Essays In Housing Markets And Financial Fragility

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Essays In Housing Markets And Financial Fragility

Abstract
This dissertation is motivated by the housing crisis of 2008. It consists of three chapters. In the first chapter, "Too Much Skin-in-the-Game? The Effect of Mortgage Market Concentration on Credit and House Prices," I propose a new theory to help explain the housing crisis. During the housing boom, a small number of institutions - the government-sponsored enterprises (GSEs) and a few banks - held most of U.S. mortgage risk. I develop a theory in which such concentration of mortgage exposure can explain features of the housing crisis. I show that large lenders with many outstanding mortgages have incentives to extend risky credit to prop up house prices. An increase in concentration can lead to a boom with worsening credit quality and a subsequent bust with widespread defaults.

In the second chapter, "Concentration and Lending in Mortgage Markets," joint with Ronel Elul and David Musto, we attempt to test the theory described in the first chapter. We provide evidence that concentration in mortgage markets can create perverse lending incentives. We exploit variation in the size of the GSEs' outstanding mortgage exposure across MSAs. Using a loan-level dataset, we provide evidence that the GSEs were more likely to engage in high-risk activities in areas where they had a large exposure to outstanding mortgages. We also provide evidence that this relationship is driven by an incentive to keep house prices high.

In the final chapter, "Housing Booms and the Crowding-Out Effect," joint with Itay Goldstein, we study the effect that investment in real estate assets has on the economy. We develop a theory in which housing price booms can sometimes lead to a crowding-out of corporate investment. We show that an increase in real estate prices does not necessarily increase aggregate investment even when firms actively use real estate assets as collateral to borrow against and invest the proceeds in positive NPV projects. We argue that at times, it can be optimal to decrease the price of housing rather than to support high housing prices to stimulate the economy and characterize when this is the case.

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ESSAYS IN HOUSING MARKETS AND FINANCIAL FRAGILITY

Deeksha Gupta

A DISSERTATION

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Finance

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For my family
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ABSTRACT

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Deeksha Gupta

Itay Goldstein

Vincent Glode

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1.1. Introduction

In the 2000s, there was an unprecedented surge in risky lending to borrowers with low FICO scores at high debt-to-income and loan-to-value ratios. These mortgages were associated with high default rates and ex-post did not seem to be profitable for credit suppliers.¹

What motivated this high-risk, seemingly unprofitable lending? A common explanation is that dispersion in mortgage holdings driven by securitization caused moral hazard problems in mortgage origination. More specifically, since credit providers could sell off mortgages, they no longer had skin-in-the-game in the mortgages they originated and therefore had a reduced incentive to monitor and originate quality mortgages.² This explanation has gained traction in macro-prudential policy following the crisis, with the Dodd-Frank act requiring a minimum level of risk retention by mortgage lenders.

However, during the housing boom, mortgage markets had a high level of concentration if we consider broader exposure to the mortgage market, such as mortgage holdings rather than just originations. In particular, the GSEs and a few banks had rising exposure to the mortgage market through the 1990s and amassed a large concentration of mortgage risk in the 2000s.³ The agencies’ share alone increased from about 7% of the U.S. mortgage market in the 1980s to over 40% in the 2000s.⁴ Additionally, about 50% of all holdings of AAA

¹Following the crisis of 2007, many private securitizers went out of business, banks such as Lehman Brothers and Bear Sterns collapsed or had to be bailed out due to their exposure to the subprime market, and Fannie Mae and Freddie Mac were placed into government conservatorship.

²For theoretical models of this mechanism see Parlour and Plantin (2008) and Vanasco (2017).

³The GSEs’ exposure to mortgages came in the form of portfolio holdings of their own loans (about half of which they held on to) and insurance guarantees on the securitized mortgages that they sold. Additionally, the agencies were the single largest investors in the private securitization market purchasing about 30% of the total dollar volume of private-label MBSes between 2003-2007 (Acharya, Richardson, Nieuwerburgh, and White (2011)) and Adelino, Frame, and Gerardi (2016a).

⁴Appendix D plots the GSEs’ market share of and total dollar exposure to the US mortgage market from 1980-2008. Although the GSEs share of the mortgage market declined between 2002 and 2006, their dollar exposure kept increasing during this time. In the model, the share and dollar exposure of mortgage lenders co-move perfectly, but it is possible to get the results of the model even if the share of mortgage lenders declines due to entry by new lenders. As long as the decline in their market power due to entry is not too large, lenders will still have incentives to increase their dollar exposure to the market. Furthermore, one can...
rated non-GSE mortgage-backed securities were concentrated amongst a few large complex financial institutions (LCFIs) (Acharya et al. (2011)). In this paper, I develop a theory of how this increase in concentration of mortgage risk can explain the surge in high-risk lending and other important characteristics of the housing boom and bust.

The model can explain key empirical features of the recent housing crisis. In particular, as mortgage markets become more concentrated, the model predicts a boom in credit characterized by increasing house prices and debt-to-income (DTI) ratios. Credit quality worsens over the life of the boom. A fundamental shock to a concentrated market can lead to a collapse in real estate prices accompanied by large-scale defaults. For a short period after the bust, lenders in concentrated markets continue to make high-risk loans. Importantly, the model can explain the timing of high-risk lending that started in the 2000s after the credit boom had already begun and the continuation of high-risk activity by the GSEs in 2007 once mortgage markets began to slow down (Bhutta and Keys (2017)).

The key idea of the model is that if credit affects house prices and house prices in turn affect the severity of default, large mortgage lenders internalize their effect on house prices and consequently on default probabilities and losses when making lending decisions. More specifically, prevailing house prices affect the profitability of previously issued mortgages since borrowers are less likely to default when house prices are high and upon default their house, which is collateral for lenders, is worth more. Lenders with a large amount of mortgages on their books therefore have an incentive to keep house prices high when they are due mortgage repayments. If lenders can influence house prices through increasing

view the market for GSE-eligible borrowers and non-GSE borrowers as segmented since only certain loans were eligible for purchase by the GSEs. The GSEs retained market power in this segment even as there was entry by private-label securitizers - primary originators would not sell a loan that was GSE-eligible to a private-label securitizer instead. In this case, we can represent the aggregate US housing market as two segmented markets each with large players (the GSEs in one and LCFIs in the other).

There is a large amount of empirical support for these assumptions. Many papers have found a connection between house prices and default. See Foote, Gerardi, and Willen (2008), Haughwout, Peach, and Tracy (2008), Palmer (2013), Ferreira and Gyourko (2015). Further, many papers also provide evidence that the availability of credit affects house prices. See Himmelberg, Mayer, and Sinai (2005), Khandani, Lo, and Merton (2009), Hubbard and Mayer (2009), Mayer (2011), Griffin and Maturana (2015), Landvoigt, Piazzesi, and Schneider (2015), An and Yao (2016) and Favilukis, Ludvigson, and Nieuwerburgh (2017).
their supply of credit, they may find it optimal to extend credit to low-quality, high-risk borrowers not because of the return they expect to make on the loan itself, but because of the boost in house prices that comes from credit provision. Lenders trade off the loss they make on the issuance of mortgages to these borrowers with the profits they make by keeping house prices high on mortgages that are due repayment.

Concentration impacts both the quantity and quality of mortgage credit. In the model, banks compete in a cournot-style framework - they decide how many mortgage loans to make taking into account their effect on house prices. In most models of industrial organization, as concentration increases, agents behave less like price-takers and the aggregate quantity supplied of the good in question decreases.\(^6\) While this “Cournot” effect is present in the model, there is a second effect of changes in concentration that is new, the “propping-up” effect. As concentration increases, individual lenders acquire larger market shares which creates an incentive to extend more credit to prop up house prices. If the propping-up effect dominates the Cournot effect, the aggregate supply of credit increases as mortgage markets become more concentrated. Furthermore, credit in more concentrated markets is generally riskier than credit in less concentrated markets. In the model, I show that it is possible for two areas with different levels of concentration to have the same level of credit provision. However, the area with higher concentration will have lower quality credit with higher default rates. The area with low concentration has credit provision due to a relatively strong Cournot effect while the area with high concentration has a weak Cournot effect but a strong propping-up effect. The marginal loan made in the area with higher concentration is riskier since banks compromise on the return they earn from the expected loan repayment due to the benefit they get from the resulting increase in house prices. If parameter values allow for equal credit provision under two different levels of concentration, in the presence of costly default a social planner would always prefer to make markets less concentrated.

This paper also contributes to an important debate on whether the housing crisis was driven

by distortions in the supply of credit or by high house price expectations by lenders and borrowers. Two central papers in this debate by Mian and Sufi (2009) and Adelino, Schoar, and Severino (2016b) examine the relationship between income growth and the growth in mortgage credit during the housing boom to address this question. In support of the credit-supply view, Mian and Sufi (2009) find that income growth decoupled from the growth in mortgage credit in the U.S. at the ZIP code-level. They point to innovations in the provision of credit to low-quality borrowers as an explanation for their findings. In support of the expectations view, Adelino et al. (2016b) find that at a borrower-level, income growth did not decouple from the growth in mortgage credit, indicating that lenders did not face distorted incentives to lend disproportionately to riskier borrowers.

Following a shock to concentration, the model can generate different correlations between income-growth and the growth in mortgage credit depending on the level of aggregation of the variables. Following an increase in mortgage market concentration, the model mortgage credit and income growth can be negatively correlated when looking across areas (such as ZIP codes), while at the same time being positively correlated when looking across borrowers. In the model, lenders have relatively more market power in affecting housing prices in areas with low income growth since in such areas without the availability of credit there is little else to drive the demand for housing and keep house prices high. Therefore, for each additional mortgage loan, the percentage increase in house prices and consequently the return to propping up house prices is high. An increase in concentration can therefore lead to a credit supply shock in areas where income growth is low, leading to a decoupling of income growth from the growth in mortgage credit. However, banks’ incentives to lend more to higher-quality borrowers do not fundamentally change. All else equal, a bank would always prefer to make a loan to a high-quality borrower, if possible, as such a loan would also serve to increase house prices. Therefore when looking at borrower-level data, the growth in income and mortgage credit can remain positively correlated.7

7In follow-up papers, Mian and Sufi (2016) and Mian and Sufi (2015), Mian and Sufi argue that some of Adelino et al. (2016b) results are driven by an improper calculation of total mortgage size and fraudulent income over-statement. In Adelino, Schoar, and Severino (2015), the authors respond to these critiques and
This paper contributes to macro-prudential policy discussion in the aftermath of the crises. From a policy perspective, it is crucial to understand the different forces that can drive housing booms and busts. With respect to the financial crisis, while steps have been taken to address the issue of securitization leading to a lack of skin-in-the-game, with the Dodd-Frank act requiring a minimum level of risk retention by lenders, concentration in the mortgage market has not been discussed much by regulators and has increased since the crisis. In 2016, *The Economist* reported that the GSEs and Federal Housing Association were funding about 65-80% of new mortgages. Further, the new regulations faced by banks have made them move out of mortgage lending. As a result, mortgage origination has become highly concentrated with new, independent firms Quicken Loans and Freedom Mortgage originating roughly half of all new mortgages. At least some of these mortgages appear to be highly risky and of questionable quality, with the report stating that 20% of all loans since 2012 have LTV ratios of over 95%. Moreover, house prices have been rising rapidly and have surpassed their peak during the boom. These patterns could be cause for concern and this paper illustrates a channel that may be driving this.

This paper puts forward a theory that can explain the deterioration in lending standards and its link to the growth of the secondary market because there was concentration in the holdings of securitized loans. Papers by Ben-David (2011), Carrillo (2013), Garmaise (2015) and Piskorski, Seru, and Witkin (2015) have shown that mortgage originators were lowering underwriting standards, becoming more lax in loan screening and not monitoring loans carefully in the years leading up to the 2008 crisis. Keys, Mukherjee, Seru, and Vig (2011) connect this phenomenon to the development of the secondary market for mortgages.

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9 Keys et al. (2011) find that loan performance was significantly worse for borrowers with a FICO score of just above 620 which conformed to a rule-of-thumb that made loans with a FICO score of 620 and above easier to securitize, than those just below. Also see Elul (2011) and Griffin and Maturana (2016).
While securitization did create a new security with potential information frictions and moral hazard concerns, it also resulted in a large concentration of mortgage market exposure with secondary market participants. In particular, the rise of securitization occurred after Salomon Brothers created a mortgage trading operation and found investors for MBS. Investor interest in MBS allowed the GSEs and some banks to grow their share of the mortgage market by becoming the key players in MBS issuance. This second effect of securitization has been largely overlooked by research into the housing crises.

Many recent papers provide support for the theory that large lenders were driving risky lending. In a paper testing this theory, Elul, Gupta, and Musto (2017) find that in 2007 as small private securitizers were withdrawing from the risky lending, the GSEs increased high-LTV mortgage purchases in MSAs in which they had high outstanding mortgage exposure. Additionally, Favara and Giannetti (2017) find that mortgage lenders in more concentrated markets internalize house price drops coming from foreclosure externalities and are less likely to foreclose on delinquent households. Dell’Ariccia, Igan, and Laeven (2012) find that the decline in lending standards was driven by large lenders and that loan denial rates were lower in areas that had a smaller number of competing lenders. Adelino et al. (2016a) find that when private securitizers designed MBS pools for the agencies, loans in GSE pools were riskier based on observable risk characteristics than loans in non-GSE pools. Nadauld and Sherlund (2013) find that securitization of sub-prime mortgages increased 200% between 2003 and 2005 and was driven primarily by the five largest broker/dealer banks resulting in a lowering of lending standards in the primary market.\footnote{Also see Jiang, Nelson, and Vytacil (2014).}

While this paper focuses on how the model applies to the housing boom and bust, the mechanism is applicable more generally. As discussed above, the model can help explain why we continue to see mortgage loans being made at high LTV ratios despite macro-prudential regulation aimed at curbing risky lending. Another related application of the model is to housing policy since 2009 aimed at stabilizing housing markets. In the aftermath of the
crisis, the government took on a large amount of mortgage exposure when the GSEs were taken into conservatorship and the Federal Reserve Bank undertook large-scale purchases of mortgage-backed securities as part of quantitative easing. Many government policies such as the Home Affordable Refinance Program (HARP), the The Home Affordable Modification Program (HAMP) and the continued purchase of mortgage-backed securities explicitly stated keeping house prices from falling as one of their goals. In 2009, when announcing some of these programs, President Obama said that “by bringing down foreclosure rates, [these policies] will help to shore up housing prices for everyone.”

I extend the model in various ways. First, I incorporate the possibility of banks propping up prices by refinancing borrowers who are close to default rather than by making loans to new borrowers. The same intuition as in the baseline model flows through - banks with large outstanding mortgage exposure may refinance a mortgage by making a loan that is negative NPV if the benefit from house price appreciation is large enough. Depending on the expected repayment a bank can get from existing versus new borrowers, refinancing or new lending can be preferable. Second, I extend the model to allow for lender heterogeneity with a few large lenders making loans alongside smaller, dispersed lenders. In this case, large lenders increase their share of the mortgage market over time, even if their market power does not change. This is because as the mortgage holdings of large lenders builds up over a boom period, they are incentivized to make riskier and riskier mortgages in an attempt to prop up prices. Such an effect is not present for smaller lenders who act like price-takers in the mortgage market. Therefore, the market share of large lenders increases over a boom as they lend relatively more than small lenders. This can help explain the rise in the exposure of banks and the GSEs to the mortgage market over the 1990s.

I additionally show that the model is robust to concentration in the mortgage market at an originator level or at a secondary market level. At an originator level, Countrywide

Financial was increasing its share of the U.S. mortgage market during the boom and accounted for about 15% of all mortgage origination in 2005. In the secondary market, the GSEs were the largest participants in the U.S. mortgage market but did not originate mortgages themselves. Rather their exposure to the mortgage market was through insurance guarantees on MBS they sold to investors, through portfolio holdings of their own loans, and through the purchase of private-labeled MBS. The key mechanism in the model simply requires concentration in mortgage holdings. The basic model setup abstracts away from the secondary market. However, I provide an equivalent version of the model in which concentration is present in the secondary market rather than the primary originator market. The key mechanism works as long as there is concentration in mortgage holdings at some level and agents with exposure to mortgage payments have some market power. If secondary market players own a large share of the mortgage market, they benefit from high house prices. If they have market power, they can offer attractive prices on the secondary market for riskier mortgages that will incentivize mortgage originators to then issue mortgages to risky borrowers. Holders of these mortgages will suffer losses on these purchases but the increase in house prices will be profitable for their outstanding mortgage exposure.

Finally, although the model is very stylized and abstracts away from many aspects of housing markets, I perform a basic calibration of the model to the 1991-2009 US housing market. The stylized model is able to match some key moments of the housing market and demonstrates that changing concentration can produce significant differences in the likelihood of a credit boom and bust, and the quantity and quality of credit expanded during the credit cycle. Specifically, when concentration is set to approximately match the GSE market share, the model is able to explain about half of the boom and bust in house prices and over 90% of lending to sub-prime borrowers during the housing boom and bust.\textsuperscript{12} In a counterfactual

\textsuperscript{12}In this paper, I focus on the private mandate of the GSEs to maximize profits for shareholders to explain high-risk lending. Although the GSEs had private shareholders, they also had a public mandate to achieve goals to support housing amongst low- and moderate-income households and in underserved areas. This private/public nature of the agencies may mean that their motivations were not purely profit-maximizing. Acharya et al. (2011) argue that it is hard to explain GSE high-risk activity because of their public mandate alone. They report that GSE adherence to their housing targets seemed to be voluntary - the GSEs missed their housing targets on several occasions without any severe sanctions by regulators. Furthermore, the
analysis of the calibrated model, I show that decreasing concentration by doubling the number of competing lenders in the mortgage market would have reduced the fraction of sub-prime lending in the housing boom and bust to 0. It would have also resulted in 30% lower growth in house prices during the boom and 80% smaller decline in house prices during the bust. This exercise suggests that the model could be quantitatively significant, although precisely estimating the magnitude of propping-up incentives during the housing boom and bust is beyond the scope of this paper.

The rest of this paper is arranged as follows. Section 1 provides a review of the literature related to this paper. Section 2 describes the main model setup. Section 3 illustrates the key mechanism of how concentration can affect credit in a simple three-period model. Section 4 discusses the main infinite horizon model and explains how the model generates housing booms and busts. Section 5 extends the model to incorporate concentration in secondary rather than primary markets, refinancing of mortgage loans and lender heterogeneity. Section 6 provides details of the calibration exercise. The last section concludes. All proofs are in the appendix to chapter 1.

1.2. Related Literature

Although the effect of concentration in markets on resulting prices and quantities is widely studied in economics, research on the effect of concentration in mortgage markets on credit and house prices jointly is relatively sparse. Scharfstein and Sunderam (2014), Fuster, Lo, and Willen (2016) and Agarwal, Amromin, Chomsisengphet, Landvoigt, Piskorski, Seru, and Yao (2017) study how competition in the mortgage market affects mortgage interest rates, but take house prices as exogenous. Poterba (1984) and Himmelberg et al. (2005) study how mortgage interest rates affect house prices, but assume perfectly competitive mortgage markets. This paper combines these ideas and studies credit and house prices when lenders internalize the impact their credit provision has on house prices.

largest housing target increases for the GSEs took place in 1996 and 2001, yet the increase in GSE high-risk activity did not take place till later. See Elenev, Landvoigt, and Van Nieuwerburgh (2016) for a theory of the quasi-government nature of the GSEs.
This paper is related to the literature on how size can affect incentives to take on risk. The main theory in this area of research is too-big-to-fail: large institutions take on excessive risks because they expect to be bailed out by the government (Stern and Feldman (2004)). In my paper, the key variable that causes institutions to take on mortgage risk is the size of their mortgage exposure rather than the size of the institution. This yields cross-sectional predictions, holding a lender fixed, and is consistent with empirical evidence. In a similar vein, Bond and Leitner (2015) develop a theory in which buyers with large inventories of assets, can make further asset purchases at loss-making prices because other market participants use prices to infer information about the underlying asset value. In their model, the buyer incurs a cost when the market value of his inventories falls too low and would therefore like to keep market prices high. In my setting, there is no asymmetric information and lenders with large outstanding mortgage make loans that are low-quality based on observable risk. This can therefore help explain the rise of sub-prime lending, which had observably higher LTV and DTI ratios and higher default rates than prime mortgages. In related work, there are other papers that have linked size to risk-taking. Boyd and Nicolò (2005) develop a theory in which banks in concentrated markets make riskier loans as higher interest rates charged by monopolistic banks make default by borrowers more likely due to increased moral hazard when borrowers face higher interest rates.\textsuperscript{13} Milbradt (2012) models how mark-to-market accounting can lead financial institutions to suspend trading. Kumar and Seppi (1992) show that uniformed investors have incentives to manipulate the spot price used to compute the cash settlement at delivery when they hold futures positions. My model focuses instead on how outstanding exposure can increase incentives to extend credit rather than cause a suspension of trade.

The paper also related to the literature on zombie-lending which documents that large Japanese banks continued to provide credit to insolvent borrowers.\textsuperscript{14} According to this literature, banks may continue to extend credit to under performing loans as it is costly for

\textsuperscript{13}For empirical evidence of concentration increasing bank risk taking, see Nicolò (2001) and Nicolò, Bartholomew, Zaman, and Zephirin.

\textsuperscript{14}See Hoshi (2006) and Caballero, Hoshi, and Kashyap (2008).
them to fall below their required capital levels, or because they wanted to avoid public criticism. In this literature, a bank may make negative NPV loans because of other externalities associated with continuing to extend credit. In my model, banks similarly have a positive externality when they make new mortgage loans through the effect of credit on house prices. The mechanism I propose arises naturally in the mortgage market because of the durability of housing. Jorda, Schularick, and Taylor (2014) and Mian, Sufi, and Verner (2017) show that a buildup in mortgage debt and real estate lending booms predict future financial crises across time and countries. This paper points to a specific feature of mortgages that creates incentive to engage in risky lending and can help explain why real estate assets are central to periods of booms and busts.

This paper also contributes to the recent debate on whether the housing boom and collapse was driven by a credit supply shock or by high house price expectations. The majority of this debate has been empirical with Mian and Sufi (2009), Favara and Imbs (2015), Griffin and Maturana (2015), Landvoigt et al. (2015) providing evidence supporting a credit supply shock and with Glaeser, Gottlieb, and Gyourko (2013) and Adelino et al. (2016b) arguing that an expectations based explanation fits the data better. The theoretical literature reconciling observations from the crisis with either view is relatively sparse, and typically requires either irrationality or misinformation to justify the housing boom. The expectations-based view often requires that buyers and lenders in housing markets hold over-optimistic views about future housing prices.15 In the case of a credit supply shock, since borrowers, securitizers and the MBS buyers faced large losses in the crisis, it is hard to explain why the credit supply shock happened without an overoptimism or misinformation about the benefits of new ways to supply credit. This paper adds to this literature by providing a theoretical framework that can reconcile many of the empirical findings driving the current debate.

15Arguments in favor of this have been made by Cheng, Raina, and Xiong (2014), Shiller (2014) and Glaeser and Nathanson (2015).
1.3. The Model

The model is an infinite horizon, discrete time model with overlapping generations. A number, $N$, of infinitely lived banks each with access to an equal share of borrowers make mortgage loans to households. Each period $t$ a new generation is born that lives for two periods and consists of a continuum $[0, 1]$ of households. Households from generation $t$ derive utility from consuming housing, $k_t \in \{0, 1\}$, when they are young, and a consumption good when they are old. Their life-time utility is given by:

$$u(k_t, c_{t+1}) = \gamma k_t + \beta c_{t+1}.$$ 

The extent to which households value housing consumption is captured by the preference parameter, $\gamma$, and $\beta < 1$ is a discount factor.\textsuperscript{16} Households have access to a storage technology which yields a return of 1.

There are two types of households: a proportion $\alpha^{nb}$ of households (“non-borrowers”) receive their endowment when they are young and the remaining households (“borrowers”) receive their endowment when they are old. “Non-borrowers” from generation $t$ are born with an endowment $\omega_t^{nb}$ at $t$. They receive a positive endowment, $\omega_t^{nb} = e^{nb}$, with probability $\phi_s^{nb}$ and 0 otherwise where $s$ is a generation-specific income shock. “Borrowers” from generation $t$ receive an endowment $\omega_t^{b}$ at $t + 1$. These households therefore need a mortgage to be able to buy a house at $t$. There are two types of borrowers: proportion $\alpha^{bh}$ of households are high-quality borrowers and the remaining are low-quality borrowers, with the former having a greater expected endowment. High and low-quality borrowers receive a positive endowment $\omega_t^{b} = e^{b}$ with probability $\phi_s^{bh}$ and $\phi_s^{bl}(< \phi_s^{bh})$ respectively and 0 otherwise.

Each generation $t$ has a generation-specific shock, $s_t \in \{R, P\}$, and can be born rich or poor with $q$ being the probability of a rich generation being born. In a rich generation, all

\textsuperscript{16}Green and White (1997), Sekkat and Szafarz (2011) and Sodini, Nieuwerburgh, Vestman, and Lilienfeldteal (2016) provide estimates of the benefits of home-ownership.
agents have a higher expected endowment than in a poor generation: \( \phi_{nb} P < \phi_{nb} R \), \( \phi_{bh} P < \phi_{bh} R \), and \( \phi_{bl} P < \phi_{bl} R \). At each time \( t \), once a generation is born, the expected endowments of its borrowers and non-borrowers are common knowledge. There is therefore no adverse selection due to information frictions in the credit market.

1.3.1. Housing Market

The housing stock, \( h_t \), depreciates at rate \( \delta \) per period where \( 0 < \delta < 1 \). Each period, competitive price-taking construction firms can produce new housing, \( n_t \), to add to the existing stock of housing. Firms have a cost of producing houses, \( c_h \), which depends on both the existing stock of housing and new houses produced. The cost to firm \( i \) of producing \( n_i^t \) new houses is \( c_h n_i^t \). This particular cost function delivers tractable solutions and captures the idea that land availability is an important factor in the cost of housing construction. Piazzesi and Schneider (2016) show that movements in the value of the residential housing stock are primarily due to movements in the value of land. Knoll, Schularick, and Steger (2017) provide evidence that rising land prices explain about 80 percent of global house price appreciation since World War II.\(^{17}\) The total supply of housing at time \( t \) is therefore given by:

\[
h_t = (1 - \delta) h_{t-1} + n_t.
\]

The demand for housing is given by the number of mortgage loans borrowers get from banks, \( h_t^b \), and the number of houses purchased by non-borrowers, \( h_t^{nb} \). I will make parameter restrictions (outlined at the end of this section) to ensure that there is some new construction every period. The price of housing, \( P_t \), is then set to clear the housing market and is given\(^{17}\) The main results of the model also hold for a more general supply function in which construction costs are affected differentially by new construction and by the existing stock of housing. More generally, the key results of the model require that house prices increase when credit supply expands.
by a linear function:

\[ P_t = ch_t. \] \(18\)

1.3.2. Mortgage Loans

At time \( t \), a household \( i \) borrows \( k^i_t P_t \) at an interest rate, \( r^i_t(s_{t+1}) \), that can be contingent on the future states of the world. At time \( t+1 \), if a household pays back its loan, it keeps its house which it can sell to use the proceeds for consumption. If the household defaults on its loan, the bank forecloses on the house and is entitled to the household’s endowment. In the model, mortgage loans are therefore similar to adjustable rate mortgages with recourse. \(19\)

1.3.3. The Household’s Problem

Each period \( t \), borrowers and non-borrowers from generation \( t \) decide whether to purchase a house. Households also have access to a storage technology which gives a rate of return of 1 at time \( t+1 \). When deciding whether to purchase a house, non-borrowers account for both the utility they get from housing consumption and the future price at which they expect to sell their home (the proceeds of which are spent on the consumption good). At time \( t \), a non-borrower with endowment \( \omega^{nb}_t \geq P_t \) will buy 1 unit of housing if:

\[ \gamma + \beta(1 - \delta)E[P_{t+1}] \geq \beta P_t. \]

Borrower households from generation \( t \) receive their endowment in the future and must borrow from banks at time \( t \) to buy housing. At time \( t+1 \), a borrower who has successfully

\(18\) Each firm solves the following problem,

\[ \max_{n^i_t} P_t n^i_t - ch_t n^i_t \]

In equilibrium, firms will produce housing until \( P_t = ch_t \).

\(19\) In a model with recourse, at time \( t \), a household with a mortgage loan from generation \( t-1 \), repays its mortgage if its net worth is larger than the repayment amount

\[ \omega^b_{t-1} + (1 - \delta)P_t \geq P_{t-1} r^i_{t-1}(s_t). \]

If the household defaults, the bank gets the maximum amount the household can repay, i.e., \( \omega^b_{t-1} + (1 - \delta)P_t \).
obtained a mortgage will either successfully repay their mortgage and can then sell their house, or default and lose their endowment and house. If a borrower’s bank charges him a state-contingent interest rate of \( r_t(s_{t+1}) \), then he will buy 1 unit of housing if:

\[
\gamma + \beta (1 - \delta) E[P_{t+1}] \geq \beta E[\min \{ P_t(1 + r_t(s_{t+1})), \omega^b_t + (1 - \delta) P_{t+1} \}].
\]

The LHS is the utility the household gains from living in the house in period \( t \) and the proceeds the household gets from selling the house at \( t + 1 \). The RHS represents the net cost of purchasing the house to the household. If the household does not have enough funds to repay its mortgage, \( \omega^b_t + (1 - \delta) P_{t+1} < P_t(1 + r_t(s_{t+1})) \), then it defaults and loses its endowment and house.

1.3.4. The Bank’s Problem

There are \( N \) infinitely lived banks that can make mortgage loans to households. Each period \( t \), banks observe the income shock of the current generation and decide how many loans to issue and at what interest rate. Each bank has access to an equal share, \( \frac{1}{N} \), of the mortgage market. The mortgage market is thus segmented implying that households borrow from their local bank and do not shop around for mortgage rates. Therefore, each bank has access to a group of borrowers without having to compete with other banks on interest rates.\(^{20}\) Although banks do not compete directly on interest rates, they interact strategically with each other due to the collective effect of their actions on house prices. This gives rise to strategic substitution effects that are similar to those in models of Cournot competition.

I solve the model in both the case when a bank cannot commit to future lending and in the case when the bank can commit to future lending. Let \( V(s_t, m^h_{t-1}, r^h_{t-1}, m^l_{t-1}, r^l_{t-1}, P_{t-1}, s_{t-1}) \) be the value function of a bank at time \( t \) where \( s_t = \{h, l\} \) represents the income shock of

\(^{20}\)Lacko and Pappalardo (2007) and Amel, Kennickell, and Moore (2008) provide empirical evidence that supports this assumption. They find that consumers tend to bank locally and do not shop around for mortgage rates.
the generation born at time $t$, $m^j_{t-1}$ represents the number of mortgage loans that the bank has made at time $t - 1$ to borrowers of type $j = \{h, l\}$ at interest rate $r^j_{t-1}$, and $P_{t-1}$ is the price of housing at time $t - 1$ (and a function of $m^h_{t-1}$ and $m^l_{t-1}$). Then at time $t$, a bank solves the following problem:

$$
V(s_{t-1}, s_t, m^h_{t-1}, m^l_{t-1}, r^h_{t-1}, r^l_{t-1}, P_{t-1}) = \max_{m^b_t \geq 0, m^l_t \geq 0, r^b_t, r^l_t} \sum_{j=\{h,l\}} m^j_{t-1} \left( \phi^b_{s_t} \min\{P_{t-1}(1 + r^j_{t-1}), e^b + (1 - \delta) P_t\} + (1 - \phi^b_{s_t}) \min\{P_{t-1}(1 + r^j_{t-1}), (1 - \delta) P_t\} \right)

- \sum_{j=\{h,l\}} m^j_t P_t \right) + \beta E \left[ V(s_t, s_{t+1}, m^h_t, m^l_t, r^h_t, r^l_t, P_t) \right]

s.t. \quad \gamma + \beta(1 - \delta) E[P_{t+1}] \geq \beta E[\min\{P_t(1 + r_t(s_{t+1})), \omega^b_t + (1 - \delta) P_{t+1}\}]

m^h_t \leq \frac{1}{N} \alpha^{bh}

m^l_t \leq \frac{1}{N} (1 - \alpha^{bh} - \alpha^{nb}).

The first term in the bank’s payoff is the amount the bank earns on loans made to borrowers from generation $t - 1$ which are due for repayment at time $t$. House prices at time $t$ affect the bank’s payoff from outstanding loans in two ways: they affect borrower net-worth which determines whether the borrower will repay or not; they also affect the bank’s payoff in case of default. The second term is the cost of new lending and the final term is the bank’s expected continuation value. The bank faces a borrower purchasing constraint - that given the repayment schedule chosen by the bank, the borrower wants to get a mortgage. The second and third constraints are the market share constraints of the bank.\(^{21}\)

\(^{21}\)Note that banks are taking into account the current and future lending decisions of all other banks when making their own decision about how many loans to make. In a slight abuse of notation, the problem as it is currently written does not make this explicit. Lending by other banks is embedded in the bank’s decision when it accounts for current and future house prices.
1.3.5. Parametric Restrictions

Given the $[0,1]$ continuum of households born every period, the maximum housing price is $c$. To help understand the following parameter restrictions, it is useful to note that given these restrictions, the price of housing in the economy will never fall below $c\phi_{P}^{nb}\alpha^{nb}$. To close out the model, I make the following parametric restrictions:

1. The private benefit of housing is large enough, i.e., $\gamma \geq \beta(c - c(1 - \delta)\phi_{P}^{nb}\alpha^{nb})$, to guarantee that non-borrowers always demand housing and there is a positive interest rate at which borrowers demand housing.

