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Essays In Finance And Macroeconomics

Haotian Xiang

University of Pennsylvania, xiang_haotian@163.com

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Abstract
This dissertation consists of two chapters that address questions in finance and macroeconomics with quantitative theories.

In the first chapter, I study how financial covenants influence firm behavior by state-contingently allocating decision rights to creditors. I develop a model with long-term debt where shareholders cannot commit to not dilute creditors in the future with new debt issuances and risky investments. Creditors intervene upon violations of covenant restrictions and restructure the debt without ex ante commitment. My quantitative analysis suggests that financial covenants significantly increase debt capacity, investment and ex ante firm value by disciplining shareholders. Nonetheless, I show that lenders' inability to commit to a restructuring plan severely impairs contractual efficiency. A further tightening of covenants, relative to the calibrated benchmark, improves their value.

In the second chapter, I investigate the impact of bank capital requirements in a business cycle model with corporate debt choice. Compared to non-bank investors, banks provide restructurable loans that reduce firm bankruptcy losses and enhance production efficiency. Raising capital requirements reduces deposit insurance distortions but also deposit tax shields. As a result, firms cut back on both bank and non-bank borrowing while going bankrupt more frequently. Implementing an optimal capital ratio of 11 percent in the US produces limited marginal impacts on aggregate quantities and welfare.

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CHAPTER 1: TIME INCONSISTENCY AND FINANCIAL COVENANTS

1.1. Introduction

Financial covenants are ubiquitous in corporate debt indentures. By granting lenders the right to accelerate debt payment when borrowers fail to maintain contracted financial ratios, these contingency clauses have long been regarded by theorists as a potential remedy for agency conflicts. Extensive empirical studies show that financial covenants are frequently violated and resulting creditor interventions play a critical role in reshaping firm investment and financing going forward.\(^1\) However, it remains unclear how these ex post decision right reallocations influence ex ante firm behavior and translate into firm value quantitatively.

To this end, I develop a tractable model of financial covenants featuring dynamic management of production risk and long-term defaultable debt under limited commitment. I then quantify the ex ante implications and efficiency of these contingency clauses. The key friction that motivates the usage of covenants is shareholders’ time inconsistency associated with long-term debt. When able to freely adjust investment and financing without having to repurchase outstanding debt, shareholders end up diluting legacy lenders. On the financing side, shareholders have the temptation to keep issuing new debt in order to further extract tax shields even if leverage and default risk are already excessive.\(^2\) On the investment side, they find it beneficial to invest in risky assets when the downside will be primarily borne by legacy lenders. Without covenants, shareholder behavior falls completely out of lenders' control once debt is in place. Debt pricing incorporates forecasts of future dilution, and thereby requires shareholders to compensate lenders ex ante.

Financial covenants provide lenders with the right to accelerate debt repayment when the financial ratio restricted by covenants is violated. Shareholders have to comply with whatever acceleration plan lenders find optimal ex post and bear the debt restructuring costs,

\(^1\)See a review by Roberts and Sufi (2009b). Appendix 1.7.1 provides an example of financial covenants from the SEC filing.

\(^2\)See theoretical analysis of this dilution problem by e.g. Admati, DeMarzo, Hellwig and Pfleiderer (2018) and in a similar sovereign debt context by e.g. Aguiar, Amador, Hopenhayn and Werning (forthcoming).
before they can retain the control over investment and financing going forward.

There are two channels through which shareholder behavior is disciplined by debt restructuring. First, the realization of a debt restructuring reshapes shareholder behavior going forward. When legacy debt is reduced after an acceleration, shareholders are forced to internalize a larger fraction of the impact of their investment and financing choices going forward. Second, shareholder behavior is also affected by the anticipation of future contractual violations. When finding debt restructuring privately undesirable, shareholders will slow down debt issuances and risky investment in order to avoid breaching covenants. In that case, precautionary motives alleviate time inconsistency even when violations do not actually realize ex post.

Lenders cannot commit to a restructuring plan. Their ex post incentive to accelerate co-moves negatively with economic fundamentals. First, when default risk is trivial and thus debt trades at a large premium, unreceived interests are safe enough to dissuade lenders from any principal acceleration. Covenant violations in these scenarios are not followed by a credit amendment. Second, when default risk is moderate, long-term lenders would like to discipline future shareholders even at the cost of forgoing some of the default premia. A debt relief takes place and the surplus from a mild debt acceleration is shared between the current equity and debt holders.\(^3\) Lastly, for a highly risky firm whose debt trades at a large discount, lenders accelerate outstanding debt aggressively upon violations, which not only generates a large total surplus but also forces current shareholders to repay some of the debt above the market price.

Quantitative applications of my model deliver the following key results. First of all, currently adopted financial covenants improve the ex ante firm value, before any investment and financing take place, by around 1%. Compared to a net benefit of debt of 4%, such an improvement suggests that the value of discipline imposed on shareholders by existing

\(^3\)As will become clear later, debt relief is possible in my model because of i) the nonlinearity of default problems and ii) the collective action of lenders in debt restructuring. See related discussions by e.g. Aguiar, Amador, Hopenhayn and Werning (forthcoming).
covenants significantly outweighs the expected incurrence of restructuring costs. In simulations, for the median firm who carries positive amount of debt and risky assets, covenants contribute to 1.5% of the total firm value. There is considerable variation across time. About 10% of the time, when fundamentals suddenly deteriorate and thus shareholders have a strong tendency to dilute legacy lenders, the presence of covenants contributes to more than 2% of firm value. The inclusion of covenants increases leverage ratio by 10% and investment rate by 20%. The average default frequency drops sharply by 30%.

A key finding of my analysis is that the value of covenants is severely impaired by lenders’ lack of commitment to enforce a certain restructuring plan. Debt relief, despite being always valuable once realized ex post, makes shareholders ex ante less cautious about debt issuances. Such an undesirable anticipation effect turns out to be quantitatively strong under the existing covenants and significantly exacerbates the dilution problem in equilibrium. A value improvement is witnessed if lenders could tie their own hands, which is of course time-inconsistent. Interestingly, resource losses incurred during the restructuring constitute a powerful punishment for shareholder misbehavior ex ante. My analyses suggest that such a “burn the boats” strategy ends up being significantly valuable.

I also investigate whether a recalibration of covenants is able to enhance their effectiveness in addressing time inconsistency. My analysis lends support to a tightening of covenants. There exits a hump-shaped relation between shareholder welfare and the covenant threshold. As covenants get tighter, a larger fraction of violations will be expected in low-risk states where discipline generated by realization and anticipation effects both becomes relatively lenient. Under an excessive tightness, the value of discipline fails to justify frequent incurrences of restructuring costs, thus making the access to covenants ex ante undesirable. My model suggests that welfare is maximized at a threshold under which covenants will be breached with a quarterly frequency of 4%, more than twice as often as what is currently observed.

This paper bridges two large strands of literature in finance and macroeconomics. First,
there is a growing literature that tries to study the time inconsistency problem of shareholders when financing themselves through long-term debt (e.g. Gamba and Triantis, 2014, Kuehn and Schmid, 2014, Crouzet, 2016, Dangl and Zechner, 2016, Gomes, Jermann and Schmid, 2016, DeMarzo and He, 2017 and Admati, DeMarzo, Hellwig and Pfleiderer, 2018)\(^4\). I contribute to this literature by modeling a potential solution to the debt dilution problem—financial covenants. A novel insight my model highlights is that the commitment problem also appears on the lender side, which plays an important role in shaping the efficiency of these contingency clauses.


Gamba and Triantis (2014) investigate the role of financial covenants when shareholders cannot commit to leverage. In their model, violations introduce an exogenous and state-
invariant restriction in shareholders’ choice. On the contrary, I model lenders’ intervention ex post and thus endogenize the violation consequences. By doing so, this paper not only uncovers the commitment problem of lenders but also is able to quantify the tradeoff underlying the calibration of covenants. Hatchondo, Martinez and Sosa-Padilla (2016) experiment with several exogenous non-financial covenants and quantify by how much they can reduce the sovereign dilution problem.


This paper proceeds as follows. Section 1.2 presents a stripped-down version of my model for illustrative purposes. I will then integrate it into a richer model in Section 1.3. Section 1.4 presents calibration and model assessments. Main counterfactual experiments are carried out in Section 1.5. Section 1.6 concludes.

1.2. Financing with Covenants

In this section, I incorporate covenants into an infinite-horizon long-term debt model with no capital and only independent and identically distributed (i.i.d.) shocks. Such a minimal-style environment allows me to transparently (analytically to a certain extent) illustrate the core mechanisms of the paper – shareholders’ time inconsistency and how the inclusion of covenants reshapes their behavior. It will then integrated into a richer model which I take to quantitative analyses.
1.2.1. Shareholders’ Problem

The model is in discrete time and recursively represented, where all agents are risk-neutral. Each period, the firm receives a profit of $Y + z$, where $z$ is i.i.d. with a cumulative density function (c.d.f.) $\Phi(z)$ and support $[-\bar{z}, \bar{z}]$.

I make two tractability assumptions, which are standard in the risky debt literature\(^6\), about rollover arrangements – the firm retires its debt stock $b$ at a rate of $\lambda$ and new debt is issued pari passu\(^7\). My modeling of financial covenants closely follows the structure adopted by industry with a real-world example presented in Appendix 1.7.1. More specifically, it requires maintaining a minimum market capitalization to debt ratio of $\kappa > 0$. When it is violated, lenders have the right to require shareholders to repay an additional $\alpha(b, z) \in [0, 1 - \lambda]$ fraction of the principal at par, which will be endogenously determined ex post by lenders in a restructuring.\(^8\) As a result, the effective debt maturity becomes state-contingent if $\alpha(.) > 0$ for some $(b, z)$.

---

\(^6\)C&I loans are typically not fully collateralized. Modeling collaterals explicitly may introduce an additional asset encumbrance problem, which can exacerbate the debt dilution. See a related theoretical analyses by Donaldson, Gromb and Piacentino (forthcoming).

\(^7\)With bankruptcy costs, imposing a strict seniority rule is insufficient to shelter lenders away from the commitment problem. A related discussion is provided by Bizer and DeMarzo (1992) and Admati, DeMarzo, Hellwig and Pfleiderer (2018). Quantitatively, the moment shareholders choose to default optimally, firm value already becomes fairly small. Making old debt more senior does not necessarily make them much safer.

\(^8\)Restricting $\alpha(.) \geq 0$ is motivated by the fact that financial covenants typically give lenders the right to accelerate but not to extend credit (See the example in Appendix 1.7.1). It is important to notice that a smaller repayment today leads to a larger repayment tomorrow, Therefore, the nature of debt acceleration is different from lenders writing down debt, which could prevent costly defaults. For the quantitative model in Section 1.3, renegotiation is costly, meaning the no-default case cannot be restored anyway.
Conditional on repaying $\alpha b$ additionally, shareholder value is given by:

$$V_e(b, z; \alpha) = (1 - \tau)(Y + z) - [(1 - \tau)c + \lambda + \alpha]b + J\left((1 - \lambda - \alpha)b\right),$$

where $\tau$ and $c$ are respectively tax and coupon rates. $J(.)$ represents the continuation value of equity.

Financial covenants are violated if:

$$\frac{V_e(b, z; 0)}{b} \leq \kappa,$$

where $\kappa$ is the violation threshold. Protected by limited liability, shareholders choose not to repay if their value becomes weakly negative after going through the debt restructuring:

$$V_e(b, z; \alpha) \leq 0.$$

The above two inequalities can be represented by two state-dependent cutoff values of the $z$ shock – $z_v$: $V_e(b, z_v; 0) = \kappa b$ and $z_d$: $V_e(b, z_d; \alpha) = 0$ – that stand respectively for covenant violation and default. It will be true both in the data and in my numerical analyses that covenant violations take place more frequently than defaults, i.e. $z_v \geq z_d$. The continuation value can thus be expressed as:

$$J(\tilde{b}) = \max_{b'} (b' - \tilde{b})Q(b') + \beta E\left[\int_{z_v}^{\bar{z}} V_e(b', z'; 0)d\Phi(z') + \int_{z_d}^{z_v} V_e(b', z'; \alpha')d\Phi(z')\right],$$  (1.1)

where $\tilde{b}$ stands for the legacy debt and $Q(.)$ the debt pricing schedule. The first term represents the gain from selling new debt while the latter two denote the discounted shareholder value next period. It reveals the fact that shareholders only internalize the price impact of their issuances on new debt, i.e. $b' - \tilde{b}$, rather than the entire stock $b'$.

---

9In reality, financial covenants typically impose restrictions on ratios such as the EBITDA-to-debt, networth, interest coverage, leverage, etc. The ratio I choose can be considered as a combination of all these different ratios.
1.2.2. Lenders’ Problem

Again, conditional on \( \alpha \) and no default, debt value is given by:

\[
V^b(b; \alpha) = (c + \lambda + \alpha)b + Q\left(b'\left((1 - \lambda - \alpha)b\right)\right)(1 - \lambda - \alpha)b,
\]

where \( b'(.) \) is equity holders’ debt policy – the solution to equation (1.1).

Nothing is recovered upon defaults, which will be relaxed in the full model. There is perfect competition at the moment of lending. As a result, lenders’ zero-profit condition pins down the debt pricing schedule \( Q(.) \):

\[
Q(b')b' = \beta\mathbb{E}\left[\int_{z_e}^{z_d} V^b(b'; 0)d\Phi(z') + \int_{z_d}^{z_d} V^b(b'; \alpha)d\Phi(z')\right]. \tag{1.2}
\]

1.2.3. Debt Restructuring

After covenants have been violated, lenders are granted all the control right and decide on how much principal to accelerate so that their collective value given repayment can be maximized. The payment acceleration policy \( \alpha(b, z) \) is determined by:

\[
\alpha(b, z) = \arg\max_{\tilde{\alpha} \in [0, 1-\lambda]} V^b(b; \tilde{\alpha}), \quad \forall z. \tag{1.3}
\]

Conditional on repayment, equity holders retain full control over the firm and are able to reissue debt in the market. Equity policy and debt pricing functions, embedded in \( V^b(.) \), are taken as given in equation (1.3). As will be illustrated later, the stickiness of debt caused by time inconsistency prevents such a one-time acceleration from being completely unwound by re-issuance. As a result, debt dynamics of the firm going forward are altered.

Acceleration makes a default more likely if shareholders find an additional principal retirement costly. Given the existence of liquidation costs, one might therefore think that lenders can do even better by imposing a more lenient acceleration for some \((b, z)\) if a default could
have been avoided. However, as long as equity holders decide to default when continuation value equals zero, which is what I have assumed, those scenarios cannot be an equilibrium because lenders always have a strict incentive to extract a little bit more. In other words, adopting an alternative restructuring problem where \( \alpha(b, z) = \arg\max_{\tilde{\alpha}} \mathbb{1}_{V^e(b, z; \tilde{\alpha}) > 0} V^b(b; \tilde{\alpha}) \) yields identical results.\(^{10}\)

Equation (1.3) also reflects an important distinction between debt adjustment through restructurings and that via market operations. When shareholders buyback their debt in the market, each lender is able to hold out her portion until the others’ have been retired and price has gone up. In equilibrium, the buyback price has to make the marginal lender indifferent and thus all benefits from risk reduction will be ultimately captured by lenders.\(^{11}\) If holdouts are possible when lenders face an acceleration at par, the maximization problem in equation (1.3) should be subject to individual lenders’ participation constraints: \( Q(b'(1 - \lambda - \tilde{\alpha})b) \leq 1. \) However, there is no need to worry about such constraints because debt restructurings require the collective action of lenders. Principal acceleration is a material amendment to the credit agreement, which in practice requires unanimous lender consent. Any lender’s holdout shall cause a failure of the restructuring and thus is inferior if the outcome improves average debt value. In other words, the “holdout effect” breaks down.

### 1.2.4. Equilibrium and Characterizations

**Definition 1.** A Markov Perfect Equilibrium is given by (i) equity holders’ policy function \( b'(.) \) with associated value function \( V^e(\cdot) \), default set \( D \) and covenant violation set \( H \); (ii) a debt pricing function \( Q(.) \) and associated value function \( V^b(.) \); (iii) a payment acceleration function \( \alpha(.) \) such that (i) given \( Q(.) \) and \( \alpha(.) \), equity holders’ decisions are optimized; (ii) given equity holders’ decisions and \( \alpha(.) \), \( Q(.) \) and \( V^b(.) \) satisfy debt holders’ zero profit condition in equation (1.2); (iii) given \( V^b(.) \), \( \alpha(.) \) solves the restructuring problem in equation

\(^{10}\)If I assume shareholders continue to operate the firm when their value equals zero, the only ex ante meaningful modification it will bring to the model is that lenders now expect a smaller default loss. Overall, how I treat these cases makes a fairly small quantitative difference: the fraction of defaults that can be avoided by a more lenient acceleration is about 2% in the simulated sample generated from the full model.

\(^{11}\)Pitchford and Wright (2011) investigate the role of individual holdouts in an extensive-form sovereign debt bargaining model.
Following the literature, I put my focus on Markov Perfect Equilibria. Since my goal here is to characterize the equilibrium rather than to establish a general theory, I assume the existence of an interior optimum and the validity of first-order conditions. Section 1.2.4.1 discusses the long-term debt problem without financial covenants, i.e. imposing $\alpha(\cdot) \equiv 0$ or equivalently $\kappa = -\infty$. I introduce them in later sections and illustrate how the problem is reshaped. Section 1.2.4.2 focuses on the realization effect given equilibrium functions. Section 1.2.4.3 discusses how the anticipation effect influences equilibrium policies and firm values.

1.2.4.1. Long-term debt and shareholders’ time inconsistency

To understand the role of time inconsistency, let’s start with a case where shareholders borrow long-term debt but can commit to future issuances. Here, I preserve the assumption that there is no commitment to repayment in order to isolate the issuance friction created specifically by debt maturity. Consider a “Ramsey” planner who maximizes the un-levered shareholder value by choosing a borrowing stream $\{b_t\}$ conditional on no previous default. The allocation has to satisfy shareholders’ optimal default rule and the zero-profit condition for lenders. As derived in Appendix 1.7.2.1, her first-order condition is given by:

$$
\beta E_t \left[ \int_{z_{t+1}}^{\tilde{z}} \tau \phi(z_{t+1}) \right] = -b_{t+1} \frac{\partial Q_{t+1}}{\partial b_{t+1}},
$$

where

$$
\frac{\partial Q_{t+1}}{\partial b_{t+1}} = -\beta E_t \left\{ \left( c + \lambda \right) + (1 - \lambda)Q_{t+2} \right\} \phi(z_{t+1}) \frac{\partial z_{t+1}}{\partial b_{t+1}}.
$$

The left-hand side (LHS) of equation (1.4) is the present value of tax shields while the right-hand side (RHS) the impact of issuances on the price of the entire debt stock $b_{t+1}$. Last-period debt $b_t$ is not a state variable and there is no endogenous debt persistency. Debt choices adjust only in response to the potential evolution of investment opportunities,
if exists. Move to equation (1.5). Because the choice of $b_{t+1}$ has no effect on $b_{t+2}$ and thus $Q_{t+2}$, the impact of issuances is reflected only by an increase in the contemporaneous default probability: $\partial z^d_{t+1}/\partial b_{t+1}$. The following proposition summarizes these results:

**Proposition 1** (Ramsey Equilibrium). *When able to commit to future issuances, shareholders internalize the impact of issuances on the entire debt. Equilibrium debt choice at any point in time does not depend on the amount of legacy debt.*

*Proof.* See Appendix 1.7.2.1

**Corollary 1.** *Policies in a long-term debt Ramsey equilibrium are time-inconsistent.*

*Proof.* See Appendix 1.7.2.2

In contrast to the Ramsey equilibrium, without commitment on issuances, shareholders re-optimize the debt structure period by period conditional on the legacy debt $\bar{b}$. The first-order condition of shareholders in the competitive equilibrium without covenants is given by:

$$
\beta E \left[ \int_{z'_d}^{\bar{z}} \tau c d\Phi(z') \right] = -(b' - \bar{b}) \frac{\partial Q(b')}{\partial b'},
$$

(1.6)

where

$$
\frac{\partial Q(b')}{\partial b'} = -\beta E \left\{ \left[ (c + \lambda) + (1 - \lambda)Q(b'') \right] \phi(z'_d) \frac{\partial z'_d}{\partial b'} 
- \int_{z'_d}^{\bar{z}} \left[ (1 - \lambda)^2 \frac{\partial Q(b'')}{\partial b''} \frac{\partial b''}{\partial b'} \right] d\Phi(z') \right\}.
$$

(1.7)

Compared to (1.4), the RHS of equation (1.6) becomes the marginal impact of issuances on the price of new debt $b' - \bar{b}$. Under the same pricing schedule, shareholders would like to borrow more than what the Ramsey planner would do simply due to a partial ignorance of the negative price impact. Of course, in a rational expectation equilibrium, the pricing schedule does adjust as lenders price the dilution problem in. The price impact in equation
Partial derivative \( \partial b'' / \partial \tilde{b}' \). Debt in equilibrium becomes history-dependent and endogenous dynamics emerge. Shareholders slowly adjust their debt even without any ad-hoc issuance cost.

