2018

Essays On Banking And Asset Pricing

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Abstract
This dissertation consists of two chapters. In the first chapter, I study both theoretical and quantitative implications of the counter-cyclical capital buffers introduced with the Basel Accord III. The proposed adjustment effectively translates into capital charges that vary over time. To this end, I develop a tractable general equilibrium model and use it to solve for optimal state-dependent capital requirements. An optimal policy trades off reduced inefficient lending with reduced liquidity provision. Quantitatively, I find that the optimal Ramsey policy requires pro-cyclical capital ratios that mostly vary between 4% and 6% and depend on the output and bank credit growth, as well as the liquidity premium. Specifically, a one standard deviation increase in GDP (bank credit) translates into 0.6% (0.1%) increase in the capital charges, while a one standard deviation increase in liquidity premium leads to a 0.2% drop. The welfare gain of implementing this Ramsey policy is relatively large.

In the second chapter, I, jointly with Scott Richard, Ivan Shaliastovich and Amir Yaron, investigate the channels of asset price variation when a representative agent owns the entire corporate sector. Utilizing novel market data on corporate bonds we measure the aggregate market value of U.S. corporate assets and their payouts to investors. Total asset payouts are very volatile, turn negative when corporations raise capital, and in contrast to procyclical cash payouts are acyclical. This challenges the notion of risk and return since the risk premium on corporate assets is comparable to the standard equity premium. To reconcile this evidence, we show that aggregate net issuances, which are acyclical and highly volatile, mask a strong exposure of total payouts’ cash components to low-frequency growth risks. We develop an asset-pricing framework to quantitatively assess this economic channel.
ESSAYS ON BANKING AND ASSET-PRICING

Tetiana Davydiuk

A DISSERTATION

in

Finance

For the Graduate Group in Managerial Science and Applied Economics

Presented to the Faculties of the University of Pennsylvania

in

Partial Fulfillment of the Requirements for the

Degree of Doctor of Philosophy

2018

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To my family and friends for their immense support.
ACKNOWLEDGEMENT

I am deeply indebted to my advisers Joao F. Gomes (co-chair), Amir Yaron (co-chair), Itay Goldstein, and Christian Opp for their insightful comments, guidance, and encouragement.

I owe immense gratitude to many friends I have made in my years at Wharton, especially to Deeksha Gupta, Elizabeth Cai, Tatyana Marchuk, Nina Karnaukh and Ryan Peters for their valuable feedback and support throughout the program.

Financial support by the Macro Financial Modeling Group dissertation grant from Becker Friedman Institute at the University of Chicago, as well as the research grant from the Rodney L. White Center for Financial Research, is gratefully acknowledged.
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CHAPTER 1 : DYNAMIC BANK CAPITAL REQUIREMENTS

1.1. Introduction

The recent financial crisis imparted renewed attention on the stability of the banking sector and on capital regulation. Both policymakers and academics have recognized that risk-based capital requirements, such as those in the 2004 Basel II Accord, tend to exacerbate business-cycle fluctuations, leading to an overly cyclical credit supply. To protect the banking sector from periods of excess credit growth and leverage buildup, the 2010 Basel III Accord introduced the countercyclical capital buffer (CCyB) framework. The proposed adjustment effectively translates into capital ratios that vary over time. Despite a few recent trials to implement CCyB regimes within countries in the European Union, there is no consensus at the practical or theoretical level about the implications of CCyBs on the macroeconomy\(^1\).

The goal of this paper is to investigate both the theoretical and quantitative implications of optimal state-dependent capital requirements. By comparison, most of the existing literature has limited itself to examining the costs and benefits of the overall level of capital requirements.\(^2\) To this end, I develop a tractable general equilibrium model and use it to characterize optimal bank capital requirements. The problem of choosing an optimal capital policy follows from the work of Ramsey (1927) and builds on the primal approach used in the optimal taxation literature\(^3\).

My model has three key features. First, high-quality production projects are scarce, especially during economic downturns. Second, banks benefit from government guarantees in

---

\(^1\)The expectation is that the buffer will be at zero through most of the business cycle and increase to the maximum 2.5% points only at the peak of the credit cycle. Even though under 2010 Basel Accord the CCyB regime will be phased in globally between 2016 and 2019, individual EU members have already introduced the CCyBs. For instance, the Bank of England maintained the buffer rate equal to 0% starting from 26 Jun 2014, and plans to increase it to 0.5% starting from 27 Jun 2018.


\(^3\)See, for example, Lucas and Stokey (1983), Chari et al. (1991) and Atkinson and Stiglitz (2015).
the form of bank bailouts, which lowers their borrowing cost, thereby creating incentives to risk-shift. Third, households value money-like assets in the form of bank debt. This implies that bank debt is priced at a premium and is a cheaper source of funding than equity. The last two features create a trade-off: while tighter capital ratios can help to mitigate a bank’s motives to risk-shift and rule out inefficient lending, at the same time they can reduce the bank’s supply of credit and the creation of safe liquid assets.

In the model, procyclical capital requirements (or, equivalently, countercyclical capital buffers) emerge endogenously as an optimal policy scheme in a Ramsey equilibrium. Because of government guarantees, banks end up financing low-quality projects and more so during periods of high economic growth, even though they are inefficient from the social prospective. The reasons are twofold. Because of better lending opportunities during expansions, banks increase the supply of credit and deposits. As a result, the premium on bank debt becomes smaller, thereby increasing the value of default option and lowering the banks borrowing costs. This translates into an even higher level of lending. In addition, during expansions banks risk-shift on a larger scale, which implies that more inefficient projects are financed in absolute terms. Consequently, heightened capital regulation can be most beneficial through credit booms. At the same time, tight capital requirements can have a contractionary effect on bank activity especially during recessions, when liquidity is most valuable to households and, as a result, equity financing is more costly for banks. This implies that optimal capital charges should be lower during economic slowdowns, thereby reinforcing the cyclicality of optimal capital ratios.

In the model, procyclical capital requirements (or, equivalently, countercyclical capital buffers) emerge endogenously as an optimal policy scheme. Presence of government induces banks to finance low-quality production projects, even though they are inefficient from the social prospective. My results show that investment in such projects builds-up

This is consistent with the “countercyclical capital buffer” instituted with Basel III. The required buffer is called countercyclical because the goal is to dampen the impact of capital regulation on the cyclicality of bank lending. The buffer itself is procyclical.
during periods of high economic growth. Because of better lending opportunities during expansions, banks increase the supply of credit. As a result, expected bailout subsidies become larger, thereby lowering banks borrowing costs and translating into an even higher level of lending. Consequently, heightened capital regulation can be most beneficial through credit booms. At the same time, tight capital requirements can have a contractionary effect on bank activity especially during recessions, when liquidity is most valuable to households. As banks reduce their deposit creation during downturns, the premium on bank debt increases and, as a result, equity financing becomes more costly for banks. This implies that optimal capital charges should be lower during economic slowdowns, thereby reinforcing the cyclicality of optimal capital ratios.

In the model, procyclical capital requirements (or, equivalently, countercyclical capital buffers) emerge endogenously as an optimal policy scheme. Because of government guarantees, banks end up financing low-quality production projects, even though they are inefficient from the social prospective. My results show that investment in such projects builds-up during periods of high economic growth, when the bailout wedge – expected government subsidy – is large. In the model, both the size of the government transfer and probability of receiving it increase in bank level of lending. As banks issue more loans, they move further into the distribution of projects’ quality and, thus, lower the quality of a marginal loan on their balance sheet. Because of better lending opportunities during expansions, banks increase the supply of credit. As a result, expected bailout subsidies become larger, thereby lowering bank borrowing costs and translating into an even higher level of inefficient lending during expansions. This implies that heightened capital regulation can be most beneficial through credit booms. At the same time, tight capital requirements can have a contractionary effect on bank activity especially during recessions, when liquidity is most valuable to households. As banks reduce their lending and deposit creation during downturns, the

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5 This paper focuses on the ongoing costs of holding equity on the balance sheet, as opposed to the costs of raising new external equity finance. One of the theories why equity is more expensive on ongoing basis has to do with the existence of the liquidity premium on safe “money-like” assets produced by banks (Krishnamurthy and Vissing-Jorgensen (2012)).
premium on bank debt increases and, as a result, equity financing becomes more costly for banks. Consequently, optimal capital charges should be lower during economic slowdowns, thereby reinforcing the cyclicality of optimal capital ratios.

My results also show that the optimal Ramsey policy used as a single policy tool is not sufficient to restore both the socially optimal level of lending and liquidity provision. To provide some intuition behind the design of optimal capital regulations, I solve separately for the “lending capital requirement” – restoring the first-best level of investment but admitting a reduced level of deposits – and the “liquidity capital requirement” – ensuring the first-level of liquidity provision but also allowing for excessive lending. Such policies focus only on one dimension of the problem, either dampening incentives of banks to risk-shift or stimulating creation of safe and liquid assets. By contrast, the Ramsey capital requirement balances reduced inefficient lending with reduced liquidity provision.

The model highlights important trade-offs one needs to consider when regulating bank capital. However, for effective policy making, it is also crucial to assess these trade-offs quantitatively. I therefore calibrate the model to best match key macroeconomic quantities and bank data variables. Using this choice of parameters, I solve numerically for the optimal policy in the Ramsey equilibrium. Specifically, I characterize the optimal state-contingent capital regulation and the set of allocations for bank lending and liquidity provision that maximize consumers’ lifetime utility. Ramsey allocations have the property that they can be implemented as a competitive equilibrium when banks encounter the prescribed capital requirements. I find that the optimal Ramsey policy requires a cyclical capital ratio that mostly varies between 4% and 6% and depends on key indicators of economic growth and asset prices. This range of values is roughly comparable with a 2.5% upper bound of the CCyB proposed by the Basel III Accord. Importantly, the optimal policy rule is not capped at 6%, and can rise above it during periods of abnormal economic growth. In addition, the

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6 This paper focuses on the ongoing costs of holding equity on the balance sheet, as opposed to the costs of raising new external equity finance. One of the theories why equity is more expensive on ongoing basis has to do with the existence of the liquidity premium on safe “money-like” assets produced by banks (Krishnamurthy and Vissing-Jorgensen (2012)).
mean Ramsey capital requirement is around 5%. This is one percentage point higher than the level of the leverage ratio, and one percentage point lower than the Tier 1 capital ratio as recommended by the Basel III Accord.

Implementing the optimal Ramsey policy delivers a permanent increase in the annual consumption and deposit holdings of households relative to the average capital ratio observed in the data. Although the exact magnitude of the welfare gain depends on the attitude of households toward risk, more than 60% is attributed to having state-dependent capital requirements. The remaining welfare improvement is achieved by setting an optimal fixed capital requirement equal to the mean Ramsey capital ratio. The optimal policy leads to a reduction in the cyclicality of bank credit – reducing inefficient lending during expansions, but increasing the supply of credit during economic slowdowns – and, as a result, achieves on average a higher level of consumption and deposit creation.

The main contribution of the paper is to characterize the optimal policy rule in terms of measurable macroeconomic and bank aggregates. While the Basel Committee recommends adjusting the capital requirements based on the “credit gap” – the deviation of the credit-to-GDP ratio with respect to its long-term trend, – I show the credit gap alone fails to capture the time variation in the optimal Ramsey policy. Instead, I find the optimal policy rule takes into account the joint behavior of the credit gap, GDP, and the liquidity premium. Specifically, the optimal policy is well approximated by the following rule:

\[
\text{capital ratio} = 5\% + 0.1\% \times \text{credit gap} + 0.7\% \times \text{GDP} - 0.1\% \times \text{liquidity premium},
\]

where the liquidity premium is a discount on the price of safe liquid assets.\(^7\) A one standard deviation increase in the credit-to-GDP ratio (GDP) translates into a 0.1% (0.7%) increase in capital charges, while a one standard deviation increase in the liquidity premium leads to a 0.1% drop. Growth in bank credit, along with the output growth, act as indicators of

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\(^7\)The three indicators are expressed in log-deviations from their steady states and normalized by their standard deviations. The GDP variable has been orthogonolized with respect to the credit gap variable.
banks' incentives to risk-shift. By contrast, the liquidity premium serves as an indicator of how expensive equity financing is for banks and how valuable liquid assets are for investors.

The paper proceeds as follows: In Section 1.2, I provide the details of bank regulation, as well as review the related literature. I develop the baseline model in Section 1.3. I characterize the first-best allocation in this economy in Section 1.4, as well as the competitive equilibria with and without capital regulations in place. In Section 1.5, I examine the lending and liquidity capital requirements within the baseline model. I provide the quantitative assessment of the model in Section 1.6, as well as present the optimal policy rule. Concluding remarks are give in Section 1.7.

1.2. Bank Regulation and Related Literature

1.2.1. The Basel Accords

The Basel Accords, developed by the Basel Committee on Bank Supervision (BCBS), consolidate capital requirements as the cornerstone of bank regulation. The 1988 Basel Accord, known as Basel I (BCBS (1988)), was criticized for the risk-insensitivity of capital charges. To address this critique, the internal ratings-based (IRB) approach was introduced with the publication of Basel II in 2004. Under risk-based regulation, the amount of capital that a bank is required to hold against a given exposure depends on the estimated credit risk of that exposure, which in turn is determined by the probability of default (PD), loss given default (LGD), exposure at default (EAD), and maturity. The key implication of the IRB approach is that riskier exposures carry a higher capital charge. The intention of this approach is to reduce bank failures and the associated systemic costs by holding the bank probability of default below some fixed target.

Both policymakers and academics have recognized that risk-sensitive capital regulations, such as that in Basel II, tend to exacerbate the inherent cyclicality of bank lending and, consequently, distort investment decisions. This is because in economics downturns losses erode bank capital, and the remaining (non-defaulted) loans are downgraded by the relevant
credit risk model, delivering higher capital charges. To the extent that it is difficult or costly for a bank to raise new capital during recessions, it will be forced to cut back on its lending activity. Kashyap et al. (2008) provide empirical evidence that equity-raising was sluggish during the recent financial crisis.

Kashyap and Stein (2004) argue that the IRB approach is largely microprudential in nature and ignores the importance of the function of bank lending. In line with the literature on capital crunches in banking, they claim that the shadow value of bank capital increases during recessions and a capital requirement that is too high when bank capital is scarce may result in reduced funding of positive net present value projects. So, if the government’s objective is both to protect the financial system against the costs of bank defaults and sustain bank-lending efficiency, the capital charges should be adjusted to the state of the business cycle (for any degree of credit-risk exposure).

An important argument that is sometimes made is that during periods of economic growth banks may hold capital in excess of the minimum regulatory requirements that could neutralize potential cyclicality problems. Repullo and Suarez (2012) develop a dynamic model of relationship lending, in which banks hold voluntary capital buffers as a precaution. They find that the capital buffers set aside during expansions are typically not sufficient to prevent credit supply shrinkage during recessions. They also document that the optimal capital requirements are higher and less cyclically-varying than the requirements of Basel II when the social cost of bank failure is high.

In an attempt to strengthen bank balance sheets against future financial upheavals, Basel III introduced the countercyclical capital buffers, which range from zero to 2.5% of risk-weighted assets. As mentioned before, the BCBS proposed to adjust the level of the buffer based on the credit gap, meanwhile acknowledging that it may not be a good indicator of stress in downturns. For example, Repullo and Saurina (2011) find that the credit gap for many countries is negatively correlated with GDP growth. This can be traced to the fact that the supply of credit typically lags the business cycle, especially in downturns.
Kashyap and Stein (2004), Gordy and Howells (2006), Saurina and Trucharte (2007), and Kashyap et al. (2008), among others, focus on the correction of risk-based capital requirements in a macroprudential direction. Using Spanish data, Repullo et al. (2010) analyze different procedures aimed at mitigating the procyclical effects of capital regulation and conclude that the most appealing one is to use a business cycle multiplier based on GDP growth. The proposed adjustment maintains the risk sensitivity in the cross-section (i.e., banks with riskier portfolios would bear a higher capital charge), but a cyclically-varying scaling factor would increase capital requirements in good times and reduce them in bad times. In line with this study, I propose a cyclical policy rule that depends positively on indicators of economic growth. In my setting, state-contingent capital requirements emerge endogenously as an optimal policy scheme. Optimal regulations promote the stability of the banking sector without contracting the supply of bank credit and deposit creation. Arguments in favor of time-varying capital regulations are also found in Kashyap et al. (2008), Hanson et al. (2011), and Malherbe (2015). Note that I abstract from the cross-sectional dimension of capital regulation and focus on the time-series dimension.

1.2.2. Literature Review

The most closely related paper is Malherbe (2015), which studies the optimal capital requirement over the business and financial cycles. In his setting, because of the general equilibrium effect and decreasing returns to scale in production a higher level of aggregate banking capital calls for a tighter capital requirement to preclude inefficient credit growth. Even though his model also finds that capital regulation should be tightened during boom episodes, the mechanism is different from mine. Liquidity channel

This research project is at the intersection of a large literature on optimal banking regulation theory and dynamic macroeconomic models of financial intermediation. The recent financial crisis has brought to the forefront the discussion of whether the capital requirements of banks should be increased (Admati et al. (2010)). On one side, Hellmann et al. (2000), Repullo (2004), and Morrison and White (2005) argue that stringent capital regula-
tion can induce prudent behavior by banks. On the other side, Diamond and Rajan (2000),
Diamond and Rajan (2001), and DeAngelo and Stulz (2013) provide theoretical evidence
that tightening capital requirements may distort banks’ provision of liquidity services, while
Dewatripont and Tirole (2012) show that stricter capital requirements may introduce govern-
ance problems. In line with the existing literature, I suggest that reduced moral hazard is
the key rationale for imposing restrictive capital regulation with reduced liquidity provision
being the main downside.

This paper is most closely related to other quantitative studies on the welfare impact of
Heuvel (2016)) and leverage constraints (Bigio (2010), Martinez-Miera and Suarez (2012),
Corbae and D’Erasmo (2014), and Christiano and Ikeda (2014)). Among the most recent
studies estimating the optimal level of fixed capital requirements are Nguyen (2014) and
Begenau (2016). Nguyen (2014) finds that consumers’ lifetime utility is maximized at 8%
capital ratio, while the optimal number estimated by Begenau (2016) is 14%. A common
feature in these two papers is that government guarantees incentivize banks to engage in
excessive risk-taking, while capital regulation helps to address these distortions. Begenau
(2016) also extends the banks’ role to providing liquidity services, which are valued by
households. I complement this branch of literature by developing a tractable framework to
quantify the benefits and costs of capital regulation over the business cycle. This allows
me to provide not only qualitative, but also quantitative recommendations on the policy
design. To the best of my knowledge, this paper is the first to solve for the stage-contingent
capital requirements within the Ramsey framework.

My paper is closely related to the papers estimating an optimal level of fixed capital re-
quirement in a dynamic general equilibrium setting. In this sense, the paper complements
existing quantitative studies on the welfare impact of leverage constraints. Van den Heuvel
(2008) was one of the first to study the welfare cost of capital requirements in a quantita-
tive general equilibrium model. Using U.S. data, he finds that the current level of capital
regulation dampens the ability of banks to create liquidity, leading to a welfare loss, and, therefore, concludes that it is too high. In his paper the bank capital structure is recovered from binding capital requirements and it is difficult to infer the welfare costs of alternative levels of bank regulation than the current one. Among the most recent studies estimating the optimal capital requirements are Nguyen (2014) and Begenau (2016). Nguyen (2014) finds that the welfare is maximized at 8% capital ratio, while the optimal number estimated by Begenau (2016) is equal to 14%. A common feature in these two papers is that government guarantees incentivize banks to engage in excessive risk-taking, while capital regulation helps to address these distortions. Begenau (2016) also extends the banks’ role to providing liquidity services, which are valued by households.

My paper is also related to prior contributions focused on welfare implications of capital constraints. Corbae and D’Erasmo (2014), Christiano and Ikeda (2014), Bigio (2010) and Martinez-Miera and Suarez (2012) reach the conclusion that a tighter leverage constraint lowers the riskiness of the financial sector, but that it also shortens bank lending capacity, thereby depressing the economic growth. Corbae and D’Erasmo (2014) study the quantitative impact of capital regulation on bank risk-taking, bank failures and market structure when there is competition among small and big banks. Bigio (2010) develops a model with endogenous liquidity mechanism and analyzes how capital requirements change the risk capacity of the economy. Christiano and Ikeda (2014) quantify the effects of leverage constraints in a framework where bankers have an unobservable effort choice. Martinez-Miera and Suarez (2012) study the role of banks in generating systemic risk taking and find that optimal capital requirements are quite high and don’t require counter-cyclical adjustments. This is one of the first paper which looks at the welfare implications of time-varying capital regulation.

There are few empirical contributions on the effects of higher capital requirements on bank lending and bank cost of capital. Looking at the data on large financial institutions, Kashyap et al. (2010) find that even if the minimum capital ratio is increased by 10%, the impact on
loan rates is likely to be modest, in the range of 25 to 45 bps. In a similar vein, Baker and Wurgler (2013) calibrate that a ten-percentage point increase in capital requirements would translate into a higher weighted average cost of capital by 60-90 bps per year. Kisin and Manela (2013) estimate the perceived costs of capital requirements by employing the data on banks’ participation in a costly loophole that helped them to bypass capital regulations. They document that a ten-percentage point increase in Tier 1 capital to risk-weighted assets leads to, at most, a 3 bps increase in banks’ cost of capital. Even though these studies shed light on the potential impact of capital regulations on the real economy, it is difficult to assess the overall welfare implications of a time-varying capital requirement in their setting.

More broadly, this paper fits a strand of macroeconomic literature on the role of financial intermediation in the development of economic crises. The transmission mechanism by which the effects of small shocks persist, amplify, and spread to the macroeconomy is first identified in the seminal works of Bernanke and Gertler (1989), Kiyotaki and Moore (1995), and Bernanke et al. (1999). The more recent studies of the balance sheet channel include Gertler and Kiyotaki (2010), Gertler and Karadi (2011), He and Krishnamurthy (2012), Di Tella (2013), and Brunnermeier and Sannikov (2014).

1.3. Model Setup

To focus on the novel mechanisms associated with dynamic bank capital requirements, I first layout the baseline model, which is kept as parsimonious as possible. I later augment the model to quantify policy recommendations on what defines optimal time variation in capital requirements of banks.

I develop a model studying the welfare implications of an endogenous capital requirement. Government guarantees can introduce distortions in bank incentives and lead to excessive risk-taking in equilibrium. Socially optimal allocation - both the first-best level of lending and first-best level of liquidity provision - can not be restored with help of only one policy tool. However, substantial welfare gains can be achieved when the government imposes a
time-varying capital requirement.

In the model, time is discrete and runs for an infinite number of periods. Financial intermediaries own the production technology and the stock of capital in this economy. They are financed either partially or entirely with deposits. The presence of deposit insurance distorts banks’ optimal choices of investment. Households consume the final goods produced in the banking sector and invest any savings in the banks’ deposits. To finance bailout expenditures, the government levies taxes on households in lump-sum fashion.

1.3.1. Banking Sector

I start by characterizing a banking sector in detail. At any point in time the economy is populated with a continuum of ex ante identical banks of measure one, indexed by \( j \in \Omega = [0, 1] \). Each bank has access to decreasing returns to scale technology and produces a final good \( y_{j,t} \), using capital as the only input,

\[
y_{j,t} = e^{\omega_{j,t} + \alpha_t l_{j,t}},
\]

where \( \alpha_t \) is an aggregate productivity shock, which follows:

\[
a_t = (1 - \rho_a) \bar{a} + \rho_a a_{t-1} + \sigma_a \epsilon_t, \quad \epsilon_t \sim iid \ N(0, 1) \tag{1.1}
\]

and \( \omega_{j,t} \) is an idiosyncratic disturbance, which is identically distributed across time and across banks:

\[
\omega_{j,t} = -\frac{1}{2} \sigma_{\omega}^2 + \sigma_{\omega} \varepsilon_{j,t}, \quad \varepsilon_{j,t} \sim iid \ N(0, 1). \tag{1.2}
\]

In the cross-section, the bank-specific shocks average to zero. This setup is isomorphic to the one, where a consumption good is produced by penniless firms, who have been credit rationed in the capital markets due to unmodeled information asymmetries, and whose only source of funding is bank loans.\(^8\) The capital used in banks’ production process

\(^8\)Suppose that each bank operates on an island \( j \); one can think of an island as an industry or a state. On each island, there is a continuum of cashless firms who require a unit investment at time \( t \) for production
can be correspondingly interpreted as the amount of bank loans issued to these types of borrowers. From this point on, I refer to $l_{j,t+1}$ as bank lending. The decreasing returns to scale assumption is crucial to capture the idea that borrowers are not homogeneous and that there is a finite number of creditworthy borrowers, or equivalently a finite number of positive net present value (NPV) production projects. This setup ensures that banks internalize that each extra lending unit (borrower) is not as productive (creditworthy) as the previous one.

A bank $j$ enters a period $t$ with capital, $l_{j,t}$, bank debt, $d_{j,t}$, and equity, $n_{j,t}$. The balance sheet equates risky assets, $l_{j,t}$, to bank debt, $d_{j,t}$, and equity, $n_{j,t}$:

\[
\begin{array}{c|cc}
\hline
\text{loans} & \text{net worth} & \text{debt} \\
\hline
l_{j,t} & n_{j,t} & d_{j,t} \\
\hline
\end{array}
\]

The bank’s revenues realized at time $t$ are equal to its earnings on the production net of the interest payments on its liabilities:

\[
\pi_{j,t} = e^{\omega_{j,t+1}} \alpha l_{j,t}^\alpha - l_{j,t} - (R_{d,t} - 1)d_{j,t}.
\]

For the statement of problem, it is useful to define equity after profits as $\tilde{n}_{j,t} \equiv \pi_{j,t} + n_{j,t}$.

Next, I assume that when a financial intermediary does not have sufficient funds to service at time $t+1$. Firms on each island are ranked according to their productivity. Specifically, a firm $i$ on an island $j$ (where $i$ denotes the firm’s ranking) produces $ae^{\omega_{j,t+1}} i^{\alpha-1}$ units of a consumption good. At time $t$, the bank on island $j$ issues loans to the first $l_{j,t+1}$ firms with the total amount equal to:

\[
\int_0^{l_{j,t+1}} 1di = l_{j,t+1}.
\]

The monopolist bank on the island can extract all surplus. In particular, at time $t+1$ the bank on island $j$ receives from the firms:

\[
\int_0^{l_{j,t+1}} ae^{\omega_{j,t+1}} i^{\alpha-1}di = e^{\omega_{j,t+1}} l_{j,t+1}^\alpha.
\]
its deposit liabilities, depositors are bailed out with probability one. In particular, banks will default on their credit obligations whenever their idiosyncratic shock $\omega_{j,t}$ is below a cutoff level $\omega_{j,t}^*$, defined by the expression:

$$\pi_{j,t} + n_{j,t} = 0 \Leftrightarrow e^{\omega_{j,t}^*+\alpha t}I_{j,t} = R_{d,t}d_{j,t}.$$ 

The net worth, $n_{j,t+1}$, available to banks at the end of period $t$ (going into period $t+1$), evolves according to:

$$n_{j,t+1} = \{\pi_{j,t} + n_{j,t}\}^+ - z_{j,t},$$

where $\{\cdot\}^+$ denotes the maximum operator $\max\{\cdot, 0\}$ and captures that banks are subject to limited liability and government guarantees. $z_{j,t}$ is the net payouts to the bank’s shareholders. A positive net transfer, $z_{j,t} > 0$, means that the equityholders receive dividends, while a negative one, $z_{j,t} < 0$, means that there is an equity issuance. Equation 1.3 demonstrates that any growth in bank equity above the deposit return depends on the premium that the financial intermediary earns on its assets, as well as the total amount of lending. To finance the difference between the capital investment and available net worth, the financial intermediary borrows an amount $d_{j,t+1}$ from households, given by:

$$d_{j,t+1} = l_{j,t+1} - n_{j,t+1}.$$ 

Finally, banks are subject to capital regulations, which require them to have a minimum

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The banking sector is subject to government guarantees for reasons that I do not explicitly model. As has been acknowledged by the academics and practitioners, without government protection, economies are prone to bank runs. Bank runs were a recurrent feature during the Great Depression in the U.S. and continue today, including recent episodes of bank failures in the U.S. (Countrywide, IndyMac), Great Britain (Northern Rock), and India (ICICI Bank). With many bank failures, there is a disruption of the monetary system and a reduction in production (Bernanke (1983), Calomiris and Mason (2003), Dell’Ariccia et al. (2008)). With the goal of protecting the economy against the potential welfare losses associated with bank runs, deposit insurance was introduced along with a number of implicit government guarantees.
amount of equity as a fraction of assets. Since loans are the only type of asset in my model, the capital requirement I instate is that equity needs be at least a fraction $\zeta_t$ of loans for a bank to be able to operate:

$$n_{j,t+1} \geq \zeta l_{j,t+1}.$$  

In each period, bank $j$ decides how many loans to issue, $l_{j,t+1}$, and makes a leverage choice to maximize the discounted sum of the equity payouts:

$$\max_{z_{j,t},l_{j,t+1},d_{j,t+1},n_{j,t+1}} E \left[ \sum_{t=0}^{\infty} \beta^t z_{j,t} \right]$$

s.t.  
$$n_{j,t+1} = \{ e^{\omega_{j,t} + \alpha_{j,t}} l_{j,t} - R_{d,t} d_{j,t} \}^+ - z_{j,t},$$  
$$l_{j,t} = n_{j,t} + d_{j,t},$$  
$$n_{j,t+1} \geq \zeta l_{j,t+1},$$  
$$n_{j,0}, d_{j,0} \text{ given.}$$

makes an investment choice - how much loans to issue, $l_{j,t+1}$, and a leverage choice - with how much debt, $d_{j,t+1}$, and how much equity, $n_{j,t+1}$, to finance their investment - to maximize the expected payouts to shareholders.

