Complex Numbers: Imagined And Realized Plots & Storylines Of Enacted Mathematics Curricula

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Complex Numbers: Imagined And Realized Plots & Storylines Of Enacted Mathematics Curricula

Abstract

Mathematics is sometimes described as a “beautiful and interconnected story” (Kaplinsky, 2019, para. 1). Precisely what is meant by the term interconnected—and how interconnectedness relates to instruction—is debatable. Some argue that interconnectedness requires the sequential ordering of mathematical ideas, like the ascending the rungs of a ladder (e.g., Abdussalaam et al., 2015; Fernandez et al., 1992; Kaplinsky, 2019). Additional clarity is warranted, however, to understand the potential influence of non-sequential orderings (Dietiker, 2012, 2013b, 2015a; Lampert, 2001; Zimba, 2011, 2012). I maintain that a better understanding of the nature of interconnectedness or coherence has implications for both instruction and the design of curriculum materials.

In the study presented here, I pursue an investigation of coherence in mathematics instruction. To do so, I apply Dietiker’s (2012, 2013b, 2015a) mathematical story framework. This framework draws on literary theory and narrative analysis, to illuminate underappreciated elements of written lessons. I draw on Dietiker’s work to theorize that mathematical plots involve connected or coherent ideas, deployed in a variety of ways to motivate students’ curiosity. At present, there are no fine-grained studies that analyze mathematical plots of multiple elementary-grades lessons, comparing nuances of sequenced ideas in both written materials and classroom instruction. To better understand ways curriculum materials offer mathematical plots, and how teachers interpret and adapt them, I undertook a case study of two, experienced elementary instructors. I found that both teachers read for mathematical plots, but for different purposes: one reads for plot complexity, to offer scaffolds to students that resequenced ideas; the other reads for moments of suspense, to stimulate students’ problem-solving by purposefully omitting key ideas. Also, curriculum materials framed mathematical plots in varying ways that both supported and undercut their implementation. My findings suggest that curriculum authors should attend carefully to mathematical plots, providing instructors with rationales for sequencing or omitting ideas. I also situate this work within several broader areas of study, particularly research on the design work of teaching (M. Brown, 2009), teachers’ participation with curriculum materials (e.g., Remillard, 2005), and fidelity of curriculum implementation (e.g., S. Brown, Pitvorek, Ditto, & Kelso, 2009).

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COMPLEX NUMBERS:

IMAGINED AND REALIZED PLOTS & STORYLINES OF ENACTED MATHEMATICS CURricula

Joshua A. Taton

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in

Education

Presented to the Faculties of the University of Pennsylvania

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Degree of Doctor of Philosophy

2019

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COMPLEX NUMBERS: IMAGINED AND REALIZED PLOTS & STORYLINES OF ENACTED

MATHEMATICS CURRICULA

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DEDICATION

To “Choc” and Tom, my grandfather and father,
who taught me the value of perseverance.
ACKNOWLEDGEMENTS

So much of my life has been devoted to pursuing academic goals that it is hard to explain how I feel, now that this consequential part of my journey is over. Tired and happy? Yes. Grateful? Undeniably. It is equally hard to conjure the names of all of those who, throughout, have walked and run alongside me. So many have helped by giving me a push, pulling me along, and—most significantly—offering an ear amidst those self-doubting hours. It is truly a daunting challenge, then, to catalogue the names of my loyal confidantes, friends, and supporters, whose contributions have made this thesis (and the learning that led to it) possible.

To begin trying, though, Dr. Janine Remillard has been my tireless guide, advocate, and mentor for well more than a decade. I met Janine at a workshop in 2007—almost by happenstance. It is difficult to fathom the impact of that near-chance meeting on my life. From the beginning, she exuded warmth and expressed confidence in me, the transformative magnitude of which are difficult to overstate. I was fortunate enough to find in Janine not only a scholar whose contributions have been pivotal in the field, but also a supervisor whose candor, abiding respect for teaching, and yearning for equity are examples to all who know her.

Janine, thank you for teaching me—among so many other things—to appreciate and embrace complexity. The challenges we face, as a society, merit the full measure of our awareness and our collective participation.

Naturally, my committee has been instrumental in this endeavor, as well. Their brilliant scholarship is indelibly stamped underneath my words, and their willingness to cheer me on has meant more than they will ever know. In particular, Dr. Abby Reisman’s forthright and goal-directed nature have been invaluable. Abby’s scholarship on teaching history, authentically, has been a true model and complements my own perspective on classroom mathematical inquiry. Likewise, early in my graduate study, Dr. Betsy Rymes expressed interest in my work—and my running! Both gave me a much-needed boost of confidence. Betsy’s perspective on narrative is woven throughout this thesis. I so appreciate her stance on the vitality of language, particularly within “front-porch society” and spaces. As she has taught me, and so many others, the lines between producers and consumers of words (and numbers!) must be blurred.
How can I possibly identify everyone else at PennGSE, to whom I owe such debts of gratitude? This is but a scant list, and I apologize to those whose names I unintentionally overlook. First and foremost, Dr. Caroline Ebby’s constant encouragement, as well as her devotion to teachers, have long been energizing and deeply humbling. Drs. Frances Rust and Sharon Ravitch have offered consistently supportive feedback, as well, managing to bolster my morale just when needed. Drs. Lois MacNamara, Ann Tiao, and Veronica Aplenc all work unceasingly on behalf of their students and have done so for me, in particular, under the tightest of deadlines. This thesis could not have been completed, truly, without their dedicated efforts; they are the metaphorical binding which holds this work together.

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there at the end. I owe you a meal (no, many!) for your invaluable feedback (while you were visiting BioSphere 2!) on a draft of the presentation for my final defense hearing.

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To my parents, Thomas and Linda Taton, you were my first and my most important teachers. The lessons you taught are with me every day. You encouraged me to reach as high as I could, while also showing me the importance of staying grounded.

My sister, Michal Fitzgerald, and my brother-in-law, Dr. Nicholas Fitzgerald, have been patient sounding boards for whom I am eternally grateful. I want to extend my love to their children, Miss Josie Mae Fitzgerald and Master T. J. Fitzgerald, who represent the future mathematicians and storytellers of our world. At the same time, I also need to recognize Dorothy and Roland Caron, and Walter and Gwendolyn Taton, who represent the cornerstones of our
family and who worked so hard for their progeny. Their persistence allowed us the privileges we now cherish.

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Philadelphia, PA
July 14, 2019
ABSTRACT

COMPLEX NUMBERS: IMAGINED AND REALIZED PLOTS & STORYLINES OF 
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Joshua A. Taton
Janine T. Remillard

Mathematics is sometimes described as a “beautiful and interconnected story” (Kaplinsky, 2019, para. 1). Precisely what is meant by the term interconnected—and how interconnectedness relates to instruction—is debatable. Some argue that interconnectedness requires the sequential ordering of mathematical ideas, like the ascending the rungs of a ladder (e.g., Abdussalaam et al., 2015; Fernandez et al., 1992; Kaplinsky, 2019). Additional clarity is warranted, however, to understand the potential influence of non-sequential orderings (Dietiker, 2012, 2013b, 2015a; Lampert, 2001; Zimba, 2011, 2012). I maintain that a better understanding of the nature of interconnectedness or coherence has implications for both instruction and the design of curriculum materials.

In the study presented here, I pursue an investigation of coherence in mathematics instruction. To do so, I apply Dietiker’s (2012, 2013b, 2015a) mathematical story framework. This framework draws on literary theory and narrative analysis, to illuminate underappreciated elements of written lessons. I draw on Dietiker’s work to theorize that mathematical plots involve connected or coherent ideas, deployed in a variety of ways to motivate students’ curiosity. At present, there are no fine-grained studies that analyze mathematical plots of multiple elementary-grades lessons, comparing nuances of sequenced ideas in both written materials and classroom instruction. To better understand ways curriculum materials offer mathematical plots, and how teachers interpret and adapt them, I undertook a case study of two, experienced elementary instructors. I found that
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The central premise of my thesis is that learning mathematics should be engaging. By engaging, I do not simply mean “fun.” It should also be intellectually stimulating, compelling, and relevant.

As I hope to clarify by presenting my research, a pedagogy of engagement is complex. Before trying to untangle its tendrils, I propose a handful of guiding principles. First, engaging mathematics lessons, fundamentally, nourish students’ curiosity. Such lessons spark growth and empower students’ resourcefulness and independence. Additionally, such lessons are responsive to student’s needs and concerns.

Second, engaging instruction necessarily exhibits clear, deliberate storylines. I expand on what I mean by the term storyline as my thesis unfolds, but I offer a proxy definition here: a storyline consists of a set of related ideas that evolve. Not all storylines are equally engaging, however. Some are more or less provocative than others. I therefore add a sufficiency criterion: engaging instruction also exhibits storylines that are imbued with affect or emotion.

In other words, an engaging mathematical storyline provokes both a cognitive and an emotional response, as well as a cognitive and an emotional trajectory. Fully defining either cognition or emotion would be a thesis unto itself. While a cognitive trajectory is, simply, the movement of ideas, constructs, or mental schemata, an emotional trajectory is much more challenging to define. Solomon (2019) offers a definition I find helpful: “Emotions are a complex experience of consciousness, bodily sensation, and behaviour that reflects the personal significance of a thing, an event, or a state of affairs” (para. 1). From this, I highlight “bodily sensation,” which reflects the commonplace synonymity of the words emotion and feeling.

Feeling is another word for sensation, and so emotions are—at least in part—physical responses that, as Solomon notes, also relate to “personal significance” (para. 1).

Why should instruction, which aims to spur cognitive growth, relate to emotion? I argue that ideas, when thoroughly devoid of emotional glue, are unlikely provoke students’ interest. They are merely primordial, free-floating. On the other hand, ideas that relate to one another and offer personal significance—and even stimulate bodily sensations—are those most likely to be retained. In my view, it should not be surprising that our most vivid memories are, often, those
tied to scents and songs. We experience these through corporeal bursts of firing synapses (see, e.g., King et al., 2019). Emotive ideas, I also believe, are those most likely to inspire students’ own exploration. They provide motivation.

Having observed, perhaps, hundreds of lessons during the past decade-plus of my career, I confess to my worry that engagement in mathematics classrooms is waning. Thinking about my own schooling—in a rural and mostly working-class corner of New England—I recall lessons delivered by my teachers that seemed carefully-crafted. These inspired me to pursue a mathematics degree at Yale University and, later, a doctorate in mathematics education at the University of Pennsylvania. This personal revelation is meant an offering of gratitude for the dedication and creativity of the teachers and administrators in my small-town, public school district, who propelled me toward the Ivy League. On the other hand, though, I have observed very few energizing and thought-provoking lessons in my last decade as a graduate student and a school administrator. I worry that the recent accountability movement, standards-oriented reforms, emphasis on data-driven results, and other performativity pressures have contributed, at least in part, to a decline in the engagement of mathematics lessons. Equally concerning—if not more so—my observations might be contextual: my professional career has mostly transpired within high-needs, large-city schools and districts. Haberman (1991) might attribute these later-in-life, differential observations to what he famously called the “pedagogy of poverty” (p. 81).

Admittedly, I am relying on scant, personal evidence for making a potentially-outsized claim. I do not intend to discredit the entirety of recent educational-reform movements, either, because the emergent techniques and scientific breakthroughs upon which they are based have contributed meaningfully to our understanding of teaching, learning, and curriculum. I merely wonder about the possible trade-offs in emphasizing performativity over a nuanced—and consequently less easily observed or measured—set of pedagogical skills. My dissertation study is therefore an attempt to discern features that make mathematics instruction more engaging, particularly what makes mathematics instruction mathematically engaging. I must clarify, though, that my study is not about classroom management nor about Lemov’s (2010, 2015) or similar sorts of techniques. It is not a longitudinal analysis. I can’t, therefore, address my overarching question—at least, not at this point—on whether mathematics lessons are becoming less engaging over time.

By presenting my findings, here, I intend to tell stories of classroom engagement and to better decipher what engagement means. I am not reporting on features of well-run and so-called fun
lessons, but instead, on features of lessons that promote mathematical engagement. As an example of such a lesson, Dietiker (2016) reports on first-graders who elicited gasps of surprise when they encountered a contradiction to their previously-held ideas. A question therefore arises—one that will remain at the conclusion of this thesis—on whether such moments are inhibited or promoted by performativity pressures. My contribution to answering this question, however modest, entails describing the nature of such lessons more concretely, including analyzing the potential role of the teacher’s guide in the design and implementation of such lessons. Teachers’ guides (sometimes known as textbooks or curriculum programs) are, after all, still considered primary resources for shaping classroom interactions and learning (Goodlad, 1984; Opfer, Kaufman, & Thompson, 2016; Perry et al., 2015; Sherin & Drake, 2009; Valverde, Bianchi, & Schmidt, 2002; Whitman, 2004).

I further suspect that an engaging mathematical storyline—whatever this descriptor might, ultimately, come to mean—is related to students’ willingness to engage in problem-solving. Besides my recollection of intriguing problems, inspiring my eventual career in education, my interest in pursuing this line of inquiry has many roots. First, as a former middle and high school mathematics teacher, I often wondered how to engage my students in creative problem-solving. I therefore sought activities and resources I presumed might help, whenever I found my textbooks and teacher’s guide wanting. I must admit that my searches were highly unstructured, and whatever I encountered and used in my classroom was likely a fortuitous happenstance. Sometimes, I’d like to think, I was successful in using resources to achieve my overarching goals. At other times, though, I was undoubtedly much less successful. I continue to ask myself, what made the difference? Because answers to these and other questions remained elusive, I pursued graduate study.

As a mathematics consultant, both in the past and currently, I have long wondered why inquiry-oriented practices are conspicuously absent within U.S. classrooms. Even within the era of the Common Core State Standards for Mathematics (CCSS-M), during which conceptual understanding and problem-solving are desired hallmarks of instruction (National Governors Association [NGA] Center & Council of Chief State School Officers [CCSSO], 2010), witnessing authentic, student-driven inquiry remains all too rare. Is there a misalignment to blame, perhaps, between the standards, themselves and mechanisms for assessing both students’ learning and

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teachers’ performance?? Are there other causes? Do curriculum programs and training need to be improved? And, if so, how?

As a supervisor of mathematics instruction, presently charged with helping teachers implement curriculum programs, I wonder if it is possible for instructional materials to be used in ways that enable students’ curiosity, astonishment, or even productive confusion? How can I work with my colleagues, teachers and school leaders alike, to help them see when and how curriculum programs can enhance students’ opportunities for problem-solving? Conversely, how can I help my colleagues understand when limitations of programs need to be overcome, or when subtle adaptations might enhance their use?

Finally, as a being in the world, I want to name my vocation proudly. Yet I cringe in anticipation, whenever I am asked to explain what I do for a living. This often occurs during on-demand car rides to and from school buildings, when the driver invariably asks about the nature of my work. As soon as the word “math” is uttered from my mouth, the responses are nearly all the same: usually, I face expressions of distaste, derision, and even anger. I want others to see, as Kaplinsky (2019) does and as I do, that mathematics is, indeed, “beautiful and interconnected” (para. 1). As celebrated Yale University mathematician, Mandelbrot (1977), writes: “Being a language, mathematics may be used not only to inform but also, among other things, to seduce” (p. 20). The bottom-line challenge—one that has long persisted in these United States and that demands a collective exasperation to address—is: How can we support teaching and the use of curriculum materials, in such a way that moves mathematics instruction away from bland exercises in answer-getting and toward cumulative moments of appreciative curiosity?
CHAPTER 1. THE PROLOGUE:

MOTIVATION AND BACKGROUND

This day forever shall be known as Diffendoofer Day….
we held a celebration, there was pizza, milk, and cake.
Like everyone, I ate too much and got a bellyache.
We laughed and whooped and hollered the entire school day long,
Then we all sang, triumphantly, “The Diffendoofer Song.”

—Dr. Seuss, Hooray for Diffendoofer Day! (1998, lines 94, 97–100)

1–1. Introduction and Assumptions: The Interconnected Story of Mathematics

Robert Kaplinsky, a popular mathematics education commentator, describes teachers as storytellers who tell the “beautiful and interconnected story” of mathematics (Kaplinsky, 2019, para. 1). In turn, he portrays mathematics itself as a progression of ideas, one idea morphing into another and gradually accruing sophistication. For example, he explains how “simple single-digit multiplication progresses into multi-digit multiplication, then the distributive property, binomial multiplication, polynomial multiplication, completing the square, etc.” (Kaplinsky, 2019, para. 2).

Kaplinsky tethers this narrative conception of mathematics to concepts, rather than to a set of algorithms. Algorithms contrast with concepts in much the same way that trains contrast with transportation. A train traverses a prescribed route to a particular destination from a given origin. It is like an algorithm. Transportation, on the other hand, signifies all manners of travel—including those yet imagined—for any set of routes. It is a concept. While the former is concrete and specific, the latter is amorphous and categorical. The teacher’s job, according to Kaplinsky, is to transport students through the concepts of mathematics, more so than the algorithms.

Teaching as Mathematical Storytelling

Kaplinsky (2019) also argues that mathematics teachers should become better storytellers to tell the story of mathematics capably. He offers several guidelines for effective mathematical storytelling that include depicting a meaningful sequence of ideas and aiming for parsimony during instruction (paras. 7–15). Superfluous details muddle the presentation, he claims. And by a meaningful sequence, he means a natural order; otherwise—if mathematical scenes are “not in an
order that’s intuitive” to students—then learners will have to “spend mental energy trying to reconcile it all” (para. 15). In other words, he argues that an intuitive sequence of ideas undergirds the larger mathematical story.

**Haphazard presentation as a barrier to understanding.** Kaplinsky (2019) rightly asserts that haphazardly ordering mathematical ideas can, at times, result in students’ confusion. This is borne out by research (e.g., Fernandez, Yoshida, & Stigler, 1992). In a sense, Kaplinsky’s overriding concern—and mine, as well—is whether students are effectively engaged in mathematical thinking throughout lessons. A disordered and choppy presentation is obvious barrier to engagement, because students’ thinking may be disrupted such that they cannot appreciate the higher purposes of the tasks at hand. Confused, they may turn away from thinking deeply.

But is disordered presentation a universal threat to understanding? Of this, I am less certain. Kaplinsky’s (2019) model of haphazard ordering is the film *Pulp Fiction* (para. 15). Here, critical, public, and my own opinions diverge from his. In 1995, Tarantino’s masterwork earned seven Oscar nominations and scores of other accolades. It might have been a challenging film, but it cannot be described as unpopular. Indeed, part of its popularity, novelty, and ongoing appeal is the unconventional and decidedly non-linear approach to presentation. *Pulp Fiction* remains a singular achievement, precisely because it confronts traditional notions of plot. In other words, as an artistic work, the film is both challenging and impactful. Its success derives, at least in part, from the challenge it offers.

And, because of its unorthodox sequencing, does *Pulp Fiction* lack coherence? Hardly. At its conclusion, to the contrary, we appreciate the myriad, unexpected connections among the characters. What holds our interest and engages us, throughout, is the puzzle of their relationships. With regard to *Pulp Fiction*, at least, I might argue against Kaplinsky (2019) that it is, indeed, a worthwhile expenditure of “mental energy to reconcile it all” (para. 15). *Pulp Fiction* lacks a pedestrian form of coherence, perhaps. But Tarantino’s writing transcends traditional ideas about plot and aspires to a higher form of coherence that, in and of itself, captivates and intrigues.

*Pulp Fiction* offers additional lessons on learning and performance. For one, a central figure of the film, Jules, inhabits the character of a righteous executioner (Bender et al., 1994). Before exacting vengeance, Jules offers a famously-dramatic rendition of a biblical verse. In so doing, he
aims to teach his victims repentance before they meet their demise. To Jules, then, teaching is a form of role-playing or even storytelling. This is an important connection.

Scholars have made a similar assertion (e.g., Clandinin & Connelly, 1992; Egan, 1989; Elbaz, 1991). The novelist Godwin (1974) claimed, after all, that “good teaching is one-fourth preparation and three-fourths theater” (p. 50). Likewise, studies of classroom engagement suggest a connection between what some have called teachers’ *enthusiastic delivery* and students’ learning (e.g., Sarason, 1999; Hidi & Renninger, 2006; Tauber & Mester, 2007). It should not be surprising, then, that teacher preparation programs offer courses, like “Teaching as a Performing Art” at the University of Massachusetts Amherst (Hart, 2007), nor that a popular, education-themed television show, *The Big Bang Theory*, also relates high-quality instruction with acting skill (Prady, Molaro, Ferrari, & Cendrowski, 2011).

**Storytelling and the risks of performativity.** There is a double-edged sword of the teacher-as-actor metaphor, however, that rests on what S. J. Ball (2003) calls the “technology of performativity.” Ball warns that modern frameworks for teaching may translate “complex social processes and events into simple figures or categories of judgement” that could become, and may already have become, “mechanisms for reforming teachers” (p. 217). Stated differently, such frameworks—which oversimplify and codify the tools, actions, and responsibilities of teachers—may constitute systems of professional control. We must mind the difference between offering teachers the tools of performance to enhance students’ learning and “changing what it means to be a teacher” in order to “produce new kinds of teacher subjects” (p. 217). By *teacher subjects*, Ball allegorizes monarchical oversight of teachers and their work. Local needs, moral judgment, and professional experience are washed away (S. J. Ball, 2003, p. 218). Such evaluation systems presume, foremost, that teacher effectiveness is a known and measurable construct. This assumption does not occupy solid empirical, nor theoretical, ground (Hiebert & Grouws, 2007).

S. J. Ball’s (2003) argument applies, reciprocally, to the dawning awareness of Jules from *Pulp Fiction*. Jules gradually learns that his executioner persona is a false one, because it was built on a purely transactional relationship. His assumed righteousness serves no higher aim than to fulfill the whims of his mercurial boss. The guiding principle of his life, in other words, is his *lack* of guiding principles. This fact is eventually discomfiting to Jules, and it reflects the vacuousness of the contemporary milieu in the U.S. (Conard, 1997). In other words, Jules
paradoxically learns about the incoherence of his life from the seemingly haphazard events he experiences, which in turn lead him to discover the foundation of values-driven living.

The film’s inchoate narrative structure therefore offers a subtle comment on our collective want of a grounding worldview. Its counter-intuitive ordering presents a teachable moment for Jules and for us: complexity and structure are brethren. Complexity without structure is untenable. And S. J. Ball’s (2003) concerns about performativity are rooted in the inverse, namely, that technologies sometimes represent structure, while depriving teachers of their autonomy. They obscure the beautiful, generative complexity of teachers’ work.

My own concerns about the technologies of performativity (S. J. Ball, 2003), and the organizations and systems that support them, is perhaps less strident than those articulated by Ball himself, or by Ravitch (2016), and other, similar critics. Nonetheless, I must explain—in the service of transparency—that I fret that teachers’ performativity, ironically, may undercut classroom engagement. I wonder, in other words, whether performativity-oriented frameworks confuse what Schlechty (2002) describes as authentic engagement with what he terms ritual (or passive) compliance. I wonder, does engagement equate with students’ attentiveness or can inquisitive hullabaloo also mark high levels of engagement?

Further, how can we be sure debates over engagement are not proxy disputes over the purposes for education? For some, education is a form of resistance, promoting critical awareness, creativity, and civic participation (Freire, 1970 / 2000). Embued with this purpose, might instructors therefore honor students’ restlessness and impertinence? Others affirm education as the primary mechanism for transmitting both culturally-embedded norms and traditional epistemologies to younger generations (Apple & King, 1983; Jackson, 1968; Hirsch, 1987; Giroux & Penna, 1983). Might instructors, committed to this stance, therefore demand students’ docility? Without a common vocabulary for classroom engagement, I fear that answers to these broad but important questions will remain elusive.


My central premise is that, regardless, mathematics learning should be engaging. Much like Kaplinsky (2019), I also argue that engaging mathematics instruction necessarily exhibits clear, deliberate storylines. I expand on what I mean by storyline as my thesis unfolds, but I offer a proxy definition here: a storyline consists of a set of related ideas that evolve. A story or narrative
consists of one or more storylines. Not all storylines are equally engaging, though. Some are more or less provocative than others. And so, I add a sufficiency criterion: engaging instruction also exhibits storylines that are embued with affect or emotion. In other words, an engaging storyline should provoke both a cognitive and an emotional response.

Why should instruction, aiming to spur cognitive growth, relate to emotion? Solomon (2019) offers an interactional definition for emotion: “a complex experience of consciousness, bodily sensation, and behaviour that reflects the personal significance of a thing, an event, or a state of affairs” (para. 1). Feeling is another word for sensation, and so emotions are—at least in part—physical responses. On the other hand, ideas that relate to one another and offer personal meaning to students—and even prompt certain kinds of bodily sensations—are those most likely to be retained. It should not be surprising, then, that our most vibrant memories are often tied to scents, songs, and stories. We experience these through corporeal bursts of firing synapses (see, e.g., King et al., 2019). Beyond solidifying retention, I also believe that emotive ideas are those most likely to inspire students’ exploration. They promote curiosity, and they are likely to inspire problem-solving.

A novel and growing area of research in mathematics education involves conceptualizing and analyzing storylines—or, more broadly, stories—within lessons. Several foundational elements of this domain of scholarship remain murky and, therefore, continued investigation is merited. In particular, several concepts and definitions remain unclear: precisely what is meant by mathematical story, how teachers perceive and interpret mathematical stories within their teacher’s guides (or textbooks, more generally), and how teacher’s guides themselves support the development of mathematical stories. My study, reported here, therefore aims to contribute to the literature in this area by documenting features of teacher’s guides and the ways two elementary-grades teachers use them to enact mathematical stories. The overall intention of such work is to offer better understanding of the role and nature of mathematical stories in the classroom. I speculate, more precisely, that if we can conceive of mathematical texts and lessons as narratives, then by their nature, we must likewise acknowledge the emotive elements of mathematics instruction. And, again, I mean something more than whether classroom environments are fun or joy-filled; considering mathematical stories as narratives, their affective elements are thought to provoke *mathematically* emotive responses.
At the same time, while I agree with the broad swath of Kaplinksy’s (2019) argument—that engagement is enabled by clear mathematical storylines—I nonetheless question what might be considered intuitive orderings of mathematical ideas. Many such orderings could—and probably do—exist, thereby complicating teachers’ decision-making. As just one example, it is common for high school mathematics texts to assume a meaning for an exponent of zero; this definition can then be employed to derive various properties of exponential expressions. An alternative ordering, though, is equally appropriate and logical: teachers could start with properties of exponential expressions and then use these to produce a meaning for an exponent of zero. More importantly for my thesis, however, I ask whether there is a pedagogical benefit to unintuitive orderings? Might such orderings, like those in *Pulp Fiction*, stimulate curiosity or surprise?

Consider another example, the following two-sentence story—told by the character, Phil Dunphy, on the television series *Modern Family*—which is offered as advice to the younger men in his family: “Never try to take a tennis ball away from a raccoon. Or, never go to play tennis with just one ball” (Burditt et al., 2019). The order of the events in this brief anecdote is certainly counterintuitive, because Phil could have relayed it, logically, in the following way:

1) I brought one tennis ball to a tennis match,

2) This was a bad idea, because a raccoon snagged the ball away from me,

3) I tried to retrieve the tennis ball from the raccoon, and

4) I was consequently injured.

Phil’s counterintuitive presentation, I argue, is precisely what holds our interest and makes it humorous. Schopenhauer (1969), after all, attributes laughter to a realization of “the suddenly perceived incongruity” (p. 59). In this case, the incongruous mix of wildlife (i.e., a raccoon) with a refined leisure activity (i.e., tennis) produces humor; even more, we laugh because of the atypical ordering of the story and the missing information supplied through our imagination. Phil’s story also has a clear moral: Be prepared. As a narrating event (Jackson, 1996) within the Dunphy storytelling-world, this anecdote has clear pedagogical intent, if not quite rising to the level of a fable. And it conveys its message successfully, by acknowledging and toying with emotional cues. Thinking about Phil’s story raises all sorts of other questions for me. I wonder about the potential impact of various ways of presenting and ordering mathematical ideas.
during classroom instruction. Below and in the chapters that follow, I try to articulate and respond to some of these questions.

1–2. Constructive Metaphors: Rungs, Ladders, Spirals, and Scaffolds

A ladder is a common metaphor or model for instruction that appears often—figuratively and sometimes literally—within modern instructional programs and digital learning resources. For example, according to Kaufman and colleagues (2017), *Eureka Math* is “one of the most widely used” programs in the U.S. today (p. 2). Within the front matter of its third-grade *Teacher Edition*, a ladder is depicted and described as “a metaphor for the teaching sequence…where each rung in the ladder represents the next step in understanding or the next skill needed to reach the objective” (Abdussalaam et al., 2015, p. 14). In addition, as the authors indicate, the *ladder* of a lesson connotes the “new complexities of each problem, as well as the sequences and progressions throughout the problems” found therein (Abdussalaam et al., 2015, p. 14). Students are to be supported by teachers in grasping one rung of the ladder, or one key mathematical idea, before moving ahead to the next.

**Epistemological Underpinnings**

More than a pedagogical method, though, the ladder model also represents a presumed epistemology of mathematics. That is, the notion of incremental-growth—built into the progression of problems and activities within a set of *curriculum materials* for teachers—also purportedly aligns with the way mathematical knowledge is itself structured and acquired. Consider, for instance, this profile of a successful mathematics tutor, John Mighton, appearing in an editorial in the *New York Times*:

> Mighton found that to be effective he often had to break things down into minute steps and assess each student’s understanding at each micro-level before moving on.…

> Breaking things down this finely allows a teacher to identify the specific point at which a

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1 Note that I define and use *curriculum* more precisely in Chapter 2. By *curriculum* and *curriculum materials* (or sometimes, alternately, *instructional materials*, *resources*, *textbooks*, *programs*, or *guides*), I generally mean written guidance on planning and implementing mathematics lessons. Note that I am glossing over an important nuance. Specifically, the authors of the CCSS-M (quoted here) refer to curricular goals or broad frameworks for developing instructional materials. To Remillard and Heck (2014), such guidance is part of the *official curriculum*, which they distinguish from textbooks and other instructional resources.

2 Throughout this thesis, I make occasional reference to what I call the *ladder approach*, a progression of mathematical ideas that follows a strictly-linear pathway of increasing complexity or sophistication.
student may need help. “No step is too small to ignore,” Mighton says. “Math is like a ladder. If you miss a step, sometimes you can’t go on.” (Bornstein, 2011, paras. 8–10)

Stated differently, mathematics is often portrayed as a discipline, whose precepts are necessarily decomposable and arranged along a hierarchical pathway. A fundamental assumption embodied by the ladder model is, of course, that mathematical understanding must be built from the ground up. Complex ideas derive from simpler ones in a linear, sequential fashion. Indeed, Bornstein also argues, “Asking children to make their own discoveries before they solidify the basics is like asking them to compose songs on guitar before they can form a C chord” (para. 7).

Another model of incremental growth in learning is offered by the authors of *Everyday Mathematics* (Bell et al., 2007). They describe the intentional spiral of their program, under which “learning is spread out over time rather than being concentrated in shorter periods” (UCSMP, n.d.-b, para. 1). In other words, a limited amount of new material is encountered in each lesson, but complexity is gradually added when that same material is “revisited repeatedly over months and across grades” (UCSMP, n.d.-b, para. 1). A simple portrayal of spiraling—perhaps an overly crude example—would be the following: Topic A.1 (e.g., multiplication of single-digit whole numbers) followed by Topic B.1 (e.g., graphing categorical data) constitutes the learning sequence of Lesson 1, while Topic A.2 (e.g., multiplication of a multi-digit whole number by a single-digit whole number) followed by Topic B.2 (e.g., analyzing graphs of categorical data) constitute the learning sequence of Lesson 2 (see Figure 1). Even though the progression of Topic A (Parts 1 and 2) is interspersed with material from Topic B (Part 1) within this hypothetical example, the activities for studying Topic A are all assumed to relate to the same idea and are thought to develop in complexity within each lesson, but still incrementally.

![Figure 1. A schematic depicting the spiraling approach of *Everyday Mathematics* (UCSMP, n.d.-b). This shows how new material is presented during each lesson and then, subsequently, revisited.](image-url)
A spiraling approach, the authors of *Everyday Mathematics* explain, “leads to better long-term mastery of facts, skills, and concepts,” because of what they call the *spacing effect* (UCSMP, n.d.-b, paras. 2–4). The spacing effect is characterized, roughly, as a brain-friendly pace for learning. Spacing contrasts with massing, which involves concentrated learning of new material within one topic area and which the authors speculate contributes to learning fatigue (UCSMP, n.d.-b, para. 5; Bruner, 1960 / 1977, p. 51). Bruner (1960 / 1977) maintains that spiraling allows for more rapid ascension into complex thought, because the learner is constantly refreshed by new ideas—particularly if those ideas are central to the discipline and if the learner is empowered to seek new knowledge (pp. 50–54). Regardless, ideas within spiraling programs are still thought to proceed in a generally hierarchical and sequenced fashion. A spiral staircase, after all, is not appreciably unlike a ladder.

I should also note that Wood, Bruner, and Ross (1976) use another structural metaphor—scaffolding—to describe how instructors can support students in achieving a complex goal. Scaffolding is perhaps an over-used and vague term in education; Bruner (1978) views scaffolding, technically, as reducing “the degrees of freedom in carrying out some task” (p. 19), so that a student can concentrate on the core ideas.

*Relationship to Mastery Learning and Potential Concerns*

The ladder model is also built into many of today’s digital-learning platforms and diagnostic assessments. Broadly, the goals of such platforms are to assess each student’s quantifiable level of skill and then to support ensuing instruction at this identified level. In this way, designers of such platforms generally say, learning can be personalized to meet students’ diverse needs. For example, the Measure of Academic Progress (MAP) is a diagnostic assessment produced by the Northwest Evaluation Association (NWEA). The MAP assessment, according to its developers, “drill[s] down to pinpoint specific gaps” in understanding that teachers can subsequently address (NWEA, 2017). Furthermore, the developers also say that the design blueprint for the MAP “arranges the skills in logical learning progressions, [so] teachers can clearly see what a student needs to learn next” (NWEA, 2017). This sort of personalization—with skill building upon skill, acquired at a student’s own pace and following an initial diagnosis—is sometimes referred to as *mastery learning* (Bloom, 1968). Motamedi (n.d.) explains that “every mastery learning program divides instruction into small units” and that learning occurs in sequence, where “sequence is described as hierarchical” (paras. 7–9).
In using such platforms, the validity of an examinee’s score is established when its standard error falls below a pre-determined cut-line (Weiss, 2004, pp. 75–77). Weiss (2004) offers psychometric support for the use of adaptive assessments for diagnostic and achievement purposes in education. He cautions, though, that the results of adaptive tests are skewed when content from multiple domains (e.g., the mathematical operations of addition and division) are arranged hierarchically within the same scale (p. 78). He consequently describes a process called content balancing that aims to remediate this potential problem by offering examinees additional items in less-represented domains or by utilizing multiple scales (pp. 78–80). Content balancing, according to Weiss, generally yields an undesirable outcome: increasing the length of an assessment (p. 79). Digital learning platforms generally aim to keep such assessments short, which therefore represents a tension between theory and implementation.

Another potential concern about adaptive tests—not addressed by Weiss (2004) but related to the balance of content—is whether the results of an adaptive test in mathematics can accurately locate an examinee’s level of skill along a learning pathway whose order is, quite possibly, an arbitrary one. In contrast to the NWEA, above, the authors of the Common Core State Standards for Mathematics (CCSS-M) write:

What students can learn at any particular grade level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, “Students who already know...should next come to learn....” But at present this approach is unrealistic—not least because existing education research cannot specify all such learning pathways. (National Governors Association [NGA] Center & Council of Chief State School Officers [CCSSO], 2010, p. 5)

More specifically, the CCSS-M authors suggest it is a choice—among many possible options—to place one lesson as the next within a prescribed sequence. Placing an examinee at some specified point might, therefore, obscure the examinee’s understanding of other domains. For example, results suggesting placement in multiplication might fail to recognize an examinee’s strong understanding of geometry. Theoretically, geometry could be arranged subsequently to multiplication within a valid learning pathway. Overall, then, this caveat offered by the CCSS-M authors undercuts a presumed epistemology of mathematics built-into many diagnostic assessments and learning platforms. Namely, this epistemology questionably maintains that content can be arranged in a reliable, specified order of difficulty. It presumes, moreover, that a sequential hierarchy is necessarily part of mathematical understanding.
Mathematical Storylines and Plots, Related to Notions of Coherence

The preceding discussion is, admittedly, somewhat muddled, because in some instances sequencing might refer to the order of problems, topics, lessons, units, or even standards. Any discussions of sequencing should transparently articulate which of these are pertinent. What some diagnostic assessments assume, and what the ladder and spiral models nevertheless connote is a linearly coherent presentation. Like Kaplinsky’s (2019), a common definition of mathematical coherence is an intuitive sequencing of related ideas developed over time (Fernandez et. al, 1992; Leinhardt, 1989). In their definition, for instance, Fernandez and colleagues (1992) use “hierarchical” and “interconnected” for describing ideas found within “lessons that make sense to students” (p. 335).

Indeed, coherence has been identified as an important feature of mathematics instruction that also promotes understanding (Fernandez et al., 1992; Greeno & MMAP, 1998; Gresalfi, 2009; Leinhardt, 1989; Leinhardt & Greeno, 1986; Sleep, 2012; Stigler & Hiebert, 1999). In her decomposition of the work of teaching for mathematical proficiency, in particular, Sleep (2012) cites “developing and maintaining a mathematical storyline” (p. 954) as a way to elevate coherence. A mathematical storyline, to Sleep, is a “deliberate progression of the mathematical ideas” also involving “connections across mathematical work, both within and across lessons” (pp. 954–955). Therefore, her definition of mathematical storyline is not unlike the common understanding of coherence, itself, described above. These terms might, in fact, be used interchangeably. But Sleep also seems to offer more specifics, possibly regarding mathematical storylines as steps along the pathway toward coherence. Storylines, as a result, might be embedded within a larger set of ideas for describing coherence but in a more holistic sort of way.

The Meaning of Connectedness

At the same time, the meaning of the connectedness or relatedness of ideas—whether a part of mathematical storylines or coherence, broadly—is not necessarily clear. Ideas may be connected to one another to varying degrees and in a variety of ways: topically, logically, sequentially, categorically, symbolically, rhetorically, semantically, and probably more. Indeed, Zimba (2011), a mathematician and architect of the CCSS-M, dramatically illustrates the complexity of coherence and the notion of connecting ideas in mathematics. Writing about the design of the CCSS-M, he describes “coherence in a nutshell” as the relatedness of ideas, tools, and techniques to “a few fundamental and familiar principles” (p. 31). These principles might be
considered *themes or preoccupations* within given domains of mathematics (Zimba, 2011, p. 7). In other words, mathematical ideas can also be regarded as coherent (and related to one another) when they fall under the same guiding concept or umbrella notion. Using his broad understanding of coherence, Zimba describes many (but certainly not all) of the surprising ways that ideas connect. As one example, he explains that middle grades students “should be able to appreciate that addition and multiplication have parallel mathematical properties, and that subtraction and division are mathematically derivative of them” (p. 17).

Zimba (2011) takes great pains, as well, to demonstrate that *coherent structures* in elementary and middle grades mathematics are “not always easy to see” nor are they “easy to describe” (p. 3). In attempting to understand the vision for coherence embedded within the CCSS-M, furthermore, Zimba argues one must consider that “the document is more than the sum of its parts” (p. 3). He asserts that it would be a mistake to regard the “delivery system” for the CCSS-M as a “master loop,” in which “Standard i” is addressed on the “ith day” of the school year and “Standard i + 1” is addressed on “Day i + 1” (p. 3). Zimba (2012) even depicts the messy tangle of a limited set of connected ideas represented in the CCSS-M via his *wire diagram*. (See Figure 2, which represents only a portion of the wire diagram.) For Zimba (2011, 2012), then, viewing coherence as a strictly linear sequence would be a severe epistemic misinterpretation. While the details of this figure are certainly unclear because of its sheer scale, compressed here, the wire diagram itself shows, undeniably, that mathematical ideas can be offered in a variety of ways.

Taking a page from Zimba’s (2011, 2012) playbook, in a manner of speaking, my thesis likewise attempts to demonstrate that the very concept of coherence in mathematics and mathematics instruction is complex and consequently merits further elaboration and investigation. I question whether our understanding of coherence is sufficiently robust and precise to be useful. In addition, as I explain throughout, differing conceptions of coherence have implications for teaching, learning, and the use of curriculum materials.

In a similar fashion, Sleep (2012) also acknowledges that the connectedness of ideas is difficult to understand (p. 955). Nevertheless, Sleep argues that teachers can develop and maintain strong within-lesson mathematical storylines “by making mathematical connections across a lesson’s activities” and “by engaging with new ideas / practices or engaging with ideas / practices in new (more challenging) ways” (p. 959). Conversely, much like Kaplinsky (2019), Sleep claims that threats to the development of mathematical storylines occur when connections are not
identified or when activities are not sequenced “within the lesson in ways that promote connections or the progression of mathematical ideas” (2012, p. 959). In her study, she found evidence of teachers making a variety of instructional moves, while using curriculum materials that either enhanced or diminished the progression of mathematical storylines during lessons.

Figure 2. A portion of Zimba’s (2012) wire diagram, depicting some of the connections among mathematical ideas of the CCSS-M (p. 5). The boxes show content standards and the lines show particular relationships among them.

Sleep’s (2012) and Zimba’s (2011, 2012) views on coherence appear to stand in tension with one another, at least to a degree. In Sleep’s work, the term storyline itself evokes an image of a single, linear, two-dimensional pathway. Zimba’s wire diagram, on the other hand, is web-like and decidedly non-linear. Acknowledging and valuing Zimba’s perspective necessarily complicates our understanding of Sleep’s findings. After all, how can we judge connectedness of ideas, when connectedness itself can be multi-valent?

Before explaining further, as a matter of convention, I note here that Sleep’s (2012) mathematical storylines are not necessarily dissimilar to the notion of mathematical stories, as articulated by Kaplinsky (2019). In this thesis, I therefore use the terms storyline and story (and, sometimes, narrative) interchangeably. At times, however, I borrow from Dietiker (2012) and refer
to stories as amalgams of one or more related storylines, which she often calls individual *story arcs* (p. 178). The context should make clear whether I am referencing an overarching story, or a narrative, or else a particular storyline.

Returning to models of storylines and their relationship to coherence, while Sleep (2012) draws heavily on the work of Lampert (2001), I maintain these two scholars also adopt somewhat different views on coherence. Sleep, as described above, uses the term storyline as an example of coherence, focused mainly on the activities occurring within lessons. Lampert (2001), on the other hand, refers to a “map of mathematical terrain” that she plans to teach (p. 184). (Imagine a ski or subway map, showing various connections between nodes of a system.) Much like Zimba’s (2012) wire diagram, Lampert’s map also represents non-linear pathways through a progression of mathematical ideas. Lampert (2001) emphasizes that—perhaps like a choose-your-own-adventure—specific routes might emerge within lessons depending on students’ responses (pp. 184–185). Much like Zimba, perhaps, Lampert also explains that mathematics is not necessarily a set progression of ideas and that a “complicated web of ideas under consideration” is perhaps a better portrayal of coherence (p. 192). In fact, she comments, “The teacher and student move according to the identification of the topics that are relevant to interpreting and solving the problem at hand” (p. 434). She adds that the connections themselves—the relationships portrayed in the maps of mathematical terrain and their natures—are also worthy of consideration for promoting a deeper understanding of the structure of mathematics (pp. 434–435).

We are left with a question: does coherence mean linear connectedness with one idea following after the next (possibly like Sleep, 2012, suggests), a web of interconnected ideas (Lampert, 2001; Zimba, 2011, 2012), or is it something else entirely? Perhaps, as Lampert (2001) suggests, coherence is not manifest through any particular representation, but rather by helping students become aware of the broad-brush relationships between mathematical ideas themselves (p. 435). In a sense, then, specific classroom tasks or activities might not be imbued with coherence, but rather, coherence might be a meta-cognitive opportunity that cuts across activities and lessons. To dig a little deeper, below, I briefly summarize some of the assumptions underlying Lampert’s maps and connect them to differing perspectives on instruction.

**Relating Coherence to Models for Instruction**

At the heart of Lampert’s (2001) non-linear mathematical terrain lies her complex model for instruction. She refers to the familiar *instructional triangle*, consisting of pairwise relationships...
between the teacher, students, and content (Cohen, Raudenbush, & Ball, 2003). At the same time, Lampert (2001) elaborates on the triangle by explaining how social and temporal complications emerge within teaching and how these also deserve attention (pp. 423–438). For example, the teacher must negotiate among the multiple ideas offered by individual students or groups of students, as they go about solving problems. This necessitates moderating between students’ contributions and helping them engage with one another.

In addition, Lampert’s (2001) model for instruction is predicated on her belief that a type of teaching, “teaching with problems,” is one that “is supposed to improve students’ performance and increase their motivation to learn” (p. 3). She contrasts this pedagogical approach, which she also calls a sense-making strategy, with more traditional approaches by explaining:

The fundamental difference between teaching with problems and other kinds of teaching revolves around the nature of content and what it means to study it in school. As it is enacted in classroom relationships while students work on problems, the content is more than a series of topics. When students engage with mathematics in a problem, the content is located in a mathematical territory where ideas are used and understood based on their relationships to one another within a field of study. (p. 431, italics in original)

To illustrate, Lampert offers a number of examples throughout her book, such as starting a fifth-grade lesson with the problem: “A car is going 50 miles per hour; how far will it go in 10 minutes?” (p. 183). Rather than solve the problem at the chalkboard for her students to replicate and copy into their notebooks, and rather than accept the first so-called correct answer to be suggested, Lampert describes how she asks students to consider the sorts of responses that might make sense. As the classroom of students pondered whether “5 miles” could be reasonable, Lampert says she “left this problem ‘hanging’ for a few days” (p. 183), while they subsequently investigated somewhat less complex but related problems.

Hiebert and Grouws (2007) might argue that Lampert’s (2001) examples depict students “struggling or wrestling with important mathematical ideas” (p. 387). This, they say, is an opportunity to learn that is tied to greater conceptual understanding. And Green (2014), summarizing the related research, coins a term for the underlying instructional approach (i.e., teaching with problems) as You, Y’all, We. This term inverts the common way of describing a heavily teacher-directed form of pedagogy, I do, we do, you do, used to represent the gradual-release model of instruction (Pearson & Gallagher, 1983). In the gradual-release model, the teacher demonstrates how to obtain solutions for an initial set of problems, then asks students to practice the same techniques as a group, and then allows students to work independently. Wu
(2014) rightly notes, of course, that employing Lampert’s *You, Y’all, We* (Green, 2014) approach is challenging and therefore demands a nuanced understanding of the mathematical content. What I aim to emphasize here, generally-speaking, is that coherent instruction does not necessarily depend upon the order of presentation and that, as Lampert (2001) says, “Content is more than a *series* [emphasis added] of topics” (p. 431).

McCallum (2018), another mathematician and architect of the CCSS-M, makes a helpful distinction between two models for instruction. Specifically, he contrasts Lampert’s (2001) sense-making model with what he terms the *making-sense* stance. The former, he says, “manifests itself in concerns about mathematical processes and practices, such as pattern seeking, problem-solving, reasoning, and communication” (para. 2). He warns that the sense-making model, insofar as it fails to promote students’ understanding of mathematical conventions, “can have the skeleton of a jellyfish” (para. 2). In contrast, the making-sense stance is concerned with the structure and logic of mathematics, which is also an important part of learning (cf. Pickering, 1995). Both stances or models, McCallum (2018) argues, are not mutually exclusive and each carries its own benefits and risks (para. 3). The extremes of either model are to be avoided, as they would leave students with wrongful impressions of the discipline of mathematics. At the same time, McCallum adds that “because time is one-dimensional, and sense-making happens over time, structuring mathematics to make sense involves arranging mathematical ideas into a coherent mathematical progression” (para. 4). He nonetheless explains that mathematical progressions can usually be ordered “in more than one way” (para. 4).

In short, because McCallum reifies the differing epistemological assumptions upon which the extreme versions of each stance rely, we might be convinced that coherence—however it might be defined—differs in accordance with an underlying instructional model. Despite this potential conflict, McCallum asserts the necessary inter-relationship of the sense-making and making-sense models, and he therefore advocates for a dual, “binocular vision” that “takes both stances at the same time” (para. 9). Coherence for McCallum, stated differently, is both a skeleton or web *and* also a progression. It requires both structure and flexibility.

**Recent Approaches: Understanding Mathematical Storylines, Plots, and the Ladder**

Even though McCallum’s theory, if complex, makes intuitive sense, there are still practical considerations that must be addressed. By leaving the car problem hanging for several days (Lampert, 2001, p. 183), we might wonder whether this choice made the instruction more or less
coherent. Lampert might argue that coherence was achieved across the unit, but McCallum (2018), and possibly Sleep (2012), might ask whether students perceived the important connections across lessons. Here, again, to judge the coherence of teaching, we must look beyond the underlying model for instruction and also consider whether we are discussing the coherence of units, lessons, topics, ideas, problems, or all of the above. For coherence to have a clear and pragmatic formulation, I contend that we must understand and distinguish among its characteristics at each of these “multiple focal lengths” (Lampert, 2001, p. 44). At each level and across levels, though, it is not immediately evident what these characteristics might be.

The research that I present here aims to address questions related to coherence. I largely concentrate on undertaking a fine-grained analysis of particular features of teaching, to better understand the nature of coherence at a localized scale. In addition, I consider the role that instructional resources play. Zimba (2011) and McCallum (2018) both imply that curriculum—generally described, for the moment, as a framework of topics to address—plays a role in guiding the directions teachers may take. It is known to shapes what gets taught and when (Goodlad, 1984; Opfer et al., 2016; Perry et al., 2015; Sherin & Drake, 2009; Whitman, 2004). What is less certain is how curriculum influences within-lesson coherence. I employ a ground-up approach in my research, looking sentence by sentence within curriculum, to better understand the nature of coherence in its presentation. Further, I explore whether coherence varies in relationship to different models of instruction, curriculum, and, perhaps even, different focal lengths (Lampert, 2001, p. 44) and lenses (McCallum, 2018) under consideration.

The mathematical story framework. Dietiker (2012, 2013b, 2015a) offers a set of tools, helpful for undertaking this sort of analysis—the mathematical story framework (MSF). I describe Dietiker’s work in greater detail in Chapters 2 and 4, but I offer a brief summary here. The MSF, primarily a model for analyzing textbooks, identifies elements of mathematical presentation and how these evolve. Using narrative theory and built from textual analysis, Dietiker’s framework identifies analogues of literary constructs—character, setting, event, plot, metaphor, genre, and so on—within the context of mathematical writing. Through the framework, we can discern the evolution of related mathematical events (whether in texts or—as I argue—during instruction). We can discern the nature of what Sleep (2012) and Kaplinsky (2019) might call, in other words, mathematical storylines or stories.
Dietiker (2012, p. 34) first builds on Bal (1985 / 2017), who distinguishes among several layers of narratives: the medium through which narratives are communicated (e.g., text, film, etc.), the sequence of events presented to a reader or audience (known as the syuzhet), and the sequence of events as they logically occur (i.e., the fabula). According to Dietiker (2012), considering the syuzhet and fabula independently from one another, in particular:

...enables analysts to recognize when the same information (the “truths” reconstructed in the fabula by the reader) is organized with different effects. These layers also provide a way to articulate the differences between the information revealed in the story as understood by the reader (the fabula) and how it is temporally revealed (the story). (p. 34)

Some of these effects, Dietiker continues, include the plot of a narrative or the:

...result of the reader’s negotiation between the two layers [syuzhet and fabula] while reading the story. This is because the plot unfolds structurally in the story layer (as the reader encounters and interprets the events in sequence), but its effect stems from the reader’s logic while he or she connects different parts of the story, reinterprets prior events, and raises questions and anticipates what is to come. (p. 40)

As analogues, Dietiker defines mathematical events as transformations of mathematical characters or objects (pp. 69–70). For example, an algebra textbook may describe obtaining the solution of the equation $3x = 27$ by applying the inverse operation, division, such that $x = 27 ÷ 3$. Division is a mathematical event. She also notes that, unlike other narratives, mathematical narratives have characters that can also be regarded as actions (p. 91). The expression $x + 5$, for instance, can be seen as a transformation of a number (by adding five) or an algebraic object unto itself (a linear binomial). The criteria for discriminating between characters and events—between objects and actions—involves the surrounding context of the ideas presented (p. 91).

Further, in the MSF, Dietiker (2012) defines the mathematical syuzhet as the presentation of mathematical events, perceived by a reader (a teacher or a student). The mathematical fabula is the presentation of mathematical events as they are logically construed. The mathematical plot signifies the “the structure and aesthetic dimensions of a mathematical story [syuzhet], with particular attention to recognizing the dynamic tension between what is known and what is desired to be learned by a reader” (Dietiker, 2012, p. 96). Stated differently, Bal (1985 / 2017) describes the plot of a narrative as that which encourages us to keep on reading or watching. As we experience the plot, we want to learn the answers to the questions: “What happens next?” and “What happens to the characters?” Likewise, Dietiker (2012) characterizes the mathematical plot as the confusion, curiosity, or surprise—generally described as the suspense—that flows from the
moment questions are raised (the beginning) until they are resolved (the end). And suspense, returning to my definition of engaging mathematical storylines, is an affective element. It is an emotion felt within the body and experienced as a type of nervousness or anticipation.

Note that, within the MSF, suspense does not preclude the resolution of a mathematical storyline; even if the answer is known or obvious, the mere raising of a question constitutes suspense. Stated somewhat differently, a mathematical narrative may provoke a reader’s interest or it may not, but the essence of the plot itself is the suspense generated by an emergent question (Dietiker, 2012, p. 183). Moreover, a storyline having a resolution doesn’t necessarily imply that the question has been answered. Instead, a mathematical storyline may end without a conclusion, so to speak, so that the underlying question remains unresolved for an extended period (or abandoned entirely). Lampert’s (2001) hanging question on car-travel might be one such example. Whether hanging storylines provoke continued curiosity or frustration is an open question.

Finally, besides the aesthetic response signaled by suspense, there is also a structural, or content-oriented, component to plot—namely, any realizations about the mathematical characters and their relationships to one another (Dietiker, 2012, p. 183).

Within the MSF, Dietiker (2012) utilizes a set of codes, which I describe further in Chapter 5, allowing mathematical events to be traced. Returning to the metaphor of the ladder, Dietiker’s codes generally mark the mathematical events within written lessons and how they progress (or fail to). While Dietiker does not use the ladder analogy, I find it a helpful one for explaining her framework and analytic approach. I elaborate below.

Possible structural progressions of mathematical narratives. Abstracting from Dietiker’s (2012, 2013b, 2015a) work, very broadly, I suggest there are three possibilities for the progression of mathematical ideas within lessons. First, they may proceed in a predictable and expected fashion—stated differently, the transformations of mathematical objects or ideas (i.e., mathematical characters) may be relatively uncomplicated. This pathway would be represented by a smooth ascension of the ladder, step-by-step or rung-by-rung. Next, mathematical storylines may progress until such point when they experience an interruption or obstacle of some kind. Interruptions may consist of an unexpected reordering of ideas, when the mathematical syuzhet—or the presentation of mathematical ideas—departs from the logical ordering of mathematical events, the underlying fabula. In this case, the ladder analog would constitute the switching of the order of one or more rungs as traversed. Yet another type of interruption would be an intentional
blockade of the mathematical fabula. Blockades can occur in several ways, but they generally connote the strategic or intentional omission of key information of some kind. In ladder-land, this might constitute the removal of one or more rungs.

Regardless of the type of blockade, rung-removal or reordering, I theorize that whether the student traverses the ladder successfully depends largely on the level of perceived challenge inherent in the pathway. Are the obstacles motivating, provoking curiosity and further inquiry? Or are they consistently defeating, like those impossible obstacles on *Ninja Warrior*? Blockades are, de facto, causative of emotional grappling. Even more, Dietiker’s (2012, 2013b, 2015a) framework shows us that mathematical storylines, and perhaps even coherence, are more complex than they are typically conceived. Storylines need not reflect the sequential connection of ideas to be impactful; coherence, likewise, might actually benefit from intentional blockades.

As noted earlier, Hiebert and Grouws (2007) might describe blockades as opportunities for “struggling or wrestling with important mathematical ideas” (p. 387). I take “important” to mean focused on core content (Bruner, 1960 / 1977). While several studies point to its importance in effectively deepening students’ conceptual understanding, the nature of productive struggle, and how to maintain it within classroom instruction, is not entirely understood. Nevertheless, I highlight the words of Bruner in describing it:

> One of the least discussed ways of carrying a student through a hard unit of material is to challenge him [sic] with a chance to exercise his full powers, so that he may discover the pleasure of full and effective functioning. Good teachers know the power of this lure. Students should know what it feels like to be completely absorbed in a problem. They seldom experience this feeling in school. Given enough absorption in class, some students may be able to fully carry over the feeling to work done on their own. (p. 50)

Observe that, here too, Bruner uses language full of affective and even physical import. He writes of the “a hard unit” and the “exercise of full powers” that represent “ways of carrying a student through” such material. He describes “the pleasure of full and effective functioning” and the lure of feeling “completely absorbed in a problem.” Students, he bemoans, “seldom experience this feeling in school.” I maintain that, in order to better understand productive struggle and classroom engagement in the context of mathematical problem-solving, we need to make explicit these tacit emotive aspects of mathematical storylines and plots and mark them within textbooks and classroom instruction. Quite simply, the affective cannot be readily separated from the cognitive.
1-4. Summary

Mathematics is sometimes described as a “beautiful and interconnected story” (Kaplinsky, 2019, para. 1). Precisely what interconnectedness means—and how it relates to instruction—remains debatable. Some imply that the interconnectedness of mathematical ideas means they must be presented in a sequential order, like the ascending rungs of a ladder (e.g., Abdussalaam et al., 2015; Fernandez et al., 1992; Kaplinsky, 2019; NWEA, 2017; Sleep, 2012). I suggest that additional clarity is warranted to ascertain the influence of non-sequential orderings on learning (Dietiker, 2012, 2013b, 2015a; Lampert, 2001; Zimba, 2011, 2012). A better understanding of the nature of interconnectedness in mathematical presentations has consequences for instruction and the design of curriculum materials.

In an effort to describe the interconnectedness of ideas within curriculum materials, Dietiker (2012, 2013b, 2015a) has offered a useful framework—the MSF. This framework draws on literary theory and narrative analysis to illuminate underappreciated elements of written lessons. Using the MSF, we can distinguish between mathematical storylines and plots, perhaps broadening notions of coherence. Though not widely used in empirical analysis of classroom instruction, I argue that the MSF could also be applied to unpacking narrative elements of enacted lessons.

Drawing on Dietiker’s (2012, 2013b, 2015a) work, I describe mathematical plots as storylines embued with affect. Mathematical plots, I theorize, represent engaging storylines that connect ideas in varying ways and potentially motivate students’ curiosity. I define mathematical storylines as a set of related mathematical ideas or events that merely evolve (cognitively). At present, these two terms are used somewhat interchangeably and also as proxies for the notion of coherence itself. There are no fine-grained studies analyzing mathematical storylines and plots of multiple lessons from an elementary classroom, comparing nuances of both written materials and classroom instruction. The goal of such analysis is to better understand: a) ways that curriculum materials offer opportunities for teachers to design engaging mathematical plots, and b) ways that teachers interpret and adapt mathematical plots. In the next two chapters, I situate this work within several broader areas of study, particularly research on the design work of teaching (M. Brown, 2009), teachers’ participation with curriculum materials (e.g., Remillard, 2005), and fidelity of curriculum implementation (e.g., S. Brown, Pitvorek, Ditto, & Kelso, 2009).
CHAPTER 2. THE CONTEXT:
THE RESEARCH PROBLEM, QUESTIONS, AND POTENTIAL IMPLICATIONS

We love you Diffendoofer School, we definitely do.
There surely is no other school that’s anything like you.
You’re gribbulous, you’re grobbulous, each day we love you more.
You are the school we treasure and unceasingly adore.

—Dr. Seuss, Hooray for Diffendoofer Day! (1998, 101–104)

2–1. Contextualizing the Research Problem: Theories of Learning and Teaching, and the Role of Curriculum Materials

Still uncertain about the whole complex of challenges related to teaching, learning, and curriculum, even at this point in my career and education, I nonetheless feel compelled to plant stakes in the ground. A second assumption of this thesis, closely related to my first on engagement, is that effective teaching consists not of telling students how to think, but instead helping them think for themselves. Like Lampert (2001) and McCallum (2018), I maintain that the sense-making model is crucial for students’ engagement and depth of understanding. Also like Lampert (2001) and McCallum (2018), I consequently worry when instruction veers too closely and ardently to the making-sense end of the spectrum. Although, I certainly agree that extreme sense-making should be avoided, as well.

Commitment to sense-making experienced a surge in the U.S. during the 1990s and early 2000s, spurred on by research and publications like the Principles and Standards for School Mathematics (National Council of Teachers of Mathematics [NCTM], 2000). I am concerned, however, that sense-making is now being relegated to a secondary position in educational policymaking and in schools. Incorporating sense-making is invariably more challenging than deploying a strict making-sense model, but sense-making is nonetheless built into many modern curriculum programs. It is certainly endorsed by the authors of the CCSS-M (e.g., McCallum, 2018). The difficulty of substantially drawing on a sense-making model in the classrooms is one reason, most likely, why schools and districts turn toward more conventional approaches. They lean toward making-sense, even while attempting to use modern, sense-making curriculum programs (e.g., Wolfman-Arent, 2014). That said, in order to face this challenge head-on, school
and district leaders must nonetheless understand the concomitant risks of encroaching too closely upon the extreme limits of the making-sense model, in addition to learning how to support teachers with sense-making instruction. Sense-making instruction offers too many potential benefits to be ignored or pushed far into the background.

**Learning Theory**

The sense-making model is supported by Piagetian (see Kolb & Kolb, 2011) and Vygotskian (1930–1934 / 1978 / 1999) theory—namely, that learning involves building knowledge in an experiential fashion by testing and discussing established beliefs. Under this very broad umbrella, loosely described as socio-constructivism, the role of the teacher involves providing conditions that promote students’ interaction with each other and with tools (including the tool of language). Integrating both sides of the teaching-learning coin, the mathematician Halmos elegantly summarizes these ideas, writing, “The best way to learn is to do; the worst way to teach is to talk” (Halmos, Moise, & Piranian, 1975, p. 466).

Despite its aesthetic heft, the parsimony of Halmos’s proclamation is a stark oversimplification. Halmos admits as much—namely, that even he cannot describe good teaching—preferring instead to judge “the performance by the product” (Halmos et al., 1975, p. 466). Much like the value-added theorists (e.g., Hanushek, 1971; Hanushek & Rivkin, 2010; Murnane, 1975), Halmos argues that effective teachers are those who consistently produce successful graduates. But he fails to enumerate the criteria for ascertaining successful “calculus students, store check-out clerks, carpenters, or auto mechanics” (Halmos et al., 1975, p. 466). Therefore, questions abound. Are we to hold those who can out-calculus and out-clerk in higher esteem? How? And how can good teachers of teachers be judged? Alice jumps into the rabbit hole.

**A Practice-Based Understanding of Teaching**

In contrast, scholarship by Ball (2000), Ball and colleagues (Ball & Bass, 2003; Ball, Hill, & Bass, 2005), as well as many others, endorses the development of a practice-based understanding of teachers’ expertise. In the words of Stein, Remillard, and Smith (2007), “Knowledge of how [emphasis added] an effect was achieved is crucial...for enhancing the field’s understanding of teaching and learning mathematics” (p. 339). Indeed, how an effect was achieved is even more
important than observing that outcomes have improved. I, too, value investigating the on-the-ground work of mathematics teaching, thereby exploring the rabbit hole.

My study therefore aims to address how mathematics teachers teach. More specifically, though, I explore the elements of teaching that I regard as the most important—those enabling students to think for themselves. Throughout the course of this thesis, then, I name and describe practices that teachers employ in developing mathematical storylines and plots—but, primarily, insofar as they support students’ attainment of mathematical proficiency through opportunities for sense-making. Unfortunately, though, far too little is presently known about the practices or moves of instruction that promote students’ understanding (Hiebert & Grouws, 2007). Therefore, as others have argued, pursuing this sort of inquiry about the work of teaching is not only worthwhile, but it is also essential to the development of the field.

Hiebert and Grouws (2007) also assert that “researchers are feeling their way through murky waters” (p. 373) without robust theories that “specify the ways in which the key components of teaching fit together to form an interactive, dynamic system for achieving particular learning goals” (p. 373). Some progress has been made, but much work remains. Ultimately, by looking intently at teachers’ work, my study primarily intends to build on a tentative theory of teaching developed by M. Brown (2009) that I describe below. Like others who have developed working theories of instruction (e.g., Brousseau, 1997; Freudenthal, 1973; Gravemeijer, 1994; Leinhardt & Greeno, 1986; National Research Council, 2001; Schoenfeld, 1998, 2011; Simon, Tzur, Heinz, & Kinzel, 2004), I hope that the theory, articulated herein, can illuminate “particular relationships, provide meaning for the phenomena being studied, rate the relative importance of the research questions being asked, and place findings from individual studies within a larger context” (Hiebert & Grouws, 2007, p. 373).

Below, before describing my study and its findings, I outline the broader research context, including M. Brown’s (2009) theory of teaching, within which I generally locate my work. To do so, I summarize principles of ambitious teaching—those aiming to promote sense-making—and I connect these principles to mathematical narratives. I then relate ambitious teaching, as a goal-oriented activity, to the design work of teaching (Brown, 2009). By definition, design involves the use of resources or tools (Wertsch, 1998). I therefore connect tool-use to teaching via textbooks or curriculum materials. Finally, I note that considerations of curriculum materials now include an understanding of mathematical narratives (Dietiker, 2012, 2013b, 2015a), as well as the progression
of mathematical ideas. From these constructs, taken together, we can see the need to consider the mathematically emotive elements of mathematics instruction.

**Ambitious Instruction and the Design Work of Teaching**

To begin, I briefly reiterate the learning goal that I am attempting to understand better: the ways in which teachers and curriculum materials interact to produce engaging mathematical storylines and plots. I contend that several factors comprise the “interactive, dynamic system” (Hiebert & Grouws, 2007, p. 373) within which this goal resides. These I describe more fully in Chapters 3 and 4. For now, though, I offer a sketch so that I can properly contextualize the problem of research my study aims to address.

**Reform-oriented instruction and sense-making.** First, instruction that involves the deployment of engaging mathematical plots also motivates mathematical problem-solving, as described previously. Pedagogical tools that enable problem-solving—and consequently, promote sense-making—align with hallmarks of *ambitious teaching*. Ambitious teaching includes a deliberate and purposeful integration of students’ ideas and engaging tasks while aiming for deeper, connected understanding of mathematics (Franke, Kazemi, & Battey, 2007; Lampert, 2001; Lampert, Boerst, & Graziani, 2011; National Research Council [NRC], 2001, p. 315). Sleep (2012) also articulates a version of ambitious teaching, built on objectives for mathematical proficiency that emphasize several “‘interwoven and interdependent’ strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition” (p. 939). These are generally drawn from the goals of the CCSS-M and based on the research-based principles of the NRC (2001). As described above, Sleep’s decomposition of teaching for proficiency includes orienting instruction toward mathematical goals by, among other strategies, “developing and maintaining a mathematical storyline” (p. 954).

Throughout this thesis, I sometimes refer to ambitious teaching as *reform-oriented*, reflecting the widespread belief among policymakers and scholars that the traditional, didactic form of instruction, customarily found in the United States, must evolve to allow students to achieve their true potential as mathematical thinkers (Hiebert & Grouws, 2007; Stigler & Hiebert, 1999, 2009; NRC, 2001). As above, reform-oriented instruction should not exclude making-sense goals (McCallum, 2018), but it necessarily incorporates sense-making (Lampert, 2001) because of the difficulty of and tendency to overlook or minimize the latter. Generally, though, reform-oriented instruction allows students more ownership in the classroom, to explore ideas and to engage in
problem-solving. Some argue that such opportunities, particularly in the low-stakes environments, also support students’ intrinsic motivation for learning (e.g., Csikszentmihalyi & Hermanson, 1995).

**Resources and the design work of teaching.** Next, ambitious teaching depends upon the successful use of resources (M. Brown, 2009; Remillard, 2016; Remillard & Taton, 2015). Some resources are contextual, based within a given classroom, school, or district, while others relate to teachers’ own capacities and knowledge. Other instructional resources, still, are those that are procured: classroom tools and curriculum materials. As noted in the Preface, curriculum materials have been, and still are, widely used in mathematics instruction in the U.S. and internationally (e.g., Begle, 1973, p. 209; Valverde et al., 2002). That said, not all teachers use curriculum materials (e.g., Freeman & Porter, 1989) and some use materials but not in ways that the authors intend (e.g., Ben-Peretz, 1990; S. Brown et al., 2009; Collopy, 2003; Remillard, 1999, 2000; Tarr, Chávez, Shih, & Osterlind, 2008). Remillard (1999) notes that teachers intending to establish and maintain reform-oriented learning environments face additional challenges, because of the inherent unpredictability of teacher-student interactions within problem-solving activity (p. 329).

Building on this prior research and theory, M. Brown (2009) offers a framework that ties together resources and their use under a novel conception of teachers’ work. In particular, he theorizes that instruction is design work, arguing:

> When teachers use curriculum materials to craft instructional episodes in order to achieve goals, when they use materials as tools to transform a classroom episode from an existing state to a desired one, they are engaging in design—whether or not they intended to do so. (p. 23)

Brown (2009) regards all “goal-directed activity” as design, or rather, he maintains that design involves “crafting something in order to solve a human problem, to change the state of a particular situation” (p. 23). A teacher’s lesson, intended to promote a change in the understanding of students, would therefore represent a designed solution to the so-called “human problem” of effecting learning. He also depicts the design work of instruction through the design capacity for enactment (DCE) framework. I describe the DCE in greater detail, later in Chapter 2 and also in Chapter 5, but for now, I simply explain it as a portrayal of a) teachers’ own resources and b) curricular resources. Together, these resources are collectively (and interactively) deployed within episodes of instruction that utilize curriculum materials.
Recognizing the complexity of design, M. Brown (2009) offers an overlaying construct, likewise grounded in the tasks of teaching, accounting for variation in how similar teachers use the same resources but differently. Brown defines this construct as pedagogical design capacity (PDC), which is the individual teacher’s ability “to perceive and mobilize existing resources” in ways that “help accomplish their instructional goals” (p. 29). According to Brown’s (2009) theory, teachers with a stronger PDC would make consistently productive classroom decisions with the curricular resources, while teachers with a less-strong PDC would do so less frequently. Brown’s empirical work supports this contention (pp. 29–31) and provides compelling evidence that teachers with putatively stronger PDC are more responsive to their settings, when they “craft instructional episodes” that aim toward specific learning objectives (p. 29).

Beyond this proof-of-concept, however, not much is known about PDC. M. Brown (2009) contends that context is an important component of PDC, because teaching is an interactive endeavor, of course, and involves the contributions of students, colleagues, district or school resources, and the like. Davis, Beyer, Forbes, and Stevens (2011) assert that teachers’ knowledge of their students, as well as knowledge of reform-oriented instruction and curriculum programs, are also aspects of PDC. Choppin (2011) shows that teachers’ familiarity with the components of written lessons, likewise, promotes the likelihood of their making what he calls effective learned adaptations (p. 335). But whether PDC can be fully decomposed into constituent parts, whether it can be measured, how it develops, its relationship to other resources—such as teachers’ mathematical knowledge for teaching (see Ball, Thames, & Phelps, 2008)—and so on, remain unclear. It is also unknown whether teachers’ interpretations of and development of mathematical storylines or plots should be incorporated within PDC.

Improving instruction through curriculum and materials. Concerns over U.S. students’ performance in mathematics are not new and are not abating. As long ago as 1845, Horace Mann characterized the performance of Boston’s public school students on a mathematics assessment as shockingly poor, remarking that “no friendly attempt at palliation can make it any better” (Caldwell & Courtis, 1924, p. 265). The launch of Sputnik in 1957, the publication of A Nation at Risk (National Commission on Excellence in Education [NCEE], 1983), the relative lack of growth on the National Assessment of Educational Progress (National Center for Education Statistics [NCES], 2018), and the relative unpreparedness of students for college and career (Ostaschevsky, 2016) have all continued to provoke concerns. (Aee, also, Lubienski, 2015 and
Ravitch, 2015.) Acknowledging that teaching does, indeed, shape students’ learning (see Hiebert & Grouws, 2007), proposals to address these persistent fears about the quality of education in the United States have largely fallen into three camps, attempts to understand and influence teachers’ knowledge and beliefs, methods or actions, and—a somewhat newer area of research—their use of materials.

Focus on curriculum has intensified in recent decades, as a result of ongoing, productive lines of investigation (see Stein et al., 2007) and public speculation about policies related to such research (e.g., Supovitz, Daly, & del Fresno, 2015). Consider, for instance, the theory of change advanced by the authors of the CCSS-M:

> For over a decade, research studies of mathematics education in high-performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country. To deliver on the promise of common standards, the standards must address the problem of a curriculum that is “a mile wide and an inch deep.” (NGA Center & CCSSO, 2010, p. 3)

Curriculum, to some, is a key determinant of students’ mathematical achievement, because of the many ways it can influence teaching (Begle, 1973, p. 209). The authors of the CCSS-M maintain that the curriculum of the U. S., at the same time, remains a roadblock to students’ learning. In their view, it is comparatively scattershot. The CCSS-M authors therefore adopt a stance on curriculum—changing teaching by changing high-level instructional frameworks and aligned resources—that is echoed by many in the education-reform movement. This approach has been described by some as a remote-control approach (Cohen & Ball, 2000; Shulman, 2004).

In Chapter 3, I explore the complex relationship between materials, teachers, and learning in greater detail. For now, it suffices to say that efforts to improve instruction via curriculum frameworks and resources have produced largely uneven results, however (e.g., Ball & Cohen, 1996; Ball & Feiman-Nemser, 1988; see also Cuban, 1993). These mixed results are perhaps attributable, at least in part, to the remote-control approach and the relatively distal influence of materials on classroom life. After all, teaching is known to be a relatively isolating profession without much direct oversight (Lortie, 1975 / 2002). And, as Remillard and Taton (2015) explain, “viewing curriculum programs and related tools as the sole solution often becomes the Achilles’ heel” of school and district leaders (p. 56). Rather than focusing on strict interpretations of fidelity of implementation, a variety of conceptualizations of teachers’ use of curriculum materials have emerged (e.g., Ben-Peretz, 1990; S. Brown et al., 2009; Remillard, 2005). These alternate
perspectives generally emphasize the importance of engaging in conversation with materials, to understand their design, underlying philosophies, and what teachers themselves bring to the table. The complex ways that teachers interpret and make use of curriculum materials, still not fully understood, are what largely determine their influence on instruction. These questions, therefore, persist within areas of open research.

**Mathematical Plots, Knowledge of Curriculum Embedded Mathematics (KCEM), and Open Areas of Research**

I nonetheless hypothesize that one component of teachers’ PDC is their ability to perceive the mathematical plot of storylines within curriculum materials and then adapt the plot to meet students’ needs. I explain this idea further in the next section of this chapter. By *mathematical plot*, I generally refer to Dietiker’s (2012) definition that encompasses both the structural and aesthetic outcomes of the interactions between the mathematical syuzhet and fabula. Furthermore, by *mathematical storylines*, I also draw from Dietiker’s framework to mean the set of related events that befall a character or characters (i.e., mathematical objects) during the uptake of a focal mathematical question. This definition particularizes my earlier, proxy definition, referring to the evolution of mathematical ideas. At the same time, from a methodological perspective, I found my definition—focusing on mathematical ideas or characters themselves—to be a helpful one. (I describe these distinctions in greater detail in the next section and in Chapter 5.) In my research, I utilize these definitions within a novel form of analysis by reviewing, side-by-side, written curriculum materials and episodes of classroom instruction.

More concretely, consider an example of a storyline from the television program *Seinfeld*: the events related to the character Kramer, as we wonder, “What will happen, if he keeps swimming in the East River?” (David et al., 1997). The plot of Kramer’s storyline consists of our surprise when he first decides to swim in polluted water, as well as our realizations about his character and his circumstances. Likewise, the plot of the entire episode consists of the surprising ways, we come to understand, the multiple storylines inter-relate. We discover, specifically, that: Kramer’s swimming improves the condition of his back, makes his mattress noisome, causes Elaine’s back-injured boyfriend to suspect an affair, leads to Elaine’s own back injury, and results in Elaine and her boyfriend joining Kramer in the now-crowded river for therapeutic purposes. Like *Pulp Fiction*, the television show *Seinfeld* is known for its novel use of multiple interwoven storylines.
to create the impression that seemingly minor actions have larger unintended effects (Smith, 1995).

**Mathematical ruses and plot twists.** Analogously, Huntley and Heck (2014) borrow from chaos theory to suggest that even minute changes in how teachers use curriculum materials might cause an outsized impact on students’ opportunities to learn. Their perspective highlights the complexity of the interactions between written materials and instruction. In so doing, Huntley and Heck also argue that existing language and frameworks do not suffice to describe either small- or large-scale adaptations of written materials. Consequently, they also call for new conceptual frameworks, reflecting “how small changes in dialogue, pacing, instructional sequence, the use of tools, or classroom interactions affect opportunities for students to achieve specific learning goals” (Huntley & Heck, 2014, p. 35).

Returning to my discussion of mathematical plots and storylines, I see these as one possibility for such a framework. There are details yet to be worked out. While Dietiker’s (2012) and Sleep’s (2012) definitions of mathematical storylines overlap in important ways, they also differ. Both conceptions of storylines generally refer to sequenced transitions of related mathematical events. Dietiker (2013b, 2015b) and Ryan and Dietiker (2018) imply, however, that mathematical storylines are perhaps more malleable than Sleep (2012) suggests. Sleep (2012) found that a loss of coherence resulted from classroom events that reduced connections among ideas. As described above, Sleep’s (2012) conception of storylines emphasizes connectedness as a way of promoting coherence; at the same time, I also note that these definitions are also somewhat under-specified.

To enact different mathematical plots, on the other hand, Ryan and Dietiker (2018) theorize that the events of mathematical storylines can be manipulated; doing so might yield differing levels of curiosity or engagement (i.e., different aesthetic experiences in plots). As evidence, Dietiker (2013b, 2015b) describes two alternate pathways within a lesson she and her colleagues developed. This lesson involves exploring the equivalence between three representations of the same value, specifically, $0.\overline{9}$, $0.999\ldots$, and $1$. A common approach with this topic involves walking students through a series of related calculations, like: 
Yet, as Dietiker explains, a very different approach asks students to confront this surprising realization by having them, first, count by decimals (0.111..., 0.222..., 0.333..., etc.) and then by fractions (\( \frac{1}{9}, \frac{2}{9}, \frac{3}{9}, \) etc.). These two counting patterns might seem disconnected from one another, until the overarching and shocking connection question is asked, “Are 0.999… and \( \frac{1}{3} \) equal to one another?” Dietiker (2015b) describes the latter approach as setting up a *ruse* (p. 18).

This sort of work on understanding plots, overall, leads us to question exactly what constitutes a *deliberate progression* (Sleep, 2012, pp. 954–955). With the MSF, it is now more difficult to discern “an order that’s intuitive” (Kaplinsky, 2019, para. 15). Even when broadly connected to one another, it seems, events need not follow a prototypical sequence of increasing complexity. Through flexible reorderings, the notion of a *plot twist* materializes, and I am interested in the role of plot twist in learning. Dietiker (2015a) defines a plot twist as a reordering of the sequence of tasks in the fabula, to produce a new syuzhet that reveals an unexpected contradiction. In the chapters that follow, I elaborate on other potential conceptions of plot twists.

**Assumptions and questions.** Again, my own focus on mathematical plots and storylines is partly motivated by a set of central assumptions about teaching and learning. First, as explained above, I assume that the various ways mathematical plots are enacted relate to students’ engagement in mathematics classrooms. Even more specifically, I assume that the variety of mathematical plots and storylines complicate our understanding of coherence and its role in instruction. Second, I hypothesize that more engaging mathematical plots could prompt different sorts of cognitive work. In particular, I presume that mathematical storylines can be deployed in ways that create suspenseful plots and are therefore can be structured to foster curiosity (Dietiker, 2013b, 2015b; Ryan & Dietiker, 2018). Consequently, such plots may enhance sense-making in the classroom. Last, I assume that curriculum materials contain mathematical storylines (Dietiker, 2012, 2015a) and incorporate the possibility of suspenseful plots. In like fashion, this assumes that

\[
3 \times 0.333... = 0.999..., \\
3 \times \frac{1}{3} = 1, \\
\frac{1}{3} = 0.333..., \text{ and so} \\
0.999... = 0.\overline{9} = 1.
\]
storylines and plots of mathematics curriculum materials influence instruction, depending upon the ways materials are interpreted by teachers and taken up in classrooms (Stein et al., 2007).

While research in this area, and related ones, is still being pursued, early findings are cause for optimism. First, research shows that storylines are intentionally built into curriculum materials (Dietiker, 2012, 2015a, 2015b). This line of inquiry also suggests that varied mathematical plots and storylines may impact students’ learning (Dietiker, 2013a, 2016; Richman, Dietiker, & Brakoniecki, 2016; Richman, Miller, Brakoniecki, & Dietiker, 2016; Ryan & Dietiker, 2018). More generally, we know that teachers perceive the coherence of curriculum materials as an intentional design feature (Dietiker, Males, Amador, & Earnest, 2018; Collopy, 2003; Remillard & Kim, 2017). Remillard & Kim (2017) demonstrate, specifically, that teachers map progressions of learning, as well as connections across representations, when they use curriculum materials. They describe the mathematical knowledge activated in this work as knowledge of curriculum embedded mathematics (KCEM). Together, even if not framed explicitly as such, Remillard’s and Kim’s research implies that PDC might incorporate mathematical plots and storylines, because teachers can perceive mathematical plots and storylines within written curriculum materials and mobilize them during instruction. Future research is needed to validate this connection, though.

