_{t-1} \) is the repayment from intermediaries of principal and interest. \( \Pi_t \) is the net lump-sum payout from the intermediaries that exit the market, which will be specified later. Euler equations hold for households in both countries:

\[ E_i \beta (\frac{C_i^{t+1} - \gamma}{C_t})^{-\gamma} R_{f,t} = 1, \quad E_i \beta (\frac{C_i^{t+1} - \gamma}{C^*_t})^{-\gamma} R_{f,t}^* = 1 \] (2.5)
2.3.2 Intermediaries

Each intermediary solves a portfolio choice problem on how much deposit to take, how many domestic risky asset shares, and how many international bonds to purchase. We exclude the holding of the foreign tree by intermediaries, since domestic assets dominate foreign assets in most countries, known as “home equity bias” (Lewis, 1999).

Intermediation is imperfect with a leverage constraint on intermediaries in both countries.

\[ V_t \geq \theta_t (P_t s_t + d_{t,t}), \quad V_t^* \geq \theta_t^* (P^*_t s_t^* + d^*_t) \]  

(2.6)

\( V_t, V_t^* \) are the market value of an intermediary. \( P_t, P_t^* \) are the prices of the Lucas trees denominated in consumption basket in respective countries. \( s_t, s_t^* \) are the holding shares, and \( d_{t,t}, d_{t,t}^* \) are the holding of the international bond. Lower-case variables indicate individual intermediary’s choice, while upper-case indicate aggregate variables. The international bond pays off riskless return \( R_b \) denominated in a half unit of the home composite good and a half unit of the foreign composite good. We assume this payoff structure to preserve symmetry between the two countries, as in Heathcote and Perri (2016). When \( d_{t,t} < 0 \), home intermediaries are effectively borrowing from foreign intermediaries to purchase home assets. \( \theta_t \) and \( \theta_t^* \) measures the tightness of leverage constraint faced by intermediaries, which are linked to the volatility in each economy. We express \( \theta_t \) as a function of the volatility as:

\[ \theta_t = \theta_0 + \theta_1 \log(\sigma_{X,t}), \quad \theta_t^* = \theta_0 + \theta_1 \log(\sigma_{Y,t}) \]

These constraints model the distinct feature of intermediaries, the value-at-risk (VaR) constraint, as we discussed in Section 2.2. \( \theta_0 \) captures leverage restrictions caused by time-invariant frictions. \( \theta_1 \log(\sigma_{X,t}) \) and \( \theta_1 \log(\sigma_{Y,t}) \) model how the constraint varies with volatility. When volatility in the economy is higher, the intermediaries’ balance sheets become riskier and they have the incentive to reduce risk taking. The constraint can be micro-founded within an optimal contracting framework, such as Adrian and Shin (2014). At the same time, the constraint can also be due to regulation, such as the Basel III’s minimum risk-weighted capital requirement ratio, non-risk-weighted leverage ratio.
requirement, and stress test. Furthermore, borrowing and lending between intermediaries across borders are settled before repaying the households. Therefore, the VaR constraint is imposed on the sum of local risky asset and international bond position, even if it is a short position. We can easily extend the constraint to include different risk weights for the local risky asset and the international bond position.

From here on, we solve the intermediary problem in the home country. The problem for the foreign intermediary is exactly identical. The value function of a representative home intermediary can be written recursively as:

$$V_t(s_t, d_t, d_{I,t}) = \max_{s_{t+1}, d_{t+1}, d_{I,t+1}} E_t \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \left[ (1 - p) n_{t+1} + p V_{t+1}(s_{t+1}, d_{t+1}, d_{I,t+1}) \right]$$

s.t.: $n_{t+1} + d_{t+1} \leq P_{t+1} s_{t+1} + d_{I,t+1}$

$$\theta_{t+1}(P_{t+1} s_{t+1} + d_{I,t+1}) \leq V_{t+1}$$

$V(s_t, d_t, d_{I,t})$ is the value of the intermediary at the end of period $t$. The value function is a function of the holdings of domestic risky asset $s_t$, international bond $d_{I,t}$, and deposit $d_t$. Households own the intermediaries, so their stochastic discount factor $\beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma}$ is used to evaluate the cash flows. In period $t + 1$, the intermediary exits the market and pays out its net worth $n_{t+1}$ with probability $1 - p$. Otherwise, it continues to operate and pays out its net worth $n_{t+1}$ with probability $1 - p$. The first constraint is the balance sheet identity. The left-hand side is equal to the intermediary’s net worth plus deposit, while the right-hand is the intermediary’s holding of risky assets. The second constraint is the leverage constraint discussed before. The dynamics of net worth for a single intermediary is given by:

$$n_{t+1} = R_{S,t+1} P_t s_t + R_{I,t+1} d_{I,t} - R_{f,t} d_t$$

where $R_{S,t+1} = \frac{P_{t+1} s_t + (1 - \alpha_t) X_{t+1}}{P_t}$ is the return on holding domestic risky assets, and $R_{I,t+1} = \frac{1 + \theta_{t+1}}{1 + \theta_t} R_{b,t}$ is the return on holding international bonds. Even though the bond has a noncontingent return of $R_{b,t}$, intermediaries face exchange rate risks.
We guess the value function is linear in all three state variables, and verify later:

\[ V_t = v_{S,t} P_s s_t + v_{L,t} d_{L,t} - v_t d_t \]

Assigning the Lagrangian multipliers to the two constraints \( \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \lambda_{t+1} \) and \( \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \psi_{t+1} \), we can obtain the first order conditions:

\[ p v_{S,t+1} + \lambda_{t+1} - \psi_{t+1} (v_{S,t+1} - \theta_{t+1}) = 0 \]

\[ p v_{L,t+1} + \lambda_{t+1} - \psi_{t+1} (v_{L,t+1} - \theta_{t+1}) = 0 \]

\[ p v_{t+1} + \lambda_{t+1} - \psi_{t+1} v_{t+1} = 0 \]

From these three first order conditions, we have the key result:

\[ v_{S,t} = v_{L,t} \geq v_t \quad (2.7) \]

\( v_{S,t} \) and \( v_{L,t} \) are the marginal value of intermediary wealth invested in the domestic risky asset and the international bond. \( v_t \) is the marginal cost for the intermediary to take deposits. When the financial constraint does not bind, all three of them are equal. When the financial constraint binds, the marginal benefit of investing in the domestic risky asset and the international bond are identical, both being larger than the marginal cost of taking deposits. The determinant of whether the financial constraint binds or not is the net worth of the intermediary. If the intermediary has ample net worth, it will exhaust investment opportunities before hitting the constraint.

We plug back the value function into the Bellman equation, and derive expressions for coefficients:

\[ v_{t,t} = E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} (1 - p + p \theta_{t+1} \phi_t^{-1}) R_{S,t+1} \right] = E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} (1 - p + p \theta_t \phi_t^{-1}) R_{L,t+1} \right] \quad (2.8) \]

\[ v_t = E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} (1 - p + p \theta_{t+1} \phi_{t+1}^{-1}) R_{f,t} \right] \quad (2.9) \]
\( \phi_t \) is the leverage ratio of the home intermediary \(^{22}\), defined to be:

\[
\phi_t \equiv \frac{n_t}{P_t s_t + d_{I,t}}
\]

Foreign intermediaries face the same problem. The pricing equation of the international bond for foreign intermediaries is:

\[
\nu_{I,t}^* = E_t \left[ \beta \left( \frac{C_{S,t+1}^*}{C_t^*} \right)^{-\gamma} (1 - p + p\theta_t^* \phi_t^{*\gamma - 1}) R_{I,t+1} \frac{Q_t}{Q_{t+1}} \right]
\] (2.10)

### 2.3.3 Aggregation

Now that we have specified the problem as well as the optimality conditions for a single intermediary. The linearity of the model simplifies aggregation. Due to the representative intermediary setting, each intermediary has the same optimality conditions and makes the same choices. We can directly replace individual variables \( n_t, s_t, d_t, d_{I,t} \) and their foreign counterparts \( n_t^*, s_t^*, d_t^*, d_{I,t}^* \) with the aggregate variables \( N_t, S_t, D_t, D_{I,t} \) in the optimality conditions.

The net worth dynamics in aggregate is different from the one for a single intermediary, due to entry and exit. The aggregate dynamics is given by:

\[
N_{t+1} = (p + \xi) (R_{S,t+1} P_t S_t - R_{I,t+1} D_t)
\] (2.11)

\[
N_{t+1}^* = (p + \xi) (R_{S,t+1}^* P_t^* S_t^* - R_{I,t+1}^* D_t^* \frac{Q_t}{Q_{t+1}})
\] (2.12)

### 2.3.4 Equilibrium

Lastly, we have market clearing conditions for good markets and asset markets.

\[
C_{X,t} + C_{X,t}^* = X_t, \quad C_{Y,t} + C_{Y,t}^* = Y_t, \quad S_t = S_t^* = 1, \quad D_{I,t} + D_{I,t}^* Q_t = 0
\] (2.13)

\(^{22}\) More precisely, \( \phi_t \) is the ratio of home intermediaries’ net worth over risky position. In our simulated economy, the properties of \( \phi_t \) barely change if we define \( \phi_t = \frac{n_t}{P_t s_t + d_{I,t} I(d_{I,t} \geq 0)} \), where \( I \) is an indicator function.
A competitive equilibrium consists of a sequence of allocations \( \{C_{Xt}, C_{Yt}, C^*_X, C^*_Y, D_t, D^*_t, N_t, N^*_t, S_t, S^*_t, D_{I}, D^*_{I}, \phi_t, \phi^*_t\} \), a sequence of prices \( \{R_{f_t}, R^*_{f_t}, P_{Xt}, P_{Yt}, P_t, P^*_t, Q_t, R_{b, I}\} \), and a sequence of intermediary valuation \( \{\nu_{S_t}, \nu_{I_t}, \nu_t, \nu^*_{S_t}, \nu^*_{I_t}, \nu^*_t\} \) such that:

(i) Households in both countries solve their optimization problems;

(ii) Intermediaries in both countries solve their constrained optimization problem;

(iii) Good markets (X and Y) clear;

(iv) Asset markets (home and foreign deposits, home and foreign risky assets, and the international bond) clear.

