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Essays On Macroeconomics And Finance

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Abstract
This dissertation consists of three chapters that address questions in macroeconomics and finance.

In the first chapter, coauthored with Nikolai Roussanov and Mathieu Taschereau-Dumouchel, we investigate the interaction between gradual technological change and business cycles. Recent empirical evidence suggests that skill-biased technological change accelerated during the Great Recession. We use a neoclassical growth framework to analyze how business cycle fluctuations interact with a long-run transition towards a skill-intensive technology. In the model, the adoption of new technologies by firms and the acquisition of new skills by workers are concentrated in downturns due to low opportunity costs. As a result, shocks lead to deeper recessions, but they also speed up adoption of the new technology. Our calibrated model matches both the long-run downward trend in routine employment and key features of the Great Recession.

In the second chapter, coauthored with Haotian Xiang, we document S-shaped dynamics of the US economy associated with the construction of the Interstate Highway System in the 1960s. We then propose a business cycle model with two steady states arising due to productive public capital and production non-convexities. Small-scale short-run public investment programs generate transitory responses while large-scale ones can produce long-run impacts. Our quantitative analysis highlights the critical role played by public investment in explaining the economic dynamics around the 1960s. However, it casts doubt on the efficiency of a large public investment expansion in the post-Great Recession era.

In the third chapter, I present a dynamic general equilibrium model in which financial interconnectedness endogenously changes over the business cycle and shapes systemic risk. To share individual risks, banks become interconnected through holding overlapping asset portfolios. Diversification reduces individual banks' default probabilities but increases the similarity of their exposures to fundamental shocks. Systemic financial crises burst at the end of credit booms when productive investment opportunities are exhausted, banks' balance sheets are weak, and their portfolios are strongly correlated. Under such circumstances, financial fragility is magnified, and even a moderate negative shock can lead to simultaneous defaults of many interconnected banks.

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ABSTRACT

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Alexandr Kopytov

João F. Gomes

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CHAPTER 1 : SHORT-RUN PAIN, LONG-RUN GAIN? RECESSIONS AND TECHNOLOGICAL TRANSFORMATION

1.1. Introduction

Over the last couple of decades rapid advances in information technology, electronics and robotics have made many jobs associated with relatively simple and repetitive tasks obsolete, as they can now be easily performed by machines. While these routine jobs have been disappearing, employment in non-routine cognitive occupations (e.g., programmers or financial analysts) and non-routine manual jobs (mainly in low-skill services), has been increasing. Both of these types of occupations are associated with tasks that have proved harder to automate, at least thus far.\(^1\)

In a recent contribution, Jaimovich and Siu (2015) (JS hereafter) have shown that this job polarization process accelerates during recessions. Over the last thirty years, employment in routine occupations experienced significant drops during economic downturns and, unlike for other types of jobs, these drops were not followed by subsequent recoveries. Strikingly, 88% of job losses in routine occupations since the mid-1980s happened during the three downturns that occurred over this span of time. In contrast, non-routine jobs experienced only small declines during these recessions, and rapidly recovered afterwards. Importantly, these patterns began during the mid-1980s, when the pace of innovation in automation technologies accelerated (e.g., see Eden and Gaggl, 2016). In prior decades, routine employment bounced back quickly as economy recovered.

We build a quantitative model to better understand these patterns, and to evaluate their importance for macroeconomic fluctuations and technology adoption. In the model, consumption goods are produced using manual services and intermediates. The manual services employ low-skill workers to perform non-routine manual tasks. Production of intermediates, in contrast, is more complex and requires a combination of non-routine cognitive tasks,

\(^1\)For more on job polarization see Acemoglu (1999), Autor, Levy, and Murnane (2003), Goos and Manning (2007), Goos, Manning, and Salomons (2014) among others.
performed by high-skill workers, and routine tasks, performed by low-skill workers, similar to Autor and Dorn (2013). While only one technology is available to produce manual services, firms can choose either an “old” or a “new” technology to produce intermediates. The new technology is more skill-intensive than the old one, and it becomes better over time due to continued improvements in information technologies and automation (Autor et al., 2003). As a result, firms progressively switch from the old to the new technology, and non-routine cognitive employment goes up as a result. Moreover, since manual services and intermediates are complements, non-routine manual employment also increases. The slow improvement in the productivity of the new technology therefore generates patterns of job polarization similar to those visible in the data.

Adopting the new technology is costly, both in terms of factors of production that must be used to reorganize the firm and in terms of the profits that are lost during the reorganization. As a result, firms in our model prefer to adopt the new technology during recessions, when factors of production are cheap and the loss in foregone profits is minimized. Recessions are also periods of skill acquisition by workers. Since wages are depressed, low-skill workers take advantage of the low opportunity costs to acquire the new skills, which will be in high demand once the firms begin using the new technology towards the end of the recession. Together, the adoption of new technology by firms and the acquisition of skills by workers take resources away from production during downturns and, as a result, amplify the effect of negative business cycle shocks. At the same time, this short-run pain creates long-lasting value in the form of a better production technology and a higher skill level.

The patterns of technology adoption and skill acquisition generated by the model have support in the data. The evidence from the Great Recession is particularly telling. Indeed, while the recession was accompanied by a decline in routine employment, post-secondary education enrollment increased markedly over the same period. At the same time, while aggregate investment fell massively, investment in new equipment and software technology was mostly unaffected or slowed down only slightly. Worldwide sales of industrial robots in-
creased sharply as the recession ended, indicating increased adoption of the new automation technology.

The model is calibrated to match standard real business cycle moments and the overall increase in the employment share of non-routine cognitive workers. Importantly, for a reasonable level of complementarity between intermediates and manual services, the model is able to explain the recent growth in the employment share of non-routine manual labor, as well as the decline in routine manual jobs. When fed with a large negative TFP shock that corresponds to the Great Recession in both its magnitude and its timing (relative to the process of technological transition), the model is able to replicate the sharp drop in the share of routine workers in the labor force that occurred between 2008 and 2010. We also show that even in the counterfactual scenario in which the economy does not suffer the Great Recession the smooth process of technological transition still delivers the employment share of routine manual workers that is observed empirically in 2012. Thus, while the Great Recession may have accelerated the process of job polarization, it does not seem to have contributed substantially to its long-term trend.

**Literature Review**

We model technological progress as involving a change in the production function, reminiscent of the general-purpose technology literature (e.g., Helpman, 1998). The new technology is relatively more high-skill-intensive, similar to Heckman, Lochner, and Taber (1998) and Goldin and Katz (1998). Relatedly, Buera, Kaboski, and Rogerson (2015) also hypothesize that the share of high-skill labor in production function has increased as a result of the recent technological change. An alternative approach would be to use the notion of capital-skill complementarity, as proposed by Griliches (1969) and Krusell, Ohanian, Rios-Rull, and Violante (2000). There, technological progress makes capital equipment more productive and cheaper, causing increase in demand for the high skill.

In our model, technology adoption requires both time and resources. In this regard, it
is similar to Jovanovic and Macdonald (1994), Andolfatto and MacDonald (1998) and, especially, to Greenwood and Yorukoglu (1997) who assume that high-skill labor is essential to adopt new technologies.

In a “pit stop” model of technology adoption, which is reminiscent of the Schumpeterian view of recessions, periods of depressed economic activity are used by firms to reorganize production or invest in organizational capital (e.g., Hall, 1991, Cooper and Haltiwanger, 1993, Aghion and Saint-Paul, 1998, and Caballero and Engel, 1999). Recently, these ideas have been brought to explain anemic employment recoveries following the three latest recessions (van Rens, 2004, Koenders and Rogerson, 2005, and Berger, 2012). Our work is also related to Restrepo (2015) who builds a model in which skill mismatch can lead to a prolonged adjustment process following a structural shock as firms struggle to find workers with the appropriate skills for new jobs.

Training is modelled in the spirit of real business cycle models augmented with human capital accumulation as in Perli and Sakellaris (1998) and DeJong and Ingram (2001). As a result, in our model investment in human capital also increases during recessions. Countercyclical investment in education is a well-established fact in the empirical literature (see, among many others, Dellas and Sakellaris, 2003, Charles, Hurst, and Notowidigdo, 2015, and Barr and Turner, 2015).

The remainder of the paper is organized as follows. Section 1.2 discusses recent empirical evidence on the interaction between routine-biased technological change and recessions. Section 1.3 describes the model. The model is calibrated in Section 1.4. Section 1.5 contains quantitative exercises. Section 1.6 concludes.

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2Schumpeter (1934) considers recessions as “industrial mutation that incessantly revolutionizes the economic structure from within, incessantly destroying the old one, incessantly creating a new one”. Caballero and Hammour (1994) study how the process of creative destruction interacts with business cycles.
1.2. Empirical evidence

In this section, empirical evidence about interaction between recent downturns and the speed of job polarization are discussed. We then show that some forms of investment in skill-complementary capital (e.g., software and information processing equipment), are only mildly pro-cyclical, in contrast with very pro-cyclical investment in structures. We use this evidence to help motivate our model, in which adoption of a new skill-intensive technology requires reorganization of the firm that disrupts production and is therefore more attractive during periods of low opportunity costs. At the same time, workers use recessions to invest in their human capital in order to satisfy the increasing demand for skill, which results in accelerated job polarization.

A large empirical literature documents that job polarization, induced by routine-biased technological change, was accelerated by the recent recessions. Hershbein and Kahn (2016) show that demand for high-skill workers rises in metropolitan statistical areas with lower employment growth. This “upskilling” effect is long lasting and does not disappear even when then labor market recovers. Moreover, firms that upskill more actively also invest more. Restrepo (2015) documents a dramatic negative impact of the Great Recession on routine cognitive jobs. Anghel, De la Rica, and Lacuesta (2014) argue that the Great Recession sped up job polarization in Spain. Zhang (2015) finds that during crises firms intensive in routine labor reduce routine employment and invest more in machines. Using a panel of Spanish manufacturing firms, Aguirregabiria and Alonso-Borrego (2001) show that firms’ decisions to reorganize production is counter-cyclical and lead to a significant shift in occupation structure towards white-collar jobs.

Most relevant for our purpose, JS argue that the three recent recessions affected routine and non-routine workers in a dramatically different way.\(^3\) They show that routine employment

\(^3\)Using FRED data, JS define routine occupations as “sales and related occupations”, “office and administrative support occupations”, “production occupations”, “transportation and material moving occupations”, “construction and extraction occupations”, and “installation, maintenance, and repair occupations”. Non-routine cognitive occupations include “management, business, and financial operations occupations”, “professional and related occupations”, “Service occupations” are non-routine manual. We use their classi-
generally drops more during recessions than non-routine employment. In addition, the three recent recessions are accompanied by no recovery in routine employment at all. Since the 1980s, per capita routine employment has been falling, not only as a fraction of total employment, but also in absolute terms. JS therefore refer to the mid-1980s as the start of the job polarization era.

The job polarization era is also marked by an overall drop in labor force participation and an increase in post-secondary education enrollment. As shown in Figure 1, labor force participation has been declining since the end of 1990s. Recessions appear to be important drivers of this decline. In particular, labor force participation fell from 66% to 63% following the Great Recession. At the same time, post-secondary education enrollment was almost flat from the mid-1970s up to the mid-1990s, but increased afterwards, with a pronounced spike around the Great Recession.\(^4\)\(^5\) In our model, both the decreasing labor force participation and the increasing education enrollment are driven by the adoption of the skill-intensive technology.

![Figure 1: Labor force participation rate (from FRED) and post-secondary education enrollment ratio. Post-secondary education enrollment ratio is defined as total fall enrollment in degree-granting institutions (from National Center for Education Statistics) over civilian noninstitutional population (from FRED).](image)

Recessions also have differential effects on different types of investment. For example, a pronounced persistent decline in private investment that happened in the aftermath of the Great Recession is to a large extent driven by an unprecedented drop in residential investment in our numerical analysis. See their paper for more details about classification and robustness.

\(^{4}\)Correlation between the labor force participation and the post-secondary enrollment ratio seems to change sign around the start of the job polarization era. Between 1963 and 1984 the correlation is 0.84, while between 1985 and 2014 it is \(-0.63\).

\(^{5}\)It is worth noticing that enrolled in post-secondary institutions do not necessarily complete their education. Thus, the dynamics of post-secondary enrollment and completion ratios might be different.
investment. In contrast, as shown in Figure 2, investment in intellectual property, such as software and R&D, as well as industrial and information processing equipment, tools that are used by skill-intensive firms, experienced a much smaller drop and recovered rapidly (see also Brynjolfsson and McAfee). In Online Appendix it is also shown that world-wide shipments of industrial robots experienced a sharp increase immediately after the Great Recession.

Figure 2: Log real per capita private investment by type (from NIPA Tables). The series are normalized to 0 in 1985.

1.3. Model

Time is discrete and goes on forever, $t = \{0, 1, \ldots\}$. The economy is populated by a representative household that consists of a unit measure of workers. A worker is either low-skill or high-skill, and a low-skill worker can become high-skill through training. On the production side, a final good is produced with two kinds of inputs. The first one is “manual services,” which can be produced using low-skill labor in non-routine manual tasks. The second input is “intermediates,” which can be produced by combining capital and the two types of labor.

Intermediates can be produced using one of two different technologies: an old technology that is low-skill intensive and a new technology that is high-skill intensive. All firms begin

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6In the model, high skill is related to ability to implement non-routine cognitive tasks and not directly to education level. Although the two are doubtlessly positively correlated, they are not the same (see JS for a discussion). Nevertheless, as discussed in Section 1.4, post-secondary education data is used for the calibration purposes.

7Our definition of two different inputs is similar to Autor, and Dorn (2013). It distinguishes between two broad types of tasks implemented by low-skill workers. Routine occupations required to produce intermediates are relatively easy to automate or offshore (e.g., secretaries, paralegals, laborers). Non-routine manual occupations in manual services, such as janitors or barbers, are not.
in period $t = 0$ by using the old technology and, as the productivity of the new technology slowly improves, progressively switch to it. Adopting the new technology requires capital and high-skill labor, and the firm must stop production while the workplace is being reorganized. Below we describe the agents in greater detail.

1.3.1. Representative household

The representative household values final consumption goods using a constant relative risk aversion utility function with coefficient $\gamma$ and discounts future utility at a rate $b$. The household consists of a unit mass of atomistic workers, each endowed with one unit of labor. A fraction $h$ of them are high-skill and the remaining $u = 1 - h$ are low-skill. Low-skill workers can either work in production ($u_p$), in which case they earn a wage $w_u$, or go to school as students ($u_s$) in order to be trained and eventually become high-skill workers. While in training, low-skill workers are out of the labor force. High-skill workers are always employed, either in production ($h_p$) for a wage $w_h$, or in schools as teachers ($h_s$). The household also owns the capital stock $k$ and either uses it to train workers ($k_s$) or rents it out to firms for production ($k_p$) at a rate $r$.

Each period, the abilities of a fraction $\delta_h$ of high-skill workers are rendered obsolete and they become low-skill. The dynamics of the mass of high-skill workers is

$$h' = (1 - \delta_h)h + \phi(k_s, h_s, u_s), \quad h' \in [0, 1], \quad (1.1)$$

where $\phi(k, h, u)$ is the training technology. As in Perli, and Sakellaris (1998), we assume that

$$\phi(k, h, u) = sk^{\beta_s} (\mu_s h^\rho_s + (1 - \mu_s) u^\rho_s)^{\frac{1 - \beta_s}{\rho_s}}, \quad (1.2)$$

where $\beta_s$ is the capital intensity of the training sector, $\mu_s$ is the high-skill intensity and $\rho_s$ relates to the elasticity of substitution between high-skill and low-skill workers (teachers and students).
The household owns the firms and receives their profits $\Pi$ every period. It also invests in new capital subject to quadratic adjustment costs $\varphi(i,k) = \frac{1}{2} \left( \frac{i}{k} - \delta_k \right)^2 k$. Capital depreciates at a rate $\delta_k$, so that its law of motion is

$$k' = (1 - \delta_k)k + i. \quad (1.3)$$

Denoting by $\Omega$ the aggregate state of the economy (which will be fully described later), the dynamic problem of the household is

$$W(h, k, \Omega) = \max_{h', k', h, h_p, u, u_p, k_s, k_p} \frac{1 - \gamma}{1 - \gamma} + b \mathbb{E} \left[ W(h', k', \Omega') | \Omega \right] \quad (1.4)$$

subject to the budget constraint

$$c + i + \varphi(i, k) = w_h(\Omega) \cdot h_p + w_u(\Omega) \cdot u_p + r(\Omega) \cdot k_p + \Pi(\Omega), \quad (1.5)$$

and the laws of motion (1.1) for high-skill workers and (1.3) for capital and to an aggregate law of motion $\Omega' = G(\Omega)$ for $\Omega$.

1.3.2. Firms and technologies

On the production side, the final good is produced by combining two inputs: intermediates and manual services. Manual services are produced by low-skill workers in non-routine manual jobs. Intermediates can be produced by either the new or the old technology, which both require high-skill and low-skill workers in non-routine cognitive and routine tasks, respectively. Final goods producers combine the two inputs into the final good, which is then consumed by the household.

Final goods producer

There is a competitive industry that produces the final consumption good by combining intermediates (from both old and new firms) with manual services. The price of the final
good is normalized to 1. The static problem of a firm in this industry is

$$\max_{y_{i,n}, y_{i,o}, y_{ms}} e^{z} \left[ \left( y_{i,n}^\theta + y_{i,o}^\theta \right)^{\frac{1}{\theta}} + y_{ms}^e \right]^{\frac{1}{\theta}} - P_o(\Omega)y_{i,o} - P_n(\Omega)y_{i,n} - P_s(\Omega)y_{ms},$$

(1.6)

where $y_{i,n}$ is the amount of intermediates produced with the new technology, $y_{i,o}$ is the amount of intermediates produced with the old technology and $y_{ms}$ is the input flow from manual services.\(^8\)

In the spirit of endogenous growth models a-la Romer (1990) and Grossman and Helpman (1991), we assume that the new technology allows intermediate firms to produce a potentially different variety of goods, implying imperfect substitutability between the two ($\theta < 1$). This assumption captures the fact that new labor-saving technologies often do not fully replace the products of old technologies. While digital sound has become much more wide-spread than its analog counterpart, demand for LPs is still nontrivial, as some music connoisseurs prefer the latter. Locally-grown organic foods are not fully substituted away by genetically modified products. Importantly, however, our results generally extend to the case of perfect substitutability, $\theta = 1$, as shown in the Online Appendix.

Aggregate total factor productivity $z$ follows an AR(1) process such that

$$z' = \rho z + \sigma_z \epsilon_z', \text{ where } \epsilon_z \sim \text{iid } N(0,1).$$

(1.7)

**Intermediates producers**

There is a unit mass of atomistic intermediates producers. These firms can operate using either the old or the new technology, indexed by $j = \{o, n\}$. We refer to firms currently operating each technology as “old firms” and “new firms,” respectively. Firms combine

\(^8\)We implicitly assume that the share parameters in the CES aggregators in (1.6) are equal to 1. This is without loss of generality given our calibration of the relative productivities of the three technologies.
capital $k$, high-skill labor $h$ and low-skill labor $u$ using the following production functions:

$$F_j(A_j, h, u, k) = A_j \left[k^{\beta(j)}(h^{\mu_j}u^{1-\mu_j})^{1-\beta}\right]^{\alpha}, \quad j = \{o, n\}, \quad (1.8)$$

where $\beta$ is capital intensity, $A_j$ is total factor productivity, $\mu_j$ captures the skill intensity of the production function, and $\alpha < 1$ is the decreasing returns to scale parameter. $\pi_j$ denotes the profits of a firm operating technology $j$ so that

$$\pi_j(\Omega) = \max_{h,u,k} P_j(\Omega)F_j(A_j, h, u, k) - w_h(\Omega) \cdot h - w_u(\Omega) \cdot u - r(\Omega) \cdot k, \quad (1.9)$$

where $P_j(\Omega)$ is the price of intermediates of type $j$.

The old and the new technology differ in two ways. First, the new technology is relatively more high-skill-intensive than the old one ($\mu_n > \mu_o$). Second, their productivities are different ($A_n \neq A_o$). At $t = 0$ the new technology is not available ($A_n = 0$) and all agents consider its arrival as a zero-probability event. All firms are using the old technology. Over time, technological progress favors the new technology such that $A_n$ grows faster than $A_o$. This induces firms to switch from the old to the new technology. Since the new technology is more high-skill intensive, technological adoption increases the demand for high-skill workers, which puts upward pressure on their wages. As a result, more low-skill workers enter the training process and the overall skill level in the economy increases. Without loss of generality, in what follows $A_o$ is normalized to 1.

Switching from the old to new technology is costly and risky. A firm that attempts to switch does not produce during the current period and successfully acquires the new technology with probability $\xi(h,k)$, $\xi \in [0, 1)$, $\xi_{hh}, \xi_{kk} < 0 < \xi_h, \xi_k$.\footnote{We assume that the technology adoption has a probabilistic nature to capture, in a tractable way, that the adoption process might be longer than one period of the model, which is taken to be one year. Brynjolfsson, Malone, Gurbaxani, and Kambil (1994) and Brynjolfsson and Hitt (2003) find that it takes several years for a firm to fully adopt computer technology.} A firm can increase its odds of successful adoption by hiring more high-skill workers $h$ or by renting more capital $k$.\footnote{The importance of high-skill labor (e.g., management and IT consultants) for technology adoption is emphasized by Nelson and Phelps (1966) and Greenwood, and Yorukoglu (1997).}
Following Andolfatto, and MacDonald (1998), the probability of successful adoption is

\[ \xi(k, h) = 1 - \exp(-\eta k^{\beta_a} h^{1-\beta_a}). \]  

(1.10)

Since a new firm never switches back to the old technology, its value is simply

\[ V_n(\Omega) = \pi_n(\Omega) + \mathbb{E} \left[ M(\Omega, \Omega') V_n(\Omega') | \Omega \right], \]  

(1.11)

where \( M(\Omega, \Omega') \) is the stochastic discount factor of the representative household.

In contrast, an old firm must decide each period whether to attempt a technology switch or to continue producing using its current technology. As a result, its value is

\[ V_o(\Omega) = \max \left\{ V_p^o(\Omega); V_a^o(\Omega) \right\}, \]  

(1.12)

where the value of production is

\[ V_p^o(\Omega) = \pi_o(\Omega) + \mathbb{E} \left[ M(\Omega, \Omega') V_o(\Omega') | \Omega \right], \]  

(1.13)

and the value of attempting to adopt the new technology is

\[ V_a^o(\Omega) = \max_{h,k} \left\{ -w_h(\Omega)h - r(\Omega)k + \xi(h, k) \mathbb{E} \left[ M(\Omega, \Omega') V_n(\Omega') | \Omega \right] + (1 - \xi(h, k)) \mathbb{E} \left[ M(\Omega, \Omega') V_o(\Omega') | \Omega \right] \right\}. \]  

(1.14)

In what follows, we denote the masses of producing new and old firms by \( m_n \) and \( m_o \), respectively. The remaining fraction, \( 1 - m_n - m_o \), are in the adoption process and are therefore not producing.
Manual services producer

There is a competitive representative firm producing manual services using low-skill workers. As in Autor, and Dorn (2013), its production function is $F_{ms}(u) = A_{ms}u$ such that it maximizes

$$\max_u P_{ms}(\Omega)F_{ms}(u) - w_u(\Omega)u,$$

where $P_{ms}(\Omega)$ is the price of services.

