Essays On Consumer Learning And Its Impact On Firm Strategies

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Essays On Consumer Learning And Its Impact On Firm Strategies

Abstract
In this dissertation, I study how the existence of consumer learning in a digital goods environment influences the profitability of various firm strategies. I develop a structural model of consumer’s learning-by-using. Adopting a Bayesian learning framework, the model describes how product experience influences willingness to pay, and also allows identification of key factors behind learning and quantification of the trade-offs that firms face. I estimate the model using a novel data set of videogame users’ play record. Using the estimated parameters, I first consider the optimal design of free trials. Digital goods providers often offer a trial version of their product in order to familiarize consumers with the product. The trial configurations considered herein include limiting duration of free usage (i.e. ‘time-locked trial’) and limiting access to certain features (i.e. ‘feature-limited trial’). I find that time-locked trials outperform feature-limited trials, and the revenue implication depends on the rate of demand depreciation during the trial period.

I then consider the optimal product unbundling strategy. As digital goods can be considered as a bundle of identical services to be consumed at different points in time, the firm can unbundle and sell each component separately over time. Offering the product in an unbundled manner allows consumers to adopt part of the product after the learning takes place, resulting in higher willingness to pay through the option value. I find that pay-per-use, an extreme form of product unbundling, outperforms traditional outright sale when there exists consumer learning, while it does not in the absence of learning. Hence the existence of consumer learning has a substantial impact on the firm’s optimal policy.

In addition to empirically studying the implications of consumer learning, I also examine an econometric problem of identifying state dependence in consumer utility. Identifying state dependence is challenging when there exists consumer heterogeneity unobservable to a researcher. I show that if consumers make two decisions at each decision occasion, one being a discrete choice from multiple alternatives and the other being a consumption intensity of the selected option, then we can nonparametrically separate state dependence and unobserved heterogeneity under mild conditions. Understanding conditions for nonparametric identification helps empirical modelers in choosing their modeling assumptions.

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ESSAYS ON CONSUMER LEARNING AND ITS IMPACT ON FIRM STRATEGIES

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A DISSERTATION
in
Economics
Presented to the Faculties of the University of Pennsylvania
in
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In this dissertation, I study how the existence of consumer learning in a digital goods environment influences the profitability of various firm strategies. I develop a structural model of consumer’s learning-by-using. Adopting a Bayesian learning framework, the model describes how product experience influences willingness to pay, and also allows identification of key factors behind learning and quantification of the trade-offs that firms face. I estimate the model using a novel data set of videogame users’ play record. Using the estimated parameters, I first consider the optimal design of free trials. Digital goods providers often offer a trial version of their product in order to familiarize consumers with the product. The trial configurations considered herein include limiting duration of free usage (i.e. ``time-locked trial'') and limiting access to certain features (i.e. ``feature-limited trial''). I find that time-locked trials outperform feature-limited trials, and the revenue implication depends on the rate of demand depreciation during the trial period.

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In addition to empirically studying the implications of consumer learning, I also examine an econometric problem of identifying state dependence in consumer utility. Identifying state dependence is challenging when there exists consumer heterogeneity unobservable to a researcher. I show that if consumers make two decisions at each decision occasion, one being a discrete choice from multiple alternatives and the other being a consumption intensity of the selected option, then we can nonparametrically separate state dependence and unobserved heterogeneity under mild conditions. Understanding conditions for nonparametric identification helps empirical modelers in choosing their modeling assumptions.
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Chapter 1

Consumer Learning and Revenue-Maximizing Trial Design

1 Introduction

Digital goods providers often offer trial versions of their products. For example, Spotify provides free trial periods of 60 days. Business and computational software such as Microsoft Office and Stata comes with a 30-day free trial. Videogame companies such as Electronic Arts and Sony offer several features from each game title for free. Reporting results from 305 software publishers, Skok (2015) finds that 62 percent of them generate some revenue from converting trial users to paid users, and 30 percent report that such conversion constitutes more than half of their revenue. A number of studies find that trial availability is positively associated with downloads of the paid version (Liu, Au and Choi 2014, Arora, Hofstede and Mahajan 2017). On the other hand, conversion rates vary significantly across firms and products. Spotify boasts that its conversion rate from the free trial to the paid version is around 27 percent, while Dropbox’s conversion rate is around 4 percent; many other firms’ conversion rates are only around 1 percent (Rekhi 2017). Brice (2009) and Turnbull (2013) both report that among the software providers they survey, the average conversion rate from a visit to the provider’s website to purchase is lower with a trial than without it. Whether and to what extent a free trial boosts revenue therefore appears to depend on factors specific to each product and market.

In this paper, I develop an empirical framework to examine how different trial designs influence consumers’ adoption decisions and firm revenue. Designing the optimal trial is not a
straightforward problem. A trial can be configured by limiting the duration of free usage ("time-locked trial") or by limiting access to certain features ("feature-limited trial"). If the trial is time-locked, the firm also needs to determine the duration of free usage. Similarly, when offering a feature-limited trial the firm needs to choose which features to include in the trial. In order to implement the optimal free trial, it is necessary to understand the trade-offs that each design entails.

I focus on one main factor through which the trial impacts revenue: consumer learning-by-using (Cheng and Liu 2012, Wei and Nault 2013, Dey, Lahiri and Liu 2013). Digital goods are typical examples of experience goods. Advertising or other external information alone may not fully inform consumers about their “match value”: their individual valuation of the product. When consumers are risk averse, the existence of uncertainty lowers their willingness to pay for the product. A free trial thus informs consumers about their true match value, increasing their willingness to pay. On the other hand, it is essentially a free substitute for the full product, creating demand cannibalization. While providing a more generous trial product fosters better consumer learning, it also increases the opportunity cost the trial imposes on the full product.

The costs and benefits of trial provision are associated with various aspects of consumer learning. In this study, I consider four main factors: (1) the initial uncertainty around consumer-product match value, (2) consumer risk aversion, (3) speed of learning relative to demand depreciation, and (4) learning spill-overs across different features of the product. Initial uncertainty and risk aversion determine the magnitude of the drop in ex-ante willingness to pay. The speed of learning influences the profitability of a time-locked trial. As consumers learn more quickly, shorter trial durations are necessary to facilitate learning, and demand depreciates less during.

---

1 A “feature” refers to a general concept regarding “part of the product”, such as game content, book chapter or news article. Since I consider a videogame, I use “content”, “feature”, and “game mode” interchangeably.

2 Match values vary across consumers due to different preference and needs, and ability to acquire product-specific skills.
the trial. Learning spill-overs across different features determine the effectiveness of a feature-limited trial. Since consumers have free access to features included in the trial, willingness to pay consists only of consumers’ valuation of features excluded from the trial. Hence, in order for a feature-limited trial to increase willingness to pay, experience with trial features needs to be informative about excluded ones.

In order to evaluate the magnitude of each factor and predict how consumers respond to different trial designs, I build and estimate a structural demand model for digital goods. Specifically, I embed the adoption decision of a durable product into a Bayesian learning framework. Consumers make purchase decisions based on their willingness to pay, which is determined by their expected utility from future consumption stream (Ryan and Tucker 2012, Lee 2013). The expectation over future utility is conditional on consumers’ beliefs about their match value. Hence, both the magnitude of uncertainty reflected in the belief and risk aversion impact their willingness to pay. The model of Bayesian learning describes the process of how a consumer updates her beliefs about her match value through product experience. In the model, each user maximizes her expected utility by choosing frequency of play, duration of each session and feature played in each session. Her uncertainty diminishes as she updates her belief. Learning spill-overs exist, in that an experience with one feature may help in updating the belief about her match value for other features. Moreover, a user may experiment with the product to explore her true match value; she takes into account future informational gains in choosing her actions. In order to capture the forward-looking nature of the decisions, I define the model as a dynamic programming problem (Erdem and Keane 1996, Che, Erdem and Öncü 2015). The solution of this problem provides a value function, which summarizes the consumer’s expected lifetime utility, determining her willingness to pay endogenously.

Consumers’ behavior under each trial design can be represented by combining the purchase
decisions and the learning process in accordance with the trial design. In the case of no trial, the purchase decision precedes the entire learning process. Willingness to pay is determined by the value function evaluated with a prior belief. If a time-locked trial is provided, the learning model with the duration specified by the trial precedes the purchase decision. Once the trial expires, consumers make a purchase decision based on their posterior belief. The posterior belief involves smaller uncertainty, increasing value from future sessions if consumers are risk averse. However, initial free sessions no longer constitute willingness to pay, creating a trade-off for the firm. On the other hand, if a feature-limited trial is provided, trial users make a purchase decision after each trial session by comparing the value from switching to the full product and staying with the trial. The firm wants to include features in the trial that create large learning spill-overs, so that trial experience also reduces uncertainty about features excluded from the trial. Meanwhile, the firm also wants to exclude high-value features from the trial to prevent demand cannibalization. In either trial case, forward-looking consumers take into account the option value of being able to make a better informed purchase decision in the future. Hence they may have a stronger incentive to experiment with the product during trial sessions.

I also account for other particularities of durable goods demand. First, consumers may wait for future price drops (Stokey 1979). In order to capture this, the purchase decision is defined as an optimal stopping problem. Consumers not only choose whether or not to purchase the product, but also determine the optimal timing of purchase (Nair 2007, Soysal and Krishnamurthi 2012). Second, in the model of learning, I account for other factors through which the past usage experience influences utility, such as novelty effect or boredom, and separately identify learning from them. Finally, I explicitly consider termination: permanent abandonment of the product. It determines the demand depreciation during the trial, influencing the trial’s profitability.

I apply my framework to trial design of a major sports videogame. The videogame includes
four features, which correspond to different content areas and are called “game modes”. I use a novel data set of lifetime session records of 4,578 users. The records consist of the duration and the game mode selected at each session. The firm did not offer any free trial during the observation period, causing a sample selection problem: I only observe users having made a purchase without trial experience. I develop a procedure to estimate the population distribution of match values from an observation of such an endogenously truncated distribution. The estimation strategy also helps applying the current framework to other environments, where the available data set is typically subject to such sample selection problem.

I find that videogame users are risk averse, and their product valuation involves significant uncertainty. For example, consider a consumer whose willingness to pay under her initial belief of her match value is $50. The 95 percent confidence interval of her true willingness to pay is [$21.80, $87.90]. Users learn quickly. One additional session of a given game mode reduces the uncertainty about the match value with the mode by up to 63 percent. Meanwhile, learning spill-overs across different game modes are small. An additional session of a mode merely decreases the match value uncertainty of the other modes by 2 percent.

Given the estimated demand model, I evaluate the revenue implications of various trial designs. I find that in this setting, time-locked trials tend to increase revenue due to fast learning. In particular, providing five free sessions is the ideal design, increasing revenue by up to 2.5 percent. Meanwhile, I find that the revenue implications vary significantly with respect to the users’ termination rate during the trial period. This implies that along with offering a free trial, the firm may want to incentivize users in order to decrease the trial termination rate. On the other hand, I find that any feature-limited trials without duration restrictions cannot increase revenue. This is due to a small number of modes in the product studied and small learning spill-overs; the opportunity cost of losing revenue from one mode always outweighs the gain from
increased valuation for other modes. However, I also find that imposing extra feature restrictions on time-locked trials can boost revenue further by up to an extra 0.7 percentage point. This is because the extra feature restrictions widen the product differentiation between the full product and the trial; consumers whose most preferred mode is excluded opt to make a purchase.

To my knowledge, this is the first empirical study that explores how the design of a free trial influences consumers’ adoption patterns. There exists an expansive theoretical literature on optimal trial provision when consumer learning exists (Lewis and Sapphington 1994, Chellappa and Shivendu 2005, Johnson and Myatt 2006, Bhargava and Chen 2012). In particular, Dey, Lahiri and Liu (2013) and Niculescu and Wu (2014) study trade-offs the firm faces in providing time-locked and feature-limited trials, respectively. In both studies, the optimality condition depends on the aforementioned demand-side factors, calling for an empirical study to measure the magnitude of such factors. The methodology I propose does so by using typical data on consumer engagement with the product. Hence, it can help firms design the optimal free trial.

Moreover, the model I develop considers consumer learning in a durable goods environment, offering a substantial departure from existing Bayesian models. Unlike a perishable goods case, in this setting purchase and consumption are distinct, separable phenomena. For example, the product I study requires an outright purchase, and consumers make only one purchase during the entire consumption stream. In such cases, the firm can manipulate when consumers make a purchase over the course of learning; “timing of purchase” emerges as a new strategy variable of the firm. Moreover, forward-looking consumers respond to different timing of purchase set by the firm by choosing how to learn during trial. For example, under a shorter free usage duration

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3Trials may impact revenues through other channels. Cheng and Tang (2010) and Cheng, Li and Liu (2015) discuss network externalities that trial users create. Jing (2016) discusses the influence of trial provision on competition. Since neither factor is relevant to the product studied, I abstract away from these alternative stories.

4There are numerous applications of Bayesian models to the repeat purchases of perishable goods, such as ketchup (Erdem, Keane and Sun 2008), yogurt (Ackerberg 2003), diapers (Che, Erdem and Öncü 2015) and detergent (Osborne 2011). Among others, physician learning about prescription drugs has a particularly large number of applications (Crawford and Shum 2005, Coscelli and Shum 2004, Chintagunta, Jiang and Jin 2009, Narayanan, Manchanda and Chintagunta 2005, Ching 2010, Dickstein 2018).
consumers may have a stronger incentive for information acquisition and experiment more with the product. It endogenously determines the distribution of willingness to pay in response to the firm policy. My model is the first that explicitly considers such interactions between firm policy and consumer actions specific to durable goods environments.

This study also augments existing empirical studies concerning free trials of durable goods. Similar to the optimal duration of a time-locked trial, Heiman and Muller (1996) study how the duration of product demonstration impacts subsequent adoption. Foubert and Gijsbrechts (2016) study implications of trial provision when the product involves quality issues. My study provides an empirical model that allows one to consider implications of different trial designs.

The study of optimal trial provision sheds a new light on the rapidly-growing “freemium” business model, where the firm offers part of its product for “free” and upsells “premium” components (Lee, Kumar and Gupta 2017). One of its main purposes is to facilitate learning from the free version and induce upsells: an objective similar to a feature-limited trial. Hence, understanding the mechanism of consumer learning helps firms determine whether to adopt a freemium strategy.

This paper is structured as follows. In Section 2, I illustrate using a simple model how the mechanism behind consumer learning affects firm revenue. In Section 3, I outline the data of videogame usage records. I also present supporting evidence for the existence of consumer learning in the environment studied. In Section 4, I build the empirical model of digital goods adoption. I describe the identification and estimation strategy in Section 5. Estimation results and model fit are discussed in Section 6. Using the estimated model I consider the optimal trial design in Section 7. Section 8 concludes and discusses possible future research areas.

\footnote{Consumers’ endogenous usage adjustment in response to firm policy is studied in other contexts, such as multipart tariff (Roberts and Urban 1988, Iyengar, Ansari and Gupta 2007, Narayanan, Chintagunta and Miravete 2007, Grubb and Osborne 2015, Goettler and Clay 2011). However, there consumers merely respond to instantaneous price change and learning is affected only as a result; consumers do not choose how to learn.}

\footnote{More broadly, this study is associated with an empirical literature concerning how consumption experience in early stages influences future repeat behavior (Fourt and Woodlock 1960).}
2 An illustrative model of consumer learning and firms’ trade-offs

In this section, I introduce a simple model of consumer learning and illustrate how each of the four factors outlined above impacts the optimal trial configuration. Consider a firm selling a videogame with two features, which provide flow utility $v_1$ and $v_2$, respectively. There is no complementarity across features and the utility from the full product is simply $v_1 + v_2$. For simplicity, I assume all consumers have the same match value and receive the same utility. The product lasts for two periods. In the second period, the utility from both features decays by $\delta$ due to boredom. At the beginning of the first period, a consumer faces uncertainty about her match value with the product. Her expected utility from each feature under uncertainty is given by $E(v_i) = \alpha v_i$, for $i = \{1, 2\}$. $\alpha < 1$ is a parameter that captures the reduction of the utility due to the uncertainty in a reduced form way. $\alpha$ is low if a user faces large uncertainty or she is very risk averse. The uncertainty is resolved once she uses the product. When there is no free trial, the willingness to pay is equal to ex-ante expected utility from the whole product over two periods.$^7$

$$U_N = E((v_1 + v_2) + \delta(v_1 + v_2)) = \alpha((v_1 + v_2) + \delta(v_1 + v_2)).$$

Aside from not providing trial (N), the firm can either offer a time-locked trial (TL) or a feature-limited trial (FL). With TL, the consumer uses the full product for free for one period and makes a purchase decision at the end of period 1. At the time of purchase, the consumer learns her true match value but has only one active period remaining. Hence her willingness to pay, which

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$^7$The consumer knows that her uncertainty will be resolved in period 2. However, at the beginning she does not know the realization yet, and hence her period 2 utility is still in the expectation in the expression of $U_N$. 

equals the incremental utility from purchasing the full product, is

\[ U_{TL} = \delta(v_1 + v_2). \]

With FL, the consumer has free access to feature 1 and chooses whether to buy feature 2. Since she can only try feature 1, she may or may not be able to resolve the uncertainty about feature 2. I assume that learning occurs with probability \( \gamma \). \( \gamma \) corresponds to the degree of learning spill-over. If two features are sufficiently similar, usage experience from one provides more information about the other and thus \( \gamma \) is high. I also assume that learning occurs well before period 1 ends. In this case, her willingness to pay is

\[
U_{FL} = \begin{cases} 
  v_2 + \delta v_2 & \text{with probability } \gamma, \\
  \alpha(v_2 + \delta v_2) & \text{with probability } 1 - \gamma.
\end{cases}
\]

In order to maximize revenue from this consumer, the firm first maximizes willingness to pay by choosing the scheme from N, TL and FL, and subsequently sets the price equal to it. When FL is provided, the price the firm sets is \( p_{FL} = v_2 + \delta v_2 \) and the consumer purchases the full product only when learning occurs. Setting price \( p_{FL} = \alpha(v_2 + \delta v_2) \) is dominated by choosing N and setting \( p_N = U_N \). In Figure 1 I plot the area in which each of \( \{U_{TL}, U_{FL}, U_N\} \) is the maximum of the three, and hence is the firm's optimal strategy. The result reflects the trade-offs described above. With large \( \alpha \) associated with small uncertainty and consumer risk neutrality, providing trial is not optimal. When \( \alpha \) is small, the optimal design depends on the relative size of \( \alpha \) and \( \gamma \). If learning spill-over \( \gamma \) is large, providing FL is optimal. On the other hand, if \( \alpha \) is sufficiently small, it follows that the ratio \( \frac{\alpha}{\delta} \) is also small; the second period utility remains high even after taking the boredom \( \delta \) into account, so is the opportunity cost of providing free access.
in the second period. Providing TL is optimal in this case.

This model abstracts away many other factors at play, such as multi-period learning and consumer heterogeneity. In addition, firms not only choose either TL or FL, but also the optimal free usage duration and features to include in the trial. In order to account for them, I develop a more realistic model in Section 4. Nonetheless, the factors discussed in this section remain the key drivers of the firm’s trade-offs in the full model.

3 Empirical environment

3.1 Product studied and data description

In this study, I apply my framework to a major sports videogame. The game title is released annually with up-to-date real league data and enhanced graphics, and every version sells millions of units. The game operates on major gaming consoles such as Sony PlayStation series and
Microsoft Xbox series. The game requires purchase of a game disk before it can be played. Hence consumers make purchase only once. The game contains four features called “game modes”. Each game mode correspond to a distinct content area, and users need to pick one mode whenever they play the game. In mode 1, users build a team by hiring players and coaches, and compete against rivals to win the championship. In mode 2, users simulate an individual sports player’s career, in order to become the MVP. In mode 3, users pick a pre-defined team and play against other teams, skipping any team management. It is the most casual mode, involving simpler tasks and requiring less skills than other modes. Finally, mode 4 allows users to compete online and be ranked against other users. No predetermined play sequence exists and users can choose any modes from the beginning.

I observe a sample of 4,578 first-time users from the version released in 2014. The sample is randomly selected among U.S. based users who registered a user account during product activation. For each user, I observe the date of activation, which I assume to be the date of purchase, and the lifetime session records. Each session record consists of the time of start and finish, and the selected game mode. I augment my data with the game’s weekly average market price, collected from a major price comparison website. The market price is the average of the prices listed on four major merchants: Amazon, Gamestop, Walmart and eBay. I assume that this market price is the purchase price.

A session, the unit of observation, is defined as a continuous play of one game mode. Upon turning the console on, the menu screen prompts users to select a game mode. Once they select one, a session starts. When they exit the mode and return to the menu screen, or shut down the console, the session ends. By definition of the session, each session consists of only one game mode.

The users in the data had no access to free trials; this is a set of users who made a purchase
without trial experience. As I show below, users in the data learn about the product by playing. Hence, no observation of free trial is necessary to identify learning. As I only observe a truncated subset of consumers, I control for the sample selection problem during the estimation procedure.

![Figure 2: Prices and sales over time](image)

Note: The prices are weekly average prices in the market. Sales are measured by the number of activations in the data.

In Figure 2, I show the history of sales and price over 35 weeks from the product release. Both follow a typical pattern of durable goods adoption: the highest price and the sales at the beginning, followed by a steady decline. In the 14th week, a lower price is offered due to Black Friday and a corresponding sales spike is observed. The 18th week is Christmas with a clear sales boost.

In Table 1, I present summary statistics of play records. Every mode is selected at a similar rate, indicating that these modes are horizontally differentiated. Each session lasts around an hour on average. Game mode 3 lasts shorter than the other modes, presumably because of its simplicity. Interval length between sessions is a measure of play frequency; shorter intervals

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8Between 2012 and 2015 the firm changed its trial design every year, presumably in order to evaluate users’ response. The no trial policy employed in 2014 is part of such experiment. Unfortunately, it is impossible to directly compare trial adopters and non-adopters using data from other years; the firm provided trials in a non-random manner, creating a sample selection problem that cannot be resolved with the current data.
indicates more frequent play. On average, users play one session in every 2.7 days. The product life is relatively short. On average, users terminate after 31 sessions. Taken together, an average user remains active for slightly less than three months, plays 30 hours in total and then terminates.

In Figure 3, I show the heterogeneity of game mode selection across consumers with different usage intensity, and its evolution over time. Each of the three bins represents users whose lifetime hours of play is in the bottom third (light users), middle third (intermediate users), and top third (heavy users) among those who remain active for at least 10 sessions. For each bin of users, each bar represents the proportion that each mode is selected in the initial 3 sessions, 4th-10th sessions, and sessions after that. Two empirical regularities are observed. First, the proportion varies across users with different intensities. For example, light users tend to play mode 3 more often than other users. This indicates that the distribution of match value over game modes may vary across users with different intensity. Second, the proportion evolves over

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Table 1: Summary statistics

<table>
<thead>
<tr>
<th>Mode</th>
<th>Choice probability</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>Hours per session</td>
<td>1.200</td>
<td>1.200</td>
</tr>
<tr>
<td>Mode 2</td>
<td>Choice probability</td>
<td>0.316</td>
<td>0.465</td>
</tr>
<tr>
<td>Mode 3</td>
<td>Hours per session</td>
<td>1.154</td>
<td>1.127</td>
</tr>
<tr>
<td>Mode 4</td>
<td>Choice probability</td>
<td>0.243</td>
<td>0.429</td>
</tr>
<tr>
<td></td>
<td>Hours per session</td>
<td>0.570</td>
<td>0.818</td>
</tr>
<tr>
<td>Mode 5</td>
<td>Choice probability</td>
<td>0.193</td>
<td>0.394</td>
</tr>
<tr>
<td></td>
<td>Hours per session</td>
<td>1.048</td>
<td>1.078</td>
</tr>
<tr>
<td>Session interval length (days)</td>
<td>2.743</td>
<td>4.358</td>
<td></td>
</tr>
<tr>
<td>Termination period (sessions)</td>
<td>30.742</td>
<td>43.795</td>
<td></td>
</tr>
<tr>
<td>Number of users</td>
<td>4,578</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sample size (users × sessions)</td>
<td>145,317</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Statistics are aggregated over all user-sessions. Choice probability is the aggregate proportion that each mode is selected. Its standard deviation is that of a user-session specific indicator variable, which is one if that mode is selected. Interval length between sessions is the number of days between two consecutive sessions. The termination period equals the number of total sessions each user played.

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9I can interpret the shorter hours of play for mode 3 in two ways. There may exist ex-ante light users, who play short sessions and prefer mode 3. The other story is that mode 3 requires less time, and users who like mode 3 tend
time. Mode 1 and 2 gain popularity, while mode 3 shrinks. This is indicative that the perceived match value evolves. These findings are consistent with consumer learning, but also indicate the necessity to introduce consumer heterogeneity to account for the systematic difference across users.

Figure 3: Evolution of game mode choice for each usage intensity
Note: Light, intermediate and heavy users are those whose lifetime hours of play are in the bottom, middle and top third of users. I exclude users who terminate within 10 sessions, in order to eliminate sample selection over time. For each bin of users, each bar represents the proportion that each mode is chosen in the first 3 sessions from the purchase, from 4th to 10th sessions, and 11th session and after.

In Figure 4, I present the evolution of session duration and session interval length for the same bins of usage intensity as in Figure 3. The usage pattern is nonstationary. On the one hand, the session durations increase over the first several sessions. Although this is consistent with learning story that the uncertainty resolution increases utility from play, there exist alternative explanations, such as skill acquisition and novelty effects. On the other, usage intensity declines to play less hours. The two stories differ in the direction of causality. As long as light users receive lower utility and exhibit lower willingness to pay, I do not need to separate the two. In the data, users who prefer mode 3 tend to buy the game further away from the release at lower prices, indicating that they indeed have lower willingness to pay.
in later sessions, likely due to boredom. These findings imply that in order to correctly identify consumer learning from the observed usage patterns, I need to control for other forms of state dependence. Users are heterogeneous both in session duration and play frequency; heavy users exhibit higher usage intensity and a slower decay than the others.

3.2 Suggestive evidence of consumer learning

In this section, I discuss two data patterns that indicate the existence of consumer learning-by-using. Other supportive evidence is discussed in the Appendix.

**High early termination rate** On average, users terminate after 31 sessions. However, there exist many early dropouts. Figure shows that 8.9 percent of users stop playing after the first session and 29.3 percent of users terminate within five sessions. Such a high early termination rate

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Footnotes:

10 Unlike role-playing games, sports games do not have pre-defined “ending” and users can keep playing it as long as they like.

11 As shown by Chamberlain (1984), purely nonparametric identification between learning and consumer heterogeneity is impossible. Hence, the validity of the argument that consumer learning exists rests on how much the data pattern “intuitively makes sense” from the perspective of each story. Reassuringly, the observed patterns fit more naturally with consumer learning. In the model, I impose restrictions on how heterogeneity can affect the evolution of utility, in order to identify learning. This assumption is often employed in the literature to disentangle state dependence from heterogeneity (Dubé, Hitsch and Rossi 2010, Ching, Erdem and Keane 2013).
Figure 5: Evolution of hazard rate of termination

Note: The hazard rate of termination at the $t$-th session is the proportion of users terminating after the $t$-th session among the active users. After 40 sessions, the hazard rate is stable.

Rate is also observed among heavy initial users: users whose first session duration is in top third. 6.6 percent of heavy initial users terminate after the first session, which is three times higher than their long-run average termination rate. Considering that most users purchase the game for around $40 to $50, some users are likely experiencing disappointment. Users who have high expectations about the match value may pay $40, only to realize their true match value is low and terminate early.\footnote{Heterogeneity in the speed of utility satiation may partly explain the patterns. Nevertheless, the fact that 9 percent of users play only one session after paying $40 strongly indicates the existence of uncertainty.}

**Experimentation across game modes** The existence of uncertainty implies that there is an option value from exploration; there is a possibility that the true match value with a mode is quite high. This prompts users to experiment with each game mode, resulting in frequent switching across modes in the early stages of consumption. In Figure 6, I show the evolution of the probability that users switch modes after each session. Two different patterns emerge. The probability that users switch from modes 1, 2, and 4 to any other mode steadily declines as users accumulate more experience: a pattern consistent with experimentation. On the other hand,
Figure 6: Evolution of probability of switch

Note: The probability that users switch from mode \( m \) at the \( t \)-th session is calculated by the number of users who selected game mode \( m' \neq m \) in the \( t+1 \)-th session, divided by the number of users who selected mode \( m \) at the \( t \)-th session. Switches from modes 1, 2 and 4 follow very similar paths and I aggregate them for exposition.

no experimentation of mode 3 seems to occur. These patterns are reasonable given that modes 3 is more casual and involves simpler in-game tasks. Indeed, as I show below, the estimated parameters support that match values with mode 3 involves smaller uncertainty.\(^{13}\)

4 A structural demand model under no trial

In the previous sections, I showed that the usage patterns evolve in a way that is consistent with consumer learning: popular game modes change over time; some users terminate very early; and users initially experiment across different modes. On the other hand, I found that usage experience may influence utility through other channels, such as boredom and novelty effects. Moreover, the existence of consumer heterogeneity is strongly indicated.

In order to identify the mechanism behind learning and to predict consumers’ behavior under

\(^{13}\)An alternative story is that consumers merely have a taste for variety at the beginning. My standpoint is that learning and love of variety are not mutually exclusive, but that experimentation is a structural interpretation of variety seeking.
different counterfactual trial designs, I build and estimate a structural model of consumers’ adoption and learning-by-using. The structural model serves two purposes. First, it allows one to identify the learning mechanism, while controlling for other contaminating effects and consumer heterogeneity. In particular, four factors that influence learning are explicitly modeled: magnitude of initial uncertainty, risk aversion, speed of learning, and learning spill-overs across different game modes. Second, the model is estimable using typical data on usage records of users without free trial experience. Hence, it provides implications for the trial profitability even without observing trial behavior.

In this section, I describe a model that corresponds to no trial case: the model I estimate with the data. Models under trial cases are discussed in Section 7. Under no trial, consumers make a purchase decision prior to any learning-by-using. The initial adoption decision is based on their product valuation, which is equal to the sum of utilities they expect from future sessions, conditional on their prior belief about their match value. The utility from each subsequent session is endogenously determined through the decisions of product usage. Because of this structure, I first describe the model of learning-by-using and then the purchase decision.

4.1 The Bayesian learning model

The model of learning-by-using characterizes users’ post-purchase play decisions. At each day, a user makes decisions according to a timeline described in Figure 7. The user first chooses whether to play a session. If she doesn’t play one, she moves the the next day. If she plays one, she selects a game mode and chooses session duration. After a session, she receives a signal informative about her true match value from the selected mode, and updates her belief. At this point, the user may decide to permanently quit playing. I refer to this as termination. Termination is an absorbing state and she never makes any decisions again. Conditional on remaining active, the user chooses whether to play another session or move to the next day. She repeats this sequence
until she terminates. In what follows, I first describe the user decisions during a session (Node B), and the decisions of play frequency and termination (Node A, C) afterward.

### 4.1.1 Selection of game modes and session duration (Node B)

At each session, users select a game mode and choose session duration. In order to allow for experimenting across modes, I assume that users are forward-looking and take into account future informational gains when selecting a game mode; users solve a discrete choice dynamic programming problem. Conditional on having selected a mode, users then choose duration of the session, which endogenously determines the flow utility from that mode.

**Game mode selection** A forward-looking user selects a game mode that maximizes the sum of her flow utility from playing and future informational return (Erdem and Keane 1996). In order to capture the nonstationary usage pattern presented in Figure 4, I assume that the problem has a finite horizon. \footnote{I assume that at $T = 100$ session all active users terminate. This is longer than the lifetime number of sessions of 93.27 percent of the users in the data.} The optimal mode selection is summarized by the following value

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**Figure 7: Timeline of the choices at each day**

Note: Double-edged nodes A through C are decision nodes.
function.

\[ V(\Omega_{it}) = \mathbb{E}[\max_{m_{it}} v(b_{it}, \nu_{imt}, h_t) + \mathbb{E}[^2_{it,t+1}) V(\Omega_{i,t+1}) | \Omega_{it}, m_{it}] + \epsilon_{imt}\sigma_i], \]

where \( \Omega_{it} = \{b_{it}, \nu_{imt}\}_{m=1}^M, h_t\}; the state variables include \( b_{it}\), \( i\)'s belief about her match value at session \( t\); \( \nu_{imt}\), the cumulative number of times that \( i\) chose mode \( m\) in the past \( t-1\) sessions; and \( h_t\), a weekend indicator, which is one for Saturday, Sunday and holidays. I denote the flow utility from the current session by \( v(b_{it}, \nu_{imt}, h_t)\). The future informational gain is summarized by the continuation payoff \( \mathbb{E}[\beta(\Omega_{i,t+1}) V(\Omega_{i,t+1}) | \Omega_{it}, m_{it}]\). \( \beta(\Omega_{i,t+1})\) is a discount factor between the current session and the next session. I discuss the definition of \( \beta(\Omega_{i,t+1})\) below. The expectation of the continuation payoff is taken over an informative signal that the user receives after the current session. I assume that there exists a choice-specific idiosyncratic utility shock \( \epsilon_{imt}\), and that \( \epsilon_{imt}\sigma_i\) follows type 1 extreme value distribution with variance \( \sigma_{\epsilon}^2\). The choice probability of each mode hence follows the logit form.

\[
P_m(\Omega_{it}) = \left( \frac{\exp \left( \frac{1}{\sigma_i} (v(b_{it}, \nu_{imt}, h_t) + \mathbb{E}[\beta(\Omega_{i,t+1}) V(\Omega_{i,t+1}) | \Omega_{it}, m_{it}]) \right)}{\sum_{m'} \exp \left( \frac{1}{\sigma_i} (v(b_{it}, \nu_{im't}, h_t) + \mathbb{E}[\beta(\Omega_{i,t+1}) V(\Omega_{i,t+1}) | \Omega_{it}, m_{it}']) \right)} \right). (1)
\]

Experimentation occurs when a user chooses a mode that generates a lower flow utility than her current best alternative to gain a higher return in the future.\(^{16}\) Since the problem is nonstationary, all the value functions and the optimal actions are a function of \( t\) in addition to the state \( \Omega_{it}\), which I suppress for notational simplicity.

\(^{15}\)A commonly imposed normalization that \( \sigma_i = 1\) is not necessary. The scale normalization is achieved by assuming that \( f(b_{it})\) has no scaling coefficients. More details are provided in the Appendix.

\(^{16}\)This trade-off is also found in the literature of Bandit models, and an index solution exists for this class of model with correlated arms (Dickstein 2018). Here I follow a dynamic programming approach, because the value function from the dynamic programming problem is a necessary input to the purchase decision.
Choice of session duration  Now I derive $v(b_{it}, \nu_{imt}, h_t)$, the flow utility from mode $m$, as a result of optimal choice of session duration. I assume that having selected game mode $m$ at session $t$, user $i$ chooses duration of the session to maximize her expected utility specified as follows.

$$v(b_{it}, \nu_{imt}, h_t) = \max_{x_{imt}} f(b_{it})x_{imt} - \frac{(c(\nu_{imt}) + x_{imt})^2}{2(1 + \alpha h_t)}.$$  

(2)

$x_{imt}$ is the session duration that user $i$ chooses. I assume that the expected utility is a quadratic function of $x_{imt}$. $f$ and $c$ are functions that represent how marginal utility from playing an extra hour is affected by the belief $b_{it}$ and the history of play $\nu_{imt}$, respectively. Accumulation of usage experience influences utility through two channels. First, due to learning, users update their beliefs about match values and their utility evolves accordingly. This is captured through $f$. Second, aside from learning, usage experience may directly influence utility. $c$ controls for such other factors. For example, any deterministic utility decay, such as satiation or boredom, implies that $c$ is increasing in $\nu_{imt}$. Likewise, due to novelty effects or skill acquisition, $c$ may decrease in $\nu_{imt}$ for some range of $t$. Separating learning from other forms of state dependence that I discussed earlier corresponds to separately identifying $c$ from learning parameters. Also, $c$ partly captures the concept of demand depreciation. If the incremental utility from additional sessions decays quickly, providing initial free sessions incurs a large opportunity cost. Finally, users tend to spend more time in weekend, indicating that they may receive higher utility. This is captured by $\alpha > 0$.

The static expected utility maximization problem has a closed form solution, resulting in the
following flow utility and the optimal session duration for each mode $m$\footnote{I assume that users choose $x_{imt}$ before playing. This ignores a possibility that learning from the current session influences the current session duration. However, this possible misspecification hardly impacts the estimate of willingness to pay. As the willingness to pay is defined by the sum of all future utility, the magnitude that learning shifts the utility from the current, single session is small relative to the entire valuation change from many future sessions.}:

$$v(b_{it}, \nu_{imt}, h_{it}) = \frac{f(b_{it})^2(1 + \alpha h_{it})}{2} - f(b_{it})c(\nu_{imt}),$$  \hspace{1cm} (3)$$

$$x^*(b_{it}, \nu_{imt}, h_{it}) = f(b_{it})(1 + \alpha h_{it}) - c(\nu_{imt}).$$  \hspace{1cm} (4)$$

Henceforth, I parametrize $f(b_{it})$ as follows.

$$f(b_{it}) = \mathbb{E}[\theta_{im}^\rho \mid \theta_{im} > 0, b_{it}],$$  \hspace{1cm} (5)$$

where $\theta_{im}$ denotes a user-mode specific true match value. $f(b_{it})$ is specified as an expectation of $\theta_{im}^\rho$ conditional on the belief. $\rho > 0$ can be interpreted as the coefficient of risk aversion. $\rho < 1$ implies that utility is concave in the true match value $\theta_{im}$, and taking expectation over $\theta_{im}^\rho$ results in utility diminution. On the other hand, $c$ can be an arbitrary function such that $c(0) = 0$.