2. Non-borrower endowment is large enough, i.e., $e^{nb} \geq c$, to guarantee that a non-borrower who receives a positive endowment can always afford to buy a house. Since non-borrowers in the model are proxying for outside housing demand, this assumption guarantees that credit is never the sole driver of house prices.\footnote{This also helps simplify the model solution as house prices will always increase with more credit. Banks do not crowd non-borrowers out of the market by making house prices too expensive.}

3. In the theoretical results, depreciation is not too low, i.e. $\phi_{P}^{nb}\alpha^{nb} > 1 - \delta$, to guarantee that there is at least some new construction every period and that the bank’s problem is thus continuous in house prices. In the calibrated version of the model, I do not restrict the parameters to satisfy this assumption.

4. Low-quality borrower endowment is small enough, i.e.,

$$\beta\phi_{R}^{bl}e^{b} + \beta(1 - \delta)c < c\phi_{P}^{nb}\alpha^{nb},$$

to guarantee that it is never profitable for banks to lend to low-quality borrowers.
to low-quality borrowers is NPV negative. Therefore, there is no reason a bank would ever make loans to low-quality borrowers unless the return from propping up prices is high enough.

**Model Robustness:** There are two key requirements for the results. First, house prices affect a household’s ability or incentive to repay a mortgage such that higher housing prices reduce the probability of default and/or the loss due to default. Second, credit provision has an effect on house prices. The model is robust to modeling mortgage loans without recourse and as fixed rate mortgages. The model is also robust to other market structures as long as banks are able to make profits in one period and offset them with losses from another. The model can also allow entry and exit so that banks lifetime profits are zero as long as they can make profits or losses period-by-period.

1.4. Three-Period Model

To demonstrate the key mechanisms of the model I start by discussing the equilibrium in a simplified three-period setting. This highlights how concentration affects both the quantity and quality of credit. It also explains how, in concentrated markets, mortgage growth can be negatively correlated with income growth across areas and positively correlated with income growth across borrowers. Uncertainty in future lending opportunities and intra-period borrower heterogeneity are not necessary to obtain the key results of the model, and therefore I abstract away from both in this simplified model. The full model keeps the intuition of the three-period model and is additionally able to produce boom and bust cycles with features that characterized the recent housing crisis.

In the first period the economy is in a rich-state with only non-borrowers and high-quality borrowers, and in the second period a poor-state hits with certainty in which there are only non-borrowers and low-quality borrowers. In the final period, no new generation is born and therefore I assume the price of housing falls to an endogenously specified liquidation value, $c\phi_P^{nb} \alpha^{nb} \geq \kappa \geq 0$. Since no high-quality borrowers are born in the second period, any
\( t = 2 \) lending will only be to low-quality borrowers. Since by assumption low-quality loans are negative NPV, banks only lend a positive amount at \( t = 2 \) if they find it profitable to prop up house prices. This setup thus clearly demonstrates when a bank is incentivized to sacrifice loan quality for the return to keeping house prices high.\(^{23}\)

I characterize the results of the model both when banks cannot commit to a level of \( t = 2 \) lending when making loans at \( t = 1 \) and when banks can commit to future lending. As I will discuss, in both cases the results are qualitatively similar but the economic intuition for why banks want to prop up prices is different. In practice, there are reasons to think that the GSEs were able to commit, at least in part, to future lending. Hurst, Keys, Seru, and Vavra (2016) provide evidence that the GSEs faced political pressure that did not allow them to make substantial changes to interest rates. These constraints could credibly allow the GSEs to commit to future activity.

The three-period model can be solved by backward induction. Since no new generation is born in the third period, banks do not lend at \( t = 3 \). In the second period, lending by any given bank \( m_2 \) is stated in the following lemma, where \( M_2^{-i} \) is lending by all other banks at \( t = 2 \).

**Lemma 1.A** In the three-period model, without commitment to future lending, a bank’s period-2 lending, \( m_2 \), is given by the following two cases.

**Case 1:** If \( \phi_R e^b \leq \gamma \beta \),

\[
m_2 = \max \left\{ 0, m_1 \frac{(1 - \delta)}{2} - \frac{\phi_P^b \alpha + M_2^{-i}}{2} + \beta \frac{\phi_P^b e^b + (1 - \delta) \kappa}{2c} \right\}.
\]

**Case 2:** If \( \phi_R e^b > \gamma \beta \),

\[
\text{In this three-period model, since } t = 3 \text{ values are } \kappa \text{ and low-quality borrowers are only born at } t = 2, \text{ the fourth parametric restriction can be simplified to } \beta \phi_P e^b + \beta(1 - \delta) \kappa < c \phi_P a^b.
\]

19
\[ m_2 = \max \left\{ 0, \frac{(1 - \phi_{bh}) m_1 (1 - \delta)}{2} - \frac{\phi^{nb}_P \alpha + M_2^{-i}}{2} + \frac{\phi^{bl}_P e^b + (1 - \delta) \kappa}{2c} \right\}. \]

In the three-period model, with commitment to future lending, a bank’s period-2 lending, \( m_2 \), is given by:

\[ m_2 = \max \left\{ 0, \frac{m_1 (1 - \delta)}{2} - \frac{\phi^{nb}_P \alpha + M_2^{-i}}{2} + \frac{\phi^{bl}_P e^b + (1 - \delta) \kappa}{2c} \right\}. \]

The loans a bank makes to low-quality borrowers, \( m_2 \), is always increasing in outstanding loans, \( m_1 \). When \( m_1 = 0 \) and the bank has no outstanding loans on its balance sheet, it will never make any loans at \( t = 2 \) to low-quality borrowers and \( m_2 = 0 \).\(^{24}\) As the amount of outstanding loans increases, \( m_2 \) can become positive. If at \( t = 1 \), a bank is unable to commit to a level of future lending, \( m_2 \), then it props up house prices to improve its return on loans that are delinquent - when the borrower is unable to return the full face-value of the loan. By increasing house prices through credit expansion, a bank is able to earn a higher return on defaulting loans since it has a claim on the house. If a bank is able to commit to future lending, it props up prices to improve its return on delinquent loans and additionally to increase the face-value it can charge on non-delinquent loans. With commitment, a bank therefore has greater incentives to prop up prices.\(^{25}\)

Loans to low-quality borrowers, \( m_2 \), is also increasing in the future expected income of low-quality borrowers, \( \phi_{P}^{bl} e^b \). It is decreasing in the housing demand coming from non-borrowers and other banks, \( \phi^{nb}_P \alpha^{nb} + M_2^{-i} \). A lower \( \phi^{nb}_P \alpha^{nb} + M_2^{-i} \) implies that an individual bank effectively has larger market power in influencing house prices since outside sources of demand are lower. In other words, a lower \( \phi^{nb}_P \alpha^{nb} + M_2^{-i} \) implies a larger elasticity of house

\(^{24}\)Since low-quality loans are assumed to be negative NPV, \( -\phi^{nb}_P \alpha^{nb} - M_2^{-i} + \phi^{bl}_P e^b + (1 - \delta) \kappa < 0 \).

\(^{25}\)When \( \phi_{R}^{bl} e^b \leq \frac{1}{c} \), bank lending at \( t = 2 \) is identical with and without commitment. In this case, \( t = 1 \) borrowers are willing to repay the bank \( \phi_{R}^{bl} e^b + (1 - \delta) P_2 \) and by setting a face-value of the loan slightly above this, banks can credibly raise house prices to improve their return on all outstanding loans at \( t = 2 \) by propping up prices. For more detail on this, see the appendix.
prices to credit. This increases the net benefit that credit expansion by the bank has on house prices.

At $t = 1$, a bank takes into account its lending at time $t = 2$ when determining how many loans to make. In period 1, a bank’s lending is stated in the following lemma, where $M_{-1}^1$ is lending by all other banks at $t = 1$.

**Lemma 1.B** In the three-period model, without commitment to future lending, a bank’s period-1 lending, $m_1$, is given by the following two cases:

**Case 1:** If $\phi_R e^b \leq \frac{\gamma}{\beta}$,

$$m_1 = \max \left\{ 0, \min \left\{ \frac{\beta}{\epsilon} \left( \phi_R e^b + (1 - \delta)P_2 \right) - \phi_R^n \alpha^{nb} - M_{-1}^1 \right\}, \frac{(1 - \alpha^{nb})}{N} \right\}.$$ 

**Case 2:** If $\phi_R e^b > \frac{\gamma}{\beta}$,

$$m_1 = \max \left\{ 0, \min \left\{ \frac{\beta}{\epsilon} \left( \frac{\gamma}{\beta} + (1 - \delta)P_2 \right) - \phi_R^n \alpha^{nb} - M_{-1}^1 \right\}, \frac{(1 - \alpha^{nb})}{N} \right\}.$$ 

In the three-period model, with commitment to future lending, a bank’s period-1 lending, $m_1$, is given by:

$$m_1 = \max \left\{ 0, \min \left\{ \frac{\beta}{\epsilon} \left( \min \left\{ \phi_R e^b, \frac{\gamma}{\beta} \right\} + (1 - \delta)P_2 \right) - \phi_R^n \alpha^{nb} + M_{-1}^1 \right\}, \frac{(1 - \alpha^{nb})}{N} \right\}.$$ 

In an equilibrium in which a bank props up house prices, if a bank lends more at $t = 1$, the elasticity of house prices is simply defined here as the percentage change in house prices for the marginal mortgage loan.
it also increases its $t = 2$ lending. This pushes up housing prices at $t = 2$ ($P_2$) in turn increasing the amount of loans a bank makes at $t = 1$. There is thus a feedback loop between $t = 1$ and $t = 2$ lending. Bank lending is also affected by the aggregate lending of other banks. The number of loans a bank makes at $t = 1$ is decreasing in the number of loans made by other banks, $M_{-i}^1$, but increasing in the number of loans made by banks in the future, $M_{-i}^2$. The more loans other banks make at $t = 1$, the higher is the price of housing at $t = 1$, making it more expensive for a bank to make mortgage loans. This causes a bank to decrease the amount it lends. The more loans other banks make at $t = 2$, the higher is the price of housing at $t = 2$, allowing banks to charge a larger interest rate on loans made at $t = 1$ and increasing their incentive to lend at $t = 1$. There is thus strategic substitution in bank lending within period but strategic complimenteries in bank lending across periods. The full characterization of the equilibrium is discussed in the following subsection.

**Numerical Example:** To help understand the mechanism, I run through a numerical example with $N = 1$. I choose the following parameters: $\alpha_{nb} = .7$, $\delta = .4$, $e^b = $200,000, $\phi^{bh} R = 1$, $\phi^{bl} P = .7$, $\kappa = $100,000, $c = $300,000. Following the second parametric restriction, $e^{nb} \geq c$. For simplicity, I assume no discounting, i.e. $\beta = 1$, and also have no non-borrower income shocks, i.e. $\phi^{nb} R = \phi^{nb} P = 1$. I also assume $\gamma \geq \phi^{bh} e^b$, so that bank lending with and without commitment are equivalent.\(^{27}\)

Imagine a bank does not take into account the effect of house prices on the profitability of its outstanding share of loans. Then in the second period, a bank will not prop-up prices. It therefore makes no loans at $t = 2$ since all loans to low-quality borrowers are negative NPV. Only non-borrowers will buy housing at $t = 2$. Therefore, housing demand in the second period is $h_2^d = .7$, and resulting house prices are $ch_2^d = $210,000. We can check that loans to low-quality borrowers are negative NPV. House prices at $t = 3$ are given by the liquidation value $\kappa = $100,000 and low-quality borrowers’ expected endowment is $\phi^{bl} e^b = $100,000.

\(^{27}\)In this example, to guarantee that loans to low-quality borrowers are negative NPV, it is sufficient that $\beta \phi^{bl} e^b + \beta (1 - \delta) \kappa < \phi^{nb} P \alpha_{nb}$.
If a bank was to lend to low-quality borrowers, it would have to pay $210,000 at \( t = 2 \) and receive an expected repayment of \((1 - \delta)k + \phi_P e^b = \$200,000\) at \( t = 3 \).

If a bank is not propping up house prices, it will make no loans in period 2. In the first period, the bank will make \( m_1 = .19 \) loans. Resulting house prices at \( t = 1 \), will be \( ch_1^t = \$268,000 \). The cost of making \( t = 1 \) loans to a bank is \$268,000 and the expected repayment from these loans is \((1 - \delta)P_2 + \phi^{bh} e^b = \$326,000\). The total profits earned by the bank are \( .19 \times (326,000 - 268,000) = \$11,213 \).

Now, let’s consider what happens if the bank takes into account its outstanding share of loans in the second period and wants to deviate to making \( t = 2 \) loans. Then if the bank’s outstanding share is \( m_1 = .19 \), a bank will find it optimal to make \( m_2 = .04 \) loans. This will increase \( t = 2 \) price to \$222,400. The bank earns an increased return of \$1,438 on its outstanding loans while making a loss of \$926 on new lending at \( t = 2 \). Banks are able to make this gain in profits at the expense of young non-borrowers. They are harmed by this increase in price and suffer an aggregate loss of \( \alpha^{nb} \times (\$222,400 - \$210,000) = \$8,680 \). This loss of young non-borrowers is transferred to banks, old non-borrowers, and construction firms.

The increase in house prices at \( t = 2 \), allows banks to make a greater return per loan they make at \( t = 1 \). Banks are now able to get an expected repayment of \$333,440 instead of \$326,000. This makes banks want to lend more at \( t = 1 \). This will in turn make the bank want to lend more at \( t = 2 \) and so on and so forth. Eventually, the bank will increase \( t = 1 \) lending to .21 and \( t = 2 \) lending to .05. House prices at \( t = 2 \) will be \$223,516 and at \( t = 1 \) will be \$271,720. The total profits earned by the banks make from \( t = 1 \) loans is \$12,836 (an increase from \$11,213). The bank earns losses on \( t = 2 \) lending totaling \$1,059 which offsets some of these profits. Young non-borrowers at \( t = 2 \) account for the rest of the transfer to banks.

\(^{28}\)Loan amounts can be calculated using Lemma 1.
1.4.1. Concentration and Credit

When concentration in mortgage holdings is low and each bank holds a small share of the market, the return to propping up prices for any individual bank is low. Banks therefore do not issue any loans to low-quality borrowers. As concentration increases, banks have access to a larger share of high-quality borrowers at \( t = 1 \). In this case, they will issue loans to risky borrowers to increase house prices and consequently the rents that they get from high-quality borrowers. Formally, we can establish the following proposition,

**Proposition 1** The three-period model has a unique equilibrium. There exists a cutoff, \( N \), such that if \( N \geq \bar{N} \), banks do not prop up houses prices and make no negative NPV loans. If \( N < \bar{N} \), banks engage in risky lending to prop up house prices and supply a positive amount of negative NPV loans.

When house prices at \( t = 2 \) are high, high-quality borrowers (who get a mortgage at \( t = 1 \)) make larger mortgage repayments to banks. This allows banks to earn greater rents from them. As the market share of banks increases, they lend to more high-quality borrowers at \( t = 1 \). This increases the effect of \( t = 2 \) house prices on their profitability. As concentration increases, banks begin to make low-quality loans at \( t = 2 \) since credit expansion keeps house prices high. As concentration decreases, banks begin to act more like price-takers in the mortgage market and no longer make loans to prop up house prices.

Despite strategic complimenterities in bank lending across time, the equilibrium is unique.\(^{29}\) The uniqueness arises due to intra-temporal strategic substitution in bank lending. If other banks pull back on lending at \( t = 2 \), an individual bank is incentivized to increase its own lending at \( t = 2 \) and not cut back on its \( t = 1 \) lending enough to give arise to multiplicity. There is therefore a unique equilibrium of the model.

\(^{29}\) Typically the presence of strategic complimenterities gives rise to multiple equilibria. The typical reason for multiplicity is as follows: when banks expect aggregate lending to be high at \( t = 2 \), they lend more at \( t = 1 \), and the high \( t = 1 \) lending would lead to the high \( t = 2 \) lending that banks anticipated. Conversely, when banks expect lending at \( t = 2 \) to be low, they lend less at \( t = 1 \) which in turn leads to low \( t = 2 \) lending as banks anticipated.
As concentration increases aggregate credit can increase or decrease. There are two competing effects. The first is a contemporaneous price effect. Large lenders internalize their effect on house prices more than small lenders. The marginal increase in price when making an additional loan affects large lenders’ cost of total lending more than that of small lenders. Lenders in a concentrated market will therefore cut back on credit more than lenders in a market with many small lenders. This effect is similar to a typical mechanism in Cournot competition in which as concentration increases, the quantity of goods supplied on the market decreases as suppliers internalize price effects more. As the number of banks decreases, this “Cournot” effect leads to a decrease in credit supply. However, since concentration also creates incentives to prop up prices, there is a second effect of change in concentration on credit, the “propping-up” effect. Concentration increases banks’ incentives to increase \( t = 2 \) prices through credit expansion and if this effect is large enough, it can cause overall lending to increase.

The following corollary summarizes the effect of concentration on mortgage lending:

**Corollary 1** *In the unique equilibrium of the three-period model, as \( N \) decreases,*

1. *credit extended by any given bank to both high- and low-quality borrowers increases,*

2. *if \( N \geq \bar{N} \) and banks are not propping up prices, aggregate credit decreases,*

3. *if \( N < \bar{N} \) and banks are propping up prices, aggregate credit can increase.*

When \( N \geq \bar{N} \) and banks are not propping up housing prices, aggregate credit is always decreasing with concentration because of the Cournot effect. As is typical in most models of competition, as the number of banks decreases, banks behave more like price-takers and are willing to issue more loans. As discussed above, when \( N < \bar{N} \), there is a second effect of concentration on credit, the propping-up effect. As banks acquire larger market shares, they issue more loans per bank at \( t = 1 \). This increases the incentive for banks to prop up prices.
and make negative NPV loans at $t = 2$. Higher $t = 2$ price further increase the incentive to issue $t = 1$ loans and so on and so forth. As concentration increases, this feedback loop can cause aggregate lending to increase.

Figure 1 illustrates the effect of concentration on house prices. As the market becomes more concentrated and the number of banks decreases, banks begin to prop up house prices. In this parametrization, credit increases with concentration in the region in which banks prop up prices, as the propping-up effect dominates the Cournot effect. As concentration decreases and $N > \overline{N}$, banks stop propping up prices and the amount of credit increases as competition in the market causes banks to behave more and more like price-takers.

Looking at Figure 1, we can see that it is possible for two areas with different levels of concentration to have the same amount of aggregate credit. However, the composition of this credit is different. In particular, the credit in the area with larger concentration is riskier - a larger fraction of lending is to high-risk borrowers. Figure 2 overlays the first graph with different credit risk characteristics - debt-to-income ratios and default rates.

As Figure 2 illustrates, although two areas with differing concentration can have the same
aggregate credit, the credit in the area with higher concentration is riskier. When banks are propping up prices, they extend credit to riskier households with high default rates and make negative NPV mortgage loans. If there is an economic cost to high mortgage default rates, this result suggests that a safer way to expand homeownership would be through increased competition rather than through creating agencies that concentrate mortgage risk. This may however come at the cost of lower income (and negative NPV) households not getting credit.

The figures above plot the debt-to-income ratio and default rates on the right y-axis against concentration. As we move along the x-axis, $N$ increases and concentration decreases. The parametrization is as follows: $\delta = .01$, $\alpha_{nb} = .2$, $\phi_{bp} = .2$, $\phi_{R} = 1$, $\phi_{P} = .7$, $\phi_{R} = 1$, $e_b = 2$, $\kappa = .45$, $\gamma = 4$, $c = 9.8$.

The Effect of Various Model Primitives

A number of factors affect banks’ incentives to prop up house prices. When the expected income of low-quality borrowers, $\phi_{P} e_b$, is high it is relatively more profitable to lend to low-quality borrowers and banks have to take a smaller loss on these loans in their effort to prop up prices. Therefore this increases the incentive to prop up house prices. When non-borrower income growth in the poor state, $\phi_{nb}^P$, is low banks have relatively more market power when it comes to affecting house prices, increasing the incentive to prop up prices. Finally, when $\delta$ is low, houses are worth more in future periods increasing how much banks and households value the future asset value of a house. As banks are incentivized to prop up
prices more when these primitives change, the threshold level of concentration necessary for banks to make high-risk loans decreases. The following corollary formalizes how $N$ changes with the various primitives of the model.

**Corollary 2** In the three-period model, $N$ increases as the expected income of low-quality borrowers increases, as the depreciation rate decreases, and as non-borrower growth in the poor state decreases. Formally,

$$\frac{\partial N}{\partial \phi_{bl}P_e} > 0, \quad \frac{\partial N}{\partial \delta} < 0 \quad \text{and} \quad \frac{\partial N}{\partial \phi_{nb}P_e} < 0.$$ 

**Asset Value of Housing**

The key property of housing that gives rise to this mechanism is that housing is a durable asset with future value. This is distinct from other goods that only serve a consumption purpose. When $\delta = 1$ and housing depreciates completely, banks and households do not care about future house prices. There are no incentives to prop-up prices, and only the Cournot effect remains. As $\delta$ decreases, the asset value of the house increases, causing banks and households to value future house prices. This creates an incentive to prop up prices when banks have a large enough exposure to the housing market. At low values of $\delta$, the incentives to prop up prices is stronger since housing is worth more upon repayment leading to a higher value of $N$. Lower values of $\delta$ also increases household net worth upon repayment and allow banks to charge larger repayments from households when making mortgage loans increasing the total amount of credit banks are willing to supply. Figure 3 shows aggregate lending for different values of $\delta$. 

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Figure 3: Propping-up and the Asset Value of Housing

The figure above plots total credit, measured by the number of households who get a mortgage, against the level of concentration in the mortgage market for different $\delta$s. As we move along the x-axis, $N$ increases and concentration decreases. The parametrization is as follows: $\alpha^{nb} = .2$, $\phi_b^b = .2$, $\phi_k^b = 1$, $\phi_R^b = .7$, $\phi_R^b = 1$, $e_b = 2$, $\kappa = .45$, $\gamma = 4$, $c = 9.8$.

Welfare

If there are no costs to default, the highest level of social welfare is achieved when the maximum number of households get credit under the assumption that bank and construction firm profits are remitted back to and are consumed by households. This is because when a household purchases a house they achieve utility of $\gamma$ from living in the house. The other payments are simply transfers between the various agents in the model. The first best in this model is when all households are home-owners. A constrained social planner who can only increase home-ownership through incentivizing banks to lend would therefore choose the level of concentration that maximizes credit provision.

We can add default costs to the model by assuming some deadweight losses per default $d$ which are not internalized by banks or households. These costs are social costs in the sense that they are not internalized by banks or households and destroy the end-of-period utility.
of all households equally. We can imagine these costs are part of the net profits remitted to households at the end of the period. If there are social costs to default, then if an increase in concentration increases credit, this improves welfare if the net gain in utility by households from consuming housing is higher than the deadweight losses due to default.

Recall from Figure 1, that it is possible to achieve the same level of aggregate credit for two different levels of concentration. An important welfare insight of the model is that in such a case, given any non-zero default costs, welfare is always higher in the area with lower concentration. Formally,

**Proposition 2** Let $d$ be social costs of $s$ default. If $\exists N_1 < N \leq N_2$ s.t. the total number of households with access to credit is equal under both $N_1$ and $N_2$, then for any $d > 0$, aggregate welfare is higher under $N_2$ than $N_1$.

This proposition is quite powerful because it is saying that for a large set of parameters, it is optimal to increase competition and decrease concentration as long as there are any costs to society of mortgage defaults, however small or large these costs might be. In this case, the constrained social planner will always choose $N_2$. The recent crisis has highlighted that there can be large costs to mortgage defaults, and in that context this model would generally advocate for a reduction in concentration.

**1.4.2. Mortgage Growth and Income Growth**

There has been an active debate on whether the housing boom and bust was driven by a supply shock or by expectations of high future house prices. Much of this debate has revolved around understanding the relationship between income growth and credit growth during the housing boom. In support of the first hypothesis, Mian and Sufi (2009) find that income growth decoupled from the growth in mortgage credit in the U.S. at the ZIP code level. They stress that such a result is in line with the supply shock hypothesis since lending seemed to have increased disproportionately to borrowers at the lower end of the income distribution. In a paper, looking at more micro borrower-level data Adelino et al.
(2016b) find that income and mortgage credit growth remained positively correlated at the borrower-level. They argue that this is more in line with high house price expectations since credit increased across the income distribution and was not disproportionately extended to high-risk low-quality borrowers.

The model can simultaneously produce different correlations between income growth and the growth in mortgage credit when looking at borrower-level data versus data that is aggregated (e.g., to a ZIP code level). In the model, an exogenous increase in concentration can cause a credit-supply shock that propagates through the expectation of higher future house prices as large lenders have an incentive to prop up house prices. This can generate a negative correlation between the growth in mortgage credit and income growth across areas (e.g. across ZIP codes) while still maintaining a positive correlation between the two at a borrower-level. Specifically, consider a change in concentration when the number of banks decreases from \( N_1 > \overline{N} \) to \( N_2 < \overline{N} \) such that banks move from not propping up prices at \( t = 2 \) to propping up prices. Define \( \Delta M \) as the change in mortgage credit following this increase in concentration. Then, we can establish the following proposition:

**Proposition 3** Following an increase in concentration when the number of banks decreases from \( N_1 \geq \overline{N} \) to \( N_2 < \overline{N} \) and banks begin to prop up house prices, mortgage credit growth is negatively correlated with non-borrower income growth in the poor state and positively correlated with borrower income growth in the poor state i.e.,

\[
\frac{\partial \Delta M}{\partial \phi^{\text{nb}}_P} < 0, \quad \text{and} \quad \frac{\partial \Delta M}{\partial \phi^{\text{bl}}_P} > 0.
\]

As concentration increases the magnitude of the credit supply shock is largest in areas that have the lowest growth in non-borrower income.\(^{30}\) All else equal, when non-borrower income growth is low at \( t = 2 \), the intra-period strategic substitution effect leads banks to extend more credit. In the absence of other sources of demand to drive up housing

\(^{30}\)Note that since \( \overline{N} \) depends on \( \phi^{\text{nb}}_P \) and \( \phi^{\text{bl}}_P \), this proposition only compares areas given an \( N_1 \) and \( N_2 \) s.t. \( \overline{N} \) for each area falls within \((N_1, N_2]\).
prices, the effective market power of banks in the housing market is higher. As a result, for every additional mortgage loan that banks extend to low-quality borrowers, the percentage increase in housing prices is greater when non-borrower income growth is low. The returns to propping up prices are therefore highest in areas with low-income growth. Looking at borrower-level income growth, the return from making mortgage loans to borrowers is higher when borrowers have larger expected income growth. All else equal, a bank would always prefer to make a loan to a high-quality borrower, if possible, as such a loan would also serve to increase house prices. The growth in mortgage credit is therefore positively correlated to the growth in borrower income, $\phi^b$.

Following an exogenous increase in concentration, this negative correlation between non-borrower income and mortgage credit growth can cause areas with low-income growth to experience larger credit-supply shocks than areas with higher income growth. Figure 4 illustrates such a case by plotting aggregate credit across an area with high versus low income growth. In the region in which banks do not prop up prices, mortgage credit growth is positively correlated to income growth. Banks only consider the return they make on the mortgage loan itself, which increases when income growth is higher. When the mortgage market is concentrated and banks are propping up prices, it is possible for credit to be higher in areas with relatively low-income growth as the return to propping up prices is higher in areas with low non-borrower income growth. In the example in Figure 4, imagine an increase in concentration which causes the number of banks to go from 8 to 6. In this case, the area with high-income growth will experience a decline in credit due to decreased competition amongst banks. At the same time, the area with low-income growth will experience a positive growth in credit due to an increase in the market power of banks.

At the borrower-level, the growth in mortgage credit and income growth can remain positively correlated. In the simplified three-period model, I provide the intuition for this result but do not show it explicitly as there is no inter-period heterogeneity amongst borrowers. Appendix B outlines an example with inter-period borrower heterogeneity that shows how
The figure above plots total credit, measured by the number of households who get a mortgage, against the level of concentration in the mortgage market for different income growths between $t = 1$ and $t = 2$. As we move along the x-axis, $N$ increases and concentration decreases. The parametrization is as follows: $\delta = .01$, $\alpha^{nb} = .2$, $\phi^{bi}_P = .2$, $\phi^{bb}_R = 1$, $\phi^{nb}_R = 1$, $c_b = 2$, $\kappa = .45$, $\gamma = 4$, $c = 9.8$. $\phi^{nb}_P$ is varied to get changes in income growth across the two plots - it is equal to .73 is the high-income growth area and .7 in the low-income growth area.

the model can simultaneously produce a negative correlation between credit growth and income growth amongst areas (such as ZIP codes) which maintaining a positive correlation at the borrower-level.

Note that the empirical facts are not unequivocally agreed upon.\textsuperscript{31} It is outside the scope and not the goal of this paper to take a stance on what empirical facts are valid and whether in fact income and mortgage credit did or did not decouple across ZIP codes and across borrowers. Therefore, the paper does not claim to explain the behavior of income and mortgage credit growth during the crisis as the empirical agreement on these facts has yet to be fully reached. Rather this paper informs the interpretation of these correlations by providing a theoretical basis for understanding these correlations and how their meanings can differ depending on the level of data aggregation.

\textsuperscript{31}See Mian and Sufi (2016), Mian and Sufi (2015) and Adelino et al. (2015)
In the three-period model, by construction in the last period house prices fall to $\kappa$ demonstrating that banks may lend to low-quality borrowers even if a crash in house prices is inevitable. This helps illustrate the incentives banks have to prop up prices in the simplest possible setup. In the infinite horizon model in Section 4, we get more realistic price processes for housing prices and credit and can assess how concentration can lead to endogenous booms and busts in house prices.

1.5. Infinite Horizon Model

I now solve the infinite-horizon model which gives rise to boom and bust cycles that can explain various empirical facts that defined the housing crisis. In the full model, there is intra-period heterogeneity: each period has both high- and low-quality borrowers. Additionally, there is uncertainty about the state of the world: with probability $q$ a rich generation is born and with probability $1 - q$ a poor generation is born in which all households have a lower expected income.

The path dependency of this problem can make it complicated to solve since in every state banks have to decide how much to lend taking into account outstanding loans and future lending. Furthermore, they also have to account for how the lending decisions of other banks will affect both current and future house prices. Given the model setup, it is possible to simplify a bank’s maximization in a similar way as the three-period model to get a tractable problem.\textsuperscript{32} In the infinite horizon model, we can show that similar to the model with three periods, once markets become concentrated banks have incentives to prop up house prices. Furthermore, because of intra-period strategic substitution amongst banks, the economy has a unique equilibrium despite strategic complimenterities in lending across periods. Initialize initial loans to 0. Then we can establish the following proposition analogous to the three-period case,

\textbf{Proposition 4} The infinite-horizon model has a unique equilibrium. There exists a cutoff, $\bar{N}$, such that if $N \geq \bar{N}$, for any possible sequence of shocks, banks do not make any loans

\textsuperscript{32}Details are provided in the appendix.
to high-risk, low-quality borrowers to prop up prices. If \( N < \overline{N} \), there are a strictly positive number of sequences of shocks in which banks will extend credit to high-risk, low-quality borrowers to prop up house prices.

The key intuition for this proposition is identical to that in the three-period case. When \( N \) is large, each individual bank has a small amount of loans on its books. Therefore banks do not benefit from making loans that are unprofitable to push up house prices as the return from propping up prices is low. As \( N \) increases and individual banks acquire larger market shares, increasing house prices allows them to profit from a greater number of loans. This increases the return from keeping house prices high. Therefore, as concentration increases, the equilibrium begins to feature loans that are made to high-risk borrowers even if the loan itself is negative NPV.

An important feature of the equilibrium is that conditional on the state of the economy, lending per bank is increasing in its outstanding loans. Formally:

\textbf{Corollary 3}  
\textit{Conditional on the state of the economy, aggregate lending} \( M_t \) \textit{is increasing in aggregate outstanding loans} \( M_{t-1} \). \textit{Lending per bank} \( m_t \) \textit{is similarly increasing in outstanding loans} \( m_{t-1} \) \textit{conditional on the state of the economy}.

This feature of the equilibrium naturally generates housing boom and bust cycles and is discussed in the remainder of this section.

\textit{1.5.1. Boom and Bust Cycles}

An exogenous increase in concentration can lead to credit booms and busts with features that match key empirical facts about the recent housing crisis.