1.3.3. Competitive equilibrium

The set of aggregate state variables $\Omega$ contains the aggregate capital stock $K$, the mass of high-skill workers $H$, the mass of firms producing intermediates which operate the new technology $m_n$, the productivity of the new technology $A_n$ and the productivity of the final goods producer $z$. A competitive equilibrium in this economy is defined below.\(^{11}\)

**Definition 1** A recursive competitive equilibrium is a collection of value functions for the firms $V_\alpha$, $V_\mu^p$, $V_\sigma$, $V_n$ and for the household $W$, and their associated optimal decisions; a collection of prices $w_h$, $w_u$, $r$, $P_o$, $P_n$, $P_{ms}$ and aggregate laws of motion $G$, such that 1) the value functions and the optimal decisions solve problems 1.4, 1.6, 1.11, 1.12 and 1.15;

\(^{11}\)Since the competitive economy is efficient, we solve the problem of a social planner maximizing the welfare of the representative household. Given the complexity of the economy, the model is solved using a perfect foresight approach. In particular, all business cycle shocks are assumed to be completely unexpected. To verify the validity of this approach, we have also solved a simpler version of the fully stochastic model globally. The perfect foresight approach does not alter the predictions of the model but significantly decreases the complexity of the computations. The role of a potentially anticipated (rather than surprise) recession is further addressed in the Online Appendix.
2) the markets for high-skill and low-skill labor and the market for capital clear:

\[ H_p = m_nh_n + m_oh_o + (1 - m_n - m_o)h_a, \] 
\[ U_p = m_nu_n + m_ou_o + U_{ms}, \] 
\[ K_p = m_nk_n + m_ok_o + (1 - m_n - m_o)k_a, \] 
\[ H = H_p + H_s, \] 
\[ U = U_p + U_s, \] 
\[ 1 = H + U, \] 
\[ K = K_p + K_s, \] 

where \( h_j, u_j, k_j, j \in \{o, n\} \) denote the demand for high-skill labor, low-skill labor and capital, respectively, of old and new firms; \( h_a, k_a \) denote the demand for high-skill labor and capital of firms adopting the new technology; \( U_{ms} \) is the demand for low-skill labor of the manual services representative producer; \( H_s, U_s, K_s \) denote respectively the amounts of high-skill labor, low-skill labor and capital used in the training process;

3) the law of motion \( G \) is consistent with individual decisions.

1.4. Parametrization

The model is parameterized to match features of the United States economy since the middle of the 1980s, the beginning of the job polarization era. One period is one year. Below, we explain how the parameters are picked and Table 1 summarizes their values.

**Business cycle shocks**

The persistence and the standard deviation of the business cycle shocks, \( \rho_z = 0.85 \) and \( \sigma_z = 0.025 \), are set to match the first order autocorrelation and the volatility of HP-filtered real GDP per capita.\(^{12}\) The persistence value is close to what is normally used in the RBC

\(^{12}\)We match the moments of the initial steady state of the economy with their data counterparts between 1947 and 1985. Recall that the job polarization era, associated in our model with the arrival of the new technology, started around the mid 1980s, as argued by JS.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Business cycle shock</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aggregate shock persistence</td>
<td>$\rho_z = 0.85$</td>
<td>Autocorrelation of output</td>
</tr>
<tr>
<td>Volatility of aggregate shock</td>
<td>$\sigma_z = 0.025$</td>
<td>Volatility of output</td>
</tr>
<tr>
<td><strong>Preferences</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk aversion</td>
<td>$\gamma = 1.0$</td>
<td>Log utility</td>
</tr>
<tr>
<td>Time discounting</td>
<td>$b = 0.96$</td>
<td>4% annual interest rate</td>
</tr>
<tr>
<td><strong>Production sector</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DRS parameter</td>
<td>$\alpha = 0.9$</td>
<td>Basu, and Fernald (1997)</td>
</tr>
<tr>
<td>Share of capital</td>
<td>$\beta = 0.3$</td>
<td>Average labor share</td>
</tr>
<tr>
<td>Share of $H$ in old technology</td>
<td>$\mu_o = 0.45$</td>
<td>R employment in 1985</td>
</tr>
<tr>
<td>Share of $H$ in new technology</td>
<td>$\mu_n = 0.83$</td>
<td>C-S dispersion in R wage share</td>
</tr>
<tr>
<td>EoS between new and old goods</td>
<td>$\frac{1}{1-\rho_s} = 4$</td>
<td>Bernard et al. (2003)</td>
</tr>
<tr>
<td>EoS between goods and services</td>
<td>$\frac{1}{1-\rho_s} = 0.167$</td>
<td>Buera et al. (2015)</td>
</tr>
<tr>
<td>Productivity of manual services</td>
<td>$A_{ms} = 5.91$</td>
<td>NRM employment in 1985</td>
</tr>
<tr>
<td>Physical capital depreciation</td>
<td>$\delta_k = 0.1$</td>
<td>10% annually</td>
</tr>
<tr>
<td>Adjustment cost parameter</td>
<td>$\chi = 0.25$</td>
<td>Investment volatility</td>
</tr>
<tr>
<td><strong>Schooling/training</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Share of capital</td>
<td>$\beta_s = 0.1$</td>
<td>Perli, and Sakellaris (1998)</td>
</tr>
<tr>
<td>EoS between $H$ and $U$</td>
<td>$\frac{1}{1-\rho_s} = 0.5$</td>
<td>Perli, and Sakellaris (1998)</td>
</tr>
<tr>
<td>Share of $H$ in education</td>
<td>$\mu_s = 0.0076$</td>
<td>Student-teacher ratio</td>
</tr>
<tr>
<td>Constant</td>
<td>$s = 0.248$</td>
<td>Post-secondary enrollment in 1985</td>
</tr>
<tr>
<td>High skill depreciation</td>
<td>$\delta_h = 0.05$</td>
<td>Heckman (1976)</td>
</tr>
<tr>
<td><strong>Technology adoption</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital share</td>
<td>$\beta_a = 0.3$</td>
<td>Same as in production sector</td>
</tr>
<tr>
<td>Ease of adoption</td>
<td>$\eta = 1.5$</td>
<td>Expected adoption lag is 3 years</td>
</tr>
<tr>
<td><strong>Technological progress</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial impact</td>
<td>$A_n^0 = 0.1$</td>
<td>Trends in NRC, NRM and</td>
</tr>
<tr>
<td>Final value</td>
<td>$A_n = 1.5$</td>
<td>R employment shares</td>
</tr>
<tr>
<td>Length</td>
<td>$T_{finish} - T_{start} = 75$</td>
<td>R employment shares</td>
</tr>
</tbody>
</table>

Table 1: Parametrization. R, NRM, NRC stand for routine, non-routine manual and non-routine cognitive, respectively. C-S stands for cross-sectional.

literature (Cooley and Prescott, 1995). The standard deviation $\sigma_z$ is somewhat larger than usual values. Since, in the model, there is no labor-leisure choice, larger fluctuations in exogenous productivity are necessary to match aggregate output volatility.\(^{13}\)

**Preferences**

The time discount rate $b$ is set to 0.96, which implies a 4% annual interest rate. The risk

\(^{13}\) As discussed in Footnote 11, the model is solved assuming perfect foresight. All shocks are therefore completely unanticipated and the value of $\sigma_z$ does not affect the impulse response functions directly. The value of $\sigma_z$ does however matter for the calibration of other parameters of the model.
aversion $\gamma$ is 1.

**Production sector**

The returns to scale parameter for firms producing intermediates is set to $\alpha = 0.9$, consistent with the estimates of Basu, and Fernald (1997). The capital share parameter is $\beta = 0.3$, consistent with aggregate data. The share of high-skill labor in the old production technology is set to $\mu_o = 0.45$ to match the fraction of routine employment in total employment at the beginning of the job polarization era. For the new technology, $\mu_n = 0.83$ in order to match the cross-sectional dispersion in routine wage share in the total wage bill across intermediates producing firms to the value found by Zhang (2015).\footnote{Zhang (2015) sorts firms based on this characteristic and finds that the spread between the highest and the lowest quantiles is 0.37. In our model, there exists a trivial cross-section of firms among intermediates producers, with old firms having a higher routine wage share. $\mu_o = 0.83$ implies that the difference in the routine wage share between new and old firms is close to 0.37. This value stays almost constant along the transition path.} The elasticity of substitution between the new and old goods in the production of consumption goods is 4, so that $\theta = 0.75$, close to the estimates of Hsieh and Klenow (2014) and Bernard et al. (2003). The elasticity of substitution between intermediates and manual services is 0.167, which implies $\epsilon = -5$, in line with estimates of Buera and Kaboski (2009) and Herrendorf, Rogerson, and Valentinyi (2013). Productivity of manual services is set to $A_{ms} = 5.91$ in order to match the employment share of non-routine manual labor in 1985. Physical capital depreciates at the rate of $\delta_k = 0.1$. The adjustment cost parameter is set to $\chi = 0.25$ to match the volatility of private investment.

**Training**

Calibrating of the training technology is not straightforward. To the best of our knowledge, there are no readily available empirical estimates of an aggregate training function that combines low and high-skill labor together with physical capital. Perli, and Sakellaris (1998) consider an RBC economy with a human capital sector. Their human capital production technology is similar to ours and we follow them by setting the capital share to $\beta_s = 0.1$ and the elasticity of substitution between high and low-skill labor to $\frac{1}{1-\rho_s} = 0.5$. The latter
value implies that high-skill labor and low-skill labor are strong complements in the training sector. The relative weight of high-skill labor is set to $\mu_s = 0.0076$ in order to roughly match the teacher-student ratio in post-secondary education.\textsuperscript{15} The productivity $s = 0.248$ is set to match the number of low-skill agents in the training process $U_s$ in the initial steady state to the fraction of the civilian noninstitutional population in post-secondary education in 1985. Finally, the skill depreciation rate is set to $\delta_h = 0.05$.\textsuperscript{16}

Technology adoption

An old firm attempting to adopt the new technology is successful with probability $\xi(k, h) = 1 - \exp(-\eta k^{\beta_h} h^{1-\beta_h})$. As for the earlier production technology, we set the capital intensity to $\beta_a = 0.3$. The parameter $\eta > 0$ governs the importance of capital and high-skill labor for the technology adoption. If $\eta$ is large, then only a few workers and small amounts of capital are required to get the transition probability close to its maximum level of 1. On the contrary, a small value of $\eta$ implies a large demand for high-skill labor and capital among adopting firms. Thus, smaller $\eta$'s are associated with larger adoption costs. We set $\eta = 1.5$. Along the transition path, the resulting probability of successful technology adoption is around 0.33.\textsuperscript{17}

Technological progress

The decision of the firms whether to adopt the new technology or not depends on the gap between $A_o$ and $A_n$. $A_o$ is normalized to 1, while the evolution of $A_n(t)$ is parameterized

\textsuperscript{15}According to the National Center for Education Statistics, this ratio was slightly below 6% in the 1980s and has increased to 7.6% by the 2010s. $\mu_r$ is set so that in the initial steady state $\frac{H_s}{U_s} = 0.07$. Due to the absence of reliable data, we ignore other forms of training besides higher education. However, as argued by Perli, and Sakellaris (1998), higher education is responsible for up to 90% of total investment in human capital.

\textsuperscript{16}In the model $\delta_h$ can be interpreted as the retirement rate, which is currently around 3% in the US. One can assume that every period a fraction $\delta_h$ of the total labor force $L$ retires and is immediately replaced by low-skill workers. At the same time, $\delta_h$ should include the rate of skill obsolescence. Existing literature estimates the depreciation rate of human capital. Despite substantial variation, $\delta_h = 0.05$ is close to what is normally found (Heckman, 1976 and Mincer and Ofek, 1982).

\textsuperscript{17}Consistent with this number, Brynjolfsson et al. (1994) and Brynjolfsson, and Hitt (2003) find that it normally takes several years for a firm to fully adopt computer technology.
as follows

\[
A_n(t) = \begin{cases} 
0, & t < T_{\text{start}}, \\
A_n^0 + (\bar{A}_n - A_n^0) \frac{1 - \exp\left(\frac{T_{\text{finish}} - T_{\text{start}}}{1 - \exp(-1)}\right)}{1 - \exp(-1)}, & t \in [T_{\text{start}}, T_{\text{finish}}], \\
\bar{A}_n, & t > T_{\text{finish}},
\end{cases}
\]  

(1.23)

where \(T_{\text{start}}\) denotes the arrival of the new technology (the mid 1980s in our case, corresponding to the start of the job polarization era in JS). Upon arrival, its productivity is \(A_n^0\) and it increases over time until it reaches \(\bar{A}_n > A_n^0\) by \(t = T_{\text{finish}}\).

The shape of the \(A_n(t)\) process is inspired by the literature on general-purposes technology (GPT). The initial impact of the new technology \(A_n^0\) can be modest. Later on, a sequence of smaller innovations enhances the productivity of the new technology. As a result, the technology reaches its peak \(\bar{A}_n\) after a (potentially long) lag \(T_{\text{finish}} - T_{\text{start}}\). This is typical of GPTs (e.g., Bresnahan and Trajtenberg, 1995, Helpman, 1998, Jovanovic and Rousseau, 2005).\(^{18}\)

In our baseline analysis, \(A_n^0 = 0.1, \bar{A}_n = 1.5, T_{\text{finish}} - T_{\text{start}} = 75\). These parameters are chosen to match the trend in the non-routine cognitive employment share reasonably well. Our choice of \(\bar{A}_n\) and \(T_{\text{finish}} - T_{\text{start}}\) is not unique to achieve this. A longer/shorter progress with a higher/lower terminal value can match the same series. Since the goal of the paper is not to predict when the growth of \(A_n(t)\) will stop, we do not take a strong stand on precise values of \(\bar{A}_n\) and \(T_{\text{finish}} - T_{\text{start}}\). Importantly, our results on interactions of routine-biased technological change with business cycles are unaffected if \(T_{\text{finish}} - T_{\text{start}}\) and \(\bar{A}_n\) are varied simultaneously to match the observed non-routine cognitive employment share.

\(^{18}\)Historically, new general purpose technologies took significant lengths of time to achieve their full potential. For example, David (1990) argues that electricity delivered a major economic boost only in the 1920s, 40 years after the first generating station came into being. Crafts (2004) finds a lag of almost 100 years for steam-related technologies. Using asset prices, Ward (2015) predicts that it will take around 50 years for the IT to be fully absorbed by the economy.
1.5. Numerical results

This section presents our main numerical results. In Section 1.5.1, we discuss the transition from the old steady state to the new one induced by the arrival of the new technology. Section 1.5.2 describes the differential impacts of business cycles on the economy in the pre- and during transition periods. Finally, Section 1.5.3 investigates the interaction between the Great Recession and routine-biased technological change.

1.5.1. Transition paths

We begin by investigating how the arrival of the new technology affects the economy without business cycle shocks. The path for the exogenous process $A_n$ is shown in Figure 3. The initial shock is small, representing the idea that a new fundamental technology is hardly productive right after its arrival. As the new technology gradually becomes better, $A_n$ increases and reaches its steady-state level after 75 years.

![Figure 3: The productivity $A_n(t)$ of the new technology.](image)

Figure 4 shows how the introduction of the new technology affects the technology acquisition decision of the firms and the occupational choices decision of the workers. Over time, firms adopt the new technology as its productivity increases (left panel). Since the new technology is relatively more skill-intensive, low-skill workers respond accordingly and start to train to acquire new skill (right panel). As a result, routine manual employment (low-skill

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19 In principle, the mass of old firms is never zero in the terminal steady state due to imperfect substitutability between the new and old technologies. It however becomes very close to zero if $A_n \gg A_o$. For our choice of parameter, the mass of old firms in the final steady state is 0.34. The transition is largely finished in 100 periods after the arrival of the new technology.
workers employed in the intermediates production) declines.

Since the two inputs in the final consumption bundle are complementary, when the productivity of intermediates go up, it is optimal also to increase output of manual services. Consequently, low-skill employment in manual services increases gradually (the dot-dashed curve in the right-hand panel of Figure 4). The model is able to generate job polarization and, as is discussed in more details in Section 1.5.3, it does a fairly good job in explaining these patterns quantitatively.

![Figure 4: Transition upon arrival of the new technology.](image)

Figure 4 illustrates other aspects of the technology adoption process. The top-left panel shows how the production of consumption goods $Y_f$ evolves over time. Despite the positive technological surprise at $t = 0$, $Y_f$ does not respond immediately. For roughly 15 years $Y_f$ is almost unchanged and starts to grow only afterwards. This is due to the GPT nature of the new technology. The adoption of such a technology requires significant investment in reorganization of the workplace and in the accumulation of the required production factors.\(^{20}\) This is illustrated by the top-right and bottom panels of Figure 5. The top-right panel shows the ratio of total adoption costs $Y_a$ to final output $Y_f$. We use two measures of $Y_a$. The first measure, $Y_{a,1}$, includes capital and high-skill labor rents in the training and adoption technologies. The second measure, $Y_{a,2}$, also takes into account forgone profits

---

\(^{20}\)This is reminiscent of the infamous Solow productivity paradox. Due to large reorganization costs, which might be mismeasured in the GDP calculations, the new technology makes a sizable impact on the economy with a significant delay. See also Brynjolfsson (1993).
due to firms being in the restructuring stage.

\[
Y_{a,1} = w_h(H_a + H_s) + r(K_a + K_s), \quad (1.24)
\]

\[
Y_{a,2} = Y_{a,1} + \frac{\partial Y_f}{\partial m_o} (\bar{m} - m_n - m_o). \quad (1.25)
\]

The calibration implies that \(Y_{a,1}\) becomes as high as 2.70% of \(Y_f\) around year 30. Around the same time, unmeasured reorganization investment, captured in our model by foregone output due to old firms going through the adoption process, account for about 0.28% of \(Y_f\). Notice that in the new steady state adoption costs are higher than in the initial steady state (1.43% vs 0.98%). In the new steady state, the number of high-skill workers is higher and more training is therefore required.

The bottom panel of Figure 5 further illustrates that the period after the arrival of the technology is marked by a diversion of resources away from the production of final good. The total mass of active firms and the overall number of workers in the production sector are shrinking for around 30 years (the dot-dashed and dashed lines, respectively). High-skill labor is required for the firms’ reorganization and training of low-skill workers. At the same time, low-skill workers start to train in larger numbers, which contributes to a drop in the labor force participation (the solid line) and to an increase in school enrollment.\(^{21}\) These two phenomena have been visible in the U.S. data for the last two decades.

In particular, the model predicts that the labor force participation drops by around 4 p.p. between 1985 and 2017, which is comparable to the number observed in the data (Figure 1, left panel). The model predicts a steady decline in the labor force participation, while in the data it was growing until the late 1990s and plummeted afterwards. Since the paper abstracts from several important aspects (e.g., labor force participation among women, which was increasing until the late 1990s), it cannot match the whole dynamics of the series.

\(^{21}\)Recall that in the model the only labor force non-participants are low-skill workers in schools \(U_s\).
(a) Output, $Y_f$

(b) Adoption cost to output, $\frac{Y_a}{Y_f}$

(c) Resource allocation

Figure 5: Top-left panel shows output of the final good sector $Y_f$. Top-right panel shows two measures of adoption costs $Y_a$ (measure 1 includes capital and high skill labor rents in the training and adoption technologies, measure 2 also takes into account forgone profits due to firms in the restructuring stage) as fractions of the final good sector production $Y_f$. The bottom panel illustrates how the allocation of resources vary over time.

Over the same period, the model-implied school enrollment ratio increased from 6.9% in 1985 up to 10.2% in 2014. This is larger than in the data, where the ratio increased from 6.9% up to 8.2% (Figure 1, right panel). There are two reasons why the model-implied increase is higher. First, in the model, schooling represents all types of training, including on-the-job training and various job training programs, while the data counterpart takes into account only formal higher education. Second, in the model all workers are either employed or in schools, and a decrease in the number of employed low-skill workers necessarily leads to an increase in the number of employed high-skill workers (with a time lag). This approach misses a recent increase in non-employment among low-skill workers (Cortes, Jaimovich, Nekarda, and Siu, 2014 and Cortes, Jaimovich, and Siu, 2016) that is unrelated to education.\footnote{\textsuperscript{22}Aguiar, Bils, Charles, and Hurst (2017) emphasize the importance of video games, streaming television, and other recreational activities that became widely accessible thanks to the improvements in information and computer technologies for reducing the labor supply of young males.} Demographic changes, such as population aging, might also play a role (Autor and Dorn, 2009).
In our model, job polarization is driven by two main forces. First, the number of low-skill workers goes down along the transition path. As a result, the supply of routine workers diminishes. Second, at each point in time, the propensity of a low-skill worker to take a routine job (i.e., a job in an old or a new intermediates producing firm) goes down. On the one hand, she is more likely to attend school. Conditional on not attending school, on the other hand, she is more likely to do a manual services job. Formally, the routine employment $R$ can be written as

$$
R = U(1 - p_{sc} - p_{nrm}),
$$

where $U$ is the total supply of low-skill workers in the economy, $p_{sc}$ and $p_{nrm}$ are the probabilities that a low-skill worker is in the training process or employed in manual services. Change in routine employment $\Delta R$ therefore can be decomposed into a composition and a propensity effect,

$$
\Delta R = \Delta U(1 - p_{sc} - p_{nrm}) - U\Delta(p_{sc} + p_{nrm}) - \Delta U(p_{sc} + p_{nrm}).
$$

Table 2 presents the decomposition of the overall decline in $R$ between changes in $U$, $p_{sc}$ and $p_{nrm}$.

<table>
<thead>
<tr>
<th>$R_{1989}$</th>
<th>$R_{2014}$</th>
<th>$\Delta R$</th>
<th>Composition</th>
<th>Propensity</th>
<th>Interaction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Schooling</td>
<td>NRM</td>
<td></td>
</tr>
<tr>
<td>$R$ 50.47%</td>
<td>38.61%</td>
<td>-11.86%</td>
<td>-11.86%</td>
<td>-5.18%</td>
<td>-3.96%</td>
</tr>
</tbody>
</table>

Table 2: Model-implied change in routine employment $R$ between 1989 and 2014. The years are chosen as in Cortes et al. (2016).

The model implies that both the composition and the propensity effects are important for job polarization, with the latter force being more significant. This is consistent with the micro evidence provided by Cortes et al. (2014) and Cortes et al. (2016).
1.5.2. Business cycles

This section compares the responses of the economy to business cycle shocks before and after the introduction of the new technology. First we investigate the impact of an adverse \( z \) shock to the economy along the transition path. Specifically, we consider a large (2.5 standard deviations) \( z \) shock happening 23 years after the new technology arrival. Assuming that the technology arrived around 1985, the timing and the magnitude of the recession in the model corresponds to the Great Recession in the data. We then compare the outcome of this first experiment with the response of the economy to the same shock before the new technology was available.\(^{23}\)

The results are shown in Figure 6. Before the arrival of the new technology, the training of workers is counter-cyclical (the dashed curves in panels (f)-(h)), as is typical of RBC models with human capital (e.g., Perli, and Sakellaris, 1998). The intuition is straightforward. During recessions, workers are relatively inefficient in production and the economy therefore uses these periods to accumulate human capital.

This process is however amplified along the transition path for two reasons. In addition to the mechanism highlighted above, the household understands that, since firms also use the recession to adopt the new technology, the future demand for high-skill workers will increase. Economic downturns are therefore a perfect period to train the workforce for this increased demand. The second reason is that training and adoption are intensive in different factors, namely low and high-skill labor, which complement each other in the production of intermediates. When firms start to adopt the new technology in recessions, they demand high-skill labor. This decreases the marginal productivity of low-skill workers who are in production, and some of them therefore move into training. It turns out that the training-

\(^{23}\) The arrival of the new technology changes the structure of the production technology. In general, this can affect the economy’s response to business cycle shocks by itself. We verify that our results are driven by the interaction between the adoption decisions and the business cycle rather than by a different production technology. In the Online Appendix we consider the impulse responses to the same \( z \) shock in the new steady state when the economy has fully transitioned. We find that these responses are much closer to their pre-transition counterparts than to the ones observed along the transition path.
adoption complementarity is the main driver behind the additional investment in human capital that is observed during the recession. As we show in the Online Appendix, when $\eta$ is high and adoption does not require substantial resources, counter-cyclical reallocation of low-skill labor towards training is much closer to the pre-transition case.

In sum, an adverse productivity shock to the final good sector leads to a more active reallocation of factors during the technological transition than before the arrival of the new technology. In particular, panels (g) and (h) show that training is now absorbing more resources. Reallocation towards adopting firms (panel (i)), which is completely absent in the initial steady state, is responsible for roughly half of the additional drop in the total production employment (panel (f)).

Panels (j)-(l) of Figure 6 illustrate the production and adoption decisions of the firms. Since the technology change requires a temporary halt in production, it is more attractive during economic downturns when profits are lower. Panel (k) shows that the mass of old firms drops by 3.5 p.p. as a result of the negative $z$ shock. This drop leads to a lagged increase in the mass of firms operating the new technology, as shown in panel (j).

The counter-cyclical adoption and training incentives are mitigated by the consumption smoothing motives of the representative household. However, this effect turns out to be relatively small for conventional values of the intertemporal elasticity of substitution.