## 2.4 Model Mechanisms

### 2.4.1 Impulse Response Functions

Figures 2.2 and 2.3 report the impulse response functions of different variables in both countries to the home country’s one-standard-deviation positive endowment shock and volatility shock. Parameters are estimated and shown in Table 2.2.

When the home country has a positive endowment shock, the dividend payment of the Lucas tree increases, and home households’ consumption growth increases. The interaction between intermediary net worth and asset price amplifies the response of return to the home tree. The marginal value of net worth \( \nu_t \) and the marginal cost of borrowing \( \nu \) both decline, as does the wedge between the two. The real risk-free rate in the home country increases slightly. The leverage ratio of intermediaries increases, because the strengthening of net worth dominates the expansion of their balance sheets. The endowment shock in the home country is transmitted to the foreign country, with all foreign variables moving in the same direction but smaller magnitude, as a result of imperfect international risk sharing. The mechanism of intermediary balance sheet synchronization is similar to Dedola et al. (2013). Since both \( \nu_t \) and \( \nu^*_t \) decrease, both intermediaries require a lower expected return on the international bond and \( R_b \) decreases. However, \( \nu_t \) in the home country declines more than \( \nu^*_t \). Consequently, the foreign currency appreciates contemporaneously and is expected to de-
preciate. We can also understand the exchange rate movement from the goods market side. An increase in the supply of home good $X$ is accompanied by a contemporaneous foreign appreciation.

The more interesting channel in our model can be seen in the impulse responses to a one standard deviation volatility shock. The volatility shock is amplified through the same interaction between intermediary net worth and asset price illustrated previously. The greater volatility will tighten home intermediaries’ financial constraints. Since the home intermediaries are not able to take as much deposit from households, home households’ consumption increases, and home real interest rate $R_f$ decreases. The marginal benefit of net worth $ν_3$ as well as the marginal cost of borrowing $ν$ both increase, and so does their difference. The transmission of the shock to the foreign country is again as previously shown: Foreign variables move in the same direction as home variables but with a smaller magnitude. Similarly, expected exchange rate change reflects the difference between home and foreign intermediaries’ valuation of the international bond. Therefore, the foreign currency is expected to appreciate.

### 2.4.2 Asset Prices

In our model, intermediaries play the central role in pricing all the assets. The augmented Euler equations for intermediaries are key determinants of exchange rates as well as prices for the home and foreign risky asset. As we show in Section 2.3.2, the asset pricing equations are equations (2.8) and (2.9).

The stochastic discount factor that prices the domestic tree and the international bond is different from the one that prices deposits. The difference depends on the wedge between the marginal value of net worth and the marginal cost of taking deposits, which relies on the tightness of the financial constraint.

The stochastic discount factor has three components: consumption growth $β(C_{t+1}/C_t)^{-γ}$, the subsequent value of the intermediary $1 - p + pθ_{t+1}φ_{t+1}^{-1}$, and the marginal value of net worth $ν_{l,t}$ or the marginal cost of taking deposits $ν_t$. When leverage constraint binds, the leverage constraint can
be rewritten as:

\[ \theta_t \phi_t^{-1} = \frac{V_t}{N_t} \]

The additional term \( 1 - p + p \theta_{t+1} \phi_{t+1}^{-1} \) is economically intuitive: With probability \( 1 - p \), the intermediary exits the market and pays out its net worth; with probability \( p \), the intermediary continues to operate, and each dollar remaining in the intermediary generates market value of \( \theta_{t+1} \phi_{t+1}^{-1} \).

### 2.4.3 Exchange Rate Puzzles

In this subsection, we review the exchange rate puzzles in the literature and analyze how our model helps resolve these puzzles.

**Backus-Smith Puzzle**

Backus and Smith (1993) show that from consumption based Euler equations, exchange rate change is perfectly correlated with consumption growth differential under the complete market. To see it more clearly, denote the home stochastic discount factor (SDF) to be \( M_{t+1} \), and the foreign SDF \( M_{t+1}^* \). Consider the return of home risk-free bond \( R_{f,t} \), the following equation hold:

\[
E_t [M_{t+1} R_{f,t}] = E_t \left[ M_{t+1}^* R_{f,t} \frac{Q_t}{Q_{t+1}} \right] = 1
\]

\( Q_t \) is the price of foreign goods in terms of home goods. If the financial market is complete, then the equation holds state by state. Therefore:

\[
\Delta q_{t+1} = m_{t+1}^* - m_{t+1}
\]

Lower-case letters are natural logarithms of variables. If we assume a constant relative risk aversion utility function, we obtain:

\[
\Delta q_{t+1} = \gamma (\Delta c_{t+1} - \Delta c_{t+1}^*)
\]
Exchange rate change is perfectly correlated with consumption growth differential. Even when the financial market is incomplete, such as in the models of Heathcote and Perri (2002) and Chari et al. (2002), the correlation between $\Delta q_{t+1}$ and $\Delta c_{t+1} - \Delta c^*_t$ is still close to 1. This is inconsistent with the weak correlation between exchange rate changes and consumption growth differentials in the data.

In our model, we have augmented Euler equations for intermediaries in both countries:

$$E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \frac{(1 - p + p\theta_{t+1}^i \phi_{t+1}^{-1})}{\nu_{t,i}} \right] = E_t \left[ \beta \left( \frac{C^*_{t+1}}{C_t} \right)^{-\gamma} \frac{(1 - p + p\theta^*_i \phi^*_{t+1}^{-1})}{\nu^*_{t,i}} \right]$$

Exchange rate change is linked to consumption growth differential plus two extra terms: the subsequent value of the intermediary, and the relative marginal value of net worth. Therefore, consumption growth is disconnected with exchange rate.

The responses of consumption and exchange rate to endowment and volatility shocks also help us understand the disconnect. As we show in the impulse response functions in Section 2.4.1, when the home country has a positive endowment shock, consumption of home households increases while the home currency depreciates. On the other hand, when the home country experiences a positive volatility shock, home consumption increases as well, but the home currency appreciates. The two forces at play offset each other and generate the weak correlation between consumption and exchange rate.

**Forward Premium Puzzle**

Uncovered Interest Rate Parity suggests that when the home country has a higher interest rate than the foreign country, the home currency is expected to depreciate in the next period so that investing in home and foreign deliver the same payoffs in expectation. However, this parity condition is rejected by data (Hansen and Hodrick, 1980; Fama, 1984). Typically, the currency with higher interest rate tends to further appreciate. This puzzle is also called the “forward premium puzzle”.

In our model, volatility shocks explain the puzzle through intermediaries’ financial constraints.
We log-normally approximate the household Euler equation, and obtain:

\[ r_{f,t} \approx -\log \beta + \gamma E_t \Delta c_{t+1} - \frac{1}{2} \gamma^2 Var_t \Delta c_{t+1} \]

As we show in the impulse response functions in Figure 2.3, when the home country experiences a positive volatility shock, the home interest rate is lower for two reasons. First of all, the variance term \( \frac{1}{2} \gamma^2 Var_t \Delta c_{t+1} \) is larger and interest rate falls through the precautionary saving effect. Second, the increased volatility tightens the constraint of home intermediaries. Home households consume more, thus lower the expected consumption growth \( E_t \Delta c_{t+1} \). The second force also makes foreign households consume less and increases the foreign interest rate.

As for exchange rates, increased home volatility tightens home intermediaries’ constraints, and widens the wedge faced by intermediaries. The financial constraint for intermediaries will in turn affect the international bond and currency market. Home intermediaries require higher expected returns on the international bonds than foreign intermediaries. Therefore, the foreign currency is expected to appreciate, even though the foreign interest rate is higher.

With a log-normal approximation, we combine home and foreign intermediaries’ Euler equations for international bond and deposit, and get:

\[ E_t \Delta q_{t+1} \approx (r_{f,t} - r_{f,t}^*) + (\log v_{t,t} - \log v_i) - (\log v_{t,t}^* - \log v_i^*) + \text{second order terms} \quad (2.14) \]

Equation (2.14) links the expected exchange rate change to the intermediaries explicitly. When the home country experiences a positive volatility shock, home intermediaries are more constrained, thus \( \log v_{t,t} - \log v_i > \log v_{t,t}^* - \log v_i^* \). If the wedges do not exist, uncovered interest rate parity holds. In our model, the wedge dominates the interest rate difference in driving exchange rates.