\textbf{Credit Boom, Rising House Prices and Rising DTI Ratios:} In the equilibrium, the amount of loans a bank makes is increasing in its outstanding loans, conditional on the state of the economy. This feature of the equilibrium naturally gives rise to an increasing time-
series of credit and house prices following a series of positive income shocks.\textsuperscript{33} This increase in credit happens even though the “fundamentals” of the shock, the underlying income of households, stays the same. The increase in credit is rather driven by the fact that lenders find it profitable to increase credit expansion to prop up house prices as the size of their outstanding exposure to the mortgage market increases. This effect is most pronounced for lenders with access to a large number of potential borrowers since they make more loans per bank, increasing the incentive to prop up prices in subsequent periods. Furthermore, they are able to make a larger number of mortgages before being limited by the market share constraint. They are therefore able to acquire larger and larger mortgage exposures, increasing their incentive to prop up prices. High concentration can therefore cause credit booms to be much larger than when concentration is low. Since the fundamentals of the economy in terms of expected borrower income remain the same, the only thing that is changing over time are house prices driven by greater credit expansion. The credit boom is therefore accompanied by an increase in house prices and rising debt to income ratios.

Figure 5 shows how a credit boom and bust differ in high concentration versus low concentration areas in an economy that has a series of consecutive high shocks followed by low shocks. In this example, the area with a higher concentration has approximately 25\% more credit expanded at the height of the boom than the area with low concentration.

\textbf{Rise in Risky Lending:} The recent housing crisis had an unprecedented increase in risky lending with mortgages being extended to borrowers with low FICO scores at high DTI and LTV ratios. Figure 6 shows the composition of credit during the boom and bust cycle. The right y-axis plots the percentage of credit extended to low-quality borrowers in the credit boom. Not only do areas with high concentration have larger credit booms, the composition of that credit is riskier. Also note that there is a boom in credit to high-quality borrowers during rich-shocks in both high- and low- concentration areas. This is by virtue of

\textsuperscript{33}For example, imagine that all banks start with 0 loans on their books and at $t = 1$ a high shock occurs and banks make $m_1 > 0$ loans. Then at $t = 2$ banks have $m_1 > 0$ loans outstanding on their books. If at $t = 2$ another high shock occurs, given that $m_1$ is increasing in $m_{t-1}$ conditional on the state of the economy, $m_2 > m_1$. And so on and so forth for any consecutive high shocks.
The figure above plots total credit, measured by the number of households who get a mortgage, across two areas with different concentration for a series of income-shocks on the x-axis. Details of the parametrization are in Appendix C.

fundamentals being better during high shocks and is in line with findings by Adelino et al. (2016b) that credit expansion increased across the income distribution during the credit boom. However, the increase in outstanding loans in high concentration areas also creates an incentive to prop up prices, leading to loans additionally being made to low-quality borrowers. The overall composition of credit is therefore riskier in high concentration areas during credit booms.

**Timing of Risky Lending during Boom:** The rise of risky lending happened in the early- to mid-2000s even though house prices had been rising since the 1990s. Most theories on the rise of risky lending do not account for the timing of the rise. For example, the securitization of mortgages was common since the 1990s but the rise in risky lending was not observed until later. As Figure 6 illustrates, the amount of risky lending increases over the life of the credit boom and does not start immediately on entering a rich state. Since low-quality loans are negative NPV, lending to risky borrowers is profitable only when the return from propping up house prices is high. A lender’s portfolio needs to be large enough for there to be an incentive to make risky loans. Since the number of outstanding loans
increases over the life of the boom, high-risk lending will begin after the start of the boom once the outstanding mortgage exposure is large enough. The exact timing of the start of risky lending depends on the amount of concentration and the profitability of high- and low-quality loans. As banks are incentivized to prop up prices, they will first saturate the high-quality market and only then move on to riskier lending.

**Continuation of Risky Lending after Boom:** Lastly, Figure 6 illustrates that once a low-shock hits the economy, lenders in the highly concentrated area continue to make risky loans for a short time. Even though fundamentals in a poor state worsen, since large lenders have a lot of outstanding loans, they have an incentive to keep lending to keep house prices high. Essentially, large lenders react less to fundamental shocks than small lenders because of the size of their balance sheet. In recent work, Elul, Gupta and Musto (2017) find that the GSEs increased their risky activity in 2007 when markets began to slow down and when private securitizers started withdrawing from the market. Additionally, Bhutta and Keys (2017) find that private mortgage insurance issuance which allowed the GSEs to securitize
loans with riskier fundamentals also increased in 2007 as the housing market was beginning its downturn. Both papers also find that this seemed to be happening in areas where other players were pulling out of markets. As house prices are likely to fall when other sources of housing demand slow down, this is in line with my model predictions.

The model thus explains many facts about the recent housing crisis. As discussed earlier, when Salomon Brothers got investors interested in MBS as an investment vehicle, Fannie Mae and Freddie Mac grew their market share tremendously having a monopoly on prime securitization and later became the biggest investors in private label MBS. This event can be viewed as an exogenous increase to the GSEs market power, giving them access to large share of potential borrowers. The model can help us understand the credit boom and bust following this increase in GSE market power.

1.6. Extensions

This section discusses some extensions to establish some additional results and robustness of the model mechanism.

Secondary Market Equivalency

During the housing boom, there was a large concentration in mortgage holdings at the GSE level and amongst a few large banks who purchased many MBS. In the model described so far, mortgage originators are assumed to be the final holders of these mortgages. The baseline model can be reframed as an equivalent problem in which mortgage holders purchase mortgages from a secondary market and do not originate any loans themselves. The key mechanism works as long as there is concentration in mortgage holdings at some level and agents with exposure to mortgage payments have some market power. If secondary market players own a large share of the mortgage market, they want to keep house prices up. If they have market power, they can offer attractive prices on the secondary market for sub-prime mortgages that will incentivize mortgage originators to issue mortgages to risky borrowers. Holders of these mortgages will suffer losses on these risky purchases but the increase in
house prices will be profitable for their outstanding mortgage exposure.

The equivalent model is as follows. A continuum $[0, 1 - \alpha^{nb}]$ of banks competitively originate mortgages and sell them to a secondary market. Each bank has access to one borrower in the continuum of borrowers. Final mortgage holders purchase mortgages on the secondary market from originators. Each holder has access to a fraction $\frac{1}{N}$ of originators and thereby to a fraction $\frac{1}{N}$ of borrowers. Assuming mortgage originators follow an originate-to-distribute model and do not hold mortgages is equivalent to assuming that their main source of funding comes from secondary markets. The originate-to-distribute model was common amongst mortgage originators during the housing boom (Purnanandam (2011)). Loutskina and Strahan (2009) and An and Yao (2016) provide evidence that the GSEs were a key source of liquidity provision for non-jumbo loans issued by banks.

A bank that originates mortgages is offered a secondary market price, $Y_t(r^j_t, \omega^{bj}_t)$, for a mortgage originated at time $t$ to a household $j$ with expected endowment $\omega^{bj}_t$ at interest rate $r^j_t$. An originator will make a mortgage loan to a borrower if:

$$Y_t(r^j_t, \omega^{bj}_t) - P_t \geq 0.$$ 

Each period holders of mortgages will choose to post secondary market prices taking into account their effect on originator decisions and how that influences housing prices. I assume that secondary market holders can purchase mortgages at different rates from different mortgage originators thereby allowing them to control how many mortgages of a type they wish to purchase. Therefore, secondary market mortgage holders maximize the following, where $m^j_{t-1}$ is the number of mortgages of type $j\{h, l\}$ they purchased in the previous period,
\[
V(s_t, s_{t-1}m_t^h, m_{t-1}^l, r_{t-1}^h, r_{t-1}^l, P_{t-1}) = \max_{m_t^h \geq 0, m_t^l \geq 0, Y_t^j, r_t^j, r_t^l} \sum_{j=\{h,l\}} m_{t-1}^j \left( \phi_{s_t}^{bj} \min\{P_{t-1}(1 + r_{t-1}^j), e^b + (1 - \delta)P_t\} \right)
\]
\[
+ \left( (1 - \phi_{s_t}^{bj}) \min\{P_{t-1}(1 + r_{t-1}^j), (1 - \delta)P_t\} \right)
\]
\[
- \sum_{j=\{h,l\}} m_t^j Y_t^j \left( \text{New Lending} \right) + \beta E \left[ V(s_{t+1}, m_t^h, m_t^l, r_t^h, r_t^l, P_t, s_t) \right] \left( \text{Continuation Value} \right)
\]
\[
\text{s.t. } Y_t(r_t^j, \omega_{t}^{bj}) - P_t \geq 0
\]
\[
\gamma + \beta(1 - \delta)E[P_{t+1}] \geq \beta E[\min\{P_t(1 + r_t(s_{t+1})), \omega_t^h + (1 - \delta)P_{t+1}\}]
\]
\[
m_t^h \leq \frac{1}{N} \alpha_t^{bh}
\]
\[
m_t^l \leq \frac{1}{N} (1 - \alpha_t^{bh} - \alpha_t^{nl}).
\]

The mortgage holder will pick \( Y_t \) so that the originator is just willing to lend, implying that \( Y_t = P_t \). Furthermore, they will choose an \( r_t \) so that borrowers repay the maximum they are willing to pay. This is equivalent to the problem faced by banks that hold onto the mortgages they originate. The same logic can be applied if mortgages are resold on the secondary market. The key requirement for the mechanism to work is that the final holder of mortgages has some market power.

**Refinancing**

Since the model has one-period mortgage contracts, it abstracts away from the bank choosing to refinance mortgage contracts to prevent defaults instead of issuing new mortgage loans. Trivially, in the model if the bank could simply give an existing borrower who is due to repay their mortgage loan a new mortgage this would be equal to writing down their debt to a face-value of zero (the endowment in the low state). In this case, it is possible to show that making a new borrower a mortgage loan rather than refinancing the existing mortgage is always a dominant strategy as the bank can additionally harness the new

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borrower’s expected endowment. If it is not possible to make loans to new borrowers, the same intuition for propping up prices applies and a bank will be willing to make a negative NPV refinancing loan if it has large enough outstanding mortgage exposure.

The baseline three-period model can be extended to incorporate the possibility of refinancing in a less trivial way. We can modify the model and assume that the generation born in period-1 lives for three periods and gets utility from consumption in either the second or the third period. The generation born at $t=1$ therefore gets utility,

$$u(k_1, c_2, c_3) = \gamma k_1 + \beta c_2 + \beta^2 c_3$$

At $t=2$, if a household from generation 1 gets an endowment of 0, with probability $\phi^b_{RP}$ they will receive an income of $e^b$ at $t=3$. This captures the idea that some households may have delayed income. At $t=2$, a bank can either choose to make a new loan to prevent defaults, or to refinance defaulting borrowers and issue them a new mortgage instead. For simplicity, I assume that since the mortgage loan has recourse, a borrower who is in default has to accept the bank’s refinancing offer at $t=2$ if the new payment demanded does not exceed the face-value of the loan multiplied by the discount factor $\beta$. In this case, a bank would prefer to refinance an existing borrower rather than lend to prop up prices to a new borrower if,

$$\phi^b_{RP} > \phi^b_P$$

If $\phi^b_{RP} < \phi^b_P$, a bank would strictly prefer to make new loans rather than refinance existing borrowers. The intuition for this is relatively straightforward. The bank will simply go for whichever option is less negative NPV. This will depend on whether a loan to a new bad-quality borrower or the existing borrower in default has higher expected future repayment. The basic intuition is applicable to both refinancing and new loans. In line with this, Elul
et al. (2017) find that the GSEs were more likely to make both high LTV refinancing and purchase loans in areas where they had large outstanding exposure.

**Lender Heterogeneity: Large and Small Lenders**

During the housing boom and bust, while the GSEs and some large LCFIs acquired a large share of the mortgage market, there were also smaller mortgage market participants. The model can be extended to evaluate the case of some large lenders and many small, dispersed lenders who behave like price-takers in the mortgage market. Specifically, I modify the model to allow small, dispersed lenders to have access to a share $s$ of households while large lenders have access to the remaining ones.

The previous results and intuition all apply here. Furthermore, in a boom-bust cycle, following a series of $h$ shocks the *effective market share* of a large lender as measured by the percentage share of all mortgages held by that lender over the total number of mortgages outstanding, is increasing over time even though access to borrowers stays the same. This is because as a large lender acquires more outstanding mortgages over the boom, the incentive to prop up prices increases, leading the lender to increase credit. This effect is not present for small lenders who are price-takers. The exposure of the large lender to the total mortgage market therefore increases over time. This extension of the model matches the empirically observed increase in Fannie Mae and Freddie Mac’s market share over the housing boom.

I also calibrate this extended model to U.S. data. The calibration is discussed in section 6.1. As concentration increases, the size of the credit boom and extension of risky credit also increase. Importantly, now the market share of large lenders increases over the boom. Figure 7 shows a credit boom and busts in areas with differing concentration. It also shows that the gain in *effective market share* of a large lender is lower when there are more competing lenders.\[^{34}\]

[^{34}]: This version of the calibration has the market share of large lenders decreasing upon the bust. The effective market share may also increase if non-borrower demand and the provision of mortgages by small price-taking lenders slows down.
1.7. Calibration Exercise

I now calibrate the stylized model to the U.S. economy. The main purpose of this model is to clearly illustrate the theory of how concentration in the mortgage market can create an incentive to extend risky credit to keep house prices high and how that can produce credit cycles with dynamics similar to that of the recent housing crisis. In doing so, the model abstracts away from various aspects of mortgage markets, housing decisions by households, details of mortgage contracts, etc. A deeper examination of the quantitative implications of concentration on mortgage credit requires a more detailed quantitative model. However, this calibration of the stylized model can help us address two questions. First, does the model produce quantitatively significant credit and house price dynamics that match the U.S. experience in the recent housing crisis? Second, does simply changing the level of concentration produce significant variation in the path of credit and house prices? If the answers to these questions are positive, then it provides support for this paper’s theory that the housing boom and bust was driven by a change in concentration.
The model is calibrated to the 1991-2009 U.S. housing market with the boom quantities aggregated across 1991-2006 and the bust quantities aggregated across 2007-2008. For this exercise, I assume that the economy experienced a sequence of rich shocks in the boom years followed by a sequence of poor shocks in the bust years. The rich and poor shocks are mapped to Federal Reserve Economic Data on the growth rate of personal income. From 1991-2006 real median personal income growth at an annual rate of 1.5% while from 2007-2009 it grew at an annual rate of -1.6%. Appendix C provides a detailed description of the data.\footnote{Appendix C tables 15-17 also provide results for calibration of the benchmark model without lender heterogeneity.}

Table 12 in Appendix C summarizes the benchmark configuration of the model parameters. Here, I discuss the parameters and results when calibrating the model with lender heterogeneity as that allows me to better target the GSE share of the mortgage market. As I lack data on the exact market share of LCFIs, I do not attempt to capture their share in the calibration. I choose $s = 0.6$ and $N = 2$ to roughly match the GSEs’ eventual market share at the height of the boom. The income shocks of high- and low-quality borrowers in the rich- and poor-states of the economy are chosen to match the default rates on prime and sub-prime loans during the boom and bust. The fraction of high-quality borrowers is chosen to match the fraction of prime versus sub-prime lending while the fraction of non-borrowers is chosen to match the fraction of cash-only house purchases.

Table 13 in Appendix C compares the model-generated quantities for the calibration to those in the data. A one-time change in concentration is able to match the house price increase and decrease well, explaining approximately 45% of the boom in house prices. A fundamental shock to the concentrated market explains about 30% of the bust in house prices. It also does a good job of matching the fraction of sub-prime borrowers during the boom explaining over 90% of the lending to sub-prime borrowers in the boom but overestimates the lending to sub-prime borrowers slightly during the bust. This may be partially driven by the fact that during 2007, the GSEs kept lending to high-risk borrowers
but private securitizers started pulling out of the high-risk market.

Table 14 in Appendix C does a counterfactual analysis. I decrease concentration by doubling $N$ from 2 to 4 while keeping the other parameters of the benchmark calibration the same. This change in concentration decreases the fraction of sub-prime lending in the boom to 0 in both the boom and bust. The boom also has 30% lower growth in house prices while the bust has about 80% smaller decline in house prices.

The model can therefore produce quantitatively meaningful magnitudes and changes in credit and prices when looking at the housing boom and bust. Moreover, changing concentration can result in significant changes to the model-implied credit boom and bust. This suggests that concentration can be an important channel that contributes to credit cycles and a more comprehensive quantitative exploration of this channel is interesting for future research.

1.8. Conclusion

This paper provides a novel theory of how concentration in mortgage markets can affect both the quantity and the quality of mortgage credit. Lenders with a large outstanding mortgage exposure have incentives to extend risky credit to prop-up house prices. An increase in concentration in mortgage markets can generate housing booms and busts with features that match the recent U.S. housing crisis.

In the aftermath of the housing crisis, policy makers have wanted to design policy to curb high-risk lending. However, the role that concentration can play in creating incentives to extend risky mortgage credit has been largely overlooked in this process. The Economist recently reported that concentration in mortgage markets has increased since the crisis as a side-effect of new regulations faced by banks which have made them move out of mortgage lending. Somewhat ironically, many of these regulations are intended to reduce risky lending. The GSEs and the FHA are currently funding between 65-80% of new mortgages, many of which appear to be highly risky. A fifth of all loans since 2012 have
LTV ratios of over 95%. This is comparable to the fraction of sub-prime lending in the years before the housing collapse. As this paper demonstrates, such high concentration can have a significant impact on both the quantity and quality of credit. It is therefore crucial to comprehensively understand the different forces that incentivize high-risk lending when designing policy.
CHAPTER 2 : Concentration and Lending in Mortgage Markets

(with Ronel Elul and David Musto)

Disclaimer: The views expressed in this paper are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of Philadelphia or the Federal Reserve System.

2.1. Introduction

The recent housing crisis has raised concerns about how housing markets should be designed and regulated to limit excessive risk taking. One characterizing feature of the US housing market during the boom and bust was the presence of institutions with large exposures to the mortgage market - the Government Sponsored Enterprises (GSEs) and large complex financial institutions (LCFIs). In 2007, when house prices started falling, these institutions had amassed massive exposure to the US mortgage market. Much of this exposure consisted of risky credit leading these institutions to either fail or require substantial government intervention to continue operations. Although there has been concern over the large size of such participants in the mortgage market, empirical research on the exact way in which size affects the quality of credit in mortgage markets is limited.

In this paper, we show that the size of mortgage exposure can affect the quality of credit. We start by documenting a striking expansion of risky lending by the GSEs, the two giants of the real estate market, in 2007. This active increase in high-risk activity is surprising as housing markets conditions had already started to deteriorate at this time and were forecasted to keep worsening. In response to the decline in housing prices and rise in defaults, smaller private-label securitizers were reducing risky lending as we would typically expect. Moreover, the termination rates on these high-risk originations by the GSEs were quite high making it hard to argue that the GSEs were expecting to profit from the future performance of these loans. In fact, in October 2006, Richard Syron, the then chairman and chief executive of Freddie Mac, October said that he was “concerned that foreclosure and loss rates are going to increase” and expected to see “a bumpy landing at a national level”
for the housing market. He also cautioned against going further out on the “risk curve” and “betting on a turnaround in pricing.”

We provide evidence that this behaviour by the GSEs can be explained through a propping-up channel (Gupta (2017)). The key idea is that if house prices affect the probability of mortgage default, institutions with large outstanding exposure to mortgage markets benefit from house prices staying high. If credit expansion increases house prices, lenders may internalize their effect on house prices. This can cause institutions with large exposure to the mortgage market, such as the GSEs, to extend high-risk credit. They are willing to sacrifice some of the return they make on the loan itself for the boost in house prices that comes from credit provision as high house prices affect the profitability of their outstanding mortgage exposure. We posit that such incentives can explain the GSEs’ ramp-up of risky lending at a time when such mortgages appeared to be the least profitable.

Using a loan-level dataset, we show that the GSEs engaged in riskier lending in MSAs in which they had a higher proportion of outstanding mortgage exposure on their balance sheet. We exploit variation in the size of the GSEs’ outstanding mortgage exposure across MSAs in 2007. This allows us to control for institution-specific characteristics that can affect lending such as risk-appetite, corporate governance, specific underwriting practices etc. Consistent with our hypothesis, we find that the GSE share of outstanding mortgage exposure in an MSA predicts new high-risk activity in that MSA.

We perform a number of tests to determine whether this relationship is driven by incentives to keep house prices high and not by other factors that would affect both the outstanding share and new risky activity. We expect propping-up incentives to be stronger when house prices are more sensitive to credit. To test this, we use Saiz’s measure of house price elasticity to see if the relationship between outstanding share and new high-risk originations stronger in areas where the housing supply curve is more inelastic since this is where we expect credit to have a stronger effect on house prices. In line with propping-up, we find that the effect of outstanding mortgage exposure on risky originations is stronger in areas where the
the housing supply curve is inelastic. Since in these regressions, we are comparing GSE high-risk activity in elastic versus inelastic MSAs conditional on their outstanding exposure, we mitigate to some extent the concern raised in Davidoff (2015) that supply constraints are correlated with unobserved demand factors. In 2007, relatively elastic MSAs experienced greater demand shocks mitigating concerns that our results are driven by demand shocks. Moreover, if demand for mortgages varied substantially across the MSAs we compare, it is hard to explain why the GSEs amassed similar outstanding exposure at the beginning of 2007. Since we are comparing MSAs in which GSE outstanding mortgage exposure is similar at the start of 2007, we expect that demand factors across these MSAs should be similar.

We also expect the incentive to prop-up to be stronger when the sensitivity of default to house prices is high. To test this, we exploit variation in the composition of the GSEs' outstanding portfolio. We measure the proportion of outstanding loans that have high loan-to-value (LTV) ratios. We expect high LTV loans to be more sensitive to default than low LTV loans, and therefore should expect to see a greater willingness to make risky mortgages in MSAs in which more of the GSE portfolio is comprised of these loans. In line with our hypothesis, we find that the GSEs are more likely to engage in high-risk lending, when a larger proportion of their outstanding portfolio is composed of high-LTV loans.

We also examine the change in Fannie Mae and Freddie Mac's lending standards in MSAs in which they had a high-outstanding share. We find that the GSEs lowered lending standards in 2007 compared to 2006 in MSAs in which they had a greater outstanding share and this effect was stronger in inelastic MSAs. To test this, We exploit a rule that if a mortgage was made to a borrower with an origination FICO score of 620 or above, it was eligible for the GSEs automated underwriting. Mortgages below the threshold, were subject to additional underwriting by the GSEs and therefore were screened more carefully. Previous research by Keys et al. (2011) has shown that there is a jump in the number of loans made at the threshold and loans above the threshold performed worse than loans just
below this threshold. Intuitively, a borrower with an origination FICO score of 619 should be very similar in terms of risk to a borrower with an origination FICO score of 621. However, because the borrower of 621 goes through an easier screening process due to automated underwriting, while the borrower with an origination FICO score of 619 has greater screening, the quality of loans right above the threshold is worse than right below. Since the automated system was standardized across the country, the proportion of loans just below the threshold to loans just above the threshold therefore tell us how strict the GSEs manual underwriting was in an MSA. The lower the proportion, the stricter their manual underwriting in the MSA. If the GSEs were increasing risky activity in MSAs in which they had a higher outstanding share, we should see a smoothing out of this proportion of the number of loans just below the threshold to the number of loans above. To test this, we look at how the proportion of loans made below the threshold (origination FICO scores 610-619) to the number of loans made just above the threshold (origination FICO scores 620-629) changed from 2006 to 2007. We indeed see a smoothing of loans around the threshold in MSAs in which the GSEs had a high outstanding share which we interpret as evidence that the GSEs relaxed their underwriting standards for loans they manually underwrote. We also find that this relationship is stronger in inelastic MSAs, in line with propping-up incentives being stronger when house prices are more sensitive to credit.

Finally, we hope to control for demand-side factors by restricting our analysis to the first half of 2007. By the end of 2006 there was a lot of guidance that house prices were set to decline and that mortgage delinquencies were expected to increase.¹ Risky lending in this period is therefore unlikely to be driven by factors such as high house price expectations, a high presence of willing investors for risky mortgages or many speculators wanting to make money off continuing increases in house prices. Indeed, we document that the private market cut back on its risky activity in 2007 possibly in response to fears of a housing bust. We therefore do not believe that the GSEs lending decisions in this time were driven by expectations that these high-risk mortgages would be profitable.

¹Further details on this guidance are provided subsequently in the paper.
Our paper has implications for the design on the housing market - in particular, how incentives to lend differ between large versus small mortgage market participants. In the aftermath of the financial crisis, a lot of focus has been on how to curb risky lending. Although some steps have been taken to address issues such as the moral hazard concerns connected to securitization and the under-capitalization of the banking sector, concentration in mortgage markets has largely gone unaddressed. Mortgage markets are more concentrated now than they were during the housing boom and bust. In 2016, The Economist reported that government organizations are funding about 65-80% of new mortgages and Quicken Loans and Freedom Mortgage are currently originating about half of all new mortgages.

A number of papers have found a connection between size and risky lending. Dell’Ariccia et al. (2012) provide evidence that worsening lending standards in the housing boom were driven by large lenders. They also find lower loan denial rates in areas with a smaller number of competing lenders. Adelino et al. (2016a) find that based on observable risk characteristic loans in GSE pools of private MBS were riskier than those in non-agency pools. Nadauld and Sherlund (2013) provide evidence that the large increase in the securitization of sub-prime mortgages can mostly be attributed to the five largest broker/dealer banks. We contribute to this literature by finding evidence that large exposure to the mortgage market can lead to risky mortgage activity and highlight a possible channel - the incentive to prop up house prices as mortgage exposure grows- that could be driving this.

Theoretically, there are a few channels that can lead to a connection between size and risky lending. As mentioned before, in this paper we test the theory that large mortgage exposure can lead to risky activity to keep house prices high. Another channel that would connect size and risky lending is if large institutions have an implicit bailout guarantee by the government. In this paper, by focusing on the GSEs and exploiting variation in GSE share in different MSAs, we can provide support for the theory of propping-up prices. Since implicit bailout guarantees should vary across- but not within-institutions, our results can not help speak to the theory of implicit-bailout guarantee that may be present for large
institutions.

The GSEs were central to the American housing market during the housing boom and bust and a large literature has studied the various ways in which the GSEs may have contributed to the fragility of mortgage markets. The agencies had a dual mandate, low-income housing targets, and the implicit subsidies they received from the government leading to lower borrowing rates and implicit bailout guarantees. All these reasons could contribute to GSE high-risk activity. To eliminate some of the concerns that these other factors are driving our results, we choose to exploit variation in the GSE exposure amongst MSAs. If for example, lower borrowing costs were the only cause of the agencies high-risk activity, their outstanding exposure should not be connected to their high-risk activity. Amongst other reasons for GSE lending, their low-income housing targets would vary across MSAs. Acharya et al. (2011) report that GSE adherence to these targets seemed to be voluntary - the GSEs missed their housing targets on several occasions without any severe sanctions by regulators. We therefore find it unlikely that our results are driven by GSE housing goals. Furthermore, if anything, we expect targets to be negatively correlated to GSE outstanding exposure.

Although there is research on how concentration in mortgage markets can affect mortgage interest rates and on how interest rates can affect housing prices, the intersection of the two is surprisingly limited. Scharfstein and Sunderam (2014) and Fuster et al. (2016) have evidence that mortgage interest rates and spreads in secondary markets are affected by competition. However, they do not look at how this can affect house prices. A large body of literature has studied the effect of credit on house prices. The user cost models of Poterba (1984) incorporated mortgage interest rates as a determinant of house prices. Empirically, papers by An and Yao (2016) Griffin and Maturana (2015), Landvoigt et al. (2015), Hubbard and Mayer (2009), Himmelberg et al. (2005), Mayer (2011), Khandani et al. (2009) and Favilukis et al. (2017) have found that house prices respond to mortgage

\footnote{See Acharya et al. (2011) for a comprehensive review.}
credit. We therefore think it quite natural to expand this literature by considering how concentration in the mortgage market impacts credit and house prices.

Our paper is one of the few which combines these two literatures and studies how large institutions may provide credit in response to their effect on house prices. In a paper close to ours, Favara and Giannetti (2015) analyze the effects of size on the probability of foreclosure and renegotiation of debt in mortgage markets. They find that mortgage lenders seem to internalize house price drops coming from foreclosure externalities and are less likely to foreclose on delinquent households in areas in which they have large outstanding exposure of mortgages on their balance sheet. We complement their analysis by looking at how internalizing pricing externalities effects the incentive of large lenders in mortgage origination. We find that there can be a darker side to large balance sheet exposure as the incentives of making risky loans can be higher in more concentrated markets.

The rest of this paper is arranged as follows. Section 2.2 describes the data, provides summary statistics and gives some background on the housing market in 2007. Section 2.3 describes the main analysis and our results. The last section concludes.

2.2. Data and Summary Statistics

We use loan-level data from Black Knight McDash (heretofore referred to as McDash). These data have been used to study the determinants of mortgage default (Elul et al., 2016), and the expansion of credit during the housing boom (Adelino, Schoar and Severino, 2016). These data are provided by the servicers of the loans, and the contributors include the majority of the top servicers. We focus on first mortgages that are originated or outstanding starting from 2005, since coverage of the McDash data was not as extensive prior to that date (particularly for subprime loans), and continuing through 2008.

The McDash data cover about two thirds percent of all mortgage originations in these years. We restrict attention owner-occupied homes and exclude multifamily properties. The McDash data set is divided into a “static” file, with values that do not change over time,
and a “dynamic” file. The static data set contains information obtained at the time of the original underwriting, such as the loan amount at origination, house value at the origination date, origination FICO score, documentation status (i.e., full-documentation versus low/no documentation of income and assets), the source of the loan (e.g., whether it was broker-originated), property location (zip code), type of loan (fixed-rate, ARM, prime, subprime, IO, Option-ARM, etc.), and whether there is a penalty for prepayment. The dynamic file is updated monthly. The most important dynamic variables for our analysis are the current principal balance and the investor type: private-securitized, Fannie Mae, Freddie Mac, portfolio, FHA. Because of the time it takes a loan to go through the securitization pipeline, many mortgages are initially recorded as portfolio loans when they first appear in the data set; therefore, we define the “investor type at origination” to be that reported at six months from loan origination. We focus attention on loans purchased by Fannie Mae and Freddie Mac (referred to as “GSE”) and Private securitized loans; the other investor types are combined into the a single residual category. Finally, we also merge in the house prices index\(^3\) at the zip code level from Corelogic HPI\(^\text{TM}\) (heretofore referred to as Corelogic), and the housing supply measures from Saiz (2010).

Summary statistics for the key variables can be found in tables 1, 2 and 3. Table 1 summarizes the GSE and private-securitized share of the mortgage market and the high-risk (high LTV, low origination FICO) composition of their portfolios at the end of 2006 at the MSA-level. Table 2 summarizes the GSEs and private-securitized share of new originations and their high-risk activity share of originations in the first half of 2007 at the MSA-level. The GSEs increased their share of high-risk mortgage activity substantially in 2007. At the end of 2006, 10% of GSE-securitized loans (purchases and refinances) consisted of mortgage loans with LTV ratios of above 80% and 15% consisted of mortgage loans with origination FICO scores of less than 660. In 2007, 30% of new GSE-securitized loans had an LTV ratio of above 80% and 21% had origination FICO scores of less than 660.

\(^3\)Using the December 2016 release.
Table 3 shows the GSEs and private-securitized share of new originations and their high-risk activity share of originations separately for the first two quarters of 2007. The average GSE loan worsened over 2007 with the high-LTV and low-origination FICO share of originations increasing in the second quarter of 2007 relative to the first. A similar deterioration in mortgage quality did not occur on the private-label side with the high-LTV share remaining constant across the two quarters and the low origination FICO share of originations decreasing from 51% in the first quarter of 2007 to 41% in the second quarter.

2.2.1. Background on the Housing Market and GSE Activity in 2007

In this section, we give an overview of the market sentiment about house prices at the beginning of 2007. In particular, we want to establish that market participants were predicting a decline in house prices and that institutions that were active in mortgage markets were reacting to this outlook of the mortgage market and reducing risky activity. We hope to make the case that it is hard to explain GSE high-risk activity at this time and that new high-risk loans did not appear to be profitable ex-ante.

In October of 2006, Moody’s released a report stating that “The U.S. housing market downturn is in full swing.” They predicted that “100 of the nation’s 379 metro areas have a significant probability of experiencing price declines” with about 20 areas experiencing double-digit crashes in house prices. In March of 2007, the NYtimes reported that the Mortgage Bankers Association had shown a record number of homes entering the foreclosure process and stated that the “troubled housing market” was expected to “weaken further”. In March 2007, the Economist further reported that about 18% of homebuyers who took out mortgages in 2006 were in negative equity and would face the “greatest difficulties”.

Investors and participants in the housing market showed signs of reacting to this outlooks. Private securitizers started cutting back on high LTV lending in mid-2006. New Century Financial’s stock fell to half its value in March of 2007. In March 2007, housing starts had fallen 33% in two months and investors were reported to be “shunning subprime and all
mortgages that seemed risky."

Furthermore, there is evidence that the agencies themselves shared this bleak outlook on the future of the housing market. As we mentioned earlier, in October 2006, Richard Syron, the then chairman and chief executive of Freddie Mac, October was “concerned that foreclosure and loss rates are going to increase” and expressed concern about “a bumpy landing at a national level” for the housing market. He also cautioned against going further out on the “risk curve” and “betting on a turnaround in pricing.” This makes it seem unlikely that the GSEs were planning to “gamble for resurrection.”