As a result of the increased technological adoption and additional workers training triggered by the recession, the drops in output, consumption and investment are all significantly more pronounced during the technological transition than before the arrival of the technology (panels (b)-(d) in Figure 6).\footnote{In Online Appendix we verify that recessions during technological transitions are still deeper, even after adjusting the output measure for training and adoption costs.}
1.5.3. The Great Recession and routine-biased technological change

We now investigate whether the model can rationalize both the long-run trend in the employment shares induced by routine-biased technological change and the importance of recessions in generating job polarization. We use the same definitions and data sources as JS. Particularly, non-routine cognitive/non-routine manual/routine jobs in their definition correspond to high-skill/low-skill manual services/low-skill intermediates jobs in the model. Figure 7 shows the results.
We consider the impact of a negative 2.5 standard deviation $z$ shock 23 years after the technology arrival for the model-implied employment shares. Again, given our timing, this shock corresponds to the Great Recession in the data.\textsuperscript{25} The top panel of the figure shows the employment share of high-skill (model) versus non-routine cognitive (data) workers. Since technological progress in our model favors the high-skill-intensive technology, the

\textsuperscript{25}The size of the shock is picked in order to match the almost 10% drop in output during the Great Recession (Fajgelbaum, Schaal, and Taschereau-Dumouchel, 2017b).
corresponding employment share is gradually growing, in line with the data. The recession induces more active training, resulting in an upward shift of the curve. Similarly, the employment share of low-skill workers in intermediates producing firms (bottom panel) is declining overall but with a sharp drop down during the downturn. Finally, low-skill services employment share (middle panel) stays almost constant for the first 15 years. Since intermediates and manual services are strong complements, during the initial transition stage, when the intermediates output is barely changed, it is optimal not to increase the manual services output as well. Later on, low-skill employment starts to grow.\textsuperscript{26}

Recent empirical evidence (e.g., JS and Hershbein, and Kahn, 2016) emphasizes the acceleration of the routine employment loss during the Great Recession. Figure 8 takes a closer look at this phenomenon. In the data, the routine employment share dropped by 1.9 percentage points between 2007Q4 and 2008Q4.\textsuperscript{27} Thus, 15\% of the overall drop observed between January of 1985 and April of 2017 happened during only 1 year, or 3\% of the total time span. In the model, a 2.5 standard deviation negative $z$ shock implies a drop of 1.43 p.p., or nearly 75\% of what is observed in the data. In the absence of the $z$ shock, the model-implied routine employment share would have declined by only 0.61 p.p. because of the gradual transition between the steady states. The model is therefore able to replicate a substantial fraction of the routine employment loss during the Great Recession. At the same time, it implies that the effect of the Great Recession was to speed up, rather than cause, the decline in routine employment, since by the year 2012 the model-implied routine shares in the model both with and without the $z$ shock essentially coincide with the empirically observed value at just over 44\%.

Overall, Figures 7 and 8 show that the model can match several important aspects of the

\textsuperscript{26}The model does not match the increase of the non-routine manual employment share during the Great Recession. A negative TFP surprise induces reallocation of high-skill workers towards adoption and teaching. As a result, the intermediates production drops. Due to complementarity between intermediates and manual services, the marginal productivity of low-skill manual service workers declines as well and the planner therefore moves them to training.

\textsuperscript{27}We consider 1 year after the start of the Great Recession, since in the model the Great Recession is approximated by 1 large negative $z$ shock.
1.6. Concluding remarks

In this paper, we analyze the interaction between routine-biased technological change and business cycles. Since economic downturns are periods of low opportunity costs, they are used by firms to optimize their production technology and by workers to adjust their skill set to a changing economic environment. Restructuring incentives are enhanced during technological transitions, associated with higher than usual demand for new skills. As a result, recessions during transitions are marked by a sizable scarcity of factors in the final good production. At the same time, routine-biased technological change is accelerated, consistent with the recent empirical evidence.
The paper provides a theoretical rationale for two major features of job polarization. First, the fraction of routine workers has been declining since at least the mid-1980s, while both non-routine cognitive and non-routine manual employment shares have been growing. Second, job polarization is concentrated in recessions. In our model, a gradual technology adoption generates the trend, while large downturns speed up the transition due to counter-cyclical restructuring incentives.

The model can be extended along several important directions. First, we assume that low-skill workers can switch between routine and non-routine manual jobs without any frictions. Although costs of training for a manual services job are presumably much smaller than for a non-routine cognitive one, they are not zero. A more detailed modelling of occupation choice might help to explain the increase in the non-routine manual employment share during the Great Recession. In particular, time to train for a manual services job is likely to make this process counter-cyclical. Thus, model-implied non-routine manual employment share should exhibit an uptick during downturns.

Furthermore, as discussed in Section 1.5.1, one could allow workers to permanently stay out of the labor force, for example, by introducing a home production sector. It would be interesting to investigate, both theoretically and empirically, how routine-biased technological change and recessions along the transition path affect labor adjustments along this margin. Another potential direction would be to enrich the model with a labor-leisure choice. If the value of leisure is affected by the new technologies, as suggested by Aguiar et al. (2017), then the model could rationalize declining labor force participation, as well as job polarization.
2.1. Introduction

The Great Recession and a subsequent slow recovery have rekindled an enthusiasm for public investment within both the academic world and the policy circle. One of the few approaches to revitalize the economy and return it to a pre-crisis path that both parties in the past presidential campaign agreed on was a massive infrastructure investment. On the Republican side, Donald Trump proposed a $1 trillion infrastructure plan that approximated 5% of the annual US domestic product. On the Democratic side, Hillary Clinton announced a $275 billion plan to rebuild US infrastructure and promised to have the plan passed in her first 100 days in the office.

A clear understanding of the potential outcome of such a large-scale short-run public investment is crucial for these policy discussions. However, existing quantitative studies of public investment consider only small shocks to productive public expenditure and consequent short-run economic responses within the standard business cycle frequency (Baxter and King, 1993 and Leeper, Walker, and Yang, 2010b). When the impact of transient public investment is nonlinear, utilizing a close-to-linear RBC framework to analyze large-scale government investment programs can be quantitatively implausible.

Is there a macroeconomic nonlinearity associated with short-run public investment. Does a large-scale public investment shock generate a long-run impact that goes beyond the business cycle frequency?

Panel (a) of Figure 9 shows that the period between the late 1950s and the early 1970s is marked by a historically high level of non-defense government investment over the postwar US, largely contributed by the construction of the Interstate Highway System. Interestingly, as presented in panel (b) of Figure 9, aggregate output per capita exhibits S-shaped
dynamics around the same time. It stays on the new growth path for decades after public investment has returned to its long-run level.\textsuperscript{1} In Section 2.2, we confirm similar dynamics across states. Moreover, we find that states with relatively larger highway spending booms witnessed more pronounced shifts in the level of per capita income.

In this paper, we analyze the impacts of short-run government investment in light of the US economy’s dynamics around the massive public investment expansion of the 1960s. In particular, we propose a business cycle model exhibiting two stable steady states. Small shocks to public investment generate standard economic responses that fade away relatively quickly while large shocks can cause a transition across steady states and thus a long-run impact.

The multiplicity of steady states in our model rests on two main pillars. First, a là Barro (1990), public capital is productive and the government investment rule is pro-cyclical. The government follows a fiscal rule under which its productive expenditure is proportional to the aggregate output. In our economy, an increase in private investment raises the aggregate output and thus leads to a build-up of public capital. An elevated stock of public capital enhances productivity and improves private incentives to invest.

\textsuperscript{1}Figure 9 presents the series up to 1990 because the mid 1990s are known to be marked by a structural change in the productivity and output growth rates, related to adoption of computer technologies (Fernald, 2016). Importantly, in the post-1990 sample, the ratio of non-defense government investment to output fluctuates around the level it reached by the mid 1970s. Online Appendix shows the series extended up to 2017.
Another key ingredient is production non-convexities. Each period, besides renting capital and hiring labor, individual firms can choose to utilize a productivity enhancing technology by paying a fixed cost (e.g., Durlauf, 1993 and Schaal and Taschereau-Dumouchel, 2015). When production factors are abundant, an increase in productivity is more attractive.

Our stationary economy features two stable steady states with different levels of output, hours, capital stocks as well as technology adoption intensity. In the high steady state, all firms choose to pay the fixed cost and operate the efficient technology. This in turn accelerates both private and public investment and thus helps to sustain capital stocks at high levels. In contrast, the economy is trapped in the low steady state when public and private capital are scarce and firms find it optimal not to adopt the efficient technology. Low aggregate productivity feeds back into low aggregates.

Despite the steady state multiplicity, the dynamic recursive equilibrium of our model is unique. We are therefore able to precisely understand how the economy responds to the two shocks inherent to our model, namely, public investment rate and productivity shocks. For small transitory disturbances to government investment, impulse responses are similar to what a standard RBC model delivers – their impacts are short-run. Similarly, productivity shocks of small scales generate only high-frequency macroeconomic responses.

Large shocks can generate a long-run impact and highly nonlinear dynamics when transitions across the steady states are involved. A sizable public investment shock significantly raises the marginal productivity in the economy starting at the low steady state. The private sector is encouraged to hire labor, accumulate capital and upgrade the technology. If the private capital stock becomes sufficiently large before the spike in public capital fades away due to depreciation, the private sector’s desire to operate under the efficient technology perseveres. In this case, the economy keeps converging to the high steady state. In other words, a successful transition requires a temporary public investment project to be sufficiently large for the private sector to respond aggressively enough within a relatively short period of time.
The timing of a public project matters for a transition. Positive productivity shocks accelerate a transition to the high steady state, while negative ones impede or can even overturn it. A sequence of large productivity shocks is able to generate a transition by themselves. It suggests that a successful public investment action in a decade with good productivity realizations, such as the 1960s, does not necessarily imply a transition at a time when the productivity behaves poorly.

We calibrate our model and conduct two quantitative case studies in different decades under the same parameter choices. First, we investigate whether the structural shift of the US economy pictured in Figure 9 was indeed caused by the large-scale public investment program or simply by positive productivity shocks. We extract public investment shocks during the transition period from the data and construct productivity shocks using the residual approach. Their impacts are then isolated. We find that the massive public investment alone can rationalize the parallel shifts to a large extent. At the same time, the transition was significantly accelerated by favorable productivity realizations. In contrast, productivity shocks by themselves cannot account for the observed macroeconomic dynamics.

Next, we take our model to the recent decade with the Great Recession and the slow recovery, when the US economy again witnessed S-shaped dynamics. Similarly, we feed in observed government investment shocks between 2007Q2 and 2017Q2 and retrieve productivity shocks via the residual approach. We investigate the following question: Could a large program of public investment, if successfully implemented right after the crisis, have helped the economy return to the high steady state? Experimenting with a counter-factual scenario where the public investment was increased by 1 trillion 2009 dollars in the post-Great Recession time (2009Q3 to 2017Q2), we arrive at a negative answer.

The reason why large government investment shocks succeeded in triggering a transition in the 1960s but might not be helpful during the slow recovery lies in the difference in exogenous

\footnote{The fact that one set of parameters governing the non-convex technology make it possible for the model to match two different episodes lends certain degree of support to our construction of the model and its quantitative realism.}
productivity. Although the technology adoption decisions of firms in our model explain a large fraction of the slowdown in measured productivity over the past decade, mapping our model to the data reveals that the exogenous productivity has not recovered yet. With the presence of these negative productivity shocks, the experimented stimulus would not be powerful enough to induce the private sector to expand. In contrast, productivity shocks did not counter-affect the role of public investment in the first case.

Our work contributes to the business cycle models with productive government capital (Baxter, and King, 1993 and Leeper et al., 2010b).\(^3\) Given the US evidence in the 1960s, by introducing highly nonlinear dynamics, our framework provides a unified laboratory to study the impact of transitory public investment shocks of both small and large scales.\(^4\)

Our model is also related to endogenous growth models with productive public capital. In his seminal work, Barro (1990) constructs an AK model, where the flow of public capital directly enters the production function. Futagami, Morita, and Shibata (1993) extend this framework and incorporate the stock of public capital into the aggregate production function. Turnovsky (2000) enriches framework of Barro (1990) by introducing elastic labor supply. These models, as usual for AK-style frameworks, rely on the knife-edge assumption of exact constant returns in the accumulatable factors. They imply high values of aggregate output elasticity with respect to private and public capital. In contrast, our calibration is in line with the existing empirical estimates.

Fernald (1999) empirically shows that the construction of the Interstate Highway System significantly contributed to the productivity growth during the 1960s and its completion was partially responsible for a consequent growth slowdown. Particularly, in contrast to the implications of standard growth models, he finds a network effect of highways: the first highway system is extremely productive, while the second one might not be.\(^5\) Furthermore,


\(^4\)One interesting example of applying a standard neoclassical framework to study large fiscal shocks is McGrattan and Ohanian (2010), who focus on the World War II. Our model, however, addresses a distinct type of public policy.

\(^5\)Relatedly, Candelon, Colletaz, and Hurlin (2013) find that investment in infrastructure tends to be
inconsistent with what a linear RBC model would predict, Röller and Waverman (2001) find that investment in telecommunications significantly affects economic growth only when a critical mass of infrastructure is established. Implications of our model are consistent with both sets of evidence.

The impact of government investment shocks is studied by Perotti (2004), Auerbach and Gorodnichenko (2012), Ilzetzki, Mendoza, and Végh (2013) in a SVAR setting. A different strand of literature tries to estimate the elasticity of the aggregate output with respect to public capital following the seminal work of Aschauer (1989). Results of subsequent research are summarized by Romp and Haan (2007) and Bom and Ligthart (2014) in recent review papers.

Our formulation of a non-convex cost and resulting persistent economic dynamics share similarities with the works of Durlauf (1993) and Schaal, and Taschereau-Dumouchel (2015). Long-lasting impacts of transitory shocks in a business-cycle environment bridges this paper with the work of Comin and Gertler (2006) and Anzoategui, Comin, Gertler, and Martinez (2016), who develop a two-sector RBC model in which R&D activities contribute to productivity variations and oscillations over medium-term cycles.

The paper proceeds as follows. Section 2.2 discusses evidence on the aggregate and state-level impacts of the infrastructure boom of the 1960s. Section 2.3 presents our model. We then characterize the model and provide further discussions in Section 2.4. Quantitative assessments are carried out in Section 2.5. Two case studies are conducted in Section 2.6. Section 2.7 concludes.

2.2. Motivating facts

To further motivate our model, this section provides additional facts related to the public capital boom in the 1960s. As is well acknowledged, a key event over that decade was the highly productive only when the initial stock of public capital is not too high.

Cooper (1987) and Murphy, Shleifer, and Vishny (1989) exploit similar settings to analyze multiple equilibria.
construction of the Interstate Highway System. The construction was authorized by the Federal-Aid Highway Act of 1956 passed under the presidency of Dwight D. Eisenhower and largely completed by 1973 (Fernald, 1999). An immediate growth impact is however unlikely. First, implementation delay of large infrastructure projects is around 3 years (Leeper et al., 2010b). Second, it takes time for private firms to take advantage of the improvement in public capital. Therefore, we use the observations from 1947Q1 to 1959Q4 when constructing pre-expansion trends.

As shown in Figure 9 in the introduction, the public investment boom of the 1960s was accompanied by S-shaped dynamics in the aggregate output per capita. Importantly, similar time-series patterns are observed in both consumption and investment per capita (Figure 10). The consistency across several major economic series, under identical detrending choices, suggests that what we have documented is indeed a systematic feature of the aggregate US economy around this period of time.

![Figure 10: Nonlinear Consumption and Investment Dynamics in the 1960s.](image)

Notes: log-linear trends are constructed using the data between 1947Q1 and 1959Q4. Details about data construction are provided in Online Appendix.

We next turn to the state level. A cross-sectional variation in sizes of public investment expansions shall be associated with differential behaviors of state-level output series. Below we verify that this is indeed the case in the data. We use state-level data on highway spending from the US Census. Since the BEA data on gross state product goes back only to 1963, we utilize state personal income as our output measure. All states, except Alaska, Hawaii and the District of Columbia, are equally split into two portfolios according to the size of their increases in the highway expenditure during the period of interest – more specifically, the difference between the average highway expenditure to state personal
income ratio from 1960 to 1972 and that over the rest of time span.

According to Panel (a) of Figure 11, both of our state portfolios experienced a sizable increase in highway spending during the 1960s. Between 1960 and 1972, highway-to-income ratios jump up respectively by 1.00 and 0.44 percentage points.\textsuperscript{7} Such increases are economically large: averaged across all states in the postwar sample, highway expenditures account for 1.59\% of personal income. Importantly, the difference between the two is both economically and statistically significant (t-value is above 6).

Panel (b) of Figure 11 shows the averages of detrended real personal income per capita across portfolios. Nonlinear dynamics we have established for the aggregate economy are seen in both series. At the same time, on average, personal income in states where highway expenditures increased relatively more substantially witnessed much larger economic booms. Panel (c) shows that the difference between the personal income series is statistically different from zero on 10\% level for at least a decade after the completion of the

\textsuperscript{7}Notice that highway spending start to rise before 1960, consistent with the fact that the Federal-Aid Highway Act was authorized in 1956. The results are largely unchanged if portfolios are formed based on infrastructure expansion sizes between 1956 and 1972.
Interstate Highway System construction.

Of course, our suggestive evidence, both on the aggregate and state levels, do not establish a causal relation between the public investment boom and observed nonlinear economic dynamics. To answer whether such nonlinearity was indeed driven by the public capital expansion or other factors that happened to contribute to productivity, we turn to our quantitative model.

2.3. Model

In this section, we lay out a simple general equilibrium model consisting of a representative household, a government and a cross-section of static firms. In Section 2.3.5, we discuss some key assumptions in our model.

2.3.1. Households

The economy is populated by a single representative family with GHH preferences (Greenwood, Hercowitz, and Huffman, 1988) maximizing its life-time utility,

$$V = \max_{\{c_t, l_t, k_{t+1}\}_{t=0}^{\infty}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \frac{1}{1 - \gamma} \left( c_t - \frac{1}{1 + \nu} l_{t+1}^{1+\nu} \right)^{1 - \gamma},$$

in which the discount factor, the inverse Frisch elasticity and the risk aversion are denoted respectively by $\beta$, $\nu$ and $\gamma$. The representative household decides on the inter-temporal capital accumulation and the intra-period labor supply. It collects capital income $R_t k_t$, labor income $W_t l_t$, and firm profits $\pi_t$. All sources of income are subject to a uniform tax rate $\tau$. The household’s budget constraint is given by

$$c_t + k_{t+1} = (1 - \tau)(W_t l_t + R_t k_t + \Pi_t) + (1 - \delta_k)l_t - T_t,$$

where $\delta_k$ denotes the depreciation rate of private capital and $T_t$ represents lump-sum taxes (negative values of $T_t$ correspond to government transfers).
2.3.2. Firms

A continuum of identical firms operate in this economy, each of which is equipped with a Cobb-Douglas production technology,

\[ y_t^i = A_t (K_t^G)^{\alpha} (k_t^i)^{\theta_k} (l_t^i)^{\theta_l} o_t^i, \]  

(2.3)

where \( \theta_k \) and \( \theta_l \) represent the output elasticities with respect to private capital and labor. The stock of public capital \( K_t^G \) enters the production function and thus directly affects marginal productivity. The output elasticity with respect to public capital is \( \alpha \). Aggregate productivity evolves exogenously as

\[ \ln A_{t+1} = (1 - \rho_A) \ln A_t + \rho_A \ln A_t + \sigma_A \epsilon^A_t, \quad \epsilon^A_t \sim N(0, 1). \]  

(2.4)

In addition to renting capital \( k_t^i \) and hiring labor \( l_t^i \) from competitive capital and labor markets every period, firm \( i \) chooses whether to raise its productivity by a factor of \( \omega > 1 \) or not, i.e. \( o_t^i \in \{ \omega, 1 \} \). If the former option is chosen, a fixed transfer cost \( f \) is incurred. It stands for expenses incurred when firms adopt a better technology to enhance production efficiency – for example hiring consulting firms and employee-training companies. Hereafter firms with \( o_t^i = \omega/1 \) will be referred to as operating with the high/low technology, respectively.

Firm \( i \) solves the following static profit maximization problem:

\[ \pi_t^i = \max \left\{ \max_{k_t^i, l_t^i} A_t (K_t^G)^{\alpha} (k_t^i)^{\theta_k} (l_t^i)^{\theta_l} - W_t l_t^i - R_t k_t^i - f, \right\} \equiv \pi_t^H^i, \]

\[ \max_{k_t^i, l_t^i} A_t (K_t^G)^{\alpha} (k_t^i)^{\theta_k} (l_t^i)^{\theta_l} - W_t l_t^i - R_t k_t^i \]

\( \equiv \pi_t^L^i. \)  

(2.5)

Conditional on the technology choice, firms adopt identical hiring and renting policies. We
therefore denote optimal capital and labor choices of firms with the high/low technology by superscripts $H/L$.

Public good provision – an expansion in $K_t^G$ – enhances marginal productivity of capital and labor. It also raises the marginal benefit of technology adoption, and as a result, a weakly larger number of firms will find it profitable to scale up their productivity. As will become clear in Section 2.4, this is the key mechanism that supports the steady state multiplicity and therefore gives rise to the nonlinearity of the model. Notice that productivity shock $A_t$ also influences the capital and labor choices as well as the technology adoption.

2.3.3. Government

Government behaviors are summarized by a set of exogenous rules. Public capital depreciates at the rate $\delta_g$ and is accumulated through public investment $G_t^I$,

$$K_{t+1}^G = (1 - \delta_g)K_t^G + G_t^I,$$

(2.6)

where the public investment to output ratio $g_t^I \equiv G_t^I / Y_t$ ($Y_t = \int y_i d\bar{\nu}$), is controlled by a mean reverting spending rule,

$$g_{t+1}^I = (1 - \rho_g)\bar{g}^I + \rho g_t^I + \sigma g_{t+1}^g, \quad \epsilon^g \sim N(0, 1).$$

(2.7)

Besides investment in public goods, government expenditures also include consumption $G_t^C$. Without aiming for a welfare analysis, we assume that the government consumption $G_t^C$ does not enter the household’s utility function. It contains, for example, wage bills of authorities, national defense spending, etc. Since our focus is public investment, we do not incorporate shocks to government consumption in order to keep the model and its solution simple. Specifically, government consumption purchases account for a fixed fraction of the
total output,
\[ \frac{G_t^C}{Y_t} = \bar{g}^C. \] (2.8)

To finance its spending on consumption and investment, the government utilizes both lump-sum and distortionary taxes. The distortionary tax rate \( \tau \) is fixed and, as a result, lump-sum taxes balance the government budget:\(^8\)
\[ G_t^I + G_t^C = \tau Y_t + T_t. \] (2.9)

### 2.3.4. Equilibrium

The model has four state variables: the stocks of private capital and public capital, the public investment to output ratio, and productivity, \( \Omega = (K, K^G, g^I, A) \). A recursive competitive equilibrium is characterized by i) a value function \( V(k, \Omega) \) and policy functions \( c(k, \Omega), k'(k, \Omega) \) and \( l(k, \Omega) \) for the household; ii) individual firm \( i \)'s decisions \( o^i(\Omega) \in \{\omega, 1\}, k^i(\Omega), l^i(\Omega), y^i(\Omega) \) and implied by them profit \( \pi^i(\Omega) \) for all \( i \in [0,1] \); iii) a set of exogenous fiscal rules \( G^I(\Omega), G^C(\Omega) \) and \( T(\Omega) \); iv) pricing functions \( R(\Omega) \) and \( W(\Omega) \); v) laws of motion for private and public capital stocks, \( K'(\Omega) \) and \( K^G'(\Omega) \); vi) mass of firms adopting the high technology \( m(\Omega) \); vii) aggregate variables \( Y(\Omega), \Pi(\Omega) \), such that:

1. \( V(k, \Omega), c(k, \Omega), k'(k, \Omega) \) and \( l(k, \Omega) \) solve (2.1) subject to (2.2), taking prices \( W(\Omega), R(\Omega) \), profits \( \Pi(\Omega) \), transfers \( T(\Omega) \), and the evolution \( \Omega'(\Omega) \) as given.

2. \( o^i(\Omega) \in \{\omega, 1\}, k^i(\Omega), l^i(\Omega), y^i(\Omega) \) and \( \pi^i(\Omega) \) solve the problem (2.5), taking prices \( W(\Omega), R(\Omega) \) as given. Moreover, \( k^i(\Omega) = k^L(\Omega), l^i(\Omega) = l^L(\Omega) \) and \( y^i(\Omega) = y^L(\Omega) \equiv A \left[ K^G \right]^\alpha \left[ k^L(\Omega) \right]^{\theta_k} \left[ l^L(\Omega) \right]^{\theta_l} \) if firm \( i \) operates the low technology and \( k^i(\Omega) = k^H(\Omega), l^i(\Omega) = l^H(\Omega) \) \( y^i(\Omega) = y^H(\Omega) \equiv A \left[ K^G \right]^\alpha \left[ k^H(\Omega) \right]^{\theta_k} \left[ l^H(\Omega) \right]^{\theta_l} \) \( \omega \) otherwise.