There is a vast literature about the relationship between stochastic volatility and forward premium puzzle. Backus et al. (2001) show that in a complete market setting with affine linear stochastic discount factors, stochastic volatility is necessary to generate time-varying currency premium.
Bansal and Shaliastovich (2013) attributes time variation in currency risk premium to volatility fluctuations in a structural model. Different from their channel, volatility affects exchange rates in our model through time-varying financial constraint faced by intermediaries. Therefore, we are proposing a new mechanism to link volatility to exchange rates that complements the existing mechanisms.

**Exchange Rate Volatility Puzzle**

Most international macro-finance models with incomplete financial market cannot generate volatile exchange rates as in the data. The exchange rate volatility in our model is close to data because the financial constraints of intermediaries amplify the endowment volatility. As a result, exchange rates are more volatile than in standard two-country models.

**Deviation from Covered Interest Rate Parity**

Covered Interest Rate Parity (CIP) is one of the most famous no-arbitrage conditions in finance. Borrowing in home currency and lending in foreign currency with a currency swap is risk free, and should yield zero profit. CIP condition holds quite well before the global financial crisis in 2007 (Akram, Rime, and Sarno, 2008), but is deviated persistently after the crisis (Du et al., 2018b). Du et al. (2018b) and Cenedese, Della Corte, and Wang (2017) illustrate that tightened banking regulation is a key driver of the CIP deviations. The requirement of the leverage ratio is a non-risk-based constraint imposed on intermediaries’ balance sheet. Even if CIP arbitrage is riskless, it expands the balance sheet. To make the expansion subject to the constraint, banks need more capital that is costly. Moreover, stringent risk-weighted capital requirement and stress tests also increase the opportunity cost of CIP arbitrage.

This phenomenon is naturally interpretable in our model with intermediaries as arbitrageurs. The return on foreign risk-free lending through a currency swap is usually called the synthetic home currency rate, denoted as $R_{cip,t}$. In the model, if the intermediaries can conduct CIP arbitrage.

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23. We assume both the synthetic rate and deposit rate are risk free and ignore the risk exposure of CIP arbitrage. Sushko, Borio, McCauley, and McGuire (2017) explain CIP deviation through counterparty risks of forward contracts. Andersen, Duffie, and Song (2017) consider funding risks and funding value adjustments as an explanation.
without any constraint, then the synthetic home currency rate will have the same Euler equation as the home currency risk-free rate.

\[ \nu_t = E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} (1 - p + p \theta_{t+1} \phi_{t+1}^{-1}) R_{cip,t} \right] \]

Therefore, the two rates are equal and CIP holds. After the tightening of regulation, intermediaries cannot freely trade currency swaps without any constraint. In this case, the marginal value of a synthetic home currency lending is \( \nu_{I,t} \), the same as other constrained investments and higher than the marginal value of deposits \( \nu_t \).

\[ \nu_{I,t} = E_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} (1 - p + p \theta_{t+1} \phi_{t+1}^{-1}) R_{cip,t} \right] \]

The currency basis \( r_{cip,t} - r_{f,t} \) becomes:

\[ r_{cip,t} - r_{f,t} = \log \nu_{I,t} - \log \nu_t > 0 \]

Hence, constrained intermediaries allow deviations from CIP. In our model, as in the real world, intermediaries are the major participants in the currency market. Households barely trade currency swaps, and cannot arbitrage away the currency basis.

Since our model has market segmentation and limits to arbitrage, it is natural for deviations from CIP to arise. Beyond the existence of the CIP deviations, the model is also able to explain some of its key cyclical properties. Avdjiev et al. (2018) and Sushko et al. (2017) document that CIP deviations are large when home currency is strong, and when volatility is large. In the model, CIP deviations correspond to the tightness of the constraint. When the constraint tightens, home currency appreciates contemporaneously and is expected to depreciate, as we discussed in the forward premium puzzle. Therefore, a stronger home currency is associated with larger CIP deviations. As for volatility, intuitively, higher volatility tightens the constraint and enlarges the CIP deviations.
2.5 Quantitative Results

After illustrating the mechanisms to resolve the exchange rate puzzles in the model, we bring the model to data and match the key facts about exchange rates quantitatively.

2.5.1 Parameter Estimation

The model is at quarterly frequency. We estimate the model with the simulated method of moments (SMM). The estimation details are in the Appendix. Benchmark parameter values are reported in Table 2.2.

Following the standard practice, we set the time discount factor, labor income share, and risk aversion at standard values, 0.995, 0.67, and 2 respectively. We assume the average growth rate to be 0 in order to match the low interest rate level\textsuperscript{24}.

Volatility and of endowment processes are estimated first, using the data in the G7 countries from 1973 to 2015 as in Colacito, Croce, Liu, and Shaliastovich (2018b). Shocks are uncorrelated across countries. The stochastic volatility processes are the main driving forces of our model. The persistence of the volatility is 0.90, and the volatility of volatility is 0.075. In our model, it is the idiosyncratic volatility that moves the relative tightness of leverage constraints. Fluctuations in the common component of volatility do not affect the risk sharing between intermediaries, so they are abstracted from the model. We impose a weak cointegration relationship between the two endowment processes with the error correction parameter $\tau$ to be 0.0005 to keep the global economy stationary.

In the SMM, we estimate six parameters, home bias $\alpha$, trade elasticity $\sigma$, survival rate $p$, initial funds $\xi$, the constant and slope of the VaR constraint $\theta_0$ and $\theta_1$ to match six key moments about intermediaries and exchange rates: the leverage ratio $\phi$\textsuperscript{25}, exchange rate volatility $sd(\Delta q)$, correlation

\textsuperscript{24} The tension between consumption growth and risk-free rate is a long-standing puzzle (risk-free rate puzzle, Weil, 1989). This paper does not attempt to provide any new insight on the resolution of the risk-free rate puzzle. For example, introducing recursive utility to separate relative risk aversion and elasticity of intertemporal substitution will match the risk-free rate even with an average growth rate of two percent (Bansal and Shaliastovich, 2013).

\textsuperscript{25} We follow the definition by Krishnamurthy and Vissing-Jorgensen (2015) of the “financial sector of all institutions supplying short-term debt. These institutions include US-Chartered Depository Institutions, Foreign Banking Offices.
between exchange rate and consumption growth differential \( \text{corr}(\Delta q, \Delta c - \Delta c^*) \), OLS estimates of currency return on interest rate differential \( \beta_{FP} \) and exchange rate change on log volatility \( \beta_{\text{vol}} \), and absolute deviation from CIP \( r_{cip} - r_f \). Despite using a new set of exchange rate related moments in the estimation, our estimates are close to the literature.

The degree of consumption home bias \( \alpha \) is as small as 0.043, close to Colacito and Croce (2013). The elasticity of substitution between the two goods is 0.587, consistent with estimates based on macro quantities (Stockman and Tesar, 1995; Heathcote and Perri, 2002). The survival rate \( p \) and initial funds rate \( \xi \) are 0.964 and 0.003, which implies an average horizon of bankers of a decade. \( \theta_0 \) is estimated to be 0.376. In Gertler and Kiyotaki (2010), these three parameters are 0.972, 0.003, and 0.383, very close to the result of our estimation. The slope of \( \theta_1 \) is 0.207, which determines the relative importance of VaR induced constraint fluctuations on exchange rates.

### 2.5.2 Quantitative Results

Table 2.3 presents the quantitative results of the model.

The first two columns report the moments in the data and in our benchmark model. The upper panel lists country-specific moments. Among the moments, consumption growth volatility is close to endowment growth volatility, which is estimated directly from data, and the leverage ratio \( \phi \) is chosen as the target moment in our estimation.

Beyond these targets, our benchmark model is able to match the mean and standard deviation of risk-free rate and the volatility and persistence of the leverage. In terms of the risky asset return, we interpret it broadly as intermediated assets, including bank loans and other fixed income securities, so bond excess returns are preferred proxies to stock excess returns. In the data, the investment-grade corporate bond holding period return (from Barclays) is 4.01 percent on average, with a volatility of 5.40 percent. Our model matches the volatility but undershoots risk premium. However, if we proxy the excess return with the long-term credit spread between Moody’s BAA corporate

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*in the US, Banks in US-Affiliated Areas, Credit Unions, Money Market Mutual Funds, Issuers of Asset-Backed Securities, Finance Companies, Mortgage Real Estate Investment Trusts, Security Brokers and Dealers, Holding Companies and Funding Corporations. Quarterly assets and liabilities data for each type of financial institution are from the Flow of Funds.*
bonds and treasury bonds as in Gertler and Kiyotaki (2010), the excess return is 1.78 percent on average, roughly the same magnitude as our model. To highlight the main mechanism, we abstract from additional features such as habit formation, long-run risks, or disaster risks, to resolve the equity premium puzzle. The model delivers additional implications on the money market: the spread between risk-free interbank lending and wholesale funding \( r_{mm} - r_f \) is equal to the wedge \( \log \nu_f - \log \nu \). Our model implies an average spread of 24 basis points, similar to the data counterpart of LIBOR-OIS spread.