Despite this negative forecast of the housing market, the GSEs increased their market share of high LTV loans in 2007. At the same time, the private market for securitization withdrew from these riskier markets. Figure 8 plots the fraction of all high-LTV (LTV > 80) loan originations that were GSE and private-market originated (purchase and refines). By the end of 2007, the GSEs grew their share of high-LTV originations to about 70%, up from 50% at the start of 2006. This ramp-up in high-risk activity was not simply the GSEs being passive and maintaining lending levels as the private-market cur back on lending. In figures 10 and 11, we plot the volume of new high-risk (LTV > 80) and lower-risk originations (LTV ≤ 80). The GSEs increased their high-LTV originations three-fold while maintaining the level of lower-risk originations. Figure 9 plots the total share of high-risk originations of all GSE originations and this share almost doubled from the beginning of 2006 to the end of 2007. The GSEs increase high-risk activity seems to have been a very active undertaking at a time when they themselves seemed to have a bleak outlook of the housing market.

These high-risk mortgages appeared to perform badly ex-post for the GSEs. About 12% of high-LTV mortgages purchased by the GSEs experienced bad terminations from origination till date. This was an increase of over 50% from 2005 when the bad termination rate was about 7%. Moreover, this number is likely understated as it includes mortgages that were likely rescued from default by government policies such as the Home Affordable Refinance Program (HARP) and the Home Affordable Modification Program (HAMP). In figures 12
and 13, we plot the fraction of GSE and private-label mortgages that were terminated badly. The termination rate on high-LTV mortgages purchased by the GSEs was rising in 2007 as the GSEs expanded risky lending. At the same time, termination rates of private-label MBSes actually started falling in mid-2006.

In 2007, market participants seemed to believe that delinquencies were on the rise and house prices were going to decline. As we would expect, following this guidance, private securitizers retreated from the market. Surprisingly, at the same time GSEs increased their high-risk activity. In the next section, we provide evidence that this ramp-up in GSE high-risk activity could have partially been driven by a want to keep house prices high in areas where the GSEs had large outstanding mortgage exposure.

2.3. Main Analysis and Results

Our main hypothesis is that the GSEs had an incentive to prop-up house prices in MSAs in which they had a high outstanding mortgage exposure. To the extent that credit affects house prices, the GSEs would therefore ramp up lending activity in areas in which they have a large number of outstanding mortgages. In this case, they may reduce lending standards and make high-risk loans - not because of the return they earn from the loan itself, but because of the benefit they get from increased house prices. The price externality associated with lending, makes the GSEs more willing to issue a risky loan. To test our hypothesis, we first run the following regression,

\[ LTV_{80}^{+} = \alpha + \beta GSEshare_{MSA} + \epsilon_{MSA} \]

\( LTV_{80}^{+} \) is the proportion of mortgages originated by the GSEs in the first half of 2007 that had LTV > 80 in MSA \( y \). GSEshare\( _y \) is the GSE share of all outstanding mortgages in MSA \( y \) at the beginning of 2007. Figure 14 shows the variation in the GSE outstanding share across MSAs. There is substantial variation that we are able to exploit in the regression,
with the outstanding share varying between approximately 20\% and 90\%.

Column 1 of table 4 presents the results of the regression for all GSE originations (purchase and refinancing). In line with our hypothesis, we find a positive and statistically significant coefficient $\beta$. The estimates in column 1 show that a percentage point increase in the outstanding share of the GSEs in an MSA implies a .48 percentage point increase in the proportion of high-risk loans made by the GSEs.

*The Sensitivity of Prices to Credit:* This result could be driven by alternative explanations - perhaps in areas where the GSEs were more concentrated, they understood the market better and therefore were more suited to making risky loans. To further test our hypothesis that risky lending was undertaken to help prop up house prices, we interact the GSEs outstanding share with Saiz’s MSA-level measures of house price elasticities. In areas where housing supply was inelastic, credit should have a greater effect on house prices. The more sensitive house prices are to credit, the greater the returns to propping-up house prices. Therefore, if the GSEs are making risky loans to help keep house prices high, we should see a greater effect of outstanding share on GSE high-risk activity in areas with inelastic housing supply. If house price responses have no effect on the GSE incentive to make loans to risky borrowers, then we should not see any differential response of how the outstanding share affects GSE lending in elastic versus inelastic MSAs. To test this, we run the following regression

\[
LTV^{80}_{MSA} = \alpha + \beta_1 GSE\text{share}_{MSA} + \beta_2 elasticity_{MSA} + \beta_3 GSE\text{share}_{MSA} \times elasticity_{MSA} + \epsilon_{MSA}
\]

$elasticity_{y}$ is the Saiz measure of elasticity in MSA $y$. This measure can vary between 0 and 12, with a higher value signifying a more elastic MSA. Figure 16 shows the variation in the Saiz measure of housing supply elasticity across MSAs. We are able to exploit a large amount of variation. According to Saiz (2010), in land-constrained cities, elasticities are
equal to and below 1. We therefore have a large number of elastic and inelastic MSAs in our sample.

Column 2 of table 4 presents the results of the regression. We find a negative and significant coefficient \( \beta_3 \) in line with our hypothesis. Therefore, column 2 of Table 4 shows that the likelihood that the GSEs will make high-risk loans for a given concentration of outstanding loans is higher in areas where we expect the sensitivity of house prices with respect to credit to be higher. This regression also has a significant and positive coefficient, \( \beta_2 \). The average elasticity across MSAs in our sample is 2.46. The estimates in column 1 show that a 1 percentage point increase in the outstanding share of the GSEs in an MSA implies a .17 percentage point increase in the proportion of high-risk loans made by the GSEs. In relatively inelastic MSAs, with elasticities close to 1, a 1 percentage point increase in the GSEs outstanding share in an MSA is associated with a .6 percentage point increase in the proportion of high-risk loans made by the GSEs.

Since we are exploiting variation in housing supply elasticity across MSAs to proxy for how sensitive house prices are to credit, our results are vulnerable to a recent argument made by (Davidoff (2015)). He argues that housing supply elasticities are correlated with unobserved housing demand as land-constrained areas (such as coastal cities) are also cities that people want to move into as there are desirable from a lifestyle perspective. Our particular focus on 2007 will likely not make our results unaffected by a greater housing demand in inelastic MSAs. In 2007, there was relatively greater housing demand in elastic rather than inelastic MSAs. We therefore do not have to worry about our results being driven by greater unobserved housing demand in inelastic MSAs (Davidoff (2015)).

We repeat our analysis for purchase and refinance loans separately. Columns (3)-(6) of table 4 report this. The effects are present in both cases but the magnitudes for refinance loans are higher. A 1 percentage point increase in the outstanding share of the GSEs in an MSA implies a 5.9 percentage point increase in the proportion of high-risk refinance loans made by the GSEs versus a 2.4 percentage point increase in the proportion of high-risk purchase
loans. In the presence of foreclosure externalities, house prices would be in danger of falling and refinancing loans can prevent this. This may be one factor driving the difference in magnitudes.

We also repeat this analysis separately for Fannie Mae and Freddie Mac instead of jointly looking at their outstanding share and new GSE originations. Tables 5 and 6 present the results of these regressions. We find similar results as before. Fannie Mae and Freddie Mac’s outstanding share predicts new high-LTV originations and this effect is stronger in inelastic MSAs.

The Sensitivity of Default to Prices: If the GSEs were attempting to prop-up house prices to avoid defaults on their outstanding portfolio of mortgages, this effect should be stronger when the outstanding mortgages they own are more sensitive to default. To test this, we include the high-LTV share of the GSE’s outstanding portfolio as an independent variable and run the following regressions,

\[ LTV_{80+}^{MSA} = \alpha + \beta_1 GSEshare_{MSA} + \beta_2 GSEportfolioLTV_{80+}^{MSA} + \epsilon_{MSA} \]

\( GSEportfolioLTV_{80+} \) is the high LTV fraction of the GSE’s outstanding portfolio. In line with propping-up, we find a positive and significant coefficient \( \beta_2 \). As before, we also run these regressions separately for purchase and refinance loans and for Fannie Mae and Freddie Mac (tables 7 and 8). The results are similar with the effects generally being stronger for refinancing loans than purchase loans. We also run the following specification to see if these effects are stronger in inelastic MSAs versus elastic MSAs,

\[ LTV_{80+}^{MSA} = \alpha + \beta_1 GSEshare_{MSA} + \beta_2 GSEportfolioLTV_{80+}^{MSA} + \beta_3 elasticity_{MSA} + \beta_4 GSEshare_{MSA} \ast elasticity_{MSA} + \beta_5 GSEportfolioLTV_{80+}^{MSA} \ast elasticity_{MSA} + \epsilon_{MSA} \]
Our results are consistent with these effects being stronger in inelastic MSAs in which house prices should be more sensitive to credit. We also obtain very high R-squares. We are able to explain almost 70% of the variation in the proportion of high-LTV loans made by the GSEs across MSAs.

*Change in Lending Standards Over Time:* Until now, all our analysis has been looking at how outstanding share at the start of 2007 predicted new high-risk lending. We now look at how lending standards *changed* within MSAs. We expect the decline in GSE lending standards to be concentrated in MSAs in which they had a large outstanding share of mortgages. We exploit a rule that if a mortgage was made to a borrower with an origination FICO score of 620 or above, it was eligible for the GSEs automated underwriting. Mortgages below the threshold, were subject to additional underwriting by the GSEs and therefore were screened more carefully. Assuming the mortgages to borrowers with origination FICOs just below the threshold had similar risk to mortgages just above the threshold, mortgages just below the threshold should be denied more often if the manual underwriting is stricter than the automated underwriting process. Keys et al. (2011) find that this is indeed the case - they show that there is a significant jump in the number of loans securitized above the threshold versus below and that the loans below the threshold seem to be better screened. Since the automated system worked similarly nationwide, the proportion of loans just below the threshold to loans just above the threshold tells us how strict the GSEs manual underwriting was in an MSA. The lower the proportion, the stricter their manual underwriting in the MSA. If the GSEs were increasing risky activity in MSAs in which they had a higher outstanding share, we should see that their manual underwriting screening is closer to their automated, laxer underwriting around the threshold. We should therefore see an increase in the proportion of loans below the threshold to loans above the threshold. This is equivalent to seeing a smoothing out of loans made around a 620 origination FICO. To test this, we run the following regression,
$$propchange_y = \alpha + \beta_1 \text{Fannieshare}_y + \beta_2 \text{elasticity}_y + \beta_3 \text{Fannieshare}_y \times \text{elasticity}_y + \epsilon_y$$

$$propchange_y = \alpha + \beta_1 \text{Freddieshare}_y + \beta_2 \text{elasticity}_y + \beta_3 \text{Freddieshare}_y \times \text{elasticity}_y + \epsilon_y$$

$propchange_y$ is the change in the proportion of loans below 620 to the loans above 620 in MSA $y$ between the first half of 2006 and the first half of 2007. The proportion of loans around the threshold is calculated by dividing the number of loans made to borrowers with origination FICO scores between 610 and 619 to the number of loans made to borrowers with origination FICO scores between 620 and 629. A positive $propchange_y$ means that relative to the first half of 2006, lending around the threshold has smoothed in the first half of 2007. $\text{Fannieshare}_y$ and $\text{Freddieshare}_y$ are Fannie Mae and Freddie Mac’s share of all outstanding mortgages in MSA $y$ at the beginning of 2007.

In line with our hypothesis, we find a positive and significant coefficient $\beta_1$ and a negative and significant coefficient $\beta_3$ for both Fannie Mae, shown in Table 10, and Freddie Mac, shown in Table 10. This implies that lending standards in 2007 fell in MSAs in which the GSEs had a large outstanding share, and this fall in lending standards is greater in relatively inelastic MSAs. As mentioned before, The average elasticity across MSAs in our sample is 2.46. Table 10 shows that a 1 percentage point increase in Freddie Mac’s outstanding share in an MSA with average elasticity is associated with a 1.3 percentage point increase in the proportion of loans made by Freddie Mac below the threshold relative to above the threshold between 2006 and 2007. In relatively inelastic MSAs with elasticities equal to one, a 1 percentage point increase in Freddie Mac’s outstanding share in an MSA is associated with a 2.9 percentage point increase in the proportion of loans made by Freddie
Mac below the threshold relative to above the threshold between 2006 and 2007 in an MSA with average elasticity. This magnitude is economically quite large and meaningful. For Fannie Mae, a 1 percentage point increase in its outstanding share in an MSA with average elasticity is associated with a .06 percentage point increase in the proportion of loans made by Fannie Mae below the threshold relative to above the threshold between 2006 and 2007. In inelastic MSAs, the magnitude of this effect increases substantially. In an MSA with an elasticity close to 1, this increases to 1.69 percentage points.

This runs counter to the narrative that the GSEs were rushing to grab market share in MSAs in which private-label markets were active and the GSEs had been pushed out, dropping lending standards to do so. We find that the agencies in fact relaxe lending standards more in MSAs in which they already had a large outstanding share of mortgages and in inelastic MSAs.

2.4. Conclusion

In this paper, we provide evidence that concentration in mortgage markets can affect the quality of credit. We test the hypothesis that institutions with a large outstanding exposure to the mortgage market have incentives to extend risky credit to prop up house prices. To discern the effect that different levels of mortgage market exposure can have on the quality of credit, we exploit variation in the size of GSE exposure across MSAs.

We find that in 2007 when the housing boom ended and house prices started falling, the GSEs increased high-risk mortgage activity in MSAs in which they had large outstanding exposure to the mortgage market. We perform a number of tests to discern whether this relationship is driven by incentives to keep house prices high and find evidence in line with our hypothesis. In particular we find the effect of outstanding share on high-risk activity is stronger in MSAs in which housing supply is relatively inelastic as it is in these MSAs that we expect credit expansion to have a strong effect on house prices. Furthermore, we also find that this effect is stronger when default rates are more sensitive to house prices. We
also provide evidence that lending standards worsened in MSAs in which the GSEs had a larger outstanding share.

Our paper has implications for the design on the housing market. We show that incentives to lend differ between large versus small mortgage market participants. In the aftermath of the financial crisis, policy makers have wanted to design policy measures that curb high-risk lending. Concentration in mortgage markets has been largely overlooked. In 2016, The Economist reported that markets are more concentrated now than they were during the housing boom and bust. Government organizations are funding about 65-80% of new mortgages and Quicken Loans and Freedom Mortgage are currently originating about half of all new mortgages. Many of these new mortgages appear to be highly risky with 20% of them having LTV ratios of over 95%. We hope to add to contribute to the macro-prudential policy discussion on how to curb high-risk lending through this paper.
2.5. Figures and Tables

GSE and Private-Label Share of High-Risk Activity

Figure 8: GSE and Private-Label Share of High-Risk Activity
The figure above plots the share of all high LTV loan originations, LTV > 80 at origination, that were GSE and private-label originated from 2005 to 2007. Source: McDash.

Proportion of High-Risk Originations of Total GSE Originations

Figure 9: Proportion of High-Risk Originations of Total GSE Originations
The figure above plots the proportion of high LTV originations, LTV > 80 at origination, as a fraction of total GSE originations from 2005 to 2007. Source: McDash.
GSE and Private-Label High-Risk Originations

The figure above plots the number of high LTV loan originations, LTV > 80 at origination, by the GSEs and private-label from 2005 to 2007. Source: McDash.

GSE and Private-Label Originations, LTV ≤ 80

The figure above plots the number of loan originations, LTV ≤ 80 at origination, by the GSEs and private-label from 2005 to 2007. Source: McDash.
Figure 12: GSE and Private-Label Termination Rates of High-Risk Loans
The figure above plots the proportion of high LTV loans, LTV > 80 at origination, that subsequently had a bad termination from 2006 to 2007, benchmarked to the first quarter of 2006. The x-axis is the quarter of origination. Source: McDash.

Figure 13: Fannie Mae, Freddie Mac and Private-Label Termination Rates of High-Risk Loans
The figure above plots the proportion of high LTV loans, LTV > 80 at origination, that subsequently had a bad termination from 2006 to 2007, benchmarked to the first quarter of 2006. The x-axis is the quarter of origination. Source: McDash.
Variation in GSE Outstanding Share by MSA

The histogram above shows the variation in the GSE’s outstanding share across MSAs at the start of 2007. Source: McDash.

Variation in Fannie Mae’s and Freddie Mac’s Outstanding Share by MSA

The histograms above show the variation in the Fannie Mae’s (left panel) and Freddie Mac’s (right panel) outstanding share across MSAs at the start of 2007. Source: McDash.
Figure 16: Variation in Housing Supply Elasticities by MSA

The histogram above shows the variation in housing supply elasticities taken from Saiz (2010). Source: McDash.
Table 1: Summary of GSE and Private-Label Mortgages Outstanding at start of 2007

<table>
<thead>
<tr>
<th>MSA-Level Mortgage Market Statistics</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Min</th>
<th>Max</th>
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</thead>
<tbody>
<tr>
<td>GSE Share of Mortgage Market</td>
<td>.63</td>
<td>.10</td>
<td>.25</td>
<td>.88</td>
</tr>
<tr>
<td>Private Share of Mortgage Market</td>
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<td>.06</td>
<td>.06</td>
<td>.41</td>
</tr>
<tr>
<td>Average LTV across MSA</td>
<td>.58</td>
<td>.06</td>
<td>.43</td>
<td>.71</td>
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<tr>
<td>High LTV (&gt; 80%) Fraction</td>
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<td>.08</td>
<td>.03</td>
<td>.39</td>
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<tr>
<td>Low Origination FICO (&lt; 660) Fraction</td>
<td>.25</td>
<td>.08</td>
<td>.03</td>
<td>.49</td>
</tr>
</tbody>
</table>

**GSE**

| Average LTV across MSA               | .54  | .07                | .31 | .68 |
| High LTV (> 80%) Fraction            | .10  | .05                | .02 | .29 |
| Low Origination FICO (< 660) Fraction| .15  | .05                | .05 | .28 |

**Private-Label**

| Average LTV across MSA               | .67  | .05                | .45 | .79 |
| High LTV (> 80%) Fraction            | .24  | .11                | .04 | .59 |
| Low Origination FICO (< 660) Fraction| .45  | .13                | .11 | .72 |

Source: McDash and Corelogic HPI™
Table 2: Summary of GSE and Private-Label Originations in first half of 2007

<table>
<thead>
<tr>
<th>MSA-Level Mortgage Market Statistics</th>
</tr>
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<tbody>
<tr>
<td>GSE Share of Mortgage Market</td>
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<tr>
<td>Private Share of Mortgage Market</td>
</tr>
<tr>
<td>High LTV (&gt; 80%) Fraction</td>
</tr>
<tr>
<td>Low Origination FICO (&lt; 660) Fraction</td>
</tr>
</tbody>
</table>

**GSE**

| High LTV (> 80%) Fraction                                                                           | .30 | .12 | .02 | .60 |
| Low Origination FICO (< 660) Fraction                                                              | .21 | .07 | .06 | .49 |

**Private-Label**

| High LTV (> 80%) Fraction                                                                           | .35 | .14 | .03 | .77 |
| Low Origination FICO (< 660) Fraction                                                              | .47 | .15 | .08 | .85 |

Source: McDash
Table 3: Summary of GSE and Private-Label Originations in first half of 2007 broken up by quarter

<table>
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<th>Mean</th>
<th>Standard Deviation</th>
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<th>Max</th>
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<tr>
<td><strong>MSA-Level Mortgage Market Statistics</strong></td>
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<tr>
<td>GSE Share of Mortgage Market</td>
<td>.68</td>
<td>.09</td>
<td>.31</td>
<td>.88</td>
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<tr>
<td>Private Share of Mortgage Market</td>
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<td>.03</td>
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<td>Low Origination FICO (&lt; 660) Fraction</td>
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<td>.08</td>
<td>.10</td>
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<tr>
<td><strong>GSE</strong></td>
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<td>High LTV (&gt; 80%) Fraction</td>
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<td>Low Origination FICO (&lt; 660) Fraction</td>
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<td>.07</td>
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<td><strong>Private-Label</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>High LTV (&gt; 80%) Fraction</td>
<td>.35</td>
<td>.15</td>
<td>.02</td>
<td>.80</td>
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<tr>
<td>Low Origination FICO (&lt; 660) Fraction</td>
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<td><strong>MSA-Level Mortgage Market Statistics</strong></td>
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<td>GSE Share of Mortgage Market</td>
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<td>Private Share of Mortgage Market</td>
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<td>.08</td>
<td>.60</td>
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<td>High LTV (&gt; 80%) Fraction</td>
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<td>.06</td>
<td>.66</td>
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<tr>
<td>Low Origination FICO (&lt; 660) Fraction</td>
<td>.34</td>
<td>.08</td>
<td>.12</td>
<td>.59</td>
</tr>
<tr>
<td><strong>GSE</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High LTV (&gt; 80%) Fraction</td>
<td>.27</td>
<td>.12</td>
<td>.01</td>
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<tr>
<td>Low Origination FICO (&lt; 660) Fraction</td>
<td>.20</td>
<td>.06</td>
<td>.06</td>
<td>.49</td>
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<td><strong>Private-Label</strong></td>
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<tr>
<td>High LTV (&gt; 80%) Fraction</td>
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<td>.02</td>
<td>.76</td>
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<tr>
<td>Low Origination FICO (&lt; 660) Fraction</td>
<td>.51</td>
<td>.16</td>
<td>.06</td>
<td>.90</td>
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</table>

Source: McDash
### Table 4: The Effect of Outstanding Loans and Price Elasticities on GSE High-Risk Originations

The table reports the results for the following regressions,

\[
LTV_{MSA}^{80} = \alpha + \beta GSEshare_{MSA} + \epsilon_{MSA}
\]

\[
LTV_{MSA}^{(e)} = \alpha + \beta_1 GSEshare_{MSA} + \beta_2 elasticity_{MSA} + \beta_3 GSEshare_{MSA} \times elasticity_{MSA} + \epsilon_{MSA}
\]

$LTV_{y}^{80}$ is the proportion of mortgages originated by the GSEs in the first half of 2007 that had LTV>80 in MSA $y$. $GSEshare_{y}$ is the GSE share of all outstanding mortgages in MSA $y$ at the beginning of 2007. $elasticity_{y}$ is the Saiz measure of elasticity in MSA $y$.

<table>
<thead>
<tr>
<th></th>
<th>Purchase Loans</th>
<th>Refinance Loans</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>GSEshare</td>
<td>.48**</td>
<td>.91***</td>
</tr>
<tr>
<td></td>
<td>(2.44)</td>
<td>(4.70)</td>
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<tr>
<td>elasticity</td>
<td>.26***</td>
<td>.13***</td>
</tr>
<tr>
<td></td>
<td>(5.75)</td>
<td>(3.57)</td>
</tr>
<tr>
<td>GSEshare x elasticity</td>
<td>−.30***</td>
<td>−.14**</td>
</tr>
<tr>
<td></td>
<td>(−4.27)</td>
<td>(−2.35)</td>
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<tr>
<td>Observations</td>
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<td>234</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.12</td>
<td>.62</td>
</tr>
</tbody>
</table>

***p < 0.01, **p < 0.05, *p < 0.1

Standard errors calculated by weighting MSAs by the number of originations

Source: McDash
Table 5: The Effect of Outstanding Loans and Price Elasticities on Fannie Mae High-Risk Originations

The table reports the results for the following regressions,

\[ LTV_{80}^{\uparrow \downarrow} = \alpha + \beta Fannieshare_{MSA} + \epsilon_{MSA} \]
\[ LTV_{80}^{\uparrow \downarrow} = \alpha + \beta_1 Fannieshare_{MSA} + \beta_2 elasticity_{MSA} + \beta_3 Fannieshare_{MSA} \times elasticity_{MSA} + \epsilon_{MSA} \]

$LTV_{80}^{\uparrow \downarrow}$ is the proportion of mortgages originated by Fannie Mae in the first half of 2007 that had LTV>80 in MSA $y$. $Fannieshare_y$ is Fannie Mae’s share of all outstanding mortgages in MSA $y$ at the beginning of 2007. $elasticity_y$ is the Saiz measure of elasticity in MSA $y$.

<table>
<thead>
<tr>
<th></th>
<th>Purchase Loans</th>
<th>Refinance Loans</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) (2) (3) (4) (5) (6)</td>
<td></td>
</tr>
<tr>
<td>Fannieshare</td>
<td>0.27 1.18*** 0.13 0.58* 0.41 1.16***</td>
<td></td>
</tr>
<tr>
<td>(0.91)</td>
<td>(2.50) (0.77) (1.61) (1.35) (2.91)</td>
<td></td>
</tr>
<tr>
<td>elasticity</td>
<td>0.24*** 0.10** 0.24***</td>
<td></td>
</tr>
<tr>
<td>(3.65)</td>
<td>(2.20) (4.36)</td>
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<tr>
<td>Fannieshare x elasticity</td>
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<td></td>
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<tr>
<td>(−2.49)</td>
<td>(−1.15) (−2.97)</td>
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<tr>
<td>Observations</td>
<td>275 234 275 234 275 234</td>
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<tr>
<td>R-squared</td>
<td>0.01 0.55 0.01 0.33 0.04 0.57</td>
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</tbody>
</table>

***$p < 0.01$, **$p < 0.05$, *$p < 0.1$

Standard errors calculated by weighting MSAs by the number of originations

Source: McDash
Table 6: The Effect of Outstanding Loans and Price Elasticities on Freddie Mac High-Risk Originations

The table reports the results for the following regressions,

\[
LTV_{80, y}^{MSA} = \alpha + \beta \text{Freddieshare}_{MSA} + \epsilon_{MSA}
\]

\[
LTV_{80, y}^{MSA} = \alpha + \beta_1 \text{Freddieshare}_{MSA} + \beta_2 \text{elasticity}_{MSA} + \beta_3 \text{Freddieshare}_{MSA} \times \text{elasticity}_{MSA} + \epsilon_{MSA}
\]

\(LTV_{80, y}^{+}\) is the proportion of mortgages originated by Freddie Mac in the first half of 2007 that had \(LTV > 80\) in MSA \(y\). \(\text{Freddieshare}_y\) is Freddie Mac’s share of all outstanding mortgages in MSA \(y\) at the beginning of 2007. \(\text{elasticity}_y\) is the Saiz measure of elasticity in MSA \(y\).

<table>
<thead>
<tr>
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<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
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</thead>
<tbody>
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<td>Freddieshare</td>
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<td>.98***</td>
<td>.32**</td>
<td>.34*</td>
<td>.95***</td>
<td>1.20***</td>
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<tr>
<td></td>
<td>(3.24)</td>
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<td>elasticity</td>
<td>.13***</td>
<td>.06***</td>
<td>.16***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(6.49)</td>
<td>(3.82)</td>
<td>(7.68)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Freddieshare x elasticity</td>
<td>–.31***</td>
<td>–.09</td>
<td>–.41***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(–3.77)</td>
<td>(–1.42)</td>
<td>(–4.84)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Observations</td>
<td>275</td>
<td>234</td>
<td>275</td>
<td>234</td>
<td>275</td>
<td>234</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.16</td>
<td>.57</td>
<td>0.04</td>
<td>.29</td>
<td>0.24</td>
<td>.64</td>
</tr>
</tbody>
</table>

***\(p < 0.01\), **\(p < 0.05\), *\(p < 0.1\)

Standard errors calculated by weighting MSAs by the number of originations

Source: McDash
Table 7: The Effect of Outstanding Portfolio Composition on GSE High-Risk Activity

The table reports the results for the following regressions,

$$LTV_{80}^{+}\_{MSA} = \alpha + \beta_1 GSEshare_{MSA} + \beta_2 GSEportfolioLTV_{80}^{+}\_{MSA} + \epsilon_{MSA}$$

$$LTV_{80}^{+}\_{MSA} = \alpha + \beta_1 GSEshare_{MSA} + \beta_2 GSEportfolioLTV_{80}^{+}\_{MSA} + \beta_3 \text{elasticity}_{MSA} +$$
$$\beta_4 GSEshare_{MSA} \times \text{elasticity}_{MSA} + \beta_5 GSEportfolioLTV_{80}^{+}\_{MSA} \times \text{elasticity}_{MSA} + \epsilon_{MSA}$$

$LTV_{80}^{+}$ is the proportion of mortgages originated by the GSEs in the first half of 2007 that had LTV$>80$ in MSA $y$. $GSEshare_{y}$ is the GSE share of all outstanding mortgages in MSA $y$ at the beginning of 2007. $GSEportfolioLTV_{80}^{+}$ is the high LTV fraction of the GSE’s outstanding portfolio. $\text{elasticity}_{y}$ is the Saiz measure of elasticity in MSA $y$.

<table>
<thead>
<tr>
<th></th>
<th>Purchase Loans</th>
<th></th>
<th>Refinance Loans</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>GSEshare</td>
<td>.29***</td>
<td>.73***</td>
<td>.14*</td>
<td>.42**</td>
</tr>
<tr>
<td></td>
<td>(2.14)</td>
<td>(3.96)</td>
<td>(1.62)</td>
<td>(2.39)</td>
</tr>
<tr>
<td>GSEportfolioLTV80$^+$</td>
<td>.13***</td>
<td>.79***</td>
<td>1.01***</td>
<td>.51**</td>
</tr>
<tr>
<td></td>
<td>(4.16)</td>
<td>(3.05)</td>
<td>(5.32)</td>
<td>(2.12)</td>
</tr>
<tr>
<td>elasticity</td>
<td>.22***</td>
<td>.09***</td>
<td>.22***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5.42)</td>
<td>(2.71)</td>
<td>(5.72)</td>
<td></td>
</tr>
<tr>
<td>GSEshare x elasticity</td>
<td>$- .25***$</td>
<td>$- .12**$</td>
<td>$- .24***$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>($- 3.81$)</td>
<td>($- 2.18$)</td>
<td>($- 3.70$)</td>
<td></td>
</tr>
<tr>
<td>GSEshareLTV80$^+$ x elasticity</td>
<td>$- .05$</td>
<td>$- .11$</td>
<td>$- .11$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>($- .53$)</td>
<td>($.89$)</td>
<td>($- 1.08$)</td>
<td></td>
</tr>
</tbody>
</table>

Observations 269 230 269 230 269 230
R-squared .46 .70 .37 .49 .37 .73

***$p < 0.01$, **$p < 0.05$, *$p < 0.1$

Standard errors calculated by weighting MSAs by the number of originations. Source: McDash
Table 8: The Effect of Outstanding Portfolio Composition on Fannie Mae High-Risk Activity

The table reports the results for the following regressions,

$$LTV_{80}^{+\text{MSA}} = \alpha + \beta_1 Fannieshare_{MSA} + \beta_2 \text{FannieportfolioLTV}_{80}^{+\text{MSA}} + \epsilon_{MSA}$$

$$LTV_{80}^{+\text{MSA}} = \alpha + \beta_1 Fannieshare_{MSA} + \beta_2 \text{FannieportfolioLTV}_{80}^{+\text{MSA}} + \beta_3 \text{elasticity}_{MSA} + \beta_4 Fannieshare_{MSA} \times \text{elasticity}_{MSA} + \beta_5 \text{FannieportfolioLTV}_{80}^{+\text{MSA}} \times \text{elasticity}_{MSA} + \epsilon_{MSA}$$

$LTV_{80}^{+\text{y}}$ is the proportion of mortgages originated by Fannie Mae in the first half of 2007 that had $LTV > 80$ in MSA $y$. $GSEshare_{y}$ is Fannie’s share of all outstanding mortgages in MSA $y$ at the beginning of 2007. $\text{FannieportfolioLTV}_{80}^{+\text{y}}$ is the high LTV fraction of Fannie’s outstanding portfolio. $\text{elasticity}_{y}$ is the Saiz measure of elasticity in MSA $y$.