At the same time, a variety of other questions remain. For one, the various ways mathematical plots and storylines are incorporated within written lessons has not yet been documented. Richman, Dietiker, and Brakoniecki (2016) contend, specifically:

Future research is needed to learn what other mathematical plot structures exist within lessons. Are these shapes of content also found in lessons of other mathematical strands, such as geometry or probability? And, importantly, how might different mathematical plots impact students both in terms of the understanding and their view of mathematics?

Taken together, these mathematical differences between “similar” lessons illustrate the myriad of options that are available to teachers as they “faithfully” enact a textbook lesson. These options, as described by the mathematical story framework, provide fertile ground for teachers, teacher educators and researchers to further explore the many different ways that high quality curricula can be effectively enacted. (p. 9)

As they imply, here, the nuanced relationships between written lessons and classroom instruction have also not been fully explored and described.

Broadly, Remillard and Kim (2017) explain, “Reading, interpreting, and reasoning about mathematics concepts as they are embedded in designed curriculum resources represent a
critically important aspect of teachers’ work that has received limited attention in the literature on knowledge for teaching” (p. 78). They continue by observing that:

Additional research and tool development are needed to advance the field. First, more research is needed to understand how teachers interpret and make use of a variety of different types and designs of curriculum resources. We believe that this work is especially important for elementary teachers, who typically do not have extensive preparation in mathematics and are, as a result, more inclined than secondary teachers to rely on curriculum resources. (p. 79)

These statements by Richman, Dietiker, and Brakoniecki (2016) and Remillard and Kim (2017) echo calls by researchers, more generally, who identify a need for analysis of specific design features in curriculum materials, as well as their embedded opportunities to learn, and the various ways these are taken up by teachers (Huntley & Heck, 2014; Remillard, Harris, & Agodini, 2014; Remillard & Heck, 2014). As noted earlier, this research is also situated within theories and frameworks on teaching, such as pedagogical design capacity (M. Brown, 2009) and what it means to use a given curriculum program (S. Brown et al., 2009).

2–2. Key Definitions

Before proceeding further, I define several key terms and offer additional, underlying assumptions. Note that, in successive chapters, many of these terms and assumptions appear with additional contextualization. In the next section, I highlight the overarching research problem I aim to address, and I delineate my particular research questions. I conclude this chapter by outlining my study and the organization of this thesis.

To begin with terms and assumption, I note that have not yet fully explained what I mean by curriculum. Even more, I have not defined teaching. While these are foundational concepts that deserve greater attention, I describe them with modest detail here and pursue an extended discussion of each, particularly of curriculum and its use, in Chapter 3. In this chapter, though, I sketch what I mean by curriculum-use, after which I then define the other constituent elements of mathematical narratives.

Curriculum

I borrow my definition for mathematics curriculum from Remillard and Heck (2014), who describe it as “a plan for the experiences that learners encounter and the actual experiences that are designed to help them reach specified learning goals for mathematics” (p. 125). Building on
prior scholarship, Remillard and Heck explain that this definition encompasses more than just the goals or topics of instruction: within their framework, curriculum incorporates both the intended and actual experiences of students in the classroom. Teachers’ intended curriculum, Remillard and Heck write, is tantamount to their stated and unstated lesson plans. They define the enacted curriculum, furthermore, as events taking place during classroom instruction. What students learn, or what Remillard and Heck call student outcomes, is sometimes referred to as the experienced curriculum (Gehrke, Knapp, & Sirotnik, 1992; Goodlad, Klein, & Tye, 1979). To differentiate among teachers’ intentions for learning, students’ classroom experiences, and teacher’s guides and textbooks, I use the terms written curriculum, curriculum materials or programs, and sometimes curricular guidance, in reference to published texts.

I note that, historically, the term curriculum materials was intended to distinguish reform-oriented resources from traditional sorts of textbooks (see Stein et al., 2007). Textbooks were primarily considered resources for exercises and example problems, while modern instructional resources now offer pedagogical guidance and additional supports. As a new term, curriculum materials was therefore used to highlight these newer features. Given that textbooks of the traditional sort are now less prevalent, due in large part to the influence of the NCTM and Common Core State Standards, I do not make this same distinction.

Teaching

According to Hiebert and Grouws (2007), teaching consists of “classroom interactions among teachers and students around content directed toward facilitating students’ achievement of learning goals” (p. 372). This definition builds on theory offered by Ball and colleagues (Cohen & Ball, 1999, 2000; Cohen, Raudenbush, & Ball, 2003) and Lampert (2001) around what is familiarly known, again, as the instructional triangle. The triangle emphasizes that teaching is relational—that it involves give-and-take between each of the depicted nodes: teachers and students, teachers and content, and students and content. The necessary give-and-take between teachers and students may seem obvious, but reciprocity between the other nodes merits additional discussion: teachers and students don’t necessarily shape (or change) content, so much as they filter and interpret it through various lenses. Of course, the content—in this case, the discipline of mathematics—certainly influences the activity that occurs in classrooms. At the same time, Lampert’s (2001) model for instruction expands and complicates these relationships even further. She highlights the social and temporal interactions, located within the triangle, such
as when teachers support student-to-student communication. She therefore describes the teacher as “an active negotiator, a broker of sorts, balancing a variety of interests that need to be satisfied in classrooms” (Lampert, 1985, p. 190).

Figure 3. The familiar instructional triangle, showing the bi-directional relationships between teachers, students, and content. This image also depicts the interior of the triangle to reflect Lampert’s (1985) description of the teacher as negotiator.

These triangular, interactional models for instruction tacitly acknowledge and align with socioconstructivist learning theory espoused by Piaget (see, e.g., Kolb & Kolb, 2011), Vygotsky (1930–1934 / 1978 / 1999), and others. Experiential learning, broadly-termed, prescribes forms of teaching exemplified by these interactional models. Interactional models for instruction also necessarily reject what Friere (1970 / 2000) describes as a banking approach. To Freire, the term banking critiques traditional modes of teaching that involve unidirectional deposits into students’ minds. Under a banking approach, the “scope of action allowed to the students extends only as far as receiving, filing, and storing” of information” (Freire, 1970 / 2000, p. 71). Wholly teacher-centered approaches to instruction have generally proven simplistic and therefore not very fruitful for promoting higher-level sorts of understanding (Hiebert & Grouws, 2007).

In contrast to banking, the instructional triangle and related lines of thinking embrace the complexity of teachers’ work and consequently offer richer possibilities for both research and practice-based interventions to improve teaching. Consider, for instance, Ball’s (1993) succinct-but-nuanced description of her own mathematics teaching:
In the service of helping 8- and 9-year-old children learn, I seek to draw on the discipline of mathematics at its best. In so doing, I necessarily make choices about where and how to build which links and on what aspects of mathematics to rest my practice as a teacher. With my ears to the ground, listening to my students, my eyes are focused on the mathematical horizon. (p. 376)

This description, accommodating the triangle, emphasizes the inevitability of teachers’ and students’ autonomy. Teachers are not machines and students are not widgets. Teachers’ work, Ball explains, involves constant negotiations and renegotiations among representations of content, forms of discourse, and ways of building community. These arenas all deserve exploration.

Yet, Hiebert and Grouws (2007) caution that even “thinly described” forms of student-centered teaching are unhelpful in research, unless the connections to students’ learning can be clearly drawn. Therefore, not only are nuanced frameworks needed, but work that articulates these sorts of fine-grained connections is also important for the advancement of the field. (Huntley & Heck, 2014). Further, Ball (1993) suggests that efforts to improve instruction should not center on easy answers, but instead on “forums for professional exchange” that incorporate “tools for interpretation and choice” (p. 396). My own research, likewise, aims to elucidate teachers’ choices, and the circumstances surrounding their choices, in hopes of building on emergent, interpretive tools for future professional conversations.

Teaching and Teachers’ Curriculum-Use

While curriculum and curriculum materials may seem conspicuously absent from the instructional triangle, they are actually latent constructs that overlay the content. Schwab (1978), in fact, notes that the curriculum is “from the start, a representation of the discipline” (p. 269). In other words, curriculum is a perspective on the node that depicts the academic subject or content.

As noted previously, M. Brown’s (2009) design lens on teaching also foregrounds instructional tools or resources (such as curriculum materials). Like Ball and colleagues (i.e., Cohen & Ball, 1999, 2000; Cohen, Raudenbush, & Ball, 2003) and Lampert (2001), Brown also views teaching as interactional among three components. To a degree, then, Brown’s (2009) DCE framework may also be regarded as triangular in nature and could therefore be superimposed upon the instructional triangle. Teacher resources in the DCE consist of a teacher’s goals and beliefs, pedagogical content knowledge (Ball et al., 2008; Shulman, 1986b), and general subject matter knowledge; these overlap the node for the teacher in the instructional triangle. Curricular resources, to Brown (2009) in the DCE, consist of procedures, domain representations, and
physical tools or objects called for within instructional texts. Together, these depict models of content shown by the instructional triangle. The interaction of teacher and curricular resources, further, represents the instructional tasks given to and taken up by students within the triangle.

M. Brown (2009), furthermore, describes curriculum materials as artifacts or tools that teachers “appropriate…within their daily craft” (p. 26). And he conceives of instructional outcomes as deriving, at least in part, from the ways teachers and curriculum materials interact. These interactions, then, represent aspects of the enacted curriculum (Remillard & Heck, 2014). Interactions with curriculum, Brown (2009) found, include teachers’ decisions to a) offload responsibility onto materials by relying heavily on the guidance offered, b) adapt materials by modifying curriculum resources in ways that “reflected contributions of both the materials” and teachers’ personal resources, and c) improvise by relying minimally, if at all, on curricular support (pp. 24–25). Put another way, teachers generally rely on curriculum materials to varying degrees, as they address content during episodes of classroom instruction.

In Chapter 3, I explore the impact of curriculum on learning and several mediating factors. For now, though, I highlight findings in the extant literature. First, policymaking and research have tended to oversimplify the relationship between curriculum and learning (Remillard & Bryans, 2004; Huntley & Heck, 2014). More complete depictions have sought to explain variation in the ways teachers use curriculum. These have, therefore, acknowledged the role of teachers’ beliefs and knowledge, as well as the influence of program design, on curriculum-instructional design decisions (e.g., Collopy, 2003; Lloyd, 1999; Thompson, 1992). Additionally, Remillard and Bryans (2004) show that teachers’ orientations toward curriculum materials (or general beliefs about them) shape their classroom decision-making around using texts. In sum, scholars have therefore recognized curriculum resources as a potential site for teacher-learning and have, consequently, called for curriculum authors to communicate directly to or with teachers about their intended philosophies and designs (Ball & Cohen, 1996; S. Brown et al., 2009; Davis & Krajcik, 2005; Remillard, 1999). Their hope, of course, is that such transparency will enable greater alignment between curriculum materials, classroom instruction, and intended learning outcomes.

**Mathematical purposing, steering, and PDC.** Even though she does not concentrate on analyzing the role of curriculum materials, Sleep (2012) offers an illuminating framework—developed from her empirical study—that nonetheless describes teachers’ interpretations and use
of resources. I summarized Sleep’s work previously, but I offer more detail here. In outlining her framework, Sleep first defines mathematical purposing as the work of articulating, and then orienting instruction toward, the lesson’s intended mathematical goals (pp. 937–938). She further distinguishes mathematical purposing from tasks involved in steering instruction or, loosely, the recursive work of responding to students while keeping the lesson’s goals in sight (p. 938). More specifically, Sleep decomposes steering into potentially overlapping tasks that include: “Spending instructional time on the intended mathematics,” “Developing and maintaining a mathematical storyline,” “Opening up and emphasizing key mathematical ideas,” and “Keeping a focus on meaning” (pp. 942–943).

Sleep’s (2012) framework illuminates elements of teachers’ work that are both curriculum-agnostic and yet relatable to curriculum-use. In particular, mathematical purposing and steering are easily associated with M. Brown’s (2009) construct of PDC. First, as one reason, when planning instruction with curriculum materials, teachers are able to identify the mathematical objectives found within written lessons, and then affiliate the activities suggested with these goals (Dietiker et al., 2018; Stein & Kaufman, 2010). This is certainly purposing work that also involves perceiving curricular affordances. As Brown (2009) also explains, teachers assimilate their goals and those embedded within materials. Assimilation, in other words, is a negotiation between the materials and teachers’ personal aims. It is yet another form of purposing. In negotiating, whether tacitly or explicitly, teachers plan to orient classroom activity toward mathematical objectives (i.e., the negotiated aims). And finally, as teachers mobilize instruction while utilizing curricular guidance, they undertake the work of steering instruction toward mathematical goals.

Sleep (2012) presumes that steering is necessarily part of any instruction that supports “the development of students’ mathematical proficiency” (p. 939). In other words, ambitious teaching requires steering. Steering also includes “developing and maintaining a mathematical storyline” (p. 954). Conversely, teachers steer instruction (even if imprecisely defined) as they develop and maintain mathematical storylines and plots. I therefore incorporate mathematical purposing and steering into my own investigation on mathematical storylines. They represent elements of teachers’ instructional practice that may also relate to teachers’ use of curriculum. What is less clear, though, is the precise nature of teachers’ steering work—what teachers do as they perceive mathematical storylines (and plots) within curriculum materials and implement them in the classroom. By analyzing mathematical storylines in texts and their uptake in episodes of
instruction, I hope to add to our understanding of steering instruction (and, consequently, PDC). Therefore, in successive chapters, I describe my analysis of teachers’ steering moves, likewise, to discern and understand better their enactment of mathematical plots and storylines.

Note that well-designed curricular guidance includes support for steering instruction (e.g., Remillard, Reinke, & Kapoor, 2019). By well-designed guidance, I refer to programs that have been field-tested, draw on empirical research, are aligned to standards, and offer educative support (Davis & Krajcik, 2005; Remillard, 2016). Therefore, when teachers undertake mathematical purposing and steer instruction while using curriculum materials, they necessarily demonstrate PDC.

**Elements of Mathematical Storylines and Plots**

In articulating the foundational elements of mathematical storylines and plots, I draw heavily upon definitions found in Dietiker’s (2012, 2013b, 2015a) mathematical story framework (MSF). That said, I also adapt and go beyond some of these terms and definitions for several reasons. For one, I tried to bridge Dietiker’s MSF and prior research on coherence, as described above. For another, as a relatively new area of scholarship on curriculum and instruction, the MSF is still under-researched and under-specified. Terms like mathematical storyline and plot, I contend, benefit from additional specificity. The definitions, articulated here, were refined over the course of undertaking my analysis. As I conducted my research, I reflected on the need for this refinement, to distinguish cleanly among the various elements of mathematical storylines and plots. My research—and the research summarized in this section—together draw on perspectives articulated Egan (1989) and Elbaz (1991), involving the work of narrative construction in teaching.

**Mathematical setting.** Mathematical settings are, to Dietiker, manifestations of mathematical representations that “create ‘spaces’ in which the mathematical characters emerge and are acted upon” (Dietiker, 2015a, p. 296). I build on this definition by describing mathematical settings as the set of all analogous, replaceable mathematical objects and their representations, considered within a given storyline. By including all representations of mathematical objects, settings will necessarily change from lesson to lesson and year to year (even for the same set of objects).

For example, if a mathematical storyline involves adding the numbers 5 and 3, plainly, then the mathematical setting is the set of single-digit whole numbers. If this storyline involved the position of 5 and 3, depicted visually, then the setting would likewise could be the line segment
(or number line) from 0 to 10. This definition also allows metaphors for objects to constitute their own storylines and settings. A storyline that involves adding two-digit whole numbers, for example, and models these numbers with base-ten blocks suggests the existence of two storylines and two settings (parallel to one another): one involves transformations in the space of two-digit whole numbers and the other involves the moves with base-ten blocks representing two-digit whole numbers.

**Mathematical character.** A mathematical character is a mathematical idea or an object. A mathematical character may consist of a number, set of numbers, function, family of functions, representation of another object or idea, operation, relationship, or even a procedure (Dietiker, 2013b, p. 15). I go slightly beyond Dietiker, though, by insisting that the defining feature of a mathematical character is that it must be treated within a mathematical text or lesson as a noun. A character, transformed over the course of a storyline, is a new character. This is not unlike how we regard characters in dramatic works; Jules in *Pulp Fiction* is a very different character at the conclusion of the film from the one he was at its inception.

**Mathematical event.** A mathematical event is an action by an agent, whether a reader, teacher, or student, that transforms a mathematical character in some fashion (Dietiker, 2013b, p. 15). Some events may actually be characters unto themselves, and context makes the distinction. For instance, adding 1 to a given number \( x \) is a transformation of the number \( x \). That said, \( x + 1 \) may also be considered a mathematical character on its own terms, transformed over the course of its own storyline (see more below).

**Mathematical storyline.** A set of related mathematical ideas or characters that evolve. Dietiker (2012) calls these *story arcs*, which she defines as a “sequenced transition from a reader’s question to its answer (if one is found)” (pp. 176–177). She also explains her preference for *story arc over storyline*, because of the visual metaphor inherent in an arc that connotes a clear beginning and ending (Dietiker, 2012, p. 177). That said, I note that in her definition that some story arcs or storylines do not have clear resolutions; I also note, as above, that prior research and theory on coherence of instruction typically use the term storyline. My own definition of storyline contrasts with Dietiker’s (2012) in that she locates story arcs mainly within the syuzhet, dependent upon the reader’s observation of transitions. My suggested approach allows for analysis of storylines within any of Bal’s (1985 / 2017) layers of narrative, either syuzhet or fabula.
In addition, my definition is somewhat more specific. The key features of my storylines are that, first, they must involve related mathematical characters. Note that relatedness may be defined in any number of ways—topically, logically, sequentially, categorically, symbolically, rhetorically, semantically, and so on. Dietiker (2012) argues for logical connection among events of a storyline, but she also recognizes that enforcing this condition also carries potential limitations (pp. 75–76). As Zimba (2011) notes, in particular, relatedness is “not always easy to see” nor is it necessarily “easy to describe” (p. 3). Relatedness, therefore, is at least partly interpretive and dependent upon any affiliated evidence that may be offered. Second, storylines show how characters evolve. Evolution, likewise, broadly refers to the transformation of a character. A character is changed, if it can be considered distinct from its preceding state and if a mathematical event (or action) links preceding and changed states. (This last condition, a linking mathematical event, prevents us from saying that each new stage of transformation launches a new mathematical storyline.)

Further, because mathematics definitions are simply re-presentations of named objects, an extended discussion of the characteristics, nature, or properties of a mathematical object is not itself a storyline. Glossaries, then, do not contain mathematical storylines (Dietiker, 2012, p. 107). Last, there are no boundaries on the duration of a mathematical storyline. A mathematical storyline may transpire over the course of a school year (or longer), across a set of lessons, and within lessons (Dietiker, 2012, p. 75). For the purposes of my analysis, I wanted to understand how teachers interpreted curricular guidance at the lesson-level, and so I concentrate on storylines that emerge and remain within particular lessons. (An exception to this rule would include written guidance for a given lesson, describing activities intended to transpire over two or more days.)

As a storyline begins, a focal question emerges about the character. Using a term from Barthes (1974), Dietiker (2012) calls this focal question a formulation. I refer to this term, sometimes calling it a main formulation, in the analysis I present in Chapters 7 to 9. I distinguish main formulations from sub-formulations, which (I discovered) are scaffolding questions that are often presented on the way to achieving a larger aim. In short, formulation question prompt the reader (or teacher, or student) to wonder about the ultimate transformation of the mathematical character. A formulation need not be raised explicitly by a text (Dietiker, 2012, p. 177). Further, a formulation need not be answered for a mathematical storyline to be deemed resolved. For the purposes of my analysis, I characterized a storyline as resolved if the formulation about the
mathematical character is either addressed (fully or in part) or is abandoned within the course of a lesson. To distinguish mathematical storylines, I developed a heuristic that I describe in greater detail in Chapter 5 (see Figure 7). In general, this heuristic involved checking whether a given mathematical character was connected to a transformed version of the character via a specific mathematical event; if so, then the character was not part of a new mathematical storyline.

**Mathematical plots and suspense.** Mathematical storylines that are embued with affect constitute a mathematical plot. Affect or emotiveness connotes a “complex experience of consciousness, bodily sensation, and behaviour that reflects the personal significance of a thing, an event, or a state of affairs” (Solomon, 2019, para. 1). Further, emotiveness is achieved in mathematical plots by manipulating the underlying fabula or logical chronology (Dietiker, 2012, p. 35) of a storyline. The syuzhet consists of the reframed fabula, although in practice, the reader actually reconstructs the fabula by making observations about syuzhet (Dietiker, 2012, p. 96).

I therefore build on Dietiker’s (2012) definition, which specifies that a mathematical plot is the interaction between the fabula and syuzhet, producing both a structural and an aesthetic response on the part of a reader (p. 96). In my definition, an aesthetic response is considered emergent from interactions between the syuzhet and fabula, but I also name the aesthetic response as an affective or emotive one and I relate mathematical plots directly to mathematical storylines (which I also define a somewhat broader fashion, as indicated above). Further, I theorize there are two main categories of fabula-manipulation: reordering of events or creating an intentional blockade in the progression of a storyline. In mathematics instruction, the overall emotive or aesthetic effect of manipulating the fabula is curiosity, confusion, or surprise—more generally described as suspense. This differs, somewhat, from other definitions of narrative suspense, which emphasize the emotions of fear and hope (e.g., Ortony, Clore, & Collins, 1998, p. 131). Following Dietiker (2012) and Bal (1985 / 2017), I have also tried to translate these emotional descriptors, appropriately and concretely, into the context of the mathematics classroom. These tend to emphasize the literal meaning of the word suspense, as an intermediary state between a form of knowing and not knowing—a stasis of hanging or dangling while awaiting resolution (as specified by Dietiker, 2012, p. 239).

Suspense, in more philosophical terms, is also complex. The commonly-discussed “paradox of suspense” suggests that narrative tension might be nullified if an audience already knows the outcome of a particular story (see Bal, 1985 / 2017, p. 93). In theory, if strictly true, this notion
would have implications for mathematical plots and storylines. Students may already know the outcome, so to speak, of mathematical plots because they might, for example, be familiar with details of algorithms in advance of their being taught.

For example, the CCSS-M assert that students fluently apply the traditional approach to multi-digit subtraction in third and fourth grades (NGA Center & CCSSO, 2010, pp. 24, 29). In reality, though, many families, teachers, and even textbooks, teach the traditional algorithm (or a version of it) much earlier, because of a broad-based lack of clarity on how to address the underlying development of concepts (Hulbert, Petit, Ebby, Cunningham, & Laird, 2017). In practice, the paradox of suspense is less of a concern, perhaps, precisely because the underlying concepts take several years to develop and because knowing the procedures does not guarantee understanding them fully (e.g., Erlwanger, 1973). Further, some narrative theorists maintain that suspense is still experienced, even if the outcome of a story is already fully known, because details may be forgotten or because we may nonetheless remain attuned to the story’s emotional cues (e.g., Yanal, 1996).

2–3. My Research Questions and the Potential Implications of my Study

Stepping back, momentarily, Davis and colleagues (2011) explain clearly that teachers “need to analyze and adapt even high-quality, reform-oriented curriculum materials to better support their own students’ learning (Barab & Luehmann, 2003; Baumgartner, 2004; Davis, 2006)” (p. 797). Among others, Ben-Peretz (1990) and Remillard (1999) explain that curriculum materials cannot specify every instructional decision needed in the classroom. Conversely, teachers may not consistently recognize “what the curriculum developers intended to convey” (Collopy, 2003, p. 306). In addition, students’ learning needs differ from classroom to classroom, and teachers’ autonomy—an important aspect of their professional identities—are other factors that motivate adaptations of materials (Ben-Peretz, 1990; Remillard & Taton, 2015). Finally, noted in the previous section of this chapter, new frameworks for understanding elements of instruction and use of curriculum are needed—particularly nuanced frameworks, sensitive enough to capture the impact of fine-grained classroom decisions. These findings, considered together, offer a sketch of the complexity of teachers’ curriculum-use and the affiliated challenges of undertaking research in this line of inquiry.
As explained previously, there is also limited scholarship analyzing the relationship between mathematical storylines of written and enacted curricula. Researchers have yet to describe the full, dynamic relationships between what instructional materials offer, what teachers perceive, and how teachers and students engage with one another vis-à-vis mathematical plots or storylines. More precisely, there is little detail (if any) offered within the literature on the various ways elementary teachers interpret mathematical plots and storylines over the course of multiple lessons and with a given set of curriculum materials. Such detail would enable mapping, not only of the ways teachers respond to the plots and storylines in their teacher’s guides, but also of the various ways that storylines and plots are embedded within materials.

**Research Questions**

Therefore, as I describe in the following chapters, I use a multi-case study approach (Yin, 2009), employing a mixture of qualitative analysis methods, to better understand nuanced features of enacted mathematical storylines motivated by curriculum materials for two elementary-grades teachers. Specifically, I explore how teachers interpret curriculum materials, as they steer instruction toward a mathematical point and enact the embedded mathematical plots and storylines. In particular, my study responds to the following research questions:

1. When regarded as narratives, how can the relationships between written and enacted lessons be characterized for two elementary mathematics teachers? Specifically:
   a. What elements of mathematical storylines and plots are preserved or altered?
   b. What are the potential signals within written materials that contribute to teachers’ interpretations of mathematical storylines and plots?

2. How do two elementary-grades teachers steer instruction toward a mathematical point, while drawing upon guidance in written lessons? Specifically:
   a. What steering moves do teachers make that relate to lesson coherence?
   b. What do teachers’ steering moves suggest about their interpretations of narrative features within written materials—namely, characters, settings, events, storylines, and plots?
3. What do teachers’ perceptions and adaptations of mathematical storylines and plots suggest about their PDC? Specifically:
   a. How are teachers’ goals, beliefs, and teaching contexts reflected in their instructional designs when drawing upon mathematical storylines and plots found in written lessons?
   b. What do teachers’ goals, beliefs, and steering moves say about the ways they participate with curriculum materials with regard to enacting mathematical storylines and plots?

Together, these questions aim to address the presentation of mathematical events—specifically uncovering whether events are sequenced in unusual ways or information is purposefully omitted. I speculate on the impact of such decisions for learning and engagement.

**Potential Implications**

This study contributes to the literature on mathematics education and curriculum-use in several ways. As noted previously, a number of scholars call for new frameworks, to investigate the fine-grained details of the relationship between written and enacted curricula (e.g., Huntley & Heck, 2014; Remillard, Harris, & Agodini, 2014; Remillard & Kim, 2017). My study contributes to an emerging field of inquiry, drawing on Dietiker’s (2012, 2013b, 2015a) MSF, to analyze this relationship. Beyond enabling a better understanding of the relationship between written and enacted curricula—including the nature of mathematical storylines and plots in elementary-grades instruction—this line of inquiry also exhibits another potential implication addressed by Huntley and Heck (2014) in their own call for new frameworks. Specifically, as Huntley and Heck suggest, stronger definitions of what it means to participate with materials could be a fortuitous outcome—namely, questions around fidelity of curriculum implementation could be addressed.

The results of my study have additional theoretical implications. Consider work by Remillard and colleagues (2014), who write of their framework outlining the opportunities to learn (OTLs) embedded within curriculum materials:

One question is whether the analytical framework used in this analysis captures all the design features that are consequential in influencing opportunities for student learning. Although the framework was informed by the existing research on curriculum design features, the research in this area is underdeveloped; it is possible that we did not consider all necessary features of consequence. In addition, among the features measured,
some might be more consequential than others, or their importance could be dependent on factors not measured. More research is needed to understand how curriculum features might be consequential for student learning. (p. 748)

My study suggests inclusion of mathematical storylines and plots within the design features of curriculum materials, representing an additional set of OTLs. A typology of OTLs related to mathematical storylines and plots might aid future analysis of curriculum materials and the elaboration of related theoretical constructs.

In addition, this sort of work might pave the way for a more complete understanding of coherence and how to assess it within written materials and enacted classroom lessons. I note that there is not, yet, an empirical or theoretical resolution of Dietiker’s (2012), Kaplinsky’s (2019), Lampert’s (2001), Sleep’s (2012), and Zimba’s (2011, 2012) differing models of coherence—those leaning toward linear presentation and those embracing the pedagogical possibilities of non-linear presentation. I therefore draw connections to PDC and KCEM, particularly as each relates to coherent instruction.

There are also implications of my work for practice. In their review of the literature on the impact of teaching on learning, Hiebert and Grouws (2007) note that regular opportunities for “struggling or wrestling with important mathematical ideas” (p. 387) theoretically support students’ conceptual understanding of mathematics. In contrast, they decry “needless frustration or extreme levels of challenge” and “feelings of despair” from “nonsensical or overly difficult problems” (p. 387). Hiebert and Grouws (2007) also observe that only a handful of studies have tried or been able to show a direct connection between productive struggle and understanding (e.g., Stein, Grover, & Henningsen, 1996; Stein & Lane, 1996).

Nonetheless, how teachers work to preserve struggle while promoting students’ progress is still not well understood. Teachers need guidance in, as Meyer (2009) says, how to “be less helpful” and how to help students become “patient with irresolution” (paraphrasing Milch, 2009, as cited by Meyer, 2009). As Ryan and Dietiker (2018) suggest, better understanding of mathematical storylines and plots, and how these operate in promoting students’ engagement, could also impact opportunities for productive struggle (see, also, Richman, Dietiker, & Riling, 2018). How teachers interpret curriculum materials, particularly mathematical storylines and plots within them, could also inform curriculum authors in designing materials to assist teachers in developing and maintaining productive struggle. Specifically, I ask, what types supports are needed to help teachers make effective use of mathematical storylines and plots in written
lessons? What sort of guidance in written lessons can enhance the impact of mathematical storylines and plots?

2–4. An Overview of my Study and the Organization of this Thesis

The research that I present in this thesis employs a case study approach (Moss & Haertel, 2016; Yin, 2009). In so doing, I make use of a variety of qualitative analysis methods. My findings are drawn from portrayals of two teachers, their school contexts, and the materials they use during instruction. I subsequently present a cross-case analysis to validate a framework for understanding some of the ways teachers interpret and draw on mathematical plots and storylines from their written materials.

Summary of Collected Data and Analytic Methods

Participant data in my study were drawn from a larger set, collected as part of a National Science Foundation (NSF)-funded research project. I was a graduate research assistant and team member on this project, the Improving Curriculum Use for Better Teaching (ICUBiT) Project (NSF DRK12 Grant No. 091841 and No. 0918126; Co-PI’s: Janine T. Remillard & Ok-Kyeong Kim). The ICUBiT Project involved studying the written and enacted curricula of a set of elementary-grades teachers. There were two overarching goals of the project, namely, to: 1) better understand the capacities of teachers that enable their effective use of curriculum materials in mathematics and 2) develop a set of analytic tools for assessing and studying these capacities (Remillard & Kim, 2009). In particular, the research team aimed to describe elements or features of teachers’ PDC and develop a tool for understanding teachers’ KCEM. The team intended to build upon existing frameworks for understanding teachers’ curriculum-use.

Twenty-five teachers were selected to participate in the research undertaken as part of the ICUBiT Project. Each had been nominated by school leaders or other colleagues on account of their experience in using a given curriculum program. Five well-known curriculum programs, three of which originated as NSF-funded programs, were represented as part of this project. For my dissertation study, I selected two teachers from among this larger set, each using different programs and in whose classes I personally collected data. My selection process also involved screening the written lessons and observation transcripts of the larger set of teachers to identify those who appeared to utilize curriculum materials and adapt them in subtle ways. To refine my methodological approach, I also conducted pilot analysis of data from these and other teachers.
The two teachers I selected also exhibited differing amounts of overall teaching experience and differing levels of mathematical knowledge for teaching (MKT). (See Hill, Rowan, & Ball, 2005, and Hill, Schilling, & Ball, 2004, for more details.) Regardless, I consider both teachers to represent exemplar cases, useful for comparison (Moss & Haertel, 2016, p. 150).

Data collection involved an approach developed by Simon and Tzur (1999) and Cobb, Zhao, and Dean (2009). This approach, known as collecting teaching sets, involved conducting interviews, as well as gathering videotaped observations of six lessons (three in the fall and three in the spring of the 2011–2012 academic year) and a curriculum reading log (CRL) for each lesson observed. The CRL is comprised of hardcopies of publisher’s written materials for each observed lesson and a protocol, used by teachers, for annotating some of their design decisions. Each teacher also participated in three types of interviews: an initial intake interview, a pre-observation interview, and a follow-up interview—one after the fall and spring rounds of observations, respectively. Additional artifacts and email records were gathered, as well, depending on the ad hoc needs of researchers and interests of participants.

I describe my analytic methods in greater detail in Chapter 5. Briefly, though, to study these teaching sets, I first applied a set of analytic codes suggested by Dietiker (2012), as I reviewed a subset of the written curriculum materials used by each teacher. I adapted Dietiker’s approach to focus on guidance offered within teacher’s guides at the sentence-level. These codes, and Dietiker’s (2012, 2013b, 2015a) affiliated MSF, allowed me to identify mathematical characters, settings, events, storylines, and features of plot. Each written lesson was read and analyzed, multiple times, with coding refinements made upon each pass. In addition, I incorporated a set of analytic codes from Labov and Waletzky (1967) and Labov (1997) to understand the progression and resolution of mathematical plots within storylines. This additional set of codes, which I explain later, was used in creating a set of graphs to depict storylines and plots. These graphs are generally akin to images of what Remillard, Cappelletti, and Dominguez de Diclo (2015) and Remillard (2018) call maps of design arcs. (I describe design arcs, as a theoretical construct and analytic tool, in greater detail in Chapters 5 and 7.) I then produced summaries of the mathematical storylines and plots within each written lesson.

After analyzing the written lessons, I then coded transcripts of the observed lessons, also using Dietiker’s (2012, 2013b, 2015a) codes and framework, as well as Labov’s and Waletzky’s (1967) and Labov’s (1997) codes. I likewise produced summaries of the mathematical storylines.
and plots of each corresponding enacted lesson. Next, I compared the images of design arcs of the written and enacted lessons to one another, to identify notable overlapping or divergent segments. I then returned to the written and enacted curricula, as I generated themes that emerged from the teachers’ interviews. More specifically, when observing areas of design arcs that suggested teachers either utilized or departed from the written guidance in the classroom, I reviewed segments of the interviews that corresponded to explanations of teachers’ decisions with regard to mathematical plots. I situated these discussions within underlying claims about the teachers’ beliefs about their programs, curriculum materials in general, instructional goals, and their teaching contexts.

**Thesis Outline**

I now pause to offer an outline of what follows. This thesis is divided, roughly, into three parts. In the first part, Chapters 1–5 offer in succession: an introduction, a review of the relevant literature, a delineation of my theoretical and conceptual frameworks, and an explication of my research methodology. In the second part, Chapters 6–8, I describe the teachers who participated in my study, the contexts in which they work, the curriculum materials that they use, and findings related to their design of instruction. In Chapters 9 and 10, I conclude this thesis by looking across these case studies, as well as the profiles of both participating teachers. I discuss what my findings say about how teachers read, interpret, and use curriculum materials. I also explore implications of my study for both practice and future research.

More specifically, in Chapters 7 and 8, I profile the two teachers in whose classrooms I conducted my research, Elsa Mackey and Torrie Blum. First, Elsa is a highly experienced fourth-grade teacher who has used one curriculum, *Everyday Mathematics*, for the great majority of her career. After teaching in a number of large-city public schools in the southeastern U.S., Elsa relocated and presently teaches at a smaller-sized, independent school located within a mid-Atlantic city. As Elsa explains when I interview her, she aims to address a number of goals as an instructor, including helping students feel mathematically competent, demonstrating applications of mathematics, and offering students opportunities for skill practice. In Chapter 9, I describe how aspects of Elsa’s teaching context and her beliefs influence how she reads for, interprets, and enacts the mathematical plots of her written curriculum materials in order to achieve these goals. Specifically, Elsa sometimes works to unravel nuanced features of the mathematical plots in the written lessons of her teacher’s guide.
Next, Torrie is in the early-to-middle phase of her teaching career. Like Elsa, she has also taught primarily in early-grades classrooms. Torrie is a third-grade teacher, using *Math Trailblazers* at a school that has participated in a pilot of the program with its authors. Consequently, she has received significant amounts of affiliated professional development. In Chapter 8, I detail her steering moves with *Math Trailblazers*. In Chapter 9, I explain how Torrie’s context, goals, and beliefs may relate to her enactment of mathematical plots. In particular, I detail her belief that *Math Trailblazers* supports conceptual understanding of mathematics; she offers her own, supplemental goal to support students’ problem-solving. Therefore, Torrie often identifies and works to maintain the subtle plot-points within her written lessons that seem to designed to promote productive struggle.

In Chapter 9, I offer a cross-case analysis. This shows that Elsa’s instructional designs meet her goals to promote students’ confidence in problem-solving. She also reads materials in a largely *efferent* (Rosenblatt, 1994) manner, but she also unpacks and steers instruction toward the underlying mathematical fabula. In contrast, Torrie’s interpretation of her curriculum materials shows both an *efferent* and *aesthetic* approach to reading. She first reads for efferent purposes, to understand the big-picture goals of lessons and the sequence of classroom events. By then seeking to interpret and mobilize aesthetic elements of the mathematical syuzhet, she also aims to enhance students’ curiosity and surprise.

Finally, in Chapter 10, I conclude by discussing the contributions of this study to theory. This includes understanding the nature of mathematical plots and storylines, as enacted from curriculum materials. In addition, I discuss implications related to frameworks for understanding teachers’ interpretations of curriculum materials, as well those on understanding teachers’ use of materials and instructional designs. This leads to a discussion of fidelity of implantation, design of materials, methodology, and limitations. I conclude my presentation of this thesis by offering considerations for future research and practice.

Now, in Chapters 3 and 4, I turn to reviewing the bodies of literature that form the conceptual and theoretical basis for my study. My review of the literature generally consists of a summary of three interconnected lines of inquiry: the nature of curriculum and curriculum materials, the impact of curriculum materials on learning, and the roles of storylines and coherence within both materials and instruction.
Before proceeding, I offer one additional note: the structure of my thesis is purposefully arranged in a somewhat non-linear structure. My outline, furthermore, is meant to reflect the script of a play or television show. There are also allusions to narratives, including resequenced narrative events, throughout. In so doing, I hope to evoke what I ultimately aim to portray: the complex structures of plots in dramatic works. I also intend to demonstrate that coherence in mathematics instruction is not necessarily marred by intentionally unusual sequences or purposefully omitted content. I would like to think that the human intellect—including and especially that of our students—is fertile enough and driven enough to not need this sort of spoon-feeding. While complex plots are more challenging, they are also more interesting.
CHAPTER 3. THE SETTING (LOCATION 1):
THE LITERATURE ON CURRICULUM MATERIALS

Our school is at the corner of Dinkzoober and Dinkzott.
It looks like any other school, but we suspect it’s not.
I think we’re learning lots of things not taught at other schools.
Our teachers are remarkable, they make up their own rules.

—Dr. Seuss, Hooray for Diffendoofer Day! (1998, lines 5–8)

3–1. What is Curriculum? What Purposes Does It Serve?

This chapter comprises what is, essentially, one portion of my theoretical framework. Here, I review the research on mathematics curriculum, define the key terms, and summarize what is known about the impact of curriculum materials on learning. Because teachers’ use of curriculum is complex, I also describe research on the factors that mediate the relationship between materials and learning. Finally, I conclude this chapter by raising questions about the fidelity of teachers’ implementation. I periodically connect my review with considerations related to coherence, insofar as these emerged within Chapters 1 and 2.

In this chapter, further, my review of the literature generally focuses on publications in mathematics education, although there are occasional references to findings in science education. While my study also concentrates on elementary mathematics instruction, my review incorporates relevant findings from middle and secondary school mathematics, as well.

The Role and Nature of Curriculum Materials within Instruction

Simply put, for decades, teachers have relied upon curriculum materials (or textbooks) as resources to support their instruction (Begle, 1973; Chingos & Whitehurst, 2012; Goodlad, 1984; Huntley & Chval, 2010; Opfer et al., 2016; Perry et al., 2015; Sherin & Drake, 2009; Whitman, 2004). I detail the nature of mathematics curriculum materials in the next section, but first I contextualize their role in classrooms, below.

Roles played by curriculum materials. Because of the relative ease of distributing textbooks to teachers (Ball & Cohen, 1996), instructional materials have long been regarded as a potential lever for improving instruction (Bruner, 1960 / 1977; Cohen & Ball, 2000; Dow, 1991; Freeman & Porter, 1989; Manouchehri & Goodman, 1998, 2000; Remillard & Taton, 2015; Shulman, 2004).
Scalability also explains why the National Science Foundation (NSF) invested more than $100 million in the development, testing, and revision of curriculum materials from 1990–2007 (Hirsch, 2007). At the same time, research has shown that teachers use curriculum materials in ways that are not necessarily intended by their authors (e.g., Collopy, 2003; S. Brown et al., 2009; Stein et al., 2007; Tarr et al., 2008). Not all such adaptations undercut learning, but those that do are termed lethal mutations (Brown & Campione, 1996). Even still, the influence of both NSF-funded and other sorts of programs on learning has been deemed tepid at best (Ball & Cohen, 1996; NRC, 2004; Polikoff, 2015; Stein et al., 2007). I explain why, later in this chapter.

In this section, I situate mathematics curriculum materials and what they offer teachers within modern conceptual frameworks and recent empirical research. Even though authors are presently transitioning away from paper-based texts and toward digital resources (Remillard & Reinke, 2017; Remillard & Taton, 2015), the interplay of mandated standards, consequential assessments, and hefty workloads will undoubtedly persist in teachers’ professional lives. These collective pressures are unlikely to completely obviate the need for instructional resources (Remillard & Heck, 2014; Stein et al., 2007; Huntley & Chval, 2010). Curriculum materials serve many important roles for teachers—such as instructional designer and learning theorist—by crafting trajectories of understanding and suggesting activities intended to promote learning.

Undertaking this sort of work—including conducting the underlying research—would be inefficient and challenging for teachers to attempt on their own (Ball & Feiman-Nemser, 1988; Remillard, 2005; Remillard & Taton, 2015; Remillard, 2016). Indeed, well-designed curriculum materials in mathematics are those that incorporate empirical findings about how students learn and contain field-tested activities. These activities, in turn, include feedback from teachers. As a result, many programs are now written in partnership with teachers (Remillard, 2016). Curriculum materials also offer guidance on unpacking mathematics content that many teachers—particularly elementary grades teachers—may not have experienced or fully appreciated as students themselves (Remillard & Taton, 2015; Remillard & Kim, 2017). In sum, Remillard and Taton (2015) explain that curriculum materials “provide guidance on the day-to-day decisions on what should be taught, in what sequence, and, in many cases, how,” and so, “curriculum programs are critical to maintaining coherence across schools and classrooms” (p. 55). Stated differently, curriculum materials undoubtedly “provide teachers access to knowledge and processes beyond their immediate experience” (Remillard & Taton, 2015, p. 58).
Curriculum as a system, not just a resource. Remillard and Heck (2014) consequently depict curriculum, not merely as a text-based resource, but as a multi-valent idea. What they refer to as instructional materials represents an element of a wide-ranging and complex system, one that encompasses a broad set of goals and influences. The system they describe, consisting of bidirectional relationships among various instantiations of curriculum, constitutes an overarching framework. They argue for examining “the process of curriculum enactment” (Remillard & Heck, 2014, p. 705), instead of looking at instructional materials alone. In so doing, Remillard and Heck maintain, researchers and policymakers should appreciate the host of decisions, designs, and approaches that collectively impact school-based implementation of curriculum (Remillard & Heck, 2014, p. 705). And by curriculum enactment, they refer to elements of an established framework, originally proposed by Gehrke and colleagues (1992) that distinguishes among the planned, enacted, and experienced forms of curriculum (pp. 54–55). Remillard and Heck expand upon and delineate important elements of this previously-articulated framework, as they articulate features of their own.

Specifically, Remillard and Heck (2014) explain that a host of enactment decisions shape the nature and uptake of curriculum materials, as it exists within the system or framework they describe. In short, their curriculum policy, design, and enactment system (p. 709) is composed of two, high-level domains. First, in one domain, Remillard and Heck describe the official curriculum as the “expectations for student learning or performance” sanctioned by governing agencies at local, state, and national levels (p. 708). Within this domain, learning standards and broad-scale policies are located (i.e., curricular aims and objectives), in addition to consequential assessments. Local policies might include, for instance, district scope-and-sequence documents. The official curriculum is interpreted by curriculum authors and policymakers, as they craft instructional resources and local guidelines or regulations, and their related decisions largely impact the nature of the content and the sequence of topics (see, also, Huntley & Chval, 2010).

Next, in the second domain, Remillard and Heck (2014) define the operational curriculum as “what actually occurs in practice through the enactment process” (p. 708). Within this domain, they locate both teachers’ intended instructional plans, as well as what actually occurs during classroom lessons. These are regarded as distinct ideas, because teachers’ intentions and actual classroom activity rarely (if ever) align. In the operational domain, teachers make a number of long-term and spontaneous decisions, as they work to accommodate curricular guidance. These include selecting, sequencing, and adapting lesson activities. Teachers must also enact classroom
tasks, in addition to observing, interpreting, and responding to students’ thinking (Remillard, 1999; Schoenfeld, 2011).

Remillard and Heck (2014) also name and describe a variety of factors that shape the interpretation and uptake of curricular goals, other guiding documents, and published instructional materials. Among these are teachers’ knowledge, goals, and beliefs, as well as the affiliated supports they receive (e.g., professional development opportunities). Features of context—such as funding decisions, time allotted for professional development, or students’ backgrounds—must also be considered (see, e.g., Davis et al., 2011). Taken together, the various elements of their framework demonstrate that research on curriculum enactment must be mindful of the various and potentially-shifting roles that materials inhabit (Remillard & Heck, 2014, p. 716). Studying teachers’ uses of materials must likewise consider the inevitable “translation and transformation” (Remillard & Heck, 2014, p. 716) that occurs throughout the entire enactment process. These adaptations are, ultimately, what determine the eventual outcomes of this negotiation and renegotiation process: the presentation of various opportunities to learn to students. Put differently, the opportunities to learn that students are offered must be situated within the system. Defined here as alignment throughout the system—from curricular aims, to consequential assessments, local adoptions, classroom lessons, and students’ experiences—perfect coherence should therefore be considered an ideal but typically unattainable aim.

**What curriculum materials offer teachers.** My focus throughout this thesis is the interaction between commercially-published instructional materials and teachers’ use of such materials. As noted in the footnotes to Chapter 1, I generally refer to such resources as *curriculum materials*. And again, the term curriculum materials generally refers to a set of tools that incorporate both content and pedagogical guidance (Stein et al., 2007). Doyle (1992) refers to these resources as the *written curriculum*, distinguishing textual guidance from related instances of classroom instruction (i.e., the enacted curriculum, described earlier). This distinction, between the written and the *enacted curriculum*, are conventions within the field that I also follow (Remillard, 1999; Stein, et al., 2007; see, also, Cal & Thompson, 2014). The term curriculum materials, I also believe, reflects the idea that materials are not isolated texts unto themselves, but are artifacts within a much larger curricular system (Remillard & Heck, 2014). The etymological roots of *curriculum*, after all, include the definition “racecourse” (“Curriculum”, 2019). As the previous discussion shows, understanding what curriculum materials offer teachers, and what role they play within instruction, necessitates this sort of contextualization.
Too much and too little. On the one hand, formal, written documents cannot anticipate every decision that teachers need to make in the classroom (Yackel, Cobb, Wood, & Merkel, 1990, p. 15; see also Ben-Peretz, 1990; Remillard, 1999; Remillard & Heck, 2014; Sherin & Drake, 2009). At the same time, a growing body of research demonstrates that curriculum materials influence classroom decisions in a variety of ways. In his foreword, Shulman neatly encapsulates this conundrum by paraphrasing an argument by Ben-Peretz (1990) that “curriculum must be understood as both far too much and far too little” (p. vii). Appreciating this too-much and too-little perspective, I soon outline some of the roles that curriculum materials play in the classroom.

Nevertheless, I first note that the potential influence of materials on classroom instruction reflects, to a degree, whether teachers take up what Davis and Krajcik (2005) describe as educative content. Building upon work by Ball and Cohen (1996), among others, Davis and Krajcik (2005) use the term educative content to describe the possible ways that materials communicate to teachers rather than through teachers. This distinction, between talking to and talking through teachers, is one articulated by Remillard (1999), who observed that materials tend to offer teachers things to say and do within the classroom but without necessarily explaining the underlying purpose. In their typology of educative content, Davis and Krajcik (2005) outline design principles that, they believe, would promote effective use of curriculum materials. These include “supporting teachers in anticipating, understanding, and dealing with students’ ideas” (p. 11) and helping “teachers determine the most salient features of an instructional representation” for the purposes of “adapting and using those representations” (pp. 10–11). In the absence of such guidance, or when such guidance is communicated unclearly, teachers’ own beliefs, experiences, and orientations will undoubtedly fill the void (see Remillard & Bryans, 2004).

Empirical scholarship on how teachers benefit from using curriculum materials. Empirical scholarship has defended the educative potential of materials (e.g., Ball and Feiman-Nemser, 1988; Collopy, 2003; Lloyd & Wilson, 1998; Remillard, 1999, 2000). Most notably, Collopy (2003) profiles a teacher who deepened her own understanding of content and pedagogy by using a set of newly-adopted and reform-oriented curriculum materials. “Ms. Ross,” the teacher in Collopy’s study, used an NSF-funded program, *Investigations in Number, Data, and Space (Investigations)*. Over the course of one school year and without any additional professional development, Ms.

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This program, commonly called *Investigations*, was developed by the Technical Education Research Center (TERC). The set of materials explored in this study were published by Dale Seymour, but the program is now published by Pearson Scott Foresman. The relevant citation is:

Ross not only developed a stronger appreciation of whole-number operations and geometry, but she also experienced a shift in her epistemological stance. In particular, she moved away from seeing mathematics as an answer-getting enterprise and toward a sense-making and reasoning craft. In addition, Ms. Ross gained insights about the numerous, potential ways her students interpreted mathematical problems and ideas. Collopy (2003) notes, though, that educative, reform-oriented curriculum materials are far from sufficient for transforming the practices of teachers; she also reports on another teacher, also using *Investigations*, whose traditional practices remained largely stable during the study.

Similarly, Lloyd and Wilson (1998) report on a teacher, “Mr. Allen,” who implemented a program, *Core-Plus*4, which “furnished a way for him to translate his [prior] understandings into new but comfortable pedagogical strategies” (p. 271). These new practices included using classroom discussions as a vehicle for connecting mathematical representations of functions and their underlying analytic concepts. Nonetheless, Lloyd and Wilson also speculate that Mr. Allen’s new approach may have been fragile—not stable over time—and they acknowledge a potential relationship between his new practices and his relatively fluid, prior understanding of the content. Mr. Allen, they note, may therefore revert back to established practices when teaching other topics with which he was less familiar (or more procedure-focused topics). In this case, his conceptions of functions and the program’s were relatively closely aligned, possibly enabling his new practices.

I pause here, momentarily, to observe that similar findings on the potential impact of educative content have also been identified in science education (e.g., Arias, Bismack, Davis, & Palincsar, 2016; Beyer & Davis, 2009; Davis et al., 2011; Schneider & Krajcik, 2002). Educative features are sometimes incorporated within classroom texts, just as with mathematics materials. When incorporated, this research demonstrates teachers’ effective deployment of strategies to promote students’ learning. But without supportive guidance, teachers are left “to make decisions based on their own experiences, pedagogical design capacity (M. W. Brown, 2009), and pedagogical content knowledge (Shulman, 1986b)” (Bismack, Arias, Davis, & Palinscar, 2014, p.

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4 The first edition of *Core-Plus* Mathematics Project text, commonly referred to as *Core-Plus*, was actually published under a different title. It was originally published by Everyday Learning and the Core-Plus Mathematics Project and then, subsequently, by Glencoe / McGraw-Hill. The relevant citation is:

506). The concomitant challenges in science education are similar to those in mathematics, namely, that teachers may not interpret such guidance as intended (S. Brown et al., 2009). Further, as Remillard and van Steenbrugge (2013) and others demonstrate, teachers may regard educative features of curriculum materials as non-compulsory and consequently exclude them from their teaching—particularly when such features appear within marginalia.

Lloyd’s and Wilson’s (1998) study, described above, builds on work by Stodolsky (1988) and also emphasizes the role of presentation in curriculum materials. Stodolsky cautions, though, that unlike Mr. Allen, teachers may discern varying, perhaps unintended, messages from the ways that mathematical work is framed. In particular, many texts foreground facts and procedures over underlying concepts and modes of inquiry. Even if inquiry-based activities are incorporated, such foregrounding may contribute to an incomplete, product-oriented understanding of mathematics held by teachers and, ultimately, by their students. Ball and Cohen (1996) therefore ask curriculum authors to explain their stances on the nature of the mathematics, in addition to offering transparent descriptions of the text’s overall goals and organization. Their call is, of course, echoed by Davis and Krajcik (2005).

Ball and Cohen (1996) also suggest that teachers may benefit from guidance related to classroom contexts. Tarr and colleagues (2008) offer empirical support for this claim, as well, in their report on a notable study in mathematics education. They identify significant differences in students’ achievement, stemming from the combination of curriculum-based supports and the nature of the classroom environment. (I describe their findings in greater detail below when reviewing the research on curriculum and students’ learning.) Davis and colleagues (2011), likewise, found that teachers’ current and prior classroom contexts influenced their use of materials. Together, these studies show that features of context not only influence curriculum-use, these features may also determine teachers’ PDC.

While curriculum materials certainly exhibit the potential to reshape teachers’ understanding of content or pedagogical practices, such results are not necessarily common. I address the reasons underlying this observation, later on, when summarizing the literature on factors that influence teachers’ curriculum-use. Furthermore, as noted by Grant, Kline, Crumbaugh, Kim, and Cengiz (2009), the educative content within curriculum materials might support some, but perhaps not all, of authors’ intended purposes. Overall, the ways in which supportive features of materials factor into teachers’ plans and instruction are not well understood. Davis and colleagues
speculate, essentially, that the frequency of educative guidance in curriculum materials relates to the likelihood teachers will observe and utilize it.

At the same time, despite these challenges, the role of curriculum in teachers’ work is certainly not pedestrian. Begle (1973) observes that the topics found within mathematics textbooks are those most likely to be taught in classrooms. Others have affirmed this finding, demonstrating its relative stability over time (Freeman & Porter, 1989; Huntley & Chval, 2010; Polikoff, 2015; Remillard, 1999, 2000; Schmidt, Houang, & Cogan, 2002). Therefore, at a relatively superficial level, curriculum materials offer teachers an outline of the intended content.

Nonetheless, Freeman and Porter (1989) also indicate that teachers make not-insignificant changes to the topics addressed in textbooks—omitting some and adding others. They also found that teachers tend to over-emphasize the example problems offered within texts, particularly for students perceived as lower-achieving (Freeman & Porter, 1989, p. 416). Other researchers have shown that it is not uncommon for teachers to change the nature of topics studied, as well, by eschewing conceptual activities for teaching procedural routines (Stein et al., 1996; Manouchehri & Goodman, 1998, 2000). Stein and colleagues (1996), in particular, showed that teachers sometimes reduce the level of cognitive demand asked of students through written lessons. Therefore, even though curriculum materials ably serve an important purpose, by setting the general boundaries of what content is to be studied within classrooms, they are much less successful in conveying how that content should be studied.

In sum, curriculum materials play a complex role in classrooms—offering teachers not only descriptions of topics to cover but also potential pedagogical approaches. Research has demonstrated that teachers can learn both mathematics content and principles of teaching from curriculum materials. In addition, as Collopy (2003) observed of teachers use of materials, teachers can learn about mathematics in addition to developing knowledge of mathematics. This distinction is an important one, first postulated by Ball (1990) and Ball and McDiarmid (1990). In other words, curriculum materials have the potential to change teachers’ understanding of the content (about mathematics), as well as their perceptions of the discipline itself and how learning occurs (of mathematics). Teachers can also deepen their understanding of students’ thinking (Collopy, 2003; Remillard, 1999, 2000). This research collectively shows, however, that how and what teachers learn from curriculum materials is not fully clear. As I discuss in greater detail below, this learning is mediated by a host of factors. These include teachers’ knowledge and beliefs (see, e.g., Remillard & Bryans, 2004). Gueudet & Trouche (2012), among others, also
explain that teachers’ learning is influenced by the ways they engage with instructional materials within school communities.

Taken altogether, then, it should not be surprising that M. Brown (2009) found empirical evidence that two teachers “who have seemingly similar knowledge, skills, and commitments” (p. 29), may enact very different lessons even when using the same written materials. Teachers necessarily interpret and adapt curriculum materials (Ben-Peretz, 1990; Remillard, 2005) but sometimes in ways that conflict with the intended approaches (Collopy, 2003; Davis et al., 2011; Freeman & Porter, 1989; Manouchehri & Goodman, 1998, 2000; Remillard, 1999, 2000; Remillard & Bryans, 2004; Stein et al., 1996; Stodolsky, 1988). Teachers’ adaptations largely determine the relationship between materials and students learning. I describe the research on this relationship in the subsequent section of this chapter. But first, I review research on the particular features of curriculum materials that serve as resources for teachers: their embedded opportunities to learn.

**Philosophical Considerations, My Own Lens, and Participating with Materials**

The previous discussion was intended, generally, to run counter to the overly-simplistic claim that “teachers frequently take and teach the textbook” (Fullan, 1982, p. 118). In contrast, modern scholarship has substantiated a more apt description—that teachers engage in a participatory relationship with curriculum materials (Remillard, 2005). Remillard (2005) calls teachers’ participation with materials a “dynamic relationship between the teacher and curriculum” (p. 221) that accounts for contributions of both person and text. Stated differently, curriculum materials aim to convey a variety of OTLs to teachers. Teachers read and filter OTLs through their own lenses as they offer learning experiences to students. These are, in turn, taken up and understood in various ways. Furthermore, well-designed and research-based materials developed through recursive processes of field-testing and revision frame OTLs with considerations of students’ cognitive development. Well-designed materials offer supportive, educative content to teachers, including transparency about their underlying principles (Ball & Cohen, 1996; Davis & Krajcik, 2005).

**Schwab and the critical theorists.** Before describing OTLs more concretely, I first embed them within broader ideas about the philosophical aims of curriculum materials. Through this discussion, I also carve out my own intellectual commitments.

First, I note that for many, the term curriculum has long meant a set of topics covered during instruction. In other words, *curriculum* and *syllabus* have been generally considered
indistinguishable. During the 1960s, however, a significant shift in thinking occurred. Scholars openly questioned the simplicity of this definition. Schwab (1969) claimed that studies of curriculum were too theoretical and too distal from the lived experiences of teachers and students. Curriculum, as a construct, needed expansion to accommodate both elements of and contexts for instruction. Apple (1979) and Giroux (1981) critiqued the notion of curriculum, furthermore, as hegemonic. Because curriculum, as a tool, seeks to preserve and pass onward accumulated knowledge, they argued that it aims to inculcate entrenched thinking. Critical theorists therefore maintain that pre-defined curriculum perpetuates marginalization; ignoring this reality, they say, obfuscates its inherently political nature. Note that, in their view, curriculum does not necessarily mean textbooks or even disciplinary norms; they acknowledge the role of the so-called hidden curriculum, which includes tacit expectations of students’ conduct (e.g., Giroux & Penna, 1983; Jackson, 1968).

These two lines of inquiry—one following Schwab and the other in critical theory—initially diverged and continue to propagate. They are not wholly incompatible, however. Like the critical theorists, Schwab and his successors also recognize the inevitability of values-driven decisions in education. For example, Schwab clearly rejected the purpose of curriculum as a pure, decontextualized transfer of knowledge, describing curriculum instead as:

…what is successfully conveyed to differing degrees to different students, by committed teachers using appropriate materials and actions, of legitimated bodies of knowledge, skill, taste, and propensity to act and react, which are chosen for instruction after serious reflection and communal decision by representatives of those involved in the teaching of a specified group of students known to the decision-makers. (Schwab, 1983, p. 240)

This definition, I argue, also politicizes curriculum and instruction. Curriculum, according to Schwab (1983), constitutes “legitimated bodies of knowledge” (p. 240), which can only be validated through community-wide agreement. Classroom activities are viewed as political constructs, as well, since the propriety of teachers’ actions depends upon local values. In short, the political genesis of his conception of curriculum is evident through the attention Schwab pays to “a specified group of students” (rather than students, generally) and their adult educators, who engage in “serious reflection” (p. 240). Schwab also suggests that education should foster the tacitly-Deweyan concerns of “skill, taste, and propensity to act and react” (p. 240) within democratic society. In so doing, particularly through highlighting action and reaction, he embraces the progressive possibility inherent within curriculum.
Critical theorists, of course, go further. They advocate for upheaval and reordering, or “fighting for concrete social encounters that generate ongoing experiences which illuminate…new and transcendent ways of thinking and behaving…” (Giroux, 1981, p. 108). Put another way, a non-hegemonic curriculum emerges through dialogic interaction between teachers and students. Together, within classrooms, teachers and students should collaboratively analyze institutions and cultural norms while also striving for social change. Differing with critical theorists, Schwab (1969) generally accepted practical constraints and necessary trade-offs (p. 14). For example, public school districts must conform to state laws and regulations or face punitive consequences. “Schwabians” do the best they can, to work within the system.

**My own experiences and their influence on my scholarship.** I find myself presently straddling both revolutionary urges and the languor of convention. On the one hand, I have served as a mathematics director of a large public school district, where I endeavored to stand against some of the de-professionalizing, performativity-oriented weapons used against teachers. In turn, I also tried to address deficit-oriented language applied to students. As one example, I resisted the entrenched use of a gradual-release model (Pearson & Gallagher, 1983) and skill-and-drill pressures within my district. On the other hand, though, I have learned through my work that there are some systems that are simply intractable, because of their sheer size and might. And others are beneficial, because they offer efficiency, clarity, and consistency. Hence, I waffle, because I can appreciate both Schwabian and critical theory.

First, under my revolutionary leanings, I must profess that I generally regard gradual-release models—particularly as they were being applied in my district—as inherently oppressive. They generally presume students lack the capacities to solve mathematical problems on their own. My resistance to gradual-release also stems from my own experiences as a classroom teacher when I was—at times—effectively ordered to read from a step-by-step mathematics textbook. I was actively precluded from partnering with or designing curriculum.

Later, as a district administrator, I saw similar practices replicated throughout our schools. I talked often with exhausted and frustrated teachers who felt, largely, that decisions were being made without their input and by technocrats. District leaders didn’t understand their students, their schools, nor best practices. Specifically, teachers were tasked by school leaders to square their use of curriculum materials with the district’s gradual-release model, even if the purchased materials were not designed with such intentions in mind.
As a white cis male of largely European descent, I worry about appropriating a metaphor not meant for my use but intended mostly about me. As an administrator, though, I acknowledge despair—or, perhaps, the opportunity—expressed by Lorde (1984) in observing that “the master's tools will never dismantle the master's house” (p. 110). If I may humbly count myself an ally of the marginalized, I therefore appreciate Gutiérrez’s (2017) call for a revolution in mathematics education, one that “reject[s] domination and exploitation in various forms” (p. 18). Among these forms of oppression, undoubtedly, are curricular policies and research that “can be seen as part of a larger assimilationist agenda that attempts to standardize all children by countering heightened racial consciousness” and that implicitly assert “what is deemed best for white children is what is deemed best for African American children” and other non-white children” (Martin, 2009, p. 18). The assimilationist agenda is very possibly manifest in a variety of ways and through a variety of policies, including through standards and instructional materials. Mathematics achievement is commonly framed in such a way that privileges in-school mathematical thinking over alternative forms; some would consequently argue mathematics achievement is constructed relative to whiteness (Gutiérrez, 2008; Martin, 2009, p. 16).

I am convinced that the status quo must change, because of what I have witnessed in schools and districts: damage inflicted, daily, upon teachers and students via overly-performative systems of control and comparison. Burnout is rampant. It is no wonder, to me, that several prominent strikes have occurred in recent years (e.g., Goldstein, 2019). Schwab (1969) might attribute cause to an observation he made decades ago, namely, that education is experiencing “a translocation of its problems and the solving of them from the nominal practitioners of the field to other[s]” (p. 3). In particular, economists, businesspeople, and thinly-credentialed administrators are increasingly serving as school leaders; these leaders, I find, fundamentally lack the requisite training and experience to make decisions that would promote teachers’ growth, while maintaining respect for the nobility of the profession and those who practice it.

By the same token, though, I also appreciate Schwab’s (1969) invocation of “the practical arts” to address educational crises, accepting “that existing institutions and existing practices be preserved and altered piecemeal, not dismantled and replaced” (p. 14). For me, Schwab represents the other side of the coin, wherein I see value in deferring to convention. Preservation of existing practices and institutions is necessary in Schwab’s view, because of “patterns of purposed action” in a number of fields that must “retain coherence and relevance to one another” for their
continued impact (p. 14). As an example of a field retaining a deeply practical bent, Schwab offers the law. Overturning precedent causes confusion and would render the entire system moot.

Curriculum materials, to a degree, are like case law: in well-designed materials, the activities and lessons have been tested, refined, and applied across contexts (Remillard, 2016). Admittedly, then, my own scholarship reflects these crossroads: I certainly believe teachers deserve more autonomy and that heavy reliance on standardized testing and routinized practices, like gradual release, can be damaging. But I also believe teachers may benefit from additional support in making use of instructional materials. After all, there are a number of high-quality programs in today’s marketplace, designed with teachers’ own goals in mind. Such programs also represent the collective wisdom of the profession and scientific, deliberative inquiry.

Participating with texts: Structure along with freedom. I therefore maintain that Schwab’s (1983) definition of curriculum, beautifully perambulatory, retains knowledge-transfer goals while also creating space for autonomy. That space is signaled by Schwab’s language: interactions fostered by “committed teachers” and reactions of their students while using “appropriate materials…chosen for instruction after serious reflection” (p. 240). To be sure, Schwab acknowledges the potential confinement of curriculum, beholden to and encompassed by the systems of which it is a part, but he nonetheless retains hope for democratic participation and individual agency within its bounds. “Curriculum,” he writes, “will deal badly with its real things if it treats them merely as replicas of their theoretic applications” (Schwab, 1969, p. 12). Stated differently, the curriculum:

...will be brought to bear not in some archetypical classroom but in a particular locus in time and space with smells, shadows, seats, and conditions outside its walls which may have much to do with what is achieved inside. (Schwab, 1969, p. 12)

Schwab (1969, 1983) undoubtedly embraces the diversity of students, teachers, and thought. I, too, embrace this complexity, and I also reject the notion that the only available paradigm is teachers’ choice to either follow or subvert curricular texts (Remillard, 2005).

Instead, like Remillard (2005), I value what teachers and materials each bring to the enterprise of instruction. Again, this has been described as a participatory lens on curriculum-use. At the same time, researchers note that curriculum materials are not always, and have not always been, thoughtfully designed (Remillard, 2005; Stein et al., 2007). Participation with texts implies, necessarily, that teachers modify curricular guidance. Participation also stems from the theory of mutual adaptation, which Berman and McLaughlin (1978) describe as a “step-by-step fine
tuning” of any school-based intervention that must account for “idiosyncratic teachers’ and situational characteristics” (p. 17). The very complexity of participating with texts is what brings me to this work.

Indeed, I argue that what participation means must be addressed, more concretely, to settle controversies and handle challenges with using curriculum materials in schools. Historically, and even today, teachers are instructed to, both, follow the text faithfully while also making adjustments for meeting students’ diverse learning needs (e.g., Huntley & Chval, 2010). Schwab (1969) would argue that such modifications, because they must address such complex needs and divided goals, therefore demand a “commitment to deliberation.” He describes this as a “complex and arduous” process that “requires consideration of the widest possible variety of alternatives” (Schwab, 1969, pp. 20–21). Regardless, understanding how teachers participate with texts, and how to offer guidance that supports their participation, represents relatively nascent work within the field.

Other Purposes of Curriculum Materials: Opportunities to Learn

While I may think of teachers’ relationship with curriculum materials a participatory one, not all researchers have the same conception. Therefore, in what follows, I review research that necessarily reflects a variety of perspectives on curriculum-use. Regardless of perspective, I nonetheless presume that curriculum materials offer OTLs for teachers and students, which are de facto artifacts of authors’ intended purposes. To ground my own analysis of curriculum materials and enacted lessons, I turn to reviewing descriptions of OTLs found within the research literature.

General opportunities to learn: Focus, coherence, and rigor in the CCSS-M. First, the writers of the CCSS-M argue that “the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country” (NGA Center & CCSSO, 2010, p. 3). They also argue for increased rigor. Note that, here, the term curriculum generally refers to what Remillard and Heck (2014) call the official curriculum. While the writers of the CCSS-M undoubtedly hope that curriculum authors will take up the charge, they also acknowledge that the standards themselves do not represent a curriculum—in the traditional sense—because they do not specify learning activities, sequences of problems and the like (NGA Center & CCSSO, 2010, p. 5; Zimba, 2012, p. 3).

I define these three terms—focus, coherence, and rigor—below, but I first contend they represent three generic OTLs. Originally, OTLs described whether students had been taught the
topics being assessed (Floden, 2002; McDonnell, 1995). (See also Elliott & Bartlett, 2016, for a review.) Ben-Peretz (1990) discusses a similar term, *curriculum potential*, to refer to the oft-tacit features of materials that shape cognitive engagement. Recently, OTLs are thought to include an expanded set of structures—situated within school environments—for promoting the likelihood of students’ learning (Greeno, 2006; Greeno & Gresalfi, 2008). These generally involve features of interactions within nested levels of context. It is commonly believed that these expanded OTL structures also exhibit differences in nature and have varying levels of quality (Hiebert, 2003). Nonetheless, as researchers note, “Opportunity to learn is widely considered the single most important predictor of student achievement” (NRC, 2001, p. 334).

I now return to focus, coherence, and rigor. *Focus* generally refers to richer coverage of fewer topics. And rigor does not mean mathematics that is more difficult, but rather reflects the pursuit of “conceptual understanding, procedural skills and fluency, and application with equal intensity” (NGA Center & CCSSO, 2019, para. 2). *Rigorous instruction* enables a deep command of mathematical ideas and tools.

I also offer a brief note, here, on coherence. Unlike focus—and, to a degree, even, rigor—*coherence* is more cumbersome to describe. The CCSS-M authors admit that “assessing the coherence of a set of standards is more difficult than assessing their focus” (NGA Center & CCSSO, 2010, p. 3). Regardless, the CCSS-M defines *coherence* as, first, a “sequence of topics and performances that are logical and reflect, where appropriate, the sequential or hierarchical nature” (Schmidt et al., 2002, p. 9, as cited in NGA Center & CCSSO, 2010, p. 3). Coherent instruction would, for instance, address two-digit, whole number multiplication before considering multiplying integers. Second, coherence in the CCSS-M also describes a set of “deeper structures” that “serve as a means for connecting the particulars” (Schmidt, Houang, & Cogan, 2002, p. 9, as cited in NGA Center & CCSSO, 2010, p. 3). Coherent instruction, framed in this way, would also relate the study of whole numbers in elementary grades to more-general principles involving rational numbers.

Traditionally, U.S. mathematics instruction has placed most of its eggs in the basket of procedural skill (Stigler & Hiebert, 1999, 2009). These reform-oriented goals, embodied by the CCSS-M instructional shifts, together intend that U. S. students attain what Skemp (1978 / 2006) describes as a *relational*—rather than an *instrumental*—understanding of mathematics. Having instrumental understanding involves knowing *how* to follow procedures but little more. It is pure procedural skill. Relational understanding, on the other hand, involves deeper understanding of
mathematical relationships and justifications for mathematical procedures (a conceptual form of understanding). It also involves fluently utilizing the appropriate tools to address specific mathematical situations. Therefore, embracing both connections and deeper structures, relational understanding also reflects Schmidt’s and colleagues’ (2002) conception of coherence. And coherent understanding, conversely, necessarily exhibits elements of rigor and focus. Therefore, focus, coherence, and rigor are somewhat interwoven.

As a brief aside, while still early in its tenure, many regard implementing the CCSS-M as an improvement on what came previously. These standards and underlying principles are thought to add much-needed consistency to the seemingly haphazard expectations and terminology offered by states themselves (Friedberg et al., 2018, p. 4). Some research has identified the positive impact of CCSS-M implementation (e.g., Schmidt & Houang, 2012). At the same time, states continue to tinker with the CCSS-M (e.g., Disare, 2017; Friedberg et al., 2018). The standards have been critiqued, as well, for bolstering the use of high-stakes assessments and for failing to take into consideration the breadth of research on child-development (e.g., Supovitz & Reinkordt, 2017; Ravitch, 2018). See Loveless (2014) for a review of the ongoing debate about the efficacy of the CCSS-M.

**Detailed OTL frameworks.** There are other, more specific descriptors of OTLs in mathematics instruction. I review this literature below. First, I summarize several noteworthy studies of OTLs that also offer implications for students’ learning. Afterwards, I concentrate on describing one particular framework for understanding OTLs that is relatively new but shows promise in explaining learning outcomes that follow from teachers’ uptake of curricular guidance.

**Key examples of OTL frameworks.** In an important study, Tarr and colleagues (2008) conceptualized the integrity of teachers’ use of mathematics programs by collecting fine-grained data on their topic-coverage, planning approach, and homework assignments. These were assessed through logs, completed by teachers, as well as classroom observations. Collectively, these factors represented OTLs found within curriculum materials. In addition, Tarr and colleagues also recorded whether teachers’ practices reflected the espoused, reform-oriented goals of the NCTM. These included students’ opportunities to make conjectures, justify their reasoning, explore multiple solution strategies, and attend to concepts. Tarr and colleagues described these goals, if achieved, as constituting a standards-based learning environment (SBLE). SBLEs, therefore, also represent OTLs within materials—expectations that are often tacit and convey a
sense of the authors’ desired pedagogy. Later, I explore the results of this investigation by Tarr and colleagues on these OTLs and students’ achievement.

Remillard (2012) draws on film studies to classify what she calls \textit{forms of address} (p. 108) found within curriculum materials. These forms, Remillard argues, represent “a set of design considerations and decisions that are not always made explicitly” (p. 110), and yet are nonetheless “relevant to how teachers engage and utilize resources” (p. 110). Remillard’s forms of address consist of five “interrelated categories”: look, structure, voice, medium, and genre. Defining each of these in detail goes beyond the intended scope of this review; however, I briefly mention three that I speculate contribute to the narrative elements of the text. Structure, first, refers to “how the resource is organized and what it contains” (p. 110) or the components and topics of the text. Voice refers to “how the authors or designers are represented and how they communicate with the teacher (Love & Pimm, 1996)” (p. 112), which includes the pronouns and ways materials talk to or talk through teachers (Remillard, 1999). Finally, genre expresses a host of familiar conceits that convey “both objectively given structures (what can be seen) and subjective schemes (ways of being understood or expectations upheld about them)” (p. 113) that connote the sense of an organizing theme. For example, Remillard explains, reform-oriented curriculum materials might represent a sub-genre within the broader genre of instructional texts. Together, forms of address relate to the ways that teachers engage with materials, including what they read and what they read for (i.e., what they are seeking to learn). In reading, ultimately, teachers discern the OTLs they subsequently translate into enacted learning experiences for students.

S. Brown and colleagues (2009) also describe a set of OTLs identified within a particular NSF-funded program, \textit{Math Trailblazers}. They conceptualized and identified OTLs, through a textual analysis, noticing “specific statements about the ways in which students were expected to engage in the activities and content” (S. Brown et al., 2009, p. 337). These statements fell into two broad categories: opportunities to reason about mathematics and opportunities to communicate about mathematics (S. Brown et al., 2009, p. 337). Under reasoning opportunities, Brown and colleagues include cues within written lessons, like offering students chances “to explore how to use a tool, representation, or strategy while solving problems,” to “develop their repertoire of strategies,” to “compare and make connections across tools, representation [sic], or strategies,” or

to “evaluate the logic of strategies or the reasonableness and accuracy of solutions” (p. 394).
Under communication opportunities, they include such cues as chances to “describe to peers, the
teacher, or in writing their use of tools, representations, or strategies,” to “respond to, explain, or
question another student’s approach to a problem,” to “refine their explanations,” or to
“generalize across problem situations” (pp. 394–395). Brown and colleagues also describe an
additional OTL of Math Trailblazers, even though they don’t describe it as such: the step-by-step
instructions for implementing classroom activities (they call this the literal curriculum). Later in
this chapter, I describe the findings of this study in greater detail, but I note here that these
researchers report that curricular guidance—around the literal curriculum and the intended
OTLs—both hindered and supported teachers’ lesson implementation.

Another notable framework: OTLs and ease-of-transition. An important follow-up
investigation by Remillard, Agodini, and Harris (2014) sought to explain differential learning
outcomes across curriculum programs in a large-scale studied published several years earlier. The
prior study was reported by Agodini, Harris, Atkins-Burnett, Heaviside, and Novak (2009), as
well as Agodini, Harris, Thomas, Murphy, and Gallagher (2010). Building on this prior research,
onetheless, Remillard and colleagues (2014) connected textual OTLs and what they generally
described as teachers’ ease-of-transition in using new materials. OTLs were assessed as not only
mechanisms for students’ learning about mathematics, but also factors in teachers’ understanding
of the programs themselves.

In their follow-up analysis, Remillard and colleagues (2014) sought to explain differences in
students’ achievement by identifying potentially-mediating factors within the programs’ designs. I
describe the details of Remillard’s and colleagues (2014) findings with regard to students’ learning
in the next section of this chapter. Regardless, first, they refined an analytic approach used by
Remillard and colleagues (2011) and found several dimensions of OTLs within these four
programs. These dimensions broadly describe programs’ (a) mathematical emphasis, (b)
instructional approach, and (c) supports for teachers. Remillard and colleagues (2014) note that,
even though “their importance has not yet been demonstrated” (p. 739) by rigorous empirical
study, these OTL dimensions varied significantly across the four programs. Therefore, because of
other factors that were controlled within original study, these dimensions likely represent key
factors to consider in relation to teachers’ curriculum-use. Because of their potential importance,
particularly in the context of my own study, I review the elements of their OTL dimensions here.
Remillard and colleagues (2014) described programs’ mathematical emphasis through their analysis of the cognitive demand of tasks (Stein et al., 1996), attention to routines that reinforced concepts and skills, and finally, focus on procedural fluency. Cognitive demand generally depicts the nature of the thinking expected of students during a mathematical activity. Stein and colleagues offer the following typology of cognitive demand: (a) memorization and reproduction of facts or definitions (memorization), (b) performing procedures without reference to underlying conceptual connections (procedures without connections), (c) performing procedures while meaningfully connecting to underlying concepts (procedures with connections), or (d) explaining or justifying approaches to problems that exhibit multiple possible answers or solution pathways (doing mathematics). Next, classroom routines suggested by programs were characterized by their expected frequency, length, and nature. Last, reinforcement of skills and concepts were analyzed by the suggested frequency and length of such opportunities, in addition to the particular mechanisms in question (quick verbal practice, worksheets, focus on arithmetic facts, focus on explanations, etc.).

The instructional approach of a program was characterized by analyzing the texts’ implied instructional model, as well as the perceived role of the teacher and expected classroom interactions. Instructional models, for one, were deemed either dialogic or direct, depending upon how they embraced principles of ambitious instruction (e.g., Franke et al., 2007) or, contrastively, positioned the teacher as the source of knowledge. Next, teachers’ expected roles—as cued within the texts—were described as “modeling or showing, explaining, guiding, and facilitating” (Remillard et al., 2014a, p. 742). Finally, the nature of classroom interactions described whether students were primarily expected to engage with the teacher, each other, or the text itself.

Last, the ways in which texts offered supports for implementation of written lessons were analyzed by assessing the nature of text-based scripts that guide teachers’ actions (Remillard & Reinke, 2012) and other types of educative content offered (Davis & Krajcik, 2005). Scripts were identified as explicit or descriptive, respectively, depending on whether texts specified the exact language for teachers to say or write (or exactly what they should do) or whether they suggested possible actions (Remillard & Reinke, 2012). Moreover, the various ways in which texts speak to teachers (Remillard, 1999) were considered by noting elements that Davis and Krajcik (2005) would classify as communicating: the program’s design rationale, mathematical concepts (for teachers’ learning), aspects of students’ thinking, and possible adaptations of classroom activities.
Summary and discussion: Coherence of instruction as an important OTL. In this section, I have summarized theoretical and empirical literature on the definition, role, and nature of curriculum and curriculum materials. In so doing, I have situated this research within a broader notion—the idea that curriculum offers “both far too much and far too little” (Ben-Peretz, 1990, p. vii). This conundrum is particularly relevant within classrooms where teachers aspire to take up reform-oriented or ambitious instruction (Davis et al., 2011; Remillard, 1999, p. 329). Because materials cannot specify everything they need, teachers necessarily interpret curricular guidance and serve as designers of curriculum (Ben-Peretz, 1990; Remillard, 1999). Remillard (2005) expands on these ideas by arguing we should consider the ways in which teachers participate dynamically with texts. Remillard’s perspective emphasizes the contributions not only of teachers, but also the guiding influences of the text, for understanding how episodes of learning are structured and implemented.

In this regard, broadly-speaking, curriculum materials can be seen as offering opportunities to learn (OTLs) through teachers’ enactment of written lessons. It is generally believed that teachers’ reading and sense-making of OTLs determine the influence of curriculum materials on both how much and, perhaps more importantly, what students learn (Stein et al., 2007). By what students learn, I refer to the nature of mathematical knowledge gained: instrumental or relational (Skemp, 1978 / 2006). Although I have alluded to studies on the impact of curriculum materials on students’ achievement, I have not yet reviewed this important area of research in great detail. I note that, even still, specific OTLs have not been rigorously tied to learning outcomes (Huntley & Heck, 2014; Remillard & Kim, 2017; Remillard et al., 2014a). Further, I have yet to summarize the various factors within Remillard’s and Heck’s (2014) framework that influence how teachers interpret and utilize curricular guidance. These are important considerations for contextualizing not only my focus on curriculum materials as resources for teaching, but also for understanding the ways that materials conceptualize, support, and potentially influence the coherence of mathematics instruction. I tackle these tasks in the next section of this chapter.