1.3.2. Household Sector

The economy is populated by a measure one of identical households. There are two types of members in each household: savers and bankers (Gertler and Karadi (2011)). Savers hold deposits at a diversified portfolio of financial institutions and return the interest earned to their household. Bankers, on the other hand, manage financial intermediaries and similarly return any earnings back to the household. The savers hold deposits at the banks that its household does not own, otherwise in the absence of tax frictions, the Modigliani-Miller theorem would hold and the irrelevance of banks’ capital structure would follow. Savers’ and bankers’ earnings are perfectly shared across the entire household to secure a representative
Let $C_t$ denote family consumption and $D_{t+1}$ denote the holdings of bank deposits. Households are risk-neutral and have a discount rate of $\beta \in (0, 1)$. I also assume that households value the non-pecuniary services provided by bank deposits and, hence, enjoy the additional flow of utility $v(D_{t+1})$, which is a concave non-decreasing function of the supply of deposits $D_{t+1}$.

Then households preferences are, therefore, given by

$$u(C_t, D_{t+1}) = C_t + \frac{D_{t+1}^{1-\eta}}{1-\eta}, \quad \eta < 1.$$ 

In each period, the household chooses a consumption level and deposit holdings to maximize their utility subject to the budget constraint:

$$\max_{C_t, D_{t+1}} E \left[ \sum_{t=0}^{\infty} \beta^t (C_t + v(D_{t+1})) \right]$$

with

$$v(D_{t+1}) = \frac{D_{t+1}^{1-\eta}}{1-\eta}, \quad \eta < 1$$

s.t.

$$C_t = R_{d,t}D_t - D_{t+1} + Z_t - T_t,$$

$$D_0 \text{ given},$$

where $T_t$ is a lump-sum tax levied by the government. Households are the owners of financial intermediaries and at the end of each period receive the net proceeds of bank activity, $Z_t$.

A unit of deposits issued in period $t$ yields a gross return of $R_{d,t+1}$ at time $t+1$.

The presence of the government guarantees implies that $R_{d,t+1}$ is a return on a riskless

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10 A household's consumption can be both positive and negative, as in Brunnermeier and Sannikov (2014). Negative consumption can be interpreted as disutility from labor.

11 See, for example, Nataliya et al. (2016). This particular utility function allows me to derive the analytical solutions for the baseline model, but it is generalized when I move to the quantitative assessment of the optimal capital requirements.
asset. The first-order conditions of 1.5 impart that the interest rate on deposits is equal to:

\[ R_{d,t+1} = \frac{1}{\beta} - \frac{1}{\beta} D_{t+1}^{-\eta}. \] (1.6)

Since households exhibit a preference for bank debt, there is a discount on its interest rate, which is the amount households are willing to relinquish in exchange for holding a risk-free asset, which gives a flow of utility compared to a risk-free asset that does not give a flow of utility. Figure 1 shows that liquidity premium is a decreasing function in the supply of bank deposits. This implies that liquid deposits become more valuable to the household when they are scarce.

Diamond and Dybvig (1983) were among the first to analyze household demand for liquidity. They find that bank deposits provide better risk-sharing possibilities to consumers, justifying the existence of the liquidity premium. Stein (2011) and Gorton and Metrick (2012) argue that collateralized short-term debt issued by banks yields a “money-like” convenience premium based on its relative safety and the transactions services that safe claims provide. Further, Krishnamurthy and Vissing-Jorgensen (2012) and Krishnamurthy and Vissing-Jorgensen (2015) document that investors value the money-like features of Treasuries the most, when the supply of Treasuries is low. The welfare implications of the presence of liquidity premium via its effects on the lending costs of banks have also been studied by Begennau (2016) and Van den Heuvel (2016).

1.3.3. Government

The role of the government is to provide deposit insurance. To finance its expenditures on bank bailouts, the government levies a lump-sum tax on households in accordance with a

\[ \lim_{D_{t+1} \to \infty} R_{d,t+1} = \frac{1}{\beta}. \]
balanced budget rule:

\[ T_t = \int_0^1 \max \left\{ R_{d,t} d_{j,t} - e^{\omega_{j,t} + n_{j,t},t} l_j^{\alpha_{j,t},t}, 0 \right\} dj. \] (1.7)

1.4. Equilibrium Characterization

**Definition.** A competitive equilibrium is a set of prices \( \{ R_{d,t+1} \}_{t=0}^{\infty} \), government policies \( \{ \zeta_t, T_t \}_{t=0}^{\infty} \), and allocations \( \{ C_t, D_{t+1}, \{ z_{j,t}, l_{j,t+1}, n_{j,t+1}, d_{j,t+1} \}_{j \in \Omega} \}_{t=0}^{\infty} \), such that:

(i) Given prices \( \{ R_{d,t+1} \}_{t=0}^{\infty} \), government policies \( \{ T_t \}_{t=0}^{\infty} \) and initial amount of savings, \( D_0 \), households maximize their life-time utility given by 1.5;

(ii) Given prices \( \{ R_{d,t+1} \}_{t=0}^{\infty} \), government policies \( \{ \zeta_t, T_t \}_{t=0}^{\infty} \) and initial capital structure \( \{ n_{j,0}, d_{j,0} \}_{j \in \Omega} \), each bank \( j \in \Omega \) maximizes the discounted sum of equity payouts given by 1.4;

(iii) The government budget constraint 1.7 is satisfied;

(iv) Market clearing conditions hold:

- resource constraint

\[ C_t + \int_0^1 l_{j,t+1} dj = \int_0^1 y_{j,t} dj, \]

- deposits market

\[ D_{t+1} = \int_0^1 d_{j,t+1} dj. \]

The nature of the bank’s problem 1.4 implies that the equilibrium is symmetric and all banks make identical decisions. The realization of the bank-specific shock \( \omega \) induces a bailout transfer from the government to a subgroup of banks, but does not affect the optimal decisions of banks. To see this result, it is useful to define the value of the bank to its
shareholders as follows:

\[ J(l_{j,t}, n_{j,t}, \omega_{j,t}, S_t) = \max \{ \pi_{j,t} + n_{j,t}, 0 \} + V(S_t), \]

where the continuation value \( V(\cdot) \) obeys the following Bellman equation:

\[
V(S_t) = \max_{n_{j,t+1}, l_{j,t+1}} \left\{ -n_{j,t+1} + E_t \left[ \beta M_{t,t+1} \int_0^{+\infty} J(l_{j,t+1}, n_{j,t+1}, \omega_{j,t+1}, S_{t+1}) dF(\omega_{j,t+1}) \right] \right\} 
\]

\[
= \max_{n_{j,t+1}, l_{j,t+1}} \left\{ -n_{j,t+1} + E_t \left[ \beta M_{t,t+1} \left( \int_{\omega_{j,t+1}}^{+\infty} (\pi_{j,t+1} + n_{j,t+1}) dF(\omega_{j,t+1}) + V(S_{t+1}) \right) \right] \right\}. 
\]  

(1.8)

The conditional expectation \( E_t \) is taken only over the distribution of aggregate productivity shocks and \( S_t \) denotes the aggregate state of the economy. To the extent that bank-specific shocks are not persistent and there is no equity issuance costs, all financial intermediaries are ex ante the same and make identical decisions. In particular, \( l_{j,t+1} = L_{t+1}, n_{j,t+1} = N_{t+1}, \) and \( d_{j,t+1} = D_{t+1}, \forall j \in \Omega. \)

To uncover the inefficiencies introduced with the presence of government subsidies, I first solve for the socially optimal allocation in this economy. Next, I characterize a competitive equilibrium when banks face no capital regulations and then compare the optimal decisions of banks in the two equilibria.

1.4.1. First-Best Allocation

A social planner chooses consumption and supply of liquid deposits to maximize households’ lifetime utility subject to the resource constraint:

\[
\max_{C_t, L_{t+1}, D_{t+1} \leq L_{t+1}} E \left[ \sum_{t=0}^{\infty} \beta^t \left( C_t + \frac{D_{t+1}^{1-\eta}}{1-\eta} \right) \right] 
\]

\[
s.t. \quad C_t + L_{t+1} = e^{\alpha t} L_t^0, \]

(1.9)
where the regulator takes into account that the competitive equilibrium is symmetric and that the deposit’s supply is bounded by the stock of capital in the banking sector. I characterize the socially optimal allocation in Proposition 1.

**Proposition 1** The first-best allocation is characterized by:

(i) Bank’s optimal finance policy:

\[
D_{t+1}^{FB} = L_{t+1}^{FB}, \quad N_{t+1}^{FB} = 0.
\]

(ii) Optimal level of bank lending, \(L_{t+1}^{FB}\), defined by:

\[
E_t [R_{t,l,t+1}^{FB}] = R_{d,t+1}^{FB}
\]

with

\[
R_{t,l+1}^{FB} = \alpha e^{\alpha t+1} (L_{t+1}^{FB})^{\alpha-1} \quad \text{and} \quad R_{d,t+1}^{FB} = \frac{1}{\beta} - \frac{1}{\beta} (D_{t+1}^{FB})^{-\eta}.
\]

Proposition 1 follows directly from the first order conditions to 1.9. The proofs of all propositions are given in Appendix A. Because households value money-like securities, it is optimal to finance all bank production with debt. The expression 1.10 pins down the first-best level of lending by equating the marginal productivity of bank capital stock to the marginal cost of lending (Figure 2). The social marginal cost of funding is equal to the rate of return at which a household is willing to hold deposits at the financial intermediaries, \(R_{d,t+1}^{FB}\). It seems implausible that the social planner would find it optimal not to keep any equity on the bank’s balance sheet. As an extension, I can introduce the social cost of bank default. In such a setup, the optimal level of a bank net worth, \(N_{t+1}^{FB} > 0\), would trade off reduced liquidity provision and reduced social cost of bank default.

I interpret the first-best level of lending, \(L_{t+1}^{FB}\), as an upper limit on the amount of positive NPV loans (creditworthy borrowers) in the economy. The model captures endogenously
that there are fewer good lending opportunities during economic slowdowns than during expansions, as I state in Proposition 2.

**Proposition 2** The socially optimal level of lending, $L_{t+1}^{FB}$, is procyclical:

$$\frac{\partial L_{t+1}^{FB}}{\partial a_t} > 0.$$  

Proposition 2 is established using the equilibrium condition 1.10 and an implicit function theorem (see Appendix A). Since the marginal productivity of lending is higher in good times, when the marginal cost is acyclical (Figure 2), it is optimal to maintain a higher level of production during expansions. This naturally advocates for a less stringent government regulations during periods of economic growth, when there are more lending opportunities.

1.4.2. Competitive Equilibrium with No Capital Regulation

To highlight the economic mechanism at the heart of the model, I now provide a detailed characterization of the bank’s lending and capital structure choice in a competitive equilibrium. In this section, suppose that financial intermediaries face no capital regulation or, equivalently, $\zeta_t = 0, \forall t$.

The banks’ first order conditions with respect to lending and the amount of equity financing are, respectively, given by:

$$E_t[R_{t,t+1}] = R_{d,t+1} - E_t\left[\int_0^{\omega_{t+1}} (R_{d,t+1} - e^{\omega} R_{t,t+1}) d\Phi(\omega)\right], \quad (1.11)$$

$$- \left(\frac{1}{\beta} - R_{d,t+1}\right) - E_t\left[\int_0^{\omega_{t+1}} R_{d,t+1} d\Phi(\omega)\right] < 0, \quad (1.12)$$

where the implicit (aggregate) rate of return on loans is equal to $R_{t,t+1} = \alpha e^{\omega_{t+1}} L_{t+1}^{\alpha-1}$ and the bailout threshold is defined by $e^{\omega_{t+1} + \alpha t+1} L_{t+1}^{\alpha} = R_{d,t+1} (L_{t+1} - N_{t+1})$. $\Phi(\cdot)$ denotes the normal distribution with mean $-\frac{1}{2} \sigma^2_{\omega}$ and standard deviation $\sigma_{\omega}$. 
The presence of government guarantees generates a bailout wedge in the bank’s cost of lending, defined by:

\[ \xi(L_{t+1}, N_{t+1}; a_t) \equiv E_t \left[ \int_0^{\omega^*_{t+1}} (R_{d,t+1} - e^{\omega}R_{l,t+1}) d\Phi (\omega) \right]. \]

This bailout wedge captures the difference between the social and private marginal cost of lending (see equations 1.10 and 1.11). The bank does not fully internalize the risk costs, since those are partially borne by the taxpayers. In particular, whenever the bank’s profits are hit by a sufficiently low idiosyncratic shock \( \omega < \omega^*_{t+1} \), their credit liabilities are covered by the government. An important property of the bailout wedge is that for a given level of lending it is decreasing in the aggregate productivity, \( \partial \xi / \partial a_t < 0 \) (Panel A of Figure 3). The economy is less likely to transition to a recession next period if it is currently in an expansion, implying a lower bailout wedge during periods of economic growth.

Similarly, I can show that the bailout wedge is decreasing in \( N_{t+1} \). The more net worth financial intermediaries are holding on their balance sheet, the safer they are, the less likely they will be bailed out and, hence, the smaller is the wedge in the lending cost.

The expression 1.12 demonstrates that equity is relatively a more expensive source of funding for banks than debt. The reasons for this are twofold. By holding equity on their balance sheets, banks forgo government subsidies and, at the same time, relinquish the liquidity premium. As a result, the financial intermediaries will tilt their capital structure towards debt and hold no equity on their balance sheets.

The funding wedge is strictly positive \( \xi(L_{t+1}, N_{t+1}; a_t) > 0 \), implying that the social marginal cost of lending is strictly greater than the private one. Therefore, there exists an excessive lending in a competitive equilibrium when the banks face no capital regulations, \( L_{t+1}^{CE} > L_{t+1}^{FB} \).\(^{13}\) It is a well documented in the banking literature, that government guarantees incentivize banks to risk-shift. Contrary to the literature, in my model banks risk-shift

\[^{13}\xi(L_{t+1}, N_{t+1}; a_t) > 0 \) as long as \( L_{t+1} > 1 \), which ensures a positive rate of return on deposits.
in terms of quantity, rather than in terms of quality. By issuing more loans, banks move further into the distribution of the projects’ quality (borrowers’ creditworthiness), meaning that more loans result in a lower quality loan portfolio. I lay out a characterization of the competitive equilibrium in Proposition 3.

**Proposition 3** *The competitive equilibrium with no capital regulation in place is characterized by:*

(i) **Optimal level of lending,** $L_{t+1}^{CE}$, defined by:

$$E_t \left[ R_{t,t+1}^{CE} \right] = R_{d,t+1}^{CE} - \xi \left( L_{t+1}^{CE}, N_{t+1}^{CE}, a_t \right), \quad (1.13)$$

with

$$R_{t,t+1}^{CE} = \alpha e^{a_{t+1}} \left( L_{t+1}^{CE} \right)^{\alpha - 1} \quad \text{and} \quad R_{d,t+1}^{CE} = \frac{1}{\beta} - \frac{1}{\beta} (D_{t+1}^{CE})^{-\eta}.$$  

(ii) **Bank’s optimal finance policy:**

$$D_{t+1}^{CE} = L_{t+1}^{CE} > L_{t+1}^{FB}, \quad N_{t+1}^{CE} = N_{t+1}^{FB} = 0.$$  

In the competitive equilibrium with the risk-shifting mechanism in place, the optimal level of lending continues to be pro-cyclical (see Proposition 4). On the one side, lending is more productive during expansions, calling for a higher level of investment. On the other side, bank cost of lending goes up in good times because of a reduced bailout wedge, pushing the optimal level of lending down (Panel B of Figure 3). Under log-normality of the aggregate productivity shock, the former channel dominates the later and, hence, the procyclicality of the optimal level of lending follows.

**Proposition 4** *In a competitive equilibrium with no capital regulations in place, the optimal level of lending is procyclical:*

$$\frac{\partial L_{t+1}^{CE}}{\partial a_t} > 0.$$  

23
Using capital regulations, the government will aim to restrict inefficient lending that arises in a competitive equilibrium. And whether tight capital requirements will be more beneficial during expansions or recessions will depend on how the level of excessive investment, $L_{t+1}^{CE}/L_{t+1}^{FR} - 1$, behaves over the business cycle. Proposition 5 provides the condition under which the lending level in a competitive equilibrium is more procyclical than the first-best allocation.

**Proposition 5** The level of overinvestment is procyclical if and only if

$$-\bar{\xi}_{a,t} < \frac{\partial \xi \left( L_{t+1}^{CE}, N_{t+1}^{CE}; a_t \right)}{\partial a_t} < 0.$$  \hspace{1cm} (1.14)

The threshold value $\bar{\xi}_a$ is defined in Appendix A. There are a number of channels in force that indicate how banks’ risk-shifting incentives change over the business cycle. First, a decreasing returns to scale production technology generates a scale effect: in proportionate terms banks risk-shift, by the same amount across the states of the economy, but since in good times they risk-shift on a larger scale, the overinvestment is higher in absolute terms. This mechanism can be demonstrated in the setup in which the bailout wedge is proportionate to the deposit rate and the deposit rate is constant. The second channel arises due to the fact that the value of default option (or, equivalently, bailout option) increases during periods of economic growth as banks’ liabilities to debtholders become larger. Since during good times the economy is more saturated with deposits, households are willing to relinquish a smaller premium on safe assets, delivering a higher deposit rate. This property of the default option results from the concave preferences for liquid assets and can be demonstrated in the setup in which banks have a constant returns to scale production technology. Third, assuming everything else is equal, the probability of exercising the default option is decreasing in the productivity shock $a_t$, meaning that the amount of inefficient lending is reduced in good times. The restriction 1.14 ensures that in the equilibrium the banks’ risk-shifting motives are procyclical.
The second channel arises due to the fact that the bailout wedge is increasing in lending, \( \partial \xi / \partial L_{t+1} > 0 \), as shown in Panel A of Figure 3. The more loans that banks issue, the bigger is the size of the government transfer in case a bailout happens. This property of the bailout wedge results from the concave preferences for liquid assets and decreasing returns to scale in production. On the one side, a higher level of investment during periods of economic growth implies a higher deposit rate, since the economy is saturated with deposits and households are willing to relinquish a smaller premium on safe assets. On the other side, decreasing returns to scale imply a lower income that is forgone in a bailout when banks increase lending. A higher deposit rate and a lower rate of return on loans, in turn, deliver a larger bailout wedge during expansions. Third, assuming everything else is equal, the bailout wedge is decreasing in the productivity shock \( a_t \), meaning that the amount of inefficient lending is reduced in good times. The restriction 1.14 ensures that in the equilibrium the bailout wedge or, equivalently, the banks’ risk-shifting motives are procyclical.

1.4.3. Competitive Equilibrium with Capital Regulation

Next suppose that equity needs be at least a fraction \( \zeta_t > 0 \) of loans for banks to be able to operate. Given that in my model equity is a relatively more expensive form of finance for banks than debt, the capital constraint is binding in each period of time (see Proposition 6).

Proposition 6 The capital constraint is binding:

\[
N_{t+1}^{CE} = \zeta_t L_{t+1}^{CE}, \quad D_{t+1}^{CE} = (1 - \zeta_t) L_{t+1}^{CE}.
\]

To demonstrate what are the implications of capital regulation on bank cost of lending, I now provide the banks’ first-order conditions with respect to lending and the amount of
equity financing, respectively, given by:

\[ E_t \left[ R_{t+1}^{CE} \right] = R_{d,t+1}^{CE} - \xi \left( L_{t+1}^{CE}, N_{t+1}; a_t \right) + \frac{1}{\beta} \lambda_t \zeta_t, \]  

(1.15)

\[ \frac{1}{\beta} \lambda_t = E_t \left[ \int_0^{\omega^*} R_{d,t+1}^{CE} dF(\omega) \right] + \left( \frac{1}{\beta} - R_{d,t+1}^{CE} \right), \]  

(1.16)

where \( \lambda_t \) is a Lagrange multiplier on the capital constraint. Imposing capital regulation introduces an equity wedge into the bank’s cost of funding. The first term of equation 1.16 captures that a better capitalized bank forgoes the government subsidies and, as a result, faces higher lending costs. A positive level of capital requirement also means that the bank’s supply of deposits is reduced, implying a higher liquidity premium and, as a result, a higher equity wedge, as captured by the second term of equation 1.16.

To better understand the costs and benefits of capital regulation, it is useful to decompose the bank’s funding costs into (i) the liquidity channel and (ii) the risk-shifting channel, which are, correspondingly, defined by:

\[ \theta \left( L_{t+1}, \zeta_t \right) \equiv R_{d,t+1} + \zeta_t \left( \frac{1}{\beta} - R_{d,t+1} \right), \]

\[ \tilde{\xi} \left( L_{t+1}, \zeta_t; a_t \right) \equiv E_t \left[ \int_0^{\omega^*} \left( (1 - \zeta_t) R_{d,t+1} - e^\omega R_{l,t+1} \right) d\Phi(\omega) \right]. \]

The obvious benefit of tightened capital requirements is reduced risk-shifting incentives for banks. As shown in Panel A of Figure 4, the bailout wedge adjusted for the presence of capital regulations \( \tilde{\xi} \) decreases as capital regulations are tightened, implying that less inefficient firms are financed. There are two reinforcing mechanisms in place: the bailout threshold \( \omega^*_t \), as well as the size of a government transfer in case of a bank failure \((1 - \zeta_t) R_{d,t+1} - e^\omega R_{l,t+1})\), falls as capital requirements become more stringent.

The level of the liquidity costs of lending \( \theta \) is mainly determined by the amount of deposits supplied to households. A higher capital requirement means that the supply of liquid assets
is reduced and, as a result, a liquidity premium is magnified. From the bank’s perspective, this implies a lower required rate of return on debt, but a higher equity wedge (Panel B of Figure 4). As long as \( \eta < 1 \), the liquidity cost of lending is strictly increasing in \( \zeta_t \). Overall, capital regulations translate into a heightened lending costs for banks, suggesting that excessive lending can be restrained by imposing a sufficiently high capital requirement.

1.4.4. Optimal Capital Requirements

The goal of a social planner when choosing an optimal level of capital requirements is to dampen bank’s risk-shifting incentives, but without restricting the bank’s supply of high-quality loans and deposits. As a matter of fact, it is not feasible to restore both first-best level of lending and liquidity provision at the same time with only the help of capital requirements. Suppose for now that the only goal of the regulator is to restore a socially optimal lending level and to achieve this goal the capital requirement is set to \( \zeta^L_t > 0 \), which is the “lending capital requirement”. This translates into a level of liquidity provision \( D^L_{t+1} \) that is below the socially optimal lending level:

\[
D^L_{t+1} = (1 - \zeta^L_t) L^F_{t+1} < L^F_{t+1} = D^F_{t+1}.
\]

Similarly, consider a social planner who aims to recover a socially optimal level of deposits and imposes a capital requirement of \( \zeta^D_t \), which is the “liquidity capital regulation”. This capital requirement, however, allows inefficient lending from a social perspective:

\[
L^D_{t+1} = \frac{D^D_{t+1}}{1 - \zeta^D_t} = \frac{L^F_{t+1}}{1 - \zeta^D_t} > L^F_{t+1}.
\]

To provide the intuition behind the design of the optimal capital regulations, I first solve for the lending and liquidity capital requirements in the baseline model. This allows me to demonstrate the cyclical properties of capital regulation in a fairly simple way.
Suppose now that the only goal of the social planner is to rule out excessive bank lending. To do so, she institutes the lending capital requirement, defined in Proposition 7.

**Proposition 7** The first-best level of bank lending, \( L^{L}_{t+1} = L^{FB}_{t+1} \), is restored when a capital requirement is set to \( \zeta^{L}_{t} \), defined by:

\[
\theta \left( L^{FB}_{t+1}, \zeta^{L}_{t} \right) - R^{FB}_{d,t+1} = \tilde{\xi} \left( L^{FB}_{t+1}, \zeta^{L}_{t}; a_{t} \right).
\]  

The expression 1.17 equates the bank’s private cost of lending to the social one, ensuring that the bank chooses the first-best level of lending when it encounters a capital requirement of \( \zeta^{L}_{t} \). The right-hand side of the equation 1.17 captures a key benefit of capital requirements – reduced risk-shifting incentives – depicted in Panel A of Figure 5 by the dash-dotted lines. The tighter the capital regulations the lower the expected government subsidies and, hence, the lower the bank’s overinvestment. As discussed earlier, a capital regulation that is too restrictive can amplify the liquidity costs of lending via an equity wedge, leading to reduced liquidity provision and potentially restricting the funding of high-quality projects. Panel A of Figure 5 shows that the spread between the private liquidity cost of lending and the social one depicted by the solid lines is increasing in \( \zeta^{L}_{t} \). The primary question I address in this paper is how optimal bank capital requirements behave over the business cycle. This essentially means answering whether the risk-shifting incentives for banks are stronger in good or bad economic environments, and whether equity financing is more costly during expansions or recessions.

The risk-shifting incentives for banks are defined by the bailout wedge \( \hat{\xi} \). Differentiating this wedge with respect to the productivity shock,

\[
\frac{d \hat{\xi} \left( L^{FB}_{t+1}, \zeta^{L}_{t}; a_{t} \right)}{da_{t}} = \frac{\partial \hat{\xi} \left( L^{FB}_{t+1}, \zeta^{L}_{t}; a_{t} \right)}{\partial a_{t}} < 0 + \frac{\partial \hat{\xi} \left( L^{FB}_{t+1}, \zeta^{L}_{t}; a_{t} \right)}{\partial L^{FB}_{t+1}} \frac{\partial L^{FB}_{t+1}}{\partial a_{t}} > 0 \leq 0,
\]
demonstrates that there are two competing effects in place. On the one hand, the probability of a bank bailout decreases with the productivity shock, implying a smaller wedge in the bank’s lending costs. On the other hand, during periods of economic growth there are more lending opportunities, indicating that more projects will be funded in the first best. Because the bailout wedge expands when lending is higher, the risk-shifting motives become stronger during expansions. Panel A of Figure 5 shows that under chosen parameter values, the latter channel dominates the former and, as a result, the bailout wedge is procyclical, suggesting that a higher capital requirement is necessary during periods of economic growth.

Households’ preference for safe assets implies that equity is costly, especially during recessions. To the extent a bank’s balance sheet shrinks during a recession, less safe assets are created by banks and the economy is less saturated with liquidity. This translates into a higher liquidity premium and, as a result, a higher liquidity cost of lending. Panel A of Figure 5 shows that for a given level of $\zeta_t$, the cost of capital requirements is countercyclical, suggesting that capital charges should be reduced during economic slowdowns.

The two effects - the countercyclical cost and procyclical benefit of capital regulation - reinforce each other and deliver a capital requirement, $\zeta^L_t$, that is high in times during periods of economic growth and low during downturns (Panel B of Figure 5).

**Liquidity Capital Requirement**

Consider now a type of capital regulation designed to restore the first-best level of liquidity provision (i.e., the first-best level of deposit supply). Proposition 8 provides a characterization of the liquidity capital requirement.

**Proposition 8** The first-best level of deposits, $D^L_{t+1} = D^F_{t+1}$, is achieved, when the social
planner sets the capital requirement equal to $\zeta_t^D$, defined by:

$$
(1 - \zeta_t^D)^{\alpha - 1} \theta \left( \frac{L_t^{FB}}{1 - \zeta_t^D}, \zeta_t^D \right) - R_{d,t+1}^{FB} = (1 - \zeta_t^D)^{\alpha - 1} \tilde{\xi} \left( \frac{L_t^{FB}}{1 - \zeta_t^D}, \zeta_t^D; a_t \right). \tag{1.18}
$$

As opposed to the lending capital requirement, the liquidity capital requirement allows inefficient bank lending in the amount of $\frac{\zeta_t^D}{1 - \zeta_t^D} L_t^{FB}$. This results in a larger bailout wedge $\tilde{\xi}$, as well as a higher liquidity cost of lending $\theta$, since the economy is more saturated with safe assets. Importantly, when a bank’s balance sheet grows, the rate of return on loans drops due to a decreasing returns to scale. This loss in a bank’s productivity can be viewed as an additional cost that arises with the liquidity capital requirement, and it is captured by the factor $(1 - \zeta_t)^{\alpha - 1} \geq 1$.

When setting the level of liquidity capital requirement, the underlying trade-off remains the same. On the one side, tight capital regulations restrict banks’ risk-shifting incentives as captured by the right-hand side of the equation 1.18. But, on the other side, it increases banks’ lending costs via the liquidity channel and, as a result, may lead to a reduction in bank loans and deposits as captured by the left-hand side of the equation 1.18. Similar to the case with the lending capital requirement, stringent capital ratios are of most use during periods of economic growth, but are also less costly, delivering a liquidity capital requirement that is procyclical.

Both lending and liquidity capital requirements focus only on one dimension of the problem – either dampening banks’ incentives to risk-shift or stimulating banks’ creation of liquid assets. In order to ensure highest lifetime utility of households in this economy, I solve for an optimal Ramsey policy that balances reduced inefficient lending and reduced liquidity provision. The Ramsey capital requirement is characterized in Section 5.
1.5. Quantitative Assessment

In this section, I quantify policy recommendations on what defines optimal time variation in bank capital requirements. First, I extend the baseline model and calibrate it to a fixed capital requirement to match key macroeconomic quantities, as well as their counterparts in the bank data. Next, I solve for the optimal policy rule using the chosen parameter specification and characterize it in terms of the relevant macroeconomic aggregates.

1.5.1. Quantitative Model

To best match the data, I generalize the households’ preferences in the quantitative model, as well as introduce a bank-independent production sector.

**Household sector.** In the model, households have constant relative risk aversion (CRRA) preferences with the risk-aversion coefficient $\gamma$. The households’ utility is defined over consumption and deposit holdings according to a constant elasticity of substitution (CES) aggregator:

$$v(C_t, D_{t+1}) = \left( \frac{C_t^\eta}{\eta} + \chi D_{t+1}^\eta \right)^{\frac{\eta}{\eta-1}},$$

where $\chi$ is a share parameter and $\tilde{\eta}$ is the elasticity of substitution between consumption and liquidity services.