**Proposition 2** (Competitive Equilibrium). *When unable to commit to future issuances, shareholders internalize the impact of issuances on the new debt. In a long-term debt competitive equilibrium, shareholders with more legacy debt find it marginally beneficial to carry a larger amount of debt, i.e. \( \partial b' / \partial \tilde{b} > 0 \).*

*Proof.* See Appendix 1.7.2.3

Since shareholders fail to internalize the price impact of issuances on past debt demand, they effectively compete against themselves across time in issuances. With equilibrium debt choices deviating from the Ramsey allocation, they lose some of their rents from the ex ante perspective.\(^{12}\)

A natural question to ask here is that whether trading one-period debt, i.e. \( \lambda = 1 \), implements\(^{13}\) the Ramsey allocation since shareholders are also forced to fully internalize price impacts period by period. After the limited issuance commitment has been neutralized, there still exists no commitment to repayment. Thus different debt maturities, even all with full issuance commitment, generate distinctive allocations as the rollover arrangement influences shareholders’ strategic decisions on whether to bear rollover gains/losses, i.e. \([Q(b') - 1] \lambda b\), at each point in time. Because there are only i.i.d. shocks, I am able to get some relatively clear results:

**Proposition 3.** A long-term debt Ramsey equilibrium where debt is always sold at (be-
\(^{12}\)DeMarzo and He (2017) analyze a long-term corporate debt model in the continuous time, where all the monopoly rents are dissipated (Coase, 1972). Imposing fixed-length periods enables the seller to make limited commitment about the path of durable stock (Stokey, 1981).

\(^{13}\)I am able to show that the Ramsey allocation can be implemented by the following incurrence covenant. Shareholders making a debt choice \( b' \) with legacy debt \( \tilde{b} \) are forced to transfer \( \tilde{b}(Q(b^{RE}) - Q(b')) \) to existing lenders, where \( b^{RE} \) is the debt choice in the Ramsey Equilibrium. Such a required compensation eliminates the incentive of shareholders to over-borrow. Apparently, writing down an “optimal contract” of this sort is infeasible in reality as it requires the knowledge of the solution to the Ramsey problem. As will become clear in the next section, payment acceleration in high-risk states captures some flavor of this “optimal contract” as shareholders in these states are punished by covenants (in expectation) when issuing debt aggressively.
low/above) par yields an identical (higher/lower) firm value compared to a short-term debt competitive equilibrium with an identical coupon rate.

Proof. See Appendix 1.7.2.4

Figure 2: Long-Term Debt and Time Inconsistency. Notes: This figure presents the impact of time inconsistency on equilibrium firm policies and values. The Ramsey policy is solved via constructing an equivalent recursive problem. Parameter and functional choices are $\beta = 0.99$, $\tau = 0.3$, $\xi = 0.25$, $Y = 0.012$, $z = 1$, $\phi(z) = 3(1 - z^2)/4$, $c = 1/\beta - 1 + 0.0019$ and $\lambda = 1/25$ for long-term debt cases.

In Figure 2a, I plot equilibrium debt policies in respectively the Ramsey equilibrium, the long-term debt competitive equilibrium without covenants, and the one-period debt model from numerical solutions. Consider an un-levered firm. In the Ramsey case, it immediately issues debt up to approximately 0.2. In contrast, debt policy is upward sloping without issuance commitment (consistent with my marginal characterization in Proposition 2), resulting in the firm being initially under-leveraged. Debt will be gradually accumulated and ultimately the firm levers up more aggressively compared to the Ramsey planner under the chosen parameters. Meanwhile, with the policy function being upward sloping, a larger $\tilde{b}$ means a higher default risk in the future. Figure 2b plots continuation firm values: $J(\tilde{b}) + Q(\tilde{b}'(\tilde{b}))\tilde{b}$. For the case of long-term debt without covenants, it is downward sloping. Of particular interests are un-levered firm values, representing shareholder welfare under different financing arrangements, where the negative impact of lack of commitment becomes
apparent. A relatively high coupon rate makes one-period debt more appealing than the Ramsey benchmark, although the difference here is small.

1.2.4.2. Realization of debt restructuring

Now I introduce financial covenants. This section characterizes the realization of a covenant violation given equilibrium policy and pricing functions. The ex post impact of debt acceleration can be decomposed into two parts. First, it transfers resources between debt and equity holders (redistribution effect). More specifically, holding shareholders’ choice of $b'$ fixed, an acceleration forces them to retire debt at face value, which might be different from the reissuing price in the market. Thus an additional loss/gain of $\left[Q(b') - 1\right]\alpha b$ is generated. Second, when debt becomes history-dependent without commitment, a shifting in the state $\hat{b}$ changes shareholders’ future debt choices and thus a firm’s continuation risk and value (efficiency effect).

On the margin, shareholders are affected by the redistribution effect. To see this, differentiate the equity value with respect to $\hat{\alpha}$ and then utilize the envelope condition to substitute out the derivative of the value function:

$$\frac{\partial V^e(b, z; \hat{\alpha})}{\partial \hat{\alpha}} = -b - b\frac{\partial J(\hat{b})}{\partial \hat{b}} = b\left[Q(b'(\hat{b})) - 1\right],$$  \hspace{1cm} (1.8)

where $\hat{b} = (1 - \lambda - \hat{\alpha})b$. Shareholders find the acceleration of an additional unit of debt beneficial if they are able to reissue it at a higher price in the market and thus get a rollover gain.

Different from shareholders, lenders are marginally affected by both the redistribution and efficiency effects. Again, consider the derivative of debt value before repayment:

$$\frac{\partial V^b(b; \hat{\alpha})}{\partial \hat{\alpha}} = b\left\{1 - Q(b'((\hat{b}))\right] - \hat{b}\frac{\partial Q(b')}{\partial b} \frac{\partial b'}{\partial \hat{b}}\right\}. $$  \hspace{1cm} (1.9)

The first term in the bracket is the transfer realized through the additional unit of debt accelerated, mirroring what is laid out in equation (1.8). The second term captures how
the value of remaining debt is marginally impacted because of the changes in legacy debt. If the upward-sloping debt policy and downward-sloping pricing function preserve after the introduction of covenants, i.e. $\frac{\partial b'}{\partial \bar{b}} > 0$ and $\frac{\partial Q(b')}{\partial b'} < 0$, the second term is positive. Debt holders in this case always benefit from the efficiency effect as the un-retired debt becomes safer.

\begin{align*}
\text{(a) Inactive violation (} b = 0.03) \\
\text{(b) Debt relief (} b = 0.15) \\
\text{(c) Debt punishment (} b = 0.23)
\end{align*}

Figure 3: State-Contingent Debt Restructuring. Notes: This figure illustrate for three different pre-violation debt levels the debt restructuring outcomes and equity/debt payoffs. Parameter and functional choices are $\beta = 0.99, \tau = 0.3, \lambda = 1/25, \xi = 0.25, Y = 0.012, z = 1, \phi(z) = 3(1 - z^2)/4, c = 1/\beta - 1 + 0.0019, \kappa = 2$.

Lenders decide on how much debt to accelerate based upon the signs and relative strengths of these two effects. Figure 3 reveals a key determinant – the pre-violation debt level. Figure 3a presents a restructuring taking place when debt is little and new debt will be traded above par even without any acceleration. From lenders’ perspective, since default risk is
fairly low, any further reduction via raising $\tilde{\alpha}$ becomes second order and is dominated by
the debt-to-equity transfer. They are uniformly worse off and choose not to take any action.
Covenant violations in these states end up being inactive, meaning no credit amendment
takes place.

As the pre-violation debt level increases, acceleration starts to take place. Figure 3b demon-
strates a scenario where debt is moderate and will be re-traded at par without any accel-
eration. Even though an acceleration still generates an undesirable redistribution to share-
holders, debt holders are willing to implement it as the potential efficiency gain becomes
relatively pronounced.\footnote{These cases arise because the default risk is nonlinear in leverage, typical for this class of models. The
sensitivity of debt price with respect to leverage grows much faster than the debt price itself in response to
an increase in leverage.} Debt relief is achieved in which equity and debt holders share
such an efficiency gain. Going back to my discussions in Section 1.2.3, such a scenario is
made possible by the break-down of the “holdout effect”. For instance, although at $\tilde{\alpha} = 0.2$
the re-traded debt price is already above 1, further acceleration can still be sustained since
individual lenders’ holdouts are not feasible.

Finally, in Figure 3c, existing debt is abundant and will be traded far below par without
acceleration. Lenders in this case have a strong desire to accelerate. First, the redistribution
effect flips its sign and starts to benefit lenders in the beginning of acceleration. Second,
with the default risk being severe, the efficiency gain becomes fairly pronounced. Lenders
keep raising $\tilde{\alpha}$ until the firm’s continuation default risk becomes so small that on the margin
the benefit from further risk reduction is offset by the transfer to equity holders. Equity
holders end up receiving a debt punishment because they have to retire debt at a higher
average price.

Although higher pre-violation debt levels lead to stronger accelerations, firms’ legacy capital
structures turn out to be identical after active restructurings. Readers might have already
noticed that the $\tilde{b}$ equating (1.9) with zero does not depend on $b$. The following proposition
formalizes this result:
Proposition 4. Define restarting legacy debt $\tilde{b}^R = \arg\max_{\tilde{b}} [Q(\tilde{b}(\tilde{b})) - 1] \tilde{b}$. Consider a firm violating covenants with debt $b$. If $b \leq \tilde{b}^R/(1 - \lambda)$, no debt will be accelerated, i.e. $\alpha = 0$. If $b > \tilde{b}^R/(1 - \lambda)$, shareholders are required to pay down the legacy debt to $\tilde{b}^R$, i.e. $\alpha = (1 - \lambda) - \tilde{b}^R/b$.

Proof. The proof requires rewriting equation (1.3) in $\tilde{b}$. □

Remark 1. When $c$ is small enough such that $\forall b', Q(b') < 1$, a full acceleration follows every violation, i.e. $\tilde{b}^R = 0$. If further $\kappa = \infty$, covenants implement short-term debt.\textsuperscript{15}

Remark 1 describes an extreme case where covenants implement one-period debt. A more general characterization of violation consequences requires knowing global properties of $b'(\tilde{b})$ and $Q(b')$ since accelerations create jumps in state variables. For such a purpose, I numerically demonstrate a capital structure restart in Figure 4 with less extreme parameters. The upward slope of debt policy and downward slope of pricing schedule preserve after the introduction of covenants. Consider a firm with a stationary capital structure. Following a violation, shareholders have to repay more and get the firm restarted with a legacy debt of $\tilde{b}^R$. Leverage experiences a persistent decline while continuation firm value rises.

1.2.4.3. Anticipation and ex ante implications

Moving from ex post to ex ante, the anticipation effect becomes no less important – equilibrium policies are influenced by the anticipation of future violations. Shareholders behave differently with the presence of covenants even if no violation is realize ex post. The key determinant is again the state-dependent restructuring payoff to shareholders. Covenant inclusions discourage debt issuances conditional on the pricing schedule when leverage is high. In those states, shareholders find debt punishment painful and thus will try to avoid breaching covenants. Because the violation threshold is written on inverse leverage, the way to achieve such a goal is to issue new debt less aggressively. In contrast, covenants

\textsuperscript{15}Recall Proposition 3. If debt is always traded below par, one-period debt is inferior to long-term debt with full commitment. In other words, when $c$ is low enough, shareholders who can commit to future issuances become worse off when creditors impose state-contingent acceleration.
encourage debt accumulation in states where leverage is moderate because shareholders are likely to get a debt relief upon back shocks.

Because the key friction is shareholders’ temptation to over-lever, the value of covenants stems from the leverage discipline they introduce. Debt relief, being always value enhancing ex post, can instead harm commitment production—shareholders become less cautious about leverage adoption when anticipating their possibility. Sometimes, that additional debt can be costlessly retired through a realization of relief, meaning shareholders successfully extract extra tax shields without paying a default loss. However, for the non-relief paths, the firm steps into the high-leverage region more rapidly and therefore experiences a default earlier.

Firm value will be enhanced by covenants only if the realization and anticipation effects together create strong discipline in future debt issuances. Factors that affect the conditional distribution of future states, such as properties of covenants and the state in which they are evaluated, all matter.\footnote{Although a realization of debt punishment always increases firm value conditional on repayment, it triggers default under certain shocks and is thus not necessarily ex post beneficial.} Figure 5 shows the solutions to a model without covenants and

\footnote{Features of the debt contract to which covenants are attached also play a role. Unreported numerical}
two with covenants but under different violation thresholds. Covenants increase firm value under high $\tilde{b}$'s as they significantly reduce future leverage no matter whether a violation actually realizes or not. Moving towards the left, debt reliefs kick in and make the conclusion ambiguous. In the benchmark case where $\kappa = 2$, including covenants overall increases debt capacity and firm value at $\tilde{b} = 0$. The initial borrowing and un-levered firm value increase. However, in the other case, the expectation of debt relief quantitatively dominates debt disciplines and thus the inclusion of covenants introduces a large dilution risk from the ex ante perspective. Initial borrowing and firm value are lower than their counterparts in the covenant-free model.

To clearly know the sign and magnitude of the ex ante value of covenants as well as their implications for equilibrium firm behavior, one needs to quantify the strengths of the realization and anticipation effects. With all the demonstrated forces in mind, I now turn to the quantitative part of the paper.

Experiments suggest that under small coupon rates, covenants are always value enhancing. Under such parameterization, debt generally trades below par and only the disciplining force is operative ex ante.
1.3. The Full Model

This section presents the full model for quantitative analyses. In addition to the model outlined in the last section, I introduce (risky) investment, persistent shocks as well as a more realistic violation/default treatment. I will carry most of the notations and economize on descriptions of ingredients that have already appeared.

1.3.1. The Environment

The firm operates two types of capital with different risk profiles: high \( k_H \) and low \( k_L \). A constant returns to scale (CRS) technology generates the following profit:

\[
\sum_{i \in \{L,H\}} (e^x + \nu_i z) k_i
\]

where the persistent income follows a standard AR(1) process:

\[
x' = (1 - \rho) \bar{x} + \rho x + \sigma \tilde{\epsilon}, \tilde{\epsilon} \sim N(0,1).
\]

\( z \) shocks, which can be interpreted as extraordinary items, capital quality shocks, accounting noises, etc., are i.i.d.

Capital \( k_H \) has a larger exposure to i.i.d. \( z \) shocks: \( \nu_H = \nu \times \nu_L \) where \( \nu > 1 \). I normalize \( \nu_L = 1 \) without loss of generosity. The p.d.f. of \( z \) is symmetric with support \([-\bar{z}, \bar{z}]\), and thus the risk choice is mean-preserving. Total capital stock \( k = k_H + k_L \). Investment is given by \( k' - (1 - \delta) k \), where \( \delta \) stands for depreciation rate. Adjusting capital incurs a cost:

\[
\Psi(\cdot) = \sum_{i \in \{L,H\}} \frac{\gamma}{2} \left( \frac{k'_i - k_i}{k} \right)^2 k,
\]

where a capital reallocation friction is embedded in the quadratic form – buying one type of capital and selling an equal amount of the other is costly.

Define \( k_H \) share \( s = k_H/k \) and further a firm’s exposure to \( z \) shocks \( a(s) = s \nu + (1 - s) \).
Conditional on $\alpha$, the value of the firm to its shareholders is given by:

$$V^e(b, k, s, x, z; \alpha) = (1 - \tau)[e^x + a(s)z]k - [(1 - \tau)c + \lambda + \alpha]b$$
$$+ J((1 - \lambda - \alpha)b, k, s, x).$$

A firm violates financial covenants when $\frac{V^e(b, k, s, x, z; 0)}{b} \leq \kappa$. A default happens if equity value becomes negative, i.e. $V^e(b, k, s, x, z; \alpha) - fk \leq 0$ where $f$ is the resource cost associated with debt restructurings. Continuation value $J(.)$ is given by

$$J(\tilde{b}, k, s, x) = \max_{b', k', s'} Q(b', k', s', x)[b' - \tilde{b}] - [k' - (1 - \delta)k] + \tau\delta k - \Psi(.)$$
$$+ \beta\mathbb{E}\left[ \int_{\tilde{z}_v}^{z_v} V^e(b', k', s', x', z'; 0)d\Phi(z') + \int_{\tilde{z}_d}^{z_d} [V^e(b', k', s', x', z'; \alpha') - fk]d\Phi(z') \right],$$  \hspace{1cm} (1.10)

where $z_v(b, k, s, x)$ and $z_d(b, k, s, x; \alpha)$ are cutoffs representing violation and default.

Move to the debt holders. Conditional on an additional payment of $\alpha b$, their value is:

$$V^b(b, k, s, x; \alpha) = (c + \lambda + \alpha)b + (1 - \lambda - \alpha)b$$
$$\times Q\left( (1 - \lambda - \alpha)b, k, s, x), k'(1 - \lambda - \alpha)b, k, s, x), s'(1 - \lambda - \alpha)b, k, s, x), x \right),$$

where $b'(.)$, $k'(.)$ and $s'(.)$ are equity policies solving equation (1.10).

Upon default, lenders recover the contemporaneous output, $(1 - \tau)[e^x + a(s)z]k$, together with the un-depreciated capital $(1 - \delta)k$. However, a liquidation cost of $\xi k$ is incurred. Compared to the simplified model, a positive default recovery creates a dilution problem – for a given amount of recovered resources, having more creditors means that each one of them ends up with less.
As usual, debt holders’ zero profit condition pins down the pricing schedule:

\[
Q(b', k', s', x)b' = \beta \mathbb{E} \left\{ \int_{z'_1}^{z'_2} V^b(b', k', s', x'; 0) d\Phi(z') + \int_{z'_3}^{z'_4} V^b(b', k', s', x'; \alpha') d\Phi(z') \right\} 
+ \left\{ \int_{z'_5}^{z'_6} [(1 - \tau)[e^{z'} + a(s') z'] + (1 - \delta) - \xi] k' d\Phi(z') \right\}.
\] (1.11)

For the convenience of carrying out counterfactual analyses on alternative covenants, I consider a more general debt restructuring problem where equity holders might have some bargaining strength. Acceleration schedule \(\alpha(b, k, s, x, z)\) is given by

\[
\alpha(b, k, s, x, z) = \arg\max_{\tilde{\alpha} \in [0, 1 - \lambda]} \theta V^e(b, k, s, x, z; \tilde{\alpha}) + (1 - \theta)V^b(b, k, s, x; \tilde{\alpha}).
\] (1.12)

where \(\theta\) stands for the bargaining power of equity holders in restructurings.

It is easy to show that the property stated in Proposition 4 extends to the full model: There exists a restarting legacy debt level \(\tilde{b}^R(k, s, x)\) from which I can back out \(\alpha(b, k, s, x, z)\). In this richer environment, \(\tilde{b}^R\) for a firm is no longer state-invariant but depends on its capital stocks, \(k\) and \(s\), as well as persistent cash flow \(x\). Again, covenant violators with identical asset-side characteristics will have to pay off part of their debt such that shareholders regain controls with the same amount of legacy debt.

1.3.2. Discussions of New Features

The full model is scale-invariant with one exogenous state variable \(x\) and two endogenous ones – legacy leverage \(\tilde{\omega} \equiv \tilde{b}/k\) and \(s\). It is achieved by the construction of production technology, adjustment costs, violation/default thresholds, and restructuring objectives. The equilibrium concept is again Markov-perfect. (Equilibrium definition of the full model and an linearity proof can be found in Appendix 1.)

1.3.2.1. Risk shifting and debt overhang

The existence of \(k_H\) introduces a risk-shifting motive. When shareholders fully internalize the impact of their asset choices, a heavy investment in \(k_H\) can hardly be beneficial as it
simply increases expected default losses.\textsuperscript{18} It is no longer the case if shareholders optimize over \( s' \) with the presence of some legacy debt. Even though a huge \( s' \) destroys the total firm value, when the legacy leverage \( \bar{\omega} \) becomes high enough relative to \( x \), shareholders can find it privately profitable because of an increase in the equity value (Leland, 1998). Lenders are sensitive to the downside risk and debt value falls sharply in consequence.