### 4.1.2 The decisions of play frequency and termination (Nodes A, C)

At decision nodes A and C, each user makes decisions of play frequency and termination. She compares value from playing to that from not playing at node A, and compares value from remaining active to that from terminating at node C. Instead of defining a full maximization problem, I take a reduced form approach to model them. Specifically, I impose the following two assumptions; (1) users' decisions are based only on the state $\Omega_{it}$ at any decision nodes located between sessions $t-1$ and $t$, and (2) decisions are influenced by an idiosyncratic shock, such that the optimal decision is representable by a probability distribution over each of the available
alternatives. This encompasses many specifications of decision rules that involve an idiosyncratic utility shock, some of which I discuss in the Appendix. I denote the probability that user $i$ plays her $t$-th session on a given day by $\lambda(\Omega_{it})$, and the probability that user $i$ remains active after session $t$ by $\delta(\Omega_{i,t+1})$. I treat these probability distributions as model primitives.

Given the structure of the decisions of frequency and termination, I derive the formula for $\beta(\Omega_{i,t+1})$: the discount factor between session $t$ and $t+1$. Assuming that users discount future utility by $\beta$ per one day, $\beta(\Omega_{i,t+1})$ is obtained as the expected discount factor between the date that session $t$ is played and the date that session $t+1$ is played. The expectation is over whether the user remains active after session $t$, and when she plays session $t+1$; because the optimal action at each decision node depends on an idiosyncratic shock that only realizes at that node, a user’s future actions are stochastic to herself. Formally, $\beta(\Omega_{i,t+1})$ is characterized as follows.

$$\beta(\Omega_{i,t+1}) = \delta \lambda + \delta(1 - \lambda) \lambda \beta + \delta(1 - \lambda)^2 \lambda \beta^2 + \ldots$$

$$= \delta(\Omega_{i,t+1}) \frac{\lambda(\Omega_{i,t+1})}{1 - (1 - \lambda(\Omega_{i,t+1})) \beta}. \quad (6)$$

The intuition is as follows. After session $t$ the user remains active with probability $\delta(\Omega_{i,t+1})$. If staying active, she plays session $t+1$ on the same day with probability $\lambda(\Omega_{i,t+1})$, on the next day with probability $(1 - \lambda(\Omega_{i,t+1})) \lambda(\Omega_{i,t+1})$, incurring the daily discount factor $\beta$, and so on.

4.1.3 State variables and their evolution

**Match value and its learning-by-using** I denote the true match value of consumer $i$ with the game by a vector $\theta_i = \{\theta_{i1}, \theta_{i2}, \ldots, \theta_{iM}\}$, where $M$ is the number of modes available in the full product. Users are heterogeneous in their match values. I assume that $\theta_i$ follows multivariate normal distribution; $\theta_i \sim N(\mu, \Sigma)$, where $\mu = \{\mu_1, \mu_2, \ldots, \mu_M\}$ is the average match value of the population and $\Sigma$ is an arbitrary variance-covariance matrix. Correlations between
match values across different modes can be either positive or negative. Heavy users play all modes more extensively than light users, generating positive correlations. On the other hand, users who like mode $m$ tend to play only mode $m$ and not other modes. This generates negative correlations.

Upon arrival at the market, the consumer does not know the realization of $\theta_i$, and has a rational expectation about it; her prior about the distribution of $\theta_i$ is equal to the population distribution of match values. In addition, she receives a vector of initial signals $\tilde{\theta}_i = \{\tilde{\theta}_{i10}, \tilde{\theta}_{i20}, ..., \tilde{\theta}_{iM0}\}$. The initial signal represents any information or impression that a user has about the product ex-ante. It creates heterogeneity in the initial perceived match value. I assume that the initial signal is independent across game modes, and is normally distributed conditional on the user’s true match value; $\tilde{\theta}_{im0} | \theta_{im} \sim N(\theta_{im}, \tilde{\sigma}_m^2)$. Henceforth, I denote the diagonal matrix of the variance of the initial signal by $\tilde{\Sigma}$. The consumer forms an initial belief as a weighted average of the prior distribution and the received signal in a Bayesian manner.

$$\theta_i | \tilde{\theta}_i \sim N(\mu_1, \Sigma_1),$$

where

$$\mu_1 = \mu + \Sigma(\Sigma + \tilde{\Sigma})^{-1}(\tilde{\theta}_i - \mu),$$
$$\Sigma_1 = \Sigma - \Sigma(\Sigma + \tilde{\Sigma})^{-1}\Sigma.$$

The initial belief is thus represented by $b_{i1} = \{\mu_1, \Sigma_1\}$.

When a user plays game mode $m$ at each session $t$, she receives a signal informative about her true match value for that mode. I assume that the signal $s_{imt}$ is normally distributed around

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18This assumption does not allow for possible bias in the initial belief. The bias in the belief, if it exists, is not separately identified from other forms of deterministic utility evolution, and hence is currently subsumed in $c(\nu_{imt})$. 

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the true match value.

\[ s_{imt} \mid \theta_{im} \sim N(\theta_{im}, \sigma_s^2). \]

I assume that the variance of the signal \( \sigma_s^2 \) remains the same over time. Introducing a time-varying signal distribution makes the model computationally intensive, and hence I opt to maintain a simple structure. Given the realized signal, the user updates the belief following the Bayesian formula.

\[
\mu_{i,t+1} = \mu_{it} + \Sigma_{it} Z_{it}' (Z_{it} \Sigma_{it} Z_{it}' + \sigma^2_s)^{-1} (s_{imt} - \mu_{imt}),
\]

\[
\Sigma_{i,t+1} = \Sigma_{it} - \Sigma_{it} Z_{it}' (Z_{it} \Sigma_{it} Z_{it}' + \sigma^2_s)^{-1} Z_{it} \Sigma_{it},
\]

where \( Z_{it} \) is a 1 by \( M \) vector whose \( m \)-th element is one and zero elsewhere.\(^{19}\)

This learning structure captures all four factors that determine firm-side trade-offs described earlier. The magnitude of initial uncertainty is determined by the variance of the initial belief \( \Sigma_1 \). Consumers know that their beliefs involve an error and hence they face a risk of mismatch. When consumers are risk averse, \( \rho < 1 \) in Equation (5) and the expected utility from future sessions is lowered. On the other hand, the speed of learning is captured by \( \sigma_s \). When \( \sigma_s \) is small, signals is more precise and uncertainty diminishes more quickly. Finally, learning spill-overs are determined by match value correlation \( \Sigma \). If the match values for two game modes are highly correlated, a signal received from one mode also helps update the belief for the other. These are the key parameters that determine the profitability of each trial design in the counterfactual exercise. Willingness to pay is likely to increase due to learning when consumers are sufficiently

\(^{19}\)This Bayesian updating structure of a normal distribution does not require to keep track of \( \Sigma_{i,t} \) in the state space. Instead, it suffices to keep the mean belief \( \mu_{imt} \) and the number of times each option is taken in the past \( \nu_{imt} \) (Erdem and Keane 1996). This allows one to reduce the effective state space to \( \Omega_{it} = \{ (\mu_{imt}, \nu_{imt})^{M=m=1, h_t} \}. \)
risk averse, magnitude of initial uncertainty is large and learning is quick relative to the utility depreciation.

**Evolution of other state variables** \( \nu_{imt} \) evolves deterministically; \( \nu_{im1} = 0 \) for all \( m \), and \( \nu_{im,t+1} = \nu_{imt} + 1 \) if \( m \) is chosen at session \( t \), and \( \nu_{im,t+1} = \nu_{imt} \) otherwise. The weekend indicator is i.i.d, and it is 1 with probability 2/7 and zero with probability 5/7. This stochastic weekend arrival helps reduce the dimension of state variables; deterministic weekend arrival requires to keep track of the day of the week in the state. This completes the description of the model for usage. This problem is solvable by backward induction. The solution consists of the optimal decision rule and the associated value function at each state \( \Omega_{it} \).

### 4.2 Purchase decisions

When there is no free trial, each consumer makes purchase decisions without any playing experience. A user’s product valuation is represented by her ex-ante value function \( V(\Omega_{i1}) \): the sum of the utility she expects from the product in the future, evaluated at the initial state \( \Omega_{i1} \).

The purchase decisions proceed as follows. I assume that the market consists of \( N \) consumers. They are heterogeneous, in that \( V(\Omega_{i1}) \) is consumer-specific. In line with the frequency of the price data, I assume that one period in the adoption model is one week. At each week \( \tau \), a fraction \( \lambda_{r}^{2} \) of the consumers randomly arrive. In light with the fact that the price of the product steadily declines over time, I allow consumers to delay their purchase to buy at lower prices (Nair 2007). Each consumer makes a purchase decision by comparing the value from buying to that from waiting for a price drop. If she makes a purchase, she quits the market and starts using the product in a way described above. If she does not make a purchase, she comes back to the market in the following week and makes the decision again. I assume that the product is available for

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26 The derivation of \( \beta(\Omega_{i,t+1}) \) in Equation (6) ignores the fact that \( h_{t} \) evolves day by day even without playing a session. In practice, I replace relevant \( \lambda(\Omega_{i,t+1}) \) in Equation (6) with its expectation over the realization of \( h_{t+1} \).
52 weeks after the release date, and hence waiting beyond 52 periods generates zero payoff. A
new version of the game is released at week 52, when the sales of older version essentially end.

The value function associated with the optimal stopping problem at week $\tau$ is

$$V_{ip}(\Omega_{i1}, p_{\tau}) = \mathbb{E}[\max\{V(\Omega_{i1}) - \eta_ip_{\tau} + \epsilon_{1i\tau}\sigma_p, \beta V_{ip}(\Omega_{i1}, p_{\tau+1}) + \epsilon_{0i\tau}\sigma_p\}],$$

where $p_{\tau}$ is the current price. If the consumer buys the product, she receives the value $V(\Omega_{i1})$
and pays $p_{\tau}$. If she does not buy at week $\tau$, she receives a continuation payoff of staying in
the market $V_{ip}(\Omega_{i1}, p_{\tau+1})$. I assume perfect foresight for the future prices, $\epsilon_{1i\tau}\sigma_p$ is i.i.d, and
follows type 1 extreme value distribution with variance $\sigma^2_p$. I do not model social learning, and
hence the value from purchase $V(\Omega_{i1})$ remains constant over time. Hence, the incentive to delay
the purchase solely comes from lower prices in the future. I assume that $\eta_i$ follows log-normal
distribution with mean $\mu_\eta$ and variance $\sigma^2_\eta$. For simplicity, I assume that $\eta_i$ is independent from
$\theta_i$. The probability that consumer $i$ makes a purchase at week $\tau$ follows the logit form.

$$P_{ip}(\Omega_{i1}, p_{\tau}) = \frac{\exp\left(\frac{1}{\sigma_p}(V(\Omega_{i1}) - \eta_ip_{\tau})\right)}{\exp\left(\frac{1}{\sigma_p}(V(\Omega_{i1}) - \eta_ip_{\tau})\right) + \exp\left(\frac{\beta}{\sigma_p}V_{ip}(\Omega_{i1}, p_{\tau+1})\right)}.$$ The model is solvable by backward induction. The consumer’s willingness to pay for the product
under no trial is defined by $\frac{V(\Omega_{i1})}{\eta_i}$: the value of the product measured in dollars.

4.3 Multiple segments

In addition to heterogeneities with respect to the true match value $\theta_i$, the belief $b_{it}$ and the price
coefficient $\eta_i$ described earlier, I allow for the existence of multiple segments $r = \{1, 2, ..., R\}$ with

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21Assuming instead that consumers form expectations based on the prices of other game titles hardly changes
the result. As in Figure 2, the prices follow a quite typical path and it is easy for consumers to forecast price
patterns.

22Since $V(\Omega_{i1})$ is already scale-normalized, I do not need to normalize $\sigma^2_p$. 

---
different population-level parameters. In particular, I allow the vector of mean match value $\mu$ and the variance of utility shock in the mode choice $\sigma_\epsilon$ to be heterogeneous. I denote segment-specific parameters with subscript $r$. I also let the variance of the initial belief be heterogeneous, denoted by $\Sigma_{1r} = \kappa_r(\Sigma - \Sigma(\Sigma + \tilde{\Sigma})^{-1}\Sigma)$ with $\kappa_1 = 1$. Introducing multiple segments allows for more flexible representation of consumer heterogeneity; heterogeneity in $\mu$ allows for the existence of ex-ante heavy and light user segments, and heterogeneity in $\sigma_\epsilon$ and $\Sigma_1$ adds flexibility in fitting game mode selection of users with different usage intensity. The probability that each user belongs to segment $r$ is denoted by $\xi_r$. This completes the model representation under no trial case.

5 Identification and estimation

5.1 Separating learning and other forms of state dependence

In my model, the history of play $\nu_{it}$ influences utility through two channels: learning, where $\nu_{it}$ influences beliefs $b_{it}$ and then utility through $f(b_{it})$, and other forms of state dependence, where $\nu_{it}$ directly influences utility through $c(\nu_{it})$. In this section, I outline the intuition behind the separate identification between these two factors. I provide a formal argument in the Appendix.

Separate identification is based on the observation that the evolution of beliefs causes non-stationary and stochastic evolution of utility. It is stochastic because beliefs evolve according to the realization of stochastic post-session signals. It is also nonstationary because users start from an uninformed state and make an irreversible, unidirectional move to an informed state. On the other hand, other forms of state dependence have to be deterministic conditional on $\nu_{int}$. If they evolve stochastically, then users’ optimal action changes over time in an unpredictable way according to their realizations: learning has to occur. Hence, separating the evolution of beliefs and other forms of state dependence is equivalent to separating stochastic and deterministic
utility evolution.

Since users choose actions based on their utility in the model, stochastic and deterministic utility evolution maps into stochastic and deterministic evolution of actions. This implies that conditional on the history of play $\nu_{imt}$, how the variance of actions across users evolves across states is solely attributed to the evolution of beliefs; $c(\nu_{imt})$ cannot influence variance because users sharing the same history have the same deterministic $c(\nu_{imt})$. Nonstationary evolution of variance also allows one to separate learning from any idiosyncratic fluctuation of actions due to factors outside of the model. On the other hand, how the average actions evolve is solely attributed to the evolution of $c(\nu_{imt})$. Evolution of beliefs due to learning cannot influence the evolution of average behavior; because of rational expectation, $E(\mu_{im,t+1}|\mu_{imt}) = \mu_{imt}$ and average belief has to remain the same over time without drift. Hence, the separate identification is achieved by utilizing first and second order moments of the actions.

Once the evolution of beliefs is identified, the parameters that characterize it are identified immediately. For example, magnitude of initial uncertainty is identified by the difference in the distribution of initial beliefs and long-run beliefs, and speed of learning is identified by the speed at which beliefs converge to the long-run average. Formal identification argument for each of the model parameters is provided in the Appendix.

5.2 Estimation

I estimate the model using simulated method of moments. Given a set of candidate parameters, I first solve the model by backward induction. In order to account for continuous state space, I use the discretization and interpolation proposed by Keane and Wolpin (1994). I then simulate sequences of actions according to the optimal policy predicted by the model; I draw a set of true match values, initial signals, and post-session signals and record the predicted actions. The set of simulated users serves as pseudo-data. The parameters are estimated such that the pseudo-data
obtained this way match most closely with the real data, according to pre-selected moments. Formally, for a vector of parameters \( \theta \) the estimator \( \hat{\theta} \) is obtained by the following minimization problem.

\[
\hat{\theta} = \arg \min_{\theta} m_k(\theta)'^{-1}m_k(\theta),
\]

where \( m_k(\theta) \) is a vector, with rows containing the difference between the data and model moments. \( \hat{V} \) is a weighting matrix.\(^{23}\)

The set of moments is selected to closely follow my identification strategy. For the model of usage, at each observed history of play \( \{\nu_{imt}\}_{m=1}^{M} \), I take as moments (1) the probability that each game mode is selected, (2) the probability that a user switches modes from the previous session, (3) the mean and variance of the duration of each session, (4) the average interval length until the next session and (5) the probability of termination. Since the number of possible paths grows as \( t \) becomes larger, there are only 172 states that I have a sufficient number of samples to satisfactorily compute these statistics. Most of them are located at the early stages of usage history.\(^{24}\) In order to augment the set of moments in later periods, I calculate the statistics above conditional only on \( t \) and not on the entire history \( \{\nu_{imt}\}_{m=1}^{M} \), and use them as moments too. In addition, in order to exploit the variation of usage patterns across users with different usage intensity, I calculate the above moments conditional on multiple bins of usage intensity. I describe the construction of the bins in the Appendix. Finally, I add as an extra set of moments the difference of the average session duration between weekdays and weekends, and the probability that users play multiple sessions within a single day. These extra moments are

\(^{23}\)As a weighting matrix, I use a diagonal matrix, whose \( \{k,k\} \) element corresponds to the inverse of the mean of the sample moment. This works to equalize the scale of the moments by inflating the moments that have a smaller scale (e.g. choice probability) while suppressing the moments with a larger scale (e.g. interval length between sessions).

\(^{24}\)I use moments from the states with more than 30 observations.
designed to aid the identification of \( \alpha \) and \( \lambda \), respectively.

The moments used to identify the model of adoption are the rate of adoption at each week \( \tau \) from the release until the 16th week, which is two weeks before Christmas. The empirical rate of adoption is equal to the proportion of consumers making a purchase at each week in the data, multiplied by the market share of the product. Market share of this product is 28.1 percent, which is calibrated using an external data set described in the Appendix. I do not use the rate of adoption on and after Christmas. This is because users activating the product on Christmas may have received it as a gift, and hence including their activation as a purchase would possibly bias the estimate of the price coefficient. In total, I have 7,375 moment conditions to estimate 47 parameters.

In order to consistently estimate the model parameters, I need to control for the sample selection problem; I only observe users who purchased the game without trial experience. In the context of simulated method of moments, the sample selection implies that in order to calculate moments regarding product usage, I need to construct a pseudo-data set of users who made a purchase: a data set comparable to the real data. I achieve this through the following procedure. At each parameter value, I first draw a set of potential consumers from the population distribution, each with willingness to pay \( V(\Omega_{41})/\eta_i \). Using the adoption model, I calculate the probability that each simulated consumer makes a purchase. Consumers with high willingness to pay receives a high probability, and vice versa. I then simulate each consumers’ adoption decision according to that probability. I use the subset of consumers predicted to make a purchase as the pseudo-data to calculate moments of the usage model. As I move through iteration rounds, both the set of users included in this subset and their actions are updated simultaneously, until the usage patterns of the selected subsample most closely match with the data. This procedure allows the estimation of population distribution of match values from the observation of users conditional
on purchase, allowing one to evaluate the impact of counterfactual policies on the population of consumers. The step-by-step description of my estimation procedure and construction of all the moments are detailed in the Appendix. There I also discuss the set of assumptions necessary to validate the counterfactual exercises based on the match value distribution obtained this way.

$c(\nu_{\text{mt}})$, $\lambda(\Omega_{it})$ and $\delta(\Omega_{it})$ are specified as a quadratic function with respect to the number of past sessions, whose coefficients can vary across users with different match values. I assume that the variance of the initial signal is proportional to the variance of the true type; $\sigma^2_m = \kappa \sigma^2_{m'}$. Also, without loss of generality I normalize the average match value of segment 1 consumers with game mode 3 to 30, and define other parameters relative to it. Concerning the number of discrete segments, I assume $R = 2$. The consumer arrival process $\lambda^a_{it}$ is specified as a uniform arrival rate $\lambda^a_u$ and the initial mass of arrival at the release date $\lambda^a_0$. I assume that the timing of arrival is independent from the location of initial beliefs. Assuming that $N$ potential consumers exist in the market, I can normalize $\lambda^a_0 = 1$ and estimate only $\lambda^a_u$ as the rate of arrival in the later weeks relative to the initial week. Market size $N$ is calibrated outside the model and it is equal to the installed base of consoles, multiplied by the share of sports games among all the game sales.

6 Estimation Results

In order to conduct model validation exercises, I randomly split the 4,578 users in the data into an estimation sample of 3,778 users and a holdout sample of 800 users. In this section, I first present the estimated parameters, and the model fit with all 4,578 users in the data. The model fit with the holdout sample is provided in the Appendix. I then use the estimated model to show how mechanism behind learning is at play in the current setting.

25While these assumptions are not very flexible, the data provide only 16 points of observation, from which I identify both the arrival process and the distribution of price coefficient. Hence, I opt for a simple process to avoid identification issues.
Table 2: Parameter estimates (usage model)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates</th>
<th>Std.error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk aversion</td>
<td>0.215</td>
<td>0.034</td>
</tr>
<tr>
<td>Holiday effect</td>
<td>1.396</td>
<td>0.023</td>
</tr>
</tbody>
</table>

Distribution of match value

<table>
<thead>
<tr>
<th>Std.errors</th>
<th>σ₁</th>
<th>69.149</th>
<th>0.665</th>
</tr>
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<tr>
<td></td>
<td>σ₂</td>
<td>83.374</td>
<td>2.464</td>
</tr>
<tr>
<td></td>
<td>σ₃</td>
<td>35.881</td>
<td>3.513</td>
</tr>
<tr>
<td></td>
<td>σ₄</td>
<td>64.096</td>
<td>2.145</td>
</tr>
</tbody>
</table>

Correlations

| ρ₁₂ | 0.510     | 0.059 |
| ρ₁₃ | 0.262     | 0.093 |
| ρ₁₄ | 0.588     | 0.035 |
| ρ₂₃ | 0.207     | 0.141 |
| ρ₂₄ | 0.515     | 0.050 |
| ρ₃₄ | 0.541     | 0.069 |

Initial signal var κ₀ 0.323 0.095
Post-session signal s.e σ₅ 28.497 0.737

(a) Common parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Segment 1</th>
<th>Segment 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean match value</td>
<td>μ₁₁</td>
<td>μ₁₂</td>
</tr>
<tr>
<td></td>
<td>24.062</td>
<td>107.106</td>
</tr>
<tr>
<td></td>
<td>16.359</td>
<td>3.604</td>
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<td>Mean match value</td>
<td>μ₂₁</td>
<td>μ₂₂</td>
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<tr>
<td></td>
<td>33.844</td>
<td>95.780</td>
</tr>
<tr>
<td></td>
<td>8.090</td>
<td>3.593</td>
</tr>
<tr>
<td>Mean match value</td>
<td>μ₃₁</td>
<td>μ₃₂</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>103.449</td>
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<tr>
<td></td>
<td>0</td>
<td>1.945</td>
</tr>
<tr>
<td>Mean match value</td>
<td>μ₄₁</td>
<td>μ₄₂</td>
</tr>
<tr>
<td></td>
<td>32.089</td>
<td>97.070</td>
</tr>
<tr>
<td></td>
<td>6.616</td>
<td>6.810</td>
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<tr>
<td>Initial uncertainty</td>
<td>k₁</td>
<td>k₂</td>
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<td></td>
<td>1</td>
<td>7.758</td>
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<td></td>
<td>0</td>
<td>1.415</td>
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<tr>
<td>Logit shock s.e</td>
<td>σₑ₁</td>
<td>σₑ₂</td>
</tr>
<tr>
<td></td>
<td>2326.129</td>
<td>3227.733</td>
</tr>
<tr>
<td></td>
<td>37.541</td>
<td>214.038</td>
</tr>
</tbody>
</table>

Proportion of seg 1 ξ₁ 0.578 0.016

(b) Segment-specific parameters

Note: μ₃₁ and k₁ are normalized. Standard error is calculated by 1,000 bootstrap simulations.

6.1 Parameter estimates of usage model

In Table 2 I present selected parameter estimates of the usage model. The standard errors are simulated using 1,000 sets of bootstrapped data set, each of which is obtained by randomly re-sampling users from the original data with replacement. In Table 2a I show the estimates of the parameters common across all users. The coefficient of risk aversion is 0.215 < 1, indicating significant risk aversion. The standard error of the match value distribution is much smaller for game mode 3; because of the simplicity of the mode, match values vary little across users.
The estimate is consistent with no experimentation of mode 3 in Figure 6. Also, correlation coefficients are all positive. A high match value for one mode implies a high match value for another. However, as I show below, the magnitude of the correlation is not high enough to generate much learning spill-over. In Table 2b I present segment-specific parameters. Each of the two discrete segments respectively captures the behavior of light users and heavy users. All the parameters for segment 2 are inflated to capture the large gap of usage intensity between light and heavy users.

![Figure 8: Evolution of estimated utility decay](image)

Note: Each line corresponds to the average evolution of \( c(\nu_{int}) \) of users who belong to each bin of usage intensity. They are calculated using 50,000 simulated sequences of actions. Bins of usage intensity are defined as in Figure 3.

In Figure 8, I present the evolution of utility due to other forms of state dependence captured in \( c(\nu_{int}) \) across users with different usage intensity. Higher \( c(\nu_{int}) \) implies lower marginal utility from an extra hour of play. Utility monotonically decays over time; the increase of utility due to skill acquisition or novelty effects does not seem to exist in this setting. The utility of heavy users tend to decay slower than others, consistent with Figure 4. As I detail in the Appendix,

\[26\]While the estimated magnitude of initial uncertainty that segment 2 faces is disproportionately high, this merely reflects the tight curvature of the utility due to small \( \rho \). Since the flow utility is quite flat at a high match value, in order to capture the fact that the uncertainty also reduces heavy users’ initial utility, the variance of the belief needs to be magnified accordingly.

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all the parameters of \( c(\nu_{imt}) \) are precisely estimated and significantly different from zero; aside from learning, other forms of state dependence exists.

### 6.2 Model fit and implications for usage pattern

![Figure 9: Model fit of game mode choice for each usage intensity](image)

Note: The data part is identical to Figure 3. The model counterpart is computed from 50,000 simulation sequences. Usage intensity is defined as in Figure 3.

In Figure 9 through 12, I present the model fit for each of the main data variations. In Figure 9, I show the model fit for the aggregate pattern of game mode selection across users with different intensity, and its evolution over time. Heterogeneities in both cross-sectional and intertemporal dimensions are well captured. Light users tend to play mode 3 while heavy users prefer mode 2. Moreover, users gradually switch from mode 3 to other game modes. The model slightly overestimates the probability that mode 3 is selected at the beginning, and that mode 4 is selected in the long-run, but the other parts fit the data quite well.

In Figure 10 I present model prediction hit rate of each user’s game mode selection. The hit rate is calculated as the choice probability that the model assigns to the mode actually selected by each user at each session, conditional on her usage history up until that point. It is obtained by integrating Equation 1 over the unobservable beliefs: \( \mathbb{E}(P_m(\Omega_{it})|\{
u_{imt}\}_{m=1}^M, h_t) \). In order to
integrate over the distribution of the belief conditional on past actions, I use simulation with
importance sampling (Fernandez-Villaverde and Rubio-Ramirez 2007). Details of this procedure
are provided in the Appendix.

Each of the lines in Figure 10 represents the model hit rate for each user at each session,
averaged across users selecting the same mode. Since no history is available to be conditioned
on at the beginning, the choice probability the model assigns to each user’s action is close to
the empirical proportion that each mode is selected. Over the first few sessions, the information
of past actions significantly improves the hit rate. As experimentation ceases around the 10th
session, the prediction hit rate reaches its peak at around 60 to 65 percent. On the other hand,
the hit rate for mode 3 remains relatively low. As shown in Figure 6, the play records of mode 3
involve more switches than the other modes in the long run; predicting future behavior is more
difficult when the user switches her choice more frequently. Nonetheless, the hit rate for mode
3 is higher than the unconditional, aggregate proportion that mode 3 is selected, which is 0.243,
indicating that the model still has a certain predictive power for mode 3.
The session durations and intervals between sessions are shown in Figure 11. The model fit of the duration of sessions and the intervals between sessions is almost perfectly captured. Since the deterministic utility evolution due to $c(\nu_{\text{int}})$ follows monotonic decline, the initial increase of the session durations is attributed solely to the utility increase due to the reduction of the uncertainty. The interval length is also captured reasonably. Since I opt for a simple functional form for $\lambda$, the bumpy pattern of the light and intermediate users are ignored and only the average is matched. The bumpy patterns, while pronounced in Figure 11, is not correlated with users’ other actions in the data. Hence, they are likely due to factors outside of the model, such as the idiosyncratic fluctuation of utility.
In Figure 12 I show the probability of termination and switching patterns. The probability of termination is underestimated around the 10th session, but the magnitude of the error is very small. The switching patterns are tracked reasonably. Two different patterns of evolution discussed in Figure 6 are both correctly matched. The estimates of other parameters and additional model fit examinations are provided in the Appendix. In particular, there I report that the model provides a reasonable fit to (1) holdout sample of 800 users, and (2) a set of users of a version released in another year.

6.3 Parameter estimates, model fit and implications for adoption model

Table 3: Parameter estimates (adoption model)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates</th>
<th>Std. error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price coef mean $\mu_\eta$</td>
<td>28.064</td>
<td>0.410</td>
</tr>
<tr>
<td>Price coef s.e $\sigma_\eta$</td>
<td>46.271</td>
<td>0.002</td>
</tr>
<tr>
<td>Arrival rate $\lambda^a_\mu$</td>
<td>0.098</td>
<td>0.003</td>
</tr>
<tr>
<td>Logit shock s.e $\sigma_p$</td>
<td>1.705</td>
<td>0.151</td>
</tr>
</tbody>
</table>

Note: Standard error is calculated by 1,000 bootstrap simulations.
In Table 3, I show parameter estimates of the adoption model. Since the price coefficient follows a lognormal distribution, the mean of the price coefficient is \( \exp(\mu \eta) \), which is at the order of \( 10^{12} \). This is reasonable given that a vast majority of potential consumers in the market don’t make a purchase. In Figure 13, I show the model fit for the weekly rate of adoption up until two weeks before Christmas. The rate of adoption is defined as the number of consumers making a purchase at each week divided by the total market size \( N \). The fit is almost perfect. The existence of the initial peak and the second peak corresponding to the lower price is captured by the heterogeneity of the price coefficient.

Using the estimated parameter values, I simulate the population distribution of willingness to pay. As I only observe users who made a purchase, the population distribution is obtained by estimating a truncated distribution of willingness to pay through the method of simulated moments, and extrapolating it using the normality assumption. Figure 14 presents this entire population distribution. The figure show the range of willingness to pay between $1 and $500, which covers 31.1 percent of the population. A majority of consumers excluded from the figure
Figure 14: Distribution of willingness to pay

Note: The figure shows the distribution of willingness to pay within the interval [$1, $500], drawn with 2,000,000 simulated samples. Willingness to pay is defined by $V(\Omega_i)/\eta_i$.

have willingness to pay lower than $1 and can be considered as never-buyers. The large number of low willingness to pay consumers appears reasonable, for the market share of this product is 28.1 percent and majority of other consumers have no intention to adopt this product. On the other hand, there exists a handful of very high willingness to pay consumers: a pattern consistent with our industry knowledge.

6.4 Examining the mechanism behind consumer learning

In this section, I illustrate the mechanism of learning identified in the current setting: magnitude of initial uncertainty, speed of learning and learning spill-overs, which in turn provides implications for the optimal trial design. In Figure 15a I show the evolution of the magnitude of uncertainty that a user faces. The uncertainty is measured by the coefficient of variation of the belief: $\sigma_{int}/\mu_{int}$. At the beginning, users face significant uncertainty. In particular, the belief of light users has a standard error that is 3.3 times higher than the mean. Reporting this in terms of willingness to pay, if a consumer has an initial willingness to pay of $p$ dollars, then the
Figure 15: Impact of learning-by-using

Note: Both panels are calculated using 50,000 simulation sequences. In Panel (a), I show the evolution of the coefficient of variation of the belief: $\sigma_{imt}/\mu_{imt}$, averaged across modes and users with different usage intensity. Intermediate and heavy users exhibit almost identical patterns and I aggregate them for exposition. In Panel (b), I present the evolution of average willingness to pay for users starting from different initial values.

95 percent confidence interval of her true willingness to pay is $[0.436p, 1.758p]$\textsuperscript{27} For example, consider a consumer with willingness to pay of $50, then her 95 percent confidence interval is $[$21.80, $87.90$]. On the other hand, heavy users face smaller uncertainty relative to their average valuation. This may indicate that users with high perceived match value engage more in pre-purchase information search. The speed of uncertainty reduction is quite fast, although the uncertainty does not collapse to zero in the short run.

The large initial uncertainty and its fast resolution indicate that willingness to pay can increase as consumers accumulate usage experience. In Figure 15b I show that indeed willingness to pay increases during the early stages of consumption. Willingness to pay reaches its peak around 5th and 6th session. The average willingness to pay after the 5th session is 13.9 percent higher than at the beginning\textsuperscript{28} Afterward, decrease in value due to the forgone session outweighs the informational gain and willingness to pay decreases as users play more sessions. Users with

\textsuperscript{27}Since the value function is nonlinear, the confidence interval of the willingness to pay is asymmetric despite the belief following a normal distribution.

\textsuperscript{28}This is the average across users with initial willingness to pay between $30 and $60, the relevant range of users featured in the counterfactual.
higher intensity reach the peak earlier because their valuation diminution due to the forgone session is larger than light users. Note that at the individual user level, learning does not necessarily increase product valuation. The error involved in the initial belief makes some consumers overly optimistic about their match value. Those consumers may be disappointed. Figure 8 shows that on average product valuation goes up due to the uncertainty resolution. This average increase in willingness to pay indicates that a free trial may increase demand and hence firm revenue. In particular, when the firm wants to provide a time-locked trial, the revenue is likely maximized around five free sessions.

**Figure 16: Marginal variance reduction from each session**

Note: The figure is computed from 50,000 simulation sequences. For each $i$ and $t$, I calculate the percentile reduction of the variance of the belief from $t$ to $t+1$, for the mode selected at $t$: $\frac{\sigma^2_{im,t+1} - \sigma^2_{im,t}}{\sigma^2_{im,t}}$. The reported solid line is its average across the simulated sequences. The dashed line corresponds to the average of variance reduction for the modes not selected at $t$, calculated in a similar way.

In order to illustrate the magnitude of learning spill-overs, in Figure 16 I decompose the impact of learning into the effect on the belief of the selected game mode (own-effect) and that of the modes not selected (spill-over). Each of the lines represents a marginal decline in the variance of the belief due to an incremental signal received at each session. It is evident that most learning comes from the strong own-effect. One additional session decreases the variance of
the own-belief by up to 63 percent, exhibiting rapid learning. On the other hand, the spill-overs play little role. The correlation of match values is not large enough for the informative signal to propagate.\footnote{Under a Bayesian learning model with normal distribution, spill-overs are virtually non-existent for correlations below 0.8 because of the nonlinearity of the spill-over process.} This indicates that provision of feature-limited trials, whose profitability relies on the magnitude of learning spill-overs, may not contribute to the revenue increase in the current setup.

7 Managerial implications: the optimal trial design

Using the estimated demand parameters, I now predict how the demand responds to various trial designs and provide revenue implications. Intuitively, free trials work as follows. If a time-locked trial is provided, consumers can first play any game modes and update beliefs about their match value. Once the trial expires, consumers make a purchase decision based on their posterior belief. Willingness to pay changes depending on how many sessions consumers have played until they make the purchase decision. Hence, the firm’s problem of choosing trial durations is equivalent to choosing at what timing consumers visit the purchase occasion during the sequence of learning.

On the other hand, in the case of feature-limited trials, the firm offers two vertically differentiated products: the full product and a feature-limited version. At each trial session, users can only choose features included in the trial. After the session, trial users make a purchase decision by comparing the value from switching to the full product and that from staying with the trial. They know that if they stay with the trial, they can visit another purchase occasion after the next trial session. In either trial case, in making trial usage decisions forward-looking consumers take into account that learning from the current session enables them to make a better informed purchase decision in the future.

In what follows, I first describe demand models under time-locked and feature-limited trials.
then simulate consumer behaviors under the estimated parameters and compare revenues under
different trial designs. I refer as “time-locked trial” to the one where consumers have access
to the full product up to a certain number of sessions. Since I assume that learning occurs
session by session, the relevant notion of a time limit is with respect to the number of sessions.
Similarly, “feature-limited trial” is the one where the firm offers a subset of game modes from the
full product for free. I do not consider such strategies as imposing restrictions within a mode;
I assume that the firm only picks modes to include, and modes included in the trial remain
identical to the ones in the full product.