\(^8\)In Online Appendix, we numerically investigate the case where the shocks to the government budget are partially absorbed by adjustments in the distortionary tax rate.
3. The mass of firms adopting the high technology is \( m(\Omega) = \int_{\Omega'}(\Omega)=\omega \, di \).

4. Individual decisions are consistent with the aggregate dynamics \( k'(K, \Omega) = K'(\Omega) \).

5. Aggregate variables \( Y(\Omega) \) and \( \Pi(\Omega) \) are given by \( Y(\Omega) = m(\Omega)y^H(\Omega) + (1-m(\Omega))y^L(\Omega) \) and \( \Pi(\Omega) = m(\Omega)(\pi^H(\Omega) + f) + (1-m(\Omega))\pi^L(\Omega) \).

6. Markets for labor, capital and consumption goods clear: \( l(K, \Omega) = m(\Omega)l^H(\Omega) + (1-m(\Omega))l^L(\Omega) \), \( k(K, \Omega) = m(\Omega)k^H(\Omega) + (1-m(\Omega))k^L(\Omega) \), and \( c(K, \Omega) + K'(\Omega) = Y(\Omega) - T(\Omega) - G^C(\Omega) - G^I(\omega) \).

7. The government budget is balanced, \( G^I(\Omega) + G^C(\Omega) = \tau Y(\Omega) + T(\Omega) \), where \( G^I(\Omega) = g^I Y(\Omega) \) and \( G^C(\Omega) = \bar{g}^C Y(\Omega) \).

8. The stock of public capital evolves according to \( K^{G'}(\Omega) = (1 - \delta_g)K^G + G^I(\Omega) \).

2.3.5. Discussions of assumptions

Before proceeding to the model characterizations, we discuss several key assumptions in the model.

Production non-convexities

The key innovation in our model, compared to a standard RBC model with public capital, is the non-convexity in the firms' production choice. In our model, the non-convexity takes a simple form of a binary technology choice. The main results of the paper are unchanged if a continuous non-convex technology choice is considered.

Voluminous literature documents non-convexities in production adjustments on the micro level. Examples of such non-convexities might include capital adjustment (Cooper and Haltiwanger, 2006), labor adjustment (Caballero, Engel, and Haltiwanger, 1997), produc-

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\( \text{43} \)
tion process adjustment (Bresnahan and Ramey, 1994, Hall, 2000), and marketing costs (Spence, 1976).

On the industry level, Cooper and Haltiwanger (1990) argue that the observed statistics of the automobile sector is consistent with a model featuring non-convexity in capital replacement. Ramey (1991) provides evidence of cost functions’ non-convexities for seven industries.

In our setting, micro-level non-convexities have important impacts on the aggregate economic dynamics. Consistent with this view, Hansen (1985) shows the importance of labor indivisibility on aggregate fluctuations. Hansen and Prescott (2005) demonstrate that occasionally binding capacity constraints can help explain business cycle asymmetries. Durlauf (1991), Durlauf (1994) and Schaal, and Taschereau-Dumouchel (2015) argue that a binary technology choice and complementarities between firms can give rise to highly persistent path-dependent responses of macro variables to productivity shocks, as suggested by the data. A series of works emphasize that firm-level non-convexities can explain economy-wide fluctuations without aggregate shocks (Bak, Chen, Scheinkman, and Woodford, 1993, Nirei, 2006 and Nirei, 2015). In the growth literature, non-convexities are utilized to explain club convergence and poverty traps (see Durlauf, 1993 and Galor, 1996 for a literature overview).

Fiscal rules

Our analyses of the impacts of public investment shocks are positive. Therefore, instead of fully specifying the government’s problem, we assume that the government follows an exogenous fiscal rule. This strategy has been widely adopted by the existing literature that tries to quantify the impact of fiscal policies (e.g., Leeper, Plante, and Traum, 2010a, Leeper

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Thomas (2002) argues that lumpy plant-level investment is not relevant for business cycle fluctuations in general equilibrium. However, as pointed out by Nirei (2015), her “...model features a continuum of firms... This choice precludes the possibility that interactions of ‘granular’ firms give rise to aggregate fluctuations...” Importance of this granularity for aggregate fluctuations is underscored by Gabaix (2011). In the setting similar to Thomas (2002), Bachmann, Caballero, and Engel (2013) demonstrate that lumpy microeconomic capital adjustments are important to generate procyclical aggregate investment sensitivity to shocks.
et al., 2010b, Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez, 2015).

Since the model requires a global solution, we try to minimize the number of state variables not essential to our study. We assume that government investment follows a stochastic process, while government consumption constitutes a fixed fraction of output. Moreover, instead of introducing government debt and associated bond smoothing rules, we assume that all government investment shocks are financed through adjustments of lump-sum taxes. In our model and case studies, the tax rate $\tau$ is fixed and thus Ricardian equivalence holds.

**Productive public capital**

We assume that the stock of public capital $K^G$ enters production function of private firms. Our way of modeling public capital goes in line with Arrow and Kurz (1970), Futagami et al. (1993), Baxter, and King (1993) and Leeper et al. (2010b) among others. An alternative modeling approach is to put a flow of public spending in the production function, as in Barro (1990). However, many public goods, including highways, are stock variables in nature. Empirical research, starting from at least Aschauer (1989), generally investigates how stock of public capital affects aggregate productivity. Working with stock variables helps us to link our model, particularly the elasticity $\alpha$, tightly with empirical estimates.

2.4. Characterizations

In this section, we establish some properties of the model and outline key intuitions behind. Under certain restrictions on parameters, the model exhibits two locally stable steady states and has a unique recursive equilibrium. Multiplicity of the steady states is addressed in Section 2.4.1. We consider a special case of a deterministic environment where we can analytically characterize the two steady states. In Section 2.4.2, we establish the existence and uniqueness of the recursive equilibrium in a stochastic environment. Lastly, in Section 2.4.3 we discuss the dynamic behavior of the model. All derivations and proofs are provided in Online Appendix.
2.4.1. Two stable steady states

In this section, our goal is to investigate non-stochastic steady states of the model. Time subscripts are therefore omitted.

Our departure from existing RBC models with productive government capital lies in the binary technology adoption choice. To see how such a formulation alters the environment, we calculate the capital and labor decisions of the $H$ and $L$ firms given prices $R$ and $W$, as well as the mass of firms operating under the high technology $m$. We then express the prices as functions of the aggregate state variables via market clearing conditions. After plugging them back into the firms’ optimal choices, we arrive at the following aggregate production function,

$$Y = \hat{A}(A, m) \left( K^G \right)^{\alpha} K^{\theta_k} L^{\theta_l},$$

$$\hat{A}(A, m) = A \left[ 1 + m \left( \omega^{1-\eta_l-\eta_k} - 1 \right) \right]^{1-\theta_l-\theta_k},$$

where we call $\hat{A}(A, m)$ the measured TFP. It depends on the exogenous productivity $A$ and the mass of firms operating under the high technology $m$, the latter of which is determined endogenously by aggregating individual firms’ binary decisions. $K$ and $L$, respectively, stand for aggregate capital and labor. Notice that for a fixed $m$, our model reduces to a standard real business cycle model with productive public capital.

To understand how $m$ is determined, consider the technology choice given in Equation (2.5). We calculate the difference between profits $\pi^H$ and $\pi^L$ for a given $m$,

$$\Delta \pi(m; K, K^G, A) = \pi^H - \pi^L$$

$$= \zeta A^{1+\nu} \left( K^G \right)^{\alpha(1+\nu)} \left[ 1 + m \left( \omega^{1-\eta_l-\eta_k} - 1 \right) \right]^{\theta_l-\eta_l} \theta_k \left( 1+\nu \right) K^{1-\eta_l+\nu} - f,$$

where scaler $\zeta = (1 - \theta_l - \theta_k)\left( \omega^{1-\eta_l-\eta_k} - 1 \right) \left[ (1 - \tau)\theta_l \right]^{\frac{\theta_l}{1-\eta_l+\nu}} > 0$.

The gain of adopting a new technology $\Delta \pi$ is strictly increasing in the private and public
capital stocks $K$ and $K^G$, together with productivity $A$. However, since $\theta_l < (\theta_k + \theta_l)(1+\nu)$, it is decreasing in $m$. Factor competition lies behind this within-period substitution effect. Due to their higher marginal productivity, $H$ firms optimally choose higher capital and labor in the competitive markets than $L$ firms. Given a predetermined capital stock and the household’s disutility from working, a larger $m$ drives up the demand for capital and labor and thus the prices $R$ and $W$. The benefit of the high technology utilization declines accordingly.\footnote{Schaal and Taschereau-Dumouchel (2015, 2016) show that if the economy features demand externality, $\Delta \pi(m)$ can become increasing or non-monotone in $m$. In their setting, firms’ technology choice is subject to coordination problem and multiple equilibria can arise. In our model, each firm is strictly worse off when more firms are utilizing the high technology, hence equilibrium choice of $m$ is unique given the state variables.}

Equilibrium $m$ is

$$m(K, K^G, A) = \begin{cases} 1, & \Delta \pi(1; K, K^G, A) > 0, \\ m^*(K, K^G, A) \in (0,1), & \Delta \pi(m^*(K, K^G, A); K, K^G, A) = 0, \\ 0, & \Delta \pi(0; K, K^G, A) < 0. \end{cases} \quad (2.13)$$

In the second case, all firms are indifferent between upgrading or not. The result can be interpreted as the outcome of a mixed-strategy equilibrium where each firm operates under the $H$ technology with probability $m^*$. It is easy to see that equilibrium $m$ is (weakly) increasing in $K$, $K^G$ and $A$.

Now inter-temporal complementarity can be clearly seen. Larger capital stocks $K$ and $K^G$ enhance marginal productivities through encouraging the technology adoption and generating a larger $m$. As $m$ elevates marginal productivity, total output and private investment increase. On the other hand, following the spending rule specified in Equation (2.7), $K^G$ also expands at a faster pace.

In the long run, endogenous state variables – $K$ and $K^G$ – fully adjust. In a deterministic world, shocks are held at their long-run means. Write the steady state mass of $H$
firms as a function of the endogenous state variables, \( m^{ss}(K, K^G) \equiv m(K, K^G, \bar{A}) \), and define \( \Delta \pi^{ss}(K, K^G) \equiv \Delta \pi(m^{ss}(K, K^G); K, K^G, \bar{A}) \). The following proposition shows the importance of our two key ingredients: productive public capital and non-convexities.

**Proposition 1** The model exhibits two stable deterministic steady states: \( \{K_H, K^G_H, m^{ss}(K_H, K^G_H) = 1\} \) and \( \{K_L, K^G_L, m^{ss}(K_L, K^G_L) = 0\} \) such that \( K_H > K_L \) and \( K^G_H > K^G_L \) if

i) \( \frac{\nu}{1+\nu} \theta_l < \alpha < 1 - \theta_k - \frac{\theta_l}{1+\nu} \),

ii) \( \Delta \pi^{ss}(K_L, K^G_L) < 0 < \Delta \pi^{ss}(K_H, K^G_H) \).

The first condition provides a boundary on the elasticity of aggregate output with respect to public capital. The lower bound is determined by the conflict between the public capital induced complementarity and the factor competition – the former makes the benefit of technology adoption an increasing function of \( m \) while the latter works in the opposite direction. When the government capital is productive (\( \alpha \) is large), the spillover effect of increase in \( m \) on accumulation of private capital through the exogenous fiscal rule is strong. If, on the other hand, labor is not responsive (\( \nu \) is large) and constitutes a large share of output (\( \theta_l \) is large), high \( m \) also induces a significant increase in wages and thus drives down the benefit of the high technology utilization. If \( \frac{\nu}{1+\nu} \theta_l < \alpha \) then the former force dominates and the multiple steady states arise. It is worth noting that the parameters associated with capital do not show up because capital is fully adjustable across the steady states. The upper bound, \( \alpha < 1 - \theta_k - \frac{\theta_l}{1+\nu} \), guarantees the boundedness of the policies and prevents explosive dynamics of the economy.

When the household utility function is not of the GHH form, a positive labor response to an increase in wages is mitigated by the wealth effect. In this case, high \( m \) is associated with a stronger factor competition. The existence of multiple steady states requires a larger value of \( \alpha \).\(^{12}\)

However, even if public good externality dominates the factor competition, the existence of

\(^{12}\)As argued by Jaimovich and Rebelo (2009) and Schmitt-Grohé and Uribe (2012), the wealth effect on labor supply is almost absent at the business cycle frequencies.
the multiple stable steady states is not guaranteed. Consider, for example, the case where a technology upgrade is extremely costly, i.e. \( f \to \infty \). It will then never be optimal for any firm to adopt the new technology and therefore \( m \) stays at zero. As argued before, in this case the model degenerates to a standard RBC model with a unique steady state. In contrast, when \( f \to 0 \), \( m \) is kept at one and again we only have a unique steady state. The second condition of Proposition 1 makes sure that the fixed cost is mild so that in the high steady state all firms would like to be equipped with the high technology, while at the low steady states no firm wants to.

Stochastic steady states are different from the steady states in a deterministic world due to risk adjustments. We verify numerically that our model preserves the steady state multiplicity in a stochastic environment.

2.4.2. Equilibrium uniqueness

Though the model exhibits two stable steady states, the recursive equilibrium, characterized by a set of policy and pricing functions, exists and is unique.

**Proposition 2** Under mild conditions, outlined in Online Appendix, there exists a unique dynamic recursive equilibrium.

Equilibrium uniqueness is important for policy quantification. In this regard, the model is distinct from previous models with externalities creating social increasing returns to scale and leading to indeterminacy of equilibria (e.g., Benhabib and Farmer, 1994 and Farmer and Guo, 1994). Proofs of the equilibrium existence and uniqueness are nontrivial since our model features aggregate non-convexities and externalities. We extend the monotone operator and lattice-theoretic technique developed by Coleman (1991) to a multi-dimensional endogenous state space.\(^{13}\)

2.4.3. Model dynamics

Steady state multiplicity implies highly nonlinear dynamics of the economy. In this section, we illustrate the implications of our model and the behavior of our economy in response to public investment and productivity shocks.

The dynamics of the model can be illustrated by the phase diagram in Figure 12. The economy features two basins of attraction. In the low steady state, private and public capital are scarce. Firms optimally choose not to utilize the high technology, \( m = 0 \). On the contrary, private and public capital are abundant in the high steady state, and all firms operate under the high technology, \( m = 1 \). Both steady states are stable. Thus, relatively small shocks cause only temporary changes. Large shocks can lead to long-lasting consequences, associated with steady state transitions.

Consider, for example, the economy at the low steady state. A massive public capital expansion incentivizes the private sector to upgrade the technology and accumulate more capital. A large increase in total output and thus the tax revenue further pushes up the amount of public investment. This positive feedback loop is counteracted by depreciation of public capital. For a large enough public investment program, the former force dominates and the economy reaches the high steady state’s basin of attraction. This steady state transition is illustrated by the dashed blue line in Figure 12.

In contrast, following a short-run investment project of an insufficient magnitude, the transition is not achieved. In those cases, private and public capital have not yet arrived at large enough levels by the moment depreciation starts to produce an impact. A decline in government capital stock suppresses marginal benefit of private investment and technology upgrade. Eventually, the economy returns back to the low steady state, as illustrated by the dot-dashed red line in Figure 12.

Similarly, insufficient government capital creation during an economic crisis or a devastating destruction of productive public resources in a war or a natural disaster can result in
unintended long-lasting consequences, since the economy might slide down from the high to low steady state.

Responses to productivity shocks inherit such nonlinearity. Similar to a government investment shock, a productivity shock alters the private sector’s incentive to accumulate capital and adopt the efficient technology and thus is able to trigger a steady-state transition. In a depressed economy with productivity growth sufficiently below its long-run trend, the private sector lacks a desire to invest. The economy starting at the high steady state might slide down to the low steady state unless government intervenes through fiscal actions that can effectively prevent the total factor productivity from a considerable drop.

2.5. Quantitative assessments

From this section, we take our model to quantitative analyses. We describe our parametrization in Section 2.5.1. In Section 2.5.2, we investigate the quantitative performance of the model.
2.5.1. Calibration

The period of the model is one quarter. Table 3 lists the parameters. Sources of the data used for calibration are outlined in Online Appendix.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preferences</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk aversion</td>
<td>$\gamma = 1.0$</td>
<td>Log utility</td>
</tr>
<tr>
<td>Frisch labor elasticity</td>
<td>$1/\nu = 3.33$</td>
<td>Higher end of macro estimates</td>
</tr>
<tr>
<td>Time discounting</td>
<td>$\beta = 0.95^{1/4}$</td>
<td>0.95 annually</td>
</tr>
<tr>
<td>Production function</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labor elasticity</td>
<td>$\theta_l = 0.56$</td>
<td>Basu, and Fernald (1997)</td>
</tr>
<tr>
<td>Private capital elasticity</td>
<td>$\theta_k = 0.24$</td>
<td>Basu, and Fernald (1997)</td>
</tr>
<tr>
<td>Public capital elasticity</td>
<td>$\alpha = 0.15$</td>
<td>Bom, and Ligthart (2014)</td>
</tr>
<tr>
<td>Depreciation rates</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Private capital</td>
<td>$\delta_k = 1 - 0.9^{1/4}$</td>
<td>10% annually</td>
</tr>
<tr>
<td>Public capital</td>
<td>$\delta_g = 1 - 0.92^{1/4}$</td>
<td>Leeper et al. (2010b)</td>
</tr>
<tr>
<td>Fiscal rule</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Government consumption</td>
<td>$\bar{g}^C = 0.235$</td>
<td>Postwar US data</td>
</tr>
<tr>
<td>Transfers</td>
<td>$\bar{z} = 0.060$</td>
<td>Postwar US data</td>
</tr>
<tr>
<td>Government investment</td>
<td>$\bar{g}^I = 0.041$</td>
<td>Postwar US data</td>
</tr>
<tr>
<td>Marginal tax rate</td>
<td>$\tau = 0.336$</td>
<td>$\tau \equiv \bar{g}^I + \bar{g}^C + \bar{z}$</td>
</tr>
<tr>
<td>GI shocks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Persistence</td>
<td>$\rho_g = 0.967$</td>
<td>Postwar US data</td>
</tr>
<tr>
<td>Standard deviation of shocks</td>
<td>$\sigma_g = 0.0011$</td>
<td>Postwar US data</td>
</tr>
<tr>
<td>Productivity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>$\bar{A} = 0.764$</td>
<td>Normalization</td>
</tr>
<tr>
<td>Persistence</td>
<td>$\rho_A = 0.94$</td>
<td>Output persistence</td>
</tr>
<tr>
<td>Standard deviation of shocks</td>
<td>$\sigma_A = 0.008$</td>
<td>Output volatility</td>
</tr>
<tr>
<td>Technology adoption</td>
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<td></td>
</tr>
<tr>
<td>Fixed cost</td>
<td>$f = 0.0051$</td>
<td>See text</td>
</tr>
<tr>
<td>Scaling up parameter</td>
<td>$\omega = 1.02$</td>
<td>Distance between steady states</td>
</tr>
</tbody>
</table>

Table 3: Parametrization.
Standard parameters

A few parameters are chosen using standard values adopted in the literature. We set the time discounting to $\beta = 0.987$. The depreciation rate of private capital is $\delta_k = 0.026$. The elasticities of output with respect to private capital and labor are set to $\theta_k = 0.24$ and $\theta_l = 0.56$, respectively. They together imply a returns-to-scale parameter of $0.8$, which lies within a range of industry-wide estimates of Basu, and Fernald (1997). For the preferences of the representative household, we adopt the log utility, i.e. $\gamma = 1$. We set $\nu = 0.3$, which implies a Frisch elasticity of labor supply of $3.33$ and is consistent with existing macro estimates (Chetty, Guren, Manoli, and Weber, 2011).\textsuperscript{14}

Stochastic processes

We calibrate $\rho_A$ and $\sigma_A$ to match the autocorrelation and volatility of the medium-term cycle component (0-200 quarters) of output. In Section 2.5.2, we describe how we measure these statistics. The resulting values are $\rho_A = 0.94$ and $\sigma_A = 0.008$. The mean $\bar{A} = 0.764$ is set to normalize capital in the low steady state to 1.

The public investment process is estimated using the postwar US data. We find the mean and persistence of the government investment to output ratio (GI ratio hereafter) to be $\bar{g}^I = 0.041$ and $\rho_g = 0.967$. The standard deviation of GI shocks is $\sigma_g = 0.0011$. The fractions of output spent on government consumption and transfers are $\bar{g}^C = 0.235$ and $\bar{z} = 0.060$. Corresponding income tax rate is $\tau \equiv \bar{g}^I + \bar{g}^C + \bar{z} = 0.336$. The quarterly depreciation rate of public capital is set to $\delta_g = 0.0206$, similar to Baxter, and King (1993) and Leeper et al. (2010b).\textsuperscript{15}

\textsuperscript{14}A robustness analysis with a higher value of $\nu = 0.6$ are provided in Online Appendix.

\textsuperscript{15}In Online Appendix, we demonstrate that a standard RBC model with public capital, even with a fairly low value of $\delta_g = 0.0127$, cannot rationalize the behavior of the US economy around the 1960s.
Non-convexity

There are two key parameters for us: $\alpha$ and $\omega$. There is not much consensus on the value of $\alpha$. Early studies find very high elasticities – for example Aschauer (1989) estimates $\alpha$ to be 0.39 – while more recent estimates tend to be lower on average, although the variability is huge. We set $\alpha = 0.15$, which is close to the average value reported by Bom, and Ligthart (2014) in their survey paper. In Online Appendix, we experiment with an $\alpha$ of 0.1, resulting in the model’s quantitative performances largely unchanged.

Our calibration of $\omega$ rests on the assumption that the US economy was in the low steady state before the start of the massive public spending of the 1960s and had largely converged to the high steady state by the early 1970s. We set $\omega = 1.02$ to match the distance between the steady state levels of output to the difference in the levels of the pre-1960 and post-1973 detrended per capita GDP. It is important to notice that we do not re-calibrate $\omega$ when taking this model to the post-2007 period. The fact that the model still provides a reasonable match of empirical series lends some support to our calibration strategy.

Being quantitatively less important, the fixed cost $f$ governs the frequency of transitions across the two steady states. Realized transitions are rare, therefore the postwar US data might not be very informative in this regard. Instead, we pick $f = 0.0051$ in order to match frequencies of extreme output growth events in the model and in the data. In our framework, a change in the mass of firms using the high technology $m$ is associated with a larger change in output. If $f$ is too high or low then $m$ is unlikely to change because it is either too costly to operate the high technology (high $f$, $m = 0$), or the scale-up is cheap and all firms use it (low $f$, $m = 1$). In this case, large fluctuations in output are less frequent since the amplification through change in $m$ does not take place. For our choice of $f$, the model economy generates a similar-to-the-data number of large output growth events. Resulting $f = 0.0051$ corresponds to 2% of average aggregate output. Online Appendix describes our approach in more details.
2.5.2. Model assessments

Before applying our framework to two case studies and conducting corresponding counterfactual analyses, it is important to examine whether our calibration is quantitatively realistic. Section 2.5.2 compares the data and model implied second moments of the major macroeconomic series. Section 2.5.2 describes the impulse response functions to the government investment and productivity shocks.