A limited commitment problem arises on the asset side. At the time of borrowing, long-term debt holders have to price shareholders’ incentive not only to over borrow ex post but also to raise the firm’s exposure to \( z \) shocks under an excessive leverage. Of course, the link between these two commitment problems is not just one-way: A heavy allocation on \( k_H \) also exacerbates the conflict of interests on the financing side. Consequences of shareholders’ incentive to extract further tax rents become more detrimental to debt values under a riskier technology because defaults are more likely ceteris paribus. Because of such feedback, equilibrium debt policies without commitment become more nonlinear.

The rate of total investment, \( i \equiv k'/k - 1 + \delta \), is pinned down by capital adjustment costs as the production technology is CRS. As long as shareholders are not able to commit to repayment, which is also the case in the Ramsey equilibrium and short-term debt model, a debt overhang problem naturally emerges. Specific to this competitive setting is the interaction between commitment friction and corporate investment along various dimensions.

Consider a firm violating covenants with a large amount of debt and \( k_H \). As legacy leverage falls due to acceleration, shareholders start to internalize a larger fraction of the price impact. Persistent declines in \( \omega \) and \( s \) alleviate expected default losses and in turn facilitate investment. However, when adjustment costs prevent a perfect and instant capital reallocation from happening, the drop in \( k_H \) might not be completely unwound by the increase in \( k_L \). Depending on quantitative strengths of these two forces, the rate of total investment

\textsuperscript{18}It can be shown analytically that \( s' \equiv 0 \) if i) there is no capital reallocation friction, i.e. \( \Psi(.) = \frac{\gamma}{2}(\frac{k' - k}{k})^2k; \) and ii) debt is one-period. In addition to the risk-shifting channel, equity holders would like to diversify their assets to a certain extent because of the capital adjustment costs. However, such a motivation enhances firm value and creates no agency problem on its own.
can be temporarily suppressed or boosted in the short run by violations. On contrary, in full commitment scenarios, future investment behavior only responds to shocks to $x$ and will thus not be altered by a shift in states.

From an unconditional perspective, if covenant inclusions can partly resolve the over-borrowing and risk-shifting problems, one should expect the under-investment to be milder on average. Firm values will be further enhanced. Overall, the existence of risk choices and debt overhang makes commitment more treasured compared to a model with only capital structure problems.

1.3.2.2. Time variation of commitment frictions

Upon an introduction of persistent shocks, the severity of commitment problem starts to exhibit rich dynamics. First, time inconsistency becomes relatively more detrimental when a deterioration in $x$ is expected. For instance, when inspecting the numerical solution to the full model, I find that for a given $s$, the equilibrium debt policies are much steeper and the under-leveraging is more severe for smaller $x$’s. Such “counter-cyclicality” roots deeply in the intrinsic property of defaults, which also underlines the feedback between risk shifting and over-borrowing that I discussed in the last section. Defaults are highly nonlinear and require complementary workings of large negative shocks and high leverage. A high $x$ means not only a handsome profit in the current period but more importantly also a huge continuation equity value. At that time, a default is unlikely even with leverage and asset risks further inflated. When $x$ worsens, shareholders’ tax shield extraction becomes more harmful from the lenders’ perspectives. Furthermore, shareholders are more willing to increase the volatility of income and shift the risk to lenders. Therefore in those scenarios, lack of commitment to future investment and financing becomes more value destructive.

Second, endogenous firm characteristics—$\omega$ and $s$—also influence the severity of the commitment problem. When existing leverage is high, shareholders will only internalize a small fraction of the default loss. Conflict of interests in those scenarios becomes relatively critical. Meanwhile, for shareholders who have already invested heavily in $k_H$, the convexity
of capital adjustment costs means that it is less costly for them to adopt an extremely high asset riskiness and thereby destroy debt holders’ value once such a strategy becomes appealing.

Overall, the severity of commitment frictions varies across time together with fundamentals of the firm. In a given state, it depends on the conditional distribution of future cash flows as well as balance sheet characteristics $\omega$ and $s$.

1.3.2.3. Restructuring cost

In reality, debt restructurings are costly. One can interpret such costs in a similar fashion to those incurred during payment defaults. There are direct charges by attorneys and accountants for rewriting contracts and in addition some indirect losses such as reputation damages caused by violation disclosure and the opportunity cost of time (Beneish and Press, 1993). In the full model, these are all summarized by the term $f_k$ in the covenant violation region of equation (1.10). A one-time incurrence of $f$ does not affect motions of state variables and therefore dynamics of non-defaulted firms. It is always ex post undesirable for shareholders and the firm as a whole since some resources are simply taken away.

However, the anticipation of $f$, similar to that of debt punishments, contributes to disciplining shareholders’ extrapolative behavior. Overall, $f$ is ex ante value destructive if the commitment problem is far from being severe and thus the anticipation effect fails to generate enough merit to justify painful realizations. For instance, in a short-term debt model where shareholders fully internalize the consequences of their choices, possible incurrences of $f$ in certain parts of the state space shall reduce ex ante welfare. In contrast, an enhancement in firm value may be witnessed if time inconsistencies are quantitatively severe enough.

1.4. Quantitative Exploration

I now proceed to quantitative analyses of the full model. This section mainly presents my calibration and model assessments.
1.4.1. Calibration

I solve the model with value function iterations (details can be found in Appendix 1.7.3.3). I adopt a quadratic approximation to the probability density function of $z$ shocks:

$$\phi(z) = \eta_0 + \eta_1 z^2.$$ 

After imposing $\phi(\bar{z}) = 0$ and $\int_{-\bar{z}}^{\bar{z}} \phi(z) = 1$, $\bar{z}$ is sufficient to pin down $\eta_1$ and $\eta_0$.

To make the model quantitatively more realistic, I impose an upper limit on the degree of risk shifting: $s' \leq \bar{s}$. One can interpret this boundary as other restrictions on firm behavior that are not in my model: regulations, career concerns, etc. Consistent with the claim made by Leland (1998), without reallocation friction, optimal $s'$ becomes bang-bang as the marginal return to risk shifting turns out to be increasing. Bounding $s'$ helps deliver quantitatively reasonable i.i.d. shocks without resorting to an unrealistically large capital adjustment cost.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.99</td>
<td>discount rate</td>
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<tr>
<td>$\delta$</td>
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<td>depreciation rate</td>
</tr>
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<td>tax rate</td>
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<td>$1/\lambda$</td>
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<td>maturity</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>$1/2$</td>
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<td>$\theta$</td>
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<td>bargaining power</td>
</tr>
<tr>
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<td>$\bar{x}$ process</td>
</tr>
<tr>
<td>$\bar{z}$</td>
<td>0.9</td>
<td>$z$ distribution</td>
</tr>
<tr>
<td>$c$</td>
<td>$1/\beta - 1 + 0.001538$</td>
<td>coupon rate</td>
</tr>
<tr>
<td>$\xi$</td>
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<td>bankruptcy cost</td>
</tr>
<tr>
<td>$f$</td>
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<td>restructuring cost</td>
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<tr>
<td>$\gamma$</td>
<td>3</td>
<td>capital adjustment cost</td>
</tr>
<tr>
<td>$\nu, \bar{s}$</td>
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<td>risk shifting</td>
</tr>
</tbody>
</table>

Table 1: Parameters

The model is calibrated under a quarterly frequency with all parameters listed in Table 7. I first directly parametrize discount rate $\beta = 0.99$, depreciation rate $\delta = 0.025$ and
corporate tax rate $\tau = 0.3$. Repayment rate is set to $\lambda = 1/12$ so that debt maturity is equal to the median value reported by Chava and Roberts (2008) for Dealscan loans. Admittedly, public firms typically have some corporate bonds, which tend to have a longer maturity. Choosing a 3-year maturity provides a conservative estimate of the covenant value.\textsuperscript{19} Tightness of equity-to-debt covenant $\kappa$ is close to what has been documented by Chava, Fang and Prabhat (2015). Again, as covenants typically shift all the discretion to lenders after they have been violated, I set $\theta = 0$.

All parameters left are calibrated. The boundary of idiosyncratic shocks $\tilde{z}$ targets at default probability. Coupon rate $c$ is set such that on average debt goes out at par in simulations, consistent with how bank loans are observed. Bankruptcy cost $\xi$ is set to match median book leverage. Restructuring cost $f$ is identified via covenant violation frequency. As a standard practice, capital adjustment cost curvature $\gamma$ is set to match the investment volatility. Parameters governing income dynamics and risk-shifting behavior—$\rho$, $\sigma$, $\nu$ and $\bar{s}$—jointly target pre- and post-violation differences in net debt issuances and investment rates between violators and non-violators. The mean of persistent income $\bar{x}$ is chosen to match the median investment rate.

1.4.2. Simulation Results

Table 2 compares unconditional sample moments generated from simulated series and their data counterparts. Inspired by a large body of empirical research on covenant violations, discussions in this section will be focused on results in that regard.

1.4.2.1. Covenant violators

Why are financial covenants violated? There are two plausible explanations. First, violations might simply be driven by bad luck – the likelihood goes up after negative shocks to $x$ even though $\omega$ and $s$ are not particularly high. For example, if covenants are imposed in a short-term debt model, this explanation will lie behind every violation. Second, it can

\textsuperscript{19}In equilibrium, lenders rarely accelerate all the debt. Therefore my results will largely preserve even if there are some corporate bonds. Moreover, in many cases, bonds become acceleratable when covenants are violated. See the example provided in Appendix 1.7.1.
Moments | Model | Median | 10% | 90%
--- | --- | --- | --- | ---
debt/assets* | 0.279 | 0.245 | 0.013 | 0.810
volatility of debt/assets | 0.029 | 0.100 | [0.013 | 0.554
investment/assets* | 0.022 | 0.024 | [0.005 | 0.083
volatility of investment/assets* | 0.024 | 0.022 | [0.006 | 0.084
market-to-book | 0.999 | 1.934 | [1.068 | 10.020
volatility of market-to-book | 0.314 | 0.693 | [0.144 | 8.511
income/assets | 0.038 | 0.018 | [−0.279 | 0.057
volatility of income/assets | 0.013 | 0.039 | [0.012 | 0.360
covenant violation frequency* | 0.017 | 0.015
default frequency* | 0.002 | 0.003

Table 2: Unconditional Moments. Notes: This table presents unconditional moments calculated from simulated sample (model median) and data (median, 10% and 90% percentiles). I simulate 2,000 firms for 1,000 quarters. Data sample spans from 1996Q1 to 2011Q4. * denotes moments used in calibration. Details about data construction are presented in Appendix 1.7.4.

also be a boom-bust story with the “leverage ratchet effect”. After a sequence of positive shocks, leverage has been piled up. At that moment, having experienced an erosion of \( x \), shareholders respond slowly in terms of buying back their long-term debt because of their resistance to transfer resources to debt holders. Meanwhile, shareholders also start to find it beneficial to increase the loading on \( k_H \) when leverage becomes excessive relative to \( x \). Under a high \( \omega \), a large exposure to \( z \) shocks and a lower level of \( x \), violations also become more likely.

Figure 6: Pre-violation Dynamics. Notes: This picture presents dynamics of state variables before covenant violations and compare them to those of the whole simulated sample. I simulate 2,000 firms for 1,000 quarters.
Which scenario is more typical for violators is a quantitative question, the answer to which depends on both the dynamics of exogenous uncertainty and how endogenous behavior responds over time. More specifically, the likelihood of receiving a sequence of bad shocks and the degree of asymmetry in leverage adjustment both matter. Figure 6 plots the dynamics of state variables averaged across firms before violating their financial covenants, which lends support to the boom-bust explanation. As the persistent income erodes after a long boom, shareholders reduce their debt slowly. Right before violations, violators still have a larger amount of debt on their balance sheet although their persistent cash flow is already lower than their counterpart’s. They also double their positions in $k_H$. The reversal in persistent income is arguably mild—less than 1/2 of a standard deviation.

In Table 3, I provide some supporting empirical evidence. Violators tend to have higher income-, net debt issuance- and investment-to-assets ratios compared to others two years before the actual breaches. These differences either revert or disappear when approaching violations. For moments that the model misses their magnitudes, correct signs are produced.

<table>
<thead>
<tr>
<th>Moments</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>vio</td>
<td>diff</td>
</tr>
<tr>
<td>$t - 1 \rightarrow t$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>income/assets</td>
<td>0.035</td>
<td>−0.002</td>
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<tr>
<td>net debt issuance/assets*</td>
<td>−0.002</td>
<td>−0.002</td>
</tr>
<tr>
<td>investment/assets*</td>
<td>0.022</td>
<td>−0.000</td>
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<tr>
<td>$t - 8 \rightarrow t - 7$</td>
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<td></td>
</tr>
<tr>
<td>income/assets</td>
<td>0.039</td>
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<tr>
<td>net debt issuance/assets</td>
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</tr>
<tr>
<td>investment/assets</td>
<td>0.025</td>
<td>0.003</td>
</tr>
</tbody>
</table>

Table 3: Pre-Violation Moments. Notes: This table presents pre-violation differences between covenant violators (vio) and non-violators (diff = violators−non-violators). I simulate 2,000 firms for 1,000 quarters and calculate the means. Empirical moments are fixed-effect regression coefficients and associated 95% confidence interval calculated from data between 1996Q1 and 2011Q4. * denotes moments used in calibration. Details about data construction are presented in Appendix 1.7.4.

1.4.2.2. After covenant violations

How firms change their behavior after covenants are violated? Table 4 reports firm statistics averaged over two quarters after violations. Covenant violators stay with a lower income-to-
assets ratio as \( x \) is persistent. Moreover, they experience declines in net debt issuances and \( k_H \) share. Because of these risk-reduction measures, their default probability going forward becomes much smaller compared to non-violators albeit \( x \) is relatively inferior. The rate of total investment drops.\(^{20}\)

In the last column, I utilize my model to isolate the causal impacts of the realization of a violation – how firm dynamics would have been different if covenant violators were not forced to retire an additional \( \alpha(\cdot) \). (Recall that the payment of \( f \) should have no impact.) Within the model, I am able to fix the state variables right before violations and subsequent shocks, and thus not bothered by the fact that distinctions between violators and non-violators are also affected by differences in economic fundamentals.

<table>
<thead>
<tr>
<th>Moments</th>
<th>Model ( \text{vio} )</th>
<th>( \text{diff} )</th>
<th>Data ( \text{diff (FE)} )</th>
<th>5%</th>
<th>95%</th>
<th>Violation impact</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t + 1 \rightarrow t + 2 )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>income/assets</td>
<td>0.035</td>
<td>-0.003</td>
<td>-0.004</td>
<td>[-0.010, 0.001]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>net debt issuance/assets*</td>
<td>-0.012</td>
<td>-0.012</td>
<td>-0.011</td>
<td>[-0.013, -0.010]</td>
<td>-0.005</td>
<td></td>
</tr>
<tr>
<td>investment/assets*</td>
<td>0.012</td>
<td>-0.010</td>
<td>-0.011</td>
<td>[-0.011, -0.009]</td>
<td>-0.002</td>
<td></td>
</tr>
<tr>
<td>( k_H ) share</td>
<td>0.004</td>
<td>-0.001</td>
<td></td>
<td></td>
<td>-0.005</td>
<td></td>
</tr>
<tr>
<td>default frequency</td>
<td>0.000</td>
<td>-0.002</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Post-Violation Moments. Notes: This table presents post-violation differences between covenant violators (\( \text{vio} \)) and non-violators (\( \text{diff} = \text{vio} - \text{diff} \)). The last column presents causal impacts of violations on endogenous behavior. I simulate 2,000 firms for 1,000 quarters. Empirical moments are fixed-effect regression coefficients and associated 95% confidence interval calculated from data between 1996Q1 and 2011Q4. * denotes moments used in calibration. Details about data construction are presented in Appendix 1.7.4.

In all, 97.8% of violations in the simulated sample are active.\(^{21}\) My results suggest that covenant violations are responsible for around half of the decline in net debt issuances. The deterioration in \( x \) are equally powerful in explaining such a decline. This piece of result is quantitatively in line with what has been concluded by Roberts and Sufi (2009a) with

\(^{20}\)Without tabulating a separate table here, I find empirically that covenant violations happening when market-to-book ratios are low tend to be associated with bigger drops in the net debt issuance and the investment rate. These results are consistent with the implications of my model.

\(^{21}\)Roberts (2015) documents that more than 75% of covenant violations in his sample lead to restructurings. Roberts and Sufi (2009a) analyze voluntary reports of covenant violation outcomes in a random sample of 10-K or 10-Q filings and conclude a lower bound of 32.2%.
a difference-in-difference design. The capital reallocation friction turns out to be quantitatively large enough to generate a drop in investment rate, even though the overhang problem has been alleviated by declines in debt flows and risk shifting. The economic magnitude is a bit smaller compared to Chava and Roberts (2008). The erosion of investment opportunities accounts for the majority of the decline in investment. Consistent with the evidence in Gilje (2016), violations cause disengagement in risk-shifting activities.

1.5. Counterfactual Experiments

This section carries out counterfactual experiments to isolate the ex ante implications of covenants. In Section 1.5.1 I quantify the impact of existing covenants \((\kappa = 0.5 \text{ and } \theta = 0)\) on shareholder values and unconditional firm behavior. Section 1.5.2 examines how shareholder welfare responds to alternative calibrations of covenants.

1.5.1. Impact of Covenant Inclusions

I first focus on the impact of covenants on ex ante firm value, or welfare, and unconditional moments. In the second section, I present how the covenant value evolves across time.

1.5.1.1. Welfare and unconditional moments

Table 5 demonstrates the impact of covenant inclusions by comparing results from the benchmark model and those from a covenant-free long-term debt model, i.e. \(\kappa = -\infty\). The welfare metric I use across different models is the shareholder value under zero debt, zero risky capital and mean level of persistent income, i.e. \(j(\tilde{\omega} = 0, s = 0, x = \bar{x}) \equiv J(0, k, 0, \bar{x})/k\).

Let’s first focus on columns 1 to 3. If shareholders have access to the covenants specified in the benchmark analyses, their welfare improves from 0.9532 to 0.9657. Covenants indeed

\(^{22}\)For tractability, the model abstracts from frictions such as equity issuance costs, which might help produce a larger short-run negative impact of acceleration on investment. Incorporating additional costs equity holders have to bear ex post in restructuring is likely to further inflate the covenant value because of a strengthening in the anticipation effect.

\(^{23}\)Under my choice of \(f\), shareholders never find it beneficial to voluntarily propose a debt restructuring to lenders. As a result, the results will be identical if I instead compare i) a long-term debt model with costly debt renegotiability where \(\theta \equiv 1\) and ii) one where covenants are present and \(\theta = 0/1\) when violated/unviolated.
Moments w/o covenants with covenants
<table>
<thead>
<tr>
<th></th>
<th>w/o recall</th>
<th>recall</th>
<th>benchmark</th>
<th>$f \equiv 0$</th>
<th>$\alpha(.) \equiv 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>welfare</td>
<td>0.9532</td>
<td>0.9582</td>
<td>0.9657</td>
<td>0.9529</td>
<td>0.9671</td>
</tr>
<tr>
<td>debt/assets</td>
<td>0.2511</td>
<td>0.2589</td>
<td>0.2789</td>
<td>0.2470</td>
<td>0.2809</td>
</tr>
<tr>
<td>investment/assets</td>
<td>0.0175</td>
<td>0.0191</td>
<td>0.0218</td>
<td>0.0172</td>
<td>0.0222</td>
</tr>
<tr>
<td>$k_H$ share</td>
<td>0.0089</td>
<td>0.0087</td>
<td>0.0053</td>
<td>0.0089</td>
<td>0.0056</td>
</tr>
<tr>
<td>default frequency</td>
<td>0.0023</td>
<td>0.0022</td>
<td>0.0017</td>
<td>0.0019</td>
<td>0.0019</td>
</tr>
</tbody>
</table>

Table 5: Impacts of Covenant Inclusions and Decompositions. Notes: This table presents simulated medians and shareholder welfare $j(0, 0, \bar{w})$ for alternative models. Column 1 reports results for a model without covenants where coupon rate is fixed to that in Table 7. Column 2 reports results for a model without covenants where coupon rate is recalibrated so that debt on average goes out at par ($c = 1/\beta - 1 + 0.00228$). Column 3 reports results for the benchmark model with covenants. Column 4 reports results for a model with covenants where $f$ is fixed to 0. Column 5 reports results for a model with covenants where $\alpha(.)$ is fixed to 0. I simulate 2,000 firms for 1,000 quarters.

produce commitment. In terms of magnitudes, such a 1.31% gain in shareholder/firm value is arguably significant considering at that point firms hardly have any default risk.\(^{24}\) Even if I increase the coupon rate for the covenant-free model such that debt still on average goes out at par, shareholder value still improves by approximately 0.78% after the introduction of covenants.