7.1 Structural demand models under a time-locked trial

If the firm provides a time-locked trial, the firm chooses the number of free sessions, which I
denote by \( \tilde{T} \). Since trial includes all the features, model of trial users is similar to that of full
product users up to \( \tilde{T} \). After \( \tilde{T} \), the trial expires and I assume that the purchase decisions
thereafter are specified in the same way as in the adoption model under no trial, but with an
updated belief. In addition, consumers may opt to purchase the full product before \( \tilde{T} \). In
order to allow this, I assume that the consumers also visit purchase occasions right after their
arrival at the market, and also at the end of each trial session. Henceforth, I denote the optimal
frequency and termination policies at each state during trial by \( \tilde{\lambda}(\Omega_{i,t+1}) \) and \( \tilde{\delta}(\Omega_{i,t+1}) \), which
may be different from the post-purchase counterpart \( \lambda(\Omega_{i,t+1}) \) and \( \delta(\Omega_{i,t+1}) \). Formally, at each
trial session \( t \leq \tilde{T} \), a user’s optimal game mode selection is specified by the following dynamic
programming problem.

\[
V_{it}^{TL}(\Omega_{it},p_{r},k_{t}) = \mathbb{E}\left[\max_{m \leq M} v(b_{it},\nu_{imt},h_{t}) + EV_{it}(\Omega_{it},p_{r},k_{t}) + \epsilon_{imt}\sigma]\right], \ t \leq \tilde{T},
\]

where \( EV_{it}(\Omega_{it},p_{r},k_{t}) = \begin{cases} E[\tilde{\delta}(\Omega_{i,t+1})V^{TL}_{i,t+1,p}(\Omega_{i,t+1},p_{r},k_{l}) | \Omega_{it},m_{it}], & \text{if } t < \tilde{T}. \\ E[\tilde{\delta}(\Omega_{i,\tilde{T}+1})V_{ip}(\Omega_{i,\tilde{T}+1},p_{r}) | \Omega_{it},m_{it}], & \text{if } t = \tilde{T}. \end{cases} \]

The flow utility remains the same as in the full product, for all the features are included in the trial. On the other hand, the consumer faces different continuation payoffs depending on whether or not they have reached \( \tilde{T} \). At \( t < \tilde{T} \), after the current session consumers visit a purchase occasion, with their trial still remaining active. I denote the value function at the purchase occasion by \( V^{TL}_{i,t+1,p} \). At \( t = \tilde{T} \), the current session is the last session playable on the trial and the user makes only purchase decisions thereafter. The continuation payoff is hence identical to the value function from the model of purchase decisions \( V_{ip} \) defined earlier, but with product valuation evaluated at state \( \Omega_{it},\tilde{T}+1 \). In both cases the continuation payoff is in the expectation over the realization of the signal the user receives from the current session. The state space involves two new elements. \( p_{r} \) denotes the price in the current week, and \( k_{t} \in \{1, 2, ..., 7\} \) denotes the current day within the week. These extra state variables influence the decision of the optimal timing of purchase and hence affect usage decisions through the continuation payoff.

Value function \( V^{TL}_{it} \) summarizes her expected lifetime utility from adopting a free trial, including the option value of future product switches.

Now consider the purchase occasion that a user visits after session \( t < \tilde{T} \). She chooses whether or not to buy the full product by comparing the value of buying to that of staying with the trial.
Her value function at the purchase occasion at $t < \tilde{T}$ is

$$V_{i,t+1,p}^{TL}(\Omega_{i,t+1}, p_\tau, k_t) = \mathbb{E}[\max\{V(\Omega_{i,t+1}) - \eta_0 p_\tau + \epsilon_1 \tau \sigma_p, V_{i,t+1}^{TL}(\Omega_{i,t+1}, p_\tau, k_t) + \epsilon_0 \tau \sigma_p\}],$$

where $\bar{V}_{i,t+1}^{TL}(\Omega_{i,t+1}, p_\tau, k_t) = \tilde{\lambda}(\Omega_{i,t+1}) \sum_{k \geq 0} (\beta(1 - \tilde{\lambda}(\Omega_{i,t+1})))^k V_{i,t+1}^{TL}(\Omega_{i,t+1}, p_{\tau+k}, k_t + k - \tilde{k})$,

$$\tilde{k} = \left\lfloor \frac{k_t + k}{7} \right\rfloor.$$

The value from buying includes $V(\Omega_{i,t+1})$: her valuation of the full product evaluated with the posterior belief. On the other hand, the value from staying with the trial, $\bar{V}_{i,t+1}^{TL}$, is characterized by taking an expectation of the trial value $V_{i,t+1}^{TL}$ specified above with respect to possible future price level $p_{\tau+k}$. $\tilde{k}$ is the number of weeks between the current and the next purchase occasion. The user takes expectations over the future prices because the date of next visit to the purchase occasion is stochastic. The next visit occurs after her next trial session, and hence the expectation is over all possible interval lengths until the next session: the same logic as in the calculation of $\beta(\Omega_{i,t+1})$ in the no trial scenario. The price is assumed to take the same value within each week $\tau$, and exhibits a discrete jump across weeks. $k_t$ records the day of the week at which each session $t$ is played. If the interval between two sessions is $k$ days, $k_t$ evolves such that $k_{t+1} = k_t + k - \tilde{k}$. This results in the form of $V_{i,t+1}^{TL}$. The solution to this dynamic programming problem provides the optimal actions and the associated value functions of a trial user at each state.

Upon arrival at the market, knowing the value of buying the full product and that of adopting a trial, the consumer chooses either to adopt the trial or to buy the full product. If the flow utility is nonnegative and I assume that trial adoption is costless, “not buying and not trying” is weakly dominated by “trying and not buying”. Hence, I do not consider the former option explicitly.

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30 Since the flow utility is nonnegative and I assume that trial adoption is costless, “not buying and not trying” is weakly dominated by “trying and not buying”. Hence, I do not consider the former option explicitly.
upon the arrival at the market is described as the maximum of the two.

\[ V_{11p}^{TL}(\Omega_{11}, p_T, 1, \epsilon) = \max\{V(\Omega_{11}) - \eta_1 p_T + \epsilon_{111} \sigma_{p}, V_{11}^{TL}(\Omega_{11}, p_T, 1) + \epsilon_{011} \sigma_p\}. \]

This completes the characterization of the consumer decisions under a time-locked trial.

In general, time-locked trials prompt users to defer the purchase. Since the trial product is identical to the full product until it expires, consumers can play an extra trial session and reduce the uncertainty further by delaying the purchase at no opportunity cost. Hence, unless there is an expected price increase, the best strategy is to not buy until the trial expires. Nonetheless, in reality consumers may purchase the full product before the trial expiration date. In the model, this is captured by the idiosyncratic utility shock \( \epsilon \).

Trial provision may also prompt users to experiment more with the product. Under no trial, the option value of experimenting is simply that the user can make better informed *game mode selections* in the future. In the trial, the option value also comes from better informed future *purchase decisions*. Hence, users have a higher incentive to experiment. This implies that learning is endogenous with respect to firm’s policy; given the trial design selected by the firm, users choose the optimal amount of learning, changing the distribution of post-trial willingness to pay.

Other aspects of demand model not discussed here, such as the consumer arrival process, are assumed to remain the same as in the no trial case. The solution to the consumer’s dynamic programming problem provides the probability that for a given trial restriction \( \tilde{T} \), a consumer with a belief \( b_{it} \) and play history \( \{\nu_{imT}\}_{m=1}^M \) makes a purchase at price \( p_T \). I denote this probability by \( Pr(\Omega_{it}, p_T, \tilde{T}) \). The aggregate demand at price \( p_T \) is equal to the probability that consumers who have not purchased the product as of week \( \tau \) make a purchase. It is obtained by taking the sum of \( Pr(\Omega_{it}, p_T, \tilde{T}) \) over all possible usage histories that end up with a purchase at price \( p_T \) for
all users arriving at different weeks, and integrating it over the distribution of match values.

\[
D^{TL}(p_\tau, \hat{T} \mid p_{\tau'}, \tau' < \tau) = \int \sum_{\tilde{\tau}} \sum_{\Omega_{it}, t \leq \tilde{T}} \lambda_{it}^{p_\tau} P_T(\Omega_{it}, p_\tau, \hat{T}) \times \sum_{\tilde{\Omega}_{it}} \prod_{\Omega_{it} \in \tilde{\Omega}_{it}} \prod_{\hat{T} \leq \tau' < \tau} (1 - P_r(\Omega_{it'}, p_{\tau'}, \tilde{T})) dF(\theta_i),
\]

where \(\tilde{\Omega}_{it}\) represents the set of all histories that reaches \(\Omega_{it}\) at period \(t\). Because of the diminishing pool of consumers, the demand at \(p_\tau\) is a function of the sequence of prices that precedes it. For a given sequence of prices and for each \(\tilde{T}\), one can calculate the firm revenue as

\[
\pi^{TL}(\{p_1, ..., p_\tau, ...\}, \tilde{T}) = \sum_{\tau} p_\tau D^{TL}(p_\tau, \tilde{T}).
\]

In general, choosing larger \(\tilde{T}\) lets consumers reduce their uncertainty further. When consumers are risk averse, this increases expected utility from future sessions. However, there is an opportunity cost; initial \(\tilde{T}\) sessions no longer constitute willingness to pay. In addition, some users may terminate during the trial period and the pool of consumers staying in the market at \(\tilde{T}\) may diminish.

### 7.2 Structural demand models under a feature-limited trial

Now I turn to the case of feature-limited trial. In this case, the firm chooses \(\tilde{M}\) game modes to include in the trial, where \(\tilde{M} < M\). Assuming that consumers visit a purchase occasion at the end of each trial session, trial users’ dynamic programming problem is defined similarly as in the

\[^31\] In practice, the firm not only chooses the number of game modes but also \textit{which} game mode is included in the trial. Since subscript \(m\) is just a label and has no cardinal meaning, one can always re-order modes so that the ones offered in the trial are labeled first.
case of time-locked trial. The value function associated with the optimal game mode selection is

\[ V_{FL}^{i,t}(\Omega_{it}, p, k_t) = \mathbb{E}[\max_{m \leq \tilde{M}} \nu(b_{it}, v_{it}, h_t) + \mathbb{E}[\delta(\Omega_{it+1})V_{FL}^{i,t+1,p}(\Omega_{i,t+1}, p, k_t)|\Omega_{it}, m_{it}] + \epsilon_{it}\sigma_{\epsilon_1}]. \]

The difference from the time-locked trial case is that the game modes available for users are now \( \tilde{M} \) instead of \( M \), whereas there is no time constraint \( \tilde{T} \). The limited access to \( m \leq \tilde{M} \) game modes prevents users from receiving signals from mode \( m' > \tilde{M} \) and impacts the way users can update their belief. On the other hand, no time limit provides users a positive value from not buying the full product at any \( t \). The value function at each purchase occasion is identical to the time-locked case, except for notational differences.

\[ V_{FL}^{i,t+1,p}(\Omega_{it+1}, p, k_t) = \mathbb{E}[\max \{V(\Omega_{i,t+1}) - \eta_{ip} + \epsilon_{it}\sigma_{p}, V_{FL}^{i,t+1,p}(\Omega_{i,t+1}, p, k_t + \epsilon_{it}\sigma_{p})\}], \]

where \( \tilde{V}_{FL}^{i,t+1,p}(\Omega_{it+1}, p, k_t) = \tilde{\lambda}(\Omega_{it+1}) \sum_{k \geq 0} (\beta(1 - \tilde{\lambda}(\Omega_{it+1}))^k V_{FL}^{i,t+1,p}(\Omega_{it+1}, p, k_t + k - \tilde{k}), \]

\[ \tilde{k} = \left\lfloor \frac{k_t + k}{7} \right\rfloor. \]

Upon arrival at the market, consumers choose either to adopt the trial or to buy the full product.

\[ V_{FL}^{i1,p}(\Omega_{i1}, p, 1) = \max \{V(\Omega_{i1}) - \eta_{ip} + \epsilon_{i1}\sigma_{p}, V_{FL}^{i1,p}(\Omega_{i1}, p, 1) + \epsilon_{i1}\sigma_{p}\}. \]

Intuitively, the trade-off that consumers face is that by buying now, consumers have full access to the product starting from the next session. On the other hand, by delaying the purchase by one more session, they can receive one more signal and reduce uncertainty at the cost of having to choose from limited features in the next session. This implies that unlike the case of time-locked trial, consumers with sufficiently high initial belief prefer to buy the full product from the
beginning; the negative impact from not having full access on the utility outweighs the option value from the trial. On the other hand, consumers who only want the game modes provided in the trial do not benefit from buying the full product, and hence are likely to remain with the trial. The solution to the dynamic programming problem provides the probability that for a given trial restriction \( \tilde{M} \), a consumer with a belief and play history \( \Omega_{it} \) makes a purchase at price \( p_\tau \). Denoting this probability by \( Pr(\Omega_{it}, p_\tau, \tilde{M}) \), I characterize the aggregate demand the firm faces at each week in the same way as in the case of time-locked trial.

\[
D^{FL}(p_\tau, \tilde{M} | p_{\tau'}, \tau' < \tau) = \int \sum \sum \lambda_\tau^0 Pr(\Omega_{it}, p_\tau, \tilde{M}) \\
\sum \prod \prod (1 - Pr(\Omega_{it'}, p_{\tau'}, \tilde{M}))dF(\theta_i).
\]

The firm revenue is determined similarly.

\[
\pi^{FL}(\{p_1, ..., p_\tau, ...\}, \tilde{M}) = \sum p_\tau D^{FL}(p_\tau, \tilde{M}).
\]

In general, the firm wants to include features in the trial that create large learning spill-overs, so that uncertainty about features excluded from the trial also diminishes due to trial experience. At the same time the firm wants to exclude from the trial the features that many consumers find high match value; the trial and the full product need to be sufficiently differentiated, in order to induce consumers to make a purchase of the full product.

### 7.3 Simulation results

Since the aggregate demand \( D^{TL}(p_\tau, \tilde{T}) \) and \( D^{FL}(p_\tau, \tilde{M}) \) have no analytical form, I compute firm revenues at each \( \tilde{T} \) and \( \tilde{M} \) using 50,000 sequences of simulated consumer actions. In order to highlight the main trade-offs of each trial design discussed earlier, I assume that the price is
held constant at $p = 52.1$, the launch price. This eliminates consumers’ incentive to wait for future price drops and hence any difference in the purchase timing between the case of no trial and trials is due to the incentive to learn. I also assume that the consumers’ optimal frequency decisions under trial remain the same as in the full product; $\tilde{\lambda}(\Omega_{it}) = \lambda(\Omega_{it})$. This is reasonable given that trial provision influences consumers’ usage decisions only through increasing option value from learning, and the choice of play frequency is independent of learning incentives.

On the other hand, as trial provision increases the option value from remaining active, users’ termination rate during trial $\tilde{\delta}(\Omega_{it})$ can be lower than the no trial counterpart $\delta(\Omega_{it})$. As I do not observe free trial, I cannot estimate $\tilde{\delta}(\Omega_{it})$ from the data. Hence I simulate revenue outcome at different values of $\tilde{\delta}$.

In Figure 17, I present revenue implications of time-locked trials. The horizontal axis is $\tilde{T}$, the number of free sessions the firm provides, and the vertical axis is the percentile revenue difference from the no trial case. Each line corresponds to different value of $\tilde{\delta}$. $\tilde{\delta}$ corresponds to the speed of demand depreciation and hence the opportunity cost of providing a free trial. Two things are worth noting. First, under any $\tilde{\delta}$ such that a free trial can increase revenue, the revenue is maximized by providing five free sessions and hence it is the best trial design. This is consistent with the patterns of evolution of willingness to pay in Figure 15b. Second, revenue implications vary significantly with respect to $\tilde{\delta}$ and trials can be profitable only when $\tilde{\delta}$ is significantly lower than $\delta$. Any trial termination rate higher than 41 percent of full product termination rate renders profitability of any time-locked trial negative. This is because significant fraction of users who terminate during the trial are indeed high willingness to pay users. For example, when $\tilde{T} = 5$, 37.6 percent of users terminating during the trial have initial willingness to pay higher than the price, and hence would likely make a purchase if there were not any trial. A trial with $\tilde{T} = 5$ is

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32 Using a different price level hardly influences the results.
33 In this model, progression of learning is determined solely by the number of sessions; conditional on someone having played $t$ sessions, how many days it took her to reach that state does not affect her beliefs at $t$. 

51
break-even when the cumulative rate of termination over five sessions is 12 percent of all users. When the rate of termination is zero, the scenario most favorable for the firm, the trial provision increases revenue by 2.54 percent. This indicates that in order to fully benefit from the trial strategy, the firm may want to incentivize users to remain active until the trial expires. In the remainder of this section, I fix \( \tilde{\delta} = 0 \), in order to ease the comparison across the profitability of different trial designs.

In Figure 18, I show how consumers change their adoption behavior in response to the provision of a time-locked trial with five free sessions. Most of the consumers whose behavior changes due to trial have original willingness to pay close to the price \( p = 52.1 \). This is reasonable because even a small change of the perceived match value is likely to flip the optimal action in that range. Consistent with the increase of revenue, the number of adopters increase post-trial. Considering users whose original willingness to pay is between $30 and $60, five trial sessions increase their willingness to pay by 15.3 percent on average. Compared to post-purchase increase of product
valuation discussed in Section 6.4, the average willingness to pay after the 5th session is higher during the trial by 1.4 percentage points. Because users have a higher incentive to experiment with the product during the trial, they reach a better informed state at the end of the 5th session.

Table 4: Revenue implications for feature-limited trials

<table>
<thead>
<tr>
<th>Feature-Limited only</th>
<th>Combined with $\bar{T} = 5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1 only</td>
<td>-12.94%</td>
</tr>
<tr>
<td>Mode 2 only</td>
<td>-12.57%</td>
</tr>
<tr>
<td>Mode 3 only</td>
<td>-14.82%</td>
</tr>
<tr>
<td>Mode 4 only</td>
<td>-14.22%</td>
</tr>
</tbody>
</table>

Note: Each cell represents the revenue from a feature-limited trial with the restriction specified by the row, measured as a percentile difference from no trial case. The revenues are calculated from 50,000 simulation sequences. The price is fixed at $p = 52.1$. I assume $\delta = 0$.

I next evaluate the revenue implication of a feature-limited trial. In the first column of Table 4, I present revenues when the firm restricts user access to only one of the game modes. The revenues are reported as a percentile difference from the no trial scenario. I find that feature-limited trials without any time limits do not increase revenue.\(^{34}\) There are two reasons behind

\(^{34}\)While not reported, when multiple game modes are provided in the trial, the revenue is even lower.
this. First, the product studied only contains four game modes. Hence, giving one away already sacrifices a significant portion of the product value. In general, a feature-limited trial is more profitable when a product includes more features. Second, this product does not exhibit large learning spill-overs. Providing one game mode hardly facilitates learning of match values with other modes.

Finally, I consider a situation where the firm combines both restrictions on time and feature access: a “hybrid” trial (Cheng, Li and Liu 2015). In this case, consumers’ dynamic programming problem is described similarly as the one from the time-locked trial, with an extra restriction that only $\tilde{M} < M$ modes are accessible during the trial. In column 2 of Table 4 I calculate revenue from adding a restriction that users can access only one mode, to the ideal time-locked trial with $\tilde{T} = 5$. The best performance is achieved when user access is limited to only mode 1. In this case, revenue increases by 3.24 percent from no trial case: 0.7 percentage point of extra revenue increase over the pure time-locked product. This is because by imposing an additional feature restriction, the firm can prompt some high willingness to pay users to make a purchase. For example, by only allowing access to mode 1, users whose most preferred mode is not mode 1 maintain high product valuation after the trial expiration and may be prompted to buy. The same set of users are less likely to buy if they are provided five free sessions of their favorite mode. The results indicate that while a feature-limited trial in itself does not make profit, it can help boost revenue from a time-locked trial by creating an additional dimension of product differentiation between the trial and the full product.

8 Conclusion

In this study, I consider the impact of trial design on firm revenue. I develop a model of consumer learning-by-using of a durable good with multiple features and identify the mechanism that in-
fluences trial profitability. I find that consumers are risk averse and the magnitude of uncertainty around consumer-product match value is large. This implies that trial provision increases consumers’ product valuation even when their utility declines over time due to other factors, such as boredom. I find that consumers learn quickly, but learning spill-overs across different game modes are small.

This study offers several substantive insights. I find that in this setting, time-locked trials perform better and providing five free sessions is the best design, increasing revenue by up to 2.5 percent. Moreover, if the firm is willing to combine both time and feature restrictions, providing only five sessions of game mode 1 boosts revenue by an extra 0.7 percentage point. On the other hand, feature-limited trials without duration restrictions are not profitable due to small learning spill-overs. I also find that the best time-locked trial is profitable only when the cumulative rate of user termination during the trial period is less than 12 percent, which is significantly lower than the observed post-trial termination rate. This indicates that the firm may want to incentivize trial users to remain active until the trial expires. In fact, the finding is consistent with empirical observations that many videogame providers offer so-called “daily rewards” to users: a user receives a reward, such as in-game currency or items, by merely logging in to the game. The reward value increases for every consecutive day that the user logs in.

The methodology presented in this study allows firms to identify the mechanism behind consumer learning from a typical data set of consumers’ engagement with the product. It hence helps firms in assessing the profitability of various trial designs. In particular, the structural approach does not require any observation of past trial. My model is applicable to other cases where consumer learning exists and firms offer products with multiple features. Digital goods satisfy these criteria: smartphone apps and subscription services. Moreover, other goods can also exhibit similar attributes. For example, gym memberships typically entail some uncertainty
(match with the instructor, offered classes, etc.) and multiple services are offered. The model helps determine whether the trial should be offered as time-locked or feature-limited (e.g. only access to yoga class).

This study contributes to the literature by offering a novel application of a Bayesian model of forward-looking consumers to a durable goods setup. One nature of durable goods environment is that purchase and consumption are separable. Providing a free trial is essentially the firm’s choosing when and how the consumers visit purchase occasions during the sequence of learning. Moreover, learning is endogenous in this model; the firm policy directly influences how consumers learn through consumers’ endogenous usage decisions. To my knowledge, this is the first empirical analysis that considers such interaction between consumer actions and the firm policy.

This study has certain limitations when applied to other environments. I assume that the number of features in the full product is fixed and known, and there is no quality learning. Appropriate modifications are necessary in order to apply the current framework to environments where the firm introduces new features frequently, or the firm wants signal product quality through trials. Indeed, I view this study as a first step toward more general understanding of consumer learning in durable goods contexts and its managerial implications. Specifically, I plan to work on two extensions of this paper in the future. First, in this study I only focus on the optimal trial design at a fixed price. The natural next step is to analyze the interaction between the optimal trial design and pricing. Although the average willingness to pay increases by 15.3 percent due to learning, under the fixed price the revenue increases by only 2.54 percent, leaving room for pricing optimization. In particular, providing a free trial enables the firm to observe each consumer’s usage patterns during the trial period, based on which the firm may be able to offer customized pricing. Second, on the technical side, adding more flexible heterogeneity to deal with spurious state dependence is necessary (Dubé, Hitsch and Rossi 2010). I impose
that there exists two discrete segments that differ in their average match values. In principle, modeling the number of segments as a free parameter and use the information criteria to select one is a suggestive strategy in modeling discrete types.

Note also that the model I develop in this study is essentially a combination of a Bayesian model and a model of purchase timing. More generally, by offering different timing of payments the firm may be able to exploit consumer learning effectively. For example, introducing subscription service or pay-per-use scheme allows consumers to cancel subscription when the realized match value is low. This option value from being able to drop out can increase willingness to pay when consumers are risk averse. I consider this extension in the next chapter.
Chapter 2

Product Unbundling under Consumer Learning-by-Using

1 Introduction

Selling multiple products as a single bundle is a common business practice. For example, digital goods contain multiple, separable features in one product; internet services are often tied with other cable services; DVD for multiple movie titles are bundled for sale. Indeed, studies have identified multiple demand-side factors that make bundling profitable. Bundling is particularly profitable when preference correlations across components are not very high (Schmalensee 1984, McAfee, McMillan and Whinston 1989, Chen and Riordan 2013); marginal cost of producing components is low relative to average willingness to pay (Adams and Yellen 1976); the number of bundled components is large (Bakos and Brynjolfsson 1999, Fang and Norman 2006); and components to be bundled have similar variance of preference across consumers (Fang and Norman 2006, Chu, Leslie and Sorensen 2011).

In practice, such accumulated knowledge may not fully explain a recent trend that digital goods providers offer their products as a subscription service. Subscription service is essentially product unbundling in the following sense. Digital goods, such as software and videogame, can be considered as a bundle of identical services to be consumed at different points in time. Selling the product through subscription is equivalent to the firm’s unbundling the product into multiple components, with each component being the access to the service during a certain period of time.\footnote{Studies show that the profitability of bundling is increasing in the number of bundled products, and at the limit where infinitely many products are bundled, bundling always dominates component pricing.}
If the firm instead adopts an outright sale strategy, it is equivalent to selling the permanent access to the service, which is a pure bundle of all components. Fast-growing entertainment companies such as Spotify and Netflix exclusively offer subscription. Software publishers such as Electronic Arts, Adobe and Microsoft are in the process of shifting from the traditional outright sale scheme to a subscription scheme. This move is common despite the fact that digital goods typically come with zero marginal cost, that the number of bundled components is large, and that the variance of preference is stable across periods: factors that are predicted to favor product bundling.\textsuperscript{36}

In this paper, I introduce a new channel that favors the firm’s decision to unbundle, particularly in the context of subscription: consumer learning. Suppose that each consumer faces uncertainty about her individual valuation, or her “match value”, for each component in the bundle. Also suppose that consumption experience with a component is informative about one’s match values with other components; one’s consumption experience in early periods is informative about her utility during later periods. Under such circumstances, different sales strategies result in consumers making purchase decisions under different information available to them. In the case of outright sale, each consumer makes the adoption decision prior to any consumption experience. Hence, the decision is based solely on her prior beliefs about her match value. On the other hand, under a subscription scheme, each consumer only makes limited commitment at the beginning. After each subscription period, she can make a subscription renewal decision based on her updated beliefs about her match value.\textsuperscript{37}

This informational difference favors the firm’s adoption of subscription strategy in two ways. First, if consumers are risk averse, smaller uncertainty about one’s match value due to learning increases the average willingness to pay in later periods. Hence, allowing consumers to purchase

\textsuperscript{36}Moreover, I find that in the environment that I study, the preference is not perfectly correlated over time, satisfying yet another condition that makes bundling profitable.

\textsuperscript{37}In this study, I assume that the firm pre-determines the entire subscription price schedule and commits to it. Hence, “renewal decision” refers to the consumers’ decisions of either accept or reject the given contract term.
a part of the product later may increase revenue. Second, under a subscription scheme, each consumer is given the right to terminate her subscription if she realizes that her true match value is low. This provides an option value to the consumer when she makes the initial adoption decision under uncertainty; she faces lower risk of being held up with the product she does not like. As a result, her willingness to pay under uncertainty also increases.

I empirically examine how the existence of consumer learning influences the profitability of outright sale and subscription strategies. Specifically, I build and estimate a structural demand model for digital goods and identify (1) how utility from product usage is correlated across time, and (2) how consumers learn their true match value as they accumulate experience with the product. The model hence enables one to quantify the trade-offs that the firm faces in adopting either outright sale and subscription, and also to examine how much of the trade-offs are attributed to each demand-side factor. The model also allows one to conduct a counterfactual exercise to find a revenue-maximizing strategy.

In the model, I embed the adoption decision of a durable product into a Bayesian learning framework. Consumers make adoption decisions based on their willingness to pay, which is determined by their expected utility from their future consumption stream (Ryan and Tucker 2012, Lee 2013). The expectation over future utility is conditional on consumers’ beliefs about their match value. Hence, both the magnitude of uncertainty reflected in the belief and risk aversion impact their willingness to pay. The model of Bayesian learning describes the process of how a consumer updates her beliefs about her match value through product experience. Each user maximizes her expected utility by choosing the frequency of usage, duration of each session and a product feature used in each session. Her uncertainty diminishes as she updates her belief. At the same time, her utility may also deterministically evolve due to other forms of state dependence, such as boredom and skill acquisition. These factors determine how utility from each session is
associated with one another. Furthermore, a user is forward-looking and internalizes her option value from future informational gain when making usage and purchase decisions. Hence, I define the model as a dynamic programming problem (Erdem and Keane 1996, Che, Erdem and Öncü 2015). The solution of this problem provides a value function, which summarizes the consumer’s expected lifetime utility, determining her willingness to pay endogenously.

I estimate the model with a data set from a major sports videogame. The data set contains lifetime session records of 4,578 users, which consist of the duration and the game feature selected at each session. The firm adopts outright sale strategy throughout the sample period. Hence, I estimate the model that corresponds to outright sale. I find that videogame users are risk averse, that their initial product valuation involves significant uncertainty, and that learning occurs quickly. As a result, the average flow utility increases by 31 percent within the first 10 sessions. On the other hand, the estimated utility correlation across sessions is in the range of 0.6 and 0.7 and the variance of utility is stable across sessions. While large uncertainty and learning favors subscription, other estimated demand factors, along with zero marginal cost, are known to favor outright sale, creating trade-offs for the firm.

Using the estimated parameters, I compare revenues from outright sale and various subscription strategies. Specifically, I consider subscription strategies that take the form of “X-session package”; each subscription period is tied with the number of sessions that consumers can play.\(^{38}\) I first consider “pay-per-use” strategy, where each subscription duration consists of one session. I find that if the firm can flexibly vary prices across sessions, pay-per-use outperforms outright sale, increasing revenue by 2.8 percent. The optimal pay-per-use price schedule is increasing across sessions. In addition to exploiting the increased average willingness to pay in later ses-

\(^{38}\text{This form of subscription is common among online game providers. Other common subscription strategies not considered in this study include ones that tie each subscription period with the number of calendar days, such as one week subscription or one month subscription. Focusing on session-based subscription strategies allows us to reduce possible contamination from factors outside of the model, such as consumers’ strategic intertemporal consumption reallocation. More details are provided in the Appendix.}\)
sions, setting increasing prices also helps increase revenue by allowing consumers with low initial perceived match value to still adopt the product. Large initial uncertainty implies that some of them realize a high true match value and keep their subscription for long time despite higher prices in the later sessions. In terms of magnitude, I find that the number of consumers who play the game at least one session under pay-per-use is higher by 198 percent than outright sale. Furthermore, 12 percent of consumers newly acquired as such remain active for at least 50 sessions, becoming a major source of revenue increase. The revenue increase is partly offset by the revenue decrease from the ones who also adopt the product under the outright sale; they spend less under pay-per-use because they enjoy lower average per-session price, and some may terminate early.

In order to examine the extent to which the existence of consumer learning influences the above policy implication, I conduct another counterfactual exercise where I assume that consumers are perfectly informed about their own match value. In that case, I find that pay-per-use decreases the firm revenue by 7.8 percent. In the absence of learning, both of the revenue-increasing effects described above no longer exist. As a result, the other demand factors and zero marginal cost that favor product bundling are pronounced.

While the study of pay-per-use with flexible pricing is useful in highlighting the trade-offs that the firm faces, the assumption that the firm can change prices at a single session level is of limited realism. Hence I also examine the profitability of two other subscription strategies. First, I consider partial unbundling, where one subscription period contains multiple sessions. I find that the profit from partial unbundling lies between that of pay-per-use and outright sale; limited flexibility in the firm strategy results in lower revenue increase than fully flexible pay-per-use pricing. Second, I again consider pay-per-use, but where the firm can only set two prices; it offers one price for initial sessions and switches to the other price at a later point in time. I
find that under the optimal two-price strategy, the revenue decreases by 1.5 percent compared to outright sale. As the firm’s strategy space is tightly limited, the firm cannot adjust prices in response to the change of willingness to pay in the market. Hence the revenue loss from interim termination of consumers outweighs the benefit from exploiting the increased willingness to pay. I conclude that in the current environment, the existence of consumer learning has a substantive impact on the firm’s optimal policy, and the major part of revenue increase comes from the firm’s ability to set different prices over time.

The finding that increasing price sequence allows consumers to “try” the product at the beginning resembles the mechanism of free trial that I study in Chapter 1. Indeed, the time-locked trial I study in Chapter 1 is a special case of general bundling strategies considered herein, where the firm first sells the bundle of the initial five sessions at zero price, and then sells the bundle of all the remaining sessions at the full price. From this perspective, the two chapters are closely associated in that in Chapter 1, I compare a special case of intertemporal unbundling (time-locked trial) with another special case of cross-feature unbundling (feature-limited trial). In this chapter, I consider a general form of intertemporal unbundling (subscription), while cross-feature domain remains fixed. On the other hand, by focusing on different subset of firm policies, these chapters provide answers to different policy questions; Chapter 1 answers to “whether and how to provide a free trial”, and Chapter 2 answers to “implication of consumer learning on product bundling”.

This study is the first empirical work that considers subscription service in the context of product unbundling. In particular, it contributes to the empirical literature of product bundling by quantifying how consumer learning influences profitability of bundling strategies. Existing studies consider profitability of bundling in various environments, albeit assuming perfectly informed consumers. Chu, Leslie and Sorensen (2011) consider a theater company’s optimal bundling of
multiple shows. In particular, they propose a practical alternative to traditional mixed bundling. Crawford and Yurukoglu (2011) consider bundling of cable TV channels while accounting for how bundling influences upstream sourcing contract. Finally, Ho, Ho and Mortimer (2012) consider bundling in the context of wholesale contract by utilizing detailed data of wholesale contract and consumer demand.

This paper is structured as follows. In Section 2, I build a simple model to highlight how consumer learning influences profitability of product bundling. In Section 3, I describe the empirical environment, and then introduce the model in Section 4. I discuss the estimation strategy and results in Section 5. In Section 6, I implement counterfactual exercises to see the profitability of outright sale and subscription. Section 7 concludes.

2 An illustrative model of consumer learning and firms’ trade-offs

In this section, I introduce a simple model of consumer learning and illustrate how learning influences profitability of outright sale and subscription. Consider a firm selling a videogame that each consumer may play for two periods. At each period \( t = \{1, 2\} \), each consumer \( i \) receives flow utility \( v_{it} \), which is either \( v_H \) (enjoy) or \( v_L \) (not enjoy). I assume that utility at each period realizes according to the following probability distribution.

\[
Pr(v_{i1} = v_H) = p_i, \quad \text{where} \quad p_i \sim U(0,1).
\]

\[
Pr(v_{i2} = v_H | v_{i1} = v_H) = p_H, \quad Pr(v_{i2} = v_L | v_{i1} = v_H) = 1 - p_H.
\]

\[
Pr(v_{i2} = v_H | v_{i1} = v_L) = p_L, \quad Pr(v_{i2} = v_L | v_{i1} = v_L) = 1 - p_L.
\]
In other words, each consumer is heterogeneous in the probability that she enjoys the game in
the first period, which is denoted by $p_i$ and it follows a uniform distribution. On the other hand,
the probability that she enjoys the second period is determined by whether she enjoyed the first
period; if she received $V_H$ in the first period, then she also receives $V_H$ in the second period with
probability $p_H$. On the other hand, if she received $V_L$ in the first period, she receives $V_H$ in
the second period with probability $p_L$.40 Using the terminology in the literature, if $p_H > p_L$,
the preference is positively correlated; those who enjoy the first period are more likely to enjoy
the second period too. Analogously, $p_H < p_L$ implies negative correlation. I assume that the
distribution of $p_i$ and the value of $p_H$ and $p_L$ are common across everyone and are common
knowledge.

Before a consumer plays the game, she faces uncertainty about her match value with the
product. I assume that while each consumer is privately informed about the realization of her
$p_i$, she does not know her utility realization at each period. Hence her adoption decision is
based on her expected utility. Her expected utility in the first period is given by
$E(v_1 | p_i) = \alpha(p_i v_H + (1 - p_i) v_L)$. $\alpha \leq 1$ is a parameter that captures the reduction of the utility due to risk
aversion in a reduced form way. $\alpha$ is low if a user is very risk averse.40 At the beginning of the
second period, the uncertainty is resolved and each consumer knows the realization of her second
period utility $v_H$ or $v_L$.

The firm produces the videogame with zero marginal cost. It can sell the product either
through outright sale, where consumers purchase the whole product upfront, or through sub-
scription, where consumers also make a renewal decision at the beginning of the second period.
I assume that only those who subscribed in the first period may continue to the second period.

39 Although $p_H$ and $p_L$ is common to everyone, the probability that each consumer receives $V_H$ at the second
period is still heterogeneous because of $p_i$.

40 Strictly speaking, this formulation is not internally consistent in that even consumers with type $p_i = 0$ who
essentially face no uncertainty still discount her utility by $\alpha$. I opt for this formulation for cleanliness of the results,
thanks to $\alpha$ entering utility linearly.
Henceforth, I assume that $v_H = 2$ and $v_L = 1$ for simplicity.

**Case 1: Subscription** I first consider the firm’s problem when the firm adopts the subscription strategy. The firm sets prices $q_1$ and $q_2$ for the initial subscription and renewal, respectively. I assume that the firm sets both prices at the beginning and commits to them. At each period, each consumer compares her flow utility and the price and makes the adoption and renewal decisions\(^{41}\) The demand in the first period is continuous in $q_1$ according to the uniform distribution of $p_i$.

\[
D_1(q_1) = Pr(\alpha(2p_i + (1 - p_i)) \geq q_1) = \begin{cases} 0 & \text{if } q_1 > 2\alpha \\ \frac{2\alpha - q_1}{\alpha} & \text{if } q_1 \in [\alpha, 2\alpha] \\ 1 & \text{if } q_1 < \alpha \end{cases}
\]

The profit from the first period is $\pi_1(q_1) = q_1 D_1(q_1)$.