Unconditional moments

<table>
<thead>
<tr>
<th></th>
<th>Medium-term cycle, 0-200 qtr</th>
<th>High frequency component, 0-32 qtr</th>
<th>Medium frequency component, 32-200 qtr</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
</tr>
<tr>
<td>Output</td>
<td>4.23</td>
<td>4.46</td>
<td>2.30</td>
</tr>
<tr>
<td></td>
<td>(2.99,6.46)</td>
<td>(1.59,2.61)</td>
<td>(2.28,6.05)</td>
</tr>
<tr>
<td>Consumption</td>
<td>3.34</td>
<td>3.62</td>
<td>1.30</td>
</tr>
<tr>
<td></td>
<td>(2.36,5.37)</td>
<td>(1.11,1.80)</td>
<td>(1.92,5.20)</td>
</tr>
<tr>
<td>Hours</td>
<td>3.75</td>
<td>3.42</td>
<td>1.78</td>
</tr>
<tr>
<td></td>
<td>(2.31,4.87)</td>
<td>(1.23,1.97)</td>
<td>(1.76,4.59)</td>
</tr>
<tr>
<td>Investment</td>
<td>12.35</td>
<td>10.61</td>
<td>8.05</td>
</tr>
<tr>
<td></td>
<td>(7.24,15.64)</td>
<td>(4.33,7.92)</td>
<td>(5.09,14.24)</td>
</tr>
<tr>
<td>TFP</td>
<td>2.46</td>
<td>2.17</td>
<td>1.52</td>
</tr>
<tr>
<td></td>
<td>(1.57,2.44)</td>
<td>(0.93,1.44)</td>
<td>(1.10,2.65)</td>
</tr>
</tbody>
</table>

Table 4: Macroeconomic Fluctuations – Model and Data. Notes: Standard deviations of macro variables in the model and in the postwar US data. The model is simulated for 280 quarters for 10,000 times. 95% confidence intervals are given between parentheses. Details about data construction are provided in Online Appendix.

As shown in Table 4, our model reproduces both high and medium frequency fluctuations of the US economy. Following Comin, and Gertler (2006), we assume that the high frequency component of a medium run cycle includes fluctuations at frequencies below 32 quarters, while frequencies between 32 and 200 quarters correspond to the medium frequency component. We use the band pass filter to separate these components (Baxter and King, 16 TFP is measured using standard Solow residual approach. Aggregate production technology is given in equation (2.10). Recall that TFP is endogenous and depends on the mass of firms utilizing the high technology.)

For high frequency fluctuations, standard deviations of output, consumption, hours implied by the model are in a good correspondence with their data counterparts. Private investment and TFP appears to be slightly more volatile in the data. The model under our calibration also generates sizable medium-run oscillations.

**Impulse responses**

**Government investment shocks.** We start by shocking the economy located at the low steady state by GI shocks of different sizes. Corresponding impulse responses are depicted in Figure 13. For the solid blue lines, the GI ratio goes up by 0.33 p.p.. For the dashed red lines the shock is 0.67 p.p.. Finally, for the dot-dashed yellow lines the shock is 1 p.p..

![Figure 13: Impacts of GI Shocks. Notes: Impulse responses of the economy starting at the low steady state to GI shocks of various sizes. X-axis is the number of quarters.](image)
A small shock does not induce any firms to start operating with the high technology, and $m$ stays at zero. The impact of the shock is transitory and similar to what is delivered by existing RBC models with public capital. During the early stages, consumption and private investment are depressed due to an increase in lump-sum taxation. Later on, when the stock of public capital goes up, output, consumption and investment increase. GHII preferences, with zero wealth effect on labor supply, imply that hours do not respond at the moment of the shock but later go up significantly, even despite an increase in consumption.

Larger shocks produce a qualitatively different impact on the economy. A large enough increase in the stock of public capital induces some firms to switch to the high technology, which pushes up firms’ demand for capital and labor. Consequently, hours and private investment increase significantly. Resulting high output can support larger public investment and hence induce more firms to adopt the new technology. As suggested by Expression (2.10), a GI shock resulting in an increase in $m$ is also associated with a growth of measured TFP $\hat{A}(A,m)$.

When firms start to operate with the high technology, the responses to shocks become more persistent, in case of the 0.67 p.p. shock, or even permanent, for the 1 p.p. shock. As mentioned earlier, a successful transition requires a transitory GI shock to be large enough – private capital should arrive at a sufficiently high level before depreciation of public capital weakens the private sector’s incentives to invest and upgrade.

Another important feature of our model is that the impact of public investment programs is state dependent. At the high steady state, all firms operate at the full capacity, $m = 1$, and even large GI shocks cannot induce additional firms’ switching (see Online Appendix for corresponding impulse responses). Hence, keep investing in public infrastructure is not a way for the government to pursue a higher growth rate in the long run.\footnote{This is reminiscent of the idea that many types of infrastructure exhibit network properties. Fernald (1999), for example, finds that while investment in highways was very productive during the period of active construction of the Interstate Highway System, afterwards additional dollar spent on roads is unlikely to generate exceptional return.}
Productivity shocks. Figure 14 plots the impulse response functions associated with $A$ shocks of 0.83% (solid blue), 1.67% (dashed red) and 2.5% (dot-dashed yellow). As in standard RBC models, investment, consumption, hours and output jump up upon the arrival of a positive productivity shock. The abundance of factors and a high productivity lead to a sharp increase in $m$ and in measured TFP (for example, in response to a 2.5% $A$ shock, measured TFP grows by almost 3.5%). Public investment goes up due to a higher output.

However, the resulting increase in the stock of public capital is fairly small even under a large $A$ shock, and thus cannot sustain an elevated level of $m$ after productivity returns to its normal levels. A huge one-time or a sequence of large $A$ shocks are required to get a successful transition. We find that a quarterly $A$ shock of at least 6.1% (more than 8.2% in measured TFP) is required to trigger the transition from the low to high steady state.
Notice that the responses to productivity shocks are much less persistent compared to those to public investment shocks. The difference mainly lies in the slow-moving nature of public capital stock, which is of course propagated by responses in \( m \).

2.6. Counterfactual Experiments

We apply our model to two quantitative case studies. In Section 2.6.1, we look into the US economy around the 1960s. In Section 2.6.2, we consider the Great Recession and a consequent slow recovery. Particularly, we are interested whether a large-scale public investment program could have helped the economy to return to its pre-Recession path.

2.6.1. Public investment in the 1960s

We first turn our attention to the 1960s. Our goal here is to study whether the public investment boom observed in the 1960s was indeed the main driver underlying the S-shaped dynamics of the US economy before 1990, documented in Figure 9. We extract GI shocks between 1960Q1 and 1972Q4 from the data and back out \( A \) shocks by matching measured TFP within the same period. In addition to a public investment boom, the model suggests that exogenous productivity \( A \) was highly favorable during the 1960s. This might be due to low oil prices or low interest rates.

As presented in Figure 15, the model offers a reasonable match of consumption and output series before 1990. However, it undershoots the investment change. In the data, the difference between the pre-1960 and post-1973 levels is almost the same for consumption and output but much larger for private investment. The model falls short on this margin because output, investment and consumption are proportional to each other across the two steady states.

We are now ready to address the relative importance of the GI and \( A \) shocks for the S-shaped dynamics via a counter-factual test. First, we keep productivity shocks while turn off GI shocks. We then repeat the exercise with GI shocks only. Figure 16 shows our
results. During the early stage, the dashed red ($A$ shocks only) and the solid blue lines (both GI and $A$ shocks) are pretty close to each other. The dot-dashed yellow lines (GI shocks only) for all series but public capital stock stay around zero. With the accumulation of government capital, however, the impact of the GI shocks becomes pronounced: the red and blue lines start to diverge. The impulse response functions, likewise, suggest that the impact of productivity shocks is immediate, while that of GI shocks unfolds gradually.

The economy switches to the high steady state in all three cases. However, the transition speeds vary dramatically. With GI surprises shut down, the transition is very prolonged, and the model significantly undershoots the responses of the major macroeconomic variables before 1990. In the absence of productivity innovations (but with the GI shocks extracted from the data), the economy converges to the high regime much faster. We conclude that the GI shocks played a more important role in the transition between the steady states around the 1960s.
Figure 16: Nonlinear Dynamics in the 1960s – Counterfactual Analyses. Notes: Roles of GI and productivity shocks are isolated.

2.6.2. Great Recession and slow recovery

The US economy after 2007 also exhibits a structural shift. The blue solid lines in Figure 17 show detrended output, consumption and private investment per capita. All series plummeted during the Great Recession and their subsequent recoveries have been either weak (for investment) or absent (for output and consumption). Moreover, the GI ratio gradually decreased to the lowest since the 1950s level after the Great Recession (see Online Appendix for corresponding figure). In this section, we first evaluate the importance of such a decline for the slow recovery. We then investigate whether a large program of public investment would have helped the US economy to return to the pre-Recession path.
We assume that the economy was at the high steady state right before the Great Recession. Similar to the previous case study, we feed the GI shocks between 2007Q4 and 2017Q2 and back out $A$ shocks with residual approach. Without recalibrating $\omega$, the model reasonably matches aggregate quantities, as can be seen in Figure 17 (red dashed lines).\textsuperscript{18,19}

How did the drop in government investment after 2010 contribute to the slow recovery? The solid blue lines in Figure 18 depict the model implied series under both productivity and GI shocks. The dashed red lines illustrate the model’s behavior without GI movement. It turns out that these two sets of lines almost coincide, which suggests that the role GI shocks have played during the slow recovery is minor.

Interestingly, even a large program of government investment would not have helped the US economy to return to the high steady state. We consider a GI shock of 1.75 percentage points right after the Great Recession (2009Q3). Being highly persistent, this program\textsuperscript{18}It turns out that these shocks push the economy to the low steady state. If we set shocks to zero after 2017Q2, the economy does not return to the pre-recession levels.\textsuperscript{19}The model undershoots the investment drop. In the data, a large fraction of investment drop is driven by a huge decrease in the residential investment, which is outside of our model.
implies a spending of about 1 trillion 2009 dollars between 2009Q3 and 2017Q2. Impacts of this program are shown by the dot-dashed lines in Figure 18. We find that such a gigantic increase in public investment would produce only limited aggregate impacts within the post-Great Recession decade. The economy is not going to return to the high steady state because of this public investment boom.

Why did a large public investment in the 1960s successfully push the economy to the high steady state, while a comparable project was unlikely to help in the aftermath of the Great Recession? It turns out that the model implied true productivity $A$ is still significantly below its trend. There is a force outside of the model that prevents $A$ from the...
recovery. Under these circumstances, public investment might be inefficient because such an undesirable aggregate productivity keeps private investment depressed. On the contrary, not only government investment was at an unprecedented level during the 1960s, but also A shocks were highly favorable.

2.7. Conclusion

In this paper, we document structural shifts of the US economy associated with a large public investment boom in the 1960s. We then build a business cycle model that can account for such highly nonlinear dynamics. Complementarity introduced by public capital and a non-convex cost associated with utilizing a more efficient technology together give rise to multiple stable steady states and the model’s nonlinearity.

The economic dynamics caused by short-run government investment programs crucially depend on their magnitudes. On the one hand, large-scale transitory shocks to public investment can cause parallel shifts in the levels of macroeconomic variables through triggering a transition across steady states. Small-scale disturbances, on the other hand, generate standard short-run economic responses.

Somewhat surprisingly, although our model highlights the merit of the large public investment in the 1960s, it casts doubt on a similar initiative in the post-Great Recession era. Our analysis suggests that the effectiveness of such a program would be severely discounted by the exogenous productivity that is currently highly unfavorable compared to the 1960s.

This paper focuses on a positive analysis of the impact of transitory public investment shocks and thus formulates government’s decisions by a set of exogenous rules. An interesting extension would be to conduct a normative investigation and examine how optimal fiscal

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20 Some potential candidates include but are not limited to a sequence of adverse financial shocks (Dominguez and Shapiro, 2013), insufficient R&D investment (Anzoategui et al., 2016), demand complementarities (Schaal, and Taschereau-Dumouchel, 2015), or uncertainty (Fajgelbaum, Schaal, and Taschereau-Dumouchel, 2017a).

21 It does not mean, however, that government investment is always useless in post-recession periods. Our model rather suggests that productive public spending can be very efficient if A is recovering after a negative shock, which does not seem to be the case for the Great Recession. We elaborate on this point in Online Appendix.
policy in this nonlinear world looks differently compared to a standard model. We leave it to future work.
CHAPTER 3 : FINANCIAL NETWORKS OVER THE BUSINESS CYCLE

3.1. Introduction

The global financial crisis of 2007–2008 has drawn academics’ and policy makers’ attention to the role played by financial interconnectedness in creating systemic risk. Prior to the crisis, a conventional view emphasized that linkages between financial institutions improve risk sharing and enhance the system’s resilience to shocks hitting individual firms. Since the crisis, a growing body of research has argued that these linkages can also magnify systemic risk by exposing institutions to common sources of risk.\(^1\) How do incentives of institutions to form interconnections change over time? Why does the financial system become fragile at certain points of the credit cycle? These are the central questions of this paper.

I construct a tractable general equilibrium model of the dynamic interplay between real aggregates, intermediaries’ balance sheets and financial interconnectedness to study the endogenous variation in systemic risk over time. In the model, banks raise funds from households and on the interbank market and extend credit to the real sector. Interbank debt is defaultable, and bankruptcy losses are priced rationally. To reduce their borrowing rates, banks diversify by investing in a finite number of risky primitive asset classes or projects. By engaging in risk sharing, banks become similarly exposed to underlying shocks and, thus, become interconnected. Because of common exposures, a negative project-specific surprise might result in widespread defaults, i.e. systemic crisis.\(^2\) Over the business cycle, time-varying credit supply and aggregate productivity govern the returns on the primitive assets, thereby affecting banks’ portfolio choices, interconnectedness, and systemic risk.

Systemic crises do not burst at random in the model. Lending expansions, driven by positive

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\(^{1}\)In a seminal work of Allen and Gale (2000), resilience of the system is increasing in the degree of interconnectedness. More recent contributions that emphasize potential fragility of densely connected systems include, among others, Elliott, Golub, and Jackson (2014), Acemoglu, Ozdaglar, and Tahbaz-Salehi (2015), and Cabrales, Gottardi, and Vega-Redondo (2017).

but temporary aggregate shocks, saturate the economy with credit and gradually exhaust productive investment opportunities. When shocks fade away, marginal products of the underlying projects are depressed because of decreasing returns to scale. Intermediaries' investments generate low payoffs, and their profit margins narrow. To mitigate rising individual default probabilities, banks diversify more actively. Doing so drives up the degree of asset commonality. The economy reaches the point of magnified financial fragility where a moderate negative shock to one of the underlying projects can cause massive simultaneous intermediaries defaults. Consistent with recent empirical evidence, in a calibrated model systemic crises are typically preceded by credit booms and rising financial interconnectedness; total factor productivity, despite being positive at the initial stages of such booms, declines later on.\(^3\)

More specifically, the financial system in this paper consists of two types of institutions, investing and noninvesting banks. Both types can invest in riskless storage. On top of that, investing banks have access to a set of more productive risky projects. Such heterogeneity gives rise to the interbank market, where investing banks borrow from noninvesting ones. Interbank debt is subject to defaults, and costs of bankruptcies of investing banks are borne by noninvesting ones. Expected default losses, rationally priced in the interbank rate, can be diminished by risk sharing. In particular, each investing bank specializes in one asset class and has to incur a monitoring cost to invest in other projects. The optimal degree of portfolio diversification trades off monitoring expenses for a lower probability of individual bankruptcy and hence cheaper interbank debt.

In normal times underlying projects are productive, and expected payoffs to the investing banks' portfolios are high. Interbank debt is cheap because of low default probabilities. The financial system is sparsely connected, because demand for diversification is limited. Strong

\(^3\)That large-scale financial crises follow credit booms is well established in the literature (e.g., Mendoza and Terrones, 2008 and Schularick and Taylor, 2012). Evolution of total factor productivity during credit booms is analyzed in Gorton and Ordoñez (2018). Recent empirical evidence points to a substantial increase in the degree of financial interconnectedness prior to the global financial crisis of 2007–2008 (Billio, Getmansky, Lo, and Pelizzon, 2012), including asset commonality (Cai, Eidam, Saunders, and Steffen, 2018). See Section 3.2 for a further discussion.
intermediaries’ balance sheets and low interconnectedness make systemic crises unlikely. The network is therefore robust, i.e. a huge negative shock is required to trigger a large-scale banking crisis.

At the end of credit booms, risky projects generate below-average returns, tightening investing banks’ profit margins. To mitigate the counterparty risk on the interbank market and prevent the borrowing rate from a sharp increase, investing banks involve in more active risk sharing. Risk sharing shifts financial intermediaries’ exposures from projects of their specialization to a broader pool of assets. While it does help investing banks to keep their individual default probabilities at modest levels, the financial system as a whole becomes fragile: affecting all entities in a similar way, even a moderate adverse project-specific shock can make many of them insolvent at once.

Large-scale banking crises, although painful, are a natural side effect of generally beneficial risk sharing. In the absence of risk sharing, the financial system is not connected. Investing banks do not simultaneously default but instead frequently go bankrupt in isolation. Moreover, facing high individual insolvency risks, they reduce their interbank borrowing and curb the supply of credit to the real sector. In fact, preventing banks from sharing risks and keeping the number of systemic crises at zero is suboptimal.4

A natural question in this regard is whether the frequency of such events occurring in a laissez-faire equilibrium is efficient. In the model, an individual investing bank internalizes how its portfolio choice and default affect the rest of the economy. Nevertheless, the decentralized allocation is characterized by too frequent systemic crises. The source of inefficiency is a pecuniary externality in presence of incomplete markets and real default costs. Agents in the economy fail to internalize that credit expansions reduce the risky projects’ returns and, crucially, enhance intermediaries’ default probabilities. Therefore, the decentralized allocation is marked by overinvestment in the risky technology, inefficiently low returns and

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4Relatedly, Allen and Gale (1998) argue that demand deposit contracts, despite giving rise to occasional bank runs, still have an overall positive effect on the economy since they allow depositors to share risks.
an excessively connected, fragile financial system.

Recent financial innovations like securitization and credit derivatives (e.g., credit default swaps and collateralized debt obligations) have facilitated diversification for individual institutions, while intensifying the degree of their common exposures and contributing to a lending boom (e.g., Shin, 2009, Stulz, 2010). The model interprets such innovations as a reduction in risk-sharing costs. This affects the economy in two ways. On the one hand, investing banks are more efficient in reducing their expected default losses. On the other hand, more accessible risk sharing expands investing banks’ borrowing capacities and improves the returns on households’ funds supplied to the financial sector. The credit supply grows, further depressing the productivities of the underlying projects and exacerbating the adverse effects of the pecuniary externality. Financial innovation in the model is destabilizing in the sense that it increases the number of systemic crises. However, the welfare implications are generally ambiguous and depend on whether risk-sharing benefits outweigh increased costs of the pecuniary externality.³

The model is calibrated to match conventional macroeconomic moments as well as the frequency of systemic financial crises of about 1.7 per century (Romer and Romer, 2017). Besides massive joint defaults of investing banks, the economy also experiences milder non-systemic crises. They are marked by bankruptcies of only those institutions whose portfolios are tilted toward projects hit by bad shocks. The two types of crises tend to happen under quite different conditions. Investing banks collapse together after prolonged credit booms during which financial interconnectedness and fragility are gradually increasing. Nonsystemic crises, on the contrary, are more likely in sparsely connected networks and are not normally preceded by unusual credit expansions. I find empirical support for these patterns in the data.

**Literature.** This paper builds on a rapidly growing literature focusing on contagion in

³The view that financial innovations improve risk sharing is traditional in the literature (Allen and Gale, 1994). More recently, a potentially destabilizing role of financial innovations has been underscored by, among others, Allen and Carletti (2006), Gennaioli, Shleifer, and Vishny (2012), Brunnermeier and Sannikov (2014).
financial networks. Early contributions by Kiyotaki and Moore (1997a), Allen, and Gale (2000), Freixas, Parigi, and Rochet (2000), and Lagunoff and Schreft (2001) present the first formal models of contagion in financial systems. More recently, the literature has identified several major channels through which granular idiosyncratic shocks might spread to the whole system, thereby causing aggregate problems (see Cabrales, Gale, and Gottardi (2016) for a review). One is the domino effect: if institutions hold claims on each other, the failure of one organization might initiate a default cascade (e.g., Acemoglu et al., 2015 and Glasserman and Young, 2015). Another channel is common exposures to the same underlying assets, as in Elliott et al. (2014) (their model also features the domino effect), Wang (2015), and Cabrales et al. (2017).

This paper’s network modeling is most similar to that of Cabrales et al. (2017). However, whereas their study analyzes financial structures emerging under a wide set of shocks distributions, I focus on financial ties arising under sufficiently thin-tailed shocks and positive linking costs. Moreover, banks in my setting endogenously choose their liabilities.

More generally, in contrast to this strand of the literature, my paper presents a dynamic model of financial interconnectedness. In principle, a static analysis is able to reveal that financial networks exhibit a robust-yet-fragile feature (Haldane, 2009), i.e. they are susceptible to occasional systemic crashes if hit by sufficiently negative shocks. However, only a dynamic model is able to shed light on why shocks that are normally absorbed by the system efficiently can in certain fragile states lead to systemic crashes.

This paper is related to the literature introducing financial frictions into macroeconomic models, initiated by the classic works of Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997b), and Bernanke, Gertler, and Gilchrist (1999). These

studies typically rely on log-linear approximations around the steady state to examine the role of financial accelerator for aggregate fluctuations. Nonlinear effects in economies with occasionally binding constraints are investigated by Mendoza (2010), He and Krishnamurthy (2012), and Brunnermeier, and Sannikov (2014). In my setting, financial crises are associated with painful recessions due to the simultaneous defaults of many institutions. This is reminiscent of Gertler and Kiyotaki (2015), who study runs on the whole banking sector.

Only a few recent papers attempt to explain boom-bust dynamics of credit around financial crises. In this respect, the works of Boissay, Collard, and Smets (2016) and Gorton, and Ordoñez (2018) are most closely related. Like in my paper, in their settings credit expansions reduce a marginal product of the underlying production technology. When it becomes sufficiently low, a crisis occurs, either due to agency issues (Boissay et al., 2016) or due to production of information revealing the quality of the collateral (Gorton, and Ordoñez, 2018; see also their earlier work, Gorton and Ordoñez, 2014). In contrast, in my paper, crises are associated with insolvencies of interconnected financial institutions, which might simultaneously default even due to a shock to one relatively narrow sector. This is reminiscent of the worldwide Great Recession initiated by the U.S. subprime mortgage crisis (e.g., Brunnermeier, 2009).7

The focus on banks’ borrowing decisions and financial interconnectedness relates this paper to Barattieri, Moretti, and Quadrini (2018). In their model, banks become interconnected by trading parts of their individual investments for a fully diversified interbank portfolio in order to expand their borrowing. My paper features two important differences. First, because of the granularity of the economy, systemic crises are triggered by project-specific surprises, and the degree of financial interconnectedness affects the probability of such events. Second, general equilibrium analysis underscores the role of endogenously changing interest rates for systemic risk.

7The boom-bust dynamics also can arise due to learning about a new financial environment (Boz and Mendoza, 2014), self-fulfilling expectations (Perri and Quadrini, 2018), or costly liquidation of long-term loans due to roll-over crises (Paul, 2018). Also related is the literature on bubbles in production economies (e.g., Martin and Ventura, 2012).
The welfare analysis of the pecuniary externality shares similarities with Lorenzoni (2008), Bianchi and Mendoza (2010), Jeanne and Korinek (2010), Bianchi (2011), and Boissay et al. (2016). My paper departs from the existing literature in that it studies the impacts of the externality on financial interconnectedness as well as its interactions with financial innovations.

The remainder of the paper is organized as follows. Section 3.2 provides motivating empirical evidence on the behavior of financial interconnectedness around the Great Recession. Section 3.3 lays out the model. Section 3.4 characterizes the model. Section 3.5 assesses the model numerically and illustrates its dynamic properties. Section 3.6 carries out the welfare analysis. Section 3.7 concludes.

3.2. Connectedness around the global financial crisis

Since the global crisis of 2007-2008, a large empirical literature has attempted to construct measures of financial interconnectedness. This task is not straightforward. Institutions might become interrelated in various ways, from holding claims on each other’s payoffs to investing in similar assets. This section argues that various measures of interconnectedness exhibit similar time-series patterns. In particular, they tend to increase prior to the global financial crisis and recede afterward, suggesting a buildup of systemic risk during seemingly tranquil times.

The literature on systemic risk typically relies on comovements in asset prices to infer ties between financial institutions. Using principal component analysis and Granger-causality networks, Billio et al. (2012) document that U.S. banks, hedge funds, broker/dealers, and insurance companies became highly interrelated in the decade preceding the global financial crisis (see also Diebold and Yilmaz, 2009, 2014). A few measures identify the tail dependency of individual institutions and the whole financial system (e.g., Huang, Zhou, and Zhu, 2009, Acharya, Pedersen, Philippon, and Richardson, 2017, and Brownlees and Engle, 2017). These measures sharply rise in turbulent times, but do not capture a buildup of systemic
risk prior to the recent financial crisis. As noted by Billio et al. (2012), this might be related to underrepresentation of large losses episodes in the precrisis data. Of particular interest here is forward-looking forward $- \Delta CoVaR$ by Adrian and Brunnermeier (2016), which has desired cyclical features.