Because of the alleviation of time inconsistency, lenders are less worried about the leverage ratchet effect and risk shifting. Prices adjust and in equilibrium firms are able to borrow more on average. Firm default probability falls sharply even though leverage becomes higher. As the expected default loss shrinks, the debt overhang problem is also alleviated, leading to a higher investment rate in the long run. To give those magnitudes some context, by a rough calculation, a firm with access to covenants will have about 20% more capital and 30% more debt after 10 years in a deterministic environment.

Now move to the last two columns. Having acknowledged the positive value of covenants, I conduct a value decomposition. More specifically, covenant violations have two consequences: endogenous debt acceleration $\alpha(.)$ and the incurrence of a restructuring cost $f$.\(^{24}\)

\(^{24}\)Welfare equals 0.9265 if shareholders are restricted to only equity financing. Simulations of my model deliver a net benefit of debt that is in line with the estimates by e.g. Korteweg (2010) and van Binsbergen, Graham and Yang (2010).
How does each component contribute to the commitment production? First, although debt acceleration always improves post-violation total firm value, it turns out to be value destructive from the ex ante perspective. By comparing shareholder welfare in columns 1 with that in column 4, one can see a decrease after introducing costless control right shifting into a covenant-free model. In addition, by looking at welfare in columns 3 and 5, one can see that covenants can become better if lenders commit not to restructure the debt, which of course is time-inconsistent. Recall discussions in Section 1.2.4.3—this result suggests that the current restructuring setup and contractual calibration produce a pretty strong expectation of debt relief in the eyes of a newborn firm without debt and risky capital. A bad incentive ex ante is provided and a speedy risk accumulation is encouraged. In contrast, although $f$ is never desirable when ex post realized, it significantly improves welfare as the anticipation effect turns out to be considerably strong. In other words, such a “burn the boats” strategy becomes highly valuable when the incentive problem is severe.

1.5.1.2. Time variation of covenant values

The value of covenants varies across time. There are two contributing factors. First, as pointed out in Section 1.3.2.2, the severity of commitment frictions varies with economic fundamentals. Second, as the default risk and commitment frictions change, outcomes of debt restructurings can also become different.

Figure 7a demonstrates the time variation of covenant values by presenting the empirical c.d.f.s calculated from my simulated sample. The first thing to notice is that the average covenant value is 50% larger than the ex ante welfare. This is because firms in the simulated sample on average have accumulated a positive amount of debt and risky assets, which have driven up the default risk and severity of commitment frictions. Moreover, the covenant value varies significantly across time. Approximately 10% of the time, covenants account for more than 2% of the firm value.

Again, I decompose all firm-quarter covenant values and plot the empirical c.d.f.’s of contributions made by, respectively, the acceleration scheme and restructuring costs. Figure 7b
shows that as the potential restructuring outcome evolves, the payment acceleration component sometimes does help enhance firm values. For instance, when a highly levered firm applies for a loan from banks, imposing state-contingent debt accelerations can be beneficial even if they are costless. In those states with a considerable amount of risk and suppressed debt prices, debt reliefs are unlikely in the near future and quantitatively dominated by the discipline generated by debt punishment. A frequency of 0.6% is not completely trivial as it is around half of the probability of violation and three times that of default. Moreover, Figure 7c suggests that the existence of $f$ plays a dominant role in commitment production and always enhances firm values under the existing calibration of covenants.

Figure 8 takes a closer look at those observations where covenant values go beyond 0.02 by tracing the dynamics of firm states 20 quarters before. It confirms that covenants become valuable when economic fundamentals deteriorate. However, different from violations (Figure 6), covenant values are most pronounced at the onset of a sudden deterioration. At that point, firms have a strong tendency to start increasing risk shifting and act slowly in debt buyback. Covenants generate significant merits by nipping severe dilution in the bud.
1.5.2. Contractual Efficiency

In this section, I move a step further and evaluate the efficiency of existing covenants in addressing time inconsistency. For such a purpose, I fix the current contracting structure and experiment on how shareholder welfare responds to adjustments in covenant calibration.

1.5.2.1. Covenant tightness

I first study the choice of violation threshold $\kappa$, which governs how frequently violations take place ex post. Figure 9a suggests that welfare is hump shaped with respect to violation frequency and the maximum is achieved under a much tighter calibration: $\kappa = 1/1.15$ and the violation probability exceeds 4%. (Recall that the benchmark violation frequency is 1.5% and $\kappa = 1/2$.) If I adjust downward the coupon rate so that debt on average trades at par, there still exists a potential efficiency improvement, though both the required tightening and the resulting gain become much smaller.\textsuperscript{25}

Behind the hump shape lies the efficient allocation of covenant violations. Suppose lenders have only one chance of costly debt restructuring, it is ex ante ideal to allocate that op-

\textsuperscript{25}The coupon rate determines the tax shield and thus plays a crucial role in the interest conflict between lenders and borrowers. When coupons are reduced, the ex post incentive of shareholders in pushing up leverage and thus default risks of legacy debt becomes milder. Therefore, when I adjust downward the coupon rate along the increase in $\kappa$, the time inconsistency becomes less harmful and thus the value of financial covenants is weakened. Expected restructuring costs start to be dominant more quickly, resulting in a lower turning point in ex ante firm value.
portunity to a state where fundamentals are highly undesirable. Ex post, lenders have a strong incentive to accelerate principal payment at shareholders’ expenses. Such an allocation is thus able to generate not only a huge risk-reduction surplus ex post but also strong discipline ex ante.

![Figure 9: Covenant Tightness and Efficiency](https://example.com/figure9.png)

**Notes:** This figure presents results from alternative models with different $\kappa$’s but all the other parameters fixed to those in Table 7. Figure 9a shows shareholder welfare $j(0, 0, \bar{x})$. Figure 9b shows the average deviation in firm value one quarter before violations (number of standard deviations below mean). Figure 9c (9d/9e) presents causal impacts of violations on the net debt issuances-to-assets ratio (investment rate/$k_H$ share) averaged over two post-violation quarters. I simulate 2,000 firms for 1,000 quarters.

As the violation threshold increases, lenders have more and more opportunities to implement a restructuring. However, violations locating in low-risk states start to account for a larger and larger proportion. This phenomenon is again driven by the nonlinearity of the defaultable debt problem. To loosely understand what happens, first send $\kappa$ to a positive value fairly close to 0. Covenant violations in that case overlap with defaults–large neg-
ative shocks together with high firm riskiness are required. It is thus unlikely to expect a violation in the “good” time. Now consider an infinitely tight covenant, i.e. $\kappa = \infty$, instead. In this scenario, one shall expect to see a large chunk of violations happening when fundamentals are fairly good. Indeed, Figure 9b shows that, as covenants get tighter, on average violations happen under a relatively higher level of firm value.

While the benefit of future restructurings is decreasing in violation frequency, the resource cost $f$ is constant. These two forces combined lead to the existence of an inner optimum. Under an extremely tight covenant, many violations take place when the lack of commitment is far from being problematic. Frequent realizations of $f$ losses impose huge harm to shareholders and the firm, which cannot be justified by the value of the commitment produced by restructurings.

Figures 9c, 9d and 9e together demonstrate that covenant violations become less consequential along the raise of $\kappa$, which happens again because violations become relatively more likely when default risk is small. Such a negative relationship between the tightness of covenants and the severity of violation consequences is consistent with the cross-sectional evidence in Demiroglu and James (2010). The activeness of violations also declines.

1.5.2.2. Allocation of decision rights

The second structural parameter governing covenants is the bargaining power $\theta$. When lenders hold all the control right in restructurings, firm value ex post is not maximized in lots of states–after default risk has been reduced to a certain level, lenders find a further retirement not privately beneficial because of the transfer to equity holders. Does this mean that equity holders should get some bargaining power and be able to enforce some transfers from lenders?

Figure 10a suggests a negative answer. Indeed, Figure 10b shows that when shareholder bargaining power is increased from 0 to 0.5, covenant violations lead to larger improvement in firm value ex post – a stronger realization effect. However, the anticipation effect changes
as well. In Section 1.5.1.1, I have shown that the acceleration mechanism fails to improve welfare exactly because equity holders tend to get too much via debt relief. With a larger bargaining power, shareholders will on average get even more, as suggested by Figure 10c, and sometimes at the expense of debt holders. My analyses suggest that the weakening of the anticipation effect quantitatively dominates the improvement of the realization effect.

1.6. Conclusion

This paper proposes a quantitative theory of financial covenants. By state-contingently introducing costly creditor intervention, these contract clauses serve as a potential solution to shareholders’ time inconsistency problem associated with long-term debt financing. My quantitative analyses show that financial covenants significantly increase debt capacity and investment, restrict asset substitution and improve ex ante shareholder welfare. Furthermore, I quantify the tradeoff underlying the calibration of covenants and demonstrate a potential improvement in efficiency.

Considering the recent boom in covenant-lite loans, it is interesting to explore whether making the covenant tightness state-contingent can significantly improve contractual efficiency further. Such an extension should be straightforward. Moreover, it is valuable to
incorporate this model into a general equilibrium framework and quantify the implications of these contingency contracts for macroeconomic quantities and fluctuations. I leave these to future work.


1.7. Appendix

1.7.1. An Example of Financial Covenants

The following paragraph presents a typical description of financial covenants in corporate financial reports and is taken from the 10-K of the HealthSouth Corporation for the fiscal year ended December 31, 2004:

“Non-compliance with these financial covenants under our credit facilities—our interest coverage ratio and our leverage ratio—could result in the lenders requiring us to immediately repay all amounts borrowed. Any such acceleration could also lead the investors in our public debt to accelerate their maturity. In addition, if we cannot satisfy these financial covenants in the indenture governing the credit agreements, we cannot engage in certain activities, such as incurring additional indebtedness, making certain payments, acquiring and disposing of assets.”

1.7.2. Proofs in Section 1.2

As noted in the main text, since my propositions are mainly for characterization purposes, I assume the existence of an inner optimum and the validity of first-order conditions.

1.7.2.1. Proposition 1

Consider a “Ramsey” problem where shareholders lack commitment to repay but could commit to the path of debt \( \{b_s\}_s \). The planning objective at time 0 is:

\[
\max_{\{Q_s, \{b_s\}, s > 1\}} J^c_0
\]

where \( J^c_s \) is recursively defined as:

\[
J^c_s = Q_{s+1}[b_{s+1} - (1 - \lambda)b_s] \\
+ \beta E_s \left\{ \int_{z_{s+1}} \left[ (1 - \tau)(Y + z_{s+1}) - [(1 - \tau)c + \lambda]b_{s+1} + J^c_{s+1} \right] d\Phi(z_{s+1}) \right\},
\]

(1.13)
subject to the participation constraint of lenders:

\[ Q_{s+1} = \beta \mathbb{E}_s \left\{ \int_{z_{s+1}}^{z_d} \left[ (c + \lambda) + (1 - \lambda)Q_{s+2} \right] d\Phi(z_{s+1}) \right\}, \]  

(1.14)

and the ex post optimal default rule of shareholders: 

\[ (1 - \tau)(Y + z^d_{s+1}) - [(1 - \tau)c + \lambda]b_{s+1} + J^c_{s+1} = 0. \] 

Initial conditions are given by \( b_0 = 0 \) and \( Q_0 = 1 \).

Define \( \Pi_{t_i,t_j} \) as the time-\( t_i \) expected probability of no default before time-\( t_j \). The first-order condition w.r.t. \( b_{t+1} \) is given by

\[ \beta_t \Pi_{0,t} b_{t+1} + \beta^{t+1} \Pi_{0,t} \mathbb{E}_t \left\{ \int_{z_{t+1}}^{z_d} \left[ -[(1 - \tau)c + \lambda] - (1 - \lambda)Q_{t+2} \right] d\Phi(z_{t+1}) \right\} 
- \mu_t \beta \mathbb{E}_t \left\{ \left[ (c + \lambda) + (1 - \lambda)Q_{t+2} \right] \phi(z^d_{t+1}) \frac{\partial z^d_{t+1}}{\partial b_{t+1}} \right\} = 0, \]  

(1.15)

and that w.r.t. \( Q_{t+1} \) for all \( t \geq 1 \)

\[ \beta^t \Pi_{0,t} [b_{t+1} - (1 - \lambda)b_t] - \mu_t + \mu_{t-1}(1 - \lambda)\beta \Pi_{t-1,t} = 0, \]  

(1.16)

and that w.r.t. \( Q_1 \):

\[ b_1 - \mu_0 = 0, \]  

(1.17)

where \( \mu_t \) is the Lagrangian multiplier.

From (1.17) and (1.16) and the fact that \( \Pi_{0,0} = 1 \), we know: \( \beta^t \Pi_{0,t} b_{t+1} = \mu_t \). Plug this together with (1.14) into (1.15), we get:

\[ \mathbb{E}_t \left[ \int_{z_{t+1}}^{z_d} \tau cd\Phi(z_{t+1}) \right] = b_{t+1} \mathbb{E}_t \left\{ \left[ (c + \lambda) + (1 - \lambda)Q_{t+2} \right] \phi(z^d_{t+1}) \frac{\partial z^d_{t+1}}{\partial b_{t+1}} \right\} 
= - \frac{b_{t+1}}{\beta} \frac{\partial Q_{t+1}}{\partial b_{t+1}}. \]  

(1.18)

Optimal choices \( b_{t+1} \) and \( Q_{t+1} \) no longer depend on legacy debt \( b_t \). The persistence of \( \{b_t\} \)
inherits that of \( \{ z_t \} \), if it exists.

1.7.2.2. Corollary 1

Consider the planner’s re-optimization at time 1 \( \{ b_{re}^e, Q_{re} \}_{s>1} \) with legacy debt \((1 - \lambda) b_1 > 0\). First-order condition w.r.t. \( Q_{re}^e \) evolves into:

\[
b_{re}^e - (1 - \lambda) b_1 = \mu_1^e.
\]

Plugging this into the first-order condition w.r.t. \( b_{re}^e \) results in a condition different from equation (1.18).

1.7.2.3. Proposition 2

The first part of the proposition is self-evident from the first-order condition in equation (1.6). To derive it, first ignore the covenants, and then shareholders’ problem becomes:

\[
J(\tilde{b}) = \max_{b'} \left\{ (b' - \tilde{b})Q(b') \right. \\
+ \beta \mathbb{E} \left\{ \int_{z_d}^{\tilde{z}} \left[ (1 - \tau)(Y + z) - [(1 - \tau)c + \lambda]b' + J((1 - \lambda)b') \right] d\Phi(z') \right\},
\]

subject to

\[
Q(b') = \beta \mathbb{E} \left\{ \int_{z_d}^{\tilde{z}} \left[ (c + \lambda) + (1 - \lambda)Q \left( b''((1 - \lambda)b') \right) \right] d\Phi(z') \right\},
\]

and \((1 - \tau)(Y + z_d) - [(1 - \tau)c + \lambda]b + J((1 - \lambda)b) = 0\).

The first-order condition derived from equation (1.19) is given by:

\[
Q(b') + \frac{\partial Q(b')}{\partial b'} (b' - \tilde{b}) - \beta \mathbb{E} \left\{ \int_{z_d}^{\tilde{z}} \left[ [(1 - \tau)c + \lambda] - (1 - \lambda) \frac{\partial J(\tilde{b})}{\partial b'} \right] d\Phi(z') \right\} = 0. \quad (1.21)
\]

The envelope theorem gives \( \frac{\partial J(\tilde{b})}{\partial \tilde{b}} = -Q(b') \). After plugging this equality together with the
pricing function of equation (1.20) into (1.21), I get
\[
\frac{\partial Q(b')}{{\partial b'}} (b' - \hat{b}) + \beta \mathbb{E} \left[ \int_{z_d}^{\bar{z}} \tau cd\Phi(z') \right] = 0. \tag{1.22}
\]

Differentiation of the pricing function is straightforward and thus omitted here. Now move to the second part of the proposition. Denote the LHS of equation (1.22) as \(H\). Use the implicit function theorem:
\[
\frac{\partial b'}{\partial \hat{b}} = -\left[ \frac{\partial H}{\partial b'} \right]^{-1} \frac{\partial H}{\partial \hat{b}} = \left[ \frac{\partial H}{\partial b'} \right]^{-1} \frac{\partial Q(b')}{\partial b'}.
\]

As I always focus on an inner solution, the second-order derivative at the optimum should be negative, i.e. \(\partial H/\partial b' < 0\). As a result:
\[
\frac{\partial b'}{\partial \hat{b}} \frac{\partial Q(b')}{\partial b'} < 0.
\]

By equation (1.7), we know \(\partial Q(b')/\partial b' < 0\). Therefore \(\partial b'/\partial \hat{b} > 0\).

1.7.2.4. Proposition 3

To see the difference between long-term debt under full commitment and one-period debt, consider an alternative recursive problem (subscripted with \(a\)) which replicates the constrained-efficient allocation under i.i.d. shocks. Conditional on no previous default, shareholders are forced always to maximize the total firm value \(F_a\) when issuing debt:
\[
F_a = \max_{b'} Q_a(b')b' \\
+ \beta \mathbb{E} \left\{ \int_{z_d}^{\bar{z}} \left[ (1 - \tau)(Y + z) - [(1 - \tau)c + \lambda]b' + F'_a - Q_a(b'')(1 - \lambda)b' \right] d\Phi(z') \right\}, \tag{1.23}
\]

where
\[
Q_a(b') = \beta \mathbb{E} \left\{ \int_{z_d}^{\bar{z}} \left[ (c + \lambda) + (1 - \lambda)Q_a(b'') \right] d\Phi(z') \right\}. \tag{1.24}
\]
and \((1 - \tau)(Y + z_d) - [(1 - \tau)c + \lambda]b + F_a - Q_a(b')(1 - \lambda) = 0\).\(^{26}\)

Plug (1.24) into (1.23):

\[
F_a = \max_{b'} \beta E \left\{ \int_{z_d}^{z} \left[ (1 - \tau)(Y + z) + \tau cb' + F'_a \right] d\Phi(z') \right\}, \quad (1.25)
\]

subject to \((1 - \tau)(Y + z_d) - [(1 - \tau)c + 1]b + F_a + [1 - Q_a(b')]\] \((1 - \lambda) = 0\). Now consider the firm value for a one-period debt problem:

\[
F_1 = \max_{b'} \beta E \left\{ \int_{z_d}^{z} \left[ (1 - \tau)(Y + z) + \tau cb' + F'_1 \right] d\Phi(z') \right\}, \quad (1.26)
\]

where \((1 - \tau)(Y + z_d) - [(1 - \tau)c + 1]b + F_1 = 0\).

Since there are no persistent shocks here, allocations become time-invariant under full commitment. This is also true for one-period debt. Now consider a Ramsey equilibrium \(\{(b_1^*)', F_1^*\}\) and a competitive equilibrium with one-period debt trading \(\{(b_1^*)', F_1^*\}\).

I) If \(Q_a(b_1^*) = 1\), \((b_1^*)' = (b_1^*)'\) and \(F_1^* = F_1^*\). The proof is by conjecture and verify. Start with the conjecture \((F_a^*)' = (F_1^*)'\) and plug it into the RHS of equation (1.25). The maximization objectives become identical, and thus \((b_1^*)'\) that solves (1.25) should also be a solution to (1.26), i.e. \((b_1^*)' = (b_1^*)'\). As a result, \(F_1^* = F_1^*\).

II) If \(Q_a(b_1^*) \leq 1\), \(F_a^* \geq F_1^*\). Again start with \((F_a^*)' \geq (F_1^*)'\). Take the solution to (1.26) and plug it into (1.25), I have \(F_a[(b_1^*)'] \geq F_1^*\). Since \(F_a^*\) maximizes (1.25), \(F_a^* \geq F_a[(b_1^*)']\). Therefore I have \(F_a^* \geq F_a[(b_1^*)'] \geq F_1^*\) and can thus verify the conjecture.

III) If \(Q_a(b_1^*) \geq 1\), \(F_a^* \leq F_1^*\). The proof resembles that in case II).

