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Essays In Macroeconomics With Financial Frictions

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Abstract

This dissertation consists of two chapters on macroeconomics with financial frictions. The first chapter studies the role of firm heterogeneity in the transmission of financial shocks to the real economy. Evidence from the recent European debt crisis shows that firms responded differently to the severe credit tightening that occurred during this period, where smaller ones adjusted their balance sheets more aggressively and performed better in economies with a more skewed firm size distribution. A model of heterogeneous firms, that face financial frictions (defaultable debt and costly equity issuance), a financial intermediation sector, and a sovereign, is proposed to explain these facts. Financial frictions are key because they generate financing structures that depend on firm size, where small firms rely more on equity than debt, which is relatively more costly. Sufficiently large increases in public debt trigger a binding lending constraint for the intermediaries that cause a crowding out of private lending and leads smaller firms to adjust more than large firms. Quantitative results show that firm heterogeneity has aggregate effects and that the model, calibrated to match Spanish firm-level data, is consistent with the empirical facts during the crisis. The second chapter studies the positive and normative implications of "liability dollarization", the intermediation of capital inflows in units of tradables into domestic loans in units of aggregate consumption, on Sudden Stops models. Liability dollarization adds three important effects driven by real-exchange-rate fluctuations that alter standard models of Sudden Stops significantly: Changes on the debt repayment burden, on the price of new debt, and on a risk-taking incentive. The optimal policy under commitment is time-inconsistent, follows a complex non-linear structure, and shows that when domestic credit or capital inflows taxes are present, capital controls are not justified. Quantitatively, an optimized pair of constant taxes on domestic debt and capital inflows makes crises slightly less likely and yields a small welfare gain, but other pairs reduce welfare sharply. For high effective debt taxes, capital controls and domestic debt taxes are equivalent, and for low ones welfare is higher with higher taxes on domestic debt than on capital inflows.

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ESSAYS IN MACROECONOMICS WITH FINANCIAL FRICTIONS

To Magdalena and Juan Ignacio

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ABSTRACT

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Enrique G. Mendoza

This dissertation consists of two chapters on macroeconomics with financial frictions. The first chapter studies the role of firm heterogeneity in the transmission of financial shocks to the real economy. Evidence from the recent European debt crisis shows that firms responded differently to the severe credit tightening that occurred during this period, where smaller ones adjusted their balance sheets more aggressively and performed better in economies with a more skewed firm size distribution. A model of heterogeneous firms, that face financial frictions (defaultable debt and costly equity issuance), a financial intermediation sector, and a sovereign, is proposed to explain these facts. Financial frictions are key because they generate financing structures that depend on firm size, where small firms rely more on equity than debt, which is relatively more costly. Sufficiently large increases in public debt trigger a binding lending constraint for the intermediaries that cause a crowding out of private lending and leads smaller firms to adjust more than large firms. Quantitative results show that firm heterogeneity has aggregate effects and that the model, calibrated to match Spanish firm-level data, is consistent with the empirical facts during the crisis. The second chapter studies the positive and normative implications of “liability dollarization”, the intermediation of capital inflows in units of tradables into domestic loans in units of aggregate consumption, on Sudden Stops models. Liability dollarization adds three important effects driven by real-exchange-rate fluctuations that alter standard models of Sudden Stops significantly: Changes on the debt repayment burden, on the price of new debt, and on a risk-taking incentive. The optimal policy under commitment is time-inconsistent, follows a complex non-linear structure, and shows that when domestic credit or capital inflows taxes are present, capital controls are not justified. Quantitatively, an optimized pair of constant taxes on domestic debt and capital inflows makes crises slightly less likely and yields a small welfare gain, but other pairs reduce welfare sharply. For high effective debt taxes, capital controls and domestic debt taxes are equivalent, and for low ones welfare is higher with higher taxes on domestic debt than on capital inflows.

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Chapter 1

Firm Heterogeneity & the Transmission of Financial Shocks During the European Debt Crisis

This paper studies the role of firm heterogeneity in the transmission of financial shocks and its macroeconomic implications. The analysis focuses on a particular event, namely the severe tightening in credit conditions for firms during the recent European debt crisis. This event was a natural experiment that provided abundant firm-level data on the effects of a sharp, sudden deterioration of credit-market conditions on non-financial firms in the countries most affected by the crisis: Greece, Ireland, Italy, Portugal and Spain (GIIPS). Existing studies have shown that an important channel explaining the deterioration of the financing conditions of firms during this event was the "crowding-out" channel, as documented by Broner et al. (2014) and more recently by Acharya et al. (2018) and Becker and Ivashina (2018). Increased government borrowing and bank holdings of public debt in these countries generated a crowding-out of private lending, reducing loanable funds available to firms and increasing their financing costs.[1]

The first part of this paper uses the Amadeus database to conduct an empirical analysis of the effects of the financial shock described above on non-financial firms in the GIIPS countries. The empirical analysis yields two important new findings regarding the relevance of firm heterogeneity in the responses of firms to the observed surge in sovereign debt: First, smaller firms adjusted their sales, liabilities, assets, and employment more than large firms. Second, smaller firms experienced smaller adjustments in these variables in countries with more skewed firm size distributions. The data also show that financing costs, measured as the ratio of debt service to total debt, increased more for smaller firms.

The second part of the paper proposes a model to explain the above facts and derives its quantitative implications. The model has three main components. First, a heterogeneous firms sector. Firms are heterogeneous in terms of productivity, capital, and debt, and face financial frictions in the form of defaultable debt and costly equity issuance, modeled as imperfect substitutes. These features endogenously generate a time-varying firm distribution. The second component is a financial intermediation sector. A representative financial intermediary lends to firms and the government, facing an occasionally binding constraint on loanable funds. The last element is the government. For simplicity, aggregate fiscal shocks and a fiscal reaction function drive public debt dynamics. The firms' financial frictions generate different financing structures, with small firms relying relatively more on equity than large firms, where size is defined by a firm's capital stock. The asymmetry in the responses between small and large firms arises from the fact that the former are more financially constrained, so they need to issue equity more often, and thus face a more expensive financing mix.

1. Another channel through which the government affected the financing conditions of firms during this episode is the pass-through of sovereign risk, as studied by Gennaioli et al. (2014a), Gennaioli et al. (2014b), Bottero et al. (2015), Sosa-Padilla (2015), Bocola (2016). According to this channel, increased sovereign risk negatively affected the balance sheet of intermediaries, who held sovereign bonds, generating a worsening in the financing conditions of firms.

When public debt is sufficiently high, the bank's constraint binds, so the amount of resources available for firm lending falls (i.e., there is a crowding-out effect), increasing firms' borrowing costs. Since firms have different financing structures depending on their size, due to financial frictions, they respond differently. Because equity issuance is increasingly costly, small firms switching (partially or entirely) from debt to equity financing adjust more aggressively than larger firms that rely relatively more on debt or internal resources. This is because issuing even more equity comes at an increasing cost, pushing small firms closer to the default cutoff, so they have to adjust more their investment and output. The size-dependent responses to the financial shock imply that economies with different firm size distributions should adjust differently. Economies with a higher fraction of small firms (i.e., less skewed firm size distributions) adjust more because small firms adjust more aggressively when financing costs increase.

Interestingly, in this model firm heterogeneity has aggregate implications. The severity of financial frictions and the magnitude of the financial shock affect the evolution of the distribution of firms, and hence the dynamics of aggregate outcomes such as output. In particular, the model predicts a larger fall in aggregate output when the firm size distribution is less skewed. We observe a similar pattern if we compare Portugal v. Spain. During the crisis, Portuguese output declined more and took longer to recover than the Spanish case. Portugal also has a firm size distribution that is significantly less skewed than Spain's.

The model is calibrated to match firm-level features of Spain. For an increase in sovereign debt consistent with the Spanish case, the model predicts that small firms adjust more than large ones, and that smaller firms would reduce less their debt and capital stocks if the economy had a larger firm-size dispersion. Regarding the aggregate effects of firm heterogeneity, the model predicts an output drop of roughly half the size of the decline observed in Spain during the European debt crisis. Comparing with the results for the same model setup but with a representative firm, output drops by 2 percentage points more (with respect to its long-run average), in the economy with heterogeneous firms. Hence, in the presence of financial frictions, firm heterogeneity amplifies the responses of aggregate variables to a tightening in credit conditions significantly.

The contributions of this paper are threefold. First, it documents the two new facts mentioned earlier using a rich European firm-level dataset, namely that during the European debt crisis small and large firms adjusted differently upon the same financial shock (where the latter adjust less aggressively), and that small firms in countries with more skewed firm size distributions adjusted less during this episode. Empirical findings related to the facts mentioned, such as the ones of Bottero et al. (2015), which uses Italian firm-bank credit relation data, show that small firms adjusted more their investment than large ones. In the same context, Arellano et al. (2017) finds that increases in Italian sovereign spreads decrease more sharply the sales growth of small firms. This paper provides evidence on more balance sheet variables for the set of countries that were most affected during this episode and also studies the role of the skewness of the distribution of firms.

Second, it contributes to the literature on heterogeneous agents in production with aggregate uncertainty. In the model, financial frictions make firms' responses to be non linearly scalable in size, and as a result firm heterogeneity has aggregate implications. This is a key departure from classic approaches, particularly Bernanke et al. (1999), in which the distribution of wealth is irrelevant. The model combines different features of the investment financing literature, such as costly equity issuance (Gomes, 2001, Cooley and Quadrini, 2001) and defaultable debt (Hennessy and Whited, 2007), and extends its traditional framework by including aggregate uncertainty via a government and financial intermediation sector. This paper is closer to the work of Khan and Thomas (2013) and Khan et al. (2014), which studies aggregate fluctuations generated by credit shocks in the context of firm heterogeneity, and it differs in two dimensions. The first one is that this paper explicitly models the financial intermediation sector, where the size of the financial shock is a function of the distribution of firms. The second difference is that the model allows for (costly) equity issuance. The imperfect substitutability between equity and debt is a key feature of the model and produces size-dependent responses to financial shocks that are consistent with the ones observed during the European debt crisis. In this line, Gilchrist et al. (2014) proposes a setting with heterogeneous firms facing similar financial frictions (but abstracting from financial shocks) to analyze the implications of uncertainty shocks on investment dynamics. In more recent work, Ottonello and Winberry (2018) studies the role of monetary policy shocks in a framework with heterogeneous firms and financial frictions, but not allowing for equity issuance and aggregate uncertainty.

The third contribution is related to the literature on the transmission of sovereign debt crises to the real economy. This paper focuses on studying the crowding-out effect that sovereign debt has on the availability of funds for non-financial firms. It shows that firm heterogeneity along with financial frictions are key to generate an amplification effect, by which a tightening of private financial conditions in response to a sovereign debt crisis has large adverse effects on aggregate output. Empirical work, such as Acharya et al. (2018) and Becker and Ivashina (2018), shows that during the crisis government debt crowded-out private credit and investment. In this regard, Broner et al. (2014) provides a theoretical framework to explain the crowing-out channel, where credit discrimination is the key driver. This paper contributes to this strand of the literature by assessing the role of firm heterogeneity on the crowing-out channel.

This paper is organized as follows. Section 1.1 provides the empirical analysis of the role of heterogeneity across firms in the effects of the financial shock triggered by the European debt crisis. Section 1.2 presents the model. Section 1.3 presents the calibration and assessment of the model's equilibrium outcomes, as well as the quantitative results regarding the transmission of financial shocks and the role of firm heterogeneity. Section 1.4 concludes.

1.1 Firm Heterogeneity & the European Debt Crisis

This section presents an empirical analysis of the effects associated on non-financial firms caused by the sharp increase in sovereign borrowing during the European debt crisis. Increased sovereign borrowing generated a crowding-out effect on firm lending, which affected the financing conditions of non-financial firms in the involved economies. This section focuses on analyzing the effects of this financial shocks at the firm level using the Amadeus database.[2] This database contains roughly 21 million firms across Europe. Firms of all sizes are required to report information about their balance sheets, so that the sample that is possible to access is more representative of the whole economy than other popular datasets, such as Compustat (which only contains publicly listed firms).

In order to study the responses at the firm level to the sovereign debt surge, which is an aggregate shock, a panel dataset is constructed. Firms in Greece, Ireland, Italy, Portugal and Spain are considered, as these countries were the most affected during the debt crisis. The analysis focuses on the time period of 2010 to 2014, in order to avoid the Great Recession.[3] The estimated specification is given by the following equation:

$$x_{i,j,t} = \beta \mathrm{B}_{j,t} + \sum_{k=2}^{5} \delta_k Q_{i,j,t,k} \mathrm{B}_{j,t} + \sum_{k=2}^{5} \phi_k Q_{i,j,t,k} \mathrm{B}_{j,t} \mathrm{Sk}_{j,t} + \kappa \mathrm{B}_{j,t} \mathrm{Sk}_{j,t} + z'_{i,j,t}\gamma + \alpha_{i,j} + \eta_{i,j,t} \tag{1.1}$$

2. The database was accessed through Wharton Research Data Services (WRDS). A brief description of how variables were treated and generated is presented in the Appendix.
3. Unfortunately, the speed at which the Amadeus database is updated does not allow to include 2015 at the moment this paper is written. Years 2015 and 2016 are not well populated in terms of observations, so they are omitted from the analysis.

where $x_{i,j,t}$ denotes the outcome x for firm i in country j at period t, $\mathrm{B}_{j,t}$ corresponds to the gross public debt-to-output ratio for country j at period t, Q_k is a size dummy that is equal to 1 if the firm belong to the kth quintile of firm size , $\mathrm{Sk}_{j,t}$ represents the skewness of the firm size distribution in country j at period t, z is a vector of controls, α is a firm fixed effect, and η is the error term.[4] The outcome variables studied are the logs of sales, total assets, total liabilities and employment, at the firm level.[5]

This specification allows us to capture the semi-elasticity of the dependent variable (measured in log) with respect to increases in government debt, but allowing for differential effects in terms of firm size and skewness of the country's firm size distribution. For example, δ_k reflects the differential effect that increases in debt have on firms of different size, while ϕ_k captures a similar effect but considering different levels of skewness of the firm size distribution.[6]

The regression equation is estimated using fixed effects at the firm level.[7] The estimation results are presented in Table 1.1. The results show that increases in sovereign debt decrease firms' sales, liabilities, assets, and employment. Also, larger firms perform better than small firms when sovereign debt increases, as it can be seen from the coefficient associated with the interaction $\mathrm{B} \times Q_k$. However, when considering the triple interaction between debt, firm size and skewness of the firm size distribution, we see that the large firms do not perform better than small ones in economies where skewness is larger. This is

4. Control variables include a set of lagged versions of all outcome variables, as well as firm size quintile dummies, and year dummies.
5. We also performed robustness checks regarding the addition of more controls, in order assess whether it is indeed government debt the variable affecting the firm-level outcomes. In particular, we include 1-year sovereign CDS spreads and also an estimation of the fraction of general government debt held by domestic banks, for each country. For the latter we follow the methodology presented in Arslanalp and Tsuda (2014). The estimation results do not change significantly.
6. The semi-elasticity of variable y with respect to variable x is given by $\varsigma_{y,x} = \frac{d \ln y}{dx}$.
7. An alternative version is estimated using random effects and adding controls for industry and country. The results do not change significantly, but the Hausman test strongly rejects the usage of a random effects specification. Another possibility is to use a pooled OLS specification. We present in the Appendix the comparison between the fixed effects, random effects, and pooled OLS specifications.

the case for all variables except for employment, where the greater the skewness the better the larger firms perform in relation to small ones.

The magnitude of the coefficients is relatively small, but their statistical significance is quite high, as suggested by their p-values. To make the analysis simpler, Table 1.2 presents the implied semi-elasticities in response to an increase of one percentage point in the ratio of gross public debt-to-output. At the average sample skewness, an increase in sovereign debt decreases the sales of small firms by 0.27%, while for large firms the decrease is smaller, 0.12%.[8] These decreases might seem small, but it is important to note that the average increase in government debt in 2012 was large. For example, in the case of Spain, in 2012 the debt-to-output ratio increased by roughly 16 percentage points. Thus, for this case, the semi-elasticity results predict that sales decreased by 4.4% for small firms, and 1.9% for large ones, a large drop in both cases. We continue to observe an asymmetry in the adjustments of small and large firms, where the former adjust more than the latter, except for the case of liabilities.

The estimated coefficients can be used to compare the responses to increases in the sovereign debt-to-output ratio between small and large firms across countries with different levels of skewness of the firm size distribution. Varying the level of skewness allows us to analyze whether these responses are different between countries. This exercise considers the contrast between the semi-elasticities to increases in public debt-to-output for Portugal (low skewness) and Spain (high skewness). The second and third rows of Table 1.2 presents the results. We see that (i) small Portuguese and Spanish firms adjust more than large firms in terms of sales, liabilities, assets, and employment, and that (ii) Portuguese firms (small and large) adjust more than Spanish firms for the same set of variables. The first result is consistent with the pattern found for the entire sample, namely that small firms adjust more than large ones. The second one reveals the role of the skewness of the firm size

8. Coefficients with a p-value above 0.10 are set to 0 when calculating the semi-elasticities.

distribution. Firms in countries with a lower skewness adjust more to an increase in the sovereign debt-to-output ratio.

Table 1.1: Panel Regression Results

Variable	Log(Sales)	Log(Liabilities)	Log(Assets)	Log(Employment)
B	-0.003	-0.003	-0.001	-0.003
	(0.000)	(0.000)	(0.000)	(0.000)
Q_2	0.327	0.718	0.519	0.161
	(0.000)	(0.000)	(0.000)	(0.000)
Q_3	0.562	1.250	0.909	0.285
	(0.000)	(0.000)	(0.000)	(0.000)
Q_4	0.754	1.712	1.237	0.368
	(0.000)	(0.000)	(0.000)	(0.000)
Q_5	0.864	2.103	1.532	0.424
	(0.000)	(0.000)	(0.000)	(0.000)
B $\times$ Q_2	-0.001	-0.001	-0.000	-0.001
	(0.000)	(0.000)	(0.000)	(0.000)
B $\times$ Q_3	0	-0.001	0.000	-0.001
	(0.000)	(0.000)	(0.474)	(0.000)
B $\times$ Q_4	0	-0.001	0.000	-0.001
	(0.036)	(0.000)	(0.000)	(0.000)
B $\times$ Q_5	0.002	0.000	0.000	0.000
	(0.000)	(0.054)	(0.000)	(0.367)
B $\times$ Q_2 $\times$ Skewness	0.000	-0.000	-0.000	0.000
	(0.058)	(0.003)	(0.000)	(0.000)
B $\times$ Q_3 $\times$ Skewness	-0.000	-0.000	-0.000	0.000
	(0.727)	(0.000)	(0.000)	(0.000)
B $\times$ Q_4 $\times$ Skewness	-0.000	-0.000	-0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)
B $\times$ Q_5 $\times$ Skewness	-0.000	-0.000	-0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.013)
B $\times$ Skewness	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.934)
Year Effects	Yes	Yes	Yes	Yes
Fixed Effects	Yes	Yes	Yes	Yes
Controls	Yes	Yes	Yes	Yes
Observations	3,195,072	3,308,816	3,313,051	3,119,977
R^2	0.58	0.736	0.911	0.585

Notes: B denotes the gross public debt-to-output ratio, Q_k is a dummy variable that is equal to 1 if the firm belongs to the *kth* quintile of firm size, and Sk is the skewness of the firm size distribution. Countries include Greece, Ireland, Italy, Portugal and Spain. Clustered standard errors at the firm level considered, p-values in parentheses.

The analysis presented above shows two important new findings regarding the relevance of firm heterogeneity and firm size distribution. First, during the debt crisis, small firms adjusted more than large firms. Second, small firms in economies where the skewness of the firm size distribution adjusted less than their counterparts in economies with lower skewness.

Table 1.2: Implied Semi-elasticities for Debt Increases (Percentages)

	Sales		Liabilities		Assets		Employment	
	Small	Large	Small	Large	Small	Large	Small	Large
Average Skewness	-0.269	-0.115	-0.273	-0.277	-0.131	0.016	-0.267	-0.263
Portuguese Skewness	-0.319	-0.150	-0.309	-0.294	-0.175	-0.017	-0.267	-0.266
Spanish Skewness	-0.300	-0.136	-0.295	-0.288	-0.158	-0.005	-0.267	-0.265

Notes: Small firms are those that belong to the first quintile of firm size, and large firms correspond to those that belong to the fifth quintile.

The last exercise of this section seeks to assess whether increases in sovereign debt indeed increased the financing cost of firms. Unfortunately, the Amadeus database does not contain information about contractual interest rates paid by firms. It is possible to construct a proxy for the average effective interest rate paid by the firm. In particular, we can compute $r_{i,t} = \frac{\text{Interest Paid}_{i,t}}{\text{Liabilities}_{i,t}}$, which proxies for the average interest rate paid by firms. Unfortunately, the database does not allow us to identify the maturity of the debt.

In order to assess if the financing costs respond to increases in sovereign debt, the following specification is estimated:

$$r_{i,j,t} = \beta \mathrm{B}_{j,t} + \sum_{k=2}^{5} \delta_k Q_{i,j,t,k} \mathrm{B}_{j,t} + \sum_{k=2}^{5} \phi_k Q_{i,j,t,k} \mathrm{B}_{j,t} \mathrm{Sk}_{j,t} + \kappa \mathrm{B}_{j,t} \mathrm{Sk}_{j,t} + z'_{i,j,t}\gamma + \alpha_{i,j} + \eta_{i,j,t} \tag{1.2}$$

where the set of independent variables is defined in the same way as in the previous regression equation. Table 1.3 presents the estimation results.

Table 1.3: Panel Regression Results, Financing Costs

Variable	$r_i \times 100$
B	0.000
	(0.000)
Q_2	-0.011
	(0.000)
Q_3	-0.017
	(0.000)
Q_4	-0.022
	(0.000)
Q_5	-0.027
	(0.000)
B $\times$ Q_2	-0.006
	(0.222)
B $\times$ Q_3	-0.027
	(0.599)
B $\times$ Q_4	-0.03
	(0.972)
B $\times$ Q_5	-0.043
	(0.637)
B $\times$ Q_2 $\times$ Skewness	0.000
	(0.007)
B $\times$ Q_3 $\times$ Skewness	0.000
	(0.000)
B $\times$ Q_4 $\times$ Skewness	0.000
	(0.000)
B $\times$ Q_5 $\times$ Skewness	0.000
	(0.000)
B $\times$ Skewness	-0.000
	(0.000)
Year Effects	Yes
Fixed Effects	Yes
Controls	Yes
Observations	2,852,913
R^2	0.001

Notes: B denotes the gross public debt-to-output ratio, Q_k is a dummy variable that is equal to 1 if the firm belongs to the kth quintile of firm size, and Sk is the skewness of the firm size distribution. Countries include Greece, Ireland, Italy, Portugal and Spain. Clustered standard errors at the firm level considered, p-values in parentheses.

The magnitudes of the coefficients are quite small, although they are statistically significant. To make the exposition clearer, average responses are also computed. The implied increase in the average interest rate for a small firm in response to a 1-percentage-point increase in the public debt-to-output ratio is of 2 basis points, while for large firms it is 0.9 basis points. For the case of Spain in 2012, the increase (roughly 16 percentage in the debt to output ratio) meant an increase of 32 and 14 basis points for small and large firms, respectively. Although the magnitude is not large, the effects are statistically significant, which reflects the fact that part of the increase in financing costs of firms is being captured in this analysis.

1.2 The Model

The model consists of 4 main components. First, is a heterogeneous firm sector, where firms differ in their debt (b) and capital stocks (k), and idiosyncratic productivity (z). Firms face financial frictions (defaultable debt and costly equity issuance) which affects how they finance their investment, leading to non-trivial financing structures which are firm-size dependent (where size is defined by the firm's capital). The time-varying distribution of firms is denoted by Γ. Second, a financial intermediation sector lends to the sovereign and firms. Financial intermediaries live 2 periods and are born with wealth $\bar{A}$. The limited wealth generates an occasionally binding constraint, that is triggered when government debt is sufficiently high. In case the constraint binds, the price at which the government (q^g) and firms (q) can borrow decreases, which is a financing cost shock for firms. Sovereign borrowing crowds-out private borrowing. Third, the sovereign. Sovereign debt (B) dynamics evolve according to a fiscal reaction function and to the realization of aggregate fiscal shocks (ε). The state variables for the sovereign are its outstanding debt and the fiscal shock ($\mathbf{S} = (B, \varepsilon)$), which will also happen to be the aggregate states of the economy. Lastly, a risk-neutral household sector that own firms and receive transfers T from the government.

The timeline of the events of the model between periods t and $t+$ is as follows. At the beginning of the period, the aggregate shock is realized. The sovereign borrows according to its fiscal reaction function, outstanding debt and realization of the shock. Firm productivity for the period is materialized, and the firm decides whether to default on its debt or not. In case of default, the firm exits forever and is replaced by a new one. If it decides to continue operating, it chooses its investment and how to finance it (debt, equity issuance, or internal resources) for the next period. The price at which the sovereign and firms borrow is determined jointly with the problem of the representative financial intermediary.

1.2.1 Firms

Firms accumulate capital, k, through investment, which can be financed by (i) internal resources of the firm, (ii) debt (b) and (iii) equity issuance. Firms face two kinds of financial frictions, defaultable debt and costly equity issuance. This setting is similar to the one of Hennessy and Whited (2007), which builds on Gomes (2001) and Hennessy and Whited (2005).[9] Firm debt is risky, because can be defaulted on. The price at which firms borrow, q, depends on their observable state variables and on the aggregate states of the economy (which will be specified below). Equity issuance, on the other hand, is costly, reflecting that the firm incurs in costs when trying to raise resources by selling equity. This is a fairly common assumption in the corporate finance literature (Gomes, 2001; Cooley and Quadrini, 2001; Hennessy and Whited, 2007; Jermann and Quadrini, 2012), and allows for the existence of a trade-off between different sources of external financing.

The production technology of a firm is given by $f(z,k) = zk^{\alpha}$ (with $\alpha < 1$), where z denotes its productivity. Firms are subject to idiosyncratic shocks to their productivity, which follow a finite state Markov process characterized by the CDF $G(z'|z)$. The law of

9. These articles provide a partial equilibrium setting featuring heterogeneous firms. Hennessy and Whited (2007) considers firms that differ in their productivity, debt, and capital, that can default on their debt and face equity issuance costs.

motion for capital is given by $k' = k(1-\delta) + i$, where i denotes investment and δ is the rate at which capital depreciates. There is an investment adjustment cost that the firm must pay, which is given by $\Psi(k',k) = \frac{\psi}{2}\left(\frac{k'}{k} - 1\right)^2 k$. Firms are subject to a proportional tax τ on profits, and incur a fixed cost F to operate.

Before production begins each period, firms decide whether to default on their debt or not. If they do, they exit the economy forever and are replaced by a firm with no debt, a level of capital $\underline{k}$, and productivity drawn from the long-run probability distribution of the Markov process specified above ($G^*(z)$).[10] Additionally, in case of default, nothing is recovered by the firm, but creditors recover a fraction θ of the firm's undepreciated capital. Firms obtain a continuation value V in case they decide not to default.

In order to finance investment, firms can rely on internal or external financing. Internal resources consist of all the resources the firm has available after producing and paying for the operational costs and outstanding debt. External financing consists of debt issuance and/or equity issuance. The price at which a firm can borrow depends on its productivity z, capital for the next period k', and borrowed amount b'. This is because the price responds to the expected future default probability of the firm, which is a function of how much it borrowed and invested today, and of its expected future productivity.[11] Hence, if borrowing is too high then the likelihood of future default might increase, and this will be reflected in the price of debt. The equilibrium price is determined by the interaction between the government, firms, and financial intermediaries, and is also influenced by aggregate states $\mathbf{S}$ and the distribution of firms Γ. Denote the equilibrium price faced by a firm with the above characteristics by $q(z,k',b';\mathbf{S},\Gamma)$. If firms decide to issue equity they incur a cost $\Lambda(e) = \lambda e^2$, where e denotes the amount of equity being issued. The functional form assumed is similar to the one used by Jermann and Quadrini (2012), and Covas and den

10. This assumption keeps the mass of firms in the economy constant.
11. Given that productivity follows a Markov process its realization today gives information about future realizations.

Hann (2012), and rationalizes the fact that there are increasing marginal costs associated to issuing equity, such as underwriter fees, as shown by Hansen and Torregrosa (1992) and Altinkilic and Hansen (2000).