I explained in Chapter 2 that coherence is an important element of mathematics instruction. As one of the key OTLs, however, it remains under-specified within the extant literature. In the absence of much-needed clarity, ad hoc approaches have emerged for describing and assessing coherence within both materials and instruction. For example, consider the definition offered by
an influential organization promoting reform, EdReports.org (n.d.). Within their assessment framework, used to evaluate the quality of curriculum materials, coherence is conceptualized, broadly, as whether or not “the sequence in which the topics are covered [is] consistent with the logical structure of mathematics” (EdReports.org, n.d., p. 4). As I noted, previously, an author of the CCSS-M explains that discerning this logical structure is difficult and open to interpretation (Zimba, 2011, 2012; cf. Lampert, 2001).

Further, the EdReports.org (n.d.) assessment framework has other, potential methodological flaws, such as commingling indicators of coherence with elements of focus and using under-defined and somewhat circular terminology (see, e.g., p. 5, Criteria 1.d. and 1.e.ii.). One well-publicized position paper therefore cautions that EdReports.org offers “current ratings and reviews [that] do not provide the types and quality of information needed to make informed choices about the extent to which particular materials support students’ learning, or teachers’ teaching…” (NCTM & National Council of Supervisors of Mathematics [NCSM], 2015, p. 1).

As I endeavored to explain in Chapter 2 and expand upon here, one of my central concerns is the relationship between coherence and teachers’ participation with curriculum materials. Materials are, of course, thought to offer coherence (Freeman & Porter, 1989; Remillard & Taton, 2015; Remillard, 2016). I therefore regard coherence as a broad-based OTL, embedded within instructional materials. This is predicated on the converse, that incoherence is a potential barrier to learning (Kaplinsky, 2019). As I argue above, since the meaning or nature of coherence remains unclear, several consequential implications arise. For example, without this clarity, how teachers might recognize the coherence of materials is questionable. Further, without this clarity, understanding teachers’ adaptations becomes more challenging; put differently, there would be a murky border of what Ben-Peretz (1990) calls the curriculum envelope, or adaptations consistent with the authors’ intentions. More specifically, if—as many seem to believe—coherence is characterized by a strict linear progression of mathematical ideas, then how could teachers make reasonable curriculum adaptations without disrupting the intended pathway?

3–2. Does Curriculum Influence Student Learning? What Factors are Involved?

At the beginning of Section 3–1, I observed that from 1990–2007 the NSF invested more than $100 million of taxpayer money to develop mathematics curriculum materials (Hirsch, 2007). 

1See Stein et al. (2007) for a review of other approaches to evaluating curriculum programs.
Also noted previously, the impact of curriculum materials on students’ achievement has been mixed (Ball & Cohen, 1996; NRC, 2004; Polikoff, 2015; Stein et al., 2007). At the same time, M. Brown (2009) and others have theorized that curriculum materials are still an important resource for teachers in designing classroom instruction. As Brown explains, teachers offload at least some of work of developing, implementing, and testing classroom tasks onto instructional materials. Therefore, to ground future discussion on the potential influence of instructional resources—particularly how teachers review and take up guidance on coherence—I summarize findings on the relationship between materials learning outcomes. These findings, it should not go without saying, do not consistently draw on the interpretive or participatory lens with which I framed the previous section. Reviewing these findings and their underlying methodologies, nonetheless, sheds light on the key ideas within the field. Therefore, such findings merit exploration. Further, since a number of variables mediate curriculum and learning, as indicated by Remillard’s and Heck’s (2014) framework, I conclude this section by describing this affiliated research.

**The Impact of Curriculum Materials on Learning**

Studies have demonstrated that curriculum materials play a role in students’ achievement, but characterizing this role is a complex undertaking. Below, I summarize prominent empirical studies on the impact of curriculum materials on learning. Many of these aim to relate specific programs to students’ results on standardized tests. In so doing, the researchers’ fundamental assumption is that curriculum materials are themselves responsible for differences in achievement. The previous section should have made clear that this assumption is problematic. It fails to account for a latent variable—teaching (Hiebert, 2003; Schoenfeld, 2006). Recognizing this insufficiency, other researchers have therefore called for ongoing study of teachers’ use of curriculum materials during instruction (Huntley & Heck, 2014; Remillard & Kim, 2017; Remillard et al., 2014a; Stein et al., 2007).

**Empirical research on program efficacy.** In 2003, Senk and Thompson published a volume on the impact of NSF-funded curriculum programs on students’ learning. (Prior to that point, studying curriculum programs and achievement was not a significant line of inquiry.) This volume provided several reasons to be optimistic. For example, in a chapter by Putnam (2003) that reviewed analysis of four commonly-used, NSF-funded elementary programs, the author stated:

Students in these new curricula generally perform as well as other students on traditional measures of mathematics achievement, including computational skill, and they generally
do better on formal and informal assessments of conceptual understanding and ability to use mathematics to solve problems. (p. 161)

Other reviews of research on NSF-funded programs offered similar conclusions (Chappell, 2003; Swafford, 2003). Stein and colleagues (2007) note, however, that none of this research “was without flaws” (p. 336). For instance, in a number of cases, such studies were conducted by the authors of the programs themselves. At that time and even into the late 2000s, research by external, independent reviewers remained scarce. I therefore update the summary by Stein and colleagues, below, by reviewing more recent findings of curriculum implementation.

Methodological comments. First, though, I offer brief comments on the methodologies for conducting this sort of comparative research. As Stein and colleagues (2007) observe, the approach that is considered the so-called “gold standard” involves randomized, controlled treatments (RCT) to account for the influence of various confounding variables. These include differences in teachers’ professional development, students’ prior achievement, school-based contextual factors, and—perhaps most importantly—differences in implementation.

Such studies are, however, notoriously difficult to complete; consequently, at the time of their review, Stein and colleagues (2007) could not identify many reports meeting this standard. They reviewed two papers that favorably compared novel programs to more traditional ones, and they described two others that also compared NSF-funded programs with traditional materials but obtained no conclusive results (Stein et al., 2007, pp. 337–338). Aside from RCTs, other empirical approaches include the use of convenience sampling accomplished through surveys or other means. Some studies use convenience sampling, which does not rise to the gold-standard level, but also try to gather additional implementation data on curriculum programs and students’ learning. Such data is collected to bolster, or triangulate, results. Empirical studies, employing all of these quantitative approaches, are reviewed below.

Before proceeding, however, I revisit one study and describe another—both not employing an RCT—that still offer compelling findings. As indicated above, research by Tarr and colleagues (2008) demonstrated that teachers using particular reform-oriented programs were more likely to establish a SBLE in their classrooms. Further, when combined with a high-level SBLE, reform-oriented programs were more likely to yield increases in conceptual understanding and problem-solving ability over more traditional programs. In a similar fashion, Boaler and Staples (2008) conducted a longitudinal, mixed-methods study of three California high schools. This was not an RCT study, either, but involved the collection of large amounts of implementation-related data.
They found that students whose teachers used a reform-oriented program—as well as classroom practices that were generally aligned with authors’ pedagogical intentions—not only learned more (and different, conceptually-oriented) mathematics but also enjoyed mathematics more.

**Conflicting results of large-scale quantitative studies.** In the previous section of this chapter, I summarized a large-scale RCT by Agodini and colleagues (2009, 2010) that likewise demonstrated curriculum materials measurably impact students’ learning. The research team found that, even after the first year of implementation, certain programs raised percentile scores by as many as 12 points for an average student. At the same time, these results were inconsistent across grade-levels. Four programs had been randomly assigned to schools within twelve, diverse districts. These were: *Scott Foresman – Addison Wesley Mathematics (SFAW)*, *Math Expressions*, *Saxon Math*, and *Investigations*. After first grade, students using *Math Expressions* outperformed those using *Investigations* and *SFAW*; after second grade, students using *Math Expressions* and *Saxon Math* outperformed those using *SFAW*. In addition, teachers were found to have implemented the written lessons generally as intended. The results, therefore, could not be explained by teachers’ variable uptake of the guidance offered by these programs.

In a similar, but longer-term, study of first- to third-grade teachers in Indiana, Bhatt and Koedel (2012) analyzed the impact of three curriculum programs on learning (*Saxon Math, SFAW*, or *Silver Burdett Ginn Mathematics [SBG]*)? They found significant differences in students’ performance on state tests. Using a sophisticated inference model, they found that both *SBG* and *SFAW* statistically outperformed *Saxon Math*. Of course, these results stand in contrast to those found by Agodini and colleagues (2009, 2010). Bhatt and Koedel (2012) speculate that this seeming contradiction is explained by the different populations of students within each study.

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7 All four programs were found in more than one-third of U.S. Grade K-2 classrooms at the time of their study (Resnick et al., 2010, as cited in Agodini et al., 2010, p. 9). All are still in use in U.S. schools and districts, as of the writing of this thesis. The relevant citations are:


8 *SBG* was published by Silver Burdett Ginn until it was purchased by Pearson. Pearson subsequently “phased out *SBG* in favor of *SFAW*** (Bhatt & Koedel, 2012, p. 403). The relevant citation is:

This general approach was mirrored in a study conducted with schools in Florida (Bhatt, Koedel, & Lehmann, 2013). One notable difference, here, is that the researchers also measured the performance of curriculum materials on various sub-topics of mathematics with first- to third-grade students. They found that one program, *Harcourt Math*, was affiliated with stronger results on the Florida state assessment, both overall and on two of the five mathematics domains (data analysis and geometry). Results on the other domains were indistinguishable from those obtained by six other traditional and reform-oriented curriculum programs used by Florida teachers at the time data was collected.

Polikoff and Porter (2014) took a different approach, trying to tie content coverage with measures of quality instruction, presuming that both together would produce greater achievement by students. They were unable to discern reliable associations, however, between these constructs. They also used the standards as images of curriculum materials, rather than studying the instructional resources themselves. Polikoff and Porter (2014) consequently speculate that methodological limitations may have contributed to the unexpectedly small correlations observed (e.g., bias in the samples of teachers or lack of sensitivity in assessments). But even after addressing possible sources of bias, a clearer explanation of their results still failed to emerge.

Another oft-cited paper, by Kane, Owens, Marinell, Thal, & Staiger (2016), reports that elementary and middle grades students—sampled across five states—fared better on state standardized tests with *Go Math!* than with other programs. To obtain these results, Kane and colleagues analyzed surveys of teachers who named the curriculum programs they used most often. The team then studied the relationship between the five most popular programs and the results of state standardized tests. They found that *Go Math!* students statistically outperformed others by as much as 0.25 standard deviations.

*General lack of data.* Nonetheless, studies with results like these—demonstrating the relative efficacy of specific mathematics programs using rigorous methods—are relatively rare. Some researchers blame a lack of usable data to conduct meaningful analysis. For example, Koedel, Li, Polikoff, Hardaway, and Wrabel (2017) note that U.S. states and local districts generally neglect to maintain and publish records of curriculum programs alongside schools’ demographic information. Better data-collection systems, Koedel and colleagues therefore argue, would

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promote better opportunities for researchers to replicate findings on program impact, using statistical methods that have been deemed well-established.

To bolster their argument, Koedel and colleagues (2017) describe one notable exception—a 2004 court decision in California that required publication of textbooks in each public school. Using this data, Koedel and colleagues consequently studied the long-term impact of four programs used in third- to fifth-grade classrooms in California: California Math, California Mathematics: Concepts, Skills, and Problem-Solving, California HSP Math, and Scott Foresman – Addison Wesley enVisionMATH California\(^\text{11}\). Of these four, using a similar model to that of Bhatt and Koedel (2012), Koedel and colleagues (2017) showed that students whose teachers used California Math scored significantly higher on the state standardized test. (An average student would have improved some six percentile points, and perhaps more—over a longer duration of exposure—with California Math.) Once again, however, the results varied across grade-levels. For example, the fourth-grade impact was mostly small and not statistically-significant. These differences, they consequently observe, are perplexing. Describing the limitations of their study, Koedel and colleagues therefore acknowledge that trying to “link specific textbook characteristics” to learning outcomes is, nevertheless, “currently hampered” by “too many potential explanatory factors” (p. 14).

**Critiques of fidelity-oriented studies.** In general, I characterize the RCT and quasi-RCT sorts of empirical studies as fidelity-oriented. They assume teachers implement programs, more or less, as the developers intend (i.e., with fidelity). Further, as Koedel and colleagues (2017) admit, researchers taking this sort of approach are sometimes thwarted by the multitude of potential factors that contribute to findings (p. 14). After all, the discussion in the previous section of this chapter sought to undercut the simplistic assumption that differences in program design translate neatly into measurable differences in learning (cf. Remillard, 2005; see also Stein et al., 2007; Remillard & Heck, 2014). Regardless, fidelity-oriented studies hold weight with a number of constituencies. Some educational policy analysts suggest that, as relatively inexpensive

\(^{11}\) The relevant citations are:
instruments of change, school and district leaders should rely on fidelity-oriented studies and prioritize the selection of curriculum programs deemed impactful, or otherwise better aligned to standards and assessments (e.g., Chingos & Whitehurst, 2012). Freeman and Porter (1989) suggest that teachers, themselves, should be held more accountable for following textbook content.

Such simplistic proclamations overlook the fact that programs are not “self-enacting” (Stein et al., 2007; Remillard & Taton, 2015). Ball and Feiman-Nemser (1988) observe, similarly, “Teaching well even from a highly prescriptive curriculum is also more complicated than many seem to appreciate” (p. 420). In contrast to many education-reform initiatives, then, distributing teacher’s guides for instructional change is a strategy doomed to failure. Doing so fails to acknowledge several key factors: not every program is suited to every environment (Stein & Kim, 2009), teachers must learn about the programmatic components and resources (Remillard & Taton, 2015), teachers’ beliefs and orientations may not align with a given program (Chávez-López, 2003; Remillard & Bryans, 2004), many other capacities are needed to enact effective instruction (Tarr et al., 2008), and even the design of a program may, itself, constitute a barrier to implementation (Ball & Cohen, 1996; S. Brown et al., 2009; Davis & Krajcik, 2005; Manouchehri & Goodman, 1998, 2000; Remillard et al., 2014a; Stein & Kim, 2009). With regard to the latter, for example, Manouchehri and Goodman (2000) call for “in-depth” guidance within texts to promote teachers’ meaningful reflection “about the content, about the possibilities for connections, and about pedagogical practices conducive to effective implementation” (p. 31). I address some of these other factors in greater detail, momentarily. In short, programs regarded ineffective might, instead, be more challenging for teachers to learn (Stein & Kim, 2009; Remillard et al., 2014).

Furthermore, even tightly-constructed empirical analyses contain potential methodological flaws. For example, in the study by Kane and colleagues (2016), only 24% of teachers surveyed had been using their current program for three or more years; further, 41% said they had switched to a new program within the last two years (including during the year of the study). Neither professional development, nor teachers’ implementation of programs, was considered. Therefore, the strong performance of Go Math!, alone, might have been an artifact of other considerations that degraded the potential, relative impact of other programs. Go Math! might have represented a unique case. In addition, Polikoff (2015) notes that such studies presume alignment between standards and the written materials (since assessments of learning are often designed in
accordance with standards). In a survey of fourth-grade programs, however, Polikoff identified significant differences between the CCSS-M and the instructional texts.

This sort of complexity, regardless, is what led the NRC (2004) to caution against fidelity-oriented research. The NRC observed that, while it may seem “deceptively simple” to determine a program’s effectiveness with a “well-designed study”:

…many factors make such an approach difficult. Student placement and curricular choice are decisions that involve multiple groups of decision makers, accrue over time, and are subject to day-to-day conditions of instability, including student mobility, parent preference, teacher assignment, administrator and school board decisions, and the impact of standardized testing. This complex set of institutional policies, school contexts, and individual personalities makes comparative studies, even quasi-experimental approaches, challenging, and thus demands an honest and feasible assessment of what can be expected of evaluation studies (Usiskin, 1997; Kilpatrick, 2003; Schoenfeld, 2002; Shafer, in press). (p. 96)

In their report, the NRC also notes that “comparative evaluation study is an evolving methodology” (p. 96) and concludes that the field “could benefit from careful synthesis and advice in order to increase its rigor, feasibility, and credibility” (p. 97). Stein and colleagues (2007), likewise, note that “student achievement in mathematics cannot be predicted solely by the type of curriculum used” (p. 359), which they also say, “points to the fallacy of assuming that the materials themselves are the primary agent in shaping opportunities for student learning and instead uncovers the important role played by the interpretive and interactive influences of teachers and students” (p. 323).

In the decade-plus since the NRC report and the review by Stein and colleagues (2007), little has changed. Studies have yet to demonstrate, conclusively, the relative impact of curriculum programs on learning—at least, in part, because of the manifold considerations that must be taken into account (Remillard & Heck, 2014; Stein et al., 2007). Again, additional frameworks for and studies of teachers’ uses of curriculum materials are still needed (Huntley & Heck, 2014; Remillard & Kim, 2017). Many programs explicitly offer a host of choices to teachers (Remillard & Reinke, 2012). And the advent of the digital marketplace means that teachers rely even less on a single set of resources when enacting lessons (Remillard & Reinke, 2017). These, collectively, complicate the enterprise of conducting fidelity-oriented studies.

In sum, Remillard and Taton (2015) observe that curriculum programs are useful tools for teachers. More specifically, Remillard and Taton (2015) explain that curriculum materials “provide guidance on the day-to-day decisions on what should be taught, in what sequence, and,
in many cases, how,” and so, “curriculum programs are critical to maintaining coherence across schools and classrooms” (p. 55). Stated differently, curriculum materials undoubtedly “provide teachers access to knowledge and processes beyond their immediate experience” (Remillard & Taton, 2015, p. 58). The success of instructional materials, however, “depends on the capacity of those using” them (Remillard & Taton, 2015, p. 58). (See also Ball & Feiman-Nemser, 1988.) Teachers must also be able to interpret and accommodate the guidance offered by curriculum authors (Ben-Peretz, 1990; M. Brown, 2009; Remillard, 2005) in ways that avoid lethal mutations (Brown & Campione, 1996). Because instructional materials, themselves, aren’t simple levers for fostering learning, I now turn to outlining research on key mediating factors.

**Factors Influencing Curriculum-Use: How Teachers Participate with Curriculum Materials**

The preceding aimed to demonstrate that judging the efficacy of a curriculum program must account for teaching. A program could be deemed ineffective, mistakenly, because teachers may not have been prepared to use it or the surrounding conditions may have been unfertile (Davis et al., 2011; Tarr et al., 2008; see also Remillard & Heck, 2014; Stein et al., 2007). Familiarity and comfort with a program certainly impact its use (see, e.g., Choppin, 2011; Remillard & Bryans, 2004). On the other hand, a program considered effective might, in fact, under-prepare students on particular topics (Bhatt et al., 2013) or fail to support conceptual understanding (Tarr et al., 2008; Boaler & Staples, 2008). Stein and Kim (2009), therefore, pose the following rhetorical question: “Rather than asking which program is better, we ask: which program is best suited to which conditions?” (p. 52). And we cannot overlook a critique offered by Polikoff and Porter (2014) who suggest that standardized tests might not be sensitive enough to distinguish the quality of instruction in relationship to instructional materials.

Acknowledging that claims about learning must be accompanied by considerations of curriculum-use, as explained above, I now review studies of teachers’ curriculum adaptations. Remillard (1999) explains that such adaptations occur either before instruction (in what she calls the *design arena*) or during instruction (in the *construction arena*). In the construction arena, teachers interpret not only the guidance of curricular texts but also, iteratively, the responses of their students (pp. 328–332). Remillard (1999) notes, further, that additional challenges are faced by teachers who aspire to maintain reform-oriented learning environments. In such classrooms, students’ unpredictable voices are supposed to take center stage, and so teachers must continually adjust activities on-the-fly (Remillard, 1999, p. 329). I explain, later, that Sherin and Drake (2009) expand upon Remillard’s (1999) *curriculum mapping framework* by including prospective
adjustments to curriculum once instruction has been completed. These two frameworks matter, I believe, because they help contextualize the phases when adaptations take place. Adaptations that occur prior to instruction, as I subsequently detail, are more likely to follow from teachers’ characteristics and their broad working contexts; adaptations that occur during instruction, are influenced by materials, as well as students’ characteristics and their reactions to teaching. Teachers’ subjective valuations, in the moment, certainly also play an crucial role (Schoenfeld, 2011).

Identifying and classifying teachers’ modifications of curriculum materials. Here, I summarize key findings on the ways teachers adapt materials. Afterwards, I address research on why teachers make certain adaptations.

To begin, Ball and Feiman-Nemser (1988) worked with pre-service teachers, who conveyed an entrenched belief that good instructors do not rely on textbooks. This belief proved to hamper these pre-service teachers’ efforts to enact lessons, particularly when they lacked sufficient knowledge and experience. Freeman and Porter (1989) found that teachers modify programs by omitting topics or focusing instruction on the practice exercises found within the textbook. This was especially the case, they found, when teachers were working with so-called “lower-achieving” students (Freeman & Porter, 1989, p. 416). Likewise, noted earlier, Stein and colleagues (1996) explain that teachers reduce the cognitive difficulty presented by written tasks. Manouchehri and Goodman (1998, 2000) also report that teachers sometimes offer students more abstract or algorithmic approaches than those suggested by the texts. Across these three studies, teachers modified curricular guidance, at least in part, because of beliefs about their students. In particular, teachers believed students required additional practice with facts and skills—because they had not yet mastered these—or that certain learning activities were, in and of themselves, too challenging.

Modifying while professing adherence. Other researchers have documented ways that teachers modify programs to varying degrees, while generally adhering (or professing adherence) to curricular guidance. Cohen (1990) describes one teacher, “Mrs. Oublier,” who proclaimed enthusiastic support of reform-oriented instruction. In particular, she welcomed new instructional materials in her school that placed an emphasis on sense-making. On the other hand, though, Cohen notes that her classroom practices constituted a “mélange of traditional and novel approaches to math instruction” (p. 312). Specifically, Mrs. Oublier used her new resources “as though mathematics contained only right and wrong answers” (p. 312) and conducted lessons “in
ways that discourage[d] exploration of students’ understanding” (p. 312). Cohen describes one notable lesson, in which she replicates an algorithmic procedure with manipulatives, instead of using them in the service of problem-solving and discussion.

Likewise, Lloyd (1999) explores the curriculum adaptations of two teachers, “Mr. Allen” and “Ms. Fay,” who both professed alignment with the reform-oriented philosophy of the program they were using but who interpreted the materials in starkly different ways. Mr. Allen found the program too open and added structure to problem-solving tasks, while Ms. Fay found it too constraining and demonstrated alternate solutions. Neither teacher changed the tasks or problems but instead modified the program during periods of cooperative work and through follow-up interactions with students.

Similarly, Remillard (1999, 2000) found that teachers’ interpretations of texts influenced their use of materials. “Catherine” appropriated tasks from the written material—generally using the suggested activities within her lessons—but she saw them as opportunities to walk her students toward correct answers rather than as vehicles for discussion. On the other hand, “Jackie” saw the program as offering, mainly, topic-level guidance. Instead of utilizing activities found within the written curriculum, she preferred to invent her own.

Choppin (2011) also reports on a teacher, “Margaret,” who made productive “knowledge-based adaptations” (p. 335) of instructional materials. Margaret changed the sequence of subtraction problems, in particular, and thereby attended to “student thinking and to the curriculum’s design rationale” (p. 335). She believed, and Choppin found, that her adaptations motivated students’ reasoning and pattern-recognition, rather than blind rule-following.

Taken together, this set of findings demonstrates that teachers may express general alignment with the sense-making principles that are embedded within reform-oriented materials. Yet, other beliefs or goals may stand in tension with a program’s design, which ultimately leads to program modifications (see, also, Davis et al., 2011). Some of these modifications undercut learning, while others enhance learning or support a different type of learning. In the cases of Ms. Fay and Jackie, for example, both teachers wanted to engage their students in more robust problem-solving and reasoning than what they believed was offered by the programs they were using.

Even when philosophical alignment is strong, modifications may still seem necessary. For example, van Zoest and Bohl (2002) profile two teaching interns, “Alice” and “Gregory,” who
express support of reform-oriented instruction and programs. Using Core-Plus materials, van Zoest and Bohl explain that:

> Gregory and Alice spent much of their planning time walking through each of the next day’s investigations in order to determine which parts students could answer without guidance, which parts might be skipped or glossed over to increase the pace of student progress, and how to avoid being overly directive while at the same time maintaining student focus on the day’s main mathematical concerns. (p. 274)

The researchers note Core-Plus materials are structured in such a way that—by adhering to the goals of the program—cooperative problem-solving is unavoidable. This means that it was easy for students (and their teachers) to perseverate and discuss, at length, elements of activities that were less central to the lesson objectives. Therefore, Alice’s and Gregory’s planning—and the subsequent adaptations they made—aimed to keep the lesson objectives at the forefront of classroom activity. These findings reflect Remillard’s (1999) assertion that reform-oriented materials present a host of challenges and dilemmas to teachers who try to use them, mainly because students’ responses are unpredictable.

**Typologies of curriculum-use.** Looking across these studies, researchers have discerned broad patterns in teachers’ use of materials. Freeman and Porter (1989) showed that teachers add, omit, or change the topics or sequences of topics found in materials. Remillard (1999, 2000), indicated above, observed that teachers either appropriate or invent tasks when designing and constructing instruction with resources. In addition, Nicol and Crespo (2006) describe a middle ground between appropriation and invention, showing that pre-service teachers also elaborate on texts by utilizing outside resources as supplements or by changing the contexts of problems. Also described earlier, M. Brown (2009) showed that teachers offload (rely on), adapt (modify), or improvise with curriculum materials, even while they teach.

Sherin and Drake (2009) review the literature on teachers’ adaptations of curriculum materials and offer an expanded framework, showing that teachers read, evaluate (i.e., critique), and adapt materials during three distinct phases of interaction before, during, and after lesson enactment. The meaning of reading and adaptation should be clear, given the preceding discussion. By evaluating instructional materials, Sherin and Drake explain, teachers determine whether the textual guidance aligns with their own understanding and beliefs; teachers also decide whether they can accommodate such guidance within their own goals for instruction. Building on this work, Stein and Kaufman (2010) found, specifically, that—before instruction—when teachers concentrate on understanding the broad mathematical objectives of lessons, they were more likely
to maintain the suggested level of cognitive demand during instruction. Again, these typologies are important insofar as researchers now consider adaptations made at various phases of teachers’ curriculum implementation and under affiliated circumstances.

**Other reasons for adaptations.** Besides those suggested above, teachers adapt or modify curriculum materials for a variety of other reasons. Teachers might change or supplant programmatic guidance, for instance, because of their conceptions of mathematics, itself, or their beliefs about teaching and learning (Collopy, 2003; Lloyd, 1999; Manouchehri & Goodman, 1998, 2000; Remillard, 1999). Specifically, teachers’ less-than-secure content knowledge is one possible reason for their skipping or modifying content (Cohen, 1999; Manouchehri & Goodman, 1998). Collopy (2003) also reports on Ms. Clark’s modifications—increasing opportunities to practice procedures—largely because of a “conflict between her identity and the beliefs targeted for change” (p. 308) by the program she used. She saw mathematics itself as a “hierarchical cannon [sic] of rules, facts, and algorithms” (p. 308) and learning mathematics as a product of rote practice. Ms. Clark also saw herself as a keeper of mathematical knowledge, so she modeled solutions for students, asking them subsequently to replicate the approaches she demonstrated.

As described earlier, how teachers read textual guidance also shapes their implementation of curriculum programs (Remillard, 1999, 2000, 2012; Sherin & Drake, 2009). Remillard (1999) shows that teachers focus on different parts of teacher’s guides as they read. They may read to find example problems or to understand how to enact particular activities. Stein and Kim (2009) found, in particular, that not all teachers read supplementary mathematical guidance offered to teachers within curriculum programs. Finally, beyond reading different parts of the text, Remillard (2012) outlines three additional considerations: what teachers read for, when they read, and who they are as readers.

Conceptions of reform movements also influence teachers’ use of curriculum materials (Cohen, 1999; Sherin & Drake, 2009; Davis et al., 2011; Manouchehri & Goodman, 1998, 2000). Cohen (1990), for instance, found that Mrs. Oublier’s incomplete understanding of the push for conceptual understanding led, at least in part, to her routinized use of manipulative tools. Remillard and Bryans (2004) argue, even further, that teachers’ orientations toward, or perspectives on, curriculum materials contribute to their use of materials. One teacher in their study, for instance, saw instructional materials as offering little more than resources for assignments; this teacher, therefore, largely overlooked the investigatory activities in his reform-oriented program. Remillard and Bryans offer a typology of orientations toward curriculum—
incorporating beliefs not only about the specific programs at their schools, but also about mathematics, teaching and learning, and curriculum generally. Their typology included characterizations of curriculum-use ranging from resistant, to skeptical, to more trusting.

Other external factors influence teachers’ use of curriculum materials. Noted earlier, Ball and Feiman-Nemser (1988) observed that traditionally impoverished views of curriculum materials may limit the degree to which teachers utilize them. Huntley and Chval (2010) explain, furthermore, that teachers adapt curriculum and instructional materials because of their perceived obligations to state, district, or school policies. For example, teachers reported changing the sequence of materials, because of requirements specified in local curriculum guides or assessment blueprints (Huntley & Chval, 2010, pp. 291–293). Among others, Davis and colleagues (2011) assert that teachers’ beliefs about their students (and, specifically, beliefs about their students’ capacities) shape curriculum adaptations. Some beliefs identified by Davis and colleagues, furthermore, were related to teachers’ present—and, notably, even their past—school contexts.

Collectively, these studies offer additional evidence for the wide-ranging influences of teachers’ curriculum-use. Understanding curriculum enactment, therefore, involves tracing the evolution of tasks and objectives along a lengthy pathway as they are encountered by teachers when reading and as they are evaluated, taken-up, and transformed. Transformations occur as teachers interpret standards and local guidance in tandem with interpreting materials (Remillard & Heck, 2014; Stein et al., 2007). Other stakeholders transform curriculum, as well (Remillard & Heck, 2014). In other words, the full breadth of transformations must be considered, as well as the underlying contexts. Doing so gives a fuller picture of whether teachers reinforce the intentions of curriculum authors, remaining within the curriculum envelope (Ben-Peretz, 1990), or whether authors’ intentions are irrevocably undercut (Brown & Campione, 1996). S. Brown and colleagues (2009) offer an intriguing third possibility, one that I cover in the next section of this chapter: teachers’ decisions might even circumvent shortcomings of curriculum materials.

### 3–3. Summary and Discussion: Notions of Fidelity

In this chapter, I have sought to accomplish four main, but generally interwoven, goals. I have sought to convey, first, that teachers’ use of instructional materials cannot be meaningfully separated from instruction itself; both are part of a broader curriculum enactment system (Remillard & Heck, 2014). In so doing, I have aimed to define the construct of curriculum and trace its conceptual evolution. Second, I have described instructional materials as helpful
resources, but I have also observed that their use depends largely on teachers’ reading and uptake of embedded guidance. Contexts within schools, districts, and systems, as well as broader sociopolitical contexts, are also important factors to consider. Third, I noted that Remillard’s (2005) participatory lens on curriculum is valuable, because it contextualizes what instructional materials offer—their embedded opportunities to learn—within teachers’ own interpretations, understanding, and capacities. In describing the text-based opportunities to learn, I explored the broad epistemological purposes of curriculum, and I opened a window into my bracketing process (Husserl, 1964). After all, to attain objectivity, as Husserl argues, I must engage with and unpack my own subjectivity. This process led me to explain that, while I am frustrated by the increased prescriptiveness of teachers’ work, generally, I simultaneously value what curriculum materials offer as tools. Finally, I summarized research on the influence of curriculum materials on students’ learning, as well as the factors that mediate teachers’ curriculum-use.

Questions that Emerge from the Participatory Lens on Curriculum-Use

Thinking about Ben-Peretz’s (1990) curriculum envelope, which represents a distillation of this chapter, raises two affiliated questions. These questions have been explored by researchers, of course, but ongoing work remains. First, in what ways can curriculum materials best convey to teachers what they offer as tools? Bruner (1960 / 1977) asserts, “A curriculum is more for teachers than it is for pupils. If it cannot change, move, perturb, inform teachers, it will have no effect on those whom they teach” (p. xv). This comment originally served to critique the failed reforms of the post-Sputnik era, also known as the New Math era, that overlooked the role of teachers in enacting curriculum (Wilson, 2003). What it suggests is that curriculum authors must speak to teachers (Remillard, 1999). The current era holds promise, insofar as curriculum authors—recognizing past missteps—are more actively engaged with teachers as partners (Ball & Cohen, 1996; Brown et al., 2009; Davis & Krajcik, 2005; Lloyd, 1999; Remillard, 1999, 2000, 2005, 2016; Tarr et al., 2008).

Even still, research hasn’t teased out, not fully, the relationship between textual guidance offered to teachers, images of practice, and classroom enactments. Therefore, a second question arises: what sorts of adaptations can teachers make that will either remain within the curriculum envelope or may transcend the envelope and will nonetheless promote students’ understanding? To address this question, researchers have therefore called for more in-depth studies and frameworks for exploring the specific textual guidance offered to teachers, how this guidance is interpreted, and how it subsequently plays out during lessons (e.g., Huntley & Heck, 2014, p. 35;
see also Remillard & Kim, 2017). The research cited in the previous sections of this chapter includes some side-by-side comparisons of written and enacted lessons, but this is not yet an established research norm. Side-by-side comparisons of written and enacted lessons are generally episodic, used as evidence for broad themes, rather than for wholesale considerations.

This second question also raises affiliated concerns regarding the fidelity of teachers’ curriculum implementation. Some analysts see curriculum materials as a plenipotentiary agent for changing instruction and improving outcomes (e.g., Chingos & Whitehurst, 2012; Freeman & Porter, 1989). This perspective, however, remains contested (e.g., Remillard, 2005). Finn (2004) claims, for example, textbooks are a “problem in American education in two ways”—first, that they are “mediocre” and, second, that teachers use them (p. 1). In a well-received TED Talk, Meyer (2010) likewise critiques mathematics textbooks for offering dehydrated reasoning opportunities, saying they are “functionally equivalent to turning on [the television program] ‘Two and a Half Men’ and calling it a day” (02:07). Meyer encourages teachers to use multimedia, instead, because “the textbook is helping you [the teacher] in all the wrong ways” (09:35). As I have attempted to explain above, this anti-curriculum mindset nonetheless mischaracterizes what modern, well-designed curriculum programs offer teachers (Remillard, 2016). It also overlooks the importance of using tools to accomplish tasks more easily or productively (M. Brown, 2009). Imagine, analogously, the absurdity of telling a chef to avoid using knives.

At the same time, it would be naïve to proclaim that a given set of curriculum materials serves all teachers and students equally well. They are far from perfect tools—even those that are well-designed. This is the essence of the participatory lens (Remillard, 2005): that both teachers and curriculum materials offer value to the relationship. In short, teachers benefit from using curriculum, but they must also modify its written lessons. Huntley and Chval (2010) state it differently, observing a somewhat mixed message offered to teachers:

…on the one hand, teachers are told to implement instructional materials in a certain manner, on the other hand NCTM encourages teachers to be flexible, which teachers may interpret to mean that they need to stray from their pacing guides. (p. 290)

This presents quite a bind, especially for those who try to evaluate teachers’ adaptations and coach teachers. It is a challenge magnified, even more, when considering the varying conceptions of coherence described in Chapter 2. If there is no widespread agreement on what it means to connect mathematical ideas, how can there be clarity on how materials should be used to promote
mathematical coherence? If coherence is a goal, then what sorts of modifications would remain in
the curriculum envelope (Ben-Peretz, 1990) and what sorts would not?

**Conceptions of Fidelity of Curriculum-Use**

To enter into such a conversation, I step back and briefly describe conceptions of
implementation fidelity. To begin, I note that fidelity has sometimes been conceived as following
the topics, as sequenced, within curriculum materials (e.g., Freeman & Porter, 1989; Tarr, Chávez,
Reys, & Reys, 2006). At the very least, this chapter should have explained that such a conception
is problematic, due in part to the complexity and number of decisions teachers make. Recall, as
well, that fidelity of implementation cannot predict students’ achievement (Stein et al., 2007, p.
359). Instead, like Remillard’s (2005), interpretivist perspectives represent another pole on the
spectrum and acknowledge reader response theory (Rosenblatt, 1988, 1994). In so doing,
interpretivist perspectives presume readers construct meaning from texts. Therefore, a straight-
line connection is impossible; materials do not merely talk *through* teachers (Remillard, 1999).

**Two forms of fidelity.** S. Brown and colleagues (2009) offer something of a middle ground,
one that has practical, methodological implications for my own work. Inspiration for their study
was drawn from an observation articulated neatly by Chávez-López (2003) that “it is possible to
‘adopt’ a textbook and use it frequently without really espousing the epistemological assumptions
that are attached to the textbook” (p. 160, as quoted by S. Brown et al., 2009, p. 383). A similar
finding has been reported by other researchers (e.g., Chval, Chávez, Reys, & Tarr, 2008;
Remillard & Bryans, 2004; Tarr et al., 2008). Taking this possible bifurcation into consideration,
Brown and colleagues (2009) therefore distinguish between two types of fidelity—the degree of
alignment between an enacted classroom lesson and a) the literal (written) lesson and b) the
authors’ intended lesson. The former describes alignment with the “steps, suggestions, and
recommendations provided in the written instructional materials” (p. 373). The latter describes
alignment with the “authors’ intended opportunities to learn” (p. 373). As described above, their
OTLs designate the supports offered around mathematical reasoning and communication (p. 377).
This distinction is said to better reflect “the character of implementation in context” (Schoenfeld,
2006, p. 17).

In their study, S. Brown and colleagues (2009) found that teachers exhibited consistency with
regard to their implementation of the intended lesson, regardless of their fidelity to the literal
lesson. They attribute this consistency to a collection of factors—including professional
development opportunities, knowledge, beliefs, etc.—or, in their words, *the teacher’s lens* (S.
Brown et al., 2009, pp. 371–372). Conversely, Brown and colleagues (2009) also identified several written lessons, followed literally by teachers, but with a) varying degrees of fidelity to the intended lesson and b) consistently low fidelity to the intended lesson. Of these latter cases, they hypothesize that the text, as designed, contributed to patterns of curriculum-use. Describing cues for the intended lesson (i.e., the OTLs), Brown and colleagues (2009) contend that “it is not clear whether the statements actually support teachers” (p. 387). They suggest additional research on “what sense teachers make of these statements” (S. Brown et al., 2009, p. 387). Further, they say, authors should attend carefully to describing the intended lesson, so that the text “ evoke[s] a cognitive structure in the reader’s mind which corresponds with the context meant” (van Dormolen, 1986, p. 151, as cited in S. Brown et al., 2009, p. 387).

**Relationship to mathematical storylines and plots.** Returning to the threads developed in Chapter 2, my own study aims to build upon the goal articulated by S. Brown and colleagues (2009). In particular, through the presentation of my findings, I aim to characterize teachers’ interpretations of the written guidance in their curriculum programs. To do so, I explore how teachers make sense of mathematical storylines and plots, which I argue are features of written materials, or cognitive structures of mathematical context (van Dormolen, 1986) that are oftentimes tacit. They represent, then, a form of the authors’ intended lesson. It remains to be seen in what way, or to what degree, storylines and plots represent the intended lesson. Moreover, Pimm (2006) suggests that storylines and plots are embedded within even austere mathematical proofs and consequently represent an epistemological journey (pp. 177–178). Stated differently, mathematical plots and storylines—regardless of the form they ultimately take within proofs—can be recreated, perhaps, as examples of the discovery (or constructive learning) process.

I suspect that relying on the literal sequence of steps places undue emphasis on a particular framing of coherent instruction. I argue that teachers might remain squarely within the curriculum envelope or productively open the envelope (Ben-Peretz, 1990), if the envelope were defined as the mathematical narrative of written lessons. I speculate this could occur even when teachers intentionally reordering the literal instructions of a lesson. I therefore hypothesize there is a need for a broader framing of fidelity, one that goes beyond attending to the sequence of steps and looks toward a larger context, like the mathematical narrative.

Before reviewing the literature on mathematical coherence and storylines in the next chapter, I note that M. Brown (2009) opens a potential, theoretical pathway for broadening the understanding of curricular fidelity. Specifically, he contrasts traditional *procedure-centric* lessons...
with novel *resource-centric* ones (p. 33). Procedure-centric lessons suggest steps for teachers to follow (i.e., the literal lesson, S. Brown et al., 2009). Resource-centric lessons, on the other hand, would represent an entirely new, as-yet created, and more flexible form—not necessarily aiming to “eschew procedures” but, rather, avoiding “the conventional practice of relying on them as the core organizing element” (Brown, 2009, p. 33). If written in accordance with Brown’s (2009) aims, resource-centric lessons would call attention to the *pedagogical affordances* of their various components and, perhaps, would include several reasonable, instructional sequences (p. 33). In so doing, Brown argues, developers might “promote mindful engagement on the part of teachers” (p. 33). My own research suggests, at the very least, that teachers sometimes interpret written lessons in a resource-centric way and re-order sequences of instructions while staying true to the intended lesson and underlying mathematical storylines.
CHAPTER 4. THE SETTING (LOCATION 2):
THE LITERATURES ON NARRATIVE AND COHERENCE

I’ve always lived in Dinkerville,
My friends all live there too.
We go to Diffenoofer School—
We’re happy that we do.

—Dr. Seuss, *Hooray for Diffenoofer Day!* (1998, lines 1–4)

4–1. Coherence in Mathematics Instruction

In this chapter, I review literature that constitutes another portion of my theoretical framework. In particular, I summarize and tie together research on coherence of mathematics instruction, mathematical narratives, and steering of instruction. As I explained in the previous chapters, I am mainly concerned with how and in what ways curriculum materials support coherent presentation of mathematical ideas.

Note, also, that in Chapter 3, I reviewed literature on the influence of curriculum materials on learning. This review, I explained, necessarily described the impact of curriculum materials along with the ways teachers use curriculum materials. This is because there is no simple one-way street from materials to student outcomes. I characterized this work, mainly, as insight regarding the interactive, participatory lens on teachers’ curriculum-use. In Chapter 5, next, I bring together key findings from its two, preceding chapters, in order to articulate my theoretical and conceptual frameworks. In Chapter 5, I also describe my research methodology.

**Pedagogical Design Capacity and Curricular Noticing**

First, though, I ground this review in a discussion of pedagogical design capacity (PDC) (M. Brown, 2009). With regard to curriculum resources, recall that PDC represents a teacher’s “skill in perceiving affordances, making decisions, and following through on plans” (Brown, 2009, p. 29). Furthermore, Brown explains, “Teachers who possess high pedagogical design capacity are able to deconstruct curriculum materials, recognize their essential elements, and reconstruct them in order to suit their needs” (p. 31). Clearly, if instructional materials offer discernably coherent pathways of learning, then teachers’ adaptations should not appreciably disrupt this pathway. Doing so would likely fall outside of the curriculum envelope (Ben-Peretz, 1990) and might
represent a lethal adaptation (Brown & Campione, 1996). Of course, making this determination is presently challenging and would necessitate future research. Brown explains, nonetheless:

To a certain extent, the elegance of a teacher’s design is a subjective determination. Yet the fact remains that not all designs are equally effective at helping teachers reach their goals, not all designs reflect the same responsiveness to the needs of a particular setting, not all designs are purposeful, and not all designs embody the same degree of utility….it is [therefore] theoretically possible to measure the quality of teachers’ designs. Further research is needed to sort out the key dimensions of PDC and find precise ways to measure these and foster their development in teachers. (p. 31)

As should have been clear from Chapters 1 and 2, I propose several new dimensions of PDC through my research, namely, coherence and its various representations within mathematical storylines and plots.

Consequently, building on the research outlined in Chapter 3, I must first discuss research on what elements of written materials teachers notice. This line of research concentrates on what teachers perceive as affordances within their curricular resources. This is a relatively new line of inquiry, and I only share relevant highlights of this work here. Specifically, Dietiker and colleagues (2018) summarize frameworks that explore teachers’ interpretations and adaptations of curriculum materials as instructional resources (e.g., Sherin & Drake, 2009). They contend, though, that “little is understood thus far about how these different descriptions of interactions collectively reveal a broader curricular process of participation with curriculum materials” (p. 522).

Dietiker and colleagues (2018) therefore build upon a prior framework, describing teachers’ professional noticing of students’ thinking (van Es & Sherin, 2008), to suggest a new framework for curriculum-use. Their curricular noticing framework (CNF) seeks to gather and extend research conceptualizing how teachers engage with instructional materials. The CNF also builds on work by van Es and Sherin (2008), and others following their lead, about teachers’ noticing. Collectively, this latter line of research has shown that what teachers notice in the classroom about students’ thinking influences their in-the-moment decision-making as they teach. Effective noticing depends upon significant knowledge. It therefore follows that effective curricular noticing also depends heavily upon teachers’ knowledge.

One particular element of the CNF, curricular attending, represents a novel way of describing teachers’ reading. Curricular attending, Dietiker and colleagues (2018) say, involves, “the skills involved in searching, looking, locating, surveying, and other ways of visually taking
in materials” (p. 525). They are careful to note that attending does not mean to suggest that teachers notice all elements of texts at all times. Teachers’ skill involves noticing elements that are particularly useful for accomplishing their instructional goals and purposes for using materials.

The CNF also offers a new, potential approach to study teachers’ use of materials. In a manner of speaking, it incorporates elements of Remillard’s (2012) reading-purposes and Remillard’s and Kim’s (2017) curriculum-activated mathematical knowledge. (I elaborate on the latter, below.) By studying the various skills of curricular attending, in particular, we may gain insight into what teachers are reading for, as well as the dimensions of mathematical knowledge that are activated as they read. At the same time, the CNF allows us to think and talk about what features of texts teachers observe when using materials—whether tacit or explicit.

Of course, as a relatively new line of inquiry, research on curricular noticing is ongoing. McDuffie, Choppin, Drake, and Davis (2018) draw on the CNF in studying the planning practices of 20 middle school mathematics teachers. They found that teachers attended, most often, to participation structures within lessons and problems for instruction and homework. Moreover, McDuffie and colleagues also found that teachers’ modifications of the elements they noticed also reflected their orientations toward curriculum materials (Remillard & Bryans, 2004). As one example, teachers with a direct orientation, or beliefs endorsing teacher-led instruction (as contrasted with dialogic beliefs), tended to view inquiry-based approaches as “beyond their students’ capabilities” (McDuffie et al., 2018, p. 181). Amador and Earnest (2018), in contrast, found that pre-service teachers in their study noticed key elements of curriculum materials (e.g., specifically when vocabulary was presented), but made adaptations because they didn’t understand the intentions behind the designs (p. 20).

Males and Setniker (2019) used eye-tracking glasses in their study of pre-service secondary school teachers reading curriculum materials. They found that participants “attended more to certain portions of each text when planning” (p. 163). Specifically, participants’ lack of experience with materials suggested that they focused on answers found in teacher’s guides, wondering whether the students’ editions also contained printed answers. Hong, Choi, Runnalls, and Hwang (2019) draw on the CNF to show that elementary teachers might be led astray in their curricular noticing; they identified key misalignments between texts and cognitive learning progressions on the concept of area that might have impeded the enacted learning trajectory.
Research on Coherence in Teaching

Near the conclusion of this section, I describe additional research on coherence as an element of curriculum materials perceptible to teachers. Here, I describe broader, more essential, and preliminary concern: whether and how, as Kaplinsky (2019) suggests, coherence plays a role in students’ learning. Coherence, I hope to demonstrate, is a core feature of effective instruction. And, yet, it also remains an under-specified notion. Indeed, the CCSS-M authors themselves observe that our knowledge on sequencing content, effectively, remains incomplete. They first explain that:

…because topic A appears before topic B in the standards for a given grade, it does not necessarily mean that topic A must be taught before topic B. A teacher might prefer to teach topic B before topic A or might choose to highlight connections by teaching topic A and topic B at the same time. Or, a teacher might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B. (NGA Center & CCSSO, 2010, p. 5)

The CCSS-M authors then indict the “existing educational research” for not being able to “specify all such learning pathways” that build knowledge deliberately (NGA Center & CCSSO, 2010, p. 5). They explain that the progression of topics in the CCSS-M was therefore built from “state and international comparisons” and the “professional judgment of educators, researchers and mathematicians” (NGA Center & CCSSO, 2010, p. 5), because empirical clarity is still lacking. Note, of course, that the CCSS-M authors are referring to the challenge of achieving topic-level coherence; understanding the coherence of activities and content found within lessons is equally challenging.

Recall that coherence also lacks an agreed-upon definition. As I suggested in Chapter 1, some regard coherence as a linear pathway (e.g., Abdussalaam et al., 2015) and others consider coherence more akin to a web (Zimba, 2011, 2012) or a network of paths on mathematical terrain (Lampert, 2001). Considering models of instruction, McCallum (2018) offers a sort of compromise—that coherence may reflect both progression and connection. In like fashion, I generally refer to the definition offered by Schmidt and colleagues (2002), reflected in the CCSS-M, that coherence constitutes: a) a “sequence of topics and performances that are logical and reflect, where appropriate, the sequential or hierarchical nature” (p. 9), and b) a set of “deeper structures” that “serve as a means for connecting the particulars” (p. 9). Keeping this definition in mind and whether it reflects other researchers’ conceptions, I now turn to reviewing studies of coherence in mathematics instruction and learning.
Early findings on coherence. Leinhardt (1989) describes lessons by expert teachers as significantly more coherent than those taught by novices. Experts’ lessons, she explains, “display a highly efficient internal structure, one that is characterized by fluid movement from one type of activity to another” along with “a logical rule-bound explanation of new material that connects well with previous material” (p. 73). Novices’ lessons, on the other hand, are “fragmented” and have “long transitions between lesson segments” with “an ambiguous system of goals that often appear to be abandoned rather than achieved” (p. 73). To a degree, Leinhardt’s understanding of coherence intermingles aspects of classroom management and academic content. Additional specificity on the nature of connecting rules is, perhaps, warranted. And research in this vein has sometimes been critiqued for not studying learning outcomes or using proxies, like experience, for expertise. Regardless, Leinhardt’s contribution is important, identifying the structure and the “rule-bound” connectedness of experts’ lessons.

While Leinhardt (1989) studied teachers’ classroom practices, Ball (1990) offers a different conception of coherence that focuses on their knowledge. Ball’s subsequent work, of course, has tied mathematical knowledge for teaching to student outcomes (Hill et al., 2005; Hill, Ball, Blunk, Goffney, & Rowan, 2007). To teach mathematics coherently, Ball argues, classroom instructors must themselves possess a connected understanding of mathematics. In her words, a connected understanding of mathematics is evidenced by understanding the relationships between “particular ideas to larger concepts” (Ball, 1990, p. 459). This definition mirrors the second part of the one offered by Schmidt and colleagues (2002). Ball (1990) found, however, that few participants in her study seemed to own such coherent mathematical knowledge. For instance, teachers mistook the mechanical rules for dividing fractions for why the algorithm makes sense. Put another way, they conflated instrumental and relational understandings of mathematics (Skemp, 1978 / 2006). Ball (2009) surmises that teachers’ experiences with “the standard school mathematics curriculum” most likely “consisted of discrete bits of procedural knowledge” (p. 459), potentially contributing to their incoherent and superficial understanding of the subject.

Fernandez and colleagues (1992) offer a framework for classroom instruction that depends upon students’ re-creations of mathematical events. In reporting on their study, they first describe a well-told and coherent story as one that “is easy to understand and remember because each event has meaning in relation to other events” (p. 335). Well-told stories, they add, are hierarchical and portray the challenges faced by protagonists, the actions taken by the set of characters, and “the new challenges that arise as a result of taking these actions” (Fernandez et
Much like coherent stories, coherent instruction is defined, accordingly, as having “clear goals that motivate and interrelate the events of the lesson” (Fernandez et al., 1992, p. 335). By “goals that motivate,” they mean evident purposes or underlying rationales. And, they argue, coherent instruction increases the chances students will be able to retell, coherently, the mathematical story of lessons they experienced (Fernandez et al., 1992, pp. 345-346).

To test the viability of this framework, Fernandez and colleagues (1992) used edited videotapes to manipulate the coherence of classroom events. In a laboratory setting, they then asked students from these same classrooms—who previously exhibited varying degrees of mathematical knowledge—to try reconstructing the events of lessons. In so doing, the students were prompted to discuss the ideas and broad mathematical goals. Overall, Fernandez and colleagues assert that “coherent lessons lead to more coherent [mental] representations” of students’ mathematical ideas. They also tie this finding to students’ learning gains and explain that, as a result, their study suggests more coherent mental representations, in turn, “lead to greater learning” (p. 363).

Reporting on their study of numeracy teaching, Askew, Brown, Rhodes, Wiliam, and Johnson (1997) found that effective teachers of elementary mathematics adopt a connectionist orientation. A connectionist orientation, they explain, is a broad constellation of practices and beliefs that emphasize conceptual links between mathematical methods and their reasonableness. Further, a connectionist orientation, they say, “places a strong emphasis on developing reasoning and justification leading to the proof aspects” of mathematics (p 32). They found the students of teachers with such an orientation made greater gains from pre- to post-tests on basic numeracy ideas and skills. In other words, while they use different terms, Askew and colleagues (1997) understanding of a connectionist orientation weaves together distinct threads from prior scholarship on mathematical coherence: one through-line portraying coherence as the logical relatedness of ideas (Fernandez et al., 1992; Leinhardt, 1989), another as connectedness to broader principles (Ball, 1990). Therefore, one plausible interpretation of this study is that learning is improved through coherent instruction, defined by both connectivity and deeper structures (cf. Schmidt et al., 2002).

Greeno and the Middle School Mathematics Through Applications Project (MMAP) group (1998) argue for an integration of principles of behaviorist, cognitive, and situative perspectives on learning. In so doing, they present elements of productive instruction, identified within research that adopts a behaviorist perspective (i.e., a perspective that assumes teachers’ classroom
actions, as stimuli, relate directly to learned associations made by students). These include coherence, achieved through the articulation of clear and explicit goals, consistent instructional routines, orderly sequences, and precision of terminology, among other characteristics. These elements, they also argue, are not incompatible with findings from cognitive- or situative-oriented research. Such research, broadly, presumes that learning happens through mental reconfiguration of schemata and that reconfiguration can occur within social contexts.

As an aside, I should note that my own perspective on learning—and how lessons are co-constructed by particular teachers and their students working tougher in particular school environments—is also a situative one. I describe narrative structures of texts and classrooms in greater detail in the next section of this chapter, but as a conversational enterprise, social narratives themselves are situative phenomena (Clandinin & Connelly, 1992; Labov & Waletzky, 1967; Labov, 1997).

More recent studies on coherence, related to teachers’ knowledge and capacity. Rowland, Huckstep, and Thwaites (2005) and Rowland (2013) employed a grounded theory approach to understand the classroom situations in which teachers drew in various ways upon subject matter and pedagogical content knowledge (Ball et al., 2008; Shulman, 1986b). In their words, they sought to understand and classify “situations in which mathematical knowledge surfaces in teaching” (Rowland, 2013, p. 22). Through their analysis of teaching practices, they ultimately developed themes that later became known as the knowledge quartet (KQ). The four dimensions of the KQ, they state, are not mutually exclusive and, as indicated above, do not represent distinct types of cognition or knowledge (Rowland, 2013, p. 22).

Their analysis revealed that significant aspects of teachers’ work involves making connections, or coherently “bind[ing] together certain choices and decisions that are made for the more or less discrete parts of mathematical content” (Rowland, 2013, p. 24). These aspects include: “making connections between procedures,” “making connections between concepts,” “anticipating complexity,” and making “decisions about sequencing” (p. 24). Elaborating further, Rowland explains that the KQ dimension connection is defined by the “conception of coherence” that “includes the sequencing of topics of instruction within and between lessons” as well as “the ordering of tasks and exercises” (p. 25, emphasis in the original). In this way, the connection dimension of KQ resembles Fernandez’ and colleagues’ (1992) notions of coherent instruction; both involve relating classroom events meaningfully to prior events. This KQ dimension, they also say, involves demonstrations of “structural connections within mathematics itself” (Rowland,
2013, p. 25). At the same time, then, Rowland and colleagues (2005) and Rowland (2013) imply that connection and coherence are not necessarily fully synonymous with sequencing and increasing complexity.

Their distinctions have important consequences. Before explaining more, I cite one example. Rowland and colleagues (2012) characterize decisions made by a pre-service teacher, “Chloe,” as generally inattentive to complexity—one aspect of the KQ dimension of connection (“AC Scenario 2,” para. 11). Chloe, they explain, asked students to solve subtraction problems using a 10 by 10 grid of numbers, starting by subtracting 19, then 9, then 11, and then 21 from any given two-digit number. By and large, students struggled to use the problem-solving strategies, involving the grid, that surfaced during classroom discussion (para. 5). When students transitioned to work independently, Chloe assigned problems subtracting 9 and 11 to students who were struggling the most; she assigned problems subtracting 19 and 21 to those struggling least. Both groups of students continued to struggle making sense of the patterns in subtraction (para. 6).

Rowland and colleagues (2012) contend that Chloe followed the curriculum framework closely, but they also suggested a more deliberate approach (para. 8). Alternately, they explain, Chloe could have paid greater attention to the underlying mathematical structure (para. 8). Specifically, when tracing the calculation through the rows and columns of the chart, subtracting 9 or 11 involves moving in opposite directions (more difficult), whereas subtracting 9 or 19 (or 11 and 21) involves moving in the same direction (easier). In sum, this example suggests that curriculum guidance should not necessarily be followed in sequence, because the KQ dimension of connection (in tandem with students’ responses) might support an alternative ordering. Chloe and her students struggled, Rowland and colleagues speculate, because she did not aim to connect the chart strategies with students’ thinking and, instead, connected magnitude of numbers with their perceived level of understanding (para. 8).

Likewise, Choppin (2011) reports on the case of a teacher who made several learned adaptations of curriculum materials during a set of Grade 6 lessons on integer operations. Learned adaptations, a term coined by Choppin, are “knowledge-based” modifications that

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12 Note that this curriculum framework, cited by Rowland and colleagues (2012), reflected the mathematics standards in place in the United Kingdom at the time the lesson was observed. This framework has been archived by the U.K. National Archives (The University of Manchester, n.d.) and so it is unclear exactly what sort of written guidance Chloe used to develop her lesson.
teachers devise after having experience with a given instructional program and that involve both “attending to student thinking and to the curriculum’s design rationale” (p. 335). In making learned adaptations, teachers adopt curriculum materials, but also “make sense of what students are doing,” and so they “go beyond simply following the prescribed curriculum in using the resources to facilitate the teaching and learning practices they envision” (p. 335).

The teacher Choppin (2011) profiles, “Margaret Wright,” made a number of learned adaptations that were “intended to provide opportunities for students to develop their own strategies and ways of thinking” (p. 349). One such adaptation involved resequencing a set of examples offered to the students—as well as providing a space and opportunity for them to conjecture about patterns involving different types of numbers—so that they could discern their own typology of subtraction problems. In her view, this approach supported students’ sense-making, because it “‘kind of forces kids into [using thinking] strategies’” (Choppin, 2011, p. 249), rather than uncritically digesting a classification provided by the text that might have led to blind rule-following.

In an important study that relates strongly to my own work, Gresalfi (2009) characterized the force of teachers’ implementation of various opportunities to learn. Force, roughly, was defined as frequency and depth of opportunity. In a case study of four classrooms, she found that prevalent opportunities to make connections across ideas were positively associated with students’ changed dispositions (p. 362). Dispositions, here, represent students’ beliefs about mathematics and their own learning. Stated differently, students’ opportunities to make connections related to their attitudes toward mathematics.

Remillard and Kim (2017) studied use of curriculum materials by analyzing texts and interviewing teachers. As with the KQ (Rowland et al., 2005; Rowland, 2013), Remillard and Kim (2017) also identify circumstances in which teachers’ mathematical knowledge is activated. Specifically, they name four dimensions of “mathematics knowledge activated by teachers when reading and interpreting mathematical tasks, instructional designs, and representations in mathematics curriculum materials” (p. 66). Collectively, they say, these dimensions constitute teachers’ knowledge of curriculum embedded mathematics (KCEM). The four, perhaps overlapping, components of KCEM that they identify are instances of curriculum-use drawing on: foundational mathematical ideas (viz., Dimension 1), representations and connections across them (Dimension 2), relative problem complexity (Dimension 3), and mathematical learning
pathways (Dimension 4). Unlike the KQ, though, the KCEM refers to teachers’ use of curriculum materials and not just episodes of instruction.

Dimension 1 of KCEM is roughly equivalent to identifying and understanding the mathematical objectives of activities or lessons. Dimensions 2 to 4, though, reflect teachers’ perceptions of mathematical coherence within lesson materials, even though Remillard and Kim (2017) do not explicitly frame them in such a fashion. Nonetheless, teachers clearly perceive coherent pathways when they identify relatedness of representations to one another, akin to the connectedness with deeper ideas (Ball, 1990; Schmidt et al., 2002). They also perceive coherence through curricular tasks that progress mathematics toward more sophisticated ideas. For instance, Remillard and Kim (2017) note that a teacher “conjectured that the partial quotient method [for division] would eventually lead [emphasis added] students to use the long division algorithm with understanding” (p. 77). This, likewise, mirrors Schmidt’s and colleagues’ (2002) conception of coherence, through the conceptual linking of ideas.

Remaining questions. At the same time, recent studies of activated knowledge in using curriculum materials continue to raise questions about what coherence means and how it is achieved. Choppin (2011) draws on Brown’s (2009) definition of PDC to argue, on the one hand, that an effective teacher may reformulate written lessons and, hence, “adopt curriculum resources in ways that build from and extend the designers’ intentions” (Choppin, 2011, p. 351). Stated differently, teachers with strong PDC, as well as a strong understanding of the design rationales of instructional materials, might depart from the sequence offered by the literal lesson (S. Brown et al., 2009) and yet still enact lessons in keeping with the authors’ underlying pedagogical philosophies. Learned adaptations might involve attending to problem complexity in different ways, as the Choppin’s (2011) example of Margaret shows. On the other hand, Remillard and Kim (2017) suggest that, as teachers interpret materials, perceiving tasks ordered by increasing complexity is perhaps an indicator of their KCEM. Remillard and Kim note, though, that future research still needs “to develop additional tools for measuring or analyzing KCEM and to uncover the impact of writing curriculum materials with KCEM in mind” (p. 75). And as does Choppin (2011), Remillard and Kim (2017) also call for additional research “to understand how teachers interpret and make use of a variety of different types and designs of curriculum resources” (p. 75).

Into this space, my own study—on which I report in this thesis—falls. I focus on coherence of lessons and written materials, as indicated here, because coherence continues to be an under-
specified notion. In addition, unpacking coherence is one vehicle for understanding adaptations teachers make to materials, especially whether adaptations introduce incoherence or “extend designers’ intentions” (Choppin, 2011, p. 351). In their position paper on open educational resources, the NCTM (2016) even observes that curriculum programs can exhibit coherence in a variety of ways, “pedagogically, logically, conceptually, in terms of learning science, and with the real world” (para. 1). The NCTM regards coherence as an asset that programs offer and cautions that open educational resources are potentially less coherent. And yet, with this breadth of possibilities, how can coherence and teachers’ adaptations of materials be effectively determined? I suggest attending to mathematical storylines and plots would help address this question.

Coherence, then, could be situated within a broader “cognitive structure…[that] corresponds with the context meant by the author” (van Dormolen, 1986, p. 151). In this case, the context would be a mathematical-epistemological one (Pimm, 2006). Recall that Fernandez and colleagues (1992) describe the connectedness of classroom events as akin to the connectedness of events in a story. Therefore, I now summarize literature on mathematical narratives, to bolster my argument.

### 4–2. Mathematical Narratives

Even though it may seem far afield, I begin this section by referencing literature on courtroom discourse. Cotterill (2002), as well as Jackson (1988, 1996), observe that narratives emerge robustly within courtroom presentations—stories of guilt, innocence, evidence, crime, victims, and so on. Even more, courtroom narratives contain many layers, largely due to complexities involving who is speaking and about whom speech is offered. These scholars, broadly speaking, therefore distinguish between narrated and narrating events—or in Jackson’s (1996) words, stories in the trial and stories of the trial (p. 27). This distinction shows that there are, on the one hand, narratives constructed by storytellers (or lawyers, witnesses, etc.); these are narrating events. On the other, the events described by these storytellers occurred on their own terms and timelines (i.e., the narrated events).

In mathematics education, narrating events constitute the discursive presentations made by teachers (and, quite likely, their students). Narrated events, in contrast, constitute the story of the mathematics. Stated differently, the narrated events of mathematics represent the epistemological generation of ideas. In what follows, I review narrative theory to explore, mainly, the narrated events of mathematics instruction—how mathematical ideas may be constructed as narratives. As I referenced previously and as I explain further below, Dietiker’s (2012, 2013b, 2015a)
mathematical story framework (MSF) largely distinguishes between narrated and narrating events found within mathematics curriculum materials. At the end of this section, when discussing the influence of mathematical storylines and plots on learning, I transition to exploring the narrating events that occur in classrooms.

**Narratological Foundations and Teachers’ Work**

More than a potential tool for understanding lesson coherence, constructing storylines or narratives are nonetheless already considered by some as central to teachers’ work. Elbaz (1991), for instance, makes an impassioned case for equating teaching with narrative construction. Drawing on the work of Clandinin and Connelly (1992) and others, Elbaz argues:

> ...story is not that which links teacher thought and action, for thought and action are not seen as separate domains to begin with. Rather, the story is the very stuff of teaching, the landscape within which we live as teachers and researchers, and within which the work of teachers can be seen as making sense. This is not merely a claim about the aesthetic or emotional sense of fit of the notion of story with our intuitive understanding of teaching, but an epistemological claim that teachers’ knowledge in its own terms is ordered by story and can best be understood in this way. This constitutes an important conceptual shift in the way that teachers’ knowledge can be conceived and studied, and it is also (in my opinion) the direction in which the field should be heading. (p. 3)

Narrative, to some, is a natural window on teachers’ own professional identities. As the literary critic Mary McCarthy (1961) observes, we have a tendency to perceive ourselves as “the hero of our own story” (p. 190). Elbaz (1991) goes further, though, and instead claims that teaching is narrating, by definition, and that teachers’ professional knowledge is a form of narrative knowledge. From this viewpoint, the epistemology of teaching involves knowledge of stories and about constructing stories. Elbaz argues that adopting a narratological lens on teaching would benefit for researchers, as well, because of its proximity to the actual work of and capacities needed for teaching.

**What is narrative?** Elbaz (1991) claims that “teachers’ knowledge in its own terms is ordered by story” (p. 3). If we assume this statement to be true, it might be easily consonant with instruction in literacy or history. But how can we reconcile Elbaz’s view in relation to science, technology, or mathematics? To do so, we would need a clearer definition of narrative and narratology in relationship to these subjects and we would also need to delineate boundaries of narrative carefully; otherwise, we might subsume too much educational theory under an overly general, and hence meaningless, abstraction.
As media and language diversify, what constitutes narrative, though, is no longer clear-cut and widely-accepted. Some, for example, might have trouble comprehending the narrative aspects of comic strips. After all, comic strips are fractured (by the frames themselves) and have a narrow scope, thereby preventing the development of narrative depth or flow. On the other hand, Bal (1985 / 2017) contends that though the medium itself is atypical comic strips still accomplish narrative work (p. 4). Bal therefore offers a contemporary definition of narrative, one that would encompass narratives found in newer media, as “a set of logically and chronologically related events that are caused or experienced by actors” (p. 5). Comic strip characters are certainly memorable, and the events that befall them are all logically, and sometimes chronologically, related within one strip.

Furthermore, Bal (1985 / 2017) explains that narratology is a field of research or an “ensemble of theories of narratives, narrative texts, images, spectacles, events—of cultural artefacts that tell a story” (p. 3). These theories, Bal (1985 / 2017) asserts, help us “understand, analyse, and evaluate narratives” (p. 3). Narratologists also explore the seeming omnipresence of narratives. Beyond comic strips, narratives are now thought to include not only novels and films, but also court cases, everyday speech, and even recipes (Labov, 1997). In so doing, scholars contend that texts, media, and artifacts should not be classified as narratives in a binary, yes or no, fashion; instead, narrativity is now regarded as a scalar quantity (see Page, 2015). Narratologists explain, then, that artifacts exhibit degrees of narrativity, depending upon the manner in which they portray characters and transpiring, related events. Recipes, as one example, exhibit narrativity (although not necessarily in the same sense as a traditional novel), because ingredients are depicted as interacting with one another, over time, until the climactic dish is said to emerge (Labov, 1997).

As additional, emotive evidence that recipes are narratives—particularly as they are read and used—consider this: what cook hasn’t reacted with surprise, when trying a new recipe and observing the moment the ingredients either finally coalesce or hopelessly clash?

Narrativity and mathematics teaching. Narrativity, as Elbaz (1991) suggests, can also be tied to broader epistemological concerns. She argues that thinking of teaching as the constructing of narratives doesn’t just “fit of the notion of story with our intuitive understanding of teaching” but it also makes “an epistemological claim that teachers’ knowledge in its own terms is ordered by story” (p. 3). To explain, I draw on Bruner (1986), who theorizes that there are two modes of knowing, narrative knowledge and paradigmatic (or scientific and mathematical) knowledge. Paradigmatic knowledge or paradigmatic thought is characterized by hypothesis, logic, and
empirical observation. In contrast, narrative thought is characterized by “human or human-like intention and action and the vicissitudes and consequences that mark their course” (p. 13). Bruner argues that narrative thought concentrates on specific times, places, and characters, while paradigmatic thought is abstract and timeless (p. 13). Bruner (1986) firmly separates these two modes, however. In other words, he disagrees with any intertwining of narrative and paradigmatic (or mathematical) thought, arguing instead that paradigmatic and narrative knowledge are “irreducible to one another” (p. 11).

**Readerly and writerly texts.** After grappling with Bruner’s distinctions, Sinclair (2005) offers a synthetic reconciliation. In this way, Sinclair helps realize Elbaz’s (1991) goal, namely, that “teachers’ knowledge can be conceived and studied” (p. 3) as narrative. First, Sinclair (2005) borrows from Barthes (1974) in observing that texts can be either readerly or writerly. As she explains, readerly texts “create an illusion of order and significance” whereas writerly texts “force the reader to produce a meaning or set of meanings” that are “not universal and absolute” (Sinclair, 2005, para. 2).

The contrast between readerly and writerly texts, in my view, is best exemplified by the novels of Dickens and Joyce, respectively. On the one hand, we as readers inhabit the world that Dickens created. We do not live in Victorian England, but we can envision it from Dickens’ creations. There is, nonetheless, emotional, chronological, and geographic—readerly—distance. On the other, I would argue, the world of Joyce inhabits us. We see ourselves and our everyday conflicts mirrored by the characters and events of *Ulysses* or *Finnegan’s Wake*. We write ourselves into Joyce, so to speak, because of its visceral familiarity. Indeed, McCarthy (1961) argues that a reader of modern fiction “wakes up in a foreign consciousness, a bundle of impressions, not knowing where he is” (p. 191).

**Mathematical proofs and textbooks.** Sinclair (2005) subsequently describes traditional mathematical proofs, which aim for brevity and precision, and she contrasts these with alternative forms of mathematical texts that she claims exhibit story-like qualities. For example, she portrays an activity involving the exploration of patterns within digits of decimal expansions of fractions—patterns that emerge when each numeral is mapped to a specific color in a representational image. Exploring these patterns, teachers and students are likely to imbue the affiliated numbers with personality, characterizing the terminating decimals as “sad” in comparison to their repeating, florid cousins. In studying these images, students are likely to understand that the behavior of repeating decimals follows from the iterated calculations that
yield the same remainder. Sinclair’s accounts of mathematical events become mathematical stories, insofar as their numerical characters exhibit emotions (para. 22) and are imbued with “personal and provisional meanings” (para. 24) that can “lead to imaginative interpretations” (para. 31).

These sorts of stories, as they emerge in mathematics textbooks (and, perhaps, teaching), depart from typical mathematical proofs, in part, because each serve distinct purposes. A proof intends to explain, as succinctly as possible, a discovery, a construction, a result—representing new knowledge—to the scientific community. In contrast, mathematical textbooks generally aim to help students see how mathematics is done—to reconstruct, in some form, the process by which the ideas were created or discovered. A mathematics text seeks to place students in the role of a working mathematician, not unlike the way Joyce seeks to place readers in the role of Stephen Daedalus. This distinction, of course, reflects the contrast between readerly and writerly text, the contrast between readerly consumption of mathematical knowledge and writerly engagement in the reconstruction of mathematics. From this observation, Sinclair (2005) consequently asks, “How do we go about creating materials for students that exemplify this kind of narrative, or ‘writerly’ possibility in mathematics?” (para. 24). She implies that many mathematics texts—despite their intended purposes—are not actually writerly, just as classic novels are not writerly.

Dietiker (2012, 2013b, 2015a) argues that Sinclair’s (2005) hypothesis is somewhat incomplete. She demonstrates that mathematics curriculum materials don’t necessarily require writerly embellishment. Dietiker (2012) argues, instead, that many mathematics texts, used for instructional purposes, can be read as mathematical stories as written. Like Sinclair, Dietiker (2012) concedes that not every mathematical text can be regarded as a mathematical story; she explains, for instance, that a mathematics glossary does not possess narrative qualities because it does not portray any sort of transformation (pp. 107). But she questions Sinclair’s belief that formalized proofs are not narratives. Drawing on Pimm (2006), Dietiker (2012) maintains that even formal mathematics proofs exhibit a form of temporality in that the objects defined within the proof undergo changes as the exposition proceeds (p. 105). With this observation, she then establishes that “to be a mathematical story, a reader must be able to recognize a sequence of mathematical events that connects a beginning with an ending” (p. 106). Before describing Dietiker’s work more specifically, I now outline additional features of narrative that support her assertions, specifically what plot might mean in the context of a mathematical story.
Conceptions of Plot

First, Labov and Waletzky (1967) and Labov (1997) offer a helpful framework drawn from their investigation of personal narratives. Within their framework, they first define what they call “a narrative of personal experience” as “a report of a sequence of events that have entered into the biography of the speaker” (Labov, 1997, Section 0, para. 1). The inclusion of biography, Labov (1997) explains, preserves the social and emotional import of a personal narrative, contrasting personal narratives with, say, a more journalistic chronicling of the “events of a parade by a witness learning out a window” (Section 0, para. 2). This contrast does not mean to imply that impartial chronicles aren’t also narratives, but rather that personal narratives served as Labov’s and Waletzky’s (1967) prototype for understanding the structure and function of everyday monologues in social situations (Labov, 1997, Introduction, paras. 3–5).

Narrative phases. Analyzing how narrators told stories, Labov and Waletzky (1967) identified several phases of prototypical narratives, or what they refer to as narrative functions. These phases of narrative occur in a general sequence: the abstract, orientation, complication, evaluation, resolution, and coda. An abstract foretells the entire story but in abbreviated fashion. The orientation establishes the fundamental characteristics of the players or the setting. Complicating action consists of a string of events leading to a climactic point that, when made clear, is known as the evaluation. The resolution consists of events that follow the evaluation, while the coda reiterates the narrative in abbreviated form. Not all narratives exhibit all phases and not all phases proceed in the exact order described (Labov & Waletzky, 1967, p. 37).

Through the elocution of these phases, a narrator charts the what we would normally describe as the plot. Labov and Waletzky (1967) and Labov (1997) typically refer to the plot (singular) of a narrative, but a sequence of narrative phases may occur multiple times and in overlapping ways within a larger story. For example, complex narratives forms are found within Seinfeldian tragi-comedies (Smith, 1995). Labov and Waletzky (1967) also offer a map (Figure 4) that depicts these phases, familiar to many grade schoolers as a plot diagram (cf. Freytag, 1863 / 1894). Freytag’s plot diagrams show the rising and falling action that surround the climax of a story. Note, briefly, that Freytag’s (1863 / 1894) diagrams were born out of Aristotelian concerns for representing the idealized forms of artistic representations, similar to Labov’s and Waletzky’s (1967) and Labov’s (1997) goal in identifying the normal form of a conversational, personal narrative.
Labov and Waletzky (1967) documented the complex speech patterns of people of color and others from marginalized groups. One important outcome of their research involved raising critical awareness of and resisting stereotypes about the intellect of those who have long been derided for communicating in ways some regard as non-standard. And in so doing, Labov and Waletzky laid foundations for what is commonly known as the narrative turn within sociological and anthropological research. Employing their framework, other researchers subsequently identified narrative structures within two-party and multi-party conversations, therapeutic interviews, and so on (Labov, 1997).

**Narrative layers.** Bal (1985 / 2017) describes another essential narratological construct, plot, by distinguishing among what she calls *layers of narrative* (p. 5) I mentioned these, briefly, in Chapter 1. One such layer, the *fabula*, consists of “a series of logically and chronologically related events that are caused or experienced by the actors” (p. 5) or characters. Another layer, sometimes known as the *syuzhet* (or *story*, more commonly) is the “particular manifestation...of a fabula” (p. 5). The fabula describes narrative events from the perspective of characters, who experience them as a logical progression. The story, on the other hand, is how the fabula is presented to an audience. As we watch movies or hear or read stories, of course, the tale may proceed from the logical end to the logical beginning (as with the Modern Family joke from Chapter 1 or the famously backwards Seinfeld episode). Events may also foreshadow those yet to come.

Narrative progress, Bal (1985 / 2017) explains, is the interaction between layers. We perceive the initial events, which raise questions in our minds. These are answered as the underlying fabula unfolds. To build suspense, to keep us questioning, the fabula is often doled out in unpredictable ways. This lack of predictability is manifest through a) deploying and attending to multiple

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**Figure 4.** The *normal form* of a narrative (Labov & Waletzky, 1967, p. 37), similar to a depiction of Freytag’s (1863 / 1894) dramatic pyramid (“Freytags pyramid.svg”, n.d.).
storylines, simultaneously, with the plot shifting from one to the next, and b) events not following one another temporally. And unpredictability is an essential feature of plots; after all, a completely fore-known story—one wholly predictable, because of close alignment between the syuzhet and fabula—would be uninteresting. As Dietiker (2012) explains, plot therefore includes an aesthetic response instead of just consisting of structure of events. In sum, plots are the recursive interactions between what is known and yet to be known or, put another way, between the questions raised by the syuzhet and the emerging answers within the fabula.

**Mathematical Narratives and Plots**

Distinctions between the fabula, syuzhet, and plot are not just academic claptrap. They help explain why some narratives—in mathematics texts and classrooms, and as part of the curriculum-design process—seem more intriguing than others. Mathematical plots that are constructed to elevate narrative tension, wherein there are significant differences between the fabula and syuzhet, promote interest. As I argued in Chapters 1 and 2, interest and engagement are potent enablers of learning (e.g., Csikszentmihalyi & Hermanson, 1995; Dewey, 1913, pp. 54–55; George, 2014).

I now summarize research in relatively novel line of inquiry on mathematical plots. I begin by reviewing existence-proof research on identifying mathematical plots in curriculum materials. I then explore mathematical plots and coherence of presentation, as the theoretical bridge between plots and learning. This bridge also adds nuance to the meaning of coherence. Finally, I conclude by describing research connecting mathematical plots and learning.

**Examples in texts and in classrooms.** Dietiker (2012, 2013b, 2015a, 2015b) provides a number of examples of narrative tension through deployment of mathematical plots in curriculum materials. Using her theory and the MSF, she analyzes written materials that exhibit particular mathematical characters, settings, events, and so on. (See, in particular, Dietiker, 2015a, 2016 for additional details on the MSF.) The mathematical plots of these texts emerge, as mathematical events are foreshadowed or stalled. This foreshadowing and stalling serve to build suspense. For example, Dietiker (2012) reviews one textbook lesson in which the reader is essentially asked, early on, “Can you predict the end state?” (p. 208). When raising this question about predictability, the text references a geometric process with uncertain outcomes. After raising this question, the authors then hint that, perhaps, predictability may not even be possible. Ironically, at

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1. I refer the reader to the definitions of these terms, offered in Chapter 2.
the same time, they equivocate and pose problems that suggest an end state might be knowable. Dietiker reports that she was left in suspense long enough—without a clear resolution—that she read through to the next lesson, ultimately, to learn the answer (pp. 208–210). Stated differently, Dietiker demonstrates that mathematical texts, even those that seem initially austere, can embody what Bruner (1986) regarded as a narrative (or non-mathematical) mode of thought.

And as a classroom example, Dietiker (2016) describes how a plot twists can emerge during instruction. Plot twists, she also demonstrates, can be rewarding for teachers and students. To do so, Dietiker studied a first-grade classroom in which a teacher displayed asymmetrical figures and elicited gasps from students. Dietiker attributes their gasps to surprise, because of their budding realization that not all shapes are symmetrical (p. 154). (Many of the shapes explored in early grades—squares, rectangles, circles, and so on—are, in fact, symmetrical.) This activity evolved in such a way, Dietiker argues, that students confronted their pre-conceived beliefs and needed to reconcile such beliefs with new information (pp. 158–163). In the enacted lesson, symmetric-only shapes were presented to students, first, before they confronted asymmetric shapes. An alternative to this plot progression could have involved discussing symmetry before undertaking a side-by-side comparison of both symmetrical and asymmetrical figures. One wonders if this latter progression would have been as intriguing for students.

Finally, Dietiker (2015b) also observes that, when designing and—importantly—field-testing written lessons, she and her co-authors were “able to purposefully test and tweak these tasks based on whether students were excited (or not)” (para. 31). They revised written lessons, based on students’ initial reactions to early drafts, to promote increased engagement in the next round. These observations suggest that excitement, derived from variations in storylines of lessons, may contribute to students’ learning. As mathematics curriculum materials convey mathematical narratives, then, they elevate tension, maintain rhythm, and build anticipation—all of which may compel “a reader to keep reading” (Dietiker, 2012, pp. 96–98) or a learner to keep on puzzling. In fact, I address findings related to mathematical narratives and learning, below. Regardless, understanding the relationship between fabula, syuzhet, and plot of mathematics texts helps us understand the aesthetic impact of mathematical stories (Dietiker, 2012; Ryan & Dietiker, 2018).