\(^{26}\)The first-order condition of this problem is given by:

\[
\beta E \left\{ \int_{z_d}^{z} \tau c d\Phi(z') \right\} = b' \beta E \left\{ [(c + \lambda) + (1 - \lambda)Q_a(b'')] \phi(z') \frac{\partial z'}{\partial b'} \right\}.
\]

One can easily prove the equivalence by conjecture-and-verify.
1.7.3. The Full Model

1.7.3.1. Equilibrium definition

**Definition 2.** A Markov Perfect Equilibrium of the full model is given by (i) equity holders’ policy function $b'(.)$, $k'(.)$, $s'(.)$ with associated value function $V^e(.)$, default set $D$ and covenant violation set $H$; (ii) a debt pricing function $Q(.)$ and associated value function $V^b(.)$; (iii) a repayment acceleration function $\alpha(.)$ such that (i) given $Q(.)$ and $\alpha(.)$, equity holders’ policies are optimized; (ii) given equity holders’ decisions and $\alpha(.)$, $Q(.)$ and $V^b(.)$ satisfy debt holders’ zero profit condition in equation (1.11); (iii) given $V^e(.)$, equity holders’ decisions and $V^b(.)$, $\alpha(.)$ solves the restructuring problem in equation (1.12).

1.7.3.2. Proof of linear homogeneity

This section sketches the proof of the linear homogeneity of the full model. First, I conjecture repayment schedule $\alpha(b,k,s,x,z)$ is homogeneous of degree 0 (HOD0) to $k$ and $b$ (conjecture i). Further, debt policy $b'(\tilde{b},k,s,x)$ and capital policy $k'(\tilde{b},k,s,x)$ are homogeneous of degree 1 (HOD1) to $k$ and $\tilde{b}$, while risk-shifting policy $s'(\tilde{b},k,s,x)$ is HOD0 to $k$ and $\tilde{b}$ (conjecture ii).

Conjecture the default cutoff is HOD0 to $k$ and $b$ (conjecture iii). Because default recovery is linear in $k$, based on conjectures i, ii, iii and the fact that the violation cutoff $z_v$ is HOD0 to $k$ and $b$ by construction, pricing function $Q(b',k',s',x)$ can be shown to be HOD0 to $k'$ and $b'$. Moreover, based on conjectures i, ii and iii, HOD0 of $z_v$ and $z_d$ and HOD0 of $Q$, I can show that the equity value function $J(\tilde{b},k,s,x)$ is HOD1 to $k$ and $\tilde{b}$. Together with conjecture i, it implies that $V^e(b,k,s,x,z;\alpha)$ is HOD1 to $k$ and $b$, which in turn verifies conjecture iii.

With conjecture i, HOD1 of $V^e$, HOD0 of $z_v$ and $z_d$, and HOD0 of $Q$, one can utilize the linearity of argmax operator and verify conjecture ii.

Because of conjecture i and the HOD0 of $Q$, it can be shown that $V^b(b,k,s,x;\alpha)$ is HOD1 to $b$ and $k$. Combining with the HOD1 of $V^e(\tilde{b},k,s,x)$, I can finally verify conjecture i.
Thanks to the linear homogeneity, I only have to solve the scaled version of the model. State space consists of \( \tilde{\omega}, s \) and \( x \), which are discretized respectively with 183, 7 and 5 points. I set the upper bound for \( \tilde{\omega} \) so that it never binds in simulations. I iterate over four functions in two loops: investment policy function \( i(\tilde{\omega}, s, x) \), leverage restart policy \( \tilde{\omega}^R(s, x) \), equity value function \( j(\tilde{\omega}, s, x) \) and debt pricing schedule \( q(\omega', s', x) \).

[1] Inner loop: Conditional on \( i(.) \) and \( \tilde{\omega}^R(.) \), iterate over \( j(.) \) and \( q(.) \) until convergence. In each step, I simultaneously update these two functions.

[2] Outer loop: Given \( i(.) \), \( q(.) \) and the policy functions \( \omega'(\tilde{\omega}, s, x) \) and \( s'(\tilde{\omega}, s, x) \) obtained in the inner loop, update \( i(.) \) analytically by the first-order condition and \( \tilde{\omega}^R(.) \) numerically by searching over a grid of 9150 points. Iterate until convergence.

1.7.4. Data Construction

<table>
<thead>
<tr>
<th>Moments</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>debt</td>
<td>( b )</td>
<td>( \text{dlttq + dlcq} )</td>
</tr>
<tr>
<td>net debt issuance</td>
<td>( b' - b )</td>
<td>...</td>
</tr>
<tr>
<td>assets</td>
<td>( k )</td>
<td>( \text{atq} )</td>
</tr>
<tr>
<td>investment</td>
<td>( k' - (1-\delta)k )</td>
<td>( \text{capxy - sppey} )</td>
</tr>
<tr>
<td>market-to-book</td>
<td>( [J(\tilde{b}) + (1-\lambda)\tilde{b}Q(b')]/k )</td>
<td>( (\text{atq} + (\text{prccq} \times \text{cshoq}) - \text{ceqq} - \text{txdbq}) / \text{atq} )</td>
</tr>
<tr>
<td>income</td>
<td>( e^x k )</td>
<td>( \text{oibdpq} )</td>
</tr>
</tbody>
</table>

Table 6: Data Construction

The main dataset I use combines i) COMPUSTAT North America Fundamental Quarterly between 1996Q1 and 2011Q4 and ii) an extended version of the covenant violation data constructed by Roberts and Sufi (2009a). Corporate default rates are taken from Exhibit 30 of Moody’s Annual Default Study: Corporate Default and Recovery Rates, 1920-2015.

I drop observations that are i) duplicated; ii) within the following industries – agriculture (sic \( \in [0000, 999] \)), utilities (sic \( \in [4900, 4999] \)), financial business (sic \( \in [6000, 6999] \)), foreign government (sic = 8888) and international affairs & non-operating establishments (sic \( \in [9000, 9999] \)); and iii) non-US. Table 6 presents how variables are constructed within the
model and in the data. All variables in the data are winsorized at top and bottom 1%.
2.1. Introduction

An unforgettable lesson policy makers and researchers have learned from the Great Recession is the value of regulating intermediary balance sheets. In the policy sphere, Basel III places more complex restrictions on banking sector leverage, while in the academic world, a new vintage of macroeconomic models with financial frictions have been developed to study the aggregate implications of bank capital regulation.\(^1\)

While voluminous macro-banking models have advanced our understanding of banks’ liabilities, a realistic characterization of their assets is largely absent in existing work. Typical models are silent about banks’ active roles in enhancing production efficiency through monitoring, debt restructuring, etc.\(^2\) Furthermore, as pointed out by Adrian, Colla and Shin (2012), firms’ debt choices over bank and non-bank finance are also ignored by current frameworks, which either force banks to be the only financing source in the economy or assume an exogenous market segmentation between financing alternatives. Without capturing a key value of banks and interactions between heterogeneous debt, a model might deliver an imprecise quantification of the aggregate impact of macroprudential policies.

I propose a business cycle model augmented with a corporate debt structure and a dynamic banking sector. Built on a formulation of Crouzet (2018), firms borrow via bank and non-bank debt, with the former being costly but special in providing debt restructuring opportunities that reduce corporate bankruptcy losses.

Modigliani-Miller is violated in this economy by a tax-bankruptcy trade-off together with widely-recognized banking sector frictions: bank dividend adjustment costs, deposit in-

---


\(^2\) In most of these models, the only role of intermediation is credit provision. Quadrini (2017) considers a model in which liabilities of banks help firm production by providing liquidity and insurance.
urance, and capital requirements. The adjustment cost of bank dividends together with capital requirements create a standard “financial accelerator” effect à la Bernanke, Gertler and Gilchrist (1999) and Kiyotaki and Moore (1997). The volatility of banks’ net worth starts to generate an additional distortion on the provision of intermediated credit.

Deposit insurance isolates banks from bankruptcy concerns and encourages the extraction of deposit tax shields. Firms push up their total leverage and rely heavily on bank finance thanks to a subsidized loan price. Associated consequences are twofold. First, banks encounter a wave of liquidations, resulting in large bankruptcy losses and a volatile equity. Second, firms over-borrow and invest in socially inefficient projects.

Raising capital requirements reduces these distortions introduced by the deposit guarantee. However, it also removes deposit tax shields. An excessively tight capital regulation leads to socially insufficient bank lending, and thus generates undesirable impacts.

The model is calibrated to the aggregate US economy. My quantitative analysis shows that, interestingly, bank and non-bank finance are complements when capital requirements are permanently tightened. The protection against bankruptcy losses provided by restructuring creates a complementarity between bank and non-bank borrowing, which turns out to dominate their perfect substitutability as production inputs. As the capital requirement becomes tight, both bank and non-bank finance are cut back. The existing macro-banking literature has focused on intermediaries’ credit supply choices and predicts a surge in alternative financing resulting from commercial banks’ regulatory arbitrage. Taking into account the uniqueness of bank loans, my analysis highlights the potential strength of debt complementarity on the credit demand side, which has been largely overlooked.

Tightening capital requirements suppresses banks’ leverage and sharply reduces their bankruptcy rate. Firms’ financing and production shrink accordingly. However, firms do not become

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3 A large number of discussions about capital regulation have the tax benefit of bank liabilities as one important consideration. See for example Kashyap, Rajan and Stein (2008), Hanson, Kashyap and Stein (2011), and Admati et al. (2013).

4 Previous work includes for example Plantin (2015), Huang (2018) and Begenau and Landvoigt (2016). See also FSOC (2012).
safer during the de-leveraging process. This is not surprising when one takes into account the uniqueness of bank loans in providing debt restructuring. Firms default on their promised debt repayment less frequently, but conditional on a distress, they are more likely to end up in a bankruptcy. In contrast to banks, firms go bankrupt more frequently as the economy deleverages.

Quantitatively, the marginal impact of raising capital requirements from the status quo on aggregate quantities and welfare is fairly small. Welfare is hump-shaped and maximized at an 11% capital requirement. Compared to a ratio of 8%, implementing the optimal policy yields a marginal welfare gain of only 0.035%. Bank finance declines by 0.58% while non-bank finance shrinks by 0.32%. Annual corporate borrowing and total output drop respectively by 0.41% and 0.18%. The banking sector becomes much safer: the probability of a bank failure decreases from 49.37 to 9.05 basis points, resulting in an 82% drop in the bank liquidation cost and a 33% drop in the volatility of bank dividend rate.


More broadly, the model I propose in this paper adds to a recent growing literature that studies how financial intermediaries affect the macroeconomy in a dynamic environment (Gertler and Kiyotaki 2010; 2015; Gertler and Karadi, 2011; Christiano and Ikeda, 2013; Brunnermeier and Sannikov, 2014; Boissay, Collard and Smets, 2016; Di Tella, 2017; Robatto, 2017). It is the first in this literature, to the best of my knowledge, where firms

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5 Gornall and Strebulaev (2018) and Harris, Opp and Opp (2017) conduct theoretical analyses of bank capital regulation in a model where firms are granted the alternative option to borrow from non-banks.
optimize a debt structure over bank and non-bank finance.


Corporate debt choice is meaningful in my model because banks provide debt restructuring opportunities that improve production efficiency. The firm’s problem in my model builds on the formulation of Crouzet (2018), who studies how loan pricing shocks in the Great Recession were transmitted to firms in a partial equilibrium Aiyagari model. De Fiore and Uhlig (2011; 2015) study corporate debt choice in an RBC environment with bank loans being unique in solving informational frictions. However, these studies do not characterize intermediaries.

Adrian, Colla and Shin (2012), Becker and Ivashina (2014) and De Fiore and Uhlig (2015) document a short-run substitutability between bank and bond finance: firms issue more corporate bonds in response to transitory bank credit supply shocks over the business cycles. My results complement these studies by showing a long-run complementarity: firms reduce non-bank finance when capital requirements are permanently raised.

More broadly, this paper is related to the growing literature on the macroeconomic implications of financial frictions. Some examples include Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1999), Mendoza (2010), Jermann and Quadrini (2012) and Christiano, Motto and Rostagno (2014). The paper proceeds as follows. Section 2.2 presents the general equilibrium model. I discuss
key mechanisms in section 2.3. Quantitative assessments of the model and counter-factual analyses are carried out respectively in sections 2.4 and 2.5. Parameter sensitivities are analyzed in section 2.6. The last section concludes.

2.2. Model

I start by presenting the corporate choice on a debt portfolio consisting of restructurable loans intermediated by banks and non-bank debt directly held by households. I then describe the non-bank and bank sectors. The government and household sectors are finally characterized. Some assumptions and their implications are discussed in section 2.2.7.

2.2.1. Firms

The production sector of the economy consists of a continuum of short-lived firms located on \( I = [0, 1] \). Firms are ex-ante identical when making financing decisions, but become ex-post different due to independent realizations of idiosyncratic shocks. Corporate decisions are made taking the stochastic discount factor of households as well as debt pricing schedules as given.

2.2.1.1. Production and Financing

Each firm \( i \in I \) born at the end of period \( t - 1 \) is endowed with a technology that has decreasing returns to scale:

\[
y_{i,t} = A_t z_{i,t} k_{i,t}^{\alpha_i}.
\]

(2.1)

The aggregate productivity shock \( A_t \) follows: \( \ln A_{t+1} = \rho_a \ln A_t + \sigma_a \epsilon_{t+1}^a \). The idiosyncratic shock \( z_{i,t} \) is i.i.d. and log-normally distributed with dispersion \( \sigma_z \) and mean \( \mu_z = -0.5 \sigma_z^2 \).

Individual firms finance their production in period \( t \) through a portfolio of bank debt \( b \) and non-bank debt \( m \) at the end of period \( t - 1 \), taking pricing schedules \( R_{t-1}^b(b, m) \) and \( R_{t-1}^m(b, m) \) as given. Ex-ante identical firms arrange their borrowing through the same debt
Though making the same decisions, firms are ex-post heterogeneous due to different realizations of the idiosyncratic productivity shock. Firm $i$’s total income at the end of period $t$ are given by:

$$\pi_{i,t} = A_t z_{i,t} (b_{t-1} + m_{t-1})^\alpha + (1 - \delta)(b_{t-1} + m_{t-1}) - \varphi b_{t-1},$$  \hspace{1cm} (2.3)

where $\delta$ is the depreciation rate of capital. For quantitative realism I assume that utilizing intermediated credit contains a proportional cost $\varphi$, which is associated with firms being monitored and complying with an extensive set of covenants.

To capture tax shields associated with debt financing, I adopt the formulation of Jermann and Quadrini (2012) and assume firms get a predetermined subsidy of $\Theta_{t-1}^f$ if ex-post no bankruptcy happens:

$$\Theta_{t-1}^f = \tau [(R_{t-1}^b - 1)b_{t-1} + (R_{t-1}^m - 1)m_{t-1}].$$  \hspace{1cm} (2.4)

2.2.1.2. Repayment

After $\pi_{i,t}$ realizes, equity holders of firm $i$ have three options. Firstly, they can fully repay their debt obligations and get the residual claim together with the tax shield. Secondly, they can choose to go bankrupt, upon which creditors recover $\chi \pi_{i,t}$ in total and then split it according to a seniority rule under which banks are more senior than non-banks.

Thirdly, they initiate a debt restructuring to banks by making them a take-or-leave offer. A restructuring is successful if banks take the offer while non-banks are fully repaid. The firm in this case avoids a bankruptcy and gets residual assets. Without loss of generality, I follow

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Hereafter I drop firm-specific subscript $i$ when there is no confusion.
Crouzet (2018) and grant firms all bargaining power during the restructuring process.\textsuperscript{7} Due to its non-transferability upon bankruptcy, the tax shield will be fully exploited by the firm in a restructuring process.

Denote firms’ debt obligations $\Pi_{t-1}^b = R_{t-1}^b b_{t-1}$ and $\Pi_{t-1}^m = R_{t-1}^m m_{t-1}$. Debt settlement outcomes, under optimal restructuring decisions, are presented in the following proposition as a simple variation of Crouzet (2018).\textsuperscript{8}

**Proposition 5.** State-contingent payoffs to firm $i$, $P_{i,t}^f$, its bank lenders, $P_{i,t}^b$, and its non-bank lenders $P_{i,t}^m$ under optimal restructuring decisions are given by:

<table>
<thead>
<tr>
<th>Panel A. $\Pi_{t-1}^b/\chi &gt; (\Pi_{t-1}^m - \Theta_{t-1}^f)/ (1 - \chi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payment</td>
</tr>
<tr>
<td>$\pi_{i,t} &gt; \Pi_{t-1}^b/\chi$</td>
</tr>
<tr>
<td>$P_{i,t}^b$</td>
</tr>
<tr>
<td>$P_{i,t}^m$</td>
</tr>
<tr>
<td>$P_{i,t}^f$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B. $\Pi_{t-1}^b/\chi &gt; (\Pi_{t-1}^m - \Theta_{t-1}^f)/ (1 - \chi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payment</td>
</tr>
<tr>
<td>$\pi_{i,t} &gt; \Pi_{t-1}^b/\chi$</td>
</tr>
<tr>
<td>$P_{i,t}^b$</td>
</tr>
<tr>
<td>$P_{i,t}^m$</td>
</tr>
<tr>
<td>$P_{i,t}^f$</td>
</tr>
</tbody>
</table>

My focus is on panel A of Proposition 5. It describes debt structures under which a restructuring happens with a positive probability: $\Pi_{t-1}^b/\chi \geq (\Pi_{t-1}^m - \Theta_{t-1}^f)/ (1 - \chi)$. This is the region consistent with our observation that restructurings are typical for the aggregate economy. A firm finds it profitable to exercise the restructuring option whenever banks’ reservation value has dropped below its loan obligation: $\chi \pi_{i,t} \leq \Pi_{t-1}^b$. A bankruptcy takes place when a full repayment to non-bank debt holders becomes infeasible even when firms

\textsuperscript{7}This assumption does not affect much firms’ ex-ante debt choices because they internalize debt prices. When banks are perfectly competitive, any rents they can extract in a restructuring because of the bargaining power allocation will be finally enjoyed by lenders. It also impose a small welfare impact ex post because only transfers are involved.

\textsuperscript{8}All proofs can be found in Appendix 2.8.1.
can benefit from a restructuring: 

$$(1 - \chi)\pi_{i,t} + \Theta_{t-1}^f < \Pi_{t-1}^m.$$ 

The restructuring region within panel A can be further broken down into two cases. It is easy to show that in the upper panel $\Pi_{t-1}^b / \chi \geq \Pi_{t-1}^b + \Pi_{t-1}^m - \Theta_{t-1}^f \geq (\Pi_{t-1}^m - \Theta_{t-1}^f)/(1 - \chi)$. First, when $\Pi_{t-1}^b / \chi \geq \pi_{i,t} \geq \Pi_{t-1}^b + \Pi_{t-1}^m - \Theta_{t-1}^f$, firms have enough resources to fully repay debt obligations $\Pi_{t-1}^b + \Pi_{t-1}^m$ but find it optimal to strategically initiate a restructuring in order to exploit their bargaining power. Second, when $\Pi_{t-1}^b + \Pi_{t-1}^m - \Theta_{t-1}^f > \pi_{i,t} \geq (\Pi_{t-1}^m - \Theta_{t-1}^f)/(1 - \chi)$, firms have to default but can go through a successful restructuring.  

Panel B shows that when bank loans constitute a relatively small fraction of corporate liabilities, restructurings never happen. To gain some intuition why such a scenario exists, consider a firm borrowing a tiny amount of money from banks but a huge chunk from non-banks. The moment it starts to find it beneficial to restructure its debt, firm cash flow should have declined to a low enough level such that $\pi_{i,t} < \Pi_{t-1}^b / \chi$. At this point, the total resource $\pi_{i,t}$ is already fairly small and thus insufficient to fully repay the large amount of non-bank liabilities. In other words, firms with these debt structures find it beneficial to go bankrupt before they can get a benefit from a restructuring.

A comparison between panel A and panel B leads to the following observation: as the debt structure tilts toward bank loans, debt restructuring probability increases. This is closely related to how bank finance complements non-bank finance, which will be illustrated in the following section through an example.

2.2.1.3. Restructuring and Debt Complementarity – An Example

Bank and non-bank finance are perfect substitutes as production inputs, but the restructuring feature of loans gives rise to a debt complementarity. Non-banks ex-ante charge firms the liquidation costs they have to bear. Borrowing more from banks increases the likelihood of debt restructuring in a default. Non-bank debt spreads are thus suppressed as...
the bankruptcy cost declines.