The lower panel shows that our model can resolve the exchange rate puzzles quantitatively. Exchange rate volatility is 8.61 percent, close to the 10 percent in the data. Consumption growth differential is weakly correlated with exchange rate change, and the regression coefficient of currency return on interest rate differential is 2.04, greater than unity. Finally, our model generates an average CIP deviation of 24 basis points as in the data. We note that the magnitude of average CIP deviation is similar to LIBOR-OIS spread. It is precisely what our model implies: the wedge \( \log \nu_f - \log \nu \) is equal to CIP deviation, as well as the spread between risk free interbank lending and wholesale funding. Moreover, the model generates the cyclicality of CIP deviations consistent with empirical evidence documented by Avdjiev et al. (2018). The change of CIP basis has a positive regression coefficient on log currency implied volatility (\( \beta_{\Delta cip, \Delta vol} = -0.26 \)) \(^{26}\) and a negative regression coefficient on the change of exchange rates (\( \beta_{\Delta cip, \Delta q} = -2.08 \)). Our model can closely match the size of \( \beta_{\Delta cip, \Delta vol} \) and the sign of \( \beta_{\Delta cip, \Delta q} \)\(^{27}\). The undershooting of \( \beta_{\Delta cip, \Delta q} \) could be a result of the specialty of the dollar discussed in Avdjiev et al. (2018).

To demonstrate the importance of volatility channel through the intermediaries, we turn off the link between the financial constraint and volatility (\( \theta_1 = 0 \)) and report the moments in Column 3. In this case, the Backus-Smith correlation is as large as 0.70. The forward premium regression coefficient \( \beta_{FP} \) is extremely large, as in this case the real interest rate barely moves. Exchange rates are not related to volatility, and the CIP deviation is 69 basis points, much larger than what we observe in the data. \( \beta_{\Delta cip, \Delta vol} \) becomes very small, while the sign of \( \beta_{\Delta cip, \Delta q} \) remains negative. We

\(^{26}\) Our definition of the basis is the negative of theirs, so the coefficient also has an opposite sign.  
\(^{27}\) Since we do not have currency implied volatility in the model, we use the log fundamental volatility \( \sigma \log(\sigma_x) \). Assuming unity elasticity between the two volatilities, the model should replicate the regression in the data.
can conclude that the volatility mechanism through intermediaries goes a long way to explain the exchange rate puzzles.

Finally, we turn off stochastic volatility completely and report the results in Column 4. The results are very similar to Column 3, showing that stochastic volatility itself does not play an important role beyond its effects through the intermediaries.

2.6 Empirical Implications

Our model provides an exchange rate determination theory through intermediaries, with equation (2.14) as the main prediction: the expected exchange rate change is driven by both interest rate differential and the relative tightness of financial constraints across the two countries. Furthermore, the financial constraint tightness is driven by the volatility in each country. In this section, we show that this relation is supported by data.

We first show that a higher dollar exchange rate volatility predicts a foreign appreciation as well as a higher currency return to borrow the dollar and invest in foreign currencies. As a more direct test on the channel of intermediaries and financial constraint, we measure the tightness of financial constraint in the US using the annual growth rate of US financial commercial paper outstanding. An increase in commercial paper is associated with looser financial constraints. We show that a higher amount of US commercial paper outstanding predicts a foreign depreciation and a lower currency return.

2.6.1 Data

We have the spot and forward exchange rate data vis-a-vis dollar for 12 countries: Australia, Belgium, Canada, Denmark, France, Germany, Italy, Japan, New Zealand, Switzerland, Sweden, and the UK. The exchange rate data are obtained from Datastream. We also obtain daily spot exchange rate, compute the realized volatility in each year, and take the average over all these countries. This measure is the average dollar exchange rate volatility, which is faced by all investors involved in dollar trade in the FX market. For direct measures of financial constraint, we use the amount of
US outstanding commercial paper for financial business, published by the Federal Reserve Board. The data spans from January 1991 to December 2015 at monthly frequency. We use the annual growth rate of commercial paper as our measure. We include several controls of average forward discounts, US price dividend ratio, US annual growth rate of industrial production, and exchange rate volatility. The control variables are from National Income and Product Accounts and CRSP.

### 2.6.2 Predictive Regressions with Dollar Exchange Rate Volatility

In this subsection, we examine the relationship between dollar exchange rate volatility, future dollar exchange rate change, and the return of borrowing dollar and investing in foreign currencies. We choose the average log dollar exchange rate volatility as a measure of financial constraint tightness in the US, as it is faced by all investors involved in dollar trade in the FX market.

The results of predictive regressions are shown in Table 2.4. The upper panel shows results for exchange rate changes, and the lower panel shows results for currency returns. Standard errors are robust to heteroskedasticity, serial autocorrelation, and overlapping observations (Hodrick, 1992). Univariate regression results in Row 1 to 3 show that a higher dollar exchange rate volatility predicts a foreign appreciation and a higher currency return. A one percent increase in dollar exchange rate volatility predicts a 20-basis-points average foreign currency appreciation and 25-basis-points currency excess returns per annum in horizons of 1 month, 3 months, and 12 months. The results are robust to including various controls, including average forward discount, US price dividend ratio, US industrial production growth. Average forward discount and industrial production growth are considered drivers of countercyclical currency risk premium (Lustig, Roussanov, and Verdelhan, 2014). Furthermore, we find that the predictive power of average forward discounts on both exchange rate changes and currency returns are weakened after controlling for exchange rate volatility.

The upper panel of Figure 2.4 reports the regression coefficients and confidence intervals of exchange rate predictability at 3 months horizon for each currency pair. All points estimates are positive and close to our results in Table 2.4, and most coefficients are statistically significant.
2.6.3 Predictive Regressions with US Commercial Paper

In this subsection, we further test the predictive power financial constraint tightness on exchange rate changes and currency returns. A tighter financial constraint manifests in the money market first and leads to a smaller amount of commercial paper. Therefore, the financial commercial paper outstanding is used as a measure of US financial constraint tightness.

Table 2.5 reports the predictive regression results of commercial paper growth for average dollar exchange rate and currency returns with different predictive horizons of 1 month, 3 months, and 12 months. The upper panel shows the results for exchange rate changes and the lower panel for currency returns. From univariate regression results in Row 1 to 3, a one-percent higher commercial paper growth rate in the US predicts a 30 basis point foreign depreciation in the subsequent month or quarter, and a 20 basis point foreign depreciation in the subsequent year. As for currency returns, a one percent higher commercial paper growth rate in the US predicts a 38 basis points lower excess return in the subsequent month or quarter, and a 26 basis points lower excess return in the subsequent year. In both exchange rate and currency predictive regressions, $R^2$ increases with predictive horizons.

The predictability of exchange rates and currency returns are also robust to controlling various variables, including average forward discount, US price dividend ratio, US industrial production growth. In this case, price dividend ratio and industrial production growth rate are considered credit demand indicators as well. After controlling for these variables, the information contained in commercial paper mostly comes from the credit supply side, or the tightness of financial constraints faced by intermediaries. We also find that the predictive power of average forward discounts is weakened after controlling for commercial paper, while dollar exchange rate volatility becomes insignificant as well.

In the lower panel of Figure 2.4, we show the univariate predictive regression coefficients and confidence intervals of exchange rates predictability at 3 months horizon for each currency pair. All point estimates are negative, and most coefficients are statistically significant.
2.7 Conclusion

Financial intermediaries are major participants in the foreign exchange market. In light of the dominance of intermediaries in the FX market and the constraints they are facing, we introduce these features into an otherwise standard international asset pricing model. An essential feature of financial intermediaries is the constraint on taking leverage. The financial constraint is tightly linked to the volatility in the economy because of the value-at-risk (VaR) rule adopted by major financial institutions.

We estimate the model using the simulated method of moments (SMM), and show that the model can resolve four exchange rate puzzles quantitatively. We resolve the Backus-Smith puzzle by replacing the standard consumption Euler equation with an intermediary Euler equation, so that consumption and exchange rates are disconnected. As for the forward premium puzzle, when volatility increases in the home country, its interest rate declines. Meanwhile, because of higher excess return required by home intermediaries, there is an expected foreign appreciation. The exchange rate volatility is closer to data, as the financial constraint amplifies the shocks in the economy. Tightened banking regulations after the global financial crises constrain the intermediaries from making arbitrage in the currency forward market and generate deviations from covered interest rate parity. Moreover, the model generates the cyclicality of CIP deviations consistent with empirical evidence. The deviations are large when home currency is strong, and when volatility is large.

Several model implications are supported by the data. As measures of intermediary financial constraints, dollar exchange rate volatility and US financial commercial paper outstanding predict exchange rate changes and currency returns. When we include the commercial and exchange rate volatility in the standard regression of currency returns on interest rate differentials, the coefficient on interest rate becomes smaller and less significant. It indicates that our mechanism is supported by data in resolving the forward premium puzzle.
### 2.8 Tables and Figures

Table 2.1: Foreign Exchange Turnovers by Counterparties

Units are in percentage point. Data source: BIS triennial survey for specific years.

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<td>Reporting dealers</td>
<td>63</td>
<td>59</td>
<td>53</td>
<td>43</td>
<td>38.9</td>
<td>39</td>
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<td>Nonfinancial customers</td>
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<td>13</td>
<td>17</td>
<td>13.4</td>
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<td>Other financial institutions</td>
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<td>33</td>
<td>40</td>
<td>47.7</td>
<td>53</td>
<td>51</td>
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</table>
Table 2.2: Parameters

The parameters in the upper panel are calibrated to the common value in the literature and empirical estimates. The parameters in the lower panel are estimated using the simulated method of moments.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Notation</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td><strong>Calibration</strong></td>
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<td>Discount factor</td>
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</tr>
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Table 2.3: Quantitative results

The model moments are obtained from the average of repeated simulations of a sample of 40 years. All moments are annualized. “Benchmark” indicated the moments of the benchmark model. “No VaR” indicated the moments of the model with constraint not related to volatility ($\theta_1 = 0$). “No SV” indicated the moments of the model with no time-varying volatility ($\sigma_X = \sigma_Y = \bar{\sigma}$).