The households’ first-order conditions imply that the rate of return on bank debt satisfies:

$$E_t [M_{t,t+1} R_{d,t+1}] = 1 - \chi \left( \frac{D_{t+1}}{C_t} \right)^{-\frac{1}{\eta}},$$

(1.19)

where $M_{t,t+1}$ is the stochastic discount factor equal to:

$$M_{t,t+1} = \left( \frac{v(C_{t+1}, D_{t+2})}{v(C_t, D_{t+1})} \right)^{\frac{1}{\eta} - \gamma} \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\eta}}.$$  

(1.20)

Similar to the baseline case, there is a discount on the deposits rate compared to the rate of
return on any other safe asset. In the quantitative model, the liquidity premium depends on the ratio of deposit holdings and consumption, rather than just the level of deposit holdings. This helps to capture the idea that higher consumption increases the marginal utility of liquidity of households.

**Production sector.** By and large, the economy’s production sector is reliant either on bank debt or access to capital markets to undertake production projects. To the extent that some borrowers lack access to capital markets, typically due to informational asymmetries, agents can be broadly classified into two groups. The first group is small businesses and households, who have their financial needs fulfilled by bank loans and mortgages. The second group is borrowers who have access to public markets and use multiple forms of debt financing. To capture this heterogeneity in borrowers, the model now includes both a banking sector and a bank-independent sector. The bank-independent sector includes firms with access to capital markets.

For the most part, the banking sector remains the same in the quantitative as in the baseline model. Each bank \( j \in \Omega \) operates a sector-specific production technology:

\[
e^{\omega_{j,t} + \bar{a}_b + a_t}^{\alpha_b}_{j,t},
\]

where the aggregate productivity shocks follows an AR(1) process:

\[
a_t = \rho_a a_{t-1} + \sigma_a \epsilon_t, \quad \epsilon_t \sim iid \mathcal{N}(0,1).
\]

The constant \( \bar{a}_b \) is introduced to better match the fraction of output produced in the banking sector. The bank-specific shock is iid across time and banks, with the law of motion given by:

\[
\omega_{j,t} = -\frac{1}{2} \sigma_\omega (a_t)^2 + \sigma_\omega (a_t) \epsilon_{j,t}, \quad \epsilon_{j,t} \sim iid \mathcal{N}(0,1).
\]

\[14\] In the quantitative model, the rate of return on any other safe asset other than bank debt is equal to \( 1/E_t [M_{t+1}] \).
Consistent with Bloom et al. (2012), idiosyncratic productivity risk varies countercyclically with the aggregate productivity; specifically, I choose \( \sigma_\omega (a_t) = \sigma_\omega e^{-\nu a_t} \).

The capital in the banking sector depreciates at rate \( \delta \) and accumulates according to:

\[
l_{j,t+1} = (1 - \delta) l_{j,t} + i_{b,j,t}.
\]

Further, the financial intermediaries incur an operating cost \( o_b \) per unit of intermediated assets. One could think of this cost as the resources spent to monitor bank borrowers. As before, the banks finance production using a mixture of equity and debt financing. Since there is neither equity issuance costs nor adjustments costs to capital, the equilibrium continues to be symmetric and all banks make identical decisions \( l_{j,t+1} = L_{t+1}, n_{j,t+1} = N_{t+1}, \) and \( d_{j,t+1} = D_{t+1}, \forall j \in \Omega. \) Hence, the output produced in the banking sector equals \( Y_{b,t} = e^{\bar{a}_b + a_t} L_t^{a_b}. \)

The bank’s first-order conditions imply that the capital constraint is binding, while the optimal level of lending satisfies:

\[
E_t [M_{t,t+1} (R_{t,t+1} + (1 - \delta - o_b))] = \theta_t - \tilde{\xi}_t,
\]

where the liquidity and risk-shifting costs of lending are, respectively, defined by:

\[
\theta_t = E_t [M_{t,t+1} R_{d,t+1}] + \zeta_t (1 - E_t [M_{t,t+1} R_{d,t+1}]), \tag{1.21}
\]

\[
\tilde{\xi}_t = E_t \left[ M_{t,t+1} \left( \int_0^{\omega^*} ((1 - \zeta_t) R_{d,t+1} - e^\omega R_{t,t+1} - (1 - \delta - o_b)) d\Phi (\omega) \right) \right]. \tag{1.22}
\]

with the loan rate being equal to

\[
R_{t,t} = e^{\bar{a}_b + a_t} L_t^{a_b - 1}. \tag{1.23}
\]

The firms in the bank-independent sector similarly employ a decreasing returns to scale and
produce consumption goods, using capital $K_{f,t}$ as the only input, $Y_{f,t} = e^{at}K_{f,t}^{\alpha_f}$. To form output, the firms rent out capital from the household sector at the rate of return:

$$R_{k,t} = \alpha_f e^{at}K_{f,t}^{\alpha_f-1}.$$  \hspace{1cm} (1.24)

The households are the owners of the financial intermediaries, as well as of the firms in the bank-independent sector. Therefore, at the end of each period, they receive proceeds from the banks’ activity $Z_t$, as well as the firms’ profits $\Pi_t$. Because the households own the capital used in production in the bank-independent sector, they are the ones who make the capital investment choices. To invest in capital, households must pay a small operational cost $o_f$. The capital in the non-banking sector depreciates at the same rate as in the banking sector and accumulates according to:

$$K_{f,t+1} = (1 - \delta)K_{f,t} + I_{f,t}.$$  

1.5.2. Calibration

Data for the aggregate sector - the banking and non-banking sectors - comes from the Financial Accounts of the United States $Z.1$ issued by the Board of Governors of the Federal Reserve\(^\text{15}\) and from National Income and Product Accounts (NIPA) from the Bureau of Economic Analysis.\(^\text{16}\) In particular, the output produced in the total economy, $Y$, is measured using the income approach as the gross value added by all sectors, except Financial Business.\(^\text{17}\) Similarly, I exclude the financial sector when measuring gross fixed capital formation, $I$, and stock of fixed assets, $K$.\(^\text{18}\) Consumption is defined as the sum of expendi-

\(^{15}\)https://www.federalreserve.gov/releases/z1/.
\(^{17}\)The sectors included are Households and Nonprofit Institutions Serving Households, Nonfinancial Noncorporate Business, Nonfinancial Corporate Business and Government. The data is collected from the Financial Accounts, S.2. Selected Aggregates for Total Economy and Sectors.
\(^{18}\)The data on investment are from the Financial Accounts, S.2. Selected Aggregates for Total Economy and Sectors, while the data on capital stock are collected from the NIPA Fixed Assets tables, specifically, Table 6.1. Current-Cost Net Stock of Private Fixed Assets by Industry Group and Legal Form of Organization and Table 7.1A. Current-Cost Net Stock of Government Fixed Assets.
tures on non-durable goods and services. All the quantities are deflated with the Consumer Price Index and normalized by the NIPA population. The data on CPI inflation are from the Bureau of Labor Statistics, with the price level being normalized to 1 as of December of 2009.

The challenging part of mapping the model to the data is to differentiate the data on output into the value added by the banking sector and the bank-independent sector. In the model, the final goods produced by financial intermediaries, \( Y_b \), are viewed as the value added by bank borrowers (predominantly, small businesses and households) and, consequently, is measured as the gross value added by Households and Nonprofit Institutions Serving Households and Nonfinancial Noncorporate Business sectors.\(^{19}\) The output produced by borrowers with access to capital markets, \( Y_f \), is measured as the gross value added by Nonfinancial Corporate Business and Government sectors.\(^{20}\) As documented in Rauh and Sufi (2010), corporate businesses both use bonds and utilize bank debt as funding sources. In this sense, I overestimate the value-added by the bank-independent sector and, as a result, underestimate the costs associated with tight capital requirements, as reduced lending translates into a smaller loss in the aggregate output. According to this measure the banking sector accounts for about 28% of the output produced in the economy. I pursue the same split between borrowers when measuring gross fixed capital formation and stock of capital in the two-sectors.

The model is calibrated at annual frequency for the 1980-2008 period. I exclude the post Great Recession period to avoid zero-lower bound concerns. The parameters are calibrated to satisfy the steady-state conditions of the model, along with the target first and second moments in the data.

**Long run behavior.** The subjective discount factor is set to \( \beta = 0.975 \), which is a value traditionally used in macroeconomics. The Cobb-Douglas function parameters \( \alpha_b \) and \( \alpha_f \)

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\(^{19}\)The data are from the Financial Accounts, S.2. Selected Aggregates for Total Economy and Sectors.

\(^{20}\)The data are from the Financial Accounts, S.2. Selected Aggregates for Total Economy and Sectors.
are chosen to match a capital-output ratio in the banking and non-banking sectors, respectively. In the data, a capital-output ratio of the financial intermediaries (or, equivalently, of firms in the banking sector) is equal to 4.9, which is roughly twice as high as a capital-output ratio of the bank-independent firms (see Table 2). As a result, the $\alpha_b = 0.78$ decreasing return to scale parameter for the banks is larger than the $\alpha_f = 0.355$ corresponding for the firms. The scalar $\bar{a}_b$ is set so that a ratio of loans to a total stock of capital in the model is identical to the one in the data. By construction, I also match a market fraction of the banking sector in terms of the produced output, as well as a capital-output ratio of the aggregate sector. The capital depreciation rate is set to $\delta = 0.075$ to produce a plausible value for the model-implied ratio of investment-to-capital. The operating cost in the banking sector $o_b$ is calibrated to match a bank’s profit-to-loan ratio. The bank data stems from the aggregated regulatory filings, the so-called call reports, as reported by the Federal Deposit Insurance Corporation (FDIC).\footnote{https://www5.fdic.gov/idasp/advSearch_warp_download_all.asp?intTab=4.} A bank’s loans are measured as the total assets net of the securities, fixed assets, and cash and due to depository institutions, while a bank’s profits are measured as the total interest and non-interest income net of the securities interest income and total interest expense. The operating cost in the bank-independent sector is determined using the standard deviation of firms’ capital investment.

**Productivity shocks.** Estimates of Solow residuals typically imply a highly persistent AR(1) process for the log productivity $a_t$. I use a standard value of 0.95 for the autoregressive coefficient and choose a $\sigma_a$ of 0.02 ($\sqrt{4 \cdot 0.01}$) to match the standard deviation of the aggregate output in the data.

**Idiosyncratic shocks.** I use the parameter $\sigma_\omega$ to determine both the risk choice of banks (i.e., level of inefficient lending) and, for the most part, the fraction of banks being bailed out each period. To the extent that there is no default in the model, I define the bailout rate as a fraction of failed banks, both assisted and not assisted by the government. The data on bank failures are from the fail bank list issued by the FDIC.\footnote{https://www5.fdic.gov/hsob/SelectRpt.asp?EntryTyp=30&Header=1&tab=bankFailures.} On average, 0.76% of
all insured institutions fail each year, delivering \( \sigma_\omega \) equal to 0.325. The parameter \( \mu_\omega \) is set to normalize the mean of idiosyncratic shock \( \omega \) to 1. The level of \( \nu \) is chosen to generate the range of the idiosyncratic volatility dispersion \( \sigma^2_\omega (a_t) \) consistent with the empirical evidence in Bloom et al. (2012). Specifically, I target a 15% increase of the volatility dispersion in recessions relative to expansions. This is on the conservative end of the values reported in Bloom et al. (2012).

**Capital adequacy ratios.** Even though my aim is to characterize optimal capital regulation that varies over the time, the model is calibrated with a fixed level of capital requirement in place, \( \bar{\zeta} \). In accordance with Basel III capital standards, FDIC-insured depository institutions are required to maintain the leverage ratio (Tier 1 capital divided by total assets) of 4% and the Tier 1 capital ratio (Tier 1 capital divided by total risk-weighted assets) of 6%.\(^{23}\) Obviously, these two ratios are different in the data, but are the same in the model. On average, depository institutions hold 7.26% of total assets and 10.20% of total risk-weighted assets in terms of core capital\(^{24}\). In the benchmark calibration, I set \( \bar{\zeta} \) to 7.26%.

**Liquidity premium.** The financial intermediaries play a special role in creating liquidity; they transform illiquid long-term assets into liquid short-term liabilities that offer non-pecuniary services to investors. In accordance with this view, when mapping \( D \) to the data, I aim to capture the short-term part of bank debt that is used to finance risky assets. Following Krishnamurthy and Vissing-Jorgensen (2015), this measure of bank debt equals the sum of all deposits and the other forms of short-term debt categories net of short-term assets and holdings of Treasuries.\(^{25}\) Further, I assume that the short-term Treasuries and

\(^{23}\)Core capital elements (Tier 1) consists of common stockholders’ equity capital, noncumulative perpetual preferred stock, and minority interests in the equity capital accounts of consolidated subsidiaries.

\(^{24}\)The data on capital adequacy ratios are collected from the “Ratios by Asset Size Group” table issued by the FDIC. Due to the availability of the data, I calculate the average leverage ratio for the sample period 1984 - 2008 and the average Tier 1 capital ration for the sample period 1990 - 2008.

\(^{25}\)An alternative approach is to map \( D \) to “core” deposits, which is the largest source of bank funding and the major source of safe and liquid assets for households (see, for example, Drechsler et al. (2016)). Core deposits include checking and money market accounts, as well as small time deposits. The cyclical properties of this measure, however, change over the sample period of 1960-2015: while deposits exhibit strong positive correlation with output growth in the earlier part of the sample, they become mildly countercyclical in the later one. Yet, over the sample period I consider in this paper deposits co-move positively with output overall. Nonetheless, I focus on a broader notion of the liquidity supply as it better captures the role of
bank debt are perfect substitutes and, as a result, require the same rate of return. Under this assumption, the liquidity premium is constructed as the spread between the rates of return on the 3-month AA Commercial Paper and the 3-month Treasury bill, similar to Krishnamurthy and Vissing-Jorgensen (2012).26,27

The preference of households for bank debt is governed by a share parameter, \( \chi \), and by elasticity of substitution between consumption and liquidity services, \( \tilde{\eta} \). The level of \( \tilde{\eta} \) is set to match the volatility of a ratio of the bank debt to consumption. Given the value of \( \tilde{\eta} \), the scalar \( \chi \) is set so that the steady-state level of the liquidity premium is equal to 0.57% (annually).

The model is quite parsimonious and requires only 10 structural parameters, besides the stochastic process for the shocks.

In Table 1, I summarize the benchmark configuration of the model parameters. Tables 2 and 3 report the implications of the chosen parameters for the first and second unconditional moments of the key variables. Overall, the model can successfully match a number of moments for the main aggregates. The model has a hard time matching the volatility of bank lending and, because of the binding capital constraint, also the volatility of bank debt. In the model, bank lending is a stock variable and is quite persistent, therefore having rather modest fluctuation over the business cycle. The low volatility of bank debt, in turn, translates into low volatility of the liquidity premium. As a potential remedy, I can introduce a stochastic process for one of the parameters governing a demand function for safe assets (\( \chi \) or \( \eta \)). For parsimony of the results, for now I consider the calibration without liquidity shocks. As reported in Table 4, the model generates the correct signs for the business cycle correlations, but overstates their magnitudes. This is a common

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26 For the period of 1997-2008, the rate of return on Commercial Paper is calculated as the average of the rate of return on Financial and Non-Financial Commercial Paper.

27 Other potential measures include (i) the spread between the rates of return on the Resolution Funding Corporation (Refcorp) bonds and the Treasury bills (Longstaff (2002)), and (ii) the spread between the interest rates on general collateral repurchase agreement (GC repo) and the Treasury bills (Nagel (2014)). These measures produce quantitatively similar results.
characteristic of the standard real business cycle model with one shock in the economy.

To understand the model dynamics, I now analyze the impulse response functions of key quantities to a one standard deviation positive total factor productivity (TFP) shock. As shown in Figure 6, following a positive shock, lending in the banking sector increases immediately by 1% and continues to grow for a number of years. This is the case because lending opportunities improve. An increase in lending, in turn, is accompanied by an increase in deposits (0.03% increase in the deposits-to-consumption ratio) and, as a result, a reduction in the liquidity premium. This result reaffirms the conjecture in the baseline model that equity financing is relatively less expensive during expansions than recessions. Moreover, this is line with the empirical evidence that the liquidity premium is countercyclical (see Table 4). At the same time, a positive shock to productivity results in a reduced probability of default in the current period. But in the subsequent periods, the probability of default spikes up. This is in response to a heightened risk-shifting by banks, as captured by the impulse response function of the bailout wedge. Note that the bailout wedge is the expected (not realized) bailout subsidy at time $t + 1$, conditional on time $t$ information and, most importantly, on the time $t$ choice of lending $L_{t+1}$.

1.5.3. Ramsey Equilibrium

The Ramsey problem characterizes a set of allocations that can be implemented as a competitive equilibrium when banks face the prescribed capital requirements. A Ramsey planner maximizes the lifetime utility of households subject to the resource constraint and imple-
mentability conditions:

$$\argmax_{C_t, L_{t+1}, N_{t+1}, D_{t+1}, K_{f,t+1}} \mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t u(C_t, D_{t+1}) \right]$$

with

$$u(C_t, D_{t+1}) = \frac{v(C_t, D_{t+1})^{1-\gamma}}{1-\gamma} - 1$$

s.t.

$$C_t = R_{d,t} D_t - D_{t+1} + R_{k,t} K_{f,t}^{\alpha_f} - I_{f,t} - o_f K_{f,t} + Z_t + \Pi_t - T_t,$$

$$E_t [M_{t,t+1} (R_{k,t+1} + (1 - \delta - o_f))] = 1,$$

$$N_{t+1} + D_{t+1} = L_{t+1},$$

$$E_t [M_{t,t+1} (R_{l,t+1} + (1 - \delta - o_b))] = \theta_t - \tilde{\xi}_t,$$

$$C_t + I_{b,t} + I_{f,t} = Y_{b,t} + Y_{f,t} - o_b L_t - o_f K_{f,t},$$

where the taxes and net proceeds from banks’ and firms’ activity are, respectively, equal to:

$$T_t = \int_0^1 \max \{ R_{d,t} D_t - e^{\omega_j,t+\alpha} L_t^{\alpha_b} - (1 - \delta - o_b) L_t, 0 \} \, dj,$$

$$Z_t = \int_0^1 z_{j,t} \, dj, \text{ with } z_{j,t} = \max \{ e^{\omega_j,t+\alpha} L_t^{\alpha_b} + (1 - \delta - o_b) L_t - R_{d,t} D_t, 0 \} - N_{t+1},$$

$$\Pi_t = Y_{f,t} - R_{k,t} K_{f,t}.$$

The rate of return on deposits, the stochastic discount factor, the liquidity and risk-shifting cost of lending, the rate of return on loans and capital are defined by 1.19, 1.20, 1.21, 1.22, 1.23 and 1.24, correspondingly. I provide further details in Appendix B. The optimal Ramsey policy – a capital requirement that implements the Ramsey allocations

$$\left\{ C_t^*, L_{t+1}^*, N_{t+1}^*, D_{t+1}^*, K_{f,t+1}^* \right\}_{t=0}^{\infty}$$

– requires a capital ratio that equals $$\zeta_t^* = N_{t+1}^*/L_{t+1}^*.$$ The set of Ramsey allocations and policies constitute a Ramsey equilibrium.

Using the chosen configuration of parameters (Table 1), I solve for the Ramsey equilibrium. Not surprisingly, the optimal policy rule for bank capital ratios $$\zeta_t^* = \zeta(S_t, S_{t-1})$$ is a function of all state variables $$S_t = \{\zeta_{t-1}, L_t, K_{f,t}, a_t\}$$ in the model, including the previous period capital requirement, capital stock in the two sectors, and the productivity shock. I find that
the Ramsey capital requirement, for the most part, fluctuates between 4% and 6%, with the mean at 5%. This range of values is based on one standard deviation of the Ramsey capital requirement above and below its mean, and is roughly consistent with an upper bound of the CCyB proposed by the Basel III Accord. Importantly, the optimal policy rule is not capped at 6%, and in periods of abnormal economic growth can go far above 6%. The steady state value of $\zeta_t^*$ is substantially lower than the Tier 1 leverage ratio observed in the data, but at the same time 1% higher than the level prescribed by the Basel III Accord. As shown in Figure 7, the effect of a positive TFP shock on the optimal policy rule is quite persistent: a one standard deviation positive TFP shock increases the capital ratio by 0.2% immediately and this effect lasts for a number of periods.

To uncover the implications of the capital regulation that varies over time, I now examine the impulse response reactions to a one standard deviation positive TFP shock in a Ramsey equilibrium. Figure 7 demonstrates that the Ramsey capital requirement indeed helps to reduce the procyclicality of bank lending, by dampening overinvestment during expansions and boosting bank credit during downturns. This reduced overinvestment, in turn, translates into a lower bailout wedge and, consequently, a lower probability of default. At the same time, the optimal policy rule dampens the procyclicality of the deposits, as there are two channels in place. The first channel is via bank lending directly: to the extent that a bank’s balance sheet may shrinks during recessions, the supply of deposits contracts, but less so in a Ramsey equilibrium. The second channel is via capital regulation itself: for a given amount of bank lending, in a Ramsey equilibrium, a larger fraction of production is financed with bank debt during recessions as compared to a competitive equilibrium with a fixed capital requirement. In this instance, the optimal policy rule goes down in a response to a negative productivity shock.

1.5.4. Optimal Policy Rule

For effective policy recommendations, it is useful to express the Ramsey capital requirement $\zeta(S_t, S_{t-1})$ as a function of key observable macro and bank quantities. To do so, I perform
the following exercise: I simulate the model with the Ramsey optimal policies in place for 1,000 times. Then, I estimate a regression of the Ramsey capital requirement on a number of model aggregates for each simulated sample:

\[ \zeta^*_t = \zeta_{ss} + \sum_{j=1}^{N} \zeta_{x_j} x^j_t + \varepsilon_t. \]

Admittedly, this policy rule is forward-looking: \( \zeta^*_t \) is a level of capital requirement set at time \( t \), but regulating a bank’s choice of capital structure at time \( t + 1 \). The results in Panel A of Table 5 demonstrates that credit gap used as the only indicator to adjust the capital charges over the business cycle does not capture the dynamics of the optimal policy rule (\( R^2 = 14\% \)). I find that that using the credit gap along with GDP (in log deviations from their steady states) delivers an \( R^2 = 99.98\% \). In particular, \( \zeta^*_t \) is characterized by the following policy rule:28

\[
4.9\% + 3 \times \left( \log \frac{L_t}{Y_t} - \log \frac{L_{ss}}{Y_{ss}} \right) + 8 \times \left( \log Y_t - \log Y_{ss} \right).
\]

In particular, a one percentage increase in the credit gap leads to a 0.03% increase in capital charges, while a one percentage point increase in GDP translates into a 0.08% increase. Equivalently, a one standard deviation increase in the credit gap leads to a 0.1% increase in capital requirements, while a one standard deviation increase in GDP translates into a 0.7% increase (Panel B of Table 5). This finding aligns with my main model prediction: in periods when banks are incentivized to risk-shift, that is in periods of high GDP and credit growth, the optimal capital requirement is increased. Interestingly, the liquidity premium plays little role if included as an additional explanatory variable in a regression (see Panel A of Table 5). However, this result does not continue to hold if the model includes liquidity shocks.

28The GDP growth variable is orthogonalized with respect to the credit gap variable, since the two indicators are strongly positively correlated.
1.5.5. Welfare Implications

Following the classic work of Lucas (1987), I calculate the potential welfare benefits (costs) associated with different types of capital regulation. To do so, I denote the expected lifetime utility of households in a world with capital regulation \( \zeta_t \) by

\[
E_{\zeta_t} \left[ \sum_{t=0}^{+\infty} \beta^t \frac{\tilde{C}_t^{1-\gamma} - 1}{1 - \gamma} \right],
\]

where the aggregate consumption is defined by \( \tilde{C}_t = v(C_t, D_{t+1}) \), accounting for the value of liquid assets to the households. The question is how much the households would be willing to pay or have to be compensated to move from the economy with a fixed level of capital requirement \( \zeta_t = \zeta_0, \forall t \) to the world with a capital ratio of \( \zeta^1 \) or to the economy where banks face the optimal Ramsey policy \( \zeta^*_t \). This compensating variation \( \varpi \) is defined as:

\[
E_{\zeta_0} \left[ \sum_{t=0}^{+\infty} \beta^t \frac{(1 + \varpi) \tilde{C}_t^{1-\gamma} - 1}{1 - \gamma} \right] = E_{\zeta_1} \left[ \sum_{t=0}^{+\infty} \beta^t \frac{\tilde{C}_t^{1-\gamma} - 1}{1 - \gamma} \right],
\]

or, equivalently:

\[
\varpi = \left( \frac{E_{\zeta_1} \left[ \sum_{t=0}^{+\infty} \beta^t \tilde{C}_t^{1-\gamma} \right]}{E_{\zeta_0} \left[ \sum_{t=0}^{+\infty} \beta^t \tilde{C}_t^{1-\gamma} \right]} \right)^{\frac{1}{1-\gamma}} - 1.
\]

Unlike the Lucas cost of business cycle, the above measure captures the changes in consumption instability across different economies, as well as the changes in steady state levels of consumption. I provide more details on the calculation of welfare costs in Appendix C.

Figure 10 depicts the variation compensation, \( \varpi \), as a function of different levels of fixed capital charges, \( \zeta^1 \in [0, 1] \), as well as the welfare result for the Ramsey economy. As the benchmark, I choose a world with an optimal fixed level capital requirement, which happens to be the stochastic steady state of the Ramsey policy. Moving from the current level of capital ratio 7.26% to the Ramsey policy translates into a 0.06% permanent increase in the annual consumption of households, although the exact magnitude of the gain depends
on the attitude of consumers towards risk.\textsuperscript{29} Two-thirds of this welfare improvement is attributed to a transition from the optimal fixed capital requirement to the time-varying one. Admittedly, the welfare gain from implementing the policy rule that is solely based on the credit gap is significantly smaller as compared to the welfare gain from having the optimal capital ratios. Implementing the optimal Ramsey policy leads to 15.5% decrease in the cyclicality of the bank credit (relative to a world with an optimal fixed level capital requirement): the banks engage in less excessive lending during periods of economic growth, but at the same time avoid contraction of lending activities during recessions. This in turn translates on average into a higher level of lending (+0.93%) and consumption (+0.11%). In addition, the increase in bank lending stimulates banks' creation of liquid assets (+0.88%). The time-varying capital ratios also lead to a reduction in the volatility of both lending (−0.13%) and deposits (−0.17%), but to an increase in the consumption volatility (+0.08%).

1.5.6. Model with Liquidity Shocks

For robustness, I also consider a model with liquidity shocks. Namely, suppose that a share parameter in the CES aggregator over consumption and deposits follows an AR(1) process:

\[
\log(\chi_t) = (1 - \rho_\chi) \bar{\chi} + \rho_\chi \log(\chi_{t-1}) + \sigma_\chi u_t, \quad u_t \sim iid \mathcal{N}(0, 1).
\]

The $\bar{\chi} = 0.01$ parameter is set to the value of a share parameter used in the benchmark calibration of the model without demand shocks. The standard deviation of the demand shock $\sigma_\chi = 0.1$ is calibrated to better match the volatility of the liquidity premium. I choose a persistence parameter of $\rho_\chi = 0.95$. As shown in Appendix Figure 9, a one standard deviation liquidity shock leads to a 0.1% decrease in the optimal capital ratio. A positive liquidity shock means that the households enjoy holding liquid assets more and, as a result, are willing to accept a lower rate of return on deposits, as captured by the impulse response function of the liquidity premium. For a given capital requirement, an increased

\textsuperscript{29}This is within the range of values documented by Lucas (1987), Alvarez and Jermann (2004), and Van den Heuvel (2008).
liquidity premium translates into lower liquidity costs of lending. At the same time, banks’ incentives to risk-shift become weaker, since the deposit rate is lower and so are the banks’ credit liabilities. The liquidity channel dominates the risk-shifting channel, therefore bank lending increases mildly to satisfy the households’ demand for liquidity.

If I continue to use the GDP and bank credit as the only two indicators to guide the dynamics of capital requirements, the $R^2$ drops to 91% (see Panel A of Table 6). Including the liquidity premium as an additional indicator increases the $R^2$ to 98%. The Ramsey policy rule can now be characterized with the following rule.\(^\text{30}\)

$$4.9\% + 2\times \left( \log \frac{L_t}{Y_t} - \log \frac{L_{ss}}{Y_{ss}} \right) + 9\times (\log Y_t - \log Y_{ss}) - 139\times ((r_{f,t} - r_{d,t}) - (r_{f,ss} - r_{d,ss})).$$

A one percentage point increase in the liquidity premium leads to a 1.4% decrease in capital charges or, equivalently, a one standard deviation increase in the liquidity premium generates a 0.1% drop in capital ratios. The liquidity premium serves as an indicator of how expensive for banks is equity financing, as well as how valuable for investors are liquid assets. In line with the baseline model, when the liquidity premium increases, banks’ lending costs go up, potentially leading to a cut in deposits. This suggests that the capital requirements should be reduced to satisfy the investors’ demand for safe assets.

1.6. Conclusions

Due to government guarantees, banks have incentives to engage in excessive risk-taking and undertake inefficient investments from a social point of view. To prevent such risk-shifting, financial regulators have instituted capital requirements. There is an ongoing debate, however, about whether they are effective, and if so, what level is most efficient. In this paper, I provide a rationale for the use of a time-varying capital requirement, employing a dynamic general equilibrium setting. A sufficiently high capital requirement can be used to rule out excessive lending, but at the same time it may limit banks from financing high-quality

\(^{30}\)The GDP growth variable has been orthogonolized with respect to the credit gap variable.
investments and lead to reduced liquidity provision. In the model, an optimal capital requirement trades off reduced risk-shifting incentives with reduced liquidity provision. Given that banks are most incentivized to risk-shift during periods of economic growth and holding equity is most costly during recessions, when the economy is the least liquid, a procyclical capital regulation emerges endogenously as a solution to the Ramsey problem.