The problem of a firm that does not default is given by:

$$V(z,k,b;\mathbf{S},\Gamma) = \max_{\{(k',b')\in\Upsilon(z;\mathbf{S},\Gamma)\}} \left\{ e + \beta \mathbb{E}_{z',\mathbf{S}'|z,\mathbf{S}} \max\{V(z',k',b';\mathbf{S}',\Gamma'),0\} \right\} \tag{1.3}$$

s.t.

$$e = (1-\tau)(zk^{\alpha} - F) - k' + k(1-\delta) - \Psi(k',k) + q(z,k',b';\mathbf{S},\Gamma)b' - b - \Phi\Lambda(e)$$

$$\Gamma' = \Omega(\mathbf{S},\Gamma)$$

where e denotes the cash flow to shareholders, Φ denotes an indicator variable that is equal to 1 when $e < 0$ (when there is equity issuance) and 0 otherwise, and Ω corresponds to the law of motion for the distribution of firms. Finally, firms choose their capital for next period and debt from sets $\mathscr{K}$ and $\mathscr{B}$, respectively, which are compact sets. As in Gomes (2001), $\mathscr{K}$ is compact because it is a closed and bounded set. It is bounded from below by 0 and bounded above by a level $\tilde{k}$, which is the largest possible capital stock that is profitable for the firm and solves $\alpha\bar{z}\tilde{k}^{\alpha-1} - \delta = 0$. Thus, we can define $\mathscr{K} = [0,\tilde{k}]$. Regarding the choice set for debt, given a price schedule $q(z,k',b';\mathbf{S},\Gamma)$, debt will be bounded below by 0 (firms cannot save via debt) and above by an endogenous level $\tilde{b}(z,k';\mathbf{S},\Gamma)$ that represents the maximum debt a firm can take, which will be a function of productivity and capital choice (and aggregate states) and such that $q(z,k',\tilde{b}(z,k';\mathbf{S},\Gamma);\mathbf{S},\Gamma) = 0$. Hence, we can define $\mathscr{B}$ in a similar way as $\mathscr{K}$. The choice correspondence $\Upsilon(z;\mathbf{S},\Gamma)$ is defined as $\Upsilon(z;\mathbf{S},\Gamma) \equiv \{(k',b') : k' \in \mathscr{K}, b' \in [0,\tilde{b}(z,k';\mathbf{S},\Gamma)]\}$. Note that this correspondence is convex, compact valued and continuous.

Notice that from the expression for the value function of the firm we can see that a threshold rule for productivity will apply. Let $\mathbf{z}(k,b;\mathbf{S},\Gamma)$ be defined as:

$$V(\mathbf{z}(k,b;\mathbf{S},\Gamma),k,b;\mathbf{S},\Gamma)=0 \tag{1.4}$$

$\mathbf{z}(k,b;\mathbf{S},\Gamma)$ defines the cutoff value of productivity for which the firm will decide to default on its obligations. It follows that we can define $\tilde{d}(z,k,b;\mathbf{S},\Gamma)$ as a binary variable that takes the value of 1 if the firm defaults today conditional on debt contracted in the previous period, and 0 otherwise:

$$\tilde{d}(z,k,b;\mathbf{S},\Gamma)=\begin{cases}1 & \text{if} \quad z<\mathbf{z}(k,b;\mathbf{S},\Gamma)\\ 0 & \text{if} \quad z>\mathbf{z}(k,b;\mathbf{S},\Gamma)\end{cases}$$

Using the decision rule $\tilde{d}(z,k,b;\mathbf{S},\Gamma)$ we can define the probability that a firm defaults in the future. Given The default probability tomorrow for a firm with productivity z that chooses capital k' and borrows b' today is given by:

$$\tilde{p}(z,k',b';\mathbf{S},\Gamma)=\mathbb{E}_{z'|z}\left[\tilde{d}(z',k',b';\mathbf{S}',\Gamma')\right]=\Pr\left(z'<\mathbf{z}(k',b';\mathbf{S}',\Gamma')|z\right) \tag{1.5}$$

1.2.2 Financial Intermediation

Risk-neutral intermediaries that live for 2 periods are born with a level of wealth $\bar{A}$, and have to decide how much to lend to the sovereign and firms.[12] The fact that intermediaries live for 2 periods, as in Aguiar et al. (2016) and Coimbra and Rey (2017), simplifies the setting because there is no need to keep track of the evolution of their level of wealth, while maintaining the key idea of the crowding-out between sovereign and firm lending.

If a firm defaults, the intermediaries recover a fraction θ of the undepreciated capital

12. Intermediaries are identical and there is a mass 1 of them.

$((1-\delta)k)$. In case of sovereign default, they recover nothing. Intermediaries face a limited liability constraint (i.e. dividends cannot be negative) in both periods, and loanable funds constraint in the first period, where dividends plus funds lent cannot exceed $\bar{A}$. The problem of the representative financial intermediary is given by:

$$\max_{\{x,x',B',\{b'(z,k,b)\}\}} x+\tilde{\beta}\mathbb{E}_{\mathbf{S}',z'|\mathbf{S},z}[x'] \tag{1.6}$$

$$\text{s.t.}$$

$$x+q^g B'(\mathbf{S})+\int q(z,k'(z,k,b),b'(z,k,b))b'(z,k,b)d\Gamma(z,k,b)\leq\bar{A}$$

$$x'=B'(\mathbf{S})(1-d^g(B(\mathbf{S}))+\int[b'(z,k,b)(1-\tilde{d}_2(z,k'(z,k,b),b'(z,k,b)))$$

$$+\tilde{d}(z,k'(z,k,b),b'(z,k,b))\theta k'(z,k,b)(1-\delta)]d\Gamma(z,k,b)$$

$$x,x'\geq 0$$

where x and x' denote the dividends of the first and second period, respectively, $b'(z,k,b)$ denote the amount lent and to a firm with characteristics (z,k,b), $k'(z,k,b)$ is the choice of capital for next period of a firm with the same characteristics, and B' denotes the size of loan going to the sovereign. The discount factor of the representative intermediary is given by $\tilde{\beta}$.

The loanable funds constraint occasionally binds because the sum of sovereign borrowing and aggregate firm borrowing is not necessarily equal the level of wealth $\bar{A}$: if it is less, the constraint does not bind, while if it is equal to the level of wealth the constraint binds. The time-varying nature of public and aggregate private debt generates periods where the constraint may or may not bind.

The first order conditions of the intermediary's problem give the pricing functions for sovereign and firm debt, as well as a slackness condition:

$$q^g(B';\mathbf{S},\Gamma) = \tilde{\beta}\frac{\mathbb{E}[(1-d^g(B'(\mathbf{S})))]}{1+\mu(\mathbf{S},\Gamma)} = \tilde{\beta}\frac{1-p^g(B'(\mathbf{S}))}{1+\mu(\mathbf{S},\Gamma)} \tag{1.7}$$

$$q(z,k',b';\mathbf{S},\Gamma) = \tilde{\beta}\frac{\mathbb{E}[1-\tilde{d}(z,k',b')(1-\theta(1-\delta)k')]}{1+\mu(\mathbf{S},\Gamma)} = \tilde{\beta}\frac{1-\tilde{p}(z,k',b')(1-\theta(1-\delta)k')}{1+\mu(\mathbf{S},\Gamma)} \tag{1.8}$$

$$\left(\bar{A} - q^g(B';\mathbf{S},\Gamma)B'(\mathbf{S}) - \int q(z,k'(z,k,b),b'(z,k,b);\mathbf{S},\Gamma)b'(z,k,b)d\Gamma(z,k,b)\right)\mu(\mathbf{S},\Gamma) = 0 \tag{1.9}$$

where $\mu(\cdot)$ represents the Lagrange multiplier of the loanable funds constraint of period 1, $d^g(B')$ is a binary variable that is equal to 1 if the sovereign defaults, and 0 otherwise, and $p^g(B')$ is the probability the sovereign defaults in the next period, given a loan B'.[13] μ depends on the aggregate states of the economy and on the distribution of firms because these two elements influence the level of sovereign borrowing and also the aggregate demand for debt of firms. This multiplier reflects the marginal utility of the intermediaries' wealth, so the higher the value it takes the more binding the constraint will be.

Equations (1.7) and (1.8) define the price at which the government and a firm with productivity z choosing k' and b' can borrow. Part of the structure of these pricing functions has standard features, such as being decreasing in the expected future probability of default or being increasing in the recovery in case of default. Notice that the Lagrange multiplier for loanable funds is present in these functions. When the loanable funds constraint binds, which is likely to happen when public debt is high enough, the price at which the government and firms can borrow decreases. The latter is equivalent to an increase in the interest rate paid on loans. The value that the multiplier takes is such that the loanable funds con-

13. The Lagrange multiplier for the limited liability constraint in period 2 is always 0, as the non-negativity constraint will never bind. To see this note that period 2 dividends are composed by the sum of non-negative payouts of assets.

straint is met with equality, so different magnitudes of μ could be observed depending on the dynamics of public debt.

1.2.3 Sovereign

The sovereign issues one-period non-state contingent bonds B at a price q^g. The primary balance of the sovereign is given by a fiscal reaction function, which depends on an aggregate fiscal shock $\varepsilon \in [\underline{\varepsilon}, \overline{\varepsilon}]$ that follows a Markov process with CDF $G^s(\varepsilon'|\varepsilon)$, and on the level of the outstanding debt in the current period. Thus, at period t, the primary balance is given by $\text{Pb}_t = n(B_t, \varepsilon_t)$. This formulation is in line with the literature on public debt sustainability, initiated by Bohn (1998), and also used in the analysis of debt crises, as in Lorenzoni and Werning (2013). As Bohn (1998) showed, a sufficient condition for the intertemporal budget constraint of the government to hold is that there is a (conditional) positive response of the primary balance to lagged debt. The fiscal reaction function can also exhibit non-linear responses to lagged debt, as it will be specified below, in order to capture the possibility of "fiscal fatigue", as in Ghosh et al. (2013), and still be consistent with fiscal solvency. For low levels of public debt the primary balance slightly increases as debt rises, for higher levels of debt there is a much larger response, but at some point the response of the primary balance weakens.

Finally, the sovereign defaults with probability p^g. This is a function of the amount of borrowing, and which is modeled by the following reduced form structure:

$$p^g_{t+1} = \min\{\max\{\alpha_s + \beta_s B_{t+1}, 0\}, 1\} \tag{1.10}$$

The likelihood of default will have a direct effect on the price at which the sovereign can borrow, as will be specified in the financial intermediation section. Given that sovereign borrowing can potentially crowd-out private investment, q^g will be a function of the distribution of firms and aggregate states.

The dynamics of debt, then, are obtained from the budget constraint of the sovereign:

$$
\begin{aligned}
q_t^g(B_{t+1};\mathbf{S}_t,\Gamma_t)B_{t+1} &= B_t - \mathrm{Pb}_t(\mathbf{S}_t) \\
B_{t+1} &\leq \arg\max_{\hat{B}_{t+1}} q_t^g(\hat{B}_{t+1};\mathbf{S}_t,\Gamma_t)\hat{B}_{t+1}
\end{aligned}
\tag{1.11}
$$

where $B_{t+1}(\mathbf{S}_t,\Gamma_t) = \mathscr{B}(\mathbf{S}_t,\Gamma_t)$ is the policy function for debt.

The budget constraint of the sovereign imposes endogenous debt limits. To see this, note that the right-hand-side of equation (1.11) is given for the current states of the economy $\mathbf{S}_t$. Given $\mathbf{S}_t$, Γ_t, the left-hand-side forms the familiar Laffer curve for debt (i.e., for B_{t+1}), but there is no guarantee that even at the level of debt that maximizes the resources raised the equality will hold. Two cases can occur. In the first one there are two values of B_{t+1} that solve equation (1.11), which is equivalent to say that these two values produce the same revenue. If this is the case, the lowest amount of debt is selected. In the second case there is no value of B_{t+1} that solves the mentioned equality. Here, debt is set to be equal to $\hat{B}_{t+1} = \arg\max q_t^g(\hat{B}_{t+1};\mathbf{S}_t,\Gamma_t)\hat{B}_{t+1}$, and the primary balance is adjusted correspondingly.

In order to maintain consistency of the sovereign's problem, the primary balance is redefined by:

$$
\mathrm{Pb}_t(\mathbf{S}_t,\Gamma_t) = \begin{cases} n(B_t,\varepsilon_t) & \text{if} \quad B_t - n(B_t,\varepsilon_t) < \hat{B}_{t+1} \\ B_t - \hat{B}_{t+1} & \text{if} \quad B_t - n(B_t,\varepsilon_t) \geq \hat{B}_{t+1} \end{cases} \tag{1.12}
$$

Lastly, government revenues at period t are given by $\hat{\tau}(\mathbf{S}_t,\Gamma_t) = \tau \times \pi_t(\mathbf{S}_t,\Gamma_t)$. In order to ensure that the fiscal reaction function for primary balance is consistent with tax revenues, a transfer T_t to households is considered. This transfer is such that $T_t(\mathbf{S}_t,\Gamma_t) =$

$\hat{\tau}(\mathbf{S}_t, \Gamma_t) - \text{Pb}(\mathbf{S}_t, \Gamma_t)$ for $t \geq 0$.

1.2.4 Households

Households in this model are assumed to be identical, risk-neutral agents that own firms and consume an endowment $\bar{M}$ plus transfers from the government T_t. The structure assumed for the households simplifies the setting of the model and allows to focus on the firm sector, which is going to be the relevant actor for the transmission of financial shocks towards the real economy.

1.2.5 Equilibrium

Given an initial distribution Γ_0, a Recursive Markov Equilibrium consists in a value functions for the firm V, policy functions $\hat{b}$, $\hat{k}$, $\hat{d}$, $\mathscr{B}$, pricing functions q, q^g, μ, and a law of motion Ω such that:

(i) Given $\mathscr{B}$, μ, q, Γ and Ω, $V(z,k,b;\mathbf{S},\Gamma)$ solves the problem of the firm and $\hat{k}(z,k,b;\mathbf{S},\Gamma)$, $\hat{b}(z,k,b;\mathbf{S},\Gamma)$ and $\hat{d}(z,k,b;\mathbf{S},\Gamma)$ are the corresponding policy functions for capital, debt, and default.

(ii) $q^s(\mathbf{S},\Gamma)$ and $q(z,k,b;\mathbf{S},\Gamma)$ solve the pricing equation of financial intermediaries.

(iii) $\mu(\mathbf{S},\Gamma)$ is consistent with the slackness condition of financial intermediaries.

(iv) Γ evolves according to $\Gamma' = \Omega(\mathbf{S},\Gamma)$.

(v) Budget constraint of the sovereign holds.

(vi) Aggregate resource constraint holds.

1.2.6 Understanding the Mechanism of the Model: A 2-Period Case

This section provides a 2-period version of the full model presented above. In order to highlight the economic intuition of the mechanism driving the model.

In the first period firms have an initial capital k_0 and productivity z_0. They decide how much to invest, borrow, produce, and whether to issue equity or not. The production technology is the same as the one specified above. Debt is defaultable, and equity issuance is costly. Firms are subject to an operational fixed cost F in both periods. Firms issue equity whenever dividends in period 1 become negative. In period 2, if firms do not default, they repay their debt and produce.[14]

The problem of the firm is given by:

$$\max_{\{e_0,e_1,k_1,b_1\}} e_0 - \Lambda(e_0)\Phi\{e_0 < 0\} + \beta \mathbb{E}_{z_1|z_0}[e_1] \tag{1.13}$$

$$\text{s.t.} \tag{1.14}$$

$$e_0 = z_0 k_0^{\alpha} - k_1 + q(b_1)b_1 - F$$

$$e_1 = z_1 k_1^{\alpha} - F - b_1$$

where e_t denote period $t = 0,1$ dividends, $\Phi =$ is an indicator variable that is equal to 1 in case the firm issues equity, and 0 otherwise, and $\Lambda(e) = e^2$ denotes the equity issuance cost function.

Define the *financing gap* of a firm as the difference between resources needed to operate and invest minus the resources available to do so. Given k_1 and b_1, the financing gap is

14. In period 2 dividends cannot be negative, as the firm would prefer to default. Thus, there is no equity issuance in period 2.

given by the following expression:

$$\text{Financing Gap} = k_1 + F - z_0 k_0^{\alpha} - q(b_1)b_1 \tag{1.15}$$

A firm is *constrained* whenever the financing gap is strictly positive, and *unconstrained* if the opposite occurs. Note that a constrained firm, conditional on k_1 and b_1, does not have enough resources to finance its investment. Hence, it needs to rely also on equity issuance. The amount of equity issued is such that the financing gap is met.

Let k_1^* and b_1^* denote the unconstrained solutions for capital and debt in period 1, respectively. The "marginally constrained firm" has an initial capital given by $k_0^* = \left[\frac{k_1^* + F - q(b_1^*)b_1^*}{z_0}\right]^{\frac{1}{\alpha}}$. Defining size as the amount of capital, we can conclude that a firm with initial size greater or equal than k_0^* will be unconstrained, while if the opposite is true then the firm will be constrained. Hence, whenever $k_0 < k_0^*$ the firm will have a positive financing gap and will issue equity.

The combined first order conditions of the firm's problem (assuming differentiability of the pricing function of debt), for a firm with initial capital k_0 and productivity z_0, yield the following conditions:

$$\beta \mathbb{E}_{z_1|z_0}\left[\alpha z_1 k_1^{\alpha-1}\right] = 1 + \Lambda'(e_0)\Phi\{e_0 < 0\} \tag{1.16}$$

$$(q(b_1) + q'(b_1)b_1)(1 + \Lambda'(e_0)\Phi\{e_0 < 0\}) = \beta \cdot \Pr(e_1 > 0) \tag{1.17}$$

These conditions are a central part of the model's mechanism because they imply that constrained and unconstrained firms choose different investment and leverage levels. To see this, consider equation (1.16) first. For a constrained firm issuing equity, the right-hand-side is strictly greater than 1 (given that $\Lambda'(\cdot) > 0$), which implies that this type of firm has a higher marginal product of capital in period 1 than an unconstrained firm. Given that the production function has decreasing returns to scale, this means that the capital accumulation for period 1 is smaller than for an unconstrained firm. Equation (1.17) implies that the marginal benefit of debt is higher for a constrained firm that is issuing equity. The intuition for this is that for every unit borrowed the firm saves the equity issuance cost, which makes debt more attractive. Comparing again with an unconstrained firm, we see that a constrained firm tends to use more debt in relation to its size, and thus has a higher leverage ratio (measured as b_1/k_0). Lastly, note that constrained firms issue equity and use debt at the same time, which implies that they have a more expensive financing mix than unconstrained firms.

The previous paragraphs showed how *small* (constrained) and *large* (unconstrained) behave in terms of their investment and how they finance it. We now explain how these features interact with the rest of the elements in the model. Assume that the sovereign borrows an amount B_1 at a price q^g, and that its initial primary balance and debt are $\text{Pb}_0 = -\varepsilon_0$ and $B_0 > 0$, respectively. Regarding the representative financial intermediary, assume an identical setting as the one specified earlier (where pricing functions are determined in a similar way). Lastly, for simplicity, assume that a fraction m of firms in the economy has initial capital $k_0^s < k_0^*$ (and hence a fraction $1-m$ has capital $k_0^l > k_0^*$). Given this, the funding constraint that the intermediary faces is $q_1^g(B_1(B_0,\varepsilon))B_1(B_0,\varepsilon) + m \cdot q(b_1^s)b_1^s + (1-m)\cdot q(b_1^l)b_1^l \leq \overline{A}$, with a corresponding Lagrange multiplier $\mu(B_0,\varepsilon,m)$.

Assume that the realization of ε is such that the lending constraint binds. Then, $q_1^g(B_1)$, $q(b_1^s)$, and $q(b_1^l)$ decrease, as $\mu(B_0,\varepsilon,m) > 0$ because the constraint binds and the demand for funds needs to be consistent with the supply of them. A decrease in the price lowers the marginal benefit of debt, measured as the resources obtained for every unit borrowed, for both types of firms. This makes them substitute debt for equity or internal resources. Initially constrained firms will issue even more equity, which directly affects more their investment in relation to initially unconstrained firms. To see this more clearly, consider again equation (1.16). Given that $\Lambda(\cdot)'' > 0$, issuing more equity increases even further the marginal product of capital for period 1, which implies that investment is decreasing more for a firm that was constrained initially. This is the key mechanism of the model. The financial frictions that firms face shape their financing structure, and these are size-dependent. Thus, firms of different sizes respond differently to the same financial shock, measured as an increase (decrease) in μ (q).

To conclude this subsection, note that the aggregate effect of the financial shock depends on the fraction of firms that are initially constrained, but there are two channels operating. The first channel is the most evident one, which is determined by the composition of firms in the economy. This is, small firms adjust more their investment than large firms, so an economy that is composed by a large fraction of small firms should have a larger response to the same shock than an economy that has a larger fraction of large firms. The second channel is a general equilibrium channel. Given that the response of the debt of firms also varies across size, economies with different firm composition will have different dynamics for the marginal utility of the intermediaries' wealth. Economies with a lower skewness in firm size distribution (i.e., a larger fraction of small firms) are more sensitive to changes in the value of μ (because small firms adjust more when the lending constraint binds), so a milder increase in μ is required to generate the same adjustment than in a higher skewness economy. Both of these channels are operating at the same time, so a quantitative analysis is required in order to see which type of economy (i.e., less skewed firm size distribution v. more skewed) will adjust more in the presence of a financial shock.

1.3 Quantitative Analysis

1.3.1 Model Solution & Calibration

The presence of aggregate risk in the model makes the distribution of firms a relevant state variable that plays a key role in pricing functions of sovereign and firm debt. Firm distribution is a high-dimensional, so keeping track of it is computationally very difficult. In order to deal with this problem, we follow the approach of Krusell and Smith (1998) and assume that agents are boundedly rational in their perception of how the distribution evolves. We assume that the dynamics of the firm size distribution are approximated by a set of moments. In particular, the distribution is summarized by the cross-sectional variance of capital, and firms consider the marginal utility of the intermediaries' wealth (a market clearing price) to be a state variable and use a linear autoregressive linear to forecast future values of these objects. This approach is similar to the one employed in Krusell and Smith (1997), but it differs on the fact that here the price follows an autoregressive rule. The algorithm used to solve the model is described in the Appendix.[15] The results from the Krusell-Smith regressions are also presented in the same section of the Appendix.

The calibration of the model proceeds as follows. We start by setting the values of a subset of parameters that can be obtained directly from the data or the literature (external calibration). We then set the rest of the parameters of the model to match specific targets from firm-level data (internal calibration), which are chosen in order to be consistent with observed moments of the cross-section of Spanish firms at a yearly frequency.[16]

External Calibration The discount factor of firms β is set to 0.96, which is a standard

15. An additional complication that arises in this setting is that the problem of the firm has kinks and non-concavities, which makes difficult to solve this problem using collocation methods or methods that rely on first order conditions. Hence, the problem of the firm is solved via value function iteration.
16. All of the moments that are targeted in the calibration are those obtained for the 2005-2008 period, in order to avoid the large distortions of the Great Recession and also to check the external validity of the model.

value in the literature. The value for the capital depreciation rate δ is 0.11, which is in line with data from EU KLEMS for depreciation of Spanish capital. The production function parameter, α, is set to be $\alpha = 0.65$, consistent with the estimates of Hennessy and Whited (2007), using cross-sectional data of non-financial, unregulated firms from Compustat. The recovery parameter for the financial intermediaries is set to match the recovery rates of subordinated debt provided by Moody's (22.9%). The recovery rate defined in the model is $\theta k(1-\delta)/b$. The average leverage ratio of Spanish firms in the Amadeus database for the 2005-2008 period is $b/k = 0.55$, and the depreciation rate of Spanish capital is 0.11. Replacing these values in the recovery rate of the model yields $\theta(1-0.11)/0.55 = 0.229$, which implies $\theta = 0.142$. The corporate effective tax rate is set to be $\tau = 0.224$, in line with data from the Spanish Ministry of Finance.

The fiscal reaction function follows the structure of Ghosh et al. (2013). They specify a reaction function of the form: $\mathrm{PB}_t = v_0 + v_1 B + v_2 B^2 + v_3 B^3 + \varepsilon$. Hence, this rule requires setting 4 parameter values. For the intercept, v_0 is set to 6.464 in order to match the average (pre Great Recession) ratio of gross public debt-to-output of Spain (according to the IMF WEO). The rest of the parameters are set to equal those found by Ghosh et al. (2013). Lastly, the parameters of rule for the sovereign's default probability are $\alpha_s = -0.036$ and $\beta_s = 0.094$. We obtain these parameter values from a linear estimation where the independent variable is the gross public debt-to-output ratio, and the dependent variable is the yearly default probability implied by CDS spreads of 1-year Spanish sovereign bonds.[17] The data sources are IMF WEO and Markit.

Internal Calibration We choose 6 parameters of the model to match moments of the firm-level data. The problem of the firm is relatively non-linear, and most moments are altered when one parameter value changes. In order to find a reasonable set of initial

17. A 1-year maturity could reflect more accurately sovereign default risk as it is closest to a (potential) default episode.

parameters, we solve a reduced version of the full model. This reduced version is a partial equilibrium model, where firms face an aggregate shock that increases the interest rate at which they borrow. We solve the simplified version 30,000 times. For every time the model is solved there is an initial random draw of a vector of parameters from a uniform distribution. Thus, for every draw of parameters, there is a corresponding set of moments. We then plot the median of different moments for given values of parameters (discretized grids). This provides useful information regarding which moments are more sensitive to each parameter.

We assume that the productivity process z follows a log-AR(1) structure:

$$\log(z') = \rho_z \log(z) + \sigma_z \varepsilon \tag{1.18}$$

where $\varepsilon \sim \mathcal{N}(0,1)$, ρ_z is the autoregressive coefficient, and σ_z is the variance of the productivity process. The productivity is discretized following Tauchen (1986). The values of ρ_z and σ_z are internally calibrated, and are set so as to target the autocorrelation of profits-to-capital ratio ($(zk^{\alpha} - F)/k$), which is equal to 0.858, and the cross-sectional standard deviation of the investment to capital ratio (i/k), equal to 0.219.

The parameter of the adjustment cost of capital, ψ, is set to be $\psi = 0.064$, in order to match the average autocorrelation of i/k observed in the data (0.186). The fixed operational cost, F, targets the default ratio of firms which is roughly 2%, and is set to $F = 0.026$. The equity issuance cost parameter, λ, is equal to 5.428 and is chosen to target the average leverage ratio of firms observed in the data, which is 0.55. Lastly, the endowment of the financial intermediary is set to be $\bar{A} = 0.251$ so the average interest rate faced by firms is roughly 4%, a value that is common in the literature.

Summary of the Calibration & Model Fit A list of all the parameter values obtained in this calibration to Spain are presented in Table 2.1:

Table 1.4: Parameter Values

Parameter	Value	Description	Target
ρ_z	0.796	AR Coef. Productivity	Autocorrelation of Profits
σ_z	0.198	Std. Dev Productivity	Std. Dev. of i/k
ψ	0.064	Investment Adjustment Cost	Autocorrelation of i/k
F	0.026	Fixed Cost	Firm Default Rate
λ	5.428	Equity Issuance Cost	Firm Leverage b/k
$\bar{A}$	0.251	Bank Endowment	Average Interest Rate
β	0.960	Firm Discount Factor	Standard
δ	0.110	Capital Depreciation	EU KLEMS
α	0.650	Capital Share	Hennessy and Whited (2007)
θ	0.142	Recovery rate	Moody's
τ	0.224	Tax Rate	Spanish Ministry of Finance
v_0	6.464	Primary Balance	Spanish Government Debt/GDP
v_1	-0.225	Primary Balance	Ghosh et al. (2013)
v_2	0.003	Primary Balance	Ghosh et al. (2013)
v_3	0.000	Primary Balance	Ghosh et al. (2013)
α_s	-0.036	Sov. Def. Prob. (I)	IMF WEO & Markit
β_s	0.094	Sov. Def. Prob.(S)	IMF WEO & Markit

The process for the aggregate fiscal shock is represented as a 2-state Markov process with high and low values ($\underline{\varepsilon}$, $\overline{\varepsilon}$). In order to calibrate the transition matrix of this process, we use the time series for the Spanish primary fiscal balance, from 1950 to 2010. This information is obtained from the Public Finances in Modern History database of the IMF (Mauro et al., 2013). We define a low-realization period whenever the primary balance is below its long-run average minus one standard deviation, and a high-realization period if the opposite occurs. Using this definition, we construct the transition probabilities by identifying the frequency with which the time series transitions between the two states. The transition matrix for the process is:

$$\mathscr{P} = \begin{bmatrix} \Pr(\varepsilon' = \overline{\varepsilon} | \varepsilon = \overline{\varepsilon}) & \Pr(\varepsilon' = \underline{\varepsilon} | \varepsilon = \overline{\varepsilon}) \\ \Pr(\varepsilon' = \overline{\varepsilon} | \varepsilon = \underline{\varepsilon}) & \Pr(\varepsilon' = \underline{\varepsilon} | \varepsilon = \underline{\varepsilon}) \end{bmatrix} = \begin{bmatrix} 0.943 & 0.057 \\ 0.250 & 0.750 \end{bmatrix}$$

In order to assess the fit of the model we compute moments from simulations of the model and contrast them with those of the actual data, which we obtain from Spanish firms in the Amadeus database. Table 1.5 presents the comparison of moments.