In the second period, consumers know their utility realization, which is either 1 or 2. Hence, the firm’s optimal strategy is either setting $q_2 = 1$ or $q_2 = 2$. If the firm sets $q_2 = 1$, then everyone who subscribed in the first period renews the subscription, resulting in the same demand as in the first period.

\[
D_2(q_1, q_2 = 1) = Pr(\alpha(2p_i + (1 - p_i)) \geq q_1) = D_1(q_1).
\]

Hence the profit from the second period is $\pi_2(q_1, q_2 = 1) = D_1(q_1)$. On the other hand, if the firm sets $q_2 = 2$, among consumers who subscribed in the first period, only the ones such that

\(^{41}\)For simplicity, in this exercise I assume that consumers do not take into account the option value that by playing the first period, they can proceed to the second period where they may receive positive surplus. This also makes it irrelevant whether consumers know the second period price at the first period.
v_{i2} = 2 keep their subscription. Hence the demand is given by

\[ D_2(q_1, q_2 = 2) = Pr(v_2 = 2, \alpha(2p_i + (1 - p_i)) \geq q_1) \]

\[ = \int_{2\alpha - q_1}^{1} p_i p_H + (1 - p_i) p_L dp_i \]

\[ = \begin{cases} 
(\frac{2\alpha - q_1}{\alpha}) \left( p_L + \frac{q_1}{2\alpha}(p_H - p_L) \right) & \text{if } q_1 \in [\alpha, 2\alpha], \\
0 & \text{if } q_1 > 2\alpha, \\
\frac{p_H + p_L}{2} & \text{if } q_1 < \alpha.
\end{cases} \]

The expression of the second line comes from the fact that there are two cases where a consumer of type \( p_i \) draws \( v_{i2} = 2 \). He may follow \( \{v_{i1} = 2, v_{i2} = 2\} \), which occurs with probability \( p_i p_H \). Alternatively, he may follow \( \{v_{i1} = 1, v_{i2} = 2\} \) which occurs with probability \( (1 - p_i) p_L \). Hence, \( Pr(v_{i2} = 2 \mid p_i) = p_i p_H + (1 - p_i) p_L \). We then take its expectation over \( p_i \)'s among those who subscribed in the first period and obtain the joint probability. Given the demand, the firm's profit is \( \pi_2(q_1, q_2 = 2) = 2D_2(q_1, q_2 = 2) \).

The firm maximizes the sum of the profit from the two periods by choosing \( q_1 \) and \( q_2 \). It follows that the firm’s optimal strategy is given as follows.

\[ q_1 = \alpha. \]

\[ q_2 = \begin{cases} 
1 & \text{if } p_H + p_L < 1, \\
2 & \text{otherwise.}
\end{cases} \]

The derivation is provided in the appendix. The associated profit is

\[ \pi_{ppu} = \alpha + \max\{1, p_H + p_L\}. \]
Intuitively, the firm serves everyone in the market in the first period by setting $q_1 = \alpha$, which is equal to the expected utility of the lowest type $p_i = 0$. In the second period, if more than half of the consumers receive $v_H = 2$, which occurs when $p_H + p_L > 1$, then the firm sets $q_2 = 2$. Otherwise, the firm again serves everyone by setting $q_2 = 1$. Note that because consumers only face uncertainty at the first period, the resultant utility decline $\alpha$ only influences the firm’s profit from the first period.\footnote{Note that because of heterogeneity in $p_i$, each consumer gains an informational rent in this environment.}

**Case 2: Outright sale** If the firm adopts the outright sale strategy, the firm sets one price $q_b$ for the access to the product over two periods. Each consumer makes the adoption decision by comparing her expected utility from the two periods with $q_b$. The expected utility of a type $p_i$ consumer from adopting the product is given as follows.

$$E(v_1 + v_2 \mid p_i) = \alpha(p_i(2 + 2p_H + (1 - p_H)) + (1 - p_i)(1 + 2p_L + (1 - p_L)))$$

$$= \alpha p_i(3 + p_H) + \alpha (1 - p_i)(2 + p_L).$$

Intuitively, she knows that if she receives $v_H$ in the first period, she also receives $v_H$ in the second period with probability $p_H$, and receives $v_L$ with probability $1 - p_H$, and so on. This implies that for a given price $q_b$, the demand is given as follows.

$$D_b(q_b) = Pr(\alpha p_i(3 + p_H) + \alpha (1 - p_i)(2 + p_L) \geq q_b)$$

$$= \begin{cases} 
\frac{\alpha(3+p_H)-q_b}{\alpha(1+p_H-p_L)} & \text{if } q_b \in [\alpha(2+p_L), \alpha(3+p_H)], \\
0 & \text{if } q_b > \alpha(3+p_H), \\
1 & \text{if } q_b < \alpha(2+p_L).
\end{cases}$$
The firm maximizes $\pi(q_b) = q_b D_b(q_b)$ with respect to $q_b$. It follows that the firm’s optimal strategy is to set a low price and let everyone adopt the product. The profit and the optimal price are

$$\pi_b = q_b = \alpha(2 + p_L).$$

The profit only depends on $p_L$ and not on $p_H$. This is because of the full market coverage. Selling to everyone implies that the price is equal to the expected utility of the lowest type with $p_i = 0$. Because the lowest type can never receive $v_H$ in the first period, $p_H$ does not appear in their expected utility. Hence, the optimal price does not depend on $p_H$.\footnote{In this environment, the uncertainty consumers face is about their own match values, which is person-specific. Hence the price does not provide any informational signal to consumers.}

In Figure 19, I illustrate how the optimal strategy in this model depends on the demand-side parameters. The figure clearly shows the trade-offs discussed above. In general, outright

![Figure 19: Optimality of different sales strategies](image)
sale dominates subscription when \( \alpha \) and \( p_L \) are high. Subscription often outperforms outright sale under low \( \alpha \) because outright sale forces consumers to make the purchase decision upfront under uncertainty. In this case, utility from both periods is subject to the decline due to risk aversion \( \alpha \). In the case of subscription, consumers know their true utility when they make the renewal decision. Hence, only the willingness to pay in the first period is subject to the decline.\(^{14}\)

The result that higher \( p_L \) favors outright sale is consistent with the literature; lower preference correlation across components makes bundling more profitable (Adams and Yellen 1976). In the extreme case where \( p_L = 1 \) and \( p_H = 0 \), all consumers, regardless of their \( p_i \), receive the same utility of \( v_1 + v_2 = 3 \). In that case, the firm sets \( q_b = 3\alpha \) and achieves the full surplus extraction. Note, however, that even when \( p_L < 0.5 \) and preference is positively correlated, outright sale still does better than subscription at high values of \( \alpha \). Negative correlation is not necessary for profitable bundling (Schmalensee 1984, Chen and Riordan 2013). Indeed, in the empirical study that I conduct in the following sections, I find that preference is positively correlated and still outright sale may outperform subscription under certain situations.

The simple model highlights how consumer learning and risk aversion can potentially make unbundling optimal even when other environments may suggest otherwise. In the following sections, I consider more realistic models that explicitly account for more flexible heterogeneity of consumers and multi-period learning, in order to implement an empirical study. Nevertheless, the trade-offs that are present in this section remain the key driver of the firm’s optimal policy.

3 Empirical environment

In this study, I apply my framework to the environment that I also studied in Chapter 1: a major sports videogame operating on videogame consoles such as Sony PlayStation series and

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\(^{14}\)Since consumers are not forward-looking in the subscription scenario, they do not benefit from the option value from future returns at the first period in this exercise.
Microsoft Xbox series. During the data period, the firm adopts the outright sale strategy and requires purchase of a game disk before the game can be played. After the purchase, at each of the sessions that they play, consumers choose among one of the four features called “game modes”. Each game mode corresponds to a distinct content area. No predetermined play sequence exists and users can choose any modes at any session. A session, the unit of observation, is defined as a continuous play of one game mode. Upon turning the console on, users select a mode in the menu screen and the session starts. When they exit the mode and return to the menu screen, or shut down the console, the session ends. By definition of the session, each session consists of only one game mode.

The videogame is released annually. I observe a sample of 4,578 first-time users from the version released in 2014. The sample is randomly selected among U.S. based users who registered a user account during product activation. For each user, I observe the date of activation, which I assume to be the date of purchase, and the lifetime session records. Each session record consists of the time of start and finish, and the selected game mode. I augment my data with the game’s weekly average market price, collected from a major price comparison website. The market price is the average of the prices listed on four major merchants: Amazon, Gamestop, Walmart and eBay. I assume that this market price is the purchase price. Because the data set is identical to the one I use in Chapter 1, see Table 1 in Chapter 1 for the summary statistics. Reduced form evidence regarding consumer learning is also presented in Section 4 of Chapter 1. The observed actions indicate that consumer learning is present in the current environment.

In this section, I show data patterns regarding the other element that also impacts the profitability of product bundling: how preference for each session is related to one another. As documented in the literature, bundling is more profitable when preference correlations between the components are not very high and the variance of preference among consumers is similar.
across components. Because actions that consumers select at each session are indicative of their preference, here I describe how the distribution of actions is associated across sessions. In Figure 20, I illustrate the intertemporal correlation of the actions. The solid line corresponds to that of the session duration, and the dashed line is that of the interval length between two adjacent sessions. Both actions represent moderately positive correlation; those who play longer in the initial session tend to play longer in the subsequent sessions too, and the same is true for the interval lengths. The correlations also exhibit a declining pattern. Utility from later sessions may change both due to learning, and other forms of state dependence, such as boredom.

![Figure 20: Correlation of actions across sessions](image)

Note: Each point on the line corresponds to the correlation between “the action selected in the first session” and “the action selected in the t-th session”.

Figure 21 shows the distribution of session durations measured in hours and that of intervals between two sessions measured in days, both at the first session and the 10th session. The distribution remains similar over time. The distributions of the durations in the later sessions have a lower density around shorter hours. This is because of the dropout of light users during early periods. The distributions of intervals between sessions are also stable. As one exception, after the first session, a larger number of consumers continue to play the second session on the
same day, creating a larger mass at zero.

![Distribution of actions in the first session and the 10th session](image)

(a) Session duration
(b) Interval between sessions

Figure 21: Distribution of actions in the first session and the 10th session

Note: The distributions for session $t$ includes all users who remain active at that $t$.

The observed patterns of consumer decisions presented in this section are merely a suggestive measure that of the underlying utility. However, they appear to indicate that aside from the existence of learning, the environment favors outright sale. Preference is not perfectly correlated across sessions; marginal cost is zero; and preference distributions are stable over time. Indeed, as I show below, the model predicts that outright sale outperforms subscription when learning does not exist.

4 **A structural demand model under outright sale**

In order to identify the mechanism behind learning and to predict consumers’ behavior under different sales strategies, I build and estimate a structural model of consumers’ adoption and learning-by-using. The model accounts for several demand-side factors. First, as already discussed, the profitability of bundling depends on how preference for each component is associated with one another. Second, identifying consumers’ risk aversion, the magnitude of uncertainty they are facing and the speed of learning is necessary to evaluate how learning influences will-
ingness to pay. Finally, consumers must be forward-looking to internalize the option value from being able to cancel the subscription in the future. Indeed, these are the factors that the model I built in Chapter 1 can account for. Utility from each session is associated with one another through learning, other forms of state dependence and consumer heterogeneity. Learning is accounted for through a Bayesian framework, and forward-lookingness is captured through dynamic programming framework. Hence, in this study I re-employ the framework used in Chapter 1. In this section, I will provide a review of the model under outright sale, where consumers first make a purchase decision and then make play decisions thereafter. I first present the Bayesian learning model that characterizes usage decisions, and then the model for purchase.

### 4.1 The Bayesian learning model

The model of learning-by-using characterizes users’ post-purchase play decisions. At each day, a user makes decisions according to a timeline described in Figure 22. The user first chooses whether to play a session. If she doesn’t play one, she moves to the next day. If she plays one, she selects a game mode and chooses session duration. After a session, she receives a signal

![Figure 22: Timeline of the choices at each day](image-url)

Note: Double-edged nodes A through C are decision nodes.
informative about her true match value from the selected mode, and updates her belief. At this point, the user may decide to permanently quit playing. I refer to this as termination. Termination is an absorbing state and she never makes any decisions again. Conditional on remaining active, the user chooses whether to play another session or move to the next day. She repeats this sequence until she terminates. In what follows, I first describe the user decisions during a session (Node B), and the decisions of play frequency and termination (Node A, C) afterward.

4.1.1 Selection of game modes and session duration (Node B)

At each session, users select a game mode and choose session duration. In order to ensure that users internalize any option values from informational gain, I assume that users are forward-looking; users solve a discrete choice dynamic programming problem. Conditional on having selected a mode, users then choose duration of the session, which endogenously determines the flow utility from that mode.

**Game mode selection**  A forward-looking user selects a game mode that maximizes the sum of her flow utility from playing and future informational return (Erdem and Keane 1996). In order to capture the nonstationary usage pattern observed in the data, I assume that the problem has a finite horizon. The optimal mode selection is summarized by the following value function.

\[ V(\Omega_{it}) = \mathbb{E}[\max_{m_{it}} v(b_{it}, \nu_{imt}, h_t) + \mathbb{E}[\beta(\Omega_{i,t+1})V(\Omega_{i,t+1}) \mid \Omega_{it}, m_{it}] + \epsilon_{imt}\sigma_{\epsilon}], \]

\footnote{I assume that at \( T = 100 \) session all active users terminate. This is longer than the lifetime number of sessions of 93.27 percent of the users in the data.}
where $\Omega_{it} = \{b_{it}, \{\nu_{imt}\}_{m=1}^{M}, h_{t}\}$; the state variables include $b_{it}$, i’s belief about her match value at session $t$; $\nu_{imt}$, the cumulative number of times that $i$ chose mode $m$ in the past $t-1$ sessions; and $h_{t}$, a weekend indicator, which is one for Saturday, Sunday and holidays. I denote the flow utility from the current session by $v(b_{it}, \nu_{imt}, h_{t})$. The future informational gain is summarized by the continuation payoff $E[\beta(\Omega_{i,t+1})V(\Omega_{i,t+1}) | \Omega_{it}, m_{it}]$. $\beta(\Omega_{i,t+1})$ is a discount factor between the current session and the next session. I discuss the definition of $\beta(\Omega_{i,t+1})$ below. The expectation of the continuation payoff is taken over an informative signal that the user receives after the current session. I assume that there exists a choice-specific idiosyncratic utility shock $\epsilon_{imt}$, and that $\epsilon_{imt}$ follows type 1 extreme value distribution with variance $\sigma_{\epsilon}^2$. The choice probability of each mode hence follows the logit form.

$$P_{m}(\Omega_{it}) = \left( \frac{\exp\left(\frac{1}{\sigma_{\epsilon}}(v(b_{it}, \nu_{imt}, h_{t}) + E[\beta(\Omega_{i,t+1})V(\Omega_{i,t+1}) | \Omega_{it}, m_{it}])\right)}{\sum_{m'} \exp\left(\frac{1}{\sigma_{\epsilon}}(v(b_{it}, \nu_{im't}, h_{t}) + E[\beta(\Omega_{i,t+1})V(\Omega_{i,t+1}) | \Omega_{it}, m'_{it}])\right)} \right). \quad (10)$$

Users face trade-offs between flow utility from the current session and the future informational gain. Since the problem is nonstationary, all the value functions and the optimal actions are a function of $t$ in addition to the state $\Omega_{it}$, which I suppress for notational simplicity.

**Choice of session duration** Now I derive $v(b_{it}, \nu_{imt}, h_{t})$, the flow utility from mode $m$, as a result of optimal choice of session duration. I assume that having selected game mode $m$ at session $t$, user $i$ chooses duration of the session to maximize her expected utility specified as follows.

$$v(b_{it}, \nu_{imt}, h_{t}) = \max_{x_{imt}} f(b_{it})x_{imt} - \frac{(c(\nu_{imt}) + x_{imt})^2}{2(1 + \alpha h_{t})}. \quad (11)$$

$x_{imt}$ is the session duration that user $i$ chooses. I assume that the expected utility is a quadratic function of $x_{imt}$. $f$ and $c$ are functions that represent how marginal utility from playing an
extra hour is affected by the belief $b_{it}$ and the history of play $\nu_{imt}$, respectively. Accumulation of usage experience influences utility through two channels. First, due to learning, users update their beliefs about match values and their utility evolves accordingly. This is captured through $f$. Second, aside from learning, usage experience may directly influence utility. $c$ controls for such other factors. For example, any deterministic utility decay, such as satiation or boredom, implies that $c$ is increasing in $\nu_{imt}$. Likewise, due to novelty effects or skill acquisition, $c$ may decrease in $\nu_{imt}$ for some range of $t$. $f$ and $c$ govern how utility from each session is correlated with one another, influencing the profitability of outright sale and subscription. Finally, users tend to spend more time in weekend, indicating that they may receive higher utility. This is captured by $\alpha > 0$.

The static expected utility maximization problem has a closed-form solution, resulting in the following flow utility and the optimal session duration for each mode $m$.

$$v(b_{it}, \nu_{imt}, h_t) = \frac{f(b_{it})^2(1 + \alpha h_t)}{2} - f(b_{it})c(\nu_{imt}),$$

(12)

$$x^*(b_{it}, \nu_{imt}, h_t) = f(b_{it})(1 + \alpha h_t) - c(\nu_{imt}).$$

(13)

Henceforth, I parametrize $f(b_{it})$ as follows.

$$f(b_{it}) = \mathbb{E}[\theta^\rho_{im} \mid \theta_{im} > 0, b_{it}],$$

(14)

where $\theta_{im}$ denotes a user-mode specific true match value. $f(b_{it})$ is specified as an expectation of $\theta^\rho_{im}$ conditional on the belief. $\rho > 0$ can be interpreted as the coefficient of risk aversion. $\rho < 1$ implies that utility is concave in the true match value $\theta_{im}$, and taking expectation over $\theta^\rho_{im}$ results in utility diminution. On the other hand, $c$ can be an arbitrary function such that $c(0) = 0$. 

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4.1.2 The decisions of play frequency and termination (Nodes A, C)

At decision nodes A and C, each user makes decisions of play frequency and termination. She compares value from playing to that from not playing at node A, and compares value from remaining active to that from terminating at node C. Instead of defining a full maximization problem, I take a reduced form approach to modeling them. Specifically, I impose the following two assumptions; (1) users’ decisions are based only on the state $\Omega_{it}$ at any decision nodes located between sessions $t - 1$ and $t$, and (2) decisions are influenced by an idiosyncratic shock, such that the optimal decision is representable by a probability distribution over each of the available alternatives. I denote the probability that user $i$ plays her $t$-th session on a given day by $\lambda(\Omega_{it})$, and the probability that user $i$ remains active after session $t$ by $\delta(\Omega_{i,t+1})$. I treat these probability distributions as model primitives.

Given the structure of the decisions of frequency and termination, I derive the formula for $\beta(\Omega_{i,t+1})$: the discount factor between session $t$ and $t + 1$. Assuming that users discount future utility by $\beta$ per one day, $\beta(\Omega_{i,t+1})$ is obtained as the expected discount factor between the date that session $t$ is played and the date that session $t + 1$ is played. The expectation is over whether the user remains active after session $t$, and when she plays session $t + 1$; because the optimal action at each decision node depends on an idiosyncratic shock that only realizes at that node, a user’s future actions are stochastic to herself. Formally, $\beta(\Omega_{i,t+1})$ is characterized as follows.

$$
\beta(\Omega_{i,t+1}) = \delta \lambda + \delta(1 - \lambda)\lambda\beta + \delta(1 - \lambda)^2\lambda\beta^2 + ... \\
= \delta(\Omega_{i,t+1}) \frac{\lambda(\Omega_{i,t+1})}{1 - (1 - \lambda(\Omega_{i,t+1}))\beta}.
$$

The intuition is as follows. After session $t$ the user remains active with probability $\delta(\Omega_{i,t+1})$. If staying active, she plays session $t + 1$ on the same day with probability $\lambda(\Omega_{i,t+1})$, on the next
day with probability \((1 - \lambda(\Omega_{i,t+1}))\lambda(\Omega_{i,t+1})\), incurring the daily discount factor \(\beta\), and so on.

### 4.1.3 State variables and their evolution

**Match value and its learning-by-using** I denote the true match value of consumer \(i\) with the game by a vector \(\theta_i = \{\theta_{i1}, \theta_{i2}, ..., \theta_{iM}\}\), where \(M\) is the number of available modes. Users are heterogeneous in their match values. I assume that \(\theta_i\) follows multivariate normal distribution; \(\theta_i \sim N(\mu, \Sigma)\), where \(\mu = \{\mu_1, \mu_2, ..., \mu_M\}\) is the average match value of the population and \(\Sigma\) is an arbitrary variance-covariance matrix.

Upon arrival at the market, the consumer does not know the realization of \(\theta_i\), and has a rational expectation about it; her prior about the distribution of \(\theta_i\) is equal to the population distribution of match values. In addition, she receives a vector of initial signals \(\tilde{\theta}_{i0} = \{\tilde{\theta}_{i10}, \tilde{\theta}_{i20}, ..., \tilde{\theta}_{iM0}\}\). The initial signal represents any information that a user has about the product ex-ante. It creates heterogeneity in the initial perceived match value. I assume that the initial signal is independent across game modes, and is normally distributed conditional on the user’s true match value; \(\tilde{\theta}_{im0} | \theta_{im} \sim N(\theta_{im}, \tilde{\sigma}_m^2)\). Henceforth, I denote the diagonal matrix of the variance of the initial signal by \(\tilde{\Sigma}\). The consumer forms an initial belief as a weighted average of the prior distribution and the received signal in a Bayesian manner.

\[
\theta_i | \tilde{\theta}_{i0} \sim N(\mu_{i1}, \Sigma_1),
\]

where \(\mu_{i1} = \mu + \Sigma(\Sigma + \tilde{\Sigma})^{-1}(\tilde{\theta}_{i0} - \mu)\),

\[
\Sigma_1 = \Sigma - \Sigma(\Sigma + \tilde{\Sigma})^{-1}\Sigma.
\]

The initial belief is thus represented by \(b_{i1} = \{\mu_{i1}, \Sigma_1\}\).

When a user plays game mode \(m\) at each session \(t\), she receives a signal informative about her true match value for that mode. I assume that the signal \(s_{imt}\) is normally distributed around
the true match value.

\[ s_{imt} \mid \theta_{im} \sim N(\theta_{im}, \sigma^2_{s}). \]

I assume that the variance of the signal \( \sigma^2_s \) remains the same over time. Introducing a time-varying signal distribution makes the model computationally intensive, and hence I opt to maintain a simple structure. Given the realized signal, the user updates the belief following the Bayesian formula.

\[
\begin{align*}
\mu_{i,t+1} &= \mu_{it} + \Sigma_{it} Z_{it}' ( Z_{it} \Sigma_{it} Z_{it}' + \sigma^2_s )^{-1} ( s_{imt} - \mu_{imt} ), \\
\Sigma_{i,t+1} &= \Sigma_{it} - \Sigma_{it} Z_{it}' ( Z_{it} \Sigma_{it} Z_{it}' + \sigma^2_s )^{-1} Z_{it} \Sigma_{it},
\end{align*}
\]

(17)

(18)

where \( Z_{it} \) is a 1 by \( M \) vector whose \( m \)-th element is one and zero elsewhere.

**Evolution of other state variables** \( \nu_{imt} \) evolves deterministically; \( \nu_{im1} = 0 \) for all \( m \), and \( \nu_{im,t+1} = \nu_{imt} + 1 \) if \( m \) is chosen at session \( t \), and \( \nu_{im,t+1} = \nu_{imt} \) otherwise. The weekend indicator is i.i.d, and it is 1 with probability 2/7 and zero with probability 5/7. This stochastic weekend arrival helps reduce the dimension of state variables; deterministic weekend arrival requires to keep track of the day of the week in the state.\footnote{The derivation of \( \beta(\Omega_{i,t+1}) \) in Equation \ref{eq:15} ignores the fact that \( h_t \) evolves day by day even without playing a session. In practice, I replace relevant \( \lambda(\Omega_{i,t+1}) \) in Equation \ref{eq:15} with its expectation over the realization of \( h_{t+1} \).} This completes the description of the model for usage. This problem is solvable by backward induction. The solution consists of the optimal decision rule and the associated value function at each state \( \Omega_{it} \).

Before moving on to the model of purchase decisions, I discuss the applicability of this learning model, which is originally developed to study free trials, to the study of profitability of subscrip-
tion. As I discussed earlier, this model can fully capture the demand-side factors that influence the profitability of outright sale and subscription strategies. However, the concept of game modes is not necessary to study subscription. In the following counterfactual exercises, I only consider unbundling of the product across time and not across modes. Hence we only need to know how utility varies across time. This implies that as long as consumers do not change consumption pattern across modes in response to different sales strategies, we can eliminate the discrete choice across game modes altogether. In that case, we estimate a distribution of flow utility from each session that subsumes both optimal mode selection and session duration, instead of estimating mode-specific utility. In this Chapter, I keep the concept of game mode in the model to maintain consistency with Chapter 1.

4.2 Adoption decisions

When the product is offered through outright sale, each consumer makes adoption decisions without any playing experience. A user’s product valuation is represented by her ex-ante value function $V(\Omega_{i1})$: the sum of the utility she expects from the product in the future, evaluated at the initial state $\Omega_{i1}$. In order to capture the fact that the product remains available in the market over one year with decreasing prices, I model the adoption decision as another dynamic programming problem, where consumers may wait for future price drops (Nair 2007).

The adoption decisions proceed as follows. I assume that the market consists of $N$ consumers. They are heterogeneous, in that $V(\Omega_{i1})$ is consumer-specific. In line with the frequency of the price data, I assume that one period in the adoption model is one week. At each week $\tau$, a fraction $\lambda_\tau$ of the consumers randomly arrive. Each consumer makes an adoption decision by comparing the value from buying to that from waiting for a price drop. If she makes a purchase, consumers can receive some information about the product beforehand, which is captured in the initial signal in the model.
she quits the market and starts using the product in a way described above. If she does not make a purchase, she comes back to the market in the following week and makes the decision again. I assume that the product is available for 52 weeks after the release date, and hence waiting beyond 52 periods generates zero payoff. A new version of the game is released at week 52, when the sales of older version essentially end.

The value function associated with the optimal stopping problem at week $\tau$ is

$$
V_{ip}(\Omega_{i1}, p_{\tau}) = \mathbb{E}[\max\{V(\Omega_{i1}) - \eta_ip_{\tau} + \epsilon_{i\tau}\sigma_p, \beta V_{ip}(\Omega_{i1}, p_{\tau+1}) + \epsilon_0\sigma_p\}],
$$

where $p_{\tau}$ is the current price. If the consumer buys the product, she receives the value $V(\Omega_{i1})$ and pays $p_{\tau}$. If she does not buy at week $\tau$, she receives a continuation payoff of staying in the market $V_{ip}(\Omega_{i1}, p_{\tau+1})$. I assume perfect foresight for the future prices, $\epsilon_{i\tau}\sigma_p$ is i.i.d, and follows type 1 extreme value distribution with variance $\sigma^2_p$. I do not model social learning, and hence the value from adoption $V(\Omega_{i1})$ remains constant over time. Hence, the incentive to delay the adoption solely comes from lower prices in the future. I assume that $\eta_i$ follows log-normal distribution with mean $\mu_{\eta}$ and variance $\sigma^2_{\eta}$. For simplicity, I assume that $\eta_i$ is independent from $\theta_i$. The probability that consumer $i$ adopts at week $\tau$ follows the logit form.

$$
P_{ip}(\Omega_{i1}, p_{\tau}) = \frac{\exp\left(\frac{1}{\sigma_p}(V(\Omega_{i1}) - \eta_ip_{\tau})\right)}{\exp\left(\frac{1}{\sigma_p}(V(\Omega_{i1}) - \eta_ip_{\tau})\right) + \exp\left(\frac{\beta}{\sigma_p}V_{ip}(\Omega_{i1}, p_{\tau+1})\right)}.
$$

The model is solvable by backward induction. The consumer’s willingness to pay for the product under no trial is defined by $\frac{V(\Omega_{i1})}{\eta_i}$: the value of the product measured in dollars.

---

48 Assuming instead that consumers form expectations based on the prices of other game titles hardly changes the result. As in Figure ??, the prices follow a quite typical path and it is easy for consumers to forecast price patterns.
4.3 Multiple segments

In addition to heterogeneities with respect to the true match value \( \theta_t \), the belief \( b_{t} \) and the price coefficient \( \eta_t \) described earlier, I allow for the existence of multiple segments \( r = \{1, 2, \ldots, R\} \) with different population-level parameters. In particular, I allow the vector of mean match value \( \mu \) and the variance of utility shock in the mode choice \( \sigma_\epsilon \) to be heterogeneous. I denote segment-specific parameters with subscript \( r \). I also let the variance of the initial belief be heterogeneous, denoted by \( \Sigma_1 = \kappa_r (\Sigma - \Sigma_0 (\Sigma + \tilde{\Sigma})^{-1} \Sigma) \) with \( \kappa_1 = 1 \). Introducing multiple segments allows for more flexible representation of consumer heterogeneity; heterogeneity in \( \mu \) allows for the existence of ex-ante heavy and light user segments, and heterogeneity in \( \sigma_\epsilon \) and \( \Sigma_1 \) adds flexibility in fitting game mode selection of users with different usage intensity. The probability that each user belongs to segment \( r \) is denoted by \( \xi_r \).

5 Estimation and results

5.1 Estimation strategy

I estimate the model using simulated method of moments. Given a set of candidate parameters, I first solve the model by backward induction. In order to account for continuous state space, I use the discretization and interpolation proposed by Keane and Wolpin (1994). I then simulate sequences of actions according to the optimal policy predicted by the model; I draw a set of true match values, initial signals, and post-session signals and record the predicted actions. The set of simulated users serves as pseudo-data. The parameters are estimated such that the pseudo-data obtained this way match most closely with the real data, according to pre-selected moments. Formally, for a vector of parameters \( \theta \) the estimator \( \hat{\theta} \) is obtained by the following minimization
\[
\hat{\theta} = \arg \min_{\theta} m_k(\theta)'\hat{V}^{-1}m_k(\theta),
\]

where \( m_k(\theta) \) is a vector, with rows containing the difference between the data and model moments. \( \hat{V} \) is a weighting matrix. 49

The set of moments is selected to follow my identification strategy. In my model, learning is identified from how variance of actions across users evolves across states; the existence of learning results in consumers’ beliefs following a stochastic process due to stochastic signal realizations. Moreover, the process is convergent to consumers’ true match value, and hence is nonstationary. This implies that the variance of the beliefs across consumers evolves in a nonstationary way, which is reflected in the nonstationary evolution of the variance of actions. I include the variance of session durations at each state as a moment to capture this. On the other hand, the other forms of state dependence, such as skill acquisition and boredom, are identified by how the average actions across users evolve across states. Hence the moments include the average session duration at each state. The set of other moments used and the identification of other model parameters are identical to Chapter 1, so interested readers are encouraged to refer to Section 5.1 and Appendix D of Chapter 1 for details. I also control for the sample selection problem in the same way as in Chapter 1. That is, I only observe usage decisions of those who purchased the game. In order to deal with this issue, when I calculate moments describing usage decisions I condition them on the set of consumers who have purchased the product.

\( c(\nu_{imt}), \lambda(\Omega_{it}) \) and \( \delta(\Omega_{it}) \) are specified as a quadratic function with respect to the number of past sessions, whose coefficients can vary across users with different match values. I assume

49 As a weighting matrix, I use a diagonal matrix, whose \( \{k,k\} \) element corresponds to the inverse of the mean of the sample moment. This works to equalize the scale of the moments by inflating the moments that have a smaller scale (e.g. choice probability) while suppressing the moments with a larger scale (e.g. interval length between sessions).
that the variance of the initial signal is proportional to the variance of the true type; $\sigma^2_{\tilde{m}_n} = \kappa \sigma^2_{m}$. 

Also, without loss of generality I normalize the average match value of segment 1 consumers with game mode 3 to 30, and define other parameters relative to it. Regarding the number of discrete segments, I assume $R = 2$. The consumers’ arrival process $\lambda_{p}^{a}$ is specified as a uniform arrival rate $\lambda_{u}^{a}$ and the initial mass of arrival at the release date $\lambda_{0}^{a}$. I assume that the timing of arrival is independent from the location of initial beliefs. Assuming that $N$ potential consumers exist in the market, I can normalize $\lambda_{0}^{a} = 1$ and estimate only $\lambda_{u}^{a}$ as the rate of arrival in the later weeks relative to the initial week. Market size $N$ is calibrated outside the model and it is equal to the installed base of consoles, multiplied by the share of sports games among all the game sales.

5.2 Estimation Results

In this section, I first present the estimated parameters, and describe the estimated demand side factors that influence the profitability of outright sale and subscription. In Table 5, I present selected parameter estimates of the usage model. The standard errors are simulated using 1,000 sets of bootstrapped data, each of which is obtained by randomly re-sampling users from the original data with replacement. In Table 5a, I show the estimates of the parameters common across all users. The coefficient of risk aversion is $0.215 < 1$, indicating significant risk aversion. The standard error of the match value distribution is reasonably high, indicating that consumers do face uncertainty about their own match value. In Table 5b, I present segment-specific parameters. Each of the two discrete segments respectively captures the behavior of light users and heavy users. All the parameters for segment 2 are inflated to capture the large gap of usage intensity between light and heavy users.\[50\]

I provide an extensive discussion of model fit and the identified usage patterns in Section 50. While the estimated magnitude of initial uncertainty that segment 2 faces is disproportionately high, this merely reflects the tight curvature of the utility due to small $\rho$. Since the flow utility is quite flat at a high match value, in order to capture the fact that the uncertainty also reduces heavy users’ initial utility, the variance of the belief needs to be magnified accordingly.
Table 5: Parameter estimates

(a) Common parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimates</th>
<th>Std.error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk aversion $\rho$</td>
<td>0.215</td>
<td>0.034</td>
</tr>
<tr>
<td>Holiday effect $\alpha$</td>
<td>1.396</td>
<td>0.023</td>
</tr>
</tbody>
</table>

Distribution of match value

| Std.errors $\sigma_1$      | 69.149    | 0.665     |
| $\sigma_2$                 | 83.374    | 2.464     |
| $\sigma_3$                 | 35.881    | 3.513     |
| $\sigma_4$                 | 64.096    | 2.145     |

| Correlations $\rho_{12}$   | 0.510     | 0.059     |
| $\rho_{13}$                | 0.262     | 0.093     |
| $\rho_{14}$                | 0.588     | 0.035     |
| $\rho_{23}$                | 0.207     | 0.141     |
| $\rho_{24}$                | 0.515     | 0.050     |
| $\rho_{34}$                | 0.541     | 0.069     |

Initial signal var $\kappa$ | 0.323     | 0.095     |

Post-session signal s.e $\sigma_e$ | 28.497 | 0.737 |

(b) Segment-specific parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Segment 1</th>
<th>Segment 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean match value $\mu_{11}$</td>
<td>24.062</td>
<td>107.106</td>
</tr>
<tr>
<td>$\mu_{21}$</td>
<td>33.844</td>
<td>95.780</td>
</tr>
<tr>
<td>$\mu_{31}$</td>
<td>30</td>
<td>103.449</td>
</tr>
<tr>
<td>$\mu_{41}$</td>
<td>32.089</td>
<td>97.070</td>
</tr>
<tr>
<td>Initial uncertainty $k_1$</td>
<td>1</td>
<td>7.758</td>
</tr>
<tr>
<td>Logit shock s.e $\sigma_{e1}$</td>
<td>2326.129</td>
<td>3227.733</td>
</tr>
<tr>
<td>Proportion of seg 1 $\xi_1$</td>
<td>0.578</td>
<td>214.038</td>
</tr>
</tbody>
</table>

Note: $\mu_{11}$ and $k_1$ are normalized. Standard error is calculated by 1,000 bootstrap simulations.
6.2 of Chapter 1. Hence in this section I present extra sets of figures relevant to the question of product bundling. In Figure 23 I evaluate the correlations of utility across sessions. The estimated correlation is around 0.7: the range where previous studies found either bundling and unbundling can be optimal depending on other factors (Schmalensee 1984). Positive correlation implies that consumers who receive higher utility from their initial session tend to also receive higher utility from the following sessions. In other words, the utility of high value consumers decays more slowly. The correlation of utility is higher than that of the actions shown in Figure 20. This indicates that actions at each period are greatly influenced by idiosyncratic factors: a finding consistent with relatively large standard error of the logit shock in Table 5. The gradual decline in correlation is partly due to learning. Conditional on the initial perceived utility, utility in the later periods evolves randomly according to stochastic arrival of the signals.

![Figure 23: Correlation of utility across sessions](image)

Note: Each point on the line corresponds to the correlation between the flow utility of a consumer in the first session, averaged across modes, and that in the $t$-th session. The correlation is calculated using 50,000 simulated sequence of actions.

In Figure 24 I show how flow utility from each game mode evolves as users accumulate usage.