A simple example will suffice to illustrate the rationale behind. Consider the following two firms: one (A) with bank and non-bank obligations of $51 and $20 and another (B) with respectively $11 and $20. Suppose the recovery rate is 50%. When the cash flow of firm A drops to $70, it can restructure with its banks and propose to them $35. Non-bank investors get a full repayment of $20 while the firm ends up with $15. In contrast, when the cash flow of firm B declines to 30, it can not avoid filing a bankruptcy. Because banks will get a full repayment in bankruptcy, firm B has to propose them at least $11. A residual of $19 is clearly not sufficient to repay non-bank investors and thus the restructuring is infeasible. In this case, banks end up with $11 while non-bank creditors suffer a loss of $16. As a result, non-bank lenders will charge firm B a much higher yield when foreseeing such a potential loss of $16. The debt structure of firm A lies in the upper panel of the table in Proposition 5 while that of firm B lies in the lower panel.

2.2.1.4. The Firm’s Problem

Firms born at the end of period \( t \) observe the pricing schedules \( R^b_t \) and \( R^m_t \) and then make their debt choices \( (b_t, m_t) \). Owned by the households, firms discount the expected return using their stochastic discount factor:

\[
M_{t+1} \equiv \beta u'(c_{t+1}) u'(c_t),
\]

(2.5)

where \( u(c_t) \) is the utility of the representative household.

The maximization program of firm \( i \in I \) is thus given by:

\[
\max_{b_t \geq 0, m_t \geq 0} E_t M_{t+1} P^f_{i,t+1},
\]

(2.6)

where \( P^f_{i,t+1} \) stands for the state-contingent firm equity payoff described in Proposition 5.
2.2.2. Non-Banks

Firms borrow directly from households in a competitive non-bank debt market. This market is subject to no frictions and regulations. Consequently, the pricing schedule of non-bank debt can be characterized by a standard zero-profit condition:

\[ E_t M_{t+1} \left( \frac{P_{i,t+1}^m}{m_t} - R^f_t \right) = 0 \quad \forall i \in I, \quad (2.7) \]

where \( P_{i,t+1}^m \) denotes the realized payoff to non-banks described in Proposition 5. The risk-free rate \( R^f_t = 1 / E_t M_{t+1} \).

To make the model simple, I assume that banks do not participate in the non-bank debt market, and as a result, the banking regulation has no direct impact on non-banks. Under such an assumption, this model describes an economy regulated under the Glass–Steagall Act or an extreme version of the Volcker Rule.

2.2.3. Banks

The banking sector is competitive and consists of a cross-section of long-lived banks located on \( J = [0, 1] \times [0, 1] \) with heterogeneous individual book equity. To prevent banks from holding a perfectly diversified portfolio of firms and thus being immune from bankruptcies, I assume each bank finances only one firm in a given period.

2.2.3.1. Regulatory Environment

The government provides banks with a full deposit insurance. Such an explicit guarantee exempts banks from paying liquidation costs associated with bank failures and helps them raise deposits at the risk-free rate. After subtracting the tax shield of deposits, the effective deposit rate all banks borrow at is given by:

\[ R^d_{j,t} = \underbrace{R^f_t}_\text{deposit rate} - \underbrace{\tau(R^f_t - 1)}_\text{tax shield} \equiv R^d_t \quad \forall j \in J \quad (2.8) \]
A capital requirement $\bar{e}$ is set up by the government to restrict bank leverage. I assume this is the only tool regulators have in hand and it is not feasible for the government, due to a lack of expertise or information, to correctly price its guarantee and ask for a risk-sensitive deposit insurance fee.

2.2.3.2. The Bank’s Problem

The individual state variable of a long-lived bank is its equity. Bank $j \in J$ with equity $n_{j,t}$ maximizes its shareholder value by deciding on dividend rate $\epsilon_{j,t}$, book equity-to-asset ratio $e_{j,t}$, and firm $i$ to lend all of its levered assets to. Its value function $V^b(.)$ is given recursively by:

$$V^b(n_{j,t}) = \max_{\epsilon_{j,t}, e_{j,t} \geq \bar{e}, i \in I} \left\{ E_t M_{t+1} V^b(n_{j,t+1}) + [\epsilon_{j,t} - \lambda(\epsilon_{j,t})] n_{j,t} \right\},$$

(2.9)

where the equity next period is:

$$n_{j,t+1} = R_{i,j,t+1}^E (1 - \epsilon_{j,t}) n_{j,t},$$

(2.10)

and the levered gross return to inside equity is given by:

$$R_{i,j,t+1}^E = \frac{1}{\epsilon_{j,t}} \max \left\{ \left( \frac{P_{i,t+1}^b}{b_t} - c_{t+1}^b - R_{i,t}^d (1 - e_{j,t}) \right), 0 \right\}.$$

(2.11)

According to Proposition 5, $P_{i,t+1}^b/b_t$ stands for the realized return to all banks who lend to firm $i$ at the end of period $t$, which banks take as given. After paying an intermediation cost $c_{t+1}^b$ and the promised deposit obligation $R_{i,t}^d (1 - e_{j,t})$, equity holders of the bank get residual assets. In other words, a bank with equity choice $e_{j,t}$ will go bankrupt in period $t+1$ if the firm $i$ it lends to encounters a low enough realization of productivity shocks such that $P_{i,t+1}^b - [c_{t+1}^b + R_{i,t}^d (1 - e_{j,t})] b_t < 0$.

Two supply-side factors are considered. Firstly, $c_{t+1}^b$ in equation (2.11) is bank’s intermediation cost. To capture its counter-cyclicality, I formulate it as a function of the aggregate
productivity:

\[ c_t^b = c^b A_t^{-\psi}. \]  

(2.12)

Secondly, it is well-recognized that the reluctance for banks to alter their dividend payout is considerably strong. It is captured in a reduced-form fashion à la Jermann and Quadrini (2012):

\[ \lambda(\epsilon_{j,t}) = \frac{\kappa}{2} (\epsilon_{j,t} - \bar{\epsilon})^2, \]  

(2.13)

where \( \bar{\epsilon} \) is the long-run payout target set to the steady state value.

Before proceeding to derive bank policies, it is useful to establish the following property of \( V^b(.) \) which is essential for the tractability of my model:

**Lemma 1.** An individual bank’s value function is linear in its equity:

\[ V^b(n_{j,t}) = n_{j,t} V^b_t \quad \forall j \in J, \]  

(2.14)

where \( V^b_t \) depends only on aggregate state variables.

There are two elements that contribute to the linearity of the value function. First, banks face a constant-returns-to-scale technology as competitive financiers. This is reflected by equation (2.10): \( n_{j,t+1} \) is proportional to \( n_{j,t} \). Second, the dividend adjustment cost is imposed on the ratio rather than the level so that the total dividend adjustment payout is proportional to \( n_{j,t} \) as well.

2.2.3.3. Bank Policies and Aggregation

Each bank \( j \in J \) makes three choices: \( \{ i, \epsilon_{j,t}, e_{j,t} \} \). Firms are ex-ante identical and thus banks find them indifferent. The decision on \( i \) does not affect the choices of \( \epsilon_{j,t} \) and \( e_{j,t} \). Although banks have different individual equity when deciding on \( \epsilon_{j,t} \) and \( e_{j,t} \), the linearity I established in Corollary 1 means that these two policies will be identical across banks.
Substitute equation (2.14) into the right-hand side of (2.9), and I can then move \( n_{j,t} \) out of the parentheses. The maximization program no longer depends on \( n_{j,t} \). I get what follows:

**Proposition 6.** Banks find firms indifferent and adopt identical leverage and dividend policies:

\[
\epsilon_{j,t} = \epsilon_t \quad \text{and} \quad \epsilon_{j,t} = \epsilon_t \quad \forall j \in J. \tag{2.15}
\]

Adopting the same leverage, equity holders of different banks investing in the same firm shall get the same realized return, i.e. \( R_{s,t+1}^E(\epsilon_t) = R_{i,j,t+1}^E(\epsilon_t), \forall \{i,j\} \in I \times J \). Plug this condition together with (2.14) and (2.15) into (2.9) and we get:

\[
V^b_t = \epsilon_t - \lambda(\epsilon_t) + (1 - \epsilon_t) \mathbb{E}_t M_{t+1} V^b_{t+1} R^E_{i,t+1}(\epsilon_t). \tag{2.16}
\]

I can now directly utilize (2.16) to characterize the optimal bank policies \((\epsilon_t, \epsilon_t)\). The leverage choice is determined by a constrained optimization:

\[
e_t = \max \left\{ \bar{\epsilon}, \ \arg \max \limits_{\tilde{\epsilon}} \mathbb{E}_t M_{t+1} V^b_{t+1} R^E_{i,t+1}(\tilde{\epsilon}) \right\}, \tag{2.17}
\]

where the second term in the bracket stands for a globally optimal leverage. Notice that it does not depend on \( i \) because firms are ex-ante identical, i.e. \( R^E_{s,t+1} \sim R^E_{k,t+1}, \forall (s,k) \in I \).

The quadratic adjustment cost not only gives realistic dynamics to the model but also provides a handy expression of the optimal dividend policy:

\[
\epsilon_t = \bar{\epsilon} - \frac{1}{\kappa} \mathbb{E}_t M_{t+1} [V^b_{t+1} R^E_{i,t+1}(\epsilon_t) - R^f_t]. \tag{2.18}
\]

My model has an aggregation result: only the first moment of the distribution of individual bank equities has an effect on the aggregate economy. Although we have cross-sectional defaults in the banking sector, policy functions can be derived as if there is a representative
bank who has equity $N_t \equiv \int n_{j,t}dj$, chooses $(\epsilon_t, \epsilon_t)$ every period and finances all firms.

Apparently, $\epsilon_t$ and $\epsilon_t$ depend on aggregate state variables, including the aggregate bank equity $N_t$ that controls the total supply of bank loans. Although the individual net worth of a single bank does not affect its policies, there is a “financial accelerator effect” on the aggregate level. The aggregate bank equity plays a role in governing the dynamics of the economy. For instance, when the aggregate bank equity becomes scarce, the equity continuation value $V_{t+1}^b$ increases. All individual banks simultaneously scale back dividend payments and weakly reduce leverage by the same magnitudes irrespective of their individual equity.

### 2.2.3.4. Evolution of Aggregate Bank Equity

I now characterize the dynamics of the aggregate bank equity. Its law of motion is provided by:

\begin{align}
N_t &= (1 - \epsilon_t)N'_t; \quad (2.19) \\
bt &= \frac{N_t}{\epsilon_t}; \quad (2.20) \\
N'_{t+1} &= \int \max\{P_{i,t+1}^b - [c_{i,t+1}^b + R_{i,t}^d(1 - \epsilon_t)]b_t, 0\}di. \quad (2.21)
\end{align}

Given an aggregate bank equity of $N'_t$ at the end of period $t$, banks choose the same dividend rates $\epsilon_t$ and thus pay out in total $\epsilon_t N'_t$ (equation (2.19)). Furthermore, they make identical book leverage choices $1/\epsilon_t$. Total bank assets in this case are levered up to $N_t/\epsilon_t$, which in equilibrium equal the demand for bank finance $b_t$ (equation (2.20)).

Production in period $t+1$ then takes place and $N'_{t+1}$ is determined as the sum of individual equities of all surviving banks. As mentioned in the last section, since leverage choices are identical across banks, all banks that have lend to firm $i$ get the same realized equity return. This means that rather than keep track of all non-defaulted banks, we just need to identify all firms with $P_{i,t+1}^b - [c_{i,t+1}^b + R_{i,t}^d(1 - \epsilon_t)]b_t \geq 0$. The aggregate bank equity equals to the
sum of individual equities of banks who have lend to these firms (equation (2.21)).

2.2.4. Government

As noted before, the government imposes a capital requirement \( \bar{e} \) on banks while insures their deposits. A lump-sum consumption tax \( T_t \) is collected in period \( t \) to finance the insurance payout. More specifically, in dealing with defaulted banks, those who have lend to firm \( i \) with \( P_{i,t}^b - c_t^b b_{t-1} - R_{t-1}^d (1 - e_{t-1}) b_{t-1} \leq 0 \), the government has to cover the difference between recovered bank assets \( \chi (P_{i,t}^b - c_t^b b_{t-1}) \) and promised deposits \( R_{t-1}^d (1 - e_{t-1}) b_{t-1} \).

I get the total lump-sum tax by summing up the insurance transfers across all the firms whose bank lenders go down:

\[
T_t = \int_{i: P_{i,t}^f - [c_t^f + R_{t-1}^d (1 - e_{t-1})] b_{t-1} \leq 0} \left\{ R_{t-1}^d (1 - e_{t-1}) b_{t-1} - \chi (P_{i,t}^b - c_t^b b_{t-1}) \right\} di. \tag{2.22}
\]

Similar to equation (2.21), I again trace defaulted banks from the firm side.

2.2.5. Households

The general equilibrium is completed by a household sector. There exists a representative agent who holds all securities and collects all incomes. It maximizes the expected lifetime utility. Per-period utility function is in the form of CRRA:

\[
u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}, \tag{2.23}\]

The aggregate resource constraint is given by:

\[
c_t + k_{t+1} = \int y_{i,t} di + (1 - \delta) k_t - l_t, \tag{2.24}\]

where \( l_t \) captures all resource losses caused by corporate and bank bankruptcies:

\[
l_t = (1 - \chi - \xi) \left\{ \int_{i: P_{i,t}^f \leq 0} \pi_{i,t} di + \int_{i: P_{i,t}^b = [c_t^b + R_{t-1}^d (1 - e_{t-1})] b_{t-1} \leq 0} (P_{i,t}^b - c_t^b b_{t-1}) di \right\}. \tag{2.25}\]
Bankruptcies produce both a direct cost and an indirect cost. A direct cost, including fees paid to lawyers, accountants and consultants, is expressed as $\xi$. It is a transfer between agents in this economy upon bankruptcies. An indirect cost of liquidations includes destructions of customer relationships, brand values, synergies, etc, which I consider to be a resource loss.

2.2.6. Equilibrium Definition

Based on the aggregation result, a recursive competitive equilibrium is defined as a set of functions for (i) firms’ borrowing decisions $b(s)$ and $m(s)$; (ii) banks’ capital structure policies $e(s), \epsilon(s)$ and the associated value function scaler $V^b(s)$; (iii) corporate debt pricing functions $R^b(s; b, m)$ and $R^m(s; b, m)$; (iv) households’ policies $c(s)$ and $k'(s)$; and (v) law of motion for the aggregate states $s' = \Psi(s)$ such that:

1. Given $R^b(s; b, m), R^m(s; b, m), c(s), k'(s), s' = \Psi(s)$ and the debt settlement outcome in Proposition 5, firm policies $b(s)$ and $m(s)$ satisfy equation (6);
2. Given $R^b(s; b, m), R^m(s; b, m), b(s), m(s), c(s), k'(s), s' = \Psi(s)$ and the debt settlement outcome in Proposition 5, banks’ policies $e(s), \epsilon(s)$ and value function scaler $V^b(s)$ satisfy equations (16), (17) and (18);
3. Given $R^b(s; b, m), b(s), m(s), c(s), k'(s), s' = \Psi(s)$ and the debt settlement outcome in Proposition 5, non-bank debt pricing schedule $R^m(s; b, m)$ satisfies equation (7);
4. Given $R^m(s; b, m), b(s), m(s), e(s), \epsilon(s), c(s), k'(s), s' = \Psi(s)$ and the debt settlement outcome in Proposition 5, bank debt pricing schedule $R^b(s; b, m)$ satisfies equation (21);
5. Households’ policies $c(s)$ and $k'(s)$ maximize their lifetime utility;
6. Debt and final good markets clear and the law of motion $\Psi(s)$ is consistent with individual decisions and the stochastic processes for $A$.

2.2.7. Discussions of Assumptions

Before proceeding, it is useful to discuss several key modeling assumptions that I have made and their implications on the results.
2.2.7.1. Fully Debt-Financed Firms

The benefit of bank loans in reducing liquidation costs increases when firms face a higher downside risk. Given this is the source of the complementarity between bank and non-bank debt, the risk profile of firms is crucial for their counterfactual borrowing behaviors when capital requirements are changed. For instance, in Crouzet (2018), firms with a smaller net-worth depend more on banks and are less likely to substitute into bond finance upon bad shocks.

Firms in my model are short-lived and thus not able to accumulate internal net-worth. However, the lack of internal net-worth does not exaggerate the complementarity between bank and non-bank debt holding bankruptcy risk fixed. By calibrating the dispersion of the idiosyncratic shock $\sigma_z$ to match the distress frequency, my model should be able to generate a realistic level of complementarity between bank and non-bank finance for the aggregate production sector. It is also important to notice that my model is able to quantitatively match firm’s dependence on banks, which lends support to the amount of production risk and thus the value of restructurable loans produced my model.

2.2.7.2. Big Firms, Small Banks

Banks in my model are small relative to firms and are restricted to finance only one firm each period. This assumption is made in order to generate bankruptcies within the banking sector. If banks are able to hold a perfectly diversified portfolio of the aggregate production sector, they are not likely to go bankrupt given their assets are safe senior debt claims.

In reality, banks do not perfectly diversify their asset holdings. For instance, Acharya, Hasan and Saunders (2006) find that for high-risk banks, diversification in loan portfolio reduces bank return while producing riskier loans. Laeven and Levine (2007) find a market valuation discount for financial conglomerates engaging in diversification in activities.\(^\text{10}\) Harris, Opp and Opp (2017) point out scenarios in which banks specialize in certain projects to extract

\(^{10}\text{See Berger, Hasan and Zhou (2010) for a comprehensive review of the literature on the focus versus diversification of banks.}\)
government bailout subsidies.

Again, what matters for the quantitative investigation of capital requirements is the bankruptcy risk of banks and thus the magnitude of the deposit insurance subsidy. As long as the failure rate of banks is realistically captured, such a “small bank” assumption on risk diversification does not necessarily matter much for the aggregate implication of the model.

Another feature associated with small banks is their perfect competitiveness. The assumption of competitive lenders is standard in defaultable debt literature. A competitive banking sector is also widely adopted by the macro-banking literature. For example, even in Repullo and Suarez (2013) where authors focus on the “lock-up” effect of relationship banking, the market is competitive at the moment of the first loan.

2.2.7.3. Financial Intermediation: Value and Costs

Social Value of Intermediation Firm’s debt structure decision in this model builds on Crouzet (2018), which emphasizes the value of bank loans in providing restructuring flexibility. Theoretical corporate debt literature highlighting such a feature of bank loans includes for instance Berlin and Mester (1992), Chemmanur and Fulghieri (1994), Thakor and Wilson (1995), Bolton and Scharfstein (1996), Gorton and Kahn (2000), Bolton and Freixas (2000; 2006).\(^\text{11}\) Given my previous assumption that restricts banks to be small, it is useful to notice that theoretical argument for bank loans’ renegotiability does not necessarily rely on banks’ large sizes compared to bond holders. For instance, Chemmanur and Fulghieri (1994) argue that the long horizon of banks brings them a strong incentive to develop a reputation for financial flexibility and results in them devoting more resources to firm evaluation and debt restructuring compared to non-banks with short horizons. This argument is also consistent with my formulation.

Empirically, private debt contracts are frequently renegotiated when financial covenants

\(^\text{11}\) The uniqueness of banks in solving informational problems has been addressed by for example Diamond (1991), Rajan (1992), Holmstrom and Tirole (1997) and Boot and Thakor (1997). Lenel (2015) presents a two-period model where bank loans are unique in solving equity-debt conflicts. Bank and non-bank debt are complements in his model.
attached are violated upon bad shocks (Roberts and Sufi, 2009a; 2009b).

Firms’ leverage and investment policies are altered as creditors and borrowers maximize joint value and try to avoid costly bankruptcies.

*Costs of Intermediation* On the one hand, firms have to pay $\varphi$ when using intermediated credit. It can be interpreted as the cost associated with firms being monitored and constrained by banks. On the other hand, banks have to pay $c^b_t$ because of monitoring activities and security holdings for hedging purposes. I spread out the cost associated with intermediation on both firms and banks simply for the model to be quantitatively realistic. As will be discussed briefly in Section 2.4.3.2, the counter-cyclicality of $c^b_t$ helps produce pro-cyclical bank dividends and a sensible time variation of the “financial accelerator”.

Banks’ dividend adjustment cost makes loans expensive in recessions. Several papers present evidence that banks are reluctant to cut dividends even entering recessions, including for example Acharya et al. (2011), Abreu and Gulamhussen (2013) and Floyd, Li and Skinner (2015).

2.3. Mechanisms

The model incorporates two sets of frictions that lead to violations of Modigliani-Miller. The first set of frictions are taxes and bankruptcy losses, which serve as important motivations to optimize capital structures for both banks and firms. The second set includes frictions considered important in the banking sector: dividend adjustment costs, deposit insurance, and capital requirements.