In this paper, I also quantify policy recommendations on which macroeconomic and bank aggregates drive the optimal time variation in capital requirements. In line with the predictions of the baseline model, the optimal Ramsey policy would respond positively to the credit gap and growth in GDP, where the two serve as indicators of a banks’ incentives to risk-shift. At the same time, the Ramsey capital requirement would adjust negatively to an increase in the liquidity premium, which indicates investor’s demand for safe and liquid assets. Two-thirds of the welfare gain generated by implementing the optimal Ramsey policy is achieved by having state-dependent capital ratios, while the remaining improvement is attributed to an optimal fixed capital requirement. This leads me to conclude that implementing the CCyBs, as recommended by the Basel III Accord, is a crucial step in promoting the stability of the financial sector.

The optimal Ramsey policy requires a capital ratio between 4% and 6% and depends on the economic growth, bank supply of credit and asset prices. Specifically, a one percentage point increase in bank lending (GDP-to-lending) translates into 0.1% (0.2%) increase in capital charges, while a one percentage point increase in liquidity premium leads to a 1.3% decrease.
This figure depicts the rate of return on deposits $R_{d,t+1}$ as a function of the deposits supply $D_{t+1}$ and the rate of return on other safe assets $\frac{1}{\beta}$. The scale of y-axis is omitted, as the numbers in this parameterized example do not have economic meaning. The parameter values are set to $\beta = 0.98$, $\eta = 0.8$, $\alpha = 0.85$, $\delta = 1$, $\rho_a = 0.97$, $\sigma_a = 0.02$, and $\sigma_\omega = 0.2$. 
This figure depicts the social marginal cost of bank lending $R_{d,t+1}$ (black dash-dotted line) and the social marginal benefit of lending during recessions $E_{t}^{B}[R_{l,t+1}]$ (red solid line) and during expansions $E_{t}^{G}[R_{l,t+1}]$ (blue solid line) as a function of bank lending $L_{t+1}$. The optimal level of lending is at the intersection of the social marginal costs and benefits. The scale of y-axis is omitted, as the numbers in this parameterized example do not have economic meaning. The parameter values are set to $\beta = 0.98$, $\eta = 0.8$, $\alpha = 0.85$, $\delta = 1$, $\rho_a = 0.97$, $\sigma_a = 0.02$, and $\sigma_\omega = 0.2$. 

Figure 2: Socially Optimal Allocation
Figure 3: Competitive Equilibrium Allocation

Panel A of this figure depicts the social marginal cost of bank lending $R_{d,t+1}$ (black dash-dotted line) and the private marginal cost of lending during recessions $R_{d,t+1} - \xi^B (L_{t+1}, N_{t+1}; a_t)$ (red dashed line) and during expansions $R_{d,t+1} - \xi^G (L_{t+1}, N_{t+1}; a_t)$ (blue dashed line) as a function of bank lending $L_{t+1}$. Panel B additionally depicts the marginal cost of lending $E_t [R_{d,t+1}]$ during recessions (red solid line) and expansions (blue solid line). The optimal level of lending at social optimum is at the intersection of the social marginal costs and benefits. The optimal level of lending in the competitive equilibrium is at the intersection of the private marginal costs and benefits. The social and private marginal benefits are identical. The scale of $y$-axis is omitted, as the numbers in this parameterized example do not have economic meaning. The parameter values are set to $\beta = 0.98$, $\eta = 0.8$, $\alpha = 0.85$, $\delta = 1$, $\rho_a = 0.97$, $\sigma_a = 0.02$, and $\sigma_\omega = 0.2$. 
Figure 4: Effects of Capital Regulations on the Bank’s Lending Costs

Panel A of this figure depicts the risk-shifting costs of bank lending as a function of capital requirements $\zeta_t$ (holding lending and productivity fixed). Panel B depicts the liquidity costs of bank lending (black solid line) and its two components: (i) the deposits rate (blue dashed line) and (ii) the equity wedge (red dash-dotted line) as a function of capital requirements $\zeta_t$. The scale of $y$-axis is omitted, as the numbers in this parameterized example do not have economic meaning. The parameter values are set to $\beta = 0.98$, $\eta = 0.8$, $\alpha = 0.85$, $\delta = 1$, $\rho_a = 0.97$, $\sigma_a = 0.02$, and $\sigma_\omega = 0.2$. 
Panel A in this figure depicts (i) the benefit of capital requirements during recessions (dash-dotted red line) and during expansions (dash-dotted blue line) and (ii) the cost of capital requirements during recessions (solid red line) and during expansions (solid blue line) as a function of $\zeta_t$. Panel B depicts the liquidity capital requirement as a function of the aggregate productivity. The scale of $y$-axis is omitted, as the numbers in this parameterized example do not have economic meaning. The parameter values are set to $\beta = 0.98$, $\eta = 0.8$, $\alpha = 0.85$, $\delta = 1$, $\rho_a = 0.97$, $\sigma_a = 0.02$, and $\sigma_\omega = 0.2$. 
Figure 6: Exogenous TFP Shock.

Benchmark Quantitative Model with Fixed Capital Requirement

The figures show the effect of a one standard deviation shock to log productivity on the variables of the benchmark quantitative model, as captured by the impulse response functions (blue solid lines). Banks are subject to a fixed capital requirement of 7.26%. All model quantities are in logs.
Figure 7: Exogenous TFP Shock.

Benchmark Quantitative Model with Ramsey Capital Requirement

The figures show the effect of a one standard deviation shock to log productivity on the variables of the benchmark quantitative model, as captured by the impulse response functions. The figures provide a comparison of the effects when banks are subject to a fixed capital requirement of 7.26% (blue solid lines) and when banks face the optimal Ramsey policy (red dash-dotted lines). All model quantities are in logs.
Figure 8: Exogenous Liquidity Shock.

Quantitative Model with Liquidity Shocks and Fixed Capital Requirement

The figures show the effect of a one standard deviation shock to log productivity on the variables of the quantitative model with liquidity shocks, as captured by the impulse response functions (blue solid lines). Banks are subject to a fixed capital requirement of 7.26%. All model quantities are in logs.
The figures show the effect of a one standard deviation shock to log productivity on the variables of the quantitative model with liquidity shocks, as captured by the impulse response functions. The figures provide a comparison of the effects when banks are subject to a fixed capital requirement of 7.26% (blue solid lines) and when banks face the optimal Ramsey policy (red dash-dotted lines). All model quantities are in logs.
The Figure depicts the variation compensation, \( \varpi \), as a function of different levels of fixed capital charges (green dots), as well as the welfare compensation corresponding to the Ramsey policy (blue square), the policy solely based on the credit gap (red diamond) and the first-best allocation (black star). The variation compensation is expressed in percentage points.
Table 1: Configuration of Model Parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subjective Discount Factor</td>
<td>( \beta )</td>
<td>0.975</td>
<td>Standard</td>
</tr>
<tr>
<td>Risk Aversion Coefficient</td>
<td>( \gamma )</td>
<td>1.000</td>
<td>Standard</td>
</tr>
<tr>
<td>Elasticity of Deposits and Consumption</td>
<td>( \eta )</td>
<td>1.200</td>
<td>St.dev. of debt-consumption ratio</td>
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<tr>
<td>Deposits Weight</td>
<td>( \chi )</td>
<td>0.010</td>
<td>Average liquidity premium</td>
</tr>
<tr>
<td>Firm Capital Share</td>
<td>( \alpha_f )</td>
<td>0.355</td>
<td>Capital-output ratio</td>
</tr>
<tr>
<td>Firm Operating Cost</td>
<td>( \sigma_f )</td>
<td>0.055</td>
<td>St.dev. of investment-capital ratio</td>
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<tr>
<td>Bank Capital Share</td>
<td>( \alpha_b )</td>
<td>0.780</td>
<td>Capital-output ratio</td>
</tr>
<tr>
<td>Bank Operating Cost</td>
<td>( \sigma_b )</td>
<td>0.065</td>
<td>Profit-to-loan ratio</td>
</tr>
<tr>
<td>Bank Output Weight</td>
<td>( \bar{a}_b )</td>
<td>-1.35</td>
<td>Capital ratio in two sectors</td>
</tr>
<tr>
<td>Capital Adequacy Ratio</td>
<td>( \zeta )</td>
<td>0.073</td>
<td>Average leverage ratio</td>
</tr>
<tr>
<td>Depreciation Rate</td>
<td>( \delta )</td>
<td>0.075</td>
<td>Investment-capital ratio</td>
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<tr>
<td>Persistence of Productivity Schock</td>
<td>( \rho_a )</td>
<td>0.95</td>
<td>Process for Solow residuals</td>
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<tr>
<td>Std of Productivity Schock</td>
<td>( \sigma_a )</td>
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</tr>
<tr>
<td>Std of Idiosyncratic Shock</td>
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<td>Bailout rate</td>
</tr>
<tr>
<td>Dispersion of Idiosyncratic Volatility</td>
<td>( \nu )</td>
<td>0.500</td>
<td>Idiosyncratic volatility dispersion</td>
</tr>
</tbody>
</table>

This table reports the benchmark configuration of the model parameters calibrated at annual frequency.
Table 2: First Aggregate Moments

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>2.5%</td>
<td>97.5%</td>
<td></td>
</tr>
<tr>
<td><strong>Aggregate Sector</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital-Output, $K/Y$</td>
<td>3.03</td>
<td>3.00</td>
<td>2.96</td>
<td>3.04</td>
</tr>
<tr>
<td>Investment-Capital, $I/K$</td>
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<td>0.07</td>
<td>0.07</td>
<td>0.08</td>
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<tr>
<td><strong>Market Fraction</strong></td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>Capital Weight, $K_b/K$</td>
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<td>0.45</td>
<td>0.43</td>
<td>0.47</td>
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<tr>
<td>Output Weight, $Y_b/Y$</td>
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<td>0.28</td>
<td>0.26</td>
<td>0.29</td>
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<tr>
<td><strong>Banking Sector</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital-Output, $K_b/Y_b$</td>
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<td>4.87</td>
<td>4.87</td>
<td>4.88</td>
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<tr>
<td>Investment-Capital, $I_b/K_b$</td>
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<td>0.07</td>
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<tr>
<td>Capital Adequacy Ratio, $N/L$, %</td>
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<td>7.26</td>
<td>7.26</td>
<td>7.26</td>
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<tr>
<td>Profit-Lending, $\pi/L$</td>
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<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
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<tr>
<td>Liquidity Premium, $R_f - R_d$, %</td>
<td>0.57</td>
<td>0.56</td>
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<td>0.59</td>
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<tr>
<td>Bailout Rate, %</td>
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<td>0.77</td>
<td>0.70</td>
<td>0.84</td>
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<tr>
<td><strong>Bank-Independent Sector</strong></td>
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<td></td>
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<tr>
<td>Capital-Output, $K_f/Y_f$</td>
<td>2.29</td>
<td>2.28</td>
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<td>2.29</td>
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<tr>
<td>Investment-Capital, $I_f/K_f$</td>
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<td>0.07</td>
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</table>

This table reports the first moments computed as averages of the data and model series. The summary statistics in the data are computed in the annual sample for the 1980-2008 period. The mean, as well as 2.5% and 97.5% values, capture the model moment distributions across the samples whose size equals the data.
Table 3: Second Aggregate Moments

<table>
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<th>Aggregate Sector</th>
<th>Banking Sector</th>
<th>Bank-Independent Sector</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
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</tr>
<tr>
<td>Consumption, $\sigma(\Delta c)$</td>
<td>1.28</td>
<td>0.88</td>
<td>0.81 0.95</td>
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<tr>
<td>Output, $\sigma(\Delta y)$</td>
<td>2.00</td>
<td>2.04</td>
<td>1.95 2.13</td>
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<tr>
<td>Investment, $\sigma(\Delta i)$</td>
<td>4.36</td>
<td>7.22</td>
<td>6.91 7.55</td>
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<tr>
<td>Output, $\sigma(\Delta y_b)$</td>
<td>2.54</td>
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<td>2.16 2.46</td>
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<td>Investment, $\sigma(\Delta i_b)$</td>
<td>9.28</td>
<td>12.81</td>
<td>12.15 13.48</td>
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<td>Lending, $\sigma(\Delta l)$</td>
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<td>1.65</td>
<td>1.47 1.85</td>
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<td>Debt-Consumption Ratio, $\sigma(\Delta d - \Delta c)$</td>
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<td>Profits, $\sigma(\Delta \pi)$</td>
<td>13.59</td>
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<td>10.11 11.22</td>
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<tr>
<td>Output, $\sigma(\Delta y_f)$</td>
<td>2.07</td>
<td>2.02</td>
<td>1.93 2.11</td>
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<tr>
<td>Investment, $\sigma(\Delta i_f)$</td>
<td>3.84</td>
<td>3.12</td>
<td>2.93 3.32</td>
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<tr>
<td>Liquidity Premium, $\sigma(R_f - R_d)$</td>
<td>0.35</td>
<td>0.06</td>
<td>0.04 0.08</td>
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</table>

This table reports the second moments computed as standard deviations of the data and model series. $\sigma(\Delta x)$ is the standard deviation of annual log growth rates of $X$. The summary statistics in the data are computed in the annual sample for the 1980-2008 period. The mean, as well as 2.5% and 97.5% values, capture the model moment distributions across the samples whose size equals the data. The statistics are expressed in percentages.
Table 4: Business Cycle Correlations

<table>
<thead>
<tr>
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<th>Model</th>
<th>Data</th>
<th>Mean</th>
<th>2.5%</th>
<th>97.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Aggregate Sector</strong></td>
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</tr>
<tr>
<td>Consumption, $\rho(\Delta c, \Delta y)$</td>
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<td>0.88</td>
<td>0.87</td>
<td>0.89</td>
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</tr>
<tr>
<td>Investment, $\rho(\Delta i, \Delta y)$</td>
<td>0.84</td>
<td>0.96</td>
<td>0.95</td>
<td>0.97</td>
<td></td>
</tr>
<tr>
<td><strong>Banking Sector</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output, $\rho(\Delta y_b, \Delta y)$</td>
<td>0.82</td>
<td>0.94</td>
<td>0.93</td>
<td>0.95</td>
<td></td>
</tr>
<tr>
<td>Investment, $\rho(\Delta i_b, \Delta y)$</td>
<td>0.70</td>
<td>0.93</td>
<td>0.92</td>
<td>0.94</td>
<td></td>
</tr>
<tr>
<td>Lending, $\rho(\Delta l, \Delta y)$</td>
<td>0.47</td>
<td>0.68</td>
<td>0.67</td>
<td>0.69</td>
<td></td>
</tr>
<tr>
<td>Deposits, $\rho(\Delta d - \Delta c, \Delta y)$</td>
<td>0.54</td>
<td>0.40</td>
<td>0.36</td>
<td>0.44</td>
<td></td>
</tr>
<tr>
<td>Profits, $\rho(\Delta \pi, \Delta y)$</td>
<td>0.15</td>
<td>0.79</td>
<td>0.77</td>
<td>0.80</td>
<td></td>
</tr>
<tr>
<td>Liquidity Premium, $\rho(R_f - R_d, \Delta y)$</td>
<td>-0.21</td>
<td>-0.04</td>
<td>-0.12</td>
<td>0.05</td>
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</tr>
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<td><strong>Bank-Independent Sector</strong></td>
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</tr>
<tr>
<td>Output, $\rho(\Delta y_f, \Delta y)$</td>
<td>0.96</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td>Investment, $\rho(\Delta i_f, \Delta y)$</td>
<td>0.59</td>
<td>0.93</td>
<td>0.92</td>
<td>0.94</td>
<td></td>
</tr>
</tbody>
</table>

This table reports the correlations of aggregate variables with the output growth in the data and in the model. $\rho(\Delta x, \Delta y)$ is the correlation of annual growth rates of $X$ with annual output growth rates; $\rho(R_f - R_d, \Delta y)$ is the correlation of annual liquidity premium (in levels) with annual output growth rates; $\rho(\Delta \pi, \Delta y)$ is the correlation of fraction of bailed out banks with the annual output growth rates. The summary statistics in the data are computed in the annual sample for the 1980-2008 period. The mean, as well as 2.5% and 97.5% values, capture the model moment distributions across the samples whose size equals the data.
Table 5: Optimal Bank Capital Policy (Benchmark Quantitative Model)

Panel A: Level-Log Regressions

<table>
<thead>
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<td>4.93</td>
<td>4.93</td>
<td>4.93</td>
<td>4.93</td>
</tr>
<tr>
<td>Credit gap</td>
<td>2.64</td>
<td>2.64</td>
<td>1.91</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(1.60, 3.48)</td>
<td>(1.60, 3.48)</td>
<td>(0.75, 2.93)</td>
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</tr>
<tr>
<td>Bank credit</td>
<td>2.51</td>
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<td></td>
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<tr>
<td></td>
<td>(2.45, 2.57)</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>GDP</td>
<td>5.45</td>
<td>8.17</td>
<td>7.91</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>(5.24, 5.72)</td>
<td>(8.16, 8.18)</td>
<td>(7.84, 7.99)</td>
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</tr>
<tr>
<td>Liquidity premium</td>
<td>-172.77</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>(-224.16, -117.29)</td>
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</tr>
<tr>
<td>$R^2$</td>
<td>0.1366</td>
<td>0.5033</td>
<td>0.8175</td>
<td>0.9998</td>
<td>0.9999</td>
</tr>
<tr>
<td></td>
<td>(0.0431, 0.2465)</td>
<td>(0.4062, 0.5927)</td>
<td>(0.7877, 0.8469)</td>
<td>(0.9997, 0.9998)</td>
<td>(0.9998, 0.9999)</td>
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Panel B: Normalized Level-Log Regressions

<table>
<thead>
<tr>
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<td>4.93</td>
<td>4.93</td>
<td>4.93</td>
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<tr>
<td>Credit gap</td>
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<td>0.09</td>
<td>0.09</td>
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<tr>
<td></td>
<td>(0.05, 0.11)</td>
<td>(0.05, 0.11)</td>
<td>(0.02, 0.09)</td>
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<tr>
<td>Bank credit</td>
<td>0.08</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.07, 0.09)</td>
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<td></td>
</tr>
<tr>
<td>GDP</td>
<td>0.16</td>
<td>0.69</td>
<td>0.67</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>(0.15, 0.18)</td>
<td>(0.63, 0.76)</td>
<td>(0.61, 0.74)</td>
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</tr>
<tr>
<td>Liquidity premium</td>
<td>-0.01</td>
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<td></td>
<td>(-0.02, -0.01)</td>
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</tr>
<tr>
<td>$R^2$</td>
<td>0.1366</td>
<td>0.5033</td>
<td>0.8175</td>
<td>0.9998</td>
<td>0.9999</td>
</tr>
<tr>
<td></td>
<td>(0.0431, 0.2465)</td>
<td>(0.4062, 0.5927)</td>
<td>(0.7877, 0.8469)</td>
<td>(0.9997, 0.9998)</td>
<td>(0.9998, 0.9999)</td>
</tr>
</tbody>
</table>

Panel A of this table presents results from level-log regressions of the form $\zeta^*_t = \zeta_{ss} + \sum_{j=1}^N \zeta_j x^*_j + \varepsilon_t$, while Panel B presents results from normalized level-log regressions, where independent variables are scaled by their standard deviations. The dependent variable is the Ramsey policy rule in year $t$ expressed in percentages (1 to 100). The independent variables include (i) credit gap $\log \frac{L_t}{Y_t} - \log \frac{L_{ss}}{Y_{ss}}$; (ii) bank credit $\log \frac{L_t}{L_{ss}}$; (iii) GDP $\log Y_t - \log Y_{ss}$; (iv) liquidity premium $(r_{f,t} - r_{d,t}) - (r_{f,ss} - r_{d,ss})$. Reported coefficients are the mean values across the samples, while the numbers reported in parentheses are the 2.5% and 97.5% values. Reported $R^2$s are the mean values across the samples, while the numbers reported in parentheses are the 2.5% and 97.5% values.
Table 6: Optimal Bank Capital Policy (Quantitative Model with Liquidity Shocks)

Panel A: Level-Log Regressions

<table>
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<tr>
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<td>4.93</td>
<td>4.93</td>
<td>4.93</td>
<td>4.92</td>
</tr>
<tr>
<td>Credit gap</td>
<td>1.71</td>
<td>1.71</td>
<td>(0.41, 2.96)</td>
<td>1.67</td>
<td>(0.61, 2.58)</td>
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<tr>
<td>Bank credit</td>
<td></td>
<td>2.20</td>
<td>(1.74, 2.64)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td></td>
<td>5.26</td>
<td>8.57</td>
<td>7.87</td>
<td>(4.62, 5.93)</td>
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<tr>
<td>Liquidity premium</td>
<td></td>
<td>-139.24</td>
<td>(-145.78, -132.49)</td>
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<td></td>
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<tr>
<td>R²</td>
<td>0.0584</td>
<td>0.3435</td>
<td>0.6577</td>
<td>0.9107</td>
<td>0.9766</td>
</tr>
<tr>
<td></td>
<td>(0.0024, 0.1551)</td>
<td>(0.2161, 0.4717)</td>
<td>(0.5562, 0.7417)</td>
<td>(0.8820, 0.9330)</td>
<td>(0.9702, 0.9820)</td>
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</tbody>
</table>

Panel B: Normalized Level-Log Regressions

<table>
<thead>
<tr>
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<th>(2)</th>
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<tr>
<td>Constant</td>
<td>4.93</td>
<td>4.93</td>
<td>4.93</td>
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<td>4.92</td>
</tr>
<tr>
<td>Credit gap</td>
<td>0.06</td>
<td>0.06</td>
<td>(0.01, 0.10)</td>
<td>0.06</td>
<td>(0.02, 0.09)</td>
</tr>
<tr>
<td>Bank credit</td>
<td>0.07</td>
<td></td>
<td>(0.06, 0.09)</td>
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</tr>
<tr>
<td>GDP</td>
<td>0.16</td>
<td>0.74</td>
<td>0.68</td>
<td></td>
<td>(-0.09, -0.08)</td>
</tr>
<tr>
<td>Liquidity premium</td>
<td></td>
<td></td>
<td></td>
<td>-0.99</td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.0584</td>
<td>0.3435</td>
<td>0.6577</td>
<td>0.9107</td>
<td>0.9766</td>
</tr>
<tr>
<td></td>
<td>(0.0024, 0.1551)</td>
<td>(0.2161, 0.4717)</td>
<td>(0.5562, 0.7417)</td>
<td>(0.8820, 0.9330)</td>
<td>(0.9702, 0.9820)</td>
</tr>
</tbody>
</table>

Panel A of this table presents results from level-log regressions of the form $\zeta_t = \zeta_{ss} + \sum_{j=1}^N \zeta_j x_j^t + \varepsilon_t$, while Panel B presents results from normalized level-log regressions, where independent variables are scaled by their standard deviations. The dependent variable is the Ramsey policy rule in year $t$ expressed in percentages (1 to 100). The independent variables include (i) credit gap $\log Y_t - \log Y_{ss}$; (ii) bank credit $\log L_t - \log L_{ss}$; (iii) GDP $\log Y_t - \log Y_{ss}$; (iv) liquidity premium $(r_{f,t} - r_{d,t}) - (r_{f,ss} - r_{d,ss})$. Reported coefficients are the mean values across the samples, while the numbers reported in parentheses are the 2.5% and 97.5% values. Reported $R^2$s are the mean values across the samples, while the numbers reported in parentheses are the 2.5% and 97.5% values.
2.1. Introduction

How risky is the capital in the U.S. corporate sector? To answer this question, the finance literature typically considers the returns and the payouts per one equity share of the aggregate stock market.\(^1\) As such, these measures ignore the proceeds from changes in the total number of asset shares (i.e., issuances and repurchases), as well as the contributions from the debt of corporations. In contrast, in our paper we focus on the aggregate investment strategy which is the claim to the entire supply of corporate capital. The aggregate strategy is appropriate for the macroeconomic and macro-finance research which features aggregate transfers of resources to and from the corporate sector at the economy-wide level. The aggregate, rather than per share, payouts and valuations are the relevant measures to assess the nature and magnitude of risks in the corporate sector, and to evaluate the risk and return implications of such models.

To measure aggregate payouts and valuations, we use the market data on prices, shares, and distributions associated with equity, debt, and total asset side of the corporations. While equity measurements are standard, we bring a novel source of corporate debt data from Barclay indices to uncover the market value of U.S. corporate bonds and their corresponding payouts to investors. The aggregate payouts include both cash (dividend and interest payments) and non-cash (share issuances and repurchases) distributions associated with debt and equity of the U.S. corporate sector. The aggregate valuations represent the total market capitalizations of equity and debt of the corporate sector.\(^2\)

We show novel empirical evidence that risk properties of aggregate payouts are quite different from those of typical per share equity dividends. First, accounting for net issuances,

\(^1\)This is a standard approach in the macro-finance literature, from the business cycle risk models of Mehra and Prescott (1985) and Campbell and Cochrane (1999) to the long-run risks of Bansal and Yaron (2004) or rare disaster of Rietz (1988) and Barro (2006).

\(^2\)The aggregate value of a corporation is commonly called the "Enterprise Value". Hence, we are examining the enterprise value of the entire US corporate sector.
total payouts are often negative, meaning that there are periods when the corporate sector receives funds from investors rather than paying them out. Indeed, in our 1975-2014 sample total payouts go below zero about 40% of the time for bonds, and 30% of the time for equity and total assets. Second, net issuances are highly volatile, and are a dominant component of the total payouts. They significantly raise the volatility of the total payouts relative to smooth cash distributions. Third, while cash payouts are strongly pro-cyclical, total payouts are generally acyclical, both at short and long horizons. For example, the correlations of consumption growth with changes in asset cash payouts increase from 20% at a 1 quarter horizon to 40% at a 5-year horizon. On the other hand, the correlations of consumption growth with changes in total asset payouts are nearly zero at all the considered horizons. Intuitively, both aggregate issuances and repurchases tend to increase during expansion periods, leading to acyclical net issuances and thus total payouts. Our market value of debt reveals that much of these adjustments take place along the debt side of the corporation.

The evidence for the acyclical nature of total payouts is especially puzzling given that asset returns are predominantly equity-like. The asset return averages 6.4%, comparable to 7.8% for the equity, and the correlation between the two is in excess of 99%. The asset returns are also considerably exposed to movements in economic growth, especially at long horizons. Taken together, the payout and return evidence challenges standard notions of risk and return in the finance literature. We develop a model that helps to explain the above features of the asset market data while accounting for the dynamics of total payouts.

The evidence of acyclical total payouts raises an important question about the economic nature and sources of risk in financial markets, i.e., what risks are being compensated? In addition, negative payouts provide a methodologically challenging aspect for standard models as well as data characterization. Indeed, it is no longer feasible to work with log growth rates because total payouts are often zero or negative. We develop a long-run risk type model that helps explain the above features of the asset market data while accounting for the dynamics of
total payouts and specifically, for the possibility of negative payouts. In the model payouts are indeed not procyclical, yet their exposure to long run growth risk generates a sizeable premium over the risk free rate. This mechanism, which underlies many long run risk model calibrations, emphasizes that it is not the business cycle risk that drives the unconditional risk premium, but a risk of a longer duration. In this regard the model highlights the tension between matching the acyclical dynamics, higher volatility, and lower persistence of total payouts relative to standard dividend cashflows and the large asset premium.

**Related Literature.** Our focus on broader notions of cashflows which account for repurchases and issuances is related to several strands of the literature. Fama and French (2001) and Grullon and Michaely (2002) are among the early papers that highlight the changing nature of firms' payouts. Dittmar and Dittmar (2004), Guay and Harford (2000), and Jagannathan et al. (2000) discuss the role of repurchases as the preferred form of distributing the transitory component of earnings, as dividend policy requires financial commitment. Bansal et al. (2005) incorporate repurchases to their alternative measure of dividends to measure cash flow risk. Boudoukh et al. (2007) find that total equity payouts, which include repurchases and issuances, provide stronger evidence of return predictability than cash dividends alone. Closest to our work are Larrain and Yogo (2008) and Bansal and Yaron (2007). Larrain and Yogo (2008) analyze, using standard VAR return decomposition, the connection between total payouts and asset price fluctuations. Importantly, their measures of payouts are based on book, rather than market values of debt, as in our work. In addition, our main focus is on understanding the cyclical and the exposure of the payouts to economic risks. Bansal and Yaron (2007) focus on total payouts in the equity market, and provide strong evidence for equity return and equity payout growth predictability.

Our findings are consistent with the evidence in Julliard and Parker (2005), Bansal et al. (2005), Hansen et al. (2008) and the basic premise of the long-run risks model of Bansal and Yaron (2004), which identifies low-frequency movements in economic growth as a key source of risk in financial markets. While these studies focus on equity markets, we show
the relevance of these growth risk channels in bond and total asset markets. In terms of related work on corporate bond returns, Chen (2010) and Bhamra et al. (2010) show the importance of low-frequency economic growth risks for the choice of the capital structure, dynamics of the leverage, and the riskiness of corporate bonds. Ferson et al. (2013) show the role of growth risks in the cross-section of equity and corporate bond returns.

Our empirical findings are also important for interpreting the expanding literature of production based asset pricing; see Jermann (1998), Lochstoer and Kaltenbrunner (2010), Croce (2014), Kung and Schmid (2014), among many others. In that literature dividend dynamics are often counter-cyclical as large TFP improvements are associated with the desire to invest and not pay dividends. In these models the notion of dividends is an encompassing one and thus more closely related to our measure of total payouts. Our analysis suggests that the mapping from model-based dividend to ones in the data should perhaps be based on total payouts. In that case, the production based models implications for dividends would accord better with the data, and the asset pricing would still be relevant with regard to the level of the observed risk premia.

The remainder of the paper continues as follows: Section 2.2 provides the data analysis. In Section 2.3 we consider an economic model, show how to address negative payouts, calibrate the model and provide quantitative results. Section 2.4 provides concluding remarks.