**Relationship of mathematical narratives to coherence and fidelity.** Dietiker (2012) states that “not every mathematical event must be related to those before and after” (p. 107). Sequences or events in mathematical narratives need not to run in linear order. Analogously, movies and other dramatic performances often foreshadow something that actually occurs much later. From the
outset, for instance, we know the eventual fate of Verona’s “star-crossed lovers” (Shakespeare, n.d., Prologue to Romeo and Juliet, 6–10). Further, some storylines are suspended, leaving us to wonder what happens to the characters, until much later in the book or movie. What matters most, when considering these sorts of fabula manipulations, is whether or not we can reconstruct—or imagine—what the actual sequence of events would be. Through imagining or interpreting, we surface questions about the characters we want answered—the essence of narrative tension and plot (Bal, 1985 / 2017, p. 83).

The same can occur, Dietiker (2012) shows, in mathematics textbooks. She explains: “In terms of mathematics curriculum, therefore, a mathematical story is the interpretation [emphasis added] of the chronological sequence of mathematical changes (‘events’) in a mathematics textbook by a reader” (Dietiker, 2015a, p. 288). It is therefore the reader’s capacity to reconstruct sequence that determines the pathway of mathematical story, not some pre-determined order that is spoon-fed by the authors. What distinguishes mathematical narratives from non-narratives (like glossaries) is whether a reader has some assurance that the mathematics “is making progress toward a goal of some kind” (Dietiker, 2012, p. 107). Thinking of mathematical narratives as connected experiences, coherence is therefore exemplified by the change experienced by mathematical characters, regardless of whether specific events logically or chronologically follow one another. In fact, under this line of thinking, if the events are not necessarily sequential, the cognitive work we undertake to attempt the reconstruction is what makes the plot—narrative, generally, or mathematical—more interesting. Our interest and our reconstruction are what cohere the events.

Note, again, that this perspective contrasts sharply with more linear interpretations of coherence (e.g., Abdussalaam et al., 2015). Returning to narrative theory from legal studies, for instance, Wagenaar and colleagues (1993) define narrative coherence as “a full and compelling account of why the central action should have developed” (as cited in Jackson, 1996, p. 24). Further, Wagenaar and colleagues (1993) also describe two forms of ambiguity that, they say, undercut coherent presentation: missing or contradictory elements (as cited in Jackson, 1996, p. 24). It seems they would not regard re-sequenced events, like those in Pulp Fiction, as examples of narrative coherence. Indeed, Jackson (1988) even insists that coherent narratives are those “arranged in a time sequence” (p. 2).

As this thesis intends to demonstrate, however, elements of narratives that are missing, contradictory, or re-sequenced are not necessarily examples of incoherence. The key to coherence isn’t sequencing, but rather the purposes, presentations, and impact of events. My perspective,
and presumably Dietiker’s (2012, 2013b, 2015a), is that coherence is a much more complex phenomenon than most generally conceive it to be.

Returning to the MSF, Dietiker (2012) also explains that mathematical stories typically consist of several mathematical *story arcs*. (As noted in Chapter 2, what she calls story arcs, I call storylines.) Story arcs are portions of narratives, relating particular characters and events, and they occur in tandem with other story arcs involving different characters and events (pp. 176–177). Multiple mathematical storylines may comprise a single lesson (Richman et al., 2016). Conversely, an overarching mathematical narrative (or lesson) may consist of multiple related storylines (like in a television episode). For example, a lesson on adding fractions may also involve storylines on equivalence, fraction models, the addition operation, a particular set of data, and so on. Analogously, a story arc in the second episode of the third season of *The Big Bang Theory* involves a cricket. The characters Sheldon, Howard, and Raj are distracted by its chirping and begin searching for it; the question raised, which the viewer wants to see resolved, is “Will they find the cricket?” Several other story arcs are also spun out of this simple event (Lorre, Prady, Reynolds, Ferrari, & Cendrowski, 2009). In this episode, the various storylines are pursued with varying intensity at various times. We don’t fully appreciate the complexity of the plot until we reflect on how the pieces, ultimately, all fit together.

Again, the same can occur in mathematics lessons—such as the one on fractions outlined earlier. And so, we learn from this perspective that obtaining a picture of a lesson’s coherence should perhaps involve looking at storylines both severally and together. Dietiker (2015a) writes:

> Although mathematical objects, processes, and representations have long been studied independently for their important roles in mathematical work, it is the coordination of all of these aspects of mathematics together in a [narrative] structural conceptual metaphor (Lakoff & Johnson, 1980) that enables one to recognize how the parts work together to make (or not make) a coherent and aesthetic whole. For example, the mathematical objects of curriculum can certainly be recognized without the story framework, but within the framework, their temporal changes, interactions, interdependence, as well as the influence of setting and action on recognizable characteristics can be recognized and evaluated. Likewise, this framework draws attention to shifts in representations and the changing complexity of procedures. (p. 299)

Dietiker, here, explains that narrative structure is what shapes the coherence of content (or its lack of coherence). If it exists, coherence may be exhibited within a given lesson, across lessons in a school year, and across multiple school years. This certainly reflects Schmidt’s and colleagues’ (2002) two-pronged definition of mathematical coherence. In other words, Dietiker (2012) also acknowledges both local and global considerations when evaluating coherence. Seeing coherence
as the strict sequencing of ideas, only, may overlook the narrative potential of other, non-sequential types of presentations (e.g., overlapping, interrupted, re-ordered, etc.).

At the same time, briefly, I emphasize my own interest remains within particular lessons and how teachers interpret lesson-level guidance, even if such lessons reference broader mathematical structures and themes (across a unit or a year). This thesis reports on my close readings of mathematics texts and their affiliated lesson enactments at a highly local level.

Further, if narrative represents the work of teaching (Elbaz, 1991), how curriculum materials support narrative-construction should also be an important area of inquiry. This suggests a different conception of fidelity of implementation (Richman et al., 2016). Indeed, a mathematical narrative or storyline, insofar as it represents the development of an underlying mathematical idea, certainly represents a “cognitive structure in the reader’s mind which corresponds with the context [emphasis added] meant by the author” (van Dormolen, 1986, p. 151). Broader than the specific activities or tasks of the written lesson, I claim, the underlying epistemological progression constitutes such context. Therefore, how materials support this underlying progression and allow for variation in storylines is an intriguing possibility.

Dietiker (2013a) argues likewise, proclaiming that the MSF:

...disrupts conventional ways of understanding the mathematical content of textbooks and invites the creation of inspiring new mathematical stories. If a novel can be appreciated for its rich characters or its sudden surprises, then why not a mathematics textbook? Although it may be unorthodox to consider mathematical objects and activity in these “novel” ways, conceptualizing the unfolding of mathematical content in a textbook as a mathematical story allows new questions to be pursued, such as what propels this mathematical story forward? How does this mathematical story build curiosity and desire to learn what will happen? What different (and new) types of mathematical stories can we find or design? (p. 19)

The goal of textual analysis, to explore teachers’ implementation and fidelity, isn’t to uncover the “correct” interpretation of a mathematical narrative in curriculum materials. Dietiker (2012) also draws on reader response theory (Rosenblatt, 1988) to explain that multiple interpretations are possible. Rather, the goal is to uncover the narrative potential of mathematical texts. As Dietiker (2012) explains, pinning down the exact narrative “structure itself is not the goal” but that, instead, “it is the reading for [emphasis added] structure that offers new insight” (p. 44). In so doing, by reading itself, the analyst gains insight into ways the text supports or undercuts teachers’ narrative presentation in the classroom.
Research on the influence of mathematical storylines and plots on learning. As indicated before, much work remains to be accomplished in this area. Evidence is just dawning that mathematical storylines play a role in learning. Explaining the theory, Dietiker (2013a) states that different mathematical plots “can affect both the experience for the reader and the nature of his or her mathematical conclusions” (p. 19).

Students’ engagement. For instance, as explained above, Dietiker (2016) reports on an empirical, qualitative study of learning in a first-grade lesson on symmetry. When the teacher dramatically reveals the non-symmetric shapes, she follows this move with the simple question, “What about these?” (p. 7). Dietiker shows that the students are motivated to investigate lines of symmetry in others shapes. They stay engaged. This eventually leads them to conclude that not all two-dimensional shapes are symmetric. Of course, some might contend that the teacher could have moved more quickly to presenting asymmetric shapes. Research has shown, though, that offering students answers to questions they may not fully understand, nor have had the chance to grapple with, can actually disrupt the learning process (see Fuson, Kalchman, & Bransford, 2005). The value of developing an idea, gradually, is therefore what underpins narrative tension and Dietiker’s (2012, 2015a) MSF.

Indeed, as another example, Dietiker (2014) reports on a set of enacted high school algebra lessons. She compared narrative structures of implemented, classroom lessons to students’ levels of captivation from classroom instruction (p. 2). Captivation was quantified through survey questions on whether activities in their classrooms held students’ interest and whether the teacher made learning enjoyable. Measures of classroom captivation were reported along with videos in a database from which lessons were selected. The theory suggests, once again, that engagement contributes to students’ learning (Dewey, 1913, pp. 54–55; George, 2014). Dietiker (2014) reports that classrooms deemed highly-captivating had fewer mathematical storylines with more mathematical events than those deemed less so. As she explains, these “high-captivate classrooms may tackle fewer mathematical questions overall but may make more points of progress while answering them” (p. 8). This structure may allow greater opportunity for ideas (and potential plot twists) to emerge.

Even still, Dietiker (2014) cautions that the survey may not adequately capture the influence of narrative structures on students’ perceived levels of engagement; instead, the survey—assessing students’ perceptions of their classrooms, overall, rather than beliefs about any
particular lesson—may have reflected their feelings about mathematics or other confounding factors (pp. 12–13). Untangling this possibility would be a locus of potential, future research.

Dietiker and her colleagues also report on teachers who use the same curriculum materials and enact lessons by re-ordering and altering questions, differently than specified by the text (Richman, Dietiker, & Brakoniecki, 2016; Richman et al., 2016). These findings are, of course, drawn from a highly granular level of analysis, differing from previous research on teachers’ curriculum-use. In addition, the focus of Dietker’s and her colleagues’ research, here, is on understanding the impact of purposeful re-sequencing on students’ engagement.

Richman and colleagues (2016), specifically, classify several types misdirection in high school algebra lessons. These misdirections, they explain, were intentional moves made by teachers that created moments during lessons when students recognized mathematical contradictions. For example, one teacher led students to believe they could guess the equation of a parabola given only one or two of its points. When contradictions of this sort emerged in classroom lessons, furthermore, they “led students to ask their own questions about the mathematics, thus increasing their investment in the outcome of the lesson” (p. 111). In the case of the parabola lesson, Richman and colleagues say that the students ended up pressing the teacher for more clues to identify the equation in question. Such misdirections were classified into several types of plot twists. Plot twists were first hypothesized and identified by Dietiker (2012, 2015a, 2016).

Impact on coherence. Unlike previous research on coherence, Dietiker and her collaborators also show that lessons with re-ordered, non-sequential questions can provoke various aesthetic or emotional responses that, in fact, could be productive for learning (Dietiker, 2016; Ryan & Dietiker, 2018; Richman, Dietiker, & Brakoniecki, 2016; Richman et al., 2016). Ryan and Dietiker (2018) report on a fifth-grade teacher, who purposefully modified the sequence of questions in her curriculum materials. She created, intentionally, a gap in the mathematical events that some might regard as incoherent. As this teacher reports, her students then observed and wondered: “‘How can we have found two different volumes for the same prism?!”’ (p. 321). The teacher had anticipated students’ potential confusion about measurement units during a geometry unit and structured instruction, so that this seeming paradox emerged. In so doing, Ryan and Dietiker report, students were motivated to pursue further inquiry—to clarify the confusion.

Consequently, Dietiker and Ryan suggest that curriculum authors and teachers pay greater attention to mathematical plots of lessons. In particular, they advocate for alternative sequences that offer plot twists (Dietiker, 2016); plot twists are, loosely, manipulations of the mathematical
fabula, akin to the misdirections described by Richman, Dietiker, and Brakoniecki (2016) and Richman and colleagues (2016). Plot twist seem to introduce incoherence, but paradoxically, may actually help the experienced curriculum exhibit greater coherence.

Building on this work, Richman and colleagues (2018) reconceptualize coherence in mathematics instruction. They first reference a widely-accepted definition of coherence as “the extent to which the events and mathematical ideas of the mathematical story (i.e., a lesson) are connected to each other” (Richman et al., 2018, p. 4). Next, they borrow from Dietiker’s MSF and argue that coherence is evident to students, instead, when they “can make a prediction about what might happen” (Richman et al., 2018, p. 4). This predictability, they argue, allows for realization of narrative tension or surprise when expectations go unmet. Coherence and interest are inhibited, though, when students “cannot recognize any connection[s]” (p. 4) and are “prevented from being able to predict what will happen later” (p. 4). This, they speculate, also inhibits learning. Thus, the notion of coherence is redefined—not as the momentary connection of mathematical events, but rather, as the student’s capacity to imagine future connection.

In developing this perspective, Richman and colleagues (2018) reflect upon what they describe as a crisis in a sixth-grade lesson. This particular crisis arose as students encountered adding-integers problems they could not solve. The teacher had fomented the crisis by leading students to believe—falsely, through a prior set of exercises—they could address all types of such problems. Only when a new problem-type emerged did students realize they did not have the proper tools to address it. As Richman and colleagues explain:

Thus, the coherence supported by the overarching questions early in the lesson reinforced the surprise and crisis….As the lesson built toward the crisis, the evident confidence that students developed was not simply the result of a series of successes on a disconnected sequence of math problems, but was, potentially, a focused feeling of mastery in answering particular overarching questions. Therefore, when the expectation of mastery was violated by the presentation of a challenging pair of tasks [emphasis added] directly related to the overarching question, the potential for surprise and subsequent curiosity was that much greater. (p. 13)

In other words, a feeling of coherence was perpetuated throughout the lesson, but not by a set of increasingly-complex problems leading, ultimately, to a generalizable idea. Instead, students grasped the generalizable idea at the outset (like foreknowing the fate of Romeo and Juliet). They believed they already knew how to add integers and that all their work was part and parcel of the same. When that belief was intentionally violated, though, students grew interested in deepening their understanding and correcting their newly-acknowledged misperception.
Like the first-grade symmetry teacher (Dietiker, 2016) and the high school algebra teacher (Richman et al., 2016), this sixth-grade teacher intentionally left students hanging (cf. Lampert, 2001). This apparent discontinuity, Richman and colleagues (2018) speculate, is what enhanced students’ curiosity and promoted their eventual appreciation of sophisticated integer-addition rules. Nonetheless, aside from the paper by Richman and colleagues (2018), that many of these reports are conference presentations suggests room to build on this empirical base.

**Summary and conclusion.** Dietiker (2012) argues that her narratological view on mathematics textbooks, using the MSF, “offers a framework that later can be used to help teachers in their curricular design work (M. Brown, 2009)…” (p. 32). Dietiker sees construction of mathematical narratives as part of curricular design. Building on this claim, I argue further that a teachers’ capacity in reading, developing, and maintaining mathematical storylines and plots in written materials is a potential indicator of PDC. Mathematical narratives, Dietiker (2012, 2013b, 2015a) has shown, represent cognitive contexts of curriculum materials (van Dormolen, 1986). The degree to which teachers notice mathematical storylines and plots in written materials, and do something with them in the classroom, certainly represents a form of resource-mobilization within instructional design (M. Brown, 2009).

Recall that I began this chapter by discussing PDC and research on what teachers’ notice from curriculum materials. I transitioned to discussing narrative—and mathematical narrative, in particular—and its relationship to a novel way of framing coherence. Research, I explained, is also paving new ground that narrative structures may influence learning. Nonetheless, this research is suggestive, not definitive. And much remains unknown about the specific practices teachers use, or the ways that teachers navigate the development of, mathematical storylines in classrooms. I therefore turn to summarizing nascent findings on how teachers mobilize resources within lessons, what Sleep (2012) might describe as *steering instruction*.

**4–3. Steering Instruction**

Earlier, I noted M. Brown’s (2009) observation that teachers mobilize ideas from instructional materials in several ways. They offload responsibility onto the materials (pulling guidance directly from the texts), they adapt guidance in various ways, or they improvise— invent—tasks of their own. Brown also notes that:
It is the skill in weaving various modes of use together and in arranging the various pieces of the classroom setting that is the mark of a teacher with high PDC, not whether they happen to be offloading, adapting, or improvising at any given moment. (p. 29)

In other words, how teachers use materials in the service of their broader goals matters more than their particular pattern of use. In this section, I review research on another way of looking at how teachers follow through on their intended plans: steering (Sleep, 2012). Steering represents the moves undertaken (or opportunities missed) by teachers as they offload or adapt curricular guidance. Teachers’ steering moves provide a window into how they perceive instructional goals and work to accomplish them. As I hope to clarify, reviewing research on steering also ties together the literature on curricular noticing and coherence—through teachers’ interpretation and deployment of mathematical storylines.

**Mathematical Purposing**

I have already mentioned Sleep’s (2012) decomposition of steering instruction into several tasks that include “developing and maintaining a mathematical storyline” (p. 943). The full set of steering tasks, identified by Sleep, include:

1. attending to and managing multiple purposes,
2. spending instructional time on mathematical work,
3. spending instructional time on the intended mathematics,
4. making sure students are doing the mathematical work,
5. developing and maintaining a mathematical storyline,
6. opening up and emphasizing key mathematical ideas, and
7. keeping a focus on meaning. (pp. 942–943)

Sleep adds that the first four tasks involve maximizing the use of instructional time, while the rest “reflect the importance of coherence and meaning” (p. 943). Steering instruction, furthermore, is part of what Sleep calls “teaching to the mathematical point” (Sleep, 2012, p. 938). The work of teaching to the mathematical point involves recursively identifying mathematical objectives and orienting students to these goals (i.e., what she calls mathematical purposing, p. 938), followed by steering instruction toward the identified objectives.

Explained previously, as well, Sleep (2012) asserts that mathematical storylines generally consist of the progression of connected ideas. She also identifies several potential barriers that
undercut the development of storylines, such as: “Difficulty identifying mathematical connections across activities…” and “Not sequencing activities within the lesson in ways that promote connections or the progression of mathematical ideas” (p. 959). The tasks of steering and these pitfalls raise two key points. First, as Sleep notes, teachers who have difficulty developing and maintaining a mathematical storyline might have difficulty noticing mathematical connections across activities; they might not perceive the underlying narrative structures being represented within the materials. Second, as I suggested earlier, Sleep’s general perspective on coherence perhaps relies on the linearity of moving from one idea to another, more sophisticated one (i.e., the ladder approach). These points merit further investigation. Regardless, research on teachers’ steering practices, described above, offer insight into how teachers perceive resources as they mobilize features of written lessons during instruction (see Remillard et al., 2019). I therefore draw on Sleep’s tasks and pitfalls of steering to consequently explore how teachers unpack mathematical storylines.

**Design Arcs**

Remillard and colleagues (2015) and Remillard (2018) conceptualize two types of teachers’ curriculum design decisions: 1) planned decisions, and 2) in-the-moment design decisions (IMDDs). Planned decisions are those made in advance of implementing a lesson, and so planned decisions are also elements of Sleep’s (2012) mathematical purposing. IMDDs, on the other hand, are reactions “to how students respond to her [the teacher’s] initial prompt[s]” (Remillard, 2018, p. 489), and so would be examples of steering practices during instruction. (I believe there is some nuance, here; to the extent that teachers’ pre-plan responses to students’ contributions, this would represent anticipated steering.) As another way to think about IMDDs, Remillard and Geist (2002) offer the term openings in the curriculum to describe the space between instructions in the written materials and when teachers need to insert their own guidance and listen to students.

**Design decisions and steering.** Based on empirical study, Remillard and her colleagues propose a tool to help identify and analyze these two types of curriculum-design decisions, a tool they call a map of instructional design arcs or design arcs (Remillard, 2018; Remillard et al., 2015). Design arcs depict “instructional episodes that typically begin with a planned instructional prompt and follow with a segment of time during which the teacher guides classroom interactions toward a particular mathematical purpose” (Remillard, 2018, p. 491). Stated differently, design arcs depict both planned decisions and IMDDs, and therefore represent teachers’ steering work (see Figure 5).
Figure 5. An example of a map of one teacher’s design arcs for a given lesson (Remillard, 2018, p. 493).

As Remillard (2018) explains, the design arc map portrays the (enacted) lesson timeline on the horizontal axis. Each arc portrays a set of comments and questions, affiliated with a particular prompt or question. The height of an arc is arbitrary. The lines on which the arc is placed indicate whether the prompt appears in the written materials, teachers’ plans, or is adapted or improvised. As a tool, the design arc map allows for broad-scale comparison of teachers’ lessons, capturing teachers’ patterns of curriculum use, and enabling further, detailed analysis.

For example, Remillard (2018) describes two minutes’ worth of a teacher’s improvisations, working to help “students to connect the value of decimal numbers to related amounts of money” (p. 494). The decisions are IMDDs, because they were not explicitly supported by the curriculum materials or the teacher’s lesson plans. She seemed motivated to build, spontaneously, on a student’s idea about ordering decimals by counting by fives and, in the teacher’s words, “‘cause they like money”’ (p. 494). In addition, Remillard catalogues the progression of detailed questions asked by the teacher, over the course of this design arc, aiming to help students recognize the counting-by-nickels pattern. While she does not say so, I argue that Remillard’s detailed analysis of the design arc potentially enables us to discern the qualitative impact of the teacher’s design decisions. The sequence of steering moves and students’ responses, as the class works toward a particular mathematical goal, reveals whether students do appreciate the mathematical context and pick up on the suggested pattern.

Methodological considerations. I pause to offer a few methodological points, which are relevant for how I use a form of design arc maps in my own analysis. First, design arcs are reminiscent of Freytag’s (1863 / 1894) plot diagrams, showing rising and falling action in a narrative. Design arc maps, then, might be effective representations of mathematical storylines and plots. Second, typically, Remillard’s (2018) and Remillard’s and colleagues’ (2015) design arcs coterminate. That is, the next design arc begins where the previous one ends. In addition, arcs
generally do not overlap; this reflects a general premise that teachers and students can only attend to one task or mathematical prompt, explicitly, at a time. I note, though, that Sleep (2012), Lampert (2001), and others proclaim that teachers often attend to multiple purposes at one time. And Dietiker (2012, 2015a) and Bal (1985/2017) explain that multiple storylines may overlap with one another over the course of a longer narrative. Therefore, if design arc maps can be used to reflect mathematical plots and storylines (and show teachers’ steering work around mathematical narratives), then the maps will need to be adjusted to incorporate multiple, simultaneous storylines and plots. Examples are given of several, overlapping storylines and plots in the subsequent chapters of this thesis, as I explain my analysis. For now, generally, overlapping storylines connote multiple ideas that are variously foregrounded and backgrounded as a mathematical narrative evolves over the course of a lesson.

Empirical study of steering. Aside from Sleep’s (2012) conceptualization from her own empirical analysis, research on steering instruction is relatively limited. Much of the literature that draws on Sleep’s framework involves interventions intended to support pre- and in-service teachers in navigating student-centered classroom discussions (e.g., Baldinger, Selling, & Virmani, 2016). In general, researchers studying teaching to the mathematical point seem to presume clarity in what is meant by maintaining a mathematical storyline among the other steering tasks. Few study the specific steering moves teachers make, to understand how they steer instruction. Notably, steering is also studied outside of the context of instructional materials. This is a key oversight and represents part of the motivation for my own research. I elaborate on an exception below, a recent study conducted by Remillard and colleagues (2019).

Indeed, Remillard and colleagues (2019) analyzed supports within four elementary curriculum programs for mathematical purposing (Sleep, 2012). In so doing, among the other purposing categories, Remillard and colleagues (2019) identified written statements that offered “connections within and across lessons related to a particular goal or including the goal while introducing, summarizing or otherwise narrating the storyline of the lesson(s)” (p. 105). For example, they considered the following note within an Everyday Mathematics lesson as storyline support: “Tell students that in this lesson they will review a counting-squares strategy to find area and then use their scale drawings from the previous lesson to find the area of the classroom” (Remillard et al., 2019, p. 109). Here, the teacher is instructed to make a connection between a previous lesson and a forthcoming classroom activity. They then tabulated the prevalence of these types of supports across programs and specific lessons. They found that curriculum authors
provided uneven support for orienting instruction toward goals—with many stated goals not receiving any guiding statements, at all, and others receiving supports of varying specificity (Remillard et al., 2019, p. 110).

Next, Remillard and colleagues (2019) also classified and tabulated teachers’ steering moves during enacted lessons. They explain that teachers “did not steer uniformly toward all the goals identified in each lesson” (Remillard et al., 2019, p. 111). They also found, broadly speaking, that when programs offered more supports for mathematical purposing toward a given goal, teachers were more likely to steer instruction toward the goal (Remillard et al., 2019, p. 113). This suggests that curriculum authors should identify a reasonable number of instructional goals and offer multiple supports for helping teachers address them.

At the same time, notably, Remillard and colleagues (2019) didn’t study teachers’ stated reasons for making given steering moves during instruction. They note, for example, that one teacher emphasized the difference between perimeter and area of two-dimensional shapes, but they did not explore the teacher’s rationale for making such a move. This sort of unpacking represents a potential research opportunity. Specifically, researchers might gain additional insight into teachers’ decision-making by triangulating steering moves, instructional guidance, and their expressed justifications for making said moves.

4–4. Summary and Conclusion

In this chapter, I have reviewed coherence in mathematics instruction. Coherence has been characterized in several ways, including: fluid transitions between activities, relationships between activities and ideas, logical sequencing of tasks, and adherence to learning pathways (see Askew et al., 1997; Ball, 1990; Fernandez et al., 1992; Leinhardt, 1989; Rowland et al., 2005; Rowland, 2013; Remillard & Kim, 2017). Over the course of this chapter, I have generally maintained that clarity in this definition is warranted, in part, because of recent empirical findings that demonstrate that prior conceptions tend to lack sufficient complexity.

My review was grounded in the construct of PDC, explaining how teachers perceive and mobilize instructional resources to address their pedagogical goals. Drawing on previous research, I claim that coherence is one element of instruction that supports learning (e.g., Fernandez et al., 1992; Greeno & MMAP, 1998; Gresalfi, 2009). I reviewed the role of coherence in classroom teaching and learning, and I then connected this literature to mathematical narratives. In so doing—borrowing from Dietiker’s (2012, 2013b, 2015a) MSF and Dietiker’s and
her colleagues’ empirical work—I aimed to problematize the presumed construction of narratives within classrooms. While narratives are thought to offer coherence to mathematics instruction by avoiding certain ambiguities and relating key ideas, Dietiker’s and her colleagues’ work suggests that carefully-planned ambiguities or discontinuities may instead heighten students’ interest and enable their learning. This work suggests coherent lessons are those in which students’ are able to predict what might happen next (even if they are wrong).

To unpack this complexity further, I argued we must acknowledge and attend to various layers of text and narration. There are, in particular, narrated mathematical events (by teachers and, even, their students), which are distinct from the way events are related (i.e., the narrating events). Narrated and narrating events, furthermore, represent interpretations of textual cues in mathematics curriculum materials—cues which, themselves, exhibit two layers: 1) an underlying, logically- and chronologically-constructed fabula, and 2) a manipulated fabula, containing potential re-orderings and intentional ambiguities, known as the syuzhet. Curriculum materials are regarded vehicles for coherence (e.g., Remillard & Taton, 2015; Schmidt et al., 2002; Stein et al., 2007), but understanding precisely how they operate in the service of coherence demands a nuanced analysis of these layers.

I concluded my review by summarizing work on how narratives are constructed in classrooms, through the discursive work of steering instruction toward a mathematical point. Design arcs are tools that represent steering and, therefore, might be modified to portray narrative structure in mathematics textbooks and enacted lessons. After all, Cal and Thompson (2014) explain that such side-by-side comparisons are important: “The link between curriculum as represented in instructional materials (i.e., the textbook) and its influence on student learning cannot be understood without examining the curriculum as designed by teachers and then enacted as part of classroom instruction” (p. 9). And, furthermore, Dietiker (2012) establishes the role of narrative in such analysis, arguing that much “can be learned about mathematics curriculum when they [sic] are read aesthetically as stories” (p. 45).

As I explained in Chapter 3, however, undertaking such side-by-side comparisons also raises the hairy question about fidelity of implementation. Recall that fidelity, as it is typically construed, refers to teachers’ faithfulness to the literal, printed instructions in curriculum materials or their inclusion of curricular topics. Scholars in the field contend, though, that this sort of fidelity is difficult to assess; further, encouraging literal fidelity overlooks the role of teachers as curriculum designers and assumes materials offer more guidance than, in reality, they actually
can (Ben-Peretz, 1990; M. Brown, 2009; Remillard, 1999, 2005). S. Brown and colleagues (2009) therefore suggest an alternative view of fidelity that incorporates teachers’ demonstrated alignment with the epistemological and pedagogical underpinnings of a given program. At the same time, teachers’ beliefs and contexts play a role in their uptake of such guidance (Berman & McLaughlin, 1978; Remillard & Bryans, 2004; Davis et al., 2011), and so Stein and colleagues (2007) argue that certain programs might be regarded as fitting better within particular schools and districts than others.

Thinking of curriculum programs as resource-centric, rather than procedure-centric (M. Brown, 2009), offers potentially-new avenues for exploring fidelity. I contend, more specifically, that mathematical narratives are subtle, often overlooked resources within instructional materials. Following Dietiker (2012, 2013, 2015a), I likewise maintain that mathematical storylines and plots introduce the potential for suspense, or narrative tension, into classrooms. Suspense—literally and etymologically meaning “to hold up” (“Suspense”, 2018)—introduces purposeful discontinuity in classroom lessons, to elevate students’ engagement. To borrow from Schopenhauer (1969), the eventual release of suspense might be satisfying and productive, akin to laughter, as “the suddenly perceived incongruity between a concept and the real object that had been thought through it in some relation…therefore occasioned by a paradoxical, and hence unexpected, subsumption…” (p. 59). Suspense, then, might be considered a novel opportunity to learn within materials. Further, a teacher’s capacities to perceive and mobilize the affordances of mathematical narratives, known as PDC, may relate to the level of engagement within classroom lessons. And the notion of fidelity, therefore, might beneficially accommodate yet another dimension: an affective component deriving from the mathematical plot, as-written.
CHAPTER 5. THE SETTING (LOCATION 3):
FRAMEWORKS AND METHODOLOGIES

It’s miserable in Flobbertown, they dress in just one style.
They sing one song, they never dance, they march in single file.
They do not have a playground. And they do not have a park.
Their lunches have no taste at all, their dogs are scared to bark.
—Dr. Seuss, Hooray for Diffendoofer Day! (1998, lines 69–72)

5–1. Epistemological Foundations

In this chapter, I briefly tie together the previous two chapters. Collectively, they represent my theoretical framework. I also extend the previous two chapters here, to offer a synthetic picture of the key relationships between these bodies of literature as they are embedded within my own particular study. This synthetic picture, focusing on the constructs that are essential to my investigation, represents my full conceptual framework. After articulating my theoretical and conceptual frameworks, which ground my research, I then describe my methodological approach and specific analytic methods. First, though, I begin by explaining the epistemological foundations upon which the subsequent descriptions of my frameworks rest.

Assumptions about Knowledge in my Research

My work depends, ultimately, on both interpretivism and a philosophical framework complicating the relationship between know-how and know-that. Know-how consists of the knowledge inherent in the capacity to act, such as balancing on a bicycle. Know-that, on the other hand, is represented by facts and information, such as the principles of physics for calculating a bicycle’s precise balance-point. Scholars relying on know-that, as a lens for social science, have tended toward positivism and post-positivist critique. Even still, know-that is not monolithic. Empiricism, for example, is an influential school of fact-based epistemological thought, accepts facts as true when supported by observational evidence. Rationalism, in contrast, holds that facts can be accepted as true, but only when deduced by human intellect and reason. Kant (1781 / 1998) famously argued against dogmatic adherence to either epistemological stripe, rationalism or empiricism, asserting the interconnected roles of both in human factual knowledge. I attempt a similar project, below, synthesizing know-how and know-that.
Synthesizing know-how and know-that and practical reasoning in education research.

Unlike Kant’s synthesis of know-that rationalism and empiricism, Ryle (1949) insists upon maintaining a clear distinction between know-how and know-that, because in his words, it is “possible for people intelligently to perform some sorts of operations when they are not yet able to consider any propositions enjoining how they should be performed” (p. 30). In other words, Ryle contends that know-how and know-that cannot be easily intertwined, because we can ride a bicycle without knowing the underlying physics.

Synthesizing forms of knowledge and making epistemic claims. Writing about education, Fenstermacher (1994) nonetheless balks at Ryle’s (1949) dichotomous stance. He argues, instead, for a sort of synthesis of know-how and know-that and for “reconsider[ing] the epistemological character of what is and can be known by and about teachers and about teaching” (Fenstermacher, 1994, p. 4), on account of prevalent and limited “epistemological perspective[s] on what teachers should know and be able to do” (Fenstermacher, 1994, p. 4). Maintaining a narrow perspective, he maintains, increases the likelihood education-related initiatives are “grounded in weak or erroneous assumptions” (p. 4), which could, in turn, undermine solutions to the problems they purport to address.

Using a series of carefully constructed analogies, Fenstermacher (1994) contends that know-how and know-that are not equivalent to one another, nor are they entirely separable; rather, he suspects they are interdependent (p. 27). He argues, for instance, that having knowledge of how to play bridge requires factual knowledge about cards. And relying on the interdependence of know-how and know-that, he then offers an approach described as practical reasoning for attaching epistemic status to claims about teaching. This is primarily why I raise these points about know-how and know-that, because practical reasoning helps me justify the warrants I make in this thesis. In outlining practical reasoning, Fenstermacher writes that “the nature of justification shifts from the presentation of evidence, analogous to the uses of evidence in formal knowledge, to the development of ‘good reasons’” (p. 48). Practical reasoning, he also explains, can be applied to surface teachers’ tacit knowledge, to make it available for reflection and justification—such as through explanations arising from interviews, hermeneutic analysis, and so on.

Practical reasoning in my own research. Therefore, in like fashion, teachers’ own voices—explanations for their decision-making—feature prominently within my own study. Practical reasoning gives their voices epistemic status. Indeed, my warrants do not stem from exhaustive, large-scale data used to make universal claims. Fenstermacher (1994) observes that universal
claims in education research are sometimes thought to “ignore, distort, reduce, reconstruct, or purge the mental language of those studied” (p. 43). Instead, heeding this caveat, I aim to present cases of practice, undertaking a practice-based inquiry of teachers’ capacities in using curriculum (Ball & Bass, 2003).

Overall, my hope is to situate teachers’ design decisions within the contexts of their own goals, belief systems, and contexts; in so doing, my cases represent images of teachers’ curriculum-use that intend to offer insight regarding the kinds of ways teachers engage with instructional resources. In particular, my goal is not to document, exhaustively, the roles of narrative structures within teachers’ classrooms, as teachers plan with and draw upon curriculum materials. Rather, my goal is to show that narrative features play a role and to open doorways for considering how curriculum and instruction might interactively support engagement and learning.

**Forms of teachers’ knowledge.** Acknowledging that know-how and know-that are intertwined and that, as an epistemological basis of research, they are conjointly relevant for study of teachers’ practice, I offer a few additional comments about teachers’ capacities to use curriculum materials. These capacities must be unpacked, because they relate to how I aim to undertake my research. In particular, when thinking about and discussing teachers’ knowledge of curriculum, I do not refer to factual, mental constructs. Instead, teachers’ knowledge of and capacities to use curriculum are related; therefore, studying their use of materials—by studying their actions—is also an investigation of teachers’ practiced-based knowledge (Ball & Bass, 2003). Therefore, I contend that studying curriculum-use also allows me to make claims about teachers’ broad perspectives on and stances toward the programs they use in their classrooms. Stated differently, the individual constructs identified within my conceptual framework (below) are not as distinct from one another as the graphic might suggest.

**Knowing- and reflecting-in-action.** First, Schön (1983) describes the history of professional work (including accounting, architecture, and even teaching) as relying heavily on technical rationality or the view that “professional activity consists in instrumental problem solving made rigorous by the application of scientific theory and technique” (Chapter 2, Section 1, para. 1). He explains, though, that technical rationality has its limits, particularly when encountering ill-defined, real-world problems or, in Schön’s words:

> In real-world practice, problems do not present themselves to the practitioner as givens. They must be constructed from the materials of problematic situations which are puzzling, troubling, and uncertain. In order to convert a problematic situation to a
problem, a practitioner must do a certain kind of work. He must make sense of an uncertain situation that initially makes no sense. (Chapter 2, Section 3, para. 7)

He therefore offers an alternate epistemology, consisting of the twin conceptions of knowing-in-action and reflecting-in-action. Knowing-in-action, as the phrase itself suggests, consists of “show[ing] ourselves to be knowledgeable in a special way” when we “go about the spontaneous, intuitive performance of the actions of everyday life” (Chapter 2, Section 4, para. 1). In like fashion, the skillful actions of an experienced professional, like a medical doctor’s honed diagnostic intuition, is a form of knowing-in-action. Knowing-in-action is loosely akin to Ryle’s (1949) know-how.

Schön (1983) notes, further, that while we often think prior to engaging in action, actions themselves constitute a form of cognition (Chapter 2, Section 4, paras. 14–24). Conversely, we can think about our actions while we undertake them, adjusting our behavior to suit the circumstances; this thought constitutes a form of reflecting-in-action (Chapter 2, Section 4, para. 15). Schön posits that professionals, including teachers, have the capacity for knowing-in-action and reflecting-in-action. These are not given. Not all professionals are, using his terminology, reflective practitioners. Schön contends that the true “professional recognizes his technical expertise is embedded in a context of meanings…which means that he needs often to reflect anew on what he knows” (Chapter 10, Section 2, para. 13). As teachers use curriculum materials, deliberatively, their use reflects their instructional capacity.

Understanding in-the-moment decision-making. Second, because technical rationality is impossible in the enterprise of education, Schoenfeld (2011) offers a “theory of people’s in-the-moment decision making” that builds on knowing-in-action and reflection-in-action as they each relate to teaching moves. Schoenfeld’s framework aims to describe how teachers “make both routine and non-routine decisions” (Chapter 2, Section 1, para. 6). By routine, Schoenfeld means actions closely related to things they “have done before” and “for which they have well established patterns of knowledge and behavior” (Chapter 2, Section 1, para. 4). In contrast, non-routine actions are responses falling outside the scope of prior experience, such as when “a teacher responds to an unusual suggestion from a student” (Chapter 2, Section 1, para. 6). In Schoenfeld’s (2011) framework, teachers effectively solve problems when they (Chapter 2, Section 1, paras. 10ff.):
1. Have a firm understanding of their goals and the relevant contexts;

2. Utilize knowledge to make decisions consistent with these goals as well as their broader orientations (or, in Schoenfeld’s words, “beliefs, dispositions, values, tastes, and preferences,” Chapter 2, Section 1, para. 3);

3. Monitor their decision-making and outcomes continuously; and

4. Proceed, recursively, by advancing sub-goals and then larger goals; roadblocks are surmounted by, first, noticing there is a problem, and then by adjusting goals or decisions.

As noted earlier, curriculum materials may help teachers with routines and provide teachers with knowledge outside their immediate experience (Remillard & Taton, 2015), but they also are unable to prescribe all classroom decisions (Ben-Peretz, 1990, p. vii; NGA Center & CCSSO, 2010, p. 5; Remillard et al., 2014; Stein et al., 2007). Therefore, Schoenfeld’s framework also describes how teachers use curriculum materials as they make interactive classroom decisions.

In addition to mapping a set of teachers’ classroom routines, demonstrating the application of his framework, Schoenfeld also explains that this recursive decision-making process may include forks or “possible branch points in the lesson” (Chapter 5, Section 4, para. 10). When encountering these, teachers rely on their subjective valuations of the range of possible outcomes (Chapter 2, Section 1, paras. 7–9), sometimes deciding on which branch to take in the blink of an eye. The role of subjective valuations during in-the-moment decision-making, Schoenfeld proclaims, involve weighing the perceived benefits, costs, and likelihoods of positive and negative outcomes against one another (Chapter 2, Section 1, paras. 7–9). Seeking to understand teachers’ subjective valuations, as they encounter and navigate branch points, is an important epistemological consideration to strengthen the warrants for any claims about teachers’ curriculum use (Fenstermacher, 1994). In other words, researchers must inquire about which of the available options teachers chose and why.

Distributed and situated knowledge. Last, knowledge is also distributed across objects and people. Wertsch (1998), in particular, contends that “almost all human action is mediated action” (Chapter 2, Section 1, para. 7). By this, he means that nearly all human activity involves using tools—if not material artifacts, then cultural tools (like language). He describes the relationship between tools and their use as exhibiting an “irreducible tension” (Wertsch, 1998, Chapter 2, Section 2) between tools and their use. Wertsch explains, more specifically, that “any attempt to reduce the account of mediated action to one or the other of these elements runs the risk of
destroying the phenomenon under observation” (Chapter 2, Section 2, para. 2). Thinking metaphorically, he might say, how can we separate the idea of NASCAR racing from a car a driver? The idea of NASCAR racing necessarily incorporates both a car (a tool) and a driver (an agent). Wertsch’s perspective draws heavily on Engeström’s (1987) cultural historical activity theory (CHAT). CHAT, broadly, explains that tool-use is not only mediated activity and also situated activity that occurs within nested social contexts at specific places and times.

Wertsch (1998) explains, further, that tools are imbued with and defined by their affordances as well as their constraints (Chapter 2, Section 5). More specifically, a given tool enables a particular action (like a cup enables drinking), but it also carries certain, related limitations. It would be unwise, for instance, to use a cup to drink coffee while riding a bicycle. A spill-proof mug, instead, would be a better tool. Affordances and constraints must be considered along with a tool, itself, particularly when considering its potential adaptations.

Likewise, when considering the features of curriculum materials and how they influence teaching, it is important to also consider both the classroom actions that instructional tools may enable and those they may constrain. And, further, considering narrative construction, there are numerous ways the same underlying story may be told, to varying effects. Therefore, narrative structures—particularly mathematical narratives embedded within curriculum materials—also exhibit affordances and constraints. Understanding how curriculum materials and mathematical narratives are deployed necessitates an awareness of these related affordances and constraints.

To continue, briefly, Pea (1993) asserts that the traditional conception of knowledge, as a cognitive and individually-held construct, is an incomplete idea. Knowledge, Pea counters, is distributed across people and tools via the “situated inventions of uses for aspects of the environment or the exploitation of the affordances of designed artifacts” (p. 50). Knowledge and tools are used to accomplish particular tasks, but tools are not used universally in the same ways. Curriculum materials, more specifically, help teachers solve problems of pedagogy, because they represent a form of knowledge unto themselves. This knowledge includes the collected understanding of mathematicians and mathematics education professionals, accumulated over time. In well-designed programs, the distributed knowledge represented in curriculum materials includes: mathematics content, field-tested activities, ideas about students’ thinking, answers to questions asked, and so on. And yet, as situated within social contexts, teachers are inevitably compelled to utilize instructional tools in novel ways to suit their own, unique purposes and the needs of their students (see, also, Berman & McLaughlin, 1978).
Designerly knowledge and its relationship to narrative construction. Last, having described teachers’ work as design (M. Brown, 2009), I now broaden this claim to explore the nature of design and make connections to narrative construction. Before going a bit deeper with design, I note that there is a vast body of design literature that extends well beyond the scope of this review. In my study, I therefore focus and draw upon modern conceptions of design as a reflective conversation (see Schön, 1983, Chapter 3, Section 2). This conversation navigates between local and contextual concerns, to address human problems. (For additional explanations of the dialogic nature of design, also see Arnheim, 1996; Csikszentmihalyi, 1996; Findeli, 1996; Krippendorff, 1996.) Cross (1982) also defends what he describes as designerly ways of knowing (p. 223) as important and fundamentally different from scientific and scholarly knowledge. Whereas scientists and scholars seek generalizable rules, Cross explains, designers seek solutions to ill-defined problems through iterative methods. Ill-defined problems are “not problems for which all the necessary information is, or ever can be, available to the problem-solver” (Cross, 1982, p. 224) but for which a solution of some kind is nonetheless imperative. Cross (1982) identifies synthesis of ideas through pattern recognition and construction as “the core of design activity” (p. 224).

Effecting students’ learning is, naturally, an ill-defined problem. The enormity of contextual and personal factors that influence cognition and learning is hard to imagine, which is why educational research remains a robust field of inquiry. And learning certainly blends local and contextual concerns, as a design goal (Schön, 1983). Indeed, consider Ball’s (1993) description of mathematics teaching as keeping one’s “ears to the ground” in addition to having one’s eyes “focused on the mathematical horizon” (p. 376). Recall, also, that Schwab’s (1969) curriculum manifests both local concerns (for particular stakeholders and decision-makers), as well as broader, socially-constructed bodies of accepted knowledge. Therefore, design is certainly an apt descriptor of teachers’ work. Cross (1982) would undoubtedly call teachers’ knowledge a form of designerly knowledge.

In addition, as noted above, the structure of a given narrative has certain affordances and constraints. Narratives are also built to serve a human problem, communicating an experience or idea. Furthermore, the variety of ways that narratives are communicated makes narrative construction an ill-defined problem. Therefore, when constructed within classrooms, narrative construction is also a designerly process. Curriculum materials that encoding narrative structures, used by teachers, are narrative tools in addressing a design problem.
Summary and Implications

Considered together, the scholarship outlined in this section explains that teachers’ effective use of curriculum materials is a form of reflective practice and problem-solving. Further, teachers’ instructional practices, derived from the tools of curriculum materials, occur within particular cultural and historical contexts (Engeström, 1987). Therefore, studying how teachers participate with curriculum materials (Remillard, 2005), as a dynamic relationship between teachers and textual tools, necessitates a multi-faceted approach to research.

As Fenstermacher (1994) and Schoenfeld (2011) suggest, understanding the basis for teachers’ decision-making is an essential element to understanding what occurs in their classrooms. The features of curriculum materials, as tools or artifacts, also need to be considered. In particular, the forms of activity that they enable, in addition to the forms they limit, represent the affordances and constraints of written materials. And, finally, as Engeström (1987) explains, contextual factors are other vital considerations of tool-use, as tools are deployed to accomplish particular, culturally-situated tasks. In classrooms with curriculum materials, specifically, these instructional tools aim to address the ill-defined problem of learning. Further, narrative construction—to the degree that narratives are represented in materials—are also a tool in a design process to influence outcomes for students.

In a later section of this chapter, I explain my data collection approach and how I analyze teachers’ beliefs and thought-processes, as well as elements of curriculum materials and the school contexts of my research participants. This methodology reflects the underlying assumptions about research on teaching and teachers’ knowledge, reflected in this section.

5–2. Theoretical and Conceptual Frameworks

I now describe the frameworks that guide my study. Much of my theoretical framework has been presented already, in fact, across Chapters 3 and 4. In addition to the epistemological foundations offered within the previous section, many of the underlying assumptions and motivations behind my work are explained in Chapters 1 and 2. These assumptions, of course, likewise constitute broad-based elements of my theoretical framework. In what follows, I review and tie together these theoretical threads more completely. Afterwards, I then name and define the constructs under investigation in my study—my conceptual framework—and I describe how they each relate to one another as depicted through a logic model. Note that, throughout this section, I call attention to particular constructs that feature in my research by use of italics.
Theoretical Framework

At the outset of this thesis, I argued that students’ engagement plays an important, and yet underappreciated, role in learning (e.g., Csikszentmihalyi & Hermanson, 1995; Dewey, 1913; George, 2014). I then related students’ engagement to teachers’ storytelling abilities (Egan, 1989; Kaplinsky, 2019). I also argued that storytelling is an under-researched and not well understood aspect of mathematics instruction, but work in this area is growing. Dietiker (2012, 2013b, 2015a), for instance, offers the mathematical story framework (MSF) as a tool for understanding the narrativity of mathematics texts. I argued that the MSF can also be utilized to study classroom lessons, and nascent scholarship with the MSF shows promise in understanding students’ level of interest. Therefore, the central aspect of my theoretical framework is: engagement in mathematics classrooms, as understood through the lens of narrative construction.

Narrative construction, I also argued, is bolstered by the coherence of affiliated storylines. Coherence is therefore another element of my theoretical framework. Curriculum materials, furthermore, are thought to support teaching via the presentation of coherent written lessons. At the same time, what is meant by coherence in mathematics instruction is not clearly defined. And the specific mechanisms that relate coherence to students’ learning are still murky. For the purposes of this study, I borrow from two sources to conceptualize coherence: first, Schmidt and colleagues (2002) define coherence as both sequential connections among ideas and their relatedness to broader principles; second, Richman and colleagues (2018) describe coherence as students’ capacities to predict future classroom events (even if these predictions prove incorrect). I do not settle on a single definition, because the field itself has not yet determined the meaning of coherence; in their own way, both definitions offer useful elements. Sometimes these elements are compatible with one another and sometimes they conflict. Indicating these alignments and tensions is a partial objective of my work. In addition, one of these definition is particularly useful for determining coherence of texts. Finally, narrative construction—like teachers’ work, generally—is regarded as a form of design work; it aims to address a particular ill-defined problem, relaying information in a coherent and engaging way.

I take instruction to be an interactive discipline, involving the mutual interactions of teachers, students, and content (Cohen, Raudenbush, & Ball, 2003; Freire, 1970 / 2000; Lampert, 2001). Teaching for mathematical proficiency involves working with students toward their attainment of a deep, connected understanding of mathematical ideas and tools. Proficiency is determined through such capacities as: procedural fluency, conceptual understanding, and the capacity to
apply or model with mathematics (NGA Center & CCSSO, 2010; NRC, 2001). Teachers also support conceptual elements of mathematical proficiency by aiming for sense-making—accommodating students’ questions, ideas, and voices in the classroom (Lampert, 2001)—as well as making-sense (McCallum, 2018). Sense-making involves explicitly attending to concepts, as well as offering opportunities for productive struggle (see, also, Hiebert & Grouws, 2007). Making-sense, on the other hand, involves drawing on the accepted conventions and algorithms of the mathematical community (McCallum, 2018; Pickering, 1995).

Further, curriculum is an image of content (Ball, 1993; Schwab, 1978), although I regard materials, as instructional tools, as being both “far too much and far too little” (Ben-Peretz, 1990, p. vii). Curriculum also constitutes a system with multiple instantiations, transformed at various points along its pathway of use (Remillard & Heck, 2014; Stein et al., 2007). Transformations or adaptations are made to suit a variety of purposes and because communication at any level is necessarily an imperfect venture. And teachers’ curriculum-use, because of reader response theory (Rosenblatt, 1988, 1994), involves a participatory and dynamic inter-relationship (Remillard, 2005).

The impact of curriculum materials on learning therefore involves features of the written lessons and program (S. Brown et al., 2009; Remillard et al., 2014), in addition to teachers’ own interpretations of texts (Ben-Peretz, 1990; Remillard, 1999, 2000, 2012; Sherin & Drake, 2009). Other influences on teachers’ curriculum-use are many and varied, including teachers’: knowledge (Manouchehri & Goodman, 1998, 2000; Remillard & Kim, 2017), school and district contexts (Huntley & Chval, 2010), beliefs and orientations (Remillard & Bryans, 2004). I review a primary set of influences, more specifically, in the next sub-section.

Finally, my own perspective on instruction and curriculum-use is influenced by my experiences as both a teacher and a district-level mathematics administrator. On the one hand, curriculum materials offer insights that are beyond individual teachers’ experience and capacities, such as designing and field-testing particular classroom activities (Remillard & Taton, 2015). On the other, implementing a curriculum program is much more complex than many assume and, therefore, should not be regarded as a simple lever for changing instruction (e.g., Berman & McLaughlin, 1978; cf. Chingos & Whitehurst, 2012). While Remillard’s (2005) participatory lens on teachers’ curriculum-use is helpful in that it emphasizes the essential role of teachers in the interpretation and enactment process, it also presents challenges for school and district
administrators. Specifically, how can administrators whether teachers are using curriculum materials as intended?

Tarr and colleagues (2008) offer one possible approach for making this determination by looking at the prevalence of SBLE characteristics within classrooms. S. Brown and colleagues (2009) build on this idea and offer another possibility by modeling the authors’ intended curriculum, using a typology of OTLs. Remillard and colleagues (2014), likewise, expand the set of OTLs that might be considered. It is not yet known whether there are other OTLs that might aid in reviewing curriculum programs and written lessons. I suggest features of mathematical plots and storylines might represent yet another set of OTLs that might aid understanding fidelity of implementation. Rather than focusing on teachers’ enactment of particular lesson steps (i.e., the literal curriculum, S. Brown et al., 2009), scholars, teachers, and school officials might look to the underlying progression of mathematical ideas. Curriculum authors might also attend to describing this progression and highlighting important points of mathematical plots, to support teachers in implementing lessons that promote problem-solving and curiosity. This would, likewise, represent a different perspective on curriculum materials, perhaps—one that embraces the resource-centric possibilities of instructional programs and not just focusing on them as procedure-centric tools (M. Brown, 2009).

**Conceptual Framework**

To unpack my conceptual framework, I now explain how my work builds on and integrates prior research frameworks. (See Figure 6.) This framework was developed, tentatively, as I began my study and engaged in pilot analysis. As I developed my findings, I refined the framework and affirmed its practical use in explaining my participating teachers’ use of curriculum materials—particularly with regard to how they perceived and constructed mathematical narratives. Below, I describe the various constructs within my framework and relate them to one another. This framework shows, in particular, how both teacher and curriculum resources support the enactment of mathematical narratives within the classroom.

**The DCE framework, teachers’ PDC, and perceiving resources.** I explained previously how M. Brown (2009) conceives of teaching as design work. He also argues that teachers’ instructional designs depend upon their individual pedagogical design capacities (PDCs). Note that, because PDC is an interactional construct that falls across both teacher and curricular resources, using the adjective individual is a bit misleading. Nonetheless, to provide additional detail on PDC, Brown offers this description:
Pedagogical Design Capacity (PDC) goes beyond the resources that are present in an instructional episode to describe the skill by which the various pieces are put into play. PDC represents a teacher’s skill in perceiving affordances, making decisions, and following through on plans. Whether such design decisions manifest as offloads, adaptations, or improvisations is a separate matter. It is the skill in weaving various modes of use together and in arranging the various pieces of the classroom setting that is the mark of a teacher with high PDC, not whether they happen to be offloading, adapting, or improvising at any given moment. Rather, PDC describes the manner and degree to which teachers create deliberate, productive designs that help accomplish their instructional goals. (M. Brown, 2009, p. 29)

I summarized what Brown means by offloading, adapting, and improvising in Chapter 3. I note that, here, Brown also observes teachers exhibit a high level of PDC when they capably assess the affordances (and, presumably, constraints) of instructional resources and then navigate using resources to accomplish their specific pedagogical goals. Therefore, to ascertain the efficacy of teachers’ instructional designs, one must also investigate teachers’ goals, in addition to their understanding of the affordances and constraints of curriculum materials.

Teachers’ use of curriculum materials, how they weave together M. Brown’s (2009) modes of use, depend on both resources offered by written lessons and teachers’ own resources. This mutualism reflects Remillard’s (2005) participatory lens on curriculum-use. Brown therefore offers the design capacity for enactment (DCE) framework, to portray how teachers’ modes of use draw from two types of resources: curricular and personal resources (pp. 26–28). He also lists several, particular resources within each of these types. Under curricular resources, he includes the procedures of written lessons, as well as domain representations, and physical-pedagogical tools. Such tools might include worksheets found within curriculum programs or manipulatives to promote students’ problem-solving. Under teacher resources, Brown depicts subject-matter or content and pedagogical content knowledge; in the context of mathematics instruction, these have been called mathematical knowledge for teaching (Ball & Bass, 2003; Ball et al., 2005; Hill et al., 2005; Ball et al., 2008). M. Brown (2009) also includes teachers’ goals and beliefs, including their beliefs about their students (Rosenthal & Jacobson, 1968; Steele & Aronson, 1995; cf. Raudenbush, 1984), mathematics and teaching and learning (Ball, 1990; Chapman, 2007; Clark & Peterson, 1986; Ma, 1999; Raymond, 1997; Shulman, 1986a; Thompson, 1992), mathematics-instruction reform (Cohen, 1990; Sherin & Drake, 2009; Davis et al., 2011; Manouchehri & Goodman, 1998, 2000), curriculum (Remillard & Bryans, 2004), and so on.

Curricular resources. To M. Brown’s (2009) DCE, I add several components. First, as noted earlier, I consider the facets of mathematical narratives embedded within materials as curricular
resources. Dietiker (2012, 2013b, 2015a) has provided empirical support for the idea that mathematics instructional materials exhibit narrativity, but narrative elements of texts have not yet been incorporated within a broader instructional framework like the DCE. Within the curricular resources of Brown’s (2009) DCE, I consequently embed Dietiker’s (2012, 2013b, 2015a) MSF. The MSF, of course, includes constructs like the mathematical fabula, syuzhet (or story), storylines, characters, events, and settings. These were defined in Chapter 2.

I broaden one of these definitions, momentarily, to point out that the altered sequences of the fabula (experienced through the syuzhet) do not depend upon altered chronologies, as they do with non-mathematical narratives. Instead, sequences of mathematical fabula are not defined “on the grounds of chronological time, but instead using a logic of justification” (Dietiker, 2012, p. 76). Of course, there is no singular form of justification, because many possible routes to the same conclusion may exist. Dietiker (2012) notes that the key criteria of logical justification involve “recognition and reorganization of the relationships between mathematical ideas” (p. 77). The relatedness of ideas—rather than their presented order—is the essential element for recognizing an underlying fabula from a mathematical syuzhet.

Note that Dietiker’s (2012, 2013b, 2015a) MSF includes additional elements of narrative theory, like genres and morals. I do not call specific attention to these granular features within my conceptual framework, but I do not mean to imply that they are not factors considered in my study. For one, I already defined genre in Chapter 3, referring to the example of reform-oriented mathematics resources as one potential sub-genre within the broader genre of instructional texts. At the same time, over the course of my study, I realized that there are potentially sub-genres within reform-oriented materials, as well. In particular, I note that some texts appear to aim for the elicitation of certain emotive aspects of learning: as I hope to make clear in Chapter 6 and beyond, I found Everyday Mathematics to operate somewhat like user’s manual and Math Trailblazers like a mystery novel or puzzle book.

For another, I refer to Dietiker’s (2012) definition of a narrative’s moral as a “resulting message or conclusion gleaned by a reader through the reading of the [mathematical] story” (p. 100). Stated differently, the moral might connote the broad educational purpose or a mathematical principle or property. As Dietiker explains, a reader might discern a moral by asking, “What is the point of the story? or What message can I take from this mathematical story and use to inform other experiences?” (p. 100, emphasis in the original). A moral might differ from an objective in that a particular storyline might be a step on the pathway to elucidating a lesson objective. A
In my own study, I discovered that exploring the mathematical moral allowed me to better understand any modifications to the mathematical fabula. As Dietiker suggests, I consistently sought to address the question: What is the point of this part of the story? How does it tie to a larger moral? In so doing, as I intend to explain in the coming chapters, I learned that I was better able to understand the function of re-sequenced mathematical events in the fabula.

In addition, while M. Brown’s (2009) DCE doesn’t specify these, I have incorporated the OTLs identified in research on curriculum (e.g., S. Brown et al., 2009; Remillard, 2012; Remillard & Reinke, 2012; Remillard et al., 2012, 2014). These were identified and defined in Chapter 3, including the look and voice of the text, as well as the text’s mathematical emphasis, preferred instructional approach, and other educative supports. Furthermore, I hypothesize there is a relationship between these sorts of OTLs, as they are typically conceived, and the novel OTLs represented by the mathematical storylines and plots. Put another way, I presume that how mathematical storylines and plots are perceived and mobilized are influenced by the look, voice, and organization of the text; conversely, opportunities for students to communicate mathematics, to problem-solve, and so on, likely influence the resulting mathematical narrative.

**Teacher resources.** On the right side of M. Brown’s (2009) DCE, teacher resources are depicted. I outline literature on these resources here. Recall that these are thought to interact with curricular resources, hence the bidirectional arrow between them, found in the DCE and my framework in Figure 6. These resources include teachers’ goals and beliefs, subject matter knowledge, and pedagogical content knowledge. I describe each of these, briefly, in what follows.

First, volumes have been written about the role of teachers’ knowledge in instruction. Beginning with Shulman (1986b, 1987)—who used the term pedagogical content knowledge (PCK) to distinguish the type of knowledge attained by a specialist (e.g., a mathematician) or member of the general public from the type of knowledge needed by teachers—scholars have sought to map the various dimensions and domains of knowledge required by teachers. Unlike pure content knowledge (Ball & Bass, 2003), efforts to improve PCK have been tied empirically to improvements in student learning outcomes (Hill et al., 2005, 2007). In particular, Hill and colleagues (2007) found that teachers’ mathematical knowledge for teaching (MKT) reliably correlates with the quality of teachers’ instruction and student learning outcomes. Of an affiliated assessment, known as the assessment of content knowledge for teaching mathematics (CKTM), Hill and colleagues write:
From our analysis of the videotape study, we can see that teachers who score well on our [CKTM] measure both avoid mathematical inaccuracies and provide instruction that is rich in representations, explanations, reasoning, and meaning. This likely translates into higher student gains. (Hill et al., 2007, p. 117)

Not surprisingly, then, PCK and affiliated research has represented an area of tremendous research interest. Building on Shulman’s (1986b, 1987) work, specific forms of knowledge for teaching mathematics have also been described (Ball & Bass, 2003; Ball et al., 2005; Hill et al., 2005, 2007; Ball et al., 2008). Recall that the KCEM and KQ represent activated forms of this mathematical knowledge within the work of teaching or while using curriculum.

Note that I do not closely examine the influence of teachers’ mathematical knowledge for teaching, even though my participants completed the CKTM assessment. As a focused case study, my sample of teachers is purposefully small. This makes generalizing about the influence of MKT challenging. That said, in the findings I subsequently present, teachers’ MKT may relate to certain missed opportunities in the classroom—opportunities potentially available from the mathematical storylines and plots of their written lessons. I found, nonetheless, that despite any relative differences in CKTM, the participating teachers in my study were able to perceive and construct mathematical storylines that effectively addressed their goals and beliefs. I note that Kim (2007) built on previous research about teachers’ knowledge and curriculum-use, and she found a similarly interactive relationship. Kim argues that teachers may overlook the importance of unfamiliar mathematical representations, in particular, but teachers and materials may nonetheless interact to promote students’ opportunities to express their thinking.

Second, teachers’ beliefs are resources they bring to bear during instruction. I consider knowledge and beliefs separable constructs, nonetheless, because they have been conceptualized and studied in ways substantially different from one another. In particular, I borrow from Clark and Peterson (1986) among others who generally regard teachers’ cognition or thought processes as broader than teachers’ knowledge. Clark and Peterson therefore describe three broad categories of teachers’ cognition, their: planning ideas, interactive thoughts and decisions while teaching, and general theories and beliefs (such as their theories about teaching and learning or beliefs about their students). Similarly, I regard MKT as one element of teachers’ practice-based cognition, but in my study, I generally regard beliefs as non-mathematical and non-pedagogical.

Closely connected to their content knowledge are teachers’ beliefs about content. For example, educational research in other content areas—aside from mathematics—shows that teachers’ orientations toward content and even their experiences with other academic disciplines
influence their classroom practices (e.g., Grossman, 1990; Wilson & Wineberg, 1988). This sort of work has also been replicated in mathematics instruction. Earlier in Chapter 3, in fact, I explored research on teacher’s beliefs about mathematics content and movements in mathematics education, insofar as these related to their use of curriculum materials (Cohen, 1990; Huntley & Chval, 2010; Lloyd & Wilson, 1998; Sherin & Drake, 2009; Davis et al., 2011; Manouchehri & Goodman, 1998, 2000). I also explained that Remillard and Bryans (2004) contend teachers’ orientations toward curriculum may influence how they use materials. Remillard and Bryans report on a set of teachers and their beliefs about the specific program used at their school, *Investigations*, and curriculum as a tool, generally. These beliefs ranged from what Remillard and Bryans characterized as adherent and trusting, to skeptical, to quietly resistant. Teachers’ uses of materials, likewise, ranged from adopting and adapting, to piloting, to intermittent and narrow.

A number of scholars have also established the connection, more broadly, between teachers’ content and pedagogical beliefs and their classroom practices (Ball, 1990; Clark & Peterson, 1986; Shulman, 1986a; Raymond, 1997; Thompson, 1992). For instance, Ball (1990) argues that teachers’ conceptions of specific mathematical ideas, as well their general conceptions of the discipline, influence their work. She found that teachers’ (even those with degrees in mathematics) often exhibit a rule-based or procedural understanding of mathematics; consequently, such teachers may struggle to offer cogent conceptual-oriented explanations of key ideas. Nearly 20 years later, Chapman (2007) still observed similar results. In contrast to studies of U.S. teachers and teaching, Ma (1999) found that Chinese teachers may have different beliefs about mathematics, what she describes as a more profound understanding of fundamental mathematics that may explain their ability to “compute correctly and to give a rationale for computational algorithms” (p. xxiv).

Much has been written on teachers’ beliefs about their students and how these influence both their classroom practices and students’ learning. For example, in a famous experiment, Rosenthal and Jacobson (1968) concluded that teachers’ expectations of their students—believing their students as either largely capable or incapable learners—largely determined students’ performance on an IQ test. Their methodological approach has been critiqued (e.g., Raudenbush, 1984). At the same time, as a manifestation of the self-fulfilling prophecy, other versions of this same line of inquiry have proven fruitful (e.g., Steele & Aronson, 1995). While I do not minimize the role of teachers’ (or students’) beliefs about academic efficacy on performance, it is not a central focus of my study. In presenting my findings, I make only occasional references to
teachers’ observations about their students, especially insofar as these appear to influence their choices related to instruction.

Rather than considering any specific beliefs in complete isolation, though, it is still necessary to consider what Thompson (1992) describes as their overall integratedness. This is because, as Raymond (1997) and others have found, teachers’ beliefs and practices may not seem to be fully aligned. When taking a broader view, though, a more comprehensive and consistent pattern of beliefs can emerge. Therefore, I include teachers’ beliefs about their students, about mathematics, about instruction, as well as their orientations toward curriculum, under the general, overarching construct labeled beliefs in the diagram of my conceptual framework.

Last, within my conceptual framework, I also separate beliefs and goals from one another (and, again, separate both from content knowledge). This is purposeful, despite the fact that beliefs and goals are often considered together. M. Brown’s (2009) DCE combines them, as well. I consider goals and beliefs independently, however, because—on the one hand—teachers’ specific goals may reinforce elements of their PDC. On the other hand, though, Davis and colleagues (2011) found that teachers’ goals sometimes exist in tension with those of the curriculum. This potential misalignment of goals and beliefs may undercut learning. Consider, also, teachers like Mrs. Oublier who may express goals about promoting conceptual understanding, for instance, but who may still retain traditional ideas about mathematics or teaching and learning (Cohen, 1990). As Cohen notes, these sorts of beliefs may actually enable practices that conflict with the teachers’ expressed goals.

Summary. In conclusion, M. Brown’s (2009) DCE concentrates on the resources that teachers and materials each bring to the table. Aside from Brown’s observations about offloading, adapting, and improvising, how teachers marshal these resources remains somewhat under-specified. Therefore, in my conceptual framework, I supplement the DCE by articulating the specific affordances of curricular resources, a larger set of OTLs, that teachers may perceive in planning and enacting instruction. I also offer additional detail on teacher resources. These collectively represent the raw materials of teachers’ PDC. To explore how resources are mobilized during instruction—the other component of teachers’ PDC—I draw upon and synthesize two additional frameworks, below: Remillard’s (1999) curriculum mapping framework and Sleep’s (2012) depiction of steering instruction toward the mathematical point.

Curriculum mapping, steering, and mobilizing resources. Before proceeding, I first situate teachers’ mobilizing of resources within Jackson’s (1996) examination of narrated and narrating
events. This distinction is important, in part, because teachers mobilize resources to relate mathematical narratives through storytelling work. To understand teachers’ construction of mathematical narratives, it is therefore important to consider not only the mathematical narratives they are working to enact (i.e., the specific mathematical characters and events) but also their narrating moves. This consideration has implications for my analytic methods.

As teachers enact mathematical narratives, Remillard (1999) offers a helpful framework that I adapt for my purposes. Remillard’s curriculum mapping framework (CMF) depicts teachers’ work as design, and it parses this work into two recursive phases (p. 322). First, the CMF shows that teachers draw on resources (those in the DCE) to develop an image of instruction that they hope to enact. In other words, teachers plan instruction by devising an intended curriculum (Stein et al., 2007). As Remillard (1999) observes, based on her empirical study, this work occurs within what she calls the design arena. This is resource-based work, involving the selection and design of classroom activities. Second, within the construction arena of the CMF, teachers actively enact lessons and thereby make adaptations and improvise, while they interact with students (Remillard, 1999, p. 322). This latter arena consists of in-the-moment design work, as teachers read materials, goals, and students’ responses, to recursively adapt plans as they interpret enactment outcomes (Remillard, 1999, p. 331). Further, Remillard (2000) found that teachers’ decisions—including those made during reading, interpreting, and using materials—are shaped by school and broader contexts.

To specify, even further, Sleep (2012) describes the moves that teachers make as they adapt and improvise within the construction arena. As noted previously, Sleep defines steering as the tasks of teaching that aim toward a specific mathematical point. Steering tasks include those moves that Sleep identifies as enabling the coherence of instruction:

5. Developing and maintaining a mathematical storyline,
6. Opening up and emphasizing key mathematical ideas, and
7. Keeping a focus on meaning. (pp. 942–943)

In her decomposition of practice, Sleep also offers strategies for undertaking and potential challenges of each task. Paraphrasing these, within her discussion of developing and maintaining mathematical storylines, some of the strategies and challenges Sleep identifies are:
Strategies

• Making mathematical connections across activities
• Framing, narrating, and summarizing the mathematical work
• Progressing the storyline by engaging with new or more challenging ideas

Problematic issues that can arise

• Not sequencing activities in ways that promote connections or progression of ideas
• Lack of framing or narration; not summarizing or closing mathematical work before moving on to a different activity on a new topic
• Not looking for broader ideas that connect lessons across the unit
• Missed opportunities to build on students’ prior knowledge
• Redundancy, over-generalizing, or narrowing tasks or questions. (p. 959)

Note, also, that Sleep’s framework incorporates mathematical purposing, which she defines as articulating the mathematical point (or objective) and orienting classroom activity toward the point (pp. 937–938). These, she explains, can occur before and during instruction (p. 938), and so I locate them each, tacitly, as elements within Remillard’s (1999) design and construction arenas helping to focus on mathematical objectives.

While Sleep (2012) found evidence that teachers’ steering generally supports their teaching for mathematical proficiency, their specific, affiliated strategies and pitfalls remain somewhat hypothetical. As noted previously, Dietiker (2012, 2013b, 2015a) theorizes, in particular, that sequencing of ideas may not need to be increasingly-challenging and linear. Empirical support for Dietiker’s contrasting theory has been offered (e.g., Richman et al., 2018). Regardless, one of the purposes of my research is to contribute to an ongoing discussion about sequencing.

Contextual variables. My goal, here, is not to review the myriad contextual factors impacting teachers’ classroom instruction. These have been documented in many other places and are not a significant focus of my study. In making this statement, though, I do not intend to underestimate the influence of broad-ranging contextual features. Instead, to focus my research, I highlight factors that have been identified as especially pertinent to teachers’ curriculum-use. Huntley and Chval (2010), for example, cite school and district policies and documents, such as scope-and-sequence documents, that compel teachers to modify and re-arrange the lessons in written curriculum materials. (Huntley and Chval also emphasize that teachers’ perceptions about these documents and policies, and not just the guidance itself, partly directs teachers’ decisions.)
Similarly, school and district goals for instruction can determine how much time and what resources are devoted to particular subjects (Marx & Harris, 2006; Morton & Dalton, 2007). Together, these may influence how teachers view the materials and resources used in their schools, particularly whether or not they regard them as helpful, unnecessary, or even optional—like items on a menu (Remillard & Taton, 2015).

Further, Davis and colleagues (2011) relate teachers’ knowledge of their students (including students’ age, family context, and socioeconomic factors) to the ways curriculum materials are interpreted and utilized. Davis and colleagues also explore the roles of supplemental instructional resources, those besides the primary set of curriculum materials used in their schools. They note that these influences, taken together, exert subtle influences on teachers’ practices, requiring a fine-grained analysis to tease out possible relationships.

Finally, I mention additional contextual features, which are crucial to teachers’ use of curriculum materials: professional development and other human capital variables. Collopy (2003) found that teachers may implement practices suggested within curriculum materials, even without significant amounts of professional development. At the same time, though, she observed that teachers’ uptake of new practices was uneven, likely due to firm, prior beliefs. She suggests that investing in professional development is necessary to improving consistent practices. Stein and Kim (2009) argue that the various demands of programs may moderate or strain the human and social capital within schools and districts. Some programs benefit from circumstances, for instance, with low teacher transience or strong social trust (Stein & Kim, 2009). Finally, Stein and Kaufman (2010) found a relationship between the quality of curriculum implementation and the amount and type of professional development offered to teachers. Greater implementation quality was affiliated with a greater focus on the big ideas of lessons (Stein & Kaufman, 2010). These findings collectively build on prior research, showing that teacher-learning is complex and that costs of investing in professional development should not be underestimated (Grossman, Weinberg, & Woolworth, 2001; Stein & Brown, 1997).

**Framework Model and Summary.** See Figure 6, which depicts the conceptual framework for my study. My framework ties together the elements of Brown’s (2009) DCE, supplementing them with Dietiker’s (2012, 2013b, 2015a) MSF. I note, in particular, that the OTLs embedded within materials, as well as the mathematical narratives they present, are incorporated into my adaptations of the DCE. Further, I segregate teachers’ goals, beliefs, and mathematical knowledge for teaching and activated mathematical knowledge within curriculum. Together, these constructs
represent the resources available for teachers to perceive as they design their intended curriculum. As teachers mobilize their plans, they recursively design and construct episodes of instruction. This design work, including teachers’ spontaneous adjustments as they enact lessons, is portrayed by Remillard’s (1999) CMF. Finally, as teachers enact instruction, they steer instruction toward their intended goals. Tasks of steering are identified by Sleep (2012).

My framework is akin to a system, where resources recursively flow into actions. It ties together capacities for using curriculum materials productively with those for constructing engaging mathematical narratives. Teachers’ resources in perceiving narratives interact with written guidance and contribute to the narratives that are, ultimately, constructed within classrooms. Keep in mind that one key assumption underlying my work is that narratives are de facto present during classroom activity (Elbaz, 1991), just as Dietiker (2012, 2013b, 2015a) found that many segments of mathematics curriculum materials can be read as narratives. More likely than not, narrative construction is more evident in classrooms aiming toward reform-oriented instruction and using reform-oriented materials. My overall stance is therefore akin to that adopted by Dietiker (2012), who writes about curriculum materials that:

...there is very little conceptualization of structures of mathematics curriculum in a way that accounts for how its parts form a whole. As math educators, we have difficulty answering basic questions about textbooks, such as “In what ways are various mathematical developments the same or different?”, “What can make this mathematical development more interesting?”, “What happens if the sequence is changed?”, and “How does what comes before and after this task affect its interpretation?”...an exploration of [narrative] structure benefits the educational community not only because it offers a new way to understand and describe mathematics curriculum, but also because it enables critique. (p. 218)

We can imagine, theoretically, an elementary grades lesson devoid of mathematical narrative, but we are likely hard-pressed to do so. Even simply practice with arithmetic facts contains narrative elements. Looking at instruction with a narrative lens, as Dietiker suggests, therefore offers an avenue not only to critique curriculum materials but also to understand better how they are mobilized during instruction (cf. Huntley & Heck, 2014). I contend that such an exploration also permits, as an ancillary benefit, better awareness of the role of mathematical storylines and plots in promoting or maintaining students’ engagement and problem-solving.
Figure 6. My conceptual framework, depicting the teacher and instructional resources related to the enactment of mathematical plots of written lessons.
5.3. Methodology

In this section, I review the methodology for conducting my research. I begin by describing my overall approach, a comparative case study analysis of two teachers. I then proceed by explaining my sources of data and data collection methods, as well as my participant selection. I then outline my specific analytic methods, connecting these to the constructs of my conceptual framework and my research questions.

Justifying Case Study and Ethnographic Approaches

Case-study research has proven to be an effective methodology for conducting educational research and, more particularly, research in mathematics education (e.g., Remillard, 2000). For example, Amador & Earnest (2018) assert:

According to Yin (2014), case study research enables the exploration of and, in fact, is necessary to understand complex phenomenon, such as the lesson planning process. Designing instruction is a complex process warranting an in-depth perspective through a methodological approach that affords opportunity to regain a holistic and real-world perspective (Yin 2014, p. 4). (p. 4)

Yin (2009) explains that “case studies are the preferred strategy when ‘how’ or ‘why’ questions are being posed, when the investigator has little control over events, and when the focus is on a contemporary phenomenon within some real-life context” (p. 1). Although Yin argues that case studies may be appropriate for explanatory (or causal) research, he notes that they are traditionally used in exploratory (or hypothesis-generation) work (pp. 3–6). He also explains that case studies are helpful in descriptive (or prevalence) research. These hypothesis and descriptive aims are certainly the sorts of questions I address in my study. I offer additional justification below. In addition, I justify my purpose in using, specifically, a multi-case design.

Rationale for case study based on my research questions and objectives. Indeed, my research questions are mainly exploratory and descriptive. On the one hand, I am trying to describe how a set of elementary-grades teachers use curriculum materials, particularly the relationship between the emergent mathematical storylines and plots of written and enacted lessons (Research Question 1). In addition, I intend to describe how teachers steer instruction, as they appear to perceive and draw upon the mathematical storylines and plots found within written lessons (Research Question 2). Through Research Questions 1 and 2, I intend to offer a characterization of ways that mathematical storylines are perceived and mobilized within instruction. On the other hand, I am also trying to generate a hypothesis about coherence and its
relationship to written and enacted mathematical storylines and plots, as well as what goals, beliefs, and capacities underlie their instructional-design decisions. This is intended to allow me to posit a relationship between PDC and mathematical narratives. Therefore, looking across teachers and across curricula, a multiple-case case study seems especially appropriate for this type of work.

Yin (2009) also observes that case studies are typical research strategies for understanding the decision-making of participants (p. 12), and so they represent an essential means for investigating teachers’ design work. Understanding the choices that teachers make, while teaching and using curriculum materials, certainly lies at the heart of my study.

There are other reasons why I argue that a case study approach is appropriate for my research. For example, I maintain that my selected participants likely represent exemplar cases (Moss & Haertel, 2016, p. 150). I addressed this point in Chapter 2. In addition, typical case studies are social-science investigations that rely on “multiple sources of evidence, with data needing to converge in a triangulating fashion, as another result” (p. 13). Considering my conceptual framework, outlined in the previous section, the phenomenon under investigation is certainly multi-valent—involving curriculum materials and their design features, beliefs, goals, knowledge, and contexts. I am also trying to understand how these mutually influence a given teacher’s approach to using curriculum materials to enact mathematical narratives. In a sense, then, I endeavor to both describe a teacher’s curriculum-use and also test potential hypotheses for what the teacher perceives within written mathematical narratives. I also explore why certain adaptations are made in classrooms. My framework and my ultimate goal, then, clearly suggest a case study approach.

Further, case study methods are appropriate when there are “many more variables of interest than data points” (Yin, 2009, p. 13). This criterion is undoubtedly met by my research objectives, given the complexity of teachers’ beliefs, the various elements of design embedded within curriculum materials, and the multitude of systemic factors. Aiming to address this sort of complexity therefore merits “prior theoretical propositions to guide data collection and analysis” (Yin, 2009, p. 13). My conceptual framework, of course, is built from previous research on similarly complex phenomena, related to teachers’ use of curriculum materials. I intend to make use of these frameworks to explain teachers’ nuanced adaptations of curriculum materials, which Huntley and Heck (2014) have identified as a crucial research need.
Finally, Yin (2009) recommends using a case study approach when “boundaries between a phenomenon and context are not clearly evident” (p. 13). Given that my theoretical and conceptual frameworks acknowledge the interactive relationship between teachers, tools, schools, and social contexts, the blurriness of such boundaries is embedded within my assumptions. As discussed above, it is difficult to disentangle teachers’ use of curriculum from how they think about curriculum materials, how districts and policymakers influence teachers’ use of materials, and how teachers and students interact when using programs as one basis for classroom activity. At the same time, despite this difficulty, attempting to pull on these threads is important for grasping the fullest measure of teachers’ curriculum-use.

Other qualitative methods and main unit of analysis. I also intend to use particular qualitative methods within my case study approach. I offer details, below, but I aim to justify their general application here. To understand teachers’ thinking—as they plan lessons with materials and as they reflect on classroom lessons—and to understand teachers’ professional activity, ethnographic data collection, qualitative coding, and theme-generation are the most appropriate analytic methods (Miles, Huberman, & Saldana, 2014, p. 8). As described below, I am analyzing documents or logs of teachers’ plans, to understand their curriculum-use and their mental images of lesson designs (Arnheim, 1993; van Dormolen, 1986). In addition, I rely on interviews to understand the choices teachers made in their classrooms; to assist with triangulation, I refer to ethnographic field notes and videotaped observations that serve to document their choices and actions. Collectively, these aim to add epistemic import to my warrants, to understand the rationales that teachers maintain for their own decisions. This endeavors to address Fenstermacher’s (1994) concerns about limitations on what I may claim regarding teachers’ cognition and choice.