Figure 19: Four interconnectedness measures. All series are normalized to 1 in 2002 and smoothed using a 1-year moving average. The shaded area represents the Great Recession (NBER timing). Online Appendix gives full nonsmoothed, nonnormalized series. Panel (a) illustrates the industry/region-based overlap in the syndicated loan portfolios of the U.S. lead arrangers. Portfolios are considered overlapping if they have common exposures to a specific borrower industry and region. Source: Cai et al. (2018). Panel (b) illustrates the ratio of nonagency mortgage-backed securities and asset-backed securities over the total assets from the 100 largest U.S. bank holding companies. Source: FR Y-9C. Panel (c) illustrates the ratio of total liabilities net deposits (noncore liabilities) over the total assets from the 100 largest U.S. bank holding companies. Source: Barattieri et al. (2018) and FR Y-9C. Panel (d) illustrates the ratio of the total liabilities of all U.S. sectors over the total liabilities of the nonfinancial sector. Source: Greenwood, and Scharfstein (2013) and U.S. Flow of Funds.

Closely related to my paper are several studies attempting to measure the degree of asset commonality directly. Cai et al. (2018) document that interconnectedness due to the overlap in syndicated loan portfolios of the largest U.S. banks has been increasing between 2001 and 2007 but dropped during the crisis (panel (a) of Figure 19). Moreover, interconnectedness

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*Ivashina and Scharfstein (2010) find that the share of syndicated loans retained by lead arrangers, a*
is positively correlated with loan portfolio diversification and systemic risk measures in the cross-section. Consistent evidence is presented by Blei and Ergashev (2014), who consider correlations between the asset portfolios of major U.S. bank holding companies.

Securitization, created to facilitate credit risk sharing, is another contributor to portfolio similarity. During the run-up of the recent crisis, banks originated massive amounts of mortgage-backed and asset-backed securities. However, a large fraction of them was not transferred to outside investors. Instead, financial institutions traded them between each other and held them both on and off their balance sheets (e.g., Shin (2009) and panel (b) of Figure 19). As a result of multiple rounds of asset exchanges, financial market participants became exposed to similar underlying shocks. Relatedly, Nijskens and Wagner (2011) show that banks that start to use credit default swaps (CDS) and collateralized loan obligations (CLO), new credit risk transfer methods that flourished prior to the Great Recession, experience increases in their market betas. Importantly, such increases come solely from higher correlation with the market, suggesting that risk sharing through holding CDS and CLO contribute to systemic risk. Similar effects of new credit risk transfer instruments are documented by Franke and Krahnen (2007) and Hänsel and Krahnen (2007).

A complementary strand of the literature considers the direct claims of financial institutions on each other. Panel (c) of Figure 19 uses the interconnectivity measure suggested by Barattieri et al. (2018). The measure is defined as banks’ liabilities held by other parties within the financial sector (noncore liabilities) over total assets (see also Hahm, Shin, and Shin, 2013 and Koijen and Yogo, 2016). A similar approach is used by Greenwood, and Scharfstein (2013). Aiming to capture the number of steps involved in credit creation, the authors construct the credit intermediation index by dividing the total liabilities of all sectors by the liabilities of the nonfinancial sector only (panel (d) of Figure 19). A larger value of the index signals more lending between financial institutions. Similar time-series patterns (growth prior to the crisis and a reduction afterward) are observed for these types of loans’ ownership concentration, has been steadily decreasing since at least 1991 but spiked in 2008.
of interconnectedness measures.

Lastly, interconnectedness of global banking network tend to rise prior to and fall in the aftermath of banking crises, both on the institutional (Hale, 2012) and on the country (Minoiu and Reyes, 2013) levels.

3.3. Model

This section presents an infinite horizon model set in discrete time. By convention, time subscripts are omitted, and next-period variables are denoted by primes. Tildes mark random variables subject to intraperiod uncertainty.

The economy consists of a risk-averse, long-lived representative household and a financial sector populated by risk-neutral and short-lived banks. The household owns all assets but can only access production technologies through intermediaries. In order to focus on frictions within the financial sector, I abstract from any household-bank frictions. The household’s role is minimal: it funds banks, supplies labor to the real sector, and makes intertemporal consumption/savings decisions.

The banking sector includes a large number of ex ante identical institutions, with a small subset of them receiving risky investment projects ex post. Ex post heterogeneity gives rise to an intraperiod interbank market in which banks with investment opportunities borrow from the remaining ones. Interbank lending is subject to costly defaults. To mitigate expected default losses priced in the interbank rate, borrowers cross-invest in each other’s projects, forming risk-sharing linkages. These ties expose institutions to shocks to a broad pool of underlying projects. Common exposures make systemic crises associated with widespread defaults possible.

Sections 3.3.1–3.3.3 lay out the benchmark model and outline the key assumptions. Section 3.3.4 discusses the assumptions in more detail.
3.3.1. Banking sector

The banking sector consists of a finite number $N$ of islands. Each island is populated with $M$ banks. Banks are risk neutral and short lived. They are ex ante identical, and, therefore, at the beginning of each period, they raise the same amount of assets $a_o$ from the representative household. All banks have access to a riskless storage technology with a constant gross return $\rho_s$. Banks are ex post heterogeneous. In particular, each period one institution per island becomes an investing bank (the remaining ones are noninvesting). The two types of institutions differ in their access to risky investment opportunities.\(^9\)

Production technology

Each island is associated with an investment project. A project of island $i \in \{1, \ldots, N\}$ (or simply project $i$) requires capital $k_i$ and labor $l_i$ to produce

\[
\tilde{y}_i = z k_i^{\eta} l_i^{1-\eta} + (1 - \delta - \tilde{x}_i) k_i,
\]

where $\eta$ is the capital share, $\delta$ is the depreciation rate, $\tilde{x}_i$ is a project-specific depreciation shock unknown at the beginning of the period, and $z$ is an aggregate productivity realized at the beginning of the period.\(^10\) $\log z$ evolves according to a standard AR(1) process:

\[
\log z' = \rho \log z + \sigma \epsilon', \quad \epsilon' \sim \mathcal{N}(0, 1).
\]

Project-specific depreciation shocks $\tilde{x}_i$, $i \in \{1, \ldots, N\}$, are treated as rare and large. Each period one and only one of $N$ projects receives a shock.\(^11\) The shock size follows a distribu-

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\(^9\)Two comments are worth making. First, the assumption of one-period banking sector is crucial to keep the number of state variables small. It would be identical to assume instead that banks are long-lived but their types are reshuffled every period, similar to Gertler and Karadi (2011). Second, the storage return $\rho_s$ is constant. As will be discussed at the end of Section 3.4.2, this is not crucial for the results.

\(^10\)Assuming different timing of shocks keeps the analysis as simple as possible. The results are unchanged if the aggregate productivity $z$ is realized at the same time as project-specific shocks. See Online Appendix for more details.

\(^11\)At the expense of tractability but without changing the qualitative results of the paper, the analysis can be straightforwardly extended to handle a richer shock structure.
tion with a sufficiently smooth cumulative distribution function $\Phi(\cdot)$. The support of $\Phi(\cdot)$ is $0 \leq x < \bar{x} \leq \infty$.

Labor is hired on a competitive market at the wage rate $w$. Solving the static labor optimization problem, the project’s operating profit is

$$\max_{l_i} \tilde{y}_i - wl_i = (R - \tilde{x}_i)k_i, \quad (3.3)$$

where

$$R = z\eta \left(\frac{(1 - \eta)z}{w}\right)^{1-\eta} + 1 - \delta. \quad (3.4)$$

$R - \tilde{x}_i$ is the return on capital invested in project $i$. Notice that $R$ does not depend on any island-specific variables.

**Interbank market and risk-sharing linkages**

While the storage technology is available to all banks, only investing ones have access to the risky underlying projects. Particularly, an investing bank from island $i$ has an expertise in project $i$ (i.e., its own project) and can invest in it at no cost. Moreover, it can allocate some of its assets to the remaining projects at an upfront cost of $f > 0$ per unit of investment, which includes any monitoring expenses. On the contrary, noninvesting banks cannot access the risky technology directly. This heterogeneity gives rise to the intraperiod interbank market in which investing banks borrow from noninvesting ones.

**Assumption 1** Interbank borrowing happens within the islands.

Assumption 1 implies that the economy features a segmented interbank market, where each segment is represented by an island. This assumption captures the idea that establishing new interbank links might be costly; these costs prevent any cross-island borrowing. An island in the model includes banks with established relationships that can contact each
other at no cost.\footnote{Notice that the degree of the interbank market segmentation (i.e., the number of islands) coincides with the number of underlying risky projects \( N \). In principle, they do not have to be the same (like in, e.g., Elliott et al., 2014). The model can be extended to handle a more general case. I verify that such a generalization does not alter the main results in any substantial way (unreported).} Section 3.3.4 discusses the assumption in more detail.

The interbank market of island \( i \) is characterized by a large number \( M - 1 \) of lenders (noninvesting banks) lending to one borrower (investing bank) at a rate of \( \rho_i \). I assume that the latter is a monopolist in this market. As a result, noninvesting banks in expectation earn the storage return \( \rho_s \).

**Assumption 2** *Investing banks extract all the surplus from the interbank borrowing.*

Since returns on the investment projects are uncertain, interbank debt is subject to defaults that are associated with real resource losses. This is rationally priced in the interbank borrowing rate \( \rho_i \). Thus, even despite their risk neutrality and positive monitoring costs, investing banks are willing to invest in projects of other islands in order to diversify their individual risks and reduce costly default probabilities. As a result of cross-island investments, an investing bank \( i \) holds a portfolio \( \{\omega_{ij}\}_{j=1}^{N} \), \( \sum_{j=1}^{N} \omega_{ij} = 1 \), where \( \omega_{ij} \geq 0 \) represents the fraction of its assets under management \( a_i \) invested in project \( j \). Here, \( a_i \) consists of assets directly raised from the household and borrowed on the interbank market. The upfront cost of portfolio formation is \( f a_i \sum_{j \neq i} \omega_{ij} \).

The \( N \times N \) matrix of portfolio holdings \( B = \{\omega_{ij}\}_{i,j=1}^{N} \) describes the structure of linkages between investing banks. In the network terminology, \( B \) is an adjacency matrix of a weighted directed graph. The elements of \( B \) represent the financial institutions’ exposures to shocks to underlying projects. For example, a large value of \( \omega_{ij} \) implies that investing bank \( i \) is strongly affected by an adverse shock to project \( j \). As long as \( \omega_{ij} > 0 \ \forall i \in \{1, \ldots, N\} \), a shock to project \( j \) affects all investing banks and can potentially lead to the simultaneous defaults of many institutions. Importantly, defaults themselves do not trigger cascading losses among investing banks, because they do not hold assets directly related to the payoffs of other intermediaries. Further comments on this modeling choice are given in Section 3.3.4.
Investing bank’s problem

The problem of investing bank $i$ can be now formulated. Because of model symmetry, island-specific indices are omitted where possible. Each investing bank starts a period with assets $a_o$, which are directly raised from the representative household. Additionally, it borrows $a_b$ at the rate $\rho$ on the interbank market and chooses an investment portfolio $\{\omega_{ij}\}_{j=1}^N$ in order to maximize its expected payoff. Assume that project $j$ is hit by an adverse shock of size $x$. At the end of the period, investing bank $i$ is insolvent when its portfolio payoff is below the face value of its debt,

$$a \left[ \sum_{s=1}^N R \omega_{is} - \omega_{ij} x \right] - \rho a_b < 0,$$

where $a = a_o + a_b$ is the total assets under management of investing bank $i$.$^{13}$ Because $\sum_{s=1}^N \omega_{is} = 1$, this expression can be rewritten as

$$x > \frac{R - \rho \mu}{\omega_{ij}},$$

where $\mu = \frac{a_b}{a}$ is the ratio of borrowed assets to total assets. Inequality (3.6) demonstrates that an adverse shock to project $j$ leads to default by investing bank $i$ if either the bank’s profit margin, captured by the spread $R - \rho \mu$, is small or its exposure $\omega_{ij}$ to such shock is large.

Investing bank $i$ maximizes an expected payoff to its portfolio under limited liability, net of

$^{13}$Notice that the diversification costs are paid by investing banks’ shareholders (the representative household) prior to any interbank relationships forming. Online Appendix elaborates on the version of the model in which $f$ is paid by investing banks ex post and thus directly affects their default cutoffs. The results are robust to this alternative modeling approach.
risk-sharing costs.

$$\max_{\rho, \mu, \{\omega_{ij}\}_{i=1}^{N}} \frac{a_o}{1 - \mu} \mathbb{E}_x \left[ \frac{1}{N} \sum_{j=1}^{N} (R - \omega_{ij} x - \rho \mu) \mathbb{I} \left\{ x \leq \frac{R - \rho \mu}{\omega_{ij}} \right\} - f \sum_{j \neq i} \omega_{ij} \right] , \quad (3.7)$$

s.t. $$\sum_{j=1}^{N} \omega_{ij} = 1, \quad \omega_{ij} \geq 0 \ \forall j \in \{1, \ldots, N\}.$$ \hfill (3.8)

Here, $$\mathbb{E}_x$$ denotes the expectation with respect to the size of project-specific shock $$x$$. The first constraint requires portfolio weights to sum up to 1. The second set of constraints restricts short-selling.

Investing bank $$i$$ is subject to the nondeviation constraint of its lenders:

$$\rho \leq \mathbb{E}_x \left[ \frac{1}{N} \sum_{j=1}^{N} \mathbb{I} \left\{ x \leq \frac{R - \rho \mu}{\omega_{ij}} \right\} + \frac{1}{\mu} \frac{1}{N} \sum_{j=1}^{N} (R - \omega_{ij} x - \theta) \mathbb{I} \left\{ x > \frac{R - \rho \mu}{\omega_{ij}} \right\} \right] . \quad (3.9)$$

In case of no default, each lender receives $$\rho$$. In case of default, lenders split the portfolio of their borrower equally. Default is subject to a loss of a fraction $$\theta \geq 0$$ of assets under management of a borrowing investing bank.

Finally, each investing bank can operate at most all assets within its island. Therefore, it is subject to the following borrowing constraint:

$$\mu \leq \bar{\mu} = \frac{M - 1}{M} . \quad (3.10)$$

In equilibrium, the fraction $$\frac{1 - \bar{\mu}}{1 - \mu}$$ of all assets in the economy is invested in the risky projects, and the remaining share $$1 - \frac{1 - \bar{\mu}}{1 - \mu}$$ goes to the riskless storage technology.

To summarize the description of the banking sector, Figure 20 shows its structure and gives the timing of the intraperiod events.
3.3.2. Representative household

The representative household starts a period with assets $a_{hh}$ and rents them to the financial sector at the rate of $r$ (panel (a) of Figure 20), defined as an average return generated by all institutions within the banking sector (see expression (3.14) below). It also supplies labor $l$ for the wage $w$. At the end of a period, the household consumes and decides on the amount
of assets in the next period $a'_{hh}$ (panel (f) of Figure 20). The household solves

$$V(a_{hh}, \Omega) = \max_{a'_{hh}, c, l} \frac{1}{1 - \psi} \left( c - \frac{1}{1 + \nu} l^{1+\nu} \right)^{1-\psi} + \beta \mathbb{E} \left[ V(a'_{hh}, \Omega') \right],$$

(3.11)

s.t. $a'_{hh} = r(\Omega) a_{hh} + w(\Omega) l - c + \chi(\Omega), \quad \Omega' = \Omega'(\Omega).$

(3.12)

The household is endowed with GHH (Greenwood et al., 1988) preferences over consumption and labor with the inverse Frisch labor supply elasticity of $\nu$, the inverse elasticity of intertemporal substitution $\psi$, and the time-discounting factor $\beta$.\(^{14}\) $\chi = \chi(\Omega)$ represents the total monitoring costs paid by investing banks to the household in order to establish risk-sharing links.\(^{15}\) Finally, $\Omega$ is the set of the aggregate state variables whose evolution $\Omega'(\Omega)$ is taken by the household as given.

3.3.3. Equilibrium

The model has three state variables: the stock of assets $A$, aggregate productivity $z$, and the size of project-specific shock $x$, $\Omega = (A, z, x)$. By symmetry, the size of a project-specific shock, but not its location, matters for the aggregates. Moreover, investing banks’ portfolio weights are symmetric, $\omega_{ij}(\Omega-x) = \omega_{ji}(\Omega-x)$, $\forall i, j \in \{1, \ldots, N\}$. Because project-specific shocks are realized after banks made their choices, a relevant set of the state variables for them is $\Omega-x = (A, z)$. Moreover, in the absence of a wealth effect on the labor supply, the wage rate does not depend on $x$ as well, $w = w(\Omega-x)$. A decentralized recursive equilibrium consists of a set of quantities and prices described below.

(i) Investing banks’ decisions $\mu(\Omega-x), \rho(\Omega-x), \{\omega_{ij}(\Omega-x)\}_{j=1}^{N}$ solve (3.8) subject to (3.9) and (3.10).

\(^{14}\)Notice that $\psi > 0$ implies that the household is risk averse, whereas bankers are assumed to be risk-neutral profit maximizers. While this assumption is not crucial for the results (see Online Appendix), it substantially simplifies both theoretical analysis and numerical solution. Section 3.3.4 discusses the assumption in more detail.

\(^{15}\)For tractability, monitoring expenses are assumed to enter the household’s budget constraint in a lump-sum fashion. Alternatively, they can be modeled as a part of the household’s labor income. This would complicate the analysis, because the aggregate supply of labor then would be split between the real (risky projects) and banking (monitoring) sectors.
(ii) The value function $V(a_{hh}, \Omega)$ and policy functions $c(a_{hh}, \Omega), l(a_{hh}, \Omega-x), a'_{hh}(a_{hh}, \Omega)$ of the household solve (3.12) taking the prices $r(\Omega), w(\Omega-x)$, the aggregate law of motion $\Omega'(\Omega)$, and transfer $\chi(\Omega-x) = A'_1 - \bar{\mu} (\Omega-x) \sum_{j \neq i} \omega_{ij} (\Omega-x)$ as given.

(iii) Capital invested in each project is $k(\Omega-x) = a_0 (A) = A' N \cdot M$.

(iv) The wage $w(\Omega-x)$ clears the labor market, $L(\Omega-x) = N \cdot l^d(\Omega-x) = l(A, \Omega-x)$, where $l^d(\Omega-x)$ solves (3.3). The return on the household's assets $r(\Omega)$ is

$$r(\Omega) = \left(1 - \frac{1 - \bar{\mu}}{1 - \mu(\Omega-x)}\right) \rho_h + \frac{1 - \bar{\mu}}{1 - \mu(\Omega-x)} \left[R(\Omega-x) - \frac{1}{N} x - \theta \frac{N^d(\Omega)}{N} - f(1 - \omega_{ii}(\Omega-x))\right],$$

(3.14)

where $R(\Omega-x)$ is given by (3.4), and $N^d(\Omega)$ is the number of investing banks in default,

$$N^d(\Omega) = \sum_{i=1}^N \mathbb{I}\left\{x > \frac{R(\Omega-x) - \rho(\Omega-x) \mu(\Omega-x)}{\omega_{ij}(\Omega-x)}\right\}.$$

(v) Individual decisions are consistent with the aggregate law of motion, $a'_{hh}(A, \Omega) = A'(\Omega)$.

(vi) Aggregate goods market clear, $A'(\Omega) = \left(1 - \frac{1 - \bar{\mu}}{1 - \mu(\Omega-x)}\right) \rho_h A + z \left(\frac{1 - \bar{\mu}}{1 - \mu(\Omega-x)} A\right)^\eta L(\Omega-x)^{1-}\eta + \frac{1 - \bar{\mu}}{1 - \mu(\Omega-x)} \left[1 - \delta - \frac{1}{N} x - \theta \frac{N^d(\Omega)}{N}\right] - C(\Omega)$.

3.3.4. Discussions of assumptions

This section discusses several key assumptions of the model.

**Household’s and bankers’ attitudes to risk**

In the model, the representative household has incentives to intertemporally smooth consumption, $\psi^{-1} < \infty$. Because its utility function is from the constant relative risk aversion family, it automatically makes the household risk averse. At the same time, banks are
assumed to be risk-neutral expected payoff maximizers. They do not weigh states of the world with the stochastic discount factor of their shareholders (the household). Although such an assumption makes their problem tractable, it might raise a question about why bankers are not acting on behalf of the household.

A vast literature studies whether managers’ and stakeholders’ objectives are aligned. Managers of firms with mixed financial structures ignore the interests of outside equity and debt holders and take excessive risks from a social perspective (Jensen and Meckling, 1976). Imperfect contracts and compensation schemes, along with agency problems, might result in excessive risk-taking even from the shareholders’ point of view. Mutual funds deviate from maximizing risk-adjusted returns due to inflow-related benefits (e.g., Chevalier and Ellison, 1997). Option-like compensation schedules can induce excessive risk taking by managers (Lambert, 1986).

The main results are largely unaffected if objectives of the household and bankers are aligned (see Online Appendix). Even though I am unable to characterize the solution analytically, the main features of the benchmark model are preserved.

**Structure of the financial system**

In the model, the financial system is two tiered. Noninvesting banks do not have access to efficient production technology and lend their funds to investing banks. The latter then invest in multiple projects on their behalf. Therefore, the interbank market exhibits a well-known core-periphery structure (e.g., Boss, Elsinger, Summer, and Thurner, 2004, in ’t Veld and van Lelyveld, 2014, Craig and von Peter, 2014, Craig and Ma, 2018). Only a few connections exist between the core and the periphery (each noninvesting bank lends to only one investing bank), and no ties exist within the periphery. The core is densely connected, because all investing banks hold diversified portfolios.

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16Risk neutrality of bankers/entrepreneurs is a typical assumption in the literature incorporating financial frictions into macroeconomic models (e.g., Carlstrom, and Fuerst, 1997, Bernanke et al., 1999).
The model abstracts from explicitly modeling why the core-periphery structure arises endogenously. I instead assume that noninvesting banks within one island are restricted to lend to only one investing bank.\textsuperscript{17} The literature provides a number of microfoundations for this structure. A partial list includes a reduction in monitoring (Craig, and Ma, 2018) costs, intermediation (Farboodi, 2015) expenses, and inventory risk (Wang, 2016).

\textbf{Types of connections and systemic risk}

In the framework of this paper, investing banks are interconnected through common portfolio holdings. Systemic crises happen when a large adverse shock to one underlying project affects all institutions and results in widespread bankruptcies, as in for example Cabrales et al. (2017). Importantly, investing banks do not hold any securities directly related to the payoffs of other intermediaries (e.g., their debt or their equity). Defaults of other institutions within the financial system thus do not cause direct problems to investing banks, which substantially simplifies the analysis. Only within islands, where bankruptcy losses propagate from the core to periphery institutions, does the domino effect (e.g., Acemoglu et al., 2015) play a role.

As discussed in the review paper of Cabrales et al. (2016), the relation between interconnectedness and probability of systemic crisis is similar regardless of the type of links (see, in particular, result 8): highly integrated systems are more susceptible to joint breakdowns. If individual players are strongly exposed to each other or to a common set of underlying shocks, they are more likely to crash together. Incorporating the domino effect into my setting, like in Elliott et al. (2014), is likely to make a densely connected interbank network even more susceptible to systemic collapses. Fragility due to commonality in asset holdings is also exacerbated by fire sales (e.g., Caccioli et al., 2014).

\textsuperscript{17}Since the model is symmetric, the interbank rates are the same across different islands. Even if noninvesting banks could contact investing banks from other islands at no cost, they would not find it profitable to do so. Generally speaking, if cross-island borrowing was possible, the equilibrium interbank rate would depend on how the surplus is split between noninvesting and investing institutions. For example, the same interbank rate would arise if the surplus was fully extracted by the latter party, like in the benchmark analysis.
The evidence for whether the domino effect is an important contributor to systemic risk is mixed (see Upper (2011) for a survey). Comparing the relative strengths of the domino and the common exposures mechanisms in the Austrian banking system, Elsinger, Lehar, and Summer (2006) find that the latter is a dominant source of systemic risk. The crucial role of common exposures for financial crises is also emphasized by Borio (2003).