---

51 See also Figure 8 in Chapter 1.
experience. Because of risk aversion and significant initial uncertainty, flow utility from the initial session is lower than its long-term counterpart. Consumers face especially large uncertainty for game mode 2 with the largest variance in the initial belief. As a result, utility from mode 2 increases by as much as 43 percent within the first 10 sessions. On the other hand, mode 3 comes with small uncertainty and utility increase is modest around 12 percent. On average across modes, flow utility increases by 31 percent in the first 10 sessions. See also Figure 15 (b) in Chapter 1, where I show that such increase in flow utility results in 13.7 percent increase in the value function.\footnote{The increase in the value function, which is the sum of the discounted future utility, is smaller than that of the flow utility. This is because as consumers play more sessions, the number of remaining sessions decreases, reducing the value of future utility by construction.} This significant increase favors the firm’s adopting subscription strategy, as it allows the firm to increase the price in response to the change in the average willingness to pay.

In Figure 25, I compare the distribution of utility between the first session and the 10th session. Besides the average increase in utility discussed above, the distribution remains stable over time.\footnote{Note that the distribution is not assumed to be stable; depending on the magnitude of prior variance, signal}

---

\textbf{Figure 24: Evolution of flow utility}

Note: Each line represents the flow utility at each session for each game mode, average across consumers. The flow utility is scaled relative to that of game mode 3 at the initial session. The average is calculated using 50,000 simulated sequence of actions.
Figure 25: Distributions of flow utility

Note: The distributions are calculated using 50,000 sequences of simulated actions. The distribution of utility at session \( t \) includes all sequences that remain active at that \( t \).

to be bundled showing similar variance of utility across consumers (Fang and Norman 2006).

Taken together, I find that preference correlations across components are positive but sufficiently lower than one, and preference distributions are stable across components. Along with zero marginal cost, these facts appear to favor outright sale. However, large consumer uncertainty and associated significant initial utility reduction strongly favor subscription strategy. I now turn to the counterfactual exercise where I evaluate how these factors shape the trade-offs that the firm faces in adopting either strategy.

6 The profitability of subscription strategies

Using the estimated parameters, in this section I conduct counterfactual exercises where I compare the profit from the optimal outright sale and various subscription strategies. Specifically, I consider subscription strategies that take the form of “X-session package”; each subscription distribution and state dependence, the distribution of utility can vary significantly at other parameter values.
period is tied with the number of sessions that consumers can play, and consumers make active
decisions to renew the subscription at the end of each subscription period. Tying the subscrip-
tion period with the number of playable sessions is a common approach among online game
providers. In the Appendix, I discuss why focusing on this type of subscription strategies al-
 lows us to eliminate contaminations from factors outside of the model in examining the effect of
consumer learning.

The first subscription strategy I consider is pay-per-use. Under pay-per-use, consumers pay
a small fee for every session that they play, which is equivalent to their making subscription
renewal decisions at every session. I also assume that the firm can flexibly change prices across
sessions. While this assumption may not accompany much realism, it allows us to highlight the
trade-offs that the firm faces due to consumer learning. The second and third strategies that I
consider aim to replicate more realistic environments, where the firm may not have an ability
to change prices at a single session level. The second strategy is partial unbundling, where each
subscription period contains more than one sessions. The third strategy is another pay-per-use,
but with an extra constraint that the firm can change price only once. The firm sets one price
over the initial several sessions, and switch to the other price for the rest of the time. In order
to see the role that consumer learning plays in determining the profitability of each strategy, I
solve the firm’s problem both under consumer learning, and also under a hypothetical situation
where I turn off consumer learning.54

6.1 Pay-per-use strategy

The demand model under pay-per-use When the firm adopts a pay-per-use strategy,
the firm sets a sequence of prices \( p = \{p_1, p_2, \ldots, p_t, \ldots\} \), where \( p_t \) is the price for session \( t \).

Upon arrival at the market, consumers see the whole price schedule \( p \) and make the subscription

54Specifically, I assume that the variance of the initial signal is zero, leaving other parameters unchanged.
decision by comparing the value from subscribing and the initial price $p_1$. Once having subscribed, consumers make renewal decisions at the beginning of every session thereafter, facing price $p_t$. Consumers’ initial enrollment decision is given by the following maximization problem.

$$V^{ppu}_p(\Omega_{i1}, p) = \mathbb{E}[\max \{ V^{ppu}(\Omega_{i1}, p) - \eta_i p_1 + \epsilon_1 \sigma_1, \epsilon_2 \sigma_2 \}],$$

where $V^{ppu}(\Omega_{i1}, p)$ is the value of enrolling in the subscription under the sequence of price $p$ and state $\Omega_{i1}$. Outside option of not subscribing is normalized at zero. Other variables and parameters are identical to the case of outright sale presented in Section 3. The value from subscribing, $V^{ppu}(\Omega_{i1}, p)$, is determined by her subsequent usage and renewal decisions. In the first session, each consumer chooses the game mode to play, knowing that she is given an opportunity to renew the subscription at the beginning of the next session.

$$V^{ppu}(\Omega_{i1}, p) = \mathbb{E}[\max_{m_{i1}} v(b_{i1}, \nu_{im1}, h_1) + \mathbb{E}[\beta(\Omega_{i2}) V^{ppu}_p(\Omega_{i2}, p) | \Omega_{i1}, m_{i1}] + \epsilon_{im1} \sigma_1],$$

In the same way as in the benchmark model in Section 3, the optimal session duration is subsumed in $v(b_{i1}, \nu_{im1}, h_1)$, and the optimal session interval and termination is subsumed in $\beta(\Omega_{i2})$.

After the initial session, the consumer alternately makes renewal decisions and play decisions until she chooses to terminate the subscription. The consumer’s value functions at $t > 1$ are hence defined similarly as follows.

$$V^{ppu}_p(\Omega_{it}, p) = \mathbb{E}[\max_{m_{it}} \{ V^{ppu}(\Omega_{it}, p) - \eta_i p_t + \epsilon_{1t} \sigma_1, \epsilon_{2t} \sigma_2 \}],$$

$$V^{ppu}(\Omega_{it}, p) = \mathbb{E}[\max_{m_{it}} v(b_{it}, \nu_{imt}, h_t) + \mathbb{E}[\beta(\Omega_{i,t+1}) V^{ppu}_p(\Omega_{i,t+1}, p) | \Omega_{it}, m_{it}] + \epsilon_{imt} \sigma_1].$$

55This formulation takes the form of optimal stopping. Hence once terminating, each consumer never re-enroll in the subscription. This assumption is reasonable in the current environment where I rarely see consumers being inactive for a prolonged time play the game again in the future.
The solution to the dynamic programming problem provides the probability that for a given sequence of price $p$, a consumer with a belief and play history $\Omega_{it}$ renews her subscription at price $p_t$. By taking its integral over the unobservable belief $b_{it}$, we have demand for each session $t$ as a function of the vector of prices $p$, with which we can calculate the optimal price under pay-per-use strategy.

Simulation Results Under Consumer Learning Since the aggregate demand has no analytical form, I compute firm revenues using 50,000 sequences of simulated actions of consumers. Implementing the simulation exercise requires several assumptions. First, in the case of pay-per-use, the price that each consumer faces at a given point is tied with the number of sessions she has played in the past, and not with the calendar day. Hence, there is no notion of “waiting for future price drop”\textsuperscript{56} In order to compare the profitability of outright sale and subscription under equal setting, I calculate the profit from outright sale assuming that the firm only set one price and consumers visit purchase occasion only once; no incentive to wait for price drops exists. Specifically, I use $p = 52.1$ as the price of outright sale strategy, which is the price that the firm sets at the launch week. Second, comparison has to be made between the optimal subscription pricing and the \textit{optimal} outright sale pricing. I have not used the supply-side moments in estimating the model, so there is no guarantee that $p = 52.1$ is the optimal outright price. Hence, I perturb the distribution of price coefficient from the estimated one, such that $p = 52.1$ is the optimal outright price\textsuperscript{57} I then calculate the optimal subscription pricing under that distribution of price coefficient. Admittedly, this is a compromised approach to correct for the mismatch between the model prediction and the reality in order to use the model I estimated in Chapter 1 to conduct the current exercises. In principle, such adjustments should be made at

\textsuperscript{56}This does not mean that the firm exercises targeted prices. Conditional on having played $t$ sessions in the past, every consumer faces the same price $p_{t+1}$.

\textsuperscript{57}I assume that the distribution of price coefficient is truncated below, and I selected the point of truncation.
the estimation stage by introducing other parameters and supply-side moments. Finally, I set 50 as the maximum number of sessions consumers may play. This means that under subscription strategy, the firm sets a vector of 50 prices. In order to make consumer willingness to pay comparable between the two strategies, I also assume that under outright sale strategy, each consumer compares her value from 50 sessions and the price in making the adoption decision.

![Figure 26: The optimal prices and retention patterns (learning)](image)

Note: The solid lines correspond to the prices and the retention rates under subscription, and the dashed lines correspond to those under outright sale. The retention rates are simulated using 50,000 sequences of consumer decisions.

In Figure 26, I compare the prices and retention rates between outright sale and subscription. In the case of outright sale, consumers pay \( p = 52.1 \) at the beginning and that is the only transaction made. In the context of Figure 26, it is equivalent to consumers’ paying per-session price of \( 52.1/50 = 1.042 \) and keep their subscription until the end. Hence the prices and retention rates can be depicted by a flat line. The optimal prices under subscription follow an increasing pattern with initial price at 33 cents and the last price at 97 cents. As a result of low initial price, the number of initial adopters is higher by 198 percent compared to outright sale. Henceforth, I call the set of consumers who adopt the product only under subscription scheme “subscription.

\(^{58}79.6\) percent of consumers in the data stop playing within the first 50 sessions.
adopter's. When consumers face large initial uncertainty, setting an increasing price path helps increase revenue through two channels. First, it allows firms to adjust price as average willingness to pay in the market increases, as described in Figure 24. Second, some of the consumers with high true match value with the product may have low belief at the beginning. By setting low prices in early periods, the firm can let them learn their true type and increase their willingness to pay significantly. The firm then exploits their increased surplus through higher prices in later periods. In general, offering subscription allows the firm to gain extra revenue from the set of consumers with low initial willingness to pay.

This revenue increase is partly offset by the decline of revenue from consumers who also make purchase under outright scheme (henceforth called “outright adopters”). Note that subscription price is always below the per-session outright price in Figure 26. Conditional on remaining active until the 50th session, each consumer pays $\sum p_t = 37.3$ dollars under subscription: 28 percent discount compared to the outright price. Moreover, some of them terminate the subscription early and pay even less. As a result, the revenue from outright adopters under subscription is lower by 31 percent. Combining these effects together, I find that subscription strategy achieves 2.9 percent increase in revenue compared to outright sale in this environment; thanks to learning, unbundling outperforms bundling.

In Figure 27, I show how consumers change their adoption patterns in response to different sales strategies. The histogram covers consumers with willingness to pay under outright sale between 10 cents and 100 dollars, which constitutes 22.2 percent of the whole market. Vast majority of consumers outside this range have willingness to pay less than 10 cents and are considered as never-buyers. Subscription adopters include the ones with fairly low willingness to pay; the initial subscription is priced at only 0.33 cents and consumers adopt when the sum of flow utility and future option value is larger than that. On the other hand, some outright
Figure 27: Consumers’ reactions to outright sale and pay-per-use (learning)

Note: Each colored area corresponds to consumers with different behavior according to different sales strategy. Starting from the segment located at the bottom, those who never adopt regardless; those who do not adopt under outright sale, but play at least one session under pay-per-use; those who adopt under outright sale, but play less than 50 sessions under pay-per-use; and those who adopt under outright sale and also play full 50 sessions under pay-per-use. The distribution is calculated using 50,000 sequences of consumer decisions.

Adopters terminate early under subscription. Their willingness to pay is located on the lower end of outright adopters, indicating that in the long-term, their flow utility is more likely to become low enough not to renew subscription. In addition, even a small decrease in their perceived utility due to learning may result in their churning.

In Figure 28 I illustrate the distribution of subscription retention length for subscription adopters and outright adopters. Most of outright adopters keep their subscription until the end, though some start terminating around the 10th session. On the other hand, those who are acquired due to low initial prices start to drop out as the price rises at the 7th session. Nevertheless, 12 percent of subscription adopters remain active until the last session; because of the large uncertainty involved in the consumers’ initial beliefs, some subscription adopters have high match values with the product. This is the segment that serves as a primary source of revenue increase in employing the subscription strategy.
Effect of consumer learning on the optimal policy  I now turn to evaluate how much of the policy implication displayed above is due to the existence of consumer learning. In order to see this, I conduct another policy simulation where I assume that consumers face no uncertainty. I find that when no learning exists, revenue under pay-per-use is lower by 7.8 percent compared to outright scheme; in the absence of learning, unbundling is not justified. In Figure 29 I compare the optimal pay-per-use price sequence and resultant retention rates under learning (solid lines) and no learning (dashed lines). The solid lines are identical to Figure 26. Two things are worth noting. First, optimal price sequence under no learning is still increasing due to selection through survival. Comparing the rate of price increase between the two environments, I find that 49.9 percent of price increase under learning is attributable to firm’s reacting to consumer learning, and remaining 50.1 percent is due to selection of active consumers. Hence, the impact of consumer learning on the optimal pricing is quantitatively substantial. Second, in the long term, the optimal prices and retention rates under learning appear to converge to those of no learning. This is reasonable given that aside from the existence of uncertainty, the two

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59Because I shut down learning, optimal outright price also changes from the observed $p = 52.1$. I find that under no learning, outright price of $p = 58.1$ maximizes the revenue, against which I compare the profitability of pay-per-use.
environments are identical. Note that the identical long-term retention rates between the two environments imply that even though consumers misunderstand their true match value under the environment of learning, the optimal price sequence is able to attract all consumers who would adopt had uncertainty not existed. In other words, the price sequence eliminates welfare losses due to consumers with high true match values misunderstanding their preference and not adopting.

Figure 30 replicates Figure 27 under the no learning environment: consumers’ adoption patterns under different sales strategies. Due to the high initial subscription price, fewer subscription adopters emerge compared to the case of learning. In the meantime, higher price implies that the loss from outright adopters is also small. With the absence of learning, this figure is similar in spirit to Figure 7 of Adams and Yellen (1976) or Figure 2 of McAfee, McMillan and Whinston (1989). The trade-offs between the extra revenue from subscription adopters and lost revenue from outright adopters depend on preference correlation across sessions. If the preference cor-
relation is low, higher utility from initial session implies lower utility from later sessions, and hence more outright adopters terminate early under subscription, reducing revenue compared to outright sale. Negative correlation also reduces revenue from subscription adopters; negative correlation implies those whose long-term utility remains high tend to have lower initial utility, and vice versa. Hence, as correlation lowers, those who subscribe initially tend to terminate early, and those who tend to keep subscription longer become less likely to subscribe initially. At the estimated parameter values with preference correlation around 0.7, outright sale outperforms subscription.

6.2 Partial unbundling

Now I consider partial unbundling, where each subscription period consists of \( \tilde{t} > 1 \) sessions. This exercise provides an environment closer to the real world practice, where consumers may use the product multiple times during a single subscription period. The model of partial unbundling is a straightforward extension of the model of pay-per-use, where consumers visit a purchase occasion every \( \tilde{t} \) sessions. In Figure 31, I compare the optimal pricing and the associated adoption rates.
under $\tilde{t} = 5$ with the ones under optimal pay-per-use. Because consumers make renewal decisions only once in five sessions, the retention rate is flat during each five-session period. The optimal per-session price under partial unbundling – the subscription price divided by five – is noticeably higher than the optimal pay-per-use price. This is because the return from offering a lower price is limited; for example, even when the firm sets the per-session price equal to the optimal initial pay-per-use price, the demand under partial unbundling is lower, for consumers are required to make a longer-term commitment than the pay-per-use case. Given the higher initial price, those who still adopt are the ones with higher perceived match value: the selection that results in higher prices in the long-term too.

I find that under the optimal pricing, the firm revenue increases by 2.6 percent compared to outright sale. In other words, while partial unbundling still increases revenue, it does not outperform pay-per-use. I also find this tendency under no learning environment; when there is no learning, partial unbundling reduces revenue by 5.2 percent compared to outright sale. Hence, in both cases the revenue from partial unbundle lies between that from pay-per-use and that from outright sale. This result appears to be consistent with the theoretical findings that in the case of multi-product bundling, either pure bundle or pure unbundle tends to be optimal and not partial bundle (Fang and Norman 2006).

6.3 Pay-per-use with two-price constraint

I next consider another pay-per-use environment, where the firm can change price only once during the entire horizon (50 periods). In other words, the firm only chooses one price for initial sessions, another price for the rest of the time, and when the price switches from one to the other. This exercise again aims to provide an environment similar to the reality, where the firm may not have an ability to charge different pay-per-use prices across sessions. In reality,

\footnote{Studying other $\tilde{t}$ is left as a future research.}
Note: The solid lines correspond to the prices and the retention rates under pay-per-use, and the dashed lines correspond to those under partial unbundling. Per-session price under partial unbundling is calculated by dividing the optimal price for each subscription period by five. The retention rates are simulated using 50,000 sequences of consumer decisions.

majority of changes in subscription prices take the form that the firm offers discounted prices for the first several subscription periods, which this exercise replicates. Figure 32 shows the comparison between the optimal two-price strategy and pay-per-use under flexible pricing. I find that the optimal two-price strategy involves a starting price similar to the flexible pricing case, and the increase to the long-term level at the 22nd session. As is evident from the figure, the firm maintains the low price for a longer time compared to the flexible pricing case. On the other hand, the price set over the later sessions, 89 cents, is close to the average price during that period under flexible pricing, 91 cents. I find that under the best two-price strategy, the firm revenue decreases by 1.5 percent compared to the outright sale strategy; even when consumer learning exists, if the firm has limited flexibility of changing price over time, subscription may not increase revenue.\textsuperscript{61}

\textsuperscript{61}Note that although this setup appears to be a general case of time-locked trial that I study in Chapter 1, here the revenue decreases while the best time-locked trial increases revenue. This is because this counterfactual is not a pure generalization of time-locked trials. In order for an environment to be a generalization of time-locked trials,
Figure 32: Comparison between two-price and flexible pricing pay-per-use (learning)

Note: The solid lines correspond to the prices and the retention rates under pay-per-use with flexible pricing, and the dashed lines correspond to those under two-price pay-per-use. The retention rates are simulated using 50,000 sequences of consumer decisions.

Overall, the counterfactual exercises indicate that in the current environment, the existence of learning indeed favors product unbundling, in that it flips the firm’s optimal strategy compared to the case of no learning. Specifically, subscription scheme is advantageous under consumer learning because (1) consumer willingness to pay tends to increase as uncertainty diminishes, as described in Figure 24, and (2) the firm has an ability to change prices over time, in order to attract consumers with low initial perceived utility, some of whom have high true match value. The firm later exploits surplus from high match value consumers. This is described in Figure 26. Moreover, major part of that revenue increase comes from the firm’s ability to change prices in accordance with the increase in willingness to pay. Partial unbundling where the firm sets 10 prices achieves 89 percent of the revenue increase compared to that of fully flexible pay-per-use with 50 prices. However, if the firm can set only two prices, the revenue cannot increase. Hence, we need to have the firm offering two bundles, one consisting of the access to the product in early periods and the other one in later periods, and consumers make adoption decisions only twice. As the current environment is pay-per-use, consumers can freely terminate the subscription after any session. Hence, consumers have much larger flexibility in the current environment than in the case of time-locked trials.
being able to change the price occasionally is a key to the success in implementing unbundling strategies.

It is worth noting that these factors that influence profitability of bundling resemble the ones that influence the profitability of time-locked trials that I study in Chapter 1. Indeed, the time-locked trial is another special case of unbundling strategies, where the firm sells the first five sessions as a bundle with zero price, and bundle all the remaining sessions as one and sells it with the full price. In Chapter 1, I compare the special case of intertemporal unbundling (time-locked trial) with another special case of cross-feature unbundling (feature-limited trial). In this chapter, I consider a general form of intertemporal unbundling (subscription), while cross-feature domain remains fixed. This difference in the focus allows us to answer to different policy questions.

7 Conclusion

The choice between outright sale and subscription can be framed as a problem of optimal product bundling. In particular, in this study I shed light on a new demand-side factor that favors subscription strategy: consumer learning. I build and estimate a model of digital goods adoption under uncertainty. Based on the estimated parameters, I find that in the environment of a major sport videogame, learning has a substantial implication for the firm’s optimal strategy. In the presence of learning the firm is better off by offering subscription, while in the absence of learning outright sale performs better. The causes of such flip in the optimal strategy are twofold. First, due to risk aversion, consumer willingness to pay tends to increase over time as uncertainty diminishes. Second, the firm has an ability to change prices over time, in order to attract consumers with low initial perceived utility, some of whom have high true match value, and later exploit surplus from high match value consumers.

I focus on digital goods environment in this study, for the move from outright sale to sub-
scription is most pronounced in that category. However, my framework is extendable to other situations where consumer learning is present and the firm has an option to adopt some forms of product unbundling. My model is directly applicable to other environments as long as the environment of interest satisfies (1) consumers use only part of the bundled components at a time, and (2) consumption experience of a component is informative about their match values with other components.

This study focuses on the informational factors that influence the profitability of subscription strategy. As a result, I assume that aside from informational difference, consumers make adoption decisions following the identical rule between the two cases; they adopt if their expected utility from the product exceeds the price. In practice, it is well known that the nature of adoption decisions may substantially differ (Scott 1976). In such cases, my study can be considered as suggesting a bound of possible revenue impact. If consumers’ psychic cost of adoption is lower under the subscription environment, then my counterfactual implication provides lower bound of the revenue increase. Moreover, there are several key restrictions that I impose on the model that are not testable. For example, Bayesian learning under Normal distribution forces the uncertainty to monotonically decrease over time. An extension to allow for nonmonotone evolution of the variance of the belief makes the model more suitable to study other environments. For example, consumers may be confused by the difference between their prior expectation and the signal they receive from their actual usage. Finally, because digital goods are durable, the firm also has an incentive to price discriminate across consumers based on their arrival times (Stokey 1979). The combination of product bundling and intertemporal price discrimination is an interesting area where no empirical studies exist. Explicitly taking into account such other factors that influence consumers’ adoption decisions under subscription environment is left as a future study.
Chapter 3:

Nonparametric Identification of State Dependence and Unobserved Heterogeneity

1 Introduction

Consumers’ current product choice is often influenced by their past choices, commonly referred to as “state dependence”. For example, consumers may have inertia and tend to stick to one product (Howard and Sheth 1969), or they may have love for variety and tend to switch products (McAlister 1982). Alternatively, consumers may learn their true preference for the product through consumption experience (Erdem and Keane 1996). In order to implement an appropriate policy, policy makers and marketers need to identify the nature of state dependence. If state dependence increases subsequent retention rate, marketing activities to attract first-time consumers are effective. On the other hand, if it decreases customer retention, the firm may reward returning consumers, in order to discourage switching. However, identifying state dependence is challenging when there exists consumer heterogeneity unobservable to a researcher. An observation that a consumer often chooses option X may result from either her past experience with X causing her to repeat, or her having an unobservable preference toward X. It is known that state dependence and unobserved heterogeneity are not nonparametrically separable even when panel data is available (Heckman 1981, Chamberlain 1984). In order to establish separate identification, one needs to impose some a priori restrictions on the model.
In this paper, I provide sufficient conditions to nonparametrically identify state dependence and unobserved heterogeneity in an environment where consumers choose one option from discrete alternatives. Specifically, I allow for arbitrary distribution of unobservable consumer types, and utility of each type of consumers at each state can take any value. Understanding conditions for nonparametric identification helps empirical modelers in choosing their modeling assumptions.

For example, in order to avoid creating spurious state dependence, one may want to allow for a flexible form of unobserved heterogeneity in her model (Dubé, Hitsch and Rossi 2010, Shin, Misra and Horsky 2012). This study offers guidelines about the extent of flexibility one can allow for without causing identification issues.

I first show that a commonly used assumption of one-period dependence – the state only consists of the action taken in the previous period and not the whole history of actions – is sufficient for nonparametric identification. The implication of one-period dependence for nonparametric identification has been left unaddressed, for existing studies adopting this assumption tend to also impose other parametric assumptions to seek for parametric identification. The result that one-period dependence alone is sufficient for identification implies that any other parametric assumptions imposed in existing studies can be relaxed to accommodate greater flexibility. The intuition is as follows. In general, in order to identify utility from each option at each state realization, we need to identify the distribution of actions at that state; the frequency that each option is selected nonparametrically identifies the relative utility of that option. Under a general form of unobserved heterogeneity, different consumers are potentially of different type, and hence we cannot pool observations across consumers. It follows that we need to identify the distribution

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62 In order to establish identification, existing studies impose various forms of parametric and nonparametric assumptions. For example, studies regarding state dependence tend to assume that the utility of a consumer in the current period is only influenced by the actions that she took in the previous period, and not the whole history of actions (Keane 1997, Seetharaman, Ainslie and Chintagunta 1999, Ackerberg 2001); studies of consumer learning typically assume that one’s consumption experience influences her current utility through a particular sufficient statistic indicated by Bayesian learning framework (Erdem and Keane 1996, Ackerberg 2003); heterogeneity is usually assumed to follow a particular parametric distribution, such as discrete or normal (Erdem, Keane and Sun 2008, Osborne 2011).
of each consumer’s actions at each state. The assumption of one-period dependence ensures that it is the case. Specifically, it ensures that the state follows a positive recurrent process; every consumer returns to every state realization in finite time. Positive recurrence implies that each consumer visits every state infinitely often during a given infinite sequence of actions, allowing us to identify the distribution of actions of every consumer at every state.

I then turn to an environment where utility depends on the entire history of actions, such as consumers’ skill acquisition and boredom. The state variable that represents the entire history necessarily follows a transient process; each consumer visits each state at most once during her infinite sequence of actions. Nonparametric identification of models with transient states is a challenge for two reasons. First, we cannot identify the distribution of actions of each consumer at each state, for we only observe one action that each consumer takes at each state realization. Second, there may exist states that some consumers never visit during their sequence of actions. At those states, we have no information to identify those consumers’ utility based on. Thus, in order to establish identification, we need to either impose some restriction on how the utility evolves across transient states, so that we use information at one state to help identify utility at another, or on the distribution of heterogeneity, so that we use information from one consumer to identify utility of another consumer.

Instead of deriving a set of restrictive assumptions, I propose an alternative approach to establish nonparametric identification under weak restrictions. I utilize the fact that in many situations where discrete choice models are used, each discrete choice by a consumer is accompanied by a continuous choice of consumption intensity of the selected option; consumers may buy pre-packaged products by selecting a brand and also the amount of purchase; they may select cell-phone plan and choose duration of calls; they may choose a TV channel and also duration to watch. As a result, researchers observe both the selected alternative and its consumption
intensity at each decision occasion.

I provide sufficient conditions to identify both transient forms of state dependence and unobserved heterogeneity using observations of discrete-continuous actions. The key assumption is that consumers choose both discrete and continuous actions to maximize their utility, and hence there exists a monotone relationship between one’s underlying utility and selected actions; consumers who value an option more are more likely to choose that option and also choose higher consumption intensity of that option. Under this assumption, I show that we can use the sequence of discrete choices to identify the set of consumers with identical type. We then obtain the distribution of the other, continuous actions of that type of consumers at each state that they visit. The identified distribution allows us to recover the utility that that type of consumers receive from the selected option at every state that they visit. Hence, even when consumers visit some state realizations only once, we can recover a subset of utility of each type of consumers.

Utility at other states are not directly recoverable due to missing observations: utility at the states that they did not visit, and that from options that they did not select at the states that they visit. It needs to be inferred from utility from other states through model restrictions. I show that one of the following conditions, along with other regularity conditions, is sufficient to recover the utility of every type at every state; (1) utility from each option only depends on the consumption history of that option, or (2) past consumptions influence current utility only through their cumulative sum, and any cross-product state dependence is additive and symmetric across products. Depending on the nature of the environment of interest, one can choose either restriction to impose on her model. The first assumption is useful when consumers choose an option from distinctively different alternatives and hence cross-product state dependence does not exist. The second assumption allows for cross-product dependence, while assuming that the state influences utility only through sufficient statistics. Using the wording above,
I establish identification by imposing restrictions on how utility evolves over transient states, while maintaining purely nonparametric form of unobserved heterogeneity.

The state dependence identified in the way described above is an aggregate effect that may result from multiple factors, such as inertia, boredom, or learning. In the second half of this paper, I turn to the question of finding the cause of state dependence. Specifically, I consider a way to separate consumer learning from other forms of state dependence. “Learning” refers to the case where consumers face uncertainty about their own type realization and that uncertainty is resolved through product experience. In other words, other forms of state dependence include any process where the state influences utility of a given, fixed type of consumers, and learning is the process where consumers’ perceived type evolves over time. Existing studies that separate the two in discrete choice environments assume either (1) learning is the only nonstationary component of the model, and other forms of state dependence are stationary (Dubé, Hitsch and Rossi 2010, Osborne 2011), or (2) there exists no heterogeneity in how other forms of state dependence influence utility (Erdem and Keane 1996, Ackerberg 2003). Under the first assumption, learning is identified by nonstationarity of the evolution of actions, while under the second assumption, it is identified from heterogeneity in how state evolution influences actions.

I show that if we observe both discrete and continuous actions at each state, we can relax both assumptions; other forms of state dependence can be nonstationary and any forms of unobserved heterogeneity are allowed. Under such a general environment, I show that conditional on a set of consumers with identical true type and history of actions, the variance of their consumption intensity evolves across states in a nonstationary way if and only if there exists learning. In other words, even when other forms of state dependence cause nonstationary evolution of the actions, the models with and without learning are distinguishable by their higher-order stationarity.

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63 In this context, one’s type realization can be interpreted as her personal valuation for the product.
This property does not provide model-free test for the existence of consumer learning. However, it is still useful in practice in that if one develops a model that assumes away consumer learning a priori, she can check validity of the assumption by testing if the estimated model satisfies the higher-order stationarity. Specifically, the test proceeds as follows. Suppose that we build and estimate a model with general forms of state dependence and unobserved heterogeneity, but assuming away consumer learning. The identification conditions provided above ensure that by taking the set of consumers who follow a particular sequence of discrete choices, we can pin down a set of consumers with identical type. Hence, using the estimated model, we can find such sequence and take the set of consumers who follow it. We then evaluate how the variance of their consumption intensity evolves. If it is stationary, then no learning exists and the model is appropriate to study the environment of interest. If it is not, then learning exists and the original model that assumes no learning is misspecified. Hence, we may reconsider a new model with consumer learning.

This study offers novel contributions to the literature concerning identification of state dependence under unobserved consumer heterogeneity in discrete choice environments. There exists empirical works that incorporate both state dependence and flexible form of heterogeneity, such as Dubé, Hitsch and Rossi (2010) and Shin, Misra and Horsky (2012). This study offers econometric justification of their approaches by showing that in their environment, adding flexible form of heterogeneity can isolate spurious state dependence without jeopardizing identification. This paper also complements existing approaches to identify consumer learning. While majority of existing studies impose either of the two assumptions discussed above, there are notable exceptions. Moshkin and Shachar (2002) assume that consumers learn about product attributes and derive identification conditions from consumers’ asymmetric responses to the changes of prod-

64 This is because in order to conduct the test, we need to pin down a set of consumers with identical true type, which is not possible without a model.
uct attributes. Similar to my study, Grennan and Town (2018) utilizes stationarity of variance of actions to identify learning. I consider identification using individual-level observations and allow for unobservable consumer heterogeneity, while they consider identification using aggregate observations. Finally, my utilizing both observations of consumers’ discrete and continuous choices resembles the strategy taken in the literature of discrete-continuous framework (Nair, Dubé and Chintagunta 2005, Narayanan, Chintagunta and Miravete 2007, Lambrecht, Seim and Skiera 2007). This study is the first that uses such observation to separate deterministic state dependence, heterogeneity and learning.

The rest of the paper is structured as follows. In section 2, I show that the assumption of one-period dependence commonly imposed in the literature overcomes the nonidentification problem when state dependence and unobserved heterogeneity coexists. In section 3, I provide an identification strategy that utilizes discrete-continuous framework in order to deal with the existence of transient states. In section 4, I provide the test for the existence of learning and suggest a way to implement it in practice. Section 5 concludes.

2 Nonparametric identification under one-period dependence

In this section, I first revisit the nonidentification results in Heckman (1981) and Chamberlain (1984); even with panel data, purely nonparametric identification of state dependence under unobserved consumer heterogeneity is not possible. I then show that the assumption of one-period dependence, where the current utility only depends on the action in the previous period, establishes identification. Throughout the section, I assume that consumers know their true type and hence assume away consumer learning.

Consider the following discrete choice environment. There exists infinitely many consumers. They are heterogeneous and each consumer $i$’s type, $\theta_i$, is distributed according to a pdf $f(\theta_i)$. 

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At each period \( t = \{1, 2, \ldots\} \) each consumer maximizes the following utility across \( M \) finite alternatives.

\[
\max_m V_m(\theta_i, h_{it}) + \epsilon_{imt},
\]

where \( h_{it} \) is a state variable that represents individual \( i \)'s history of consumption up to period \( t - 1 \). \( V_m \) represents utility from choosing option \( m \) by a consumer of type \( \theta_i \) at state \( h_{it} \). Different \( V_m \) across different \( h_{it} \) results from the state dependence. \( \epsilon_{imt} \) is an idiosyncratic utility shock. I assume that it satisfies the following property.

**Assumption 1.** \( \epsilon_{imt} \) is iid, and has strictly positive, symmetric density around zero over unbounded support.

I assume that the choice set \( M \) is the same across all states. I denote the optimal choice at each state by \( m^*(\theta_i, h_{it}) \), or \( m_{it}^* \) when its dependence on the state is apparent from the context. For example, if \( \epsilon_{imt} \) follows Type 1 Extreme Value distribution, then the choice probability takes the familiar logit form.

\[
P(m_{it}^* = m|\theta_i, h_{it}) = \frac{\exp(V_m(\theta_i, h_{it}))}{\sum_{m'} \exp(V_{m'}(\theta_i, h_{it}))}.
\]

In this general representation, \( \theta_i \) is a mere label representing that each consumer’s utility is different from one another and has no cardinal meaning. Since discrete choice framework requires normalization of utility level and scale, I assume without loss of generality that \( V_M(\theta_i, h_{it}) = 0 \) for all \( \theta_i \) and \( h_{it} \). The normalization is required at every state, for consumers do not compare options across different states.

Our objective is to identify \( V_m(\theta_i, h_{it}) \) for every \( \theta_i \) at every \( h_{it} \) and \( f(\theta_i) \). Suppose that we observe infinitely many consumers, each with infinite sequence of discrete actions \( m_{i}^{*} = \ldots \).
\{m^*_1, ..., m^*_t, ... \}. Because of the unbounded support of \( \epsilon_{int} \), the observations are not degenerate, in that for all state \( h_{it} \) and option \( m \), there exists a set of consumers who visits that state and chooses that option. Nevertheless, it is known that under the general specification above, such observation of discrete choices alone is not sufficient to identify the model.

**Proposition 1.** We cannot identify both \( V_m(\theta_i, h_{it}) \) at every \( \{\theta_i, h_{it}\} \) and \( f(\theta_i) \) from the observation of infinitely many \( m^*_i \).

This proposition is an application of general results in Heckman (1981) and Chamberlain (1984) to the current environment. The idea is that we can always assign a unique \( \theta_i \) to each set of consumer who follow each of different histories \( m^*_i \). In other words, we can call \( \theta_i \) as “the type of consumers such that they follow \( m^*_i \)”. Once the type is assigned as such, then for each \( \theta_i \), any \( V_m(\theta_i, h_{it}) \) that rationalizes their observed action \( m^*(\theta_i, h_{it}) \) at each \( h_{it} \) is not identified from one another. Thus, we cannot identify the model solely from the observation of discrete actions. Intuitively, we can always attribute all the observed variations to the realization of consumer type, leaving no variations to identify \( V_m \) for each type.

More generally, identification of \( V_m \) in a discrete choice environment requires observation of the *distribution* of choices across alternatives; the frequency that each option \( m \) is selected at state \( h_{it} \) nonparametrically identifies \( V_m \). In the current environment, the existence of a general form of unobserved heterogeneity requires to observe such distribution of actions *for each type of consumers*, which identifies \( V_m \) of that type of consumers. However, without imposing any restrictions on the model, we cannot identify the distribution of actions of every type of consumers at every state realization, causing underidentification.

In order to resolve this issue, existing studies have imposed an extra set of assumptions. The one that is predominantly used is one-period dependence of the following form.
Assumption 2.

\[ V_m(\theta_i, h_{it}) = V_m(\theta_i, m_{i,t-1}^*) = V_m(\theta_i) + \Delta V_m(\theta_i), \quad \text{if } m_{i,t-1}^* = m. \]

\[ = V_m(\theta_i) \quad \text{otherwise.} \]

In other words, utility at period \( t \) depends only on the action taken at \( t - 1, m_{i,t-1}^* \), and not on the entire history \( h_{it} \). In addition, if \( m_{i,t-1}^* = m \), then at period \( t \), only the utility from choice \( m \) is influenced and not \( m' \neq m \). Intuitively, this specification assumes that for each alternative, there exists a type-specific baseline utility \( V_m(\theta_i) \) and the experience with product \( m \) in the previous period deterministically influences the utility from that product by \( \Delta V_m(\theta_i) \). \( \Delta V_m(\theta_i) > 0 \) corresponds to loyalty or inertia, and \( \Delta V_m(\theta_i) < 0 \) corresponds to love for variety.