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|                      | Exchange rates moments |        |        |       |
| $sd(\Delta q)$      | 10.50 | 8.61      | 8.35   | 8.25  |
| $corr(\Delta q, \Delta c - \Delta c^s)$ | -0.13 | -0.10     | 0.70   | 0.70  |
| $\beta_{FP}$        | 2.20  | 2.04      | 7.82   | 7.91  |
| $\beta_{vol}$       | 0.21  | 0.09      | 0.00   | 0.00  |
| $r_{cip} - r_f$      | 0.24  | 0.24      | 0.69   | 0.69  |
| $sd(r_{cip} - r_f)$  | 0.27  | 0.11      | 0.17   | 0.17  |
| $\beta_{\Delta cip, \Delta vol}$ | -0.26 | -0.18     | 0.01   | 0.00  |
| $\beta_{\Delta cip, \Delta q}$   | -2.08 | -0.07     | -0.22  | -0.21 |
Table 2.4: Volatility and Exchange Rates

The table reports estimates from OLS regressions of future exchange rate changes and currency excess returns on currency volatility and other controls. \( \Sigma_{t=1}^{h} \Delta y_{t+h} = \beta_0 + cp_t \beta_1 + AFD_t \beta_2 + pd_t \beta_3 + \Delta ip_t \beta_4 + vol_t \beta_5 + u_t \). \( \Delta y_t \) is either exchange rate changes or currency excess returns. \( cp_t \) is the annual growth rate of commercial paper outstanding. \( AFD_t \) is the average forward discount. \( pd_t \) is price-to-dividend ratio. \( \Delta ip_t \) is the annual growth of industrial production. The t-statistics are based on heteroscedasticity and autocorrelation consistent (HAC) standard errors (Hodrick, 1992). Data are monthly from 1980M1 to 2015M12.

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Table 2.5: Commercial Paper Outstanding and Exchange Rates
The table reports estimates from OLS regressions of future exchange rate changes and currency excess returns on currency volatility and other controls. \( \sum_{i=1}^{h} \Delta y_{t+i} = \beta_0 + \text{vol} \beta_1 + AFD \beta_2 + pd \beta_3 + \Delta ip \beta_4 + u_{t+h}. \) \( \Delta y_i \) is either exchange rate changes or currency excess returns. \( \text{vol} \) is the average dollar realized volatility. \( AFD \) is the average forward discount. \( pd \) is price-to-dividend ratio. \( \Delta ip \) is the annual growth of industrial production. \( h \) shows the predictive horizon of months. The t-statistics are based on heteroscedasticity and autocorrelation consistent (HAC) standard errors (Hodrick, 1992). Data are monthly from 1991\( M_1 \) to 2015\( M_{12} \).

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Figure 2.1: Cross-border Banking Claims and Liabilities
The figure plots weighted average of 21 countries’ cross border banking claims (liabilities) over external portfolio claims (liabilities). Cross-border banking claims (liabilities) are from BIS locational banking statistics, and external portfolio claims (liabilities) are from Lane and Milesi-Ferretti (2007), updated until 2011. We use the share of a country’s external portfolio claims (liabilities) as the weight.
Figure 2.2: Impulse Response Functions to a Positive Home Endowment Shock

The figure reports the impulse responses to a positive one-standard-deviation home endowment shock. Variables include consumption growth ($\Delta c$), risky asset price ($P_s$), risky asset return ($R_s$), risk-free rate ($R_f$), international bond return ($R_b$), marginal value of investment for intermediaries ($\nu_s$), marginal cost of taking deposit for intermediaries ($\nu$), and leverage ($\phi$). Impulse responses of both home and foreign variables are shown in the same figure. Also, the responses of exchange rate ($Q$) are reported.
Figure 2.3: Impulse Response Functions to a Positive Home Volatility Shock

The figure reports the impulse responses to a positive one-standard-deviation home volatility shock. Variables include consumption growth ($\Delta c$), risky asset price ($P_s$), risky asset return ($R_s$), risk-free rate ($R_f$), international bond return ($R_b$), marginal value of investment for intermediaries ($\nu_s$), marginal cost of taking deposit for intermediaries ($\nu_l$), and leverage ($\phi$). Impulse responses of both home and foreign variables are shown in the same figure. Also, the responses of exchange rate ($Q$) are reported.
Figure 2.4: Exchange Rate Predictability: Individual Countries

The figures present the univariate regression evidence of predictability of future dollar value by volatility and growth of commercial paper outstanding. The dependent variables are the log changes in real dollar values against individual currencies. The figure shows the OLS coefficients on exchange rate volatility (upper panel) and commercial paper outstanding (lower panel) and the associated HAC 95% confidence intervals. Data are monthly from 1980M1 to 2015M12.
Appendices
Appendix for Chapter 1

.1 Proofs

.1.1 Proof of Proposition 2

Using the definition in (1.12) and (1.13):

\[
L_D^j - L_H^j = \frac{\lambda}{1 + \lambda} \left[ \max(w_h^j + w_c^j + w_f^j - 1, 0) + \max(-w_f^j, 0) \right] 
- \frac{\lambda}{1 + \lambda} \left[ \max(1 - w_h^j - w_f^j - w_c^j, 0) + \max(w_f^j, 0) \right] - w_c^j
\]  (15)

There are three cases in which the funding shock is relevant.

Case 1: \(-w_h^j - w_c^j - w_f^j + 1 < 0, w_f^j < 0\). Under this case:

\[
L_D^j - L_H^j = -w_c^j + \frac{\lambda}{1 + \lambda} (w_h^j + w_c^j - 1) = -\frac{1}{1 + \lambda} [w_c^j - \lambda (w_h^j - 1)]
\]

Case 2: \(-w_h^j - w_c^j - w_f^j + 1 < 0, w_f^j > 0\). Under this case:

\[
L_D^j - L_H^j = \frac{\lambda}{1 + \lambda} (w_h^j + w_c^j + w_f^j - 1) - \frac{\lambda}{1 + \lambda} w_f^j + w_c^j = -\frac{1}{1 + \lambda} [w_c^j - \lambda (w_h^j - 1)]
\]

Case 3: \(-w_h^j - w_c^j - w_f^j + 1 > 0, w_f^j < 0\). Under this case:

\[
L_D^j - L_H^j = -\frac{\lambda}{1 + \lambda} w_f^j - \frac{\lambda}{1 + \lambda} (1 - w_h^j - w_c^j - w_f^j) - w_c^j = -\frac{1}{1 + \lambda} [w_c^j - \lambda (w_h^j - 1)]
\]
We see that in three cases, the HJB equation shown in proposition 1 has the same form. Take the first order condition with \( w^j_c \), we always have:

\[
V^{j}W^{j}(-i) + V^{j}_{W} + \eta \frac{\phi}{1 - \phi} \frac{1}{1 + \lambda} > 0
\]  
(16)

First, marginal utility of wealth decreases with \( W \), so \( V^{j}_{W} < V^{j}_{W} + \eta \frac{\phi}{1 - \phi} \frac{1}{1 + \lambda} \) for \( W + \eta \frac{\phi}{1 - \phi} \frac{1}{1 + \lambda} > W \). Under Assumption 1, equation (16) always holds. Therefore, agent \( j \) chooses to hold enough liquid assets to protect herself from being exposed to the funding shock in all three cases, i.e.: \( \max(L^j_D - L^j_H, 0) = 0 \).

.1.2 Proof of Proposition 3

There are four possible cases: \( w^j_f > (>)0, w^j_f + w^j_h + w^j_c - 1 > (>)0 \).

**Case 1:** \( w^j_f < 0, w^j_f + w^j_h + w^j_c - 1 > 0 \)

\[
L^j_D = \frac{\lambda}{1 + \lambda} (w^j_h + w^j_c - 1), L^j_H = w^j_c
\]

From \( L^j_H \geq L^j_D \), we solve for:

\[
w^j_c \geq \lambda (w^j_h - 1)
\]

**Case 2:** \( w^j_f > 0, w^j_f + w^j_h + w^j_c - 1 > 0 \)

\[
L^j_D = \frac{\lambda}{1 + \lambda} (w^j_h + w^j_c + w^j_f - 1), L^j_H = \frac{\lambda}{1 + \lambda} w^j_f + w^j_c
\]

From \( L^j_H \geq L^j_D \), again we solve for:

\[
w^j_c \geq \lambda (w^j_h - 1)
\]

**Case 3:** \( w^j_f < 0, w^j_f + w^j_h + w^j_c - 1 < 0 \)

\[
L^j_D = -\frac{\lambda}{1 + \lambda} w^j_f, L^j_H = w^j_c + \frac{\lambda}{1 + \lambda} (1 - w^j_f - w^j_c - w^j_f)
\]
From $L^j_H \geq L^j_D$, again we solve for:

$$w^j_c \geq \lambda (w^j_h - 1)$$

**Case 4:** $w^j_f > 0, w^j_f + w^j_h + w^j_c - 1 < 0$

Under this case, the agent does not borrow from either domestic agents or foreign agents, so optimal cash holding must be 0.