From my study, I hope to understand the work of teaching by studying individual teachers, as representatives of the profession. The teacher (while teaching) is my primary unit of analysis (see Yin, 2009, for details on proposing a main unit of analysis). In particular, I have engaged in typological theorizing (George & Bennett, 2005, pp. 233–262). In so doing, I articulate what I have since learned, through my research, represents key differences between two teachers’ practices. My two participants are therefore adjacent types working in exemplar circumstances (George & Bennett, 2005; Moss & Haertel, 2016, p. 153). I also intend to explore how teachers use curriculum materials and deploy other resources in their decision-making. As I explain, below, this also implies there are embedded units of analysis in my research design.
Data Sources and Data Collection

In what follows, I provide details on the sources of my data and my data collection methods. I begin this section by contextualizing my participant data within a broader research project on which I worked as a graduate student. This project, described in Chapter 2, is known as the ICUBiT Project. I then describe the specific sources and tools I used to collect data from the participants represented in my own study.

The ICUBiT Project. My study is part of a larger project, the ICUBiT Project. This work was supported by the National Science Foundation under grants No. 0918141 and No. 0918126 (Co-PIs: Remillard & Kim). The goals of the ICUBiT Project were to: 1) better understand the capacities of teachers that enable their effective use of curriculum materials in mathematics; and 2) develop a set of analytic tools for assessing and studying these capacities (Remillard & Kim, 2009). The research team, of which I was a member, sought to identify components of teachers’ PDC and to develop a tool for understanding teachers’ KCEM. My own study grew from the team’s investigation of teachers’ steering moves in enacting instruction, while drawing on mathematics curriculum materials.

The data collected in my study represents a subset of the data collected during the ICUBiT Project. As I noted in Chapter 2, there were 25 elementary teachers (in third- to fifth-grade classrooms) selected to participate in this research project. They had each been nominated by school leaders or colleagues for their experience with one of five, well-known curriculum programs. Together, their schools represented a diverse group of contexts, including public and independent schools located in urban, suburban, and rural settings. All participating teachers agreed to be videotaped during instruction and interviewed both before and after teaching. In addition, each teacher completed two assessments (the CKTM and an early version of an assessment of KCEM), and each submitted two types of logs (described below). Additional artifacts were collected on an ad hoc basis. I should note that I personally collected all of the data used in my study from the two teachers represented in my case study analysis.

Participant selection. From the set of teachers participating in the ICUBiT Project, I selected two teachers, each of whom represents a single case of curriculum-use. (See Table 1 for a list of these participants and key characteristics about each.) Participants in the ICUBiT Project were selected, in part, because their schools exhibited a commitment to one of five curriculum programs, including offering professional development opportunities for using these materials. As indicated by Table 1, below, these teachers are experienced users of the programs shown. Brown’s
and Choppin’s (2011) empirical findings suggest that teachers’ understanding of the affordances and constraints of the features of these programs may develop over time and therefore, teachers with experience were recruited to the study.

<table>
<thead>
<tr>
<th>TEACHER</th>
<th>SCHOOL TYPE</th>
<th>SETTING</th>
<th>CURRICULUM PROGRAM</th>
<th>GRADE LEVEL</th>
<th>OVERALL YRS EXPERIENCE (W/ PROGRAM)</th>
<th>CKTM</th>
</tr>
</thead>
<tbody>
<tr>
<td>ELSA MACKEY</td>
<td>Independent (PK–12)</td>
<td>Urban</td>
<td><em>Everyday Mathematics</em></td>
<td>4</td>
<td>17 (13)</td>
<td>Lower</td>
</tr>
<tr>
<td>TORRIE BLUM</td>
<td>Independent (PK–8)</td>
<td>Suburban</td>
<td><em>Math Trailblazers</em></td>
<td>3</td>
<td>6 (4)</td>
<td>Higher</td>
</tr>
</tbody>
</table>

Table 1. Key variables describing the teachers participating in my study (identified with pseudonyms). These include: each of their school types and settings, curriculum programs used at their schools, grade levels taught (at the time of my observations), years of teaching experience overall (and with the given program), and relative CKTM score (Hill et al., 2004).

I first profile Elsa Mackey in Chapter 7 and then describe her curriculum-use vis-à-vis mathematical narrative-construction. I devote Chapter 8 to Torrie Blum, including profiling her experience, beliefs, and goals in the same fashion as I profile Mackey’s. From the larger ICUBiT Project set, I selected Mackey and Blum as my focal participants for several reasons. As Table 1 shows, both had considerable teaching experience at the time I collected my data, including significant experience in their schools and with the specific programs shown. Stated differently, as they themselves admitted, both teachers had taught the lessons I observed many times previously. Mackey had even taught with *Everyday Mathematics* in several other contexts, including other regions of the U.S. As Davis and colleagues (2011) suggest, this gave her a unique perspective on the role of context in her adaptations of the program. And while Blum’s experience is more modest than Mackey’s, I note that Blum taught at a school that was involved in helping curriculum authors and publishers field-test and revise the *Math Trailblazers* program. Therefore, Blum had additional opportunities to learn about *Math Trailblazers* than a typical teacher using this program and, perhaps because of her experiences in helping with revisions, I found her to be especially thoughtful and reflective when describing its design features and intentions.

In addition, both teachers worked in school settings that offered considerable autonomy to teachers. Despite this autonomy, both teachers still relied heavily on the written guidance found within the primary curriculum programs used at their schools. In contrast, other teachers in similar settings—I learned from undertaking pilot analysis of other ICUBiT Project data—tended to utilize a variety of resources within even single classroom lessons. (This smorgasbord
approach, it seemed to me, tended to diminish the coherence of lessons.) Having autonomy while also adhering to suggested activities, I surmised, would offer me the greatest opportunity to portray the nuanced curriculum adaptations made by each teacher as they worked to take advantage, perhaps, of the coherence built into materials. The pilot analysis I undertook, testing my analytic methods, appeared to confirm this general hypothesis.

I mention briefly, as well, that whether or how MKT relates to PDC is unknown. To account for the possibility that teachers’ MKT is at least partly responsible for their design-decisions when using curriculum materials, I also selected two teachers with differing CKTM scores. Note, again, that the CKTM instrument is a reliable, validated assessment of teachers’ understanding of common content knowledge (CCK) and special, pedagogically-oriented content knowledge (SCK) in various areas of mathematics (Hill & Ball, 2004; Hill et al., 2004; Schilling et al., 2007; Schilling & Hill, 2007). While I no longer have access to each teacher’s individual score, the ICUBiT Project team classified Mackey’s CKTM as among a lower tier of participating teachers and Blum’s as within a higher tier.

**Curriculum programs.** In Chapter 6, I also profile the two curriculum programs under question, *Everyday Mathematics* and *Math Trailblazers*. Here, again, I note that my selection of Mackey and Blum was not happenstance; to me, it was important that their schools used two different types of NSF-funded mathematics programs. As I explain in Chapter 6, drawing on a theory by M. Brown (2009), *Everyday Mathematics* has been described by as a procedure-centric program (Stein & Kim, 2009). In contrast, *Math Trailblazers* could be characterized as a resource-centric program. I described the nature and types of supports offered by these sorts of programs in Chapter 3.

At the same time, each of these program-types may also impose certain demands on schools and teachers. Procedure-centric programs may be considered somewhat easier to implement in schools or districts with high turnover and limited opportunities for regular professional development. Yet, unlike conventional belief, procedure-centric and spiraling programs (like *Everyday Mathematics*) may actually expose students to a “patchwork view of the mathematical terrain” (Stein & Kim, 2009, p. 51). This outcome depends, of course, on teachers’ implementation of the program along with affiliated supports. In environments with low mutual trust, Stein and Kim argue, such programs may also undercut programmatic coherence, because teachers are likely to “take an entrepreneurial approach to what and how they teach as opposed to committing their allegiance to a common curriculum” (p. 51). Resource-centric programs, according to Stein
and Kim, also impose certain demands in schools with high turnover. This is because they are complex and, therefore, require and also “represent a long-term investment in teachers” (p. 51).

Since both programs are NSF-funded, reform-oriented programs, they aim to support students’ conceptual understanding and offer considerable problem-solving opportunities. As Remillard (1999) and others have observed, these sorts of programs are also challenging for teachers, because they place such emphasis on unleashing students’ voices and ideas in classrooms. Consequently, I presumed that studying classrooms using NSF- and reform-oriented materials would be especially productive in my research, since teachers would necessarily undertake significant steering work to deploy mathematical storylines and plots. Researchers have described the interactions in such programs as being either teacher-student or student-student rather than curriculum-student (see, e.g., Remillard et al., 2014; Stein & Kim, 2009). I consequently believed there would be greater opportunity to observe teachers’ modifications, as they interacted with students (and as students interacted with each other) than with programs that have been deemed teacher-directed, highly-scripted, or teacher-proof.

*Multiple-case study design.* I pause here, momentarily, to explain my decision to employ a multiple-case study design. According to Yin (2009), multiple-case studies are appropriate to achieve greater robustness by replicating instances with similar results (what Yin calls a *literal replication*) or to “produce contrasting results but for predictable reasons” (p. 46). Yin calls the latter a *theoretical replication* and suggests that results should be “predicted explicitly at the outset of the investigation” (p. 51). (See also adjacent types of case studies, described by Moss & Haertel, 2016, p. 153). Given the contrast between programs (procedure-centric v. resource-centric) and given the contrast in teachers’ CKTM scores, I initially speculated that Blum’s mathematical narratives might be somewhat richer than Mackey’s. In what ways and for what expressed reasons, I could not say at the outset. But, generally, as my review of the literature suggests, I suspected that resource-centric programs offered greater flexibility in the sequencing of mathematical events. This flexibility, I further hypothesized, might be used to greater impact by a teacher with a strong grasp of the underlying mathematical connections.

In Chapters 7 and 8, I explain in what ways these predictions were borne out. Broadly, I found that both teachers made pedagogical moves supporting students’ proficiency through construction of mathematical narratives. And, yet, from what I observed, I speculate that Blum’s lessons will achieve longer-lasting residue and positive associations for students, nurturing them as mathematical thinkers and problem-solvers.
**Sources of data: Teaching sets.** Data collection involved an approach developed by Simon and Tzur (1999) and Cobb, Zhao, and Dean (2009). This approach, known as collection of teaching sets, consists of conducting interviews and making observations of lessons, over the course of a school year. The ICUBiT Project team also administered two assessments and collected additional, key artifacts. Next, I describe each of these sources of data in greater detail, and I conclude by articulating how these sources conform to my overall research design. Before doing so, I briefly add that, as a member of the ICUBiT Project team, I participated extensively in the development and refinement of data-collection protocols.

**Curriculum logs and other artifacts.** Prior to my classroom observations (described below), each teacher was asked to complete a Curriculum Reading Log (CRL) for each lesson. The CRL was developed by the ICUBiT Project team for the purpose of understanding the role of curriculum materials in teachers’ lesson planning. The CRL consists of a photocopy of the pages in the teacher’s guide that correspond to an upcoming lesson; the teacher is asked to indicate, using three different highlighting colors, material in the guide that they: a) read intently, b) intend to use in teaching the lesson, and c) intend to use in a modified form. During an initial orientation meeting, reading was defined for ICUBiT Project teacher-participants as an attempt to gather and understand key details within the text; in other words, reading was contrasted with skimming. On their CRLs, teachers are also asked to indicate any supplementary materials from outside sources that may have informed their lesson-planning or that they intend to use during instruction.

I also collected artifacts—like worksheets—from teachers who offered them to show samples of the types of resources used during instruction. Last, email correspondence with teachers was also saved. This correspondence mainly consisted of verifications of observation dates and times, but also included my broad questions about intended plans or considerations that should be taken into account. Such considerations included logistical details, such as the best place to locate the camera. On occasion, teachers noted changes to the schedule (e.g., fire drills) that might impact the lesson observation and whether or not it would be considered typical.

Finally, I collected a host of other artifacts to contextualize my findings regarding both the written and enacted curricula represented in my study. With regard to the written curriculum, I collected descriptions of each program from publicly-available websites and prior research studies. With regard to the enacted curriculum, I aimed to ground teachers’ lessons within their work environments. I therefore gathered publicly-available information about each school’s mission, goals, student population, academic program, and so on.
Assessments. As indicated previously, each teacher was asked to complete the CKTM, as well as the curriculum embedded mathematics assessment (CEMA). The CEMA is an instrument developed and tested by the ICUBiT Project team, to assess teachers’ activated mathematical knowledge in using curriculum materials (see Kim & Remillard, 2011). This activated mathematical knowledge is known as KCEM and is described by Remillard & Kim (2017). In the chapters that follow, rather than referring to teachers’ CEMA scores, I draw on the dimensions of KCEM and qualitative evidence (similar to the approach taken by Remillard and Kim, 2017). I have taken this approach, because the CEMA underwent several rounds of testing and refinement during the course of the ICUBiT Project.

Classroom observations of instruction: transcripts and ethnographic field notes. For each teacher, a set of six lessons was videotaped: three in the fall of 2011 and three more in the spring of 2012. Collecting a set of fall and spring lessons was intended to help ascertain the relative consistency of teachers’ use of curriculum materials, perhaps indicative of a design fingerprint. Also, per the Institutional Review Board, video cameras were aimed at teachers or the whiteboard in the classroom; in other words, all reasonable efforts were made to avoid capturing images of students or their work. Teachers’ voices were amplified by a wireless microphone, which they carried throughout the lesson. See Appendix D, which details the observation protocol.

Each videotaped lesson was then transcribed, and I assisted in transcribing and in training the transcribers. Transcribers were instructed to record, as accurately as possible, the teacher’s spoken words and any student responses (to the extent discernable). Background conversation was not captured in the transcripts. Keep in mind, of course, that transcribing is considered a form of analysis (Gee, 2014, Chapter 5, Section 6). In my analysis, I therefore modified transcripts, on occasion, so that they included thicker description (Geertz, 1973). In so doing, I consulted the videotapes to validate these modifications. For example, in one lesson, Mackey asks: “Which number can go on the square to put this sentence true?... if I am eating the big number which one of these numbers should go here?” (observation transcript, 12/12/2011). I consulted the videotape to learn, and modified the transcript to indicate, that Mackey had a) pointed to a multiple-choice problem on the whiteboard board and b) nodded in assent after a student’s response.

I also recorded ethnographic field notes during each classroom observation, in addition to taking notes during interviews. Observation field notes described the physical layout of the classroom (including a seating chart) as well as a catalogue of key, timed events during the lesson. Field notes captured elements of lessons, potentially, that were not easy to capture on
videotape, including contextual variables relevant to the instruction. Observation field notes were also instrumental in preparing for and conducting post-observation interviews (explained below). Notes taken during interviews were mainly intended as duplicative—in the event of the failure of the recording device or excessive, obscuring background noise.

**Interviews.** All interviews were semi-structured (Weiss, 1994). As Weiss (1994) indicates, semi-structured interviews contain pre-written or scripted sorts of questions, in addition to broader topic outlines. They are intended to balance the need for consistency across members of the research team (and interviewees) along with flexibility for addressing topics of interest to both the interviewer and interviewee (Weiss, 1994). In addition, semi-structured interviews allow for opportunities to ask clarifying questions (Patton, 2002; Weiss, 1994).

Teachers were interviewed, once, after soliciting their participation in the ICUBiT Project (see Appendix A). This introductory interview followed a brief orientation about the research aims of the project and considerations related to informed consent—outlining potential risks to participation, expectations and benefits, data protections, and so on. During the interview, teachers were asked about their work history, including their experiences with the curriculum program used in their school or district. Teachers were also asked to explain their beliefs about curriculum and and to describe their general patterns of curriculum-use, including whether they used supplemental resources. They were also asked how they would describe their teaching style or philosophy. Teachers were also interviewed, briefly, prior to each lesson (see Appendix B). These were informal interviews, sometimes conducted via email (as noted above), to help orient the researcher to the classroom space and the upcoming lesson.

At the conclusion of both the fall and spring sets of observations, an extended follow-up interview was also conducted (see Appendix C). These interviews contained a number of predetermined questions, asking the teacher whether the observations represented so-called “typical lessons” and how successful the lessons were thought to be. In addition, follow-up interviews explored key moments within the lessons, to elicit a portrait of decisions made by the teacher—as she interacted with students and drew on curriculum materials to enact instruction. CRLs, videotapes of lessons, and field notes of classroom observations were collectively used to prepare for conducting follow-up interviews. Moments of lessons were identified, when teachers appeared to make significant modifications of curricular guidance or held tightly to such guidance (i.e., offloaded); these moments were probed during the follow-up interviews. Videotapes and field notes therefore acted as resources for stimulating the teacher’s recall of events (Sherin &
Drake, 2009). The teacher was also asked about the role of curriculum materials in planning and designing instruction and whether or not, or how, this role had changed over time (Drake & Sherin, 2006).

Comment on research design: using critical and embedded cases. Last, I offer an important note on the overall design of my study. From my pilot analysis of all six lessons, I selected two or three lessons that I deemed critical cases. Yin (2009) explains that case study research often involves identifying and reporting on specific cases that allow for testing an articulated theory—i.e., confirming, challenging, or extending it. Again, my succinct, overarching theory is that: teachers (implicitly and explicitly) perceive the mathematical narratives—storylines and plots—within their written curriculum materials; further, depending upon their beliefs and capacities, they adapt and construct narratives to address particular goals. For each teacher, the lessons I selected represented critical cases, because they seemed to represent specific instances when the teaching stayed relatively close to the curricular guidance. In addition, teachers in these lessons did not rely very much on additional, supplemental resources. These cases, I therefore believed, would best represent teachers’ perceptions of the embedded mathematical narratives within materials and adaptations of narratives made during instruction.

In addition, since these lessons represent sub-units of analysis—whereas my main unit of analysis are teachers and their PDC—my research design also includes an embedded case study (rather than consisting of a purely holistic approach). Yin defines embedded case studies as those “when, within a single case, attention also is given to a subunit or subunits” (p. 41). Subunits might consist of particular projects, meetings, locations, and so on. Holistic designs are generally used when no meaningful subunits can be identified, although a potential pitfall of using a holistic design is analysis that remains too abstract. In contrast, embedded designs allow investigators to explore phenomena with significant operational detail. On the other hand, as Yin cautions, care must be taken to return to describing the main unit of analysis rather than languishing within the subunit (p. 44).

In my analysis, I found it particularly important to discuss how teachers—through their instructional practices—operationalized the mathematical narratives within specific lessons (and even within specific moments of lessons). This operationalizing is, of course, the essence of a teacher’s PDC. Therefore, I use an embedded approach and take care to avoid its main pitfall.
Data Analysis

In this section, I review my analytic methods and how these were used in responding to my research questions. Throughout, I refer obliquely to my conceptual framework (see Figure 6). At the conclusion of this section, regardless, I present a logic model for my study and explicitly tie together my research questions, data, and conceptual framework.

Research question 1 (RQ1): Characterizing relationships between mathematical narratives of written and enacted lessons. I pursued RQ1 in an iterative fashion that involved two phases. In the first, I coded written and enacted lessons and produced maps of design arcs. I then used these maps as analytic tools, to help address RQ2. In the second phase, I returned to RQ1 by generating themes from RQ2 and answering RQ1 more concretely. (Because of this, in Chapters 7 and 8, I respond to RQ2 before responding to RQ1.) I describe the tools I used during both phases in addressing RQ1, below.

Before proceeding further, though, Dietiker (2013b) cautions that the mathematical fabula, specifically, “can be represented in a variety of ways, [but] each with a loss of information” (p. 16). The same is true, certainly, for the mathematical syuzhet or both together. Dietiker (2012) shows, nonetheless, that metaphors—including visual representations—are helpful for understanding the development of content. She draws on work by Sfard (1998), who explains:

Because metaphors bring with them certain well-defined expectations as to the possible features of target concepts, the choice of a metaphor is a highly consequential decision. Different metaphors may lead to different ways of thinking and to different activities. We may say, therefore, that we live by the metaphors we use. (Sfard, 1998, as cited by Dietiker, 2012, p. 20)

Dietiker (2012) then proceeds by arguing that conceptualizing the mathematics curriculum as narrative calls attention to both the connections among its parts and its sequencing of content. This is the basis of her entire project, of course, and she suggests that adopting such a stance has implications for understanding learning. More concretely, even representations of mathematical narratives exhibit analogous limitations. I maintain, though, these are offset by opportunities to grasp an overview of key elements of storylines and plots through a visual map.

Written curriculum materials: OTLs and forms of address. My analysis of the written curriculum materials of Everyday Mathematics and Math Trailblazers was a multi-step process. To develop a portrait of each program, particularly how its OTLs interacted with embedded narrative structures, I reviewed: the history of each program’s development, authors’ expressed
design intentions, broad components of lessons (including front-matter descriptions), and key findings about the program from previous research. Generally-speaking, I aimed to characterize the OTLs found in the framework offered by Remillard and colleagues (2011, 2014), as well as the types of scripts described by Remillard and Reinke (2012).

Together, these helped me to describe the forms of address found within these instructional texts (Remillard, 2012). As noted above, I concentrated on analyzing the voice, structure, and genre of mathematics texts and narratives. (See Chapter 3.) Overall, then, this approach involved hermeneutic content analysis, to generate a portrait of how a reader might come to understand or engage with text in its intended context of use (Patterson & Williams, 2002). Further, the collected forms of address within these mathematics texts constitute a profile of writerly engagement (Barthes, 1974, as cited by Dietiker, 2012, p. 172). Analyzing mathematical texts in this fashion characterizes the design features offered by the text—with which readers actively engage to make sense of their reading.

Written curriculum materials: Codes for plots and storylines. In addition, I employed two sets of codes to analyze the narrative structures (the plots and storylines) of written lessons. Focusing on the within-lesson level, as explained previously, I first drew on Dietiker’s (2012, 2013b, 2015a) definitions in the MSF to identify mathematical settings, characters, and events. I then developed a coding manual with descriptors, inclusive and exclusive criteria, and exemplars (Bernard & Ryan, 2010). I generally used the set of a priori plot elements and affiliated codes suggested within Dietiker’s (2012) MSF. Recall that these were drawn from Barthes (1974) framework for analyzing plot. Summarizing my coding manual, these plot elements and codes are shown in Table 2.

I made a small number of changes to the plot codes in the MSF. Because the MSF does not readily indicate the rising and falling action, contributing to a reader’s (or student’s) levels of suspense, I supplemented this framework with several, additional components. First, because I largely coded at the sentence-level within mathematical texts, I noticed that formulated questions were often addressed through a series of scaffolding questions. For instance, a primary (or main) formulated question in a written lesson, aiming toward a mathematical objective or big idea, might be: How do you round multi-digit numbers, following the typical convention? Questions that scaffold an answer to this question might be: What is the place-value to which you intend to round? and Is the digit to the right of this place-value 5 or larger? Scaffolding questions in
written and enacted lessons, I found, were helpful for understanding steering. I call these scaffolding questions *sub-formulations* and added a code for them in my coding guide.

In addition, I adapted and incorporated Labov’s and Waletzky’s (1967) and Labov’s (1997) framework about the structure of *narratives of personal experience* (or oral narratives that are generally biographical). Within this framework, they describe clauses that, roughly, correspond to utterances about events in a narrative\(^\text{14}\). They also describe an overarching, canonical structure for personal narratives that, they found, includes sequences of clauses corresponding to the following narrative phases (Labov, 1997):

1. **Abstract**: A brief overview of an entire narrative
2. **Orientation**: Sequences of clauses providing information about the setting, characters, and so on
3. **Complication**: Sequences that report “a next event in response to a potential question, ‘And what happened [then]?’” (Labov, 1997, Definition 3.3)
4. **Evaluation**: A clause or clauses that offer “information on the consequences of the event for human needs and desires” (Labov, 1997, Definition 4.1)
5. **Resolution**: Sequences of clauses that constitute “the set of complicating actions that follow the most reportable event” (Labov, 1997, Definition 11.1)
6. **Coda**: A brief recapitulation of an entire storyline

These phases generally describe the familiar pathway of dramatic structure that runs from the *exposition*, to the *rising action*, to the *climactic moment* (or *climax*), to the *falling action*, to the *denouement* (Freytag, 1863 / 1894).

I must offer a few additional notes on Labov (1997). First, Labov explains that the *evaluation* is typically found within the complication section of a narrative; it is the moment when the audience of a personal narrative becomes aware of the consequences of a key event within the full breadth of the story. In mathematics instruction, because a reader of a mathematics text isn’t necessarily relaying a narrative to an audience (e.g., as with teachers engaged in planning), I have chosen to reframe the evaluation from an emic perspective: in my analysis, the evaluation

\(^{14}\) Labov (1997) offers an axiomatic system with technical, linguistic-based definitions for describing the structure of personal narratives. I paraphrase these definitions, here, but refer the reader to the original for additional detail.
<table>
<thead>
<tr>
<th><strong>PLOT ELEMENT (ANALYTIC CODE)</strong></th>
<th><strong>BRIEF DESCRIPTION</strong></th>
<th><strong>PURPOSE / NOTES</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ABSTRACT (AT)</strong></td>
<td>Brief overview of an entire storyline</td>
<td>Hints at eventual answer</td>
</tr>
<tr>
<td><strong>ORIENTATION (ON)</strong></td>
<td>Sequences providing narrative information</td>
<td>Sets the stage of a storyline</td>
</tr>
<tr>
<td>Thematization (Tn)</td>
<td>Description of the setting or character</td>
<td>Helps identify focus of storyline</td>
</tr>
<tr>
<td>Proposal (Pl)</td>
<td>Possibility of a question raised</td>
<td>Hints at an emergent question</td>
</tr>
<tr>
<td>Main Formulation (Fn+)</td>
<td>An overarching, broad question</td>
<td>Tied to a learning objective / idea</td>
</tr>
<tr>
<td><strong>COMPLICATION (CN)</strong></td>
<td>Sequences raising suspense (no answers)</td>
<td>Motivate continued reading</td>
</tr>
<tr>
<td>Sub-formulation (Fn⁻)</td>
<td>A specific or scaffolded sub-question</td>
<td>Clearly related to an F⁺</td>
</tr>
<tr>
<td>Promise (Pe)</td>
<td>Indication an eventual answer will be given</td>
<td>Keep interest in abeyance</td>
</tr>
<tr>
<td>Snare (Se)</td>
<td>Attempt to mislead the reader (student)</td>
<td>May promote anxiety or confusion</td>
</tr>
<tr>
<td>Equivocation (En)</td>
<td>Ambiguity, containing partial truth &amp; untruth</td>
<td>May promote anxiety or confusion</td>
</tr>
<tr>
<td>Jamming (Jg)</td>
<td>Suggestion a question is unanswerable</td>
<td>May frustrate or diminish interest</td>
</tr>
<tr>
<td><strong>EVALUATION (EN)</strong></td>
<td>Sequences offering consequences or purpose</td>
<td>Typically in the complication</td>
</tr>
<tr>
<td><strong>RESOLUTION (RN)</strong></td>
<td>Answering or closing sequences</td>
<td>Apply to both F⁺ and F⁻</td>
</tr>
<tr>
<td>Suspended answer (Sa)</td>
<td>Unanswered or significantly-delayed answer</td>
<td>Storyline (scene) shift or end</td>
</tr>
<tr>
<td>Partial answer (Pa)</td>
<td>Incomplete answer</td>
<td>Provides <em>some</em> closure to F⁺⁻</td>
</tr>
<tr>
<td>Disclosure (De)</td>
<td>Explicit answer</td>
<td>Provides <em>full</em> closure to F⁺⁻</td>
</tr>
<tr>
<td><strong>CODA (CA)</strong></td>
<td>Brief recapitulation of an entire storyline</td>
<td>Summarizes storyline</td>
</tr>
</tbody>
</table>

Table 2. My analytic codes for capturing elements of mathematical storylines and plots.
consists of the moment when (or if) the protagonist (a mathematical character in the narrative) recognizes the meaning or importance of the befalling events.

Second, *resolution* is also considered part of the complicating action. The *complicating action*, furthermore, is the part of the narrative that exhibits chronological (or, in the case of mathematical narratives, logical) ordering. The other phases of narrative structure are generally atemporal. Third, note that Labov (1997) describes the complicating sequences as providing an answer to a temporal question: *What happened next?* But Labov does not imply a story must be told in chronological order; remember that narrative formalists argue the sequence, the syuzhet, can be a reordered version of the fabula—as long as the audience can mentally reconstruct the chronological (or, in the case of mathematical narratives, logical) sequence of the events.

Next, Labov (1997) defines the *resolution* as a set of clauses that give the narrative a sense of closure and that typically follow the evaluation or climax. Again, in the context of mathematical narratives, I redefine the resolution as any set of clauses that offer an answer to the main formulation (or the primary question raised about the character in a mathematical storyline). This answer may be a complete answer (a *disclosure*) or an answer with incomplete detail (a *partial answer*). To mark when storylines do not offer a resolution or when they shift away from the question at hand for an extended period of time (defined as three or more mathematical events), I also include a “non-answer” answer (a *suspended answer*) in my coding scheme. Last, not all narratives exhibit *abstracts* or *codas*.

Labov’s and Waletzky’s (1967) and Labov’s (1997) framework for personal narratives and the codes that I adapted from their phases of narrative structure were tremendously helpful. For one reason, they aided in identifying the beginning and end of mathematical storylines within texts and classroom lessons. I eventually learned to recognize descriptive statements about mathematical characters as *orientations*—signaling a new mathematical storyline. As an example, this is the orientation of a new storyline in a *Math Trailblazers* lesson:

Use the transparencies of the *Hundreds Template Transparency Master*. Lay four flats on the grids and ask students to do the same…. (Grade 3, Unit 6, Lesson 3, p. 57)

I considered this an orientation, because there is no mathematical question that has been raised. Instead, both sentences together describe a new mathematical setting (a hundreds template or grid), on which new mathematical characters (*flats*, or plastic pieces that are 10 cm by 10 cm with 100 squares etched onto them).
Of course, I also developed and used a heuristic to identify storylines and narrative phases (see Figure 7); this heuristic consisted of the following sub-vocalized question and answers, while I read mathematical texts (or reviewed lesson transcripts):

1. Does this (new) clause represent an event—a mathematical action—that connects a new stage of a mathematical character to the character’s previous state?

2. If yes—i.e., the new clause connects evolved stages of the same mathematical character—then the event represented is likely part of the same storyline.

3. If no—i.e., the new clause refers to an event potentially about a *different* character—then is the character part of a new storyline?
   a. If yes, then what is the main formulation about the new character in this new storyline?
   b. If no, then any questions about the new character should relate to the main formulation already stated.

This heuristic was very helpful, particularly during enacted lessons when there was no clear shift from one mathematical storyline to another.

In addition, through my analysis using these phase codes, I learned that mathematical narratives also exhibit a canonical structure mirroring the canonical structure of personal narratives. Specifically, after some opening and framing material (describing a mathematical setting or character), mathematical storylines often exhibit an initial question raised about a particular character (i.e., a main formulation). Through a series of scaffolding questions that are asked (and, often, subsequently answered), or what I call sub-formulations, the storyline is then brought to a resolution. This structure aided my overall understanding of narrative construction and teachers’ departures from its path.

Finally, Labov’s and Waletzky’s (1967) and Labov’s (1997) narrative phases also helped me create design arcs that mirrored Freytag (1863 / 1894) diagrams. I assigned an arbitrary height on a y-axis to each narrative phase and by double-coding mathematical events, as they arise upon a timeline, I was able to create storyline arcs. (The lines themselves are cubic splines of the code pairs, plotted on a coordinate grid.) These storyline arcs, the design arcs, seemed to convey a sense of the flow of mathematical narratives more strongly than did the plot diagrams in
Dietiker’s (2012) portrayals of mathematical storylines and plots. Dietiker acknowledges, in fact, there are many different ways to represent the mathematical fabula (p. 76). I therefore offer more details on the construction of my design arcs in Chapter 7 and below.

Figure 7. My heuristic for deciding on whether a mathematical event is part of a new mathematical storyline or continues an ongoing storyline.

Written curriculum materials: Re-reading, unit of analysis, metaphors, and other analytic details. In applying these two sets of codes, those from the MSF and those representing narrative phases, I learned that some elements of mathematical plots revealed themselves only after reading and re-reading the text (or, in the case of classroom observations, the enacted lesson transcript). This is because, for instance, it is difficult to tell whether a snare—a deliberate attempt to mislead a reader—occurs until a complete understanding of the narrative (and its resolution) are attained. Multiple re-readings, to uncover these subtle plot-points, are therefore suggested in applying the MSF (Dietiker, 2012, p. 177). Although Dietiker doesn’t point this out, there are clear implications, here, involving teachers’ rereading to uncover plot during lesson preparation (see my implications, discussed in Chapter 10).
When coding written lessons, my unit of analysis consisted of individual *lexia*15. Dietiker (2012) defines lexia as: “a portion of text for which there is something to note, in which some meaning that [sic] has been established for the reader” (pp. 173–174). She continues to explain that, following Barthes (1974), lexia may be flexible in length—depending on the interpretation of events by a reader. Because I aimed to compare written and enacted curricula at a fine-grained level—looking for turns of phrase that might indicate subtle changes in mathematical storylines and plots—I chose to define my lexia, mainly, as individual sentences within written lessons. On rare occasions, compound sentences (with different mathematical events, characters, or settings) were broken apart into multiple lexia and coded separately. Conversely, I discovered in my pilot analysis that, sometimes, sets of sentences in written lessons were generally redundant to one another. I considered sentences redundant, if they offered a more-particular elaboration on a character’s traits. For example, these two sentences in *Math Trailblazers* were considered redundant and coded as if they were a single lexia:

Students can reason that four people sharing a pizza fairly will each receive more pizza than six people sharing the same size pizza. That is, ¼ of a pizza is more than ½ of the same size pizza because fourths are larger than sixths. (Grade 3, Unit 9, Lesson 6, p. 97)

This sort of redundancy was determined when successive sentences did not shift to describing a new mathematical character, did not portray events befalling a character, or did not shift to describe aspects of settings.

Next, from my pilot analysis and Dietiker’s (2012) methodology, I recognized that the lexia for a given mathematical character—as well as its affiliated events—can be aggregated. In so doing, these constitute a single mathematical storyline. As Dietiker (2012) and Richman, Dietiker, and Brakoniecki (2016) show, multiple storylines can start at different points in a lesson, and so they can partly or even completely overlap. In addition, I learned from my analysis that multiple, overlapping storylines of written lessons often occur when authors use metaphors to describe mathematical ideas. I define *metaphors* in mathematical narratives as descriptions of mathematical characters or events are *not* the most abstract objects or ideas under question16. For instance, base-ten blocks are a representation of and a narrative metaphor for multi-digit whole

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15 The term *lexia* follows from Barthes (1974), and I take it mean segments of texts (or multiple words). This is why, I believe, the plural Latinate form lexia is used to convey a singular fragment, rather than the more-common *lexium*.

16 There is, potentially, a problem of infinite regress here: integral numbers and operations might be considered, themselves, representative elements of abstract mathematical objects known as *groups* (in group theory). Therefore, in defining mathematical metaphors, I generally confine the level of abstraction to the highest one that could conceivably be appreciated by students at a given grade-level.
numbers. Metaphors, I have found, tend to draw comparisons to broader constructs while helping students focus on concrete models. Recognizing these metaphors, I argue, is a form of Remillard’s and Kim’s (2017) KCEM, namely, Dimension 2 (“representations and connections across them”).

As an example of mathematical metaphors, in Unit 6 (Lesson 3) of the third-grade editions of *Math Trailblazers*, the written lessons ask teachers and students to model three-digit numbers on a template using base-ten pieces. I affiliated the transformations of the base-ten pieces with one mathematical storyline. Implicitly, the whole numbers they represent are transformations in another setting and are therefore considered a separate storyline. This duality of storylines (metaphorical and referential) offer several advantages when analyzing enacted lessons. In particular, as tools, metaphors can call attention to key ideas while also narrowing the reference frame. Therefore, understanding how teachers recognize and utilize the affordances and constraints of narrative metaphors also involves a nuanced appreciation of their interpretations of materials to achieve specific ends. As hinted above, it also offers a more focused window on Dimension 2 of teachers’ KCEM.

Finally, I note that Dietiker (2012) defines *momentary questions* as those that are asked and answered, right away, by successive lexia (p. 199). Dietiker also complicates the role of momentary questions, depending on how they function within the broader storyline. My adjustments to the MSF allow for this distinction to be clarified, because if momentary questions are tied to a main formulation (an essential question about a mathematical character), then they are part of the complicating action of a storyline, as scaffolding questions or sub-formulations. They are therefore not isolable. Momentary questions are themselves brief storylines, because a character is nonetheless transformed in the process of offering and resolving their formulation, even if such storylines are not particularly interesting or engaging.

*Enacted lesson observations and design arcs.* To code enacted lessons, I used an approach similar to that described above but within lesson transcripts. I coded transcripts using my adapted version of the MSF, and I referred to videotapes on regular occasions to clarify any ambiguities or note any relevant visual media (e.g., a problem written on the whiteboard).

After multiple rounds of coding both materials and observation transcripts—aiming to refine my coding approach and affirm the identified storylines and plots—I began by building maps of design arcs that allowed for side-by-side comparisons of written and enacted lessons. Building design arcs involved, first, placing storylines on a coordinate plane with a consistent and
comparable horizontal axis (accumulated number of mathematical events for the written lesson and proportion of lesson duration for the enacted lesson). Storylines were then transformed into arcs by assigning arbitrary heights to the structural codes for narrative phases (Labov & Waletzky, 1967; Labov, 1997).

Following an approach similar to that of Remillard and colleagues (2015), I also incorporated markers on design arc maps that, at least visually, aimed to represent particular aspects of mathematical plots. (See, e.g., Figure 9.) As an example, I tagged complicating action that elevated suspense, or instances of jamming, snaring, or equivocation. For enacted lessons, I also marked periods of time when students were engaged in seatwork, either individually or in groups. These periods of time, I found, represented teachers’ steering work on local, student-specific goals (rather than on class-wide instructional goals). Due to recording limitations, seatwork was also more difficult to analyze. Therefore, I generally omitted such periods from my analysis of storylines and plots.

I also used different-colored lines, purple, for design arcs that were invented (or added) by teachers and were not represented in any substantive fashion in written materials. Blue arcs represented those storylines that appeared in both the written and enacted lessons. Confirming prior research about NSF-funded programs, generally, I also found that—even when teachers drew heavily on the activities suggested within the written lessons—they nonetheless added a number of new mathematical storylines to serve a variety of purposes. I elaborate on these in successive chapters.

After completing the maps of design arcs—which I consider depictions of mathematical storylines and plots found in written and enacted lessons—I engaged in a side-by-side, comparative, visual analysis. These comparisons allowed me to notice, broadly, changes to the sequence, selection, and extent of learning activities—looking from the written to the enacted lessons. In addition, by coding plot-points, I could surmise whether teachers modified the key elements of plots found in written lessons. Rather than comparing literal lesson instructions (S. Brown et al., 2009), I explored the relationship between underlying mathematical fabula, in addition to characters, settings, metaphors, and so on. These, to me, seemed more grounded in the mathematics and students’ experiences with the narration, rather than the sequence of particular questions in activities. These observations, generally-speaking, also helped me to focus on segments of lessons that I examined more closely in addressing RQ2.
Potential warrants made. After pursuing the first phase for addressing RQ1, I wrote analytic memos that described the ways teachers drew on curriculum materials. These memos were detailed, describing the mathematical characters, events, and plot-points across the written and enacted lessons. After developing maps of design arcs and looking more closely at teachers’ steering moves (for RQ2), I returned to these analytic memos and developed a profile of each teacher’s curriculum-use with regard to mathematical narratives. In characterizing their patterns of use, I surmised what dimensions of KCEM they activated (especially Dimensions 2–4), as well as describing their general relationship to narratives in written lessons.

Research question 2 (RQ2): Describing teachers’ steering moves, aiming toward a mathematical point and enacting mathematical storylines and plots. To address RQ2, I began by reviewing the maps of design arcs generated during the first phase of analyzing RQ1. As noted above, I looked for differences and similarities in sequence, duration, or plot elements of mathematical storylines. I highlighted segments of lessons with significant differences or noticeable conformity, and I used these observations to drive my deeper analysis of written guidance and steering moves.

Written curriculum materials: CRLs. To investigate the adaptations teachers made with curriculum materials, more particularly, I first studied their CRLs. Recall that the CRLs asked teachers to indicate elements of the written lessons that they read deeply, in addition to elements that they planned to include or to modify during instruction. To use the CRLs in my analysis, I noted of teachers’ written comments, focusing on segments from RQ1 that suggested significant steering work. In addition, teachers occasionally used CRLs to assist in their own lesson-planning. In so doing, they sometimes added supplementary notes to help explain their teacher-intended curriculum. I noted, for instance, that Mackey’s CRL indicated she both read and planned to implement this activity, and she also added a note to keep in mind a key conversion factor that is used in one of the problems, namely, that there are 144 in$^2$ in 1 ft$^2$ (See Figure 8.) These notes were captured in my analysis for RQ3, more so than for RQ2. The notes were more relevant for understanding and explaining the goals behind teachers’ steering moves, rather than the nature of their written-enacted relationship.

Enacted lessons: Videotapes of observations, lesson transcripts, and field notes. Having reviewed teachers’ CRLs, I annotated lesson transcripts for teachers’ steering moves as they deployed mathematical narratives. I also used my field notes to triangulate and supplement the observations I made from the lesson transcripts. In reviewing the transcripts, I was not looking to
catalogue the frequency of teachers’ steering moves (nor the related strategies or pitfalls) described in Sleep (2012). Instead, my annotations sought to identify—and develop qualitative descriptors of—the teaching moves used to preserve or alter key plot-points of written lessons. As M. Brown (2009) explains of the efficacy of teachers’ instructional designs, “It is the[ir] skill in weaving various modes of use together and in arranging the various pieces…not whether they happen to be offloading, adapting, or improvising at any given moment” (p. 29).

Figure 8. Elsa Mackey’s CRL for Everyday Mathematics (Grade 4, Lesson 8–4, p. 676). Her CRL shows an annotation on Problem 5, as well as highlights indicating her planned use of “Estimating Your Skin.”

In aggregating the themes that I observed across written and enacted lessons, I used constructivist grounded theory (Charmaz, 2006). In so doing, I sought to characterize how teachers adapted storylines and plots and consider the contexts of enactment. On occasion, to make better sense of teachers’ and students’ comments, I also relied on tools of classroom discourse analysis (CDA) (Rymes, 2009). These include exploring the various interactional dimensions that relate to classroom discourse, in addition to unpacking the cues found in classroom speech and participants’ interpretations of one another’s utterances (Rymes, 2009).

Interview transcripts. On rare occasions, I also made use of the follow-up interviews I had conducted with teachers, in order to gain clearer understanding of their steering moves. Generally, follow-up interviews focused on the reasons for teachers’ design decisions. Occasionally, though,
if I had difficulty understanding the nature of the specific adaptations made by teachers, I sought and found evidence of teachers’ modifications within the transcripts. For example, Mackey explained to me that her annotated CRL indicated her intention to skip a particular activity in an Everyday Mathematics lesson; I confirmed this observation during the course of our interview. In general, though, follow-up interview transcripts were used in addressing RQ2, only to help triangulate findings from other sources of data.

Potential warrants made. In the process of my analysis for RQ2, I made claims about teachers’ steering moves. Again, I focused on particular segments of lessons when teachers appeared to offload responsibility on the curriculum materials or depart from them (indicated by the design arcs). Remillard and colleagues (2019) found that written lessons suggest varying numbers and types of steering moves related to specific learning objectives; teachers use these in differing ways, as well. Teachers and programs also exhibited certain patterns in how they deployed steering moves (Remillard et al., 2019, p. 111). In my analysis, I took a narrower approach and mainly focused on describing how teachers progressed the storyline and plot, in addition to understanding how mathematical questions built upon one another (or didn’t).

In my analytic memos, I described transcript segments as exhibiting either low or high suspense. This determination largely depended on whether and in what ways teachers’ enacted lessons utilized certain elements of mathematical plots. (In particular, I focused on prevalence of resequenced events or other sorts of complicating codes.) Likewise, I noted within teachers’ CRLs whether the materials they used supported higher or lower segments of suspense. Both sorts of warrants followed from and were supported by evidence in my collected codes and memos. As noted above, I then used these findings to characterize, broadly, the relationship between written and enacted lessons (RQ1).

Research question 3 (RQ3): Understanding teachers’ PDC. To address RQ3, I aimed to understand and describe teachers’ motivations around enacting particular storylines and plots. In particular, I sought to understand in what ways the mathematical narratives they constructed reflected their goals, beliefs, and school contexts. By addressing this question, I also aimed to demonstrate the importance of considering mathematical narratives, as a component of teachers’ PDC. Here, I also used grounded theory. I generated themes by reviewing findings from RQ1 and RQ2, first, and then aimed to situate these within an analysis of teachers’ interviews and profiles of their teaching contexts. Below, I describe my analytic approach with each source of data used in addressing this research question.

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Artifact analysis. Portraits of teaching contexts were developed from a hermeneutic analysis of the publicly available material on their schools. In particular, I aimed to get a sense of the mission of each school, as expressly indicated or through any profile of students or graduates offered on its website. In some cases, this also involved reading the characterization of the school by an introductory or welcome letter, written and posted by a school leader. I also reviewed school websites on its approach to mathematics instruction, describing the curriculum materials it has selected, and so on. Finally, I also reviewed profiles of the student population. In some cases, this also involved navigating external websites to which schools submitted data for marketing and recruitment purposes (e.g., Private School Review). To develop these portraits, I wrote and revised analytic memos, summarizing my findings.

Interview analysis. During interviews, I also asked teachers to reflect on their expectations of and goals for instruction at their schools. To address context, the introductory interview protocol included such questions as:

7. What do you believe is the major emphasis or the philosophy of these curriculum materials?

15. Are there other resources elsewhere [e.g., provided by the district or the department, or researched by the teacher] that you regularly consult and that are not part of the curriculum for developing your lesson plans? If so, how do you use these materials?

16. When you have a question about the curriculum or curriculum materials [i.e., “curriculum” broadly defined, here, as including all purchased curricular resources and any district/departmental or other materials], what do you do? How does your school or district support your use of the curriculum [again, “curriculum” here is broadly defined]?

These questions, among others, allowed teachers to describe how curriculum materials functioned as intended supports within their schools or districts. Specifically, participants explained whether they were expected to use the selected programs and in what ways. (See Appendices A to C for details of the interview protocols.)

I also used a grounded theory approach, here, to code responses to questions that elucidated elements of teachers’ working contexts—including perceived expectations within their school communities, insights about students’ learning needs, beliefs about curriculum, and general pedagogical philosophies. These codes were aggregated into a theoretical overview, describing each teacher’s goals and beliefs along these dimensions.
Potential warrants made. Together with teachers’ relative CKTM scores, these data allowed for a description of a) the teacher resources in M. Brown’s (2009) DCE, and b) the teacher-context interface (Davis et al., 2011). (Recall that Davis and colleagues identified context and beliefs about students within context, as an element of teachers’ PDC.) From this theory-development, my analysis substantiated claims about the relative efficacy of each teacher’s instructional designs. Further, I drew on these results to make an additional claim that mathematical narratives (perceived and mobilized) should be regarded as elements of PDC.

Summary and logic model. To explain more concretely, my analytic approach connects with my conceptual framework and addresses my research questions through the following logic model (see Table 3). In Yin’s (2009) words, using a case study strategy “begins with ‘a logic of design...a strategy to be preferred when circumstances and research problems are appropriate rather than an ideological commitment to be followed whatever the circumstances’ (Platt, 1992a, p. 46).” The logic model, shown in Table 3, therefore aims to unpack the details of my research design by first articulating a broad need (within the left-most column). Each need is drawn from my review of the literature. Next, proceeding from left to right, I describe an affiliated research question, a summary of the data collected to address the question, the related elements of my conceptual framework, and a theorized outcome. Each of the theoretical outcomes drives my responses to the research questions that follow in subsequent chapters.
<table>
<thead>
<tr>
<th>Research Need</th>
<th>Paraphrased Research Question</th>
<th>Collected Data</th>
<th>Related Framework Element(s)</th>
<th>Theoretical Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>A better understanding of coherence in written &amp; enacted mathematics lessons; fidelity of curriculum implementation.</td>
<td>How to characterize the relationship between mathematical narratives of written &amp; enacted lessons?</td>
<td>Teacher’s guides, Videotapes of observations, Lesson transcripts, Field notes</td>
<td>DCE (curricular resources), CMF (construction arena)</td>
<td>A sketch of each teacher’s overall relationship with narrative structures of curriculum materials and dimensions of KCEM activated in classroom narrative construction</td>
</tr>
<tr>
<td>A better understanding of the fine-grained and in-the-moment design decisions made with curriculum materials.</td>
<td>What are teachers’ steering moves, as they enact mathematical plots and storylines?</td>
<td>CRLs, Videotapes of observations, Lesson transcripts, Field notes, Interview transcripts</td>
<td>CMF (construction arena) &amp; Steering Moves</td>
<td>A profile of each teacher’s classroom adaptations of narratives in curriculum materials and intended plans for instruction</td>
</tr>
<tr>
<td>An understanding of whether and how mathematical narratives contribute to PDC; the roles of their beliefs and goals in constructing mathematical narratives.</td>
<td>What do teachers’ enactments of mathematical plots and storylines say about their PDC and underlying goals, beliefs, &amp; contexts?</td>
<td>Interview transcripts, School websites</td>
<td>DCE (teacher resources), CMF (construction &amp; design arenas), &amp; Contexts</td>
<td>An assessment of each teacher’s PDC and the role of narrative construction within PDC; an explanation of underlying reasons for each teacher’s choices in constructing mathematical narratives</td>
</tr>
</tbody>
</table>
CHAPTER 6. THE CHARACTERS:
PARTICIPANTS AND CURRICULUM MATERIALS

Of all the teachers in our school,
I like Miss Bonkers best.
Our teachers are all different,
But she’s different-er than the rest.

We also have a principal, his name is Mr. Lowe.
He is the very saddest man that any of us know.
He mumbles, “Are they learning this and that and such and such?”
His face is wrinkled as a prune from worrying so much.


6–1. Introduction

The present chapter provides a crucial foundation for what follows. Specifically, I begin Chapter 7 by profiling one of the participants of my case study, Elsa Mackey. Elsa is an experienced user of *Everyday Mathematics* (Bell et al., 2007). I then describe her school context before exploring how she uses written curricular guidance to enact mathematical storylines and plots. In so doing, I draw upon observations made here in Chapter 6. Likewise, I begin Chapter 8 with a profile of Torrie Blum. Torrie is an experienced user of the second program that I describe in this chapter, *Math Trailblazers* (TIMS Project, 2008). I proceed in Chapter 8 by describing Torrie’s implementation of this program, again referring to material in the present chapter.

Here in Chapter 6, I describe these two curriculum programs in greater detail. In order to contextualize the parts of my thesis that follow, I offer a short history of the development of each program, as well as an overview of each program’s components and OTLs (see Table 10 for a summary). I also provide an overview of the research that has been conducted on each program. I conclude this chapter by looking across both programs and making a few comparative claims.

6–2. Everyday Mathematics

Beginning with *Everyday Mathematics*, I draw on research from the previous chapters to ground this review of its features. In particular, I utilize the frameworks of OTLs offered by
Remillard and collaborators (Remillard et al., 2011, 2014; Remillard & Reinke, 2012). In using these frameworks, I refer to prior research on these programs; when such research was not available, as indicated, I draw on my own textual, hermeneutic analysis. After this section on *Everyday Mathematics*, I use a similar approach for describing *Math Trailblazers* in the next. In the following chapters, I report on teachers’ adaptations of the embedded mathematical storylines and plots of written lessons, grounded in the OTLs and design features summarized here.

**A Short History of Everyday Mathematics**

*Everyday Mathematics* is sometimes known, colloquially, as “Chicago Math.” This moniker derives from the University of Chicago School Mathematics Project (UCSMP), under which the program was initially developed (Usiskin, n.d.). The original UCSMP leaders and program authors included recognized faculty experts, such as Sharon Senk, Zalman Usiskin, and Max Bell. Bell, in particular, “was a pioneer in the desire to teach applications of mathematics” (Usiskin, n.d.) and inspired the real-world problem-solving theme of *Everyday Mathematics*. The UCSMP was initially funded in 1983 by the Amoco Foundation, among other sources, to analyze curriculum materials and approaches used in other nations (Usiskin, n.d.).

In 1989, the Amoco Foundation, the Carnegie Corporation, and the NSF began funding the UCSMP, to use the gathered research in writing a new program for U.S. teachers and students. After rounds of field-testing and revision, publication of the first edition of the elementary program of *Everyday Mathematics* was completed in 1997. After a similarly rigorous development process, the second edition was published by the end of 2002 and the third by the end of 2007 (Usiskin, n.d.). McGraw-Hill Education assumed publishing of the third edition (Usiskin, n.d.), which is the edition on which I am reporting.

Usiskin (n.d.) also writes that when “the Common Core State Standards were first announced, UCSMP was pleased that many of the developments that were championed in its materials were represented in the mathematical practices and content standards.” An edition of the *Everyday Mathematics* elementary program, more completely aligned to the CCSS-M, was published by the end of 2011. Throughout its history, corresponding editions for middle and high school mathematics were also developed along analogous timeframes (Usiskin, n.d.).

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17 I do not report on the methodology of this analysis in Chapter 5, because it was mainly informal. I used similar approaches to those in the ICUBiT Project but did not engage in strict coding and theme generation.
Lesson Components and OTLs

The third edition of *Everyday Mathematics* (studied here) was built by the UCSMP from a set of guiding principles. These included a desire to “move [away] from nearly exclusive emphasis on naked number calculation [and] to developing conceptual understanding and problem-solving skills in arithmetic, data, probability, geometry, algebra, and functions” (UCSMP, n.d.-a). Lessons were also designed to connect daily content with students’ prior learning, to encourage partner and small-group work and communication, and (as noted above) to include real-world applications of mathematics (UCSMP, n.d.-a). Finally, several components of the program, including the *Math Boxes*, described below, aimed to provide opportunities for practicing basic arithmetic skills.

*Everyday Mathematics* is also well-known for its so-called *spiraling* approach, in which the primary topics of lessons may change from day to day (UCSMP, n.d.-b). Students return to ideas from one topic (e.g., fraction equivalence) after an interim on a different topic (e.g., probability). In the authors’ view, “spiraling leads to better long-term mastery of facts, skills, and concepts” (UCSMP, n.d.-b). They speculate that spiraling is effective, because students may attend to key ideas that appear less regularly or make conceptual connections across topics (UCSMP, n.d.-b).

Last, a set of content strands and affiliated goals were developed for *Everyday Mathematics*, emphasizing the importance of estimation, conceptual understanding, and a using a variety of models, representations, and strategies (including algorithms) for solving arithmetic problems (UCSMP, ca. 2005).

Key components and design features. Looking more closely at the design of *Everyday Mathematics* (3rd ed.), there are several features worth noting. In what follows, I draw upon my own analysis (where noted) and prior research. Specifically, I rely on Remillard’s and colleagues’ (2014) framework for understanding curricular OTLs. These OTLs include the mathematical emphasis, overall instructional approach, and types of educative supported offered to teachers (see Chapter 3). I also utilize Remillard’s and colleagues’ textual analysis of *Everyday Mathematics* from the ICUBiT Project, which includes depictions of the OTLs found in a random sample of third- to fifth-grade lessons (see Remillard et al., 2011; Remillard & Reinke, 2012).

Each written lesson in the *Teacher’s Lesson Guide* (TLG) of *Everyday Mathematics* begins with an overview page. For each component of the lesson, this page highlights the mathematical objectives, as well as the key activities, skills, vocabulary, and materials. There are three major lesson components in the TLG. These are called 1) *Teaching the Lesson*, 2) *Ongoing Learning and
Practice, and 3) Differentiation Options. Briefly, Ongoing Learning and Practice is the portion of the lesson, primarily, in which teachers are tasked with attending to procedural fluency (Remillard et al., 2014, p. 740). Here, students complete Math Boxes, problems requiring skills taught in previous lessons. (This is also a manifestation of the spiraling design of Everyday Mathematics.)

A handful of problems offered within the Math Boxes preview upcoming content, likely intended to be used as a diagnostic assessment (E. Mackey, personal interview, 10/28/2011). Note, there is no specified amount of time devoted to Ongoing Learning and Practice, nor the Differentiation Options. A teacher who participates in my thesis research, Elsa Mackey, indicates her belief that each comprises, roughly, one-third of a full lesson (personal interview, 10/28/2011).

Just before Teaching the Lesson, there are daily warm-up activities found under a heading entitled Getting Started. I have included Getting Started activities in my analysis, because my preliminary review showed these occupied significant portions of enacted lessons. I also discovered Getting Started activities commonly serve as a conceptual link to the material that follows. I speculate that such activities may, to a degree, represent what Dietiker (2012, 2015a) describes as the abstract (or a pre-summary) of the mathematical storyline that launches the plot. Activities included under Getting Started generally consist of three pieces: Mental Math and Reflexes problems, usually graduated in difficulty and related to basic numeracy; a Math Message, intended to lay a foundation for the main topic of the lesson; and a Study Link Follow-Up, essentially reviewing the prior lesson’s homework. Mental Math and Reflexes constitutes a daily routine, which is intended to reinforce previously-taught skills and concepts (Remillard and colleagues, 2014, p. 749). My preliminary analysis showed that Elsa tended to use the Mental Math and Reflexes and Math Message elements, which is why I included her enactment of them.

For my study, I concentrate on the main component of the written lesson, known as Teaching the Lesson. It constitutes the bulk of classroom activity, targeting the key mathematical objectives. The latter two components, Ongoing Learning and Practice and Differentiation Options, generally involve students’ independent (rather than whole-class) work. From a methodological perspective, these were more challenging to analyze, because the teacher’s role shifts away from developing key ideas with the whole classroom of students and toward prodding and answering questions from individuals. I therefore hypothesized and confirmed that mathematical storylines might not be readily observed during these independent-work portions. Furthermore, pilot analysis using the ICUBiT Project dataset suggested teachers may not attend
as carefully to, or may even skip, the guidance offered in these latter two components. This sort of
guidance is sometimes perceived as optional (see, e.g., Remillard & van Steenbrugge, 2013).

Within Teaching the Lesson, teachers are offered guidance on reviewing the Math Message
and implementing the main learning activities. This guidance contains suggestions on assessment,
in addition to addressing the differentiated needs of learners (such as needs of students who are
English-language learners). Some lessons include succinct guidance about learning progressions
under headings called Links to the Future. As one example, the fourth-grade version of Everyday
Mathematics explains to teachers that students can use base-ten blocks as manipulative tools in
problem-solving but that such tools should not be used in fifth-grade (TLG-4, p. 262). Within
written lessons, there also are occasional notes about mathematical ideas and concepts. These
often reference a separate guide, known as the Teacher’s Reference Manual (TRM), that teachers
can use to support their understanding of the content; however, research on use of Everyday
Mathematics suggests that teachers generally do not utilize material found in the TRM (Stein &

OTLs and elements of the intended lesson. Remillard and colleagues (2011) found that
guidance offered within Everyday Mathematics lessons generally concentrates on directing action
or talking through teachers rather than talking to teachers. Recall that the distinction between
talking through and to reflects Remillard’s (1999) observation that materials sometimes tell
teachers precisely what to do (talking through) instead of explaining design decision to them
(talking to). Of a random sample of numeracy lessons, approximately 75% of written material
(counted at the sentence level) also falls into the directing action category (Remillard et al., 2011,
p. 15). As I explain in greater detail, below, educative content that talks to teachers can have
meaningful impact on lesson implementation (Stein & Kaufman, 2010). The TRM, the
supplementary reference noted above, appears to be the primary resource in Everyday
Mathematics for offering educative mathematics content to teachers.

At the same time, Ongoing Assessment, Adjusting the Activity, and Links to the Future,
collectively offer material on helping teachers understand students’ thinking and the overall
progression of content. Nonetheless, these elements do not always support teachers in responding
to students’ misconceptions or—stated differently—they do not offer always offer sufficient
guidance on how to steer instruction (Sleep, 2012). For example, on a lesson involving geometry,
students are deemed to be making “adequate progress” in finding areas of two-dimensional
shapes, drawn on grids, “if their strategy includes counting whole squares and combining partial
squares” (TLG-4, p. 677). If students are not making adequate progress, though, there are no clear suggestions offered on how to respond.

Remillard and colleagues (2011) also describe the teacher’s role in Everyday Mathematics, on the one hand, as guiding. They say that guiding “gives the teacher a significantly less didactic role than telling [emphasis added], but still, the teacher is framed as the primary shaper of classroom interactions” (p. 10). In contrast, they define telling as a traditional form of instruction, generally not reflected in Everyday Mathematics lessons. By telling, the teacher directs procedures that students are expected to replicate (Remillard et al., 2011, p. 10). Furthermore, they explain that guiding elements of Everyday Mathematics give “the teacher questions to ask designed to prompt student thinking along with answers to expect” (Remillard et al., 2011, p. 10). Students’ roles in Everyday Mathematics, on the other hand, involve both independent and collaborative work, as well as chances to explain their solutions, find patterns, interpret and use models, and so on. Altogether, I therefore borrow from Remillard and colleagues (2014), to characterize the instructional approach of Everyday Mathematics as blended, falling between the classifications of dialogic and direct approaches. This characterization indicates that the teacher is expected to facilitate students’ production of ideas (through classroom discourse) and also, at times, utilize more didactic sorts of moves. Classroom interactions, in addition, are typically structured as a mix of teacher-student and student-student conversations. The written lessons signal conversations intended to occur among students with small icons that say “partner activity” and “small-group activity.”

Through the ICUBiT Project, Remillard and colleagues (2011) report that a random sample of the third- to fifth-grade mathematical tasks in Everyday Mathematics exhibit the following proportions of cognitive demand:

<table>
<thead>
<tr>
<th>Level of Cognitive Demand</th>
<th>Proportion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Memorization (Mem)</td>
<td>22%</td>
</tr>
<tr>
<td>Procedures without Connections (PwoC)</td>
<td>22%</td>
</tr>
<tr>
<td>Procedures with Connections (PwC)</td>
<td>50%</td>
</tr>
<tr>
<td>Doing Mathematics (DM)</td>
<td>6%</td>
</tr>
</tbody>
</table>

*Table 4.* Proportions of levels of cognitive demand (Stein et al., 1996) found within a sample of Everyday Mathematics lessons.
Note that *Everyday Mathematics* tasks analyzed by Remillard and colleagues (2011) were those focal activities within lessons. In other words, they reviewed tasks expected to occupy the most amount of time in the classroom. (In *Everyday Mathematics*, again, these are generally found in *Teaching the Lesson*.) Recall from Chapter 3 that Stein’s and colleagues’ (1996) framework generally portrays tasks as requiring factual, procedural, or more conceptual sorts of understanding. The first two categories, Mem and PwoC are generally considered lower demand tasks, while the last two categories, PwC and DM, are higher demand. Table 4 shows that, in the random sample studied by Remillard and colleagues (2011), lessons in *Everyday Mathematics* are nearly split between lower demand and higher demand tasks. From prior research, this would suggest that *Everyday Mathematics* lessons exhibit moderate to moderate-to-high levels of cognitive demand (Stein & Kim, 2009).

Within another framework on elements of curriculum programs, Remillard and Reinke (2012) contrast two types of textual scripts offered to teachers: explicit and descriptive. Explicit scripts offer the exact “words a teacher might use to ask a question, start a discussion, introduce an idea, or respond to a student’s idea” (p. 744). Descriptive scripts are more general and offer paraphrased suggestions. Writing about *Everyday Mathematics* (3rd ed.), Remillard and Reinke note that lessons consist of blended scripts with both explicit and descriptive instructions. In their words, the authors of *Everyday Mathematics* nonetheless “relied on explicit scripts somewhat selectively and in moderation” (Remillard & Reinke, 2012, p. 7). This suggests, of course, the majority of scripted guidance in *Everyday Mathematics* is descriptive in nature. As one example of explicit scripting, on the other hand, during a lesson about perimeter, fourth-grade teachers are instructed to “Ask questions about the perimeter of a work triangle. What is the smallest perimeter of a work triangle that meets the experts’ recommendations?” Teachers, here, are expected to read this question—and those that follow—verbatim.

Remillard and Reinke (2012) also explain that *Everyday Mathematics* makes extensive use of customization options. To customize lessons, they note, the authors provide a significant number of illustrative examples, sets of questions from which teachers are encouraged to sample. Authors also include contingency scripts of if-then sorts of pathways to navigate. These choices depend on teachers’ interpretations of students’ responses. Of course, there are also optional tasks or activities within *Everyday Mathematics* lessons (like the Differentiation Options). Remillard and Reinke note, though, that there is typically very little guidance provided about how and when to use the various customization options available.
I offer these descriptions of *Everyday Mathematics*, because design features may influence teachers’ use of materials. Indeed, as Stein and Kaufman (2010) found with regard to *Everyday Mathematics* (see below), and as Remillard and colleagues (2014) also found, such features can even impact the quality of teachers’ implementation and students’ learning. More research is still needed, though, to uncover additional features that may influence instruction and learning (see, e.g., Richman, Dietiker, & Brakoniecki, 2016; Huntley & Heck, 2014; Remillard et al., 2014; Stein & Kaufman, 2010). Overall, the design of *Everyday Mathematics*—the authors’ intended curriculum—suggests that activities are to be directed by a mathematically-authoritative teacher, who is voicing instructional guidance for students (Stein & Kim, 2009). At the same time, the program also consistently offers several pathways and options from which teachers may choose. These options, I later argue, interact with mathematical plots and storylines to constitutes a portrait of teachers’ enactments of *Everyday Mathematics*.

**Additional Research on Everyday Mathematics**

Two studies, of note, explore features of *Everyday Mathematics* and teachers’ enactments. Both compare *Everyday Mathematics* with another curriculum program, *Investigations*. Both studies are also predicated on the notion that interactions between the design of curriculum programs and teachers’ decision-making merit investigation, to surface the potential link between the design of materials and students’ achievement (Remillard, Lloyd, & Herbel-Eisenmann, 2009; Huntley & Heck, 2014).

In the first of these two studies, Stein and Kim (2009) found that the majority *Everyday Mathematics* tasks are predominantly (79%) PwoC tasks, whereas *Investigations* tasks are predominantly (89%) DM tasks. At the same time, they also found *Investigations* offers more written guidance than *Everyday Mathematics* for implementing its higher-demand tasks. More specifically, *Everyday Mathematics* lessons did not offer as many explanations for their design decisions nor as many cues for interpreting students’ thinking. Note that higher-demand tasks are generally considered more challenging for teachers to implement, because of the inherent unpredictability of students’ responses (Stein et al., 1996). Higher-demand tasks therefore benefit from additional explanation.

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1[Recall that the full name of the *Investigations* program is *Investigations in Number, Data, and Space.*]
Stein and Kim (2009) explain that, even if *Investigations* is thought to be a more challenging program, then nonetheless:

> The educative possibilities offered by the *Investigations* curriculum, however, suggest the need to pay attention to more than just demand on teacher learning exerted by the curriculum and to also attend to the opportunities for teacher learning that are embedded in the curriculum. (p. 667)

In other words, they argue, the implementation guidance offered by programs, and how teachers attend to such guidance, provides a richer picture of the relative difficulty of using them—more so than by merely reviewing cognitive demand, alone. In contrast to *Investigations*, then, *Everyday Mathematics* is characterized as a relatively low-demand and low-support program (see Stein & Kaufman, 2010, p. 666). Looking to M. Brown (2009), they also describe it as a procedure-centric program (Stein & Kim, 2009, p. 50), partly because of its focus on directing teachers’ actions rather than offering design transparency.

In the second study, Stein and Kaufman (2010) build on this prior work, investigating how teachers interpreted and used guidance found within *Everyday Mathematics* and *Investigations*. They also collected data on aspects of teachers’ professional development, mathematical knowledge for teaching, experience, and overall perceptions of both programs. Stein and Kaufman subsequently compared the quality of implementation at two locations: Greene, a large urban public-school district (using *Investigations*), and Region Z, a network of schools within the public-school system of New York City (using *Everyday Mathematics*). At the time of the study, both locations were in their second or third years of using these programs. Among a host of interesting findings, Stein and Kaufman report that *Investigations* teachers implemented lessons with significantly higher quality than did *Everyday Mathematics* teachers.

Further, Stein and Kaufman (2010) found that teachers who read for, discussed, and focused on the big mathematical ideas (or main objectives) tended to implement the programs with higher quality. This differential pattern occurred, regardless of program used and regardless of teachers’ knowledge. In sum, they explain:

> Unlike teacher capacity, then, how teachers use the curriculum appears to shape the quality of their lessons. In particular, when teachers talked about or reviewed big mathematical ideas that students were supposed to be learning in both Greene and Region Z, they tended to have higher quality lessons. (Stein & Kaufman, 2010, p. 681)

Last, teachers found *Investigations* lessons to be laid out more clearly than *Everyday Mathematics* lessons. The teachers using *Everyday Mathematics* claimed that the layout and design of the
program may have been hindrances to identifying the big ideas of the lesson. At the same time, Stein and Kaufman speculate that professional development offered at Greene may have provided more in-depth support for teachers in understanding how to use *Investigations* and, in particular, identifying the key mathematical objectives. Nonetheless, they note that more “work is needed to tease out the interrelationship between the features of the curriculum and teachers’ instruction” (Stein & Kaufman, 2010, p. 687). I suggest, of course, that how big ideas develop within lessons—through mathematical plots and storylines—represent possible, additional features.

6–2. *Math Trailblazers*

I now turn to describing *Math Trailblazers*. In this section, I provide a short history of the program and its author team. In addition, I outline its key lesson components, OTLs, and design features. I conclude this section by reviewing notable studies on *Math Trailblazers*.

**A Short History of Math Trailblazers**

*Math Trailblazers* originated at the University of Illinois at Chicago (UIC) and, more specifically, within the Teaching Integrated Mathematics and Science Project (TIMS). The TIMS Project was founded by UIC faculty members Howard Goldberg and Philip Wagreich in the 1980s, focused on improving teaching and learning in elementary mathematics and science classrooms (UIC & LSRI, 2015). Goldberg is a particle physicist with an interest in mathematics and science education (Kendall Hunt, n.d.) and Wagreich is a mathematician, who participated in efforts to review and revise the NCTM *Standards* in the late 1990s (Education Development Center [EDC], Inc., 2001, p. 4). In the early 1990s, the TIMS Project team, which also included well-known mathematics educator Cathy Kelso, received a five-year NSF grant to develop the first edition of *Math Trailblazers*; the overarching goal of this grant to produce curriculum materials incorporating the NCTM *Standards* (UIC & LSRI, 2015). The first edition was released in 1996–1997 (UIC & LSRI, 2015).

Four editions of *Math Trailblazers* have been published for teachers and students from kindergarten to fifth grade. The latest Common Core-aligned version, was released in 2014, and all four editions appear under the Kendall Hunt label (Kendall Hunt, n.d.). Following the first edition, teachers’ feedback provided insights for second-edition revisions, released in 2004 (Kendall Hunt, n.d.). The third edition, published in 2008, constituted a major update following a sizable field-test study. The field-test study involved interviews of some 1,500 teachers using *Math Trailblazers* and their students, as well as hundreds of video observations (Kendall Hunt, n.d.).
The teacher using *Math Trailblazers* in my study, Torrie Blum, even noticed as much; during one of our interviews, she observed the third edition incorporates suggestions—modifications that she had already made in her own classroom—for separating the work on addition and subtraction and for using number lines with subtraction (personal interview, 12/02/2011). The fourth edition also incorporates suggestions from this field-test study and other studies, in addition to revisions to accommodate the CCSS-M (Kendall Hunt, n.d.).

**Lesson Components and OTLs**

There are several key tenets underlying the design of the third edition of *Math Trailblazers*, which is the version studied in my research. According to program author, Wagreich, “The big idea of *Math Trailblazers* is doing mathematics in a meaningful way” (EDC, Inc., 2001, p. 4). Meaningful mathematics, he explains, involves “a context that [makes] sense to” students (EDC, Inc., 2001, p. 4). Contexts in *Math Trailblazers* are drawn not only from science but also other real-world situations. Wagreich describes these contexts as being purely mathematical or fantasy-related, in addition to many that draw on stories in language-arts texts (EDC, Inc., 2001, p. 4). Beyond these contextual components, *Math Trailblazers* also aims to motivate students to discover mathematical principles through experimentation, which includes the cognitive processes of reading, writing, and collecting and analyzing data (EDC, Inc., 2001, p. 4). For example, one well-known lesson in *Math Trailblazers*, involves bouncing a ball from varying heights to identify an underlying linear relationship and make predictions (EDC, Inc., 2001, p. 4). Physical tools, manipulatives, are therefore key components of the program.

Wagreich adds he and his co-authors “worked very hard to get a balanced curriculum” (EDC, Inc., 2001, p. 6). By balanced, he explains that he means an “interwoven” treatment of various domains of mathematical content, as well as a synthesis of activities that promote deeper conceptual understanding and skill development (EDC, Inc., 2001, pp. 5–6). He explains the motivation for such a balance:

> There are certain basic facts that everybody needs to know: you need to know your multiplication facts; you need to learn how to graph. But if you teach these skills detached from any other thinking, in an isolated way, it’s harder for kids to learn them because the skills don’t have any meaning for them. Even the procedural aspects of learning mathematics can be made easier if you’re teaching them in a way that has connected them with mathematical meaning. (EDC, Inc., 2001, p. 5)

Wagreich cites research showing that “in the long run,” students using *Math Trailblazers* have virtually the same opportunities to commit the basic arithmetic facts and procedures to memory as
do their peers experiencing more traditional programs (EDC, Inc., 2001, p. 5). This level of mastery is expected no later than grade four, regardless of the type of program used. The difference with *Math Trailblazers*, he argues, is the meaning and depth of understanding attached to this fact-based knowledge (EDC, Inc., 2001, pp. 5–6).

**Key components and design features.** In this sub-section, I look closer at the specific components of *Math Trailblazers*. As before, with *Everyday Mathematics*, I draw upon Remillard’s and colleagues’ (2014) framework of OTLs embedded within the design of curriculum materials and Remillard’s and colleagues’ textual analysis in the ICUBiT Project (see Remillard et al., 2011; Remillard & Reinke, 2012). Again, through this review, I intend to offer context on how teachers may interpret and interact with *Math Trailblazers* (Ben-Peretz, 1996; M. Brown, 2009; Remillard, 1999, 2005; Remillard et al., 2014).

The Unit Resource Guide (URG) of *Math Trailblazers* is the teacher’s primary text for implementing lessons. In addition, the program offers a *Teacher Implementation Guide* (TIG), which is described as “background information and support for teachers on the pedagogy and content of the program” (EDC, Inc., 2001, p. 3). As with the TRG in *Everyday Mathematics*, the TIG in *Math Trailblazers* is a potentially-helpful resource but one not commonly read by teachers (S. Brown et al., 2009). The URG consists of nearly 20 units per grade with approximately 10 lessons per unit (often fewer). Many lessons are intended to be taught across several days (or class periods), which is specified in the lesson overview. At the same time, my review showed that minimal guidance is offered on breaking activities across multiple days.

An overview page begins with a short paragraph summarizing the lesson activities. (A similar, and somewhat redundant, page appears at the end of each lesson; this concluding page is entitled *At a Glance*.) The introductory paragraph on the initial overview page describes actions students will take during the lesson, very generally; this contrasts with other programs that summarize mathematical objectives or ideas at the outset. Mathematical objectives are listed, too, but as a set of approximately a half-dozen points under the heading *Key Content*. Homework assignments, materials lists, and suggestions on *Daily Practice Problems* (DPPs) follow this overview page. The DPPs are similar to the *Math Boxes* in *Everyday Math*, representing skill-based practice problems or word problems on key ideas taught previously.

The written guidance that teachers are expected to follow is located underneath a heading called *Teaching the Activity*. After this section, a short debrief discussion (or example problem) is found under *Summarizing the Lesson*. My analysis concentrates on the activities and discussions...
for Teaching the Activity and also Summarizing the Lesson. The text is largely narrative in nature, offered through a sizable number of paragraphs with full, descriptive sentences. For Math Trailblazers lessons in my sample, in fact, I coded an average of 88 mathematical events (roughly equivalent to the number of sentences). This compares to an average of 28 mathematical events for my coded Everyday Mathematics lessons. Margin notes are minimal but do include pictures of the Student Guide (i.e., the student textbook), the Discovery Assignment Book (i.e., the student workbook), as well as Content Notes and TIMS Tips. Content Notes are brief comments on mathematical ideas (e.g., URG-4, Unit 6, Lesson 1, p. 32), and TIMS Tips are usually suggestions for managing classrooms (URG-4, Unit 6, Lesson 1, p. 37). TIMS Tips are so-named, because they are derived from the TIMS Project field-tests of lesson activities (EDC, Inc., 2001, p. 4). I describe an additional set of margin notes for differentiation support, called Meeting Individual Needs, later on.

In addition, one noteworthy feature of Math Trailblazers is the inclusion of thought bubbles that describe ways students may consider the problems (EDC, Inc., 2001, p. 8). For example, “Maya” is a cartoon figure, who is portrayed as reasoning (via a depicted thought bubble) about a method for finding 9 × 4 by first finding 10 × 4 and then subtracting 4 from this product (URG-4, Unit 7, Lesson 3, p. 66). Math Trailblazers also offers significant support for teachers in understanding students’ thinking by providing formative assessment exercises. These include samples of students’ actual work, as well as affiliated scoring rubrics. As one example from fourth-grade, a formative assessment shows a student’s solution to a complex arithmetic problem and an affiliated scoring rubric (URG-4, Unit 7, Lesson 3, pp. 70–71). The rubric asks teachers to assess not only the accuracy of students’ work and the depth of their understanding (URG-4, Unit 7, Lesson 3, pp. 70–71). (These are consequently called Solving and Knowing rubrics.) Rubrics are generally not meant to classify students’ skill-mastery, but rather to support the teacher’s ongoing understanding of their progress (e.g., URG, Grade 4, Unit 7, Lesson 3, p. 66).

OTLs and elements of the intended lesson. In their sample of third- to fifth-grade lessons, Remillard and colleagues (2011), found that approximately 65% of the guidance in Math Trailblazers directs teachers’ actions (p. 15). Of the programs studied by Remillard and colleagues (2011), this proportion—sentences directing teachers’ actions—was the smallest in Math Trailblazers. In contrast to several other programs, Math Trailblazers lessons offered more information about the intentions of the lesson (or the design rationale), including how students might think about the content, the foundational mathematical principles, and supports for
Describing the teacher’s role, Remillard and colleagues (2011) find that *Math Trailblazers* promotes *facilitating* lessons. In their words, activities “position the teacher in a much less central way” that do most programs (Remillard et al., 2011, p. 10). In *facilitating* lessons, teachers are guided to provide opportunities for students “to explore, make observations, develop their own approaches to solving problems” (Remillard et al., 2011, p. 10). As an example, the URG for Grade 3, Unit 2, Lesson 8 states that teachers should:

> Encourage students to develop their own strategies as they solve Questions 4–10 in pairs or groups. Remind students that they need not find the exact total cost of the fruit in order to solve the problems…. After students solve the problems, discuss their solution strategies…. Let students teach each other through discussion. (p. 108)

This example shows that teachers are encouraged to offer students tips in their problem-solving work but not to demonstrate solutions for students. Opportunities for students to collaborate are also suggested. The teacher-student and student-student interactions are, therefore, *dialogic* (Remillard et al., 2014). Students are positioned as largely independent learners, tasked with inventing their own strategies and making generalizations—by reflecting on their own and their peers’ work (Remillard et al., 2011, p. 12).

Turning to the other key OTLs, Remillard and colleagues (2011) report on the level of cognitive demand within a random sample of third- to fifth-grade tasks in *Math Trailblazers* (3rd ed.). They found tasks were distributed as shown in Table 5. Given that the tasks from these randomly-sampled lessons reflect only the two higher-demand categories, prior research suggests that lessons might be more difficult for teachers to implement (Stein & Kim, 2009).

On the other hand, as the research comparing *Everyday Mathematics* and *Investigations* shows, higher-demand tasks can be implemented effectively, particularly when the program offers sufficient support for lesson enactment (Stein & Kaufman, 2010). As noted above, Remillard and colleagues (2011) found that *Math Trailblazers* contains at least as much support for lesson enactment—helping teachers understand students’ ideas and thinking, explaining the purpose and
mathematics behind suggested activities, and so on—as does Investigations. Therefore, it is likely that Stein and Kim (2009) would also classify Math Trailblazers as a high-demand and high-support curriculum program.

<table>
<thead>
<tr>
<th>Level of Cognitive Demand</th>
<th>Proportion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Memorization (Mem)</td>
<td>–</td>
</tr>
<tr>
<td>Procedures without Connections (PwoC)</td>
<td>–</td>
</tr>
<tr>
<td>Procedures with Connections (PwC)</td>
<td>73%</td>
</tr>
<tr>
<td>Doing Mathematics (DM)</td>
<td>26%*</td>
</tr>
</tbody>
</table>

* Some of these sampled activities consisted of two parts, one part that involved DM and another that involved PwC; here, I follow a convention of describing lessons with the highest level of cognitive demand evidenced.

Table 5. Proportions of levels of cognitive demand (Stein et al., 1996) found within a sample of Math Trailblazers lessons.

In addition, using the framework presented by Remillard and Reinke (2012), I categorize the types of scripts offered to teachers as a blend of descriptive and explicit. Lessons are constituted, largely, by descriptive scripts because they do not stipulate the language to be deployed mimetically within classroom conversations. Instead, the text paraphrases lines of inquiry that teachers and students could pursue. For instance, in a third-grade lesson on sorting tangram pieces into various categories, the URG directs teachers to “ask questions like: Pick one of the groups in your sort. What do the shapes in this group have in common?” (URG-U, Unit 11, Lesson 1, p. 22, italics in the original). Remillard and Reinke would, most likely, also describe the descriptive scripts in Math Trailblazers as elaborated, since the text offers a significant amount of detail that “communicate[s] a sense of tone and intent of the designed instruction” (p. 9).

Furthermore, as with lessons in Everyday Mathematics, explicit scripts in Math Trailblazers appear to be used infrequently. When used, they generally specify questions for students, but the nature of the questions seems intended to prompt classroom conversation rather than concrete answers. These sorts of questions therefore support teachers in adhering to the facilitating role. For example, the set of questions in the third-grade tangram lesson (referenced above) asks students to make general observations, rather than to provide specific answers to heavily-prescribed. These are often written as “why” sorts of explanatory questions.