While many existing studies adopt this specification, to my knowledge they focus on parametric identification of their models by imposing other parametric assumptions, such as consumer heterogeneity following a normal distribution or utility being linear in states. As a result, the implication of one-period dependence for nonparametric identification has been left unaddressed. Here I show that the assumption of one-period dependence alone is indeed sufficient to nonparametric identification of the model.

**Proposition 2.** Under Assumptions 1 and 2, both \( V_m(\theta_i, m_{i,t-1}^*) \) and \( f(\theta_i) \) are identified from the observation of infinitely many \( m_{i,t}^* \).

The proof is provided in the Appendix. Proposition 2 implies that under one-period dependence, any extra assumptions imposed in the existing studies can be relaxed and we can still identify the model. This result provides the econometric justification of empirical studies that allow for flexible forms of heterogeneity in their models (Dubé, Hitsch and Rossi 2010, Shin, Misra and Horsky 2012). Their models are identified regardless of the flexibility of added consumer
heterogeneity.

The intuition of the identification is as follows. Assumptions 1 and 2 ensure that every consumer returns to every state realization $m_{i,t-1}^* = m \in \{1, ..., M\}$ in finite time. That is, state evolution is positive recurrent. This implies that every consumer visits every state infinitely often during her $m_i^*$. Hence, we can identify the distribution of actions of every consumer at every state. The frequency that each consumer selects each option $m$ at that state identifies her $V_m(\theta_i, m_{i,t-1}^*)$. The difference in the frequency within consumer across states identifies state dependence, and that within state across consumers identifies heterogeneity. By assigning the same $\theta_i$ to consumers with identical $V_m(\theta_i, m_{i,t-1}^*)$ for all $m$ at all $m_{i,t-1}^*$, we also identify how many types of consumers exist and the number of each type of consumers, $f(\theta_i)$.

Two insights are worth noting. First, when $M > 2$, we have an overidentified system.\(^{65}\) Hence, some cross-product state dependence can also be identified; $V_m(\theta_i, m_{i,t-1}^*) = V_m(\theta_i) + \Delta V_{m,m'}(\theta_i)$ is allowed even when $m_{i,t-1}^* = m' \neq m$. This implies that we can insert a flexible form of heterogeneity to the specifications employed in the studies of umbrella branding, such as Erdem and Sun (2002), and identify it. Second, the same identification argument is applicable to a more general X-period dependence; the current utility depends on actions in the past $\tau > 1$ periods. In other words, models with finite dependence of any length are nonparametrically identified.

3 Identification of transient state dependence using observations of discrete-continuous choices

The identification approach based on the recurrence property is not applicable when a transient state exists: the case where consumers visit each state realization at most once. Transient

\(^{65}\)For each type $\theta_i$, we observe $M - 1$ choice probabilities at each state $m_{i,t-1}^*$, resulting in $M(M - 1)$ moment conditions. Assumption 2 leaves $2(M - 1)$ parameters to be identified for each $\theta_i$: there exists $M - 1 V_m(\theta_i)$ and $M - 1 \Delta V_m(\theta_i)$. Hence, when $M > 2$, we have an overidentified system.
states emerge when utility depends on the entire history of actions. For example, during the consumption stream of a drama series, consumers’ intent to watch the next episode may depend on how the storyline has developed in the sequence of previous episodes; utility from playing a videogame may follow an inverse U-shape due to initial skill acquisition and long-term satiation. Models to study these environments contain state variables that represent consumers’ experience with the product, which follows a transient process.\textsuperscript{66} When transient states exist, we cannot identify the distribution of actions of each consumer at each state, for we only observe one action that each consumer takes at each state realization. Moreover, some consumers may never visit some state, because if a consumer skips some state realization once, then by definition of transiency that consumer never gets a chance to visit that state again. At the states that consumers never visit, we have no information to identify their utility with. Hence in order to establish identification, we need to either impose some restriction on how the utility evolves across transient states, so that we use information at one state to help identify utility at another, or on the distribution of heterogeneity, so that we use information from one consumer to identify utility of another consumer.

In this section, I show that if consumers make two decisions at each state realization, one discrete choice from multiple options and the other continuous choice of consumption intensity, then we can identify transient state dependence and unobserved heterogeneity under mild assumptions. In essence, even when consumers visit each state at most once, if we observe two actions at each state, we can pin down each consumer’s type using one action and obtain the distribution of the other action conditional on consumer type, which is sufficient to identify the utility of that type of consumers at that state. In this section, I assume that consumers know their true type \( \theta_i \) and there is no learning.

\textsuperscript{66}An alternative approach to deal with state transiency is to include some moderating effect to make the state evolution recurrent, such as the assumption that consumers forget what they learned.
In order to let consumers make both discrete and continuous choices at each state, I modify the consumers’ problem as follows. At each period $t$, each consumer $i$ solves the following maximization problem.

$$\max_m V_m(\theta_i, h_{it}) + \epsilon_{imt},$$

where $V_m(\theta_i, h_{it}) = \max_x u_m(x, \theta_i, h_{it})$.

In other words, utility from each discrete option is endogenously determined by the decision of consumption intensity. I denote the optimal consumption intensity by $x^*_m(\theta_i, h_{it})$, or simply $x^*_{imt}$.

In order for the observation of $x^*_{imt}$ to be informative about her $V_m(\theta_i, h_{it})$, I impose the following assumption on the relationship between $x^*_m$ and $V_m$.

**Assumption 3.** At each $h_{it}$, $u_m(x, \theta_i, h_{it})$ is such that there exists a unique $x^*_m(\theta_i, h_{it})$ and a strictly monotone relationship between $x^*_m(\theta_i, h_{it})$ and $V_m(\theta_i, h_{it})$.

Assumption 3 asserts that if $x^*_m(\theta_i, h_{it}) > x^*_m(\theta'_i, h_{it})$, then $V_m(\theta_i, h_{it}) > V_m(\theta'_i, h_{it})$; the type of consumers who selects higher consumption intensity receives higher utility. This assumption plays a key role in the identification. Because consumers make discrete choices based on their $V_m$, the monotonicity between $x^*_m$ and $V_m$ implies that the type of consumers with higher utility from option $m$ is more likely to choose option $m$ and choose higher $x^*_m$. Hence both discrete and continuous choices contain information of $V_m(\theta_i, h_{it})$. This allows us to use observation of one action to identify the type of a consumer, and evaluate her utility using the other action. Moreover, the assumption is not restrictive given that in many environments, consumers who are more likely to choose one alternative are also likely to exhibit higher consumption intensity of that alternative, such as the purchase of pre-packaged products (Nair, Dubé and Chintagunta)}

Even if $\theta_i$ is continuously distributed, this assumption does not map into the supermodularity of $u_m(x, \theta_i, h_{it})$ between $x$ and $\theta_i$ at each $h_{it}$. Because different $\theta_i$ type can follow different utility evolution over $h_{it}$, we could have both $x^*_m(\theta_i, h_{it}) > x^*_m(\theta'_i, h_{it})$ and $x^*_m(\theta'_i, h'_{it}) > x^*_m(\theta_i, h'_{it})$ for some $\theta_i \neq \theta'_i$ and $h_{it} \neq h'_{it}$.
The monotonicity implies that there exists a strictly monotone function \( g_{m,h_{it}} \), such that

\[
V_m(\theta_i, h_{it}) = g_{m,h_{it}}(x_m^*(\theta_i, h_{it})).
\]

\( g_{m,h_{it}} \) represents the relationship between various \( x_m^*(\theta_i, h_{it}) \) selected by different \( \theta_i \) consumers and their \( V_m(\theta_i, h_{it}) \) for a given \( h_{it} \) and \( m \). The identification of \( V_m(\theta_i, h_{it}) \) for all types and states is thus equivalent to the identification of \( x_m^*(\theta_i, h_{it}) \) and \( g_{m,h_{it}} \) for all types and states. In order to normalize the utility from one option, I assume that \( V_M(\theta_i, h_{it}) = x_M^*(\theta_i, h_{it}) \) for all \( \theta_i \) and \( h_{it} \). In other words, the benchmark utility of each consumer at state \( h_{it} \) is scaled by her consumption intensity of product \( M \), and utility from other products is defined relative to it. If there is an option not to consume any product, we can instead set \( V_M(\theta_i, h_{it}) = 0 \) for all \( \theta_i \) and \( h_{it} \) as an outside option and the following analysis applies.

The current model implies that at a given \( h_{it} \), consumers with the same type would choose identical \( x_{it}^* \) with one another. In practice consumers may choose different \( x_{it}^* \) for idiosyncratic reasons. In order to capture this, I assume that at each period, each consumer \( i \)'s actual consumption intensity is determined as \( \tilde{x}_i(\theta_i, h_{it}) = x_{it}^*(\theta_i, h_{it}) + e_{imt} \). \( e_{imt} \) is an error term that is distributed independently from \( \theta_i \) such that it has an unbounded support, \( E(e_{imt}) = 0 \) and it has stationary variance over time.

Now suppose that we observe infinitely many consumers, each with an infinite sequence of discrete choices \( m_{i}^* = \{m_{i1}^*, ..., m_{it}^*, ...\} \). We also observe the sequence of consumption intensity \( \tilde{x}_i = \{\tilde{x}_{im1}, ..., \tilde{x}_{imt}, ...\} \), such that the \( t \)-th element of \( \tilde{x}_i \) corresponds to \( \tilde{x}_m(\theta_i, h_{it}) \) for \( h_{it} = \{m_{i1}^*, ..., m_{i,t-1}^*, \tilde{x}_{im1}, ..., \tilde{x}_{im,t-1}\} \) and \( m = m_{it}^* \). In other words, we observe consumer \( i \)'s

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\(^{68}\) This is without loss of generality. By construction of discrete choice environment, if we multiply all \( V_m \) by an arbitrary \( k > 0 \) or add an arbitrary constant \( l > 0 \), then we obtain the same choice probability. Hence, the utility from one has to be fixed at an arbitrary value. I choose \( x_M^*(\theta_i, h_{it}) \) as the value for expositional clarity.

\(^{69}\) I denote \( \tilde{x}_m(\theta_i, h_{it}) \) by \( \tilde{x}_{imt} \) when its dependence on the state is evident from the context.
consumption intensity only at the state she visited and only for the option she selected at that state.

In order to establish identification with such observations, I employ the following assumption.

**Assumption 4.** \( \forall \theta_i, \exists \bar{m}_{it}^* = \{m_{i1}^*, ..., m_{it}^*\} \) such that \( P(\theta_i \mid \bar{m}_{it}^*) = \frac{P(m_{it}^* \mid \theta_i)}{\sum_{i'} P(m_{it}^* \mid \theta_{i'})} \) converges to one as \( t \to \infty \).

I also use one of the following assumptions.

**Assumption 5.** For all \( \theta_i \), at each \( h_{it} \),

\[
V_m(\theta_i, h_{it}) = V_m(\theta_i, h_{int}),
\]

where \( h_{int} \) is a state variable that only tracks consumer \( i \)'s history of consumption of option \( m \);

\( h_{int} = \{\bar{x}_{im1}, \bar{x}_{im2}, ..., \bar{x}_{imt_{m}}, ..., \bar{x}_{im_{t_m}}\} \), \( \bar{t}_m \) is the number of times that type \( \theta_i \) consumer selected option \( m \) during periods \( 1, ..., t - 1 \).

**Assumption 6.**

1. For all \( \theta_i \), at each \( h_{it} \),

\[
V_m(\theta_i, h_{it}) = V_m(\theta_i, \bar{x}_{int}) + \Delta V(\theta_i, \bar{x}_{i,-m,t}),
\]

where \( \bar{x}_{int} = \sum_{\tau<t} \bar{x}_{im\tau}1\{m_{\tau}^* = m\} \) and \( \bar{x}_{i,-m,t} = \sum_{\tau<t} \sum_{m'\neq m} \bar{x}_{im\tau}1\{\bar{m}_{\tau}^* = m'\} \). \( 1\{\cdot\} \) is an indicator function which is one if inside the bracket is true, and zero otherwise.

2. \( \Delta V(\theta_i, 0) = 0, \forall \theta_i \).

3. \( V_m \) and \( g_m, \{\bar{x}_{int}, \bar{x}_{i,-m,t}\} \) are continuous at \( \bar{x}_{int} = 0 \).

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\(^{70}\)Assumption 4 is a restriction on the endogenous variable \( m_{it}^* \) and not on model primitives. In general, restrictions on primitives make it easier to check if the model one develops satisfies the identification condition. Hence, I provide below an example of model primitives that satisfy Assumption 4.
Assumption 7.

1. For all $\theta_i$, at each $h_{it}$,

$$V_m(\theta_i, h_{it}) = V_m(\theta_i, \bar{x}_{imt}) + \Delta V(\theta_i, \bar{x}_{i,-m,t}),$$

where $\bar{x}_{imt} = \sum_{\tau < t} \bar{x}_{im\tau}$ and $\bar{x}_{i,-m,t} = \sum_{\tau < t} \sum_{m' \neq m} \bar{x}_{im'\tau}$.

2. $\Delta V(\theta_i, 0) = 0, \forall \theta_i$.

3. $\Delta V$ and $g_m, \{\bar{x}_{imt}, \bar{x}_{i,-m,t}\}$ are continuous at $\bar{x}_{i,-m,t} = 0$.

I show that these assumptions are sufficient to nonparametrically identify transient state dependence and unobserved heterogeneity.

**Proposition 3.** If Assumptions 1, 3, 4, and one of Assumptions 5, 6 or 7 are satisfied, then both $f(\theta_i)$ and $V_m(\theta_i, h_{it})$ for all $\theta_i$ and $h_{it}$ are identified by the observations of infinitely many $m^*_i$ and $\bar{x}_i$.

The proof is provided in the Appendix. Here I provide intuitions of the restrictions each assumption imposes and how they lead to identification. Broadly, the identification argument consists of two parts. First, we use the observation of each consumer’s $m^*_i$ to identify her $\theta_i$. In practice, consumers who follow identical $m^*_i$ are assigned the same type. In the proof, I show that under Assumption 4, the type assigned as such corresponds to $\theta_i$ in the model. Once the set of consumers with identical $\theta_i$ is identified, the distribution of $\bar{x}_{it}(\theta_i, h_{it})$ – the distribution of consumption intensity among those who share the same $m^*_i$ – identifies $V_m(\theta_i, h_{it})$ of type $\theta_i$ consumers from the selected option $m^*_i$ at each $h_{it}$ that they visit during their $m^*_i$.

Intuitively, Assumption 4 ensures that the set of consumers with the same $m^*_i$ has the same $\theta_i$ in the following way. If we assign a type to each individual according to her sequence of discrete

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71 More specifically, the average of $\bar{x}_m(\theta_i, h_{it})$ identifies $V_m(\theta_i, h_{it})$. This is because $E(\bar{x}_{imt}) = x^*_{imt}$. 

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choices $m^*_i$, in general such type does not necessarily represent the true $\theta_i$ in the model. Because of idiosyncratic utility shocks $\epsilon_{imt}$, consumers with different $\theta_i$ may follow the same sequence of discrete choices. Assumption 4 asserts that for every type $\theta_i$, there exists a sequence of discrete actions $\tilde{m}^*_{it}$, such that by considering the set of consumers taking that sequence, we can pin down their $\theta_i$ with probability approaching to one as the sequence becomes longer. In other words, while multiple types of consumers may take $\tilde{m}^*_{it}$ for any finite $t$, as $t$ tends to infinity, other types of consumers are more likely to leave the sequence, leaving only type $\theta_i$ on the sequence at the limit.\textsuperscript{72} I provide an example of environment where Assumption 4 is satisfied below.

The second part of identification argument deals with the fact that we only observe $\tilde{x}_{imt}$ at a subset of state realizations. For any $h_{it}$ that type $\theta_i$ consumers did not visit, or for any $m$ that they did not select at the state they visited, we do not observe their $\tilde{x}_{imt}$ and hence we cannot draw inference of $V_m(\theta_i, h_{it})$ from it. Assumptions 5 through 7 impose restrictions on $V_m$, so that such limited information about $\tilde{x}_{imt}$ is sufficient to recover $V_m(\theta_i, h_{it})$ of type $\theta_i$ at all $h_{it}$. In order to understand what restrictions are necessary to achieve this, first note that in general, such missing observations stem from the fact that for each type of consumers, we only observe one particular order in which the discrete actions are selected, $m^*_i$. For example, if $m^*_i$ includes selection of option 1 at $t = 1$, and then option 2 at $t = 2$, by the time we observe the consumption intensity of option 2 of that type of consumers they have already consumed option 1. Hence we have no information about what their consumption intensity of option 2 would be if they had not consumed option 1. In order to identify their utility from option 2 without experience with option 1, we need to assume that for some subset of consumers, the experience with option 1 does not influence their utility from option 2. In other words, we need to impose restriction on how utility from option $m$ is influenced by the experience with option $m' \neq m$. This is what

\textsuperscript{72}Note that Assumption 4 only ensures the \textit{existence} of one such sequence for each $\theta_i$, which is sufficient for identification. There may exist infinitely many other $m^*_i$’s that do not pin down $\theta_i$ uniquely.
Assumptions 5 through 7 do.

Assumption 5 requires that utility from each option is influenced only by the consumption history of that option. This eliminates such missing observations due to cross-product state dependence as the example above. Indeed, this assumption ensures that all types of consumers visit all the state realizations during their $m^*_i$ and hence we observe their $\tilde{x}_{int}$ at all states. This is because the state $h_{int}$, which determines the utility from option $m$, only evolves as a result of consumer $i$'s choice of option $m$. Hence, the evolution of $h_{int}$ is independent of the order that consumer $i$ selects other options. Hence, what identifies $V_m(\theta_i, h_{int})$ is how the consumption history of option $m$ influences future consumption level of option $m$ of that type of consumer: information that we have for all $\theta_i$ and $m$. Hence, we no longer need to infer utility at one state from the actions at another state.

Assumption 6 and 7 incorporate possible cross-product state dependence. In this case, I impose that the magnitude of spill-overs needs to be the same across $m$, and both the own-effect and the spill-over has to depend only on a cumulative sum $\{\tilde{x}_{int}, \tilde{x}_{i,-m,t}\}$ and not on the whole history. I also impose continuity of $V_m$ in $\{\tilde{x}_{int}, \tilde{x}_{i,-m,t}\}$ around zero. Assumption 6 and 7 differ in how continuity is imposed. Under Assumption 6, the own-effect of state dependence is assumed continuous. On the other hand, under Assumption 7, cross-product state dependence is continuous. Intuitively, these continuity assumptions assert that consumers who have little experience (i.e. low past consumption intensity) with an option receives similar utility to the ones who have no experience, and their utility is identical at the limit. The assumption implies that in the example above, we can identify the utility from option 2 without experience with option 1 by considering the set of consumers who selected low consumption intensity of option 1.

Because Assumptions 5 through 7 impose different restrictions on the utility, in practice one can use the one that is most appropriate for the environment of interest. Assumption
5 is appropriate for situations where consumers choose an option from distinctively different alternatives and hence cross-product state dependence does not exist. For example, consider a consumer’s TV watching behavior. Her utility from watching ESPN can flexibly depend on her past experience with ESPN, but may not depend on her past experience with Fox News.

On the other hand, Assumption 6 and 7 is appropriate for environments where cross-product dependence is likely. For example, videogame users may acquire skills through experience, and skills acquired in one game may be transferable to another. In this case, the magnitude of cross-product dependence needs to be the same across $m$, and both the own-effect and the cross-effect has to depend only on a cumulative sum $\{\bar{x}_{imt}, \bar{x}_{i,-m,t}\}$ and not on the whole history. A videogame user’s skill may only depend on how many hours she spent with the game in total, and not on her play intensity at each individual session in the past. As long as she has played five hours in total, whether she played one session that lasted five hours, or she played five one-hour sessions results in the same skill level.

**Example 1 (Sufficiency for Assumption 4)**

Unlike Assumptions 5, 6 and 7, Assumption 4 is a restriction on the endogenous variable $m^*_i$ and not on model primitives. In general, restrictions on primitives make it easier to check if the model one develops satisfies the identification condition. Hence, I consider a familiar logit case and derive a condition on model primitives that is sufficient for Assumption 4. Suppose that $\epsilon_{imt}$ follows Type 1 Extreme Value distribution. Also assume for simplicity that the state space only consists of past discrete choices and not past usage intensity. In other words, $h_{it} = \tilde{m}_{i,t-1} \equiv \{m_{i1}^*, ..., m_{i,t-1}^*\}$. In this environment, the following restrictions on $V_m$ is sufficient for Assumption 4.

**Lemma 1.** Assumption 4 is satisfied if for every $\theta_i$, there exists $m^*_i$ and $\bar{l}$, such that starting
from history $\bar{m}_{i,t-1}^*$, for all $t > i$ and for all $\theta_i' \neq \theta_i$,

$$V_{m_i^*}(\theta_i, \bar{m}_{i,t-1}^*) - \max_{m' \neq m_i^*} V_{m'}(\theta_i, \bar{m}_{i,t-1}^*) > V_{m_i^*}(\theta_i', \bar{m}_{i,t-1}^*) - \min_{m' \neq m_i^*} V_{m'}(\theta_i', \bar{m}_{i,t-1}^*),$$

where $\bar{m}_{i,t-1}^*$ is element one through $\bar{t} - 1$ of $m_i^*$, and $m_{it}^*$ is the $t$-th element of $m_i^*$.

The proof is provided in the appendix. It states that the condition for Assumption 4 is satisfied if conditional on having followed the sequence $m_i^*$ up to period $\bar{t}$, type $\theta_i$ consumers receive much higher utility from option $m_{it}^*$ than any other types. Hence they are far more likely to follow $m_i^*$. In the long-term, type $\theta_i$ is the only type that take $m_i^*$ with probability approaching to one.

Here is one example of utility specification that satisfies the condition of Lemma 1. Consider for each of the $M$ discrete options, there exists a type of consumers who become “loyal” to that option. In other words, there exists $M$ discrete types of consumers, each with the following utility.

$$V_m(\theta_i, \bar{m}_{it}^*) = V_0 + \{\theta_i = \theta_m\}f\left(\sum_{\tau < t} 1\{m_{i\tau}^* = m\}\right) + \{\theta \neq \theta_m\}g\left(\sum_{\tau < t} 1\{m_{i\tau}^* = m\}\right)$$

$$+ g\left(\sum_{\tau < t} 1\{m_{i\tau}^* \neq m\}\right),$$

where $f(0) = 0$, $g(0) = 0$ and $f'(x) > 0$, $g'(x) < 0 \forall x \geq 0$.

In other words, all consumers start from the same baseline utility $V_0$, and the utility of different type of consumers evolve differently as they accumulate consumption experience. The monotonically increasing function $f$ and decreasing function $g$ represents loyalty development; if type $m$ consumers consume product $m$, then it increases her utility from option $m$ in the next period through $f$ while it decreases utility from option $m' \neq m$ through $g$. Hence, consumption
experience with \( m \) makes type \( m \) consumer more likely to choose \( m \) in the future.

In order to see how this specification satisfies Lemma 1, consider a sequence \( m_i^* \) that always takes option 1 at every period. Then, starting from any period \( t > 1 \), utility of type 1 consumers from each option can be represented as follows.

\[
V_1(\theta_1, \{1, 1, \ldots, 1\}) = V_0 + f(t).
\]

\[
V_m(\theta_1, \{1, 1, \ldots, 1\}) = V_0 + g(t), \forall m \neq 1.
\]

Similarly, utility of type \( m \neq 1 \) consumers from each option is:

\[
V_m(\theta_m, \{1, 1, \ldots, 1\}) = V_0 + g(t), \forall m.
\]

Hence,

\[
V_1(\theta_1, \{1, 1, \ldots, 1\}) - \max_{m \neq 1} V_m(\theta_1, \{1, 1, \ldots, 1\}) = f(t) - g(t) > 0
\]

\[
= V_1(\theta_m, \{1, 1, \ldots, 1\}) - \max_{m \neq 1} V_m(\theta_m, \{1, 1, \ldots, 1\}),
\]

holds for all type \( m \neq 1 \) consumers and Lemma 1 holds.

**Example 2 (Finite evolution of state)**

Proposition 3 establishes nonparametric identification under any forms of transient state dependence. If we impose some restrictions on the evolution of states, it is possible to relax some of the assumptions imposed in Proposition 3. An environment that is worth a specific note from this perspective is where the state stops evolving after some finite \( \tilde{t} \). In other words, for any \( t > \tilde{t} \),

\[\text{Recurrence of state evolution discussed in Section 2 is one of such examples.}\]
the relevant state stays at $h_{it} = h_{i\tilde{t}} = \{m_{i1}^{*},...,m_{i\tilde{t}}^{*}\}$ This may hold when marginal impact of past usage decreases sufficiently over time, and hence additional experience with the product no longer influences the future utility. For example, consumers may acquire skill of a videogame through initial $\tilde{t}$ sessions and they reach the upper-bound of possible skill level at some point. Similarly, consumers' long-term loyalty to a product, once developed, does not depend on the daily consumption activities in the long-term.

When state evolution is finite, identification of the model does not require Assumptions 5 through 7 anymore. This is because under finite evolution of state, for every type of consumers, we can observe their visit to every state realizations. The intuitive argument is as follows. Suppose that the state evolution stops at period $\tilde{t}$. First note that under Assumption 4, in order to pin down the set of consumers with identical type, it suffices to take a set of consumers who follow an identical sequence of discrete choices starting from period $\tilde{t} + 1$. Assumption 4 ensures the unique determination of the type from consumers' long-term discrete actions, and hence excluding the observation of the initial finite $\tilde{t}$ periods from the conditioning set does not affect its property. This in turn implies that each of the consumers determined as the same type can follow an arbitrary sequence of discrete choices up to $\tilde{t}$. Because of the unboundedness of $\epsilon_{int}$ (Assumption 1), when we have infinitely many observations, for every sequence of $\{m_{i1}^{*},...,m_{i\tilde{t}}^{*}\}$ we always have some consumers of that type following that sequence. It follows that we observe every type of consumers visiting every state realizations.

The assumption of finite state evolution is a quite powerful alternative to Assumptions 5

\[\footnote{Note that this is different from a typical finite dependence, where relevant state at period $t$ consists of actions in the immediately preceding $t' < t$ periods; the state is defined by a moving window. Indeed, the case of moving window is a simple extension of one-period dependence in Chapter 2 and identification is established by an observation of discrete choices alone using its recurrence property. In example 2, state is always "actions in the first $\tilde{t}$ periods" and the window never moves. Hence the state evolution is still transient even when the evolution stops at some $\tilde{t}$.} \]

\[\footnote{Note that in either case, consumption at any future $t$ still depends on $h_{i\tilde{t}}$. The states' stopping evolving does not mean that consumers forget what they did. Rather, what they developed at the beginning persists forever regardless of how they consume the product afterward.} \]

\[\footnote{This property of Assumption 4 is quite general and not specific to the current environment. I show here that it is particularly useful when combined with the assumption finite evolution.} \]
through 7, for we can identify cross-product dependence without any restriction. Moreover, $\tilde{t}$ can be arbitrarily long as long as it is finite. The practical implication of this example is that if we have a data set with finite length, we can split the data into early periods and later periods. We then use the latter to identify type and use the former to identify state dependence.

4 Implications of consumer learning

In this section, I explicitly take consumer learning into account in the model. I first present that there exists a model property that holds if and only if consumer learning exists. Hence, models with and without learning are distinguishable from each other. I then show that if one develops a model that assumes away consumer learning a priori, she can check validity of the assumption by testing if the estimated model satisfies that property. I also propose a way to implement such a test in practice.

I again consider the model that I developed in Section 3, and define consumer learning in that environment as follows. For a given true type $\theta_i$, each consumer starts from an initial belief $\theta_{i0}$ and behave as if she is type $\theta_{i0}$. As she accumulates consumption experience, her belief $\theta_{it}$ evolves and converges to her true type. i.e.

$$P(\theta_{it} = \theta_i \mid \theta_i) \to 1,$$

as $t \to \infty$. The model with no learning is a special case of this representation, where $P(\theta_{it} = \theta_i \mid \theta_i) = 1$ for all $t$. For presentational simplicity, henceforth I assume that $\theta_i$ and $\theta_{it}$ only take discrete points, but one could analogously consider learning under continuous distributions.

I first show that models with and without consumer learning exhibit distinct predictions about how the variance of consumption intensity of a given type of consumers evolves over time.
Proposition 4. Conditional on a set of consumers with the same true type \( \theta_i \), following a given history \( h_{it} = \bar{h}_{it} \) and taking the same option \( m_{it}^* \) at \( t \), the variance of consumption intensity

\[
\text{Var}(\bar{x}_m(\theta_{it}, h_{it}) \mid \theta_i, h_{it} = \bar{h}_{it}, m = m_{it}^*) \]

varies across \( t \) in a nonstationary way if and only if there exists learning.

Proof.

\[
\text{Var}(\bar{x}_m(\theta_{it}, h_{it}) \mid \theta_i, \bar{h}_{it}, m_{it}^*) = \text{Var}(x_{m_{it}^*}^*(\theta_{it}, h_{it}) + \epsilon_{int} \mid \theta_i, \bar{h}_{it})
\]

\[
= \sum_{\theta_i'} P(\theta_{it} = \theta_i' \mid \theta_i, \bar{h}_{it}, m_{it}^*) (x_{m_{it}^*}^*(\theta_i', \bar{h}_{it}) - E(x_{m_{it}^*}^*(\theta_i', \bar{h}_{it}) \mid \theta_i))^2
\]

\[
+ \text{Var}(\epsilon_{int} \mid t).
\]

The first line is the definition of \( \bar{x}_{int} \) and second line takes the variance of \( x_{m_{it}^*}^*(\theta_{it}, h_{it}) \) for \( m = m_{it}^* \) and \( h_{it} = \bar{h}_{it} \) conditional on true \( \theta_i \). If there is no learning, the first term of the second line is zero, for \( P(\theta_{it} = \theta_i \mid \theta_i) = 1 \) and hence \( E(x_{m_{it}^*}^*(\theta_i', \bar{h}_{it}) \mid \theta_i) = x_{m_{it}^*}^*(\theta_i, \bar{h}_{it}) \). Hence, \( \text{Var}(\bar{x}_m(\theta_{it}, h_{it}) \mid \theta_i) = \text{Var}(\epsilon_{int} \mid t) \), which is stationary over time. On the other hand, if there exists learning, the first term is strictly positive for finite \( t \). Moreover, it converges to zero because \( P(\theta_{it} = \theta_i \mid \theta_i) \to 1 \).

Hence, the variance follows a nonstationary path such that it converges to a stationary process \( \text{Var}(\epsilon_{int} \mid t) \) as \( t \to \infty \).

Proposition 4 states that even when any form of state dependence may cause nonstationary evolution of \( \bar{x}_m(\theta_i, h_{it}) \) of consumers with a given type \( \theta_i \), its variance remains stationary unless there exists learning. Hence, we can distinguish between models with and without learning by examining how the variance of consumption intensity for a given \( \theta_i \) evolves across states \( h_{it} \). The intuition is as follows. If no learning exists, even if any nonstationary state dependence flexibly evolves.
depends on consumer type, conditional on a set of consumers of the same type, every consumer on average follows the same sequence of state dependence, and any deviation from the average is solely due to idiosyncratic factors, whose variance is stationary. On the other hand, if learning exists, the set of consumers with the same true type may face different perceived type in early periods and choose consumption intensity accordingly. As a result, the variance of consumption intensity in early periods also reflects such misperception, creating nonstationary evolution of variance.

This property can be used to empirically determine whether we need a model of consumer learning, or a model without learning suffices, in order to study an environment of interest. Suppose that one assumes a priori that there exists no consumer learning in the environment of interest, and builds and estimates a model accordingly. The model can include any other forms of nonstationary state dependence and unobserved heterogeneity such that identification is ensured through Proposition 3. Using the estimated model, we can test the validity of the assumption of no learning by empirically replicating Proposition 4. We first pick a set of consumers who shares the same \( \theta_i \); using the estimated \( \hat{V}_m(\theta_i, h_{it}) \) for all \( \theta_i \) and \( h_{it} \), we can calculate \( \hat{P}(\theta_i | \hat{m}_{it}^*) \). We use it according to an empirical analog of Assumption 4 to determine which \( \hat{m}_{it}^* \) to take, in order to take the set of consumers with identical \( \theta_i \).

Several notes are in order. First, there may exist situations where we do not need to rely on the estimated \( \hat{P}(\theta_i | \hat{m}_{it}^*) \) to find the right \( m_{it}^* \) to pin down the set of consumers with identical \( \theta_i \). For example, consider the logit example presented in Section 3. In that environment, regardless

\[128\]
of the specific shape of $f$ and $g$ and the value of $V_0$, we know a priori that by taking any sequence $m^*_i$ such that after some $\bar{t}$ it keeps selecting only option $m$, we can pin down the set of type $m$ consumers. In such cases, we can implement the test first and ensure the validity of the assumption before we estimate the model.

Second, thanks to the observation of both discrete and continuous actions, this approach to separate learning and other forms of state dependence is more general than other existing approaches. Existing studies either assume (1) learning is the only nonstationary component of the model, and other forms of state dependence are stationary (Iyengar, Ansari and Gupta 2007, Dubé, Hitsch and Rossi 2010, Osborne 2011), or (2) there exists no heterogeneity in how state evolution influences utility (Erdem and Keane 1996, Ackerberg 2003); we do not need to impose either restrictions. This implies that when one observes both discrete and continuous actions, adopting the current approach is subject to lower risk of mistaking confounding factors as learning. For example, other things being equal, the test based on the variance stationarity can rule out any confound due to other forms of nonstationary state dependence, while the test based on the former assumption cannot.\footnote{It does not mean that the test is always valid. If the model one develops is not flexible enough, the variance of actions may evolve in a nonstationary way due to factors outside of the model. In that case, all tests proposed in the literature, including mine, are invalid.}

5 Conclusion

Policy makers need to understand the nature of state dependence in order to implement an appropriate policy. Nevertheless, identifying state dependence has been a challenge due to its contamination with unobserved consumer heterogeneity. In this paper, I consider strategies to nonparametrically identify state dependence and unobserved heterogeneity. I first show that the assumption of one-period dependence is sufficient for the identification. I then consider identification of models with transient states utilizing discrete-continuous framework.
I also show that when consumers learn about their true preference, their actions evolve in a unique way; the variance of their consumption intensity conditional on their true type and past history follows a nonstationary path. Using this property, we can check if we need to include consumer learning in our model specification in order to study the environment of interest. I propose a way to implement such a test in practice. That is, we first build and estimate a model of state dependence without accounting for learning, use the estimated model to pin down the set of consumers with identical true type, and see if the variance of their consumption intensity is stationary. If no learning is detected, the model is correctly specified, and if learning exists, the model is rejected and we may build a new model that explicitly takes learning into account.

The results of nonparametric identification presented in this paper indicate that many assumptions imposed in the existing studies can be relaxed to allow for a greater model flexibility. Hence, unlike a typical parametric form imposed in existing studies, such as Normal distribution, we can identify more flexible, nonparametric form of unobserved heterogeneity. For example, if one admits one-period dependence, then the coefficient of such “previous consumption term” can be heterogeneous according to a fully nonparametric distribution. In order to avoid creating any spurious state dependence, adding as much flexibility is thus suggested.

This study considers a hypothetical situation where a researcher observes infinitely many consumers with infinitely long sequence of actions. In practice, data available to her may only have short horizon and limited sample size. It is left as a future study to explore the implication of the current identification results in an environment with finite sample. Nevertheless, the identification argument based on infinite sample still provides an upper bound in the identifiable model specification in finite sample environments; if a model is not identifiable with infinite observations, then it is never identifiable with finite sample.

This study suggests one approach to nonparametrically identify a general form of state depen-
dence and unobserved heterogeneity. Ultimately, because of the nonidentification result presented in Section 2, purely nonparametric identification does not exist and we need to impose some restrictions on the model. Apparently, most appropriate restrictions depend on the environment of interest and available observations. This study offers an identification strategy that is appropriate at the cases where one observes consumers’ discrete-continuous actions. There are many other alternative scenarios. For example, we may observe multiple discrete choices, such as consumer’s decisions in market A and market B. Identification strategies that should be employed vary across cases and hence, theories of identification at other popular environments are called for.
Appendix

A Sample of consumers used in this study

The sample of users consists of a set of first-time users who made a purchase of version 2014 on Sony PlayStation 3, PlayStation 4, or Microsoft Xbox360 console. “First-time users” refer to consumers who had no experience with versions released in preceding three years. This is the set of consumers who had no trial access. On the other hand, some users who play this game on Microsoft Xbox One console had access to a trial version; the firm provided a 6-hour time-locked trial to consumers who owned Xbox One console and subscribed to the firm’s loyalty program. I opt not to use these samples, because the perfect overlap between the trial access and the enrollment to the loyalty program creates a complicated sample selection problem between the consumer match value and the trial access. Rather, I use the sample of users where no such issue exists, identify consumer learning and recover the effect of trial provision in a structural way.