3.4. Characterization

This section outlines how the interconnectedness of the financial system and systemic risk interact with the dynamic choices of the representative household. Section 3.4.1 characterizes the structure of investment banks’ portfolios. Section 3.4.2 analyzes the static interbank problem and establishes several comparative statics results regarding the dependence of interconnectedness and systemic risk on the aggregate state variables. Section 3.4.3 discusses the interplay between the representative household’s intertemporal decisions and the interbank outcomes. Online Appendix contains all proofs.

3.4.1. Network structure

I start by characterizing the structure of investing banks’ portfolios. In what follows, the distribution of project-specific shock size $\Phi(x)$ is assumed to be sufficiently concave,

**Assumption 3** $x \frac{\partial^2 \Phi}{\partial x^2} + 2 \frac{\partial \Phi}{\partial x} < 0$.

As shown in Online Appendix, Assumption 3 holds for several relevant loss distributions, including Pareto.

Under Assumption 3, diversification is useful for investing banks to reduce individual default probabilities. Proposition 3 follows.

**Proposition 3** As long as linking cost $f$ is sufficiently small, the constraints $\omega_{ij} \geq 0$ do not bind $\forall j$. Moreover, investing bank $i$ invests a fraction $\omega_{ii} = \alpha \geq \frac{1}{N}$ of assets under its management in project $i$ and invests equal shares of the remaining assets in other projects, $\omega_{ij} = \frac{1-\alpha}{1-\alpha N} \leq \frac{1}{N}$, $\forall j \neq i$. 86
An investing bank fully internalizes how its expected bankruptcy losses are priced into the interbank rate $\rho$, so it is willing to minimize them (subject to the diversification cost). The default probability of investing bank $i$ is

$$p_{ind}^d = \frac{1}{N} \mathbb{E}_x \sum_{j=1}^{N} \mathbbm{1} \left\{ x > \frac{R - \rho \mu}{\omega_{ij}} \right\} = \frac{1}{N} \sum_{j=1}^{N} h_1(\omega_{ij}), \text{ where } h_1(\omega_{ij}) = 1 - \Phi \left( \frac{R - \rho \mu}{\omega_{ij}} \right).$$

(3.15)

Assumption 3 guarantees that $h_1(\omega_{ij})$ is convex. Since $\sum_{j=1}^{N} \omega_{ij} = 1$, $p_{ind}^d$ is minimized when a portfolio is evenly exposed to the underlying shocks. In other words, the default probability can be reduced through diversification.

All projects yield the same diversification benefits. For investing bank $i$, allocating resources toward all but its own project $i$ is subject to equal nonnegative costs. Hence, the fraction of assets invested in project $i$ is (weakly) larger than in all the remaining ones, $\omega_{ii} = \alpha \geq \omega_{ij} = \frac{1-\alpha}{N-1}$, $\forall j \neq i$. All off-diagonal elements of the adjacency matrix $B$ equal to $\frac{1-\alpha}{N-1}$, and a vector of $\alpha$ is on its main diagonal:

$$B = \begin{pmatrix}
\alpha & \frac{1-\alpha}{N-1} & \ldots & \frac{1-\alpha}{N-1} \\
\frac{1-\alpha}{N-1} & \alpha & \ldots & \frac{1-\alpha}{N-1} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{1-\alpha}{N-1} & \frac{1-\alpha}{N-1} & \ldots & \alpha
\end{pmatrix}. \tag{3.16}$$

$\alpha$ captures the strength of investing banks’ cross-exposures. If $\alpha$ is close to one, portfolios are weakly diversified, and the network defined by $B$ is sparsely connected. Investing banks go bankrupt predominantly because of shocks to their own projects. On the contrary, when $\alpha$ approaches its minimum value of $\frac{1}{N}$, ties between investing banks are strong. Shocks to individual projects affect all institutions in a similar way. Systemic crises are more likely.

Below, $IC(\alpha) = \frac{1-\alpha}{1-1/N} \in [0,1]$ is referred to as the interconnectedness of the financial system. Corollary 1 follows.
Corollary 1 The number of investing banks in default \(N^d\) is

\[
N^d = \begin{cases} 
0, & x \leq \frac{R - \rho \mu}{\alpha}, \\
1, & x \in \left(\frac{R - \rho \mu}{\alpha}, \frac{R - \rho \mu}{N - 1}\right), \\
N, & x > \frac{R - \rho \mu}{N - 1}.
\end{cases}
\]  

(3.17)

Hereafter, events with \(N^d = 1\) and \(N^d = N\) are called nonsystemic and systemic financial crises, respectively.

3.4.2. Interconnectedness and systemic risk

This section analyzes the solution to the static investing banks’ problem. By Proposition 3, investing bank \(i\) optimally chooses \(\omega_{ii} = \alpha \geq \frac{1}{N}\) and \(\omega_{ij} = 1 - \alpha \leq \frac{1}{N}\ \forall j \neq i\). Therefore, the default probability of an investing bank is

\[
p_{\text{ind}}^d = \frac{1}{N} \left(1 - \Phi\left(\frac{R - \rho \mu}{\alpha}\right)\right) + \frac{N - 1}{N} \left(1 - \Phi\left(\frac{R - \rho \mu}{\frac{1 - \alpha}{N - 1}}\right)\right),
\]  

(3.18)

where \(p_{\text{own}}^d\) and \(p_{\text{oth}}^d\) are the probabilities of becoming insolvent because of shocks to its own and other projects, respectively. Clearly, \(\frac{\partial p_{\text{own}}^d}{\partial \alpha} > 0\) and \(\frac{\partial p_{\text{oth}}^d}{\partial \alpha} < 0\). Diversification makes defaults due to its own shocks less likely at the cost of elevated probability to go bankrupt because of shocks to other projects. Under Assumption 3, the former effect dominates, \(\frac{\partial p_{\text{ind}}^d}{\partial \alpha} > 0\), and diversification reduces expected default losses.

While diversification helps individual institutions protect themselves against costly defaults, it also increases the chances of a systemic collapse. When the degree of portfolio overlap is sufficiently high, a shock to one project is more likely to trigger the simultaneous failure of all investing banks. Because all institutions allocate at least a fraction \(\frac{1 - \alpha}{N - 1}\) of their assets toward each project, a joint default materializes when \(x > \frac{R - \rho \mu}{\frac{1 - \alpha}{N - 1}}\), which happens
with probability
\[ p_d^{syst} = 1 - \Phi \left( \frac{R - \rho \mu}{\frac{1-\alpha}{N-1}} \right). \] (3.19)

Clearly, \( \frac{\partial p_d^{syst}}{\partial \alpha} < 0 \). Systemic crises are more likely in a densely connected network ceteris paribus.

As diversification is costly, there is always a certain degree of heterogeneity across investing banks’ portfolios. Hence, there exists a range of project-specific shock sizes such that a shock within this range causes bankruptcy of only the investing bank whose project of specialization is hit (Corollary 1). The probability of such a nonsystemic crisis is
\[ p_d^{nonsyst} = \Phi \left( \frac{R - \rho \mu}{\frac{1-\alpha}{N-1}} \right) - \Phi \left( \frac{R - \rho \mu}{\alpha} \right). \] (3.20)

It is straightforward to see that \( \frac{\partial p_d^{nonsyst}}{\partial \alpha} > 0 \). Highly connected systems feature more homogeneous portfolios of individual institutions, resulting in fewer incidences of nonsystemic events.

In equilibrium, the variables affecting default probabilities, \( \alpha \) and the spread \( R - \rho \mu \), are set endogenously. Below, I investigate comparative statics of these variables with respect to changes in the aggregate state variables \( A \) and \( z \). I start by analyzing the case in which the underlying projects are sufficiently productive, so that investing banks are willing to borrow all funds within their islands, \( \mu = \bar{\mu} \). At the end of this section, I consider the region of the state space in which the underlying projects are unproductive and the inequality (3.10) is slack, \( \mu < \bar{\mu} \).

In what follows, I make an additional assumption about the shape of the project-specific shock size distribution,

**Assumption 3’** \[ x^2 \frac{\partial^3 \Phi}{\partial x^3} + 4x \frac{\partial^2 \Phi}{\partial x^2} + 2 \frac{\partial \Phi}{\partial x} > 0. \]
Assumption 3′ guarantees that diversification becomes more attractive when investing banks’ balance sheets weaken. In particular, \( \frac{\partial^2 p_{ind}}{\partial \xi \partial \alpha} < 0 \), where \( \xi = R - \rho \mu \); a reduction in the spread \( \xi \) makes the individual default probability \( p_{ind}^\xi \) more sensitive to changes in \( \alpha \), that is, diversification. Online Appendix verifies that Assumption 3′ holds for several loss distributions, including Pareto.

Under Assumptions 3 and 3′, Proposition 4 establishes how investing banks’ choices are affected by changes in the aggregate state variables \( A \) and \( z \).

**Proposition 4** Assume that condition (3.10) holds as equality, \( \mu = \bar{\mu} \). Then \( \alpha(A, z) \) is decreasing in \( A \) and increasing in \( z \), and the spread \( \xi(A, z) = R(A, z) - \rho(A, z)\bar{\mu} \) is decreasing in \( A \) and increasing in \( z \).

The aggregate state variables affect the interbank allocation through projects’ return \( R \). Clearly, aggregate productivity \( z \) is positively related to \( R \). As long as \( \mu = \bar{\mu} \), an increase in the amount of assets \( A \) translates into higher investment in the risky projects. Since capital and labor are complementary in the production function, the labor demand goes up. Wages rise, and the return \( R \) goes down (see Equation (3.4) and Online Appendix for formal proofs).

From the perspective of investing banks, \( R \) is a nonrandom part of their returns on assets. Holding everything else equal, lower \( R \) reduces the spread \( R - \rho \bar{\mu} \) and increases the chance of costly defaults. Noninvesting banks, lenders on the interbank market, require compensation for the additional risk. The interbank rate \( \rho \) goes up. To prevent their costs of funding from a sharp rise, investing banks (borrowers) diversify more.

Therefore, a change in projects’ return \( R \) affects the fragility of the financial system in two ways, which are illustrated by Figure 21. First, lower \( R \) narrows the spread \( R - \rho \bar{\mu} \) and makes both systemic and nonsystemic crises more likely (the latter is true by Assumption 3). Second, cross-exposures of investing banks become stronger. Importantly, the degree of portfolio overlap rises precisely at the time when insolvency risks are already large.
More dispersed investments prevent the probability of individual default $p_{ind}^{d}$ from steeply increasing. On the other hand, joint collapses become more likely. Corollaries 2.A and 2.B follow.

**Corollary 2.A** As long as $\mu = \bar{\mu}$, the probability of systemic default $p_{syst}^{d}(A, z)$ is increasing in $A$ and decreasing in $z$. Individual and nonsystemic default probabilities, respectively $p_{ind}^{d}(A, z)$ and $p_{nonsyst}^{d}(A, z)$, and the interbank rate $\rho(A, z)$ are generally nonmonotone in $A$ and $z$.

**Corollary 2.B** Assume that $\mu = \bar{\mu}$. Define a threshold in the amount of assets $A^{*}(z, x)$ as

$$x = \frac{R(A^{*}(z, x), z) - \rho(A^{*}(z, x), z)\bar{\mu}}{\frac{1 - \alpha(A^{*}(z, x), z)}{N-1}}. \tag{3.21}$$

Simultaneous defaults of all investing banks happen if and only if $A > A^{*}(z, x)$. Moreover, $A^{*}(z, x)$ increases in $z$ and decreases in $x$.

Figure 22 summarizes the results of comparative statics with respect to the total amount of assets $A$ and aggregate productivity $z$. As long as condition (3.10) holds as an equality, the fraction of assets invested in the risky projects is $\frac{1 - \bar{\mu}}{1 - \mu} = 1$ (panel (a), the horizontal parts of the lines), the variables respond to changes in $A$ and $z$ in line with Proposition 4 and Corollary 2.A. When the return to the underlying projects $R$ is sufficiently low, the constraint $\mu \leq \bar{\mu}$ is slack. In this regime, the risky projects are unproductive, and investing banks optimally choose to reduce their borrowing. As a result, a nonzero amount of the economy’s total assets $A$ is allocated to the storage technology. A further increase in $A$ barely affects the amount of assets invested in the risky projects and hence the return $R$. The default probabilities, spread, interbank rate, and interconnectedness all flatten.\(^{18}\) Importantly, even in the states with $\mu < \bar{\mu}$ (high $A$, low $z$), the financial system stays densely connected, and systemic risk remains elevated.

\(^{18}\)Recall that in case of default lenders equally split the recovered value of their borrower’s portfolio. Return to lenders in those states of the world is proportional to $\frac{\alpha(a + \frac{a + \rho o}{\bar{\mu}})}{\bar{\mu}} = \frac{1}{\bar{\rho}}$ (see expression (3.9)). Therefore, a reduction in $\mu$ increases the expected return to noninvesting banks, who start to charge a lower rate $\rho$ on the interbank market. As a result, default probabilities $p_{syst}^{d}$ and $p_{ind}^{d}$ decrease somewhat.
Finally, it is worth commenting on the role of the constant storage return for Proposition 4, and Corollaries 2.A and 2.B. Because the productivity of the banks’ outside option does not depend on the state of the economy, a reduction in projects’ return $R$ necessarily narrows the spread $R - \rho \bar{\mu}$. As long as the spread shrinks, investing banks become riskier and start to diversify more actively. Interconnectedness goes up, further increasing the probability of systemic crisis. While the assumption $\rho_s = const$ is sufficient for these results to hold, it is far from necessary. For example, Online Appendix verifies that even if the storage return varies one-to-one with $R$, the analog of Proposition 4 can be established.

3.4.3. Model dynamics

Even though the interbank problem is static, the financial institutions’ optimal choices, being functions of the state variables, are changing over time in a nontrivial way. This section discusses how the representative household’s intertemporal choices and the time-
(a) Share of risky investment, $\frac{1-\mu}{1+\mu}$

(b) Interconnectedness, $IC$

(c) Spread, $R - \rho \mu$

(d) Systemic default prob., $p^d_{syst}$

(e) Individual default prob., $p^d_{ind}$

(f) Interbank rate, $\rho$

Figure 22: Comparative statics with respect to total amount of assets $A$ and aggregate productivity $z$. Varying aggregate productivity $z$ interact with the interbank outcomes.

Figure 23 presents a set of stylized asset accumulation policies $A'(A, z, x)$ for two values of aggregate productivity $z$, $z_{mean} < z_{high}$, and two values of project-specific shock size $x$, $x_{good} < x_{bad}$. The discontinuities in policies at $A^*(z_{mean}, x_{bad})$ and $A^*(z_{high}, x_{bad})$ are associated with joint failures of all investing banks, where the threshold $A^*(z, x)$ is defined in Corollary 2.B.\textsuperscript{19}

Consider the economy starting at point $O$, which is the steady state for $z = z_{mean}$ and $x = x_{good}$. At this point, the total amount of assets is relatively low, and the risky projects are productive. Risk sharing is limited because of low default probabilities of individual investing banks. In states with highly productive projects and a sparsely connected financial network, systemic collapses are unlikely. An adverse realization of a project-specific shock, $x = x_{bad}$, brings the economy to $B_O$. This is not exceptionally painful, because the financial system manages to avoid costly joint failures, $A_O < A^*(z_{mean}, x_{bad})$.

\textsuperscript{19}For expositional purposes, the borrowing constraint (3.10) is assumed to bind, so that Corollary 2.B holds. Further, $x_{good}$ is treated as sufficiently small, so systemic defaults do not occur for any combinations of $A$ and $z$, which are shown in Figure 23. Finally, policy discontinuities associated with nonsystemic crises are not shown, because they are small in comparison with those driven by systemic collapses.
Consider now the economy at point $C$. Here, the aggregate productivity is the same as at $O$, $z = z_{\text{mean}}$. However, the amount of assets is significantly larger, $A_C > A^*(z_{\text{mean}}, x_{\text{bad}}) > A_O$, which translates into a lower return $R$ and higher default probabilities for investing banks. They diversify actively, making their portfolios highly similar to each other. Chances of systemic collapse surge. The same project-specific shock that has only a modest negative impact on the economy at $O$ is more detrimental at $C$. It causes painful simultaneous bankruptcies of all investing banks, bringing the economy from $C$ to $B_C$.

How does the economy move from $O$ to $C$? A positive surprise to aggregate productivity $z$ (in Figure 23, $z$ increases from $z_{\text{mean}}$ to $z_{\text{high}}$) drives up the return on risky projects, thus incentivizing the household to save. The economy starts to converge to point $O'$. Since $\log z$ is an AR(1) process, after a period of productivity boom $z$ eventually reverts to $z_{\text{mean}}$, bringing the economy to point $C$, where assets are abundant but not particularly productive. As discussed above, at $C$ the financial system is more fragile than at $O$. If a project-specific shock turns out to be bad at this state ($x = x_{\text{bad}}$), all investing banks go bankrupt at once because of strong common exposures.

Why does the household keep accumulating assets even though expected default losses become high? One reason is consumption smoothing. At the same time, the agents in the
economy do not internalize that investment in risky projects reduces $R$ and thus make the financial system more fragile. This gives rise to a pecuniary externality, which results in excessive amount of investment, inefficiently low return $R$, over-connected financial network and too frequent systemic crises in the decentralized equilibrium. The externality is formally analyzed in Section 3.6.

3.5. Quantitative analysis

This section presents a quantitative analysis of the model. The model is calibrated in Section 3.5.1. Section 3.5.2 presents the impulse responses to aggregate and project-specific shocks. Section 3.5.3 describes the economy’s behavior around systemic and nonsystemic financial crises. Section 3.5.4 emphasizes the importance of connectedness for financial fragility via several counterfactual experiments. Finally, Section 3.5.5 investigates whether the model matches untargeted moments related to the frequency and the severity of financial crises.

3.5.1. Parameterization

The period of the model is 1 year. Table 5 lists the parameters. Online Appendix describes the moment construction in more detail. Online Appendix presents the sensitivity analysis with respect to several key variables.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preferences</td>
<td></td>
<td>Banking sector</td>
<td></td>
</tr>
<tr>
<td>Inverse IES</td>
<td>$\psi = 5$</td>
<td>Number of islands</td>
<td>$N = 10$</td>
</tr>
<tr>
<td>Inverse Frisch elasticity</td>
<td>$\nu = 0.6$</td>
<td>Banks per island</td>
<td>$M = 670$</td>
</tr>
<tr>
<td>Time discounting</td>
<td>$\beta = 0.97$</td>
<td>Diversification cost</td>
<td>$f = 0.005$</td>
</tr>
<tr>
<td>Production technology</td>
<td></td>
<td>Storage return</td>
<td>$\rho_s = 1.009$</td>
</tr>
<tr>
<td>Capital share</td>
<td>$\eta = 0.33$</td>
<td>Default loss</td>
<td>$\theta = 0.1$</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>$\delta = 0.087$</td>
<td>Project-specific shocks</td>
<td></td>
</tr>
<tr>
<td>Aggregate shocks</td>
<td></td>
<td>Tail index</td>
<td>$\gamma = 3$</td>
</tr>
<tr>
<td>Persistence</td>
<td>$\rho_x = 0.83$</td>
<td>Minimum value</td>
<td>$x_{\text{inf}} = 0.088$</td>
</tr>
<tr>
<td>St. dev. of innovations</td>
<td>$\sigma_x = 0.019$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Parameterization.
Preferences. The time discounting factor is $\beta = 0.97$. The Frisch elasticity of labor supply is $\nu^{-1} = 1.67$. The intertemporal elasticity of substitution is $\psi^{-1} = 0.2$. These values are standard in the literature.

Production technology. The capital share is $\eta = 0.33$. The model treats project-specific shocks as capital quality shocks directly affecting the total amount of assets in the economy, as in for example Gertler, and Karadi (2011). Therefore, the total depreciation rate consists of a constant part $\delta$ and a time-varying part associated with $\bar{x}$ shocks. $\delta$ is set to 0.087, which implies that on average the household replaces 10% of assets every period.

Banking sector and project-specific shocks. In the model, large project-specific shocks trigger defaults of financial institutions. Many studies show that extreme realizations of losses have moderately heavy tails (e.g., Jansen and de Vries, 1991, Gabaix, 2009), well described by the Pareto distribution with the tail exponent between 2 and 5 (Ibragimov et al., 2011). I therefore assume that

$$
\Phi(x) = \begin{cases} 
1 - \left(\frac{x_m}{x}\right)^\gamma, & x \geq x_m, \\
0, & x < x_m,
\end{cases}
$$

where $x_m > 0$ is the lower bound of the support and $\gamma = 3$ is the tail index (Gabaix, 2009). Importantly, Assumptions 3 and 3' hold for this distribution.

In the baseline analysis $x_m$ is set to 0.088. The number of risky projects $N$ is 10. $N$ captures the level of the financial system’s granularity. Both $N$ and the size of project-specific shocks governed by $x_m$ directly affect the frequency of systemic defaults in the model. Higher $N$ implies that diversification is more effective in protecting banks against

---

20The banking sectors of many developed economies are dominated by a few large institutions (Laeven, Ratnovski, and Tong, 2014); for example, the 10 largest U.S. bank holding companies account for almost 70% of all assets (FR Y-9C filings). As to the granularity of the portfolios of financial institutions, Billo et al. (2012) document that the first 10 principal components of the returns of major financial institutions and insurers are responsible for up to 83% of all variation. Using FR Y-9C reports, Duarte and Eisenbach (2018) study the strength of fire-sale spillovers; they identify 18 distinct asset classes in the portfolios of U.S. bank holding companies, of which 6 are almost riskless (i.e., they have a risk weight below 20%).
project-specific shocks, making financial crises rarer. Larger project-specific surprises work in the opposite direction. Given $N$, $x_m$ is set so that the frequency of systemic financial crises in the model is around 1.7 per century. This is in line with Romer, and Romer (2017) (more on this in Section 3.5.5). I verify that the results are unchanged for different values of $N$ as long as $x_m$ is simultaneously recalibrated to match the same target.

I set $f = 0.0050$ to match the ratio of monitoring expenses over the loan value reported in Craig, and Ma (2018). Using a revealed preferences approach, they estimate that monitoring costs account for 23% of expected banks’ gross loan value net of funding costs. More specifically, Craig, and Ma (2018) consider a setting in which lending banks (noninvesting banks in my model) lend funds to borrowing banks through core intermediary institutions. The authors find that borrowing banks lose 23% of their values generated on monitoring. In my framework, investing banks directly lend to the real sector, playing the role of a conglomerate of borrowing and intermediary organizations. The value of an interbank loan of size $a_b$ is $(R - \frac{1}{N}E_x x) a_b$, the funding cost is $\rho a_b$, and the amount lost on monitoring is $f(1 - \alpha)a_b$. $f$ is calibrated so that $\frac{f(1 - \alpha)}{R - \frac{1}{N}E_x x - \rho}$ is on average 23%.

The storage return $\rho_s$ and the number of banks per island $M$ are calibrated using the balance sheets and income statements of the largest U.S. bank holding companies’ from the FR Y-9C filings. The storage return governs the investing banks’ cost of borrowing $\rho$. The number of banks per island $M$ is related to the borrowing limit $\bar{\mu} = \frac{M-1}{M}$. $\rho_s - 1 = 0.92\%$ and $M = 670$ are parameterized to match the average net interest income over assets $R - \frac{1}{N}E_x x - \rho \mu$ (2.6% in the data) and the average interest income over assets net of interest expenses over liabilities $R - \frac{1}{N}E_x x - \rho$ (2.4% in the data).

Finally, the fraction of assets lost in default is $\theta = 0.1$, a value close to Bernanke et al. (1999).

**Aggregate shocks.** Using the postwar U.S. data on GDP, hours, and investment from FRED, I construct a series of Solow residuals and log-linearly detrend it. The same exercise
is then repeated within the model. $\rho_z$ and $\sigma_z$ are picked to match the data and model-implied persistence and the standard deviation of innovations in the AR(1) process (3.2). I obtain $\rho_z = 0.83$ and $\sigma_z = 0.019$. Online Appendix describes the procedure in more detail. I also verify that the model performs reasonably well in terms of matching the second moments of the major macroeconomic series.21

3.5.2. Impulse response functions

I start investigating quantitative features of the model by considering impulse response functions to aggregate and project-specific innovations. Figure 24 shows impulse response functions to a positive aggregate shock hitting the economy at its long-run mean. An expansionary aggregate surprise improves the returns on risky projects. Default risks recede, and investing banks cut risk-sharing expenses. Interconnectedness goes down. Naturally, the household increases its accumulation of assets. The stock of assets is slow moving and reaches its peak with a lag (around period 12). At this point, assets are abundant, and $z$ has almost returned to its long-run mean. The risky projects are on average not exceptionally productive, and investing banks spend more on diversification. Interconnectedness rises. As a result, systemic crises become more likely. Consistent with Section 3.4.3, fragility of the financial system is elevated after a boom in productivity.