Table 1.5: Targeted and Non-targeted Moments

Targeted	Model	Data
Default rate	0.017	0.020
Leverage (b/k)	0.550	0.547
Cross-sectional Std. Dev. of i/k	0.291	0.219
Autocorrelation of i/k	0.200	0.186
Autocorrelation of π/k	0.772	0.858
Non-targeted		
Investment-capital ratio i/k	0.151	0.128
Equity Issuance Frequency	0.108	0.098
Profits-assets ratio π/k	0.178	0.103
Cross-sectional Std. Dev. π/k	0.126	0.149
Cross-sectional Std. Dev. b/k	0.203	0.262
Autocorrelation of b/k	0.750	0.903

Except for the cross-sectional volatility of the investment to capital ratio, the model does a reasonable job in terms of fitting targeted moments of Spanish firm-level data. Regarding non-targeted moments, the model performs relatively well in terms of approximating non-targeted moments. The fact that the model performs reasonably well in terms of matching these firm-level features suggests that it is a good benchmark for studying the transmission of financial shocks to the real economy.

Policy Functions & Model Mechanism The 2-period model presented in section 1.2 suggested that small firms, which are more financially constrained than large ones, would reduce more their investment in comparison to large firms as a response to the financial shock. Using the solved model we can assess if the policy functions produce results that are consistent with these predictions.

Define the financing gap of the full model by:

$$\begin{aligned}\mathrm{FG}(z,k,b;\mathbf{S},\Gamma) = k'(z,k,b;\mathbf{S},\Gamma) + b + \Psi(k'(z,k,b;\mathbf{S},\Gamma),k) - (1-\tau)(zk^{\alpha} - F) \\ - k(1-\delta) - q(z,k'(z,k,b;\mathbf{S},\Gamma),b'(z,k,b;\mathbf{S},\Gamma);\mathbf{S},\Gamma)b'(z,k,b;\mathbf{S},\Gamma)\end{aligned} \tag{1.19}$$

where $x'(z,k,b;\mathbf{S},\Gamma)$ corresponds to the policy function of variable x as a function of the state variables of the firm.

As in section 1.2, the financing gap reflects the difference between the resources the firm needs to operate, minus the resources the firm can raise without issuing equity. We say that a firm is constrained if the financing gap is strictly positive, and it needs to issue equity in order to operate. We say a firm is unconstrained if the financing gap is negative.

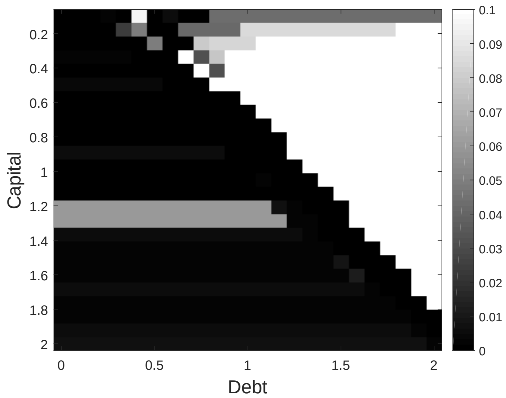
We use the policy functions obtained from the solution of the model to construct the financing gap. The state-space of the model is highly-dimensional, so we focus on one particular set of states: we set the productivity to be equal to the average of the process, and the sovereign debt-to-output ratio is set to the long-run average of Spain. We then proceed to compare the financing gaps and policy functions between the high and low realization of the aggregate shock, which reflects an increase in the financing costs of firms. Figure 1.1

presents 3 heat maps that show the differences in the financing gap, capital for next period k', and leverage b'/k, between the low and high financing cost scenarios.

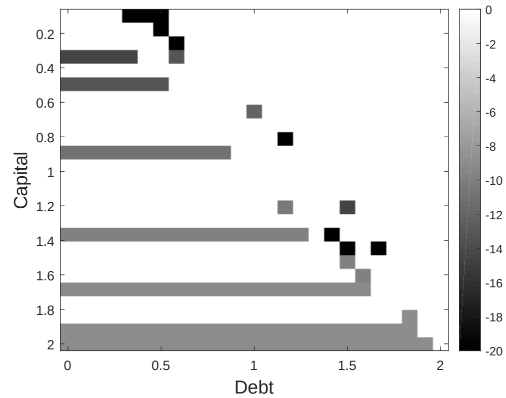
Panel (a) of Figure 1.1 shows the differences in the financing gaps when the financial costs increase. A positive value denotes an increase in the financing gap. The heat map shows that an increase in the financing costs increases the financing gap mostly for highly indebted firms (white part of the map), but also for some medium-sized firms and small firms that were initially highly levered. Panel (b) shows the percentage changes in capital k' when the financing costs increase. The firms that reduce their capital the most for next period are small and indebted (the same group for which the financing gap increased the most), and also some medium-sized ones. For these groups capital decreases by roughly 20%. Large firms also reduce their capital, but by a much lower amount, around 8%. Panel (c) shows the percentage changes for leverage. We observe a similar pattern than in the case of capital.

The results presented in Figure 1.1 are in line with the predictions of the 2-period model. Constrained firms reduce their investment the most upon an increase in financial costs. These firms also tend to be small and using a large amount (relative to their size) of debt. It is important to note that this analysis is fixing variables of the state space. For a more in-depth understanding, it is necessary to allow for these variable to endogenously adjust. This kind of analysis is presented in the next subsections.

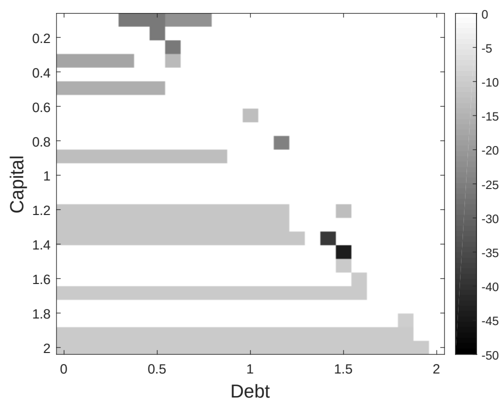
Figure 1.1: Policy Function Heat maps - Increase in Financing Costs



(a) Changes in Financing Gap



(b) Percentage Change in Capital



(c) Percentage Change in Leverage

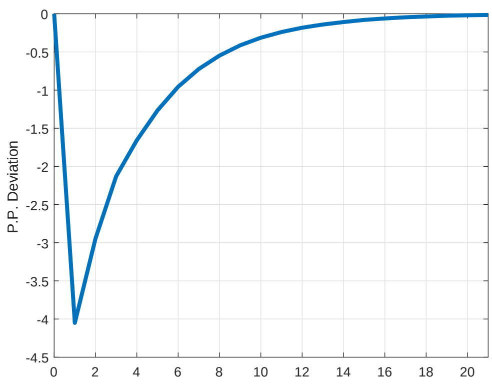
1.3.2 Aggregate and Firm-Level Dynamics

This subsection focuses on analyzing the responses of the economy upon a sharp increase in sovereign debt. We perform two sets of exercises. The first set focuses on aggregate behavior, assessing to what extent the dynamics of the model are consistent with those of the data. The second one shows the responses conditioning on firm size, to see if the model is able to deliver patterns where small firms adjust more than large ones, as observed in the data.

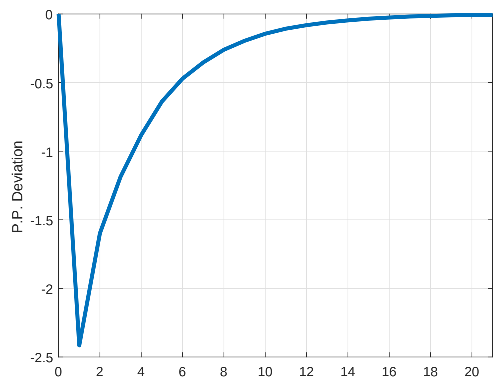
We examine aggregate and firm-level dynamics by computing impulse response functions to an increase of sovereign debt consistent with the increase in the ratio of gross debt-to-GDP observed in Spain in 2012 (from 69.5% to 85.7%). The procedure to compute the impulse response functions is the one presented in Gilchrist et al. (2014), and is described in the Appendix. Figure 1.2 presents the impulse response functions (IRFs) for aggregate output, capital, debt, and leverage. We see that output drops by roughly 4%, which is larger than the observed decline in Spanish real output in 2012, which was close to 3%. The economy reduces its leverage, by around 2.5%, which is consistent with the evolution of capital and debt, where the latter adjusts more than the former. These responses show that a sharp increase in sovereign debt can generate important fluctuations in output. However, these responses may not be necessarily the ones observed on the equilibrium path of the model, given that an exogenous sequence was fed to the model in order to generate them.

The observed dynamics in the IRFs are generated by a specific shock at particular initial conditions of the economy. In order to assess whether these observed dynamics are also observed without setting specific initial conditions, we perform an event-study analysis using the simulated dataset. We define the event as a situation in which sovereign debt is one standard deviation above its long-run mean. We show the dynamics of the relevant variables (which are aggregated according to the time-varying distribution) in a 10-period window centered on the event date. These dynamics correspond to the average across all the identified events in the simulations. Figure 1.3 presents the results of the event study.

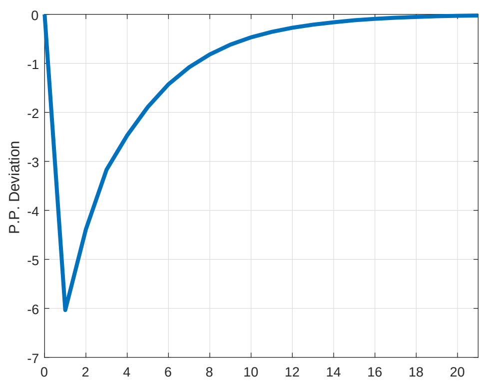
Figure 1.2: Impulse Response Functions - Aggregates



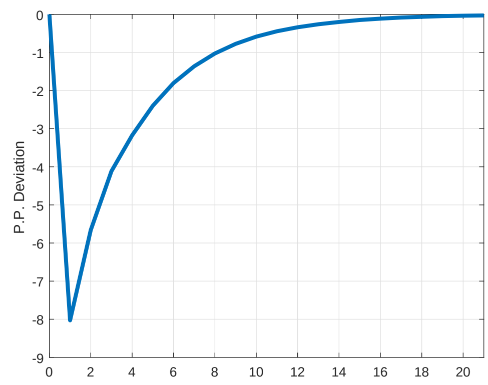
(a) Output



(b) Leverage



(c) Capital

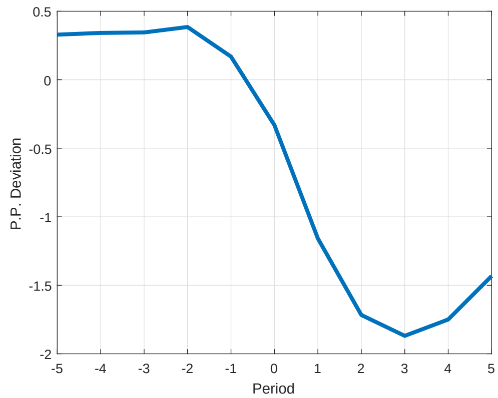


(d) Debt

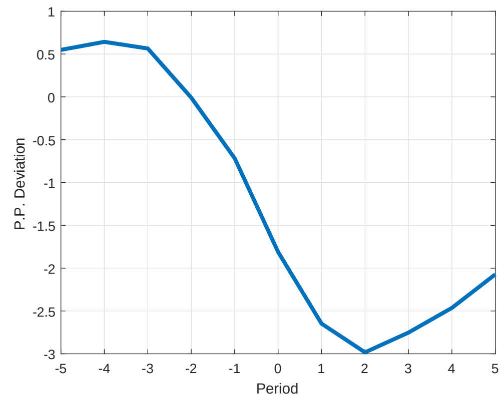
The results of the event study show that around the event date the sovereign debt-to-output ratio increases by around 15 percentage points, which is close to the actual increase during 2012 for the Spanish case. Output is about 0.3% below its long-run average at the event and continues dropping to roughly 2% below its mean, 3 years after. This gap of 1.7 percentage points corresponds to the decline in output after the event and is roughly half of the observed decline in real Spanish output. The dynamics of capital also follow a similar pattern to the one of output (given the specification of the production function of the model, where output and capital are proportional at the firm-level). Interestingly, we observe that firm debt (i) adjusts strongly around the event, and (ii) starts reacting even before the large increase in sovereign debt. This forward-looking behavior is consistent with firms internalizing an increasing path in sovereign debt and thus a potentially binding lending constraint for financial intermediaries in the future. In this way firms start adjusting accordingly, especially after the increasing path in sovereign debt is realized. The marginal utility of the intermediaries' wealth takes a bit of time to react, as it starts increasing one period before the event, but it then rises sharply to being roughly 8% above its long-run average.

We study next firm-level responses. The purpose is to assess if the model is able to generate patterns similar to those observed in the firm-level data. Recall from the empirical section that during the European debt crisis small firms reduced more their sales, assets, and liabilities in comparison to large firms. Figure 1.4 presents the IRFs to the same shock specified for the results in Figure 1.2. The results show that indeed small firms adjust much more aggressively: their output, leverage, capital, and debt drops more than for large firms (differences of 6.5, 6.25, 8, and 13 percentage points, respectively). These results support the theoretical predictions of the 2-period model of section 1.2 and the analysis of the policy functions performed in this section: small firms reduce more their investment than large firms when their financing costs increase. This is because small firms rely more on equity. When the cost of debt increases due to the financial shock, these firms are forced to substitute debt for equity, but given that it is increasingly costly and that they were already issuing equity, using this external financing source comes at a large cost. Another factor that makes the responses of small and large firms to differ is that the fixed cost of production pushes small firms closer to the default cutoff. This affects the price at which they can borrow, and also how far they can go with issuing equity.

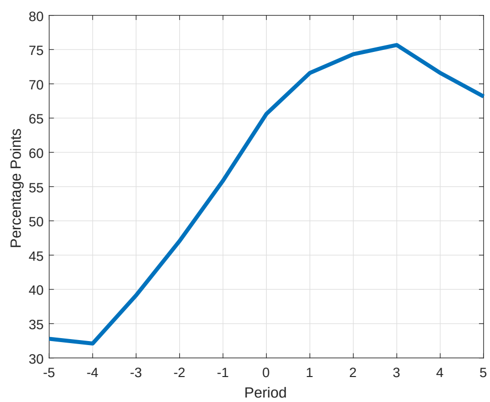
Figure 1.3: Event Study



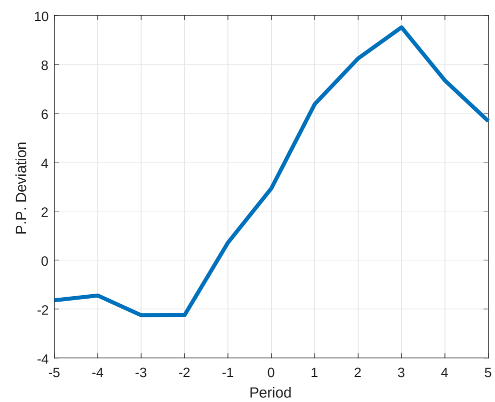
(a) Output



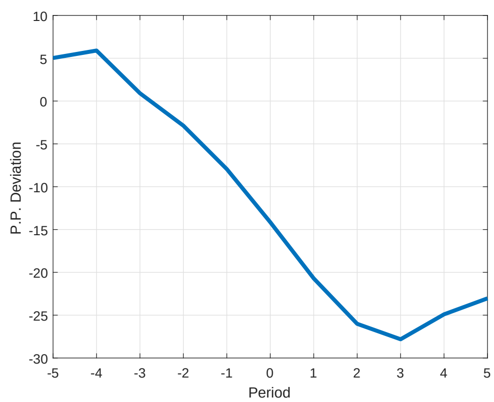
(b) Capital



(c) Sovereign Debt/Output

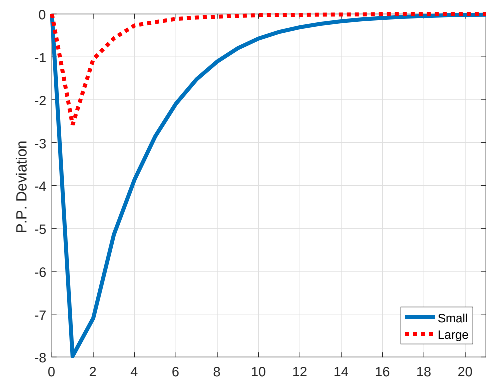


(d) Marginal Utility of the Intermediaries' Wealth μ

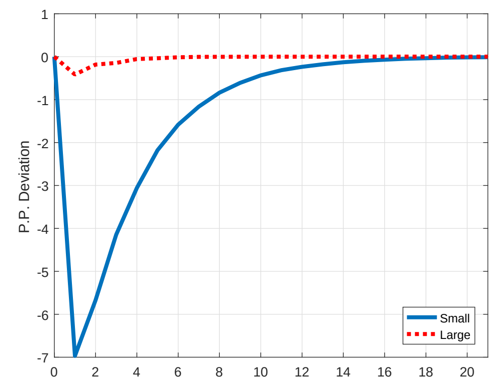


(e) Debt

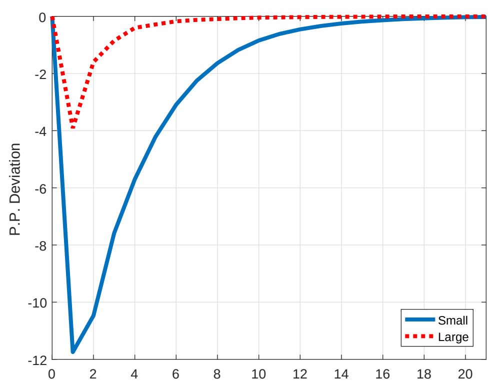
Figure 1.4: Impulse Response Functions - Firm-Level



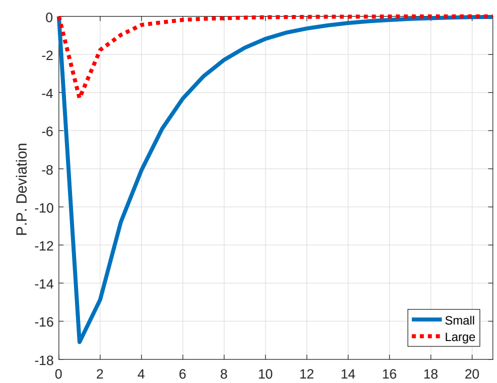
(a) Output



(b) Leverage



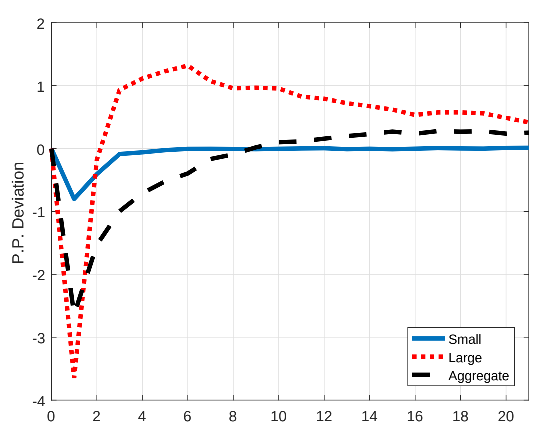
(c) Capital



(d) Debt

As shown in section 1.2, the key elements of the model that allow it to generate a pattern where small firms adjust more than large firms upon a financial shock are the costly equity issuance and the imperfect substitution between external financing alternatives. To illustrate this point, consider what would happen if equity issuance is not allowed. This is a common feature in corporate finance models, as it simplifies the computation. Ottonello and Winberry (2018), for example, find that firms in the US (listed in Compustat) respond to monetary policy shocks, but in their case, less levered firms are more sensitive. In the context of this model, more levered firms are those who face a larger financing gap and want to avoid excessive equity issuance, so they will be more responsive to financial shocks. Thus, whether there is equity issuance or not in the model can potentially affect the responses that firms will have in terms of their sizes. Figure 1.5 presents the response of output when equity issuance is not allowed. When there is no equity issuance, the responses are less marked in terms of small, large and the aggregate economy. Also, surprisingly, the type of firm that adjusts most is flipped: when there is no equity issuance, large firms adjust more than small firms, a pattern that is opposite to what is observed during the European debt crisis. The core of this paper is not to explain these differences, but it is an interesting result from the modeling point of view because assuming that firms can not issue equity is not innocuous. When equity issuance is not possible, small firms internalize that by borrowing too much they might be pushed to the default cutoff in the case financing costs sharply increase, as they will not able to rollover their debt. These firms are constrained, and they have no other source of external financing, so they borrow less. Thus, financing cost shocks have less severe effects on them than on large firms, which can rely on internal resources, as they will be unconstrained with a higher likelihood.

Figure 1.5: Impulse Response Function of Output - No Equity Issuance



1.3.3 The Relevance of Firm Heterogeneity

This subsection studies the role of firm heterogeneity on the responses of the economy to an increase in sovereign borrowing. In particular, two economies are contrasted, one with heterogeneous firms and one with a representative firm. Additionally, the responses of two economies composed by heterogeneous firms but with different distributions are computed, to assess the role of firm size distribution on the economy.

As seen in the previous section, small and large firms face different financing conditions and respond differently to the same kind of stimulus. It is important to note that asymmetric responses in terms of firm size do not guarantee that the composition of firms will matter for aggregates. For example, in Krusell and Smith (1998), the policy functions of agents in the model are almost linear and with the same slope. In addition to this, a large fraction of agents is rich enough to be in the linear part of the policy function. Thus, redistribution of wealth virtually has no effect on aggregate consumption. Another example is the case of Bernanke et al. (1999), where firms are linearly scalable in their net worth, rendering the role of the distribution of firms irrelevant. This section studies whether in this setting the heterogeneity of firms and the distribution are relevant.

Two exercises are performed in order to assess the relevance of firm heterogeneity. In the first, firm heterogeneity is nearly shut down by setting the variance of the productivity component equal to 20% of the original, and re-calibrating the rest of the parameters of the model.[18] By proceeding in this way, the "no heterogeneity" framework resembles a model with a representative firm. Then, we compute impulse response functions to a sharp increase in sovereign borrowing. The second exercise consists in computing impulse response functions that are conditional on different levels of skewness of the firm size distribution. This provides an alternative to assess whether firm size distribution is

18. The targeted moments are the same, except for the cross-sectional variance of the investment to capital ratio.

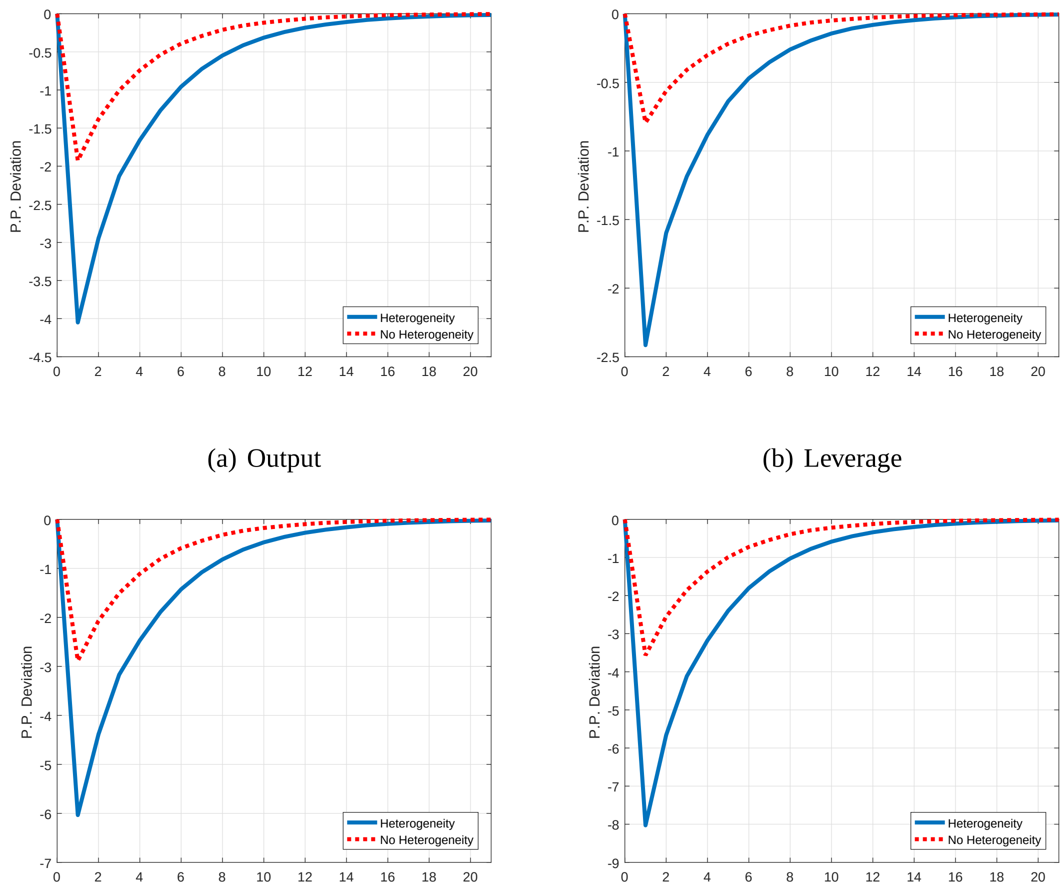
quantitatively important, and also to study if economies with larger skewness adjust more or less than those with a smaller skewness.

Figure 1.6 presents the IRFs of aggregate responses for the baseline economy ("heterogeneity") and an economy calibrated to have a standard deviation of productivity equal to $\sigma_z^{\text{No Het}} = 0.2 \times \sigma_z$, a substantially lower degree of heterogeneity. The economy with firm heterogeneity responds much more to the increase in government debt. Fluctuations in output in the baseline case are nearly twice than those in the no-heterogeneity case, while similar differences are observed for debt and capital. Regarding leverage, the differences are even larger. The baseline economy adjusts nearly 3 times more than in the no-heterogeneity scenario. These differences show that the composition of firms in the economy quantitatively matters, and that firm heterogeneity is a substantial source of amplification in terms of the responses of key macroeconomic aggregates.

The next exercise compares the responses of economies with high and low skewness of the firm size distribution.[19] In the first one, there is a bigger share of large firms. Intuitively, given that large firms adjust less when facing a financial shock, we should expect to see that an economy with greater firm size skewness adjusts less in aggregate terms. Figure 1.7 presents the comparison of the IRFs of these two economies. The IRFs for the "high skewness" case is the benchmark, while the "low skewness" consists on the responses to the same shock as the benchmark economy but conditioning the variance of capital (aggregate moment on the policy function) to be 50% of the baseline scenario.

19. The economy with low skewness has a skewness equal to 50% of the high skewness case.

Figure 1.6: Impulse Response Functions - Heterogeneity v. No Heterogeneity



(a) Output

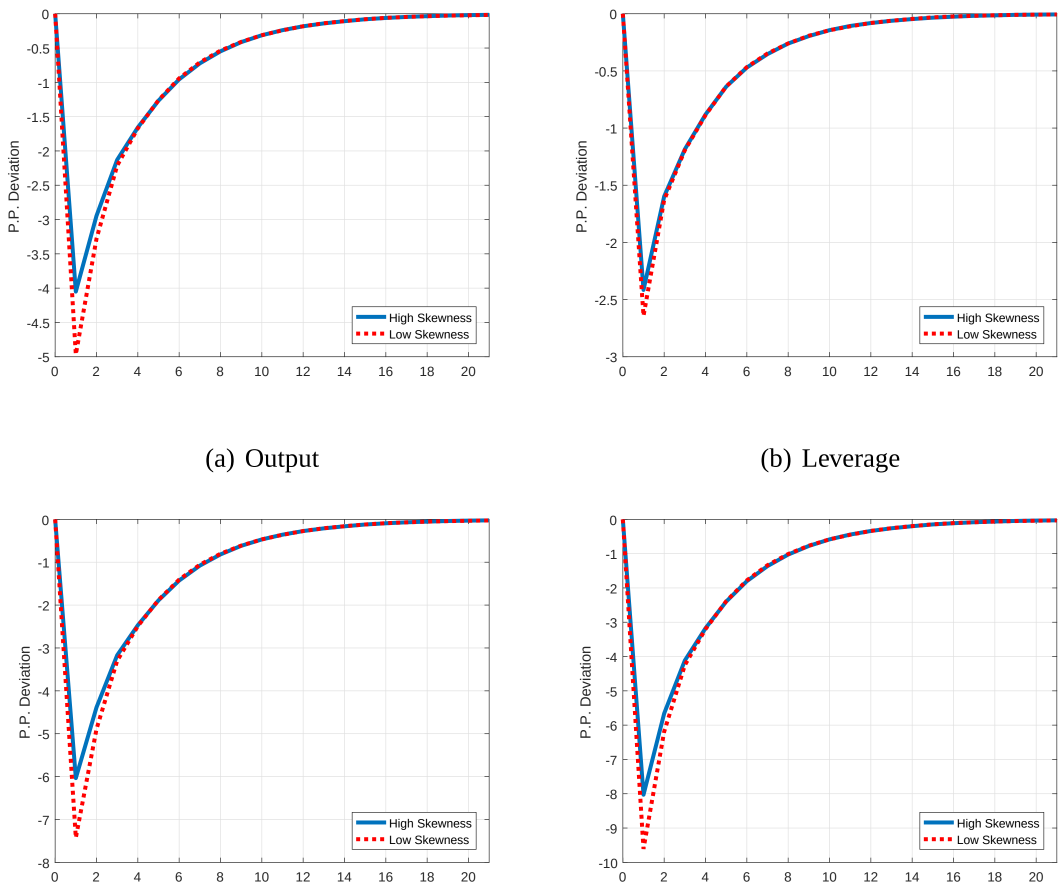
(b) Leverage

(c) Capital

(d) Debt

Output and capital respond roughly 25% more in the economy with lower skewness. The differences are smaller in the case of leverage and debt. These results suggest that the differences come from the right tail of the distribution of firm size, as it is mainly where the two economies differ. A less skewed distribution still includes a similar density of small firms, but it contains a significantly lower density of large firms. Thus, differences in the skewness of distributions can account for observed gaps in the responses of two different economies.