2.2. Empirical Analysis

2.2.1. Payouts and Valuations

In this section we lay out key relationships between valuations and payouts which underlie our empirical analysis. Unlike the majority of the literature which considers a per share investment strategy in equity, our main focus is on the aggregate strategy which is the claim to the entire supply of corporate capital. The payoff on this aggregate strategy includes standard cash distributions in the form of dividends and interest payments, and, importantly, non-cash distributions, such as share issuances and repurchases associated with
the equity and debt sides of the corporate balance sheet.\footnote{Larrain and Yogo (2008), Boudoukh et al. (2007) and Bansal and Yaron (2007) also consider broader notions of payouts which incorporate share issuances and repurchases.}

define the key return and payout variables which pertain to cash, issuances, repurchases, and total distributions. As in Bansal and Yaron (2007), we first analyze the return to two trading strategies: the hold-one-share strategy, where an investor holds one share forever, and the aggregate holding strategy, where a representative agent holds the entire supply of capital. These trading strategies have identical returns, but have different measured payouts to investors.

Consider a standard gross return on holding one share of an asset between period $t$ and $t + 1$, $R_{t+1}$:

$$R_{t+1} = \frac{P_{t+1} + CF_{t+1}}{P_t},$$

(2.1)

where $P_t$ is the price per share, and $CF_t$ is the cash payout per share of an asset. Cash payout refers to cash dividends or coupon/interest payments for equity or bond, respectively.

Multiplying the numerator and denominator by the number of outstanding shares $N_t$, we can rewrite the gross return equation in the following way:

$$R_{t+1} = \frac{V_{t+1} + N_t \cdot CF_{t+1} - (N_{t+1} - N_t) \cdot P_{t+1}}{V_t},$$

(2.2)

where $V_t = N_t \times P_t$ is the total market capitalization. The right-hand side of the above equation can be interpreted as the return to the aggregate investment strategy. Its value is given by the total market capitalization $V_t$, and its payoff corresponds to the total resource distribution $D_{a,t+1}$ in the form of aggregate cash distributions and share issuances and repurchases:

$$D_{a,t+1} = D_{t+1} - NI_{t+1},$$

(2.3)

where $D_{t+1} = N_t \cdot CF_{t+1}$ is the aggregate cash payout, and $NI_{t+1} = ISS_{t+1} - REP_{t+1}$ is the aggregate net issuance. The issuances $ISS_{t+1}$ and repurchases $REP_{t+1}$ capture the
transfer of resources in and out of the firm, respectively. The outflow at date $t + 1$ is given by,

\[ REP_{t+1} \equiv - \{ N_{t+1} - N_t \}^- P_{t+1} \geq 0. \]  \hspace{1cm} (2.4)

It is positive when there is a repurchase of the existing shares, i.e. when $N_{t+1} - N_t \leq 0$. Similarly, define issuances as:

\[ ISS_{t+1} \equiv \{ N_{t+1} - N_t \}^+ P_{t+1} \geq 0. \]  \hspace{1cm} (2.5)

This represents the inflow of resources into the corporation following a new issuance of shares when $N_{t+1}$ is greater than $N_t$. As can be seen from the above equations, total payout to the aggregate strategy is directly affected by net issuances, above and beyond standard cash distributions. Share repurchases increase total aggregate payout, while share issuances reduce it.

Notably, equations 2.1 and 2.2 define very different investment strategies: the former is a per share strategy typical for an individual investor, while the latter is an investment into the aggregate supply of firm’s capital. These strategies have very different valuations and payouts, even though by construction they have identical returns (see also Bansal and Yaron (2007) and Larrain and Yogo (2008)). They also differ in applicability: the aggregate strategy is particularly relevant in the context of macroeconomic and macrofinance literature which features transfers of resources between the representative firm and the representative agent at the economy-wide level. The aggregate, rather than per share, payouts and valuations should then be used to assess the nature and magnitude of risks in the corporate sector, and to evaluate the risk and return implications of such models. In subsequent sections we describe our approach to measure aggregate payouts and valuations associated with debt, equity, and asset side of the U.S. corporate sector, and highlight their economic implications relative to the standard measurements.

\[ ^5 \text{See also } ? \text{ and } ? \text{ for a related discussion of these strategies.} \]
The total payout, $D_{a,t}$ includes both outflows from firms to investors in the form of cash payouts and repurchases, and inflows into the corporate sector in the form of issuances. A present value representation of returns implies that

$$V_t = \sum_{j=1}^{\infty} \exp \left\{ - \sum_{k=1}^{j} \log R_{t+k} \right\} D_{a,t+j},$$

(2.6)

with

$$R_{t+1} = \frac{V_{t+1} + D_{a,t+1}}{V_t}.$$ 

(2.7)

That is, the current market capitalization reflects the discounted sum of a net transfer of aggregate resources from all the firms.

In this paper, we measure total payouts to equity, debt, and total assets of the firm. Total assets refers to the sum of the two components of the corporate sector - equity and debt. Debt, in its turn, is the sum of the long-term and short-term debt.

2.2.2. Data and Empirical Measurements

We use market data on prices, shares, and distributions to measure market valuations and payouts. The latter includes both cash (dividend and interest payments) and non-cash (share issuances and repurchases) distributions associated with debt, equity, and asset side of the U.S. corporate balance sheet. For accurate and relevant measurements, we emphasize the importance of using the market price data, whenever possible. This is especially pertinent to debt-related variables. The market data for bond valuations and distributions are not as widely available as for equities, so the majority of studies in the literature have resorted to book rather than market valuations, which can affect empirical measurements.6

6It has been common either to use book values to capture debt valuations or approximate market values by imputing the maturity distribution of long-term debt as pioneered by Brainard and Shoven (1980). See also Bernanke and Campbell (1988), Hall et al. (1988), Richardson and Sloan (2003), and Larrain and Yogo (2008).

To measure equity-related variables, we use the Center for Research in Security Prices (CRSP) Monthly Stock File. The dataset provides equity price per share (prc) and share

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data \((\text{shrout})\) at an individual security level, as well as holding period returns including and excluding dividends, \(\text{ret}\) and \(\text{retx}\), respectively. We include only common stock listed on NYSE, AMEX, NASDAQ, and NYSE Arca stock exchanges.\(^7\) Similar to Boudoukh et al. (2007) and Larraín and Yogo (2008), we measure individual stock \(i\) net issuance in month \(t\) as a change in shares outstanding valued at the month-end share price:\(^8,9\)

\[
n_{it} = prc_{it} \times \text{shrout}_{it} - prc_{it-1} \times \text{shrout}_{it-1} \times (1 + \text{retx}_{it}).
\]

Different from Boudoukh et al. (2007) but similar to Larraín and Yogo (2008), we also account for changes in the entity structure due to initial public offerings, mergers, acquisitions, and exchanges, which is necessary to fully capture total transfers of resources in and out of the corporations. Specifically, we use firm’s market capitalization on the first trading month, \(\text{shrout} \times \text{prc}\), to measure net issuances during the IPO. We use CRSP delisting data to identify securities with delisting codes of 2xx and 3xx, and use their delisting price \((\text{dlprc})\) and the delisting return \((\text{dlretx})\) to account for the repurchases during mergers and acquisitions. We aggregate the firm-level data and compute market valuations, dividends, returns, and net issuances at the total market level.\(^10\)

We face several challenges associated with measuring the payouts and valuations of the debt side of the corporate balance sheet. U.S. corporations issue a wide variety of debt instruments, and most of the trade takes place at the over-the-counter (OTC) dealer’s

\(^7\)We have checked that including preferred stocks and/or excluding government-sponsored enterprises (GSEs) does not have a material effect on our results.

\(^8\)This is equivalent to measuring net issuances as the value of the change in the number of shares over the period, appropriately adjusted by the cumulative adjustment share and price factors \(\text{cfacshr}\) and \(\text{cfacpr}\) that account for splits and other corporate events:

\[
n_{it} = (\text{shrout}_{it} \times \text{cfacshr}_{it} - \text{shrout}_{it-1} \times \text{cfacshr}_{it-1}) \times \frac{\text{prc}_{it}}{\text{cfacpr}_{it}}.
\]

\(^9\)Valuing net issuances at the average of beginning-of-month and end-of-month prices, as in Boudoukh et al. (2007), instead of the month-end prices as in Larraín and Yogo (2008) does not impact our results.

\(^10\)Bansal and Yaron (2007) and Welch and Goyal (2008) measure aggregate net issuances directly in the market index data as \(\text{MCAP}_t - \text{MCAP}_{t-1} \times (1 + \text{VWRETX}_t)\), where \(\text{MCAP}\) is the market capitalization and \(\text{VWRETX}\) is the value-weighted return excluding distributions. The index and firm-based approaches treat differently firm exits for reasons other than mergers, acquisitions and exchanges; e.g., defaults and bankruptcies would show up as negative net issuances using the index data, but not in our approach using the firm level data. Empirically, however, the two measures are quite similar.
market. As such, there is no convenient centralized platform to obtain market valuations for firm’s debt. Further, there is a double-counting concern: corporate loans made by financial institutions (banks), which are financed by the debt and equity of the banks themselves, should be excluded from the analysis.

To tackle these issues, we bring novel bond market value data from Barclays Indices. The Barclays Indices are widely used throughout the financial industry because of their accuracy and wide range of market coverage. Reported market capitalizations and month-to-date index returns are updated on daily basis, and our data is taken on the last trading day of the month when bond prices are hand marked by traders. Reported total returns are decomposed into coupon returns and price returns which facilitates calculation of monthly coupon cash flows and net issuances.\(^{11}\)

Barclays Indices represent many types of debt instruments, varying from debentures and asset-backed bonds to commercial paper issues, and our goal is to measure all of the outstanding corporate debt. To capture long duration debt, we include the following sub-indices of the Barclays U.S. Universal Index: Corporate Investment Grade (IG), Corporate High Yield (HY), 144A Ex Aggregate, Commercial Mortgage-Backed Securities (CMBS) and Fixed Rate Asset-Backed Securities (ABS). All of the bonds represented in the above sub-indices have fixed-rate coupon, are fully taxable, include both senior and subordinate debt, and must have at least one year-to-final maturity.\(^{12}\) Additional details for the characteristics of the bonds are given in the Appendix D, Table D.1. We further augment our debt measure with corporate issues of taxable municipal bonds, in particular Industrial Development Revenue Bonds (IDR), Pollution Control Revenue Bonds (PCR), and U.S. Convertibles Composite Index, since those are outside of the Universal Index.

To measure debt of short duration, we include the following Barclays sub-indices: Asset-

\(^{11}\)Unlike for equities, we do not have access to individual bond data, so the payout and valuation computations are done at the index level.

\(^{12}\)The Universal Index excludes bonds that has less than 1 year to maturity as they become money market eligible. ABS and CMBS must have a remaining average life of at least one year, while bonds that convert from fixed to floating rate will exit the sub-indices one year prior to conversion.
Backed Securities Floating Rate (ABS FRN), Floating Rate Notes (FRN), Floating Rate Notes High Yield (FRN HY) and Short-Term Corporate Index (STI). The floating-rate securities in the above sub-indices may have longer maturity, but their interest rate durations are typically less than 1 year. More details are given in the Appendix D (Table D.1). We further augment our measure with short-term debt instruments, such as commercial paper and certificates of deposits, using Compustat/CRSP Merged Database. Specifically, "Debt in Current Liabilities" (item 34, dlc) serves as a good proxy for the market value of outstanding short-term debt. Short duration notes payable tend to be quite insensitive to changes in interest rates, hence, book values provide reasonable assessments of their valuations. As in Richardson and Sloan (2003), we employ income statements and measure net issuance directly as "Current Debt Changes" (item 301, dlcch). To construct coupon cash flows, we use one month repo rate, which is obtained from Bloomberg.

We face two issues of double counting: banks and insurance companies. Banks make direct non-marketable loans to corporations. Buying all of the banking sectors equity and debt includes the rights to the cash flow from the direct loans. To avoid double counting we exclude the value of bank borrowing from the market liabilities of the non-financial corporate sector. The issue of insurance companies is more difficult because they invest heavily in marketable corporate debt. We have not been able to find time-series data on the market value of insurance company corporate debt holdings. Instead, we assume that corporate bond holdings of insurance companies can be partially offset by their bond issuance. As such, we exclude bonds issued by insurance companies within the major Barclays indices, in particular, IG, HY, Convertibles and FRN. The remaining Barclays indices are not disaggregated by the corporate sector and, hence, bonds issued by insurance companies can not be excluded. However, as of December 2014 IG, HY, Convertibles and FRN indices constituted around 70% of the market value of all Barclays indices included in our measure.

13 An alternative approach is to employ the balance sheet data and measure net issuance as the change in "Debt in Current Liabilities" (item 34, dlc). As discussed in Richardson and Sloan (2003), this method suffers a number of limitations. Specifically, debt can be added to the balance sheet through mergers and acquisitions rather than through the issuance of new debt. Also, open market repurchases of bonds can involve cash payments that differ from the carrying value of the debt.
of debt market and as such the measurement error should be minimal.\footnote{Other liabilities of insurance companies, such as policy liabilities, do not represent ownership claims, and thus do not need to be included in our asset and payouts measures. We also tried excluding the insurance companies all together by removing their equity, which did not have any impact on our key results.}

In addition to the asset prices, we also use aggregate macroeconomic data. We collect the data on GDP and consumption, defined as the sum of expenditures on non-durable goods and services, from the BEA tables. The data on CPI inflation come from the Bureau of Labor Statistics. The price level is normalized to 1 in December of 2009. All the nominal quantities are deflated by the CPI to obtain real measures.

Our benchmark sample covers the period from 1975 until 2014, due to the availability of the bond data from Barclays. For some of our supplemental analysis we also use equity only data that go back to the 1930s.

2.2.3. Empirical Evidence

Market Prices and Returns

We start our empirical analysis by describing the key properties of market capitalizations and returns to equity, debt, and assets of the U.S. corporate sector.

Figure 11 shows the evolution of the components of corporate debt. As can be seen on the top panel, investment grade bonds made up the entirety of our measure of long-term debt in the beginning of the sample. The role of other debt instruments, especially high-yield bonds and 144A issues, has significantly increased over time and helped fuel growth in the corporate debt market. By the end of the sample, the real market value of the long-term corporate debt has reached 7.09 trillion (in December 2009 dollars), and more than half of it is comprised of non-IG bonds. By the end of the sample, we estimate that the real market value of the long-term corporate debt has reached 6.5 trillion (in December 2009 dollars), and more than half of it is comprised of non-IG bonds.

The bottom panel of Figure 11 shows that short-term non-bank corporate debt is nearly
entirely made up of short-term loans (STL), with Floating rate notes (FRN) and asset-backed securities (ABS) entering in early 2000s into the dataset. Unlike the long-term corporate debt whose value has been generally growing over time, the market value of short-term debt increased from 0.29 trillion in 1975 to its maximum of 8.72 trillion in 2007, then precipitously fell during the Financial Crisis. It remains at its mid-1990s value of about 3.87 trillion at the end of 2014.\textsuperscript{15}

Figure 12 shows the dynamics of the market capitalization of aggregate debt, equity, and assets, where the latter corresponds to the sum of the first two. Aggregate equity is on average twice as large as debt, which leads to a typical estimate of debt-to-equity ratio of 1/2. Asset and equity values tend to be more volatile and more procyclical than debt, and also experience a larger growth over time. Our estimates suggest that the real market value of U.S. corporate assets grew from 3.79 trillion in mid-1975 to nearly 37.60 trillion at the end of 2014, which is comprised of 2.89 to 26.63 trillion increase for equity and 0.79 to 10.97 for bonds.

To assess the riskiness of aggregate equity, debt, and assets, we provide basic summary statistics for the returns on these claims. Table 7 shows that in our sample, the average real equity return is 7.8\%, with a standard deviation of 16.3\%. The debt return is smaller on average, and is much less volatile: its mean is 2.9\%, and its standard deviation is 5.4\%.

The asset return is the weighted average of the two, with the weight tilted more to equities which represent a larger fraction of the asset value. As shown in Table 7, the average asset return is 6.4\%, and its volatility is 12\%. Further, the asset and equity returns are nearly perfectly correlated. Hence, the aggregate assets of the U.S. corporate sector are quite risky, and the magnitude and nature of risk is comparable to that of equities.

\textsuperscript{15}This is consistent with the evidence in Kacperczyk and Schnabl (2010) who document a significant decline in the commercial paper during the Financial Crisis. They suggest substitution to other sources of financing, adverse selection and the inability of issuers to issue the commercial paper, and institutional constraints as potential reasons for the collapse.
Payouts

We next consider the empirical evidence for the payouts to debt, equity, and assets of U.S. corporations. Following the discussion in Section 2.2.1, for each of these components the aggregate payouts can be broken down into cash (total dividends or interest payments) and non-cash (net share issuance) distributions.

Figure 13 shows the time series of the total payouts and their cash and net issuance components. Naturally, cash payouts are positive. On the other hand, net issuances are not restricted to be of any sign. At times when firms opt for a net distribution of resources through repurchases, net issuances are negative, and they become positive when firms attract capital through new share issuances. In our sample, net issuances of equity, debt, and corporate assets are mostly positive: the firms generally draw resources from the investors. Figure 13 also shows that net issuances are much more volatile than cash distributions, and thus are a dominant component of total payouts. Compared to cash distributions, total corporate payouts are very volatile, and can actually be negative: in our sample, they go below zero about 40% of the time for bonds, and 30% of the time for equity and total assets.

To analyze formally statistical properties of the aggregate payouts, we need to convert them into stationary variables: in levels, payouts are a random walk. However, the standard method of using logarithms to define growth rates does not work in our setting because payouts can be less than or equal to zero. Instead, we rely on alternative measures of growth in which we normalize the level change by the consumption level $C_t$, e.g. $\frac{\Delta D_{a,t}}{C_t} = \frac{D_{a,t} - D_{a,t-1}}{C_t}$, or by the aggregate output $Y_t$, e.g. $\frac{\Delta D_{a,t}}{Y_t} = \frac{D_{a,t} - D_{a,t-1}}{Y_t}$. At the same time, we continue to use the standard log growth rate to measure consumption and output growth,

\footnote{We have also examined alternative normalizations by a previous period consumption level, $\frac{\Delta D_{a,t}}{C_{t-1}}$, by the average consumption level in the current and previous period, $\frac{\Delta D_{a,t}}{\frac{1}{2}(C_{t-1} + C_{t})}$, by the constant non-linear trend of consumption, $\tilde{\Delta} D_{a,t} \equiv \frac{\Delta D_{a,t}}{\exp(g)}$ with $g = \frac{1}{T} \sum_{t=1}^{T} \Delta c_t$, and by excluding the scaling all together. The results are very similar to our benchmark. Further, in Section 2.2.4 we show that using normalized changes versus log growth rates does not make any difference for our key correlation results for positive cash distributions.}
\[ \Delta c_t \equiv \log \left( \frac{C_t}{C_{t-1}} \right) \] and \[ \Delta y_t \equiv \log \left( \frac{Y_t}{Y_{t-1}} \right), \] respectively. One of the advantages of our growth measure is that it is additive. For example, because the level of asset payouts is the sum of equity and debt, our measure of growth rate in asset payouts is also equal to a simple sum of growth rates in equity and debt payouts.

Figure 14 shows the time-series of growth in cash payouts \( \Delta D \), net issuance \( \Delta NI \), and the total payouts \( \frac{D_a}{C} \) for equity, debt and total assets. Table 8 documents the key summary statistics for these variables. The presented evidence highlights the significance of our novel payout components related to firms’ debt and net issuances, which are missed by the typical measures of per-share cash payouts on equity.

First, one can see that debt payouts contribute a significant fraction to the fluctuations in asset payouts. For the cash component, the growth in debt payouts is twice as volatile as the growth in equity cash payouts, so that the asset cash payout growth mostly reflect variations in debt payments: the correlation between the growth rates in debt and total asset cash payouts is 93%. The volatilities of growth rates in equity and debt net issuances are more comparable (3.0 and 3.8, respectively), so the fluctuations in asset net issuances are more evenly split between equity and debt. The growth rates in total payout on assets have a 60% correlation with growth in total payouts on debt, and 35% with total payouts on equity.

Second, it is evident from Figure 14 and Table 8 that growth rates in net issuances are much more volatile than growth rates in the corresponding cash payouts. For example, the volatility of net equity issuance growth is 10 times larger than the volatility of the equity cash dividends, and for debt, the volatility of changes in net issuances is 6 times larger than that of bond cash payouts. This implies that total payouts, which are equal to the difference between cash and net issuances, are very volatile, and are driven predominantly by shocks to net issuances. Indeed, the volatilities of the changes in total payouts are very similar to those of net issuances, and are several times larger than the volatilities of the corresponding cash flows.
Overall, the evidence suggests that our total asset payouts have very different properties relative to typical measures of corporate distributions, such as equity dividends. It is also quite distinct from other, earnings-based measures popular in the literature. Earnings capture profits generated by the firm during the period, and aggregate earnings are often used to measure total performance of the corporate sector. Aggregate earnings, however, are conceptually distinct from the asset payouts. They are an accounting, rather than an economic measure based on actual distributions. They contain retained earnings, which represent the capital not paid out to the investors. Most importantly, earnings do not incorporate asset repurchases and issuances, which we showed are the dominant part of the asset payouts. Because of that, aggregate earnings are more aligned with asset cash distributions, rather than asset total payouts. Indeed, in our sample, growth rate in aggregate earnings (EBIT) has a 70% correlation with changes in asset cash payouts, while its correlation with changes in total asset payouts is actually negative, -20%.

Economic Growth Risk Exposure

Our earlier analysis shows that total asset payouts are very volatile relative to equity payouts, and asset returns are about as risky as equity returns. We next assess the economic nature of risk in asset returns and payouts through their exposure to high and low frequency variations in macroeconomic growth. The exposure to fluctuations in economic growth is one of the main tenets of the macro-finance research, from the business cycle risk models of Mehra and Prescott (1985) and Campbell and Cochrane (1999) to the long-run risks approach of Bansal and Yaron (2004). Early power utility models of Mehra-Prescott to modern theories based on habit formation (CC) and long-run risks (BY). It is a natural starting point for the analysis of the economic nature of risk in the financial markets.

First, we consider the business-cycle behavior of the payouts. Table 9 shows contemporaneous correlations of the growth rates in payouts with consumption or output growth. We show the results for the benchmark annual sample from 1975 to 2014, for the 1975 to 2006
sample which excludes the Financial Crisis, and at a quarterly frequency. Notably, cash, net issuances, and asset payouts have very different business cycle properties. Cash payouts tend to be procyclical: in the benchmark sample, the correlations of consumption growth with changes in cash payouts range from 0.21 for debt to 0.34 for assets; these correlations increase to 0.34 and 0.44, respectively, using output growth to measure cyclicality. The correlations remain positive excluding the Financial Crisis and at a quarterly frequency, and are typically above 10%. These results are consistent with the evidence in the literature that per share equity dividend growth rates are positively correlated with aggregate growth, and extend it to cash payments on debt and total assets.

Table 9 further shows that changes in net issuances are acyclical and even counter-cyclical. The correlation estimates for net issuances are virtually always smaller, in absolute value, than those for the cash payouts, and are negative in almost half of the cases. For example, in our benchmark sample the correlation of consumption growth with equity net issuances is -0.17, and it is 0.14 and 0.00 for debt and total assets, respectively. The fact that net issuances drive a large part of the variations in total payouts implies that the aggregate payouts are much less procyclical than the cash payouts. In fact, our estimates suggest that the total payouts on the assets of the corporate sector are essentially acyclical. In the benchmark sample, the correlation of consumption growth with total asset payouts is 0.07, and it drops to zero excluding the Crisis and in quarterly data. The reduction in the cyclicality of total payouts is due to both accounting for debt and net issuance data. Indeed, in the benchmark sample the equity payout correlations with consumption growth drop from 0.34 to 0.20 when we account for the net issuances of equity, and it drops further to 0.07 when we also incorporate debt.

The correlation results suggest that total asset payouts are essentially unrelated to the short-run (i.e., quarterly or annual) fluctuations in economic growth. On the other hand, our earlier evidence suggests a considerable exposure of the returns to long-run economic growth fluctuations (see Figure 17). To expand the evidence beyond the short run, we consider
the term structure of the payout cyclicality, and compute the multi-horizon correlations of consumption growth with changes in the payouts, e.g.

\[
\rho_h \equiv \text{Corr} \left( \frac{\Delta D_{a,t}}{C_t} + \frac{\Delta D_{a,t+1}}{C_{t+1}} + \ldots + \frac{\Delta D_{a,t+h}}{C_{t+h}}, \Delta c_t + \Delta c_{t+1} + \ldots \Delta c_{t+h} \right),
\]

for \( h \) equal to 0, 1, ..., 20 quarters. We focus on quarterly growth rates to address the short sample concerns, and compute the GMM standard errors. We plot these correlations as a function of the horizon \( h \) for cash payouts, net issuances, and total payouts on equity, debt and assets in Figures E.1 and 16.

Our results for \( h = 0 \) confirm our short-run evidence in Table 9. The cash payouts are procyclical. The correlations are statistically significant for all cases except for the correlation of debt payout with consumption, which is marginally significant (the debt correlation is significant for the output growth measure). The correlations of aggregate growth with changes in the net issuances or total payouts are economically and statistically indistinguishable from zero. Interestingly, our cyclicality evidence remains similar in the medium and long run. The correlations for cash payouts remain stable, positive, and statistically significant up to the considered 5 year horizon. Growth in net issuances is acyclical, and the correlations for the total payouts are smaller, in absolute value, than those for the cash components. While the equity and asset payout growth correlations with economic growth tend to be positive at all the frequencies, they are indistinguishable from zero. Thus, the evidence suggests that cash payouts are exposed to economic risks at short and long frequencies, but changes in net issuances do not seem to respond to high or low frequency movements in the fundamentals. Because they are so volatile, net issuances drive most of the fluctuations in aggregate payouts, and significantly reduce the cyclicality of the total payouts on equity, debt, and assets.

We use a similar approach to assess the exposure of asset valuations to low and high frequency movements in aggregate economic growth. We consider the term structure of return cyclicality, and compute multi-step correlations of equity, debt, and asset returns with con-
sumption or output growth, e.g.

\[ \rho_h \equiv Corr \left( r_t + r_{t+1} + \ldots + r_{t+h}, \Delta c_t + \Delta c_{t+1} + \ldots \Delta c_{t+h} \right), \]

for \( h \) equal to 0, 1, ..., 20 quarters. We plot these correlations as a function of the horizon \( h \) for equity, debt, and assets returns in Figure 17. Nearly all of the correlations are positive, which suggests that the returns are risky with respect to growth fluctuations. The estimates tend to be smaller and insignificant in the short run. The correlations increase in the medium and long run, and most of them become significant, especially when we use output growth to measure cyclicality. For example, the asset return correlations with consumption growth are below 20% at a quarterly horizon, and increase to 30% at annual or lower frequencies. The asset return correlations with output growth are statistically and economically indistinguishable from 0 on a quarterly frequency, and reach 45% at a 3-year horizon. These findings indicate that equity, bond, and total asset valuations are strongly exposed to economic growth concerns, especially at lower frequencies.

2.2.4. Robustness and Extensions

In this section we expand and verify our main empirical results. We document the importance of using market relative to book values to measure debt, extend and economically interpret the findings for acyclical of net issuances, present further evidence for the long-term behavior of payouts using spectral analysis, consider a global perspective, and provide additional robustness checks with respect to measurements and samples.

**Book versus market values.** One of the key novel features of our analysis is the reliance on the market rather than book values to measure prices and payouts. The availability of reliable bond market data from Barclays Indices imposes limitations in terms of the sample size and the coverage; however, we find that using it has important implications for the measured payouts. Indeed, we follow Larrain and Yogo (2008) and construct book values of debt from the Flow of Funds. We plot the time series of cash, net issuance, and total payouts
from debt on Figure 18. The corresponding market and book payout growth rates are positively correlated and share common patterns, however there are several key differences. The book value quantities are much smoother, and are two to three times less volatile than the market value ones. For example, the unconditional volatility of the total payouts on debt goes up from 1.14 to 3.55 once we switch from book to market measurements. They also have different cyclical properties. Figure 19 contrasts the term structures of payout cyclicalitiy when we use book versus market values of debt. We continue to use market values to measure equity payouts to focus on an incremental impact of debt measurements. The Figure shows that the book value cash payouts from debt and therefore, total assets, are much less procyclical than the market-based ones, and are essentially acyclical in the short and the long run. Hence, using book values to measure debt would suggest that that asset payouts are not exposed to economic growth risk, while market-based estimates strongly point to a large sensitivity of (the cash components of) the payouts to low-frequency growth fluctuations.\footnote{The Flow of Funds data include both the public and private sector. This raises a concern whether the difference in results reflects an inclusion of private firms, or the market versus book measurements. We instead use Compustat database to measure book value of debt, and obtain very similar results that the book-value-based correlations of debt cash payouts are much lower, by about 0.2 correlation units, than the market-based quantities.}

**Acyclicality of Net Issuances.** Our evidence suggests that acyclicality of net issuances is responsible for the a-cyclicility of total payouts, in spite of a strong procyclicalility of cash payments. To help interpret the acyclicality of net issuances themselves, it is helpful to consider the issuances and repurchases components separately. This, however, requires individual firm data, which we can only obtain for equities from the CRSP Monthly Stock File. With this caveat in mind, we focus on the term structure of cyclicality for the equity payouts, and split the equity net issuances into issuances and repurchases.\footnote{Following the literature, we attribute firm’s net issuances to issuances (repurchases) if the number of shares outstanding increases (decreases) over the month, and aggregate the firm-level measures to aggregate index issuances and repurchases.} Figure 20 shows the results for the benchmark sample from 1975 to 2014, while in Figure 21 we consider a longer sample which starts in 1949. The results from both samples are consistent with our
main findings: cash payout growth rates are procyclical, especially in the long run; changes in net issuances are acyclical, and total payout growth appears acyclical as well.