I impose a few extra restrictions to select observations to use during estimation. First, I focus on the set of consumers making a purchase within 35 weeks of product release. Since a new version of the title is released annually, consumers making a purchase at later weeks may switch to the newer version and terminate the play of the older version earlier than they would without the newer version. Eliminating the last 17 weeks from the sample is sufficient, for vast majority of users terminate their play within 17 weeks. Second, in creating the moments for the adoption model I only use the data on purchases up to 16 weeks from the product release, which is two weeks before the Christmas. Consumers activating the product on Christmas are likely to have received it as a gift. Hence, their activation should not be counted as a purchase when estimating the price coefficient. Since the purchase is made by the diminishing pool of consumers, the number of purchase at each week is a function of the history of purchases that
precedes that week. Hence, dropping Christmas period implies dropping all the post-Christmas periods as well.

B Additional evidence of consumer learning in the data

Practice mode as the initial choice Aside from the four main game modes used in the analysis, the game also features an extra mode called “practice mode”. In practice mode, users repeatedly conduct tasks necessary to play well in other modes. This helps new users gain an understanding about the basics of the game, and develop playing skills. On the other hand, practice mode in itself does not provide much excitement. It neither offers full matchups, nor any team-building and players’ career simulation. Since users typically play practice mode only at the very beginning, I do not explicitly treat the mode as a feature and drop all practice mode sessions from the sample. However, the raw data including practice mode indicate that first-time users tend to start from practice mode. In Table 6 I present the proportion of game mode selection in the initial session, including practice mode. Nearly 40 percent of first-time users choose practice mode. Given the nature of practice mode, such observation implies two things. First, new users are not familiar with the game, and thus the assumption of match value uncertainty appears reasonable. Second, users do not choose to start with one of the main game modes and learn on the way. Instead, they are willing to incur the cost of forgone flow utility to gain the return, either informational or skill, in the future. In other words, there is a strong indication that users are forward-looking.

Initial increase of the duration of sessions In Figure 4 in Section 1.3, I showed that session durations increase over time in the initial few sessions. In Figure 11 I showed that the learning of risk averse consumer can capture such pattern quite well; the uncertainty reduction
increases the expected utility and hence users play longer. Moreover, I found that such initial increase of the hours spent is not attributable to novelty effect and skill acquisition. As I showed in Figure 8, \( c(\nu_{imt}) \) is monotonically increasing.

In this section I introduce three other stories that can cause the initial increase of durations, and argue that learning still plays a role even after controlling for them. The first alternative story is sample selection; users with short initial duration tend to drop out earlier, and hence the duration of users who survive is longer. In Figure 8 in Section 1.1, I condition on the set of consumers who remains active for 10 sessions, and hence the initial increase comes from the evolution of actions of a user and not from sample selection. The second story is the selection of game mode; users tend to choose the mode that requires less time, such as mode 3, and switch to more time-consuming modes in later periods. In order to consider such possibility, in Figure 33, I show the average session duration conditional on each game mode for users who remain active until the 10th session. The duration still increases within a mode in first few sessions, indicating that there exists a factor that increases utility within a mode.

The last alternative is the existence of day-level time constraint. If users have daily time constraint that is constant across days, and allocate the available time to more game modes at the beginning for experimenting purpose, the session duration is naturally shorter. I observe users tend to play multiple sessions per day in early periods, and hence this story applies to my data. However, as shown in Figure 34, even after aggregating the usage up to daily level, I

<table>
<thead>
<tr>
<th>Mode</th>
<th>Choice probability</th>
<th>Hours of play Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>0.134</td>
<td>0.603</td>
<td>0.819</td>
</tr>
<tr>
<td>Mode 2</td>
<td>0.129</td>
<td>0.859</td>
<td>1.091</td>
</tr>
<tr>
<td>Mode 3</td>
<td>0.245</td>
<td>0.3</td>
<td>0.595</td>
</tr>
<tr>
<td>Mode 4</td>
<td>0.107</td>
<td>0.63</td>
<td>0.872</td>
</tr>
<tr>
<td>Practice</td>
<td>0.384</td>
<td>0.212</td>
<td>0.436</td>
</tr>
</tbody>
</table>

Note: The table represents user behaviors in the very first session, aggregated across all users.
still observe the shorter initial duration. Hence, the story of time constraint alone cannot fully explain the initial lower usage intensity.

C Model specification in detail

Structural interpretation of frequency choice and termination In the main text I assumed that the decisions of play frequency and termination are represented in a reduced form way by a probability denoted by $\lambda(\Omega_{it})$ and $\delta(\Omega_{it})$. In this section I show that a decision process where consumers compare payoffs from each of the options and receive an idiosyncratic utility shock generates policy functions that are consistent with this representation.

Consider a user’s decision at the beginning of the day (node A in Figure 7). She chooses whether to play a session or not by comparing the value from playing and that of not playing. The value from playing is simply $V(\Omega_{it})$, the value function defined at node B. On the other hand, the value of not playing is computed in the following way. Suppose that if the user does not play today, then starting from tomorrow she follows a policy such that she plays a session
with probability $\lambda(\Omega_{it})$ on a given day. Then the value from not playing today is the expected discounted sum of value from playing session $t$ at some future date, where the expectation is taken over when the user plays the game next time. Denoting this expected discount factor by $\beta'(\Omega_{it})$, the value of not playing today is $\beta'(\Omega_{it})V(\Omega_{it})$, where $\beta'(\Omega_{it}) = \frac{\beta\lambda(\Omega_{it})}{1-(1-\lambda(\Omega_{it}))\beta}$.

Here I assume that the user receives an idiosyncratic utility shock for each of the options. Denoting the realization of the shock by $\epsilon_f$, her optimal policy is defined as

$$\max\{V(\Omega_{it}) + \epsilon_{f1}, \beta'(\Omega_{it})V(\Omega_{it}) + \epsilon_{f2}\}. \quad (19)$$

If we assume that $\epsilon_{f1}$ and $\epsilon_{f2}$ follows type 1 extreme value distribution, then the user’s optimal policy is represented as $\lambda(\Omega_{it}) = \frac{\exp(V(\Omega_{it}))}{\exp(V(\Omega_{it}))+\exp(\beta'(\Omega_{it})V(\Omega_{it}))}$. On the other hand, if we assume $\epsilon_{f1}$ follows Normal distribution with zero mean and variance $\sigma^2$ and $\epsilon_{f2} = 0$, then $\lambda(\Omega_{it}) = \Phi\left(\frac{V(\Omega_{it})-\beta'(\Omega_{it})V(\Omega_{it})}{\sigma}\right)$. The nonparametric representation of $\lambda(\Omega_{it})$ employed in this study
encompasses these as a special case. Similar argument applies to $\delta(\Omega_{it})$.

**An extension of frequency choice**  In Section 5, I defined $\lambda$ as the probability that a user plays the game at each day. There I assumed that $\lambda$ only depends on $\Omega_{it}$. However, this also implies that the probability of playing a session in a day does not depend on any calendar day notion, such as the number of sessions the user already played *on the same day*. In general we expect that the probability of playing another session decreases in the number of sessions played on the same day. Hence, in the empirical analysis I let the probability that “a user plays one session” and that “the user plays another session conditional on already playing at least one on the same day” be different. I denote the former as $\lambda_1(\Omega_{it})$ and the latter as $\lambda_2(\Omega_{it})$. This distinction would change the representation of the discount factor as follows.

$$
\beta(\Omega_{i,t+1}) = \delta \lambda_2 + \delta(1-\lambda_2)\lambda_1 \beta + \delta(1-\lambda_2)(1-\lambda_1)\lambda_1 \beta^2 + ... \\
= \delta \left( \lambda_2 + (1-\lambda_2) \frac{\beta \lambda_1}{1-(1-\lambda_1)\beta} \right).
$$

Recall that $\beta(\Omega_{i,t+1})$ is located in the continuation payoff such that a user already played one session in the day. This implies that in the path of continuation, the probability that she plays the next session on the same day is always $\lambda_2$ and that she does not is $(1-\lambda_2)$. On the other hand, on the next day and after, any session she plays is always the first session of the day, and the probability that she plays a session is $\lambda_1$. During the simulation of sequences to calculate moment conditions, I use these $\lambda_1$ and $\lambda_2$ in accordance with the definition; I calculate a user’s action using $\lambda_1$ if she is at the beginning of a day, and using $\lambda_2$ if she played one session on the

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80 In these special cases the optimal policy only depends on $\Omega_{it}$ through $V(\Omega_{it}) - \beta'(\Omega_{it})V(\Omega_{it})$. The nonparametric policy is hence consistent with more general representation of payoffs, such as inclusion of arbitrary utility from choosing outside option.

81 Precisely speaking, this representation is inconsistent with the representation of $\beta(\Omega_{i,t+1})$ in Equation (6), where users do not take into account the realization of future $\epsilon_f$. To the extent that those stochastic utility shocks are merely summarizing factors outside of the model, forward-lookingness with respect to such factors is not substantial.
same day. During the estimation procedure, I estimate $\lambda_1(\Omega_u)$ and $\lambda_2(\Omega_u)$ directly from the data.

**Note on model normalization** Discrete choice problems require two normalizations of utility to control for the indeterminacy of level and scale. In this model the imposed normalizations are as follows; (1) flow utility from not playing is zero, both before purchase (not purchasing) and after purchase (not playing a session), and (2) both utility from gameplay and session duration are influenced by the belief $b_{it}$ only through $f(b_{it})$, where $f$ is monotone with respect to $\mu_{imt}$ for a given $\sigma^2_{imt}$ and has no scaling parameters. The first assumption follows the standard practice in the literature and normalizes the level of the flow utility $v(b_{it}, \nu_{imt}, h_t)$. The second assumption normalizes the scale of the flow utility by that of the observed session duration. Both the session duration and the flow utility in the initial period are determined by $f(b_{it})$ and that it is monotone. Hence, there is a one-to-one mapping from the session duration to the associated utility level. Moreover, $f(b_{it})$ has no scaling parameter and hence utility has the same scale as the session duration, which is “hour”. Once this assumption provides a scale of the utility and the value function, no extra normalization on the variance of the idiosyncratic shocks, both for game mode selection and for purchase, is necessary.

## D Identification

### D.1 Separating learning and other forms of state dependence

The main identification challenge in this study is to separate the parameters of learning $\{\Sigma, \tilde{\Sigma}, \sigma^2_s\}$ from other forms of state dependence $c(\nu_{it})$. Here I provide a formal argument for the separate identification. The key to the identification is that the evolution of $c(\nu_{it})$ is deterministic.

---

**82** In other words, if I write $f(b_{it}) = c + b * E[\theta_{im} | \theta_{im} > 0, \mu_{imt}, \sigma^2_{imt}]$, the normalization is $b = 1$.

**83** More precisely, it suffices to assume that for each of the sessions $t$, the utility from playing is zero at one of the state realizations.
conditional on the observed \( \nu_{it} \), while learning involves stochastic evolution of the utility. This implies that the evolution of the variance of the actions across states identify learning, while the evolution of the average actions identify other forms of state dependence. For simplicity, I consider a case with a single segment; \( R = 1 \). I denote the states observable to a researcher by \( \Omega_{it} = \{ \{ \nu_{imt} \}_{m=1}^{M}, h_t \} \). The difference from \( \Omega_{it} \) is that \( \Omega_{it} \) does not include the belief \( b_{it} \), which is unobservable to a researcher. Formally, the identification of learning parameters \( \{ \Sigma, \tilde{\Sigma}, \sigma_s^2 \} \) takes the following two steps. I first identify the variance of beliefs at each state, \( \text{Var}( \mu_{imt} | \Omega_{it} ) \), from the observation of the variance of the session duration, \( \text{Var}( x_{imt}^* | \Omega_{it} ) \). I then identify \( \{ \Sigma, \tilde{\Sigma}, \sigma_s^2 \} \) using the variation of \( \text{Var}( \mu_{imt} | \Omega_{it} ) \) across states.

Identification of \( \text{Var}( \mu_{imt} | \Omega_{it} ) \) at each \( \Omega_{it} \) from \( \text{Var}( x_{imt}^* | \Omega_{it} ) \). The identification of \( \text{Var}( \mu_{imt} | \Omega_{it} ) \) goes as follows. In general, the session duration \( x_{imt}^* \) is a function of \( b_{it} = \{ \mu_{it}, \Sigma_{it} \} \) and \( \nu_{imt} \) through Equation (4). However, since everyone at \( \Omega_{it} \) has the same usage history, \( \nu_{imt} \) does not influence \( \text{Var}( x_{imt}^* | \Omega_{it} ) \). Moreover, Equation (9) indicates that users who share the same usage history must have the same \( \Sigma_{it} \) too; evolution of \( \Sigma_{it} \) only depends on the history of choices, and not on the realization of past signals. Hence, the distribution of the session duration among users at \( \Omega_{it} \) solely reflects the distribution of their \( \mu_{it} \); there is a one-to-one mapping from \( \text{Var}( \mu_{imt} | \Omega_{it} ) \) to \( \text{Var}( x_{imt}^* | \Omega_{it} ) \). Moreover, this mapping is monotone, and hence we can invert it to identify \( \text{Var}( \mu_{imt} | \tilde{\Omega}_{it} ) \) from \( \text{Var}( x_{imt}^* | \tilde{\Omega}_{it} ) \). The monotonicity comes from the fact that \( f(b_{it}) \) is a known function for a given set of parameters and it is monotone in \( \mu_{imt} \) at each \( \tilde{\Omega}_{it} \).

Note that in practice, I only observe the distribution of the duration of session for the selected game modes; I observe truncated distributions and not the population-level distribution. However, the belief follows normal distribution and the point of truncation is determined by a fully parametrized model. Since normal distribution is recoverable, observation of arbitrarily
truncated distribution, together with the model that specify the point of truncation, is sufficient to identify the population distribution.

**Identification of $\Sigma$, $\hat{\Sigma}$, and $\sigma_s^2$ from $\text{Var}(\mu_{imt} | \bar{\Omega}_{it})$**

The identification of $\text{Var}(\mu_{imt} | \bar{\Omega}_{it})$ at each observed state $\bar{\Omega}_{it}$ means that now we know how the variance of the mean belief evolves over time. This gives us sufficient information to identify most of the parameters that characterize learning: $\Sigma$, $\hat{\Sigma}$, and $\sigma_s^2$. In order to see this, consider $\text{Var}(\mu_{i1})$, the variance of the beliefs for all modes at the initial session, and $\text{Var}(\mu_{im2} | m_{i1} = m)$, the variance of the belief for mode $m$ at $t = 2$, at the state where mode $m$ was also selected at $t = 1$.

\[
\text{Var}(\mu_{i1}) = \text{diag}(\text{Var}(\mu + \Sigma(\Sigma + \hat{\Sigma})^{-1}(\hat{\theta}_{i0} - \mu)))
= \text{diag}(\Sigma(\Sigma + \hat{\Sigma})^{-1}\Sigma).
\]

\[
\text{Var}(\mu_{im2} | m_{i1} = m) = \text{Var}\left(\mu_{im1} + \frac{\sigma_{im1}^2}{\sigma_{im1}^2 + \sigma_s^2}(s_{im1} - \mu_{im1})\right)
= \left(\frac{\sigma_{im1}^2}{\sigma_{im1}^2 + \sigma_s^2}\right)^2 (\sigma_s^2 + \sigma_m^2) + \left(1 - \left(\frac{\sigma_{im1}^2}{\sigma_{im1}^2 + \sigma_s^2}\right)^2\right) \text{Var}(\mu_{im1}),
\]

where $\text{diag}(\cdot)$ denotes diagonal elements of the argument. $\sigma_{im1}^2$ is the $\{m, m\}$ element of $\Sigma_1$, and $\text{Var}(\mu_{im1})$ is the $m$-th element of $\text{Var}(\mu_{i1})$. Both Equation (20) and (21) consist of $\{\Sigma, \hat{\Sigma}, \sigma_s^2\}$, and are not mutually collinear with respect to these parameters. Hence, these equations impose distinct restrictions on the relationship among $\{\Sigma, \hat{\Sigma}, \sigma_s^2\}$. Similarly, the distribution of session duration for game mode $m$ at sessions where game mode $m' \neq m$ is played at the previous session provides a restriction on the correlation between $m$ and $m'$. In the same way, the variance of the belief at each state is not collinear with one another and serves as an individual constraint. Since the number of possible state realizations grows without bound as $t$ goes up while the number of parameters is finite, one can pin down $\{\Sigma, \hat{\Sigma}, \sigma_s^2\}$.

There are other variations that helps identify $\hat{\Sigma}$. For example, $\hat{\Sigma}$ not only determines the variance of the belief, but also determines the magnitude of the error involved in the initial belief. For example, if disproportionately
The intuition is as follows. In the initial session, each user forms the initial belief using her prior and the initial signal; the distribution of beliefs $\mu_1$ reflects $\Sigma$ and $\bar{\Sigma}$. As she learns her true match value $\theta_i$, the distribution of $\mu_\bar{\iota}$ converges to that of $\theta_i$, which reflects only $\Sigma$. Moreover, the speed of convergence is determined by the precision of the signal $\sigma^2_s$. Hence, by observing the variance of the belief at the early stage of consumption, that in the long-run, and the speed of convergence, one can identify $\Sigma$, $\bar{\Sigma}$ and $\sigma^2_s$.

**Identification of $\mu$ and $c(\nu_{int})$** Identification of $\mu$ and $c(\nu_{int})$ comes from the knowledge of average session duration $\mathbb{E}(x^*_{int} \mid \bar{\Omega}_i)$. Specifically, the average match value of the population $\mu$ is identified from $\mathbb{E}(x^*_{im1})$. At $t = 1$, $c(0) = 0$ and hence $x^*_{im1} = f(b_{i1})$ from Equation (4). Since $f$ is monotone in $\mu_{im1}$, the average duration in the initial session, $\mathbb{E}(x^*_{im1})$, identifies $\mathbb{E}(\mu_{im1})$. The identification of $\mathbb{E}(\mu_{im1})$ for each $m$ immediately implies the identification of $\mu$. This is because $\mu_{i1} = \mu + \Sigma(\Sigma + \bar{\Sigma})^{-1}(\bar{\theta}_{i0} - \mu)$ and hence $\mathbb{E}(\mu_{i1}) = \mu$. Intuitively, under rational expectation the mean of the initial belief equals that of the true match value. Over time, the average session duration evolves due to $c(\nu_{int})$. Learning cannot influence the evolution of the average duration because of the rational expectation. Hence, $\mathbb{E}(x^*_{int} \mid \bar{\Omega}_i)$ at each $\bar{\Omega}_i$ relative to $\bar{\Omega}_{i1}$ identifies $c(\nu_{int})$.

**D.2 Identification of other parameters and multiple segments**

Other model parameters to be identified are utility parameters $\rho$, $\alpha$, probability of termination and play frequency $\lambda(\Omega_{it})$, $\delta(\Omega_{it})$, the distribution of price coefficient $\mu_{\eta}$, $\sigma^2_{\eta}$, consumer arrival process $\lambda^c$, the variance of idiosyncratic shocks $\sigma_{cr}$, $\sigma_p$ and the distribution of multiple segments $\xi_r$. I set the daily discount factor $\beta$ at 0.999.\(^85\)

---

\(^85\) Large number of users buying the product early at high price play very little, it indicates that the magnitude of the error involved in the initial belief is large.

\(^85\) Daily discount factor of 0.999 corresponds to annual discount factor of 0.95. The identification of discount factor is known to be quite difficult (Magnac and Thesmar 2002). In general it requires an exogenous factor that only influences future payoffs and not the current payoff, which the current data set does not offer.
The identification of $\rho$ relies on the intertemporal switching across game modes during the initial experimenting periods. Since all the other learning parameters and other channels $c(\nu_{imt})$ are identified solely from the observation of session durations, the only remaining parameter to fit the initial switching patterns is $\rho$. Intuitively, given the belief users experiment with smaller number of modes if $\rho$ is small. When $\rho$ is small, consumers are more risk averse and hence trying a new, unfamiliar game mode is more costly.

$\sigma_\epsilon$ is identified by the difference between relative hours spent on each game mode and the choice probability of that mode. Consider two game modes A and B, and users on average spend 2 hours on A and 2.1 hours on B: a situation that implies that utility users receive from these modes are similar. If utility is not weighted by $\sigma_\epsilon$, the choice probability has to be such that B is chosen with slightly higher probability than A. If B is selected far more often than A in the data, then it follows that $\sigma_\epsilon$ is low and that the size of idiosyncratic shock is very small, so that its realization hardly flips the choice even when the utility difference is modest. $\lambda_1(\Omega_{it})$, $\lambda_2(\Omega_{it})$ and $\delta(\Omega_{it})$ are identified from the distribution of termination probability and play frequency at each observed state $\bar{\Omega}_{it}$. $\alpha$ is identified by the difference in session durations between weekdays and weekends.

The identification of the distribution of $\eta_i$, $\mu_\eta$ and $\sigma^2_\eta$, comes from the rate of purchase at periods where the price is on a declining trend. Given that users are forward-looking, heterogeneity in the timing of adoption identifies the distribution of user patience; some users are willing to wait for price drops, while others make a purchase even when they know the future price is lower. The patience in the adoption model comes from $\eta_i$. If $\eta_i$ is low, then the return from future price decline is low, so is the incentive to wait. Hence, the rate of price decline and the number of purchases made during that period identifies $\mu_\eta$ and $\sigma^2_\eta$.

Provided that the variations in the timing of purchase under declining price are already used
to identify $\mu_\eta$ and $\sigma^2_\eta$, remaining variations to identify the consumer arrival process are limited. The identification of $\lambda^a$ relies on the number of purchases at periods where price is increasing, as well as the total market share of the product. When the price is increasing, the forward-lookingness does not play any role; the return from waiting is low. Hence, the purchase rate is solely determined by the average price coefficient of consumers who remain in the market. Conditional on the distribution of price coefficient, this is a function of $\lambda^a$. The market share of the product is also a function of $\lambda^a$. Hence, I use them to identify $\lambda^a$. $\sigma_p$ serves as a residual buffer between the model prediction and the data. If $\sigma_p = 0$, then the timing of purchase is deterministic for a given willingness to pay. Hence, the proportion of consumers making a purchase at each week must match with the corresponding truncated CDF of the distribution of $V(\Omega_{i1})/\eta_i$. Any difference from that identifies the magnitude of idiosyncratic shock $\sigma_p$. In practice, I did not encounter any issues in identifying parameters in the model for purchase.

Finally, even when the market consists of multiple segments of consumers with different population-level parameters, the argument provided above remains valid. When multiple discrete segments exist, the distribution of match value becomes a discrete mixture of normal distributions. For a given weight $\xi_r$, the behavior of users corresponding to the segment assigned by $\xi_r$ identifies the parameters for that segment. For example, suppose there exists two segments, one with low mean utility and the other with high mean utility, and the probability that a consumer belongs to high segment is 0.2. Then I identify parameters associated with high and low segment from the behavior of top 20 percent and bottom 80 percent of consumers, respectively. Having obtained the best fit between the data and the model prediction for a given $\xi_r$, $\xi_r$ is determined to best match among them.

\footnote{A flexible form of arrival process is not separately identified from the heterogeneity of price coefficient. If I pick a sequence of arrival such that “the rate of arrival at $\tau$ equals the rate of purchase at $\tau$”, then I can justify all the variation in the timing of the purchase solely by the arrival process and $\eta_i = 0$ for all $i$; everyone buys at the week of arrival.}
E Details of the estimation procedure


1. I first pick a set of candidate parameter values \( \Theta \). In order to pick the starting value, I calculate the value of the objective function described in Section 6 at 1,000,000 random parameter values, and pick the smallest one.

2. Given \( \Theta \), I solve the model of usage and purchase described in Section 4 for each segment \( r \). I solve for the value functions by backward induction. In order to ease the computational burden, I use the discretization and interpolation method (Keane and Wolpin 1994). For each session \( t \), I randomly pick 15,000 points from the state and evaluate the value function at these points. I then interpolate the values at other points by fitting a polynomial of the state variables. The variables included as regressors are as follows: \( \mu_{imt}, \nu_{imt}, \exp(\mu_{imt}/100), \mu_{imt} \times \nu_{imt}, \mu_{imt} \times \mu_{im't}, h_t, \mu_{imt} \times h_t \) for all \( m \) and \( m' \neq m \).

Before estimating the full model, I tested the validity of the polynomial approximation by solving both the approximated value function and the full solution of a 10-period dynamic programming problem. The approximation was quite close to the full solution, with \( R^2 \) being over 0.97 at every \( t \).

3. Once I obtain the value function at each state for each segment \( r \), I first create moments for the purchase model. I draw 2,000,000 individuals, each of whom belongs to segment \( r \) with probability \( \xi_r \), which is the proportion of the segment in the market. I draw their true match value \( \theta_i \) following \( N(\mu_r, \Sigma) \) and the initial signal \( \tilde{\theta}_i \) from \( N(\theta_i, \tilde{\Sigma}) \), and create the initial belief \( \mu_{i1} \) and \( \Sigma_{1r} \). I also draw their timing of arrival at the market according to \( \lambda_{ir}^a \). Using the initial belief and the timing of arrival, I simulate their purchase decisions.
following the policy function computed in the model of purchase. The simulated purchase decisions are used to create moments of the pattern of purchase, which are matched with the data.

4. In order to create moments for usage model, I take the subset of simulated individuals who are predicted to make a purchase in the previous step. This is the sample of users comparable to the real data. For this sample of users, I draw a sequence of usage. For each user and for each session, given the drawn state $\Omega_{it}$ I draw her actions according to the policy functions, and draw a signal realization. Using the signal I update her beliefs and calculate the state $\Omega_{i,t+1}$. The sequences of actions obtained this way are used to create moments of the usage patterns.

5. I calculate the value of the objective function and update the parameters. I repeat these steps until the convergence is achieved.

6. In order to check if the global minimum is attained, I conduct this exercise with multiple starting values.

**Construction of moments** From the usage model, I use the following set of moments.

1. The choice probability of each game mode at session $t$, conditional on the history of game mode selections and average duration of the past $t-1$ sessions: $\mathbb{E}(m_{it} \mid \nu_{it}, \bar{x}_{it})$, where 
   
   $\bar{x}_{it} = \frac{\sum_{t' < t} x_{it'}}{t-1}$ and $x_{it'}$ is the hours of play at session $t'$. (372 moments)

2. The average and variance of the duration of session $t$, conditional on the history of game mode selections and average duration of the past $t-1$ sessions, and on the mode selected at session $t$: $\mathbb{E}(x_{int} \mid \nu_{it}, \bar{x}_{it}, m_{it}), Var(x_{int} \mid \nu_{it}, \bar{x}_{it}, m_{it})$. (164 moments)

3. The probability that the game mode selected at session $t+1$ is different from the one at
session $t$, conditional on the history of game mode selections and average duration of the past $t-1$ sessions, and on the mode selected at session $t$: $Pr(m_{i,t+1} \neq m_{it} \mid \nu_{it}, \bar{x}_{it}, m_{it})$. (82 moments)

4. The average interval length between session $t$ and session $t+1$, conditional on the history of game mode selection and the average duration of the past $t$ sessions: $\mathbb{E}(d_{i,t+1} \mid \nu_{i,t+1}, \bar{x}_{i,t+1})$. $d_{i,t+1}$ denotes the interval length between session $t$ and $t+1$. (123 moments)

5. The probability that a user terminates after session $t$, conditional on the history of game mode selection up to session $t$ inclusive, and the duration of session $t$: $\mathbb{E}(term_{it} \mid \nu_{it} + m_{it}, x_{it})$. $term_{it}$ is one if user $i$ terminates after session $t$, and zero otherwise. (93 moments)

6. The average session duration and the choice probability of each game mode at session $t$, and the average interval length between session $t$ and $t+1$ for each bin of users with different lifetime hours of play: $\mathbb{E}(x_{imt} \mid t, X_i)$, $\mathbb{E}(m_{it} \mid t, X_i)$ and $\mathbb{E}(d_{it} \mid t, X_i)$, where $X_i = \sum_{t' \leq \tilde{t}_i} \sum_m x_{imt'}, \tilde{t}_i$ is the number of sessions user $i$ played until she terminates. (5,250 moments)

7. The probability of termination for each bin of users with different average hours of play in the initial five sessions from purchase: $\mathbb{E}(term_{it} \mid t, X_{wi})$, where $X_{wi} = \sum_{t' = 1}^{5} \sum_m x_{imt'}$. (900 moments)

8. The probability that the game mode selected in session $t+1$ is different from the one at session $t$, for each game mode selected in session $t$: $Pr(m_{i,t+1} \neq m_{it} \mid t, m_{it})$. (120 moments)

9. The average duration of session $t$ conditional on weekday and weekend: $\mathbb{E}(x_{imt} \mid t, h_t)$. (60 moments)

10. The probability that users play multiple sessions within a day: $Pr(d_{it} = 0 \mid t)$. (30 moments)
In order to condition the moments on continuous variables (e.g. cumulative hours of play in the past sessions), I create 10 bins for each of them and compute conditional expectations in each bin. In addition, I create moment 3 and 4 for each subset of samples who survived at least 5 sessions, 10 sessions and 20 sessions. For some users at some sessions, the record of the session duration is missing. I exclude those sessions from the calculation of moments involving session durations. The missing duration is simply due to the technical difficulty of keeping track of the timestamp of play, and no systematic correlation between the pattern of missing data and the pattern of usage was observed. Hence, it does not introduce any bias in the estimates. I only use the first 30 sessions as the moments to ease the computational burden. As shown in Section 3, most of consumption dynamics, and hence the implied consumer learning, stabilize within the first 10 sessions. Hence, the variation from the initial 30 sessions is sufficient to identify both the mechanism behind learning and the distribution of the true match value.

The moments used to identify the adoption model are the rate of adoption at each week from week 1 through 16. In the model, the rate of adoption is calculated by the number of simulation paths making a purchase at each week, divided by the number of total simulation paths. In the data, the rate of adoption corresponds to the proportion of consumers making a purchase at each week in the data, multiplied by the market share of the product, whose derivation is described below.

Note that the moment conditions closely follow the identification argument provided earlier. For example, the evolution of belief is identified by the average duration in the initial session and the evolution of the variance of the session duration. This is accounted for through moment 1. The identification of the coefficient of risk aversion \( \rho \) comes from initial switching pattern, which is accounted for by moment 2. In general, the evolution of the behaviors across states are the identifying variations of the parameters, and hence all the moments are conditioned on the finest
possible bins of histories that maintain a certain number of observations in each bin.

**Derivation of market share**  Here I describe the derivation of market share of the product, which is used in computing the empirical rate of adoption. I assume that the total market size for the sports games is proportional to the share of sports games among all the videogame software sales. The average share of sports games between 2007 and 2015 is 16 percent. The total market size of all games is assumed to be equal to the installed base of PlayStation 3, PlayStation 4 and Xbox 360, which is 99.42 million units. Hence, the market size for sports games is $99.42 \times 0.16 = 15.91$ million. This number corresponds to $N$ in the current study. The sales of the focal game that is compatible to the above consoles are 4.47 million units. Therefore, the market share of this title is $4.47/15.91 = 0.281$, which I use as the market share of the game.

**Parametrization of $f$, $c$ and $\lambda$**  In this section, I provide details of parametrization employed in the current study. In the main text, I assumed that $f$ is parametrized as follows.

$$f(b_{it}) = E[\theta_{im}^c \mid \theta_{im} > 0, b_{it}].$$

In practice, I find that when $\rho$ is very small, $f$ defined as such is almost flat with respect to perceived match value, creating significant computational slowdown. In order to address this, I use a transformed version of it, specified as follows.

$$f(b_{it}) = (E[\theta_{im}^c \mid \theta_{im} > 0, b_{it}]P(\theta_{im} > 0 \mid b_{it}))^{\frac{1}{\rho}}.$$

This transformation makes the computation faster by an order of magnitude. Since this functional form does not have a closed form, in practice I compute $f(b_{it})$ for each $\rho$, $\mu$ and $\sigma^2$ using Gauss-

\footnote{Xbox One is excluded from the market size because the consumers with Xbox One are excluded from the current analysis.}
Legendre quadrature.

\( c \) is specified as a quadratic function of the past number of sessions as follows.

\[
c(\nu_{imt}, b_{it}) = (\gamma_1 - \gamma_2 f(b_{it}))\nu_{imt} + (\gamma_3 - \gamma_4 f(b_{it}))t - \gamma_5 \nu_{imt}^2.
\]

In order to capture the observed pattern that the evolution of usage intensity is heterogeneous, I allow the coefficients to depend on the perceived match value through \( f(b_{it}) \). Since allowing \( c \) to be a flexible function of \( b_{it} \) introduces an identification issue, I assume that \( c \) depends on \( b_{it} \) only through \( f(b_{it}) \), thereby maintaining identifiability.\(^{88}\)

Similarly, \( \lambda_1(\Omega_{it}) \), \( \lambda_2(\Omega_{it}) \) and \( \delta(\Omega_{it}) \) are parametrized as follows.

\[
\begin{align*}
\lambda_1(\Omega_{it}) &= \phi_{l1} + \phi_{l2} \left( \frac{\bar{\mu}_{it} - \bar{\mu}}{\bar{\sigma}} \right) - \left( \phi_{l3} - \phi_{l4} \left( \frac{\bar{\mu}_{it} - \bar{\mu}}{\bar{\sigma}} \right) \right) t + \phi_{l5} t^2, \\
\lambda_2(\Omega_{it}) &= \lambda_1(\Omega_{it}) + \phi_{l6}, \\
\delta(\Omega_{it}) &= \phi_{d1} + \phi_{d2} \left( \frac{\bar{\mu}_{it} - \bar{\mu}}{\bar{\sigma}} \right) - \phi_{d3} t + \phi_{d4} t^2,
\end{align*}
\]

where \( \bar{\mu}_{it} = \frac{\sum m \mu_{imt}}{M}, \bar{\mu} = \frac{\sum m \mu_m}{M}, \bar{\sigma} = \frac{\sum m \sigma_m}{M} \).

Both \( \lambda \) and \( \delta \) are quadratic with respect to the number of past sessions \( t \), and its intercept and slope depends on the current belief. The term \( \frac{\bar{\mu}_{it} - \bar{\mu}}{\bar{\sigma}} \) represents the normalized location of the belief of user \( i \) relative to the average belief of the population. This specification allows for a possibility that a user with higher perceived match value plays the game more frequently and has lower probability of termination, and she is increasingly so as she accumulates more experience. \( \lambda_2 \), the probability that a user plays multiple sessions in a day, is different from \( \lambda_1 \) only by an additive constant. As discussed in the previous section, this extra constant term captures that even heavy users do not often play multiple sessions within a day. In Table 7, I present the

\(^{88}\)The identification issue arises when both \( f \) and \( c \) are nonparametric. In practice, I impose a particular parametric form on \( f \) and hence having more flexible \( c \) function as a function of \( b_{it} \) is likely to maintain identification.
Table 7: Parameter estimates of decay, termination rate and intervals between sessions

| Parameter estimates of decay, termination rate and intervals between sessions |
|-------------------------------|-----------------|-----------------|-----------------|-----------------|
| \( c(\beta) \) | \( \gamma_1 \) | 0.042 | 0.003 | \( \lambda(\Omega) \) | \( \varphi_{11} \) | 0.456 | 0.001 |
| \( \gamma_2 \) | 0.025 | 0.005 | \( \varphi_{12} \) | 0.085 | 0.011 |
| \( \gamma_3 \) | 0.036 | 0.004 | \( \varphi_{13} \) | 0.065 | 6*10^{-5} |
| \( \gamma_4 \) | 0.072 | 0.001 | \( \varphi_{14} \) | 0.052 | 0.001 |
| \( \gamma_5 \) | 0.005 | 0.0002 | \( \varphi_{15} \) | 0.015 | 0.002 |
| \( \delta(\Omega) \) | \( \varphi_{d1} \) | 0.896 | 0.0001 | \( \varphi_{d2} \) | 0.024 | 0.001 |
| \( \varphi_{d3} \) | -0.017 | 5*10^{-8} | \( \varphi_{d4} \) | -0.008 | 2*10^{-8} |

Note: Standard error is calculated by 1,000 bootstrap simulations.

parameter estimates of the functions presented in this section.

F Other figures of model fit

In Figure 35, I show the evolution of the probability that each game mode is selected. Unlike Figure 9, this shows the choice probability at every single session, while not conditional on usage intensity. The choice pattern is tracked quite well. The choice probability of game mode 2 and 3 (1 and 4) are slightly underestimated (overestimated), but the magnitude of the error is small.

In Figure 36, I show the histogram of the duration of the very first session. The first session is chosen merely for expositional purpose and the fit for the other sessions are similar. Notably, the model tracks the shape of the distribution flexibly, even though the belief follows normal distribution. This is because of the existence of multiple types. The low segment creates a mass below 1 hour, and the high segment creates one around 2 hours.