Innovations to aggregate productivity affect financial fragility (i.e., the probability of systemic crisis), but defaults themselves are triggered by adverse realizations of project-specific shocks. Given the state of the economy $(A, z)$, a project-specific surprise can cause three distinct bankruptcy events depending on its size (Corollary 1). If size $x$ is small, all investing banks are solvent; the financial network effectively protects institutions against project-specific shocks. For a sufficiently large $x$, all investing banks collapse at once because of common exposures. Finally, as long as $x$ is not too small or too large, only the bank that

---

21Under this calibration, the underlying projects are on average productive relative to the storage, so that in the economy around its steady state all resources are allocated to the risky technology, $\mu = \bar{\mu}$ (the constraint (3.10) binds). Substantial deviations from the steady state, associated with a reduction in projects’ return $R$, might bring the economy to the states in which (3.10) is slack. In simulations $\mu < \bar{\mu}$ 13% of time.
specializes in a directly hit project fails to repay its debt. Transitions between the three regimes are associated with policy discontinuities, similar to those shown in Figure 23.

Figure 25 illustrates the economy’s responses to two project-specific surprises of slightly different sizes. For \( x = x_1 \), a nonsystemic crisis bursts. Only one investing bank defaults; the remaining ones are on the edge of collapse but still solvent. This shock has quite limited impacts on the economy, because real default losses are not exceptionally large (in particular, fraction \( \theta N \) of assets is lost). A marginally worse shock \( x = x_1 + \epsilon \) causes a systemic crisis: all investing banks collapse due to such a project-specific surprise.\(^{22}\) Real losses are magnified by a factor of \( N \). The economy experiences a deep recession. Because the stock of assets is directly affected by the crisis, the recovery is slow even though the downturn is caused by a nonpersistent shock.

\(^{22}\) \( x \geq x_1 \) and \( x \geq x_1 + \epsilon \) with a probability of 1.16% and 0.99%, respectively. \( \epsilon \approx 0.021 \), implying that the amount of assets lost directly due to the shocks (but not defaults) differs by 0.21% only.
3.5.3. Economy around financial crises

The previous section has illustrated the distinct impacts of the aggregate and project-specific shocks: the former ones affect the economy’s exposure to the latter ones, which are the immediate triggers of financial crises. The goal of this section is to investigate how the economy, hit by these two types of shocks simultaneously, behaves around the events of financial crises. To do so, the model is simulated for 1,000,000 periods. The incidences of defaults of investing banks are identified. I start by analyzing systemic crises, when all investing banks become insolvent at once. The solid lines in Figure 26 represent the average paths of both exogenous and endogenous variables around those events.

Systemic financial crises do not hit the economy at random. They tend to happen after booms in productivity and credit. In the run-up of a typical boom, a prolonged series of positive aggregate surprises incentivizes the household to save. The amount of assets and investment in the risky projects grow. At this stage, risky projects are still sufficiently productive because of above-average z, and the spread $R - \rho\mu$ is only marginally below its long-run mean. Diversification incentives are modest, and interconnectedness stays close to
Figure 26: Average paths of the economy around systemic and nonsystemic crises \((t = 0)\). The model is simulated for 1,000,000 periods. Series in panels (a)–(g) are expressed as the percentage deviations from their long-run averages. Series in panels (h) and (i) are expressed as a percentage.

its steady-state level.

The probability of a joint collapse sharply rises at the second stage of a boom. Because of its \(AR(1)\) structure, the aggregate productivity reverts to its mean and even falls below it prior to \(t = 0\), the period of systemic crisis. Consumption smoothing and the pecuniary externality prevent the household from quickly responding to such changes in \(z\). The stock of assets stays high, and the return on risky projects deteriorates. Aiming to mitigate the rise in the interbank rate, investing banks diversify more actively and increase their cross-exposures. If one of the projects is hit by a sufficiently bad shock at this moment, the whole financial system crashes.

Prior to systemic crises, below-average values of the aggregate productivity contribute to
the financial fragility. At the same time, $z$ is the main driver of credit booms preceding joint collapses. In the run-up of a typical systemic crisis, aggregate productivity is normally above its average. Panel (a) of Figure 27 shows the distributions of log $z$ at the moment of crisis ($t = 0$) and for the 10 periods prior to it ($t = -10$). They are clearly different from a symmetric zero-centered unconditional distribution: log $z$ tends to be negative at $t = 0$, whereas the opposite is true at $t = -10$.

At the end of credit booms, the spread $R - \rho \mu$ is narrow and interconnectedness $IC$ is high. Under these circumstances, a project-specific shock that in normal times does not trigger a systemic crisis might result in the simultaneous bankruptcy of all investing banks. I find that 55% of project-specific shocks initiating systemic crises in simulations would not cause such an event if the economy was at its steady state. Panel (b) of Figure 27 shows that a substantially smaller project-specific shock is required to cause a joint default in a densely connected network. In fact, 88% of all systemic crises occur when interconnectedness is above its long-run mean.

Unlike systemic collapses, when all investing institutions become insolvent at once, nonsystemic financial crises are marked by default of only one institution. The dashed lines in Figure 26 represent the average paths of the economy around these events. In contrast to systemic defaults, nonsystemic ones tend to happen when interconnectedness is below
average and the spread is elevated. Generally, narrower spreads make both systemic and nonsystemic crises more likely ceteris paribus. However, investing banks respond to lower profitability by spreading their resources among the underlying projects more evenly. The increased degree of portfolio homogeneity reduces the chance of a nonsystemic event. The second force dominates: 60% of all nonsystemic crises happen when interconnectedness is below its long-run average.

3.5.4. Interconnectedness and financial crises

In the model systemic and nonsystemic crises are more likely in a strongly and weakly connected financial system, respectively. The network’s dynamic responses to changing macroeconomic conditions shape the economy’s exposure to financial distress. This section illustrates how the level of, and the time variation in, interconnectedness affect the frequencies of the two types of crises via a series of counterfactual experiments.

The level of interconnectedness is governed by the diversification cost parameter $f$. A decrease in $f$ eases risk sharing and thus can be interpreted as a financial innovation (Allen, and Gale, 1994). A reduction in $f$ affects the economy in two ways. First, at each state $(A, z)$, investing banks naturally become more interconnected. Because of cheaper diversification, in the states of the world in which the constraint (3.10) is slack they also borrow more. Second, lower $f$ improves the return on household’s assets due to diminished intermediation costs (Equation 3.14). The household saves more, pushing down the productivities of the underlying projects. In response, investing banks further increase diversification (panel (a) of Figure 28, solid line). As a result, the economy features fewer nonsystemic crises, while being more susceptible to massive intermediaries defaults (panels (b) and (c) of Figure 28, solid lines).

Next, I investigate the relative importance of the two effects of a reduction in $f$ (i.e., cheaper

\footnote{Given the state $(A, z)$, better risk sharing makes individual defaults less likely. The impact on the systemic crisis probability is in general ambiguous. On the one hand, joint collapses are more probable in densely connected systems. On the other hand, diversification increases investing banks’ profit margin thanks to a lower borrowing rate. Numerically, I find that the former force dominates: $p_d^{\text{ syst}}$ is a decreasing function of $f$ state by state.}
risk sharing and lower intermediation costs) for probabilities of systemic and nonsystemic financial crises. To do so, I assume that $f$ consists of two parts, $f = f_1 + f_2$. Both affect investing banks’ portfolio decisions, whereas only $f_1$ affects the return on household’s assets. In the benchmark economy $f_2 = 0$. For the purpose of isolating the role of interconnectedness, I vary $f_1$ and simultaneously change $f_2$ so that the average interconnectedness remains at its initial level (panel (a) of Figure 28, dashed line). In this experiment, a reduction in the intermediation cost only leads to elevated stock of assets and narrower investing banks’ profit margins but leaves the mean network density intact. The sensitivity of systemic crises’ frequency to $f_1$ declines (panel (b) of Figure 28, dashed line). In the absence of adjustments in the level of interconnectedness, narrower spreads also make nonsystemic crises more likely (panel (c) of Figure 28, dashed line).

In the benchmark model, flexibility in portfolio adjustments helps investing banks keep their expected bankruptcy losses relatively low when the productivities of the underlying primitive assets deteriorate. In particular, they shift portfolio compositions from their own projects, thereby reducing their individual default probabilities and increasing systemic risk. When barred from changing interconnectedness beyond its long-run average, investing banks remain predominantly exposed to their own projects. They also choose to borrow less on the interbank market (the constraint (3.10) is slack more often). Without time variation...
in interconnectedness, the frequency of nonsystemic crises jumps up almost twice, whereas systemic defaults become 10% rarer (panels (b) and (c) of Figure 28, square markers).

3.5.5. Financial crises statistics

This section investigates whether the model matches some untargeted moments related to the frequency and the severity of financial crises. I first verify that the model-implied frequencies of systemic and nonsystemic events are in line with the data. I then examine whether the model can numerically account for well-documented credit boom-bust dynamics around financial crises (e.g., Schularick, and Taylor, 2012).

In a long sample compiled by Jorda, Schularick, and Taylor, 2016 (JST) and covering 17 advanced economies since 1870, systemic banking crises happen on average 4.0 times per 100 years. However, it is unclear whether the definition of “systemicness” used by JST immediately applies in my setting. For example, in many cases their classification relies on Reinhart and Rogoff (2009), who identify systemic banking crises as episodes associated with “closure, merging, or takeover by the public sector of one or more financial institutions.” In the model, a failure of one investing bank is defined as a nonsystemic event. Using OECD Economic Outlook reports, Romer, and Romer (2017) (RR) construct a more detailed measure of financial distress severity for 24 countries (including all 17 from JST’s dataset) but use a shorter sample starting from 1967. Their measure suggests that the frequencies of systemic and nonsystemic crises are, respectively, 1.7% and 2.4% annually. The subsample of JST’s data over the same years implies an annual frequency of banking crises of 3.1%. In comparison with JST, RR appear to identify more episodes of financial distress overall, although fewer of them are classified as systemic.

\[A \text{ reduction in the weight of own project } \alpha \text{ by } d\alpha \text{ is associated with a modest increment in the degree of common exposures, because } d\alpha \text{ is evenly spread among all other projects. Consequently, holding interconnectedness fixed affects the probability of nonsystemic events more substantially.}\]

\[\text{Following RR, I categorize an event as systemic if its measure reaches the value of 7. All other episodes in which the measure is above 0 is considered nonsystemic. Online Appendix gives more details.}\]

\[\text{For example, JST’s chronology implies that Finland, Belgium, and the Netherlands experienced systemic crises in 2008, whereas RR consider them to be nonsystemic crises. At the same time, some episodes (e.g., France in 1995, Australia in 2008) are treated as nonsystemic by RR but are absent from JST. See RR for a more detailed comparison between the chronologies.}\]
The minimum value of project-specific shocks $x_m$ is calibrated to generate around 1.7 joint failures of investing banks per 100 years. The model-implied annual frequency of nonsystemic crises is 2.5%, close to RR’s number. Importantly, this moment is not a calibration target.

In the model, systemic defaults tend to happen at the end of credit booms, when the simultaneous crash of all investing organizations is likely because of strong common exposures and weak balance sheets. Table 6 compares the model and data-implied amplitudes of boom-bust cycles. In comparison with all financial crises, systemic ones are preceded by larger increases and followed by more significant reductions in credit and output, both in the model (columns 1 and 2) and in the data (columns 3 and 4, RR’s chronology). The busts sizes are more pronounced in the model. One reason is that the paper abstracts from any government interventions aimed at mitigating real losses during default events, while in reality bailouts are widespread.

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>RR</th>
<th>JST</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All</td>
<td>Systemic</td>
<td>All</td>
</tr>
<tr>
<td>Credit boom</td>
<td>1.75</td>
<td>3.04</td>
<td>1.36*</td>
</tr>
<tr>
<td>Credit bust</td>
<td>-3.27</td>
<td>-5.95</td>
<td>-1.96**</td>
</tr>
<tr>
<td>Output boom</td>
<td>1.00</td>
<td>1.21</td>
<td>1.34**</td>
</tr>
<tr>
<td>Output bust</td>
<td>-1.94</td>
<td>-3.12</td>
<td>-2.20**</td>
</tr>
<tr>
<td>Frequency</td>
<td>4.2</td>
<td>1.7</td>
<td>4.4</td>
</tr>
</tbody>
</table>

Table 6: Boom/bust is defined as an average 2 years growth of the HP-filtered variable of interest prior/subsequent to crises. The smoothing parameter is $\lambda = 6.25$. All numbers are in percent. Model figures are based on 1,000,000 simulations; credit is defined as the amount of assets invested in risky projects, $A_{t-1} - \bar{\mu}_1 - \bar{\mu}_2$. “JST” and “RR” stand for crisis chronologies by Jorda et al. (2016) and Romer, and Romer (2017), respectively. Credit and output data are from Jorda et al. (2016). “RR sample” represents a subsample of a long sample used by Jorda et al. (2016) (“Full sample”), for which financial crises severity measure of Romer, and Romer (2017) is available. **, * mark significance of the difference between the mean of the variable and zero at 1% and 10% levels, respectively.

The last two columns report the same statistics using JST’s crisis chronology. The amplitude

---

To use credit and output data from JST, I restrict the analysis to 17 countries. Because of this, the frequencies of the financial distress episodes in columns 3 and 4 are slightly different from those reported in the text (1.7% vs. 1.8% for systemic and 2.4% vs. 2.6% for nonsystemic crises).
of credit boom-bust cycles is larger around RR’s episodes of systemic financial distress (columns 4 and 5), confirming that RR’s systemic crises are on average more severe than that of JST. Finally, in the full sample (column 6), the amplitude is somewhat larger than in more recent data.

3.6. Welfare analysis and policy implications

In the model, painful systemic crises tend to happen when the amounts of assets and, hence, interbank borrowing are above their long-run means. Under such conditions, the returns on risky projects are on average low, the financial sector is strongly interconnected, and systemic risk is high. A natural question in this regard is why the household optimally chooses to have the stock of assets at such a high level and does not actively dissave in spite of an elevated financial fragility. One reason is consumption smoothing. At the same time, taking the return on assets as given, the household does not internalize how its intertemporal consumption/savings decisions affect the financial fragility. This gives rise to a pecuniary externality, which has nonzero impact on the household’s welfare because of market incompleteness and real costs of financial distress in default states of the world (Greenwald and Stiglitz, 1986). In Section 3.6.1, I solve the social planner’s problem to analyze the impacts of the externality on the size of credit booms, interconnectedness, the frequency of financial crises, and the household’s welfare. Section 3.6.2 discusses policy implications. Online Appendix provides additional details about the planner’s problem.

3.6.1. Planner’s problem

In the first-best case, financial frictions are completely absent from the economy. In particular, investing banks are financed only through equity, so there are no costly bankruptcies. Investing banks do not spend on risk-sharing connections. Such an allocation, however, requires unrestricted planning abilities and is therefore hardly feasible. As is standard in the literature (e.g., Bianchi, 2011), I instead consider a constrained planner. It makes intertemporal consumption/savings decisions for the household but allows the interbank and
labor markets to clear competitively. Formally, the constrained planner solves

\[
V^{SB}(A, z, x) = \max_{A', C} \frac{1}{1 - \psi} \left( C - \frac{1}{1 + \nu} L(A, z)^{1+\nu} \right)^{1-\psi} + \beta \mathbb{E} \left[ V(A', z', x') \right],
\]

s.t. \( A' + C = \left( 1 - \frac{1 - \mu(A, z)}{1 - \bar{\mu}} \right) \rho_s A + \)

\[
\left( 1 - \frac{1 - \mu(A, z)}{1 - \bar{\mu}} \right) \rho_s A + \left( 1 - \frac{1 - \mu(A, z)}{1 - \bar{\mu}} \right) L(A, z)^{1-\eta} + \frac{1 - \mu(A, z)}{1 - \bar{\mu}} A \left[ 1 - \delta - \frac{1}{N} x - \theta 1 - \frac{N^d(A, z, x)}{N} \right],
\]

where \( L(A, z), \mu(A, z) \) and \( N^d(A, z, x) \) are set as in the decentralized equilibrium (DE) defined in Section 3.3.3. The constrained planner’s allocation is denoted by \( SB \) (second best).

Unlike the representative household, the planner internalizes how its intertemporal savings decisions affect financial fragility. There are two main differences between the DE and SB asset accumulation policies. First, when facing increased systemic risk, the planner dissaves more aggressively. Figure 29 shows that, in comparison with the benchmark DE case, a typical systemic crisis in the SB economy is preceded by a smaller credit boom and triggered by a worse shock. Prior to nonsystemic crises, on the contrary, the planner accumulates more assets than does the representative household (not reported).

![Figure 29: Average paths of the benchmark (DE) and constrained efficient (SB) economies around systemic crises (t = 0). The models are simulated for 1,000,000 periods. All series are expressed as a percentage deviation from their long-run averages.](image-url)
Second, the SB and DE economies feature different long-run steady states. On the one hand, the planner is willing to have fewer assets and a more sparsely connected financial system on average to avoid painful systemic crises. On the other hand, nonsystemic crises become more likely under such conditions. Moreover, the planner also internalizes that the diversification expenses are non deadweight losses and are eventually rebated to the household, so, in the SB case, they do not dampen savings. While it is generally unclear which effect dominates, the SB economy turns out to have, on average, 6% fewer assets.

To quantify the differences between the SB and DE economies, I compute the compensating variation, that is, the percentage by which consumption in the DE allocation should be changed in order to achieve the same welfare as in the SB allocation. The compensating variation of moving from allocation $i$ to allocation $j$ is denoted by $\kappa^{i\rightarrow j}$ and is defined as

$$
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u \left( (1 + \kappa^{i\rightarrow j})C^i_t, L^i_t \right) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u \left( C^j_t, L^j_t \right).
$$

(3.26)

Notice that $\kappa^{i\rightarrow j}$ accounts for the transition from $i$ to $j$. Table 7 reports $\kappa^{DE\rightarrow SB}$, along with the frequencies of systemic and nonsystemic crises, long-run means of assets, consumption, output, labor, and interconnectedness in the DE and SB economies (rows 1 and 2). The planner chooses to work and consume less. Output and asset stock are lower, and the financial system is more sparsely connected. The frequency of systemic events diminishes quite substantially, from 1.7% to 1.1%. At the same time, the number of nonsystemic crises increases marginally. The main macroeconomic series are less volatile under the constrained planner (Table ?? in Online Appendix). The welfare gain from moving to the SB allocation is 0.05% of permanent consumption, similar to Bianchi, and Mendoza (2010). It is worth noting that accounting for the transitional dynamics is important for the welfare calculation. During the initial stages of the $DE \rightarrow SB$ transition, the household dissaves, contributing the lion’s share to the overall welfare gain.
Table 7: Columns report, respectively, the long-run averages of assets, consumption, labor, output, and interconnectedness; the annual frequencies of systemic and nonsystemic financial crises; and the compensating variation in $\kappa_{DE \to i}$ for decentralized (DE), second-best (SB), decentralized with the optimal flat tax on savings ($DE_{optimal \ flat \ tax}$), and decentralized with optimal diversification cost ($DE_{optimal \ f}$) allocations.

By definition, $\kappa_{DE \to DE} = 0$. To compute $\kappa_{i \to j}$, I simulate 10,000 paths of the economies $i$ and $j$ for 500 periods, starting from the ergodic distribution of $i$.

3.6.2. Policy implications

I now discuss policies aimed at restoring constrained efficiency. The SB allocation can be decentralized by a state-contingent tax $\tau(A, z, x)$ on savings $d_{hh}$, whereas tax proceeds are rebated to the household in a lump-sum fashion. This policy prevents a buildup of the interbank debt prior to systemic crises, avoiding the states of overconnected networks and inefficiently high financial fragility. The optimal tax is set so that the planner’s policies satisfy the representative household’s intertemporal Euler equation.

**Proposition 5** A state-contingent tax $\tau(A, z, x)$ on the household’s savings can restore constrained efficiency.

I find that the optimal tax is positive on average (0.38%), equalizing the long-run steady states of the DE and SB economies. It is positively correlated with $A$ and negatively correlated with $z$ and $x$. Prior to systemic events, the tax rate rises sharply (Figure 30), preventing larger credit booms and higher financial fragility. In the aftermath of systemic crises, $\tau$ drops below its long-run mean. In those states of the world the representative household, which does not internalize that the diversification expenses are eventually rebated to it, rebuilds the asset stock more slowly than would the planner (see also panel (c) of Figure 29).

Since state-contingent policies might be difficult to implement in practice, I also study the
welfare implications of a simple flat tax on savings. I find that the optimal fixed tax rate is quite close to the long-run average of the state-contingent tax. The welfare gain, however, is about 60% of what is achieved by a time-varying policy (row 3 of Table 7). Unlike the state-contingent tax, the flat one only corrects the steady-state levels but, by definition, cannot prevent large fluctuations in credit and in interconnectedness.

Finally, I investigate the welfare implications of financial innovations associated with a reduction in the cost of establishing risk-sharing connections. As discussed in Section 3.5.4, a decrease in $f$ affects the economy in two ways. First, it incentivizes the household to accumulate more assets and thus exacerbates the oversaving problem. Second, it eases risk sharing and makes the financial system more interconnected. Holding everything else equal, more efficient diversification reduces expected default losses, which is generally welfare improving.\textsuperscript{28} At the same time, it makes systemic crises more frequent. This might be undesirable from the household’s point of view. Because bankers are risk neutral, whereas the household is risk averse, the latter experiences a larger utility loss in the states of joint defaults. Although how the household’s welfare is affected by financial innovations is unclear in principle, I find that an increase in $f$ by about 25% is optimal from the household’s perspective. The welfare gain, however, is only slightly above 0.01% of permanent

\textsuperscript{28}A decrease in $f$ improves the expected profitability of the investing banks’ portfolios. In the states of the world in which the constraint (3.10) is slack, investing organizations expand their borrowing and invest more in the risky projects. Because banks also do not internalize their impacts on the projects’ returns and hence financial fragility, this is subject to a welfare loss. Online Appendix provides further discussion.
consumption (row 4 of Table 7).\textsuperscript{29}

The policies considered in this section are mainly directed toward reducing the probability of systemic crises. Although the optimal state-contingent tax does help the economy recover faster after episodes of financial distress, it does not affect the immediate losses brought by breakdowns of the financial system. In this respect, interventions aimed at mitigating default losses, such as bailouts, might be desirable. Financial institutions, rationally expecting such policies, are likely to change their behavior ex ante (Acharya, 2009 and Farhi and Tirole, 2012). For example, within my framework, investing banks might choose to become overconnected in expectation of broad-based bailouts during episodes of systemic crises. However, as suggested by recent work (Bianchi, 2016 and Allen, Carletti, Goldstein, and Leonello, 2017), the strength of a collective moral hazard problem might crucially depend on specific policy tools. A formal cost-benefit analysis of such interventions is beyond the scope of this paper.

3.7. Concluding remarks

This paper has presented a general equilibrium model in which fragility arises because of dynamically evolving links connecting financial institutions. The overlap between banks' portfolios shapes the economy’s exposure to financial crises associated with costly intermediaries’ defaults. During periods of credit expansion, banks evenly spread available funds across the underlying projects, thereby reducing their individual bankruptcy probabilities but magnifying systemic risk. Episodes of large-scale financial distress tend to happen after credit booms, when banks are densely connected and their balance sheets are weak.

To keep the analysis tractable, I have considered a setting in which banks internalize how their default decisions affect the rest of the economy and specifically other parties within the financial sector. Therefore, the model is likely to provide a lower bound on the importance

\textsuperscript{29}Online Appendix considers the version of the model in which risk attitudes of the household and of bankers are aligned. In that economy, all agents weigh the same events equally, and, thus, at each state, banks' portfolio choices are optimal from the household’s perspective. I still find that an increase in $f$ is welfare improving.
of interconnectedness for fragility. It would be interesting to analyze how externalities (e.g.,
due to fire sales, a domino effect, anticipated government interventions, or noninternalized
social costs of severe banking distress) affect the financial architecture and contribute to
the evolution of systemic risk. These questions are left for future exploration.


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