My goal here is to illustrate the workings of the model. I first present how firms and banks make their decisions in section 2.3.1. I then discuss the role of capital requirements in section 2.3.2.

---

12 As Roberts (2015) documented, a large number of renegotiations are observed in the good time. These renegotiations take place as a way to complete contracts under ex-ante incomplete information, which is not the margin I consider here.
2.3.1. Optimal Policies

I first characterize the optimal decision of banks in this economy given firm behaviors. I then describe the debt choice of firms given optimal bank policies. My discussions will be largely centered on the deterministic steady state where I can show the properties of the model more transparently. How the bank dividend adjustment cost affects optimal policies in a stochastic environment will be briefly discussed at the end of each section.

2.3.1.1. Bank Policies

Taking lending returns \( P_{i,t+1}^{b} / b_t \), \( \forall i \in I \) – as given, banks decide on \( i, e_t \) and \( \epsilon_t \). Proposition 6 states that banks find firms indifferent when providing loans. Therefore, I focus on the two other choices – \( e_t \) and \( \epsilon_t \) – in the following discussion.

In the deterministic steady state, the adjustment cost by construction disappears and the continuation value of the bank equity is fixed to unity. Deposit tax shields create a wedge between the required return to depositors and the effective cost of deposits to banks. With a government guarantee, banks are able to extract such a wedge without being charged for potential bankruptcy losses by depositors. Banks, therefore, have a strict incentive to lever up until the capital requirements bind, i.e. \( e_t = \bar{e} \).

Meanwhile, equity holders of banks earn a risk-free rate because there is no adjustment cost and banks are perfectly competitive. This means that the payout ratio \( \epsilon_t = 1 - \beta \). We have the following proposition:

**Proposition 7.** In the deterministic steady state, capital requirements bind, i.e. \( e_t = \bar{e} \).

The bank dividend rate \( \epsilon_t = 1 - \beta \). The value of the aggregate bank equity \( V_t^b = 1 \).

In a dynamic environment with the presence of an adjustment cost, the aggregate bank equity can sometimes become scarce and the continuation value can become greater than one. Under those circumstances, failures are more costly for banks. Therefore, banks’ incentive to keep pushing up leverage is weakened while their willingness to pay dividends drops. Capital requirements can still be binding in this case as long as the increase in the
continuation value does not overturn the dominant impact of deposit insurance. In contrast, during the good time when the aggregate bank equity is abundant, leverage and dividend policies tend to be more aggressive. Capital requirements become even more restrictive in these scenarios. It turns out that capital requirements always bind in my quantitative analyses.

2.3.1.2. Corporate Debt Choice

Since firms internalize the impact of debt choices on debt prices, I first characterize how pricing schedules look like before moving into firm’s problem. Given banks’ optimal policies described in the last section, loan pricing schedule in the deterministic steady state – $R^b(.)$ – is given by the zero-profit condition of equity holders of banks:\footnote{This can be easily derived through equations (2.19), (2.20) and (2.21) by setting $N_{t+1} = N_t$ together with bank policies $e_t = \bar{e}$ and $\epsilon_t = 1 - \beta$ as in Proposition 7.}

$$E_t M_{t+1} \left( \frac{P^b_{i,t+1}}{b_t} - c^b_{t+1} - R^d_t (1 - \bar{e}), 0 \right) - R^f_t = 0 \quad \forall i \in \mathbf{I}. \quad (2.26)$$

To compare with the non-bank debt pricing schedule in equation (2.7), I re-write the above equation as:

$$E_t M_{t+1} \left( \frac{P^b_{i,t+1}}{b_t} - R^f_t 
+ \left[ (1 - \bar{e})\tau(R^f_t - 1) + \max \left\{ c^b_{t+1} + R^d_t (1 - \bar{e}) - \frac{P^b_{i,t+1}}{b_t}, 0 \right\} - c^b_{t+1} \right] \right) = 0 \quad \forall i \in \mathbf{I}. \quad (2.27)$$

The first two terms in the bracket mimic the pricing schedule for non-bank debt. The second line describes how loan pricing is different. Intermediated credit enjoys two types of subsidies. The first term in the second line represents the deposit tax shields. The second one represents the deposit insurance transfer: the gap between banks’ expenditure on intermediation activities and deposits $c^b_{t+1} + R^d_t (1 - \bar{e})$ and loan return $P^b_{i,t+1}/b_t$ in banks’ failure states. However, conducting intermediation activities is expensive and the loan yield
has to cover such a cost $c^b_{t+1}$. Under my calibration, the intermediation cost dominates the subsidies, making bank finance relatively more expensive on the supply side.

With pricing equations (2.7) and (2.27), I am ready to characterize firm policies in the deterministic case. The total amount of borrowing $b_t + m_t$ is mainly governed by the decreasing returns to scale technology together with the tax-bankruptcy trade-off. Bank dependence of the debt structure, $s_t \equiv b_t/(b_t + m_t)$, is encouraged by loans’ benefit in reducing liquidation losses while discouraged by the costs paid both on the demand side ($\varphi$) and the supply side (second line in (2.27)) of intermediated credit.

Formally, substitute (2.7) and (2.27) into firms’ objective described jointly in equation (2.6) and the Panel A of Proposition 5, where my simulated economy will be located at. I arrive at the following steady-state expression for the expected firm payoff:

$$
\mathbb{E}_z P^f(b, m, z) \big| R^b(b, m), R^m(b, m)
$$

$$
= \beta \left\{ \int_{-\infty}^{\infty} \pi(z) d\Phi(z) - R^f(b + m) + [1 - \Phi(z^f)]\Theta^f - \int_{-\infty}^{z^f} (1 - \chi) \pi(z) d\Phi(z) \right\} - [c^b - \tau(R^f - 1)(1 - \bar{e})]b - \int_{-\infty}^{z^b} \left\{ ((1 - \bar{e})R^d + c^b)b - \chi \pi(z) \right\} d\Phi(z) \right\},
$$

(2.28)

where $z^f$ and $z^b$ are respectively bankruptcy cutoffs of the idiosyncratic shock for firms and banks: $P^f(b, m, z^f) \big| R^b(.), R^m(.) = 0$ and $P^b(b, m, z^b) \big| R^b(.), R^m(.) = [c^b + R^d(1 - \bar{e})]b = 0$.\(^{14}\) The first line on the right hand side is common to all defaultable debt models – firms get all the production income (net of the fixed cost $\varphi$ and lenders’ opportunity cost $R^f(b + m)$)

\(^{14}\)The equation is expressed by assuming that a full repayment to banks will not cause their failures, i.e. $P^b(b, m, z^b) \big| R^b(.), R^m(.) = \chi \pi(b, m, z^b)$. Define the idiosyncratic shock cutoffs for debt restructuring $z^r : \pi(b, m, z^r) = \Pi^b/\chi$ and firm default $\pi(b, m, z^d) = \Pi^b + \Pi^m - \Theta^f$. It will be true under my calibration that:

$$z^r > z^d > z^f > z^b.$$

Apparently, these cutoffs vary across time when there is aggregate uncertainty. Above inequalities also imply that two layers of bankruptcy losses are incurred for firms with $z \leq z^b$. I provide a detailed derivation of equation (2.28) in Appendix 2.8.1.
together with corporate tax shields but have to pay for the losses associated with their bankruptcies. Moreover, they also have to cover additional costs incurred by banks.

Figure 11 plots firms’ expected payoffs expressed in equation (2.28) with respect to a set of debt structures in the neighborhood of the deterministic steady state debt choice \( (k = 4.145, s = 0.379) \). Firms’ objective is locally concave with respect to both \( k \) and \( s \). Figure 11 shows that the optimal bank dependence \( s \) increases together with the scale of total financing \( k \). Given the decreasing returns to scale, firms enter troubles more frequently under a larger production scale. They thus have a willingness to depend more heavily on banks in order to exploit the restructuring benefit of bank loans.

![Figure 11: The Firm’s Problem. Notes: This figure plots the firm’s expected payoff in a deterministic environment under the debt structures in the neighborhood of the steady state debt choice computed with first-order conditions – \( (k = 4.145, s = 0.379) \).](image)

With the aggregate uncertainty, the bank dividend adjustment cost gives rise to the “financial accelerator effect”: the aggregate bank equity starts to influence corporate debt choice and thus production. When bank balance sheets are hurt in a recession, loans become relatively more expensive as banks start to ask for a strictly positive expected return to compensate the possible loss of the continuation value in bankruptcies. In contrast,
when the aggregate bank equity is abundant, banks are willing to lend even with a negative expected return rather than pay out dividends just to avoid incurring the adjustment cost. Variations in total finance and bank dependence shall be amplified by the “financial accelerator effect”.

2.3.2. *Deposit Insurance and Capital Requirements*

The costs associated deposit insurance are incurred on both the banking and the firm sides. First, failed to internalize the impact of their leverage decisions on deposit price, banks absorb excessive deposits and become fragile. Large liquidation costs associated with bank failures are incurred. Second, firms borrow aggressively and rely heavily on banks. Households’ consumption becomes insufficient as the deposit insurance taxes consumption to subsidize investment. Although firms adopt higher leverage and enter distress more frequently, unlike banks, corporate liquidations do not necessarily have to be more frequent thanks to a debt structure tilting towards restructurable loans. In fact, as will be shown in section 2.5.2, tightening capital requirements decreases firms’ default probability but increases their bankruptcy probability. Quantitatively, strengths of these forces associated with liquidations are driven largely by bankruptcy losses and tax shields.

While liquidation losses and production distortions are present even without aggregate shocks, in a stochastic environment, the “financial accelerator effect” is exacerbated as the aggregate bank equity becomes more volatile because of excessive bank failures. The impact of this dynamic channel crucially depends on the linearity of the model.

Capital requirements help constrain banks from taking leverage. Firstly, bank liquidation losses and distortions on the production side are alleviated. Second, the “financial accelerator” amplification declines.

However, a too aggressive capital regulation might lead to insufficient bank leverage taking as it reduces deposit tax shields in addition to insurance transfers. Again, consider the second term of equation (2.27). Raising $\bar{e}$ weakly suppresses not only the deposit insurance
transfers \max\{c_{t+1}^b + R_t^d (1 - \bar{e}) - P_{t+1}/b_t, 0\} \) but also deposit tax shields \((1 - \bar{e})\tau (R_t^d - 1)\) state by state. Since the former term is convex while the latter is linear with respect to \(\bar{e}\), the reduction in deposit tax shields will ultimately become dominant and make bank finance luxury.\(^{15}\)

2.4. Quantitative Assessments

I first describe the parameter choices and the method with which the model is solved. Fitness of the model is then assessed.

2.4.1. Parameters

The period of the model is a year. Split into two groups, parameter choices are presented in Table 7. The first group contains fairly standard parameters in the literature: discount rate \(\beta\), risk aversion \(\gamma\), aggregate productivity persistence \(\rho_a\) and dispersion \(\sigma_a\), corporate tax rate \(\tau\), and capital depreciation rate \(\delta\). The capital curvature is set to be 0.5 following Jermann and Yue (2018).

Bank assets in this economy contain only corporate loans, which are assigned with a 100% risk weight under Basel Accords. Banks finance themselves through deposits and equity. Therefore, various risk-based capital ratios and the total leverage ratio collapse to one in this model. I map \(\bar{e}\) to the total risk-based capital ratio. Basel I and II require the total capital ratio to be no less than 8%. Basel III requires a combined Tier 1 and Tier 2 capital ratio of at least 8% for a bank holding company to be considered adequately capitalized. As a result, I set \(\bar{e} = 0.08\).

I set \(\chi\) to be 0.38, median of asset recovery rates of firms going through a Chapter 7 bankruptcy documented by Bris, Welch and Zhu (2006).\(^{16}\) The direct cost of liquidations is considered to be small. I set \(\xi = 0.06\) in accordance with the estimate of Altman (1984).

\(^{15}\)Corporate tax shields proportional to interest rates will partly weaken firms’ incentive to move away from banks when loans become more expensive.

\(^{16}\)Similar values are also reported by Acharya, Bharath and Srinivasan (2007) and Corbae and D’Erasmo (2017). For example, Corbae and D’Erasmo (2017) report a median recovery rate of 49.09% in Chapter 11 and 5.80% in Chapter 7. With the probability of Chapter 11 equal to 79.15%, a rough calculation gives a bankruptcy recovery rate of 40%.
<table>
<thead>
<tr>
<th>Value</th>
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<td>$\varphi$</td>
<td>0.25</td>
<td>firm compliance cost</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.1</td>
<td>dividend adjustment cost</td>
</tr>
<tr>
<td>$c^b$</td>
<td>0.06</td>
<td>bank lending cost</td>
</tr>
<tr>
<td>$\psi$</td>
<td>3.4</td>
<td>bank lending cyclicality</td>
</tr>
</tbody>
</table>

Table 7: Parameters. Notes: This table reports benchmark parameter choices. The upper panel includes parameters following existing literature and regulatory requirements. The lower panel includes parameters that are calibrated.

The second set of parameters are calibrated to match empirical moments between 1988, the year when the Basel Capital Accords were created, and 2015. The variance of idiosyncratic productivity shock $\sigma_z$ is set so that the frequency of debt restructuring in my model matches that of covenant violation in the data.\footnote{The majority of covenant violations lead to debt restructuring (Roberts, 2015).} Firms’ covenant compliance cost $\varphi$ is identified via bank failure rates. The intermediation cost $c^b$ is set to match the mean of loan spreads. For the second moments, the bank dividend adjustment cost $\kappa$ and the cyclicality of their intermediation cost $\psi$ jointly target the volatility of commercial banks’ dividends and that of loan spreads.

### 2.4.2. Solution Method

I adopt third-order perturbation with pruning (Andreasen, Fernández-Villaverde and Rubio-Ramírez, 2013). Local methods are faced with two challenges. First, although capital requirements bind in the deterministic steady state, they can become occasionally binding with aggregate risk: $\arg \max_{\bar{e}} E_t M_{t+1} V_{t+1}^{b} R_{t+1}^{E} (\bar{e}) \geq \bar{e}$. Second, as illustrated in Propo-
sition 5, firms’ payoffs are characterized by two sets of equations depending on their debt choices.

In the US, the aggregate equity ratio adopted by commercial banks does not vary across business cycles (Adrian and Shin, 2010) and is fairly close to the equity requirement.\(^{18}\) Moreover, debt restructurings between banks and firms take place regularly. Inspired by these two observations, I solve the model with the conjectures that capital requirements always bind and corporate debt choices always fall into the panel \(A\) of Proposition 5.\(^{19}\) I verify these two conjectures ex-post by examining the simulated path. Throughout all simulations in both the benchmark and counter-factual analyses, neither one of the these conjectures has been violated for more than 0.1% of the time. More details can be found in Appendix 2.8.2.

The fact that capital requirements are still binding in recessions, similar to typical models with the financial accelerator, also suggests that the model is fairly linear. Relatedly, my results will largely be unchanged when I solve the model using first-order perturbation.

2.4.3. Model Assessments

What is first laid out in this section are the comparisons between unconditional sample moments generated from the simulated series of key variables in the model and their data counterparts. Impulse response functions are then presented in order to illustrate the dynamic behaviors of the model.

2.4.3.1. Unconditional Moments

Table 8 shows that the model does a reasonable job in matching firm, bank and macro moments of the US since the establishment of the Basel Accords.

\(^{18}\)Given the complexity of the capital regulation, it is challenging to show empirically the tightness of each capital requirement (Cecchetti and Kashyap, 2016). For the US commercial banking sector, the average total risk-based capital ratio between 1990 (when FDIC sample starts for this variable) and 2015 is 9%. The time-series average of the un-adjusted equity ratio between 1988 and 2015 is 9%. Kisin and Manela (2016) document that the largest US banks utilized a loophole to bypass capital requirements.

\(^{19}\)Conjecture-verify approaches have been widely adopted in solving medium-scale macro models with collateral constraints (e.g. Gertler and Kiyotaki, 2010 and Jermann and Quadrini, 2012) and capital requirements (e.g. Begenau, 2015).
Table 8: Unconditional Moments. Notes: This table compares annual moments generated from the simulated series and their data counterparts. Series between 1988 and 2015 are utilized to construct empirical moments. The model is simulated for 5000 periods before the calculation of unconditional moments. Moments with * have been utilized in the calibration. Details can be found in Online Appendix 2.8.3.

The firm side of the model is simplified for tractability reasons. Whether it is realistic shall be important for the credibility of the counter-factual predictions. The fact that the model is able to approximately match the debt structure, default probability and recovery rates in defaults, without targeting specifically, lends support to my specification of firm problem and what has been shown in Proposition 5.20

The variation in portfolios tend to be tiny when the model is linear while that in tail statistics depends sensitively on the shape of the shock distribution. The model finds it difficult to match the standard deviations of bank dependence, firm default probability and

20Due to the availability of aggregate data, I regard public firms with a BB/Ba rating as an representative of the aggregate production sector. As clear from Table 8, in a sample provided by Rauh and Sufi (2010), the asset-weighted average bank dependence of BB-rated firms is fairly close to the loan-to-liability ratio of the US non-financial businesses in the Flow of Funds.
bank failure probability. However, the failure to replicate these moments should not be a major concern in the following welfare analysis as the model is close to linear.

2.4.3.2. Impulse Responses

![Figure 12: Impulse Responses. Notes: This figure shows the impacts of a positive shock to productivity, \( \ln A_t \), of one standard deviation (1.6%). Generalized impulse response functions initialized at the mean of the ergodic distribution are plotted.](image)

Although second moments might not be quantitatively important for the welfare analysis, it is still interesting to see whether the dynamic aspects of the model are realistic. I consider a positive shock of one standard deviation (1.6%) to the aggregate productivity \( \ln A_t \). Impulse response functions are plotted in Figure 12. The aggregate consumption, output, investment and capital increases in response to the shock. The counter-cyclical lending cost helps the model generate the observed pro-cyclical bank dividends. Only under such pro-cyclicality, the adjustment cost is able to produce a sensible financial accelerator effect – over-lending in the boom and slow recoveries from recessions. The pro-cyclicality of bank dependence

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21I don’t have a time series of the probability of covenant violation. Moreover, given a default rate, the bankruptcy rate and credit recovery rates are closely linked in my model. Because of the difference in data quality, I use recovery rates for the purpose of assessments.

22Without the time variation in the intermediation cost, the aggregate bank net-worth tends to highly stable because bank loans are safe senior claims. Variations in productivity and thus the demand for loans are much larger. In that case, inconsistent with the data, banks would like to cut dividends in booms and pay out in crises.
is in line with the evidence presented by Adrian, Colla and Shin (2012) and Becker and Ivashina (2014).

The consistency between the model implications and the US empirical evidence, with noted exceptions above, strengthens the credibility of the counter-factual welfare analysis carried out in the coming sections.

2.5. Implications of Capital Requirements

In this section, counterfactual analyses are carried out to investigate aggregate implications of capital requirements. More specifically, I solve and simulate the model for different levels of capital requirements ranging from 7% to 15% with all the other parameters fixed to Table 7. I then compare across the unconditional moments of the simulated series.

2.5.1. Bank and Non-Bank Debt

![Figure 13: Debt Choices. Notes: This figure presents how debt quantities and debt prices vary when capital requirements change between 7% and 15%. The model is simulated for 5000 periods before the calculation of unconditional moments.](image)

To comply with a tighter capital requirement, banks start to charge a wider loan spread because of reductions in deposit insurance subsidies and deposit tax shields. With loans
becoming more expensive, firms cut back on bank finance. In line with the empirical literature on the impact of an increase in capital requirements, the magnitudes of changes in price and quantity are fairly small. Starting from the status quo (8%), a one percentage point increase in the required equity ratio transmits to a loan spread increase of 0.85 basis points and a bank lending drop of 0.27%.

Bank and non-bank debt turn out to be complements. The share of non-bank finance increases by 2.76 basis points when the capital ratio increases by one percentage point. However, the debt substitution at the micro level is dominated by the complementarity at the macro level. The amount of non-bank finance drops by 0.15% as the total borrowing responses more drastically than the bank dependence $s$. Although firms de-lever, the non-bank debt yield increases as the restructuring role of banks is constrained.

2.5.2. Several Frictions

The first four plots in Figure 14 depict respectively probabilities of debt restructuring, firm default, firm bankruptcy, and bank failure (Recall footnote 14). When the capital requirement is tightened, firms and banks cut back on borrowing. The frequency of strategic restructuring remains stable, while firms’ default probability shrinks sharply. Bank failures are almost eliminated when the regulatory constraint is raised beyond 14%.