<table>
<thead>
<tr>
<th></th>
<th>Purchase Loans</th>
<th>Refinance Loans</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Fannieshare</td>
<td>-.03</td>
<td>.64</td>
</tr>
<tr>
<td></td>
<td>(-.14)</td>
<td>(1.53)</td>
</tr>
<tr>
<td>FannieportfolioLTV$_{80}^+$</td>
<td>1.49***</td>
<td>1.21***</td>
</tr>
<tr>
<td></td>
<td>(5.03)</td>
<td>(3.82)</td>
</tr>
<tr>
<td>elasticity</td>
<td>.18***</td>
<td>.04</td>
</tr>
<tr>
<td></td>
<td>(3.02)</td>
<td>(.92)</td>
</tr>
<tr>
<td>Fannieshare $\times$ elasticity</td>
<td>-.24*</td>
<td>-.04</td>
</tr>
<tr>
<td></td>
<td>(-1.74)</td>
<td>(-.39)</td>
</tr>
<tr>
<td>FannieshareLTV$_{80}^+$ $\times$ elasticity</td>
<td>-.23**</td>
<td>.00</td>
</tr>
<tr>
<td></td>
<td>(-2.08)</td>
<td>(.03)</td>
</tr>
<tr>
<td>Observations</td>
<td>269</td>
<td>230</td>
</tr>
<tr>
<td>R-squared</td>
<td>.47</td>
<td>.66</td>
</tr>
</tbody>
</table>

***$p < 0.01$, **$p < 0.05$, *$p < 0.1$

Standard errors calculated by weighting MSAs by the number of originations. Source: McDash
Table 9: The Effect of Outstanding Portfolio Composition on Freddie Mac High-Risk Activity

The table reports the results for the following regressions,

\[
LTV_{80}^{+\text{MSA}} = \alpha + \beta_1 \text{Freddieshare}_{\text{MSA}} + \beta_2 \text{FreddieportfolioLTV}^{+\text{MSA}} + \epsilon_{\text{MSA}}
\]

\[
LTV_{80}^{+\text{MSA}} = \alpha + \beta_1 \text{Freddieshare}_{\text{MSA}} + \beta_2 \text{FreddieportfolioLTV}^{+\text{MSA}} + \beta_3 \text{elasticity}_{\text{MSA}} + \beta_4 \text{Freddieshare}_{\text{MSA}} \times \text{elasticity}_{\text{MSA}} + \beta_5 \text{FreddieportfolioLTV}^{+\text{MSA}} \times \text{elasticity}_{\text{MSA}} + \epsilon_{\text{MSA}}
\]

\(LTV_{80}^{+\text{y}}\) is the proportion of mortgages originated by Freddie Mac in the first half of 2007 that had \(LTV > 80\) in MSA \(y\). \(GSEshare_{y}\) is Freddie’s share of all outstanding mortgages in MSA \(y\) at the beginning of 2007. \(\text{FreddieportfolioLTV}^{+\text{y}}\) is the high LTV fraction of Freddie’s outstanding portfolio. \(\text{elasticity}_{y}\) is the Saiz measure of elasticity in MSA \(y\).

| | Purchase Loans | | Refinance Loans | |
|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Freddieshare | .67*** | 1.02*** | .30*** | .49*** | .89*** | 1.16*** |
| | (4.48) | (4.79) | (3.57) | (2.56) | (5.42) | (5.93) |
| FreddieportfolioLTV80+ | 1.06*** | .61*** | .90*** | .30*** | 1.02*** | .72*** |
| | (4.47) | (3.39) | (4.90) | (1.52) | (4.65) | (3.86) |
| elasticity | .11*** | .04 | .14*** |
| | (6.93) | (2.52) | (8.10) |
| Freddieshare x elasticity | −.30*** | −.13** | −.37*** |
| | (−4.63) | (−2.33) | (−5.41) |
| FreddieshareLTV80+ x elasticity | −.05 | .20** | −.06 |
| | (.61) | (1.97) | (−.66) |
| Observations | 269 | 230 | 269 | 230 | 269 | 230 |
| R-squared | .43 | .68 | .27 | .41 | .49 | .72 |

***\(p < 0.01\), **\(p < 0.05\), *\(p < 0.1\)

Standard errors calculated by weighting MSAs by the number of originations. Source: McDash
Table 10: The Effect of Outstanding Loans on Lending Standards for Fannie Mae

The table reports the output for the following two regressions,

\[ \text{propchange}_y = \alpha + \beta_1 \text{Fannieshare}_y + \epsilon_y \]

\[ \text{propchange}_y = \alpha + \beta_1 \text{Fannieshare}_y + \beta_2 \text{elasticity}_y + \beta_3 \text{Fannieshare}_y \times \text{elasticity}_y + \epsilon_y \]

\( \text{propchange}_y \) is the change in the proportion of loans below 620 to the loans above 620 in MSA \( y \) between 2006h1 and 2007h1. The proportion of loans around the threshold is calculated by dividing the number of loans made to borrowers with origination FICO scores between 610 and 619 to the number of loans made to borrowers with origination FICO scores between 620 and 629. A positive \( \text{propchange}_y \) means that relative to the first half of 2006, lending around the threshold has smoothed in the first half of 2007. \( \text{Fannieshare}_y \) is Fannie’s share of all outstanding mortgages in MSA \( y \) at the beginning of 2007. \( \text{elasticity}_y \) is the Saiz measure of elasticity in MSA \( y \).

\[ \begin{array}{ccc}
(1) & (2) \\
\text{propchange} & \text{propchange} \\
\hline
\text{Fannieshare} & .24 & 2.80^{**} \\
& (.36) & (2.25) \\
\text{elasticity} & .46^{***} & \\
& (3.43) & \\
\text{Fannieshare} \times \text{elasticity} & -1.11^{***} & \\
& (-3.03) & \\
\text{Observations} & 273 & 232 \\
\text{R-squared} & 0.00 & .07 \\
\end{array} \]

\(*^{***} p < 0.01, ** p < 0.05, * p < 0.1\)

Source: McDash
Table 11: The Effect of Outstanding Loans on Lending Standards for Freddie Mac

The table reports the output for the following two regressions,

\[ \text{propchange}_y = \alpha + \beta_1 \text{Freddieshare}_y + \epsilon_y \]

\[ \text{propchange}_y = \alpha + \beta_1 \text{Freddieshare}_y + \beta_2 \text{elasticity}_y + \beta_3 \text{Freddieshare}_y \times \text{elasticity}_y + \epsilon_y \]

\( \text{propchange}_y \) is the change in the proportion of loans below 620 to the loans above 620 in MSA \( y \) between 2006h1 and 2007h1. The proportion of loans around the threshold is calculated by dividing the number of loans made to borrowers with origination FICO scores between 610 and 619 to the number of loans made to borrowers with origination FICO scores between 620 and 629. A positive \( \text{propchange}_y \) means that relative to the first half of 2006, lending around the threshold has smoothed in the first half of 2007. \( \text{Freddieshare}_y \) is Freddie’s share of all outstanding mortgages in MSA \( y \) at the beginning of 2007. \( \text{elasticity}_y \) is the Saiz measure of elasticity in MSA \( y \).

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
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<tbody>
<tr>
<td>propchange</td>
<td>.47</td>
<td>3.91**</td>
</tr>
<tr>
<td>propchange x elasticity</td>
<td>(.67)</td>
<td>(2.13)</td>
</tr>
<tr>
<td>elasticity</td>
<td>.22*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.87)</td>
<td></td>
</tr>
<tr>
<td>Freddieshare x elasticity</td>
<td></td>
<td></td>
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<tr>
<td>Observations</td>
<td>270</td>
<td>230</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.00</td>
<td>.01</td>
</tr>
</tbody>
</table>

***\( p < 0.01 \), **\( p < 0.05 \), *\( p < 0.1 \)

Source: McDash
3.1. Introduction

Many policies aim to subsidize investment in housing. A unique feature of subsidization schemes in the housing market, as opposed to other markets, is that policy makers often care not just about the amount of investment in housing but also about the impact of policy on the level of house prices. In fact, following the collapse of the housing market in 2008, many government interventions such as the large scale repurchase of mortgage-backed securities were made with the explicit goal of supporting housing prices. The level of housing prices matters since real estate equity is considered to be an important tool for investment and wealth accumulation. The rationale behind this argument is that households and firms obtain a durable asset when they purchase real estate and they can use this asset to obtain funds to invest and build up their wealth. In a 2013 policy brief, the White House explained the importance of these policy measures in stimulating the economy, “Housing wealth is growing again, with owners’ equity up $2.8 trillion since hitting a low at the beginning of 2009. This in turn has contributed to increased economic activity through consumer spending, small business investment, and more.”

The typical channel through which high house prices lead to increased investment and wealth accumulation is the collateral channel. Households and firms are able to borrow against the value of real estate assets that they own and spend or invest the proceeds. This leads to policy goals by the government taking the form of supporting real estate investment and keeping house prices high in an attempt to keep the value of real estate equity high. However, empirical research on the effect of high house prices on investment and wealth accumulation has found mixed results. Chaney, Sraer, and Thesmar (2012) and Gan (2007) find a positive effect of increases in collateral value on investment of firms who own real estate assets. On the other hand, recent work by Chakraborty, Goldstein, and MacKinlay
(2014) and Jorda et al. (2014) find that banks in areas that experienced a significant real estate price boom, increased mortgage lending while simultaneously decreasing commercial lending making the net effect of high house prices on investment ambiguous.

The mixed empirical evidence suggests that the collateral effect is not able to capture all the dynamics of how increased house prices affect investment. Furthermore, standard economic theory on subsidization policies typically cares about the final quantity of the good in the market in question and not the level of prices.\(^1\) Since in housing markets the level of prices is often a policy goal in itself, this approach may not apply to evaluating subsidization schemes in the housing market. These issues call for a comprehensive framework for studying the effect of house prices on investment and household wealth accumulation, the value of policies trying to support high house prices and the form that housing market policies should take.

In this paper, we develop a model to understand the effect of house prices on investment and highlight a novel channel that can cause high house prices to crowd-out investment in the economy. In the model, a firm borrows from banks to invest in real estate and in its own projects. The firm is financially constrained because of an inability to commit to repaying loans using future income and therefore may not be able to invest in all positive investment opportunities. The firm can use real estate as collateral to help it obtain loans from a bank and can thereby relax its financial constraint. Real estate thus serves a dual purpose as an asset in the model - giving the firm returns on investing and also relaxing the firm’s borrowing constraint.

The market price of housing generates externalities on investment and subsequently household wealth accumulation when the firm is financially constrained and cannot invest in all productive NPV opportunities. An increase in housing prices allows firms to borrow more against home-equity thus generating a positive externality on the funds available for investment. However, an increase in housing prices also causes the cost of investment in real

\(^1\)In other markets in which subsidies are common, such as staple foods or healthcare, the final policy goal is usually about broad access to the good. Prices matter in only so far as how they affect the final level of access to the good in question.
estate to increase taking away funds that could be used to invest in the firm’s own projects. This generates a negative externality on investment. When the firm is a price taker and is financially constrained, these externalities prevent the first-best level of investment from being achieved.

The key novel insight of the model is that an increase in real estate prices does not necessarily increase investment even when the firm actively uses its real estate assets as collateral to borrow against and invests the proceeds in positive NPV projects. In our model the typical narrative of rising real estate prices helping to relax borrowing constraints by increasing the value of collateral and thus encouraging greater lending and investment is indeed true. However in addition to this, we also show that the effect of an increase in real estate prices on aggregate investment can be negative due to a possible crowding-out of firm project investment as firms are incentivized to invest more in real estate. While an increase in real estate prices relaxes borrowing constraints, it also increases the amount of money that has to be spent on new real estate purchases. This second effect takes resources away from investment in firm projects in the presence of financial constraints and if big enough can decrease the aggregate level of investment in the economy, subsequently reducing household wealth.

This negative investment effect of a boom in asset prices is in opposition to the collateral effect that is usually discussed in the literature. Our model is able to reconcile the differing empirical findings of the collateral effect on investment since the effect of high housing prices on investment can be positive or negative depending on the magnitude of the crowding-out effect relative to the collateral effect. In particular, when the existing amount of real estate equity is low before a house price boom and the future returns from investing in new real estate assets are high, the crowding-out effect is relatively stronger than the collateral effect and house price booms are bad for aggregate investment.

Our paper relates to recent studies on the inefficiencies that can arise from asset price booms when agents do not internalize their effect on the price of assets. Lorenzoni (2008) develops
a model in which credit booms lead to inefficient borrowing because atomistic entrepreneurs do not internalize pecuniary fire sale externalities. In this case, once a crisis hits there is excessive contraction in investment and asset prices. Bianchi and Mendoza (2010) propose a model in which agents over-borrow when they do not internalize how their borrowing decisions affect the price of an asset that is also used as collateral. Private agents do not internalize that fire sale of assets can cause a debt-deflation spiral leading to a large decline in asset prices and a shrinking of the economy’s ability to borrow.

In all these papers, externalities arise because financial constraints depend on the market value of assets and agents do not internalize how their choices affect these market values. In these papers, negative effects of agent actions on investment are not observed during the asset price boom. Rather, the negative effects on investment are realized when a bust occurs due to fire sale externalities. In contrast, in our model, the negative effects of price booms on investment are realized during the boom phase itself, because investment in firm projects can be crowded-out due to high house prices leading to under-investment during the boom period. Our mechanism can thus help explain the findings in Chakraborty et al. (2014) since the authors find that under-investment in some firms occurred during the house price boom. In recent work, Hurst et al. (2016) find that the housing price boom reduced college enrollment as young men instead chose to work in the construction sector, also supporting a crowding-out story. Papers in which negative investment effects are realized only during the bust of asset prices, are not able to explain these results.

Additionally, the literature has so far focused on how collateral constraints can generate inefficiencies when agents are atomistic price-takers. Our paper focuses on inefficiencies that come through both the collateral constraint and the budget constraint. The crowding-out channel arises because firms don’t internalize that investing in real estate will cause house prices to increase, tightening their budget constraints and leaving less money to invest in their own projects. In our model the representative household cannot redistribute the profits of construction companies to firms that are purchasing real estate causing this new budget
constraint externality. In our model, we do not have heterogeneous agents and simply require that the representative household is constrained in that they cannot immediately redirect the funds that the firm spends on real estate back to the firm. Intuitively, this means that when a firm buys a house, it does not immediately receive the amount it paid for the house back to fund more investment. We believe this to be a constraint in a representative agent framework that captures a realistic friction that is often present when firms make investment decisions and as we demonstrate in the paper, this can give rise to important externalities through the budget constraint.

In our model, there may be either over- or under-investment depending on whether the crowding-out effect or the collateral effect is stronger. In a similar vein, Davila and Korinek (2017) develop a model in which heterogenous atomistic agents face financial constraints and also obtain either over- or under-borrowing. Alongside firesale externalities, they explore a second set of externalities which can lead to under-borrowing and investment in equilibrium called distributive externalities. These arise because agents do not internalize how equilibrium price changes affect the consumption and investment decisions of other agents which may prevent optimal risk-sharing in the economy. In our framework, we abstract away from fire sale externalities and also do not have distributive externalities as all agents are risk-neutral and there is no uncertainty. Instead, we propose a new externality that can also give rise to under-investment, i.e. the crowding-out effect.

We additionally analyze the optimal design of housing subsidization programs when governments wish to build household wealth and stimulate investment in the economy. Subsidization policies are often targeted at increasing the demand for mortgages by either incentivizing banks to lend more to the mortgage market or incentivizing the purchase of real estate. Examples include the mortgage interest tax deduction, tax credits to first-time home buyers, low insurance payments to qualify for government guarantees on mortgages in the United States and right-to-buy and help-to-buy schemes in the United Kingdom. Contrary to this 

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2This friction is similar to a constraint in empirical banking papers in which the representative household does not internalize that they are both the equity and debt holders of banks.
common practice, our analysis demonstrates that in many cases, it can be preferable to actually tax investment in real estate and subsidize the supply of housing instead. Such subsidies include but are not limited to subsidizing construction companies directly, a tax-credit to the sellers of houses and decreasing red-tape around land rights.

An important insight of the model is that demand- and supply-side mortgage subsidization policies affect real estate and firm project investment in distinct ways. Interestingly this result goes against the traditional economic insight that it does not matter what side of the market is taxed or subsidized - the real effects of such interventions are the same. The difference between the two interventions arises in the model because of their opposite effects on the price of housing. While demand-side subsidies increase the price of housing by shifting the demand curve out, supply-side subsidies decrease the price of housing by shifting the housing supply curve out. In the presence of financial constraints and price externalities from housing, the real effects in the economy are not price-insensitive.

Supply- and demand- side subsidies are not equivalent in the model because they affect the agents’ budget constraints differently when agents are financially constrained through their effect on housing prices. When the collateral effect is stronger than the crowding out effect, an increase in real estate prices helps encourage investment by relaxing borrowing constraints since the value of the agents’ collateral increases. This makes demand-side subsidies preferable. When the crowding-out effect dominates, a decrease in prices makes housing cheaper for the agent to invest in thus aiding the financially constrained agent. This channel makes supply-side subsidies preferable. Interestingly, we find that in our setting, a social planner will want to combine expansionary supply-side policies like subsidies to construction companies with contractionary demand-side policies such as a tax on household real-estate investment and vice-versa. In particular, supply-side subsidies are optimally combined with demand-side taxes when the return from real-estate investment is high or alternatively when the government wishes to increase purchases of new houses over increasing existing home-owner equity. This leads to a rather counter-intuitive implication
of our framework - when the government believes that there are high returns from investment in housing, it is optimal to tax investment in housing and subsidize the production of housing due to the externalities generated by housing prices.

The model can also shed light on existing interventions in the housing market. Our model predicts that policies such as the mortgage interest rate tax deduction, which affect the demand-side of the housing market, may be ineffective in increasing home-ownership rates since they increase the price of purchasing housing. Glaeser and Shapiro (2002), Hilber and Turner (2014) and Hanson (2012) find evidence consistent with this. On the other hand, the model predicts demand-side interventions which increase the price of housing should help existing home-owners increase investment and consumption due to an increase in home-owner equity. Keys, Piskorski, Seru, and Yao (2014) find evidence consistent with this when examining demand-side interventions (lower mortgage rates) that targeted indebted households. They find that households experience an increase in home-owner equity following such interventions which they use to increase durable spending.3

Related Literature: The negative real crowding-out effect of price booms we find in our paper relates to Tirole (1985) in which asset prices bubbles crowd out productive real investment by raising interest rates and reducing firm incentives to invest. In a similar vein, Farhi and Tirole (2012) find that the rise in interest rates might further restrict credit availability for financially constrained firms. In our paper, we do not require a bubble to produce the negative real effects accompanying a price boom. We simply require that the agents is financially constrained. Our paper also contributes to the macro literature which tries to understand the role of asset prices for the real economy and how price changes amplify shocks to investment. Seminal papers in this field such as Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) discuss the amplification of negative shocks to asset prices when these assets also serve as collateral for financially constrained agents. Our proposed mechanism suggests that asset price booms may also serve to cause negative

3Also see Agarwal, Amromin, Chomsisengphet, Piskorski, Seru, and Yao (2015) and Agarwal, Amromin, Ben-david, Chomsisengphet, Piskorski, and Seru (2016).
shocks to investment.

Theoretically, our model highlights a substitution effect amongst investments in a world with financial constraints which resembles the papers on internal capital markets by Stein (1997) and Scharfstein and Stein (2000). In these papers, a financially constrained headquarters makes an investment decision on how to allocate resources across divisions. A positive shock to the investment opportunities in one division diverts resources away from other divisions. In our model, agents face similar constrains and allocate resources to mortgages at the expense of firm investment following price appreciations.

We also contribute to the way different policies affect household debt. Policies that decrease the price of housing lead to the household having a smaller debt burden than policies that increase the price of housing. This is because the crowding-out effect encourages investment by effectively making the household richer while the collateral effect channel increases investment by allowing the household to borrow more. In our framework, there are no negative consequences of taking on debt. However, many recent papers have highlighted the role high household debt plays in generating fragility and deeper recessions (Mian and Sufi (2011), Mian, Sufi, and Verner (2015), Shularick and Taylor (2012)). Our paper can contribute to this strand of literature by highlighting the effect different housing policies have on household debt.

Finally our paper contributes to a large and body of literature in economics on optimal subsidies and taxation policies. To the best of our knowledge, there are not many papers that consider the differences in subsidizing the supply versus the demand side of housing markets. Glaeser, Gyourko, and Saiz (2008) argue that we must consider the supply side of the housing market to understand fluctuations in housing cycles. They present a model in which areas with more elastic housing supply have fewer and shorter housing bubbles. They also do an empirical analysis in which they find that areas with a more inelastic housing supply were the ones that experienced a large run-up in housing prices in the 1980s. Our model predictions are in line with theirs, also predicting that housing prices in areas with
a more elastic supply will be less affected by shifts in housing demand. However, we focus on the optimal housing policy given different elasticities of the supply curve and actually consider supply-side policies which may not cause a price boom at all in order to increase homeownership. Another paper worth mentioning is by Romer (2000), in which he argues that government policy programs aimed at increasing innovation should focus on subsidizing the supply of scientists and engineers rather than the demand for them. Romer makes the simple observation that if the supply of scientists and engineers is inelastic, subsidizing their labor demand may simply push wages up without increasing the equilibrium amount of innovation by much. He therefore recommends policies that would make the supply of such labour more elastic. While this is obviously true in our model, we find a difference in demand- and supply- side subsidies even if we keep the elasticity of the supply curve constant. This is due to the fact that financial constraints respond differently to increases versus decreases in price.

The rest of the paper is arranged as follows. Section 3.2 outlines the main model. Section 3.3 discusses the features of the model equilibrium. Section 3.4 discusses a policy that can achieve the first-best level of investment and welfare. Section 3.5 compares demand and supply based policy interventions and discusses how the model can be applied to evaluate existing policy interventions in the housing market. The last section concludes. All proofs are in the Appendix to Chapter 3.

3.2. The Model

There are two dates (1, 2), a firm which can invest in its own projects or real estate, a construction company which can build new houses, a representative bank which can make loans to the firm to undertake investment and and a representative household who is the final owner of the construction company and the firm. All agents are risk-neutral. At \( t = 1 \), the firm owns liquid funds, \( \omega \), and an existing stock of real estate, \( B \). The firm can use this stock along with any new real estate purchases, \( x_m \), as collateral to borrow an amount \( l \) from banks. The firm can use its liquid capital and the bank loan to invest in its own
projects or real estate. It additionally has access to a storage technology that has a return of 1.

At \( t = 1 \), a representative bank can make a loan, \( l \), to the firm at interest rate \( r_l \) to invest in its own projects and real estate. Loans need to be collateralized because of moral hazard in the repayment of loans. The firm can use its real estate stock as collateral and can commit to repaying a portion of the value of this stock \( \phi(B + x_m)P \) at \( t = 2 \), where \( P \) represents the price of housing in the economy and \( \phi < 1 \). \( \phi \) represents the degree of pledgability of collateral. This formulation of the collateral constraint is similar to that in Gertler and Karadi (2011) and in Gertler and Kiyotaki (2015).\(^4\)

Investing in its own project gives the firm a return of \( r_f(x_f) \) at \( t = 2 \) for every \( x_f \) units invested at \( t = 1 \). The function \( r_f \) has the following standard properties, \( r'_f(x_f) > 0 \) and \( r''_f(x_f) < 0 \) for all \( x_f \), \( r'_f(0) = \infty \) and \( r'_f(\infty) = 0 \). Investing in real estate gives the firm a return \( r_m(x_m) \) at \( t = 2 \) for every \( x_m \) units invested at \( t = 1 \). The function \( r_m \) has the following standard properties, \( r'_m(x_m) > 0 \) and \( r''_m(x_m) < 0 \) for all \( x_m \), \( r'_m(0) = \infty \) and \( r'_m(\infty) = 0 \). The price per-unit of housing, \( P \), is determined by demand and supply in the housing market. The representative construction firm takes the price of housing as given and has a strictly increasing and convex cost of housing production given by \( K(x_m) \).

The return from real estate can be interpreted as any return the firm makes from the use of these assets in its production or sales activities. As an investment good, the return on real estate can also reflect beneficial tax treatment of owning property. Alternatively, the return from housing can be viewed as innovations in rental income which take the form of savings for the firm when it owns its real estate assets rather than leasing.

All consumption takes place at \( t = 2 \) and profits are rebated back to the household. The firm’s profits are rebated to the household at \( t = 2 \) once the investment returns are realized.

\(^4\)In particular, each period \( t \), the firm can abscond immediately with the funds it borrows. In this case the bank is able to recover a fraction \( \phi \) of the firm’s real estate stock. For the bank to lend to the firm therefore, \( l \leq \phi(B + x_m)P \).
while the construction company’s profits can be rebated to the household in \( t = 1 \) or \( t = 2 \). If the profits are rebated at \( t = 1 \), we assume that the household cannot channel them to the firm to additionally invest. This is an important assumption of our model - resources cannot be shuffled across the construction company and the firm. Intuitively this means that when a firm purchases real estate assets, it does not immediately receive back the funds it just spent on that purchase. We additionally assume that all agents are price-takers in the economy. They therefore do not internalize their effect on \( P \) when making decisions.

**The Firm:** The firm borrows an amount \( l \) from banks and maximizes its profits, \( \pi_f \). It therefore solves the following portfolio allocation problem, where \( x_m, x_f \) and \( x_s \) are the units of real estate, firm project investment and storage purchased:

\[
\max \pi_f = \max_{(x_f, x_m, x_s, l) \geq 0} \quad r_f(x_f) + r_m(x_m) + x_s - l(1 + r_l)
\]

s.t \( x_f + P x_m + x_s \leq \omega + l \)

s.t \( l \leq \phi (B + x_m) P \)

The first three terms in the firm’s \( t = 2 \) wealth represent the value of the firm’s investment portfolio and the last term is the repayment that the firm must make at \( t = 2 \) to the bank. The first constraint is the firm’s budget constraint while the second constraint is its borrowing constraint.

**The Construction Company:** The construction company decides how many new houses to construct at \( t = 1, x_m \), to maximize its profits, \( \pi_c \). It solves the following maximization problem:

\[
\max \pi_c = \max_{x_m \geq 0} \quad P x_m - K(x_m)
\]

**The Bank:** The representative bank competitively sets interest rates at \( t = 1 \). Since there is no uncertainty in the economy, at \( t = 2 \) the firm’s profits are deterministic conditional on
its chosen investment portfolio at $t = 1$. The bank will therefore never lend more than the firm’s ability to repay and default will never occur. Competition between banks therefore drives the equilibrium rate of interest on loans, $r_t$, to 0.

**The Household:** The household is the final holder of the firm, construction company and the bank in the model. Since competition drives bank profits to 0, the household’s final utility is given by the sum of the profits of the firm, $\pi_f$, and the construction company, $\pi_c$.

### 3.2.1. Equilibrium

An equilibrium of this economy is given by, (i) The firm’s portfolio allocation $x_m, x_f$ and $x_s$ given the price of housing $P$, (ii) The construction company’s choice of housing production $x_m$ given the price of housing $P$, (iii) Price $P$ such that the housing market clears.

### 3.3. Equilibrium Analysis

We begin the analysis of the equilibrium in this model by outlining the first best level of investment in the economy and a benchmark case in which the firm does not face any financial constraints. In this benchmark case, the economy achieves the first best level of investment and consumption. We then discuss the equilibrium when the firm is financially constrained and show how price externalities in the housing market generate inefficiencies in investment and consumption.

#### 3.3.1. First Best

The first best level of investment in this economy maximizes the total resources available to the household for consumption at $t = 2$. The unconstrained social planner wanting to achieve this investment allocation solves,

$$
\max_{x_f, x_m \geq 0} \quad r_f(x_f) + r_m(x_m) - x_f - K(x_m)
$$
The social planner has the following first order conditions,

\[ r'_f(x_f) = 1 \]
\[ r'_m(x_m) = K'(x_m) \]

The first best level of investment in this economy are given by the quantities of \( x_m \) and \( x_f \) when the marginal return from investing is equal to the marginal cost of undertaking the investment. We define \( x^*_m \) and \( x^*_f \) as the first best levels of investment in real estate and the firm’s projects, i.e. \( \frac{r'_m(x^*_m)}{K'(x^*_m)} = 1 \) and \( r'_f(x^*_f) = 1 \).

3.3.2. Benchmark Equilibrium without Financial Constraints

We start by outlining a benchmark case in which the firm is not financially constrained. In the model, this is equivalent to the firm being able to borrow from the bank without the need for posting collateral as long as it has sufficient funds at \( t = 2 \) to cover its \( t = 1 \) loan. Therefore conditional on the expected return from an investment being above 1, the firm can borrow the funds available to invest in it. The presence of the storage technology ensures the firm never invests with an expected return of below 1 and therefore the firm has no limit to the amount it can borrow. In the benchmark case, the firm solves the following problem:

\[
\max_{(x_f, x_m, x_s, l) \geq 0} \quad r_f(x_f) + r_m(x_m) + x_s - l(1 + r_l) \\
\text{s.t.} \quad x_f + Px_m + x_s \leq \omega + l
\]

Using the fact that \( r_l = 0 \), the first order conditions yield the following equilibrium quantities of real estate and firm project investment in the economy:

\[ r'_f(x_f) = 1 \]
\[ \frac{r'_m(x_m)}{P} = 1 \]
Assuming households do not borrow simply to store (i.e. they have a weak preference for not taking a loan), storage is used in equilibrium if $\omega \geq Px_m + x_f$. If $\omega < Px_m + x_f$, then the household will borrow an amount $l = Px_m + x_f - \omega$ from banks to fund its additional investment.

The first order condition for the construction company gives us the equilibrium quantity of housing supplied:

$$K'(x_m) = P$$

Market clearing implies that the level of investment chosen by the firm is equal to the first first best level of investment in the economy. Therefore, a firm that is not financially constrained invests in $x_m^*$ units of real estate and $x_f^*$ in its own projects.

From the above analysis, we can see that to achieve the first best level of investment the unconstrained firm wants to borrow:

$$l = \max\{0, K'(x_m^*)x_m^* + x_f^* - \omega\}$$

In the model with financial constraints, the borrowing constraint on the firm allows it to borrow a maximum of $\phi K'(x_m)(x_m + B)$. Therefore even in the equilibrium with financial constraints the first best level of investment can be achieved if:

$$K'(x_m^*)x_m^* + x_f^* - \omega \leq \phi K'(x_m^*)(x_m^* + B)$$

When the above inequality holds, the firm is able to borrow enough to fund all productive investment opportunities and achieves the first best level of investment. When the above inequality does not hold, the firm has more positive NPV investment opportunities than the funds necessary to invest in these opportunities. This is the pertinent case for when the
level of house prices generate externalities on investment. Formally, we define the firm as having “limited funds” when the following equation is satisfied:

\[ K'(x^*_m) x^*_m + x^*_f > \phi(B + x^*_m)K'(x^*_m) + \omega \]  

(Limited Funds)

The above equation implies that the firm is financially constrained and cannot invest in all positive investment opportunities even if it borrows to its full capacity. Therefore the firm’s borrowing constraint will bind in equilibrium. This is a key assumption that drives the main results of the model. Since this is the pertinent case when the decentralized equilibrium is inefficient, henceforth we will assume that the firm is constrained by limited funds. We will discuss its importance in the following section and throughout the paper.

3.3.3. Equilibrium with Financial Constraints

We now move to the equilibrium analysis of the main model in which the firm is financially constrained. At \( t = 1 \), the firm solves the following investment problem:

\[
\begin{align*}
\max_{(x_f,x_m,x_s,l) \geq 0} & \quad r_f(x_f) + r_m(x_m) + x_s - l(1 + r_l) \\
\text{s.t} & \quad x_f + Px_m + x_s \leq \omega + l \\
\text{s.t} & \quad l \leq \phi(B + x_m) P
\end{align*}
\]

If the Limited Funds assumption is satisfied, the firm’s borrowing constraint binds in equilibrium and \( l = \phi(B + x_m) P \). In this case, the household will never invest in the storage technology since it has unexploited NPV projects at \( t = 1 \) that yield a return strictly greater than 1. Using the fact that competition between banks and the deterministic setting causes \( r_l = 0 \), we can simplify the portfolio allocation problem to the following:

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\[
\max_{x_f, x_m} \geq 0 \quad r_f(x_f) + r_m(x_m) - \phi (B + x_m) P
\]
\[
s.t \quad x_f + (1 - \phi) P x_m \leq \omega + \phi BP
\]

This yields the following first order conditions:

\[
r_m'(x_m) = \lambda P (1 - \phi) + \phi P
\]
\[
r_f'(x_f) = \lambda
\]

\(\lambda\) is the lagrange multiplier on the budget constraint. We can combine the two FOCs to obtain the following equation that determines the amount of investment given the price of housing:

\[
r_m'(x_m) = P r_f'(x_f) (1 - \phi) + \phi
\]

(3.1)

Market clearing requires that \(P = K'(x_m)\).