Math Trailblazers also offers a broad spectrum of approaches to customization (Remillard & Reinke, 2012). These, too, are oriented toward supporting teachers in adopting a facilitating role.
Also using Remillard’s and Reinke’s typology, my review shows that there are occasional illustrative examples (viz., as above, “…ask questions like…”). These appear to a lesser extent, perhaps, than those found in *Everyday Mathematics*. *Contingency scripts* also appear within each lesson I studied, often multiple times per lesson. As an example, during the tangram lesson, teachers are asked to review students work and “if there are discrepancies,” to have students explain (URG-3, Unit 11, Lesson 1, p. 23).

Finally, my sample of *Math Trailblazers* lessons included fewer alternative activities, or what Remillard and Reinke call *pedagogical options*, than those found in my sample of *Everyday Mathematics* lessons. When these appear in *Math Trailblazers*, they are typically presented as opportunities for differentiation. For instance, in a lesson on rounding, an alternate approach is suggested for students who are not yet comfortable working with three-digit numbers (URG-3, Unit 6, Lesson 3, p. 60). This appears within a differently typeset call-out box. During the same lesson, some students are allowed to use “front-end estimation” (URG-3, Unit 6, Lesson 3, p. 61). This pedagogical option, unlike the previous one, appears within the main body of text. As Remillard (2012) notes, the look of a program is likely to influence how and what teachers read. Employing very few pedagogical options, regardless, *Math Trailblazers* lessons give the impression they are to be implemented *in toto*, as-is.

By the same token, given that the program positions teachers as facilitators—encouraging them to make continual adjustments based on students’ ideas—one might argue that the lessons, themselves, are written to emphasize the variety of pedagogical options. For all practical purposes, then, *Math Trailblazers* lessons could be considered lengthy sets of pedagogical options. M. Brown (2009) might describe it as program leaning toward a resource-centric approach. Altogether, the OTLs of *Math Trailblazers* suggest that enacted lessons—the authors’ intended curriculum—involves teachers, first, posing thought-provoking problem situations to students and, next, allowing students to work on their own, with their peers, and using the tools at their disposal. The teacher’s role therefore involves a lot of steering work (Sleep, 2012).

**Additional research on Math Trailblazers**

As noted previously, *Math Trailblazers* was designed in a highly-iterative fashion. Results of field-testing and research on the second and third editions were incorporated into the revised third and fourth editions. Now, I briefly describe several additional studies, exemplifying the wide interest in NSF-funded programs like *Math Trailblazers*. In Chapter 3, I reviewed research on the efficacy of such programs, which is, indeed, an ongoing interest. I summarize one such study,
below. I also review two studies with *Math Trailblazers* that have implications on the curriculum-development process.

First, as an efficacy-focused study, Carter and colleagues (2003) report on a sample of third-grade students in Illinois, whose teachers used *Math Trailblazers* for two years. Participating schools were selected from communities regarded as both high-needs and more affluent. The researchers found that students’ achievement on a basic-skills test performed no worse than students at the same schools before *Math Trailblazers* was implemented. In addition, Carter and colleagues review several case studies, showing students’ improved reasoning and problem-solving capacities with *Math Trailblazers*.

Next, recall from Chapter 3 that S. Brown and colleagues (2009) studied teachers’ use of *Math Trailblazers* in developing their framework for fidelity of implementation. They showed that fidelity to the steps of the literal lesson did not determine teachers’ fidelity to the authors’ intended lesson (nor vice versa). The latter type of fidelity, they explain, incorporates the desired OTLs for classroom instruction. Brown and colleagues also found patterns among teachers and lessons with regard to both types of fidelity. Overall, they argue that various elements of particular *Math Trailblazers* lessons may have either supported or inhibited teachers’ uptake of written support. Brown and colleagues suggest, then, that authors (and researchers) attend carefully to indicators of the intended lesson and how teachers may interpret such guidance.

Last, Superfine, Kelso, and Beal (2010) describe their study of the curriculum-design and research process. They explain that their investigation “examines the research and revision process of a commonly used research-based mathematics curriculum—*Math Trailblazers* (MTB)—to provide understanding about how this [curriculum-] development process actually works and how the process can be improved” (p. 910). They build on prior theoretical work, to illustrate how *Math Trailblazers* could be an exemplar among research-based programs in that it was developed via a highly-iterative process with extensive field-testing and revision.

At the same time, Superfine and colleagues (2010) note that certain elements were underrepresented in the design of *Math Trailblazers*, such as “why and under what conditions the curriculum is effective” (p. 927). Moreover, still other elements—such as teachers’ professional development and affiliated assessments—have not been evaluated at all. Superfine and colleagues conclude by saying their overall aim was not to describe the efficacy of *Math Trailblazers*—nor to suggest particular assets or flaws with regard to its development—but instead “to make visible [emphasis added] the research and revision process” (p. 931). Instead, simply by pursuing this
aimed-for transparency, they both hoped to inform researchers and to “help local policy makers and administrators make more informed decisions” about the program-selection (p. 931).

6–4. Summary and Conclusion

In this chapter I reviewed two curriculum programs represented in my study: Everyday Mathematics and Math Trailblazers. In so doing, I offered a short history of each program’s development, as well as an outline of key components, design features, OTLs, and affiliated research. Through this review and using established research frameworks, I note that Everyday Mathematics can be described, relatively, as a low-demand and low-support program (Stein & Kaufman, 2010, p. 666). In contrast, Math Trailblazers can be described, relatively, as a high-demand and high-support program. At the same time, both programs originated as NSF-funded and reform-oriented instructional materials. These sorts of programs place students’ ideas on center stage, which requires additional steering work (Sleep, 2012). Such programs are also regarded, then, as more challenging for teachers to implement, because of the relative uncertainty involved in lesson enactment (e.g., Henningsen & Stein, 1997).

Because both programs are reform-oriented programs that necessarily carve out significant roaming space for teachers and their students—or, as Ben-Peretz (1990) might say, a wide curriculum envelope—they are important for my study. Both programs, of course, necessitate teachers’ adaptations, and both aim to develop concepts through activities, representations and applications, and over extended periods of time. Therefore, both represent instructional resources—theoretically, at least—that deploy mathematical storylines and plots for teachers to read and interpret.

On the one hand, Everyday Mathematics represents a procedure-centric approach to lesson design (Stein & Kim, 2009, p. 50). On the other, Math Trailblazers might be characterized as leaning resource-centric. In addition, differences in OTLs and the amount and types of other educative supports suggest potential differences in teachers’ implementations. Stein and Kaufman (2010) found that teachers using a program like Math Trailblazers enacted lessons that were deemed higher-quality than those using Everyday Mathematics. Cues for big ideas were found more prominent in the program similar to Math Trailblazers. One question, pursued through my investigation, remains whether or not this translates into differences in presentation of mathematical storylines and plots. Likewise, another central question is whether teachers perceive storylines and plots, mobilize them during instruction, and modify them in response to students.
CHAPTER 7. ACT I: ELSA MACKEY’S MATHEMATICAL STORYLINES AND PLOTS—
TANGIBLE, COHERENT, AND FABULA-ORIENTED

We have three cooks, all named McMunch, who merrily prepare our lunch….

We were eating their concoctions, telling jokes and making noise,
when Mr. Lowe appeared and howled, “Attention, girls and boys!”

He began to fuss and fidget, scratch and mutter, sneeze and cough.
He shook his head so hard, we thought his eyebrows would come off.

—Dr. Seuss, Hooray for Diffendoofer Day! (1998, lines 53, 57–60)

7–1. Introduction: Elsa Mackey and Golden Hawk Preparatory School

Because Davis and colleagues (2011) assert contextual features influence teachers’
curriculum-use (and comprise elements of their pedagogical design capacity [PDC]), I begin
Chapters 7 and 8 with a profile of each participant and her school. Through my conceptual
framework, I then explore the relationship between teacher and curricular resources in
mathematical narrative-construction. Recall that the design features of instructional programs
(Chapter 6) also comprise elements of PDC, as teachers perceive and mobilize the affordances of
the written lessons (M. Brown, 2009). Of course, teachers’ past experience with and orientations
toward curriculum influence their use of materials, as well (e.g., Choppin, 2011; Remillard &
Bryans, 2004). I report on these in Chapter 9.

In this opening section, I describe Elsa Mackey and Golden Hawk Preparatory School. In my
initial meeting with Elsa, she proclaimed support for conceptual-oriented teaching and endorsed
the conceptual-oriented basis of Everyday Mathematics. At the same time, Elsa indicated she
reads materials, to find specific activities or problems to implement successfully. She explains she
usually omits those that present challenges. As I explain in greater detail in Chapter 9, for Elsa,
conceptual understanding and students’ engagement in her classroom are closely related. This
perspective manifests itself in multiple ways in her classroom lessons, which I describe below.

Elsa Mackey’s Background and Experience

Elsa Mackey is a highly-experienced elementary school teacher, beginning her 17th year in the
classroom. Aside from two years of teaching third grade—and her first- and second-grade
student-teaching experiences—she has only taught fourth-grade students. Early in her career, Elsa worked in the southeastern U.S., including at an “inner-city public school” (personal interview, 10/28/2011). She subsequently moved to the mid-Atlantic region and has taught for several years at Golden Hawk Preparatory School (Golden Hawk). Just like “Maggie,” the teacher profiled by Davis and colleagues (2011), Elsa’s teaching background is diverse. This is an important observation, because Davis and colleagues attribute Maggie’s curricular adaptations, in part, to her prior experiences with less-confident students (Davis et al., 2011, p. 804).

Elsa’s current school, Golden Hawk, is a self-described “college preparatory, coed, [religiously-affiliated] day school serving grades pre-kindergarten through 12th grade.” The school is an independent (or private) and co-educational school with a selective admissions process. It is located within or nearby to a large population center. The school’s population is relatively small with only 600 students and a student:teacher ratio of about 10:1. In addition, its students have diverse backgrounds: 38% are students of color and 8% are international students. Golden Hawk boasts a rich academic and athletic program with state-of-the-art resources. Since 2009, and possibly earlier, elementary teachers have used Everyday Mathematics (3rd Edition) as their primary mathematics resource.

Elsa was recommended as an ideal participant for the ICUBiT Project, because of her significant experience with Everyday Mathematics. During our initial interview, Elsa told me that—except for her first four years of teaching—she has always used Everyday Mathematics (E. Mackey, personal interview, 10/28/2011). This included her time in the southeastern U.S. Even still, I regard Elsa’s professional learning opportunities with Everyday Mathematics as modest. According to Elsa, Golden Hawk held several days of training, years ago, but hasn’t offered many refresher opportunities (personal interview, 10/28/2011). She also had training early in her career, offered by a representative “for the company” (personal interview, 10/28/2011). This representative, she says, revealed “secrets of the book” (personal interview, 10/28/2011). She regards her colleagues as occasional resources for answering questions, as well. And, as I later explain in greater detail, Elsa’s orientation to Everyday Mathematics seems generally activity-focused (Remillard & Bryans, 2004, p. 367).

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1 This profile of the school and its student population was obtained from Golden Hawk’s website on January 6, 2018. I am not aware of any significant changes to this profile from the time of my initial data collection to now.
The Organization of the Remainder of This and Succeeding Chapters

The remainder of this chapter is organized around responses to my research questions. Embedded within are descriptions of Everyday Mathematics text and Elsa’s classroom lessons. Note that I am sequencing the following sections in accordance with my overall analytic approach. That is, I begin with addressing RQ2 and unpacking maps of design arcs. These, in turn, help me explore finer-grained elements of Elsa’s steering moves. Understanding Elsa’s approach to steering, broadly, then allows me to characterize the relationship between Elsa’s written and enacted curricula (RQ1). As explained in the previous chapter, these findings were generated from my analytic coding and memoing. In Chapter 9, I use my findings from RQ1 and RQ2, along with my profile of Elsa’s background and beliefs, to explain her teacher-intended curriculum and decision-making (RQ3). Of course, I concentrate on describing Elsa’s choices in constructing mathematical narratives. In Chapter 9, I also explore Elsa’s work, as it contrasts with that of my other participant, Torrie Blum.

7–2. RQ2—Elsa’s Steering Moves: Focus and Broad Coherence

In this section, I unpack the maps of design arcs for Elsa’s written and enacted lessons. As I conducted my analysis, these maps helped me isolate potentially-key elements of Elsa’s steering. I consequently characterize her steering moves in this section on RQ2, first, to understand better her overall relationship with the written lessons she uses, subsequently, in addressing RQ1. I begin by orienting the reader to my design arc maps.

Reading the Maps of Design Arcs

The maps, depicting two of Elsa’s classroom lessons and their written counterparts, are shown in Figure 9 and Figure 10. In explaining how to interpret the maps, I refer to Figure 9 on Lesson 8–3 in Everyday Mathematics. This lesson was taught on February 28, 2012 with Elsa’s fourth-grade students. Again, my design arc maps intend to offer a high-level and side-by-side overview of the narrative structure of written and enacted instantiations of a lesson—and, specifically, their emergent mathematical plots and storylines.

Paired maps their axes. The left-hand map (Figure 9a) represents the overall narrative structure of the written lesson. In comparison, the right-hand map (Figure 9b) depicts the related, enacted lesson. The scales on the horizontal axes are constructed, to the extent possible, so that a
direct comparison across maps is meaningful. In other words, the relative positions and durations of the storylines (as arcs) are broadly relatable—with the caveats noted, below, however.

The ordered number of mathematical events is shown on the horizontal axis (in Figure 9a). Because English text proceeds from top to bottom, I assume readers of written lessons experience the syuzhet in a sequential fashion, as events appear one after the other. (Recall the fabula is the mental reconstruction of logically-sequenced mathematical events.) I learned from Dietiker’s (2012, 2013b, 2015a) mathematical story framework (MSF), and my own analysis, that mathematical events occur, roughly, at the sentence-level; therefore, the horizontal axis typically refers to the ordinal number of a sentence in a written lesson. Further, the horizontal scale on the enacted map (Figure 9b) represents the proportion of the classroom lesson at any given point (in minutes), running from 0 to 100%. Individual points are given by ordered pairs that—for their x-coordinates—show the percentage of the total lesson at which a mathematical event occurs. For, say, (37, 2.5), the indicated event occurs 37% of the way through the entire lesson. Only mathematical events are plotted on the maps, and so, some lessons may be said to “begin” after the class period begins or “end” before the period’s conclusion.

The vertical axis is, essentially, arbitrary. It runs from 0 to 3, as a ratio-level scale, but the meaning of 0 and the vertical distance between any two points does not connote any particular unit. I adopted this convention, admittedly, for pragmatic purposes. For one reason, it aided consistency across lessons and it eased conversions between my codes and the final diagrams. (I used spreadsheet formulas to plot the maps and their arcs.) Nevertheless, there is an intended, qualitative purpose for the vertical scale. As curves, design arcs have a vertex, intended to be located above 1 on the vertical axis. (A few arcs do not exhibit such vertices, but for technical reasons20.) Tracing a single arc from left to right, as a lesson proceeds, it ascends to a point and then descends. Ascending and descending intervals correspond, loosely, to the rising and falling action of the storyline. Arcs that terminate above the horizontal axis convey one of two meanings: if it terminates at, approximately, \( y = 0.3 \), the corresponding storyline was partly answered; if it terminates at about 0.7, the corresponding storyline was not resolved. (The former storylines were

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20 Because of the way design arcs were drawn, using spreadsheet formulas and scripting, some design arcs exhibit unusual features like vertices that have y-coordinates less than 1. This occurred, generally, because the ordered pairs of points assigned to my codes were connected via cubic splines; these were interpolations, determined by a process involving matrix inversion, and not all matrices are readily invertible.
coded with Pa and the latter with Sa; see Chapter 5.) Last, arcs that terminate directly on the horizontal axis represent storylines that have been completely answered (a disclosure).

These conventions are built from assumptions about procedure-centric instructional resources. In using procedure-centric materials, teachers are presumed to follow instructions, step-by-step (M. Brown, 2009; S. Brown et al., 2009). Recall that Stein and Kim (2009) identified Everyday Mathematics as a procedure-centric program.

**Storylines, plots, and map annotations.** Next, I describe how storylines (i.e., here, design arcs) are plotted on the maps. All the lexia of a given mathematical storyline (in the syuzhet) receive an ordered pair of codes for plotting on the coordinate plane. The $x$-coordinate represents the accumulated number of the event (e.g., the 5th event has an $x$-coordinate of 5) or the proportion of the total lesson (as described above). The $y$-coordinate, likewise, indicates Labov’s and Waletzky’s (1967) and Labov’s (1997) narrative phases (as applicable):

- $0 = \text{abstract, orientation, disclosure}$
- $1 = \text{complicating action}$
- $2 = \text{evaluation}$
- $0.7 = \text{suspended answer}$
- $0.3 = \text{partial answer}$

In other words, the $y$-coordinate describes whether the main formulation (or primary question) about a given mathematical character is resolved. Further, due to my interpolation approach—using cubic splines—vertices of arcs tend to fall between $y = 1$ and $y = 2$ on the vertical scale. When arcs rise above $y = 2$, they typically indicate an evaluative sequence$^{21}$.

Next, blue arcs represent storylines from the written lessons. Blue arcs in the enacted maps, likewise, represent storylines clearly tied to written lessons. In both written and enacted lessons, blue arcs exhibit the same main character(s) and key mathematical events. Storylines in written lessons are also labeled—e.g., with Figure 9a, MS1 to MS4. Storylines in purple are those that

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$^{21}$ In a handful cases, for the technical reasons specified in Footnote 20, vertices do not appear within the intended range; when extremely divergent vertices were observed—in other words, when the related matrix, to find the coefficients of the cubic spline, is near-singular or singular—manual adjustments to the vertex were made. These manual adjustments brought the vertex within the expected range. And, to clarify, this method was used to plot the relatively large number of storylines represented in my dataset in an efficient, automated fashion.
Figure 9. The design arc diagrams, showing the written storylines and plot of Lesson 8–3 and Elsa’s enacted lesson drawing on these resources. The dashed line represents a storyline that was first dropped but returned to, later, in the lesson.

Figure 10. The design arc diagrams, showing the written storylines and plot of Lesson 8–4 and Elsa’s enacted lesson drawing on these resources.
are improvised or invented by teachers during instruction; they are not labeled. As noted in my conceptual framework, mathematical storylines may overlap, either partly or completely.

Storylines that incorporate significant amounts of steering guidance (either written or enacted) tend to be the longer than those that do not. On occasion, storylines may begin but not conclude before another emerges. (Analogously, a dramatic performance may shift to a new scene, while the characters in the prior scene remain in the background for an extended period.) When I observed three or more mathematical events in a new storyline—without the previous one resolved—I considered the storyline suspended and marked it accordingly. If the storyline resumed, but later, the intervening period was marked with a dotted line (see, e.g., Figure 9b). As Remillard and colleagues (2019) explain, further, the amount of written steering guidance does not necessarily correspond to the amount of steering in an enacted lesson. Comparisons of the length of storylines, therefore, should be considered carefully. Stated differently, a relatively brief storyline, as written, may require (or may be given) a sizable steering when enacted. Remillard and colleagues did find a loose correlation with this sort of complexity, however.

Finally, I also annotated other key events on my maps of design arcs. These included markers for partially-answered or suspended storylines (i.e., Pa and Sa codes) and markers of plot twists (Dietiker, 2016; Ryan & Dietiker, 2018). Plot twists in complicating action add suspense—confusion, surprise, or curiosity—to mathematical storylines. I indicated plot twists with complicating-action codes (Pe, Sn, En, or Jg for promises, snares, equivocations, or jammings). When relevant, I also indicated components of lessons (e.g., Teaching the Lesson), as well as portions of lessons involving students’ independent work time, among other notes.

Structural Comparisons: Written and Enacted Instantiations of Lessons 8–3 and 8–4

My analysis is built from the analytical codes described in Chapter 5 that mark the emergent mathematical storylines of written and enacted lessons. Recall that, as Dietiker (2012) explains, the purpose of this sort of analytic coding isn’t to identify the single, defined set of mathematical storylines or plot features within a given lesson:

As with any interpretive framework, interpretations will vary among different readers; however, as opposed to being a threat of analysis, multiple interpretations instead are additional evidence that new relational qualities can be recognized and described when mathematics curriculum is read as mathematical stories. (Dietiker, 2012, p. 184)

Stated differently, again, identifying a fixed structure of mathematical storylines isn’t the primary focus of my analysis. As I noted earlier, instead, Dietiker explains that “it is the reading for
structure that offers new insight” (p. 44). Citing Rosenblatt (1994), Dietiker also contrasts two modes of reading, *efferent reading* (for information) and *aesthetic reading* (for emotional experience). The latter is my general approach: as my analysis deepened, I aimed to understand the suspense offered by lessons. In addition, I used Dietiker’s (2012) re-reading approach to mitigate potential biases. Specifically, I re-analyzed each lesson in my dataset, several times, and I revised my coding as needed. In so doing, I noted which parts remained stable. I also avoided analyzing written and enacted lessons, together, so that each was considered independently.

**Background on the unit.** I now compare the written and enacted versions of Lessons 8–3 and 8–4. I selected these two lessons as representative of Elsa’s practice for several reasons. For one, they represented lessons in which she drew heavily upon the written materials and relied less on materials from outside the program. For another, both included activities to support students’ conceptual understanding. (Other lessons focused, more concretely, on refining students’ procedural fluency.) Therefore, I thought these lessons would invite students’ contributions during classroom discussions and, consequently, accommodate Elsa’s opportunities to interpret and adapt their embedded mathematical storylines and plots.

Before proceeding, I situate these lessons within the unit. In Unit 8, students investigate the perimeter and area of planar figures. In Lesson 8–1, students explore kitchen arrangements by measuring and drawing the perimeters of the *work triangles* connecting the stove, sink, and refrigerator. In Lesson 8–2, students measure the perimeter of their classroom. Lesson 8–3 involves concepts of area, including measuring the area of the classroom. Areas of irregular shapes are explored in Lesson 8–4. Beginning with Lesson 8–5 and for the remainder of Unit 8, students formalize their understanding of area by making sense of formulas for rectangles, parallelograms, and triangles. The mathematics becomes increasingly abstract during the unit, and furthermore, each lesson is connected to a real-world problem.

*Structural comparisons of the written and enacted lessons of Lesson 8–3.* Looking at the maps of design arcs, broadly, I note in Figure 9a that Lesson 8–3 is a relatively short-length lesson, as written, meaning it has a relatively small number of mathematical events and few storylines. The main objectives of Lesson 8–3 are: “To review basic area concepts; to provide practice estimating the area of a polygon by counting unit squares and using a scale drawing to
find area” (TLG-4, p. 670). There are four storylines. Note that a storyline on finding the area of students’ classrooms (MS3) contains two, shorter storylines that run parallel to MS3—one on finding the area in square feet (MS2) and another on converting this measurement to square yards (MS4). Each of the written storylines is summarized in Table 6.

In the written lesson, as marked on the map (Figure 9a), there are two periods for students’ independent work (i.e., working instead of participating in whole-class narration by their teacher). These periods are signaled by the descriptors “independent activity” or “partner activity.” When students are released to work alone or in small groups, regardless, I considered this independent work time. Independent work time is commonly indicated within the text, as well. For example, in Lesson 8–3, the teacher is instructed to: “Have students complete Problems 1–4 [in their Math Journals] independently. They can complete Problems 5–7 with a partner” (TLG-4, p. 672).

Figure 9b, portraying the enacted lesson, shows that Elsa devoted considerable attention to MS1. And just like the written lesson, Elsa’s enactment of MS1 also ends with a partial answer. Yet, the written lesson also includes an equivocation that does not appear within the implemented lesson. Elsa also appears to add a sizable number of improvised or invented storylines (M. Brown, 2009). In contrast to the written lesson, moreover, there is only one period of independent work for students. This period occurs in the second portion of MS1, extending for nearly half of the lesson. She also omits several written storylines.

The maps of Lesson 8–3 suggest, overall, that we should investigate several elements of Elsa’s instruction and steering, to develop a nuanced understanding of her instructional design. In particular, we should explore her enactment of MS1, including students’ independent work time. We should also seek to describe the storylines Elsa added to and removed from the written lesson. These questions and potential answers are explored further, below.

**Structural comparisons of the written and enacted lessons of Lesson 8–4.** Now, though, I offer an overview of the structural features of Lesson 8–4. This next, succeeding lesson builds on ideas from Lesson 8–3 but is more complex. It aims to support students in making sense of unit conversions, as well as finding the areas of irregular two-dimensional shapes. Its main objectives

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22 In this chapter, rather than offering a full, in-text citation, I use a shorthand to reference pages of *Everyday Mathematics* by referring to the fourth-grade *Teacher’s Lesson Guide* (TLG-4) and the page number.
<table>
<thead>
<tr>
<th>Mathematical Storyline</th>
<th>Character(s)</th>
<th>Setting</th>
<th>Main Formulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>MS1</td>
<td>Polygons (rectangular &amp; non-rectangular)</td>
<td>2-D Grid</td>
<td>“Can you find the area of polygons, making use of gridlines superimposed upon them? How?”</td>
</tr>
<tr>
<td>MS2</td>
<td>2-D Polygon</td>
<td>Scale Drawing</td>
<td>“What is the area of the drawing of your classroom (measured in square inches)?”</td>
</tr>
<tr>
<td>MS3</td>
<td>A Whole Number</td>
<td>Customary (U.S.) Area Measurement</td>
<td>“Using your scale drawing, what is the area of your classroom (measured in square feet)?”</td>
</tr>
<tr>
<td>MS4</td>
<td>A Whole Number</td>
<td>Customary (U.S.) Area Measurement</td>
<td>“What is the equivalent measurement of the area of your classroom in square yards?”</td>
</tr>
</tbody>
</table>

Table 6. Mathematical storylines and key narrative dimensions of Lesson 8–3 of *Everyday Mathematics*. Storylines highlighted in blue text are those enacted (whole or in part) by Elsa.

<table>
<thead>
<tr>
<th>Mathematical Storyline</th>
<th>Character(s)</th>
<th>Setting</th>
<th>Main Formulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>MS1</td>
<td>Square</td>
<td>2-D Grid</td>
<td>“How many square inches are there in one square foot?”</td>
</tr>
<tr>
<td>MS2</td>
<td>A Whole Number</td>
<td>Customary (U.S.) Area Measurement</td>
<td>“What would you guess is the total surface area of your skin? Can you estimate the surface area?”</td>
</tr>
<tr>
<td>MS3</td>
<td>Amorphous, Closed Figure</td>
<td>2-D Grid (1 in x 1 in squares)</td>
<td>“How can you find the area of a two-dimensional figure with a curved boundary?”</td>
</tr>
<tr>
<td>MS4</td>
<td>A Whole Number</td>
<td>Customary (U.S.) Area Measurement</td>
<td>“How do you convert square inches to square feet?”</td>
</tr>
<tr>
<td>MS5</td>
<td>The Number 144</td>
<td>1 ft x 1 ft squares (multiple)</td>
<td>“What multiple of 144 is closest to the number of square inches in your estimate of surface area?”</td>
</tr>
<tr>
<td>MS6</td>
<td>A Whole Number</td>
<td>Customary (U.S.) Area Measurement</td>
<td>“How do you convert square feet to square yards?”</td>
</tr>
</tbody>
</table>

Table 7. Mathematical storylines and key narrative dimensions of Lesson 8–4 of *Everyday Mathematics*. Storylines highlighted in blue text are those enacted (whole or in part) by Elsa.
are: “To demonstrate how to estimate the area of a surface having a curved boundary; and to provide practice converting from one square unit to another” (TLG-4, p. 675). Lessons 8–3 and 8–4 are similar, then, except that the borders of the shapes in Lesson 8–3 are all line segments. These two lessons also differ, more specifically, in how they address measurement units.

Lesson 8–4 consists of six mathematical storylines (labeled MS1 to MS6 in Figure 10a). These storylines are also summarized in Table 7. Upon quick inspection, I observe that the storylines of this enacted lesson appear to reflect those of the written lesson, more closely, than those in Lesson 8–3. At the same time, however, I note a changed sequence of storylines from MS4 to MS6. More work must be done to verify and explain this initial impression. In addition, MS4 appears to be implemented somewhat differently than suggested in the written lesson. Specifically, while most of the key elements of plot in the written lesson (e.g., the partially-answered storylines) are also represented in the enacted lesson, MS4 contains jamming that does not appear in the enacted lesson. It appears that Elsa has added a handful of storylines to the enacted lesson, as well.

Collectively, the maps of Lesson 8–4 suggest exploring the relationship between written and enacted storyline for MS4 (and the sequence from MS4 to MS6). In particular, we may want to investigate the instance of jamming in MS4 and Elsa’s adaptations. Finally, as with Lesson 8–3, we may want to describe the storylines Elsa appears to add to the enacted lesson. Likewise, these are explored in the next sub-section of this chapter.

Elsa’s Steering Moves in Lessons 8–3 and 8–4

Using the maps of design arcs for the written and enacted instantiations of Lessons 8–3 and 8–4, I am now prepared to dig more deeply into key episodes of Elsa’s steering work. In summary, throughout these lessons, Elsa generally remains focused on the intended mathematics, and she offers regular opportunities for students to do mathematical work. I mention these criteria, because non-mathematical time has been tied to the incoherence of lessons (Fernandez et al., 1992). Regarding storylines, Elsa also narrates and frames key ideas as they emerge within lesson activities. Therefore, we might characterize her steering as both attending to strategic use of instructional time and, more broadly, coherent presentation (Sleep, 2012). At the same time, however, nuanced elements of her steering suggest that the sequence of ideas may have undercut the mathematical residue left by these lessons. I explain more, below.

Elsa’s steering moves within Lesson 8–3: Key ideas opened, despite some fracturing. In the description that follows, I outline the steering moves Elsa makes during her implementation of
Lesson 8–3. As suggested by my structural analysis of design arc maps, I concentrate on elemental aspects of MS1. Generally-speaking, in this lesson, Elsa works to open-up and reinforce foundational ideas for her students. Despite this, she also concentrates on a narrow portion of the lesson’s objectives, which leads to a somewhat fractured classroom experience.

The progression of storylines related to MS1. The main formulation, or primary question, of the mathematical storyline MS1 involves two-dimensional shapes—depicted on a grid—and finding their areas. I would describe the affiliated task, furthermore, as a PwC-type task. Indeed, students must compute areas, but they must also grapple with the meaning of area. In so doing, students are asked to count both the complete and partial grid-squares, occupied by a given shape. As noted previously, Elsa adds storylines, too, many of which are affiliated with MS1. I therefore review these in tandem with my review of the progression of MS1.

Rather than describe the entirety of Elsa’s enactment of MS1, because it constitutes nearly the whole lesson, I offer highlights that characterize her steering. First, MS1 begins with the Math Message, asking students to read a brief description of area (as contrasted with perimeter). Students are also instructed, “Be ready to describe a situation in which you would need to know the area of a surface” (TLG-4, p. 671). Elsa, likewise, launches the lesson by asking the class to “Read page 133 and the question I have for you while you’re reading this is if you could describe a situation in which you need the know the area of a surface” (observation transcript, 02/28/2012). At the same time, I note the TLG does not ask the teacher to address such applications during the ensuing discussion; instead, the teacher is asked to concentrate on the definition of area, units of measurement, and a strategy students will use, shortly: counting squares.

After a few minutes, Elsa draws a 3 × 2 unit rectangle on the board (as in the SRB, p. 133). She turns to students and asks for circumstances when they would need to find area. One student refers to Elsa’s drawing as a representation of a chair, covered by fabric. How this explanation relates to area is a bit unclear, however. At this point, students may not understand whether Elsa is asking how to find the area of a 3 × 2 rectangle or to describe area situations. Students remain silent until another offers, “When you’re making clothes?” (observation transcript, 02/28/2012). Elsa’s asks, “When you’re making clothes how?” (observation transcript, 02/28/2012).

During her follow-up explanation, the student confuses perimeter and area, and Elsa capitalizes upon this opportunity. She first defines the perimeter as the length required to trace around the shirt, and then Elsa distinguishes perimeter from area by referring to the “square yards of fabric—of plaid, my shirt, for example—to cover the whole thing” (observation transcript,
Elsa focuses on the definitions, even more, by subsequently relating a personal anecdote about her brother was painting her house and miscalculating the amount of paint:

…so, he [my brother], he figured out how much area, like, one side of the house was. Right? He, like, multiplied it by four, because there were four sides [emphasis added]. And he thought to himself, “OK, like this is how much we need to cover [the four walls] with paint.” And he looked the paint can, and he looks on the back of the paint can, and it says how many square feet are covered with that one paint can. So he determined that we need two-and-a-half cans of paint, to paint this whole house. So, like, I was, like, OK, sure. Alright. So, I went to the store, bought the paint, bought two-and-a-half cans, it was like maybe 75 to 80 dollars for it….Well, clearly something went wrong with my brother’s calculation—to this day, we’re, like, not really sure what happened—but we needed more, like, eight or nine paint cans to cover the whole house. And we think, we think what happened, is he just added the perimeter of the feet [one wall of] the house, rather than figuring out the area. (observation transcript, 02/28/2012)

In addition to noting Elsa’s work in distinguishing area and perimeter, I have emphasized a portion of Elsa’s anecdote involving multiplication. I sense that Elsa’s own understanding of area is not necessarily conceptual.

I surmise that, as a result, Elsa concentrates on helping students internalize related formulas. I also note this contrasts with the authors’ intentions, because the written lesson (including p. 133 of the SRB) emphasizes that area can be found by merely counting the grid-squares covered by a two-dimensional shape. Indeed, Elsa then steers the conversation toward an algorithm for finding the area of a 2 cm × 2 cm square tile on the classroom floor. When students remain silent, once again, she supplies an answer:

So, I guess what I am getting at is that in order to figure out a square centimeter you’d have to multiply the length times the width. OK? And the length and the width in that case are the same. (observation transcript, 02/28/2012)

Elsa then presents yet another example—trying to drive her point home—contrasting the perimeter of a dinner table (and the number of guests that could be seated around it) with its area.

These two examples—the floor tile and dinner table—provide fodder for Elsa’s transition to a new activity: offering students a series of multiple-choice questions. These broadly involve definitions of area and perimeter, as well as formulas for each. For example, one asks:

Which phrase best describes the definition of area? (A) side by side; (B) length times width; (C) side plus side; (D) side minus side. (observation transcript and videotape, 02/28/2012)

Elsa’s CRL for Lesson 8–3 includes this comment: “* Add Here Prom. Board flipchart – review perimeter + area” (see Figure 11). (“Prom. Board flipchart” is Elsa’s shorthand for using the...
interactive whiteboard and its affiliated slideshow feature.) As students answer these questions, Elsa guides them into explaining their responses more concretely. After one question, she observes, “A couple of us are a little confused…So, what I just want to review with you is that perimeter is where you add all of the sides….So it should be (C)…” (observation transcript, 02/28/2012). Elsa also asks “Matt” to find the perimeter of a $3 \times 2$ rectangle; after he responds “10,” she asks him to explain the details of this result (observation transcript, 02/28/2012). He offers two ways: $6 + 4$ and $3 + 2 + 3 + 2$ (observation transcript, 02/28/2012).

Steering moves, added and removed storylines, and the written lesson. In general, throughout Elsa’s enactment of MS1—investigating strategies for finding the area of two-dimensional shapes—she steers instruction by calling attention, repeatedly, to the difference between area and perimeter (i.e., opening up and emphasizing key ideas). She also aims to connect examples on area to an underlying formula, namely, multiplying length and width. It isn’t clear from Elsa’s enactment, though, whether she realizes this formula only applies to rectangles. Nonetheless, she also steers instruction by seeking to connect the mathematics to tangible, real-world applications (thereby keeping a focus on meaning). These steering moves serve to add mathematical coherence to her lesson.

In addition, Elsa develops storylines by connecting the mathematical ideas across problems, and she consistently narrates and summarizes the work undertaken by students. In fact, at the end of her lesson, she asks several students to recapitulate the important ideas about area and perimeter, and she offers a summary of her own that connects to her earlier focus on mathematical applications (observation transcript, 02/28/2012). On the other hand, it is less clear whether Elsa’s sequencing helps ideas progress in a meaningful way. Her goals remain persistent, but mathematical ideas are not necessarily deepened. The focus remains on appreciating the difference between area and perimeter, but as formulas or definitions.

One exception occurs, perhaps, after students complete the multiple-choice questions. She sets them to work, individually, on Problems 1–4 in their Student Journal. As they begin, Elsa notes: “You might notice that some of the boxes are divided in half. And so you might need to add some of those halves together” (observation transcript, 02/28/2012). This guidance is intended to call attention to the underlying concepts of area. Elsa then circulates around the classroom, but she discovers a need to offer substantial help to her students. It seems to me students’ struggles were predicated on what came before: they may have been led astray by Elsa’s focus on definitions and the rectangular-area formula. Few seem to be using counting squares as a an
appropriate strategy. In addition, I observe one student who has found the correct area for Problem 2, but for the wrong reasons: she multiplies 5 cm and 2 cm to obtain 10 cm², as Elsa has instructed her to do; but, she explains her work to me, incorrectly using the diagonal side-length and believing it to be 2 cm long (instead of finding the height and recognizing the diagonal side-length is, approximately, 2.24 cm) (field notes, 02/28/2012). I suspect this student may not have been alone in having such fundamental misperceptions.

As explained above, I noted Elsa adds a number of brief storylines while enacting MS1. Looking more closely, I observe these are the real-world examples (like the dinner table example) and multiple-choice questions. Together, these seem to reinforce Elsa’s broad organizing structure, to consistently unpack the relationship between area and perimeter through real-world applications.

In contrast, the written lesson suggests a very different approach for MS1 (TLG-4, p. 671). After an orienting discussion on the basic concepts of area, the teacher is instructed to assign page 227 of the students’ Math Journal, containing two-dimensional shapes superimposed on grid squares. After completing Problems 1 through 4 on their own, students may then work in pairs to complete the remaining three problems on this page. Interestingly, only Problem 1 is a rectangle that can be solved by multiplying the length and width. Problem 2 is a non-rectangular parallelogram, and Problems 3 and 4 are right triangles. Problems 5 through 7 are triangles and non-rectangular parallelograms that require more complex approaches for finding their areas.

Teachers are next guided (by an Ongoing Assessment call-out box) to review students’ work on page 227 and to gauge their success in aggregating completely- and partially-covered grid-squares. In short, it seems MS1 is a problem-solving opportunity for students, to confront a
potentially-suspenseful and confusing situation—non-rectangular shapes that partially overlap with grid-squares (i.e., a potential equivocation). Rather than instruct them on solving these problems, perhaps via a direct instruction approach, the teacher’s role is to assign these problems and to observe and assess students’ solution approaches.

Therefore, reading the enacted lesson from an aesthetic perspective (Rosenblatt, 1994), this shift—from practicing the use of formulas to counting partial grid-squares—seems somewhat jarring. Stated differently, the suspense offered by the Math Journal problems contrasts sharply with Elsa’s pre-communicated goals on multiplying lengths and widths. I suspect that, despite the focus on key ideas (e.g., the definition of area) and a connected set of mathematical exercises, students experience a measure of incoherence in the presentation of this mathematical storyline. At the same time, however, I note teachers are provided little guidance on how to address students’ challenges with finding the areas of figures like these. I discuss these observations, further, in the next section.

I observe, finally, that Elsa omits mathematical storylines MS2 to MS4. When I inquire why, during our follow-up interview, she admits that—having taught this lesson many times—there are complicated logistics involved. These contribute to frustrations that, she indicates, have now led her to omit the affiliated activities. To complete the activities, indeed, students must measure the lengths of walls in their classroom, circumnavigating desks and other furniture. Elsa also notes students tend to measure inaccurately and struggle to create scale drawings, as requested within the written lesson (E. Mackey, personal interview, 03/01/2012).

**Summary and conclusion.** I must offer a brief caveat before proceeding: the purpose of my analysis is not to concentrate on missed opportunities for deepening students’ conceptual understanding. One might argue that there were several within this lesson, especially if taken outside of the context of students’ prior learning experiences and Elsa’s ongoing assessment of their understanding. Even though, at this point in the school year, I had visited Elsa’s classroom several times, I did not have the advantage of knowing the students and their work. Elsa’s choices to address formulas, more specifically, may have been founded in her awareness of students’ multiplicative reasoning capacities (which I could not assess). Instead of identifying potential opportunities to enhance students’ understanding, my general aim, here, was to uncover the mathematical storylines Elsa utilized during her lessons. In addition, I sought to describe the overall effect of consequential adaptations she made when steering instruction. Again, ultimately, this sort of analysis aims to understand the coherence of the mathematical presentation.
In Chapter 9, I also explore how Elsa’s goals and beliefs potentially influenced her decisions. For the time being, I observe her steering moves (and the written materials) interacted in ways that both supported and potentially undermined their coherence. On the one hand, Elsa demonstrated a consistent focus on key ideas of the storyline and main objectives of the lesson. On the other, the materials didn’t offer her much support in grappling with the problems from the *Math Journal*. Consequently, she was unable to set up this activity to inspire students’ curiosity, puzzlement, and productive struggle. She used it, instead, as a summative-type assessment of the prior segments of the lesson—a purpose for which the activity was not designed.

Finally, even though students’ productive struggle may have been limited, the cognitive demand of the activity was not, likewise, reduced (Hiebert & Grouws, 2007; Stein & Lane, 1996; Stein et al., 1996). For one reason, throughout the activity related to MS1—and the lesson as a whole—students had opportunities to discuss and use formulas and definitions with meaning (see, e.g., the vignette about Matt above). Elsa maintains the PwC level of demand. Instead, as she navigated MS1, Elsa seemed to have a different purpose in mind for the activity than indicated by the authors; I believe that this difference, combined with the lack of an explicit curricular rationale, conspired to produce a less-suspenseful storyline.

**Elsa’s key steering moves in Lesson 8–4: Focused and coherent, despite less suspense.**

Recall that one of the main objectives of Lesson 8–4, as written, is for students to be able to “demonstrate how to estimate the area of a surface having a curved boundary…” (TLG-4, p. 675). Another involves “practice converting from one [type of] square unit to another…from square inches to square feet and from square feet to square yards” (TLG-4, p. 675). Having reviewed the lesson multiple times, I would add another, unstated objective: that students come to appreciate the purpose behind—i.e., the reasons for—converting among measurement units.

I observed Elsa’s implementation of Lesson 8–4 on March 1, 2012. As noted above and in Table 7, there are six mathematical storylines within the written text of *Teaching the Lesson*. As with Lesson 8–3, Elsa inserts additional mathematical storylines, not suggested by the written materials. She omits none, however. Here, I concentrate on describing Elsa’s enactment of three storylines, MS4 to MS6. These storylines—I learned from the structural analysis—represent episodes when Elsa may have deviated in notable ways from the written lesson. In particular, she makes a consequential adjustment to a plot twist found in this lesson.

**Progression of storylines in the enacted lesson.** Before describing the key mathematical storylines of my analysis, MS4 to MS6, I offer a brief summary of the *Math Message*. This
activity, intended to be short in duration, poses management challenges for Elsa and her students and therefore becomes extended. Essentially, the Math Message asks students to patch together an 8.5 in \(\times\) 11 in piece of grid paper into a 1 ft \(\times\) 1 ft square; measuring, taping, and general dexterity appear to be their main difficulties, causing the activity to balloon in length. I mention this activity, regardless, because the 1 ft \(\times\) 1 ft square features in subsequent portions of the lesson. As they conclude the Math Message activity, Elsa reminds her students—without much discussion—that they can find the number of square inches within a square foot by multiplying the length by the width, thereby obtaining 144 in\(^2\) (observation transcript, 03/01/2012).

Next, during the second and longest phase of the lesson, Elsa and her students work together in enacting MS2 (of which MS4, MS5, and MS6 are constituent parts). The activity affiliated with this overarching storyline involves estimating the surface area of a person’s skin (i.e., a complete epidermal layer). During the activity’s setup, Elsa solicits ideas on how to accomplish this feat; one student suggests counting multiples of one hand covering a person’s skin-surface (and finding the area of the hand). Elsa acknowledges this suggestion is a “really, really, great way of making that connection for us” (observation transcript, 03/01/2012), and she implies the suggestion is similar to one the class will soon use. She then builds on this suggestion, guiding students to guesstimate the surface area (in square feet) by mapping their square-foot Math Message cutouts, iteratively, onto themselves.

Elsa subsequently redirects to follow the instructions on page 230 of their Math Journal (TLG-4, p. 676). These instructions ask students to use a rule-of-thumb for estimating the surface area of their skin (in square inches) by measuring the surface area of one hand and multiplying the result by 100 (TLG-4, p. 676). As in Lesson 8–3, students are expected to find the area of one hand by counting complete and partial grid-squares; I, therefore, classify this activity as a PwC-type task. As the students proceed with completing page 230, Elsa reminds them to keep their fingers close together as they trace their handprints (as instructed by the written lesson). And she tells them to follow the steps indicated, including where to write their results. I therefore note that the guidance given to teachers, here, is relatively straightforward and step-by-step, which Elsa follows closely as she implements this activity.

Here, Elsa begins to develop MS4, an investigation of students’ rule-of-thumb estimates in square inches and converting these estimates to square feet. As students begin to work, Elsa tells them: “But, then, if you look at the questions below, they want you to figure out square inches, square feet, and then square yards. OK, and, uh, and I will be walking around to kind of help you
with that” (observation transcript, 03/01/2012). After a few minutes of independent work time, she interrupts them, having observed a pattern in their responses:

Just look up here….Some of you are at the point where you are at [Problem] Number 5. And you have a number that’s…you have a number that’s in the thousands and you’d be realizing that you have to convert it to how many square feet. Right? And that’s kinda tricky because you’re going to have to take that number and divide it by 144. Because there are 144 in² in one square foot. If you’d like to make it a little bit easier and use a calculator, you may. But…if you get an answer that has a decimal point, I want you to round your answer, as it should be an estimate. (observation transcript, 03/01/2012)

Elsa had observed students were writing thousands of square inches in Problem 5, even though this result was several orders of magnitude away from their prior guesstimates. After offering this support, Elsa then circulates and gives students explicit instructions—for example, “So you got 1,200 in², divide it by 144” (observation transcript, 03/01/2012).

As the written lesson suggests, Elsa progresses the storyline MS4 by summarizing the information students have, only thus far, gathered. In doing so, however, they are not supposed to have progressed beyond Problem 4. This is an important observation, when reading the lesson in an aesthetic mode (Rosenblatt, 1994). In progressing the storyline beyond Problem 4, nevertheless, Elsa steers by asking students to share ranges of their results. After hearing several roughly-similar values, Elsa asks, “Why is it that we divide it [i.e., 100 times the area of a hand in square inches] by 144…can you explain that?” (observation transcript, 03/01/2012). A student responds with a reasonable explanation, and Elsa then reiterates the key points:

…one foot equals 12 in. Right? But one square foot equals 144 in². That was the tricky part. And some of you just wanted to divide by feet [sic] and not square feet [sic]. And then when you’re able to do that, how many square feet did you get? (observation transcript, 03/01/2012)

In the second full sentence in this quote, Elsa misspeaks—saying “feet” and “square feet,” when she clearly means inches and square inches. Nonetheless, she indicates to students a potential misconception, namely, that they might be tempted to divide their rule-of-thumb estimates by 12 and not by 144. Note that, even though she has asked students to perform the conversion, Elsa has not yet asked students to compare their original guesstimates (in square feet) with their rule-of-thumb estimates (in square inches).

Elsa asks students to reflect upon the work they have done, as she brings the lesson to a close:

OK, let me ask you a question. Take a moment and look up at your estimate at the top [of page 230]. How many square feet? If you were close to your estimate—I would say within, like, two or three feet either way—stand up. If you were—wait, wait—if you
were close to your estimate, within one or two feet either way, stand up. (observation transcript, 03/01/2012)

In other words, as the lesson is about to conclude, Elsa asks students to compare their earlier guesstimates with their rule-of-thumb estimates. Of course, this comparison is suggested within the written guidance for mathematical storyline MS4 (see Error! Reference source not found.). This comparison is supposed to occur, though, immediately after students have completed the rule-of-thumb estimate and summarized their results. In contrast to the written lesson, then, Elsa asks students to perform the conversion, first, and then summarize their results. Unlike the written lesson, she also asks students to make their results public by having them stand up, if their guesstimates were close; for those students whose guesstimates were “way off,” she asks them to share the pair of numbers involved and reflect upon the nature of the differences (observation transcript, 03/01/2012).

I want to reiterate the sequence of mathematical events in MS4, because the order seems important and results in a subtle but powerful point. The Math Journal does not prompt explicit reflection about potential differences between guesstimates and rule-of-thumb estimates; in fact, students are not supposed to work beyond Problem 4 on their pages. I therefore take the abrupt transition and lack of scaffolding about the comparison, as intentional. In other words, I regard students’ recording of values, as intending to provoke curiosity (and possibly confusion). When students summarize their results (after completing Problem 4), they should confront the sizable differences in the values obtained. The authors of Everyday Mathematics suggest as much, informing the teacher that she should prompt students for the comparison after they have completed Problems 1–4 and, as a whole class, summarized the results obtained, so far. The text then explains: “The guesstimate and the rule-of-thumb estimate cannot be immediately compared…” (TLG-4, p. 677). I describe the potential impact, below.

Briefly, though, in order to explain this complex sequence of events more fully, I omitted description of an intervening pair of mathematical storylines. I describe these, here, before shifting back to discussing the aesthetic impact of MS4. After working on the rule-of-thumb estimates and before Elsa initiates a wrap-up discussion, she draws on guidance from the written lesson to ask students to convert their skin-area estimates into an equivalent number of square yards (MS6). Together, the class explores this problem and relates it to the prior day’s discussion on measuring fabric. They next investigate the relative sizes of multiples of 144 in² (MS5). The
enacted sequence, here, is different than the written sequence; in the lesson’s text, MS5 and MS6 are intended to overlap. I describe the potential intention behind this design, below, as well.

Steering moves and the written version of MS4. Elsa made a number of important steering moves during the lesson, and my purpose is not to catalogue and quantify them all here. Instead, I offer a brief, qualitative overview of her general steering approach. First, throughout this lesson, Elsa demonstrated consistent and careful attention to reviewing her students’ work, looking for key misperceptions and helping them attend to precision in their answers. As a side note, students’ participation, throughout, was active and disperse, as well. While less important for maintaining coherence, she therefore spent time on the intended mathematical work and managed multiple goals at the same time, to keep her students on-task.

In addition, Elsa asked students to do the mathematical work, largely by themselves, particularly when finding the surface areas of their irregularly-shaped hands. She also invites them to carefully explain one of the main objectives of the lesson (understanding the conversion between square feet and square inches). While she certainly offered scaffolded support, when needed, she still encouraged them to grapple with and make sense of (or open up) key mathematical ideas. She even calls explicit attention to the conversion objective (observation transcript, 03/01/2012).

Finally, with regard to storylines, Elsa consistently framed and narrated students’ work—including pausing at important moments to solicit their results. She even asks them to summarize their progress. This approach deftly consolidated important points within the storyline. Ideas certainly grew more complex, as MS4 progressed, and tasks and problems were undoubtedly connected. At the conclusion of the lesson, moreover, Elsa similarly tied together several important threads that had been woven throughout. In our follow-up interview, Elsa observes, “…I felt like they all got it in the end. I don't think there was anybody who didn't figure that out. And I was really excited about that because it's not always the case” (personal interview, 03/01/2012). As a classroom observer, I would be hard-pressed to disagree with this assessment.

In sum, Elsa’s steering work undoubtedly made strategic use of instructional time (Sleep, 2012, p. 943). The coherence of her lesson—using the criteria offered by Sleep (2012)—seemed strong, as well. Nevertheless, I discern something missing from her lesson—something that seems ineffable, were it not for Dietiker’s (2012, 2013b, 2015a) framework. The MSF, I contend, providing us with the appropriate language. Indeed, throughout my analysis of this lesson, I intended to suggest two pivotal questions: 1) Did students understand that the purpose of unit-
conversion is to compare two different ways of expressing the same quantity (an unstated, but nonetheless clear, objective)? and 2) What was the long-term residue of this activity (Davis, 1992; Hiebert et al., 1997)? Without surveying, interviewing, or otherwise assessing Elsa’s students, it is difficult to know. But I speculate that Elsa and certain elements of the written lesson unintentionally undercut how it feels to think like a mathematician. In other words, the written lesson contained elements of suspense that were subtle and, had they been preserved, might have contributed to students’ satisfaction in puzzling through an answer.

I defend this claim by exploring how MS4 contains an instance of jamming (Dietiker, 2012, p. 175). That is, it contains events that—to a novice reader—give the impression an answer is impossible. For example, 1,152 in$^2$ appears quite distinct from 8 ft$^2$, even though these values are actually equivalent amounts. Jamming occurs across four mathematical events in MS4 that I am labeling herein as A, B, C, and D. In sequence, the written curriculum asks students (through their teacher’s actions) to: (A) guesstimate the number of square feet, then (B) estimate the number of square inches (using the rule-of-thumb), and finally (D) confront the difference—in order to motivate the need for (C) a conversion of units. In contrast, however, the mathematical fabula of the written storyline MS4 proceeds A-B-C-D (see Figure 10). This would be the logical reconstruction: one would convert units first and then compare measurements.

I interpret this subverted sequence in the written lesson as intentional, creating an intellectual need for conversion (Fuller, Rabin, & Harel, 2011; Harel, 2013; Meyer, 2009, 2015). Confronting the difference in measured quantities—for, importantly, the same object—creates a form of cognitive dissonance that suggests a purpose for converting units. Again, the authors of Everyday Mathematics even admit as much: “The guess and the estimate cannot be immediately compared…” (TLG-4, p. 677). Elsa, instead, alters the order of these events by having students, first, convert among measurement units (C) before comparing the differences between their guesstimates and rule-of-thumb estimates (D). Stated differently, the mathematical syuzhet of Elsa’s lesson (i.e., the events in the storyline implemented in her classroom) aligns more closely with the sequence of events in the underlying mathematical fabula (i.e., the logical flow of events, A-B-C-D).

Elsa also changes the sequence of MS5 and MS6. She walks students through the process for converting square feet to square yards with MS6—generally following the authors’ guidance—but before asking them to contemplate multiples of 144 in$^2$ with MS5. Therefore, MS5 and MS6 were segregated from one another and their order was reversed; in contrast, the written lesson
overlaps MS5 and MS6 but also presents them in the order shown in Table 7. The overall effect of this adaptation is hard to gauge, except I presume the authors wanted to suggest conversions can be achieved fluidly and simultaneously, evoking the notion of equivalent values. By concretely segregating these storylines, I speculate Elsa may have given students the false impression these are separate calculations and, hence, result in non-equivalent values. This is pure hypothesis, however.

7–3. RQ1—Elsa’s Written-Enacted Relationship: Fabula-Oriented

In this section, I draw on my analysis of Elsa’s steering moves, to characterize the relationship between the narrative construction of her written and enacted lessons. I proceed by looking across both of Elsa’s implemented lessons and describing her enactment of mathematical storylines and plots. I relate these observations to the potential role and nature of coherence in her mathematical presentation and her activation of dimensions of KCEM. Broadly construed, I regard Elsa’s written-enacted relationship as fabula-oriented. As I explain, below, the character of Elsa’s implementation is generally aligned with the underlying mathematical fabula of lessons. In Chapter 9, I also offer insight into reasons for Elsa’s narrative-construction choices, and I contrast Elsa’s with those of the other participant in my case study.

Mathematical Storylines—Key Elements: Characters and Settings

Over the course of my study, as explained previously, I found Elsa’s use of curriculum materials to be largely activity-focused (Remillard & Bryans, 2004). While she drew heavily on the topics suggested by the lessons and used many of the activities, she generally read the teacher’s guide as a menu of options. She therefore selected some activities, omitted others, and extended the lengths of selected activities to fill gaps. While not a part of my analysis, I observed this was often the case with the Differentiation Options component of the lesson; when implementing this component, typically during the last third of the class period, she gave students a set of optional tasks from which they, themselves, could choose (e.g., field notes, 10/28/2011, 11/07/2011, 02/28/2012). These included supplemental activities from outside resources.

Looking across both lessons, reviewed here, Elsa added a number of mathematical storylines. On occasion, these were generated by her students, such as the discussion in Lesson 8–3 on tailoring a shirt. Mainly, though, Elsa added mathematical storylines to offer students additional opportunities to practice skills. For instance, during Lesson 8–4, she interrupted students’ work on the skin surface-area activity, to help them practice, mentally, how to multiply whole numbers by...
Likewise, Elsa devoted a significant amount of practice-time in Lesson 8–3 to using of a formula for the area of a rectangle. Stein and colleagues (2007) remark that a common belief about reform-oriented programs is the notion they do not offer enough skill-based practice to students, and so these choices may have reflected Elsa’s perceptions of potential shortcomings of Everyday Mathematics.

Of course, Elsa also omitted storylines from her enacted lessons. These were fewer in number than those that she added, and the omitted storylines generally occurred in a complex stretch of Lesson 8–3. Similar adding-or-omitting adaptations identified in previous research, have been attributed, generally, to teachers’ comfort level with the underlying mathematics (e.g., Manouchehri & Goodman, 1998, 2000) or familiarity with the activity itself (Choppin, 2011). In the case of Lesson 8–3, though, Elsa argued the activities she omitted had, in the past, posed classroom-management challenges. The overall impact of these omissions, it seemed to me, involved reducing students’ opportunities to work with scale drawings (a complex topic) and limiting their preparation for the unit conversions in Lesson 8–4. (These were the objectives behind storylines within activities Elsa omitted in Lesson 8–3.)

Nonetheless, when she used Everyday Mathematics activities during Teaching the Lesson, she tended to preserve the mathematical characters and settings embedded within the written guidance. She spent significant classroom time, for example, on MS6 in Lesson 8–4, which involved converting square feet into square yards. The mathematical character, here, is the number of square feet representing a person’s skin surface-area. The mathematical setting is the units of the U.S. Customary System. And the mathematical events are those that transform the character (a number of square feet) into its changed state (a number of square yards). As the written lesson suggested, Elsa modeled this conversion by using students’ square-foot cutouts and taping nine at a time on the whiteboard to portray a square yard.

To Dietiker (2012), this sort of gradual development of ideas is the essence of a productive storyline. She explains:

If a mathematical story can be described as answering questions immediately after they are raised, then the lesson offers little for the reader to look forward to (except being asked more questions). However, mathematical questions that are opened and sustained throughout a mathematical story can offer a reader suspense and wonder and motivate him or her to keep reading (similar to “will Romeo and Juliet live happily ever after?”). (Dietiker, 2012, p. 99)
This perspective, of course, aligns with Sleep’s (2012) steering task for coherent instruction—particularly, opening up and emphasizing key mathematical ideas. At the same time, we learn from Dietiker (2012) and from Elsa’s lessons that opening up key ideas may be a necessary condition for coherence and engagement, but it is not a sufficient one. I explain more, below.

**Mathematical Plots—Key Elements: Events, Fabulae, and Syuzhets**

Across Elsa’s implemented lessons, the essential mathematical events of her enacted mathematical storylines generally follow the sequence suggested within the written materials. Of course, I make this claim while also acknowledging that reform-oriented materials cannot possibly offer enough specificity to guide every classroom event. Indeed, not only did Elsa respond to her students’ thinking—in ways that weren’t predicted by the text—but she also incorporated their ideas into her lessons.

While analysis of students’ responses is beyond the scope of this thesis, I briefly note that she necessarily added mathematical events to her enactment of storylines, to navigate them in tandem with her students (and also to assess their understanding, periodically). I noted the fabric example above. As another example, when implementing Lesson 8–3, Elsa reviews the definition of area with students and observes that they may be confusing it with the definition of perimeter. To make these ideas more concrete, she and her students therefore navigate a problem on finding the perimeter of a table. In so doing, she steers instruction so that “Jacob” articulates his approach to adding the lengths of the sides (observation transcript, 02/28/2012).

Other than adding (and omitting) mathematical storylines in her enactments, as noted above, Elsa does not make significant adjustments the sequence of events suggested by the text. There are three notable exceptions, however. First, in Elsa’s enactment of MS1 in Lesson 8–3, she reduces the likelihood of students experiencing an equivocation by pre-instructing them to count and aggregate partially-complete squares. I argue, again, that her prior focus on formulas likely contributed to students’ difficulties in this portion of the lesson. The written lesson, in contrast, suggests she monitor students’ work, to see if they develop this partial-count strategy on their own from reading the Math Message. Second, Elsa reorders MS5 and MS6 in Lesson 8–4. These storylines, I argue, were intended to be enacted simultaneously (or nearly so), to evoke the idea of equivalence. Last, and perhaps most significantly, Elsa re-sequences the events of the syuzhet in the skin surface-area activity (MS4 of Lesson 8–4). In so doing, she instructs students in converting units prior to comparing their guesstimates and rule-of-thumb estimates. I maintain
that this impacts students’ opportunities to grapple with the differences in the measurements and to develop an understanding of the main purpose of unit conversion.

Together, these adaptations suggest that Elsa is focused on the mathematical fabula that underlies the lessons. I therefore describe her implementation as \textit{fabula-oriented}. In other words, she appears to interpret mathematical storylines in the written materials in such a way as to identify and illuminate the logical relationships between mathematical ideas. During instruction, she works to preserve these logical relationships. At the same time, I note that the written lessons in \textit{Everyday Mathematics} also present ideas in a generally logical fashion. Storylines of written lessons do not contain many plot elements (in the syuzhet) that undo, upend, or reorient the fabula. When the authors do manipulate the fabula, however, they offer little explicit guidance on how to preserve the suspense that potentially could emerge. Consequently, both Elsa’s approach and the materials themselves generally align with regard to presentation of mathematical plots.

\textbf{Discussion: Coherence and Activation of Curriculum-Embedded Mathematics Knowledge}

Considering the coherence of Elsa’s enacted lessons, and their relationship to the affiliated written lessons, there are also a number of important observations to make. Here, I remind the reader that my definition of coherence is complex and somewhat unsettled. On the one hand, I draw on the definition offered by Schmidt and colleagues (2002) that reflects the breadth of research on coherence in mathematics instruction; this definition emphasizes both the logical sequencing of ideas, as well as the connectedness between ideas and higher-order constructs. On the other, I refer to a more-recent definition by Richman and colleagues (2018) about the \textit{predictability} of lesson events at any given moment. I compare and contrast these definitions, as they apply to my summary of Elsa’s instructional designs below.

\textit{Coherence and written-enacted relationships}. I note, first, that in Lesson 8–3, Elsa introduced a number of practice problems on area and perimeter before having students work on the problems in their \textit{Math Journal}. These problems were multiple-choice problems, aiming to support students in grappling with the differences between area and perimeter and to practice applying an area formula. Not only are these problems internally connected, or sequenced in a reasonable way, but they also relate to a broader objective within the lesson. In particular, Elsa seems to be noticing a potential point of confusion, emphasized within the \textit{Math Message} reading. They might be regarded, then, as emblematic of Schmidt’s and colleagues’ (2002) notions around coherent presentation.
At the same time, I wonder whether multiple-choice questions offer enough engaging mathematical sustenance on which students can chew. Dietiker (2012) notes, though, that this type of problem—and practice problems, more generally—do not necessarily mean that the lesson segment is incoherent. She explains:

Although this [sort of problem] does not make (for me) an interesting mathematical story, interpreting this task in this way [as a storyline] does have the required elements of a mathematical story: at least one mathematical event connecting a beginning with an ending….[yet] it is reasonable to assume that this disruption may obscure the relationships with the rest of the mathematical story….it also could halt a reader’s sense of mathematical progress and threaten his or her ability to anticipate what is ahead.

(Dietiker, 2012, pp. 102-103)

Put another way, it isn’t the nature of the inserted multiple-choice problems, themselves, that may “halt a reader’s sense of mathematical progress” (Dietiker, 2012, p. 103) but whether the problems are leveraged in such a way that allows students to appreciate broader ideas. Dietiker therefore explains the rhythm of a mathematics text—loosely defined as the balance and juxtaposition of momentary (i.e., quick-answer) and elaborated questions (pp. 234–235). Too many multiple-choice questions and what Dietiker might describe as the melody of the lesson will appear staccato and insistent. Too many elaborated questions and the lesson will appear languorous.

In this case, in Lesson 8–3, because of Elsa’s focus on the formula for the area of a rectangle, the lesson becomes unbalanced. Its rhythm is staccato. Students’ perception of the larger aims may have indeed been obscured; therefore, they struggled unduly with the Math Journal problems, rather than seeing them as an opportunity for generative, slower-paced problem-solving. As explained above, I also noticed that at least one student misapplied the formula in working on these problems. In a sense, this student’s aesthetic sense may have been activated by the staccato rhythm of the extended set of multiple-choice questions, like a musical earworm.

Second, with regard to the skin surface-area activity in Lesson 8–4, I note that Elsa’s reordering of the events may be regarded—as work that progresses mathematical ideas and strengthens coherence. Traditional indicators of coherence are, indeed, present throughout this storyline. If we regard coherence in a novel way, recognizing the predictability of future mathematical events and the rhythm of the lesson (Dietiker, 2012; Richman et al., 2018, p. 4), then our assessment potentially changes. (Recall that students do not need to make an accurate prediction, but instead, this criterion simply asks whether the events of the lesson allow for a prediction.) Indeed, as I review Elsa’s enactment, I can imagine students wondering, “Why am I doing this?” What, after all, is the point of using a complex rule-of-thumb to measure surface area
in a different way? (Students could have simply used either the rule-of-thumb or their guesstimate and converted across units with either value.) This purpose-oriented question persists, even after students have been instructed to make the unit conversion.

The written lesson, in contrasts, suggests a setup. Students should confront a seeming paradox: How can the same object (i.e., the epidermal layer) have two different measurements that are wildly different values? After all, during the intended follow-up discussion, the authors add the comment that the two results, the guesstimate and rule-of-thumb estimate, “cannot be immediately compared” (TLG-4, p. 677). Therefore, when this storyline proceeds, as written, a student might make the following claims:

1. I’ve found the surface area of my skin by guesstimating with square-foot cutouts
2. I’ve also estimated by finding a multiple of the surface area of my hand
3. These are just two different ways of obtaining the same result…right?

One imagines that fourth-grade students, new to the ideas of unit-conversion, would naturally predict the results of the guesstimate and rule-of-thumb estimate to be similar. This prediction is upended, however, when the values are compared side-by-side. Richman and colleagues (2018) might offer this sort of explanation:

*Therefore, when the expectation of mastery was violated by the presentation of a challenging pair of tasks directly related to the overarching question, the potential for surprise and subsequent curiosity was that much greater. (p. 13, emphasis added)*

Here, in Lesson 8–4, it isn’t “the expectation of mastery” (Richman et al., 2018, p. 13) that was violated but, rather, the expectation of equivalence. And, as enacted, Elsa’s instructions to convert the measurement units before comparing them—much later in the lesson—effectively eliminates this potential for surprise. There is no opportunity for students to make the prediction (#3, above), because Elsa told them these values represented the same amount and needed conversion. Of course, she also explained to them, step-by-step, how to convert these values.

Notably, in both cases (MS1 in Lesson 8–3 and MS4 in Lesson 8–4), the written materials offered little explicit guidance on understanding and utilizing their inherent opportunities for productive struggle (or, put another way, the opportunities to elevate narrative suspense). With MS1 of Lesson 8–3, a potential equivocation about shapes and partial-squares was not emphasized in such a way that Elsa could launch and sustain students’ problem-solving. Instead, teachers were instructed to simply monitor whether students realized the counting partial-squares approach on their own. And the authors didn’t anticipate that teachers might offer students a more-familiar
tactic, giving them a formula to use. With MS4 of Lesson 8–4, likewise, the intended sequence of events was presented, but two key purposes were left unexplained—the purpose behind: a) preventing students’ from working beyond Problem 4, nor b) having them first observe and reflect upon the two different results they had obtained before asking them to convert. Without these explanations, it seems almost inevitable that Elsa would have resequenced the mathematical events, to flow in a fashion she found more logical.

**Knowledge of curriculum embedded mathematics (KCEM).** Considered together, Elsa’s steering work in these two lessons and her *fabula-oriented* adaptations, allow for a few claims about her KCEM. I note that, as shown by Remillard and Kim (2017), a full analysis would also require that Elsa articulate her own understanding of these dimensions in her curriculum-use. Therefore, for now, my claims are built purely from my classroom observations. Nevertheless, first, I note her focus on the distinction between perimeter and area, in addition to ideas behind converting square feet into square yards (for example). These show Elsa’s activation of Dimension 1 of KCEM (foundational mathematical ideas) in her use of materials. Furthermore, in consistently scaffolding the work for her students, Elsa also demonstrates activation of Dimension 3 (problem complexity).

Even still, her focus on area was mainly formulaic throughout MS1 of Lesson 8–3. This suggests, perhaps, that her reading of the written lesson in Everyday Mathematics did not activate a concrete (counting-oriented) representation of area—modeled on the squares of an underlying grid. Representations and connections among ideas constitute Dimension 2 of KCEM. In addition, during MS4 of Lesson 8–4, Elsa appeared to overlook the complexity of students’ thinking with regard to unit-conversion. In particular, she may have missed an essential facet of any authentic form of mathematical inquiry: the *intellectual need* for engaging in such work (Fuller et al., 2011; Harel, 2013; Meyer, 2009, 2015). Fourth-grade students are still unfamiliar with the notion of equality, particularly as equality relates to different forms of expression and measurement (see Alibegovic et al., pp. 104–106). This shows, then, that Elsa’s use of the written lesson may not have activated Dimension 4 of KCEM (mathematical learning pathways). Elsa’s KCEM partly explains her use of these Everyday Mathematics resources. They also suggest opportunities for curriculum authors to call attention to the mathematical knowledge needed in implementing these undoubtedly complex activities from Unit 8.
7–4. Summary and Conclusion

As I have intended to convey above, Elsa’s instruction demonstrates many essential elements of coherence, as she works to enact mathematical storylines and plots. She makes effective use of steering moves, to make use of instructional time, as well, and to attend to important mathematical ideas. Moreover, the written text supports her in implementing lessons, particularly their fabula-oriented elements. On the other hand, though, we have also seen that Elsa’s adaptations may have lessened opportunities for suspense that appeared—admittedly in implicit ways—within the mathematical plots of written guidance. In particular, during Lesson 8–4, Elsa unraveled a plot twist on converting units. (I offer more detail on what Elsa noticed about this plot twist in Chapter 9). The overall effect was a smooth transition across a complex set of mathematical events, a smoothing that perhaps lacked some of the rhythmic variance of alternative approaches.

Note that Hiebert and colleagues (1997) identify residue as the insight students gain from mathematical tasks about: a) the structure of mathematics, itself, or b) strategies and methods for solving problems (pp. 22-23). They state that “much of the content in current curricula, as presented in popular textbooks, is appropriate as long as students are allowed to make the mathematics problematic” (p. 25, emphasis in the original). Tasks and activities should allow for grappling and problem-solving with an affiliated intellectual need (Fuller et al., 2011; Harel, 2013; Meyer, 2009, 2015). This statement appears to have been offered in response to common criticisms of curriculum programs, particularly at that time, that: a) they emphasized making-sense over sense-making (cf. McCallum, 2018) or b) that sense-making opportunities were implemented in surface-level ways that obviated problem-solving (Cohen, 1990). Hiebert and colleagues (1997) therefore contrast “some tasks that are being proposed as innovative and reform-minded would be inappropriate” (p. 26), while noting that computation problems, seemingly simple, can actually leave important and lasting residue when enacted appropriately.

Therefore, I also argue, it isn’t the nature of the tasks presented to students that we should consider through this sort of analysis. Further, I also argue against focusing too intently on whether Elsa follows or subverts the text (Remillard, 1999). Indeed, the broader question remains not whether Elsa’s students learned; I would argue there is ample evidence they did. I would ask, instead, did students experience problem-solving in such a way that left a hefty patina about mathematics? I might argue that, by not having the chance to discover the need to count half-squares (in Lesson 8–3) or to convert among measurement units (in Lesson 8–4), students had
fewer opportunities than were otherwise available to strengthen their problem-solving skills. In addition, despite what they learned, I wonder whether they engaged or strengthened their overall understanding of the nature of mathematics.

Stated differently, borrowing from the language of the CCSS-M, I wonder whether students had rich-enough opportunities to reason abstractly and quantitatively (CCSS-M Standard for Mathematical Practice #2) or attend to precision (CCSS-M Standard for Mathematical Practice #6). (I don’t want to belabor this point, but I am drawing from CCSS-M to help clarify my prior claims.) With regard to the former, students may not have had the opportunity to make sense of the underlying concept of area as it relates to a given model. And with the latter, students may not have recognized that precisely equivalent amounts may have different numerical expressions.

Note that the CCSS-M Standards for Mathematical Practice (SMPs) are intended to “describe varieties of expertise that mathematics educators at all levels should seek to develop in their students” (NGA Center & CCSSO, 2010, p. 6). Altogether, then, my observations suggest that curriculum authors might want to consider and call attention to the abrupt transitions or plot twists within their written lessons. That way, they could signal to teachers how structures of mathematical storylines serve to enhance suspense and motivate problem-solving and a deeper appreciation for the structures and methods of mathematical inquiry.
CHAPTER 8. ACT II: TORRIE BLUM’S MATHEMATICAL STORYLINES—
ORIENTED TOWARD PROBLEM-SOLVING AND APORIAL

He wrung his hands, he cleared his throat, he shed a single tear:
Then sobbed, “I’ve something to announce, and that is why I’m here.”

“All schools for miles and miles around must take a special test,
To see who’s learning such and such—to see which school’s the best.
If our small school does not do well, then it will be torn down,
And you will have to go to school in dreary Flobbertown.”

—Dr. Seuss, Hooray for Diffendoofer Day! (1998, lines 61–66)

8–1. Introduction: Torrie Blum and Heritage Gardens School

As with Chapter 7, I begin this chapter by profiling the second participant in my multiple-case study, Torrie Blum. I also describe her school, Heritage Gardens School. Like Elsa, I learned that Torrie also endorses conceptual-oriented instruction and the design of Math Trailblazers. In addition, Torrie perceives subtle plot-points found in Math Trailblazers lessons; she works to enact these points in her lessons, and she even creates additional opportunities, building upon the resources for enhancing her students’ engagement. Her adaptations also promote her students’ opportunities to engage in problem-solving and inquiry.

Torrie Blum’s Background and Experience

Torrie Blum is a highly self-reflective elementary school teacher in the early-to-middle part of her career. At the time of our initial interview, she was amidst her sixth year of teaching. She spent the previous three-and-a-half years as a third-grade teacher at Heritage Gardens School (Heritage Gardens). And prior to her current placement, she taught first- and second-grade students at another school.

I describe Torrie as particularly self-reflective for several reasons; I elaborate on some of these in Chapter 9. For now, though, I broadly describe her stance on professional learning. First, I observed during our initial meeting that Torrie evinces a commitment to growth, through and within her work, particularly whenever encountering a challenge. Contrasting her own learning about mathematics instruction with other types of learning, she says, “It’s not like, um, like studying a topic, you know” (T. Blum, personal interview, 11/02/2011). By studying a topic, I take
her to mean a set of related facts, like learning the common ingredients in Italian-style cooking. This type of studying, Torrie seems to say, satisfies an immediate curiosity.

In contrast, Torrie says, “With the teaching of math, it feels like there has to be something driving that interest” (T. Blum, personal interview, 11/02/2011, emphasis added). In other words, unlike studying a topic, Torrie’s stance on learning about mathematics instruction is motivated by her desire to unlock new ways of thinking for her students. To Torrie, this larger and more complex goal implies that—for her to develop continually as a teacher—hefty doses of reflection and fearless experimentation are required (T. Blum, personal interview, 11/02/2011).

**Heritage Gardens School and Math Trailblazers**

Torrie teaches at Heritage Gardens, a small, religiously-affiliated, and co-educational independent school. While similar to Golden Hawk, including having a nearby location, Heritage Gardens is nonetheless unique among independent schools in its area. For one reason, it boasts an extraordinarily low student:teacher ratio of 5:1 with an overall population of about 110 students. Consequently, it is well-known for its nurturing environment. It serves students from pre-kindergarten through eighth grade, and it is located in a suburb of a large U.S. city.

Heritage Gardens also has a rich and long history. Indeed, it was founded in the late 19th century as one of the first schools among its peer institutions in the mid-Atlantic. It has a quaint, picturesque, single-building campus that also includes adjacent gardens, parks, and athletic fields. The school also offers substantial support for students with mild-to-moderate specialized learning needs, particularly in reading or with ADHD (Private School Review [PSR], 2019). The educational philosophy of Heritage Gardens, according to its website, is rooted in intellectual rigor, the arts, and service to others. In addition, its mission expressly promotes students’ independence, as well as to their capacities to collaborate, question, reason, and communicate. Kindness is an expressly-stated virtue of the school community.

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2: This profile of Heritage Gardens School, as of the 2018-2019 academic year, was obtained from the website Private School Review (PSR). I am not aware of any significant changes to the basic facts about Heritage Gardens, since the original data collection for my thesis. PSR’s (2019) stated mission is: “To educate and energize families about private schools and the opportunities they offer. By providing information that is insightful and up-to-date, we help families make better educational choices.” Further, its data-collection method is described as follows: “Schools directly update the information on our site at regular intervals, to help provide the most current data for families. Schools also respond to inquiries sent from our site, so that families can conveniently use our standardized forms to ask schools questions and receive free informational materials” (PSR, 2019).

24 This description of Heritage Gardens is drawn from two sources: first, the “Mission & Philosophy” page on its website; second, the “Welcome from Head of School” page. Rather than quote directly from this website, I paraphrased and summarized, to preserve the school’s anonymity.
Furthermore, Heritage Gardens embraces a pedagogical approach—grounded in its philosophy and mission—aptly described by the Head of School on its About Us website. Here, the Head of School depicts a pre-kindergarten lesson that involves counting blocks, facilitated by a teacher who expresses genuine curiosity about her students’ thinking. She works with students individually, asking a series of carefully-structured questions to uncover students ideas about quantity and addition. At the same time, this teacher encourages students to share their blocks and to solve problems collaboratively. The Head of School concludes this anecdote by emphasizing the importance of students’ inventing their own strategies in mathematics and by implying that analogous instructional approaches proliferate in other subjects. Nevertheless, in elementary mathematics, the school uses Math Trailblazers (3rd Edition)² as its primary instructional resource. Prior to my classroom visits, teachers had been using this program for about seven years and, consequently, exhibited a strong understanding of its underlying design principles. In fact, Heritage Gardens had been selected to pilot Math Trailblazers materials; teachers have been field-testing and offering feedback on drafts of the fourth edition, provided by the publishers.

Because of their participation in field-testing, teachers at Heritage Gardens have had atypically rich opportunities to learn about the program. Moreover, they have been encouraged by the publishers and authors to think about and express opinions on its affiliated resources. This makes Torrie an especially interesting research participant, because she has been primed to discuss the features of Math Trailblazers and to explore how she makes use of them. Stated differently, Torrie may not have as much classroom or curricular experience as Elsa has, but Torrie is likely more-informed as a user of curriculum than her years would otherwise indicate. As I describe in greater detail in Chapter 9, I characterize Torrie’s orientation toward Math Trailblazers, generally, as adherent and trusting (Remillard & Bryans, 2004, p. 367).

One additional note: at Heritage Gardens, Torrie partners regularly with a math coordinator, Delia. As the math coordinator, Delia generally oversees mathematics instruction from pre-kindergarten into middle school—much like a traditional department chair. Delia occasionally helps Torrie in addressing students’ individual learning needs by working with them one-on-one or in small groups. She also talks with Torrie about the design and intentions of Math Trailblazers lessons, occasionally providing other resources to use as supplements. Delia, I believe, was

responsible for establishing the partnership between teachers at Heritage Gardens and the authors and publishers of *Math Trailblazers*.

8–2. RQ2—Torrie’s Steering Moves: Meaning and Thinking

In this section, I review the maps of design arcs for Torrie’s written and enacted lessons. As with the previous chapter, my goal here is to isolate elements of Torrie’s steering work that I subsequently explore and describe in responding to RQ2. Afterwards, in the next section, I describe her overall relationship with the written lessons she uses in addressing RQ1.

**Structural Comparisons: Written and Enacted Instantiations of Lessons 6–2 and 9–6**

Recall that the design arc maps are intended to portray the mathematical plots and storylines that emerge from both Torrie’s written and enacted lessons. As before, I review these maps with an aim toward unpacking the aesthetic experiences for students (Rosenblatt, 1994).

**Background on the units and characterizing the program.** Before reviewing the maps, though, I contextualize the written lessons within the overall progression of learning. Unlike Elsa’s lessons from Unit 8, reviewed in Chapter 7, I have selected two of Torrie’s that are split across different units in the third-grade version of *Math Trailblazers*: Unit 6 on “Adding Larger Numbers” and Unit 9 on “Parts and Wholes” (TIMS, 2008). To situate these lessons, Lesson 6–2 and Lesson 9–6, I summarize the development of content in both of these units below.

Understanding the context of each lesson, I maintain, is helpful for making sense of the suggested activities. Nonetheless, because I am coding and analyzing within, not across, lessons, the discontinuity in the sequence of presented lessons should not mar the eventual outcome.

**Additional comment on methods.** As with Elsa’s pair of lessons, Lesson 6–2 and Lesson 9–6 seem illustrative of Torrie’s overall approach to using materials. Specifically, Torrie generally begins each lesson with a warm-up problem, using materials outside of *Math Trailblazers*, but she then transitions back to the program soon thereafter. For the remainder of her lesson, typically, she draws heavily on the written guidance for implementing classroom activities. As I explain, below, Torrie makes a number of strategic adaptations, as well, calling attention to particular mathematical ideas or strategies.

I should also note that—for both Elsa and Torrie—I analyzed several more lessons than those described here. The others were either largely similar or else deviated in significant but explainable ways (e.g., administering a test). My selection approach is consonant with Yin’s
(2009) description of an embedded, representative, multi-case study design. Again, the intention behind this research design is to develop a profile of each teacher’s narrative construction and curriculum-use, ultimately, to understand the broader phenomena of narrativity and coherence in mathematics instruction.