Combine the four cases together with $w^j_c \geq 0$, we can conclude that:

$$w^j_c = \max\{\lambda (w^j_h - 1), 0\}$$

(17)

### 1.3 Proof of Corollary 2

The proof starts from the equation (1.28) and (1.30). For simplicity, we only consider that for households. The same procedure applies for bankers, except for that $\mu_s - r$ should be replaced by $\mu_s - r - \lambda i$.

As in the main text, we denote portfolio share as $w$, the diffusion of asset return as $\Sigma = \begin{bmatrix} \sigma_s & \sigma_f \end{bmatrix}$, and the diffusion of state variable $\Sigma_\Omega$, with each column representing one state variables’ exposure to various Brownian shocks. We suppress $j$ in the following proof. Equation (1.30) can be inverted into:

$$\mu - r = \gamma \Sigma' \Sigma w - \frac{1 - \gamma}{1 - \psi} \Sigma_\Omega \Sigma \nabla G^\prime$$

(18)

$$= \gamma \frac{d}{dt} \text{cov}(dR, \frac{dW}{W}) - \frac{1 - \gamma}{1 - \psi} \frac{d}{dt} \text{cov}(dR, \frac{dG}{G})$$

(19)

For given price of consumption basket $\tilde{P}$, we can write out the first order condition with respect to the consumption basket (though redundant given equations (1.26) and (1.27)):

$$G = (\tilde{PC})^{\psi-1}$$

(20)
Therefore, equation (19) can be rewritten as:

$$\mu - r = -\frac{d}{dt}\text{cov}\left(\frac{1 - \gamma}{1 - \psi} \frac{d\bar{Pc}}{Pc} - \gamma \frac{dW}{W} - (1 - \gamma) \frac{d\bar{P}}{P}, dR\right)$$  (21)

### 2 Solution Details

In this section, we describe the details of how to solve the model numerically. We rely on Chebyshev approximations to the unknown functions. We start with a set of conjectured functions: value functions $G^h, G^b$, exchange rate $Q$, dividend yield of local stock $F$, and household portfolio share $w^h, w^f$. We only show the details for the case of fixed nominal interest rates, while extending the model to incorporate stochastic interest rate is straightforward with an additional state variable $i$.

The optimality conditions for consumption of traded and nontraded goods are shown in Corollary 1, equations (1.26) and (1.27). Combined with the CES aggregation equation (1.18), we can obtain:

$$c^j_x = G^j \alpha^\theta \left[ \alpha^\theta + (1 - \alpha)^\theta Q^{1-\theta} \right]^{\frac{\psi^j}{\psi - 1}}$$  (22)

$$c^j_y = c^j_x \left( \frac{1 - \alpha}{\alpha Q} \right)^\theta$$  (23)

The market clearing condition of nontraded good implies:

$$\omega c^h_x + (1 - \omega)c^b_x = \frac{X}{W}$$  (24)

Define the dividend of local stock $Z = X + QY$, then:

$$\frac{dZ}{Z} = \left[ \mu_s + \frac{Q\bar{\tau}}{1 + Q\bar{\tau}} \left( \mu_q + \sigma_q' \sigma_s \right) \right] dt + \left[ \sigma_s + \frac{Q\bar{\tau}}{1 + Q\bar{\tau}} \left( \frac{\bar{\sigma}_o}{Q} \sigma_s + \frac{Q\bar{\sigma}_x}{Q} \sigma_s \right) \right]' dB \equiv \mu_s dt + \sigma_s' dB$$  (25)

As $F = \frac{Z}{P}$, the return process for local stock is:

$$dR_s = (F + \mu_P) dt + \sigma_P' dB$$  (26)
where:

$$\sigma_P = \sigma_z - \frac{F_0}{F} \sigma_\omega - \frac{F_X}{F} \sigma_\chi$$  \hspace{1cm} (27)$$

$$\sigma_\omega = (w^h_h - w^b_h)(\sigma_z - \frac{F_0}{F} \sigma_\omega - \frac{F_X}{F} \sigma_\chi) + (w^h_f - w^b_f)(\sigma^* + \frac{Q_0}{Q} \sigma_\omega + \frac{Q_X}{Q} \sigma_\chi)$$  \hspace{1cm} (28)$$

$$\sigma_\chi = \chi \sigma^*$$  \hspace{1cm} (29)$$

Thus, we can solve for $\sigma_\omega$ from equations (25), (28), and (29):

$$\sigma_\omega = \frac{(w^h_h - w^b_h)\left[\sigma_z + \left(\frac{Q^*}{1+Q^*} \frac{Q_X}{Q} \chi \sigma^* - \frac{F_X}{F} \chi \sigma^*\right) + (w^h_f - w^b_f)(\sigma^* + \frac{Q_0}{Q} \chi \sigma^*)\right]}{1 - (w^h_h - w^b_h)\left(\frac{Q^*}{1+Q^*} \frac{Q_0}{Q} - \frac{F_X}{F}ight) - (w^h_f - w^b_f) \frac{Q_0}{Q}}$$  \hspace{1cm} (30)$$

Now that we can solve for all other volatilities, $\sigma_q$, $\sigma_f$, $\sigma_z$, and $\sigma_P$.

Next we derive the expected excess return of the two assets from the portfolio holding of households from Corollary 1:

$$\mu_s - r = \gamma_h \sigma'_s \sigma_s + \gamma_h \sigma'_f \sigma_f - \frac{1 - \gamma_h}{1 - \psi_h} \left(\frac{G^h_0}{G^h} \sigma'_0 \sigma_s + \frac{G^h_X}{G^h} \sigma'_X \sigma_s\right)$$  \hspace{1cm} (31)$$

$$\mu_f - r = \gamma_h \sigma'_f \sigma_f + \gamma_h \sigma'_s \sigma_f - \frac{1 - \gamma_h}{1 - \psi_h} \left(\frac{G^h_0}{G^h} \sigma'_0 \sigma_f + \frac{G^h_X}{G^h} \sigma'_X \sigma_f\right)$$  \hspace{1cm} (32)$$

According to market clearing condition, the portfolio holding of bankers are:

$$w^b_h = \frac{1 - \chi \frac{X}{W} - \omega w^b_h}{1 - \omega}, w^b_f = \frac{\chi \frac{X}{W} - \omega w^b_f}{1 - \omega}$$  \hspace{1cm} (33)$$

where $\frac{X}{W}$ is given by equation (24).
Then we can derive the drift of state variables:

\[ \mu_\omega = \omega (1 - \omega) \left[ c^h_x - c^b_x + Q(c^h_y - c^b_y) + (w^h_x - w^b_x)(\mu_s - r) + (w^h_y - w^b_y)(\mu_f - r) + w^b_x i + \Pi^b - \Pi^h \right] + \kappa (\bar{\omega} - \omega) \]

(34)

\[
\mu_x = \tau - \left( \frac{1 - \alpha}{\alpha Q} \right)^\theta + \chi r^* \]

(35)

Thus we can derive the drift of exchange rate \( \mu_q \), dividend \( \mu_z \), and the drift of local stock price change \( \mu_P \):

\[
\begin{align*}
\mu_q &= \frac{Q_\omega}{Q} \mu_\omega + \frac{Q_\chi}{Q} \mu_x + \frac{1}{2} \left( \frac{Q_\omega \omega}{Q} \sigma_\omega \sigma_\omega + \frac{Q_\chi \chi}{Q} \sigma_\chi \sigma_\chi \right) + \frac{Q_\omega \chi}{Q} \sigma_\omega \sigma_\chi \\
\mu_z &= \mu_x + \frac{Q_\tau}{1 + Q_\tau} (\mu_q + \sigma_q' \sigma_q) \\
\mu_P &= \mu_z + \sigma_P \sigma_P - \sigma_z \sigma_P - \mu_F
\end{align*}
\]

(36)

(37)

(38)

where:

\[
\mu_F = \frac{F_\omega}{F} \mu_\omega + \frac{F_\chi}{F} \mu_x + \frac{1}{2} \left( \frac{F_\omega \omega}{F} \sigma_\omega \sigma_\omega + \frac{F_\chi \chi}{F} \sigma_\chi \sigma_\chi + \frac{2 F_\omega \chi}{F} \sigma_\omega \sigma_\chi \right)
\]

(39)

The expected return to local stock is \( \mu_s = F + \mu_P \), and real risk free rate is \( r = \mu_s - (\mu_s - r) \). Finally, we check whether the following conditions from (40) to (44) are satisfied.