Figure 1.7: Impulse Response Functions - Low v. High Skewness



(a) Output

(b) Leverage

(c) Capital

(d) Debt

The results of these two exercises show that firm heterogeneity and the distribution of firms are relevant quantitatively. Firm heterogeneity generates responses of output that are twice the size of those of an economy with no firm heterogeneity. Additionally, the skewness of the firm size distribution accounts for up to 25% of the observed differences (in the data across low and high skewness cases) firm in the responses of output.

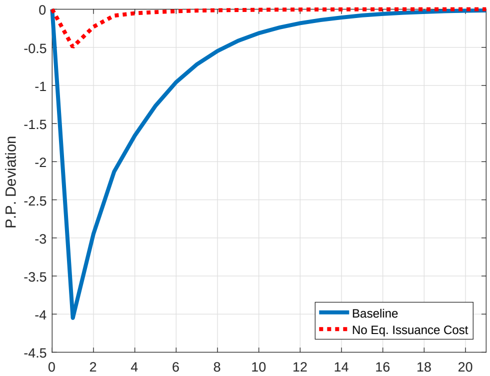
1.3.4 The Role of Financial Frictions

This subsection studies the role that financial frictions play in the aggregate responses to a financial shock caused by a sharp increase in sovereign borrowing. In particular, it focuses on the role that costly equity issuance has on the responses of firms to the financial shock. Section 1.2 showed how financial frictions in the model generate different responses for large and small firms when financing costs increase. The imperfect substitution between equity and debt was a key element to generate those asymmetric responses. Figure 1.8 shows the IRFs for an economy with no equity issuance costs, but with defaultable debt.[20]

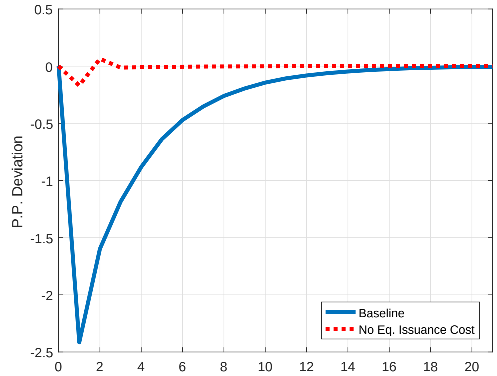
Figure 1.8 presents the responses of the baseline economy and the economy without equity issuance costs. The differences are striking. In the costless equity issuance case, the drop in output, leverage, capital, and debt is barely noticeable when comparing it to the baseline case. For example, we see that output in the baseline case drops by roughly 4%, while in the costless equity issuance scenario it drops by 0.5%. Similar patterns can be drawn from the rest of the variables. These results suggest that financial frictions play a key role in the amplification of the responses to financial shocks. The intuition for this is similar to the one provided in the 2-period model presented in section 1.2. Debt and equity are imperfect substitutes because debt is defaultable and equity issuance is costly. Thus, switching debt for equity (which is triggered by the financial shock) comes at a cost, which generates size-dependent adjustments in the investment of firms. If equity issuance is costless, then there is no additional cost for the firm when switching from debt to equity, which implies that the adjustments in investment triggered by financial shocks should be significantly lower. This is corroborated by the results presented in Figure 1.8.

20. Some degree of financial friction is needed in order to generate responses to the financial shock. In a completely frictionless environment firms would automatically choose their optimal level of capital, and finance investment with any of the two external financing alternatives.

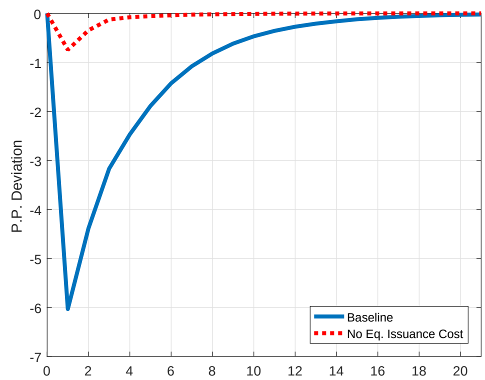
Figure 1.8: Impulse Response Functions - No Equity Issuance Cost



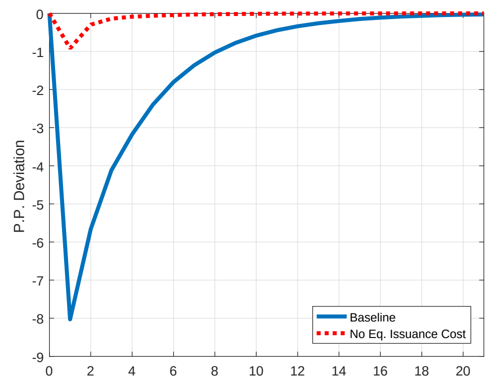
(a) Output



(b) Leverage



(c) Capital



(d) Debt

Lastly, notice that firm heterogeneity loses its amplification in the absence of financial frictions. Recall from Figure 1.6 that firm heterogeneity amplifies the responses of the variables of interest to the financial shock. In the costless equity issuance scenario, aggregate responses to the financial shock are negligible (in relation to the baseline case), which leaves little space for the amplification that firm heterogeneity generates.

1.4 Conclusions

This paper examines the role that firm heterogeneity and financial frictions play in terms of the propagation of financial shocks to the real economy. The recent European debt crisis provides a natural experiment to study these issues. During this crisis, a sudden deterioration of the credit conditions faced by firms in the GIIPS countries generated asymmetric responses at the firm level, which differed according to firm size. Empirical evidence suggests that one channel through which this deterioration occurred is a "crowding out" channel, by which increased government borrowing and bank holdings of sovereign debt generated a crowding-out of private lending, reducing funds available to firms and increasing their financing costs.

The empirical section of this paper documented two new facts regarding the response of firms in the GIIPS countries during the debt crisis using a rich panel dataset at the firm-level. It is observed that (i) smaller firms adjusted their sales, liabilities, assets and employment more than large firms, and (ii) smaller firms experienced smaller adjustments in these variables in economies with more skewed firm size distributions. The data also show that financing costs, measured as the ratio of debt service to total debt, increased more for smaller firms.

This paper proposed a model to explain the above two facts, and derived its quantitative implications. The model has three main components. First, firms that are heterogeneous in terms of productivity, capital and debt, and face financial frictions in the form of defaultable debt and costly equity issuance. Second, financial intermediaries that lend to firms and the government, facing an occasionally binding constraint on loanable funds. Third, a government that causes the occasionally binding constraint to bind when public debt increases sharply. When this occurs, the resources available for firm lending decreases, which generates an increase in their financing costs. Firms have different financing structures conditional on their size because debt and equity are imperfect substitutes. In particular, small firms will tend to rely more on equity than debt and thus face a more expensive financing mix. The financial shock will force firms to substitute debt for internal resources or equity. Small firms issue even more equity (which is increasingly costly), which forces them to adjust their investment more aggressively than large firms.

In this framework firm heterogeneity has aggregate implications. In the presence of financial frictions, the financial shock affects the evolution of the distribution of firms, and hence the dynamics of aggregate outcomes such as output. The model predicts that an economy with a less skewed firm size distribution faces larger drops in output upon a financial shock. The intuition for this result is that economies with a higher fraction of small firms (i.e., less skewed firm size distribution) will tend to adjust more because small firms are more sensitive to the financing cost shock. This pattern resembles the one observed in European debt crisis, where Portugal (an economy with low skewness on firm size distribution) faced larger drops in output than Spain (an economy with larger skewness than Portugal).

The model was calibrated to firm-level and aggregate data for Spain. Quantitative results show that for an increase in sovereign debt consistent with the Spanish case, experienced during the European debt crisis, the model predicts that small firms adjust more than large ones, and that smaller firms would have adjusted less if the economy had a larger firm-size dispersion. Regarding the aggregate effects of firm heterogeneity, the model generates an output drop of roughly half the size of the drop observed in Spain. Comparing with the results for the same model setup but with a representative firm, output drops by 2 percentage points more (with respect to its long-run average), in the economy with heterogeneous firms. Thus, firm heterogeneity along with the presence of financial frictions significantly amplifies the responses of aggregate variables to a tightening in credit conditions.

Chapter 2

Positive and Normative Implications of Liability Dollarization for Sudden Stops Models of Macroprudential Policy[21]

A nontrivial fraction of financial intermediation in emerging markets is characterized by what Calvo (2002) labeled "liability dollarization:" Banks intermediate capital inflows denominated in hard currencies (i.e. units of tradable goods) into domestic loans denominated in national currencies (i.e. units of national consumer prices). In South Korea or Mexico, for example, dollar inflows are lent out typically in domestic currency units, and the same happens with a sizable share of Euro and Swiss Franc inflows in the emerging markets of Eastern Europe. A report by the Bank for International Settlements showed that in 2007, just before the global financial crisis, the ratio of foreign currency liabilities to total liabilities of commercial banks in emerging markets was about 40 percent in Latin America, 25 percent in Europe, and 15 percent in Asia, Africa and the Middle East, and the median ra-

21. Coauthored with Enrique G. Mendoza. This paper was prepared for the IMF's Eighteenth Jacques Polak Annual Research Conference.

tio of external liabilities to gross loans in emerging markets was about 36 percent.[22] Using IMF data, Eichengreen and Hausmann (1999) reported that in 1996, just before the Asian Crisis, the ratios of foreign liabilities to total assets in commercial banks ranged from 143 percent in Indonesia to 775 percent in Thailand.

The workhorse Sudden Stops model (SS) that has been widely used to study macroprudential policy in emerging markets to date abstracts from liability dollarization, because it is built upon the canonical *Dependent Economy* framework of International Macroeconomics.[23] This framework includes income and consumption of tradable and nontradable goods but assumes that debt is denominated in units of tradables. SS models consider a stochastic variant of this setup in which domestic agents borrow by selling non-state-contingent bonds denominated in units of tradables, facing a credit constraint by which their debt cannot exceed a fraction of their income, part of which originates in the nontradables sector.[24] The key element of SS models is that the collateral provided by nontradables income is valued at the market-determined price of nontradable goods relative to tradables, which yields two central implications: First, it introduces the Fisherian debt-deflation amplification mechanism, by which a binding collateral constraint triggers a feedback mechanism linking reduced borrowing capacity, decreased consumption of tradable goods, and collapsing relative prices. Second, it introduces a "macroprudential" pecuniary externality, by which agents do not internalize in good times the effect of their borrowing decisions on relative prices and borrowing capacity in bad times when the credit constraint binds. These two features of the SS setup are related, because the magnitude of the pecuniary external-

22. *Capital Flows and Emerging Market Economies*, CGFS Papers No. 33, Bank for International Settlements, January 2009.
23. This framework originated in the seminal articles by Salter (1959), Swan (1960), and Díaz-Alejandro (1965).
24. This setup originates in the work of Mendoza (2002). Studies that explore the models' normative implications, and in particular the implications for macroprudential policy, include Bianchi (2011), Benigno et al. (2016), Korinek (2011), Schmitt-Grohé and Uribe (2017), Bianchi et al. (2016), and Hernández and Mendoza (2017).

ity is determined by the size of the Fisherian amplification effect on prices (see Mendoza (2016)). Quantitative studies (e.g. Bianchi (2011), Bianchi et al. (2016)) have shown that both financial amplification and pecuniary externalities are large in SS models, and that optimal macroprudential policy reduces significantly the frequency and magnitude of Sudden Stops.

The assumption that domestic debt is in units of tradables simplifies theoretical and quantitative work with SS models significantly, but it also rules out liability dollarization by construction. Intermediaries in these models can be viewed as either domestic or international banks that raise funds by issuing liabilities in tradables units at the world real interest rate, and lend them to domestic agents in the same units and at that same rate but requiring them to post collateral.[25] Interestingly, previous strands of the literature on emerging markets crises did introduce liability dollarization, particularly with the aim of studying aggregate implications of balance sheet effects and bank failures resulting from large devaluations (e.g. Choi and Cook (2004) and Céspedes et al. (2004)). This literature, however, did not focus on the implications of liability dollarization for private (nonfinancial) borrowers, which are the main focus of this study.

This paper shows that modeling the effects of liability dollarization on domestic borrowers alters significantly the results derived from SS models. In particular, we propose a model of Sudden Stops with liability dollarization (SSLD) in which intermediaries raise funds abroad in units of tradable goods but lend them out to domestic agents in units of the country's aggregate consumption good, represented by a CES composite of tradables and nontradables. As in standard SS models, the value of newly issued debt cannot exceed a fraction of the market value of income in units of tradables. In order to focus on the effects of liability dollarization on domestic borrowers, we assume that there are no other frictions

25. This implies that the standard SS models of macroprudential policy do not justify the use of capital controls as an instrument to discriminate foreign v. domestic credit. See subsection 4.1 for details.

in financial intermediation. Banks simply arbitrage the cost of raising funds abroad v. the expected return of domestic loans. Bank liability is unlimited and there are no restrictions on equity issuance or dividends, so that bank failures do not play a role in crisis dynamics in the model.

Liability dollarization introduces three key effects that operate via real-exchange-rate movements. First, if a country experiences a real appreciation, the higher "ex-post" value of the real exchange rate increases the domestic debtors' burden of repaying outstanding debt, because repaying debt denominated in units of aggregate consumption takes up more resources at a higher real exchange rate (a higher relative price of CES consumption in units of tradables). Second, if instead of a higher current real exchange rate, the future real exchange rate is expected to increase, arbitrage by intermediaries facing a given opportunity cost of funds raises the price they are willing to pay for newly issued domestic debt denominated in units of aggregate consumption, which lowers the interest rate on this debt and strengthens borrowing incentives. Third, even with unchanged current and expected real exchange rates, the negative conditional co-variance between future marginal utility and future real exchange rates incentivizes risk-taking (i.e. additional borrowing) because it creates a negative premium on the domestic real interest rate, which lowers the marginal cost of borrowing for domestic agents. Moreover, these three effects interact with precautionary savings incentives by altering the volatility of income and the expected return on non-state-contingent domestic assets.

The three effects of liability dollarization produce major differences in the positive and normative predictions of the SSLD model relative to SS models. Regarding positive findings, we show that under perfect foresight, the debt-repayment-burden effect has two major implications: It makes Sudden Stops milder and multiple equilibria harder to obtain. Multiplicity requires significantly higher limits on debt-to-income ratios and income realizations that fall within a narrower range of relatively high values. These results are illustrated with quantitative examples for a widely used calibration for the case of Argentina borrowed from the work by Bianchi (2011), and also for a calibration similar to that used in the recent study on multiplicity in SS models by Schmitt-Grohé and Uribe (2018).

The normative analysis yields three important results. First, by studying the optimal policy of a social planner acting with commitment under uncertainty, we identify a new pecuniary externality distorting the competitive equilibrium, which we label the "intermediation externality." This externality co-exists with the macroprudential externality of SS models, and results from the planner's incentives to respond to the three effects of liability dollarization, which are not internalized by private borrowers because they operate through market-determined prices (i.e. real exchange rates). Moreover, this externality is present even without credit constraints, and it can increase or reduce the social marginal cost of borrowing relative to its private counterpart. Hence, it can result in either under-borrowing or over-borrowing in the unregulated competitive equilibrium.

Second, we show that the optimal policy under commitment is time-inconsistent (i.e. lacks credibility). The planner has the incentive to pledge higher future consumption to sustain higher expected real exchange rates, and thus reduce ex-ante real interest rates at present. Once the future arrives, however, high real exchange rates are undesirable because of the higher debt repayment burden.

Third, the planner's optimal policy *does not* justify the use of capital controls, because it can be decentralized with an effective debt tax in which capital controls and taxes on domestic borrowing are equivalent. As in studies of optimal macroprudential policy in SS models with debt-to-income constraints, the planner cannot alter equilibrium allocations when the credit constraint binds. When it does not bind, it tackles the intermediation and macroprudential externalities by equating the social marginal cost of borrowing with its private counterpart, and this requires only an effective tax on debt. This debt tax can be managed equally with just taxes on capital inflows or on domestic credit, or any mix of both that supports the optimal effective debt tax rate. Hence, a differential tax treatment on foreign v. domestic credit is not justified.

Since the optimal policy lacks credibility and yields a complex, nonlinear schedule of effective debt taxes, we conduct a quantitative exploration of the effectiveness of a simple policy strategy that uses constant taxes on domestic debt and capital inflows, using again the calibration for Argentina taken from Bianchi (2011). In this analysis, the budgetary effects of both taxes are transferred to domestic private agents, which allows capital controls to prop up borrowing capacity when credit is constrained, and hence capital controls and domestic taxes need not be equivalent. The results show that the welfare-maximizing pair of constant taxes features a higher tax on domestic debt than on capital inflows, lowers the probability of Sudden Stops slightly, and yields a small welfare gain (while other tax pairs of similar magnitudes can reduce welfare sharply). Thus, under the best arrangement of constant taxes, the use of capital controls is justified, but capital inflows are taxed at a lower rate than domestic credit. Moreover, for tax pairs that support high effective debt taxes, capital controls and domestic debt taxes are again equivalent, and for other pairs welfare is higher with higher taxes on domestic debt than on capital inflows.

The rest of the paper proceeds as follows: Section 2.1 describes the model and characterizes the unregulated competitive equilibrium. Section 2.2 provides a theoretical and quantitative characterization of the differences between the SS model and SSLD models. Section 2.3 studies optimal financial policy of a social planner acting under commitment and conducts the quantitative analysis of constant taxes on domestic debt and capital inflows. Section 2.4 provides conclusions.

2.1 Model Structure

2.1.1 Private agents

Consider a small open economy where a representative agent consumes tradable goods (c^T) and nontradable goods (c^N). Preferences are given by a standard expected utility function with a constant-relative-risk-aversion (CRRA) period utility function that depends on a CES composite good c_t:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \quad u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}. \tag{2.1}$$

where:

$$c_t = \left[\omega \left(c_t^T\right)^{-\eta} + (1-\omega)\left(c_t^N\right)^{-\eta}\right]^{-\frac{1}{\eta}}, \eta > -1, \omega \in (0,1). \tag{2.2}$$

$\mathbb{E}(\cdot)$ is the expectation operator, β is the discount factor, γ is the coefficient of relative risk aversion, and $1/(1+\eta)$ is the elasticity of substitution between c_t^T and c_t^N.

The relative price of nontradable goods in units of tradables is denoted p_t^N, and the relative price of the composite good c_t in units of tradables is denoted p_t^c. Following standard practice (see Obstfeld and Rogoff (1996) p. 227), we apply the Duality Theory of consumer choice to characterize this price as the price index that corresponds to the minimum expenditure $c_t^T + p_t^N c_t^N$ such that $c_t = 1$. The price index is given by:

$$p_t^c = \left[\omega^{\frac{1}{1+\eta}} + (1-\omega)^{\frac{1}{1+\eta}} \left(p_t^N\right)^{\frac{\eta}{1+\eta}}\right]^{\frac{1+\eta}{\eta}}. \tag{2.3}$$

This relative price is also the economy's consumer-price-based measure of the real exchange rate, because foreign prices are normalized to 1 for simplicity and purchasing power parity in tradables holds, and hence the ratio of domestic to foreign consumer prices is the same as p_t^c. Notice also that p_t^c is a monotonic, increasing function of p_t^N.

The agent receives a stochastic endowment of tradable goods y_t^T and a fixed endowment of nontradable goods $\bar{y}^N$, and can trade non-state-contingent bonds b_t^c denominated in units of c_t at a price q_t^c with financial intermediaries. Choosing the price of tradables as the numeraire, the agent's budget constraint is:

$$q_t^c p_t^c b_{t+1}^c + c_t^T + p_t^N c_t^N = p_t^c b_t^c + y_t^T + p_t^N \bar{y}^N. \tag{2.4}$$

The left-hand-side of this expression shows the uses of the agent's income in units of tradables: purchases (sales) of bonds that require (generate) resources by the amount $q_t^c p_t^c b_{t+1}^c$ when $b_{t+1}^c > 0$ ($b_{t+1}^c < 0$), plus total expenditures in consumption of tradables and nontradables. The right-hand-side shows the sources of the agent's income: Income from maturing bond holdings $p_t^c b_t^c$ (or repayment of debt if $b_t^c < 0$), the realization of the endowment of tradables y_t^T, and the value of the nontradables endowment in units of tradables $p_t^N \bar{y}^N$. The stochastic process of the tradable endowment follows a standard Markov process to be specified later.

Borrowing requires collateral in the same way as in standard SS models. Hence, only a fraction of the agent's income is pledgeable as collateral, and as a result, the agent cannot borrow more than a fraction κ of total income in units of tradables:

$$q_t^c p_t^c b_{t+1}^c \geq -\kappa(y_t^T + p_t^N \bar{y}^N). \tag{2.5}$$

The representative agent chooses the stochastic sequences $\{c_t^T, c_t^N, b_{t+1}^c\}_{t\geq 0}$ to maximize (2.1) subject to (2.4) and (2.5), taking b_0, $\bar{y}^N$, and $\{p_t^N, p_t^c, q_t^c, y_t^T\}_{t\geq 0}$ as given.

2.1.2 Financial Intermediation

We assume that there are deep-pockets, risk-neutral financial intermediaries who float bonds in international markets at a world-determined price q^* (i.e. the inverse of the gross world real interest rate, R^* which is kept constant for simplicity). They use the resources raised this way to fund purchases of the bonds that domestic agents issue in order to borrow. These intermediaries price domestic bonds according to this standard no-arbitrage condition:

$$q_t^c = \frac{q^* \mathbb{E}_t \left[p_{t+1}^c \right]}{p_t^c}. \tag{2.6}$$

The price of domestic bonds issued at date t has associated with it the *ex-ante* domestic real interest rate in units of c given by $R_{t+1}^c \equiv 1/q_t^c = \frac{R^* p_t^c}{\mathbb{E}_t \left[p_{t+1}^c \right]}$. This is the rate at which domestic bonds are contracted at date t, and the expected real exchange rate ($\mathbb{E}_t \left[p_{t+1}^c \right]$) is one of its key determinants. Similarly, the ex-ante domestic real interest rate in units of tradables is defined as $R_{t+1}^T \equiv 1/(q_t^c p_t^c)$, and it is also determined by the expected real exchange rate. In contrast, the value of the real exchange rate realized at $t+1$ (p_{t+1}^c) determines the ex-post real interest rate paid in units of tradables, which is given by $\tilde{R}_{t+1}^T \equiv \frac{R_{t+1}^c p_{t+1}^c}{p_t^c}$ with an associated implied price of $\tilde{q}_t^T \equiv \frac{q_t^c p_t^c}{p_{t+1}^c}$. As we show later, the difference between these ex-ante and ex-post interest rates plays a central role in driving the effects of liability dollarization.

Except for liability dollarization, this is a frictionless characterization of financial intermediation. Banks have unrestricted access to world capital markets, intermediation does not incur any costs other than the funding cost R^*, banks can pay negative dividends, and can always cover a shortfall between income from loans paid by domestic agents and repayment to foreign creditors with additional external borrowing. These assumptions are made so that we can isolate the effects of liability dollarization on the borrowers (i.e. the nonfinancial private sector), which have not been considered in the Sudden Stops literature before, and they also simplify both the theoretical analysis and the numerical solution of the model. On the other hand, modifying the model to add intermediaries exposed to financial distress because of liability dollarization is of course an important item for further research.

2.2 Competitive Equilibrium & Comparison with Standard Models

The competitive equilibrium of the SSLD model is given by sequences of allocations $\{c_t^T, c_t^N, b_{t+1}^c\}_{t\geq 0}$, and prices $\left\{p_t^N, p_t^c, q_t^c\right\}_{t\geq 0}$ that solve the optimization problem of the representative agent, satisfy the no-arbitrage condition of the financial intermediaries, and satisfy also the market-clearing condition of the nontradables sector ($c_t^N = \bar{y}^N$) and the resource constraint of the tradables sector ($c_t^T = y_t^T - q_t^c p_t^c b_{t+1}^c + p_t^c b_t$).

The equilibrium conditions are the following:

$$p_t^N = \left(\frac{1-\omega}{\omega}\right)\left(\frac{c_t^T}{c_t^N}\right)^{\eta+1} \tag{2.7}$$

$$u_T(t) = \beta \mathbb{E}_t\left[u_T(t+1)\tilde{R}_{t+1}^T\right] + \mu_t \tag{2.8}$$

$$q_t^c p_t^c b_{t+1}^c \geq -\kappa\left[y_t^T + p_t^N \bar{y}^N\right], \qquad \text{with equality if } \mu_t > 0, \tag{2.9}$$

$$q_t^c p_t^c = q^* \mathbb{E}_t\left[p_{t+1}^c\right] \tag{2.10}$$

$$c_t^N = \bar{y}^N \tag{2.11}$$

$$c_t^T = y_t^T - q_t^c p_t^c b_{t+1}^c + p_t^c b_t, \tag{2.12}$$

where μ_t is the non-negative Lagrange multiplier on the credit constraint, and $u_T(t) \equiv u'(c_t)\partial c_t/\partial c_t^T$. Notice also two implications of these equilibrium conditions that will play an important role in the analysis that follows: First, (2.3), (2.7) and (2.11) imply that at equilibrium the price of nontradables and the price of consumption are increasing functions of tradables consumption, denoted $p^N(c_t^T)$ and $p^c(c_t^T)$ respectively. Second, if the credit constraint binds, c_t^T is in fact independent of the value of μ_t and is given by the solutions to the following nonlinear equation in c_t^T formed by conditions (2.7), (2.9) holding with equality, (2.11) and (2.12):

$$c_t^T = (1+\kappa)y_t^T + \kappa p^N(c_t^T)\bar{y}^N + p^c(c_t^T)b_t^c. \tag{2.13}$$

2.2.1 Comparison with Standard SS Models

We compare the above equilibrium conditions with those pertaining to the standard model of Sudden Stops in order to isolate the effects of liability dollarization.[26] In particular, since debt is non-state-contingent, liability dollarization introduces three key effects that result from the fact that borrowers are affected by real-exchange-rate fluctuations that induce movements in ex-ante and ex-post real interest rates, and these effects are present even without the collateral constraint (although their magnitude does change with credit constraints).

1. *Fluctuations in the debt repayment burden*: At any date t, the burden of repaying debt ($b_t^c < 0$) contracted at $t-1$ is $p_t^c b_t^c$. Hence, variations in the realized real exchange rate alter the repayment burden. A real appreciation (depreciation) increases (reduces) it, inducing non-insurable fluctuations in income disposable for tradables consumption. This income effect can also be interpreted in terms of changes in the ex-post real interest rate in units of tradables. A real appreciation (depreciation) increases (reduces) the burden of debt repayment because it increases (reduces) the

26. The equilibrium conditions in the standard models differ in that R^* replaces $\tilde{R}_{t+1}^T$ in condition (2.8), the terms $q_t^c p_t^c b_{t+1}^c$ and $p_t^c b_t^c$ are replaced with $q^* b_{t+1}$ and b_t where b_t are bonds in units of c_t^T, and condition (2.10) is removed.

ex-post real interest rate. Moreover, these fluctuations increase effective income volatility and thus strengthen incentives for precautionary savings, which weaken incentives to borrow.

2. *Fluctuations in the price of new domestic debt:* The intermediaries' pricing condition (2.6) implies that, for a given world interest rate, the price of newly-issued domestic bonds in units of tradables $q_t^c p_t^c$ rises when the real exchange rate is expected to appreciate, which implies that the ex-ante domestic interest rate in units of tradables R_{t+1}^T falls. Since we are studying an economy with debt ($b_{t+1}^c < 0$), this fall in the intertemporal relative price of c_t^T triggers income and substitution effects pushing for debt to increase. The lower interest rate also weakens self-insurance incentives.