Consistent with a rise in the non-bank debt yield, corporate bankruptcies are more frequently observed when the restructuring flexibility of bank finance is weakened. Although corporate bankruptcy losses increase when banks become constrained, within the range I plot, they are quantitatively dominated by the drop in banks’ bankruptcy losses. Unreported results show that when capital requirements go beyond 16.7%, total bankruptcy losses start to increase.

An improvement in the bank capital adequacy sharply reduces the bank failure probability and makes the aggregate bank dividend much less volatile. The “financial accelerator”

\[23\text{\textsuperscript{For example, Kisin and Manela (2016) estimate that a one percentage point increase in capital requirements would lead to no more than a 0.3-basis-point increase in banks’ cost of capital and a 0.15 percent reduction in the quantity of lending.}}\]
friction is alleviated and the dividend adjustment cost drops. However, the absolute scale of such a drop is fairly almost trivial.

2.5.3. Macroeconomy and Welfare

Starting from a low capital ratio, tightening the leverage restriction plays a corrective role in removing distortions brought by the deposit insurance – large bankruptcy losses, a strong “financial accelerator effect” and an over-investment problem. However, when the banking sector becomes sufficiently safe, to keep raising capital requirements starts to restrict production due to a reduction in deposit tax benefits. In consequence, turning points in output and consumption are witnessed.

Lifetime utility exhibits an inverted-U shape and achieves maximum at 11%, about 3 percentage points higher than what is currently implemented under the Basel Capital Accords. I perform a Lucas (1987)-style calculation to evaluate the welfare implications of the capital regulation. Compared to the status quo, implementing the optimal capital ratio yields a
Figure 15: Macroeconomy and Welfare. Notes: This figure presents how output, investment, consumption and utility vary when capital requirements change between 7% and 15%. The model is simulated for 5000 periods before the calculation of unconditional moments.

welfare gain of 0.035%. Aggregate corporate borrowing drops by 0.41% and output declines by 0.18%. Such a small welfare gain is in line with other business cycle analyses such as Begenau (2015).

2.6. Further Analyses

In Table 9, I illustrate how parameter choices governing the welfare trade-off I made in section 2.4.1 affect the optimal capital requirement. The magnitudes of the marginal benefits of raising capital requirements – reducing liquidation costs and the “financial accelerator” distortion – are controlled respectively in $\xi$ and $\kappa$. The tax rate $\tau$ affects the turning point where the economy transits from an over-investment/intermediated region to an under-investment/intermediated region.

The first parameter I alter in this exercises is $\xi$, which controls how socially expensive bankruptcies are. Two alternative values I experiment with are 0.12 and 0.18. Given an asset recovery rate of 0.38, they represent respectively a liquidation resource cost of 50%...
Table 9: Alternative Parameters. Notes: This table shows how optimal capital requirements vary under parameter choices different from Table 7. Values with * are used in the benchmark analysis. Last three columns present changes in welfare, bank and non-bank finance when capital requirements increase from 0.08 to the optimum presented in column 3.

<table>
<thead>
<tr>
<th>Value</th>
<th>Optimal CR</th>
<th>Welfare Gain (0.01%)</th>
<th>Δb (%)</th>
<th>Δm (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ξ 0.06*</td>
<td>0.11</td>
<td>3.48</td>
<td>-0.58</td>
<td>-0.32</td>
</tr>
<tr>
<td>0.12</td>
<td>0.10</td>
<td>1.48</td>
<td>-0.45</td>
<td>-0.25</td>
</tr>
<tr>
<td>0.18</td>
<td>0.09</td>
<td>0.04</td>
<td>-0.27</td>
<td>-0.15</td>
</tr>
<tr>
<td>κ 0.10*</td>
<td>0.11141</td>
<td>3.4845</td>
<td>-0.5767</td>
<td>-0.3244</td>
</tr>
<tr>
<td>0.15</td>
<td>0.11148</td>
<td>3.4957</td>
<td>-0.5774</td>
<td>-0.3247</td>
</tr>
<tr>
<td>0.20</td>
<td>0.11155</td>
<td>3.5070</td>
<td>-0.5782</td>
<td>-0.3249</td>
</tr>
<tr>
<td>τ 0.35*</td>
<td>0.111</td>
<td>3.48</td>
<td>-0.58</td>
<td>-0.32</td>
</tr>
<tr>
<td>0.30</td>
<td>0.112</td>
<td>3.36</td>
<td>-0.59</td>
<td>-0.34</td>
</tr>
<tr>
<td>0.25</td>
<td>0.113</td>
<td>3.14</td>
<td>-0.60</td>
<td>-0.35</td>
</tr>
</tbody>
</table>

and 44%. When ξ becomes smaller, bankruptcies of banks and firms become more expensive for households. As a result, the optimal capital requirement should be tighter in order to prevent bankruptcies.24

The second panel captures the role capital requirements play in alleviating the “financial accelerator” distortion. The adjustment cost causes persistent booms and recessions. However, with the presence of an over-investment problem in the deterministic steady state, certain slow recoveries from mild recessions can turn out to be welfare improving. My experiment suggests that the “financial accelerator” is overall welfare-destructive.25 As the dividend adjustment cost goes up, the high leverage of banks distorts their credit provisions more heavily. Consequently, the optimal capital requirement rises.

A too harsh capital requirement can result in socially insufficient investment and production because of an elimination of the deposit tax shields. As the tax rate τ increases, the economy is going to enter the under-investment/intermediated region at a faster speed. Capital regulation should therefore be less aggressive so as not to restrict banks from creating unique values.

24Recall that the increase in corporate bankruptcies is quantitatively dominated by a decline in bank failures for capital requirements between 7% and 15%.

25For a given level of capital requirements, the lifetime utility of households reduces when κ increases. These results are available upon request.

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In terms of the magnitudes, bankruptcy costs produce fairly strong impacts. Influences of the tax rate choice is much milder. Since the model is close to linear, second moments contribute trivially to welfare.

2.7. Conclusion

This paper has presented a macro-banking model with an endogenous corporate debt choice between bank and non-bank finance. Intermediated credit is costly for firms but provides debt restructuring options that reduce bankruptcy losses. The model is calibrated to the US data to study the impact of capital regulation.

Raising capital requirements alleviates distortions induced by the deposit insurance – frequent bank liquidations, distorted bank lending and excessive corporate investment. However, it also removes deposit tax shields and can thus result in socially insufficient bank financing.

Interestingly, because of the restructuring flexibility of bank loans, bank and non-bank credit serve as complements on the aggregate level. As capital requirements become tight, firms suffer a decline in production efficiency and go bankrupt more frequently. Non-bank finance drops.

Welfare is hump-shaped and maximized when the capital ratio is set to 11%. When capital requirements are raised from 8% to the optimum, a lifetime consumption gain of 0.035% can be achieved. Aggregate corporate borrowing and output drop respectively by 0.41% and 0.18%.

Incorporating financial shocks, addressed for example by Jermann and Quadrini (2012), Christiano, Motto and Rostagno (2014) and Bassett et al. (2014), into this economy might yield richer and more realistic dynamics. This can partially be achieved by making \( c_t^b \) a separate stochastic process rather than a function of \( A_t \). It could also be interesting to extend this framework to quantify the implications of bank liquidity requirements.


2.8. Appendix

2.8.1. Proofs

2.8.1.1. Proposition 5

This proposition is a straightforward extension of Crouzet (2018) with an assumption that tax shields are non-transferable upon bankruptcies. The proof is neglected to save space.

2.8.1.2. Lemma 1 and Proposition 6

Conjecture \( V^b(n_t) = n_t V^b_t \) and substitute it into the right-hand side of bank \( j \)'s value function in equation (2.9):

\[
V^b(n_{j,t}) = \max_{\epsilon_{j,t}, \epsilon_{j,t} > \bar{\epsilon}_i} \left\{ E_t M_{t+1} V^b_t \left( b_{i,j,t+1}^b (1 - \epsilon_t) n_{j,t} + [\epsilon_{j,t} - \lambda(\epsilon_t)] n_{j,t} \right) \right\}.
\]

I have verified the conjecture and proved Corollary 1.

Moreover, the maximization program on the right-hand side no longer depends on the individual state variable \( n_{j,t} \). Since \( P_{b,x,t}^b \sim P_{b,v,t}^b \), \( \forall (x,v) \in I \), two banks get the same expected return regardless of which firm they choose to finance individually as long as their leverage and dividend policies are identical. Therefore, the optimal policies \( \epsilon_{j,t} \) and \( \epsilon_{j,t} \) are the same across all banks. I have proved Proposition 6.

2.8.1.3. Proposition 7

1) \( V^b_t = 0 \)

Since there is neither aggregate uncertainty in the deterministic steady state, \( M = \beta \), nor a dividend adjustment cost \( \lambda(\bar{\epsilon}) = 0 \), the law of motion for aggregate bank equity described in equation (2.16) is reduced down to:

\[
V^b_t = \epsilon_t + (1 - \epsilon_t) \beta E_t V^b_{t+1} R^E_{t,i,t+1}.
\]

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In a steady state where both bank and non-bank debt exist, \( \epsilon_t \) can neither be 0 nor infinity. To guarantee a well-defined banking sector:

\[
\beta E_t V^b_{t+1} R^E_{i,t+1} = 1 \Rightarrow V_t^b = 1.
\]

2) \( \epsilon_t = 1 - \beta \)

Let’s move to the optimal dividend policy. Since banks are indifferent between paying dividends or retaining earnings, the equilibrium dividend rate \( \epsilon_t \) is given by equation (2.19):

\[
N_t = (1 - \epsilon_t)N'_t = (1 - \epsilon_t)N'_{t+1},
\]

from which it is clear that we are done if \( N'_{t+1} = R^f_t N_t \).

Consider the law of motion of bank equity in the deterministic steady state:

\[
N'_{t+1} = \int_{z^b_{t+1}}^{\infty} \Pi_t^b d\Phi(z) + \int_{z^r_{t+1}}^{z^b_{t+1}} \chi \pi_{i,t+1}(z) d\Phi(z) - \int_{z^b_{t+1}}^{\infty} [(1 - \epsilon_t)R^d_t + c^b_t]b_t d\Phi(z),
\]

where the debt restructuring cutoff is given by \( \pi(z^r_{t+1}) = \Pi^b_t + \Pi^m_t - \Theta^f_t \). Bank failure cutoff is given by: \( \chi \pi(z^b_{t+1}) = [(1 - \epsilon_t)R^d_t + c^b_t]b_t \). Bank’s expected return can be expressed as:

\[
E_t R^E_{i,t+1} = \int R^E_{i,t+1}(z) d\Phi(z) = \frac{N'_{t+1}}{\epsilon_t b_t} = \frac{N'_{t+1}}{N_t} = R^f_t,
\]

where the last two equalities come from respectively equation (2.20) and the result we have got in part 1) of this proof.

3) \( \epsilon_t = \bar{\epsilon} \)
Write out banks’ objective function in the steady state:

\[ E_t R_{i,t+1}^E = E_t \frac{1}{e_t} \max \left\{ \frac{P_{b,t+1}^b}{b_t} - c^b - R_t^d (1 - e_t), 0 \right\} \]
\[ = \frac{1}{e_t} \int_{z^b}^{\infty} \left[ \frac{P_{b,t+1}^b(z)}{b_t} - c^b - R_t^d (1 - e_t) \right] d\Phi(z) \]

and differentiate it w.r.t. \( e_t \):

\[ \frac{1}{e_t^2} \left\{ [1 - \Phi(z^b)] R_t^d e_t - \int_{z^b}^{\infty} \left[ \frac{P_{b,t+1}^b(z)}{b_t} - c^b - R_t^d (1 - e_t) \right] d\Phi(z) \right\} = \frac{[1 - \Phi(z^b)] R_t^d - E_t R_{i,t+1}^E}{e_t} \]

Since we know from part 2) that \( E_t R_{i,t+1}^E = R_t^f \), the above derivative equals to:

\[ \frac{[1 - \Phi(z^b)] R_t^d - R_t^f}{e_t} < 0, \]

where the last step goes through because of the tax shield associated with deposits: \( R_t^d < R_t^f \).

2.8.1.4. Derivation of Equation (2.28)

In Panel A, the objective function of a firm is:

\[ \frac{1}{\beta} \left\{ \int_{z_{i+1}}^{\infty} (\pi_{i,t+1}^b - \Pi_t^m + \Theta_t^f) d\Phi(z) + \int_{z_{i+1}}^{z_{i+1}^f} [(1 - \chi) \pi_{i,t+1} - \Pi_t^m + \Theta_t^f] d\Phi(z) \right\}, \]

where the firm bankruptcy cutoff is given by \((1 - \chi) \pi(z_{i+1}) = \Pi_t^m - \Theta_t^f \).

The zero profit condition of non-bank investors in steady state is given by:

\[ \int_{z_{i+1}}^{\infty} \Pi_t^m d\Phi(z) = [1 - \Phi(z_{i+1}^f)] \Pi_t^m = R_t^f m_t. \]

As established in Proposition 7, banks earn zero excess profit in steady state, i.e., \( N_{t+1}^t - \)
\[ R_t^f N_t = 0, \text{ and keep maximal leverage } \bar{e}. \text{ We have the following:} \]
\[
\int_{z_{t+1}^f}^{\infty} \Pi_t^b d \Phi(z) + \int_{z_{t+1}^b}^{z_{t+1}^f} \chi \pi_{i,t+1}(z) d \Phi(z) - \int_{z_{t+1}^b}^{\infty} [(1 - \bar{e}) R_t^d + c^b] b_t d \Phi(z) = R_t^f b_t \bar{e} \\
\iff - \int_{z_{t+1}^f}^{\infty} \Pi_t^b d \Phi(z) - \int_{z_{t+1}^b}^{z_{t+1}^f} \chi \pi_{i,t+1}(z) d \Phi(z) \\
= \int_{-\infty}^{z_{t+1}^f} \left\{ [(1 - \bar{e}) R_t^d + c^b] b_t - \chi \pi_{i,t+1} \right\} d \Phi(z) - \left[ R_t^d + (R_t^f - R_t^d) \bar{e} + c^b b_t \right].
\]

After substituting pricing equations into the firm’s objective function, we have the expression written as equation (2.28):
\[
\beta \left\{ \int_{z_{t+1}^f}^{\infty} (\pi_{i,t+1} - \Pi_t^b) d \Phi(z) + \int_{z_{t+1}^f}^{z_{t+1}^b} (1 - \chi) \pi_{i,t+1} d \Phi(z) - R_t^f m_t + [1 - \Phi(z_{t+1}^f)] \Theta_t^f \right\} \\
= \beta \left\{ \int_{-\infty}^{\infty} \pi_{i,t+1} d \Phi(z) + [1 - \Phi(z_{t+1}^f)] \Theta_t^f + \int_{-\infty}^{z_{t+1}^f} \left\{ [(1 - \bar{e}) \bar{R}_t^d + c^b] b_t - \chi \pi_{i,t+1} \right\} d \Phi(z) \\
- \int_{-\infty}^{z_{t+1}^f} (1 - \chi) \pi_{i,t+1} d \Phi(z) - \{ R_t^f m_t + [R_t^f - R_t^d \bar{e} + c^b b_t \right\} \right\}. 
\]

2.8.2. Verifying Conjectures

After the model is solved, the conjecture that simulated path stays in the panel \( A \) of Proposition 5 is verified using the following condition
\[
J_t^f \equiv \frac{R_t^b b_t}{\chi} - \frac{R_t^m m_t - \Theta_t^f}{1 - \chi} > 0.
\]

The conjecture that CR always binds is verified by making sure banks have an incentive to still push up leverage when the capital requirements are already hit. This is achieved by examining the derivative of the bank’s objective function in equation (2.9) with respect to
More specifically, the condition to verify is $J_t^2|_{e_t = \bar{e}} < 0$.

These two conjectures hold in simulations pretty well. Throughout all 5000-period simulations of the model under different capital requirements between 7% and 15%, $J_t^1 > 0$ has never been violated once while the maximum number of periods in which $J_t^2 < 0$ has been violated is 1.

2.8.3. Moment Constructions

In this section, I provide a more detailed description about how moments presented in Table 8 are constructed within the model and from the data.

2.8.3.1. Model

The following table presents the construction of variables within the model. I simulate the model for 5000 periods and then calculate the unconditional sample moments of simulated series.

2.8.3.2. Data Sources

Calculations are all based on annual US data between 1988 and 2015.

1. Aggregate data

Aggregate financial data are from non-financial business (L.102) in Flow of Funds of United States. Bank loan is loans; liability (FL144123005). Non-bank debt is total liabilities (FL144190005) minus loans; liability.

Aggregate consumption $C$ stands for personal consumption in non-durables (PCNDA) and services (PCESDA); $Y$ for real gross domestic output (GDPCA); $I$ for gross private domestic
<table>
<thead>
<tr>
<th>Moments</th>
<th>Model Counterparts</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A. Firm statistics</strong></td>
<td></td>
</tr>
<tr>
<td>Bank dependence</td>
<td>$b_{t-1}/k_t$</td>
</tr>
<tr>
<td>Covenant violation prob.</td>
<td>$\Phi(z_r^f)$</td>
</tr>
<tr>
<td>Default prob.</td>
<td>$\Phi(z_d^f)$</td>
</tr>
<tr>
<td>Bank debt default recovery</td>
<td>$\chi \int_{-\infty}^{z_r^f} \pi_{i,t} \Phi(z_r^f)/[R_{t-1}^b, b_{t-1}]$</td>
</tr>
<tr>
<td>Non-bank debt default recovery</td>
<td>$\Phi(z_f^b)/\Phi(z_d^f)$</td>
</tr>
<tr>
<td><strong>Panel B. Bank statistics</strong></td>
<td></td>
</tr>
<tr>
<td>Failure prob.</td>
<td>$\Phi(z_b^f)$</td>
</tr>
<tr>
<td>Loan spread</td>
<td>$R_b^f - R_f^f$</td>
</tr>
<tr>
<td>Deposit rate</td>
<td>$R_f^f$</td>
</tr>
<tr>
<td>Net dividend rate</td>
<td>$\epsilon_t$</td>
</tr>
<tr>
<td><strong>Panel C. Macro Moments</strong></td>
<td></td>
</tr>
<tr>
<td>$Y$</td>
<td>$\exp(x_t)k_t^\alpha - (1 - \chi)\exp(x_t)k_t^\alpha {1 - \Phi(\frac{\mu + \sigma_e^2 - z}{\sigma_e})} + \chi {1 - \Phi(\frac{\mu + \sigma_e^2 - z}{\sigma_e})}$</td>
</tr>
<tr>
<td>$I$</td>
<td>$k_t^{\epsilon} - (1 - \delta)k_t + (1 - \chi){1 - \phi_{s,t}} \Phi(z_f^s) + \chi(1 - \phi_{s,t}) - \chi'_{s,t}\Phi(z_f^d)k_t$</td>
</tr>
<tr>
<td>$C$</td>
<td>$Y_t - I_t$</td>
</tr>
</tbody>
</table>

Table 10: Moment Constructions within the Model. Notes: Restructuring, default, firm bankruptcy and bank bankruptcy cutoffs are respectively defined by: $\chi\pi_{i,t}(z_r^f) = \Pi_{t-1}^b, \pi_{i,t}(z_d^f) = \Pi_{t-1}^b + \Pi_{t-1}^m - \Theta_{t-1}^f, (1 - \chi)\pi_{i,t}(z_f^b) = \Pi_{t-1}^m - \Theta_{t-1},$ and $\chi\pi_{i,t}(z_b^f) = [c_b^d + R_{t-1}^d(1 - \bar{\epsilon})]b_{t-1}$.

fixed investment (FPIA) plus consumption on durables (PCDGA).

2. Bond market data

Bond market data are calculated from “Annual Default Study: Corporate Default and Recovery Rates, 1920-2015”. Default rates are taken from Ba, Exhibit 30. Without a detailed time series, the recovery rate is directly taken from Ba, Exhibit 21.

3. Bank data

Banking sector data are from “Quarterly Income and Expense of FDIC-Insured Commercial Banks and Savings Institutions” and “Failures and FDIC Assistance Transactions”. Failed/unprofitable rate is number of failed institutions divided by that of unprofitable institutions.

The loan spread is calculated following Hanson, Kashyap and Stein (2011):

$$\frac{\text{interest income of domestic+foreign office loans + Lease financing receivables}}{\text{net loans and leases}} - \frac{\text{interest expense of domestic+foreign office deposits}}{\text{deposits}}$$
which is then annualized by summing over four quarters.