**Constrained Social Planner Allocation:** To establish the inefficiencies in the decentralized equilibrium, we solve the constrained social planner’s problem in this economy. The constrained social planner chooses the optimal investment for the firm that maximizes the representative household’s final wealth. The constrained social planner in this economy takes into account how housing prices affect the financial constraints of the firm but faces the same borrowing and budget constraint as the firm. Additionally, the planner is also unable to reallocate resources from the construction company to the firm to use for investment\(^5\). The social planner therefore solves:

\[
\max_{x_f, x_m, x_s} \geq 0 \quad r_f(x_f) + r_m(x_m) + x_s - l(1 + r_l) + \pi_c(x_m)
\]
\[
s.t \quad x_f + P(x_m) x_m + x_s \leq \omega + l
\]
\[
s.t \quad l \leq \phi (B + x_m) P(x_m)
\]

\(^5\)Note that the inability of the planner to reallocate resources from the construction company to the firm is important since the construction profits do not show up in the right hand side of the firm’s budget constraint.
When the Limited Funds assumption is satisfied, the firm is financially constrained in equilibrium, it’s borrowing constraint binds and \( x_s = 0 \). Additionally using the fact that \( r_t = 0 \), this problem simplifies to:

\[
\begin{align*}
\max_{(x_f,x_m) \geq 0} & \quad r_f(x_f) + r_m(x_m) - \phi(B + x_m)P(x_m) + P(x_m)x_m - K(x_m) \\
\text{s.t} & \quad x_f + P(x_m)(1 - \phi)x_m \leq \omega + \phi BP(x_m)
\end{align*}
\]

The FOCs for the constrained social planner are:

\[
\begin{align*}
 r_f'(x_f) &= \lambda \\
 r_m'(x_m) + P + P'x_m - K'(x_m) &= \lambda(P(1 - \phi) + P'(1 - \phi)x_m - P' \phi B) + \phi P + \phi P'(B + x_m)
\end{align*}
\]

To compare the constrained social planner’s allocation to that of the household’s in the decentralized equilibrium, we impose market clearing prices faced by the firm in equilibrium i.e. \( K'(x_m) = P \). Then, the social planner’s optimal allocation is given by:

\[
\begin{align*}
 r_m'(x_m) &= P(r_f'(x_f)(1 - \phi) + \phi) + P'x_m(r_f'(x_f) - 1) - P' \phi(B + x_m)(r_f'(x_f) - 1) \quad (3.2)
\end{align*}
\]

Comparing (3.2) to the decentralized allocation in (3.1), we see that the two differ when \( r_f'(x_f) > 1 \). This is the case when the Limited Funds assumption holds and the firm is not able to undertake all positive NPV projects. Comparing the social planner’s allocation in (3.2) with the firm’s in (3.1), there are two additional terms on the RHS. The term \( P'x_m(r_f'(x_f) - 1) \) captures the crowding-out effect. As the firm demands more real estate, the price of housing rises and it has to pay a greater amount for all units of real estate.

\[\text{The presence of the storage technology ensures that } r_f'(x_f) \geq 1.\]
leaving it with less funds to invest into firm projects. This decreases the optimal $x_m$ chosen by the social planner. The term $P' \phi(B + x_m)(r'_f(x_f) - 1)$ captures the collateral effect. As the firm demands more real estate, the price of housing rises and loosens the firm’s borrowing constraint, giving it more funds to invest. This increases the optimal $x_m$ chosen by the planner. Internalizing the price effects of housing therefore makes the constrained social planner’s optimum differ from that of the firm’s when the household is financially constrained. Therefore, the first welfare theorem does not hold in this case.\(^7\)

Let $x^e_m$ be the decentralized equilibrium demand for housing and $x^s_m$ be the optimal amount of housing in the constrained social planner equilibrium. Based on the above analysis, we can establish the following proposition on how the decentralized equilibrium allocation differs from that of the constrained social planner. Given the generality of our functional forms, for ease of exposition it is easier to express the condition in terms of high-level observables rather than primitives.\(^8\) In a corollary to this proposition, we explain how primitives of the model affect investment in the decentralized equilibrium.

**Proposition 5** If $x^s_m > \phi(B + x^s_m)$, the crowding-out effect is larger than the collateral effect. In this case, the decentralized equilibrium features inefficiently high investment in housing and $x^s_m < x^e_m$.

Conversely, if $x^s_m < \phi(B + x^s_m)$, the crowding-out effect is smaller than the collateral effect. In this case, the decentralized equilibrium features inefficiently low investment in housing and $x^s_m > x^e_m$.

The inefficiency in the decentralized equilibrium in this model arises because the firm is acting like a price-taker and does not internalize the effect real estate demand has on prices.

\(^7\)The above analysis accounts for how a social planner would change the portfolio allocation picked by the firm when the construction company acts competitively and picks $K'(x_m) = P$. A similar analysis can be done if the social planner was choosing how much the construction company produces with the firm acting competitively and picking $r'(x_m) = P(r'_f(x_f)(1 - \phi) + \phi)$.

\(^8\)Davila and Korinek (2017) argue that this is usual in normative problems and note that that even Ramsey's characterization of optimal commodity taxes relies on demand elasticities that are endogenous to the level of taxes.
The price of housing affects both the value of the firm’s collateral and the amount of funds the firm has to fund investment opportunities. When the existing amount of real estate is high (a high $B$), an increase in the price of housing affects the firm’s ability to borrow. The social planner internalizes this effect and therefore wants to increase the demand for real-estate investment to push up the aggregate price of housing. When the existing amount of housing in the economy is low (a low $B$) or the firm is not able to use its housing as collateral efficiently (a low $\phi$), an increase in the amount of housing causes house prices to be inefficiently high and resources have to be diverted away from investment in firm projects when the firm faces borrowing constraints. The social planner internalizes this effect and therefore wants to decrease the demand for real-estate investment to decrease the aggregate price of housing.

Focusing on the primitives of the model, as $B$ and $\phi$ increase, the firm can use housing more efficiently as collateral to find other investment projects and has to give up less investment in its own project per dollar spent on real estate purchases. We can establish the following corollary to the above proposition on how the decentralized equilibrium allocation differs from that of a planners based on model primitives:

**Corollary 4** $\exists B$ s.t. when $B > B$, the constrained social planner wants to increase the decentralized equilibrium investment in housing and when $B < B$ the constrained social planner wants to decrease the decentralized equilibrium investment in housing.

Similarly, $\exists \phi$ s.t. when $\phi > \phi$, the constrained social planner wants to increase the decentralized equilibrium investment in housing and when $\phi < \phi$ the constrained social planner wants to decrease the decentralized equilibrium investment in housing.

As discussed above, investing in real estate requires the firm to use funds that it can otherwise invest in its own projects. This externality tightens the budget constraint of the firm and can decrease investment in firm projects. At the same time, investing in real estate increases the collateral value of the firm, allowing it to borrow more and giving it
additional resources to invest in its projects. This can lose the firm’s budget constraint. The net effect on firm investment depends on whether the crowding-out or the collateral effect dominates. We can establish the following corollary to the above proposition about the level of investment into the firm’s projects in the economy:

**Corollary 5** If \( x_m^s > \phi (B + x_m^s) \) then increasing housing investment crowds investment out of firm projects. Conversely if \( x_m^s < \phi (B + x_m^s) \) then increasing housing investment crowds investment in to firm projects.

Using the budget constraint of the firm we can express the investment in the firm’s projects in terms of the investment in real estate:

\[
x_f = \omega + \phi BP(x_m) - (1 - \phi) P(x_m)x_m
\]

To study the effect that that increasing housing investment has on firm project investment we take the derivative of firm investment w.r.t \( x_m \):

\[
\frac{\partial x_f}{\partial x_m} = \phi B \frac{\partial P}{\partial x_m} - (1 - \phi) \frac{\partial}{\partial x_m} (P x_m)
\]

(3.3)

An increase in the price of housing affects the firm’s investment into its own projects in two ways as discussed above - the **collateral effect** and the **crowding-out effect**. The **collateral effect** is captured by the first term on the right hand side of (3.3). An increase in the price of housing loosens the firm’s borrowing constraint as the existing stock of real estate is now worth more. The firm can therefore borrow more against its future income and invest more in its projects at \( t = 1 \). The **crowding-out effect** is captured by the second term of (3.3). The boom in the price of houses causes the firm to spend relatively more on real estate purchases and therefore it must compensate by reducing the amount spent on investing in its projects. Which effect dominates depends on relative increase in the value
of collateral \( \frac{\partial}{\partial x_m} \phi BP \) versus the relative increase in the amount that the firms needs to pay for the extra investment in real estate \( (1 - \phi) \frac{\partial}{\partial x_m} (P x_m) \). If the existing housing in an economy is high, a small increase in price can relax borrowing constraints enough to lead to a crowding-in of investment into the firm’s projects.

We can see that when \( \phi = 0 \) and the firm cannot borrow at all against it’s real estate stock, the collateral effect disappears and only the crowding-out effect remains. Increasing investment in real estate therefore always leads to a substitution away from firm project investment. When \( \phi = 1 \), the crowding-out effect is zero and only the collateral effect remains. In this case, increasing investment in housing always allows the firm to increase investment into its own projects.

While the collateral effect has been widely discussed and studied in the literature, the crowding-out effect that we propose is novel. This is because we focus on not just externalities arising from the collateral constraint but also the budget constraint. The fact that resources cannot be reallocated simultaneously between the construction company and firm is important to this result. Otherwise for every dollar the construction company makes in revenue, the dollar can be reallocated to the firm to invest, so that house prices have no net effect on the budget constraint. In practice, we feel that the assumption that resources cannot be frictionlessly moved around is extremely realistic as a firm which is making investment decisions is unlikely to think that it will be reallocated funds it spends on various purchases.

3.4. Policy

One of the key takeaways of our model is that when the firm is financially constrained, price effects in the real estate market are not welfare-neutral. Therefore when we consider government interventions, how they affect the price of housing is critical. In our setting housing price movements can create externalities on the firm’s investment in its projects. Therefore when examining policies that tax or subsidize investment in real estate we must
not only consider the resulting level of real estate investment, but also the resulting price of housing as this affects the level of investment in firm projects.

To study different housing market policies in our setting, we consider government subsidies or taxes to the real-estate sector and how they affect the total amount of investment in the economy. This can be done by a demand side-subsidy (tax) which increases (decreases) the $t = 2$ per-unit return on real estate by an amount $r_g$. In practice most government interventions such as the mortgage interest rate tax deduction to increase real-estate investment tend to fall into demand-side interventions. We also consider supply-side subsidies (taxes) in the form of a per-unit subsidy (tax), $b$, that the government can give to construction companies as an alternative intervention. The government must have a balanced budget and households are taxed an amount $\tau$ at $t = 2$ to cover the cost of the subsidy. We assume that all subsidy/taxation payments are made at $t = 2$ and that construction companies can operate at a loss between period 1 and 2 without any additional costs.

A demand-side intervention, $r_g$, changes the firm maximization problem as follows:

$$
\max_{(x_f,x_m,x_s,l) \geq 0} \quad r_f(x_f) + r_m(x_m) + r_g x_m + x_s - l(1 + r_l)
$$

$$
s.t \quad x_f + P x_m + x_s \leq \omega + l
$$

$$
s.t \quad l \leq \phi (B + x_m) P
$$

If the Limited Funds assumption is satisfied, taking the firm’s first order conditions the amount of equilibrium investment given the price of housing is now determined by the following equation:

$$
r'_m(x_m) + r_g = P(r'_f(x_f)(1 - \phi) + \phi)
$$

A supply-side intervention, $b$, changes the construction company’s maximization problem as follows:
\[ \max_{x_m \geq 0} \quad (P + b)x_m - K(x_m) \]

Market clearing now requires that that \( P + b = K'(x_m) \).

In the previous analysis, we show that when the limited funds assumption holds and the firm is financially constrained, it chooses a different allocation than that chosen by a constrained social planner who takes into account how investing in real estate changes the price of housing. In this case, we can find a demand-based subsidization scheme that can restore the constrained social planner optimum. The following proposition states this:

**Proposition 6** A demand-based subsidization scheme in which \( r^*_g = K''(x^*_m)(\phi B - (1 - \phi)x^*_m)(r'_f(x^*_f) - 1) \) and \( b^* = 0 \) restores the socially optimum level of housing and firm project investment chosen by the constrained social planner.

A demand tax, \( r_g \), essentially allows us to shift the firm’s demand for investment in real estate to match the constrained social planner allocation. When the crowding-out effect dominates, the social planner wants to reduce equilibrium investment in housing which can be accomplished by taxing investment in real estate. Alternatively, when the collateral effect dominates, we can restore the constrained social planner optimum by subsidizing investment in real estate. A particularly interesting aspect of this proposition is that if this investment-driven collateral effect is not large enough in the economy (a low \( B \) or \( \phi \)), the optimum policy for the government would be to have a negative tax on real estate investment. The size of the government subsidy or tax is larger when quality of investment opportunities in the firm sector \( (r'_f) \) is higher. This is because better firm projects increase the social cost of the externality on investment coming from housing prices. The size of the government subsidy or tax is also larger when the supply curve is more inelastic \( (K''(x^*_m)) \). This arises because of the non-neutrality of prices in the model. The price effects of housing drive the investment externalities and a more inelastic supply curve leads to greater price movements as we change the amount of real estate investment.
We can thus restore the constrained social planner optimum by taxing or subsidizing real estate investment. However, in our economy we can actually do better than the constrained optimum and achieve the first best by exploiting the externalities of real estate prices on investment. A tax or subsidy on the demand for housing shifts the household demand for housing out via a subsidy when the collateral effect dominates, and shifts household demand in via a tax when the crowding-out effect dominates. When $b = 0$, such a policy is restricted to a housing quantity/price combination that is permitted by the housing supply curve, i.e. $K'(x_m) = P$. A tax or subsidy on the supply side of the housing market, allows for additional shifts of the housing supply curve as well as the demand curve. This increases the combination of price/quantity combinations that are permissible in equilibrium.

We can show that a combination of demand- and supply-side interventions can help restore the first-best level of investment and consumption. Formally, we can establish the following proposition:

**Proposition 7** The first-best level of welfare can be achieved by a subsidy pair

$$
\{r_g, b\} = \left\{ \frac{x_f^* - \omega}{\phi B - (1 - \phi)x_m^*} - r_m'(x_m^*), \frac{x_f^* - \omega}{\phi B - (1 - \phi)x_m^*} - r_m'(x_m^*) \right\}
$$

This proposition is an important result of our paper and yields a very surprising insight. In the presence of price externalities from housing, the optimal way to build wealth in the economy is to combine expansionary housing supply subsidies with contractionary housing demand taxes and vice-versa. From the proposition we can see that in the optimal subsidy scheme $r_g = -b$. This is simply due to the linearity of the demand and supply subsidies. In equilibrium it has to be the case that $K'(x_m^*) - b = P = r_m'(x_m^*) + r_g$. Since at the optimal level of housing investment $K'(x_m^*) = r_m'(x_m^*)$, this implies that $-b = r_g$.

Since price externalities are the key source of inefficiency in the model, the optimal policy needs to target price movements which require opposite subsidies to demand and supply. The government’s problem can thus be thought of as wanting to achieve a certain price
level at the optimal level of housing investment. Whenever the collateral effect is stronger than the crowding-out effect at $x_m^*$, the government taxes supply and subsidizes demand, both of which put upward pressure on the price of housing. The price is increased until the collateral value of housing is high enough such that the firm can borrow enough to fund the optimal level of investment into its own projects. Conversely, when the crowding-out effect is stronger than the collateral effect at $x_m^*$, the government needs to put downward pressure on the price of housing to relax the firm’s budget constraint and provide it with enough funds to undertake the optimal level of investment. In this case, the government taxes demand and subsidizes supply, both of which put downward pressure on the price of housing.

When the crowding-out effect is stronger at $x_m^*$, i.e. when $(1 - \phi)x_m^* > \phi B$, the government wants to push house prices down which can be achieved through supply-side subsidies and demand-side taxes. Thus, when $x_m^*$ is quite high relative to $B$ and the government wants to increase new housing purchases, we should be pushing housing prices down rather than trying to support housing prices. This involves measures such as a tax on real estate investment and rebates to construction companies.

This result highlights that when the return to new housing investment is high (high $x_m^*$), it is preferable to try and reduce the price of housing, rather than to make borrowing for firms easier by allowing them to take on more leverage. Other arguments in the literature also support household’s taking on less leverage. We get the same in our model but through a completely different channel. We abstract away from the risk created by taking on more leverage and show that even from a wealth accumulation perspective taking on leverage may not be optimal for households and firms. This is driven by the fact that if additional leverage is generated due to an increase in prices, that rise in prices also reduces the funds available for a firm to invest in other productive opportunities. Reducing the costs of downpayments of houses can instead free up funds and increase a firm’s ability to invest. Demand subsidies in the model induce taking on leverage while supply subsidies cause a
reduction in the leverage required. The crowding-out effect works as a counter to taking on more leverage.

**Implementation:** The optimal policy in our model requires knowledge of \( x_f^* \) and \( x_m^* \) which depend upon the exact functional forms \( r_m \) and \( r_f \). These quantities are hard to determine without a full structural model. In the model, when the crowding-out effect is stronger than the collateral effect, housing prices should optimally be decreased. We can show that the relative magnitude of the crowding-out effect and the collateral effect depend on three parameters of the model, \( B, \phi \) and \( x_m \), where \( x_m \) is the observed level of new investment in housing. These three parameters are sufficient to determine which effect is relatively larger, we simply have to compare \( (1 - \phi)x_m \) to \( \phi B \). This determines whether we should subsidize the supply of housing and tax the demand or vice-versa. This approach is limited in that it cannot tell us the exact magnitude of the subsidies and taxes. However, these three statistics are sufficient to guide us in the direction of the intervention and have the benefit of being easily observable. This approach follows Chetty (2009) and derives formulas based on sufficient statistics to guide policy without the need to estimate deeper parameters of the model.

In the next section, we discuss supply- and demand-side interventions in more detail and explain why the two are not equivalent in our framework. We also explain how the insights from our theory can help us evaluate existing interventions in the housing market.

3.5. Comparing Demand and Supply Subsidies

Traditional economic theory has long established that under general conditions, it does not matter whether we subsidize (tax) supply or demand from a welfare perspective. The gains to consumers and suppliers are the same and depend only on the relative elasticities of the supply and demand curves. However in our model, we find that taxing the supply and demand sides of the market can be quite different due to the effects that are caused by the increase or decrease in the price of housing when households and firms are financially
constrained and when there is an inter-temporal opportunity cost of capital. In our model, this cost is the presence of a second sector in the economy since this seems to be an empirically relevant case. However, this cost can be much more general, and could include labor income costs, costs associated with the moral hazard of lending such as monitoring costs, etc.

In this economy, the introduction of subsidies \( (r_g > 0) \) that increase the demand for real estate always lead to an appreciation in the price of housing. The price for housing increases because each additional home is more expensive to produce giving rise to an upward sloping supply curve in the housing market. For the housing market to clear and respond to the increase in demand, the price of housing must consequently appreciate. Conversely, the introduction of subsidies \( (b > 0) \) that increase the supply for housing have an opposite effect on price to that of demand subsidies. They lead to a decrease in the price of housing. Supply side interventions hence do not cause the boom in housing prices that demand subsidies do.

In most literature on externalities, the socially optimum level of the good in question is affected by the existence of externalities. This level is typically independent of price movements. However, in our paper the externalities themselves are generated due to prices and therefore this causes demand- and supply- subsidies to have different welfare implications. This will be discussed more and formalized in the rest of this section. Since typically externalities lead the social planner to choose a particular optimal level of a good, in this case housing, we will compare supply and demand subsidies holding fixed the level of housing investment in the economy. This will help clarify the main forces in the model that are driving the difference between these two policy interventions.

3.5.1. Demand and Supply Equivalence

As discussed earlier, in classic economic theory the welfare implications of taxing or subsidizing the supply and demand side of the market are the same. We will therefore start this section
with a discussion of when subsidizing the supply and the demand side of the market in our model are welfare equivalent. We will then discuss which frictions cause the welfare implications of these two policies to differ.

If the firm is not financially constrained, then demand and supply subsidization are equivalent policies. For any level of real estate investment that can be achieved in the economy, subsidizing either the supply or demand generate the same utility for the representative household. This is because the firm will simply invest until all the productive investment opportunities in the economy are realized. In the following propositions, $U$ refers to the utility of the representative household. We can then establish the following proposition:

**Proposition 8 (Demand and Supply Equivalence)** Suppose the firm is unconstrained. Then for any $r'_g$ generates $x'_m$, $x'_f$ and $U'$, there exists a $b'$ that also generates $x'_m$, $x'_f$ and $U'$. The converse is also true.

Proposition 8 states the conditions under which supply and demand subsidies are welfare equivalent. The firm in this model is financially constrained which prevents all productive investment opportunities from being realized in equilibrium. Once this constraint is taken away, the costs of being financially constrained i.e. the investment externalities that investing more in housing causes on firm projects, disappear as well. The welfare gains from supply and demand subsidies therefore come from how they each affect the firm’s ability to borrow and invest.

From proposition 8, we see that without financial constraints, subsidizing the demand- and supply-side are welfare equivalent. The firm’s constraints prevent it from investing in all productive investment opportunities. To understand the difference in the two policy interventions, we therefore need to look at their effect on financial constraints and subsequent investment in the economy. Having established when demand and supply subsidies produce the same effects, we now establish two more propositions that explain why they differ in our model.
In the model, supply and demand respond to future government subsidies at $t = 2$. Demand subsidies increase the demand for housing resulting in an increase in the price of housing while supply subsidies increase the supply of housing resulting in a decrease in the price of housing. Prices therefore respond to future subsidies. However, a key part of the benefits and costs of subsidies are provided to households and firms through price movements. Supply subsidies may allow the household to invest more by lowering the price of housing at $t = 1$. This effectively makes the household relatively richer at $t = 1$ since taxes to pay for the subsidies are paid in the future. Alternatively demand side subsidies directly increase the value of collateral that households have by increasing the price of housing thus helping them to borrow more.

In the absence of a collateral effect in the model demand subsidies will thus lose their advantage over supply-side subsidies. The following proposition formalizes this result:

**Proposition 9** If $\phi = 0$, subsidizing (taxing) the supply (demand) side of the housing market pareto dominates subsidizing (taxing) the demand (supply) side of the housing market. That is, for any demand-subsidy $r_g^+$ that is associated with utility $U$, there exists a supply-subsidy $b^+$ that generates higher $U' > U$ and for any supply-tax $b^-$ that is associated with utility $U$, there exists a demand-tax $r_g^-$ that generates higher $U' > U$.

This proposition states that without a collateral effect, supply subsidies (demand taxes) are always preferable to demand subsidies (supply taxes). The key assumptions driving this result are financial constraints and an inter-temporal advantage of having more capital early. In the case of our model, the advantage is being able to invest in positive NPV projects of firms. When $\phi = 0$. In this case the borrowing capacity of the firm does not change with the price of housing. Therefore the introduction of demand subsidies will push up price and always cause a reallocation of household investment in favor of mortgages and away from firm investment due to a negative crowding-out effect. Supply subsidies, on other hand, will free up household funds to invest in more projects by pushing the cost of housing down and have a positive crowding-out effect.
We can establish an analogous proposition for demand-based subsidization schemes. Namely:

**Proposition 10** If $\phi = 1$, subsidizing (taxing) the demand (supply) side of the housing market pareto dominates subsidizing (taxing) the supply (demand) side of the housing market. That is, for any supply-subsidy $b^+$ that is associated with utility $U$, there exists a demand-subsidy $r^+_g$ that generates higher $U' > U$ and for any demand-tax $r^-_g$ that is associated with utility $U$, there exists a supply-tax $b^-$ that generates higher $U' > U$.

When $\phi = 1$, the firm is able to borrow against the full value of its real estate stock. Therefore investment in housing does not require the firm to substitute away from funds that it would otherwise use for investment in its projects. When $\phi = 1$, the household can effectively borrow to fund all its new housing purchases. This neutralizes the crowding-out effect of price movements while keeping the stronger collateral effect that demand subsidies have since they push up the price of housing.

It is worth noting that when $\phi = 1$, and the firm can borrow up to the full value of its housing stock from the bank and therefore the firm will choose the optimal level of housing. However, if the limited funds assumption continues to hold, then the firm will still not be able to pick the optimal level of investment in its own projects, still creating room for government intervention.

### 3.5.2. Evaluating Existing Interventions

The above propositions highlight the usefulness of front-loading benefits and back-loading costs of policies when there are intertemporal opportunity costs of capital. They can also be used to evaluate the effectiveness of different interventions which affect the same side of the market. As we discussed, Chaney et al. (2012) find that high housing prices increase investment for real estate-owning firms through a loosening of their borrowing constraints. In more recent work, Chakraborty et al. (2014) find that real estate price booms can crowd-out investment in firms as more bank loans are used to purchase housing. In line with these results, our theory predicts that policies which encourage house price booms would
be beneficial for investment when the firms in the area with price booms already own real 
estate (high $B$) but would crowd-out investment when firms do not already have sizeable 
real estate assets.

Many interventions trying to support house prices are often targeted at households rather 
than at firms. The theory can also provide insight into housing market policies that affect 
household rather than firm-level investment. The firm in our model can be reinterpreted 
as a household where the housing return is a convenience yield from consuming services 
attributable to home-ownership, and the investment in firm projects can be interpreted as 
household investment in non-real estate assets, such as the stock and bond market. Under 
this interpretation, we can use insights from the model to help judge the effectiveness of 
various interventions in the housing market. Consistent with the model predictions, Hilber 
and Turner (2014) find that the mortgage interest rate tax deduction does not seem to be 
an effective way to increase home-ownership rates. They further find that the mortgage 
interest rate tax deduction seems to increase house prices and document a negative effect of 
this increase on homeownership rates amongst downpayment-constrained households. This 
effect seems to be particularly strong in areas where the supply of housing is relatively 
inelastic. Our model can shed insight into these empirical findings as a demand-side policy 
that back-loads benefits, such as the mortgage interest rate tax deduction, negatively affects 
new housing investment by households who face financial constraints since such policies 
push up the price of housing. Glaeser and Shapiro (2002) and Hanson (2012) also provide 
evidence showing that the mortgage interest rate tax deduction seems to have little effect 
on increasing home-ownership, particularly amongst financially constrained households.

Recent research has also looked at policies targeting home-owners in the aftermath of the 
crisis. Agarwal et al. (2016) and Agarwal et al. (2015) evaluate the effectiveness of the 
Home Affordable Modification Program and the Home Affordable Refinancing Program. 
They find evidence that these programs increased house prices and had positive effects 
on foreclosure rates, delinquencies on consumer debt and allowed indebted homeowners to
increase spending on durable consumption. Keys et al. (2014) find that lower mortgage rates in the aftermath of the housing crisis also led to a decrease in defaults and an increase in spending on durable consumption particularly amongst constrained households. Since these programs were targeted at existing home-owners rather than encouraging new home-ownership, our model predicts that demand-side interventions that increase house prices should prove effective in these cases. For households with existing home equity, an increase in housing prices should relax collateral constraints allowing them to more easily pay down debt and increase spending on durable consumption. In line with our model, the empirical evidence suggests that these effects are mostly present in constrained households.

There are unfortunately less programs targeted at supply-side interventions in the housing market and consequently less research investigating the effects of these on homeownership rates and household consumption. However in a recent paper Sodini et al. (2016) study an intervention in the Swedish housing market in which previously municipally owned buildings were made available for purchase to residents at steep discounts relative to market prices. This is similar to a supply-side intervention in our economy as the intervention leads to an increase in housing supply via a lowering of the price of housing. They find that in this case home-owners increase investment into the stock market in line with predictions from our model, that decreasing the price of purchasing housing can free-up home-owner wealth to invest in other sectors.

3.6. Concluding Remarks

In this paper we develop a comprehensive framework for studying the effect of housing policy on investment and wealth accumulation. We find that an increase in real estate equity does not necessarily lead to efficient investment because high housing prices can generate externalities on investment. In doing so, we highlight a novel theoretical channel that can help explain empirical evidence on negative effects on investment during the boom phase of housing prices. We also find that supply and demand subsidies are not equivalent in the presence of price externalities. When the return to investing in real estate in the
economy is high, the optimal policy involves an expansion of supply subsidies and a tax on housing demand. We summarize below some of the key insights of the paper.

(i) **Investment:** We find that a housing price increase is only good for investment if the existing stock of real estate ownership by firms is large. In this case, firms can use real estate as collateral effectively and increasing house prices provide them the ability to increase investment profitably. When the existing stock of housing is low, increasing the price of housing can lead to negative externalities on investment and crowd-out investment from firm projects *even when the firm actively uses its real estate assets as collateral to fund investment*. In such a case, policies aimed at reducing the price of housing are preferable for investment.

(ii) **Price Externalities:** A novel feature of our model is looking at price externalities in a general equilibrium framework with externalities arising because of both a collateral constraint and a budget constraint. When externalities exist because of prices the standard result of the irrelevance of using supply or demand subsidies no longer applies when subsidies and taxes are paid out and collected in the future. This is because supply and demand curve movements move prices in the opposite direction.

(iii) **Debt:** Different policies in our model have different implications for the level of household debt. A positive *crowding-out effect* effectively gives the firm more funds at \( t = 1 \), leading it to achieve higher levels of investment without taking on additional debt. In fact, due to downward price movements associated with a positive crowding-out effect, debt may actually decrease. On the other hand, a positive *collateral effect* allows the firm to achieve higher levels of investment by increasing its debt capacity. In our framework, since there is no uncertainty or default, there are no negative consequences to taking on more debt. However, many recent papers have found that a buildup in debt can cause increased economic fragility. Our analysis contributes to this literature by explaining how different housing policies can have different effects on household and firm debt. In particular, if the government wishes to expand investment
in real estate which is a common policy objective for many governments, focusing on supply subsidies that have a positive crowding-out effect and negative collateral effect, may be a more sustainable intervention.
Proof of Lemma 1. The three-period model can be solved by backwards induction. Since no new generation is born at \( t = 3 \), banks do not lend at \( t = 3 \). Additionally, the price of housing at \( t = 3 \) is given by the liquidation value \( \kappa \). At \( t = 2 \), a bank solves,

\[
\max_{m_2 \geq 0} \left[ m_1 \left( \phi_{bh} \min \{ P_1(1 + r_1), e^b + (1 - \delta)P_2 \} + (1 - \phi_{bh}) \min \{ P_1(1 + r_1), (1 - \delta)P_2 \} \right) - m_2 P_2 \right. \\
+ \beta m_2 \left( \phi_{p} \min \{ P_2(1 + r_2), e^b + (1 - \delta)\kappa \} + (1 - \phi_{p}) \min \{ P_2(1 + r_2), (1 - \delta)\kappa \} \right) \\
\left. \right. \text{Repayment Outstanding Loans} \quad \text{New Lending} \\
\left. \right. \text{Repayment of New Loans} \quad \text{s.t.} \gamma + \beta(1 - \delta)\kappa \geq \beta E \left[ \min \{ P_2(1 + r_2), \omega_2^b + (1 - \delta)\kappa \} \right] \\
0 \leq m_2 \leq \frac{1}{N}(1 - \alpha_{nb}).
\]

In the following analysis, I refer to the first constraint faced by the bank as the borrower purchasing constraint. The above problem can be simplified by focusing on the bank’s choice of interest rate. Since banks have monopoly power over their borrowers when setting interest rates and loans have full recourse, a bank will charge the maximum interest rate that borrowers are willing to pay.

I start by considering the bank’s choice of interest rate at \( t = 1 \). If a bank can not commit to future lending, there are two cases depending on the relative values of \( \phi_{bh}^R \) and \( \gamma \).

Case 1: If \( \phi_{bh}^R e^b \leq \frac{\gamma}{\beta} \) at \( t = 1 \) a bank will charge borrowers,
\[ P_1(1 + r_1) > e^b + (1 - \delta)P_2. \]

In this case, a proportion \( \phi_{bh} \) of bank borrowers receive a positive endowment, have net worth equal to \( e^b + (1 - \delta)P_2 \) and repay the bank \( e^b + (1 - \delta)P_2 \) while the remaining do not get an endowment, default and the bank gets \( (1 - \delta)P_2 \). In expectation, borrowers repay the bank \( \phi_{bh} e^b + (1 - \delta)P_2 \) which satisfies their purchasing constraint. In the model without commitment to \( t = 2 \) lending when banks make \( t = 1 \) loans, banks can only credibly prop up house prices to improve their return on loans when the borrower cannot repay the full face-value of the loan. The bank will therefore choose a face-value that is strictly higher than \( e^b + (1 - \delta)P_2 \).

**Case 2:** If \( \phi_{bh} e^b > \frac{\gamma}{\beta} \), a bank will charge,

\[ P_1(1 + r_1) = \frac{\gamma}{\beta \phi_{bh}} + (1 - \delta)P_2. \]

In this case, a proportion \( \phi_{bh} \) of bank borrowers receive a positive endowment, have net worth equal to \( e^b + (1 - \delta)P_2 \) and repay the bank in full \( \frac{\gamma}{\beta \phi_{bh}} + (1 - \delta)P_2 \) while the remaining do not get an endowment, default and the bank gets \( (1 - \delta)P_2 \). Borrowers repay the bank \( \frac{\gamma}{\beta} + (1 - \delta)P_2 \) in expectation which just satisfies their purchasing constraint. Note that in this case, a bank cannot charge a higher face-value because borrowers with a positive endowment would be able to repay the higher face-value and this would violate their purchasing constraint. A bank will therefore only be able to improve its return on all defaulting loans from a proportion \( (1 - \phi_{bh}) \) of borrowers.

By similar reasoning, the interest rate a bank will charge borrowers at \( t = 2 \) is s.t. \( P_2(1 + r_2) \geq e^b + (1 - \delta)\kappa \). The assumption on low-quality loans being unprofitable implies that \( \phi_{P} e^b < \frac{\gamma}{\beta} \).
The equilibrium solution in both cases is similar. In Case 1, I can write an equivalent maximization problem for the bank at time 2 only incorporating the portion of its profits from outstanding loans that will be affected by its \( t = 2 \) lending,

\[
\max_{m_2 \geq 0} \begin{align*}
m_1(1 - \delta)P_2 - m_2P_2 + \beta m_2 \left( \phi^{bl}_p e^b + (1 - \delta)\kappa \right) \\
\text{s.t. } 0 \leq m_2 \leq \frac{1}{N}(1 - \alpha^{nb}).
\end{align*}
\]

Define \( M_2^{-i} \) as the total lending by all other banks at \( t = 2 \). Then, taking the FOC, the optimal number of loans issued by the bank at \( t = 2 \), \( m_2 \), is given by,

\[
m_2 = \max \left\{ 0, \frac{m_1(1 - \delta) - \phi^{nb}_p \alpha^{nb} - M_2^{-i} + \beta \frac{\phi^{bl}_p e^b + (1 - \delta)\kappa}{\phi^{nh}_p}}{2} \right\}.
\]

Note that \( m_2 \) is always less than \( \frac{1}{N}(1 - \alpha^{nb}) \) so the maximum constraint on \( m_2 \) never binds. This is because the assumption on low-quality loans always being negative NPV gives implies that \( -\phi_p^{nb} \alpha^{nb} - M_2^{-i} + \beta \frac{\phi^{bl}_p e^b + (1 - \delta)\kappa}{\phi^{nh}_p} < 0 \) and \( m_1 \) always have to be less than \( \frac{1}{N}(1 - \alpha^{nb}) \) due to bank’s \( t = 1 \) lending constraint.