3. *Risk-taking borrowing incentive:* The marginal cost of borrowing faced by domestic agents falls because of the positive co-movement between consumption and the real exchange rate. This result is evident if we re-write the Euler equation for domestic bonds as follows:

$$u_T(t) = \beta R^* \mathbb{E}_t \left[u_T(t+1) \right] + \beta \mathrm{Cov}_t(u_T(t+1), \tilde{R}_{t+1}^T) + \mu_t. \tag{2.14}$$

The marginal cost of borrowing in the right-hand-side of this expression differs from the standard SS model with debt in units of tradables only because of the term $\beta \mathrm{Cov}_t(u_T(t+1), \tilde{R}_{t+1}^T)$. This term is negative because $u_T(t+1)$ is decreasing in c_{t+1}^T and tradables consumption and the price of consumption are positively correlated. Hence, the marginal cost of borrowing is lower than in the standard model. In fact, since c_{t+1}^T and p_{t+1}^c are *perfectly* correlated, $\mathrm{Cov}_t(u_T(t+1), \tilde{R}_{t+1}^T) = \frac{-\sigma_t(u_T(t+1))\sigma_t(p_{t+1}^c)}{q^* \mathbb{E}_t[p_{t+1}^c]}$, where $\sigma_t(u_T(t+1))$ and $\sigma_t(p_{t+1}^c)$ are conditional standard deviations of marginal utility and the real exchange rate respectively. As a result, the risk-taking borrowing incentive strengthens when tradables marginal utility is more variable and/or the coefficient of variation of the real exchange rate ($\frac{\sigma_t(p^c_{t+1})}{\mathbb{E}_t[p^c_{t+1}]}$) rises. This result also implies that, for given standard deviations of marginal utility and prices, the risk-taking incentive weakens when the real exchange rate is expected to appreciate. Hence, an expected real appreciation triggers opposing effects on borrowing incentives: It weakens the risk-taking incentive but strengthens the effect on the price of new debt.

Because of these three effects, Sudden Stops will differ in magnitude and frequency across SSLD and SS models with identical parameters, and they may occur at different levels of debt and income. If the constraint binds in both models for a given value of y^T_t, however, allocations and prices will differ only due to differences in the repayment burden of the outstanding debt. This is because, when the constraint binds, c^T_t is determined by the non-linear equation (2.13), and this equation differs across the two models only because of the debt repayment terms. Still, along the equilibrium path of each model, the magnitude and frequency of Sudden Stops will differ depending on the outstanding debt (b^c_t) and the size of income shocks (y^T_t) needed to trigger the constraint in each economy. The outstanding debt that triggers Sudden Stops is endogenous and depends on the history of previous income shocks and optimal debt decisions, which differ in the two models because of the three effects described above. Similarly, the frequency of Sudden Stops will differ depending on the long-run probability with which each economy reaches states with enough debt to trigger a Sudden Stop.

SS models also have the property that while they can support unique equilibria, so that a particular sequence of income causes a shift from a unique unconstrained equilibrium to a unique constrained one, they can also support equilibrium multiplicity, so that for a given income level there can be both constrained and unconstrained equilibria and which one prevails depends on a "sunspot" variable (see, for example, Schmitt-Grohé and Uribe

(2017, 2018)). Because multiplicity emerges due to the possibility of supporting more than one equilibrium when the credit constraint binds, due to the responses of collateral values and optimal debt choices, it is reasonable to expect that liability dollarization should also lead to differences across the SS and SSLD models in the conditions required for multiplicity and in the characteristics of multiple equilibria.

In the remainder of this Section, we compare the characteristics of Sudden Stops and multiple equilibria in the two models in a perfect-foresight environment, in which the solutions of the models can be fully characterized analytically. For a comparison of quantitative solutions of stochastic versions of the models see Mendoza and Rojas (2017).

2.2.2 Perfect Foresight Analysis

Under perfect foresight, ex-ante and ex-post real exchange rates and interest rates are the same. As a result, the Euler equation and resource constraint of the SSLD model can be rewritten as follows (assuming the standard no-Ponzi-game condition):

$$u_T(t) = \beta R^* \left[u_T(t+1)\right] + \mu_t \tag{2.15}$$

$$\sum_{t=0}^{\infty} R^{*-t} c_t^T = \sum_{t=0}^{\infty} R^{*-t} y_t^T + p_0^c b_0^c, \tag{2.16}$$

where $\sum_{t=0}^{\infty} R^{*-t} y_t^T \equiv W_0$ is the tradables non-financial wealth of the economy. These two conditions, together with (2.7), (2.9) and (2.11) characterize fully the SSLD equilibrium under perfect foresight. Following Mendoza (2005), we simplify the analysis by assuming that $\beta R^* = 1$, $b_0^c < 0$ (i.e. the economy starts with some debt), initial tradables income is lower than in the future so that agents would want to set $b_1^c < 0$, and we study wealth-neutral shocks such that y_0^T changes keeping W_0 constant, hence inducing agents to borrow more.[27] For a sufficiently large cut in y_0^T, the collateral constraint binds, but for smaller shocks it does not.

If the collateral constraint does not bind, and since $\beta R^* = 1$, tradables consumption is constant at the standard value of textbook models of consumption smoothing:

$$\overline{c}^T = (1-\beta)(W_0 + p_0^c b_0^c). \tag{2.17}$$

It is straightforward to verify that the conditions that characterize the perfect-foresight equilibrium of the SS model are almost identical and yield an analogous solution for tradables consumption, except for one difference: In the term that represents financial wealth in the right-hand-side of (2.17), financial wealth is given by the term b_0 in the SS model (with bonds denominated in tradables), v. $p_0^c b_0^c$ in the SSLD model. These two terms will differ in general, because for given values of the exogenous initial conditions b_0^c and b_0, the equilibrium value of the initial price p_0^c determines whether the burden of repayment of the initial debt is higher in the SSLD or the SS case. However, taking the equilibrium price of nontradables $p^{N,SS}$ from an unconstrained solution of the SS model for a given b_0, we can compute the implied value of the consumption price index $p^{c,SS}$ and then define a threshold initial debt level in the SSLD model such that $\tilde{b}_0^c \equiv b_0/p^{c,SS}$. At this debt level, the SSLD and SS models yield identical unconstrained perfect-foresight equilibria because both financial and nonfinancial wealth are the same.

If $b_0^c < \tilde{b}_0^c$, the SS model yields higher tradables consumption and prices than the SSLD model.[28] Consumption rises by less than the reduction in debt because the increase in p^c increases the debt repayment burden and hence offsets some of the effect of the lower debt.

27. A wealth-neutral income shock at t=0 is defined by income levels (y_0^T, y_1^T) such that $y_1^T - \overline{y}^T = R(\overline{y}^T - y_0^T)$ and $y_t^T = \overline{y}^T$ for $t \geq 2$. Hence, wealth remains constant at $W_0 = \overline{y}^T/(1-\beta)$.

28. Similarly, keeping $\tilde{b}_0^c$ and b_0 unchanged when making parametric changes that affect wealth (for example, temporary or permanent, unanticipated changes in the tradables income stream) results in different equilibria that depend on the changes in initial prices and debt repayment burden.

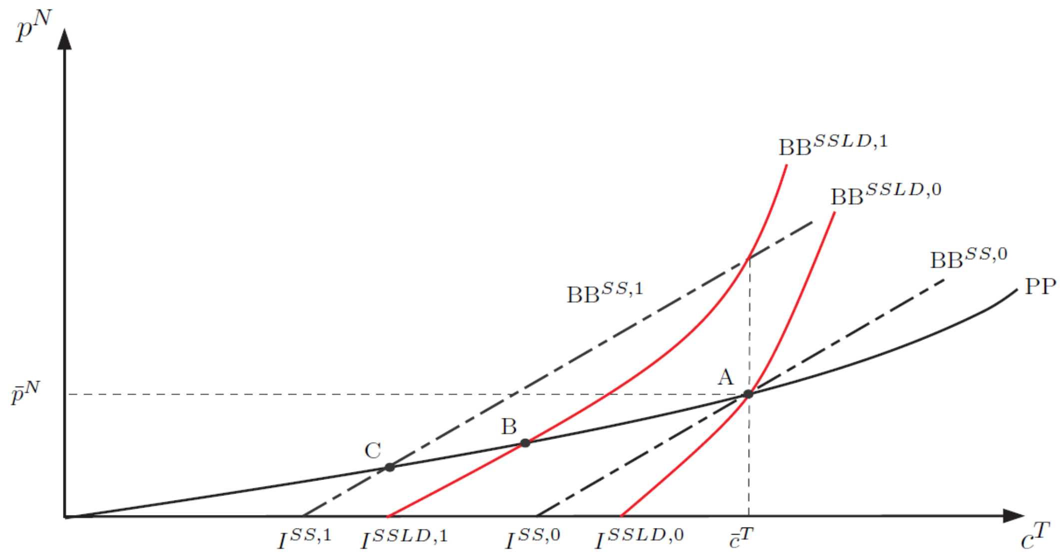
Because of perfect foresight, the effects of liability dollarization operating via the price of newly issued debt and the risk-taking incentive are ruled out. Only the effect operating via the debt repayment burden is at work. Note that when $b_0^c > \tilde{b}_0^c$ the results presented above go in the opposite direction.

If the reduction in y_0^T is sufficiently large to make the collateral constraint bind at $t = 0$, a Sudden Stop occurs. Condition (2.12) implies that c_t^T falls, because access to debt to sustain tradables consumption is constrained. Then it follows from condition (2.7) that p_t^N falls to clear the nontradables market. This generates a further tightening of the collateral constraint, because it reduces the value of collateral provided by the nontradables endowment in condition (2.9). Formally, the date-0 allocations and prices are determined by condition (2.13). This condition is again almost the same that determines tradables consumption in a Sudden Stop in the SS model, except for the debt repayment term $p^c(c_0^T)b_0^c$, which in the SS model is just b_0. By the same argument as before, the Sudden Stop equilibrium prices of the SS model could be used to set the exogenous value of b_0^c so as to make the SS and SSLD solutions when the constraint binds the same. But to compare Sudden Stops across the two models, assume instead that the initial condition is set at the value $\tilde{b}_0^c$, which sustains identical *unconstrained* equilibria. In this case, the Sudden Stop equilibria of the two economies differ.

(a) Sudden Stops with unique equilibria

Figure 2.1 illustrates the determination of equilibria in both models in the (c^T, p^N) space for a scenario in which the equilibria are unique, in a manner analogous to Figure 2 in Mendoza (2005). The *PP* curve is the marginal rate of substitution in consumption of tradables and nontradables, which given isoelastic preferences and the constant endowment of nontradables yields a convex function that maps c^T into p^N (this curve is the same in the SS and SSLD models). The various BB curves show the value of p^N that corresponds to a value of c^T such that the collateral constraint holds with equality and the tradables resource constraint is satisfied (i.e. equation (2.13) solved for p_0^N as a function of c_0^T), for the SS and SSLD models and for each under different values of y_0^T. In each case, equilibrium is reached where the *PP* curve and the relevant BB curve intersect.

Figure 2.1: Sudden Stops under Perfect Foresight: Unique Equilibria



Mendoza (2005) showed that in the SS model, the BB^{SS} curves are increasing, linear functions of c_0^T with an horizontal intercept given by $I^{SS} \equiv (1+\kappa)y_0^T + b_0$ and a slope of $m^{SS} \equiv 1/(\kappa\bar{y}^N)$. For the SSLD model, we show in the Appendix that the BB^{SSLD} curves are also increasing in c_0^T with an horizontal intercept given by $I^{SSLD} \equiv (1+\kappa)y_0^T + \omega^{1/\eta}\tilde{b}_0^c$ and a slope of $m^{SSLD} \equiv [1 - p^{c\prime}(t)\tilde{b}_0^c]/(\kappa\bar{y}^N)$.[29] Notice that again the difference between the SS and SSLD models is due to differences in the repayment burden of the initial debt. The term $\omega^{1/\eta}$ in the intercept of the SSLD model is the lower bound of p^c that is reached

29. As shown in the Appendix, the BB^{SSLD} curves are concave if the elasticity of substitution between c^T and c^N is greater or equal to 1, convex if it is less or equal than 1/2, and switch from concave to convex as c^T rises if the elasticity is between 1/2 and 1. Under reasonable parameter values for emerging markets and any elasticity between 0 and 1, however, BB^{SSLD} is either strictly convex or nearly linear with a slightly concave segment for very low c^T.

when $p^N = 0$. Since at $\tilde{b}_0^c$ we have the same debt in units of tradables in the SS and SSLD models at the higher prices supported in the unconstrained equilibrium, the intercept of the SSLD model must be to the right of the one in the SS model (since the same debt is valued at the minimum price in the intercept of the SSLD model).

The BB^{SS,0}, BB^{SSLD,0} curves are for a threshold income level $\hat{y}_0^T$ such that with a wealth-neutral shock that reduces date-0 income that much, the credit constraint allows for just enough debt to still support the unconstrained equilibrium (i.e. below this threshold the constrain becomes binding).[30] Hence, by construction (and again assuming the initial SSLD debt is $\tilde{b}_0^c$) these two *BB* curves intersect *PP* at point A and yield the same equilibrium values of c^T and p^N in the two models. The BB^{SS,1}, BB^{SSLD,1} curves are for $y_0^T < \hat{y}_0^T$, which shifts the BB curves to the left, triggering the credit constraint and causing a Sudden Stop.

The main point of Figure 2.1 is to show that, when equilibria are unique, Sudden Stops under perfect foresight are milder in the SSLD model. The Sudden Stop equilibria are reached at points B and C for the SSLD and SS model respectively.[31] Since the BB^{SSLD,1} curve is always steeper than the BB^{SS,1} curve and has a higher horizontal intercept, and since for given y_0^T the two curves always intersect at point A, BB^{SSLD,1} must cut the PP curve to the right of where BB^{SS,1} cuts it.[32] This implies that in the Sudden Stop of the SSLD economy, tradables consumption and relative prices are higher than in the SS economy. Both equilibria are Sudden Stops, because financial amplification via the deflation of

30. The threshold income in the SSLD model is $\hat{y}_0^T = \frac{\bar{c}^T - \bar{p}^c \tilde{b}_0^c - \kappa \bar{p}^N \bar{y}^N}{1+\kappa}$, where $\bar{c}^T$, $\bar{p}^c$ and $\bar{p}^N$ are the unconstrained equilibrium allocations and prices.

31. The Figure shows the values of c_0^T, p_0^N such that the budget and credit constraints hold with equality and p_0^N equals the corresponding marginal rate of substitution. It is also straightforward to show that the Euler equation holds with a Lagrange multiplier of $\mu_0 = u'(c_0^T) - u'(c_1^T) > 0$ and that $\mu_t = 0$ for $t \geq 1$. In the SS model, since W_0 is unchanged, $y_1^T > \bar{y}^T > y_0^T$, and $y_t^T = \bar{y}^T$ for $t \geq 2$, it follows that the fact that $c_0^T < \bar{c}^T$ implies $c_1^T > \bar{c}^T$. Moreover, $c_t^T = c_1^T$ for $t \geq 2$. This, together with $y_1^T > \bar{y}^T$, implies that $b_2 > b_1 > -\kappa(y_1^T + p_1^N y^N)$ and $b_t = b_2$ for $t \geq 2$.

32. The fact that $p^{c\prime}(t)\tilde{b}_0^c < 0$ implies that $m^{SSLD} > m^{SS}$, and the fact that $\omega^{1/\eta}\tilde{b}_0^c < b_0$ implies that $I^{SSLD} > I^{SS}$.

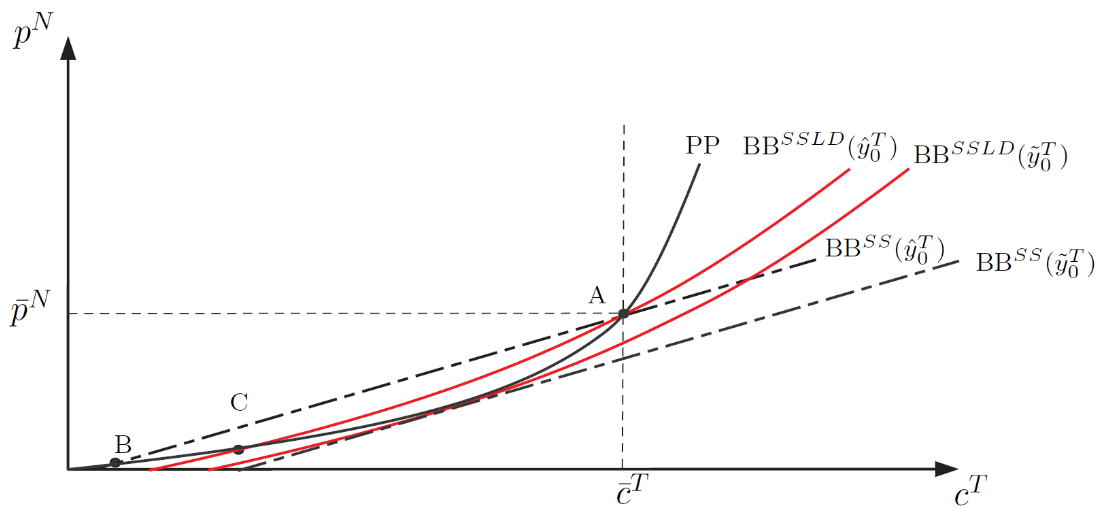
the value of collateral causes a sudden drop from the unconstrained stationary consumption and prices at point A, but the drops in the SSLD model are always milder. The intuition is simple: In the SSLD economy, the fall in the real exchange rate associated with a Sudden Stop reduces p_0^c, and hence the burden of repaying $\tilde{b}_0^c$ in terms of tradable goods falls, providing additional resources for consumption of tradables. Notice that around point A the BB curves are steeper than the PP curve, which as we show below is a sufficient condition for the equilibria to be unique.

It is also worth noting that there could be a second intersection of the BB and PP curves in Figure 1 if we extended its domain far enough. The second intersection, however, is not an equilibrium. This is because, as is evident from the Figure, c_0^T would be higher than $\bar{c}^T$, and given the specification of the wealth-neutral shock the intertemporal resource constraint would imply $c_1^T < c_0^T$, which would imply a negative Lagrange multiplier ($\mu_0 = u'(c_0^T) - u'(c_1^T) < 0$). Moreover, the unconstrained equilibrium cannot co-exist with the unique Sudden Stops equilibria, because as Figure 1 shows, the value of p_0^N at which the nontradables market would clear if tradables consumption is $\bar{c}^T$ is too low for the resource constraint to be satisfied with the credit constraint binding (i.e. at $\bar{c}^T$ the PP curve is below the BB^{SS,1} and BB^{SSLD,1} curves).

(a) Sudden Stops with multiple equilibria

Figure 2.2 illustrates the equilibrium of both models when there are multiple equilibria. Intuitively, multiplicity can emerge if the parameters of the PP and BB curves are such that the curves intersect twice for $c^T \leq \bar{c}^T$. BB curves that are flatter than the PP curve around point A are necessary (but not sufficient) for this to happen. In the Figure, the BB curves for $\hat{y}_0^T$ are constructed in the same way as the $BB^{SSLD,0}$ and $BB^{SS,0}$ curves of the previous Figure: They correspond to a wealth-neutral reduction in y_0^T such that the credit constraint is marginally binding (i.e. it sustains the same amount of debt as in the unconstrained equilibrium) assuming again that the initial debt of the SSLD economy is set at $\tilde{b}_0^c$. Hence, point A has the same meaning as before. It shows the unconstrained stationary equilibrium that is identical in the SSLD and SS economies. What has changed is the curvature and slope of the PP and BB curves, and in particular the BB curves are significantly flatter. As a result, at an initial income of $\hat{y}_0^T$, the SS and SSLD economies now have two equilibrium solutions each: The unconstrained outcome at point A and the Sudden Stop outcomes (points B and C for the SS and SSLD economy respectively).

Figure 2.2: Multiple Equilibria under Perfect Foresight



Note, however, that BB curves flatter than the PP curve around point A are not sufficient for equilibrium multiplicity, because this also depends on the value of y_0^T. In Figure 2, for $y_0^T < \hat{y}_0^T$, the BB curves would shift left and only one Sudden Stop equilibria would survive in each economy (the unconstrained equilibrium is no longer attainable, just as in Figure 1). When $y_0^T = \hat{y}_0^T$, we have the two equilibria as explained above. For $y_0^T > \hat{y}_0^T$, the BB curves shift *to the right* and three equilibria exist, because there would be two intersections with PP to the left of A, plus the unconstrained equilibrium is also attainable since income *exceeds* $\hat{y}_0^T$.[33] In contrast, when the BB curves are steeper than PP at point A, the unconstrained equilibrium is the unique equilibrium for any income higher than $\hat{y}_0^T$. As y_0^T continues to rise, however, y_0^T reaches another threshold value $\tilde{y}_0^T$, different in the two models, at which once again the unconstrained equilibrium and only one Sudden Stop equilibrium are possible (see Figure 2.2), and for $y_0^T > \tilde{y}_0^T$ multiplicity disappears and only the unconstrained equilibrium survives, because the BB curves shift to the right enough to never intersect with PP.[34]

It follows from the above discussion that equilibrium multiplicity requires: (i) BB curves flatter than PP around point A, and (ii) wealth-neutral income shocks in the specific interval $\hat{y}_0^T \leq y_0^T \leq \tilde{y}_0^T$. As we explain next, equilibrium multiplicity is harder to obtain in the SSLD model because, for the same parameter values and initial conditions, multiplicity requires higher values of κ and narrower income intervals.

Condition (i) can be formalized as follows: *A BB curve steeper than the PP curve around $\bar{c}^T$ is a sufficient condition for a unique equilibrium when a Sudden Stop occurs.* Since the PP curve is given by condition (2.7), its slope can be expressed as $\frac{(1+\eta)p_t^N}{c_t^T}$, which at the common unconstrained equilibrium of the two models equals $\frac{(1+\eta)\bar{p}^N}{\bar{c}^T}$. Consider next the slope of the BB curve in the SS model, which is $1/(\kappa\bar{y}^N)$. It follows from equating these two slopes that the threshold value of κ that guarantees a unique equilibrium in the SS model is $\hat{\kappa}^{SS} = \frac{\bar{c}^T}{(1+\eta)\bar{p}^N\bar{y}^N}$. Any κ lower than this implies that Sudden Stop equilibria for any $y_0^T \leq \hat{y}_0^T$ are unique. This condition is the same one derived in Mendoza (2005).

The slope of BB in the SSLD model is $\frac{1-p_0^{c\prime}b_0^c}{\kappa\bar{y}^N}$, and hence the threshold value of κ for uniqueness of Sudden Stop equilibria under liability dollarization is $\hat{\kappa}^{SSLD} = \frac{\bar{c}^T(1-\bar{p}^{c\prime}b_0^c)}{(1+\eta)\bar{p}^N\bar{y}^N}$.

33. In all the Sudden Stops equilibria of Figure 2, $c_0^T < \bar{c_T} < c_1^T$ and the Euler equation holds with a Lagrange multiplier $\mu_0 = u'(c_0^T) - u'(c_1^T) > 0$. This is again because, given constant wealth, the intertemporal resource constraint implies that c_1^T rises as the credit constraint makes c_0^T fall.

34. $\tilde{y}^T$ is the income level that makes the BB curves tangent to the PP curve, which is different for the two models.

However, since we are assuming that the initial debt of the SSLD model is the value $\tilde{b}_0^c$ that supports the same unconstrained equilibria in both models, it follows that $\hat{\kappa}^{SSLD} = \hat{\kappa}^{SS}(1 - \overline{p}^{c\prime}\tilde{b}_0^c) = \kappa^{SS}\left(1 - \xi_{p^c,c^T}\frac{\overline{p}^c\tilde{b}_0^c}{\overline{c}^T}\right)$, where ξ_{p^c,c^T} is the elasticity of the CES price index with respect to tradables consumption, and is given by $\xi_{p^c,c^T} = \frac{1+\eta}{1+\left(\frac{\omega}{1-\omega}\right)^{\frac{1}{1+\eta}}(p^N)^{-\frac{\eta}{1+\eta}}}$. Hence, since $\tilde{b}_0^c < 0$ and $\xi_{p^c,c^T} > 0$ because $\overline{p}^{c\prime} > 0$, we obtain that $\hat{\kappa}^{SSLD} > \hat{\kappa}^{SS}$. Thus, for identical parameters and the same initial conditions, multiplicity is harder to find in the SSLD model because it maintains uniqueness at higher values of κ (i.e. the range of values of κ for which Sudden Stop equilibria are unique is larger in the SSLD model). Moreover, $\hat{\kappa}^{SSLD}$ exceeds $\hat{\kappa}^{SS}$ by a percentage given by the absolute value of $\xi_{p^c,c^T}\frac{\overline{p}^c\tilde{b}_0^c}{\overline{c}^T}$, which in turn depends on preference parameters and the values of W_0, y_0^T and $\tilde{b}_0^c$.

If $\kappa \geq \hat{\kappa}^{SSLD} > \hat{\kappa}^{SS}$, it is possible to have multiple equilibria in both models, but as we explained earlier, this is only a necessary condition. Multiplicity needs also condition (ii) requiring tradables income to be in the interval $[\hat{y}_0^T, \tilde{y}_0^T]$, where as noted above $\tilde{y}_0^T$ is different in the SS and SSLD models, while $\hat{y}_0^T$ is the same (because of the assumption that the initial debt of the SSLD model is $\tilde{b}_0^c$). Hence, assuming that κ is high enough to support multiplicity, the difference in the width of this interval across the two models indicates the extent to which multiplicity is easier or harder to obtain in one model v. the other. A wider (narrower) interval for the SSLD model suggests that multiplicity is easier (harder) to obtain, although the deterministic nature of the analysis does not take into account the probability of tradables income falling into a particular interval. Visually, Figure 2.2 suggests that the multiplicity interval of tradables income is narrower for the SSLD model, but this is easier to illustrate with quantitative examples, as we show below.

In the case of the SS model, the above analysis of equilibrium multiplicity has an equivalent formulation in terms of the analysis conducted by Schmitt-Grohé and Uribe (2018). They derived the same condition on a threshold value of κ required for multiplicity, and showed that multiplicity requires in addition that the initial bond holdings are within an interval of relatively high values (i.e. multiplicity requires relative *low* initial debt). The value of b_0 (for $b_0 < 0$) must be high enough so that at the unconstrained equilibrium the threshold condition on κ holds, but not so high that the credit constraint does not bind at date 0 (since higher b_0 implies also higher b_1 at the unconstrained equilibrium). Intuitively, the consistency of the two approaches follows from noticing that parametric differences in b_0 keeping y_0^T constant can be alternatively represented as parametric differences in y_0^T keeping b_0 constant by capitalizing initial income differences into changes in bond holdings. Hence, an interval of relatively high b_0 (i.e. *low* debt) that sustains multiplicity translates into an interval of relatively *high* income values. We used the latter approach here because it allows us to make Sudden Stop outcomes in the SS and SSLD models comparable by keeping the initial debt set at the value such that the unconstrained equilibria are the same.

2.2.3 Quantitative Examples

We examine next the results of quantitative examples that illustrate the main theoretical findings we have presented. In particular, we illustrate the characteristics of Sudden Stops in the SSLD economy, compare Sudden Stops between the SS and SSLD models when Sudden Stop equilibria are unique, and show that multiplicity is harder to obtain in the SSLD model. We set parameter values following the calibration proposed by Bianchi (2011), which was designed to match properties in annual data for Argentina. Since we solve for perfect-foresight equilibria, we ignore the parts of his calibration that relate to the stochastic processes of tradables and nontradables income, and set $\bar{y}^N = 1$ for all t and $y_t^T = 1$ for $t > 0$. Table 2.1 lists the parameter values.

Table 2.1: Parameters

Parameter	Value
γ	2
η	0.205
ω	0.31
β, q^*	0.91
$\bar{y}^N$	1.00

Bianchi (2011) set the coefficient of relative risk aversion to $\gamma = 2$, a standard value in DSGE models. He also set $\eta = 0.205$, so that the elasticity of substitution between tradables and nontradables $(1/(1+\eta))$ is 0.83, which is the upper bound of a range of existing estimates. This elasticity is key for determining the elasticities of both p^c and p^N to changes in sectoral consumption allocations, which play a central role in determining the effects of Sudden Stops and the magnitude of the externalities driving the design of optimal financial policy, as we show in the next Section. $\omega = 0.31$ is set so as to match a tradables consumption share of 32 percent. Bianchi also set the discount factor to $\beta = 0.91$, so that his stochastic SS model can match Argentina's average net foreign asset position-GDP ratio of -0.29 from the data constructed by Lane and Milesi-Ferretti (2001). We take this same value of β and since we are assuming $\beta R^* = 1$ this implies $q^* = 1/R^* = \beta$.