G Model validation exercises

Model fit to holdout sample  Among 4,578 users in the data, a randomly selected 800 users are not used in the estimation and serve as holdout sample. In Figure 37, I present the model
Figure 35: The model fit of the game mode selection

Note: The choice probability is computed by the number of users who select each game mode at each session, divided by the number of users who remain active. The model counterpart is calculated using 50,000 simulation sequences.

fit with this sample. In order to ease comparison, the figures presented here are identical to the ones presented earlier, except that the data part is replaced by that of the holdout sample. The model maintains a good graphical fit to most of the data patterns. Adoption pattern presented in Figure 37 fits less well due to the existence of secondary peak around the 5th week that does not exist in the estimation sample. On the other hand, all usage patterns exhibit a reasonable fit. The average prediction hit rate for the game mode selection is 0.534, and LR+ is 3.438. Both of them are very close to corresponding ones from the estimation sample, which is 0.545 and 3.587, respectively. The model also maintains good out-of-sample predictive power for the session duration and the intervals between sessions. The ratio of standard error of prediction errors between out-of-sample and in-sample is 1.041 for the session duration, and 1.033 for the intervals between sessions. In other words, the magnitude of errors involved in the out-of-sample prediction of the durations and the intervals is only 4.1 and 3.3 percent higher. They

\[ \tilde{x}_{it} - E(x^* | \omega_{it}) \in \{v_{out}\}_{m=1}^{M}, h_t \), where \( \tilde{x}_{it} \) is the observed session duration. The error for intervals between sessions is defined similarly.\]
Figure 36: The model fit of the distribution of duration of the initial session
Note: Users whose duration is less than 6 minutes are dropped.

indicate that the model estimates capture the underlying mechanism common across all users, rather than merely reflect some particular variation of the estimation sample.

Model fit to users from another year Throughout the paper, I use a set of consumers making a purchase of version released in 2014 as an estimation sample. Another model validation exercise is to ask whether the model estimated as such can predict actions of users from some other years. While the quality of graphics and the real-league data contained in the game are updated every year, main features mostly stay the same across versions. Hence, it is reasonable to expect similar consumer behaviors across years. In order to explore this, I compare the model prediction with a set of first-time consumers making a purchase of version 2015. This set of users serves as an ideal holdout sample. Version 2015 features exactly the same number of game modes with the same name, allowing one to calculate the same measure as for version 2014. Moreover, the set of first-time users of version 2014 and that of version 2015 are mutually exclusive, providing an opportunity for pure out-of-sample fitting exercise.
Figure 37: Model fit to holdout sample

Note: The data part is calculated using holdout sample of 800 users. The model counterpart is identical to the figures presented before.
Figure 38: Model fit to user actions from another year

Note: The data part is calculated using 5,211 first-time users activating version 2015. The model counterpart is identical to the ones presented in the main text.
In Figure 38, I present the model fit with this sample. Overall graphical fit is surprisingly good. In particular, the fit of the session durations presented in Figure 38a is almost as good as that of the estimation sample. The pattern of game mode selection presented in Figure 38c is less ideal. This is reasonable because specific characteristics of each mode provided in version 2014 and 2015 can be slightly different from each other. It is also notable that the probability of termination is in general higher in version 2015, as represented in Figure 38d. Although this may indicate the existence of possible quality issue for version 2015, such speculation is completely outside the model. Finally, the good fit of adoption pattern presented in Figure 38f is remarkable given that the model prediction is calculated using the history of prices for version 2014. This indicates both the demand structure and the price pattern are quite similar between these two years. The average prediction hit rate for the game mode selection is 0.533, and LR+ is 3.431. These are very close to the in-sample ones. The ratio of standard errors of the prediction errors between out-of-sample and in-sample is 1.025 for the session duration, and 0.977 for the intervals between sessions. In other words, the errors involved in the out-of-sample prediction is 2.5 percent higher for the session durations, and 2.3 percent lower for the intervals. Overall, these results are indicative that the model is not merely useful to explain user behaviors from the specific version of the product I study, but also capture a universal tendency underlying the consumer behaviors.

H Importance sampling simulation to forecast individual usage pattern

In order to evaluate model prediction for each user’s actions, it requires an expectation of the action specified by the model over the unobserved belief, conditional on usage history. For example, prediction of a user’s game mode selection is given by $\mathbb{E}(P_m(\Omega_{it})|\{\nu_{imt}\}_{m=1}^M, h_t)$. I calculate this integral through simulation. In general, simulating a conditional expectation requires a suf-
ficiently large number of simulation draws that satisfy the conditioning requirement. This is because the simulated expectation is nothing but the average of such samples. However, as we simulate a sequence of actions with a long horizon, the number of possible histories increases quickly, making it difficult to secure sufficient sample size at each state.

In order to deal with this issue, I employ importance sampling approach suggested by Fernandez-Villaverde and Rubio-Ramirez (2007). The idea is that for each $i$ and at each period $t$, I replace simulation sequences that do not explain the observed actions at $t$ very well with ones that do it better. By repeating this replacement at every $t$, when one evaluates the conditional expectation at $t+1$, all sequences in the pool are likely to have the history that user $i$ actually follows. Hence, the pool consists of more sequences that satisfy the conditioning requirement.

Formally, the simulation proceeds as follows. Suppose that I intend to calculate $\mathbb{E}(P_m(\Omega_{it}) \mid \{\nu_{imt}\}_{m=1}^M, h_t)$ for all $t$. For each individual user in the data, I first draw her true type $\theta_{is}$ from the population distribution of match value, draw her initial signal $\tilde{\theta}_{i0s}$ and calculate the initial belief $b_{i0s}$. Subscript $s$ denotes each simulation draw. At each session $t$, given the drawn belief $b_{i0s}$ and other relevant state the model provides the probability that the mode user $i$ selected in the data is selected. I denote this probability by $P_m(b_{i0s}, \{\nu_{imt}\}_{m=1}^M, h_t \mid \theta_{is})$. By taking its average over simulation draws, I have an estimator of $\mathbb{E}(P_m(\Omega_{it}) \mid \{\nu_{imt}\}_{m=1}^M, h_t)$ at $t$.

Moving on to period $t+1$, in order to compute $P_m(b_{i,t+1,s}, \{\nu_{im,t+1}\}_{m=1}^M, h_{t+1} \mid \theta_{is})$, it requires $b_{i,t+1,s}$: a set of draws corresponding to the belief at period $t+1$. While crude frequency estimator suggests that I simply draw a set of signals $s_{its}$ for the chosen action and update $b_{its}$ to get $b_{i,t+1,s}$, importance sampling inserts an additional step; I replace sequences that exhibits low likelihood of explaining the user’s session $t$ action with the ones with high likelihood. Specifically, I first weight each of the draws $b_{its}$ by $\frac{P_m(b_{i0s}, \{\nu_{imt}\}_{m=1}^M, h_t \mid \theta_{is})}{\sum_{s'} P_m(b_{i0s'}, \{\nu_{imt}\}_{m=1}^M, h_t \mid \theta_{is'})}$. This weight corresponds to how well each draw $b_{its}$ explains the behavior at period $t$, relative to other draws $b_{i0s'}$. The draw

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90In the original paper, this method is called “Particle filtering”. 

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that fits the data well at period $t$ receives higher weight. I then re-draw the set of beliefs from this re-weighted set of $b_{its}$ in the same way as boot-strapping with replacement. Those with high weight may get drawn multiple times, while those with low weight may not get drawn at all. This re-draw provides a re-weighted set of $b_{its}$, from which I construct $b_{i,t+1,s}$ in the same way as in the crude estimator. This additional step guarantees that at each $t+1$, the belief $b_{i,t+1,s}$ is drawn such that $b_{its}$ explains the action taken at session $t$ well. Hence, it is more likely that many of those sequences satisfy the conditioning requirement $\{\{\nu_{imt}\}_{m=1}^M, h_t\}$ for all $t$.

I Assumptions necessary to validate the counterfactual exercises

In the counterfactual exercises, I draw on predictions about how consumers not making a purchase without trial would behave under a trial: a set of users I do not observe in the data. I estimate the population distribution of match values from its truncated subset and conduct the counterfactual based on it. The validity of the predictions based on such extrapolated distribution rests on two assumptions; (1) willingness to pay follows a parametric distribution specified in the model and hence I can calculate the population distribution of willingness to pay from the truncated one, and (2) all consumers follow the same decision rule in making purchase decisions specified in the model. In the environment studied, these assumptions are reasonable. In general, the precision of such extrapolation remains high as long as the magnitude of extrapolation is fairly small outside the sample range. In my study, willingness to pay on average increases by 15.3 percent due to trial, implying that those who are induced to purchase due to the trial are the ones whose willingness to pay is already close enough to the observed samples. Hence there is no reason to believe that their distribution of willingness to pay is drastically different, or they behave in a systematically different way.

91 Once $b_{its}$ is replaced by a new sequence, the corresponding true type $\theta_{is}$ is replaced as well, so that each re-drawn $b_{its}$ has a correct corresponding $\theta_{is}$.
In this section, I derive the optimal prices in the model presented in Section 2.

**Case 1: pay-per-use** In the case of pay-per-use, the firm can either choose \( q_2 = 1 \) or \( q_2 = 2 \) in the second session. I calculate the optimal \( q_1 \) corresponding to each scenario and then derive the global optimum.

First consider the case where the firm sets \( q_2 = 1 \). In that case, the firm’s total profit can be written as follows.

\[
\pi(q_1, q_2 = 1) = \pi_1(q_1) + \pi_2(q_1, q_2 = 1) = \left(\frac{2\alpha - q_1}{\alpha}\right)(q_1 + 1).
\]

Note that \( D(q_1) = 1 \) at \( q_1 = \alpha \). Hence, the firm has no incentive to set \( q_1 < \alpha \). The first-order condition is

\[
2\alpha - 1 - 2q_1 = 0 \Rightarrow q_1 = \frac{2\alpha - 1}{2}.
\]

However, \( \frac{2\alpha - 1}{2} < \alpha \). Hence the firm’s optimal price is bounded below by \( q_1 = \alpha \). The associated profit is \( \pi(\alpha, 1) = \alpha + 1 \).

Next consider the case of \( q_2 = 2 \). In this case, the firm’s total profit can be written as follows.

\[
\pi(q_1, q_2 = 2) = \pi_1(q_1) + \pi_2(q_1, q_2 = 2) = \left(\frac{2\alpha - q_1}{\alpha}\right)\left(q_1 + 2p_L + \frac{q_1}{\alpha}(p_H - p_L)\right).
\]

By taking the first-order condition, we have

\[
\alpha + p_H - 2p_L - \left(\frac{p_H - p_L}{\alpha} + 1\right)q_1 = 0 \Rightarrow q_1 = \frac{\alpha(\alpha + p_H - 2p_L)}{\alpha + p_H - p_L}.
\]
Again, $\frac{\alpha(\alpha + p_H - 2p_L)}{\alpha + p_H - p_L} < \alpha$, and hence the firm’s optimal price is bounded below by $q_1 = \alpha$. The associated profit is $\pi(\alpha, 2) = \alpha + p_H + p_L$.

To summarize, the firm’s profit is $\pi_{ppu} = \max\{\pi(\alpha, 1), \pi(\alpha, 2)\} = 1 + \max\{p_H + p_L\}$. The firm chooses $q_2 = 1$ if and only if $p_H + p_L < 1$.

**Case 2: outright sale** In the case of outright sale, the firm’s total profit can be written as follows.

$$\pi(q_b) = q_b D(q_b),$$

where $D(q_b) = \begin{cases} \frac{\alpha(3+p_H)-q_b}{\alpha(1+p_H-p_L)} & \text{if } q_b \in [\alpha(2 + p_L), \alpha(3 + p_H)], \\ 0 & \text{if } q_b > \alpha(3 + p_H), \\ 1 & \text{if } q_b < \alpha(2 + p_L). \end{cases}$

First, assume that $q_b \in [\alpha(2 + p_L), \alpha(3 + p_H)]$ and solves the first-order condition.

$$\alpha(3 + p_H) - 2q_b = 0 \Rightarrow q_b = \frac{\alpha(3 + p_H)}{2}.$$  

Clearly, $\frac{\alpha(3+p_H)}{2} < \alpha(3 + p_H)$ is satisfied. However, $\frac{\alpha(3+p_H)}{2} > \alpha(2 + p_L)$ requires $p_H > 1 + 2p_L$. Since $p_H \in [0, 1]$, this condition is never satisfied. Thus we conclude that there is no optimum such that $q_b > \alpha(2 + p_L)$. Hence $q_b = \alpha(2 + p_L)$ is the unique optimum, with the associated profit of $\alpha(2 + p_L)$.

**K Alternative subscription strategies in the counterfactual**

In this study, I consider subscription strategies that take the form of “X-session package”; each subscription period is tied with the number of sessions that consumers can play, and consumers
make active decisions to renew the subscription at the end of each subscription period. While this form of subscription is common among online game providers, other firms offer another form of subscription where each subscription period is tied with calendar days, such as weekly subscription or monthly subscription. When we study the effect of consumer learning, focusing on the session-based subscription allows us to reduce contamination from factors outside of the model for two reasons. First, calendar-day based subscription creates incentives of consumers to intertemporally reallocate their consumption to reduce subscription costs. For example, consumers may only sign up for the monthly subscription of a videogame only during the month that they have much time to play it. Alternatively, consumers may squeeze in more consumption near the end of one subscription period and cancel the subscription afterward. Such intertemporal substitution incentive is not identifiable from the current data, where only outright sale is observed. Second, such calendar-day-based subscription often automatically renews without active decision of consumers. Such consumers’ inattention or inertia of enrollment as a default choice is again not identifiable from the current data. Allowing for the existence of these contaminating factors outside of the model would significantly bias the counterfactual results. Hence I focus on session-based subscription where neither of these issues is present.

L Proof of Proposition 2

The objects to identify are \( V_m(\theta_i, m^*_{i,t-1}) \) for every \( \theta_i \) and \( m^*_{i,t-1} \), and \( f(\theta_i) \). Consider a consumer with a sequence of choices \( m^*_i \). During that sequence, for each option \( m \), we observe infinitely many periods where \( m^*_{it} = m \). This is because Assumptions 1 and 2 ensure that the state is positive recurrent; the unbounded support of \( \epsilon_{imt} \) (Assumption 1) implies that at all period \( t \), each consumer selects every option with strictly positive probability regardless of her \( V_m(\theta_i, m^*_{i,t-1}) \). Hence the expected duration between her choices of option \( m \) is finite for all \( m \); the state follows
a positive recurrent process.

Denote by $t_{im}$ the set of periods that the consumer chooses option $m$. Then the set of actions taken at periods $t_{im} + 1$ corresponds to the set of actions taken by some type of consumer at state $m_{i,t-1}^* = m$. For now, assign her an arbitrary type $\theta_i$. Because we have infinitely many periods in the set $t_{im} + 1$, we have a distribution of actions $m^*(\theta_i, m_{i,t-1}^*)$ for every $m_{i,t-1}^* = m$.

First consider the distribution of $m^*(\theta_i, M)$: actions at periods $t_{iM} + 1$. Because of the normalization, we know that $V_M(\theta_i, M) = 0$. Hence, consumers choose actions by comparing $V_m(\theta_i)$ and zero. Accordingly, the probability that each $m$ is selected identifies $V_m(\theta_i)$ for each $m$. For example, if $\epsilon_{imt}$ follows a Type 1 Extreme Value distribution, then $P(m_{it}^* = m \mid m_{i,t-1}^* = M) = \frac{\exp(V_m(\theta_i))}{\sum_{m'} \exp(V_{m'}(\theta_i))}$.

Having identified $V_m(\theta_i)$, now consider $t_{i1} + 1$. Now the consumer’s utility from choosing option 1 is $V_1(\theta_i) + \Delta V_1(\theta_i)$ while that from choosing others remain the same $V_m(\theta_i)$. Hence the probability that she chooses option 1 at periods $t_{i1} + 1$ identifies $\Delta V_1(\theta_i)$. In the same way, we can identify $\Delta V_m(\theta_i)$ for each $m$. This completes the identification of utility of consumers that follow $m_{i}^*$.

Having identified utility of every consumer at every state in this way, we can assign a unique type to a set of consumers who share the identical utility for all $m$ at all $m_{i,t-1}^*$. This identifies $f(\theta_i)$ and completes the identification of the model.

M Proof of Proposition 3

Under assumption 5, the objects to identify are $V_m(\theta_i, h_{imt})$ for every $\theta_i$ and $h_{imt}$, and $f(\theta_i)$. Under assumption 6 and 7, the objects to identify are $V_m(\theta_i, \bar{x}_{imt})$ and $\Delta V(\theta_i, \bar{x}_{i,-m,t})$ for every $\theta_i$, $\bar{x}_{imt}$ and $\bar{x}_{i,-m,t}$, and $f(\theta_i)$.

\footnote{Recall that $\theta_i$ is a mere label and has no cardinal impact on $V_m$.}
In the same way as in the previous nonidentification result, first assign one \( \theta_i \) to each individual who follows the same \( m_i^* \). Then for each set of consumers with assigned type \( \theta_i \), we observe one sequence of discrete actions \( m_i^* \) and consumption intensity \( \tilde{x}_m(\theta_i, h_{it}) \) at each \( h_{it} \) and \( m \) on the path of \( m_i^* \).

In order to understand the role that Assumptions 5 through 7 play, first note that the observation of \( \tilde{x}_m(\theta_i, h_{it}) \) immediately identifies \( x_m^*(\theta_i, h_{it}) \). This is because:

\[
E(\tilde{x}_m(\theta_i, h_{it})|\theta_i, h_{it}) = x^*(\theta_i, h_{it}) + E(e_{imt} | \theta_i, h_{it}) = x^*(\theta_i, h_{it}).
\]

Moreover, if we have \( x_m^*(\theta_i, h_{it}) \) for all \( \theta_i \) at all \( m \) and \( h_{it} \), then the identification of \( V_m(\theta_i, h_{it}) \) is a straightforward application of a standard discrete choice model, where different consumers face different “product attribute” \( x_m^*(\theta_i, h_{it}) \) for each \( m \) and choose one option \( m_i^* \). The relationship between \( x_{imt}^* \) and \( m_i^* \) across consumers with different \( \theta_i \) at each \( h_{it} \) identifies \( g_{m,h_{it}} \), and hence identifies \( V_m(\theta_i, h_{it}) = g_{m,h_{it}}(x_m^*(\theta_i, h_{it})) \).

Hence, if we observe \( \tilde{x}_m(\theta_i, h_{it}) \) for all \( \theta_i \) at all \( m \) and \( h_{it} \), we can identify the universe of \( V_m(\theta_i, h_{it}) \).

However, we only observe \( \tilde{x}_{imt} \) at limited state realizations. Accordingly, we cannot identify the universe of \( x^*(\theta_i, h_{it}) \) directly from the data. Assumptions 5, 6 and 7 impose restrictions on \( V_m \), so that such limited information about \( x^*(\theta_i, h_{it}) \) is sufficient to recover \( V_m(\theta_i, h_{it}) \) for all \( m, \theta_i \) and \( h_{it} \). I first present the identification argument when Assumption 5 holds. I then consider the cases of Assumption 6 and 7 later.

**Case 1: Assumptions 1, 3, 4 and 5 hold** Assumption 5 provides the most straightforward identification of \( V_m(\theta_i, h_{it}) \) at all \( \theta_i, m \) and \( h_{it} \) by assuming \( V_m(\theta_i, h_{it}) = V_m(\theta_i, h_{imt}) \). For each \( \theta_i \) type consumer, we observe \( \tilde{x}_m(\theta_i, h_{imt}) \) at each \( h_{imt} = \{\tilde{x}_{im1}, ..., \tilde{x}_{im\tilde{t}}\} \). Note that because \( \tilde{x}_{imt} \) involves an idiosyncratic component, for a given \( m_i^* \), each consumers’ \( \tilde{x}_{imt} \) at each \( t \) can

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\(^{93}\)Note that although we assign \( \theta_i \) according to each consumer’s infinite sequence \( m_i^* \), at any finite \( t \), consumers with different types can still follow the same \( h_{it} \).
vary. Hence, for the set of type $\theta_i$ consumers, different individual can follow different history of $h_{imt}$. As a result, for each $\theta_i$ we observe $\tilde{x}_m(\theta_i, h_{imt})$ at the universe of $h_{imt}$. Since $V_m(\theta_i, h_{imt})$ only depends on $h_{imt}$, we can identify the universe of $V_m(\theta_i, h_{imt})$ according to the argument above.

In order to see why Assumption 5 allows us to observe the universe of $\tilde{x}_{imt}$, consider a consumer with a sequence $m_i^*$, such that he selected product $m'$ in the very first period. By construction, when he consumes product $m$ for the first time, he has already consumed $m'$. Accordingly, if we let $V_m$ depend on the entire history $h_{it}$, we only observe $\tilde{x}_m(\theta_i, h_{it})$ at $h_{it}$ that includes he record of $m'$ and identify $V_m(\theta_i, h_{it})$ at such $h_{it}$ only. Because type $\theta_i$ is assigned based on $m_i^*$, among $\theta_i$ type consumers, everyone follows the same sequence and hence we never know $V_m(\theta_i, h_{it})$ of $\theta_i$ type without experience of $m'$. Such an absence of visit to some $h_{it}$ occurs because past consumption of $m'$ influences utility of $m$. If only the consumption of $m$ influences utility of future $V_m$, then regardless of her consumption experience of $m'$, when the consumer chooses $m$ for the first time her utility remains $V_m(\theta_i, \phi)$.

This argument establishes identification of $V_m(\theta_i, h_{it})$ at all $m$, $\theta_i$ and $h_{it}$ for a given assignment rule of $\theta_i$, which is to assign $\theta_i$ according to $m_i^*$ of each consumer. It remains to show that defining $\theta_i$ this way corresponds to $\theta_i$ defined in the model. In general, different consumers with the same $\theta_i$ can follow different sequence of discrete choices due to randomness by $\epsilon_{imt}$, and as a result, consumers with different $\theta_i$ can take the same path. Hence, in order to justify the current assignment rule of $\theta_i$, we need to rule out he possibility that one sequence of discrete choices $m_i^*$ maps into multiple $\theta_i$. Assumption 4 ensures that it is indeed the case. Assumption 4 states that for each $\theta_i$, there exists a sequence of discrete actions $m_i^*$ such that only $\theta_i$ type consumer takes it in the long-term. In other words, for any finite $t$, consumers with different $\theta_i'$ may take the sequence $m_i^*$, but as we take $t \to \infty$, other consumers move away from this sequence fast.
enough, leaving only $\theta_i$ consumers. Hence, by taking the set of consumers following $m_i^*$, we can take the set of consumers with type $\theta_i$ with probability approaching to one. This completes the identification of $V_m(\theta_i, h_{it})$ and $f(\theta_i)$ under assumptions 1, 3, 4 and 5.

**Case 2: Assumptions 1, 3, 4 and 6 or 7 hold** In the case of assumption 6 and 7, we cannot identify $x^*_m(\theta_i, h_{it})$ independently from $g_{m,h_{it}}$. Hence, we first fix any monotone $g_{m,h_{it}}$ for all $m$ and $h_{it}$ as given. Then, through the relationship that $x^*_m(\theta_i, \bar{x}_{imt}, \bar{x}_{i,-m,t}) = g_{m,h_{it}}^{-1}(V_m(\theta_i, \bar{x}_{imt}) + \Delta V(\theta_i, \bar{x}_{i,-m,t}))$, we can identify $V_m(\theta_i, \bar{x}_{imt}) + \Delta V(\theta_i, \bar{x}_{i,-m,t})$ at states $\{\bar{x}_{imt}, \bar{x}_{i,-m,t}\}$ that type $\theta_i$ consumers visit. Note that the sufficient statistics condition makes the order in which each option is selected irrelevant; given that total consumption intensity is $\{\bar{x}_{imt}, \bar{x}_{i,-m,t}\}$, other factors do not influence utility, such as the current value of $t$ or and how many times each option was selected in the past.

Under this assumption, among consumers following the same $m_i^*$ and hence getting assigned the same $\theta_i$, different consumers face different $\{\bar{x}_{imt}, \bar{x}_{i,-m,t}\}$. This is because consumption intensity at each period $\bar{x}_{imt}$ involves idiosyncratic shock. In particular, by considering the set of type $\theta_i$ consumers at three different points in time, we can identify their $V_m$ at majority of state realizations. First, consider periods where they have already consumed all the available options at least once in the past.\footnote{Because of Assumption 1, all types of consumers consume all options infinitely often during their sequence $m_i^*$.} In those periods, we observe $\bar{x}_m(\theta_i, \bar{x}_{imt}, \bar{x}_{i,-m,t})$ for all $\bar{x}_{imt} > 0$ and $\bar{x}_{i,-m,t} > 0$ and hence identify $V_m(\theta_i, \bar{x}_{imt}) + \Delta V(\theta_i, \bar{x}_{i,-m,t})$ at all $\bar{x}_{imt} > 0$ and $\bar{x}_{i,-m,t} > 0$.

Second, consider their consumption intensity at the very first period. Denote by $m_{\theta_i}$ the option that that type of consumers selected at the first period. Since no previous experience exists at the initial period, using the observation of $\bar{x}_{im_{\theta_i},0}$, we can identify $V_m(\theta_i,0)$ for $m = m_{\theta_i}$.

Finally, for $m \neq m_{\theta_i}$, when type $\theta_i$ consumer selected that $m$ for the first time, they have already experienced at least one other option, but not the option $m$ itself yet. Hence, the usage intensity of option $m \neq m_{\theta_i}$ when it was selected for the first time, $\bar{x}_m(\theta_i,0, \bar{x}_{i,-m,t})$ maps into
\[ V_m(\theta_i, 0) + \Delta V(\theta_i, \bar{x}_{i,-m,t}) \text{ for } \bar{x}_{i,-m,t} > 0. \]

In summary, from the observation of \( \bar{x}_{imt} \) of type \( \theta_i \) consumers at three different points in time, we can identify the following utility components of that type of consumers.

1. \( V_m(\theta_i, \bar{x}_{imt}) + \Delta V(\theta_i, \bar{x}_{i,-m,t}) \) for all \( \theta_i, m, \bar{x}_{imt} > 0 \) and \( \bar{x}_{i,-m,t} > 0 \),
2. \( V_m(\theta_i, 0) \) for all \( \theta_i \) and \( m = m_{Q\theta_i} \),
3. \( V_m(\theta_i, 0) + \Delta V(\theta_i, \bar{x}_{i,-m,t}) \) for all \( \theta_i, m \neq m_{Q\theta_i} \) and \( \bar{x}_{i,-m,t} > 0 \).

Indeed, we have identified all the components of utility, except that some \( V_m(\theta_i, \bar{x}_{imt}) \) and \( \Delta V(\theta_i, \bar{x}_{i,-m,t}) \) are only identified as a sum. The third condition in Assumption 6 and 7 provides different approaches to break the summation and establish identification of each components.

In the case of Assumption 6, for each \( \theta_i \) we consider period \( i'(m_0, \theta_i) \), where \( \theta_i \) consumers select \( m_{Q\theta_i} \) again after they consumed some \( m' \neq m_{Q\theta_i} \) at least once. Then, order the set of consumers so that they follow a decreasing order in \( \bar{x}_{i,m_0,i'(m_0,\theta_i)} \) and consider the sequence of \( x^*_m(\theta_i, \bar{x}_{i,m_0,i'(m_0,\theta_i)}, \bar{x}_{i,-m_0,i'(m_0,\theta_i)}) \) as \( \bar{x}_{i,m_0,i'(m_0,\theta_i)} \to 0 \).

\[
\lim_{\bar{x}_{i,m_0,i'(m_0,\theta_i)} \to 0} x^*_m(\theta_i, \bar{x}_{i,m_0,i'(m_0,\theta_i)}, \bar{x}_{i,-m_0,i'(m_0,\theta_i)})
= \lim_{\bar{x}_{i,m_0,i'(m_0,\theta_i)} \to 0} g^{-1}(V_m(\theta_i, \bar{x}_{i,m_0,i'(m_0,\theta_i)}) + \Delta V(\theta_i, \bar{x}_{i,-m_0,i'(m_0,\theta_i)}))
= g^{-1}(V_m(\theta_i, 0) + \Delta V(\theta_i, \bar{x}_{i,-m_0,i'(m_0,\theta_i)})).
\]

The first equality is by definition of \( V_m(\theta_i, h_{it}) \), and the second equality is due to the continuity of \( V_m \) and \( g_{m, \bar{x}_{imt}, \bar{x}_{i,-m,t}} \) at \( \bar{x}_{imt} = 0 \). This implies that the difference between this limit and \( V_{m_0}(\theta_i, 0) \) identified earlier identifies \( \Delta V(\theta_i, \bar{x}_{i,-m_0,i'(m_0,\theta_i)}) \). The separation between \( \Delta V \) and \( V_m \) completes the identification.

Intuitively, this limit is utilizing the idea that those who consumed \( m_0 \) in the past but very
little amount should have the same utility as those who never consumed $m_0$. Hence, if their utility from consuming $m_0$ at some later $\tilde{t}(m_0, \theta_i)$ is different from that at the initial period, the difference is attributable to the spill-over effect $\Delta V$.

Alternatively, if Assumption 7 holds, then take $\tilde{t}(m_1, \theta_i)$ where type $\theta_i$ consumers select any $m \neq m_0$ for the first time. I denote such $m$ by $m_1$. Because $\tilde{t}(m_1, \theta_i)$ is the first time they consume any $m \neq m_0$, it follows that $\bar{x}_{i,m_1,\tilde{t}(m_1,\theta_i)} = 0$ and $\bar{x}_{i,m_1,\tilde{t}(m_1,\theta_i)} = \bar{x}_{i,m_0,\tilde{t}(m_1,\theta_i)}$. Hence the observed consumption intensity of product $m_1$ at $\tilde{t}(m_1, \theta_i)$ can be denoted by $\bar{x}_{m_1}(\theta_i, 0, \bar{x}_{i,m_0,\tilde{t}(m_1,\theta_i)})$.

Now order each individual, so that their $\bar{x}_{i,m_0,\tilde{t}(m_1,\theta_i)}$ follows a decreasing order, and consider sequence of $x_{m_1}^*(\theta_i, 0, \bar{x}_{i,m_0,\tilde{t}(m_1,\theta_i)})$ as $\bar{x}_{i,m_0,\tilde{t}(m_1,\theta_i)} \to 0$.

\[
\begin{align*}
\lim_{\bar{x}_{i,m_0,\tilde{t}(m_1,\theta_i)} \to 0} x_{m_1}^*(\theta_i, 0, \bar{x}_{i,m_0,\tilde{t}(m_1,\theta_i)}) &= \lim_{\bar{x}_{i,m_0,\tilde{t}(m_1,\theta_i)} \to 0} g^{-1}(V_{m_1}(\theta_i, 0) + \Delta V(\theta_i, \bar{x}_{i,m_0,\tilde{t}(m_1,\theta_i)})) \\
&= g^{-1}(V_{m_1}(\theta_i, 0) + \Delta V(\theta_i, 0)) \\
&= g^{-1}(V_{m_1}(\theta_i, 0)).
\end{align*}
\]

The first equality is by definition of $V_m$. The second equality is because of continuity of $g_{m,\bar{x}_{i,m_0,\tilde{t}(m_1,\theta_i)}}$ and $\Delta V$ at $\bar{x}_{i,m_0,\tilde{t}(m_1,\theta_i)} = 0$ specified in Assumption 7. The final equality holds because $\Delta V(\theta_i, 0) = 0$. This implies that by subtracting this limit by $V_{m_1}(\theta_i, 0) + \Delta V(\theta_i, \bar{x}_{i,m_1,\tilde{t}(m_1,\theta_i)})$ identified above, we can identify $\Delta V(\theta_i, \bar{x}_{i,m_1,\tilde{t}(m_1,\theta_i)})$ and separate identification is done.

The intuition is that for those who consumed $m_0$ at the initial period but very little should have almost zero spill-over utility change when they consume $m_1$ at period $\tilde{t}(m_1, \theta_i)$. As a result, by comparing such consumers’ utility with others who consumed significant amount of $m_0$, we can identify the magnitude of utility spill-over $\Delta V$.

Once all $V_m(\theta_i, \bar{x}_{i,m_0})$ and $\Delta V(\theta_i, \bar{x}_{i,m_1,\tilde{t}(m_1,\theta_i)})$ are identified at all $\theta_i$, $\bar{x}_{i,m_0}$ and $\bar{x}_{i,m_1,\tilde{t}(m_1,\theta_i)}$, $g_{m,\bar{x}_{i,m_0,\tilde{t}(m_1,\theta_i)}}$ and
is identified by utilizing the same variations as above. The only difference is that unlike the case of Assumption 5, we have not identified $x_m^*(\theta_i, \bar{x}_{imt}, \bar{x}_{i,-m,t})$ at all $\{\bar{x}_{imt}, \bar{x}_{i,-m,t}\}$. Instead, for each type $\theta_i$ consumer, we have identified $x_m^*(\theta_i, \bar{x}_{imt}, \bar{x}_{i,-m,t})$ only at each state that type $\theta_i$ consumers visited, and for other states that they did not visit, we instead have $V_m(\theta_i, \bar{x}_{imt})$ and $\Delta V(\theta_i, \bar{x}_{i,-m,t})$ through the procedure above. Hence, the discrete choice problem we consider at each state $\{\bar{x}_{imt}, \bar{x}_{i,-m,t}\}$ is such that for alternatives whose $x_m^*(\theta_i, \bar{x}_{imt}, \bar{x}_{i,-m,t})$ is known, we specify the utility by $g_m,\bar{x}_{imt},\bar{x}_{i,-m,t}(x_m^*(\theta_i, \bar{x}_{imt}, \bar{x}_{i,-m,t}))$, and the utility from other alternative is specified by $V_m(\theta_i, \bar{x}_{imt}) + \Delta V(\theta_i, \bar{x}_{i,-m,t})$ identified above. This discrete choice problem associates $x_m^*(\theta_i, \bar{x}_{imt}, \bar{x}_{i,-m,t})$ and $m^*(\theta_i, \bar{x}_{imt}, \bar{x}_{i,-m,t})$ and identify $g_m,\bar{x}_{imt},\bar{x}_{i,-m,t}$. This completes the identification of the model.

Theoretically, this identification strategy can be generalizable in several aspects. First, the identification based on Assumption 7 goes through even when we make $\Delta V$ each option $m$ specific: $\Delta V_m(\theta_i, \bar{x}_{i,-m,t})$. In that case, instead of taking only $m_1$, for all $m \neq m_0$ we take the first time that option is selected and do the exercise above to identify $\Delta V_m(\theta_i, \bar{x}_{i,-m,t})$. However, such implementation may not be practically useful, for some $m \neq m_0$ may be selected much later in the sequence and by that time, taking $\bar{x}_{i,-m,t} \to 0$ is impractical. Second, although in Proposition 3 we assumed that history $h_{it}$ influences utility only through $\bar{x}_{imt}$ and $\bar{x}_{i,-m,t}$, we can instead allow the entire history $h_{imt}$ and $h_{i,-m,t}$ to influence utility, where $h_{i,-m,t}$ is defined analogously to $h_{imt}$. In that case, continuity of $V_m$ and $g$ in the case of Assumption 6, and that of $\Delta V$ and $g$ in the case of Assumption 7, has to be with respect to all arguments in the vector $h_{imt}$ and $h_{i,-m,t}$.
Proof of Lemma 1

The condition implies that for a given history $\bar{m}_{\bar{t}-1}^*$ at period $\bar{t}$, for all $t > \bar{t}$ and for all $\theta'_i \neq \theta_i$,

$$\frac{\exp(V_{m_i^*}(\theta_i, \bar{m}_{t-1}^*))}{\exp(\max_{m' \neq m_i^*} V_{m'}(\theta_i, \bar{m}_{t-1}^*))} > \frac{\exp(V_{m_i^*}(\theta'_i, \bar{m}_{t-1}^*))}{\exp(\min_{m' \neq m_i^*} V_{m'}(\theta'_i, \bar{m}_{t-1}^*))}.$$

This in turn implies that

$$\frac{\exp(V_{m_i^*}(\theta_i, \bar{m}_{t-1}^*))}{\sum_{m' \neq m_i^*} \exp(V_{m'}(\theta_i, \bar{m}_{t-1}^*))} > \frac{\exp(V_{m_i^*}(\theta'_i, \bar{m}_{t-1}^*))}{(M - 1) \exp(\max_{m' \neq m_i^*} V_{m'}(\theta'_i, \bar{m}_{t-1}^*))}.$$

Rearranging LHS and RHS yields

$$\exp(V_{m_i^*}(\theta_i, \bar{m}_{t-1}^*)) \sum_{m' \neq m_i^*} \exp(V_{m'}(\theta'_i, \bar{m}_{t-1}^*)) > \exp(V_{m_i^*}(\theta'_i, \bar{m}_{t-1}^*)) \sum_{m' \neq m_i^*} \exp(V_{m'}(\theta_i, \bar{m}_{t-1}^*)).$$

Adding $\exp(V_{m_i^*}(\theta_i, \bar{m}_{t-1}^*)) \exp(V_{m_i^*}(\theta'_i, \bar{m}_{t-1}^*))$ to both sides yields $P(m_i^* | \theta_i, \bar{m}_{t-1}^*) > P(m_i^* | \theta'_i, \bar{m}_{t-1}^*)$ for all $t$ and $\theta'_i$. This is a sufficient condition for $P(\theta_i | \bar{m}_{t}^*) = \frac{P(\bar{m}_{t}^* | \theta_i) \sum_{\theta'_i} P(\bar{m}_{t}^* | \theta'_i)}{\sum_{\theta'_i} P(\bar{m}_{t}^* | \theta'_i)}$ to be monotonically increasing after $\bar{t}$, which ensures that $P(\theta_i | \bar{m}_{t}^*) \rightarrow 1$ as $t \rightarrow \infty$. 

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References