Unit overview and structure of the program. To begin the topic overview, in Unit 6, third-graders experience and use several approaches for adding two-digit whole numbers. In Lesson 6–1, they explore a number line for finding sums, in addition to working with partial sums to decompose two-digit numbers into constituent place-value parts. In Lesson 6–2, students expand on these strategies by using base-ten blocks as models for adding. Lesson 6–3 involves techniques of estimating sums, so that students can evaluate the accuracy of their adding. Next, in Lesson 6–4, students connect strategies and models with more sophisticated numerical approaches for adding two- and three-digit numbers. Finally, Lessons 6–5 and 6–6 conclude the unit, offering additional practice to strengthen the efficiency and accuracy of students’ computation through the traditional algorithm.

Shifting to Unit 9, students begin by naming and informally comparing fractions through explorations of real-world situations in Lesson 9–1. In Lessons 9–2 and 9–3, students practice working with plastic fraction-pieces to understand and name fractions. Lesson 9–4 introduces another model for fractions, a bar model. In Lesson 9–5, students explore the relationships between fraction-pieces, fraction bars, and fractions on number lines. Students compare fractions in Lesson 9–6 and practice concepts from across the unit in Lesson 9–7. Generally, this unit aims to help students understand fractions as numbers, in addition to offering them practice in naming, representing, and comparing fractional values. Informal notions of fraction equivalence are discussed throughout Unit 9, while more formal ideas and operations with fractions are reserved for later grades.

In what follows, I analyze the design arc maps of Torrie’s written and enacted lessons, to describe their narrative structures and sequences of events. Note that in Chapter 6, I characterized Math Trailblazers as leaning toward a resource-centric design. Indeed, Math Trailblazers portrays elemental components of classroom activities and, on occasion, articulates the pedagogical or mathematical purposes behind them. A resource-centric design, generally, implies greater fluidity in organization—potentially accommodating several different sequences of classroom events. Nevertheless, nearly every paragraph of written lessons begins with a directive to teachers: “Ask…,” “Tell…,” and so on. These proceed in a somewhat linear fashion. In addition, the
program overview of *Math Trailblazers* does not characterize its design in resource-centric terms. As a result, and because teachers read lessons in a top-down fashion, I generally interpret *Math Trailblazers* lessons in a procedure-centric way. Therefore, I contend that maps of written lessons, shown below, reliably express the intended sequence of mathematical storylines and plots.

**Structural comparisons of the written and enacted lessons of Lesson 6–2.** Studying the design arc maps in Figure 12a, I observe that Lesson 6–2 has a significant number of mathematical events, nearly 40, and six mathematical storylines. These storylines are summarized, briefly, in Table 8. Lesson 6–2 builds on the addition strategies presented in Lesson 6–1 by asking students to model two- and three-digit addition problems, set in a real-world context, with base-ten pieces. Several mathematical objectives are shown under *Key Content* on the first page: measuring the area of irregular shapes, understanding place-value and adding multi-digit numbers, comparing and ordering four-digit numbers, estimating sums and differences, and using base-ten blocks as models (URG-3, Unit 6, Lesson 2, p. 33). Given this large number of objectives in this lesson, it should not be surprising it requires two or three classroom sessions (URG-3, Unit 6, Lesson 2, p. 33). Regardless, the written lesson consists of only one part with no guidance offered to the teacher on how or when to split the lesson across multiple days. Unlike those of Lesson 9–6, then, the maps of Lesson 6–2 shown in Figure 12 do not contain any annotations portraying lesson segments.

In addition, the first written storyline, MS1, stretches over the majority of the lesson with four shorter storylines subsumed underneath. In a manner of speaking, this primary mathematical storyline is the vehicle through which many of the objectives are addressed. It begins with the teacher telling students they will be assisting in making costumes for a play and, to do so, will need to work in groups of three to trace (or model) their coats. Furthermore, in the map of the written lesson (i.e., Figure 12a) there are three complicating plot-points worth noting:

1. A promise of an answer (Pe) in MS2;
2. An episode of jamming (Jg) in the simultaneous storylines, MS3 and MS4; and
3. An equivocation (En) in MS5.

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2 For teachers, the URG of *Math Trailblazers* (3rd ed.) is modular—i.e., there are separate, repaginated books for each unit. Therefore, I adopt a common convention for citing pages in the URG by listing the unit, lesson, and page number for each within-text reference (e.g., URG, Unit 6, Lesson 2, p. 33). Further, because of the unique circumstances at Heritage Gardens, as a field-test site, Torrie’s CRLs contain a mix of third-edition and draft, fourth-edition texts. While I report page numbers in my citations—to document my evidence—I have taken care to obscure the editions of any images of *Math Trailblazers* materials shown herein.
Figure 12. The design arc maps, showing the written storylines and plot of Lesson 6–2 and Torrie’s enacted lesson drawing on these resources.

Figure 13. The design arc maps, showing the written storylines and plot of Lesson 9–6 and Torrie’s enacted lesson drawing on these resources.
We can see, further, that only one storyline is fully-resolved and that the others are either partly-resolved or are not resolved (i.e., they are coded with Pa or Sa).

Comparing the map of the written lesson to that of the enacted, several similarities and key differences emerge. First, as the Lesson Overview suggests, the enacted lesson is broken across Day 1 (Figure 12b, bottom) and Day 2 (Figure 12b, top). Looking across both days, each of the mathematical storylines from the written lesson is represented in the enacted lesson, except MS6. Like Elsa, it appears that Torrie has also added a number of storylines (in purple). In contrast, Torrie’s added storylines are concentrated at the beginning and end of each day.

This structural, top-level analysis suggests we should investigate several elements of Torrie’s instruction and steering. Namely, the maps indicate we should explore her enactment of the complicating plot-points. As with Elsa, we should also seek to understand and describe the storylines Torrie added to and omitted from the written lesson. I review Torrie’s steering moves for Lesson 6–2, after likewise reviewing the structure of the maps for Lesson 9–6.

**Structural comparisons of the written and enacted lessons of Lesson 9–6.** Lesson 9–6 is complex lesson with a sizable number of mathematical events and 16 storylines. (See Figure 13 and Table 9 for additional details.) Despite this structural complexity, the lesson involves two broad objectives: a) recognizing that the number of equal-size pieces, into which a circle (or other whole object) is divided, determines the relative size of each piece, and b) developing strategies for comparing fractions (URG-3, Unit 9, Lesson 6, p. 93). As the overview indicates, Lesson 9–6 is intended to be taught over approximately two days. It is consequently presented in two parts, which are labeled accordingly on the design arc map (see Figure 13a). The first part involves making sense of, and comparing, unit fractions to one another (e.g., ¼). The second part involves comparing non-unit fractions (e.g., 2/3) to the fraction one-half.

The map of the enacted lesson (Figure 13b) contains elements of Part 1 and the summary of the written lesson. As Figure 13b shows, however, the enacted lesson does not appear to contain storylines drawn from Part 2 of the written lesson. Furthermore, Torrie appears to enact six of the sixteen storylines from the written lesson. She also adds several mathematical storylines. With regard to mathematical plots, Torrie seems to incorporate two equivocations near the beginning of the lesson; these are not necessarily reflected in written text (cf. Figure 13a). Next, Torrie may have enacted storylines MS1 and MS3 simultaneously—whereas, in the written lesson, these storylines appear somewhat distinct. Finally, the map of Torrie’s enacted lesson suggests she may
have eliminated or greatly reduced students’ opportunities for working independently during MS1 and MS3.

This sort of structural analysis likewise provides motivation for investigating and describing Torrie’s enactment of MS1 to MS3. This should include analyzing any new episodes of equivocation. As with the other lessons I have analyzed, storylines that Torrie added and removed should also be described. I note, specifically, that Torrie did not enact Part 2 Lesson 9–6 on the day following her enactment of Part 1. Again, I offer some of the rationale behind Torrie’s design-decisions in Chapter 9 of this thesis. In this chapter, the focus remains on unpacking her implementation and the affiliated written guidance.

<table>
<thead>
<tr>
<th><strong>Mathematical Storyline</strong></th>
<th><strong>Character(s)</strong></th>
<th><strong>Setting</strong></th>
<th><strong>Main Formulation</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>MS1</td>
<td>Amorphous, Closed Figure</td>
<td>2-D Surface</td>
<td>“What is the surface area of the irregular figure that represents a tracing of your coat?”</td>
</tr>
<tr>
<td>MS2</td>
<td>Line of Symmetry</td>
<td>2-D Closed Figure</td>
<td>“How can we use the line of symmetry, to simplify our work in finding the area?”</td>
</tr>
<tr>
<td>MS3</td>
<td>Base-ten Pieces</td>
<td>2-D Closed Figure</td>
<td>“How many multiples of each type of base-ten piece will cover half of the coat tracing?”</td>
</tr>
<tr>
<td>MS4</td>
<td>Multi-digit Whole Numbers</td>
<td>Base-ten Pieces</td>
<td>“What is the sum of the multi-digit numbers represented by the base-ten pieces?”</td>
</tr>
<tr>
<td>MS5</td>
<td>Areas of Coats Multi-digit Whole Numbers</td>
<td>Base-ten Whole Numbers</td>
<td>“What is the proper order of smallest to largest whole number, representing the areas of coats?”</td>
</tr>
<tr>
<td>MS6</td>
<td>Areas of Coats Multi-digit Whole Numbers</td>
<td>Base-ten Whole Numbers</td>
<td>“What is the approximate difference in cm² between the areas of the smallest and largest coats?”</td>
</tr>
</tbody>
</table>

Table 8. Mathematical storylines and key narrative dimensions of Lesson 6–2 of Math Trailblazers. Storylines highlighted in blue text are those enacted (whole or in part) by Torrie Blum.

Torrie’s Steering Moves in Lessons 6–2 and 9–6

Below, I describe Torrie’s steering work in her implementation of Lessons 6–2 and 9–6. Once again, I do not intend to quantify and tabulate all of her steering moves but, instead, to present a picture of her work. In general, as I explain, I found that Torrie made effective and strategic use of instructional time, while also promoting the coherence of her instruction. I note, furthermore, that Torrie does something more than maintain a high level of cognitive demand and establish
productive classroom norms. Some of her particular steering moves, in fact, pique students’ curiosity and promote their active engagement in independent problem-solving.

**Torrie’s steering moves in Lesson 6–2: Students doing the mathematical work and a focus on meaning.** I observed Torrie teaching Lesson 6–2 on November 28, 2011 and November 30, 2011. Again, the main learning activity of the primary mathematical storyline, MS1, involves tracing students’ coats on chart paper. Students are to work collaboratively in small groups on this task. After tracing their coats, they are asked to cover the tracing with base-ten pieces, entirely, and tabulate the number of each type of base-ten piece. Students then use this tabulated data to add the accumulated values of the base-ten pieces and thereby find the area of the coat-tracing. The cognitive demand of this main task, to me, is representative of a DM-type activity.

<table>
<thead>
<tr>
<th>Mathematical Storyline</th>
<th>Character(s)</th>
<th>Setting</th>
<th>Main Formulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>MS1</td>
<td>Unit Fractions</td>
<td>Rational Numbers</td>
<td>“How are unit fractions compared?”</td>
</tr>
<tr>
<td>MS2</td>
<td>Fourth, Sixth, Halves, Thirds</td>
<td>Fraction-Pieces</td>
<td>“How does Jimmy make sure that each family member gets a fair-share of pizza?”</td>
</tr>
<tr>
<td>MS3</td>
<td>Fourth, Sixth, Halves, Thirds</td>
<td>Fraction-Pieces</td>
<td>“A member of which family gets more pizza?”</td>
</tr>
<tr>
<td>MS4</td>
<td>One-half &amp; One-third</td>
<td>Rational Numbers</td>
<td>“Which fraction is larger, ½ or ¼?”</td>
</tr>
<tr>
<td>MS5</td>
<td>One-half &amp; One-third</td>
<td>Fraction-Pieces</td>
<td>“How can you show which fraction is larger, ½ or ⅗, with fraction-pieces?”</td>
</tr>
<tr>
<td>MS6</td>
<td>Non-unit Fractions (i.e., Bars)</td>
<td>Fraction Strips</td>
<td>“Which fractions are smaller than one-half? Larger than one-half?”</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MS14</td>
<td>Non-unit Fractions</td>
<td>Rational Numbers</td>
<td>“Which fraction is larger?”</td>
</tr>
<tr>
<td>MS15</td>
<td>Non-unit Fractions</td>
<td>Fraction-Pieces</td>
<td>“How can you show which fraction is larger, using fraction-pieces?”</td>
</tr>
<tr>
<td>MS16</td>
<td>Non-unit Fractions (i.e., Bars)</td>
<td>Fraction Strips</td>
<td>“How can you show which fraction is larger, using the ‘Comparing Fractions to One-Half’ chart?”</td>
</tr>
</tbody>
</table>

Table 9. Storylines and dimensions of Lesson 9–6. Blue text indicates those storylines enacted by Torrie.
Promise of an answer in MS1. As noted in the structural analysis of the written map above, several mathematical storylines exhibit key plot-points during the complicating action. The first such moment occurs at the beginning of MS2. Without any prefatory description, teachers are instructed by the Math Trailblazers authors, “Encourage students to use their knowledge of symmetry to make the measuring go faster” (URG-3, Unit 6, Lesson 2, p. 37). In so doing, the authors establish what Barthes (1974) and Dietiker (2012, 2015a) describe as “the promise of an answer (Pe).” Promised answers express an epistemological self-awareness of a theoretical outcome without offering detail allowing for a solution. Put another way, promised answers merely allude to a resolution of a raised question.

To enact this prompt, teachers might ask such question as, Is there anything you could do, to make it easier, so you don’t have to count so many base-ten pieces? or How would you describe the shape of the coat-tracing—is it symmetrical or not? And does that help you, at all? Students might then be motivated to discuss geometric features of the coat-tracing. My reading of the written text, here, suggests an intention to promote students’ own inquiry and exploration. A more directive approach would have entailed something akin to the following: Tell students that, since the coat-tracing is symmetrical, they can cut the tracing in half, vertically, count the number of base-ten pieces that cover only one side, and double this count to obtain the whole area.

Likewise, Torrie generally works to maintain the inquiry-oriented nature of this problem-solving hint. As she introduces her students to the task, she asks:

Now, do you think that there is something about that coat, that might—kind of, drawing over there—that would help you be even more efficient than covering the whole coat? Is there something that we know about math, that would help us be more efficient than covering up the entire coat?... And I am just going to put in your mind, it’s one of the techniques that you use for “Joe the Goldfish.” (observation transcript, 11/28/2011, emphasis added)

I note, briefly, that Torrie references efficiency, a mathematical habit of mind that she also discussed with students as they completed a warm-up problem, moments earlier. Regardless, she suggests that students need not cover the entire coat, and she makes a not-so-veiled reference to a so-called “technique” used with “Joe the Goldfish.” I subsequently learn that “Joe the Goldfish” is the title of Lesson 5–6 in Math Trailblazers, part of an earlier unit on geometry and area. During Lesson 5–6, students are tasked with making a raincoat for a goldfish, which requires them to think about the symmetry of Joe’s left and right ventral sides.
Even still, students struggle with Torrie’s question. To a degree, Torrie eases the challenge by asking, next, “Raise your hand if you were going to say, you'd measure half of it and then to do what, ‘Lyra?’” (observation transcript, 11/28/2011). “Lyra” finishes the idea and confirms that half the result could be doubled. Torrie then asks how students might describe the line represented by the zipper of the coat, and another student responds by calling it the line of symmetry.

It seems Torrie may have funneled (Wood, 1998) students’ responses toward an answer she, herself, already had in mind. And, yet, an earlier classroom episode complicates this sequence of questions. Indeed, at the very beginning of this activity, Torrie introduces the goal—saying that students will be measuring the area of their coats—and she solicits ideas on how results could be obtained. Responding suddenly, Lyra says, “I think what we’re going to do is, we are going to measure the area of one side of the coat and then double it, and then add a little bit” (observation transcript, 11/28/2011). Torrie’s immediate reaction is wide-eyed surprise (field notes and video tape, 11/28/2011). She shifts conversation away from what Lyra has offered: “Ohhhh-kaaayyyy! [prolonged exclamation] We’re going to start off with Step 1” (observation transcript, 11/28/2011).

Then, as the written lesson suggests, Torrie opens the activity by discussing the mechanics of tracing coats. The text says that “there are several practical points to discuss, such as: The coats should be zipped [and]…Do not use markers when tracing…” (URG-3, Unit 6, Lesson 2, p. 36). Therefore, Torrie’s purpose, here—by asking students how they can find the areas of their coats—may have been to prompt students to consider and make suggestions on the process not the mathematics. To me, this explains Torrie’s manifest surprise at Lyra’s astute—and presumably unexpected—proclamation. In effect, Lyra has offered a spoiler, revealing elements of the plot that the teacher was hoping to keep in abeyance. Taken together, I consider this set of exchanges as evidence that Torrie wants to preserve the opportunity for problem-solving suggested in the written lesson. After all, rather than asking straightforward questions, she insinuates: “Is there something that we know about math…” and “I am just going to put in your mind…” (observation transcript, 11/28.2011). Torrie also ignores Lyra’s suggestion, decisively, only referring to it after she feels students have been adequately prepared to tackle the task. Even if she wasn’t entirely successful, Torrie nonetheless aimed to preserve the promise of an answer suggested within the mathematical plot of Lesson 6–2.

**Jamming in MS3 and MS4.** When asking students about using symmetry, Torrie segues to a discussion of error. First, she asks if it’s possible to find the exact line of symmetry on the coat-
tracings. (Students respond, no, it’s not.) Torrie and her students then discuss, briefly, how to estimate the location of the line of symmetry. Next, of a coat’s area, Torrie asks:

Are we going to be finding this, exact? Anything, exact? What are—how are we going to, I mean—think about when we did with, uh, counting up those partial, the whole squares and the partial squares. We had some room for error. So there’s going to be some room for error, here, “Marcus.” It’s not going to be the exact. (observation transcript, 11/28/2018)

This discussion is signaled, but subtly, within the written lesson. The text merely indicates that imprecision in covering the coat “should be discussed at some point” (URG-3, Unit 6, Lesson 2, p. 37). Notably, Torrie not only raises this caveat, but she also extends it and connects it to students’ work on symmetry.

I interpret this as an instance of jamming for several reasons. First, the text literally tells the reader that an exact answer cannot be found. While this might seem trivial to those unfamiliar with elementary students, becoming comfortable with imprecision is a milestone in young students’ mathematical development (see, e.g., Alibegovic et al., p. 62) Further, teachers of elementary mathematics in the U.S. are, typically, well-acquainted with their students’ discomfort with inexact answers. This discomfort might be cultural, because in the U. S., we often regard mathematics as an enterprise in finding only so-called right answers (Boaler, 2013). In the context of U.S. instruction, then, the mere suggestion of inexactitude is likely to provoke confusion or frustration for students.

Last, written guidance on estimating sums in the next lesson, Lesson 6–3 of Math Trailblazers, indicates that third-grade students may feel compelled to find exact answers to addition problems. Having taught Unit 6, previously—in addition to her own experiences with third-grade students—Torrie likely anticipates students’ potential struggles with inexact answers. And, unlike plot-points that intend to elevate suspense, we might regard this instance of jamming as an unusual one, in that it perhaps intends to reduce students’ levels of frustration.

Equivocation in MS5. The text of Lesson 6–2 establishes a potential equivocation in MS5. In other words, the written lesson offers a sequence incorporating an idea that, mathematically, is perceived simultaneously as both true and untrue. In so doing, I surmise that mathematics texts intend to prompt students to reconsider their reasoning and tease-out the appropriate chain of logic. (In truth, equivocation in mathematics texts is not a well-studied phenomenon, so this claim is admittedly speculative.) In short, as written, the activity of MS5 establishes conditions for
considering two whole-number values—where one may seem larger than the other, but where the reverse is actually the case. Students have to confront and make sense of this seeming-paradox.

To explain further, I summarize the series of events in the written lesson (URG-3, Unit 6, Lesson 2, p. 38), followed by the enacted classroom events. As written, after students complete the activity with base-ten blocks, teachers are instructed to have students record their sums—i.e., the values representing the areas of their coat-tracings—on notecards (URG-3, Unit 6, Lesson 2, p. 38). The teacher then helps students tape these notecards on the whiteboard; in so doing, students are supposed to collaborate and decide upon the sequence for displaying them, aiming to arrange the notecard-values in ascending order (URG-3, Unit 6, Lesson 2, p. 38). The teacher is not necessarily expected to help them decide upon the order or to verify the accuracy of the result, although this point is subtly made within the written lesson. The teacher may certainly help them work together and physically place the notecards on the board. Afterwards, the teacher helps the class tape the corresponding coat-tracings on the board, placing one each above the individual notecards; at that point, students collectively inspect the coat-tracings (as visual models), to ensure the notecards are organized from smallest to largest area (URG-3, Unit 6, Lesson 2, p. 38).

Next, the text instructs, “Ask students for suggestions to explain any big discrepancies” (URG-3, Unit 6, Lesson 2, p. 38). Since the coat-tracings should progress from smallest to largest—which can be verified, easily, by inspection—any discrepancies suggest that the affiliated numerical values are improperly ordered. As students perform this inspection, they are likely to encounter the equivocation, as setup by the text: How can this value be larger than that value? After all, this coat is smaller than that coat! In other words, students are expected to confront, potentially, the presumed untruth of the notecards via the evident truth of the coat-tracings. Reading the text from in an aesthetic fashion (Rosenblatt, 1994) reveals that this sequence is intended to promote surprise or confusion; in so doing, students must make sense of their method for arranging the multi-digit values.

Torrie enacts MS5 on the second day of Lesson 6–2 (November 30, 2011). The related activity is implemented in Torrie’s classroom after she facilitates a warm-up problem, offers a summary of the previous lesson, and then provides additional time for students to work on the coat-tracing and adding activity. Afterwards, even though Torrie does not follow the guidance in the written lesson precisely, she achieves a similar effect to that intended. Perhaps she has a different purpose in mind, thereby explaining her adaptations. In particular, she first assists students with their adding and then gathers their notecards. Differing from the text, somewhat, Torrie works with
students to ensure the notecards are arranged properly, in ascending order. She then asks students to compare the ordered values to the relative sizes of the coat-tracings. In so doing, Torrie’s students discover errors in their addition—because, they note, one coat is only marginally larger than the rest but is quantified as the largest by some 5,000 cm².

Through this adaptation, Torrie’s students nonetheless engage in error analysis: rather than analyzing potential errors in ordering multi-digit numbers, students uncover potential errors in their addition calculations. There are, of course, tradeoffs to Torrie’s approach. As she implemented this storyline, only egregious addition errors will be identified. As the lesson is written, on the other hand, students might grapple with more subtle errors in addition or ordering. Nonetheless, Torrie’s enactment still allows for a potential equivocation to emerge. And this equivocation is still aligned with the main objectives of the activity. It also promotes students’ opportunities for inquiry and collaboration.

Added storylines, removed storylines, and steering moves. I insinuated, above, that some of Torrie’s added storylines include her use of an outside resource for conducting a warm-up activity. She calls the warm-up a “stumper problem” (observation transcript, 11/28/2011). And during our subsequent interview, Torrie explains she pulls stumper problems from resources by mathematics education consultant and author, Marcy Cook. In each of the lessons I observed, Torrie also facilitates a brief closing discussion—typically, this consists of students volunteering to explain something new that they learned.

There are two main exceptions, however, within Torrie’s implementation of Lesson 6–2. First, on Day 1, after students discuss symmetry and before they explore estimation approaches, Torrie offers students a tool to assist them in their coat-tracing activity: a three-column table that she has created for tabulating their results. Before they begin working on tracing their coats, she instructs them in how to use this table, while tallying the number of each type of base-ten piece (i.e., “bits” = ones, “skinnies” = tens, and “flats” = hundreds). She also explains that using this table is optional (observation transcript, 11/28/2011). And on Day 2, as students add the values of the base-ten pieces that cover their coat-tracings and begin writing sums on notecards, Torrie offers them support in adding together the digits. She guides them in working place-by-place, and this work includes another, ancillary storyline on the meaning of zero in any given place.

Finally, as the second day of the lesson nears its end, Torrie asks students to estimate the differences in areas of successive coats. This storyline lasts for a few minutes, as Torrie and the class proceed from one coat to the next, working together. I presume this added storyline is
inspired by guidance in the written lesson on estimating the difference between the areas of the smallest and largest coats; since the characters of this enacted storyline are markedly different than those in the written lesson, however, I considered it separately as an added storyline. Likewise, Torrie omits a mathematical storyline affiliated with the estimation-of-difference activity—namely, that students are to discuss which place-values are most relevant when comparing four-digit numbers.

To characterize Torrie’s steering moves, throughout this lesson, she invests heavily in students’ opportunities to work independently and collaboratively. In particular, she supports them in tracing their coats, covering the tracings with base-ten pieces, and then adding together their tabulated results. To me, this sort of steering work is emblematic of making sure students are doing the thinking (Sleep, 2012). During discussions, even if she isn’t always successful, Torrie also tries to elicit students’ problem-solving strategies.

As the students work, furthermore, Torrie continually prompts them to make sense of the work: together, she and the class explore the relative magnitudes of the base-ten pieces, ways to tabulate their results, and investigate the validity of the ordered numbers. This, clearly, keeps a focus on meaning. Finally, with regard to mathematical storylines, activities and questions are undoubtedly connected to one another, and Torrie even works to relate students’ work to ideas from other lessons. Toward the end of Day 2, however, Torrie offered significant help to students who were struggling with adding together the base-ten pieces. This might constitute a narrow form of storyline-related questioning that Sleep (2012) describes as “‘dragging’ students through the mathematics” (p. 959). She might be critiqued, as well, for changing the focus of the notecard storyline, MS5, away from the intended goal involving ordering. And, yet, she might have actually improved the coherence of this activity in relation to coat-tracing; after all, the focus of the coat-tracing activity was on adding multi-digit whole numbers. Torrie’s adaptation allowed students to investigate the validity of their addition work, rather than shifting to a new concept that hadn’t yet appeared in the lesson (i.e., ordering).

I would therefore describe Torrie’s steering work, overall, as making effective use of instructional time and also generally-coherent. In addition, she took advantage of her several plot-points in the lesson, to stimulate students’ sense-making and problem-solving. To me, this represents something more than working to maintain high levels of cognitive demand or to promote inquiry-based sociomathematical norms (Yackel & Cobb, 1996). I intend to explain what I mean in the next section of this chapter and in Chapter 9.
**Torrie’s steering moves in Lesson 9–6: Opening up and emphasizing key ideas.** I observed Torrie’s implementation of Lesson 9–6 on March 5, 2012. The written lesson begins by asking teachers to, first, have students read several paragraphs in their workbooks about “Jimmy’s Pizza Shop” (URG-3, Unit 9, Lesson 6, p. 97). This page explains that Jimmy, a pizza-shop owner, cuts his pizza, so that all members of a family can eat exactly one piece. Data are presented about orders Jimmy fulfilled one night; many of the ensuing portions of the lesson are built around this data. After an introductory exploration, the primary activity asks students to solve problems about the fractional amounts of pizza allotted to several families of different sizes and then, consistently, “show or tell how you know” (e.g., URG-3, Unit 9, Lesson 6, p. 98). I therefore regard this primary lesson activity as a DM-type activity.

**Equivocation in MS2.** At the beginning of the implemented Lesson 9–6, likewise, Torrie prompts students to read several pizza-shop paragraphs in their texts. Without further instruction, she then asks them to draw a pizza, cut into pieces, for the six-member Franklin family. Torrie then displays several students’ drawings on a document camera, and she and her students subsequently discuss several ways of drawing sixths as precisely as possible (observation transcript, 03/05/2012). Torrie then charges students with drawing pizzas for the other families at Jimmy’s pizza shop. As students begin working, she emphasizes, “The most important thing are equal parts!” (observation transcript, 03/05/2012).

In contrast, the written lesson instructs the teacher to have her students match fraction-pieces with the fractional amounts eaten by members of each family (URG-3, Unit 9, Lesson 6, p. 97). Fraction-pieces are plastic manipulatives, tools that represent common or unit fractions as circular segments. The purpose of these questions, it seems, is to help students make sense of various ways of equipartitioning circles. Presumably, the fraction-pieces focus students’ attention on interpreting the circumstances of the problem, while reducing some of the cognitive load of doing the equipartitioning themselves. The written lesson, furthermore, does not offer the teacher detailed guidance on enacting this discussion, nor does it mention the use of a document camera. Torrie’s work in this opening discussion therefore represents a slight pedagogical adaptation (M. Brown, 2009) of the curricular guidance, even though she maintains focus on the same mathematical character, setting, and trajectory of events—namely the fractions of pizzas allotted to four families: the Franklins, Wus, Deweys, and Larsons.

Torrie brings a new element into mathematical storyline MS2, however, as she enacts the lesson. She challenges students to draw a representation of the pizza for the three-member Larson
family, observing, “And now I’ve just given you a curve ball” (observation transcript, 03/05/2012). Her students ask for clarification, but Torrie defers providing an answer, saying, “As you start to draw, you will see” (observation transcript, 03/05/2012). Torrie then circulates, helping students as they work. Indeed, while drawing, one student suddenly proclaims, “Ha! It’s a peace sign!” Some murmur or nod, knowingly (field notes and video tape, 03/05/2012).

Several other students are struggling, and so Torrie assists them by asking, “Are they equal?” (observation transcript, 03/05/2012). She follows this question with another: “Can you see—right now, looking at this—what you could do to make it…that would make it be equal parts?” (observation transcript, 03/05/2012). Torrie concentrates, here, on helping students reflect on and make use of their prior experience with drawing equal-sized sixths. Notably, Torrie consistently presses her students to tackle the problem on their own, rather than allowing them to use fraction-pieces. Fraction-pieces, essentially, would have consisted of pre-fabricated answers. The impact of using fraction-pieces would have made the task akin to a multiple-choice question.

When reconvening the students, to discuss their drawings of thirds, Torrie proclaims:

But this is a curve ball, right? Because we’re used to doing it in halves [first], and then we have on either side to do it in quarters. We have that [drawn as] two-and-two [on either semi-circular portion]. Doing it in sixths, we have three-and-three. (observation transcript, 03/05/2012)

As a class, students then participate in an extended discussion of how to draw thirds. In her enactment of this storyline, Torrie acknowledges a difficulty not anticipated by the written text—namely, that odd numbers of fractional, circular pieces pose a unique challenge for third-graders. By undertaking this approach, Torrie offers students additional opportunities to make sense of and construct their understanding of fractions as portions of wholes. She recognizes her students are not yet comfortable with the idea of equipartitioning various differently-shaped objects. In addition, she raises an equivocation with her so-called “curve ball.” On the one hand, students are aware that circles can be divided into equal-sized fractional pieces (a mathematical truth); on the other hand, though, some students may think this is an impossible task (a mathematical untruth), because their knowledge of equipartitioning hasn’t yet extended to comprehending fractional pieces that aren’t factors of one-half.

Comparing fractional pieces in MS1 and MS3. The written lesson suggests, next, that students work in pairs to compare the families’ fractional pizza-pieces. Torrie introduces these mathematical storylines by telling students that “we’re trying to move away from our pieces and
Initially, students are challenged by Torrie’s request to prove which fractional piece, a sixth or a fourth, is larger. Eventually, though, one student offers an idea:

Well, [we know] that there are four people in the Wu family, so you cut them into fourths. And there are six people in the Franklin family, so you cut them into sixths...But since a full-sized pizza is cut into six equal parts, then the Wus get more, because they have two less people than the Franklins. (observation transcript, 03/05/2012)

Other students agree. Even though this response is preternaturally complete, Torrie refuses to accept it and move ahead. She notes that some students look puzzled, and so, instead, she and the students document this line of thinking by writing the two fractions, as numbers, $\frac{1}{4}$ and $\frac{1}{6}$. Though not required in the written lesson, they also write the corresponding inequality symbol.

Torrie then pauses and makes an observation. On the document camera, she displays a student’s work (see Figure 14), and says:

This is true, right? The number 6 is larger than the number 4. So the Franklins have six people and the pizza was divided into six slices. And the Wus had four people and the pizza was divided into four. So all of a sudden, “Marcus,” when we’re going to use these numbers to represent fractions, this doesn’t make sense. Does anybody know what I’m talking about? (observation transcript, 03/05/2012)

For the next several minutes, Torrie and the students explore this seeming paradox or equivocation, that—in contrast to whole numbers—a larger numeral (in the denominator) indicates a smaller-sized fraction. During their exploration, Torrie and the students discuss several important and interconnected facts: the number of pieces into which the Franklin’s pizza is divided (i.e., six), the fraction of a circle the pieces represent (i.e., sixths), the number of pieces of the Wu’s pizza (i.e., four), and the fraction of a circle represented by a Wu member’s piece (i.e., fourths). Throughout the discussion, furthermore, they employ not only fraction notation but also the plastic fraction-pieces (observation transcript, 03/05/2012).
Eventually, after several minutes, “Manuel” asks the following: “Is a less number in, like, whole numbers, is the one in whole numbers more in fractions…Is that always true?” (observation transcript, 03/05/2012). Torrie immediately ascertains the import of this student’s question and explores another example the class. In this next example, she relies heavily on the fraction one-half in her comparison. Afterwards, another student responds to Manuel’s earlier question by observing, “Um, and I think I know why. Because if you have a bigger denominator, that means it’s in more pieces and the pieces will be smaller for all of them” (observation transcript, 03/05/2012). Torrie asks several other students to explain this idea in their own words; several do so, successfully. And, as the written lesson suggests, Torrie and her class complete this portion of the lesson by comparing the sizes of the Larson’s and the Dewey’s slices (observation transcript, 03/05/2012).

In contrast to Torrie’s approach, the written lesson suggests that—with this particular storyline—she ask a series of heavily-scaffolded questions. For example, the authors suggest asking, “What fraction of the pizza do the Larsons get? The Deweys?” (URG-3, Unit 9, Lesson 6, p. 98). It appears that, through this overall line of questioning, students are expected to make the connection, gradually, that the larger the number in the denominator, the smaller the size of the piece. Torrie appears to move more deliberately than the written lesson suggests, allowing for several opportunities to reiterate the key principle; at the same time, Torrie offers students
additional opportunities to model and discuss their thinking, because she notices—as they work and explain their ideas—that they aren’t fully on the same page.

Torrie’s approach in the remainder of Part 1 of the lesson, however, remains consistent with the written guidance. Students then explore patterns in a table, recording the number of pieces and the fraction for each family’s pizza. And during this succeeding discussion, Torrie addresses another student’s misconceptions (observation transcript, 03/05/2012). Specifically, one student reviews the rows of the table and observes, “It gets smaller every time, like, \( \frac{1}{4} \) to \( \frac{1}{5} \), to \( \frac{1}{6} \). And…the number part, it goes down, too…like, 6 to 4, to 3, to 2” (observation transcript, 03/05/2012). Torrie listens and immediately responds: “OK, so hang on a second. Hang on” (observation transcript, 03/05/2012). Together, she and the class refer back to the pizza problem and the sizes of the pieces; soon thereafter, this same student publicly refines her observation.

Unlike Lesson 6–2 and the other Math Trailblazers lessons I reviewed, the written guidance in Lesson 9–6 contains fewer suspenseful plot elements. I cannot conclusively say why. Perhaps the authors recognized that fractions tend to provoke confusion. Therefore, in contrast to other lessons, Lesson 9–6 may have been designed, intentionally, as a relatively straightforward one. Torrie, regardless, capitalizes on her so-called “curve balls,” or equivocations, whenever they emerge during her implementation. In so doing, she encourages students to engage in problem-solving. I note that students are actively engaged, throughout the lesson, and that conversation is consistently lively and participation seems equitable (field notes and video tape, 03/05/2012).

Added, removed, and resequenced storylines. As in her previous lesson, Torrie added several storylines to those suggested in the written lesson. Most of these were incidental to the main objectives at hand. One, for instance, involved asking students about the colors of the plastic fraction-pieces. This was facilitated as an orientation to the pieces, not suggested within the written lesson. Torrie conducted this orientation before the students used the pieces in modeling the fraction-comparison problem. Another involved recalling the symbols for expressing inequalities. At the same time, this was not a trivial addition: doing so allowed students to make sense, using precise mathematical symbols, the relationship between the sizes of denominators and the sizes of fractions (see Figure 14). Finally, Torrie also added storylines, asking students to draw and describe their thinking, when representing fractions with other unit-fraction examples she improvised.
Of the mathematical storylines Torrie omitted, all fell in Part 2 of Lesson 9–6. Part 2 involves comparing non-unit fractions to one-half, using a table—similar to the one in Part 1—to investigate patterns in numerators and denominators. I noted, above, that she did not return to Part 2 on the ensuing day; it isn’t apparent, to me, why Torrie abbreviated Lesson 9–6 in this fashion. Instead, after Part 1, Torrie’s students play a concluding game, called “Fraction Hex.” Interestingly, “Fraction Hex” involves making comparisons of non-unit fractions. As I reviewed Lesson 9–7, furthermore, I note that Torrie’s students did not seem to have difficulty with the activities therein that also involved working with non-unit fractions (field notes, 03/05/2012). Perhaps, then, Torrie had determined students didn’t need additional time on a second day with Part 2 of Lesson 9–6.

Looking at the design arc map (see Figure 13b), it also appears Torrie has resequenced mathematical storylines MS1 through MS3. Looking closer at both the written and enacted lessons shows that this may not be the case. First, as written, MS2 and MS3 are more-specific mathematical storylines that are subsumed under a broader formulation in MS1, namely—*How are unit fractions compared?* In other words, MS1 is intended to be enacted simultaneously with MS2 and MS3. By having them draw fractional pieces of pizzas, instead, Torrie extracts the equipartitioning task (MS2) from the overarching storyline. This, as noted above, serves to focus their work on what follows and to give them practical experience at apportionment. Then, MS1 and MS3 are generally enacted, as written. The overall effect, then, is not a dramatic reordering of storylines or activities but a focusing.

Finally, as noted above, Torrie may have reduced the independent work time for completing the worksheet, because she generally worked through the problems with them. The written lesson directs her to allow students to work independently on completing the affiliated worksheet. That said, we might also say Torrie interspersed students’ independent thinking-time with whole-class instruction to help scaffold and support their evolving understanding. Without this support, particularly among students needing scaffolding on reading texts, they may not have been as successful in making sense of the mathematics.

**Steering moves.** Throughout this lesson, Torrie orients instruction toward a particular goal: recognizing that larger denominators are found in smaller fractional pieces. Notably, this goal is not explicitly stated within the written lesson, although it is clearly the focus of much of the classroom activity. With her students, Torrie prods, probes, reiterates, and models in order to work toward this broad and complex goal. Consequently, Torrie’s primary steering move in this lesson
involves opening up and emphasizing key mathematical ideas (Sleep, 2012). She also undeniably makes thoughtful use of instructional time, and through her questioning techniques, she asks students to do the thinking work.

Further, with regard to developing and maintaining mathematical storylines, Torrie makes connections within and across the questions and activities; for instance, when a student struggles with articulating her reasoning about sizes of fraction pieces, Torrie and the class return to using fraction-pieces to model elements of the pizza problem. She narrates and frames each part of their work together, so that students can see the relationships between the problems and the overall goal. Torrie might be critiqued for not developing and progressing ideas beyond unit-fraction comparisons. At the same time, she incorporates increasingly sophisticated notation and challenges students to explain their thinking in various ways (including moving away, gradually, from drawing or using fraction-pieces). Finally, the wrap-up game, “Fraction Hex,” does allow students to grapple with non-unit fractions. As introduced, this activity may have been somewhat incoherent—since students had yet to experience Part 2 of the lesson. At the same time, I did not observe students struggling to compare non-unit fractions in the game, and they also enjoyed playing it with each other (field notes, 03/05/2012).

In sum, Torrie’s steering maintained many of the subtle elements of the mathematical plots that appeared within the written lessons of Math Trailblazers. Even more, during her implementation of Lesson 9–6, she enacted plot twists of her own design that were not found within the curriculum materials. Torrie’s plot twists, or so-called “curve balls” (observation transcript, 03/05/2012), not only enhanced the suspense of the lesson but also maintained focus on the key concepts. In Chapter 9, I argue that Torrie’s adaptations of the written lesson likewise reflected her goals, beliefs, and perceptions of her teaching context. In general, I believe that they also created a lively buzz in her classroom and promoted mathematical habits of mind.

8–3. RQ1—Torrie’s Written-Enacted Relationship: Aporial

Just as in Chapter 7, here, I look across Torrie’s lessons to describe her enactment of mathematical plots and storylines. At the end of this section, I relate my observations about her narrative construction to the role and nature of coherence in her instruction. I also link these observations to a high-level assessment of the dimensions of KCEM that Torrie seems to have consistently activated in her work. Broadly, with regard to construction of mathematical narratives, I characterize Torrie’s written-enacted relationship as *aporial*. As I later explain, this
implies she has an awareness of the mathematical syuzhet and makes her own adaptations to enhance students’ engagement and inquiry. Once again, I speculate on potential reasons underlying her decisions in Chapter 9, comparing and contrasting these with Elsa’s.

Mathematical Storylines—Key Elements: Characters and Settings

As I explained at the outset of this chapter, I found Torrie’s approach to using materials—i.e., her orientation to Math Trailblazers—as adherent and trusting (Remillard & Bryans, 2004). By this, I generally mean she offloaded much of the design work on the materials themselves (M. Brown, 2009). Aside from the stumper problems from Marcy Cook, described earlier, her offloading strategy was particularly evident with regard to classroom activities and how these related to the broad mathematical objectives. She enacted tasks from Math Trailblazers, largely, as intended.

M. Brown (2009) is careful to note, though, that offloading does not imply an absolution of responsibility. Indeed, as Davis and colleagues (2011) explain: “Teachers need to analyze and adapt even high-quality, reform-oriented curriculum materials to better support their own students’ learning (Barab & Luehmann, 2003; Baumgartner, 2004; Davis, 2006)” (p. 797). Torrie, in this regard, offers an exemplar case. She uses the activities suggested by the program’s authors, including typically preserving the same mathematical characters and settings (and events). Even still, Torrie also adjusts the particular questions asked of students, to help them reflect upon and make sense of their mathematical thinking. Because students’ ideas are so prominent in Torrie’s classroom, it stands to reason that she engages in significant amounts of steering work—specifically, to keep them focused on the intended mathematics and to address key concepts.

Like Elsa, Torrie added storylines to her implemented lessons. In contrast, though, her added storylines tended to consist of warm-up activities or debrief discussions. Torrie also added storylines that called attention to, or scaffolded, challenging ideas. Elsa offered scaffolding sorts of storylines to her students, as well; in the lessons I observed, however, Elsa’s added storylines tended to consist, more often, of skill-based practice.

From the data I collected, it is difficult to get a sense of the purpose behind Torrie’s omission of mathematical storylines. Nonetheless, these omitted storylines appear to have been concentrated in Part 2 of Lesson 9–6. I speculate that she may not have had time to implement Part 2 or else felt her students had been sufficiently prepared without it. In another lesson I analyzed, Lesson 6–3 (not reported here), Torrie omitted a sizable number of mathematical
storylines. At the same time, I noted that Lesson 6–3 mainly involves procedural fluency with rounding multi-digit numbers; in so doing, as written, it contains some 40 mathematical storylines within many different exercises. Not only is it unlikely that a teacher would address all of these storylines during a single lesson, I suspect that the lesson was intentionally designed to offer a plethora of examples, so that teachers would draw from only those they found most relevant for their students. Therefore, I think Lesson 6–3 represents an atypical case and that Torrie’s omitted storylines are relatively few in number.

Mathematical Plots—Key Elements: Events, Fabulae, and Syuzhets

Over Torrie’s enacted lessons, in addition, she generally preserved the sequence of primary mathematical events. I noted that in Lesson 9–6, she isolated elements of mathematical storyline MS%, which resulted in a slight shuffling of the suggested questions. Furthermore, importantly, Torrie recognized and drew upon elements of mathematical plots from the complicating action. This involved working to address the instances of jamming, equivocation, and promises of answers found within the written lesson. While Torrie may not have always been fully successful in her implementation of these mathematical plots, it is clear from my analysis that her teacher-intended lesson included these plot-points. (I offer additional detail on this claim in Chapter 9.)

Students’ contributions and mathematical syuzhets. As did Elsa, Torrie also listened carefully to students and adjusted instruction in accordance with their ideas. I explained in Chapter 7 that analyzing students’ responses is generally beyond the scope of this thesis. Nonetheless, briefly, remember that I described Torrie’s steering work with a student, who attempted to reiterate the main objective of Lesson 9–6 but made a notable misstatement. As another example of Torrie’s steering work, keeping the focus on the intended mathematics and thereby strengthening the storyline, I mention an exchange during Lesson 6–2. When collecting data on students’ coat-tracings, one student suggests finding the median of the sums obtained. Torrie responds, “That’s actually really complicated…but it involves some pathways that I don't want to spend [time] on right now” (observation transcript, 11/30/2011).

Last, I note that Torrie even supplemented complicating action within mathematical storylines by introducing plot-points that had not been included in the written lesson. She constructed two instances of equivocation, in particular, during Lesson 9–6. One involved trying to subdivide a circle into thirds, and the other involved exploring a seeming-paradox on the relative sizes of numerals in the denominator and their corresponding fractional amounts. I argue that each
positively impacted students’ level of engagement, in addition to enhancing their problem-solving opportunities.

In like fashion, I mention one other episode from Lesson 6–3 (not reported here). During her enactment of a mathematical storyline on rounding, Torrie appears to perceive an equivocation within the written lesson—namely, that some values (e.g., 198) have a single “closest” rounding-value (e.g., 200), no matter whether one rounds to the nearest tens or hundreds. This contrasts with students’ prior experiences, having already identified different closest values according to the target place-value. While circulating through the classroom and supporting students in making sense of this idea, “Semaj” is struggling but says to Torrie, “Wait, wait. Uhhh…Wait, I want to figure this out on my own, though” (observation transcript, 11/30/2011).

The notion of aporia in mathematics instruction. I offer this episode as additional evidence—a particularly noteworthy and unusual example—of students’ engagement and desire to persist in solving problems. I characterize Torrie’s overall approach to reading and planning with Math Trailblazers, consequently, as aporial. This term draws on the ancient Greek word for “puzzlement” (Woodhouse, 1910b; Wikipedia, 2019b). In so doing, I mean to call attention to Torrie’s attuned appreciation of the mathematical syuzhet and plot-points in the written lessons, as well as her work to enhance the suspense by modifying the fabula. Again, to me, her steering work in this regard includes—and also represents addressing more than—cognitive demand or sociomathematical norms.

Note that aporia sometimes carries with it notions of helplessness. This is derived, in part, from its etymological roots, which signal an impasse or, literally, a blocked passage (Woodhouse, 1910b; Wikipedia, 2019b). I use the term in a sense closer to that conveyed by Derrida (1992), however, who views aporia as a post-modern, post-structuralist condition of being. He argues that elements of modern life are paradoxes—nations that, simultaneously, contain impossibility together with possibility (Derrida, 1992, p. 29). For example, Derrida describes the notion of a gift-giving as aporial:

The truth of the gift (its being or its appearing such, its as such insofar as it guides the intentional signification of the meaning-to-say) suffices to annul the gift. The truth of the gift is equivalent to the non-gift or the non-truth of the gift. (p. 27)

A gift, in other words, cannot be a gift if not given selflessly. Any reciprocation, even a polite thanks, obviates selflessness. Despite this inevitably aporetic condition, however plausible, gift-
giving still persists in modern life. Contemporary life dictates that, regardless, we all grapple with post-modern guilt and admixture of intentions that are inherent to gift-giving.

I use aporia in a way reminiscent of another paradoxical term in mathematics education, namely, productive struggle (see Hibert & Grouws, 2007). Struggle is productive insofar as it isn’t entirely debilitating—that is, through the course of struggle, epistemic progress is made. But epistemic progress likely engenders additional struggle, perhaps to the degree that the progress isn’t readily seen or measured. Likewise, aporia is intended to connote an analogous metaphysical (rather than epistemic) condition. Aporia is an affective or aesthetic sense, representing the condition of being confused. Within this condition, simultaneously, exists the opposite condition of possibility. This is felt, perhaps, as want or a yearning. Possibility immersed within confusion, the obverse affective state, is a form of confidence. It might be regarded as comfort or tranquility that an answer is, nonetheless possible. And its existence is supported by Semaj’s willingness to find the answer on his own. Fruitful puzzlement might, therefore, be a synonym for aporia—an aesthetic (not epistemic) analogue of productive struggle.

Nonetheless, by using this term, I do not mean to assert that Torrie’s students are any more engaged, overall, than Elsa’s. I make no affiliated claims about the relative quantity of learning across both classrooms. What I do mean to imply, though, is the qualitative difference in the type of engagement—and the nature of what students are engaged in undertaking—across both classrooms. Throughout her lessons, Torrie regularly sought to provoke surprise and, perhaps, confusion in her students. This occurred, even while she and her students tackled the problems suggested within the written lessons. Torrie’s enacted lessons didn’t depart dramatically from the authors’ intentions; instead, she imbued activities with a feeling of what if? Her interpretations of mathematical plots in Math Trailblazers lessons yielded to her students’ desires to engage in productive struggle.

Discussion: Coherence and Activation of Curriculum-Embedded Mathematics Knowledge

Here, I build on the previous subsections by discussing Torrie’s steering work and its implications on the overall coherence of her mathematical presentation. I also connect Torrie’s steering work to indicators of her KCEM. Indicators of KCEM, of course, offer insight into Torrie’s knowledge of curriculum and mathematics, a teacher resource for the effective implementation of instructional resources.
Coherence and written-enacted relationships. As my analysis and report hopes to show, I would be challenged to identify moments of incoherence, traditionally-defined, in Torrie’s lessons. In several cases, even, her steering work might have addressed potential shortcomings in the coherence of Math Trailblazers. For instance, in Lesson 6–2, the written guidance on ordering notecards might be regarded as drawing attention away from the primary focus of the mathematics: adding multi-digit numbers.

Even still, Elsa enacted coherent mathematics instruction, as well. How are we to make sense of the perceived contrast, then, between Elsa’s and Torrie’s curriculum implementations? I suggest an answer to this question lies within the novel definition of coherence, offered by Richman and colleagues (2018). More specifically, if mathematical coherence also consists of students’ capacities to make predictions about subsequent mathematical events, including their perceptions of the lesson’s rhythm, then Torrie’s lessons seem to exhibit a quality that Elsa’s seem to lack. In Lesson 6–2, students might predict:

1. I’ll probably need to make sense of symmetry and do something with it, as I tabulate base-ten pieces on my coat-tracing;
2. My ultimate answer won’t be entirely accurate; and
3. The relative sizes of the coat-tracings should align with the increasing values for area written on the notecards.

Likewise, in Lesson 9–6, students might predict: 1) I can divide a circle into three pieces, and 2) the fraction one-sixth is larger than the fraction one-fourth, because the whole number six is larger than the whole number 4. Whether or not these predictions are accurate is another point, entirely. But, either way, there is certainly a qualitative difference between Elsa’s and Torrie’s experienced lessons with regard to this predictability criterion.

Last, in Torrie’s enactment, she appears to have a flexible understanding of the sequence of mathematical events. That is, she doesn’t seem to interpret mathematical events as necessarily progressing from simpler to more complex—in a strictly linear, ladder-like fashion. Instead, Torrie’s lessons incorporate purposeful moments of seeming-incoherence: what I referred to, earlier, as barriers and resequenced rungs. She uses these moments to stimulate interest. And, notably, the design of Math Trailblazers supports her in doing so; written lessons also incorporate plot elements that seem intended to promote suspense—students’ surprise, confusion, or curiosity—by manipulating the logically-arranged mathematical fabula. In Chapter 9, I offer more detail on Torrie’s beliefs about instruction and, furthermore, how she uses Math Trailblazers to
address her particular goals. My analysis reveals, in addition, that some of the plot-points in *Math Trailblazers* (and in *Everyday Mathematics*) may not have been enacted in ways envisioned by the authors. This suggests, perhaps, that curriculum development should attend more carefully to notions related to mathematical narratives.

**Knowledge of curriculum embedded mathematics (KCEM).** Thinking about Torrie’s steering work, as well as the relationship between her written and enacted curricula, I offer a few comments on her KCEM. First, as Torrie enacts lessons in *Math Trailblazers*, she certainly attends to foundational mathematics (e.g., the meaning of the denominator of fractions) that are also represented within the written, curricular guidance. This represents the activation of Dimension 1. Further, when Torrie utilizes the materials and asks students to draw and explain their representations of fractions in Lesson 9–6, we see Dimension 2 (connections across representations). Likewise, she makes heavy use of base-ten pieces in her implementation of Lesson 6–2.

With regard to Dimension 3, relative problem complexity, Torrie acknowledges (and builds upon) the relative challenge of drawing one-third in Lesson 9–6. Furthermore, with a student who is struggling to explain the relationship between the whole number in the denominator and the relative size of the fractional piece, Torrie adjusts the example she has been using (sixths and fourths) to a more straightforward case (halves and fourths).

Finally, consider Dimension 4, mathematical learning pathways. Remillard and Kim (2017) assert:

> Recognizing learning pathways in a curriculum includes understanding how a particular mathematical goal is situated within a set of ideas that develop over time. Teachers identify mathematical learning pathways when they recognize the seeds of a particular concept introduced in a previous grade and the eventual fruits of the same concept in a later grade. (p. 75)

They further affiliate Dimension 4 of KCEM with the activation of pedagogical content knowledge, like Shulman’s (1986b) *lateral curriculum knowledge* or Ball’s and colleagues’ (2008) *horizon knowledge*. Considered together, this type of knowledge generally refers to understanding the long-term pathways of mathematical learning.

In Lesson 6–2, Torrie certainly draws on the seeds of algorithmic, multi-digit addition by having students tabulate base-ten pieces. (This is particularly true, when encouraging students to use the place-value tool she designed.) This skill is not required in third grade. That said, without
further investigation, it is difficult to know in what way Torrie relates the content of third grade to that of other grades. I do not have examples in my dataset of her clearly building on second-grade (or prior) content nor insinuating progression to fourth-grade (or later) content. As with Elsa in Chapter 7, knowing more about Torrie’s KCEM requires an in-depth analysis of her own explanations of mathematical progressions. While I allude to some of these ideas in Chapter 9, a full analysis is beyond the scope of this thesis.

In sum, Remillard and Kim (2017) note that roots of KCEM are found in Sleep’s (2012) tasks of steering instruction toward the mathematical point. I therefore argue that Torrie’s steering work strongly suggests a vibrant KCEM. I deepen this claim in Chapter 9 through her interview data.

8.4. Summary and Conclusion

Torrie’s instruction, like Elsa’s, demonstrates many elements of coherent presentation. In particular, thinking of traditional notions of coherence, Torrie connects ideas across problems and activities; she also helps students appreciate broader concepts. With regard to Sleep’s (2012) steering moves and coherence, Torrie makes strategic use of instructional time, stays focused on important mathematics, opens up key ideas for her students, and maintains strong mathematical storylines. In contrast to Elsa, however, Torrie both notices and embraces elements of mathematical plots that serve to heighten students’ interest in problem-solving. Generally, the written lessons support her enactment of these plot-points, even if they occasionally hint at them in subtle ways. Torrie even adds elements of her own, when the written materials seem to lack these sorts of suspenseful moments. Her adaptations demonstrate a certain flexibility with the events of mathematical storylines, sequencing and presenting them in ways that intentionally deviate from a strictly-linear approach.

Reflecting on the residue that these lessons may have left for students, I again focus on the SMPs of the CCSS-M. These are the habits of mind that authentic mathematical inquiry intends to promote. As this chapter aims to show, through her participation with Math Trailblazers, Torrie’s students certainly model with mathematics (SMP #4) in finding the surface areas of their coat-tracings. They also make sense of problems and persevere in solving them (SMP #1), as the anecdote from Semaj shows. And in looking at denominators and sizes of fractional pieces, they reason quantitatively and abstractly (SMP #2). I could go on. In general, I would argue that Torrie’s students are supported by both their teacher and the curriculum materials used at Heritage Gardens to think like mathematicians. And, as noted above, the classroom humdrum,
collaborative give-and-take, and general enthusiasm for problem-solving are all indicative of their budding and positive relationship with the discipline of mathematics. One can only hope this third-grade patina is durable enough to carry them through the slings and arrows in mathematics that will, inevitably, follow.

In sum, mathematics tutor John Mighton may have said that, in problem-solving, “no step is too small to ignore” (Bornstein, 2011, para. 10). From my imagined viewpoint on their hypothetical confrontation, Torrie Blum might be forgiven for responding: “But giant leaps must be encouraged, too.”
But [w]hen the test was handed out. “Yahoo!” we yelled. “Yahoo!”
For it was filled with all the things that we all knew we knew.
There were questions about noodles, and poodles, and frogs and yelling,
About listening and laughing, and chrysanthemums and smelling.
There were questions about other things we’d never seen or heard,
And yet we somehow answered them, enjoying every word.

—Dr. Seuss, Hooray for Diffendoofer Day! (1998, lines 75–80)

9–1. Introduction

In Chapter 7, I described Elsa’s approach to constructing narratives with her curriculum materials, Everyday Mathematics, as fabula-oriented. By this, I meant that—in the lessons I observed—Elsa tended to unravel mathematical plot twists. Put another way, Elsa seemed to perceive potentially-complex sequences of mathematical events that were perhaps intended to elevate narrative suspense, and she reordered them in such a way that retained the character of their logical orientation (i.e., the mathematical fabula).

And in Chapter 8, I described Torrie’s approach as aporial. I coined this term for curriculum-use, borrowing from Derrida (1992), to convey a sense of possibility inherent within a sense of frustration. In my observations of Torrie’s instruction, I noted she seemed to perceive opportunities within written lessons to elevate narrative suspense—students’ surprise, confusion, or curiosity—and she attempted to enact those opportunities faithfully. Even more, Torrie supplemented mathematical storylines with her own plot twists, if she found that the written guidance was somewhat lacking. Stated differently, Torrie appeared to seek out opportunities to induce a sense of fruitful puzzlement in her students.

Both teachers, Elsa and Torrie, can be said to have enacted coherent instructional episodes. While I noticed occasional missed opportunities to deepen students’ conceptual understanding, this observation does not alter the fact that hallmarks of coherence were obviously present in their lessons. For one, while not necessarily a traditional feature of coherence, both teachers made strategic use of their instructional time (Sleep, 2012). I mention time-on-task, nonetheless,
because non-instructional time has been shown, understandably, to have a marked impact on a lesson’s coherence (Fernandez et al., 1992). Consistently, during both of their lessons, I observed hardly any unproductive disruption or non-mathematical time.

For another, classroom activities and tasks were tied to one another, and they were all clearly related to broader mathematical objectives. These represent traditional notions of coherence. Using Sleep’s (2012) criteria for coherent instruction, which includes opening up key ideas, progressing mathematical storylines, and keeping a focus on meaning, my analysis deepens. On occasion, such as with Elsa’s use of multiple-choice problems on area and perimeter, the mathematical ideas may not have progressed deliberately. Progression, I have learned, doesn’t necessarily imply a strictly linear pathway for learning; coherent lessons should nonetheless convey a sense of broadened complexity or abstractness. Elsa’s multiple-choice questions tended to focus on formulaic or definitional aspects of area and perimeter, instead, rather than exhibiting a rhythm that pressed toward sophistication.

Torrie, too, might be critiqued for undercutting the meaning of base-ten pieces in the “Coat of Many Bits” activity. As the lesson neared its conclusion, she was somewhat prescriptive in offering guidance to students on finding the sums of their tabulated results—rather than asking them to grapple with counting and regrouping the base-ten pieces as a physical-cognitive exercise. Given that this episode occurred late in Day 2 of this lesson, it is possible that Torrie’s motivation for her somewhat algorithmic treatment was time-related. Indeed, teachers must often attend to multiple purposes with finite resources as they support students’ mathematical development (Lampert, 2001; Sleep, 2012).

Despite these potential criticisms, the broader question at hand is whether Elsa’s and Torrie’s teaching left a lasting mathematical residue for their students. I claim that, on the one hand, Elsa’s fabula-oriented use of curriculum materials may have diminished students’ opportunities to engage in authentic mathematical inquiry. On the other hand, Torrie’s aporial use of curriculum materials may have enticed students into problem-solving. Regardless, looking back at M. Brown’s (2009) design capacity for enactment (DCE) framework, I note that, thus far, I have omitted a significant set of explanatory or mediating factors for the teaching I observed: each teacher’s resources—specifically, their goals and beliefs. In addition, as Davis and colleagues (2011) found, there are potential contextual factors that should be considered.

Therefore, in this chapter, I address RQ3, broadly summarized as: How are teachers’ goals, beliefs, and contexts related to their instructional designs and construction of narrative, and what
do these say about the way they participate with curriculum materials and their pedagogical design capacity (PDC)? To pursue this question, I look across both teachers’ use of instructional materials in enacting mathematical narratives. In so doing, here, I aim to present potential explanatory factors for their decision-making. These, I argue, are rooted in their goals, beliefs, and orientations about curriculum materials, mathematics instruction, their students, and so on. I proceed by reporting, mainly, on my analysis of my interviews with Elsa and Torrie. I begin with profiling Elsa’s beliefs about curriculum and goals for instruction and potential contextual influences. I then tie these to particular instances of teaching I observed. I continue with a similar profile of Torrie before discussing broader, cross-cutting implications about their participating with materials and PDC.

9–2. Influences on Elsa’s Construction of Mathematical Narratives

This section concentrates on highlighting Elsa’s interview responses. Through our conversations, she expresses a set of beliefs about curriculum materials, instruction, and her students that likely guide her instructional decision-making. In addition to reviewing these, during our interviews, I reference moments of Elsa’s instruction and ask her to comment on her choices. This strategy aims to address epistemological concerns by Fenstermacher (1994) and Schoenfeld (2011) that I might over-generalize from my observations of teachers’ work. By contextualizing my observations within their own stated goals and beliefs, I better understand the phenomenology of their intended curricula.

In what follows, I first review Elsa’s beliefs about Everyday Mathematics and general instructional goals. I then turn to outlining potential contextual features. Together, I argue, these relate to the episodes of instruction that I recollect from Chapter 7.

Elsa’s Beliefs About Curriculum and Her Instructional Goals

The following discussion addresses Elsa’s beliefs about Everyday Mathematics. In particular, I explore her prior experiences in professional development and how these relate to her approach in differentiating instruction. I also explain Elsa’s perspective on the underlying philosophy of the program. In many ways, these relate to her overall goals, as an instructor; I review these, here, as well. Afterwards, I describe Elsa’s decision-making, connected to her beliefs and goals.

Elsa’s beliefs about Everyday Mathematics. Elsa’s beliefs about Everyday Mathematics are related to her experiences with the program, her professional development opportunities, and a
broader set of cultural and personal ideas about mathematics instruction. I begin my summary of Elsa’s beliefs by noting a potentially-formative experience during a professional development session, very early in her career (more than a decade prior to our interview).

Secrets of the book, related to problem-solving and opportunities for differentiation. In her professional development session, Elsa recalls, a representative “for the company” relayed “secrets of the book” (personal interview, 10/28/2011). One of these secrets, she explains further, involved the independent-practice problems typically given at the end of a lesson, *Math Boxes*. The training representative explained that these, intentionally, contain material that has not yet been taught; they are designed, then, as pre-assessment problems. Elsa remembers a technique, one she continues to use, for reducing students’ stress when encountering these problems. She obscures the pre-assessment problems with sticky notes on students’ workbook pages, thereby allowing them to experience success “instead of it starting on the first box…and then falling apart” (E. Mackey, personal interview, 10/28/2011). She asks her students to ignore any problems to which they do not believe they can respond correctly.

Despite this perceived limitation of *Everyday Mathematics*, Elsa endorses what she believes are its numerous opportunities for differentiating instruction. As with its *Math Boxes*, Elsa says she draws on and adapts the program to address her students’ learning needs.

To explain further, she contrasts *Everyday Mathematics* with a Houghton-Mifflin program that she used at the beginning of her career. She found this program had “something that was missing” (E. Mackey, personal interview, 10/28/2011). She states:

> But, it [the Houghton-Mifflin program] was much more like, “Okay, today I’m going to teach you how to do x, y, or z, this is the scale, now you’re going to go practice it.” And it had almost no problem-solving pieces or if it had problem-solving pieces, it was like, a workbook page or something like that. I didn’t feel like there was a lot that went with it. (E. Mackey, personal interview, 10/28/2011)

As a result, in Elsa’s view, her students couldn’t really explain their mathematical thinking in words or in writing. She says:

> And, like, I would say to my kids, “Tell me how you got that answer.”
> 
> “Well, I just get it.”
> 
> “Well clearly, you haven’t mastered it, because you can’t tell me, or you can’t write it.” (E. Mackey, personal interview, 10/28/2011)
Elsa continues by proclaiming her belief that students should not only read and write, mathematically, but also describe their thinking through drawings (personal interview, 10/28/2011).

To address this need—to support her students in developing problem-solving and explaining capacities—Elsa supplemented the Houghton-Mifflin program with word problems from *Math Exemplars* (Exemplars, Inc., 2019). On occasion, she still uses tasks from *Math Exemplars* or other, similar resources like Marcy Cook’s, she says. But Elsa notes, even still, that she hasn’t currently “felt like I needed something like that [Math Exemplars], because I feel like the *Everyday Math* weaves it in” (personal interview, 10/28/2011). In addition, Elsa continues, these problem-solving opportunities are also ready-made resources for differentiation. To her, differentiation seems to mean enrichment for students who are mathematically confident or, in her words, are “really bright students” (E. Mackey, personal interview, 10/28/2011). Therefore, she says, while she “definitely like[s] that section” of *Differentiation Options in Everyday Mathematics*, she admits, too, that “they could be beefed-up a little bit more” and that “even some of the enrichment they have isn’t quite hard enough” (E. Mackey, personal interview, 10/28/2011).

Overall, though, Elsa feels confident in her ability to use the *Everyday Mathematics* program. Aside from learning about the “secrets” of *Everyday Math*, she says, “I don’t feel like I have a lot of questions about the curriculum, per se, that I’m not clear about” (personal interview, 10/28/2011). She will occasionally discuss elements with colleagues, but Elsa also admits that such occasions—and opportunities for collaboration, generally—are rare (personal interview, 10/28/2011).

*Beliefs about the philosophy of Everyday Mathematics.* Asked to describe her understanding of the program’s philosophy or its overall emphasis, Elsa says:

> The major philosophy of *Everyday Math*, I think the idea is, to apply mathematical thinking in helping children gain an understanding of how this would be applicable to the rest of the world or their surroundings of how they might use it. Because I feel like it does do a pretty good job of connecting a concept to how a child might use it, as opposed to other times [without *Everyday Mathematics*], a kid would say, “Well, why do I have to learn this?” You know, I feel like the program sets them up for doing that. I don’t know if that’s a philosophy, but I think it is. (personal interview, 10/28/2011)

Given the title of the program, it isn’t entirely surprising that Elsa would interpret *Everyday Mathematics* as intentionally highlighting the practical, real-world utility of mathematics. She continues by describing its *World Tour* component, which—she maintains—is intended to present
mathematics within social contexts and to broaden students’ cultural horizons (E. Mackey, personal interview, 10/28/2011). Interestingly, though, despite her earlier claims about understanding mathematical concepts, Elsa doesn’t appear to consider the models, representations, or conceptual-oriented features of *Everyday Mathematics* as part of its underlying design philosophy.

In addition, Elsa explains that she occasionally supplements *Everyday Mathematics* with worksheets and websites, because of its relative lack of opportunities for students to practice traditional methods or algorithms and to memorize arithmetic facts (personal interview, 10/28/2011). In particular, she describes a self-created quiz book on multiplication facts and use of a website called *multiplication.com*. This site has a tag line (“Master the multiplication facts now!”) and affiliated resources intended for memory-building, through quizzes, worksheets, online flash cards, and other strategies.

**Elsa’s general instructional goals.** Elsa describes her goals in several different ways. First, she proclaims her explanations are intended to match students’ individual learning styles. Elsa feels this makes learning fun and relevant for students, in addition to reducing their overall anxiety (personal interview, 10/28/2011). Explaining, further, she says:

> So I think there is a lot of anxiety with kids in math, so trying to remove that piece for them by saying, “You know”—like, I’d even kid with them, I’d be, like—“Seriously, you’re not making me work very hard today. I mean, I know I’m not getting paid that much, but it actually is my job to help you.” And, I think, especially—you know, not to be sexist but—for girls in math, you know, to say to them, “Okay, I know your mom wasn’t good at math, and my mom wasn’t good at math, and I’m not good at math, but we’re going to get through this. And it’s because of the way I’m showing it to you.” Um, I think that is like a main thing—not to give up. And to realize that if they’re not understanding it, it’s not because they’re not capable, it’s because I’m not teaching to their learning style or their strength. (E. Mackey, personal interview, 10/28/2011)

She perceives her role, then, as adjusting her explanations to better suit students’ needs. But, tacitly here, she also admits her own, gendered anxieties about mathematics by linking students’ struggles with her own and even her mother’s.

Another of Elsa’s goals, as an instructor, involves wanting students to understand why algorithms make sense and their relationship to alternative methods—such as a partial-products approach to performing multiplication (personal interview, 10/28/2011). And recall, above, that Elsa wants her students to develop their capacities for explaining their thinking through writing and pictures, particularly when tackling word problems.
Potential Contextual Influences

I now situate Elsa’s beliefs within her own perceptions of contextual features of Golden Hawk. Note that I am not making any empirical claims about Golden Hawk, as a community, but rather I intend to describe the phenomenology of Elsa’s experiences. Other Golden Hawk faculty members, of course, might have different perceptions. Broadly utilizing an epistemological approach suggested by Fenstermacher (1994) and Schoenfeld (2001), I therefore can only make warrants about Elsa’s experience, insofar as she, herself, explains her thinking.

First, Elsa proclaims her enjoyment of the independent-school ethos, because (unlike some of her public-school experiences) adapting curriculum materials is entirely within her purview. Of Golden Hawk’s culture, embracing teacher-autonomy, she explains:

You can really sort of create what you like, as long as you’re kind of covering the same things…you can add other things in there [to the Everyday Mathematics program], if you want to and have your own style and flair for it. (E. Mackey, personal interview, 10/28/2011).

I note, here, that Elsa also obliquely describes her role as a teacher as instructing while exhibiting an individual style or flair. In other words, she perceives her work as necessarily involving personal expression. Not surprisingly, then, I observe Elsa’s classroom is decorated by scores of ornate and colorful geckos, which she tells me are reminders of her travels to Central and South America (field notes and personal interview, 10/28/2011).

Elsa also describes what she believes is a significant challenge of Everyday Mathematics, namely that “it’s very language-based” (personal interview, 10/28/2011). In particular, she explains that its overall approach involves heavy use of word problems, foregrounded, to drive the exploration of ideas. She therefore explains her belief about why her students sometimes struggle with the program:

It’s a very language-based curriculum. Um, for a school like ours, it fits really nicely because it’s the feel of our school. And that’s the kind of thinking that we’re expecting children to do. So, um, I just think sometimes it can be a little challenging for kids. Um. Anybody who maybe has like a—some sort of learning issue, even if it’s processing difficulty—um, they sometimes can have a hard time reading the directions and understanding what they’re supposed to do. Um, and applying that. I had a student, one year when I was teaching in [the southeastern U.S.] who had a non-verbal learning disability and she, like, just couldn’t do it. And she could do other math curriculum. But, it was just so language-based and all of that. So that was challenging. But I think, for the most part, they really have to think and attend to it while they’re working. (E. Mackey, personal interview, 10/28/2011, emphasis added)
While she believes that *Everyday Mathematics* “fits nicely” with Golden Hawk, because of its rigorous approach and general focus on literacy skill-building, Elsa also believes that it presents a barrier to some of her students (including students with whom she worked in public education, earlier in her career). Specifically, Elsa explains that her students exhibit a wide range of prior successes and confidence around mathematics. She describes a sizable group of students in her fourth-grade class at Golden Hawk as having specialized learning needs, particularly with regard to reading or general “processing” (E. Mackey, personal interview, 10/28/2011).

At the same time, she characterizes students’ struggles with *Everyday Mathematics* as more attentional, and less conceptual, in nature. She says that her students “don’t always read everything on the page” and that “they miss a lot of things” when trying to work independently (interview transcript, 10/28/2011). This perception also generally explains her typical response. Specifically, Elsa offers her students pointers on how to interpret word problems. “I go through and have them circle words and highlight words;” she explains (interview transcript, 10/28/2011). Further, if students have trouble with particular tasks, she says, she first tries “saying it in a different way” (E. Mackey, personal interview, 10/28/2011).

In some cases, Elsa says, her guidance is more straightforward; she tells students such things as: “Well, you need to think about it for a second” (personal interview, 10/28/2011). She sometimes asks, moreover, “Why don’t you read it [the problem or instruction] back to me? What are they asking you?” (E. Mackey, personal interview, 10/28/2011). Finally, Elsa also encourages her students to utilize resources they have available to help themselves. She sometimes “won’t answer questions for them” (personal interview, 10/28/2011) unless they’ve reviewed any affiliated pages in their *Student Reference Book* (SRB). The SRB is, essentially, a reference manual containing definitions and worked examples.

Overall, I note that Elsa’s perceptions of her students’ needs—and of Golden Hawk’s philosophies—dovetail with her stated goals. Specifically, she believes they need support with reading about and explaining their mathematical thinking. Her perceptions also stand in tension, somewhat, to her understanding of *Everyday Mathematics*. She critiques the program for exhibiting, perhaps, an overly-heavy focus on word problems, as the underlying motivation for students’ learning experiences.
**Explanation of Instructional Decision-Making and Mobilized Storylines**

I take a twinned approach in exploring Elsa’s instructional decision-making. First, I review Elsa’s stance on reading and using the *Everyday Mathematics* teacher’s guide. As Remillard (2012) explains, what they encounter in reading materials—and what they read for—is “relevant to how teachers engage and utilize resources” (p. 110). Second, I portray Elsa’s particular uses of curriculum, insofar as I observed them, set within the broader context of her beliefs and goals (as described above).

**Elsa’s general use of the teacher’s guide.** Elsa reads and uses the *Teacher’s Lesson Guide* (TLG) of *Everyday Mathematics* in ways generally consistent with her beliefs and goals. In “any ways, of course, she also follows the authors’ intentions. Elsa notes, for one reason, that her lessons include 20 minutes of whole-class instruction followed by 40 minutes of paired, small-group, or independent work (personal interview, 10/28/2011). This latter period of time consists of games, practice problems, and remediation or enrichment activity. Her approach is largely intended to strengthen her class’s understanding of grade-level mathematics while also meeting students’ personalized needs. And, indeed, the components of *Everyday Mathematics* are broadly arranged, so that teachers follow this intended structure and format (if not the precise timing).

Further, describing how she relies on the TLG, Elsa succinctly explains, “So I read over it, and I decide what I like in it” (personal interview, 10/28/2011). She also reads the problems on students’ workbook pages, she says, “to see what types of examples they gave, to make sure that I covered all of the types possible” (E. Mackey, personal interview, 10/28/2011). Elsa offers a particular circumstance of how she uses the TLG during instruction:

> And I’ll glance at it and look down and be, like, “Oh, yeah, I want to make sure that I do a problem where it’s a ‘greater than’ and a ‘less than’ rather than an ‘equal sign’ or whatever.”...So, yeah, I mean, I usually have it up on my table—up front—or in my hand. And then, when the kids finish, I have them bring their work over, and then I check it. And I circle the ones they might have missed, and then give them the opportunity to go back and try again on them. (personal interview, 10/28/2011)

During lessons, then, besides checking to ensure she has covered all problem-types, Elsa also uses the TLG as an answer key. And allowing students to revise their work is a strategy, to a degree, for reducing students’ anxieties about traditional one-shot-only approaches in mathematics instruction.

Whenever she reads what she describes as the “background information” on students’ thinking and mathematics concepts, Elsa indicates she thinks about “what [specific] kids are
“going to struggle” and “what kids are going to need some enrichment” (personal interview, 10/28/2011). This reading informs her planning around which students will work together, which activities she will enact, and whether she will supplement *Everyday Mathematics* with outside materials. In this way, as explained earlier, she reads with a particular aim toward differentiating activities for her students, based on her understanding their prior successes and confidence. This also evidences her activity-focused lens on curriculum materials (Remillard & Bryans, 2004).

**Elsa’s lesson-enactment, tied to her goals and beliefs.** Here, I review each of Elsa’s goals and beliefs, reviewed above, and provide an instance of how they may influence her instructional designs or decision-making. I aim to connect these with her construction of mathematical narratives, although these classroom episodes have broader implications, as well.

**Real-world applications.** First, recall Elsa’s claim that the main philosophy of *Everyday Mathematics* involves portraying the real-world relevance of mathematics. In Lesson 8–3, Elsa seems to perceive that students may confuse the terms area and perimeter. She therefore inserts several multiple-choice practice problems, because—in her words—the previous lesson on area “was too short,” and so she “wanted to beef it up a little bit and add some review in there” (personal interview, 03/01/2012). These sorts of momentary storylines (Dietiker, 2012) are framed as decontextualized problems, although several contain specific measurement units. Nonetheless, Elsa motivates this activity through an anecdote about her brother, painting Elsa’s house and potentially confusing perimeter and area. Throughout this phase of her lesson, Elsa also introduces problems about building a door and setting a dining table. She even incorporates a student’s suggestion about the fabric needed to tailor a shirt, to address her imprecision with using these terms.

Asked why she relies heavily on the real-world applications found in *Everyday Mathematics*, supplementing them with her own, Elsa explains:

> I feel like it relaxes them. And if I can give them some examples of how this might have applied—because, honestly, what kid is going to think about the efficiency of their kitchen?—but if some teacher talks about, “Oh, this is how I regulated my kitchen, and this is what I did,” they’ll remember that. So that's why I do that.... The hope is that the kids will make the connection....So you remember the specifics about what the story was, and then you were able to connect with the lesson. (personal interview, 03/01/2012)

Several of Elsa’s goals merge, here, including making the mathematics personally relevant (via someone students know), helping students remember key ideas, and reducing their anxiety. Of course, her understanding is certainly not very far afield from the authors’ own design intentions.
To summarize, in my observations of her work, Elsa adapts the mathematical storylines of the written lessons, consistently, by connecting them to notions her students might find more tangible.

**Differentiating instruction.** Another of Elsa’s goals involves appropriate differentiation to meet her students’ needs. Once again, I refer to an episode from her enactment of Lesson 8–3. During our follow-up interview, I ask Elsa how she interpreted guidance in the written lesson on having students count whole and partial grid-squares to find the area of two-dimensional shapes. She explains that she thought the language in the lesson was a scaffold, offered to help students who were struggling (personal interview, 03/01/2012). She explains that she used this guidance:

> …in talking with some of the ones who were having a hard time figuring out area, counting the squares, and then how you would use the portions of the squares to fill them in to count them. (E. Mackey, personal interview, 03/01/2012)

In other words, she circulated around the room to monitor their work—following episodes of whole-class instruction. With students who were struggling, she instructed them personally in counting whole and partial grid-squares.

It is unclear to me whether Elsa recognizes that, to a degree, her multiple-choice problems introduced a certain amount of incoherence. They did not provide insight into the counting strategy called for in the students’ *Math Journal*. At the same time, given the balance of time allotted to whole-class and personalized instruction in Elsa’s stated, typical lesson—only 20 minutes for whole-class discussion—it is possible that she regards whole-class instruction as offering an overview of the topic; the finer details, she may believe, are to be addressed through differentiated instruction. During independent work time, then, she may engage in local construction of mathematical narratives.

**Reading and writing mathematics.** Closely affiliated with her goals for personalizing instruction, Elsa also intends to support her students in strengthening her problem-solving skills. These skills, in her view, also include reading and writing about mathematics. As an example of this type of support, Elsa allows students several quiet minutes to read page 133 of the SRB (in Lesson 8–3), rather than simply telling them the information presented (observation transcript, 02/28/2012).

Furthermore, when asking students to work on page 227 of their *Math Journal*, Elsa offers this support (also in Lesson 8–3):
Could you circle on your paper the word “area?” Because I want to make sure you don’t confuse this with perimeter. The second thing that I want to point out to you is that the answer, the answer says “area equals blank centimeters square.” So you can either do this by counting them—if that’s easier—but you also might want to think about multiplying the shape. OK? (observation transcript, 02/28/2012)

Here, she offers them a coding or annotating strategy, to assist in reading mathematics texts and keying-in on important details. She also offers them a reminder on the mathematical strategy she hopes they will use—a formula or counting grid-squares.

And, likewise, during her enactment of Lesson 8–4, Elsa instructs students to read and follow the instructions on page 230 of their Math Journal (observation transcript, 03/01/2012). She asks them to do so, rather than reading each step aloud to them or telling them the instructions. As she explained, above, Elsa then encouraged them to work collaboratively on making sense of the task, or to use each other as resources. She monitored their work and periodically called out key pieces of information, such as noting that “one square foot = 144 in?” (observation transcript, 03/01/2012).

Summary. Collectively, Elsa’s design decisions are rooted in a strong set of consisted beliefs about curriculum and the teaching-and-learning of mathematics. Considering how she describes her prior experiences in a public-school setting, it therefore makes sense that she aims to reduce students’ anxiety about mathematics, overall. In addition, in a college-preparatory environment, like Golden Hawk, it is not uncommon for students to believe—rightly or wrongly—that their future academic success hinges on their performance in mathematics.