Given the above parameter values, and $y_t^T = 1$ for all t, we solve for the unconstrained equilibrium of the SS model for a value of b_0 such that the initial net foreign asset position-GDP ratio $(b_0/(y_0^T + \bar{p}^N \bar{y}^N))$ matches Argentina's average, which implies $b_0 = -0.868$. Then, using the value of the consumption price index at this equilibrium, $p^{c,SS}$, we solve for $\tilde{b}_0^c = b_0/p^{c,SS}$, which is the initial condition for bonds in the SSLD model that supports the same unconstrained stationary equilibrium as the calibrated SS model. This guarantees that the initial net foreign asset position-GDP ratio of the SSLD model, which is given by $p_0^c \tilde{b}_0^c/(y_0^T + \bar{p}^N \bar{y}^N)$, is also -0.29.

Consider next the wealth-neutral shocks to y_0^T (which assume future tradables income changes as needed to keep W_0 unchanged). As y_0^T falls agents borrow more (bond holdings fall) at $t = 0$ in order to maintain the same unconstrained consumption stream. For a given value of κ, this new debt choice approaches the maximum allowed by the credit constraint ($-\kappa[y_0^T + p_0^N \bar{y}^N]$) as initial income falls. As long as the constraint does not bind, the SS and SSLD economies stay at the unconstrained equilibrium, but when it binds the allocations and prices at date 0 move to a Sudden Stop equilibrium, which may or may not be unique as we showed earlier.

We study first the case in which Sudden Stop equilibria are unique in both economies. To this end, we set $\kappa = 0.29$ and solve for the equilibria of the two models for wealth-neutral income shocks in the interval (0.85,1). Figure 2.3 presents the results for the SSLD economy as percent deviations from the unconstrained equilibrium, and shows also results for a hypothetical scenario in which p^c and p^N are kept fixed at the unconstrained levels.

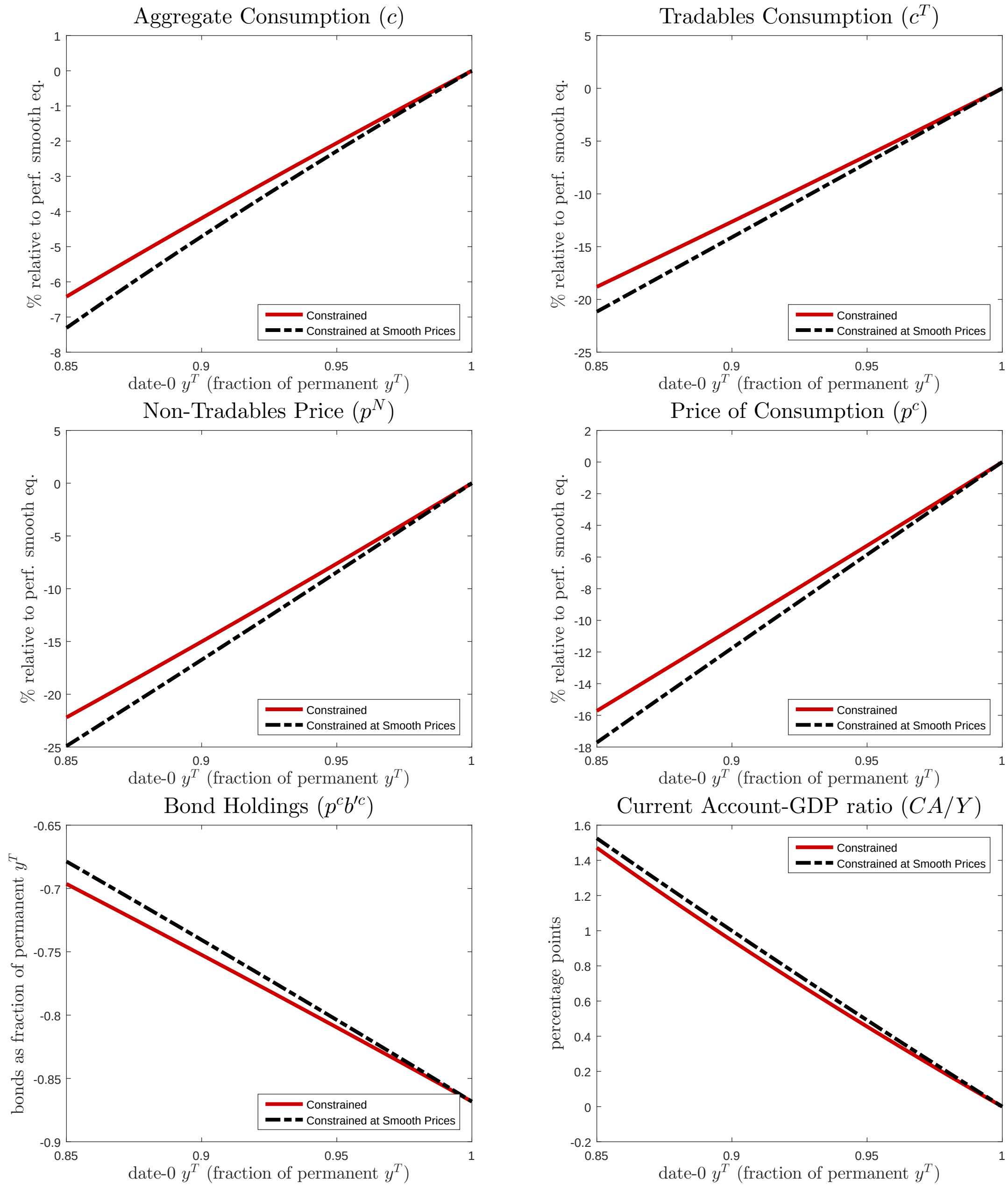
When income is sufficiently high (for y_0^T near 1), the credit constraint does not bind and therefore the plots show zero deviations from the unconstrained equilibrium. As y_0^T falls the constraint becomes binding triggering Sudden Stops, and the plots show that aggregate and tradables consumption, as well as the price of nontradables and the real exchange rate, display sizable declines. For example, for a 5 percent shock ($y_0^T = 0.95$), c_0 and c_0^T fall by 2 and 6.3 percent respectively, and p_0^N and p_0^c fall by 7.6 and 5.3 percent respectively, bond holdings in units of tradables rise 580 basis points and there is a current account reversal of 45 basis points.

The comparison of Sudden Stops vis-a-vis the scenario with constant prices helps illustrate the extent to which the Fisherian deflation effect from the credit constraint (reducing c_0^T) and the debt-repayment-burden effect from liability dollarization (increasing c_0^T) offset each other. Keeping prices constant removes both effects, so that the declines in consumption that result are driven only by the exogenous drop in income and the fall in the exogenous component of borrowing capacity (κy_0^T), which are at work in both scenarios. Hence, if the two effects were of equal magnitude (in absolute value), the results for the SSLD case and the smooth prices case would be identical. The fact that the latter yields slightly larger Sudden Stops indicates that in the results for the SSLD economy the deflation and repayment-burden effects nearly offset each other, but the latter is slightly larger (in absolute value). Hence, through the debt-repayment-burden effect, liability dollarization is providing a very good hedge against the loss of resources caused by the endogenous component of borrowing capacity ($\kappa p_0^N \bar{y}^N$). This result may not hold for different parameterizations, and it will also fail more generally in stochastic SSLD models in which the new-debt-price and risk-taking incentive are also at work.

Figure 2.4 compares Sudden Stop equilibria between the SSLD and SS economies for $y_0^T = 0.97$ (a 3 percent shock) for values of κ in the (0.21,0.303) interval. Recall that both models have identical parameters and identical initial conditions, and hence the unconstrained solutions are identical.

In line with the theoretical results derived earlier, the SSLD model always produces milder Sudden Stops, because of the debt-repayment-burden effect. For the upper bound of κ, the credit constraint does not bind, and hence both economies remain at the unconstrained equilibrium, which is identical between the two. As κ decreases and borrowing capacity tightens, both economies move to unique Sudden Stop equilibria, but as the plots illustrate, the Sudden Stops of the SSLD economy are significantly smaller. For a drop in the debt-to-income limit of 5 percentage points, reducing κ from 0.3 to 0.25, the SS (SSLD) model produces declines in c_0 and c_0^T of 18 and 45 (4.7 and 14) percent respectively, and drops in p_0^N and p_0^c of 51.7 and 38.7 (16.6 and 11.7) percent respectively. The current account rises by 20 percentage points of GDP in the SS model, compared with 4.1

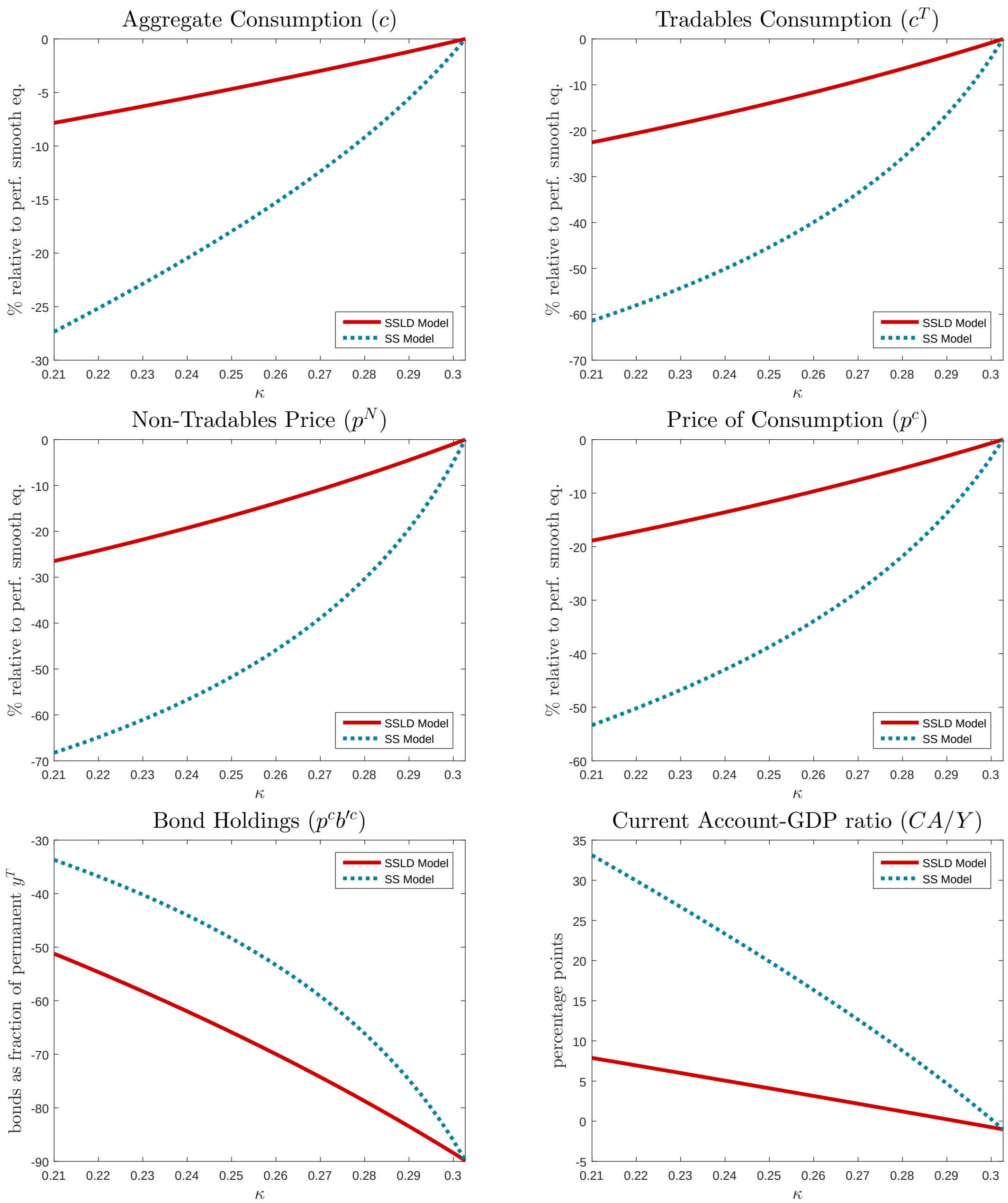
Figure 2.3: Sudden Stops in Response to Income Shocks in the SSLD Model



in the SSLD model.[35] The collapse in the real exchange rate (p_0^c) moderates the Sud-

35. Mendoza and Rojas (2017) show that the finding that Sudden Stops are milder extends to quantitative comparisons of stochastic SS and SSLD models, in which the debt-price and risk-taking effects of liability dollarization are present, and that for the same calibration the SSLD model performs better at matching the observed empirical regularities of Sudden Stops.

Figure 2.4: Sudden Stops in Response to Changes in κ in the SS and SSLD Models



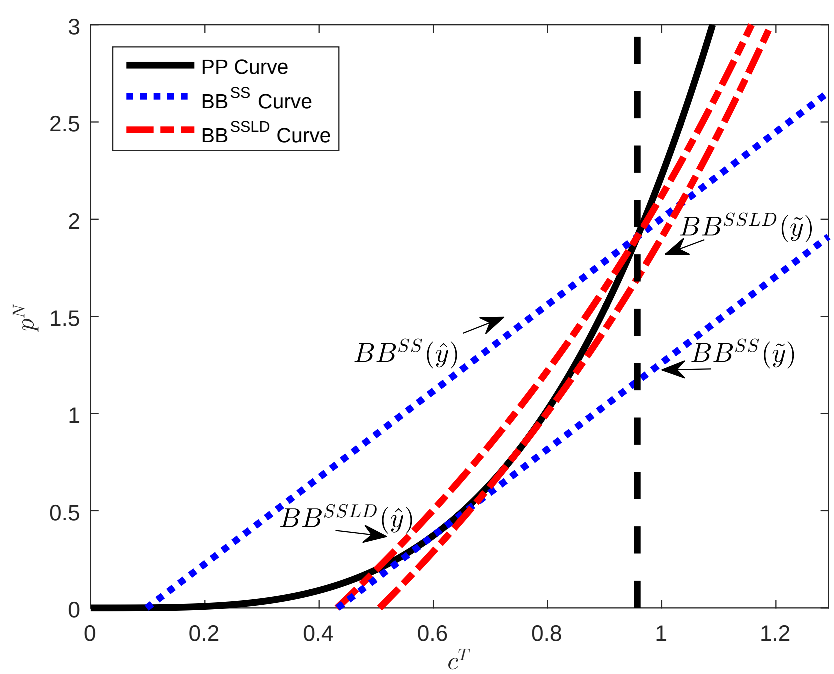
den Stops of the SSLD economy significantly, whereas in the SS economy its even larger collapse does not affect the debt repayment burden. Notice also that the gap between the equilibrium responses in both models widens as κ decreases, indicating that the debt-repayment-burden effect works as an endogenous hedge that partially weakens Sudden Stops and by larger amounts for larger corrections in borrowing capacity.

Consider next the case of equilibrium multiplicity. Using the same parameter values as in the previous experiments, the threshold values of κ that are necessary (but not sufficient) for multiple equilibria in each model are $\kappa^{SS} = 0.38$ and $\kappa^{SSLD} = 0.68$. Moreover, the factor by which the latter exceeds the former is $1 - \xi_{p^c,c^T} \frac{\bar{p}^c \tilde{b}_0^c}{\bar{c}^T} = 1.79$. Hence, the SSLD continues to yield unique Sudden Stop equilibria at κ values up to roughly 1.8 times the threshold value of the SS model. Thus, the liability dollarization model requires significantly higher κ values to produce multiplicity.

As explained earlier, in addition to $\kappa \geq \hat{\kappa}$, multiplicity requires income shocks to be within a particular range. Under the calibrated parameter values we proposed, however, the required income intervals do not intersect and hence multiplicity cannot be generated in both the SS and SSLD models simultaneously. Hence, in order to produce Sudden Stop outcomes with multiplicity we altered the model's parameters. We lowered the elasticity of substitution between c^T and c^N to 0.285 ($\eta = 2.5$), which is much lower than the range from literature estimates cited by Bianchi (2011), and halved the initial debt position (i.e. $b_0^c = -0.149$). With these parameter changes and setting $\kappa = 0.45$ we can support multiple equilibria in both models, as shown in Figure 2.5.

This Figure shows the equilibrium determination of the SS and SSLD models in our quantitative examples for two values of initial income, the threshold value $\hat{y}_0^T$ at which the credit constraint sustains just enough debt to support the unconstrained equilibrium, and the upper bound $\tilde{y}_0^T$ above which the only equilibrium the models can support is the unconstrained equilibrium. For each case, the Figure shows the corresponding BB curves of the SS and SSLD models, as well as the PP curve. As explained earlier, when income equals either $\hat{y}_0^T$ or $\tilde{y}_0^T$ the models support two equilibria, the unconstrained equilibrium and one Sudden Stop equilibrium, and for income levels inside this interval the models support three equilibria, the unconstrained one and two Sudden Stop ones. The value of $\hat{y}_0^T$ (0.397) is the same for the two models because we solve them with identical parameters and initial conditions. The values of $\tilde{y}_0^T$ are different because of the effects of liability dollarization, and they are $\tilde{y}^{T,SS} = 0.627$, and $\tilde{y}^{T,SSLD} = 0.463$ for the SS and SSLD model respectively. Hence, the range of income in which multiple equilibria exist under this calibration is much narrower in the SSLD model than in the SS model (about 1/3rd the size, 0.07 v. 0.23). Moreover, as noted earlier, multiplicity occurs when tradables income is "relatively high," in the sense of being (weakly) higher than the income level at which the credit constraint is only marginally binding. Hence, Sudden Stops triggered by multiplicity coincide with high income.

Figure 2.5: Multiple Equilibria in the SS and SSLD Models



The difficulty in generating multiplicity with the SSLD model can be illustrated further by studying an alternative numerical example calibrated to match the scenario with multiplicity in the SS model studied in Schmitt-Grohé and Uribe (2018). Their calibration sets $\eta = 1$, which implies an elasticity of substitution of 0.5, still inside the range of empirical estimates but closer to the lower bound than in our baseline calibration, and $\omega = 0.26$ (v. 0.31 in our experiments). Using the same initial debt we used here, these parameter values imply threshold values for κ of $\kappa^{SS} = 0.192$ and $\kappa^{SSLD} = 0.456$. Hence, at their calibrated value of $\kappa = 0.3$, there is multiplicity in the SS model, in line with their findings, but not in the SSLD model. Under liability dollarization, κ would need to be nearly 2.4 times larger than the threshold of the SS economy, or 1.5 times their calibrated value, in order to be in the region in which multiplicity is possible.

2.3 Normative Analysis

In this Section, we study the normative implications of introducing liability dollarization in Sudden Stops models, focusing on unique equilibria for simplicity. Following Bianchi and Mendoza (2017), we characterize optimal financial policy following a primal approach by analyzing the allocations attainable to a social planner who chooses the debt of private agents under commitment subject to the resource, market-clearing, and collateral constraints, and letting goods markets and financial intermediaries operate competitively.[36] We then explore the implications of this optimal policy for the design of domestic credit regulation (i.e. domestic debt taxes) v. capital controls. As we show below, the optimal policy is time-inconsistent, because of the planner's ability to affect the ex-ante domestic interest rate for debt contracted at date t (or the expected real exchange rate) with the consumption of tradable goods planned for date $t+1$. Moreover, capital controls and debt taxes are equivalent, so under the optimal policy the SSLD model *does not* support the use of capital

36. The last assumption is equivalent to assuming that the planner cannot contract debt directly with foreign lenders in units of tradables, and instead borrows from the same intermediaries as private agents.

controls as a policy aimed at discriminating external v. domestic credit. Finally, since the optimal policy lacks credibility and follows a complex, nonlinear schedule, we study quantitatively the effectiveness of a simpler policy that sets time-invariant tax rates on domestic credit and capital inflows in a stochastic, infinite-horizon environment.

2.3.1 Optimal Policy under Commitment

Assuming that the regulator chooses bond holdings for private agents, their optimization problem reduces to a simple static problem of choosing tradables and nontradables consumption subject to a budget constraint that includes the income from tradables and nontradables, and a lump-sum transfer (tax) from the planner that represents the amount of resources generated by borrowing (repaying). The private sector no longer chooses b^c_{t+1} and does not face the collateral constraint. Hence, the first-order conditions of the agents' optimization problem no longer include the Euler equation for bonds (equation (2.8)) and the borrowing constraint (equation (2.9)), but the optimality condition for sectoral consumption allocation (equation (2.7)) still holds, since the market for nontradables still clears competitively. In addition, the no-arbitrage condition of financial intermediaries must also hold, because these intermediaries are also still operating competitively.

Using the nontradables market-clearing condition to substitute for c^N_t and the intermediaries' no-arbitrage condition to substitute for $q^c_t p^c_t$, the social planner's problem can be written as follows:[37]

$$\max_{\{c_t^T, b_{t+1}^c\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t(c_t^T, \overline{y}^N)) \tag{2.18}$$

s.t.

$$q^* \mathbb{E}_t \left[p^c(c_{t+1}^T) \right] b_{t+1}^c + c_t^T = p^c(c_t^T) b_t^c + y_t^T \quad \forall t \tag{2.19}$$

$$q^* \mathbb{E}_t \left[p^c(c_{t+1}^T) \right] b_{t+1}^c \geq -\kappa (y_t^T + p^N(c_t^T)\overline{y}^N) \quad \forall t. \tag{2.20}$$

It is important to note that in this problem, as was the case in the competitive equilibrium, c_t^T is independent of the value of μ_t when the constraint binds, and is determined by the same non-linear equation (2.13), which in this case follows from combining conditions (2.19) and (2.20).

As we show below, the solution to the above problem displays time-inconsistency. The key feature of the problem behind this result is that in the planner's resource and borrowing constraints, the real exchange rate expected for $t+1$ ($\mathbb{E}_t \left[p^c(c_{t+1}^T) \right]$) affects disposable resources and borrowing capacity at date t. As a result, the planner's optimal plans for a given future date affect consumption allocations and borrowing capacity in the past.

The time-inconsistency result can be derived fomally by simplifying the planner's Euler equation for bonds to reduce it to the following expression:[38]

$$\lambda_t = \frac{u_T(t) + \mu_t \kappa p^{N\prime}(t)\overline{y}^N - p^{c\prime}(t) b_t^c \left(\mathbb{E}_{t-1}[\lambda_t] + \frac{\mathrm{Cov}_{t-1}(\lambda_t, p^c(t))}{\mathbb{E}_{t-1}[p^c(t)]} \right)}{1 - p^{c\prime}(t) b_t^c}. \tag{2.21}$$

The planner's marginal utility of wealth at date t (λ_t) depends on two terms determined

37. The planner's problem is written in short notation for simplicity. Given the Markov process of y_t^T, the expectations are taken over histories of realizations, with the date-t probability of a history y^{Tt} denoted by $\pi_t(y^{Tt})$ and the associated consumption and bonds allocations denoted by $c_t^T(y^{Tt})$ and $b_{t+1}^c(y^{Tt})$ respectively.

38. See Mendoza and Rojas (2017) for full details.

at $t-1$. First, $\mathbb{E}_{t-1}(\lambda_t)$, which reflects the effect of changes in the amount of resources generated by debt contracted at $t-1$ on the marginal utility of wealth expected for date t. This is possible because, by affecting real exchange rate expectations, the planner alters q^c_{t-1} and hence the amount of tradable goods that a given amount of debt issued at $t-1$ yields. Second, $\text{Cov}_{t-1}(\lambda_t, p^c(t))/\mathbb{E}_{t-1}[p^c(t)]$, which has a similar form as the private risk-taking incentive identified earlier in condition (2.14), except the planner's covariance is with respect to the social marginal utility of wealth one period ahead, instead of the marginal utility of tradables consumption. The private risk-taking incentive reduces the expected marginal cost of borrowing between t and $t+1$. The planner, in contrast, considers how the covariance term alters the debt chosen at $t-1$ and thereby the debt repayment burden of date t. Through these two feedback effects, the planner's choice of consumption and debt at t affects price expectations and the planner's covariance term at date $t-1$, which in turn alter λ_t by affecting the debt chosen at $t-1$ and hence the burden of debt repayment at date t.

These feedback effects produce time-inconsistency because, as of a given date t, the planner has the incentive to pledge higher consumption at $t+1$, so that a higher expected real exchange rate props up q^c_t and reduces the ex-ante real interest rate, strengthening borrowing incentives and borrowing capacity via the effects of liability dollarization discussed earlier. Ex-post, however, delivering on this pledge is suboptimal, because higher prices at $t+1$ imply a higher ex-post real interest rate, and thus a higher burden of debt repayment in that period. This time-inconsistency mechanism is at work regardless of whether the constraint binds or not, but it interacts with the constraint because by affecting borrowing incentives it affects the likelihood that the constraint can bind at equilibrium.[39]

Bianchi and Mendoza (2017) obtained a similar result showing the time-inconsistency

39. Notice that, when $\mu_t > 0$, pledging higher c^T_{t+1} could make the constraint less tight by increasing the price of bonds, but this does not generate additional resources for consumption, which are still be given by $-\kappa(y^T_t + p^N_t \bar{y}^N)$.

of optimal macroprudential policy under commitment in a model in which assets serve as collateral, but the mechanism driving the time-inconsistency that we described above is different. In Bianchi and Mendoza, the asset-pricing condition connecting current asset prices to future consumption leads the planner to prop up asset prices when the collateral constraint binds by pledging *lower* future consumption, which is not optimal to do ex-post. In contrast, in this model (and also in the SS model), since p_t^N and c_t^T are independent of the planner's future plans when the constraint binds at t, the planner cannot prop up the value of collateral with its future plans.

It is important to notice that time-inconsistency emerges here even though the collateral constraint is defined in terms of a limit on the debt-to-income ratio (a flow constraint), instead of a debt-to-assets ratio (a stock constraint, as in Bianchi and Mendoza (2017)). Hence, time-inconsistency of optimal financial policy under commitment can exist in Fisherian models with either stock or flow collateral constraints. What is necessary is to have a vehicle that allows the planner to affect past prices and allocations, or borrowing capacity, with current consumption and debt choices. In contrast, optimal policy is time-consistent in standard SS models, because there is no vehicle for this to happen.

The planner's first-order conditions can be re-arranged to produce an alternative expression for the planner's Euler equation for bonds that equates the social marginal costs and benefits of borrowing, which is useful for characterizing the inefficiencies affecting the competitive equilibrium. As shown in Mendoza and Rojas (2017), the resulting expression is:

$$u_T(t) = \beta \mathbb{E}_t \left[\left[u_T(t+1) + \mu_{t+1} \kappa \bar{y}^N p^{N\prime}(t+1) \right] \tilde{R}^T_{t+1} \Psi(t+1) \right] \\ + \mu_t \left(\psi(t) - \kappa \bar{y}^N p^{N\prime}(t) \right), \qquad (2.22)$$

where:

$$\Psi(t+1) \equiv \left(\frac{\psi(t)}{\psi(t+1)}\right) \tag{2.23}$$

$$\psi(t) \equiv 1 - p^{c\prime}(t)b_t^c + p^{c\prime}(t)b_t^c\left(\frac{\mathbb{E}_{t-1}[\lambda_t]}{\lambda_t} + \frac{\mathrm{Cov}_{t-1}\left(\lambda_t, p^c(t)\right)}{\lambda_t\mathbb{E}_{t-1}[p^c(t)]}\right). \tag{2.24}$$

The comparable Euler equation in the competitive equilibrium is:

$$u_T(t) = \beta\mathbb{E}_t\left[\tilde{R}_{t+1}^T u_T(t+1)\right] + \mu_t^{CE}, \tag{2.25}$$

where μ_t^{CE} denotes the multiplier of the collateral constraint in the competitive equilibrium.

Consider first the last terms in the right-hand-side of both Euler equations, which include the multipliers μ_t and μ_t^{CE}. These terms are only present if the collateral constraint binds at date t, but in this case the specific values of the multipliers are irrelevant for the planner's allocations, because, as we noted earlier, c_t^T is independent of μ_t and μ_t^{CE} when credit is constrained. Note, however, that allocations and prices will differ for the planner and the unregulated equilibrium, because by internalizing the externalities the planner will alter the likelihood of the constraint becoming binding and will generally arrive at states in which the constraint binds with different b_t^c.

Compare now the first terms in the right-hand-sides of the above Euler equations. These terms indicate that the product $\tilde{R}_{t+1}^T u_T(t+1)$ is part of both the social and private marginal costs of borrowing, but the social marginal cost includes other terms that reflect the effect of the pecuniary externalities at work in the model. The term $\mu_{t+1}\kappa p^{N\prime}(t+1)\bar{y}^N$ is familiar from the standard SS models. It captures the macroprudential externality operating via the effect of the price of nontradables (i.e. the value of collateral) on borrowing capacity, and is strictly positive whenever states with $\mu_{t+1} > 0$ have positive probability at $t+1$ as of date t, because p^N is increasing in c^T. In states in which the collateral constraint is expected to bind, the social marginal cost of borrowing includes the shadow value of the loss in borrowing capacity caused by the fall in the price of nontradables if a Sudden Stop occurs, because the planner internalizes how the debt chosen at t affects the size of the price drop at $t+1$. Hence, this macroprudential externality is an overborrowing externality, since it implies that the marginal cost of borrowing for private agents is lower than the social marginal cost.