At \( t = 1 \) a bank takes into account its lending at time \( t = 2 \) and solves,

\[
\max_{m_1 \geq 0} \begin{align*}
- m_1 P_1(m_1) + \beta m_1 \left( \phi^{bh}_R e^b + (1 - \delta)P_2(m_1) \right) - \beta m_2(m_1)P_2(m_1) \\
+ \beta^2 m_2(m_1) \left( \phi^{bl}_p e^b + (1 - \delta)\kappa \right) \\
\text{s.t. } m_1 \leq \frac{1}{N}(1 - \alpha^{nb}).
\end{align*}
\]

I now solve for \( m_1 \) when \( m_2 > 0 \) and when \( m_2 = 0 \). The first order condition for both, gives the following choice of \( m_1 \) for the bank,
\[ m_1 = \max \left\{ 0, \min \left\{ \frac{\beta}{c} \left( \phi^{bh} e^b + (1 - \delta)c(\phi^{nb} \alpha^{nb} + M^{-i}_2 + m_2) \right) - \phi^{nb} \alpha^{nb} - M^{-i}_1, \frac{1 - \alpha^{nb}}{N} \right\} \right\}. \]

Note that the above solution is under the assumption that at \( t = 2 \), other banks take \( m_2 \) as given. This approach assumes that deviations at \( t = 1 \) from equilibrium are not observable.

An alternative approach involved other banks taking \( m_2 \) as a function of \( m_1 \) at \( t = 2 \), in which case at \( t = 1 \) when a bank chooses \( m_1 \), it would also take into account its decision on \( M^{-i}_2 \). The model solution if equilibrium deviations are observable is similar but less tractable.

In Case 2, I can similarly write an equivalent maximization problem for the bank at time 2 only incorporating the portion of its profits from outstanding loans that will be affected by its \( t = 2 \) lending,

\[
\max_{m_2 \geq 0} \left( 1 - \phi^{bh}_R \right) m_1(1 - \delta)P_2 - m_2P_2 + \beta m_2 \left( \phi^{bl}_P e^b + (1 - \delta)\kappa \right)
\]

s.t. \( 0 \leq m_2 \leq \frac{1}{N}(1 - \alpha^{nb}) \).

Taking the FOC, the optimal number of loans issued by the bank at \( t = 2 \), \( m_2 \), is given by,

\[ m_2 = \max \left\{ 0, \frac{(1 - \phi^{bh}_R)m_1(1 - \delta) - \phi^{nb} \alpha^{nb} - M^{-i}_2 + \beta \frac{\phi^{bl}_P e^b + (1 - \delta)\kappa}{c}}{2} \right\}. \]

The constraint \( m_2 \leq \frac{1}{N}(1 - \alpha^{nb}) \) will not bind for the same reason as in Case 1. At \( t = 1 \) a bank takes into account its lending at time \( t = 2 \) and solves,
\[
\max_{m_1 \geq 0} -m_1 P_1(m_1) + \beta m_1 \left( \frac{\gamma}{\beta} + (1 - \delta)P_2(m_1) \right) - \beta m_2(m_1)P_2(m_1) \\
+ \beta^2 m_2(m_1) \left( \phi_P e^b + (1 - \delta)/(\gamma) \right)
\]

s.t. \( m_1 \leq \frac{1}{N}(1 - \alpha^{nb}) \).

Taking the first order condition, the bank’s choice of \( m_1 \) when \( m_2 > 0 \) is given by,

\[
m_1 = \max \left\{ 0, \min \left\{ \frac{\frac{\beta}{c} \left( \frac{\gamma}{\beta} + (1 - \delta)c(\phi_P e^b + M_2^i + m_2) \right) - \phi_R e^{nb} - M_1^i}{2 - \frac{\beta}{\gamma}(1 - \phi_R e^{nh})(1 - \delta)^2}, 1 - \alpha^{nb} \right\}, \frac{1}{N} \right\}
\]

When \( m_2 = 0 \), the bank’s choice of \( m_1 \) is given by,

\[
m_1 = \max \left\{ 0, \min \left\{ \frac{\frac{\beta}{c} \left( \frac{\gamma}{\beta} + (1 - \delta)c(\phi_P e^b + M_2^i + m_2) \right) - \phi_R e^{nb} - M_1^i}{2}, 1 - \alpha^{nb} \right\}, \frac{1}{N} \right\}
\]

With commitment, the solution is similar. If \( \phi_R e^{nb} \leq \frac{\gamma}{\beta} \), at \( t = 1 \) a bank will charge borrowers,

\[
P_1(1 + r_1) \geq e^b + (1 - \delta)P_2.
\]

The bank can set the interest rate equal to \( e^b + (1 - \delta)P_2 \). The bank now solves for both \( m_1 \) and \( m_2 \) at \( t = 1 \). It therefore solves,
\[
\max_{m_1, m_2 \geq 0} - m_1 P_1(m_1) + \beta m_1 \left( \phi_R^{b_1} e^b + (1 - \delta) P_2(m_2) \right) - \beta m_2 P_2(m_2) \\
+ \beta^2 m_2 \left( \phi_P^{b_2} e^b + (1 - \delta) \kappa \right)
\]

\[
s.t. \quad m_1, m_2 \leq \frac{1}{N} (1 - \alpha^{nb}).
\]

The first order conditions give,

\[
m_1 = \max \left\{ 0, \min \left\{ \frac{\beta}{c} \left( \phi_R^{b_1} e^b + (1 - \delta) c (\phi_R^{nb} \alpha^{nb} + M_2^{-i} + M_2^{-i}) - \phi_R^{nb} \alpha^{nb} - M_2^{-i} \right), \frac{1 - \alpha^{nb}}{N} \right\} \right\}
\]

\[
m_2 = \max \left\{ 0, \frac{m_1(1 - \delta) - \phi_R^{nb} \alpha^{nb} - M_2^{-i} + \beta \phi_R^{b_1} e^b + (1 - \delta) \kappa}{2} \right\}
\]

If \( \phi_R^{b_1} e^b > \frac{\gamma}{\beta} \), at \( t = 1 \) a bank will charge borrowers,

\[
P_1(1 + r_1) = \frac{\gamma}{\beta \phi_R^{b_1} e^b} + (1 - \delta) P_2.
\]

The bank now solves for both \( m_1 \) and \( m_2 \) at \( t = 1 \). It therefore solves,

\[
\max_{m_1, m_2 \geq 0} - m_1 P_1(m_1) + \beta m_1 \left( \frac{\gamma}{\beta} + (1 - \delta) P_2(m_2) \right) - \beta m_2 P_2(m_2) \\
+ \beta^2 m_2 \left( \phi_P^{b_2} e^b + (1 - \delta) \kappa \right)
\]

\[
s.t. \quad m_1, m_2 \leq \frac{1}{N} (1 - \alpha^{nb}).
\]

The first order conditions give,
\[ m_1 = \max \left\{ 0, \min \left\{ \frac{\beta}{\gamma} \left( \frac{\phi^{nb}R^b + \phi^{nb}M^{-i} + m_2}{2} \right) - \phi^{nb} \alpha^{nb} - M_1^{-i}, \frac{1 - \alpha^{nb}}{N} \right\} \right\}. \]

\[ m_2 = \max \left\{ 0, \frac{m_1(1 - \delta) - \phi^{nb} \alpha^{nb} - M_2^{-i} + \beta \frac{\phi^{eh} + (1 - \delta) \kappa}{c}}{2} \right\}. \]

\[ \text{Proof of Proposition 1.} \] I start the proof by showing that an equilibrium in symmetric strategies exists and then show that this equilibrium is unique. I work through Case 1 in which the bank can not commit to future lending. Case 2 and the equilibrium with commitment can be proved similarly.

I first consider equilibria in which all banks are propping up prices \((m_2 > 0)\), and then equilibria in which no bank is propping up prices. I later show that these are the only two possible equilibria, as an equilibrium in which some banks prop up prices and some do not, does not exist.

First, I consider equilibria in which all banks prop up house prices \((m_2 > 0)\). Then, using Lemma 1, at \(t = 2\), a bank’s choice of \(m_2\) and \(m_1\) are given by,

\[ m_2 = \frac{m_1(1 - \delta) - \phi^{nb} \alpha^{nb} - M_2^{-i} + \beta \frac{\phi^{eh} + (1 - \delta) \kappa}{c}}{2}. \]

\[ m_1 = \min \left\{ \frac{\beta}{\gamma} \left( \phi^{eh} + (1 - \delta) \phi^{nb} \alpha^{nb} + M_2^{-i} + m_2 \right) - \phi^{nb} \alpha^{nb} - M_1^{-i}, \frac{1 - \alpha^{nb}}{N} \right\}. \]
If $m_1 = 0$, $m_2$ can never be greater than 0, therefore we can drop the minimum constraint from $m_1$.

In a symmetric equilibrium, $M^i_2 = (N - 1)m_2$ and $M^i_1 = (N - 1)m_1$. Substituting these into the above expressions, $m_1$ and $m_2$ are given by,

$$m_2 = \frac{m_1(1 - \delta) - \phi_P^{nb}\alpha^{nb} + \beta^{\phi_P^{lb}e^b + (1-\delta)\kappa}}{N + 1}.$$

$$m_1 = \min \left\{ \frac{1}{N + 1 - \beta(1-\delta)^2} \left( \frac{\beta}{2} \left( \phi_P^{nb}\alpha^{nb} + \frac{(N - 1)\left(-\phi_P^{nb}\alpha^{nb} + \beta^{\phi_P^{lb}e^b + (1-\delta)\kappa}\right)}{N + 1} \right) 
+ \frac{1}{c} (\phi_R^{lb}e^b + (1 - \delta)\kappa) + \frac{\beta}{c} \phi_R^{lb} \right), \frac{1 - \alpha^{nb}}{N} \right\}.$$

For $N \geq 1$ The denominator of the RHS is increasing in $N$, while the numerator is decreasing in $N$ since by assumption $-\phi_P^{nb}\alpha^{nb} + \beta^{\phi_P^{lb}e^b + (1-\delta)\kappa} < 0$. Therefore $m_1$ is decreasing in $N$.

This is an equilibrium as long as banks want to make $m_2 > 0$ at $t = 2$. This is the case when,

$$m_1(1 - \delta) - \phi_P^{nb}\alpha^{nb} + \beta^{\phi_P^{lb}e^b + (1-\delta)\kappa} > 0.$$

Rearranging,

$$m_1 > \frac{\phi_P^{nb}\alpha^{nb} - \beta^{\phi_P^{lb}e^b + (1-\delta)\kappa}}{1 - \delta}.$$

Define $\bar{N}$ as the value of $N$ at which $m_1 = \frac{\phi_P^{nb}\alpha^{nb} - \beta^{\phi_P^{lb}e^b + (1-\delta)\kappa}}{1 - \delta}$. This is given by,
\[
\phi^{nb} \alpha^{nb} - \beta \frac{\phi^b_{Reb} + (1-\delta)\kappa}{c} = \min \left\{ \frac{\beta}{c} \left( \phi^{hb}_{Reb} + (1-\delta)c\phi^{nb}_{P} \alpha^{nb} \right) - \phi^{nb}_{Reb} \alpha^{nb}, \frac{1 - \alpha^{nb}}{N} \right\}.
\]

This can be rearranged to give the following expression for \(N\),

\[
N = \min \left\{ \frac{\beta}{c} \left( (1-\delta)\phi^{hb}_{Reb} + (1-\delta)^2c\phi^{nb}_{P} \alpha^{nb} + \phi^{bl}_{Pe} + (1-\delta)\kappa \right) - \alpha^{nb}((1-\delta)\phi^{nb}_{Reb} + \phi^{nb}_{P}), \frac{(1-\alpha^{nb})(1-\delta)}{\phi^{nb}_{P} \alpha^{nb} - \beta \frac{\phi^{bl}_{Pe} + (1-\delta)\kappa}{c}} \right\}.
\]

Since \(m_1\) is decreasing in \(N\), when \(N < \overline{N}\), an equilibrium in symmetric strategies in which banks prop up prices exists.

**Uniqueness:** I now show that when all banks are propping up prices, the symmetric equilibrium is the unique equilibrium. We can write any given bank’s \(t = 1\) and \(t = 2\) optimal lending in terms of prices,

\[
m_2 = m_1(1-\delta) - \frac{P_2}{c} + \frac{\beta}{c} \left( \phi^{bl}_{Pe} + (1-\delta)\kappa \right).
\]

\[
m_1 = \min \left\{ \frac{\beta}{c} \phi^{bl}_{Pe} + \frac{\beta}{c} (1-\delta)P_2 - \frac{P_1}{c}, \frac{1 - \alpha^{nb}}{N} \right\}.
\]

Since all banks face the same \(P_1\) and \(P_2\), if this equilibrium exists, all players must be behaving symmetrically. When banks are behaving symmetrically the equilibrium solution is unique (calculated above). Therefore, the symmetric equilibrium is the unique equilibrium when banks are propping up house prices.

I now consider equilibria in which banks do not prop up house prices. Similar to the case
before, I first establish an equilibrium in symmetric strategies is which banks do not prop up prices and then show that this is the unique equilibrium.

When the bank is not propping up prices, \( m_2 = 0 \). In this case \( P_2 = c\alpha^{nb} \phi_P^{nb} \). At \( t = 1 \), a bank solves,

\[
\max_{m_1} - m_1 P_1(m_1) + \beta m_1 (\phi_R^{bb} e^b + (1 - \delta) \alpha^{nb} \phi_P^{nb}).
\]

The FOC is given by,

\[
-c m_1 - c(\alpha^{nb} \phi_R^{nb} + m_1 + M^{-i}_1) + \beta \phi_R^{bb} e^b + \beta(1 - \delta) \alpha^{nb} \phi_P^{nb} = 0.
\]

\[
m_1 = \max \left\{ 0, \min \left\{ \left. \frac{\beta \phi_R^{bb} e^b + \beta(1 - \delta) \alpha^{nb} \phi_P^{nb} - \alpha^{nb} \phi_R^{nb} - M^{-i}_1}{2}, \frac{1 - \alpha^{nb}}{N} \right\} \right\}.
\]

In a symmetric equilibrium,

\[
m_1 = \max \left\{ 0, \min \left\{ \left. \frac{\beta \phi_R^{bb} e^b + \beta(1 - \delta) \alpha^{nb} \phi_P^{nb} - \alpha^{nb} \phi_R^{nb} - M^{-i}_1}{N + 1}, \frac{1 - \alpha^{nb}}{N} \right\} \right\}.
\]

This is an equilibrium as long as banks do not want to make \( m_2 > 0 \) at \( t = 2 \). This is the case when,

\[
m_1(1 - \delta) - \phi_P^{nb} \alpha^{nb} + \beta \frac{\phi_R^{bb} e^b + (1 - \delta) \kappa}{c} \leq 0.
\]

This is satisfied whenever \( N \geq \bar{N} \).

**Uniqueness:** Uniqueness follows as it did before when banks are not propping up prices.
We can write any given bank’s $t = 1$ and $t = 2$ optimal lending in terms of prices,

$$m_2 = 0.$$ 

$$m_1 = \max \left\{ 0, \min \left\{ \frac{\beta}{c} \phi_{P} e^{b} + \frac{\beta}{c} (1 - \delta) P_2 - \frac{P_1}{c}, \frac{1 - \alpha_{nb}}{N} \right\} \right\}.$$ 

Since all banks face the same $P_1$ and $P_2$, if this equilibrium exists, all players must be behaving symmetrically. When banks are behaving symmetrically the equilibrium solution is unique (calculated above). Therefore, the symmetric equilibrium is the unique equilibrium when banks are propping up house prices.

To complete the proof, I also need to rule out the case when some banks are propping up prices and some are not. I prove this by contradiction. Imagine bank $i$ is propping up prices and bank $j$ is not. Then at $t = 1$,

$$m_i^j = \min \left\{ \frac{\beta}{c} \phi_{P} e^{b} + \frac{\beta}{c} (1 - \delta) P_2 - \frac{P_1}{c}, \frac{1 - \alpha_{nb}}{N} \right\}.$$ 

$$m_j^i = \max \left\{ 0, \min \left\{ \frac{\beta}{c} \phi_{P} e^{b} + \frac{\beta}{c} (1 - \delta) P_2 - \frac{P_1}{c}, \frac{1 - \alpha_{nb}}{N} \right\} \right\}.$$ 

These expressions imply that $m_i^j = m_j^i$. However, if this is the case, $m_2 = m_1$ which is a contradiction. Therefore, in any equilibrium either all banks will be propping up prices, or no bank will be propping up prices.

For Case 2 without commitment, the proposition can be proven in a similar way. In this case, $\overline{N}$ is given by the following expression,
\( N = \min \left\{ \frac{\beta}{\gamma} \left( (1 - \delta) \min \left\{ \phi_{\emptyset}^{h b} \phi_{R}^{h b} e^{b} + (1 - \delta)^{2} \phi_{P}^{a} \alpha^{n b} + \phi_{P}^{b} e^{b} + (1 - \delta) \kappa \right\}, \phi_{P}^{a} \alpha^{n b} - \beta \frac{\phi_{P}^{b} e^{b} + (1 - \delta) \kappa}{\gamma} \right\}, \right. \\
\left. \frac{\alpha^{n b} \left( 1 - \delta \right) (1 - \phi_{R}^{h b}) \phi_{R}^{n b} + \phi_{P}^{n b} \right\}, \frac{(1 - \alpha^{n b}) (1 - \phi_{R}^{h b}) (1 - \delta)}{\phi_{P}^{a} \alpha^{n b} - \beta \frac{\phi_{P}^{b} e^{b} + (1 - \delta) \kappa}{\gamma}} \right\}. \\
\right) \\
\]
\[ M_1 = \max \left\{ 0, \min \left\{ N \frac{\beta \phi_{b}^{h} e^{b}}{c} + \beta (1 - \delta) \alpha_{n}^{b} \phi_{P}^{n} - \alpha_{n}^{b} \phi_{R}^{n}, (1 - \alpha_{n}^{b}) \right\} \right\}. \]

This is increasing in \( N \).

If \( N < \overline{N} \), the economy is in an equilibrium in which banks prop up house prices. The total credit at \( t = 1 \) extended by a single bank is given by,

\[
m_1 = \frac{1}{N + 1 - \beta (1 - \delta)^2 - \beta (1 - \delta)^2 \frac{N - 1}{2(N + 1)}} \left( \frac{\beta}{2} \frac{1 - \delta}{2} \left( \phi_{P}^{n} \alpha_{n}^{b} + \phi_{P}^{n} e^{b} + (1 - \delta) \kappa \right) \right) + \beta \frac{1 - \delta}{2c} \left( \phi_{P}^{n} e^{b} + (1 - \delta) \kappa \right) + \frac{\beta}{c} \phi_{R}^{n} e^{b} - \phi_{R}^{n} \alpha_{n}^{b}. \]

\( m_1 \) increases as \( N \) decreases (from the proof of Proposition 1). The total credit at \( t = 2 \) extended by a single bank is given by,

\[
m_2 = \frac{m_1 (N)(1 - \delta) - \phi_{P}^{n} \alpha_{n}^{b} + \beta \frac{\phi_{P}^{n} e^{b} + (1 - \delta) \kappa}{c}}{N + 1}. \]

As \( N \) decreases, \( m_2 \) increases. Therefore, the total credit extended by a single bank increases as \( N \) decreases.

The total credit in the economy is given by,
\[ M_1 + M_2 = \frac{N}{N + 1 - \beta \frac{(1 - \delta)^2}{2} - \beta (1 - \delta)^2 \frac{N - 1}{2(N+1)}} \left( 1 + \frac{1 - \delta}{N + 1} \right) \left( \beta \frac{1 - \delta}{2} (\phi_P^n \alpha^n b + \right.
\left. \frac{N - 1}{N + 1} \left( -\phi_P^n \alpha^n b + \beta \phi_P^n e^b \frac{(1 - \delta)\kappa}{c} \right) \right) + \beta^2 \frac{1 - \delta}{2c} (\phi_P^n e^b + (1 - \delta) \kappa + \phi_R^n e^b \n\left. - \phi_R^n \alpha^n b) + \frac{N}{N + 1} \left( -\phi_P^n \alpha^n b + \beta \phi_P^n e^b \frac{(1 - \delta)\kappa}{c} \right). \]

Taking the derivative w.r.t \( N \),

\[- \left( (1 - \delta)^2 \beta + (1 - \delta) - 1 \right) N^2 - 2N - (1 - \delta) - 1 \left( \beta \frac{1 - \delta}{2} \phi_P^n \alpha^n b + \beta^2 \frac{1 - \delta}{2c} \n\left. \right) (\phi_P^n e^b + (1 - \delta) \kappa) + \beta \phi_R^n e^b - \phi_R^n \alpha^n b) \right) - \frac{-\phi_P^n \alpha^n b + \phi_P^n e^b \frac{(1 - \delta)\kappa}{c}}{(N + 1)^2 \left( 2(1 - (1 - \delta)^2 \beta) N + 1 \right)^2} \n\left. \right) ((1 - \delta)^2 \beta + 1 - \delta - 4) N^4 + (4(1 - \delta)^2 \beta - 2(1 - \delta) - 12) N^3 \n\left. \right) + (-1 - \delta)^3 \beta^2 + (2(1 - \delta)^3 + 5(1 - \delta)^2) \beta - 6(1 - \delta) - 12) N^2 \n\left. \right) + 2((1 - \delta)^2 \beta - 2(1 - \delta) - 4)N + 1 - \delta). \]

The second term in the above expression is always negative and this can cause the value of the total derivative to be negative. Therefore aggregate credit can increase as \( N \) decreases.

The proof in Case 2 and the case with commitment follow similarly. \( \blacksquare \)

**Proof of Corollary 2.** Consider Case 1 without commitment. From Proposition 1, \( \mathcal{N} \) is given by,

\[ \text{For } N \geq 1, \text{ the multiplier on } \phi_P^n \alpha^n b \text{ and } \phi_P^n \alpha^n b \text{ are always negative. The mathematics of when exactly this expression is negative is tedious and does not add anything much to understanding the main mechanism in the paper but can be made available on request. An example of this can be seen in the graphical illustration in the paper.} \]
\[
N = \min \left\{ \frac{\beta}{c} \left( (1 - \delta) \phi_R e^b + (1 - \delta)^2 c \phi_P \alpha^{nb} + \phi_P e^b + (1 - \delta) \kappa \right) - \alpha^{nb} \left( (1 - \delta) \phi_R + \phi_P \right), \phi_P \alpha^{nb} - \beta \phi_P e^b + (1 - \delta) \kappa \right\}
\]

\[
\frac{(1 - \alpha^{nb})(1 - \delta)}{\phi_P \alpha^{nb} - \beta \phi_P e^b + (1 - \delta) \kappa}.
\]

It is straightforward from the above expression that,

\[
\frac{\partial N}{\partial \phi_P e^b} > 0.
\]

As \( \delta \) increases, the denominator in the expression for \( \overline{N} \) increases. At the same time the numerator of the above expression is decreasing. To see this clearly for the first term of the minimization, taking the derivative of the numerator of \( N \) w.r.t. \( \delta \), we get,

\[
-\frac{\beta}{c} \left( \phi_R e^b + 2(1 - \delta) c \phi_P \alpha^{nb} + \kappa \right) + \alpha^{nb} \phi_R
\]

(A.1)

Recall from Proposition 1, that when banks are not propping up prices,

\[
m_1 = \max \left\{ 0, \min \left\{ \frac{\beta \phi_R e^b}{c} + \beta (1 - \delta) \alpha^{nb} \phi_P - \alpha^{nb} \phi_R, \frac{1 - \alpha^{nb}}{N} \right\} \right\}.
\]

Therefore, in the relevant range, \( \frac{\beta \phi_R e^b}{c} + \beta (1 - \delta) \alpha^{nb} \phi_P - \alpha^{nb} \phi_R > 0 \) since \( m_1 \geq 0 \). This implies that (A.3) is negative. Therefore \( \overline{N} \) is decreasing in \( \delta \).

Similarly as \( \phi_P \) increases, the denominator of \( \overline{N} \) is increasing. Additionally in the first term of the minimization, the numerator is decreasing: taking the derivative of the numerator of
\( \bar{N} \) w.r.t. \( \phi_{P}^{nb} \), we get,

\[
\alpha^{nb}(\beta(1 - \delta)^2 - 1) < 0.
\]

Therefore \( \bar{N} \) is decreasing in \( \phi_{P}^{nb} \).

The proof in Case 2 and the case with commitment follow similarly. ■

**Proof of Proposition 2.** For any default costs, \( d > 0 \), the total welfare under \( N_1 \) is

\[
\begin{align*}
(M_1(N_1) + M_2(N_1) + (\phi_{P}^{nb} + \phi_{R}^{nb})\alpha^{nb})\gamma + \beta e^b(\phi_{R}^{bh} + \phi_{P}^{bh})(1 - \alpha^{nb}) + 2\beta\alpha^{nb}e^{nb} \\
- \beta d(M_1(N_1)(1 - \phi_{R}^h)M_2(N_1)(1 - \phi_{P}^h))
\end{align*}
\]

The total welfare under \( N_2 \) is

\[
\begin{align*}
(M_1(N_2) + (\phi_{P}^{nb} + \phi_{R}^{nb})\alpha^{nb})\gamma + \beta e^b(\phi_{R}^{bh} + \phi_{P}^{bh})(1 - \alpha^{nb}) + 2\beta\alpha^{nb}e^{nb} \\
- \beta d(M_1(N_1)(1 - \phi_{R}^h))
\end{align*}
\]

Since \( M_1(N_2) = M_1(N_1) + M_2(N_1) \), \( \phi_{R}^b > \phi_{P}^b \) and \( d > 0 \), welfare is always higher under \( N_2 \) than \( N_1 \). ■

**Proof of Proposition 3.** Consider Case 1. Since \( \bar{N} \) depends on \( \phi_{P}^{bh} \) and \( \phi_{P}^{nb} \), this proposition only compares areas given an \( N_1 \) and \( N_2 \) s.t. \( \bar{N} \) for each area falls within \( (N_1, N_2] \). Following an increase in concentration from \( N_1 \geq \bar{N} \) to \( N_2 < \bar{N} \), the change in total lending is given by,
\[ \Delta M = \left( N_2 + \frac{(1 - \delta)N_2}{N_2 + 1} \right) \]
\[ + \frac{N_2(1 + 1 - \delta)}{N_2 + 1} \left( \frac{\beta(1 - \delta)^2}{2(N_2 + 1)} - \frac{\beta(1 - \delta)^2}{2N_2 + 1} \right) + N_2 \left( \frac{\phi_{p}^{nb} \alpha_{p}^{nb} + \phi_{bl}^{b} e^{b} + (1 - \delta) \kappa}{c} \right) - N_1 \frac{\phi_{b}^{b} e^{b}}{c} + \beta(1 - \delta) \alpha_{p}^{nb} \phi_{p}^{nb} - \alpha_{nb}^{nb} \phi_{R}^{b} \right). \]

Taking the derivative w.r.t \( \phi_{p}^{nb} \),

\[ \frac{\partial \Delta M}{\partial \phi_{p}^{nb}} = N_2 \left( 1 + \frac{1 - \delta}{N_2 + 1} \right) \beta \frac{1 - \delta}{2} \left( \alpha_{p}^{nb} - \frac{(N_2 - 1) \alpha_{p}^{nb}}{N_2 + 1} \right) - N_2 \alpha_{nb}^{nb} \frac{N_2}{N_2 + 1} - N_1 \beta \frac{(1 - \delta) \alpha_{p}^{nb}}{N_2 + 1} < 0. \]

If \( \frac{\partial \Delta M}{\partial \phi_{p}^{nb}} < 0 \), then income growth and the growth in mortgage credit can be negatively correlated. For this to be the case, we require that,

\[ N_2 \left( 1 + \frac{1 - \delta}{N_2 + 1} \right) \beta \frac{1 - \delta}{2N_2 + 1} - \frac{N_2 \alpha_{nb}^{nb}}{N_2 + 1} - N_1 \beta \frac{(1 - \delta) \alpha_{p}^{nb}}{N_2 + 1} < 0. \]

Simplifying,

\[ (1 - \delta) \frac{N_2 + 2 - \delta}{N_2^2 + 1 + 2N_2 - \beta(1 - \delta)^2 N_2 + 1} - \frac{1}{\beta} \frac{N_2}{N_2 + 1} - N_1 \frac{1 - \delta}{N_1 + 1} < 0. \]

The denominator of the coefficient multiplying the first term is greater than the numerator. Therefore the first term is strictly less that the value of the second term and this expression is always less than 0. Therefore, \( \frac{\partial \Delta M}{\partial \phi_{p}^{nb}} < 0. \)
Looking at (A.2), we can see that all the terms multiplying $\phi_P^{bl}$ are always positive. Therefore, 
$$\frac{\partial \Delta M}{\partial \phi_P^{bl}} > 0.$$ 
The proof in Case 2 and the case with commitment follow similarly. □

**Proof of Proposition 4.** I start the proof by simplifying the problem in a similar way to the three-period model by focusing on the bank’s choice of interest rate. Given state-contingent repayments, state by state, a bank will charge the maximum interest rate such that borrowers are willing to pay. Consider the interest rate a bank will charge borrowers from generation $t$ when the state of the world is $s_t \in \{R, P\}$. As in the three-period model, depending on the relative values of $\phi_{sl}^{bh}, \phi_{sl}^{bl}$ and $\gamma$, there are various possible cases. Here, I will work through the interest rate problem in the case in which $\phi_{R}^{bh} e^b < \frac{\gamma}{\beta}$ without commitment. The other cases are similar.

If $\phi_{sl}^{bj} e^b < \frac{\gamma}{\beta}$, where $j = \{h, l\}$ represents borrower-type, a bank will charge:

$$P_{t-1}(1 + r_{t-1}(s_t)) > e^b + (1 - \delta)P_t(s_t).$$

In this case, a proportion $\phi_{sl}^{bj}$ of bank borrowers get a positive endowment and pay the bank $e^b + (1 - \delta)P_t(s_t)$ while the remaining do not get an endowment and pay the bank gets $(1-\delta)P_t(s_t)$. In expectation, borrowers repay the bank $\phi_{sl}^{bj} e^b + (1-\delta)E[P_t(s_t)]$ which satisfies their purchasing constraint. When $\phi_{R}^{bh} e^b < \frac{\gamma}{\beta}$, because of the assumptions on the income of high- versus low-quality borrowers and in rich versus poor states, $\phi_{sl}^{bj} e^b < \frac{\gamma}{\beta}$, $\forall s_t, \forall j$.

As in the three-period case, in the model without commitment, banks can only credibly prop up house prices to improve their return on loans when the borrower cannot repay the full face-value of the loan. State-by-state the bank will therefore choose a facevalue that is strictly higher than $e^b + (1 - \delta)P_t(s_t)$.

In this case, define $m_t = m_t^h + m_t^l$. Then, we can write the bank’s maximization problem
at any time $t$ as:

$$V(s_t, m_{t-1}) = \max_{m^h_t \geq 0, m^l_t \geq 0} m_{t-1}(1 - \delta)P_t - \sum_{j \in \{h,l\}} m^j_t P_t + \beta \sum_{j \in \{h,l\}} m^j_t \phi^{bj}_s e^b + \beta E[V(s_{t+1}, m_t)]$$

s.t. $m^h_t \leq \frac{1}{N} \alpha^{bh}$, $m^l_t \leq \frac{1}{N} (1 - \alpha^{bh} - \alpha^{nb})$.

The above problem is independent of interest rates chosen by the bank. Therefore each period a bank only decides on the amount of loans they wish to issue to each type of borrower. Further, it is independent of the income of generation $t - 1$. The housing price only depends on the total number of loans outstanding to generation $t - 1$. In this case, the problem is also independent of the type of loans made to high- versus low-quality borrowers at $t - 1$ but in other cases, it is not.