Interestingly, while changes in net issuances are acyclical, both of its components are quite procyclical in the data: the correlations of changes in issuances and repurchases with consumption growth are positive, and in many cases statistically significant. A potential explanation for these findings is that our measures aggregate across firms which may have different needs for capital. In good times, some firms in the cross-section face good investment opportunities and thus acquire more capital through issuances. Other firms may opt for the distribution of profits to the investors, which can be done either through cash dividends or through the repurchases. The repurchases may be the preferred form of distributing the transitory component of earnings, as dividend policy requires financial commitment (e.g., Lintner (1956)), consistent with the evidence in Dittmar and Dittmar (2004), Guay and Harford (2000), and Jagannathan et al. (2000). In both cases, issuances and repurchases increase. This makes them procyclical separately, while on net basis the two effects offset each other, which leads to acyclical net issuances at the aggregate level.

In Table ?? we show the portion of the cospectrum due to cycles above and below 4 years. For cash payouts on equity, debt, and assets, a significant portion of their covariance with consumption is due to low frequency variation: for example, for assets, out of the total correlation of 0.33, 0.26 comes from cycles above 4 years. The cospectrum for net issuances is close to zero at both short and long frequencies. Finally, for equity and asset total payouts, the cospectrum is negative for cycles below 4 years, but it is positive for low-frequency cycles above 4 years. While the magnitudes are small, the evidence suggests that total payouts inherit some of the long-run growth exposure of the cash payouts. The lower panel of the Table shows that the results are similar when we calculate the co-spectrum of the payout growth with the output growth.

**Wavelet Analysis.** The high volatility of acyclical net issuances makes it challenging to identify the risks associated with total payouts. To help uncover the long-run properties
of the series, we perform a decomposition of the correlation between the payout changes and the economic growth using a discrete wavelet transform. Specifically, we estimate a sample wavelet correlation between the two on a scale by scale basis, where each scale is associated with specific frequency interval (e.g. the wavelet correlation for the wavelet scale 16 corresponds to periods of 32-64 quarters). In this part of the analysis we use quarterly growth rates adjusted by x12 ARIMA model which helps reduce the seasonality patterns in the data. All other computational details are provided in Appendix F.

In Table 10 we show the estimates of wavelet correlations at different frequencies and the associated 5% confidence interval. For cash payouts on equity, debt, and assets, a significant portion of their correlation with consumption is due to low frequency variation: for example, for assets, the wavelet correlation equals to 55% associated with period of 32-64 quarter, and 21% at 2-4 quarter frequency. The wavelet correlation for net issuances is statistically close to zero at both short and long frequencies. Finally, consistent with the benchmark evidence, the correlations for the total payouts are much lower than for the cash components, and are measured with a substantial noise. For asset payouts, the estimated correlations are positive at lower frequencies beyond 16 quarters.

**Global Perspective.** Our main results are conducted from the U.S. perspective and analyze the exposures to the economic risks in the U.S. consumption and output data. To the extent that investors are diversified in international markets, they may instead care about the exposures to global macroeconomic shocks. To confirm the robustness of our results, we construct several alternative measures of aggregate economic risks using the international macroeconomic data. We collect quarterly GDP data from major industrialized countries, and measure global output as a value-weighted GDP across countries. Alternatively, to reduce the impact of the U.S., we remove the U.S. from the GDP sample or use equal weights. Figure E.1 in the Appendix E shows the term structures of asset payout when we use global GDP to measure aggregate output. The results are very similar to the benchmark findings: growth rates in cash payouts are strongly procyclical, while growth rates in net issuances...
and total payouts are acyclical. For brevity, the Figure only reports the results for the total assets; our findings for equity and debt are very similar to the benchmark as well.

**Alternative Samples and Measurements.** We perform several other robustness checks to assess the validity of our results. Specifically, we consider: i) equity payouts in longer samples going back to 1949 or 1930; ii) using different sampling frequencies, such as annual growth rates at quarterly frequency, and seasonally adjusted quarterly growth rates either by band-pass, x12 ARIMA model, or by looking at year-to-year changes. The results reported in Table E.1-E.3 are consistent with our benchmark findings.

Finally, for our empirical analysis we use normalized changes in payouts to measure their growth rates, e.g. $\frac{\Delta D_{at}}{C_t} = \frac{D_{at} - D_{at-1}}{C_t}$. This is necessitated by the fact that payouts are often negative, and thus typical log growth computations can not be performed. To assess whether the use of non-traditional growth measures can have an impact on our findings, we consider cash dividends, which are always positive, and contrast the term structures of the correlations based on our growth measure,

$$\rho_h \equiv Corr \left( \frac{\Delta D_t}{C_t} + \frac{\Delta D_{t+1}}{C_{t+1}} + \ldots + \frac{\Delta D_{t+h}}{C_{t+h}}, \Delta c_t + \Delta c_{t+1} + \ldots \Delta c_{t+h} \right),$$

with the one based a more standard log growth measure,

$$\rho_h \equiv Corr \left( \Delta d_t + \Delta d_{t+1} + \ldots \Delta d_{t+h}, \Delta c_t + \Delta c_{t+1} + \ldots \Delta c_{t+h} \right).$$

Figure E.2 shows that the two term structures are virtually identical. Thus, using normalized changes to measure growth rates does not seem to cause statistical issues for our results.

2.3. Model

Our novel empirical evidence suggests that total asset payout growth is acyclical at short and low frequencies. However, corporate assets demand a risk premium and are signifi-
cantly exposed to economic growth risk, especially in the long run. To explain this puzzling empirical evidence, we argue that total assets payouts are dominated by acyclical net issuances which mask economic growth risk of the cash payouts. We develop a long-run risks valuation framework to quantitatively assess the plausibility of our economic explanation. Independently, we make a methodological contribution to the literature by providing an alternative log-linearization framework of Campbell and Shiller (1988) to the cases with negative payouts.

2.3.1. Economic Setup

Preferences. We consider a discrete-time endowment economy, in a spirit of Bansal and Yaron (2004) and a subsequent long-run risks literature. The preferences of the representative agent over the future consumption stream are characterized by the Kreps and Porteus (1978) recursive utility of Epstein and Zin (1989) and Weil (1989):

\[
U_t = \left(1 - \beta \right) C_t^{1-\gamma} + \beta \left( E_t U_{t+1}^{1-\gamma} \right)^{\frac{1}{1-\gamma}}, \tag{2.9}
\]

where \( C_t \) is consumption, \( \beta \) is the subjective discount factor, \( \gamma \) is the risk-aversion coefficient, and \( \psi \) is the elasticity of intertemporal substitution (IES). For ease of notation, the parameter \( \theta \) is defined as \( \theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}} \). Note that when \( \theta = 1 \), that is, \( \gamma = 1/\psi \), the recursive preferences collapse to the standard case of expected power utility, in which case the agent is indifferent to the timing of the resolution of uncertainty of the consumption path. When risk aversion exceeds the reciprocal of IES (\( \gamma > 1/\psi \)), the agent prefers early resolution of uncertainty of consumption path, otherwise, the agent has a preference for late resolution of uncertainty.

Epstein and Zin (1989) show that the asset pricing restriction for any asset return \( r_{j,t+1} \) satisfies a standard Euler condition

\[
E_t \left[ \exp \left\{ m_{t+1} + r_{j,t+1} \right\} \right] = 1, \tag{2.10}
\]
where $m_{t+1}$ is the log of the intertemporal marginal rate of substitution (IMRS), defined as

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \triangle c_{t+1} + (\theta - 1) r_{c,t+1}. \quad (2.11)$$

$\Delta c_{t+1} = \log (C_{t+1}/C_t)$ is the log growth rate of aggregate consumption, and $r_{c,t}$ is a log return on the asset which delivers aggregate consumption as dividends (the wealth portfolio).

**Consumption dynamics.** As in Bansal and Yaron (2004), the consumption growth rate contains a small predictable component $x_t$ which determines the conditional expectation of consumption growth, and the volatility of fundamental shocks is time-varying and is captured by the state variable $\sigma_t^2$:

$$\Delta c_{t+1} = \mu_c + x_t + \sigma_t \eta_{t+1}, \quad (2.12)$$

$$x_{t+1} = \rho_x x_t + \varphi_x \sigma_t \epsilon_{t+1}, \quad (2.13)$$

$$\sigma_{t+1}^2 = \sigma_0^2 + \nu (\sigma_t^2 - \sigma_0^2) + \sigma_w \omega_{t+1}. \quad (2.14)$$

The parameters $\rho_x$ and $\nu$ capture the persistence of the expected growth and volatility news, and $\sigma_0, \varphi_x,$ and $\sigma_w$ govern the unconditional scales of shocks to realized and expected consumption and consumption volatility, respectively.

**Corporate sector payouts.** To model the corporate sector, we focus on the total asset side of the balance sheet and provide a parsimonious exogenous specification for the cash and net issuances components of the aggregate asset payouts. For simplicity, we do not consider the issues of optimal capital structure and issuance and repurchase decisions, and leave these model extensions for future research.

Following Hansen et al. (2008), Bansal and Yaron (2007) and Bansal et al. (2005), cash payouts are co-integrated in logs with the consumption level:

$$\log \left( \frac{D_t}{C_t} \right) \equiv s_t. \quad (2.15)$$
The co-integrating residual $s_t$ is stationary, persistent, and is exposed to the low-frequency growth risk:

$$s_{t+1} = \mu_s + \rho_s (s_t - \mu_s) + \phi_s x_t + \varphi_s \sigma_t \epsilon_{t+1}.$$  \hspace{1cm} (2.16)

Parameters $\mu_s, \rho_s,$ and $\varphi_s$ determine the unconditional level, persistence, and the volatility of the cash payout dynamics, and $\phi_x$ govern the sensitivity to the expected growth risks.

To accommodate net issuances, we first define the adjusted net issuance $H_t$ as

$$H_t \equiv C_t + NI_t = C_t + ISS_t - REP_t.$$  \hspace{1cm} (2.17)

Economically, we expect the adjusted net issuances to be always positive: the repurchase component of net issuances is a capital distribution from firms to investors, supplemental to cash dividends and coupons, all of which are used to finance consumption expenditures. Hence, even ignoring issuances, we expect $C_t > REP_t,$ and therefore $H_t > 0.$ 19 Then, we assume that the log of the adjusted net issuances to consumption is driven by i.i.d. shocks:

$$\log \frac{H_t}{C_t} = \mu_h + \varphi_h \sigma_t \epsilon_t,$$  \hspace{1cm} (2.18)

where $\mu_h$ and $\varphi_h$ capture the unconditional level and volatility of the net issuances. Unlike cash flows, net issuances are not directly exposed to low-frequency fluctuations in economic growth. Further, different from the cash flow dynamics, the process for net issuances is specified in levels and not in logs, because they can be negative. 20

The four shocks $\eta_{t+1}, \epsilon_{t+1}, \omega_{t+1}$ and $\epsilon_{t+1}$ are i.i.d standard Normal. We allow for the

19 This is also strongly supported by the data. In our sample, equity repurchases are on average below 10% and never exceed 40% of the level of consumption. Adding issuances, the issuances net of repurchases never fall below 15% of the total consumption at equity, debt, or asset levels.

20 Our approach is different from Boudoukh et al. (2007) who add a constant to net yield to make it positive. Adjusting net issuances by the level of consumption allows us to impose co-integration between the key aggregate quantities while guaranteeing positivity of adjusted net issuances.
correlation between the transitory shocks to consumption growth and cash payout growth:

\[ \text{Cov} (\eta_{t+1}, u_{t+1}) = \alpha. \]  

(2.19)

2.3.2. Model Solution

For tractability, we consider an approximate solution to the model based on the log-linearization of the consumption and asset return.

Valuation of consumption claim and the IMRS. The log-linearization of the consumption return is standard and follows from Campbell and Shiller (1988). Specifically,

\[ r_{c,t+1} = \log \left( \frac{V_{c,t+1} + C_{t+1}}{V_{c,t}} \right) \approx \kappa_{0,c} + \kappa_{1,c}v_{c,t+1} + \Delta c_{t+1} - v_{c,t}, \]  

(2.20)

where \( v_{c,t} = \log \left( \frac{V_{c,t}}{C_{t}} \right) \) is the valuation of the consumption claim, and \( \kappa_{0,c} \) and \( \kappa_{1,c} \) are the linearization coefficients which are determined in equilibrium by the unconditional level of the consumption asset valuation. Under the log-linearized consumption return, the value of the consumption claim, the consumption return, and hence, the stochastic discount factor are linear in the underlying states of the economy, and can be solved in a closed form. As shown in Appendix G and elsewhere in the literature, the value of the consumption claim is given by:

\[ v_{c,t} = A_{0,c} + A_{1,c}x_t + A_{2,c} \sigma_t^2, \]  

(2.21)

and the equilibrium log stochastic discount factor satisfies,

\[ m_{t+1} = m_0 + m_x x_t + m_\sigma \sigma_t^2 - \lambda_\eta \sigma_t \eta_{t+1} - \lambda_e \varphi e \sigma_t e_{t+1} - \lambda_w \sigma_w \omega_{t+1}, \]  

(2.22)

The exposures of the consumption asset and the market prices of risks are pinned down by the model and preference parameters, and are provided in Appendix G. The economic content of the long-run risks model is that when agents have a preference for timing of
uncertainty resolution, the short-run, long-run, and volatility risks \((\eta, e, \text{ and } \omega, \text{ respectively})\) are priced, and determine the risk compensation in asset markets. Specifically, for \(\gamma > 1\) and \(\psi > 1\), the consumption claim requires a positive risk premium because the consumption asset return is low in bad times of low realized or expected consumption growth \((\lambda_\eta, \lambda_e > 0 \text{ and } A_{1,c} > 0)\), or high consumption volatility \((\lambda_w < 0 \text{ and } A_{2,c} < 0)\). Quantitatively, the risk premium is dominated by the compensations for the expected consumption and volatility risks, which are magnified due to a large persistence of these shocks.

**Valuation of corporate assets.** The payouts from the corporate sector are determined by the cash and net issuance components in (2.15) and (2.18), respectively. Notably, unlike for typical consumption and dividend claims, these payouts can be negative when firms need capital and issue a large amount of equity or debt. Hence, we can not use a standard Campbell and Shiller (1988) approximation, and derive an alternative approach to log-linearize the return. We rewrite the return on the corporate assets in (2.2) as follows:

\[
1 + R_{d,t+1} = \frac{V_{d,t+1} + D_{t+1} - NI_t}{V_{d,t}} = \frac{V_{d,t+1} + D_{t+1} + Ct_{t+1} - H_{t+1}}{V_{d,t}} = \frac{C_{t+1}}{C_t} \cdot \frac{1 + \frac{V_{d,t+1}}{C_{t+1}} + \frac{D_{t+1}}{C_{t+1}} - \frac{H_{t+1}}{C_{t+1}}}{V_{d,t}}
\]

where \(V_{d,t}\) is the value of the asset. Notably, all the ratios in the last equation are positive: consumption, prices, cash payouts, and adjusted net issuances are all above zero. We can then log-linearize the expression above around the unconditional log values of \(\overline{vc}_d, \overline{dc}\) and \(\overline{hc}\) to derive the log-linear approximation for the asset return:

\[
r_{d,t+1} \approx \kappa_{0,d} + \kappa_{1,d}vc_{d,t+1} + \Delta ct_{t+1} + \kappa_{2,d}dc_{t+1} + \kappa_{3,d}hc_{t+1} - vc_{d,t}, \tag{2.23}
\]

where \(vc_{d,t} = log \left( \frac{V_{d,t}}{C_t} \right)\) is the log asset value to consumption ratio, \(dc_{t} = log \left( \frac{D_t}{C_t} \right)\) is the log ratio of the asset cash payouts to consumption, and \(hc_{t} = log \left( \frac{H_t}{C_t} \right)\) is the ratio of the
net issuance to consumption. The expressions for the log-linearization coefficients \( \kappa \) are provided in the Appendix G.

Our log-linear approximation in (2.23) nests a standard one for the consumption asset in (2.20). Indeed, when there are no net issuances and cash payouts are equal to consumption, then \( h c_t = d c_t = 0 \). When net issuances are part of the payouts, the total payout can now be negative and can no longer be used to scale valuations and define growth rates. This is why we have to switch to consumption to scale all the quantities, and rewrite the payouts in terms of positive cash and the adjusted net issuance components. Finally, our approach is also different from the linearizations in Larrain and Yogo (2008) and Bansal and Yaron (2007). These papers effectively log-linearize the returns around the positive issuance and repurchase components of the net issuances. The disadvantage of this method is that it requires modeling issuances and repurchases separately. Recall that due to the data limitations, we can not separate the issuances and repurchases at the asset level, and thus prefer modeling the issuances net of repurchases directly.

\[
\kappa_{1,d} = \frac{\exp \{ vc_d \}}{1 + \exp \{ vc_d \} + \exp \{ dc \} - \exp \{ hc \}}.
\]  

(2.24)

Using our log-linearization solution to asset returns in (2.23), we can now use the corporate payout dynamics in (2.15)-(2.18) and the equilibrium stochastic discount factor in (2.22) to solve for the equilibrium asset valuations. The corporate valuations are linear in the economic states:

\[
v_{c,d,t} = A_{0,d} + A_{1,d} x_t + A_{2,d} \sigma_t^2 + A_{3,d} s_t. \]  

(2.25)

Similar to the consumption asset, for typical model parameters corporate assets are risky: they fall in bad times of low economic growth \( (A_{1,d} > 0) \) or high consumption volatility \( (A_{2,d} < 0) \). The asset prices also increase at times of a positive gap between cash payouts and consumption \( (A_{3,d} > 0) \): because the gap \( s_t \) is persistent, it signifies higher cash payments to investors in the future.
2.3.3. Implications for Payouts and Valuations

We calibrate the model, and assess whether it can quantitatively account for our empirical evidence. As is common in this literature, we calibrate the model at a monthly frequency, and use simulations to target the data at an annual horizon. Specifically, we time-aggregate the simulated monthly output from the model and construct annual growth rates and payout changes, asset returns, and valuation ratios. We report the median and percentiles for the model statistics based on 10,000 Monte-Carlo simulations with $40 \times 12$ monthly observations each that match the length of the historical data. We also show the population values that correspond to a long-sample of 10,000 annualized observations.

**Consumption and corporate payouts.** Table 11 reports the parameter values for the model. In a spirit of the long-run risks literature, the consumption calibration features persistent low-frequency movements in the expected growth and consumption volatility. The persistence of the expected growth component is set at 0.985, and that of the volatility shocks at 0.999. The scales of the expected growth and volatility shocks are rather small to account for the empirical properties of the macroeconomic fundamentals in the data.

Table 12 shows that our model can match salient properties of the consumption data. The Table reports the mean, standard deviation, and the persistence of the consumption growth at 1, 2, and 5 lags in the data and in the model. The data moments are computed for the benchmark 1975-2014 sample, as well as for a long sample going back to 1929. The median model values are close to the data, and in all the cases the data values are within the confidence interval of the model.

We next calibrate the dynamics of the cash payouts and net issuances. The cash payouts are moderately persistent ($\rho_s = 0.96$) and are exposed to the expected growth fluctuations ($\phi_s = 6$). Recall that in the model the ratio of net issuances to consumption is unpredictable and driven by its own i.i.d. shock, so we only need to set its overall level and scale. As shown in Table 13, the model can successfully capture the key moments of cash payout, net
issuance, and total payout dynamics in the data. Changes in net issuances are several times more volatile that changes in cash payouts. This leads to a highly volatile total asset payout growth dominated by shocks to net issuance. Its volatility of 3.15 is comparable to 4.36 in the model. The Table also shows that, unlike cash dividends which are always positive, net issuances and total payouts can go negative. In the model, net issuances become negative about 5% of the time, while total payouts turn negative 25% of the time. These estimates are consistent with the data.

Changes in annual cash payouts are mildly persistent both in the model and the data. Changes in net issuances actually have a negative persistence in the data. This is also captured by the model structure because i.i.d. shocks to the levels of net issuances lead to negative autocorrelation for the changes. Changes in total payouts behave like net issuances, and have a negative persistence both in the data and in the model. The Table also shows that the model can capture well the short-run cyclicality evidence in the data. Cash payouts are positively correlated with annual consumption growth: the correlation is 0.30 in the data and 0.34 in the model. Net issuances are acyclical, and total payouts are essentially acyclical as well: their correlation with consumption growth 0.00 in the data relative to 0.02 in the model. The confidence intervals on the asset payout correlations are quite large, which is consistent with the idea that the net issuances introduce a substantial noise in measuring the exposures of aggregate payouts to economic risks.

Perhaps surprisingly, the model does not match the means of net issuances and aggregate payouts, even though these are effectively governed by the exogenous parameters in the model (the data value is within the 5% confidence interval of the model). The unconditional mean net issuance changes in the data is actually larger than the mean of changes in cash payouts, which implies that the mean of total payout changes is negative (see Table 13). Taking these estimates at the face value, this suggests that future payouts from the corporate sector are negative on average, which would lead to negative asset valuations. This is counterfactual in the data, and is ruled out in our model solution approach which
forces prices to be positive. Because there is a substantial statistical uncertainty about the estimates of the mean, we instead target a lower value for the average net issuances which is below the average of the cash payouts. Under these values, the average total payouts are positive, and their present value is positive as well.\[^{21}\]

**Asset prices and economic risk.** To study the implications for the asset prices, we calibrate the preference parameters to standard values in the literature. The risk aversion is set at 10, and the IES parameter is 1.5. This configuration implies a preference for early resolution of uncertainty and a strong substitution effect. These margins play an important role to generate sizeable risk compensations and realistic dynamics of the asset prices (see Bansal and Yaron (2004)).

Table 14 shows the model implications for the key asset-pricing moments, such as the mean and standard deviation of the risk-free rate and the asset return. The model replicates quite well a relatively low level and volatility of the risk-free rate in the data. The level is 0.86\% in the data relative to 1.64\% in the model, and its volatility is 1.78\% in the data and 0.82\% in the model. The asset returns are risky: the asset risk premium is 6.42\% in the data relative to 6.25\% in the model, and the asset return volatility is about 12\% both in the model and in the data.

What is the nature and sources of risk in corporate assets? The key source of risk in our economy is the news to the expected consumption growth, which accounts for about a half of the compensation for the total asset risk premium, with volatility and short-run consumption risks explaining the rest. This expected growth risk compensation reflects the exposure of the cash component of the aggregate payouts to low-frequency fluctuations in expected consumption through the parameter $\varphi_s$. Indeed, zeroing out net issuance components from the total payouts by setting $\varphi_h = 0$ does not materially affect the model implied asset risk.

\[^{21}\] Asset payouts which are positive on average is a necessary but not a sufficient condition for the convergence and positivity of asset valuations. Due to aggregate risk compensation, states with negative payouts in general contribute more to total asset valuations than states with positive payouts. This places further discipline on the payout parameters to ensure that the solution to asset prices exists.
premium.\footnote{The volatility of net issuances is exposed to consumption volatility, so that net issuances affect the asset exposure to the volatility risks. Net issuances also impact the values of the steady states for the log-linearization of returns. These effects are quite small.}

The term structures of cyclicality help further assess and validate the expected growth risk channel for the payouts and valuations. As shown in Figure 22, asset returns co-move positively with consumption growth at short and long horizons in the data, and the model can quantitatively capture these correlations. The model also replicates very well the correlation patterns across different components of the payouts and at different frequencies. Both in the data and in the model, the cash payouts are procyclical at all horizons, while the net issuances and total payouts are effectively acyclical. The correlations are very similar in the model and in the data.

Hence, even though corporate payouts are dominated by shocks to net issuances which are uncorrelated with the aggregate economy, the large exposure of cash payouts to low-frequency growth fluctuations makes total payouts risky, and generates a large risk premium for asset returns.

2.4. Conclusions

We measure the market value of U.S. corporate assets and their payouts to investors. Our measure of total payout includes not only the cash dividends and interest payments (cash payouts), but also net transfers in the form of repurchases and new issuances of equity and debt.

We document several novel empirical findings. First, total asset payouts often turn negative, meaning that there are periods when investors finance the corporate sector. Second, net issuances are highly volatile, and are a dominant component of the total payouts. Third, while cash payouts are procyclical, total payouts appear acyclical. This holds for equity, debt and especially for asset payouts. This evidence challenges standard notions of risk and return, because asset returns are risky and comparable to equities.
We develop a long-run risk model to account for the empirical evidence. In the model, net issuances are acyclical and highly volatile, which masks the exposure of cash components of total payouts to low frequency economic risks. The model matches acyclical dynamics of the total payout, while generating a sizeable asset risk premium.

There are several extensions of our paper that would be fruitful to pursue in future work. On the empirical side, it would be interesting to consider properties of the valuations and payouts across firms, and not just at the aggregate level. Theoretically, it would be useful to develop an economic model which endogenizes the payout decisions. As a next step, one can calibrate or estimate the economic environment and quantify the role of economic risks for payout policy and asset valuations. We leave these extensions for future research.
The Figure shows the market value of the components of the long-term and short-term corporate debt. The data are real annual observations from 1975 to 2014, and are expressed in trillions of December 2009 dollars.
The Figure shows the market values of equity, debt, and assets. Grey bars indicate the NBER recessions. The data are real annual observations from 1975 to 2014, and are expressed in trillions of December 2009 dollars.
The Figure shows equity, debt, and asset payouts. The payouts include cash, net issuances, and total payouts. Grey bars indicate the NBER recessions. The data are real annual observations from 1975 to 2014, and are expressed in trillions of December 2009 dollars.
Figure 14: Changes in Equity, Debt, and Asset Payouts

The Figure shows changes in equity, debt, and asset payouts, scaled by the consumption level. The payouts include cash, net issuances, and total payouts. Grey bars indicate the NBER recessions. The data are real annual observations from 1975 to 2014.
The Figure shows multi-horizon correlations between equity, debt, and asset payouts and consumption growth. The payouts include cash, net issuances, and total payouts. The data are real quarterly observations from Q1.1975 to Q4.2014. The standard errors are Newey-West adjusted.
The Figure shows multi-horizon correlations between equity, debt, and asset payouts and output growth. The payouts include cash, net issuances, and total payouts. The data are real quarterly observations from Q1.1975 to Q4.2014. The standard errors are Newey-West adjusted.
The Figure shows multi-horizon correlations between equity, debt, and asset excess returns and measures of economic growth, such as consumption (left panels) and output (right panels). The data are real quarterly observations from Q1.1975 to Q4.2014. The standard errors are Newey-West adjusted.
The Figure shows changes in debt payouts, scaled by the consumption level, and computed using the market (solid line) or book (dashed line) values. The payouts include cash, net issuances, and total payouts. Grey bars indicate the NBER recessions. The data are real annual observations from 1975 to 2014.
The Figure shows multi-horizon correlations between equity, debt, and asset payouts and consumption growth. Debt payouts are computed using the market (solid line) or book (dashed line) values. The payouts include cash, net issuances, and total payouts. The data are real quarterly observations from Q1.1975 to Q4.2014. The standard errors are Newey-West adjusted.
The Figure shows multi-horizon correlations between equity payouts and consumption growth. The pay- 
outs include cash, net issuances, issuances, repurchases, and total payouts. The data are real quarterly 
observations from Q1.1975 to Q4.2014. The standard errors are Newey-West adjusted.
The Figure shows multi-horizon correlations between components of equity payout and consumption growth. The payouts include cash, net issuances, issuances, repurchases, and total payouts. The data are real quarterly observations from Q1.1949 to Q4.2014. The standard errors are Newey-West adjusted.
The Figure shows multi-horizon correlations between asset payouts and consumption growth, and excess asset returns and consumption growth in the data and in the model. The payouts include cash, net issuances, and total payouts. The data (solid line) are real quarterly observations from Q1.1975 to Q4.2014. Model median (circles) and 5-95% confidence interval (dashed line) are based on a long simulation of the model.
The Table reports summary statistics for equity, debt, and total asset returns. The mean and standard deviation are in percentage terms. The data are real annual observations from 1975 to 2014.