The term $\Psi(t+1)$ captures a second pecuniary externality that is particular to the SSLD model, namely the intermediation externality. As the planner's Euler equation show, $\Psi(t+1)$ distorts the ex-post real interest rate paid in units of tradables. In turn, $\Psi(t+1)$ is a ratio composed of the intermediation externality's effects operating via the three mechanisms discussed earlier (repayment debt burden, price of newly issued debt, and risk-taking incentive) at date t relative to $t+1$, which are summarized in the terms $\psi(t)$ and $\psi(t+1)$ respectively. Using the expression that defines $\psi(t)$, it follows that the effect operating via the debt repayment burden is captured by the term $1-p^{c\prime}(t)b_t^c$, which is strictly greater than 1 because $b_t^c<0$ and $p^{c\prime}(t)>0$. The planner internalizes that additional borrowing at date t increases $p^c(t)$, which increases the burden of repaying outstanding debt. The effects operating via the price of new debt and the risk-taking incentive are captured by the term $\frac{p^{c\prime}(t)b_t^c}{\lambda_t}\left[\mathbb{E}_{t-1}(\lambda_t)+\frac{\text{Cov}_{t-1}(\lambda_t,p^c(t))}{\mathbb{E}_{t-1}[p^c(t)]}\right]$. This term reflects the fact that the planner internalizes the intermediaries' no-arbitrage condition, and hence takes into account how the social marginal cost of borrowing responds to the effect of changes in date-t consumption on price expectations, the price of debt and incentives to borrow at $t-1$. In particular, the planner takes into account how these effects alter the amount of resources in units of tradables that debt contracted at $t-1$ generates and the social valuation of the risk-taking incentive at $t-1$.

If $\Psi(t+1)$ is greater (smaller) than 1, the intermediation externality increases (reduces) the social marginal cost of borrowing relative to the private marginal cost of borrowing, and hence it operates as an overborrowing (underborrowing) externality. Unfortunately, while it is possible to show that $\psi(t) > 0$ (see Mendoza and Rojas (2017)), the size of $\Psi(t+1)$ cannot be determined unambiguously. Using condition (2.24), however, we can infer that, everything else constant, $\Psi(t+1)$ is greater (smaller) than 1 if the outstanding debt in period t is larger (smaller) than in period $t+1$, and/or if the incentives to borrow at $t-1$ are weaker (stronger) than at t (i.e. if the planner's real-exchange-rate expectations at t for $t+1$ exceed those at $t-1$ for t). Under these conditions, the intermediation externality operates as a second overborrowing (underborrowing) externality. Moreover, unlike the macroprudential externality that is only present when $\mu_{t+1} > 0$ has positive probability as of date t, the intermediation externality is always present, regardless of whether the constraint is expected to bind or not (although the actual value of $\Psi(t+1)$ does depend on whether the constraint is expected to bind).

It is worth noting that the intermediation externality vanishes if we assume either complete asset markets or perfect foresight. Under perfect foresight, the covariance term in condition (2.24) vanishes and since $\mathbb{E}_{t-1}[\lambda_t] = \lambda_t$, it follows that $\psi(t) = 1$ for all t and hence $\Psi(t) = 1$.[40] The same happens under complete markets because λ_t becomes time- and state-invariant.

In order to decentralize the planner's allocations as a competitive equilibrium, we consider possibly using two policy instruments: Capital controls (i.e. taxes on the intermediaries' inflows of foreign capital) and domestic debt taxes (taxes on domestic borrowing). Capital controls are modeled as a tax θ_t that raises the interest rate at which intermediaries borrow from abroad above R^* (i.e. it lowers the price of bonds sold abroad below q^*). With

40. One effect of the intermediation externality does remain, and it operates via the debt repayment burden of the exogenous date-0 debt $p_0^c b_0^c$. This can be seen in condition (2.24) because for $t = 0$ there is no matching term $\mathbb{E}_{-1}[\lambda_0]$ to cancel the two terms with $p^{c\prime}(t) b_t^c$. Hence, the planner has the incentive to increase p_0^c to reduce the debt repayment burden at $t = 0$.

this tax in place, the intermediaries' no-arbitrage condition becomes:

$$q_t^c = \frac{q^*}{(1+\theta_t)} \frac{\mathbb{E}_t\left[p_{t+1}^c\right]}{p_t^c}. \tag{2.26}$$

The revenue generated by this tax is rebated to intermediaries as a lump-sum transfer, which can also be a lump-sum tax if $\theta_t < 0$. Notice that the tax is known at the moment of issuing bonds, and is paid with the bond repayment.

The tax on domestic debt is denoted τ_t. If this tax is used, the budget constraint of the representative agent becomes:

$$q_t^c p_t^c b_{t+1}^c + c_t^T + p_t^N c_t^N = p_t^c b_t^c (1+\tau_t) + y_t^T + p_t^N \bar{y}^N + T_t, \tag{2.27}$$

where T_t is a lump-sum rebate of the revenue generated by this tax (or a lump-sum tax if $\tau_t < 0$).

If both taxes are used, the agent's Euler equation for bonds can be expressed as:

$$u_T(t) = (1+\tau_t)(1+\theta_t)\beta\mathbb{E}_t\left[u_T(t+1)\tilde{R}_{t+1}^T\right] + \mu_t^{CE}. \tag{2.28}$$

This condition implies that, unless decentralizing socially optimal allocations requires different taxes on capital inflows and domestic debt for reasons other than distorting the private agents' intertemporal decision margin, taxing one is equivalent to taxing the other. What matters is the combined effective tax rate $(1+\tau_t^{ef}) \equiv (1+\tau_t)(1+\theta_t)$, and the particular values of each tax are undetermined. One such instance is the standard SS model, in which intermediation is inessential and the only inefficiency affecting the unregulated decentralized equilibrium is the macroprudential externality leading agents to underestimate the social marginal cost of borrowing. In this case, the optimal policy can be implemented equally with only domestic debt taxes, only capital controls or any mix of both that yields the same τ_t^{ef}.[41] Thus, the standard SS model of Sudden Stops *does not* provide a justification for capital controls as a policy to discriminate domestic v. foreign credit flows.

The presence of the intermediation externality in the SSLD model is not sufficient to break the above equivalence result. The optimal policy under commitment still requires only an effective debt tax, and any combination of τ_t and θ_t that yields the same τ_t^{ef} yields a competitive equilibrium with identical allocations as those that solve the planner's problem. Hence, optimal policy under commitment in the SSLD model shares the property of the SS model that it *does not* justify the use of capital controls.

Since allocations and prices are independent of μ_t when the constraint binds at t, the relevant use of the tax is when $\mu_t = 0$. In this case, the optimal tax is the one that equalizes the social and private marginal costs of borrowing given by the right-hand-sides of conditions (2.22) and (2.25):

$$\tau_t^{ef} = \frac{\mathbb{E}_t\left[\left(u_T(t+1) + \mu_{t+1}\kappa\bar{y}^N p^{N\prime}(t+1)\right)\tilde{R}_{t+1}^T \Psi(t+1)\right]}{\mathbb{E}_t\left[\tilde{R}_{t+1}^T u_T(t+1)\right]} - 1. \tag{2.29}$$

The numerator of this expression includes terms that correspond to the macroprudential and intermediation externalities. Since the intermediation externality can yield social marginal costs of borrowing higher or lower than their private counterparts, in principle the tax could be negative (i.e. a subsidy). In addition, unlike the optimal taxes of the SS model, in the SSLD model the taxes are used at date t even if the collateral constraint has zero probability of becoming binding at $t+1$, as long as $\Psi(t+1) \neq 1$. Hence, it is not only a "macroprudential" tax, but a broader financial policy aimed at tackling the intermediation externality.

41. As explained earlier, intermediation in the SS setup can be interpreted as frictionless domestic banks that borrow and lend in tradables units, or as the nonfinancial private sector borrowing directly from abroad. Either way, the optimal policy needs to tackle only the inefficiency driving a wedge between the social and private marginal costs of domestic borrowing, and hence domestic debt taxes and capital controls are equivalent.

Moreover, this optimal tax policy inherits the time-inconsistency of the social planner's problem, and hence it lacks credibility.[42]

One important caveat of the above equivalence result is that it holds in part because of the stylized formulation of financial intermediation. If issuing domestic loans has a variable cost, for instance, the marginal cost of issuing domestic bonds would be subtracted from the right-hand-side of (2.26) and this would imply that setting θ_t at a given rate results in a larger increase in effective borrowing costs that setting τ_t at the same rate. Hence, extending the model to introduce realistic frictions in financial intermediation may not only introduce non-neutral balance sheet effects on banks as the real exchange rate moves, but may also provide a justification for capital controls.

If we switch to the standard SS model by imposing on the above optimal tax result the assumption that debt is issued in units of tradables, the expression reduces to $\tau_t^{ef} = \frac{\mathbb{E}_t\left[\mu_{t+1}\kappa\overline{y}^N p^{N\prime}(t+1)\right]}{\mathbb{E}_t\left[u_T(t+1)\right]}$, which is the optimal debt tax of the standard SS model (e.g. Bianchi (2011), Bianchi et al. (2016)). The optimal tax of the SSLD model also preserves the result from the standard SS model that the value of the tax is indeterminate when $\mu_t > 0$, because as explained above the planner's allocations are independent of μ_t when the collateral constraint binds. Therefore, any debt tax consistent with the collateral constraint being binding in the decentralized equilibrium with debt taxes can support the planner's allocations when $\mu_t > 0$. In quantitative applications, the convention in the literature is to determine if a zero tax is consistent with this outcome, and if so the tax is assumed to be zero when the constraint binds.

42. See Mendoza and Rojas (2017) for an analysis of optimal time-consistent policy for a conditionally-efficient regulator that takes as given the pricing function of private debt of the unregulated competitive equilibrium. In this case, the equivalence breaks. Capital controls support the debt pricing function, effectively implementing a policy that targets the expected rate of real appreciation, and domestic debt taxes are set as needed to support the optimal τ_t^{ef} given the optimal θ_t.

2.3.2 Quantitative Evaluation of Simple Rules for Debt Taxes & Capital Controls

The optimal financial policy under commitment has two shortcomings. First, it lacks credibility because of time inconsistency. Second, the optimal effective debt tax that would implement it would follow a non-linear schedule with complex variations over time and across states of nature, as dictated by the various terms in the optimal tax schedule defined in (2.29). In light of these shortcomings, we explore the effectiveness of a simpler policy strategy that consists of time-invariant taxes on domestic debt and capital inflows. In particular, we conduct a quantitative analysis of the extent to which this strategy can reduce the magnitude and severity of Sudden Stops and increase social welfare relative to the unregulated competitive equilibrium of the SSLD economy.

To conduct this analysis, we calibrate a stochastic version of the model and solve it numerically for the unregulated competitive equilibrium. Then we solve for competitive equilibria under different time-invariant values of τ and θ, and implement an algorithm that searches for the welfare-maximizing pair of these constant taxes.[43] We use a baseline calibration with most parameters set at the same values as in the perfect foresight analysis of Section 2.2.2. The only modifications are that we add the calibration of the stochastic process of the tradables endowment, for which we adopt again the one proposed by Bianchi (2011), and we reset the value of R^* also to match Bianchi's ($R^* = 1.04$). The stochastic model requires $\beta R^* < 1$ to have a well-defined stochastic steady state, because with $\beta R^* = 1$ agents accumulate an infinitely large stock of precautionary savings. The Markov process for y^T is constructed to approximate Bianchi's estimated AR(1) time-series process for the cyclical component of tradables GDP in Argentina, for which he obtained an autocorrelation coefficient $\rho_{y^T} = 0.54$ and a standard deviation $\sigma_{y^T} = 0.059$. We then used the

43. We solve the competitive equilibrium with and without taxes using the same time-iteration algorithm with fixed grids as in Mendoza and Rojas (2017).

quadrature method proposed by Tauchen and Hussey (1991) to construct a Markov chain with 9 realizations centered around $\mathbb{E}[y^T] = 1$. We keep the value of $\kappa = 0.29$, which is the same we used to generate Sudden Stops in the perfect foresight experiments. This κ value also yields a frequency of Sudden Stops in the SSLD model that is close to empirical estimates (3.83 percent v. 3.32. percent in emerging markets data as reported in Mendoza (2010)), and lower than in Bianchi's SS model (5.5 percent).

When solving for competitive equilibria with constant taxes, we assume that the revenue (cost) generated by positive (negative) values of θ is rebated (charged) to private agents as part of their lump-sum transfers (taxes) T_t. We do this because, if they are passed on to intermediaries, the frictionless formulation of financial intermediation that we adopted renders a cut in θ equivalent to parametric increases in q^*, and hence subsidizing capital inflows is equivalent to lowering the world interest rate arbitrarily, making very large subsidies that can be painlessly paid for optimal.[44]

Under the above assumption, transfers to private agents are given by $T_t = -\tau p_t^c b_t^c - \theta q_t^c p_t^c b_{t+1}^c$, which together with the agents' budget constraint (2.27) and the pricing condition (2.26) yields the same resource constraint for tradables as in the equilibrium without taxes (2.12), thus removing the income effects induced by τ and θ. In the equilibrium conditions that result, the two taxes appear as before in the Euler equation for bonds forming the wedge that defines τ^{ef}, but now θ also appears in the collateral constraint as a term that contributes to make the constraint less tight. This is so because a higher θ moves borrowed resources further away from their constrained maximum, since it reduces $q_t^c p_t^c$ (see eq. (2.26)), which increases $q_t^c p_t^c b_{t+1}^c$ for $b_{t+1}^c < 0$. This effect can also be interpreted as if

44. If intermediaries pay the lump-sum taxes to finance these subsidies, the taxes cause a harmless fall in dividends, because there is no limit on bank liability and no constraint requiring bank dividends to be positive. Lowering θ so as to approach -1 would then be optimal, because while the subsidy-adjusted value of collateral $(-(1+\theta)\kappa(y_t^T + p_t^N \bar{y}^N))$ allows only for an infinitesimally small amount of debt, the amount of resources in units of tradables that this debt generates $(-(q^*\mathbb{E}_t(p_{t+1}^c)/(1+\theta))b_{t+1}^c)$ grows infinitely large.

θ effectively increases the fraction of income pledgeable as collateral, since the collateral constraint can be re-written as $q^*\mathbb{E}_t(p^c_{t+1})b^c_{t+1} \geq -\kappa(1+\theta)(y^T_t + p^N_t \bar{y}^N)$.

If follows from the above arguments that, if we remove the collateral constraint, capital controls and domestic debt taxes levied on an otherwise competitive economy are again equivalent. Only τ^{ef} matters, and decomposing it into τ and θ is irrelevant. If the collateral constraint is present and is occasionally binding (*and* under the assumption that the budgetary impact of the capital controls is allocated to private agents instead of banks), the equivalence breaks because θ has an effect separate from τ when the constraint binds: It increases (reduces) borrowing capacity as θ rises (falls). Hence, the regulator now has an instrument that can alter allocations when the constraint binds. If using it can increase social welfare, this can justify using capital controls as a policy specifically aimed at targeting capital inflows.

Two important caveats about this result: First, it does not necessarily follow that a policy of setting θ high enough for the constraint never to bind while keeping τ^{ef} at zero is the best policy. This is because the intermediation externality is present even when the constraint does not bind, and this can make a competitive equilibrium with some degree of credit frictions more desirable than one without credit constraints (i.e. welfare in an SSLD economy where the constraint never binds is not necessarily higher than one where it can bind, while in the SS model this is always the case). Second, the above result does not alter the result that the equivalence between capital controls and domestic debt taxes holds for implementing the social planner's equilibrium under commitment, because the planner has no incentive to act when the constraint binds, since it cannot alter allocations.[45]

In the quantitative experiments we discuss next, τ and θ take possible values from

45. Intuitively, the planner acting under commitment is constrained-efficient in terms of being subject to the collateral constraint and the pricing conditions of goods and asset markets, and the latter in particular means the planner is committed not to distort the intermediaries no-arbitrage condition, which the ad-hoc constant θ distorts.

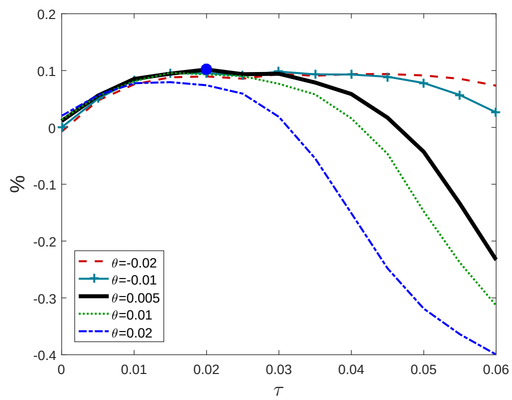
discrete grids. We solve the tax-distorted competitive equilibrium of the SSLD model for each available pair (τ, θ) and search for the pair that yields the largest welfare gains relative to the unregulated competitive equilibrium. These welfare gains are computed as compensating variations in consumption constant across dates and states of nature that equate expected lifetime utility in the competitive equilibrium with taxes with that in the unregulated competitive equilibrium.

Figure 2.6 shows three plots that illustrate the welfare effects of fixed taxes on domestic debt and capital flows. Panel (a) shows the welfare effects of varying τ in the [0,0.06] interval for $\theta = [-0.02, -0.01, 0.005, 0.01, 0.02]$ (where 0.005 is the value that maximizes welfare with respect to θ). Panel (b) shows the welfare effects of varying θ in the [-0.03,0.06] interval for $\tau = [0.01, 0.02, 0.03, 0.04]$ (where 0.02 is the value that maximizes welfare with respect to τ). Panel (c) uses the same data of Panel (a), but plotted as a function of the value of τ^{ef} corresponding to each (τ, θ) pair.

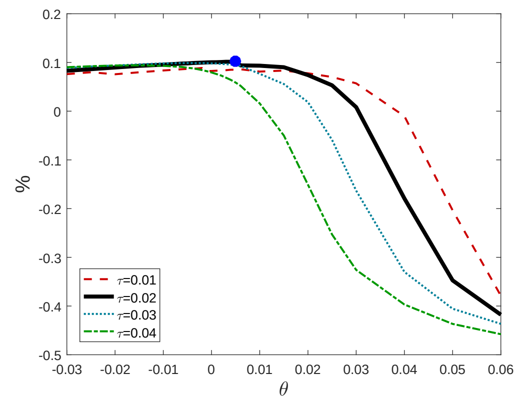
To understand the intuition behind these plots, keep in mind that without policy intervention the economy is affected by the macroprudential and intermediation externalities. Constant taxes can in principle weaken these externalities, but nothing guarantees a welfare-increasing outcome for arbitrary (τ, θ) pairs. Whether this is the case or not depends on the extent to which the distortions introduced by these taxes tackle the externalities v. the costs of these distortions themselves.

The distortions that the constant taxes introduce are determined by the following effects. First, there are the two distortions evident from the optimality conditions mentioned earlier: 1) both higher τ or higher θ increase the effective real interest rate in the Euler equation for bonds, thus increasing the marginal cost of borrowing; 2) higher θ increases borrowing capacity by increasing the effective fraction of income pledgeable as collateral. There are also two precautionary-savings

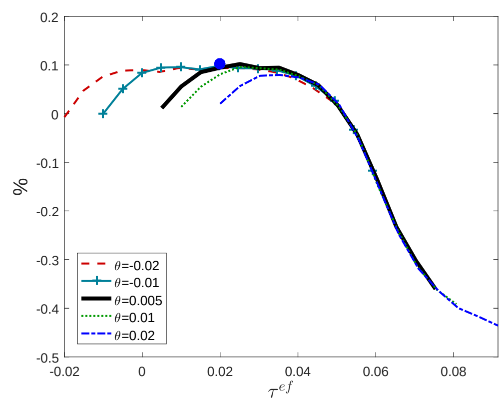
Figure 2.6: Welfare Effects of Constant Taxes



(a) Varying τ for given θ



(b) Varying θ for given τ



(c) Varying τ^{ef} for given θ

Note: The circles identify the maximum value of welfare gains and the corresponding maximum τ and θ points for maximizing welfare gains across all (τ, θ) pairs.

effects that are dynamic implications of the first two effects: 3) the interest-rate effect of higher θ or higher τ strengthens precautionary savings incentives; and 4) the collateral effect of higher θ reduces the need for precautionary savings. Finally, there is also a Sudden Stops effect: 5) as a result of the previous effects, changes in τ and/or θ affect the frequency and magnitude of financial crises, and these effects are non-monotonic (e.g if θ is so high that the constraint never binds, or is set at $\theta = -1$ so that no debt is allowed, the Fisherian deflation mechanism disappears).

When considering precautionary savings effects, it is worth recalling that the typical stationary asset demand curve of incomplete-markets models, which plots average bond holdings at different interest rates, is generally concave with a vertical asymptote at an ad-hoc debt limit and an horizontal asymptote where the interest rate equals the rate of time preference (see Ljungqvist and Sargent (2004)). This has two important implications for the effects of θ and τ. First, because of the concavity, changes of equal size in θ or τ have much stronger effects on the average debt position around a high interest rate than a low one. Second, there is an asymmetry between the two instruments in how they alter the stationary debt position: changing τ or θ implies similar movements along the asset demand curve, but changing θ also alters credit limits and thus shifts the asset demand curve.

Panel (a) of Figure 2.6 shows that for $\tau < 0.02$ welfare rises with τ and is about the same across the five values of θ. In this region, a higher debt tax is beneficial because, via the effects mentioned above, it reduces the adverse effects of the macroprudential and intermediation externalities. The separate effect of θ on borrowing capacity does not make much difference, because although the lower values of θ reduce borrowing capacity, the higher debt taxes are already aiming to reduce debt in the economy. In contrast, as τ increases above 0.02, welfare starts to decline for each value of θ, as now taxing debt has a rapidly growing distortionary effect on borrowing decisions that exceeds the benefits of weakening the externalities. Moreover, welfare is much lower for higher θ, because higher θ implies higher τ^{ef} for the same τ, so the distortion on the borrowing decisions is larger. In addition, higher θ makes the distortion on the marginal cost of borrowing more painful, because it increases borrowing capacity which weakens incentives for precautionary savings (i.e. strengthens the desire to borrow).

For each curve corresponding to a given value of θ in Panel (a), there is a range of values of τ for which welfare is nearly independent of τ. This is an implication of the shape of the welfare curves in Panel (b), which show that, for a given value of τ, there is always a threshold value of θ below which welfare is only marginally increasing in θ. In this region, the interest rate and borrowing capacity effects of θ push against each other, with low values of θ reducing the marginal cost of borrowing and precautionary savings because of the former, but also reducing borrowing capacity and increasing precautionary savings because of the latter. The net result is that welfare rises only slightly with θ. On the other hand, for θ higher than the threshold value, welfare begins to fall sharply as θ rises, because now the marginal cost of borrowing is rising too much relative to the costs of the externalities, and the increased borrowing capacity is irrelevant. Note also that for a given θ in this region, welfare is sharply lower at higher τ, because this implies higher τ^{ef} and hence a stronger distortionary effect of debt taxes.

Panel (c) of Figure 2.6 illustrates three important results of the regime with constant taxes. First, there is a region of tax pairs for which debt taxes and capital controls are equivalent, and hence only the effective debt tax matters. In particular, when the τ, θ values yield $\tau^{ef} \geq 0.038$, a given τ^{ef} yields the same welfare regardless of the value of θ. This is because at sufficiently high τ^{ef} incentives to borrow are weakened enough to make the effect of θ on borrowing capacity irrelevant, and as explained earlier, in the absence of this mechanism the two instruments are equivalent. But for $\tau^{ef} < 0.038$ the equivalence breaks. For a given τ^{ef} in this region, welfare is lower at higher values of θ. This is a key result, because it shows that when the two instruments are not equivalent, it is preferable to generate a given τ^{ef} with a mix that uses (weakly) lower capital controls, because in this region a higher borrowing capacity with a higher θ makes taxing debt more costly. Second, as in Panel (a), for each value of θ there is an interval of values of τ^{ef} that generates roughly similar welfare effects and this interval is wider for lower θ, which is again due to the flat region of welfare effects identified in Panel (b). This result is important because it shows that, if the choice is only over constant taxes and θ is set relatively low, regulators have more "margin of error" for setting τ without reducing welfare sharply. Third, it is easy for constant taxes to produce outcomes that reduce welfare relative to the unregulated competitive equilibrium, by as much as as 0.45% for $\tau^{ef} = 0.09$. Any (τ, θ) pair that yields a value of τ^{ef} above 0.045 is worst than leaving the economy unregulated and fully exposed to Sudden Stops, and this is true for all the values of θ. Considering in addition that, as noted below, even the welfare-maximizing constant taxes yield small welfare gains and modest declines in the frequency and severity of Sudden Stops, this result highlights the importance of careful quantitative evaluation of macro-oriented financial regulation.

The welfare-maximizing pair of tax rates is $\tau^* = 0.02$ and $\theta^* = 0.005$, which implies $\tau^{*ef} = 0.025$ and yields a welfare gain of only 0.1 percent.[46] Table 2.2 sheds light on the effectiveness of this policy for reducing the magnitude and frequency of Sudden Stops by comparing key moments of the unregulated competitive equilibrium v. the equilibrium with the welfare-maximizing pair of taxes. The long-run averages of consumption and the debt ratio are about the same, the latter just a notch smaller with the constant taxes. Since mean consumption is about the same, we can infer that the differences in welfare are largely influenced by differences in how the macroprudential and intermediation externalities affect regular business cycles, the frequency and magnitude of Sudden Stops, and the frequency with which the collateral constraint binds (even without a Sudden Stop).[47] With the constant taxes, the probability of the collateral constraint being binding falls by over 350 basis points (from 35.4 to 31.8 percent) and the probability of Sudden Stops falls by roughly 60 basis points (from 3.83 to 3.23 percent), but in fact when Sudden Stops do happen consumption falls slightly more.

46. This is only 1/5th the size of the gain that Mendoza and Rojas (2017) found for an optimal, time-consistent policy of time-varying tax rates.
47. As in the literature (see Mendoza (2010)), we define Sudden Stops as states in which the constraint binds and the current account increases by more than two standard deviations.

Table 2.2: Effectiveness of Constant Taxes

Long-run Moments[1]	(1)	(2)
	DE	CT
Average $(P^c b^c/Y)$ %	-29.41	-29.07
Welfare Gain[2] %	n/a	0.10
Prob. of Sudden Stops[3] %	3.83	3.23
Prob$(\mu_t > 0)$ %	35.38	31.84
Domestic Debt Tax Rate τ %	n/a	2.00
Capital Inflows Tax Rate θ %	n/a	0.50
Average c	0.989	0.989
Average change of c in Sudden Stops %	-4.60	-4.87

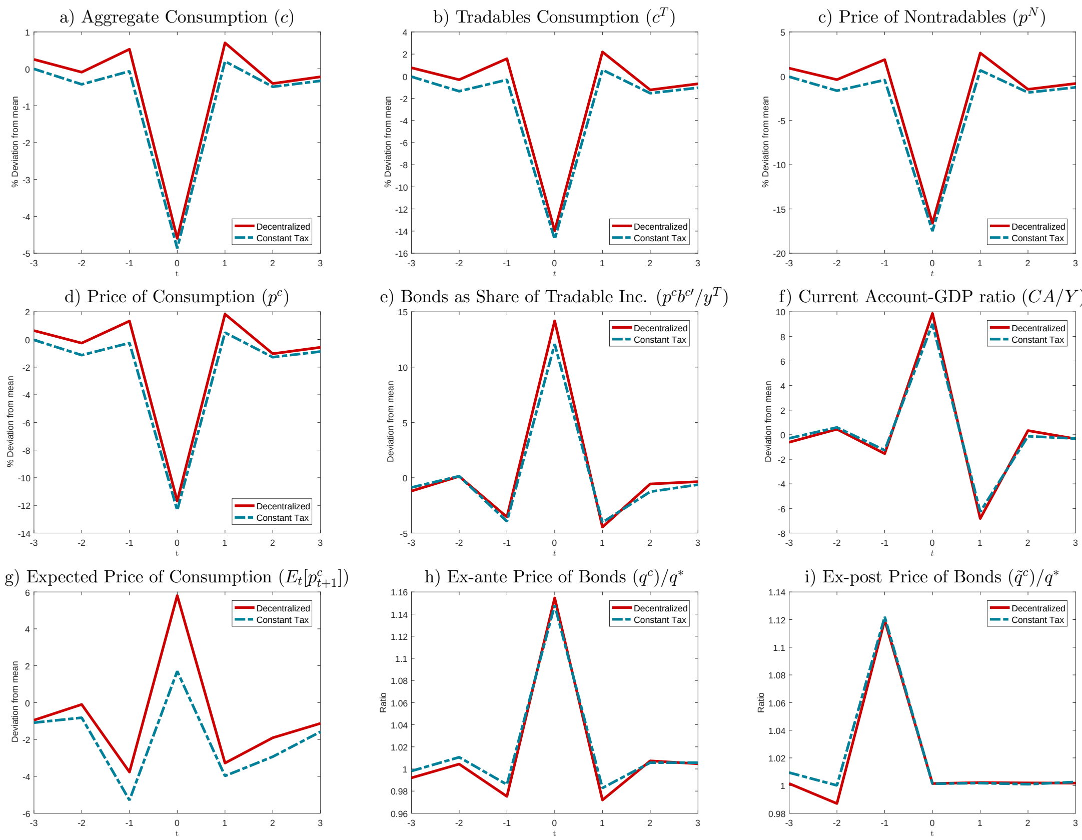
[1] DE denotes the unregulated decentralized economy and CT the economy with constant taxes and capital controls.
[2] Welfare gains are computed as compensating variations in consumption constant across dates and states that equate welfare in the economy with regulation with that in the unregulated decentralized equilibrium. The welfare gain W at state (b^c, y^T) is given by $(1+W(b^c,y^T))^{1-\sigma}V^{\mathrm{DE}}(b^c,y^T) = V^i(b^c,y^T)$. The long-run average is computed using the ergodic distribution of the unregulated economy.
[3] A Sudden Stop is defined as a period in which the constraint binds and the current account raises by more than two standard deviations in the ergodic distribution of the decentralized economy.