Using the envelope theorem,

$$\frac{\partial E[V(s_{t+1}, m_t)]}{\partial m^h_t} = \frac{\partial E[V(s_{t+1}, m_t)]}{\partial m^l_t} = (1 - \delta)E[P_{t+1}]$$

This gives the following FOCs for a bank,

$$\frac{\partial V(s_t, m_{t-1})}{\partial m^h_t} = m_{t-1}(1 - \delta)c - P_t - m^h_t c + \beta \phi^{bh}_s e^b + \beta(1 - \delta)E[P_{t+1}].$$

$$\frac{\partial V(s_t, m_{t-1})}{\partial m^l_t} = m_{t-1}(1 - \delta)c - P_t - m^l_t c + \beta \phi^{bl}_s e^b + \beta(1 - \delta)E[P_{t+1}].$$

The equilibrium lending by a bank is given by,
\[ m_h^t = \max \left\{ 0, \min \left\{ \frac{m_{t-1}(1 - \delta) c - P_t + \beta \phi_{s_t} e^b + \beta (1 - \delta) E[P_{t+1}]}{c}, \frac{\alpha^{bh}}{N} \right\} \right\}. \]

\[ m_l^t = \max \left\{ 0, \min \left\{ \frac{m_{t-1}(1 - \delta) c - P_t + \beta \phi_{s_t} e^b + \beta (1 - \delta) E[P_{t+1}]}{c}, 1 - \alpha^{bh} - \alpha^{nb} \right\} \right\}. \]

Given a choice of lending to high-quality borrowers over low-quality borrowers, it is always dominant for a bank to make a loan to a high-quality borrower. Therefore, if,

\[ \frac{m_{t-1}(1 - \delta) c - P_t + \beta \phi_{s_t} e^b + \beta (1 - \delta) E[P_{t+1}]}{c} \leq \frac{\alpha^{bh}}{N} \]

\[ m_l^t = 0. \]

I now show an equilibrium is always symmetric if all banks start with the same level of initial loans \( m_0 \). Consider a bank’s lending at \( t = 1 \).

\[ m_h^1 = \max \left\{ 0, \min \left\{ \frac{m_0(1 - \delta) c - P_1 + \beta \phi_{s_1} e^b + \beta (1 - \delta) E[P_2]}{c}, \frac{\alpha^{bh}}{N} \right\} \right\}. \]

\[ m_l^1 = \max \left\{ 0, \min \left\{ \frac{m_0(1 - \delta) c - P_1 + \beta \phi_{s_1} e^b + \beta (1 - \delta) E[P_2]}{c}, 1 - \alpha^{bh} - \alpha^{nb} \right\} \right\}. \]

If all banks have the same \( m_0 \), then the above equations will be identical for all banks and they will choose the same \( m_1 \). Similarly at \( t = 2 \), we can show that if all banks have the same \( m_1 \), they will choose the same \( m_2 \) and so on and so forth. Given the symmetric
equilibrium solution, I can rewrite equilibrium lending as,

\[
\frac{M_t^h}{N} = \max\left\{0, \min\left\{\frac{M_{t-1}^h (1 - \delta)c - cM_t^h - c\alpha^{nb}\phi_{st}^h + \beta\phi_{st}^c + \beta(1 - \delta)E[P_{t+1}]}{c}, \frac{\alpha^{bh}}{N}\right\}\right\}.
\]

(A.3)

\[
\frac{M_t^l}{N} = \max\left\{0, \min\left\{\frac{M_{t-1}^l (1 - \delta)c - cM_t^l - c\alpha^{nb} + \beta\phi_{st}^c + \beta(1 - \delta)E[P_{t+1}]}{c}, \frac{1 - \alpha^{bl}}{N}\right\}\right\}.
\]

(A.4)

In the above equation, \(\alpha^{bl} = 1 - \alpha^{bh} - \alpha^{nb}\). Given these first order conditions, I can write an equivalent maximization problem of a representative bank in this economy. This is given by,

\[
V(s_t, M_{t-1}) = \max_{M_t^h \geq 0, M_t^l \geq 0} \left\{ M_{t-1}^h (1 - \delta)P_t + \sum_{j=\{h,l\}} \left( -\frac{M_t^j}{N} P_t - \frac{N - 1 - M_t^j}{2} c - \frac{(N - 1)}{N} M_t^j \phi_{st}^c \alpha^{nb} c + \beta M_t^j \phi_{st}^c \right) \right. \\
\left. - \frac{(N - 1)}{N} M_t^h M_t^l c + \beta E [V(s_{t+1}, M_t)] \right\} \\
s.t. \ M_t^h \leq \alpha^h \\
s.t. \ M_t^l \leq 1 - \alpha^h - \alpha^{nb}.
\]

The first order conditions for this representative bank give the same aggregate lending as those of the individual banks. I can show that the above maximization of the equivalent representative bank is a contraction mapping. Define
\[
U(s_t, M_{t-1}, M^h_t, M^l_t) = \frac{M_{t-1}}{N}(1 - \delta)P_t + \\
\sum_{j=\{h,l\}} \left( -\frac{M^j_t}{N}P_t - \frac{N - 1}{N} \frac{M^2_t}{2} c - \frac{(N - 1)}{N} M^j_t \phi^h_{s_t} \alpha^b c + \beta M^j_t \phi^b_{s_t} c \right) - \frac{(N - 1)}{N} M^h_t M^l_t c
\]

Since \(M^h_t, M^l_t\) and \(M_{t-1}\) are bounded, \(P_t\) is bounded and therefore \(U\) is bounded. Define an operator,

\[
(TV)(s_t, M_{t-1}) = \max_{0 \leq M^h_t \leq \alpha^h, 0 \leq M^l_t \leq 1 - \alpha^h - \alpha^b} \left\{ U(s_t, M_{t-1}, M^h_t, M^l_t) + \beta E[V(s_{t+1}, M_t)] \right\}.
\]

Take \(V\) to be bounded. Since \(U\) is bounded by assumption, then \(TV\) is also bounded. \(TV\) satisfies monotonicity. Suppose \(V < W\). Let \(g_h(M_{t-1}, s_t)\) and \(g_l(M_{t-1}, s_t)\) be the optimal policy functions (not necessarily unique) corresponding to \(V\) for \(M^h_t\) and \(M^l_t\) respectively. Then for all \(M_{t-1} \in [0, 1 - \alpha^b]\),

\[
TV(s_t, M_{t-1}) = \\
U(s_t, M_{t-1}, g_h(M_{t-1}, s_t), g_l(M_{t-1}, s_t)) + \beta E[V(s_{t+1} + g_h(M_{t-1}, s_t) + g_l(M_{t-1}, s_t))] \\
\leq U(s_t, M_{t-1}, g_h(M_{t-1}, s_t), g_l(M_{t-1}, s_t)) + \beta E[W(s_{t+1}, g_h(M_{t-1}, s_t) + g_l(M_{t-1}, s_t))]. \\
\leq \max_{0 \leq M^h_t \leq \alpha^h, 0 \leq M^l_t \leq 1 - \alpha^h - \alpha^b} \left\{ U(s_t, M_{t-1}, M^h_t, M^l_t) + \beta E[W(s_{t+1}, M^h_t + M^l_t)] \right\} \\
= TW(s_t, M_{t-1})
\]

\(TV\) also satisfies discounting. Let \(a > 0\). Then,
\[ T(V + a)(s_t, M_{t-1}) = \]
\[ = \max_{0 \leq M_h^t \leq \alpha bh, 0 \leq M_l^t \leq 1 - \alpha nh - \alpha nb} \{ U(s_t, M_{t-1}, M_h^t, M_l^t) + \beta E[V(s_{t+1}, M_t) + a]\} \]
\[ = \max_{0 \leq M_h^t \leq \alpha bh, 0 \leq M_l^t \leq 1 - \alpha bh - \alpha nb} \{ U(s_t, M_{t-1}, M_h^t, M_l^t) + \beta E[V(s_{t+1}, M_t)]\} + \beta a \]
\[ = TV(s_t, M_{t-1}) + \beta a \]

The model therefore satisfies Blackwell’s conditions and is bounded and is therefore a contraction mapping with modulus \(\beta\). Therefore, an equilibrium of this economy exists and can be found through value function iteration.

The Hessian matrix of \(U\) is given by,
\[
\begin{pmatrix}
-\frac{N+1}{N}c & -\frac{N-1}{N}c \\
-\frac{N-1}{N}c & -\frac{N+1}{N}c
\end{pmatrix}
\]

The determinant of the Hessian is given by,
\[
\left( \frac{N + 1}{N}c \right)^2 - \left( \frac{N - 1}{N}c \right)^2 < 0.
\]

The Hessian matrix is negative semi-definite since the determinant is less than 0 and \(-\frac{N+1}{N}c < 0\). Therefore, for all \(M_h^t\) and \(M_l^t\), \(U\) is a strictly concave function. Let \(S_{M_t}\) be the set of all possible values of \(M_t\). Then since \(S_{M_t}\) is convex, the correspondence which gives the set of all feasible allocations given \(M_t\) is convex, and \(U\) is continuous and bounded, there is a unique policy function associated with the above problem and \(V^*\) is strictly concave. The equilibrium is therefore unique.

From (A.3) and (A.4), it is straightforward that conditional on the state of the economy, \(M_t\) is linearly increasing in \(M_{t-1}\). Furthermore, due to linearity, conditional on state, \(M_h^t\)
is strictly increasing in $M_{t-1}^h$ when $M_{t-1}^h < \alpha^{bh}$.

An equilibrium in which banks do not make any loans to low-quality borrowers to prop-up prices is equivalent to an equilibrium in which $\forall t$ and $\forall s$,

$$\left(\frac{M_{t-1}^h}{N}(1-\delta)c - cM_t^h - c\alpha^{nb} + \beta\phi^{bh}_{s_t} + \beta(1-\delta)E[P_{t+1}]\right)\frac{N}{c(N+1)} \leq 0$$

Since $M_t$ is linearly increasing in $M_{t-1}$ (and strictly increasing in the range $M_{t-1} \in [0, \alpha^{bh}]$), then $\exists T$ s.t. a series of $T$ consecutive $R$ shocks that will eventually give $M_T = \alpha^{bh}$ (as long as $V(R, M_0) > 0$). Since the incentive to prop up prices is highest when outstanding loans are the highest, if at $T+1$, in either state, a bank does not want to prop up prices given that they expect no other banks to be propping up prices, then for any series of shocks banks make no loans to low-quality borrowers. Substituting in $M_{t-1} = \alpha^{bh}$ into the above condition, such an equilibrium exists when,

$$\left(\frac{\alpha^{bh}}{N}(1-\delta)c - cM_t^h - c\alpha^{nb} + \beta\phi^{bh}_{s_t} + \beta(1-\delta)E[P_{t+1}]\right)\frac{N}{c(N+1)} \leq 0$$

Since banks will always make high-quality loans before low-quality loans, this implies that $M_t^h = \alpha^{bh}$. Substituting this in,

$$\frac{\alpha^{bh}}{N}(1-\delta)c - c\alpha^{bh} - c\alpha^{nb} + \beta\phi^{bh}_{s_t} + \beta(1-\delta)E[P_{t+1}] \leq 0$$

The LHS is decreasing as $N$ increases. Note that $P_{t+1}(M_{t-1}, s_t)$ is increasing in outstanding loans per bank, $\frac{M_{t-1}}{N}$, and therefore as $N$ increases, $E[P_{t+1}]$ decreases. Therefore, $\exists N$ s.t. when $N \geq \bar{N}$, an equilibrium in which banks do not prop up house prices exists. When $N < \bar{N}$, bank will make loans to low-quality borrowers and prop up prices. The other cases can be worked through similarly. □
Proof of Corollary 3.

The proof for this is contained in the proof for Proposition 3. Please refer to that above.

■
Appendix B

Income Growth and Mortgage Growth with Borrower Heterogeneity

In the three-period model in the main text it is clear that non-borrower income growth can cause there to be a negative correlation between credit and income-growth when looking at an area following an increase in concentration. However, to additionally be able to show that a positive correlation can still exist between credit and income growth at the borrower-level, we need to allow for borrower level heterogeneity. To illustrate the two results simultaneously, it is enough to allow borrower-level heterogeneity at $t = 2$ and look at $t = 2$ credit when concentration changes. At $t = 2$, let there be a proportion $\alpha_{bh}$ of high-quality borrowers who get an endowment with probability $\phi_{R}^{bh}$. Then we get the following graphs for credit at a borrower-level versus an area-level as concentration changes.

![Figure 17: Income and Mortgage Credit Growth: Borrower Heterogeneity](image)

The figure on the left plots total credit, measured by the number of households who get a mortgage, against the level of concentration in the mortgage market for different income growths between $t = 1$ and $t = 2$. The figure on the right plots credit received by high-quality borrowers versus by low-quality borrowers inside the area with lower-income growth against the level of concentration. As we move along the x-axis, $N$ increases and concentration decreases. The parametrization is as follows: $\delta = .01, \alpha^{nb} = .2, \phi_{P}^{nh} = .2, \phi_{R}^{bh} = 1, \phi_{R}^{nh} = 1, \epsilon_{k} = 2, \kappa = .45, \gamma = 4, b = 9.8, \alpha^{bh} = .01, \phi_{P}^{nh} = .41$. $\phi_{P}^{nh}$ is varied to get changes in income growth across the two plots - it is equal to .73 in the high-income growth area and .7 in the low-income growth area.
Appendix C

Calibration of Benchmark Model

**Data Description:** The fraction of sub-prime borrowers is from a research report by the Financial Inquiry Commission using data from Inside Mortgage Finance. U.S. house price changes are calculated from Federal Reserve Economic Data. The home-ownership rate is taken from the U.S. Census Bureau. The default rates on prime and sub-prime loans are taken from a research report by the U.S. Census Bureau. The fraction of cash-only house purchases are from RealtyTrac. The private benefit of home-ownership is hard to measure in the data and therefore I choose a value of $\gamma$ so that it does not determine any equilibrium quantities.

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of high-quality borrowers</td>
<td>$\alpha^{bh}$</td>
<td>.5</td>
<td>Loans to prime borrowers</td>
</tr>
<tr>
<td>Fraction of non-borrowers</td>
<td>$\alpha^{nb}$</td>
<td>.06</td>
<td>Cash only house purchases</td>
</tr>
<tr>
<td>Low-quality shock rich state</td>
<td>$\phi^{hl}_R$</td>
<td>.9</td>
<td>Default rate sub-prime loans boom</td>
</tr>
<tr>
<td>Low-quality shock poor state</td>
<td>$\phi^{hl}_P$</td>
<td>.75</td>
<td>Default rate sub-prime loans bust</td>
</tr>
<tr>
<td>High-quality shock rich state</td>
<td>$\phi^{bh}_R$</td>
<td>.98</td>
<td>Default rate prime loans boom</td>
</tr>
<tr>
<td>High-quality shock poor state</td>
<td>$\phi^{bh}_P$</td>
<td>.94</td>
<td>Default rate prime loans bust</td>
</tr>
<tr>
<td>Discount Factor</td>
<td>$\beta$</td>
<td>.99</td>
<td>Standard</td>
</tr>
<tr>
<td>Depreciation</td>
<td>$\delta$</td>
<td>.02</td>
<td>Standard</td>
</tr>
<tr>
<td>Number of banks</td>
<td>$N$</td>
<td>2</td>
<td>Mortgage market concentration</td>
</tr>
<tr>
<td>Borrower endowment</td>
<td>$e_b$</td>
<td>2.4</td>
<td>House prices, sub-prime fraction</td>
</tr>
<tr>
<td>Construction cost</td>
<td>$c$</td>
<td>5.6</td>
<td>House prices, sub-prime fraction</td>
</tr>
<tr>
<td>Likelihood of rich state</td>
<td>$q$</td>
<td>.5</td>
<td>House prices, sub-prime fraction</td>
</tr>
</tbody>
</table>

To limit the number of free parameters, $\gamma \geq 2.6$, $\phi^{nb}_R = \phi^{nb}_P = 1$. 

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Table 13: Aggregate Moments: Benchmark Model

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of sub-prime borrowers in boom</td>
<td>.12</td>
<td>.12</td>
</tr>
<tr>
<td>Fraction of sub-prime borrowers in bust</td>
<td>.05</td>
<td>.05</td>
</tr>
<tr>
<td>Default rate sub-prime loans in boom</td>
<td>.10</td>
<td>.12</td>
</tr>
<tr>
<td>Default rate sub-prime loans in bust</td>
<td>.25</td>
<td>.25</td>
</tr>
<tr>
<td>Default rate prime loans in boom</td>
<td>.02</td>
<td>.02</td>
</tr>
<tr>
<td>Default rate on prime loans in bust</td>
<td>.06</td>
<td>.06</td>
</tr>
<tr>
<td>Fraction of cash only house purchases boom</td>
<td>.09</td>
<td>.13</td>
</tr>
<tr>
<td>Fraction of cash only house purchases bust</td>
<td>.10</td>
<td>.22</td>
</tr>
<tr>
<td>House price increase boom</td>
<td>.52</td>
<td>1.43</td>
</tr>
<tr>
<td>House price decrease bust</td>
<td>-.11</td>
<td>-.20</td>
</tr>
<tr>
<td>Change home-ownership rate boom</td>
<td>.07</td>
<td>.05</td>
</tr>
<tr>
<td>Change home-ownership rate bust</td>
<td>.07</td>
<td>.02</td>
</tr>
</tbody>
</table>

In the model without commitment, since the banks charge a face-value slightly higher than $e^b + (1 - \delta)P_2$, I consider a mortgage as delinquent when borrowers repay the banks less than $e^b + (1 - \delta)P_2$.

Table 14: Counter factual Analysis: Benchmark Model

<table>
<thead>
<tr>
<th></th>
<th>N=2</th>
<th>N=4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of sub-prime borrowers in boom</td>
<td>.12</td>
<td>0</td>
</tr>
<tr>
<td>Fraction of sub-prime borrowers in bust</td>
<td>.05</td>
<td>0</td>
</tr>
<tr>
<td>House price increase boom</td>
<td>.52</td>
<td>.22</td>
</tr>
<tr>
<td>House price decrease bust</td>
<td>-.23</td>
<td>-.03</td>
</tr>
</tbody>
</table>
Calibration of Model with Dispersed Lenders

Data Description: The data is as before. The GSE market share is calculated from the Federal Reserve and Federal Housing Finance.

Table 15: Configuration of Model Parameters: Lender Heterogeneity

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of high-quality borrowers</td>
<td>$\alpha^h$</td>
<td>.5</td>
<td>Fraction of prime/sub-prime borrowers</td>
</tr>
<tr>
<td>Fraction of non-borrowers</td>
<td>$\alpha^{nb}$</td>
<td>.06</td>
<td>Fraction of cash only house purchases</td>
</tr>
<tr>
<td>Low-quality shock rich state</td>
<td>$\phi^R_{lb}$</td>
<td>.9</td>
<td>Default rate sub-prime loans boom</td>
</tr>
<tr>
<td>Low-quality shock poor state</td>
<td>$\phi^P_{lb}$</td>
<td>.75</td>
<td>Default rate sub-prime loans bust</td>
</tr>
<tr>
<td>High-quality shock rich state</td>
<td>$\phi^R_{hb}$</td>
<td>.98</td>
<td>Default rate prime loans boom</td>
</tr>
<tr>
<td>Low-quality shock poor state</td>
<td>$\phi^P_{hb}$</td>
<td>.95</td>
<td>Default rate prime loans bust</td>
</tr>
<tr>
<td>Discount Factor</td>
<td>$\beta$</td>
<td>.99</td>
<td>Standard</td>
</tr>
<tr>
<td>Depreciation</td>
<td>$\delta$</td>
<td>.02</td>
<td>Standard</td>
</tr>
<tr>
<td>borrower endowment</td>
<td>$e_b$</td>
<td>1.4</td>
<td>house price boom/bust</td>
</tr>
<tr>
<td>Elasticity</td>
<td>$c$</td>
<td>5.8</td>
<td>house price boom/bust</td>
</tr>
<tr>
<td>Number of banks</td>
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<td>Mortgage market concentration</td>
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<tr>
<td>Borrower share of dispersed banks</td>
<td>$s$</td>
<td>.6</td>
<td>Mortgage market concentration</td>
</tr>
<tr>
<td>Likelihood of rich state</td>
<td>$q$</td>
<td>.5</td>
<td></td>
</tr>
</tbody>
</table>

To limit the number of free parameters, $\gamma \geq 1.4$, $\phi^R_{lb} = \phi^P_{lb} = 1$. 

144
Table 16: Aggregate Moments: Lender Heterogeneity

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of sub-prime borrowers in boom</td>
<td>.11</td>
<td>.12</td>
</tr>
<tr>
<td>Fraction of sub-prime borrowers in bust</td>
<td>.07</td>
<td>.05</td>
</tr>
<tr>
<td>Default rate sub-prime loans in boom</td>
<td>.10</td>
<td>.12</td>
</tr>
<tr>
<td>Default rate sub-prime loans in bust</td>
<td>.25</td>
<td>.25</td>
</tr>
<tr>
<td>Default rate prime loans in boom</td>
<td>.02</td>
<td>.02</td>
</tr>
<tr>
<td>Default rate prime loans in bust</td>
<td>.06</td>
<td>.06</td>
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<tr>
<td>Default rate on prime loans in bust</td>
<td>.02</td>
<td>.02</td>
</tr>
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<td>Fraction of cash only house purchases boom</td>
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<td>Fraction of cash only house purchases bust</td>
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<tr>
<td>House price increase boom</td>
<td>.63</td>
<td>1.43</td>
</tr>
<tr>
<td>House price decrease bust</td>
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<td>-.20</td>
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<tr>
<td>Change home-ownership rate boom</td>
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<td>Change home-ownership rate bust</td>
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<td>GSE Market Share '07</td>
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<td>.42</td>
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</table>

In the model without commitment, since the banks charge a face-value slightly higher than $e^b + (1 - \delta)P_2$, I consider a mortgage as delinquent when borrowers repay the banks less than $e^b + (1 - \delta)P_2$.

Table 17: Counter Factual Analysis: Lender Heterogeneity

<table>
<thead>
<tr>
<th></th>
<th>N=2</th>
<th>N=4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of sub-prime borrowers in boom</td>
<td>.11</td>
<td>0</td>
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<tr>
<td>Fraction of sub-prime borrowers in bust</td>
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<td>House price increase boom</td>
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<td>House price decrease bust</td>
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<td>-.01</td>
</tr>
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</table>
Appendix D

Figure 18: GSE share of all outstanding US mortgage debt
The above figure plots Fannie Mae and Freddie Mac’s share of all outstanding US mortgage debt. Source: Federal Reserve and Federal Housing Finance.

Figure 19: GSE total dollar exposure to the US housing market
The above figure plots Fannie Mae and Freddie Mac’s total dollar exposure to the US housing market. Source: Federal Reserve and Federal Housing Finance.
A.2. Appendix to Chapter 3

Proof of Proposition 5. To begin the proof first note that, $P'(x_m) = K''(x_m) > 0$ in both the decentralized and the SP equilibrium. The equilibrium condition for the SP equilibrium requires,

$$r'_m(x_{sp}^m) = K'(x_{sp}^m)(r_f(x_{sp}^f)(1 - \phi) + \phi) + K''(x_{sp}^m)x_{sp}^m(r_f(x_{sp}^f) - 1) - K''(x_{sp}^m)\phi(B + x_{sp}^m)(r_f(x_{sp}^f) - 1)$$

The equilibrium condition for the decentralized equilibrium requires,

$$r'_m(x_{dc}^m) = K'(x_{dc}^m)(r_f(x_{dc}^f)(1 - \phi) + \phi)$$

Substituting the SP equilibrium quantities into the RHS of the decentralized equilibrium,

$$K'(x_{sp}^m)(r_f(x_{sp}^f)(1 - \phi) + \phi)$$

If $x_{sp}^m > \phi(B + x_{sp}^m)$, then,

$$r'_m(x_{sp}^m) > K'(x_{sp}^m)(r_f(x_{sp}^f)(1 - \phi) + \phi)$$

The LHS of the above equation is decreasing in $x_m$ which the RHS is increasing in $x_m$. Therefore $x_{sp}^m < x_{dc}^m$.

Conversely, if $x_{sp}^m < \phi(B + x_{sp}^m)$, then,

$$r'_m(x_{sp}^m) < K'(x_{sp}^m)(r_f(x_{sp}^f)(1 - \phi) + \phi)$$

The LHS of the above equation is decreasing in $x_m$ which the RHS is increasing in $x_m$. Therefore $x_{sp}^m > x_{dc}^m$. ■
Proof of Proposition 6. When \( r^*_g = K''(x^*_m)(\phi B - (1 - \phi)x^*_m)(r'_f(x^*_f) - 1) \) and \( b^* = 0 \), the household’s first order condition is,

\[
 r'_m(x_m) + K''(x^*_m)(\phi B - (1 - \phi)x^*_m)(r'_f(x^*_f) - 1) = P(r'_f(x_f)(1 - \phi) + \phi)
\]

When \( b = 0 \), \( P = K'(x_m) \) in equilibrium. Substituting that in,

\[
 r'_m(x_m) = K'(x_m)(r'_f(x_f)(1 - \phi) + \phi) + K''(x_m)x_m(r'_f(x_f) - 1)
\]

\[- K''(x_m)\phi(B + x_m)(r'_f(x_f) - 1) \]

For this equation to hold, \( x_m = x^*_m \) and \( x_f = x^*_f \) since it is identical to (3.2). For the government to have a balanced budget, \( \tau = r_g x^*_m \). Substituting this into household utility, we see that the utility is the same as that of the constrained social planner.

Proof of Proposition 7. At optimal,

\[
x^*_f = \omega + \phi BP - P(1 - \phi)x^*_m \tag{A.5}
\]

This gives us

\[
P = \frac{x^*_m - \omega}{\phi B - (1 - \phi)x^*_m} \tag{A.6}
\]

At that level household doesn’t invest anymore in \( x_f \) since \( r'(x^*_f) = 1 \). Looking at the household’s FOC, we also require that,

\[
\frac{r'_m(x^*_m) + r_g}{P} = 1 \tag{A.7}
\]

This gives us an \( r_g \) of

\[
r_g = \frac{x^*_f - \omega}{\phi B - (1 - \phi)x^*_m} - r'_m(x^*_m) \tag{A.8}
\]

and it gives us a \( b \) of,

\[
K'(x^*_m) = \frac{x^*_m - \omega}{\phi B - (1 - \phi)x^*_m} + b \tag{A.9}
\]
therefore $b$ is,

$$b = K'(x^*_m) - \frac{x^*_f - \omega}{\phi B - (1 - \phi)x^*_m}$$  \hspace{1cm} (A.10)

We know that at $x^*_m$, $K'(x^*_m) = r'_m(x^*_m)$ and therefore we can rewrite $b$ as,

$$b = r'(x^*_m) - \frac{x^*_f - \omega}{\phi B - (1 - \phi)x^*_m} = -r_g$$  \hspace{1cm} (A.11)

Therefore, expansionary supply-side policy (positive $b$) have to be accompanied by contractionary demand-side intervention (negative $r_g$) to achieve the optimum. $b$ is positive when,

$$r'(x^*_m) - \frac{x^*_f - \omega}{\phi B - (1 - \phi)x^*_m} > 0$$  \hspace{1cm} (A.12)

Rewriting,

$$r'(x^*_m) > \frac{x^*_f - \omega}{\phi B - (1 - \phi)x^*_m}$$  \hspace{1cm} (A.13)

Under this subsidy scheme the household’s utility is given by,

$$U = r_m(x^*_m) + r_f(x^*_f) + r_g x^*_m - \tau - l + (P^b + b)x^*_m - K(x^*_m)$$

where $l = \phi(B + x^*_m)P^b = \omega - P^b x^*_m - x^*_f$. For the government to have a balanced budget, $\tau = (b + r_g)x^*_m$. Substituting this into household utility and using the fact that $P^b = K'(x_m) - b$, the above simplifies to,

$$U = r_m(x^*_m) + r_f(x^*_f) - K(x^*_m) - x^*_f + \omega$$

We see that the utility is the same as that of the unconstrained social planner.

\[\blacksquare\]

**Proof of Proposition 8.** When the household can invest as it likes and is not financially
constrained, the household chooses \( x_f^* \) s.t. \( r'(x_f^*) = 1 \) \( \forall b, r_g \). This can be proven by contradiction. Suppose the household chooses an \( x_f < x_f^* \). Then taking a loan of \( x_f - x_f^* \) and deviating to \( x_f^* \) will provide higher terminal wealth and therefore an \( x_f < x_f^* \) cannot be optimal. Suppose the household chooses an \( x_f > x_f^* \). Then reducing its loan by \( x_f - x_f^* \) and deviating to \( x_f^* \) will provide higher terminal wealth and therefore an \( x_f > x_f^* \) cannot be optimal for the household. In the analysis that follows, superscript \( d \) refers to a demand-side quantities, while superscript \( s \) refers to supply-side quantities.

A demand side intervention \( r_g \) that generates \( x'_m \) will be associated with household utility \( U^d \) given by,

\[
U^d = r_m(x'_m) + r_f(x_f^*) + r_gx'_m - \tau^d - l^d + P^dx'_m - K(x'_m) \tag{A.14}
\]

Substituting in for \( \tau^d \) and \( l^d = P^dx'_m + x_f^* - \omega \), this simplifies to

\[
U^d = r_m(x'_m) + r_f(x_f^*) - x_f^* + \omega - K(x'_m)
\]

Similarly, a supply side subsidy \( b \) that generates \( x'_m \) is associated with household utility \( U^s \) that is given by,

\[
U^s = r_m(x'_m) + r_f(x_f^*) - \tau^s - l^s + (P^s + b)x'_m - K(x'_m)
\]

Substituting in for \( \tau^s \) and \( l^s = P^sx'_m + x_f^* - \omega \), this simplifies to

\[
U^s = r_m(x'_m) + r_f(x_f^*) - x_f^* + \omega - K(x'_m)
\]

Therefore, the representative household has the same utility under both supply and demand subsidies. \( \blacksquare \)
Proof of Proposition 9. In the analysis that follows, superscript $d$ refers to a demand-side quantities, while superscript $s$ refers to supply-side quantities. Say we want to achieve a level of $x'_m$. Then a demand side intervention will require $r_g$ such that,

$$r'_m(x'_m) + r_g = K'(x'_m)r'_f(r'^d_f)$$ (A.15)

A supply side intervention will require $b$ such that,

$$r'_m(x'_m) = \left( K'(x'_m) - b \right) r'_f(r'^s_f)$$ (A.16)

Using the fact that $l = 0$ and that $\tau = r_gx'_m$, the utility of the household under demand-side intervention is then given by,

$$U^d = r_m(x'_m) + r_f(x'^d_f) - K(x'_m) + \omega - x'^d_f$$ (A.17)

Similarly, the utility of the household under supply-side intervention is given by,

$$U^s = r_m(x'_m) + r_f(x'^s_f) - K(x'_m) + \omega - x'^s_f$$ (A.18)

Using the budget constraint and that fact that in equilibrium $P^d = K'(x'_m)$ and $P^s = K'(x'_m) - b$, we see that,

$$x'^d_f = \omega - K'(x_m)x'_m$$

$$x'^s_f = \omega - (K'(x'_m) - b)x'_m$$ (A.19)

We can rewrite (A.23) as,

$$U^s = r_m(x'_m) + r_f(x'^d_f + bx'_m) - x'^d_f - bx'_m - K(x'_m)$$
Since \( r'(x^d_f + bx'_m) \geq 1 \) and \( r'' < 0 \), when \( b > 0 \), \( r_f(x^d_f + bx'_m) - x^d_f - bx'_m > r_f(x^d_f)x^d_f \). Therefore \( U^s > U^d \). Conversely, when \( b < 0 \) \( r_f(x^d_f + bx'_m) - x^d_f - bx'_m < r_f(x^d_f)x^d_f \). Therefore \( U^d > U^s \) and demand taxes pareto dominate supply taxes. ■

**Proof of Proposition 10.** In the analysis that follows, superscript \( d \) refers to a demand-side quantities, while superscript \( s \) refers to supply-side quantities. Say we want to achieve a level of \( x'_m \). Then a demand side intervention will require \( r_g \) such that,

\[
    r'_m(x'_m) + r_g = K'(x'_m)r'_f(r^d_f) \quad (A.20)
\]

A supply side intervention will require \( b \) such that,

\[
    r'_m(x'_m) = (K'(x'_m) - b)r'_f(r^s_f) \quad (A.21)
\]

Using the fact that \( \tau = r_gx'_m \), the utility of the household under demand-side intervention is then given by,

\[
    U^d = r_m(x'_m) + r_f(x^d_f) - K(x'_m) + \omega - x^d_f - K'(x'_m)(x'_m - B) \quad (A.22)
\]

Similarly, the utility of the household under supply-side intervention is given by,

\[
    U^s = r_m(x'_m) + r_f(x^s_f) - K(x'_m) + \omega - x^d_f - (K'(x'_m) - b)(x'_m - B) \quad (A.23)
\]

Using the budget constraint and that fact that in equilibrium \( P^d = K'(x'_m) \) and \( P^s = K'(x'_m) - b \), we see that,

\[
    x^d_f = \omega - K'(x'_m)x'_m + K'(x'_m)(x'_m + B) = \omega + K'(x'_m)B \\
    x^s_f = \omega - (K'(x'_m) - b)x'_m + (K'(x'_m) - b)(x'_m + B) = \omega + (K'(x'_m) - b)B \quad (A.24)
\]
Following the same steps as in the proof of proposition 9, we can show that in this case demand subsidies pareto dominate supply-side subsidies and supply taxes pareto dominate demand taxes. ■


T. Davidoff. Supply Constraints Are Not Valid Instrumental Variables for Home Prices Because They Are Correlated With Many Demand Factors. pages 1–31, 2015.


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