The definition of dividend yield:

\[
F = \frac{X (1 + Q\bar{\tau})}{P} = \frac{X}{W} (1 + Q\bar{\tau}) \frac{1}{1 - \chi Q\bar{\xi}}
\]

(40)

Consistency of foreign asset return:

\[
\mu_f - r = r^* + \mu_q + \sigma_q' \sigma^* - r
\]

(41)

Portfolio holding for bankers:

\[
\mu_s - r - \lambda i = \gamma_b \sigma_s' \sigma_s + \gamma_b \sigma_f' \sigma_f - \frac{1 - \gamma_b}{1 - \psi_b} \left( \frac{G^b_\omega}{G^b} \sigma_\omega \sigma_s + \frac{G^b_\chi}{G^b} \sigma_\chi \sigma_s \right)
\]

(42)
\[ \mu_f - r = \gamma_b \sigma' \sigma_f + \gamma_f \sigma' \sigma_f - \frac{1 - \gamma_b}{1 - \psi_b} \left( \frac{G^b_{\omega}}{G^b} \sigma' \sigma_f + \frac{G^b_{\chi}}{G^b} \sigma' \sigma_f \right) \]  

(43)

HJB equations for both agents:

\[
0 = \frac{1}{1 - \frac{1}{\psi_j}} \left( \frac{(c^j)^{1-\psi_j}}{(G^j)^{1-\psi_j}} - (\rho + \kappa) \right) + \mu_w^j - \frac{1}{2} \gamma_j (\sigma^j_w)' \sigma^j_w + \frac{1}{1 - \psi_j} \left( \frac{G^j_{\omega}}{G^j} \mu_{\omega} + \frac{G^j_{\chi}}{G^j} \mu_{\chi} \right) 
\]

\[+ \frac{1}{2(1 - \psi_j)} \left[ \frac{\psi_j - \gamma_j}{1 - \psi_j} (G^j_{\omega} + G^j_{\omega \omega}) \right] \sigma^j_{\omega} \sigma_{\omega} + \frac{1}{1 - \psi_j} \left[ \frac{\psi_j - \gamma_j}{1 - \psi_j} (G^j_{\chi} + G^j_{\omega \chi}) \right] \sigma^j_{\omega} \sigma_{\chi} 
\]

\[+ \frac{1}{1 - \psi_j} \left[ \frac{\psi_j - \gamma_j}{1 - \psi_j} (G^j_{\omega} + G^j_{\omega \omega}) \right] \sigma^j_{\omega} \sigma_{\omega} + \frac{1}{1 - \psi_j} \left[ \frac{\psi_j - \gamma_j}{1 - \psi_j} (G^j_{\chi} + G^j_{\omega \chi}) \right] \sigma^j_{\omega} \sigma_{\chi} \]  

(44)

where:

\[ \mu^j_w = r - c^j_c - Q(c^j_c + w^j_h (\mu_r - r) + w^j_j(\mu_f - r)) \]

\[\sigma^j_w = w^j_h \sigma + w^j_j \sigma_f \]  

(45)

\[ c^j = \left[ \alpha (c^j_c)^{\frac{\sigma}{\mu}} + (1 - \alpha) (c^j_j)^{\frac{\sigma}{\mu}} \right]^{\frac{\mu}{\sigma - 1}} \]  

(46)

In total, we have six unknown functions and six equilibrium conditions to solve for them.

### 3 Simulation Details

In our model, the low-interest-rate countries have higher bank leverage, so that bankers in these countries accumulate wealth faster than households and the ergodic distribution of \( \omega \) will be highly tilted toward the bankers compared to those high-interest-rate countries. To make the ergodic mean of \( \omega \) similar, we impose a wealth redistribution between bankers and households that keep the ergodic distribution of \( \omega \) concentrating in the region of interest with a substantial currency risk premium. In the simulation, we specify \( \Pi^h \) and \( \Pi^b \) as:

\[ \Pi^h = -\delta + \frac{\delta \bar{\omega}}{\omega}, \Pi^b = -\delta + \frac{\delta (1 - \bar{\omega})}{1 - \omega} + w^b_i \]  

(47)

In every period, \( \delta \) fraction of each agent’s wealth is taxed and redistributed. \( \bar{\omega} \) fraction is redistributed to households and \( 1 - \bar{\omega} \) is redistributed to bankers. We set \( \delta \) to be larger in low interest
rate countries, so that the ergodic mean of $\omega$ across countries are closer.

In our simulation in section 1.3.3, $\delta$ for each economy is set as 0.02, 0.014, 0.012, 0.008, 0.004, and 0, respectively.

We report the results without the introduction of the lump-sum redistribution in Appendix 4.2, and find they do not affect the qualitative features of policy functions. The average carry trade return is smaller without the lump-sum redistribution.

The model is simulated using the solutions obtained in section 1.3.2. We use a two-point approximation of the standard Brownian motions: $dB = 1$ or $-1$ with equal probability $\frac{1}{2}$. We simulate 1000 periods for each economy and discard the first 100 periods for the computation of ergodic mean.

### 3.1 Impulse Response Functions

We solve the model globally and there is no deterministic steady state. To obtain the impulse response functions, we simulate $N = 2,000$ parallel economies. In each economy, we first simulate $T_1 = 120$ periods with randomly drawn Brownian shocks. For the period $T_1 + 1$, we set the domestic endowment shock to be 1 (a positive endowment shock) for all economies. Then each economy evolves freely for $T_2 = 20$ periods. We take the average of each variable across the 2,000 parallel economies for $T_2$ periods, and subtract their average values in period $T_1$, the ergodic mean.

### 4 Additional Results with the Model

#### 4.1 Current Account Cyclicality

In this section, we look at the impulse response of current account surplus (in terms of the traded good) scaled by $X$ in the model with fixed interest rates, as in section 1.3.2. In our model, the current account surplus is equal to $\overline{CA} = Y - C^h - C^b$. It is a monotonic function of exchange rate $Q$:

$$CA = \frac{\overline{CA}}{X} = \tau - (\frac{1-\alpha}{\alpha Q})^\theta$$

(48)
Therefore, the impulse responses of CA mirrors the responses of $Q$. The current account surplus increases when there is a positive endowment shock to the economy.

In the international business cycle literature, it has been widely known that current account is countercyclical, while our model predicts a procyclical current account surplus. The driving force of current account countercyclicity is investment, which is abstracted away from our model. If we redefine output as the sum of nondurable good and service consumption plus net export, the correlation between net export and output is positive for most advanced economies. The correlation between consumption, net export, and the sum of the two (defined as output) are shown in Table E.1.

4.2 Fixed Interest Rates without Redistribution

In this appendix, we repeat the numerical solution to the model with $\Pi^b = 0, \Pi^b = w^b_i$. The purpose is to show that the change in redistribution does not change the policy functions or impulse responses, while making the currency return spread between $i = 0$ and $i = 0.05$ economy smaller. All qualitative results remain.

We show the counterpart of Figure 1.2 and Figure 1.3 in Figure D.1 and Figure D.2, the counterpart of Figure 1.4 in Figure D.3, and the counterpart of Table 1.2 in Table D.1.

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28. I thank Yang Liu for sharing the data.
Figure D.1: Leverage, Local Stock Return Exposure, and Excess Stock Return

Panel B: Local Stock Return Exposure

Panel C: Excess Local Stock Return

Note: This figure shows the solutions of bank leverage, excess local stock return, and local stock return exposure for economies with fixed nominal interest rate $i = 0$ and $i = 0.05$ in each panel, respectively. The graphs are on the same scale. $\omega$ represents the households’ wealth share, and $\chi$ represents the scaled net foreign asset position.
Figure D.2: Exchange Rate Exposure and Currency Risk Premium

Panel A: Exchange Rate Exposure

Panel B: Currency Risk Premium

Panel C: Ergodic Distribution

Note: This figure shows the solutions of exchange rate exposure and currency risk premium for economies with fixed nominal interest rate $i = 0$ and $i = 0.05$ in Panel A and B. Panel C shows the ergodic distribution of the two state variables. The graphs are on the same scale. $\omega$ represents the households’ wealth share, and $\chi$ represents the scaled net foreign asset position.
Figure D.3: Impulse Responses to a Positive Endowment Shock

Note: This figure shows the impulse responses of various variables to a one standard deviation positive endowment shock in the two economies with \( i = 0 \) and \( i = 0.05 \). The solid blue line represents the \( i = 0 \) (low-interest-rate) economy and the dashed red line represents the \( i = 0.05 \) (high-interest-rate) economy. Impulse responses are obtained by simulation of \( N = 2,000 \) parallel economies and taking their average.
Table D.1: Simulation Results for A Cross-Section of Economies with Fixed Interest Rates