Examining how the constant taxes affect Sudden Stop dynamics sheds more light on their effectiveness. To study Sudden Stop dynamics, we follow the same procedure as in Mendoza and Rojas (2017), which is based in generating a long time-series simulation of the economies with and without taxes, identifying Sudden Stop events in the simulated data of the current account, and constructing seven-year event windows centered on the date when Sudden Stops occur. Figure 2.7 shows Sudden Stop event windows for the model's key variables in the unregulated economy and in the economy with the welfare-maximizing pair of constant taxes.

This Figure shows that the constant-taxes regime does poorly in terms of macroeconomic performance when Sudden Stops hit (i.e. at $t = 0$). Aggregate and tradables consumption, as well as the relative price of nontradables and the consumption prince index, show slightly larger declines with the constant taxes in place. The current account reversal and the ex-ante and ex-post prices of domestic bonds are about the same with or without taxes. In contrast, the expected real exchange rate is significantly less volatile, but this is a straightforward implication of the capital controls and the no-arbitrage condition of intermediaries (taking into account that bond and consumption prices are similar in the regulated and unregulated economies). The Figure also shows that consumption is less volatile overall with the constant taxes, which suggests that the 0.1 percent welfare gain that these taxes produce is due to both lower probabilities of Sudden Stops and binding credit constraints and a smoother consumption process.

Figure 2.7 also illustrates that Sudden Stop episodes in the unregulated economy are broadly in line with the empirical regularities of Sudden Stops: Large declines in consumption, the price of nontradables and the real exchange rate, and a sizable reversal in the current account-GDP ratio. Mendoza and Rojas (2017) show that these Sudden Stops are milder than in the standard SS models but they are actually a closer quantitative match to the observed features of Sudden Stops in emerging markets.

Figure 2.7: Sudden Stop Events: Unregulated Economy v. Economy with Constant Taxes



Note: All variables except those measured as output ratios are plotted as percent deviations of their corresponding long-run averages. Variables measured as output ratios are shown as differences relative to the long-run average of the corresponding ratio and expressed in percent.

2.4 Conclusions

We modified the workhorse model of Sudden Stops and macroprudential policy in emerging markets by introducing liability dollarization. Frictionless banks intermediate foreign liabilities in units of world tradable goods into domestic loans denominated in units of aggregate consumption, which is a composite good that combines tradables and nontradables. A collateral constraint limits the resources that can be generated by borrowing not to exceed a fraction of the market value of total income in the same units, so that the equilibrium relative price of nontradables enters as a determinant of borrowing capacity. Liability dollarization introduces three effects absent from the workhorse model that work through fluctuations in the real exchange rate: ex-post real exchange rates alter the burden of repaying existing debt, expected real exchange rates alter domestic bond prices and ex-ante real interest rates, and the negative correlation between marginal utility and real exchange rates provides a risk-taking incentive by lowering the marginal cost of borrowing.

We provided analytical results and quantitative experiments (based on a widely-used calibration for Argentina) showing that under perfect foresight only the first of the three effects operates and two key results follow: Sudden Stops are milder than in standard SS models and multiplicity of equilibria with Sudden Stops is harder to obtain. In the SSLD model, unique equilibria are sustained for higher debt-to-income limits and the range of income levels that support multiplicity is narrower. Quantitatively, multiplicity in the SSLD model requires particular preference parameters that deviate from typical calibrations, much higher limits in debt-to-income ratios than those used in standard Sudden Stops models, and even then it is present for significantly narrower income ranges.

We also conducted a normative analysis of the optimal financial policy of a constrained-efficient regulator acting under commitment. The competitive equilibrium is distorted by two pecuniary externalities: First, the macroprudential externality typical of standard Sudden Stops models, which is present at date t only when the credit constraint is expected to bind with some probability at $t+1$, because the planner internalizes the effects of the date-t borrowing decision on the size of the $t+1$ crash in collateral values. Second, an intermediation externality induced by the three effects of liability dollarization, which is present regardless of the credit constraint, because private agents do not internalize the effects of their borrowing decisions on actual and expected real exchange rates. Optimal policy tackles both externalities, but is also time-inconsistent: At date t, the planner has the incentive to pledge higher consumption for $t+1$ to create expectations of real appreciation and reduce interest rates, but ex-post at $t+1$ an appreciated real exchange rate is undesirable because it increases the private agent's burden of debt repayment. Moreover, decentralizing this optimal policy does not justify the use of capital controls, because capital controls and domestic debt taxes play equivalent roles. Both are needed only to alter the marginal cost of borrowing of private agents, regardless of the source of credit. In addition, the rule governing optimal effective debt taxes is a complex, non-linear rule.

Since the optimal policy is complex and lacks credibility, we examined the potential for constant tax rates on capital inflows and domestic debt to produce welfare-improving outcomes relative to the unregulated competitive equilibrium. If the budgetary implications of capital controls are charged to private agents instead of banks, there can be a role for capital controls because they can alter borrowing capacity, whereas domestic debt taxes cannot. Quantitatively, the equivalence between capital controls and domestic debt taxes reappears at relatively high values of effective debt taxes, and for low values of effective debt taxes, welfare is higher when capital controls are set at lower rates than domestic debt taxes. The welfare-maximizing constant taxes are in the region where the two instruments are not equivalent, yielding a 2 percent debt tax v. a 0.5 percent tax on capital inflows. However, there is only a modest gain in welfare of 0.1 percent, which is largely due to a reduced frequency of Sudden Stops and binding credit constraints, while macro dynamics around Sudden Stops do not improve markedly, and in fact consumption and price declines are slightly larger when a Sudden Stop hits. These results also show that quantitative evaluation of the policy mix of debt taxes and capital controls is critical, because slight variations can produce regulated environments that leave private agents significantly worse off than in the unregulated economy.

The importance of liability dollarization in emerging markets highlights the relevance of our findings for the analysis of Sudden Stops. From a positive standpoint, we found quantitatively significant effects of liability dollarization operating via real exchange rate fluctuations. Moreover, in Mendoza and Rojas (2017) we found that the mechanism making Sudden Stops milder with liability dollarization also makes them more consistent with the stylized facts of Sudden Stops than in the standard model. From a policy perspective, we established here that the optimal policy under commitment with or without liability dollarization does not justify capital controls, only a tax on the rate at which domestic agents borrow that can be implemented equally by taxing capital inflows or domestic debt. Our analysis of simple macroprudential policies also fails to make a good case for capital controls, as the welfare-maximizing policy uses mainly domestic debt taxes. Introducing liability dollarization is also critical for demonstrating that the optimal policy under commitment is time-inconsistent, and hence lacks credibility, because in the standard model this issues does not arise. Hence, liability dollarization makes credibility problems that are pervasive in other key areas of economic policy relevant for macroprudential policy as well.

An important limitation of our analysis is that it abstracted from modeling frictions in financial intermediation. We focused only on the effects of liability dollarization on domestic non-financial private agents, which had not been studied before, but in an environment in which risk-neutral banks are nearly frictionless and yield a simple no-arbitrage condition for pricing domestic non-state-contingent debt. Further research should follow the earlier literature on liability dollarization and emerging markets crises to introduce more significant frictions in financial intermediation, and in particular the possibility of bankruptcy and/or non-neutral bank balance sheet effects as a result of the kind of real-exchange-rate fluctuations we examined here.

Appendices

Appendix A

Appendix to Firm Heterogeneity & the Transmission of Financial Shocks During the European Debt Crisis

A.1 Data Appendix

The details of procedure for data cleaning and preparation are presented in this section. I follow steps similar to those of Gopinath et al. (2017). For data cleaning the steps are:

(i) Drop firm-year observations that have missing information on total assets, operational revenues, sales and employment.

(ii) Drop firm-year observations with missing, 0 or negative values for operational revenues and total assets.

(iii) Drop observations with missing industry information.

(iv) Drop firms who have negative total assets in any year, and if employment, sales or tangible fixed assets are negative.

Also, firms from the financial sector and government are dropped from the sample (NACE codes 65, 66, 67, and 75). Variables used in the analysis are constructed in the following way:

(i) Age: Reporting year minus year of incorporation plus 1. Observations with negative values are dropped.

(ii) Liabilities: Total shareholders funds and liabilities minus shareholders funds. Observations with negative values are dropped. Also a consistency check is performed: observations with liabilities different from the sum of current and non current liabilities are dropped.

(iii) Capital Stock: tangible fixed assets plus intangible fixed assets. Observations with negative values of intangible fixed assets are dropped, as well as those with zero tangible fixed assets, with tangible fixed assets that exceed total assets and cases with negative depreciations.

(iv) Equity: shareholders funds. We drop observations whose ratio of shareholders funds and total assets are below percentile 0.1.

(v) Leverage: ratio between liabilities and total assets. We drop observations for which the leverage ratio is below or above the lowest and highest 0.1 percentiles, respectively.

(vi) Size: share of firm's assets over total assets for a given year.

(vii) Average Effective Interest Rate: Observations with negative values for the ratio $\frac{\text{Interest Paid}}{\text{Liabilities}}$ are dropped.

A.1.1 Additional Specifications

This subsection presents robustness checks regarding the specification of the regression model presented in section 1.1. We estimate the regression model using a random effects specification and also using (pooled) OLS, and contrast the results with those of the baseline fixed effects specification.

Tables A.1 and A.2 present the results of the random effects and OLS specifications. Unlike in the case of the fixed effects specification, the random effects and OLS specifications allow for country and industry effects (in addition to the same controls as in the baseline specification).

We see that the effects of government debt on sales, liabilities, and employment and not substantially different between the fixed effects and random effects specifications, but there is a difference for the case of assets of the firm (the effect of government debt loses statistical significance in the random effects specification). When comparing the results from the OLS specification with the baseline, we see that the results are quite different. The latter is because of the nature of the underlying assumptions of each specification. The OLS specification assumes that there is no unobserved heterogeneity (i.e., the firm-specific characteristics are uncorrelated with the regressors), which is unlikely in a setting like the one presented in this paper.[48]

Overall, the results show that the random effects specification, despite not being preferred to the fixed effects specification according to the Hausman test, are not significantly different from those of the fixed effect model.

48. The F-statistic of the fixed effects specification provides a way of jointly testing if all individual firm-specific effects are 0. The null hypothesis is that firm-specific effects are 0. The associated p-values are 0.000 for all the estimated regressions, suggesting that the fixed effect specification should be preferred to the pooled OLS one.

Table A.1: Panel Regression Results: Additional Specifications - Sales, Liabilities, Assets, and Employment

Variable	Log(Sales)			Log(Liabilities)			Log(Assets)			Log(Employment)		
	FE	RE	OLS	FE	RE	OLS	FE	RE	OLS	FE	RE	OLS
B	-0.003	-0.002	-0.002	-0.003	-0.001	-0.000	-0.001	0.000	0.000	-0.003	-0.000	-0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.294)	(0.207)	(0.000)	(0.000)	(0.020)
Q_2	0.327	0.254	-0.019	0.718	0.708	0.51	0.519	0.596	0.426	0.161	0.091	0.064
	(0.000)	(0.000)	(0.044)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Q_3	0.562	0.448	0.015	1.250	1.14	0.791	0.909	0.987	0.674	0.285	0.171	0.121
	(0.000)	(0.000)	(0.137)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Q_4	0.754	0.674	0.129	1.712	1.547	1.056	1.237	1.370	0.921	0.368	0.271	0.206
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Q_5	0.864	0.934	0.408	2.103	2.171	1.59	1.532	1.914	1.384	0.424	0.417	0.352
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
B $\times$ Q_2	-0.001	0.000	0.002	-0.001	-0.001	-0.000	-0.000	-0.001	0.000	-0.001	-0.001	-0.000
	(0.000)	(0.473)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.706)
B $\times$ Q_3	0.000	0.000	0.002	-0.001	-0.001	-0.000	0.000	-0.001	-0.001	-0.001	-0.001	-0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.474)	(0.000)	(0.000)	(0.000)	(0.000)	(0.763)
B $\times$ Q_4	0.000	0.001	0.001	-0.001	-0.001	-0.000	0.000	-0.001	-0.001	-0.001	-0.001	-0.000
	(0.036)	(0.000)	(0.000)	(0.000)	(0.000)	(0.033)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
B $\times$ Q_5	0.002	0.002	0.002	0.000	0.000	-0.000	0.000	-0.000	-0.001	0.000	-0.000	-0.000
	(0.000)	(0.000)	(0.000)	(0.054)	(0.336)	(0.005)	(0.000)	(0.000)	(0.000)	(0.367)	(0.007)	(0.000)
B $\times$ Q_2 $\times$ Sk	0.000	0.000	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	0.000	0.000	0.000
	(0.058)	(0.000)	(0.000)	(0.003)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.013)	(0.074)
B $\times$ Q_3 $\times$ Sk	-0.000	0.000	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	0.000	0.000	-0.000
	(0.727)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.408)	(0.573)
B $\times$ Q_4 $\times$ Sk	-0.000	0.000	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	0.000	-0.000	-0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.004)	(0.000)
B $\times$ Q_5 $\times$ Sk	-0.000	-0.000	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	0.000	-0.000	-0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.013)	(0.000)	(0.000)
B $\times$ Sk	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	(0.000)	(0.012)	(0.012)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.934)	(0.000)	(0.000)
Year Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Fixed Effects	Yes	No	No	Yes	No	No	Yes	No	No	Yes	No	No
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country Effects	No	Yes	Yes	No	Yes	Yes	No	Yes	Yes	No	Yes	Yes
Industry Effects	No	Yes	Yes	No	Yes	Yes	No	Yes	Yes	No	Yes	Yes
Observations	3,195,072	3,195,072	3,195,072	3,308,816	3,308,816	3,308,816	3,313,051	3,313,051	3,313,051	3,119,977	3,119,977	3,119,977
R^2	0.58	0.699	0.712	0.736	0.914	0.923	0.911	0.967	0.970	0.585	0.8816	0.885

Notes: B denotes the gross public debt-to-output ratio, Q_i is a dummy variable that is equal to 1 if the firm belongs to the *ith* quintile of firm size, and Sk is the skewness of the firm size distribution. Countries include Greece, Ireland, Italy, Portugal and Spain. Clustered standard errors at the firm level considered, p-values in parentheses.

Table A.2: Panel Regression Results: Additional Specifications - Proxy for Average Effective Interest Rate

Variable	Average Effective Interest Rate × 100		
	Fixed Effects	Random Effects	Pooled OLS
B	0.000	0.000	0.000
	(0.000)	(0.041)	(0.577)
Q_2	-0.011	-0.015	-0.017
	(0.000)	(0.000)	(0.000)
Q_3	-0.017	-0.022	-0.022
	(0.000)	(0.000)	(0.000)
Q_4	-0.022	-0.026	-0.026
	(0.000)	(0.000)	(0.000)
Q_5	-0.027	-0.032	-0.031
	(0.000)	(0.000)	(0.000)
B × Q_2	-0.006	0.000	0.000
	(0.222)	(0.000)	(0.000)
B × Q_3	-0.027	0.000	0.000
	(0.599)	(0.000)	(0.000)
B × Q_4	-0.03	0.000	0.000
	(0.972)	(0.000)	(0.000)
B × Q_5	-0.043	0.000	0.000
	(0.637)	(0.000)	(0.000)
B × Q_2 × Sk	0.000	0.000	0.000
	(0.007)	(0.000)	(0.000)
B × Q_3 × Sk	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)
B × Q_4 × Sk	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)
B × Q_5 × Sk	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)
B × Sk	-0.000	-0.000	-0.000
	(0.000)	(0.000)	(0.000)
Year Effects	Yes	Yes	Yes
Fixed Effects	Yes	No	No
Controls	Yes	Yes	Yes
Country Dummies	No	Yes	Yes
Industry Dummies	No	Yes	Yes
Observations	2,852,913	2,852,913	2,852,913
R^2	0.001	0.019	0.020

Notes: B denotes the gross public debt-to-output ratio, Q_i is a dummy variable that is equal to 1 if the firm belongs to the *ith* quintile of firm size, and Sk is the skewness of the firm size distribution. Countries include Greece, Ireland, Italy, Portugal and Spain. Clustered standard errors at the firm level considered, p-values in parentheses.

A.2 Computational Appendix

A.2.1 Solution Method

The solution method follows Krusell and Smith (1997) and Krusell and Smith (1998). Unlike these articles, where the distribution of wealth is approximated by the average wealth in the economy, in this paper we assume that the distribution of firms is well approximated by the cross-sectional variance of capital. The individual states for firms are (z,k,b) and the aggregate states are $(B,\sigma^2,\mu,\varepsilon)$.

We conjecture log-linear specifications for variance and the marginal utility of the intermediaries' wealth:

$$\log\sigma^{2\prime} = \alpha_\sigma + \beta_\sigma \log\sigma^2 + \gamma_\sigma \log(1+B) + \delta_\sigma \log(1+\mu) + \kappa_\sigma \varepsilon \tag{A.1}$$

$$\log(1+\mu') = \alpha_\mu + \beta_\mu \log\sigma^2 + \gamma_\mu \log(1+B) + \delta_\mu \log(1+\mu) + \kappa_\mu \varepsilon \tag{A.2}$$

The steps of the solution algorithm are the following:

1. Start with a guess for the parameters of the forecasting rule given by the vector

$$\Theta^{\text{guess}} = (\alpha_\sigma, \beta_\sigma, \gamma_\sigma, \delta_\sigma, \kappa_\sigma, \alpha_\mu, \beta_\mu, \gamma_\mu, \delta_\mu, \kappa_\mu) \tag{A.3}$$

2. Using the forecasting rules solve the problem of sovereign using its budget constraint.

3. Using the forecasting rules and the policy function of the sovereign (which is obtained from solving the budget constraint), solve the problem of the firm. For this:

   (a) Guess a value for the value function $\tilde{V}$.

(b) Using this guess, construct a default indicator and also the pricing function $\tilde{q}$ for the firm.

(c) Using $\tilde{V}$ and $\tilde{q}$, solve the problem of the firm using value function iteration. The pricing function is updated after every maximization step (instead of waiting for convergence on V), which increases the speed of the algorithm.[49]

(d) Using the solution to the firm's problem V, construct an update for the pricing function, denote it q. Notice that a default decision can be directly computed from V, so q can be inferred from it and from aggregate laws of motion. If $||V - \tilde{V}|| <$ tol and $||q - \tilde{q}|| <$ tol, then proceed to the next step. Otherwise go back to step (b) and use V as a guess, construct $\tilde{q}$ using this new guess.

4. Simulate an economy with I firms and the sovereign, for T periods. At every period of the simulation find the value of μ that solves the slackness condition of the intermediaries' problem. For this start by assuming that the constraint does not bind (this is, $\mu = 0$). If the firm aggregate demand plus the sovereign demand for debt do not exceed the level of wealth of the intermediaries', then set $\mu = 0$ and proceed to the next period. Otherwise, perform a bisection algorithm to find the value of μ that makes the slackness condition to hold.

5. Using the completed sequence of simulated variables update forecasting rules and Find Θ^{new}. If $||\Theta^{\text{guess}} - \Theta^{\text{new}}|| <$ tol, stop. Otherwise, go back to step 1 and use $\tilde{\Theta} = \varphi\Theta^{\text{new}} + (1-\varphi)\Theta^{\text{guess}}$ as the new guess.

The coefficients obtained are presented in Table A.3:

49. Solutions of the model under this approach and one where the pricing function is updated after convergence of V are the same.

Table A.3: Regression Results

Coefficient	σ^2	μ
α_i	-0.245	-0.570
	(0.040)	(0.022)
β_i	0.801	0.243
	(0.022)	(0.012)
γ_i	-0.037	0.091
	(0.011)	(0.006)
δ_i	0.311	-5.629
	(0.752)	(0.826)
κ_i	0.000	0.004
	(0.002)	(0.001)
R^2	0.967	0.912

Note: Standard errors in parentheses.

A.2.2 Construction of Impulse Response Functions

1. Simulate two economies with the same sequence of cross-sectional idiosyncratic productivity for firms (N firms) until a date $\hat{t}$, with government debt fixed at its long run average.

2. In period $\hat{t}$, there is an increase in sovereign debt. Let x_{it}^0 denote the value of variable x for firm i in period t in an economy with no aggregate shock, and x_{it}^1 the equivalent but in the economy with the aggregate shock. Define the impulse response of firm i in period t as $\tilde{x}_{it} = \frac{x_{it}^1 - x_{it}^0}{x_{it}^0} \times 100$. The response of the economy will be given by $\tilde{x}_t = \sum_i^N \tilde{x}_{it}/N$.

3. In order to mitigate potential history dependence, perform this procedure until there are M sequences of responses. The impulse response function is the average of the M sequences.

Appendix B

Appendix to Positive and Normative Implications of Liability Dollarization for Sudden Stops Models of Macroprudential Policy

B.1 Properties of the BB^{SSLD} Curve

The BB^{SSLD} curve determines the value of p^N that corresponds to a value of c^T such that both the resource and the collateral constraint hold with equality (see equation (2.13) in the text). Formally, the BB^{SSLD} curves is given by the following function:

$$p^N(c^T) = \frac{c^T - (1+\kappa)y^T - p^c(c^T)b^c}{\kappa \bar{y}^N}$$

We omit time subscripts for simplicity, but notice b^c corresponds to the outstanding debt at the beginning of the period. Given the parametric restrictions on c^T, κ, y^T, $\bar{y}^N$ and b^c, the fact that $p^c(c^T)$ is continuous implies that the BB^{SSLD} curve is continuous.

The horizontal intercept of the BB^{SSLD} curve is found by evaluating the above expression when $p^N = 0$. Using equation (2.3) to determine the consumption price index when $p^N = 0$, it follows that the intercept is the value of tradables consumption such that $c^T = (1+\kappa)y^T + \omega^{\frac{1}{\eta}} b^c$.

To obtain the slope of the BB^{SSLD} curve, we take the first derivative of the above expression, which yields $\frac{\partial p^N}{\partial c^T} = \frac{1-\frac{\partial p^c}{\partial c^T} b^c}{\kappa \overline{y}^N}$. The sign of the slope depends on the signs of $\frac{\partial p^c}{\partial c^T}$ and b^c. Using equation (2.3), it follows that because of the CES structure of preferences, $\frac{\partial p^c}{\partial c^T} = (1+\eta)\frac{1-\omega}{\omega}\left[\omega + (1-\omega)(c^T)^{\eta}\right]^{\frac{1}{\eta}}(c^T)^{\eta-1} > 0$. Moreover, since we are interested in economies with debt ($b^c < 0$), it follows that $\frac{\partial p^N}{\partial c^T} > 0$. Hence, the BB^{SSLD} curve is increasing in c^T

To determine whether the BB^{SSLD} curve is concave or convex, we analyze its second derivative, which is the following:

$$\frac{\partial^2 p^N}{\partial {c^T}^2} = -\frac{\frac{\partial^2 p^c}{\partial {c^T}^2} b^c}{\kappa \overline{y}^N}$$

The sign of this derivative is the same as the sign of the second derivative of p^c with respect to c^T. This derivative can be expressed as:

$$\frac{\partial^2 p^c}{\partial {c^T}^2} = (1+\eta)\frac{1-\omega}{\omega}\left(\omega + (1-\omega)(c^T)^{\eta}\right)^{\frac{1}{\eta}}(c^T)^{2(\eta-1)}\left[\frac{1-\omega}{\omega + (1-\omega)(c^T)^{\eta}} + (\eta-1)(c^T)^{-\eta}\right]$$

All the terms in the right-hand-side of this expression are positive, except for the last term in square brackets, which has an ambiguous sign. Hence, the sign of this derivative is determined by the sign of the term $\left[\frac{1-\omega}{\omega+(1-\omega)(c^T)^{\eta}} + (\eta-1)(c_T)^{-\eta}\right]$. We can characterize the conditions determining the sign of this term by first reducing it to this expression:

$$\left[\frac{1}{\tilde{\omega}+(c^T)^\eta}+\frac{\eta-1}{(c^T)^\eta}\right] \tag{B.1}$$

where we used the definition $\tilde{\omega} \equiv \omega/(1-\omega)$. Analyzing this expression, it follows that:

$$\left[\frac{1}{\tilde{\omega}+(c^T)^\eta}+\frac{\eta-1}{(c^T)^\eta}\right] \gtreqqless 0 \quad \Leftrightarrow \quad \eta \gtreqqless \frac{\tilde{\omega}}{\tilde{\omega}+(c^T)^\eta} \tag{B.2}$$

Notice the expression in the right-hand-side of the last inequality is always a positive fraction, but its magnitude varies with c^T, ω and η, which is what makes the direction of the inequality ambiguous. Still, given that $\eta > -1$ from the CES functional form, and that $c^T > 0$ and $0 < \omega < 1$, we can establish the following three results:

1. *The p^c and BB^{SSLD} functions are strictly concave when the elasticity of substitution betwen tradables and nontradables is greater or equal to 1*: If $-1 < \eta \leq 0$ (i.e. $1/(1+\eta) \geq 1$) then $\eta < \frac{\tilde{\omega}}{\tilde{\omega}+(c^T)^\eta}$, and hence $\left[\frac{1}{\tilde{\omega}+(c^T)^\eta}+\frac{\eta-1}{(c^T)^\eta}\right] < 0$ and thus both p^c and BB^{SSLD} are strictly concave, for any positive c^T, ω.

2. *The p^c and BB^{SSLD} functions are strictly convex when the elasticity of substitution betwen tradables and nontradables is less or equal than 1/2*: If $\eta \geq 1$ (i.e. $1/(1+\eta) \leq 1/2$) then $\eta > \frac{\tilde{\omega}}{\tilde{\omega}+(c^T)^\eta}$, and hence $\left[\frac{1}{\tilde{\omega}+(c^T)^\eta}+\frac{\eta-1}{(c^T)^\eta}\right] > 0$ and both p^c and BB^{SSLD} are strictly convex, for any positive c^T, ω.

3. *The p^c and BB^{SSLD} functions are concave (convex) for sufficiently low (high) c^T when the elasticity of substitution is between 1/2 and 1*: If $0 < \eta < 1$ (so that $1/2 < 1/(1+\eta) < 1$), the sign of $\left[\frac{1}{\tilde{\omega}+(c^T)^\eta}+\frac{\eta-1}{(c^T)^\eta}\right]$ changes from negative to positive as c^T rises. Around $c^T = 0$ we obtain $\eta < \frac{\tilde{\omega}}{\tilde{\omega}+(c^T)^\eta} = 1$ and as c^T increases $\frac{\tilde{\omega}}{\tilde{\omega}+(c^T)^\eta}$ falls. Hence, near $c^T = 0$ the derivatives are negative and the curves are concave. By

continuity, for sufficiently low c^T the curves are concave, and for sufficiently high c^T the curves turn convex.

Bianchi (2011) notes that estimates of the elasticity of substitution in tradables and nontradables consumption for emerging markets are in the [.4,.83] interval. Hence, most of this range falls in the region where the third result above applies. Quantitatively, however, in our baseline calibration for the perfect-foresight experiments taken from Bianchi's work (which uses $\eta = 0.205$, $1/(1+\eta) = 0.83$) and for other exercises using reasonable values of b^c and y^T and any $0 < \eta < 1$, we found that BB^{SSLD} is either convex or nearly linear, except for a slightly concave segment for very low c^T. In all of these experiments, the concavity is visible only for $c^T < 0.05$ compared with a perfect-foresight unconstrained equilibrium of $c^T = 0.92$ using our baseline calibration, or a Sudden Stop outcome of $c^T = 0.83$ for a wealth-neutral negative shock of nearly 20 percent from an initial income of $y^T = 1$. Hence, assuming convex or nearly linear BB^{SSLD} curves when the elasticity of substitutions is less than unitary is innocuous.

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