Table 7: Asset Returns in the Data

<table>
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<tr>
<th></th>
<th>Equity</th>
<th>Debt</th>
<th>Asset</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
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<td>2.94</td>
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<tr>
<td>Std</td>
<td>16.33</td>
<td>5.36</td>
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<tr>
<td>AC(1)</td>
<td>-0.08</td>
<td>0.20</td>
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*Cross-Correlations:*

<table>
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<tr>
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<tr>
<td>Asset</td>
<td>0.99</td>
<td>0.54</td>
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Table 8: Asset Payouts in the Data

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<th>Equity</th>
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<th>Asset</th>
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<tbody>
<tr>
<td>Mean</td>
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<td>0.27</td>
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<tr>
<td>Std</td>
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<td>0.78</td>
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<tr>
<td>AC(1)</td>
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<td>0.37</td>
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Cross-Correlations:

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<th>Asset</th>
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<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>Asset</td>
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<td>0.93</td>
<td></td>
</tr>
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</table>

(b) Net Issuance

<table>
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<th>Equity</th>
<th>Debt</th>
<th>Asset</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.17</td>
<td>0.19</td>
<td>0.36</td>
</tr>
<tr>
<td>Std</td>
<td>3.02</td>
<td>3.78</td>
<td>3.32</td>
</tr>
<tr>
<td>AC(1)</td>
<td>-0.13</td>
<td>-0.29</td>
<td>-0.36</td>
</tr>
</tbody>
</table>

Cross-Correlations:

<table>
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<tr>
<th></th>
<th>Equity</th>
<th>Debt</th>
<th>Asset</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt</td>
<td>-0.54</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asset</td>
<td>0.29</td>
<td>0.64</td>
<td></td>
</tr>
</tbody>
</table>

(c) Total Payout

<table>
<thead>
<tr>
<th></th>
<th>Equity</th>
<th>Debt</th>
<th>Asset</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.03</td>
<td>-0.06</td>
<td>-0.10</td>
</tr>
<tr>
<td>Std</td>
<td>3.08</td>
<td>3.55</td>
<td>3.15</td>
</tr>
<tr>
<td>AC(1)</td>
<td>-0.09</td>
<td>-0.36</td>
<td>-0.35</td>
</tr>
</tbody>
</table>

Cross-Correlations:

<table>
<thead>
<tr>
<th></th>
<th>Equity</th>
<th>Debt</th>
<th>Asset</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt</td>
<td>-0.56</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asset</td>
<td>0.35</td>
<td>0.58</td>
<td></td>
</tr>
</tbody>
</table>

The Table reports summary statistics for the changes in equity, debt, and asset payouts, scaled by the consumption level. The payouts include cash payouts, net issuances, and total payouts. The mean and standard deviation are in percentage terms. The data are real annual observations from 1975 to 2014.
Table 9: Asset Payout Cyclicality

(a) Annual Data (1975-2014)

<table>
<thead>
<tr>
<th></th>
<th>$\frac{\Delta NI}{C}$</th>
<th>$\frac{\Delta NI}{Y}$</th>
<th>$\frac{\Delta NI}{a}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>0.34</td>
<td>-0.17</td>
<td>0.20</td>
</tr>
<tr>
<td>Debt</td>
<td>0.21</td>
<td>0.14</td>
<td>-0.11</td>
</tr>
<tr>
<td>Asset</td>
<td>0.30</td>
<td>-0.00</td>
<td>0.07</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\frac{\Delta NI}{Y}$</th>
<th>$\frac{\Delta NI}{C}$</th>
<th>$\frac{\Delta NI}{a}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>0.44</td>
<td>-0.23</td>
<td>0.27</td>
</tr>
<tr>
<td>Debt</td>
<td>0.34</td>
<td>0.21</td>
<td>-0.16</td>
</tr>
<tr>
<td>Asset</td>
<td>0.44</td>
<td>0.03</td>
<td>0.08</td>
</tr>
</tbody>
</table>

(b) Annual Data (1975-2006)

<table>
<thead>
<tr>
<th></th>
<th>$\frac{\Delta NI}{C}$</th>
<th>$\frac{\Delta NI}{Y}$</th>
<th>$\frac{\Delta NI}{a}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>0.24</td>
<td>-0.07</td>
<td>0.09</td>
</tr>
<tr>
<td>Debt</td>
<td>0.06</td>
<td>0.09</td>
<td>-0.08</td>
</tr>
<tr>
<td>Asset</td>
<td>0.14</td>
<td>0.01</td>
<td>0.02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\frac{\Delta NI}{Y}$</th>
<th>$\frac{\Delta NI}{C}$</th>
<th>$\frac{\Delta NI}{a}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>0.35</td>
<td>-0.17</td>
<td>0.19</td>
</tr>
<tr>
<td>Debt</td>
<td>0.20</td>
<td>0.03</td>
<td>0.00</td>
</tr>
<tr>
<td>Asset</td>
<td>0.30</td>
<td>-0.18</td>
<td>0.23</td>
</tr>
</tbody>
</table>

(c) Quarterly Data (1975-2014)

<table>
<thead>
<tr>
<th></th>
<th>$\frac{\Delta NI}{C}$</th>
<th>$\frac{\Delta NI}{Y}$</th>
<th>$\frac{\Delta NI}{a}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>0.16</td>
<td>0.02</td>
<td>0.00</td>
</tr>
<tr>
<td>Debt</td>
<td>0.20</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>Asset</td>
<td>0.22</td>
<td>0.01</td>
<td>0.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\frac{\Delta NI}{Y}$</th>
<th>$\frac{\Delta NI}{C}$</th>
<th>$\frac{\Delta NI}{a}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>0.15</td>
<td>-0.11</td>
<td>0.13</td>
</tr>
<tr>
<td>Debt</td>
<td>0.32</td>
<td>-0.06</td>
<td>0.08</td>
</tr>
<tr>
<td>Asset</td>
<td>0.27</td>
<td>-0.11</td>
<td>0.13</td>
</tr>
</tbody>
</table>

(d) Quarterly Data (1975-2006)

<table>
<thead>
<tr>
<th></th>
<th>$\frac{\Delta NI}{C}$</th>
<th>$\frac{\Delta NI}{Y}$</th>
<th>$\frac{\Delta NI}{a}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>0.17</td>
<td>0.06</td>
<td>-0.05</td>
</tr>
<tr>
<td>Debt</td>
<td>0.08</td>
<td>-0.06</td>
<td>0.06</td>
</tr>
<tr>
<td>Asset</td>
<td>0.17</td>
<td>-0.00</td>
<td>0.01</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$\frac{\Delta NI}{Y}$</th>
<th>$\frac{\Delta NI}{C}$</th>
<th>$\frac{\Delta NI}{a}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>0.10</td>
<td>-0.06</td>
<td>0.07</td>
</tr>
<tr>
<td>Debt</td>
<td>0.22</td>
<td>-0.10</td>
<td>0.11</td>
</tr>
<tr>
<td>Asset</td>
<td>0.19</td>
<td>-0.11</td>
<td>0.11</td>
</tr>
</tbody>
</table>

The Table reports correlations between changes in payouts, scaled by the aggregate level of the economy, and measures of economic growth. The payouts include cash payouts, net issuances, and total payouts, and are computed for equity, debt, and assets. The left and right panels use consumption or output, respectively, to measure the aggregate level of the economy. The data are real, and correspond to (a) annual observations from 1975 to 2014; (b) annual observations from 1975 to 2006; (c) quarterly observations from Q1.1975 to Q4.2014; (d) quarterly observations from Q1.1975 to Q4.2006.
Table 10: Wavelet Correlation between Asset Payout and Consumption Growth

<table>
<thead>
<tr>
<th>Scale</th>
<th>Cash Payout</th>
<th>Net Issuance</th>
<th>Total Payout</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 - 4 Quarters</td>
<td>0.05 (-0.18, 0.26)</td>
<td>-0.02 (-0.24, 0.20)</td>
<td>0.02 (-0.20, 0.24)</td>
</tr>
<tr>
<td>4 - 8 Quarters</td>
<td>0.15 (-0.17, 0.44)</td>
<td>0.00 (-0.31, 0.31)</td>
<td>0.01 (-0.31, 0.32)</td>
</tr>
<tr>
<td>8 - 16 Quarters</td>
<td>0.35 (-0.11, 0.69)</td>
<td>0.04 (-0.41, 0.47)</td>
<td>-0.03 (-0.47, 0.42)</td>
</tr>
<tr>
<td>16 - 32 Quarters</td>
<td>0.63 (0.01, 0.90)</td>
<td>-0.14 (-0.71, 0.53)</td>
<td>0.19 (-0.50, 0.73)</td>
</tr>
<tr>
<td>32 - 64 Quarters</td>
<td>0.15 (-0.53, 0.71)</td>
<td>0.21 (-0.49, 0.74)</td>
<td>-0.14 (-0.71, 0.54)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Scale</th>
<th>Cash Payout</th>
<th>Net Issuance</th>
<th>Total Payout</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 - 4 Quarters</td>
<td>0.03 (-0.19, 0.25)</td>
<td>-0.00 (-0.22, 0.22)</td>
<td>-0.01 (-0.23, 0.21)</td>
</tr>
<tr>
<td>4 - 8 Quarters</td>
<td>0.03 (-0.28, 0.34)</td>
<td>0.06 (-0.25, 0.37)</td>
<td>-0.07 (-0.38, 0.24)</td>
</tr>
<tr>
<td>8 - 16 Quarters</td>
<td>0.23 (-0.24, 0.61)</td>
<td>0.23 (-0.24, 0.61)</td>
<td>-0.22 (-0.61, 0.24)</td>
</tr>
<tr>
<td>16 - 32 Quarters</td>
<td>0.52 (-0.16, 0.87)</td>
<td>0.24 (-0.46, 0.76)</td>
<td>-0.13 (-0.70, 0.54)</td>
</tr>
<tr>
<td>32 - 64 Quarters</td>
<td>0.71 (0.15, 0.93)</td>
<td>0.35 (-0.36, 0.80)</td>
<td>-0.24 (-0.76, 0.46)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Scale</th>
<th>Cash Payout</th>
<th>Net Issuance</th>
<th>Total Payout</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 - 4 Quarters</td>
<td>0.21 (-0.01, 0.41)</td>
<td>-0.04 (-0.25, 0.18)</td>
<td>0.04 (-0.18, 0.26)</td>
</tr>
<tr>
<td>4 - 8 Quarters</td>
<td>0.25 (-0.06, 0.52)</td>
<td>0.01 (-0.30, 0.32)</td>
<td>0.02 (-0.29, 0.33)</td>
</tr>
<tr>
<td>8 - 16 Quarters</td>
<td>0.34 (-0.12, 0.68)</td>
<td>0.27 (-0.19, 0.64)</td>
<td>-0.21 (-0.60, 0.25)</td>
</tr>
<tr>
<td>16 - 32 Quarters</td>
<td>0.58 (-0.08, 0.89)</td>
<td>-0.13 (-0.70, 0.55)</td>
<td>0.32 (-0.38, 0.79)</td>
</tr>
<tr>
<td>32 - 64 Quarters</td>
<td>0.55 (-0.12, 0.88)</td>
<td>0.06 (-0.59, 0.67)</td>
<td>0.11 (-0.56, 0.69)</td>
</tr>
</tbody>
</table>

The Table reports wavelet correlations between changes in payouts and consumption growth. The payouts include cash payouts, net issuances, and total payouts, and are computed for equity, debt, and assets. The panels report the estimates of wavelet correlation for different scales, with 5% confidence intervals reported in the brackets. The data are real quarterly observations from Q1.1975 to Q4.2014, seasonally adjusted by x12 ARIMA model.
Table 11: Configuration of Model Parameters

<table>
<thead>
<tr>
<th>Preferences</th>
<th>$\delta$</th>
<th>$\gamma$</th>
<th>$\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9992</td>
<td>10</td>
<td>1.5</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Consumption</th>
<th>$\mu_c$</th>
<th>$\rho_c$</th>
<th>$\varphi_c$</th>
<th>$\sigma_0$</th>
<th>$\nu$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0024</td>
<td>0.985</td>
<td>0.038</td>
<td>0.005</td>
<td>0.999</td>
<td>0.000001</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cash Payout</th>
<th>$\mu_s$</th>
<th>$\rho_s$</th>
<th>$\phi_s$</th>
<th>$\varphi_s$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.65</td>
<td>0.96</td>
<td>6</td>
<td>5</td>
<td>-0.15</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Net Issuance</th>
<th>$\mu_h$</th>
<th>$\varphi_h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.045</td>
<td>18</td>
<td></td>
</tr>
</tbody>
</table>

The Table reports the configuration of model parameters. The model is calibrated at a monthly frequency.

Table 12: Model Implications: Consumption

<table>
<thead>
<tr>
<th>Data</th>
<th>1929-2014</th>
<th>1975-2014</th>
<th>Model</th>
<th>Med</th>
<th>2.5%</th>
<th>5%</th>
<th>95%</th>
<th>97.5%</th>
<th>Pop</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E (\cdot)$</td>
<td>2.97</td>
<td>2.69</td>
<td></td>
<td>2.89</td>
<td>1.21</td>
<td>1.53</td>
<td>4.20</td>
<td>4.41</td>
<td>2.91</td>
</tr>
<tr>
<td>$\sigma (\cdot)$</td>
<td>2.20</td>
<td>1.67</td>
<td></td>
<td>1.99</td>
<td>1.22</td>
<td>1.29</td>
<td>3.03</td>
<td>3.25</td>
<td>2.19</td>
</tr>
<tr>
<td>$AC (1)$</td>
<td>0.52</td>
<td>0.27</td>
<td></td>
<td>0.43</td>
<td>0.11</td>
<td>0.15</td>
<td>0.67</td>
<td>0.71</td>
<td>0.54</td>
</tr>
<tr>
<td>$AC (2)$</td>
<td>0.23</td>
<td>0.20</td>
<td></td>
<td>0.19</td>
<td>-0.19</td>
<td>-0.14</td>
<td>0.50</td>
<td>0.55</td>
<td>0.35</td>
</tr>
<tr>
<td>$AC (5)$</td>
<td>0.04</td>
<td>-0.17</td>
<td></td>
<td>0.04</td>
<td>-0.31</td>
<td>-0.24</td>
<td>0.32</td>
<td>0.38</td>
<td>0.21</td>
</tr>
</tbody>
</table>

The Table reports the data and model properties of real consumption growth. The summary statistics in the data are computed in the annual samples from 1929 to 2014 and from 1975 to 2014. The median and 2.5%, 5%, 95%, and 97.5% values capture the model moment distributions across the small samples whose size equals the data. Population values correspond to a long simulation of the model. Means and volatilities are expressed in percentage terms.
Table 13: Model Implications: Asset Payouts

(a) Cash Payout

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
<th>Median</th>
<th>2.5%</th>
<th>5%</th>
<th>95%</th>
<th>97.5%</th>
<th>Pop</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(\cdot)$</td>
<td>0.27</td>
<td>0.21</td>
<td>0.04</td>
<td>0.07</td>
<td>0.39</td>
<td>0.43</td>
<td>0.22</td>
<td></td>
</tr>
<tr>
<td>$\sigma(\cdot)$</td>
<td>0.78</td>
<td>0.61</td>
<td>0.37</td>
<td>0.39</td>
<td>1.00</td>
<td>1.08</td>
<td>0.65</td>
<td></td>
</tr>
<tr>
<td>$AC(1)$</td>
<td>0.37</td>
<td>0.27</td>
<td>-0.03</td>
<td>0.02</td>
<td>0.50</td>
<td>0.54</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>$AC(2)$</td>
<td>-0.21</td>
<td>0.00</td>
<td>-0.29</td>
<td>-0.26</td>
<td>0.27</td>
<td>0.31</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>$AC(5)$</td>
<td>-0.20</td>
<td>-0.06</td>
<td>-0.34</td>
<td>-0.30</td>
<td>0.19</td>
<td>0.24</td>
<td>-0.04</td>
<td></td>
</tr>
<tr>
<td>$corr(\cdot, \Delta c)$</td>
<td>0.30</td>
<td>0.34</td>
<td>-0.04</td>
<td>0.02</td>
<td>0.56</td>
<td>0.62</td>
<td>0.36</td>
<td></td>
</tr>
<tr>
<td>% of Neg Payouts</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
</tbody>
</table>

(b) Net Issuance

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
<th>Median</th>
<th>2.5%</th>
<th>5%</th>
<th>95%</th>
<th>97.5%</th>
<th>Pop</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(\cdot)$</td>
<td>0.36</td>
<td>0.15</td>
<td>-0.09</td>
<td>-0.05</td>
<td>0.36</td>
<td>0.40</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>$\sigma(\cdot)$</td>
<td>3.32</td>
<td>4.29</td>
<td>2.51</td>
<td>2.82</td>
<td>6.39</td>
<td>6.85</td>
<td>4.36</td>
<td></td>
</tr>
<tr>
<td>$AC(1)$</td>
<td>-0.36</td>
<td>-0.48</td>
<td>-0.68</td>
<td>-0.66</td>
<td>-0.26</td>
<td>-0.22</td>
<td>-0.50</td>
<td></td>
</tr>
<tr>
<td>$AC(2)$</td>
<td>0.08</td>
<td>-0.01</td>
<td>-0.37</td>
<td>-0.33</td>
<td>0.29</td>
<td>0.36</td>
<td>-0.01</td>
<td></td>
</tr>
<tr>
<td>$AC(5)$</td>
<td>-0.03</td>
<td>-0.00</td>
<td>-0.33</td>
<td>-0.29</td>
<td>0.28</td>
<td>0.33</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>$corr(\cdot, \Delta c)$</td>
<td>-0.00</td>
<td>0.02</td>
<td>-0.23</td>
<td>-0.19</td>
<td>0.23</td>
<td>0.26</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>% of Neg Payouts</td>
<td>5.00</td>
<td>5.00</td>
<td>0.00</td>
<td>0.00</td>
<td>12.50</td>
<td>15.00</td>
<td>4.27</td>
<td></td>
</tr>
</tbody>
</table>

(c) Total Payout

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
<th>Median</th>
<th>2.5%</th>
<th>5%</th>
<th>95%</th>
<th>97.5%</th>
<th>Pop</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(\cdot)$</td>
<td>-0.10</td>
<td>0.07</td>
<td>-0.19</td>
<td>-0.14</td>
<td>0.29</td>
<td>0.33</td>
<td>0.07</td>
<td></td>
</tr>
<tr>
<td>$\sigma(\cdot)$</td>
<td>3.15</td>
<td>4.36</td>
<td>2.57</td>
<td>2.82</td>
<td>6.51</td>
<td>6.91</td>
<td>4.40</td>
<td></td>
</tr>
<tr>
<td>$AC(1)$</td>
<td>-0.35</td>
<td>-0.47</td>
<td>-0.67</td>
<td>-0.65</td>
<td>-0.25</td>
<td>-0.19</td>
<td>-0.48</td>
<td></td>
</tr>
<tr>
<td>$AC(2)$</td>
<td>0.14</td>
<td>-0.01</td>
<td>-0.37</td>
<td>-0.32</td>
<td>0.29</td>
<td>0.35</td>
<td>-0.01</td>
<td></td>
</tr>
<tr>
<td>$AC(5)$</td>
<td>-0.02</td>
<td>-0.00</td>
<td>-0.34</td>
<td>-0.28</td>
<td>0.27</td>
<td>0.33</td>
<td>0.01</td>
<td></td>
</tr>
<tr>
<td>$corr(\cdot, \Delta c)$</td>
<td>0.07</td>
<td>0.03</td>
<td>-0.22</td>
<td>-0.19</td>
<td>0.23</td>
<td>0.29</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>% of Neg Payouts</td>
<td>30.00</td>
<td>25.00</td>
<td>5.00</td>
<td>10.00</td>
<td>47.50</td>
<td>51.25</td>
<td>24.87</td>
<td></td>
</tr>
</tbody>
</table>

The Table reports the data and model properties of changes in asset payouts, scaled by the consumption level. The payouts include cash payouts, net issuances, and total payouts. The summary statistics in the data are computed in the annual sample from 1975 to 2014. The median and 2.5%, 5%, 95%, and 97.5% values capture the model moment distributions across the small samples whose size equals the data. Population values correspond to a long simulation of the model. Means and volatilities are expressed in percentage terms.
Table 14: Model Implications for Asset Prices

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Median</th>
<th>2.5%</th>
<th>5%</th>
<th>95%</th>
<th>97.5%</th>
<th>Pop</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(r_f)$</td>
<td>0.86</td>
<td>1.64</td>
<td>0.31</td>
<td>0.53</td>
<td>2.43</td>
<td>2.61</td>
<td>1.65</td>
</tr>
<tr>
<td>$\sigma(r_f)$</td>
<td>1.78</td>
<td>0.82</td>
<td>0.42</td>
<td>0.46</td>
<td>1.36</td>
<td>1.49</td>
<td>1.04</td>
</tr>
</tbody>
</table>

**Risk-Free Return:**

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Median</th>
<th>2.5%</th>
<th>5%</th>
<th>95%</th>
<th>97.5%</th>
<th>Pop</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(r_d)$</td>
<td>6.42</td>
<td>6.25</td>
<td>0.24</td>
<td>1.78</td>
<td>10.52</td>
<td>11.81</td>
<td>6.13</td>
</tr>
<tr>
<td>$\sigma(r_d)$</td>
<td>12.00</td>
<td>12.08</td>
<td>7.72</td>
<td>8.24</td>
<td>19.68</td>
<td>22.23</td>
<td>12.95</td>
</tr>
</tbody>
</table>

The Table reports the data and model properties of the real risk-free rate and the asset return. The summary statistics in the data are computed in the annual sample from 1975 to 2014. The median and 2.5%, 5%, 95%, and 97.5% values capture the model moment distributions across the small samples whose size equals the data. Population values correspond to a long simulation of the model. Means and volatilities are expressed in percentage terms.
APPENDIX A: Proofs

Proposition 1

The social planner’s problem 1.9 can be equivalently stated as:

\[
L = \max_{L_{t+1}, D_{t+1}} E \left[ \sum_{t=0}^{\infty} \beta^t \left( e^{\alpha t} L_t^\alpha - L_{t+1} + \frac{D_{t+1}^1}{1-\eta} + \varphi_t (D_{t+1} - L_{t+1}) \right) \right],
\]

where \( \varphi_t \) is the Lagrange multiplier on the deposits supply constraint.

The first-order conditions are given by:

\[
\left( \frac{\partial L}{\partial D_{t+1}} \right) : \quad (D_{t+1}^{FB})^{-\eta} + \varphi_t < 0,
\]

\[
= 0 \quad if \quad D_{t+1}^{FB} > 0,
\]

\[
\left( \frac{\partial L}{\partial L_{t+1}} \right) : \quad -1 - \varphi_t + \beta E_t \left[ \alpha e^{\alpha t+1} (L_{t+1}^{FB})^{\alpha-1} \right] \leq 0,
\]

\[
= 0 \quad if \quad L_{t+1}^{FB} > 0,
\]

\[
\left( \frac{\partial L}{\partial \varphi_t} \right) : \quad D_{t+1}^{FB} - L_{t+1}^{FB} \leq 0,
\]

\[
= 0 \quad if \quad \varphi_t > 0.
\]

The first-order condition A.1 along with A.3 delivers a corner solution for the socially optimal level of bank debt, \( D_{t+1}^{FB} = L_{t+1}^{FB} \). This in turn implies that \( N_{t+1}^{FB} = 0 \). Provided that the following condition holds:

\[
\lim_{L_{t+1}^{FB} \to 0} \left\{ -1 + (L_{t+1}^{FB})^{-\eta} + \beta E_t \left[ \alpha e^{\alpha t+1} (L_{t+1}^{FB})^{\alpha-1} \right] \right\} = +\infty,
\]

I have an interior solution for the socially optimal level of lending, \( L_{t+1}^{FB} > 0 \).

The second-order condition with respect to lending,

\[
\left( \frac{\partial^2 L}{\partial L_{t+1}^2} \right) : \quad \beta E_t \left[ \alpha (\alpha - 1) e^{\alpha t+1} (L_{t+1}^{FB})^{\alpha-2} \right] < 0,
\]

ensures that \( L_{t+1}^{FB} \) is a global maximum of the social planner’s problem. Rearranging equation A.2 and using that \( \varphi_t = (L_{t+1}^{FB})^{-\eta} \) completes the proof.■
In the equilibrium, the deposit rate is equal to

Without loss of generality, I restrict attention to the case when the deposit rate is strictly positive.

where

Solution bank’s objective function is strictly decreasing in the amount of equity financing, delivering a corner

Using an implicit function theorem and condition A.4, the procyclicality of

Proposition 2

Define:

Using an implicit function theorem and condition A.4, the procyclicality of \( L_{t+1}^{FB} \) follows from:

where I use that:

\[
\frac{\partial R_{d,t+1}^{FB}}{\partial L_{t+1}^{FB}} = \eta \frac{1}{\beta} (L_{t+1}^{FB})^{-\eta-1} > 0 \quad \text{and} \quad \frac{\partial E_t[e^{a_{t+1}}]}{\partial a_t} = \rho E_t[e^{a_{t+1}}] > 0. \]

Proposition 3

Under symmetric equilibrium, each bank solves:

The first-order conditions are given by:

The following condition holds:

Without loss of generality, I restrict attention to the case when the deposit rate is strictly positive.

In the equilibrium, the deposit rate is equal to

Provided that the following condition holds:

\[
\lim_{L_{t+1}^{CE} \to 0} \left( E_t \left[ \alpha e^{a_{t+1}} \left( L_{t+1}^{CE} \right)^{-1} \right] - R_{d,t+1}^{CE} \right) = +\infty,
\]

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I have an interior solution for the optimal level of lending, \( L^cE_{t+1} > 0 \).

For ease of exposition, define the random variable \( u_{t+1} = e^{\omega t + \alpha t + 1} \), which is, conditionally on time \( t \) information, log-normally distributed with mean \( \bar{u}_t = -\frac{1}{2} \sigma^2 + (1 - \rho_a) \hat{a} + \rho_a \hat{a}_t \) and standard deviation \( \sigma_u = \sqrt{\sigma_a^2 + \sigma^2} \). The change of variables allows me to re-write the first-order conditions with respect to lending as follows:

\[
\text{FOC} \left( L_{t+1} \right) = \int_{v_{t+1}^*}^{+\infty} \left( \alpha u_{t+1} L_{t+1}^{\alpha - 2} - R_{d,t+1} \right) dF_t \left( u_{t+1} \right),
\]

where the bailout threshold equals to

\[
v_{t+1}^* = \frac{R_{d,t+1} (L_{t+1} - N_{t+1})}{L_{t+1}^\alpha}.
\]

The second-order condition with respect to lending implies that the objective function is neither concave, nor convex in \( L_{t+1} \):

\[
\left( \frac{\partial^2 L}{\partial L_{t+1}^2} \right) : \int_{v_{t+1}^*}^{+\infty} \alpha (\alpha - 1) u_{t+1} L_{t+1}^{\alpha - 2} dF_t \left( u_{t+1} \right) + \frac{\partial v_{t+1}^*}{\partial L_{t+1}} \left( R_{d,t+1} - \alpha v_{t+1}^* L_{t+1}^{\alpha - 1} \right) f_t \left( v_{t+1}^* \right) \leq 0,
\]

where

\[
\frac{\partial v_{t+1}^*}{\partial L_{t+1}} = \frac{R_{d,t+1} (L_{t+1}^\alpha + \alpha L_{t+1}^{\alpha - 1} (L_{t+1} - N_{t+1}))}{(L_{t+1})^{2\alpha}} > 0.
\]

Given that \( N_{t+1}^cE = 0 \), \( \forall L_{t+1} \in [0, +\infty) \), I can re-write the SOC as follows:

\[
\text{SOC} \left( L_{t+1} \right) = \int_{v_{t+1}^*}^{+\infty} \alpha (\alpha - 1) u_{t+1} L_{t+1}^{\alpha - 2} dF_t \left( u_{t+1} \right) + R_{d,t+1}^2 \left( 1 - \alpha \right)^2 \left( L_{t+1}^\alpha - L_{t+1}^\alpha \right) f_t \left( v_{t+1}^* \right).
\]

The definition of the bailout threshold delivers that \( R_{d,t+1} = v_{t+1}^* L_{t+1}^{\alpha - 1} \), which allows me to further iterate the SOC:

\[
\text{SOC} \left( L_{t+1} \right) = (1 - \alpha) L_{t+1}^{\alpha - 2} \left( -\alpha \int_{v_{t+1}^*}^{+\infty} u_{t+1} dF_t \left( u_{t+1} \right) + v_{t+1}^* (1 - \alpha) f_t \left( v_{t+1}^* \right) \right).
\]

Note that:

\[
\text{SOC} \left( L_{t+1} \right) < 0 \iff \alpha > \alpha \left( v_{t+1}^* \right) \equiv \frac{v_{t+1}^2 f_t \left( v_{t+1}^* \right)}{\int_{v_{t+1}^*}^{+\infty} v_{t+1} dF_t \left( u_{t+1} \right) + v_{t+1}^* f_t \left( v_{t+1}^* \right)}. \quad \text{(A.5)}
\]

I have that:

1.

\[
\lim_{v_{t+1}^* \to 0} \alpha \left( v_{t+1}^* \right) = \frac{\{0\}}{E_t \{v_{t+1} \} + \{0\}} = 0;
\]
2. \[
\begin{align*}
\lim_{v_{t+1} \to +\infty} \alpha (v^*_t) &= \lim_{v_{t+1} \to +\infty} \frac{1}{1 + \frac{\int_{v_{t+1}}^{+\infty} v_{t+1} dF_t (v_{t+1})}{v_{t+1}^2 f_t (v^*_t)}} = 1, \\
\text{since} \\
\lim_{v_{t+1} \to +\infty} \frac{\int_{v_{t+1}}^{+\infty} v_{t+1} dF_t (v_{t+1})}{v_{t+1}^2 f_t (v^*_t)} &= \lim_{v_{t+1} \to +\infty} \frac{-v_{t+1} f_t (v^*_t)}{v_{t+1}^2 f_t (v^*_t) \left(1 - \frac{\log v_{t+1} - \bar{v}_t}{\sigma^2_v}\right)} = \\
&= \lim_{v_{t+1} \to +\infty} \frac{1}{\frac{\log v_{t+1} - \bar{v}_t}{\sigma^2_v}} = 0;
\end{align*}
\]

3. \[
\frac{\partial \alpha (v^*_t)}{\partial v_{t+1}} > 0,
\]

since \[
\text{sign} \left\{ \frac{\partial \alpha (v^*_t)}{\partial v_{t+1}} \right\} = \text{sign} \left\{ v^*_t f_t (v^*_t) \left(1 - \frac{\log v_{t+1} - \bar{v}_t}{\sigma^2_v}\right) \int_{v^*_t}^{+\infty} v_{t+1} dF_t (v_{t+1}) + v^2_{t+1} f_t (v^*_t) \right\} = \\
= \text{sign} \left\{ \left(1 - \frac{\log v_{t+1} - \bar{v}_t}{\sigma^2_v}\right) \int_{v^*_t}^{+\infty} v_{t+1} dF_t (v_{t+1}) + v^2_{t+1} f_t (v^*_t) \right\},
\]

and provided that:

(a) \[
\left(1 - \frac{\log v_{t+1} - \bar{v}_t}{\sigma^2_v}\right) \int_{v^*_t}^{+\infty} v_{t+1} dF_t (v_{t+1}) + v^2_{t+1} f_t (v^*_t) > 0, \quad v^*_t < \exp \{ \bar{v} + \sigma^2_v \},
\]

(b) \[
\frac{\partial}{\partial v_{t+1}} \left(1 - \frac{\log v_{t+1} - \bar{v}_t}{\sigma^2_v}\right) \int_{v^*_t}^{+\infty} v_{t+1} dF_t (v_{t+1}) + v^2_{t+1} f_t (v^*_t) = \\
= -\frac{1}{\sigma^2_v v_{t+1}} \int_{v^*_t}^{+\infty} v_{t+1} dF_t (v_{t+1}) < 0,
\]