<table>
<thead>
<tr>
<th>i</th>
<th>0</th>
<th>0.01</th>
<th>0.02</th>
<th>0.03</th>
<th>0.04</th>
<th>0.05</th>
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<tbody>
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<tr>
<td></td>
<td>Panel A: Asset Prices (in percent)</td>
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<td></td>
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</tr>
<tr>
<td>$\mu$</td>
<td>3.538</td>
<td>3.617</td>
<td>3.767</td>
<td>3.986</td>
<td>4.183</td>
<td></td>
</tr>
<tr>
<td>$r$</td>
<td>1.695</td>
<td>2.023</td>
<td>2.201</td>
<td>2.327</td>
<td>2.134</td>
<td>2.392</td>
</tr>
<tr>
<td>$\mu_s - r$</td>
<td>1.843</td>
<td>1.594</td>
<td>1.474</td>
<td>1.438</td>
<td>1.852</td>
<td>1.791</td>
</tr>
<tr>
<td>$\sigma_s$</td>
<td>6.327</td>
<td>5.274</td>
<td>4.634</td>
<td>4.105</td>
<td>3.674</td>
<td>3.161</td>
</tr>
<tr>
<td>$\sigma_q$</td>
<td>3.782</td>
<td>2.891</td>
<td>2.496</td>
<td>2.222</td>
<td>1.897</td>
<td>1.225</td>
</tr>
<tr>
<td>$\mu_f - r$</td>
<td>1.109</td>
<td>0.789</td>
<td>0.620</td>
<td>0.502</td>
<td>0.708</td>
<td>0.457</td>
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<tr>
<td></td>
<td>Panel B: Portfolio Choices</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w^b_h$</td>
<td>1.557</td>
<td>1.527</td>
<td>1.484</td>
<td>1.421</td>
<td>1.385</td>
<td>1.367</td>
</tr>
<tr>
<td>$w^f_h$</td>
<td>-0.008</td>
<td>0.050</td>
<td>0.101</td>
<td>0.144</td>
<td>0.197</td>
<td>0.228</td>
</tr>
<tr>
<td>$w^b_f$</td>
<td>0.272</td>
<td>0.388</td>
<td>0.491</td>
<td>0.581</td>
<td>0.667</td>
<td>0.785</td>
</tr>
<tr>
<td>$w^f_f$</td>
<td>0.007</td>
<td>-0.060</td>
<td>-0.108</td>
<td>-0.146</td>
<td>-0.174</td>
<td>-0.139</td>
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<tr>
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<td>Panel C: State Variables</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$\omega$</td>
<td>0.425</td>
<td>0.457</td>
<td>0.483</td>
<td>0.498</td>
<td>0.534</td>
<td>0.626</td>
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<td>$\chi$</td>
<td>-0.073</td>
<td>-0.084</td>
<td>-0.111</td>
<td>-0.120</td>
<td>-0.161</td>
<td>-0.221</td>
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<td></td>
<td>Panel D: Return Exposures</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{FX}$</td>
<td>1.178</td>
<td>0.809</td>
<td>0.645</td>
<td>0.529</td>
<td>0.390</td>
<td>0.178</td>
</tr>
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<td>$\beta_s$</td>
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<td>1.166</td>
<td>1.024</td>
<td>0.906</td>
<td>0.810</td>
<td>0.695</td>
</tr>
<tr>
<td>$\beta_{FX,s}$</td>
<td>0.865</td>
<td>0.666</td>
<td>0.581</td>
<td>0.522</td>
<td>0.452</td>
<td>0.302</td>
</tr>
<tr>
<td>$\beta_{FX,b}$</td>
<td>0.581</td>
<td>0.445</td>
<td>0.386</td>
<td>0.346</td>
<td>0.299</td>
<td>0.199</td>
</tr>
</tbody>
</table>
.5 Additional Empirical Results

.5.1 Consumption, Net Export, and Output

Table E.1 shows the correlation between growth rates of nondurable good and service consumption \( c \), net export \( nx \), and the sum of the two, \( y \).

Table E.1: Correlation of Consumption, Net Export, and Output

<table>
<thead>
<tr>
<th></th>
<th>( corr(c,y) )</th>
<th>( corr(c,nx) )</th>
<th>( corr(nx,y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>0.95</td>
<td>-0.29</td>
<td>0.26</td>
</tr>
<tr>
<td>Canada</td>
<td>0.43</td>
<td>-0.24</td>
<td>0.77</td>
</tr>
<tr>
<td>France</td>
<td>0.68</td>
<td>-0.18</td>
<td>0.59</td>
</tr>
<tr>
<td>Germany</td>
<td>0.63</td>
<td>-0.15</td>
<td>0.67</td>
</tr>
<tr>
<td>Italy</td>
<td>0.51</td>
<td>-0.31</td>
<td>0.66</td>
</tr>
<tr>
<td>Japan</td>
<td>0.84</td>
<td>0.00</td>
<td>0.54</td>
</tr>
<tr>
<td>UK</td>
<td>0.74</td>
<td>-0.08</td>
<td>0.61</td>
</tr>
<tr>
<td>Australia</td>
<td>0.42</td>
<td>-0.23</td>
<td>0.79</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.10</td>
<td>-0.24</td>
<td>0.94</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.72</td>
<td>-0.12</td>
<td>0.61</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.56</td>
<td>-0.10</td>
<td>0.77</td>
</tr>
<tr>
<td>New Zealand</td>
<td>0.56</td>
<td>-0.13</td>
<td>0.75</td>
</tr>
<tr>
<td>Norway</td>
<td>0.11</td>
<td>-0.27</td>
<td>0.92</td>
</tr>
<tr>
<td>Portugal</td>
<td>0.29</td>
<td>-0.41</td>
<td>0.76</td>
</tr>
<tr>
<td>Spain</td>
<td>0.36</td>
<td>-0.74</td>
<td>0.36</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.49</td>
<td>-0.11</td>
<td>0.81</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.38</td>
<td>-0.09</td>
<td>0.89</td>
</tr>
<tr>
<td>G7</td>
<td>0.67</td>
<td>-0.18</td>
<td>0.59</td>
</tr>
<tr>
<td>G10</td>
<td>0.40</td>
<td>-0.24</td>
<td>0.76</td>
</tr>
<tr>
<td>Mean</td>
<td>0.51</td>
<td>-0.22</td>
<td>0.69</td>
</tr>
</tbody>
</table>

Note: This table shows the correlations of consumption, output, and net export growth rate for 17 countries, as well as the average of the correlations for G7, G10, and G17. Consumption only consists of nondurable goods and services, and output is equal to the consumption of nondurable goods and services plus net exports. Data span 1970 to 2014 at quarterly frequency.

.5.2 Panel Regressions with Emerging Economies

Table E.2 in this subsection shows the results analogous to Table 1.4 with 44 countries including emerging economies of Bulgaria, Chile, Czech Republic, Egypt, Hong Kong, Hungary, India, In-
Table E.2: Bank capital ratio, forward discount, and currency return with emerging economies

<table>
<thead>
<tr>
<th></th>
<th>Forward discount</th>
<th>Currency return</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(1)</td>
</tr>
<tr>
<td>Capital ratio</td>
<td>0.385**</td>
<td>0.330**</td>
<td>0.289**</td>
<td>0.267**</td>
<td>0.419**</td>
</tr>
<tr>
<td></td>
<td>(0.033)</td>
<td>(0.057)</td>
<td>(0.074)</td>
<td>(0.047)</td>
<td>(0.128)</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.534**</td>
<td>0.541**</td>
<td></td>
<td></td>
<td>0.178</td>
</tr>
<tr>
<td></td>
<td>(0.065)</td>
<td>(0.031)</td>
<td></td>
<td></td>
<td>(0.481)</td>
</tr>
<tr>
<td>GDP</td>
<td></td>
<td></td>
<td>-0.388*</td>
<td>-0.323**</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.235)</td>
<td>(0.032)</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.079</td>
<td>0.428</td>
<td>0.099</td>
<td>0.446</td>
<td>0.064</td>
</tr>
</tbody>
</table>

Note: Fama-Macbeth regression results of the forward discount (left panel) and the currency return (right panel) on the bank capital ratio (the inverse of leverage, in percentage). In both panels, column (1) report the univariate regression coefficients. Column (2) controls for inflation, column (3) controls for the log GDP (size) of each country, and column (4) controls for both inflation and log GDP. Data are monthly including 44 countries, from Jan 1990 to Dec 2016. Both advanced economies and emerging economies are included. Annual measures of the bank capital ratio and the GDP share are used repetitively for months within a year. Standard errors are Newey-West adjusted with 120 lags. ** indicates statistical significance at 5% level. * indicates statistical significance at 10% level.

donesia, Israel, Malaysia, Mexico, Philippines, Poland, Russia, Saudi Arabia, Singapore, Slovakia, South Africa, Taiwan, Thailand, Turkey, and Ukraine.

### 5.3 ProCyclical Carry Trade

Figure E.1 in this section show results analogous to Figure 1.6, while replacing the average bank stock return with average country MSCI indices in the cross-section. The results are similar with using the average bank stock return.
Figure E.1: Currency Beta on Global Stock Returns

Note: This figure plots the relationship between a country’s average forward discount (the upper panel) and the average bank capital ratio (the lower panel) and the exchange rate beta with respect to the average stock return for the G10 currencies (vis-a-vis dollar). Data range from November 1983 to December 2016. Euro exchange rate is used for “DEM” after 1999.
Appendix for Chapter 2

.1 Model Estimation

The equilibrium model is estimate by simulated method of moments (SMM). Estimation methods are detailed in Fernández-Villaverde, Rubio-Ramírez, and Schorfheide (2016).

Denote the moment from data sample by \( \hat{m}_T(Y) \) and mode-implied moments \( E[\hat{m}_T(Y)|\theta_0, M_1] \) under model \( M_1 \) and parameter \( \theta_0 \). Define the discrepancy

\[
G_T(\theta|Y) = \hat{m}_T(Y) - E[\hat{m}_T(Y)|\theta_0, M_1]
\]

Our estimator \( \hat{\theta}_{smm} \) minimizes the criterion function of weighted discrepancy

\[
\hat{\theta}_{smm} = \arg\min_\theta G_T(\theta|Y)^TWG_T(\theta|Y)
\]

Suppose there is a unique \( \theta_0 \) that \( G_T(\theta|Y) \to 0 \) almost surely, then the estimator is consistent.

Our model has six estimated parameters is exactly identified by six moments. The weight matrix adjust for the difference of the units. When computing the model-implied moments, we simulated \( \lambda = 80 \) short samples of 40 years data (\( T = 160 \)).

\[
E[\hat{m}_T(Y)|\theta_0, M_1] = \frac{1}{\lambda} \sum_{i=1}^{\lambda} \hat{m}_T(Y^i|\theta_0, M_1)
\]
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