In particular, her technique on covering challenging Math Boxes connects, rather cleanly, with her obscuring of the central plot twist in Lesson 8–4. Asked about this adaptation, specifically, Elsa admits:

I skipped around a lot. I don’t know if it was just today that I was doing it, but I was all over the map. And it might be interesting next time, for you if, well, I don’t know, but it might be interesting to have people number it [the written lesson] in the order that they read it. So I was, like, all over the map. (personal interview, 03/01/2012)

I should add, here, that my interview did not contain questions about the specific order in which teachers implement activities. Further, the ICUBiT Project was not designed to uncover the progression of mathematical storylines. Even still, when I press Elsa for more detail about why she felt her instruction during this lesson was, in her words, “all over the map,” she responds:

…I first started scanning through it to see, I think, I was looking for what I thought would be hard to do. And then what, there were some things that I read that I was, like, “No, I’m not doing this.” (personal interview, 03/01/2012, emphasis added)
Therefore, considering the broader context, I believe that Elsa’s steering work nonetheless suggests a pattern in how she perceives and mobilizes mathematical events. As she herself indicates, it seems Elsa reads for the relative problem-complexity of curriculum materials, in addition to the foundational ideas represented in texts (i.e., Dimensions 1 and 3 of KCEM). And she mobilizes these resources, perhaps even subconsciously, with an aim toward helping her students enjoy mathematics. For her (and, perhaps, for many of her students), this means reducing their levels of confusion, surprise, or curiosity—i.e., narrative suspense.

9–3. Influences on Torrie’s Construction of Mathematical Narratives

Following a similar structure and approach to that, above, I turn to exploring potential influences on Torrie’s construction of mathematical narratives. These include her beliefs about *Math Trailblazers*, her broader instructional goals, and elements of her teaching context. I summarize these before linking them to her instructional decision-making, as she perceives and mobilizes the mathematical storylines and plots found within her instructional resources.

**Torrie’s Beliefs About Curriculum and Her Instructional Goals**

Analogously to that above, this subsection plumbs Torrie’s beliefs about *Math Trailblazers*. In particular, I explore her multi-faceted description of the program’s philosophy and instructional approach. I also characterize her primary goals, as an instructor. And, afterwards, I describe Torrie’s decision-making, connected to her beliefs and goals.

*Torrie’s beliefs about Math Trailblazers.* As I explained above, Torrie and her colleagues at Heritage Gardens have had significant opportunities to explore the design principles of *Math Trailblazers*. As a result, she gives thoughtful and elaborated answers to corresponding questions during our interviews. I summarize Torrie’s understanding of the philosophy of *Math Trailblazers* and her perceptions of students’ and teachers’ roles, below, before next discussing her broader goals for instruction.

*Beliefs about the philosophy of Math Trailblazers.* To describe the philosophy of *Math Trailblazers*, Torrie uses a variety of terms. For instance, she describes it as a “hands-on” program (personal interview, 11/02/2011). In her view, *Math Trailblazers* involves many active-learning tasks that employ physical tools or manipulatives for solving problems.

In addition, Torrie characterizes *Math Trailblazers*, generally, as “conceptual” (personal interview, 11/02/2011). In her words, the program aims to help students “understand it deep before
moving on” (T. Blum, personal interview, 11/02/2011). This implies, as Torrie says, “It’s not about, ‘Oh, here’s the algorithm—off you go.’ And just memorize that and, then, you know” personal interview, 11/02/2011, emphasis noted). Instead, over time, the program’s activities, tools, and models—for example, base-ten blocks or expanded notation—coax sense-making and understanding from students. In her view, this deep understanding involves being able to “do the same problem in two different ways” (T. Blum, personal interview, 11/02/2011).

Finally, she contrasts the philosophy of Math Trailblazers, as she perceives it, with her own, past experiences as a student of mathematics. In her recollection, her teachers often told her, “Here’s how you do it, now go home and practice 25 of them [problems]. We’re going to go over 15 of them [problems] in school…” (T. Blum, personal interview, 11/02/2011). Not surprisingly, perhaps, she admits that using Math Trailblazers was “very intimidating the first year” (T. Blum, personal interview, 11/02/2011). She had to confront a radical shift in pedagogy away from what she had known throughout her schooling. Torrie also had to recognize that she, herself, might not have had, for instance, a “deep understanding about addition—multiple-digit addition” (personal interview, 11/02/2011). These are complex challenges that not all teachers are able to acknowledge, much less be willing to try overcoming them.

**Expected roles.** Elaborating, further, Torrie outlines the program’s expected roles for teachers and students. Of students, she explains that there is a “whole language” and a “whole, sort of, um, culture” around students’ problem-solving (T. Blum, personal interview, 11/02/2011). The culture and language of Math Trailblazers are intended to evoke students’ independently-developed strategies for solving problems. Through the activities, Torrie says, “They’re discovering it [the solution] on their own…,” (personal interview, 11/02/2011). As a result, she also says, “you just, as the teacher, have to observe and record, as opposed to requiring” (T. Blum, personal interview, 11/02/2011, emphasis discerned). By requiring, I take Torrie to mean expecting students to replicate an approach that the teacher has already modeled—i.e., teacher-centered, direct-instruction.

She also describes her own role, the teacher’s, as involving “mentoring and modeling” (T. Blum, personal interview, 11/02/2011). Furthermore, she explains, the program expects her “to watch [students] and to get ideas and to try them out, and then to see that person [or student] again, again, and again” (personal interview, 11/02/2011). Having observed Torrie’s instruction, several times, I believe I understand what she means by “see that person again, again, and again”:
actively circulating throughout the room to monitor, continuously, students’ ongoing individual and small-group work.

**Torrie’s general instructional goals.** Not dissimilar from Elsa and *Everyday Mathematics*, Torrie identifies two, main limitations of *Math Trailblazers*. These, Torrie explains, have contributed to the adaptations she and her colleagues have made—addressing some of her overarching pedagogical goals. First, she notes that she and her colleagues “have had to, amend—add-in—some more rote memorization of the facts” (T. Blum, personal interview, 11/02/2011). In other words, the faculty at Heritage Gardens believes the program needs more opportunities for practicing and committing basic arithmetic facts to memory. Again, this is not an uncommon concern about reform-oriented or NSF-funded programs (Stein et al., 2007). Notably, despite this concern, I did not see Torrie utilize the *Daily Practice Problems*, which are intended to support students’ procedural, skill-based fluency (EDC, Inc., 2001, pp. 5-6). This may have been an aberration from her usual approach, or perhaps the program’s stated philosophy on arithmetic computation hasn’t been fully internalized by the faculty at Heritage Gardens.

Second, regardless, even though Torrie describes *Math Trailblazers* as a conceptual program, she supplements it with word problems from Marcy Cook. Again, she describes these problems as stumper problems, typically used as a warm-up at the beginning of class. Somewhat differently than Elsa’s, her motivation for using these involves promoting students’ general problem-solving skills and “perseverance” or “grit” (T. Blum, personal interview, 11/02/2011).

Indeed, Torrie contrasts the stumper problems with what she calls “integrated, woven problem-solving” (personal interview, 12/20/2011). So-called “integrated” problems, for Torrie, are those already found within curricular units and lessons; the strategies for solving them are part and parcel of the program’s content objectives. With the stumper problems, on the other hand, Torrie aims to promote what she calls “classical problem-solving” of “isolated problems” (personal interview, 12/20/2011). This might involve, for instance, using “elimination to solve a [novel] problem” or “just guess and check” (T. Blum, personal interview, 12/20/2011). In sum, solving stumper problems, for Torrie, involves building a toolkit of heuristics and strategies for tackling never-before-seen sorts of questions.

Torrie has identified problem-solving as a crucial goal in her classroom, because as she says, “problem-solving is very elusive and a lot of kids just have no idea where to begin—and that’s a problem” (personal interview, 12/20/2011). Asked whether these stumper problems have made a difference, Torrie reflects and responds:
In providing this response, Torrie explains that she has been using stumper problems, since Delia (the math coordinator) suggested she attend a workshop on “way[s] to get kids excited about math” (personal interview, 12/20/2011). This workshop occurred two years previously, and so this is Torrie’s second year of using stumper problems. This year, she feels that she has better classroom routines in place for facilitating such problems; in addition, she is now drawing on—and was inspired by—language from a New York Times article on Angela Duckworth’s research on grit (Tough, 2011) (T. Blum, personal interview, 11/02/2011).

Last, also like Elsa and Everyday Mathematics, Torrie believes a significant “barrier to entry” in Math Trailblazers is its emphasis on reading (personal interview, 11/02/2011). In her view, the program is especially demanding to students for whom “languages can be difficult” (T. Blum, personal interview, 11/02/2011). Torrie explains she tries to support students with making sense of problems, not only by repeated exposure (such as with the stumper problems) but also through one-on-one interaction. Addressing students’ language-learning needs, then, is an overarching goal of Torrie’s instruction.

**Potential Contextual Influences**

Beyond the comments about Heritage Gardens, offered in Chapter 8, there are a few supplemental points to make or emphasize. Here, I compare and contrast these points with elements of Elsa’s potential contextual influences.

In particular, one notable difference between Elsa’s and Torrie’s schools involve supports for students with diverse learning needs, particularly in reading. As indicated above, Torrie’s school has a very small student:teacher ratio (PSR, 2019), while Elsa’s has a larger such ratio and a sizable guidance team (E. Mackey, personal interview, 10/28/2011). In addition, Heritage Gardens is known to attract students with diverse learning needs, particularly in literacy (PSR, 2019). In contrast, even though both schools may employ similar perspectives and approaches to meeting students’ personalized goals, they allocate resources in a different fashion. Likewise, Golden Hawk has a relatively small proportion of learners with diverse needs (E. Mackey, personal interview, 10/28/2011). Stated differently, the range of students’ needs at Golden Hawk may be wider than that at Heritage Gardens.
Next, I offer a contrasting point about Torrie’s background, as compare to Elsa’s. Torrie, unlike Elsa, has only taught in small, independent schools. The other school at which Torrie taught, prior to her arrival at Heritage Gardens, is one that has a similar philosophy and student population. Specifically, Torrie’s previous school is also known for it’s nurturing and interdisciplinary program, as well as significant amounts of teacher-autonomy. In fact, the mathematics curriculum at Torrie’s previous school, I believe, is largely teacher-created.

Last, the influence of the authors and publishers of *Math Trailblazers* is not to be discounted. In addition to receiving significant professional development support, Delia is a consequential bridge between the publishers and authors and Torrie and her colleagues. Moreover, Delia’s perspective on curriculum-use is much like Remillard’s (2005) own—namely, that teachers should use programs to suit their students’ needs but that active participation with materials necessarily means interpreting and modifying written guidance, as appropriate. Delia therefore encourages teachers to deepen their understanding of *Math Trailblazers*—to incorporate tools from among its many rich offerings—while also utilizing outside resources to address particular goals (like Marcy Cook’s problems). Torrie offers an anecdote, to portray her professional relationship with Delia, broadly, as follows:

Um, and you know, we have a math coordinator who’s great. And she is well-versed in math. So, she is also saying, “Hey, check this out.” And I’m like, “Oh, yeah?” And then I dutifully look through it, and then send it back [without incorporating Delia’s idea]. And then I was teasing her, because, I think, “Finally…[laughs a little, indistinctly speaking].”…[She’s been] asking me to do [something] for several years and [imitating Delia], “You did something with it this year!” So, there’s that, too, in teaching. That sometimes you—it’s a great idea and you just look at it and—go, “Ugh, too much.” But you know it’s good. And, thankfully, there’s someone outside you, saying, “This is good, this is good.” And, then, you say, “Oh, now I get it.” (personal interview, 11/02/2011)

Not surprisingly, as I later explain in greater detail, Torrie describes what she characterizes as “layers” of her use of *Math Trailblazers* (personal interview, 11/02/2011). In particular, she proclaims that can, in her words, “do your basic stuff and get the lesson done” (personal interview, 11/02/2011). As teachers become ready for more—similar to her explanation of working with Delia—the program offers additional, embedded ideas that become more and more apparent (personal interview, 11/02/2011).

Generally-speaking, Torrie’s goals and beliefs—including her beliefs about her students’ needs—seem to align with the program’s design. They also reflect the high-level types of adaptations she makes, such as incorporating Marcy Cook’s problems into her lessons. While
there are differences between the program’s goals and those of Heritage Gardens with regard to arithmetic facts, *Math Trailblazers* also seems to fit within the school’s context.

**Explanation of Instructional Decision-Making and Mobilized Storylines**

Torrie’s use of *Math Trailblazers* is grounded, heavily, in the way she reads and interprets the program’s resources. She takes a particular, language-focused stance that I aim to depict here. After describing how she reads *Math Trailblazers* lessons, in addition to elements of her curriculum-use, I then describe episodes of instruction.

**Torrie’s approach to preparing and reading.** To begin, Torrie explains that she generally skims *Math Trailblazers* lessons, doing so several times, rather than reading them deeply. Her goal is to obtain a high-level sense of both the language and the content expectations found within the written guidance. She describes this skimming as, perhaps, the second stage of an iterative process in her learning about the program. Indeed, Torrie admits the first time she read and taught with *Math Trailblazers*, she said to herself, “I don’t get it” (personal interview, 11/02/2011). Explaining further, she says: “It wasn’t the [sort of] language that I understood” (personal interview, 11/02/2011). From her early attempts to implementing activities, making mistakes along the way, she observes, “I saw how important it [the language] was, later on. And I’m like, ‘Wow. I’m going to have to emphasize that next year’” (T. Blum, personal interview, 11/02/2011, emphasis discerned). At that point, she read more deeply. Now that she is more familiar with the program and its specific activities, she doesn’t feel as compelled to engage in as much intense study. Such an iterative approach may represent, at least in part, what she means by the “layers” of learning about *Math Trailblazers* (T. Blum, personal interview, 11/02/2011).

Torrie’s approach to reading is largely consonant with the authors’ expectations. In the program overview, in fact, the authors write that “there is too much information in the *Teacher Implementation Guide* [TIG] and *Unit Resource Guide* for any teacher to digest all at once” (TIMS, 2004, p. 3). They suggest, then, that teachers “select portions of the manual to examine at different points” and to look for the “big picture of the program’s components and features” (TIMS, 2004, p. 3). The authors also reassure teachers that “this information will eventually become second nature to you” (TIMS, 2004, p. 3).

Torrie generally agrees. She says, “I don’t think I ever sit down and, you know, read it [fully]” (T. Blum, personal interview, 11/02/2011, emphasis noted in her speech). In Torrie’s view, the written lessons are difficult to read thoroughly, because they are so “text-heavy” (personal
interview, 11/02/2011). Asked, furthermore, how the written lessons nonetheless influence her planning work, she says, “I think I [have] made it my own at this point…” (personal interview, 11/02/2011). While the mathematical goals come from Math Trailblazers, she says that her understanding of the lessons comes from “doing it, living it” (T. Blum, personal interview, 11/02/2011). On the surface, these statements appear to suggest that Torrie may not plan carefully with Math Trailblazers. From my observations, however, I believe she has honed her understanding of the program’s goals and features, such that she has largely internalized the intended flow of lessons.

Torrie also recognizes “different questions every year” (personal interview, 11/02/2011) that she explores, gradually as needed, within Math Trailblazers. More specifically, Torrie explains that her students may ask questions, or present certain needs, she doesn’t immediately know how to address (T. Blum, personal interview, 11/02/2011). She remains introspective, though, observing that “next year, you have the same—different students—[but] same need” (T. Blum, personal interview, 11/02/2011). Consequently, she continues, “And then you go looking for it [a solution], and you go, like, ‘Oh! Look at that!’” (personal interview, 11/02/2011). She often finds solutions within the program itself, as she becomes aware of students’ challenges in learning particular content. For these and other reasons, then, Torrie likewise describes Math Trailblazers as “richer than one can absorb in one year” (personal interview, 11/02/2011). Furthermore, Torrie’s discovery-oriented style of curriculum-use is emblematic of an interpretive reading style, not unlike that described by reader response theory (Rosenblatt, 1988, cited by Dietiker, 2012, p. 43). As Dietiker (2012) explains, readers’ identities shift through their lived experiences, and so too will their understandings of a given text as it is read and reread.

Finally, asked how she uses the teacher’s guide during or after instruction, Torrie says that she refers to it “as a security blanket” but only every ten days or so (personal interview, 11/02/2011). From my line of questioning, perhaps feeling that she is being judged about her curriculum-use, Torrie reiterates her stance that the written guidance in Math Trailblazers does, in fact, drive her instruction. She clarifies, “But I feel like I haven’t given—I’ve given—a little short shrift to it. Because, really, it is what I do” (T. Blum, personal interview, 11/02/2011, emphasis noted in her speech).

In short, I believe Torrie’s own perceptions of what and how she reads may differ from an outside observers. To an outside observer, she may not appear to be reading or using the materials intently. Instead, as my preceding analysis intends to suggest, I would borrow from Remillard
and argue that the question at hand is less about what Torrie reads and more about how she reads. Rather than reading to commit specific language to memory, Torrie may be reading for a deeper understanding of the foundational mathematical knowledge (Dimension 1 of KCEM). As I noted in Chapter 8, I also suspect Torrie is reading to understand and grapple with the relative complexity of problems (Dimension 3) within Math Trailblazers. Indeed, Torrie herself indicates as much, saying that she reads the URG to understand the “sequencing” of content and for “tying into the overall—like, here are—the unit’s goals” (personal interview, 11/02/2011).

**Torrie’s lesson-enactment, tied to her goals and beliefs.** As explained, above, Torrie has a number of instructional goals; these include her desire to address students’ individual learning needs, to teach conceptually, and to support her students’ procedural fluency. Here, rather than reviewing how each of these are manifest within her instruction, I concentrate on unpacking what I perceive to be moves related to her overriding motivation.

*An overarching goal.* In particular, broadly-speaking, Torrie appears focus on promoting students’ productive struggle. From her use of the stumper problems, to her use of equivocations in the classroom, she seems to want her students to see themselves as competent problem-solvers. To reiterate from an earlier quote, Torrie observes, “problem-solving is very elusive and a lot of kids just have no idea where to begin—and that’s a problem” (personal interview, 12/20/2011). Mathematics is sometimes frustrating, she tells her students, and that is a perfectly natural feeling to have; at the same time, she says to them, “No, it’s not impossible” (personal interview, 11/02/2011). She tells them, moreover, they need to “be able to look at a problem you’ve [sic] never seen before and have to look at it cold. And to have some set of skills to think, ‘Well, what’s my first step gonna be?’” (T. Blum, personal interview, 12/02/2011). Through their problem-solving, she therefore wants her students to develop grit (personal interview, 11/02/2011).

Given this context, Torrie’s decision-making is certainly understandable. She perceives opportunities for elevating suspense within written lessons and tries to take advantage of these (as in Lesson 6–2). If specific mathematical storylines are lacking in this regard, like those at the beginning of Lesson 9–6, she supplements the written guidance by trying to throw her students curve balls. For example, she asks students to try drawing a circle divided into thirds and she prompts them to grapple with the seeming-paradox of denominators. Since problem-solving necessarily involves explaining one’s answers, she also asks them to justify their thinking often.

*Characterizing Torrie’s curriculum-use and instruction.* Near the conclusion of a follow-up interview, Torrie and I discussing differentiation strategies. During our discussion, she tells me
that she recently completed an online class—specifically on techniques of differentiation (interview transcript, 12/02/2011). But she also describes it as “horrible” and “useless” (interview transcript, 12/02/2011). Straying from the interview protocol, I probe, “Why was the?—I’m just curious….” (interview transcript, 12/02/2011). Her response, I find, is very telling and illuminates her overall stance on curriculum and instruction.

Indeed, Torrie explains that the instructor “had an outline” that was very detailed and had been shared with students in the class. It showed “how we’re going to go, how we’re going to spend the next fifty minutes” (interview transcript, 12/02/2011). Torrie tells me that she disagrees, strongly, with this instructor’s approach. She proclaims, instead, “It shouldn’t be so linear” (interview transcript, 12/02/2011). Torrie explains further:

…as teachers, you know what you have to get through. I’m trusting you’re going to get through it. Like, you did *that*. I have *this* question, but we’re going to skip [it], because you’re doing—you’re talking—about *that* now. So we’re just going to skip there and figure that out. And you glance through [your plans and observe], “OK, you got what you wanted. Here’s one more question we haven’t answered. So you weren’t didactic—is that the word? You know, where you just stuck to some formula…. You had your list…you started with the first one. OK. And then we—it—evolved. If you saw your list, you could go back. (interview transcript, 12/02/2011)

I should note that, to this point, we had not discussed how teachers could modify written lessons. This was a largely spontaneous assertion on Torrie’s part, since we had only discussed elements of Torrie’s enacted lessons. Here, in sum, she equates didactic and formulaic and she describes her frustration with the instructor for not being responsive to students’ questions and ideas (e.g., “…you’re talking about that now”) (T. Blum, personal interview, 12/02/2011). Torrie’s preferred model for instruction, furthermore, might involve a high-level “list” of topics but a flexible structure, allowing the learning to evolve organically (personal interview, 12/02/2011).

Torrie explains that the former, more organic way, is her own instructional approach. This does not mean, though, that her lessons are free-for-all experiences, she is careful to explain (personal interview, 12/02/2011). She says:

The structure is important. And things do lead naturally to other things. You can’t just skip something, I don’t think, and be as effective. And I think that—this is a thought—that, hopefully, these people [curriculum writers or researchers] have been very thoughtful about: this, to this, to this. But, sometimes, [Delia] will say, “You know, this lesson is not—now, let’s talk about that lesson.” With her 30 years’ experience, she can see it. “You don’t have to go ‘A, B, C.’ You really have to add an ‘H’ in there.” And so she can help create that. Or, if I had that many years of experience, I would be able to
Here, unlike her previous statements, Torrie embraces the idea of continuity and progression. Yet, at the same time, she observes that ordering of ideas or events need not be so strict. Therefore, her claims about the structure of mathematics appear to be related to broad principles, rather than to particular activities. To me, her statement appears emblematic of her flexible approach to narrative construction in mathematics.


Taking a step back, to see the larger picture of Elsa’s and Torrie’s work in using materials, I offer three cross-case findings. These findings synthesize ideas from the previous two sections. I focus on Elsa’s and Torrie’s reading, self-perceptions of their roles, and features of materials. I also relate these findings to commentary on productive struggle and coherence.

Reading and Interpreting Materials

First, most fundamentally, Elsa and Torrie both read curriculum materials with an eye to understanding their mathematical plots and storylines. On the one hand, Elsa read Everyday Mathematics lessons, somewhat transactionally, to find activities and problems with which her students would be successful. Otherwise, she drew on outside resources. At the same time, when reading, she modified the sequence of events or other elements of plot, to address her primary goal of lessening students’ anxiety. To a large degree, her reading of Everyday Mathematics was efferent or informational. When considering emotive aspects of mathematics classrooms, she activated Dimension U of KCEM (relative problem complexity) in order to moderate the level of challenge that her students experienced.

Torrie, on the other hand, has read Math Trailblazers lessons differently as her implementation of the program evolved. Initially, she read written guidance to get a sense of the particular language of Math Trailblazers lessons, in addition to trying to understand the big ideas and overall flow of activities. As she gained experience, though, her reading style seemed to shift. Most recently, as she read, she deepened her appreciation of the language, while also trying to understand nuanced elements of classroom activities. In this way, her reading transitioned from a more efferent to a more aesthetic mode. I characterize her recent reading as primarily aesthetic, because she looked for and found ways to enhance the suspense of mathematical plots enacted in
her lessons. This aesthetic mode of reading also aligned with her overarching goal to promote students’ perseverance in problem-solving. Torrie had a particular definition for problem-solving, too, on which she drew—namely, gathering crumbs of information and using a wide-range of broad heuristics, to tackle not-before-seen types of mathematics problems.

These modes of reading for mathematical plot represent a new finding. My characterization of Elsa’s and Torrie’s interpretation of materials in a narrative fashion adds to the nascent literature on curricular noticing (Dietiker et al., 2018). Prior to this study, in fact, the elementary teachers’ use of curriculum materials has not been tied to the deployment of mathematical storylines and plots. Instead, previous studies have focused on uncovering the mathematical plots of written materials alone (e.g., Dietiker, 2012, 2015a), the aesthetic responses of students during elementary mathematics instruction (e.g., Dietiker, 2016), or the influence of mathematical plots in secondary classrooms (e.g., Richman et al., 2018).

Roles as Instructors

Second, Elsa’s and Torrie’s differing conceptions of their roles as instructors, combined with their differing beliefs and goals about curriculum materials, at least partly explain how they read and enacted mathematical storylines and plots. More specifically, Elsa’s expressed beliefs about her role as a guide to mathematical understanding. In so doing, she engaged her students in low-stress ways, and she tried to make mathematics personally-relevant to them. These beliefs also contributed to her mode of reading and how she utilized plot-points found within her written materials.

Torrie, in contrast, saw herself primarily as an observer and recorder in the classroom. As her students worked independently and together on challenging problems, she encouraged them to strengthen their problem-solving toolkits. Torrie’s professional image, I believe, led her to identify, preserve, and supplement features of mathematical plots in a particular way. Specifically, she sought and found elements of written lessons that promoted her students’ confusion, curiosity, or surprise. When these were wanting in the materials, Torrie modified mathematical storylines to include elements of plot and enhance the suspense experienced within the enacted lesson.

This finding, about the influence of these teachers’ roles on their decision-making, builds on previous research on goals and beliefs and uptake of curricular guidance (e.g., Remillard & Bryans, 2004). This also supports findings by Richman and colleagues (2018) that mathematical
plots influence students’ classroom experiences. At the same time, this finding is novel in that it ties teachers’ self-conceptions of their work to the construction of mathematical narratives.

**The Design of Materials**

Of course, the teacher-curriculum relationship is bidirectional (M. Brown, 2009; Remillard, 2005) and embedded within school contexts (Davis et al., 2011). Therefore, I have also reported on features of curriculum materials and teachers’ contexts that seemed to promote Elsa’s and Torrie’s goals in the classroom. Likewise, some elements of mathematical storylines and plots undercut or stood in tension with teachers’ primary objectives.

In reform-oriented programs, like *Everyday Mathematics* and *Math Trailblazers*, models and representations are key metaphors for enhancing students’ conceptual understanding. Both teachers recognized and drew on representations (Dimension 2 of KCEM) as they enacted mathematical storylines and plots. Both, also, had a strong sense of the mathematical characters and settings (Dimension 1 of KCEM). These were represented in the key ideas of lessons on which they kept instruction focused (Sleep, 2012).

In the case of Elsa, though, some curricular, narrative features worked against her personal objectives. These tensions explained some of her modifications. In particular, she seemed to unravel a plot twist in Lesson 8–4, because she thought it would be too challenging for students. Consequently, she walked them through the steps of unit-conversion, rather than taking advantage of an opportunity to make sense of the underlying purpose of having two different estimation methods. Her perceptions of students’ struggles in mathematics and her prior professional learning opportunities—both from her earlier teaching experiences—may have contributed to this modification. This is similar to a finding by Davis and colleagues (2011) of a teacher’s practices with regard to an inquiry-based science curriculum—that conflicted with her goals in helping students organize their science notebooks. In Chapter 10, I argue that features of *Everyday Mathematics* could have been reworked, perhaps, in such a way that would have aligned with Elsa’s foundational goals. I offer a suggestion for ways the authors could have called attention to the purpose of having two different estimation methods and the related plot twist.

Torrie, on the other hand, used a program that included several plot twists. Even still, Torrie experienced some difficulty in responding to the symmetry-spoiler. She was challenged to give her students enough support in making sense of her question without revealing too much of the
surprise. She also may have overlooked potential opportunities for helping students engage with big-picture ideas related to ordering multi-digit numbers.

These observations collectively suggest that Torrie was skilled at interpreting the plot-points of mathematical storylines in written lessons, and yet, she may have benefitted from additional curricular guidance. In particular, she may have benefited from support in establishing the connection between curiosity-provoking mathematical questions and potentially surprising responses. Stated differently, Torrie grasped subtle cues within mathematical plots within Math Trailblazers lessons, but the consistency with which these were deployed and targeted toward the underlying concepts was occasionally lacking. This last point expresses a potential implication for curriculum authors that I take up in Chapter 10.

Finally, incorporating elements of plots and storylines into my analysis of teachers’ work proved helpful for understanding productive struggle (Hiebert & Grouws, 2007) within the context of Elsa’s and Torrie’s classrooms. Based on prior research (e.g., Remillard et al., 2011; Stein & Kim, 2009) and my own analysis of its OTLs, Everyday Mathematics has been characterized as a moderately-demanding and moderately-supportive program. This characterization connotes the general regularity sense-making opportunities coupled with guidance for teachers in implementing tasks that are considered more challenging. On the other hand, from my own analysis, I would describe Math Trailblazers as a high-demand and high-support program. (See Stein & Kim, 2009, for more information on these sorts of characterizations.) This implies that regular sense-making opportunities are also coupled with significant amount of guidance on students’ thinking or suggestions on implementing cognitively demanding tasks.

In my classroom observations, the embedded productive struggle and support of each program contributed to different sorts of implementations. Elsa maintained the level of cognitive demand found within the tasks of Everyday Mathematics lessons. At the same time, she interpreted mathematical plots in such a way that perhaps diminished the potential for confusion, curiosity, or surprise. I speculate that this influenced students’ long-term motivation to persist in problem-solving, as well as their appreciation of broad mathematical habits of mind and mathematics itself.

Torrie likewise maintained the cognitive demand of Math Trailblazers tasks, which reached toward a somewhat higher levels than those in Everyday Mathematics. As noted above, Torrie also sought to not only preserve, but also supplement, plot-points in her lesson implementation.
Her approach, I argue, aligned with her goals to promote students’ perseverance and to augment and hone the tools in their problem-solving toolkits. I suggest that a variety of factors led to these adaptations, including the inquiry-oriented culture at Heritage Gardens. Torrie also saw curriculum materials (including Math Trailblazers) as tools to be used flexibly. Therefore, she read the written lessons of Math Trailblazers to identify not only the progression of mathematical ideas and problems-complexity, but also for language in setting up and navigating activities. She consequently reordered sequences of mathematical events and omitted information, to heighten her students’ experienced suspense.

This contrast between cognitive demand and suspense suggests that written tasks and mathematical plots contribute differentially to productive struggle. These two factors, task-level and plot-suspense, may not be fully orthogonal. But my findings suggest, nonetheless, that they are distinct. Both teachers implemented tasks, as designed, but one’s adaptations lessened the suspense of implemented lessons while the other’s enhanced it.

Finally, thinking about the sub-genres into which I might locate each program, I have similarly contrasting observations. As written—and as Elsa implemented its lessons—Everyday Mathematics tended to focus on the procedures and applications of mathematical understanding. Therefore, I might consider Everyday Mathematics within a sub-genre of mathematics instructional texts that is function-focused, perhaps akin to user’s manuals. In contrast, as Torrie taught with Math Trailblazers, she took advantage of its embedded opportunities for promoting students’ curiosity and, even, their confusion. Therefore, I might consider Math Trailblazers within a sub-genre akin to mysteries.

**Implications for PDC and Coherence**

I present two additional findings, stemming directly from my cross-case analysis. In particular, first, my framework and the analysis intend to make the case that dimensions of mathematical narratives are constituent components of PDC. Elsa and Torrie both perceived elements of mathematical plots within their curriculum materials and flexibly deployed resources to achieve their goals. From reviewing Table 10, it appears likely that Torrie’s goals were closely affiliated with the intended pedagogy of Math Trailblazers, and so her PDC might be considered generally stronger. In the lessons I observed, Elsa did not take as full advantage of the opportunities within Everyday Mathematics to promote students’ sense-making in accordance with its intended OTLs.
Second, with regard to understanding coherence, my analysis portrays two teachers who implement coherent instruction as it is traditionally defined. Because of the differences in the rhythms of their lessons, my analysis also suggests that the definition of coherence needs to be deepened. In addition to Dietiker’s (2012) notion of rhythm, I refer to a definition of coherence by Richman and colleagues (2018) that involves students’ capacities to make predictions. Both, I argue, should be taken into account when discussing or assessing the coherence of teachers’ instruction and use of materials. Despite the adaptations Torrie made, reordering sequences and introducing barriers, her instruction should not necessarily be regarded as less coherent than intended. Conversely, because Elsa reordered sequences to make them more logical, her instruction should not necessarily be regarded as more coherent than intended. In short, ladder-like coherence—a strictly-linear pathway of step-by-step instruction—may not be the strongest bellwether of effective, coherent mathematics instruction.

9–5. Summary and Conclusion

In this chapter, I have tried to demonstrate that Elsa and Torrie each represent particular and important cases of curriculum-use and narrative construction. Elsa, on the one hand, enacts mathematical storylines intended to reduce her students’ levels of anxiety. This leads her to stick closely to the underlying fabula of the written lesson, reducing somewhat the potential for narrative suspense. She indicates, in addition, that she reads lessons in an efferent fashion (Rosenblatt, 1994), to discern what will be challenging for her students and to unfurl plots with high levels of difficulty. Torrie, on the other, enacts mathematical storylines to strengthen her students’ problem-solving. This leads her to seek out and implement mathematically-suspenseful instructional sequences. At this point, she reads materials for aesthetic purposes (Rosenblatt, 1994). Early on, though, Torrie read Math Trailblazers materials for efferent purposes, to better understand the nature of classroom activities.

At the same time, contextual features and elements of each program’s designs influenced both Elsa’s and Torrie’s narrative construction. In the Everyday Mathematics lessons, a procedure-centric presentation tended to exhibit fewer plot twists. Plot twists were occasionally embedded within mathematical storylines, but they surfaced in somewhat oblique ways. In the Math Trailblazers lessons, the presentation was also somewhat procedure-centric; at the same time, there were resource-centric elements—such as the occasional descriptions of students’ thinking and problem-solving strategies. These, potentially, encourage teachers to construct mathematical
narratives in more flexible ways. Plot twists were observed with greater frequency in *Math Trailblazers* than in *Everyday Mathematics*. Even still, the purpose behind plot twists in *Math Trailblazers* remained generally implicit.

Conceptualizing mathematics instruction as narrative construction allows for a number of key observations. First, distinguishing narrated from narrating events (Jackson, 1996) allows for understanding the simultaneous influence of textual- and teacher-enabled components of lessons. Each contributes to coherence, plot, and suspense, and each must be considered separately and together. This distinction is made, more particularly, between mathematical fabula and syuzhet—enabling for the identification of plot-points within mathematical storylines. Further, textual elements contributing to narrative construction include the designed OTLs of written lessons. From these, teachers may discern intended pedagogical philosophies and intended curricula, as well as plot-points and storylines themselves.

In Table 10, I summarize what I have learned about Elsa’s and Torrie’s lessons, as well as the influences on their decision-making, using my conceptual framework. I describe, specifically, the OTLs of each program, in addition to my high-level characterizations of narrative structures. I also highlight (in italics) specific elements of curricular and each teacher’s own resources—namely, aspects of their goals, beliefs, and knowledge. These highlighted elements are those that I consider especially prominent in each teacher’s instructional decision-making.

As Table 10 shows, Elsa and Torrie represent a portrait in contrasts. Despite the fact that both teach in relatively similar contexts—Elsa in a private, religious school for students in grades PK–12 and Torrie, likewise, in a private, religious school for students in grades PK-8—they each interpreted and utilized curriculum materials in largely divergent ways. Elsa searched for activities and, as noted above, sometimes unfurled plot twists. Torrie searched for big ideas, and observed her students’ thinking after introducing (or trying to sustain) plot twists.

Furthermore, referring to M. Brown’s (2009) DCE, Elsa tended to offload responsibility for identifying mathematical topics on her curriculum materials, but she adapted activities to include real-world applications and to address students’ learning needs, as perceived. Elsa also sought to
<table>
<thead>
<tr>
<th>Conceptual Framework Element</th>
<th>Elsa Mackey &amp; Everyday Mathematics</th>
<th>Torrie Blum &amp; Math Trailblazers</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Curricular Resources</strong>---</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cognitive Demand</td>
<td>PwC (Lessons 8–3 &amp; 8–4)</td>
<td>DM (Lessons 6–2 &amp; 9–6)</td>
</tr>
<tr>
<td>Voice</td>
<td>Directing action (high amounts) &amp; talking through</td>
<td>Directing action (lesser amounts) &amp; talking to</td>
</tr>
<tr>
<td>Instructional Approach</td>
<td>Blended: dialogic &amp; direct</td>
<td>Dialogic</td>
</tr>
<tr>
<td>Teacher’s Role</td>
<td>Guiding</td>
<td>Facilitating</td>
</tr>
<tr>
<td>Presentation</td>
<td>Procedure-centric; blended scripts &amp; customization</td>
<td>Procedure- &amp; resource-centric; blended scripts &amp; customization</td>
</tr>
<tr>
<td>Level of Narrative Suspense (Plot Twists)</td>
<td>Lower</td>
<td>Higher</td>
</tr>
<tr>
<td><strong>Teacher Resources</strong>—</td>
<td></td>
<td></td>
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<tr>
<td>Goals</td>
<td>Strengthen procedural fluency</td>
<td>Strengthen procedural fluency</td>
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<tr>
<td></td>
<td>Utilize real-world applications</td>
<td>Differentiate instruction</td>
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<td></td>
<td><em>Differentiate instruction &amp; reduce anxieties</em></td>
<td>Develop conceptual understanding</td>
</tr>
<tr>
<td></td>
<td>Strengthen reading &amp; writing</td>
<td><em>Promote problem-solving</em></td>
</tr>
<tr>
<td>Beliefs</td>
<td>Curriculum—activity-focused; offers real-world applications</td>
<td>Curriculum—adherent &amp; trusting; offers problems</td>
</tr>
<tr>
<td></td>
<td>Instruction—engage students; seek personal relevance</td>
<td>Instruction—observe &amp; assess students’ work; facilitate discussion &amp; problem-solving</td>
</tr>
<tr>
<td></td>
<td><em>Students—must pay attention to details; anxious about math</em></td>
<td>Students—need to develop grit</td>
</tr>
<tr>
<td>CKTM</td>
<td>Lower</td>
<td>Higher</td>
</tr>
<tr>
<td>KCEM</td>
<td>Activates Dimensions 1 &amp; 3</td>
<td>Activates Dimensions 1-3 (and, possibly, 4)</td>
</tr>
<tr>
<td><strong>Use of Resources</strong>—</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Events &amp; Plots</td>
<td>Fabula-oriented</td>
<td>Aporetic</td>
</tr>
<tr>
<td>Reading Approach</td>
<td>Efferent for activities</td>
<td>Efferent, at first, <em>for big ideas</em>; aesthetic, thereafter</td>
</tr>
<tr>
<td>Steering Instruction</td>
<td>Address key ideas &amp; connections; offload topics &amp; adapt tasks</td>
<td>Uncover meaning &amp; thinking; offload tasks &amp; adapt language</td>
</tr>
</tbody>
</table>

Table 10. Summary of findings, related to my conceptual framework.
highlight important ideas for students, like the difference between area and perimeter. In contrast, Torrie tended to offload responsibility for classroom activities onto her curriculum materials, but she adapted the sequence of events and language she used. In particular, she asked students to draw fractions of pizzas before grappling with the numerical relationships. Torrie also sought to help her students uncover big-picture ideas and to make conceptual meaning from activities.

Possible contributors to these styles of enactment are highlighted in Table 10. From my interviews and observations, it seemed that Elsa activated Dimensions 1 and 3 of KCEM in her use of curriculum, while Torrie seemed to activate Dimensions 1 through 4 in her enactment work. Torrie’s perspectives on instruction and her students—that she should observe their work in persevering on challenging problems—seemed to be powerful aspects of her belief system. Elsa’s overriding beliefs appeared to be her understanding of curriculum as offering activities (to be adapted, somewhat ironically) and of her students’ as needing to attend to details. Hence, she offered students practice problems, concentrated heavily on terminology, and asked them to read their texts carefully.

Other elements of their curriculum programs also resonated differently with each teacher. Torrie seemed especially attuned to the dialogic, facilitating model of Math Trailblazers lessons and its DM-type tasks; this may have resulted, at least in part, from her rich professional development opportunities. Elsa seemed to gravitate toward teaching the procedures of mathematics, guiding students more directly in following steps to solve PWC-type mathematics problems. Elsa sought to explain the purpose behind procedures, although these didn’t always address the most abstract mathematical notions for building deeper understanding.

Overall, though, these differences potentially mask other, important similarities that Elsa and Torrie share. In particular, both are experienced teachers drawing on reform-oriented programs. And both argue that these materials support their goals of deepening students’ conceptual understanding, even though both have different conceptions of what this means. Both perceive their programs as having limitations with regard to procedural fluency, and both considered their programs language-intensive. Of course, both Elsa and Torrie teach in schools that offer significant autonomy to teachers. Finally, both teachers implement lessons that generally maintain the expected level of cognitive demand (Stein et al., 1996; Stein & Lane, 1996) and the other OTLs embodied within their written materials. Together, this makes Elsa and Torrie important cases for understanding how teachers read and adaptations mathematical narratives.
Regardless, overall, my conceptual framework served as a useful tool for comparing and contrasting key variables involved in teachers’ construction of mathematical narratives. The framework draws on and expands prior frameworks related teachers’ curriculum use. In particular, M. Brown’s (2009) DCE shows the resources in pay in the design of mathematical narratives; my framework adds specificity in reviewing these resources. Further, my framework makes use of Remillard’s (1999) curriculum mapping framework and Sleep’s (2012) framework for steering instruction to the mathematical point. My approach adds detail to teachers’ recursive design of instruction, through steering mathematical storylines and plots; I also connect this work to conceptions of coherence. All told, my conceptual framework shows the complex and multi-dimensional nature of the construction of mathematical narratives.
CHAPTER 16. THE EPILOGUE:
REFLECTIONS, CONTRIBUTIONS, AND IMPLICATIONS

“You’ve saved our school! You’ve saved our school!” He jubilantly roared.

“We got the very highest score!” He wrote it on the board….

“Ahem! Ahem!” coughed Mr. Lowe.

“You all deserve a bow. I thus declare a holiday—it starts exactly now.

Because you’ve done so splendidly in every sort of way,

This day forever shall be known as Diffendoofer Day.


10–1. Reflections on Purpose

I undertook this study because I gradually perceived the omnipresence of narrativity in communication. As Labov and Waletzky (1967) realized, decades ago, intimate conversations are often constructed personal narratives. My own dawning awareness of this idea led me to think about layers of meaning—that what we mean is sometimes (always?) different from how we say it. And that traditional narratives—novels, plays, television episodes, and films—have complex structures, involving digressions, metaphors, wordplay. These are often clues to unlocking hidden secrets of meaning. These hidden secrets are, themselves, rewards for an astute reader’s (or writer’s!) attention. They often bridge the gap between the frailties and flaws of letters and words, as expressed, and looming but otherwise ephemeral thought.

While written language has existed for hundreds of years, considering the span of civilization, it is as yet a novel and imprecise technology. Speech, both verbal and non-verbal, experienced its dawn in a much earlier time. Regardless, from person to person and from generation to generation, the same challenge has remained and it has not abated: how to convey meaning. Both fortunately and unfortunately, there are no easy prescriptions. On the one hand, formulaic expression—as with recipes or, perhaps, academic conventions—can ease the burden by narrowing the available pathways and channeling ideas through established norms of prior thinkers. But for complex, non-linear thought, formulaic expression is an impossibility. On the other, the multiplicity of ways to conveying meaning is an invigorating exercise in creativity that
is simultaneously imbued with limitless freedom. With such freedom, though, comes the sometime-paralysis of too much choice.

Thinking about all of this, I wondered: Why do we, as a society, tend to constrain expression in mathematics text? The advantages of formulaic expression are undeniable, particularly with the economy of mathematical terminology and symbology. Yet, as it is typically construed, mathematics remains unaccessible to many. Proofs and explanations try to capture too much with too little. Consequently, students who try to enter the stream of mathematical knowledge are often left without a raft; once immersed, they yearn for the shoreline and to climb out of the water. Or else they drown. I ask the same question, likewise, with regard to mathematics curriculum materials: Why do we try, as educators, seek to constrain how our fellow teachers in expressing themselves during mathematics instruction? Why must mathematical learning be conveyed in a tightly-constructed flow—avoiding all eddies, divergences, rapids, pools, and waterfalls?

I therefore sought a type of curriculum program—reform-oriented programs—and a type of school that, I thought, would allow for insights about the variety of expressive structures available to teachers of mathematics. Elsa and Torrie were undoubtedly exemplars, reflecting the independence offered to teachers at their schools and flexible ways of using their rich instructional programs. Leslie Dietiker and her mathematical story framework (MSF), in addition to Janine Remillard and her participatory lens, became the key elements of my codebooks, through which I could interpret and appreciate the events in Elsa’s and Torrie’s classrooms. I learned, through my analysis, that narrative structures not only exist within written mathematical texts but also within classroom episodes that draw on texts. And that comparing the two, text and instruction, reveals subtleties of rhythm that, I suspect, influence students’ uptake of content.

I learned, more particularly, that suspense often lurks within the mathematics classroom. And, yet, recognizing and embracing it is easier said than done. Elsa seemed to want her students to become stronger mathematical thinkers and problem-solvers—and she made strong moves in teaching conceptually—but her outweighing concern was whether students would feel defeated by too-difficult challenge. This sometimes slowed the rhythm of her lessons and funneled instruction. Torrie, likewise, seemed to want her students to grapple with mathematics, and she was unafraid to mix-up her pitches—sometimes throwing students fastballs, sometimes curves. While her classroom was often abuzz with mathematical activity, she nonetheless received a number of vacant stares. Sometimes, to stretch my tired metaphor even further, her pitches missed
the strike zone. Sometimes her questions, intending to provoke curiosity, needed shaping and refinement in order to hit their targets.

And, finally, the ways in which Elsa’s and Torrie’s programs both helped and inhibited their delivery was unmistakable, when considered through the narrative lens. Both programs made heavy use of metaphors, running storylines in parallel to one another. Both offered opportunities for plot twists, as they related mathematical events. On the other hand, Elsa’s program offered fewer such plot twists; when they were offered, they were obscured because of a lack of a clearly articulated mathematical purpose. The process-oriented, step-by-step way in which lessons tended to be written may have limited opportunities for more expressive narrations. And Torrie’s program offered a number of plot twists, but these were sometimes buried within strings of narrative text. Despite her experience and qualifications, even Torrie found the density and volume of reading barely manageable. Like Elsa’s program, Torrie’s also obscured the mathematical purpose of its narrative elements.

As I reflect on the experiences of conducting my study, nearing the end of its own lengthy narrative, I also consider the ways in which—I hope—it may contribute to the field. In addition, I am cognizant of a number of limitations that should not be overlooked. I take up these charges in the succeeding portions of this chapter.

10–2. Contributions and Implications

I organize this presentation of hoped-for contributions (and potential implications) around two central themes. First, I offer contributions to theory and research methodology. Second, I offer more pragmatic contributions for K-12 teachers of mathematics and authors of curriculum programs. Afterwards, in the next section, I discuss various needs for future research that build upon my study.

Theoretical Contributions

I believe that my research contributes to theory in three ways. First, it broadens and deepens our understanding of teachers’ capacities and knowledge. With regard to M. Brown’s (2009) pedagogical design capacity (PDC), I argue that narrative dimensions should be considered among its constituent elements. In particular, the fabula and syuzhet—along with mathematical characters, settings, metaphors, and the like—are resources embedded within materials that teachers deploy during instruction. We might, therefore, assess the quality and character of
teachers’ and materials’ storytelling, just as we evaluate that of, say, film. This contribution also adds fine-grained empirical evidence to a body of prior scholarship on narrative thought in education (Clandinin & Connelly, 1992; Egan, 1989; Elbaz, 1991).

With regard to Remillard’s and Kim’s (2017) conception of KCEM, outlining the types of mathematical knowledge activated in using curriculum materials, my study both builds on this prior research and particularizes it. Dimensions of KCEM were identified in Elsa’s and Torrie’s narrative construction, and so, KCEM is affiliated with the narrative resources in texts and teaching. Moreover, the complexity of narratives in curriculum programs and instruction also suggests a need to supply detail to the KCEM framework. Dimension 4 is presently construed the recognition of the long-term storyline of mathematical narratives. Drawing on my study, sub-domains of Dimension 4 might include awareness of the locally-restructured mathematical fabula—in other words, within lessons, whether the mathematical syuzhet has changed the sequence of mathematical events, or whether it omits pertinent information, and to what narrative effect. Likewise, in activating teachers’ appreciation of the complexity of problems, my work shows that Dimension 3 might well be served by adding sub-components about the complexity of mathematical plots—which are accessible to students, which are more challenging, in what ways are plots scaffolded, and so on. Finally, Dimension 2 could be reframed, because connections across representations are undoubtedly metaphors in mathematical narratives. Therefore, drawing on prior work on the role of metaphor in mathematics education (see, e.g., Dietiker, 2012, p. 20), Dimension 2 might be enhanced by incorporating markers of the affordances and constraints of given metaphors as tools for mathematical thinking.

Next, my study offers support for reframing coherence. For a long time, the notion of coherence has been regarded as a necessary component of effective mathematics instruction. At the same time, it has been essentially portrayed as a given. My work, building on an earlier study by Richman and colleagues (2018), explains that coherence deserves to be regarded with a more critical eye. The definition of coherence, traditionally explained, merits additional clarification. I argue that what we mean, collectively, by coherence in presentation is not at all clear. When we consider alternative models for instruction, especially various sense-making models (Lampert, 2001) that do not follow a linear, connected pathway and do not necessarily embrace the I Do, We Do, You Do model (Pearson & Gallagher, 1983), the notion of coherence becomes especially murky. I therefore submit that Sleep’s (2012) framework for teaching to the mathematical point offers important insights about teachers’ steering work. At the same time, some of its elements
(such as developing and maintaining a mathematical storyline) might benefit from additional detail, as noted above. And, consequently, I suggest that coherence—as an overarching idea—merits additional empirical study.

Finally, building on a powerful underlying motivator for my study, my research also contributes to the idea of participating with curriculum materials (Remillard, 2005). Far from just an epistemological commitment, the participatory lens has significant implications on both theory and practice. On the one hand, the participatory lens liberates teachers from being told they must follow a program with fidelity; fidelity, considering the role of teachers’ reading and the imprecision of authors’ communication, is necessarily an unattainable goal. On the other, though, if teachers are so liberated, in what ways can teachers be understood to, and said to, draw on curricular guidance? Instructional resources are, and will undoubtedly continue to be, helpful tools that both expand teachers’ knowledge and incorporate wisdom that goes beyond their frames of experience. Therefore, I argue that participation with texts also merits expansion to include participation with narrative features of texts. This notion, I maintain, offers new ways to consider teachers’ work. We might ask different, more helpful questions, rather than simply asking: Did teachers follow the program? Instead, we might ask such complex and nuanced questions as: Did they use the program to help build a mathematical narrative that allowed for students to make predictions about potential mathematical outcomes?

Stated differently, my study suggests that participation with texts might be reframed in such a way as to tie teachers’ work, more concretely, to the underlying mathematical presentation (and its fabula-syuzet relationship). This connection would be ever-more important in an era of resource-centric programs (M. Brown, 2009). Of course, generally, this notion also raises the question of fidelity of implementation. My work, therefore, also suggests we might currently be overlooking an important dimension of fidelity. Specifically, S. Brown and colleagues (2009) have already identified two dimensions: fidelity to the literal, written steps of instruction and fidelity to the authors’ intended curriculum. They found, further, that these are not reducible to one another, nor are they necessarily orthogonal. To these dimensions, I add a possible third: fidelity to the mathematical plot. This third dimension, I note, is an affective dimension; it asks: does the instruction utilize the elements of suspense embedded within written lessons, to promote students’ curiosity, surprise, and even confusion? And, if written materials do not exhibit much suspense—as Dietiker (2012) suggests many programs do not—then in what ways can we note
that the teacher, like Torrie, may go beyond the page and thereby heighten the emotive classroom experience for students?

**Methodological Contributions**

I next explore the potential methodological contributions of my work. First, I note that my conceptual framework is certainly not novel, but it combines and deepens prior conceptions from research on curriculum-use. I therefore argue that it usefully highlights the multi-dimensional nature of narrative construction with curriculum. As I have done, I can be utilized to trace the different influences on teachers’ instruction.

That said, any framework that seeks to heed calls for added nuance around teachers’ curriculum-use—like those made by Huntley and Heck (2014) and Remillard and Kim (2017)—must also acknowledge and incorporate the broader curricular system. Within this line of research, it is impossible to avoid a systems-oriented perspective. Each element of the framework articulated by Remillard and Heck (2014) exists in tension with the others. Conversely, understanding facets of any single component, such as the enacted curriculum, must be framed along with any affiliated constructs. These mutually exert influence.

Therefore, while considering the bidirectional and nested influences of the curricular system, my small-scale study nonetheless intends to test the validity of my framework on the enacted curriculum. Emerging frameworks are useful, insofar as they illuminate aspects of instruction otherwise hidden. At the same time, the true measure of a new framework, ultimately, is determinable only via future study. Additional data, testing the usefulness of my framework in characterizing teachers’ curriculum-use and narrative construction, is needed. Even still, I am cautiously optimistic that my multi-case studies of Elsa’s and Torrie’s instruction, presented here, offer important insights about their work. In particular, my conceptual framework—and the affiliated methodology—offers a comprehensive approach for conducting side-by-side comparisons of texts and teaching.

Even more specifically, I learned that analyzing enacted lessons through a narrative lens is challenging. There are few guideposts on which to rely for conducting such analysis. I therefore argue that my approach, articulated in this thesis and particularly in Chapter 5, also contributes to the methodological literature on researching the enacted curriculum. I found, for instance, that Labov’s and Waletzky’s (1967) and Labov’s (1997) narrative phases were helpful for distinguishing among storylines in enacted lessons. Of course, Dietiker’s (2012, 2013b, and 2015a)
MSF was also instrumental for understanding and marking mathematical events and plot-points. Furthermore, my analysis of the structure of narratives—building on the theory of design arcs (Remillard et al., 2015)—is also a novel methodological contribution. Analyzing structure via design arcs allowed for insight about the high-level structures of written and enacted lessons; these enabled further, detailed investigations of classroom discourse. As I explored both written and enacted lessons, at a highly local level, I also developed and articulated tools for defining and comparing lexia. I am interested to learn in what ways researchers could use and build upon these methods to conduct additional, fine-grained analysis of classroom instruction.

**Practical Implications—for Teachers and Curriculum Authors**

The remainder of this section is organized around two, closely-related practical implications. First, what does my conceptual framework suggest about the malleability of teachers’ implementing mathematical storylines and plots? In other words, do the cases of Elsa and Torrie indicate whether patterns of enactment are fixed or could be transformed, over time? And, if malleable, what sorts of supports would be needed within curriculum materials, to help them make intentional, narrative-construction decisions? Considering both of these implications is relevant for teachers’ learning, design of curriculum materials, and our collective understanding of the relationship between resources and instruction.

**Patterns of narrative construction.** Regarding the former, I note that my sample of lessons is small. Nonetheless, the sheer number of influences on teachers’ narrative construction—indicated within my framework—indicates the potential that patterns may be set through mutual reinforcement. For example, Elsa’s goals around reducing students’ anxiety were, I found, potent influences on her classroom interactions with students. Her classroom demeanor was consistent; she regularly sought to reassure students that, together, “we’re going to get through this” (E. Mackey, personal interview, 10/28/2011). Beliefs around mathematics, viewing it in an oppositional way, as Elsa suggests, are likely generational and may be influenced by gender associations (e.g., Beilock, Gunderson, Ramirez, & Levine, 2010). Thinking of mathematics as largely formulaic may reinforce and be reinforced by a desire to avoid the messiness of problem-solving, due to prior struggle and perceived helplessness. While these beliefs and patterns may or may not have been fully present in Elsa’s instruction, daily, they appeared to rise to prominence on more than one occasion during my classroom observations. Moreover, the design of *Everyday Mathematics*—as one weighted heavily toward procedures—also seemed to reinforce the somewhat linear presentation that Elsa adopted.
In contrast, note that Torrie described an entire culture and language with regard to the problem-solving found in *Math Trailblazers*. Torrie appeared to participate actively within this culture and to use its language, rather than resist its influences. Therefore, likewise, I would argue that Torrie’s patterns of narrative construction are likely to persist. Overall, Torrie’s underlying motivation, it seems, was to empower her students to inhabit, fully, the roles of successful problem-solvers. This drove much of her classroom interaction. At the same time, she also described her own prior challenges in mathematics. And it is also clear that Torrie (and her colleagues at Heritage Gardens) have invested in surmounting these sorts of challenges—avoiding reversion to apprenticed patterns of instruction (Lortie, 1975 / 2002). Indeed, Torrie learned about and with *Math Trailblazers*, to enact different plots than what she may have experienced as a student.

**Potential transformations and implications for curriculum authors.** Regarding the second implication, the supports needed to transform teachers’ narrative construction, my comments are largely hypothetical. Nonetheless, there were elements of mathematical plots that both Elsa and Torrie overlooked that would have also aligned with their overarching instructional goals. Therefore, because of this alignment, I speculate that additional supports could have enhanced the suspense of their enacted lessons. And Torrie’s example of learning about different types of mathematical plots, even if not explicitly framed as such, is an additional example of transforming established patterns of engagement.

**Suggested revisions.** In particular—thinking of potential opportunities to support teachers’ growth—during Lesson 8–4, Elsa missed the opportunity for students to confront the seeming dissonance between their estimates and guesses. Doing so would have promoted confusion and, potentially, motivated their understanding of unit-conversion. Elsa clearly wants her students to appreciate the real-world relevance of mathematics, namely, that mathematics should be useful. In addition, she wants to address potential anxieties that students may face. I think a different type of curricular guidance, acknowledging these influences, could have supported Elsa’s narrative construction. Imagine, for example, if the text had explained:

In this activity, students are supposed to first compare their guesses and estimates, in order to realize that these numbers cannot be easily compared. They should be surprised, or maybe even crestfallen, that their guesses and their estimates are so different.

This confusion can motivate them to make sense of their results. Therefore, in order to help them understand that the need for and importance of converting among different measurement units—in order to make valid comparisons of real-world values—ask
students, first, why they think these results are so different? What are the possible explanations for what went wrong?

Collect and validate a number of different, reasonable suggestions. If no one offers “different units” as an idea, ask them to identify the units in their guess, and in their estimate, and whether they learned anything from the “Math Message” warmup activity about the sizes of different units?

This type of guidance, I submit, acknowledges Elsa’s potential concerns while offering her a pathway to implement the plot twist. In other words, Elsa is the type of teacher who might benefit from additional information about the purposes for plot twists, related to productive struggle (and how to minimize unproductive struggle).

Likewise, during Lesson 6–2, Torrie offered substantial hints to students about improving their efficiency through use of symmetry. She clearly wants her students to grapple and to recognize useful structures in mathematics. Therefore, I submit the following potential revision to Math Trailblazers, to support Torrie with making her discussion of symmetry more goal-directed and problem-solving oriented:

*Using symmetry.* Students may find covering the coat and tabulating the values of all of the base-ten pieces tedious; they may start to get frustrated. Observe whether some students are trying to cover their entire coats with base-ten pieces, particularly on both sides of the “zipper-line.” If so, this is an opportunity to help students strengthen their problem-solving skills and their understanding of geometry (from Unit 5).

As you circulate around the room, to monitor students’ working, casually ask how long they think this work will take? Interrupt students who are covering both sides of the zipper, but only after they have had time to consider your question about the time needed to complete the activity.

Ask them whether they can find a strategy to make their work more efficient? They may suggest ideas such as, “Use only ‘flats’ and ignore the ‘skinnies’ and ‘bits’” or “Cover part of the coat with 4 ‘flats,’ and then estimate how many groups of 4 will be needed to cover the rest.” They may have other ideas, as well.

These are valid strategies. Ask students whether using them will change the accuracy of their results—and, if so, is there anything else they could try? Maybe something that takes into consideration, or uses, the *shape* of the coat? If they have trouble responding, then ask “Could symmetry help?” At that point, if they are unsure what you mean, ask if they can divide the region in half and whether that would save them time. How? Why? Be sure they can explain, and ask them to share what they learned during the concluding discussion.

This aims to promote Torrie’s goals around students’ thinking. At the same time, it follows Meyer’s (2009) advice to “be less helpful” and avoiding funneling questions (Wood, 1998).
This sort of guidance is certainly an investment in print and paper, and Torrie has already observed that *Math Trailblazers* is text-rich. That said, this description could be considered canonical advice—that is, teachers could refer to it as an exemplar of a type of discussion they could learn to implement with regularity. And with curriculum materials moving toward digital delivery, this sort of guidance could be uncovered on an as-needed basis by using hyperlinks or branching. In a similar vein, consider Dixon’s (2018) distinction between *just-in-time* and *just-in-case* scaffolding. Could curriculum materials be designed to offer just-in-time support for deploying mathematical plots—particularly complex ones?

Collectively, then, this discussion highlights a possible implication of my research. Could curriculum authors intentionally use Dietiker’s MSF and the methods outlined in my thesis, to better understand the plots that emerge from their designs? And could this awareness motivate more careful attention to setting-up or building moments of suspense and surprise? In other words, could the written-enacted analyses, like the ones I’ve reported on here, support authors in modifying and revising their lessons? In this regard, several possibilities are suggested by Dietiker (2015). Future work still needs to be undertaken in this area, however.

*Reading and rereading.* Torrie’s case exemplifies work by Choppin (2011) on teachers’ learned adaptations made through program familiarity. In looking at her pattern of reading and interpreting materials, Torrie explains she read *Math Trailblazers* during her first year for, primarily, informational (i.e., efferent) purposes. Afterwards, she explains how new layers opened for her. I suggest that her later readings were for aesthetic purposes (Rosenblatt, 1994). The authors of *Math Trailblazers* offered support, encouraging teachers to read and reread the materials (TIG-3, p. 3). *Eureka Math* is another program that encourages teachers to read and reread its modules and lessons, as they prepare for instruction (Abdussalaam et al., 2015, p. 14). What other sorts of guidance could be incorporated within written lessons, to draw on reader response theory and to encourage multiple close readings of curriculum texts? In what ways could mathematical storylines and plots be highlighted to distinguish efferent from aesthetic readings? While answers to these questions remain unknown, they suggest practical applications of the MSF and my conceptual framework.

**10–3. Future Research**

I have already alluded to several, future avenues of research in the previous two sections. For example, I suggested forms of curricular guidance that might call attention to embedded
mathematical storylines and plots. Whether and how teachers engage with this sort of guidance—in addition to understanding ways of presenting such guidance effectively—certainly represent potential lines of future inquiry.

In addition, I have suggested that further research on teachers’ reading is worthwhile. Remillard (2012) has identified many important elements of programs that teachers read. Among others, Remillard and van Steenbrugge (2013) have identified elements of curriculum materials, such as margin notes, that are laid out in such a way that teachers may be likely to overlook them. What alternate formats exist, including using digital media, to call attention to the most important ideas for implementing curriculum materials effectively? Of course, I maintain that additional research on cues for mathematical storylines and plots is an essential component of such research.

Next, little is known about the different types of plots or other narrative dimensions (like metaphor) that are presently embedded within curriculum materials. I found that Math Trailblazers was constructed, somewhat, like a mystery novel—as a sub-genre of mathematics curriculum texts. In the lessons I analyzed, I noted a number of instances of equivocation. Likewise, I found a significant instance of jamming in Everyday Mathematics. What is the prevalence of such plot-points across grade-levels and within programs? Just as Everyday Mathematics tasks have been described as, primarily, PwC-type tasks, does it also express patterns of narrative structures? This is another avenue for research, one that is particularly important if my theory that teachers’ patterns of narrative construction is substantiated. In particular, if teachers’ patterns or inclinations are durable, then how these align with or contradict patterns found in programs is important for understanding a program’s fit (Stein et al., 2007).

I also undertook, but did not report on, investigations of the density and duration of mathematical storylines and plots. My approach was similar to that of Richman, Dietiker, & Brakoniecki (2016). In particular, I computed the mean number of storylines per lesson, mean number of events per storyline, mean number of storylines per event, and so on. While there appeared to be patterns across both written and enacted curricula, I chose to focus on other elements of my analysis in this report. Therefore, a future line of inquiry might involve such quantitative analysis, perhaps even offered in connection with the qualitative findings in this thesis.

As Remillard and colleagues (2014) note, furthermore, research on OTLs is ongoing. I suggest, here, that elements of mathematical plots and storylines should also be considered OTLs within curriculum materials. I have endeavored to tie narrative structures to other OTLs within
the written lessons, as well. Future research on OTLs might, therefore, identify other contributing factors within curriculum programs that aid, inhibit, or otherwise shape narrative construction.

My line of inquiry offers potential insights on teachers’ uses of curriculum materials, showing that teachers’ implementations of lessons may reflect relatively high levels of fidelity to the literal and intended curricula but variable alignment with their inherent mathematical plots and storylines. As noted above, this suggests the potential inclusion of an added dimension of fidelity to the framework offered by S. Brown and colleagues (2009), one that might help researchers and local officials determine more concretely whether and in what senses teachers are using selected programs. Stated differently, teachers’ adaptations of curriculum materials might be reviewed from another lens besides literal and intended fidelity, namely, fidelity to the mathematical plot and storyline. This also suggests that teachers’ adaptations, even if divergent from the literal or intended lesson, might still be deemed appropriate adaptations.

Finally, there is a vitally important stakeholder that is largely overlooked in my report: students. Future research should explore the role of students in narrative construction. Consider, for example, this observation from Elsa:

I think every time you plan what you’re going to do, you end up doing something completely differently, because a kid asks a question or something. Like what “Autumn” said today about the hands [being used to estimate the surface area of skin], I didn’t think that was going to come out and that seemed great to me. So I just went with it. (E. Mackey, personal interview, 03/01/2012)

Here, Elsa is describing the contribution of a student, “Autumn,” who proposed estimating the surface area of skin by counting the number of times a hand would cover a body. Autumn’s suggestion was offered even before the rule-of-thumb estimate. Saying that she “just went with it,” Elsa is explaining that she validated and connected Autumn’s idea to her subsequent description of proceeding with the rule-of-thumb estimate.

Regardless, Elsa’s observation reinforces the idea that classroom narratives are not solely determined by teachers. Of course, curriculum materials and teachers are only two of many influences on classroom activity. Students, themselves, may suggest lines of inquiry that teachers and students choose to pursue (or not). These lines of inquiry may even raise plot-points, elevating classroom suspense. Students may certainly hint at answers they know (without explaining, i.e., promising an answer), or they may suggest an answer is impossible (i.e., jamming). In fact, while not part of my dataset, I recently observed a first-grade lesson in which the teacher asked for different ways to compose the sum of eight. Students suggested sums, like 5
One student, though, practically leapt from her chair to offer the sum $4 + 2 + 2$. At her suggestion, the other first-grade students visibly gasped, laughed, and chattered for several minutes. It was energy upon which, I suggested, the teacher could have capitalized. Students at that age, very likely, hadn’t yet seen or contemplated sums with three addends.

10–4. Limitations of the Study

There were several limitations to my study. First, as explained in Chapter 5, I conducted a multi-case study with two participants. Therefore, it is difficult to generalize beyond what is presented here. I cannot make claims about all teachers of mathematics, nor about all users of Everyday Mathematics or Math Trailblazers. Nor can I make claims about these programs, generally, because different teachers may utilize different elements of these programs or adapt them in different ways. And, as noted, I only report on two representative lessons within each program. Future research should include more teachers with a greater number of lessons.

At the same time, through the design of my study, I intended to select two teachers with notable characteristics (relative autonomy, experience, etc.). Likewise, I sought to portray two lessons for each teacher that seemed emblematic of their practice—across the set of lessons I observed in the winter and spring of 2011–2012. Via this approach, I aimed to find teachers whose interpretations and adaptations of mathematical narratives was particularly robust. Therefore, in theory, Elsa and Torrie may represent somewhat idealized, or exemplar, cases among U.S. teachers (see Moss & Haertel, 2016, p. 150).

Second, my analytic approach for studying enacted lessons is largely untested. As I noted, while the MSF has been utilized—for several years—to analyze written lessons within curriculum programs, it has not been used very widely to study classroom instruction. Likewise, it hasn’t been used in comparing and contrasting episodes of written and enacted instruction. The same can certainly be said of my structural analysis using design arcs. The methods I used to determine my units of analysis, to attempt coding them in valid and reliable ways, and to extract themes from my codes are certainly inspired by Dietiker’s and her colleagues’ work—but they are largely of my own creation. Future work, using my methods and my framework, should be conducted to continue testing their utility.

Next, looking more deeply, there is also a potential bias in my analytic approach. I analyzed the written lessons, first, before studying Elsa’s and Torrie’s classroom instruction. Some might argue, then, that I identified elements of mathematical storylines and plots in classroom
instruction that may have been subtly influenced by my review of the written lessons. I attempted to mitigate this potential bias by reading and coding all of the written lessons, several times, before subsequently turning to the enacted lessons. Likewise, I reviewed the transcripts and videotapes of enacted lessons, multiple times, to consistently challenge and validate my previous rounds of coding. I made adjustments at each round—at least three times each per written and per enacted lesson—until I deemed my coding stable.

That said, as Dietiker (2012) notes, reading for narrative structure is less about identifying the “correct” interpretation under a positivist or structuralist paradigm. Instead, since readings will invariably differ by reader (just as interpretations of movies differ by viewer, it is the act of interpreting that calls attention to important potential features of lessons. In like fashion, several movie critics may see different things when watching a film, but this variance does not invalidate the phenomenologies of their experience; what they see may resonate in different ways with different film lovers. So, too, with narrative criticism of written and enacted instruction.

Finally, I hinted at, but likely paid to little attention to, the role of teachers’ knowledge in narrative construction. I noted that Torrie’s CKTM was in the higher tier of ICUBiT Project teachers; likewise, Elsa’s was in the lower tier. Again, I must emphasize that this classification does not imply that Elsa was any less successful as a teacher. Instead, I admit the possibility that Torrie’s relatively higher MKT perhaps contributed to the finer points of mathematical concepts (and plots) she sought to extract. Even still, Torrie had difficulty connecting representations of addition using base-ten blocks with a non-algorithmic strategy. For several reasons, she should not necessarily be regarded as a extreme outlier with regard to her MKT. Nonetheless, overall, future research should perhaps include another round of interviews or surveys—trying to more directly tie instances of perceived and enacted plots with teachers’ pure content and pedagogical content knowledge.

10–5. Concluding Thoughts

At the conclusion to her thesis, Dietiker (2012) offers her vision for mathematics curriculum as “a complex narrative able to stimulate imagination and curiosity in readers” (p. 247). She also references an apocryphal quote of Buckminster Fuller’s: “To change something, build a new model that makes the existing model obsolete. That, in essence, is the higher service to which we are all being called” (as cited by Dietiker, 2012, p. 247). My study and this thesis represents the higher service to which I feel I have been called. And in likeminded fashion with Dietiker, my
vision for mathematics instruction involves stimulating the curiosity of students. While she imagines novel, complex, and innovative mathematics curriculum materials, I envision sense-making classrooms, awash in activity, and electrified by problem-solving.

I see my work as drawing heavily from and building upon that of Ben-Peretz (1990) and Remillard (2005). Ben-Peretz sought to free teachers from what she perceived as the “tyranny of curricular texts” (p. 9). And Remillard sought to reconceptualize teachers’ relationship with materials as a participatory one, utilizing the best of teachers’ and curricular resources. Similarly, I have described teachers’ deviations from curricular guidance that, on the one hand, I argue, faithfully represented their beliefs and goals. On the other, their work addressed essential ideas found within their programs. And each, in their own way, acknowledged the role of emotion in the mathematics classroom. Elsa wants to reassure her students of their capability. Torrie wants to excite them as problem-solvers. In this regard, my model aims to capture at least some of the elements of their complex relationship with their curriculum materials and their students, as they incorporate affect within their classrooms.

Indeed, I have sought to identify and profile two teachers, Elsa and Torrie, who endeavor greatly to do right by their students. Their overall decision-making, I maintain, cannot be maligned. They have no ill-intents. I certainly hold tremendous, abiding respect for their work and for their obvious love of their students. The enduring question, to me, is what can we do—as a community of educators, education researchers, and policymakers—to nourish and sustain the efforts of these tireless instructors, while also promoting their students’ mathematical proficiency? In an era of high-stakes accountability and data-driven decision-making, how can we make the best of modern tools, while also fearlessly acknowledging that learning will never be purely scientific and that humans are not widgets? How can we recenter mathematics classrooms, so that they are goal-directed, as well as humanistic and lively places? Even after conducting this research, I have no firm answers. But I have an enduring faith that the truth is out there.
APPENDIX A: ICUBiT PROJECT—
INTRODUCTORY INTERVIEW PROTOCOL

Preparation

The purpose of this interview is to gather basic information about the teacher, his/her background and experience with and use of curriculum. Additionally, you may also be describing the Table of Contents Record and the Curriculum Reading Log and setting-up times for observation (depending on scheduling; see note 6 on TCR & CRL, below).

Make sure to:

1. Bring two copies of the consent form.
2. Review the interview protocol below.
3. Bring a working audio recorder.
4. Bring copies and examples of the TCR and CRL.
5. Bring appropriately colored highlighters for the CRL.
6. Plan ahead how you will introduce the TCR and CRL (p. 3 of this protocol), as there may not be enough time to do so at the end of the interview: you may want to schedule two separate sessions (one for the interview and one for explaining the TCR & CRL OR you may want to provide a general introduction in a phone call / email OR you may want to explain the TCR & CRL before the interview.

Introduction (5 minutes)

I am working with a team of researchers at Western Michigan University and the University of Pennsylvania who are interested in understanding more about how teachers read and use math curriculum materials when planning instruction. We really appreciate your willingness to participate in our project.

I’d like to audiotape the interview. However, I need you to give me formal permission to do so by signing this consent form. [Review the consent form with the teacher and sign.]

Before we start, do you have any questions for me?

First, I am going to ask you some background questions your teaching and curriculum use. Then I am going to ask you some specific questions about your current curriculum materials and you use of it.

[Note to interviewer: Generally, “curriculum materials” means all of the materials distributed with a purchased curriculum package—e.g., teachers’ lesson guide, mathematics resource guide,
etc. When noted, the questions ask for teachers’ perceptions of the teacher’s guide, specifically, or other materials that may not be included in the purchased curriculum package. You may have to explain these distinctions during the course of the interview.]
Interview (15–30 minutes)

Background

1. How many years have you taught?
2. Which grades have you taught?
3. Which curriculum packages have you used in the past?
4. What are you using now?

Current Curriculum Materials

Opinion about curriculum materials / package

5. For how long have you been using these (current) curriculum materials?
6. What aspects of the curriculum materials do you like? Dislike?
7. What do you believe is the major emphasis or the philosophy of these curriculum materials?
8. How do these ideas/curricular goals compare to your own ideas and goals?
9. [If the teacher hasn’t addressed other curriculum packages prior to now, ask this question] How do these curriculum materials compare to others you have used in the past?

How curriculum materials are used

10. What does a typical lesson look like for you?
11. How do you prepare for a lesson and how do you use the teacher’s guide in doing so?
   a. [Follow-up questions] Does the teacher’s guide help you in understanding the mathematical focus of a lesson? How does it do so, or not do so?
   b. Does the teacher’s guide help you in organizing the timeline of a lesson (i.e., what you and the students will do at given moments of the lesson)? How does it do so, or not?
12. Are there other specific things, the teacher’s guide elps you to do? Do you use the teacher’s guide during instruction? How? If not, why not?
13. Do you go back to the teacher’s guide after you teach? If so, what do you do with it?
14. Do you refer to (or consult) any other resources that are part of [the purchased curriculum package, e.g., Everyday Math] when planning or teaching a lesson? If so, How?
15. Are there other resources elsewhere [e.g., provided by the district or the department, or researched by the teacher] that you regularly consult and that are not part of the curriculum for developing your lesson plans? If so, how do you use these materials?
16. When you have a question about the curriculum or curriculum materials [i.e., “curriculum” broadly defined here, as including all purchased curricular resources and any district/departmental or other materials], what do you do? How does your school or district support your use of the curriculum [again, “curriculum” here is broadly defined]?
Introduction to the Instruments (5-10 minutes)

1. Explain the Table of Contents Record
   a. Show examples
   b. Ask if they have any questions or concerns.
2. Explain the Curriculum Reading Log.
   a. Provide appropriately colored highlighters
   b. Show examples
   c. Explain that we will send an email reminder (a few days before the observation) and that we would like to collect the pages of the CRL that are relevant for the observation before doing the observation itself.

Ask if teacher has any questions or concerns.

Set-up Times for You to Observe (5 minutes)

1. Schedule 3 times where you can come in to observe the teacher. These 3 observation times should be within the span of a ____ week period.
2. Get a sense of which lessons the teacher will be teaching during those periods.
3. Remind the teacher that you would also like to return in the spring; in the spring, you will perform another set of observations, as well as require use of the TCR/CRL, and there will be another set of interviews. You intend to contact the teacher about 3 weeks before the second round of data collection, unless the teacher needs additional notice.

[Remember to close with] Thank you very much for your participation. Please don’t hesitate to contact us/me if you have any questions in the meantime. See you soon.
APPENDIX B: ICUBiT PROJECT—
PRE-OBSERVATION INTERVIEW PROTOCOL

Preparation

The purpose of this interview is to build a rapport that makes the teacher feel comfortable to teach as they normally would. It is also to inform the teacher of what you will be doing and where you will be doing it.

Make sure to:

1. Bring two copies of the consent form.
2. Review the interview protocol below.
3. Bring a working audio recorder/video recorder.

Introduction (5 minutes)

As you know from our previous meeting I am working with a team of researchers at Western Michigan University and the University of Pennsylvania who are interested in understanding more about how teachers read and use math curriculum materials when planning instruction. We really appreciate your willingness to participate in our project and I look forward to our three observations this week.

I'd like to audiotape the interview. However, I need you to give me formal permission to do so by signing this consent form. [Review the consent form with the teacher and sign.]

Before we start, do you have any questions for me?

Interview (10–15 minutes)

1. I want to remind you that I am not here to judge how well you teach so please just be yourself. Where do you prefer I sit?
2. Is there anything that I need to know about your class or you want me to be aware of before I observe the lesson?
3. Is it okay if I walk around and look at what students are doing? What about talking with students?
4. What math topics are you going to be teaching this week?
5. Are the three lessons I will be observing related to one another? In what way?
6. Is there anything you think students will have particular difficulty with?
7. Is there anything you would like me to do before we get started?
APPENDIX C: ICUBiT PROJECT—
FOLLOW-UP INTERVIEW PROTOCOL

Follow-up interview questions are in three main categories: (A) about the observed week in general, (B) about the week in specific, and (C) about the CRL in relation to the week observed. The purpose of the interview is to investigate how the curriculum influenced their instructional decision-making. Use the listed questions and prepared clips/incidences to probe what teachers were/are thinking when they made/make a decision.

Preparation

1. Review the lessons in the curriculum
2. Watch the classroom tapes of the week
3. Pick video clips/incidences and get them ready for use during the interview
4. Review the teacher’s CRL of the week
5. Use all the listed above together to check to see if there are any specific things to come up [to check with the teacher during the interview]

Interview

(A) About the Week in General

1. I observed three lessons two weeks ago when you taught ----. How typical was the week in terms of teaching? In terms of using the curriculum materials?
2. Is there anything unusual or specific of the week that I need to know about?
3. How did you feel about the students’ responses to the lessons?
4. Do you remember anything that you did differently from what you planned? If so, what did you do and why?
5. What would you do differently next time? Why?

(B) About the CRL

1. Here is what you highlighted in a copy of the lessons (CRL).
   a. Tell me about parts you highlighted in yellow. What parts do you usually read and why?
   b. (When applicable) I noticed that sometimes you read [a section in the lesson, e.g., Ongoing Assessment] and sometimes not. Tell me about it. Is there a particular reason for that?
   c. Tell me about parts you highlighted in blue. How do you determine what to use in your lesson from the parts you read?
d. You have some parts highlighted in orange – meaning parts that influenced your planning or parts that you adapted. Tell me about these parts.

2. You also have some parts not highlighted in this copy. What parts do you usually not read and why?

3. Is this how you read the curriculum regularly? Or, is this reading very particular to this week? Why?

4. I noticed you highlighted [choose a portion/ Portions in the CRL on Codes 2 (rationale), 3 (student thinking), and/or 4 (mathematics)]. Did this help your planning or teaching? How?

5. How has your curriculum use changed over your career? [Probe specifics. Ask this question in the first follow-up interview only.]

(C) About the Week in Specific (with or without video clips)

1. As I looked at the videotapes of your lessons, I noticed [choose a few moments related to Codes 2, 3, and 4, such as emphasizing a particular mathematical idea for this question and repeat the same set of questions]. Tell me about what happened. What made you decide to do that?

2. Ask a combination of the following questions:
   a. I also noticed you skipped this part of what you planned. Tell me about what you were thinking. What made you decide to do that?

   b. I also noticed that you added this part that was not planned. Tell me about what you were thinking. What made you decide to do that?

   c. I noticed again that you used --- instead of --- that was suggested in the curriculum. Tell me about how you made that decision.

3. We just talked about ways of using curriculum, such as skipping, adding, and changing to the lesson. How typical was this?

4. Does the curriculum provide any guidance about making these kinds of altering lessons - adding or skipping parts of the lesson, or choosing options?

5. Are there any other ways you use the curriculum that are different from those you described today?

6. Is there anything else you would like to add?
**APPENDIX D: ICUBiT PROJECT— CLASSROOM OBSERVATION TEMPLATE**

<table>
<thead>
<tr>
<th>Date:</th>
<th>Teacher:</th>
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<tbody>
<tr>
<td>Time of Observation:</td>
<td>Grade:</td>
</tr>
<tr>
<td>School:</td>
<td>Observer:</td>
</tr>
<tr>
<td>Number of students present:</td>
<td></td>
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<tr>
<td>Curriculum:</td>
<td></td>
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<tr>
<td>Lesson:</td>
<td></td>
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<tr>
<td>Mathematical Focus:</td>
<td></td>
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</tbody>
</table>

**Classroom Map**

Draw a classroom map representing the way student desk/tables are set up and the location that the teacher occupies in the room.

**Classroom Description**

In a narrative, describe the classroom (1/2 page). Include any relevant information that is recorded on the board and available resources. Describe in order the major pieces of the lesson, noting the amount of time spent on each activity.
**Description of Events**

In the following table, keep track of any relevant occurrences in the class. Make sure to record the time of transitions between tasks and pay special attention to student actions and talk, as these will not be recorded on camera. Do not include any information that could be used to identify any students.

<table>
<thead>
<tr>
<th>Time (on recording)</th>
<th>Occurrence</th>
<th>Notes</th>
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**Other**

Is there anything from your observation that pertains to the ICUBiT Project that you’d like to comment on?
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