(c) \[
\lim_{v^*_t \to +\infty} \left(1 - \frac{\log v_{t+1} - \bar{v}_t}{\sigma^2_v}\right) \int_{v^*_t}^{+\infty} v_{t+1} dF_t (v_{t+1}) + v^2_{t+1} f_t (v^*_t) = \\
= \lim_{v^*_t \to +\infty} \left(1 - \frac{\log v_{t+1} - \bar{v}_t}{\sigma^2_v}\right) \int_{v^*_t}^{+\infty} v_{t+1} dF_t (v_{t+1}) + \frac{v^2_{t+1} f_t (v^*_t)}{\int_{v^*_t}^{+\infty} v_{t+1} dF_t (v_{t+1})} = 0.
\]
Condition A.5, along with the properties of $\alpha \left( v_{t+1}^* \right)$, implies that there exists $v_{t+1}^{**}$, such that:

$$\begin{align*}
SOC \left( L_{t+1} \right) &= \begin{cases} < 0, & v_{t+1}^* \in [0, v_{t+1}^{**}] \\ = 0, & v_{t+1}^* = v_{t+1}^{**} \\ > 0, & v_{t+1}^* \in (v_{t+1}^{**}, +\infty) \end{cases},
\end{align*}$$

where $v_{t+1}^{**}$ is defined by:

$$\alpha = \alpha \left( v_{t+1}^{**} \right).$$

Provided that $v_{t+1}^*$ is monotonically increasing in $L_{t+1}$, there exists a corresponding $L_{t+1}^{**}$, such that:

$$\begin{align*}
SOC \left( L_{t+1} \right) &= \begin{cases} < 0, & L_{t+1} \in [0, L_{t+1}^{**}] \\ = 0, & L_{t+1} = L_{t+1}^{**} \\ > 0, & L_{t+1} \in (L_{t+1}^{**}, +\infty) \end{cases}.
\end{align*} \quad (A.6)$$

Consequently, I have that for $L_{t+1} \in (L_{t+1}^{**}, +\infty)$

$$\frac{\partial}{\partial L_{t+1}} FOC \left( L_{t+1} \right) > 0,$$

which together with

$$\lim_{L_{t+1} \to +\infty} FOC \left( L_{t+1} \right) = 0,$$

delivers that:

$$FOC \left( L_{t+1} \right) < 0, \quad \forall L_{t+1} \in (L_{t+1}^{**}, +\infty).$$

This implies that the objective function is strictly decreasing in the convex region, i.e., when $L_{t+1} \in (L_{t+1}^{**}, +\infty)$. Finally, provided that the following conditions hold:

$$\lim_{L_{t+1} \to 0} FOC \left( L_{t+1} \right) = +\infty,$$

$$\frac{\partial}{\partial L_{t+1}} FOC \left( L_{t+1} \right) < 0, \quad L_{t+1} \in [0, L_{t+1}^{**}),$$

and

$$FOC \left( L_{t+1}^{**} \right) < 0,$$

there exists a unique $L_{t+1}^{CE} \in (0, L_{t+1}^{**})$, such that:

$$FOC \left( L_{t+1}^{CE} \right) = 0.$$

To sum up, the bank’s objective function is concave in lending in the region $L_{t+1} \in (0, L_{t+1}^{**})$ and strictly decreasing when $L_{t+1} \in (L_{t+1}^{**}, +\infty)$. Hence, $L_{t+1}^{CE}$ is a unique global maximum. ■
Proposition 4

Define

\[ F \left( L_{t+1}^{CE}, a_t \right) = E_t \left[ \alpha e^{a_{t+1}} \left( L_{t+1}^{CE} \right)^{\alpha-1} \right] - R_{d,t+1}^{CE} + \xi \left( L_{t+1}^{CE}, N_{t+1}^{CE}; a_t \right) \]  

(\text{A.7})

\[ = \int_{\nu_{t+1}^*}^{+\infty} \left( \alpha \nu_{t+1} \left( L_{t+1}^{CE} \right)^{\alpha-1} - R_{d,t+1}^{CE} \right) dF_t (\nu_{t+1}) = 0 \]

Taking derivative of A.7 with respect to lending delivers

\[
\frac{\partial F \left( L_{t+1}^{CE}, a_t \right)}{\partial L_{t+1}^{CE}} = \int_{\nu_{t+1}^*}^{+\infty} \left( \alpha (\alpha - 1) \nu_{t+1} \left( L_{t+1}^{CE} \right)^{\alpha-2} - \frac{\partial R_{d,t+1}^{CE}}{\partial L_{t+1}^{CE}} \right) dF_t (\nu_{t+1})
+ \frac{\partial R_{d,t+1}^{CE}}{\partial L_{t+1}^{CE}} \left( R_{d,t+1}^{CE} - \alpha \nu_{t+1}^* \left( L_{t+1}^{CE} \right)^{\alpha-1} \right) f_t (\nu_{t+1}^*)
= \text{SOC} \left( L_{t+1}^{CE} \right) \frac{\partial R_{d,t+1}^{CE}}{\partial L_{t+1}^{CE}} (1 - F_t (\nu_{t+1}^*)) < 0,
\]

since the rate of return on deposits is increasing in bank lending

\[ \frac{\partial R_{d,t+1}^{CE}}{\partial L_{t+1}^{CE}} = \eta \frac{\beta}{\alpha} \left( L_{t+1}^{CE} \right)^{-\eta-1}. \]

Before differentiating A.7 with respect to productivity shock, it is useful to re-write \( F \left( L_{t+1}^{CE}, a_t \right) \) as follows:

\[ F \left( L_{t+1}^{CE}, a_t \right) = \left( \alpha E_t \left[ \nu_{t+1} | \nu_{t+1} \geq \nu_{t+1}^* \right] \left( L_{t+1}^{CE} \right)^{\alpha-1} - R_{d,t+1}^{CE} \right) (1 - F_t (\nu_{t+1}^*)), \]

where

\[ E_t \left[ \nu_{t+1} | \nu_{t+1} \geq \nu_{t+1}^* \right] = e^{\tilde{\nu}_{t} + \frac{1}{2} \sigma_{\nu}^2} \Phi \left( \frac{\log \nu_{t+1}^* - \tilde{\nu}_{t}}{\sigma_{\nu}} \right) = e^{\tilde{\nu}_{t} + \frac{1}{2} \sigma_{\nu}^2} \frac{1 - \Phi \left( \frac{\nu_{t+1}^* - \tilde{\nu}_{t}}{\sigma_{\nu}} \right)}{1 - \Phi \left( \tilde{\nu}_{t} \right)}, \]

where, for ease of notation, I define \( \tilde{\nu}_{t} = \frac{\log \nu_{t+1}^* - \tilde{\nu}_{t}}{\sigma_{\nu}} \). Now,

\[
\frac{\partial E_t \left[ \nu_{t+1} | \nu_{t+1} \geq \nu_{t+1}^* \right]}{\partial \tilde{\nu}_{t}} = e^{\tilde{\nu}_{t} + \frac{1}{2} \sigma_{\nu}^2} \frac{\left( 1 - \Phi \left( \tilde{\nu}_{t} - \sigma_{\nu} \right) \right) \Phi \left( \tilde{\nu}_{t} \right) + \frac{1}{\sigma_{\nu}} \left( 1 - \Phi \left( \tilde{\nu}_{t} \right) \right)}{1 - \Phi \left( \tilde{\nu}_{t} \right)}
\]

\[ = \frac{1}{\sigma_{\nu}} e^{\tilde{\nu}_{t} + \frac{1}{2} \sigma_{\nu}^2} \frac{1 - \Phi \left( \tilde{\nu}_{t} - \sigma_{\nu} \right)}{1 - \Phi \left( \tilde{\nu}_{t} \right)} \left( \sigma_{\nu} - \frac{\Phi \left( \tilde{\nu}_{t} \right)}{1 - \Phi \left( \tilde{\nu}_{t} \right)} + \frac{\Phi \left( \tilde{\nu}_{t} - \sigma_{\nu} \right)}{1 - \Phi \left( \tilde{\nu}_{t} - \sigma_{\nu} \right)} \right). \]

Provided that the following conditions hold:
\[ \lim_{\tilde{v}_t \to +\infty} \left( \sigma_v - \frac{\phi(\tilde{v}_t)}{1 - \Phi(\tilde{v}_t)} + \frac{\phi(\tilde{v}_t - \sigma_v)}{1 - \Phi(\tilde{v}_t - \sigma_v)} \right) = \left\{ \sigma_v - \left\{ 0 \right\} + \left\{ 0 \right\} \right\} = \]
\[ \lim_{\tilde{v}_t \to -\infty} \left( \sigma_v - \frac{-\tilde{v}_t\phi(\tilde{v}_t)}{-\Phi(\tilde{v}_t)} + \frac{-\phi(\tilde{v}_t - \sigma_v)}{-\Phi(\tilde{v}_t - \sigma_v)} \right) = 0, \]

\[ \lim_{\tilde{v}_t \to -\infty} \left( \sigma_v - \frac{-\tilde{v}_t\phi(\tilde{v}_t)}{-\Phi(\tilde{v}_t)} + \frac{-\phi(\tilde{v}_t - \sigma_v)}{-\Phi(\tilde{v}_t - \sigma_v)} \right) = \left\{ \sigma_v - \left\{ 0 \right\} + \left\{ 0 \right\} \right\} = \sigma_v, \]

\[ \frac{\partial}{\partial \tilde{v}_t} \left( \sigma_v - \frac{\phi(\tilde{v}_t)}{1 - \Phi(\tilde{v}_t)} + \frac{\phi(\tilde{v}_t - \sigma_v)}{1 - \Phi(\tilde{v}_t - \sigma_v)} \right) = \]
\[ \frac{\partial}{\partial \tilde{v}_t} (\sigma - \lambda (-\tilde{v}_t) + \lambda (-\tilde{v}_t + \sigma)) = \lambda'(-\tilde{v}_t) - \lambda'(-\tilde{v}_t + \sigma) < 0, \]

where \( \lambda(\tilde{v}_t) \equiv \frac{\phi(\tilde{v}_t)}{\Phi(\tilde{v}_t)} \) is an inverse of Mill’s ratio, with \( \lambda'(\cdot) < 0 \) and \( \lambda''(\cdot) > 0 \).

I have that:

\[ \text{sign} \left\{ \frac{\partial E_t [v_{t+1} | v_{t+1} \geq v^*_t]}{\partial \tilde{v}_t} \right\} = \text{sign} \left\{ \left( \sigma_v - \frac{\phi(\tilde{v}_t)}{1 - \Phi(\tilde{v}_t)} + \frac{\phi(\tilde{v}_t - \sigma_v)}{1 - \Phi(\tilde{v}_t - \sigma_v)} \right) \right\} = \oplus. \]

This implies that:

\[ \frac{\partial F(L_{t+1}^{CE}, a_t)}{\partial a_t} = \alpha \frac{\partial E_t [v_{t+1} | v_{t+1} \geq v^*_t]}{\partial \tilde{v}_t} \frac{\partial \nu_t}{\partial a_t} (L_{t+1}^{CE})^{\alpha-1} (1 - F_t (v^*_t)) > 0. \]

Using implicit function theorem and equation A.7, the pro-cyclicality of \( L_{t+1}^{CE} \)

\[ \frac{dL_{t+1}^{CE}}{da_t} = -\frac{\partial F(L_{t+1}^{CE}, a_t)}{\partial a_t} \frac{\partial \nu_t}{\partial L_{t+1}^{CE}} > 0. \]
**Proposition 5**

Employing the results from the derivation of Propositions 2 and 4, I have that:

\[
\text{sign}\left\{ \frac{d}{d a_t} L_{t+1}^{CE} \right\} = \text{sign}\left\{ \frac{d L_{t+1}^{CE}}{d a_t} L_{t+1}^{FB} \right\} = \text{sign}\left\{ \frac{d L_{t+1}^{CE}}{d a_t} L_{t+1}^{FB} - \frac{d L_{t+1}^{FB}}{d a_t} L_{t+1}^{CE} \right\}
\]

\[
= \text{sign}\left\{ E_t \left[ \alpha \rho e^{a_{t+1}} \left( L_{t+1}^{CE} \right)^{\alpha-1} \right] + \frac{\partial \xi(L_{t+1}^{CE}, N_{t+1}^{CE}; a_t)}{\partial a_t} L_{t+1}^{FB} \right\}
\]

This implies that the level of the excessive investment is pro-cyclical if and only if

\[
\frac{\partial \xi(L_{t+1}^{CE}, N_{t+1}^{CE}; a_t)}{\partial a_t} < -\xi_{a,t},
\]

where

\[
\xi_{a,t} = E_t \left[ \alpha \rho e^{a_{t+1}} \left( L_{t+1}^{FB} \right)^{\alpha-1} \right]
\]

I can also show that the bailout wedge for any level of lending and equity financing is monotonically decreasing in \( a_t \). To do this, I re-write the bailout wedge as follows:

\[
\xi(L_{t+1}^{CE}, N_{t+1}^{CE}; a_t) = E_t \left[ \int_0^{\omega_{t+1}} (R_{d,t+1} - \alpha e^{a_{t+1}} L_{t+1}^{a-1}) d\Phi(\omega) \right]
\]

\[
= \int_{v_{t+1}}^{+\infty} (R_{d,t+1} - \alpha v_{t+1} L_{t+1}^{a-1}) dF_t(v_{t+1})
\]

\[
= (R_{d,t+1} \alpha E_t \left[ v_{t+1} \mid v_{t+1} \leq v_{t+1}^* \right] L_{t+1}^{a-1}) \Phi(v_{t+1}),
\]

where

\[
E_t \left[ v_{t+1} \mid v_{t+1} \leq v_{t+1}^* \right] = e^{\tilde{v}_t + \frac{1}{2} \sigma_v^2} \frac{\Phi(\tilde{v}_t - \sigma_v)}{\Phi(\tilde{v}_t)}
\]

Now,

\[
\frac{\partial E_t \left[ v_{t+1} \mid v_{t+1} \leq v_{t+1}^* \right]}{\partial \tilde{v}_t} = e^{\tilde{v}_t + \frac{1}{2} \sigma_v^2} \left( \frac{\Phi(\tilde{v}_t - \sigma_v)}{\Phi(\tilde{v}_t)} + \frac{1}{\sigma_v} \phi(\tilde{v}_t - \sigma_v) \phi(\tilde{v}_t) + \frac{1}{\sigma_v} \phi(\tilde{v}_t - \sigma_v) \phi(\tilde{v}_t) \right) \Phi(\tilde{v}_t)^2
\]

\[
= \frac{1}{\sigma_v} e^{\tilde{v}_t + \frac{1}{2} \sigma_v^2} \left( \frac{\Phi(\tilde{v}_t - \sigma_v)}{\Phi(\tilde{v}_t)} + \phi(\tilde{v}_t) - \phi(\tilde{v}_t - \sigma_v) \right) \Phi(\tilde{v}_t)^2
\]

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Provided that the following conditions hold:

(i) 
\[
\lim_{\tilde{v}_t \to -\infty} \left( \sigma_v + \frac{\phi(\tilde{v}_t)}{\Phi(\tilde{v}_t)} - \frac{\phi(\tilde{v}_t - \sigma_v)}{\Phi(\tilde{v}_t - \sigma_v)} \right) = \left\{ \sigma_v + \{0\} - \{0\} \right\}  
\]

(ii) 
\[
\lim_{\tilde{v}_t \to -\infty} \left( \sigma_v + \frac{\phi(\tilde{v}_t)}{\Phi(\tilde{v}_t)} - \frac{\phi(\tilde{v}_t - \sigma_v)}{\Phi(\tilde{v}_t - \sigma_v)} \right) = \left\{ \sigma_v + \{0\} - \{0\} \right\} = \sigma_v, 
\]

(iii) 
\[
\frac{\partial}{\partial \tilde{v}_t} \left( \sigma_v + \frac{\phi(\tilde{v}_t)}{\Phi(\tilde{v}_t)} - \frac{\phi(\tilde{v}_t - \sigma_v)}{\Phi(\tilde{v}_t - \sigma_v)} \right) = \frac{\partial}{\partial \tilde{v}_t} (\sigma - \lambda(\tilde{v}_t) \lambda(\tilde{v}_t - \sigma)) = \lambda'(\tilde{v}_t) - \lambda'(\tilde{v}_t - \sigma) > 0, 
\]

I have that 
\[
\text{sign} \left\{ \frac{\partial E_t [v_t+1 | v_t+1 \leq v^*_t+1]}{\partial \tilde{v}_t} \right\} = \text{sign} \left\{ \left\{ \sigma_v + \frac{\phi(\tilde{v}_t)}{\Phi(\tilde{v}_t)} - \frac{\phi(\tilde{v}_t - \sigma_v)}{\Phi(\tilde{v}_t - \sigma_v)} \right\} \right\} = \oplus. 
\]

This implies that
\[
\frac{\partial \xi (L_{t+1}, N_{t+1}; a_t)}{\partial a_t} = -a_t \frac{\partial E_t [v_t+1 | v_t+1 \leq v^*_t+1]}{\partial \tilde{v}_t} \frac{\partial \tilde{v}_t}{\partial a_t} L_{t+1}^{\alpha-1} \Phi(\tilde{v}_{t+1}) 
\]
\[
+ \xi (L_{t+1}, N_{t+1}; a_t) \frac{\phi(\tilde{v}_{t+1})}{\Phi(\tilde{v}_{t+1})} \frac{\partial \tilde{v}_t}{\partial a_t} \frac{\partial \tilde{v}_t}{\partial a_t} < 0. 
\]

**Proposition 6**

In a competitive equilibrium with capital regulation in place, each bank solves:

\[
\mathcal{L} = \max_{L_{t+1}, N_{t+1} \leq L_{t+1}} E \left[ \sum_{t=0}^{\infty} \beta^t \left( \{ e^{\omega_t + a_t} L_t^\alpha - R_{d,t} L_t + R_{d,t} N_t \}^+ - N_{t+1} + \lambda_t (N_{t+1} - \zeta_t L_{t+1}) \right) \right],
\]

where \( \lambda_t \) is the Lagrange multiplier associated with the capital constraint. The corresponding first-order conditions are given by:

\[
\left( \frac{\partial \mathcal{L}}{\partial N_{t+1}} \right) : -1 + \lambda_t + \beta R_{d,t+1}^C E_t \left[ \int_0^{\omega_{t+1}} R_{d,t+1}^C \Phi(\omega) \right] \leq 0, 
\]

\[
= 0 \quad \text{if} \quad N_{t+1}^C > 0. 
\]
\[
\left( \frac{\partial L}{\partial L_{t+1}} \right) : -\lambda_t \zeta_t + E_t \left[ \alpha e^{\alpha_{t+1}} \left( L_{t+1}^{CE} \right)^{\alpha-1} \right] - R_{d,t+1}^{CE} + \xi \left( L_{t+1}^{CE}, N_{t+1}^{CE} ; a_t \right) \leq 0,
\]
\[
= 0 \text{ if } L_{t+1}^{CE} > 0,
\]
\[
\left( \frac{\partial}{\partial \lambda_t} \right) : N_{t+1}^{CE} - \zeta_t L_{t+1}^{CE} \geq 0,
\]
\[
= 0 \text{ if } \lambda_t > 0.
\]

Suppose first that the capital constraint is slack, i.e., \( N_{t+1}^{CE} > \zeta_t L_{t+1}^{CE} \). This means that the Lagrange multiplier is equal to zero, which in turn implies that the bank’s objective function is strictly decreasing in \( N_{t+1}^{CE} \), implying a corner solution for the level of equity financing \( N_{t+1}^{CE} = 0 \). This is a contradiction. ■

**Proposition 7**

Substituting equation 1.17 into the bank’s first order conditions with respect to lending 1.15, I obtain that:
\[
E_t \left[ \alpha e^{\alpha_{t+1}} \left( L_{t+1}^{CE} \right)^{\alpha-1} \right] = R_{d,t+1}^{CE},
\]
which implies that \( L_{t+1}^{CE} = L_{t+1}^{FB} \). In an equilibrium with lending capital requirement in place, the deposit rate is equal to:
\[
R_{d,t+1}^{CE} = \frac{1}{\beta} - \frac{1}{\beta} \left( (1 - \zeta_t) L_{t+1}^{CE} \right)^{-\eta},
\]
and the bailout rate is defined by:
\[
e^{a_{t+1} + \alpha_{t+1}} \left( L_{t+1}^{CE} \right)^{\alpha} = R_{d,t+1}^{CE} (1 - \zeta_t) L_{t+1}^{CE}.
\]

Given that the following conditions hold:

\[
(i)
\]

\[
\frac{\partial}{\partial \zeta_t} \left( \theta \left( L_{t+1}^{FB}, \zeta_t \right) - R_{d,t+1}^{FB} \right) = (1 - \zeta_t) \frac{\partial R_{d,t+1}^{CE}}{\partial \zeta_t} + \left( \frac{1}{\beta} - R_{d,t+1}^{CE} \right)
\]
\[
= -\eta \frac{1}{\beta} \left( (1 - \zeta_t) L_{t+1}^{FB} \right)^{-\eta} + \frac{1}{\beta} \left( (1 - \zeta_t) L_{t+1}^{FB} \right)^{-\eta}
\]
\[
= (1 - \eta) \frac{1}{\beta} \left( (1 - \zeta_t) L_{t+1}^{FB} \right)^{-\eta} > 0,
\]

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\(\partial \xi (L^{FB}_{t+1}, \zeta; a_t) = E_t \left[ \int_0^{\omega_{t+1}^*} \left( -R_{d,t+1}^L + (1 - \zeta_t) \frac{\partial R_{d,t+1}^L}{\partial \zeta_t} \right) \Phi (\omega) \right] + E_t \left[ \frac{\partial \omega_{t+1}^*}{\partial \zeta_t} (1 - \zeta_t) (1 - \alpha) R_{d,t+1}^L \phi (\omega_{t+1}^*) \right] < 0, \)

with
\[
\frac{\partial \omega_{t+1}^*}{\partial \zeta_t} = -\frac{1}{1 - \zeta_t} + \frac{1}{R_{d,t+1}^L} \frac{\partial R_{d,t+1}^L}{\partial \zeta_t} < 0,
\]

there is a unique solution to equation 1.17. ■

**Proposition 8**

Substituting equation 1.18 into the bank’s first order condition with respect to lending 1.15, I obtain that:
\[
E_t \left[ \alpha e^{a_{t+1}} \left( L^{FB}_{t+1} \right)^{\alpha - 1} \right] = R_{d,t+1}^{FD},
\]

which implies that \(L^{FD}_{t+1} = \frac{L^{FB}_{t+1}}{1 - \zeta_t^D} \) and \(D^{FD}_{t+1} = D^{FB}_{t+1} \). In an equilibrium with liquidity capital requirement in place, the deposit rate is equal to:
\[
R_{d,t+1}^{FD} = \frac{1}{\beta} - \frac{1}{\beta} \left( (1 - \zeta_t^D) L^{FD}_{t+1} \right)^{-\eta} = R_{d,t+1}^{FB},
\]

and the bailout rate is defined by:
\[
e^{\omega_{t+1}^* + a_{t+1}} \left( L^{FD}_{t+1} \right)^{\alpha} = R_{d,t+1}^{FD} (1 - \zeta_t^D) L^{FD}_{t+1},
\]
or, equivalently:
\[
e^{\omega_{t+1}^* + a_{t+1}} \left( L^{FB}_{t+1} \right)^{\alpha} = R_{d,t+1}^{FD} (1 - \zeta_t^D) L^{FB}_{t+1}.
\]
Given that the following conditions hold:

(i)

\[ \frac{\partial}{\partial \zeta_t} \left( (1 - \zeta_t)^{\alpha-1} \theta \left( \frac{L_{t+1}^{FB}}{1 - \zeta_t}; \zeta_t \right) - R_{d,t+1}^{FB} \right) = (1 - \alpha) (1 - \zeta_t)^{\alpha-2} \theta \left( \frac{L_{t+1}^{FB}}{1 - \zeta_t}; \zeta_t \right) \\
+ (1 - \zeta_t)^{\alpha-1} \left( \frac{1}{\beta} - R_{d,t+1}^{CB} \right) > 0, \]

(ii)

\[ \frac{\partial}{\partial \zeta_t} \left( (1 - \zeta_t)^{\alpha-1} \tilde{\xi} \left( \frac{L_{t+1}^{FB}}{1 - \zeta_t}; \zeta_t \right) \right) = -\alpha (1 - \zeta_t)^{\alpha-1} E_t \left[ \int_0^{\omega_{t+1}^*} R_{d,t+1}^{CB} d\Phi(\omega) \right] \\
+ E_t \left[ \frac{\partial \omega_{t+1}^*}{\partial \zeta_t} (1 - \zeta_t)^{\alpha-1} (1 - \alpha) R_{d,t+1}^{CB} \phi(\omega_{t+1}^*) \right], \]

with

\[ \frac{\partial \omega_{t+1}^*}{\partial \zeta_t} = -\frac{\alpha}{1 - \zeta_t} < 0, \]

(iii)

\[ \theta \left( L_{t+1}^{FB}; 0 \right) = R_{d,t+1}^{FB} \quad \& \quad \tilde{\xi} \left( L_{t+1}^{FB}; a_t \right) > 0, \]

there is a unique solution to equation 1.18. □
APPENDIX B: Ramsey Equilibrium

A Ramsey planner maximizes the lifetime utility of households subject to the resource constraint and implementability conditions:

$$\max_{C_t, L_{t+1}, N_{t+1}, D_{t+1}, K_{f,t+1}} E \left[ \sum_{t=0}^{\infty} \beta^t u(C_t, D_{t+1}) \right]$$

with

$$u(C_t, D_{t+1}) = \frac{v(C_t, D_{t+1})^{1-\gamma}}{1-\gamma} - 1$$

s.t.

$$C_t = R_{d,t} D_t - D_{t+1} + R_{k,t} K_{f,t}^{\alpha_f} - I_{f,t} - o_f K_{f,t} + Z_t + \Pi_t - T_t, \quad (B.1)$$

$$E_t [M_{t,t+1} (R_{k,t+1} + (1 - \delta - o_f))] = 1, \quad (B.2)$$

$$N_{t+1} + D_{t+1} = L_{t+1}, \quad (B.3)$$

$$E_t [M_{t,t+1} (R_{l,t+1} + (1 - \delta - o_b))] = \theta_t - \xi_t, \quad (B.4)$$

$$C_t + I_{b,t} + I_{f,t} = Y_{b,t} + Y_{f,t} - o_b L_t - o_f K_{f,t}, \quad (B.5)$$

where the taxes and net proceeds from banks’ and firms’ activity are, respectively, equal to:

$$T_t = \int_0^1 \max \left\{ R_{d,t} D_t - e^{\omega_{j,t} + \alpha_t} L_t^{\alpha_b} - (1 - \delta - o_b) L_t, 0 \right\} dj, \quad (B.6)$$

$$Z_t = \int_0^1 z_{j,t} dj, \quad \text{with} \quad z_{j,t} = \max \left\{ e^{\omega_{j,t} + \alpha_t} L_t^{\alpha_b} + (1 - \delta - o_b) L_t - R_{d,t} D_t, 0 \right\} - N_{t+1}, \quad (B.7)$$

$$\Pi_t = Y_{f,t} - R_{k,t} K_{f,t}. \quad (B.8)$$

The rate of return on deposits, the stochastic discount factor, the liquidity and risk-shifting cost of lending, the rate of return on loans and capital are, correspondingly, defined by 1.19, 1.20, 1.21, 1.22, 1.23 and 1.24. The optimal Ramsey policy requires a capital ratio that equals:

$$\zeta_t = \frac{N_{t+1}}{L_{t+1}}. \quad (B.9)$$

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The Ramsey problem can be further simplified. By substituting expressions B.6, B.7 and B.8 in the household’s budget constraint B.1, I obtain the resource constraint B.5, therefore implying that I can omit it.

**Proposition 9** The allocation rules \( \{C_t, D_t, \{z_{j,t}, l_{j,t+1}, n_{j,t+1}, d_{j,t+1}\}_{j \in \Omega}, K_{f,t+1}\}_{t=0}^\infty \) and policies \( \{\zeta_t, T_t\}_{t=0}^\infty \) in a competitive equilibrium satisfy conditions B.1-B.9. Furthermore, given the allocation rules and policies that satisfy B.1-B.9, we can construct the price rules \( \{M_{t,t+1}, R_{d,t+1}, R_{l,t+1}, R_{k,t+1}\}_{t=0}^\infty \) that together with the given allocations and policies constitute a competitive equilibrium.

**Proof.** The first part of the proof is quite straightforward and follows from the fact that the competitive equilibrium is symmetric. For the second part of the proof, I use the Ramsey allocations to construct the prices as follows:

\[
M_{t,t+1} = \left( \frac{v(C_{t+1}, D_{t+2})}{v(C_t, D_{t+1})} \right)^{\frac{1}{\eta}-\gamma} \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\eta}},
\]

\[
R_{d,t+1} = \frac{1}{E_t[M_{t,t+1}]} \left( 1 - \chi \left( \frac{D_{t+1}}{C_t} \right)^{-\frac{1}{\eta}} \right),
\]

\[
R_{l,t+1} = \alpha b e^{\alpha t_{l+1} L_{t+1}^{\alpha b-1}},
\]

\[
R_{k,t+1} = \alpha f e^{\alpha t_{k+1} K_{f,t+1}^{\alpha f-1}}.
\]

The expressions for the SDF, deposits rate and rate of return on capital along with the budget constraint B.1 and condition B.2 ensure that the households choose their optimal allocations for consumption and deposits. Similarly, the expression for the rate of return on loans together with the capital constraint B.9, balance sheet constraint B.3 and condition B.4 ensures that the banks choose their optimal lending level and capital structure. The net distributions to the banks’ shareholders B.7 are defined by the same expressions in the competitive and Ramsey equilibria. The government budget constraint B.6 and resource constraint B.5 are identical. ■

Alternatively, the problem of a Ramsey planner can be restated in accordance with the pri-
mal approach, by substituting in prices, namely, $R_{d,t+1}$, $R_{l,t+1}$ and $R_{k,t+1}$. Given the policy rules $\{c_t^*, T_t^*\}_{t=1}^\infty$ that solve the Ramsey problems, the households, banks and firms would optimally choose the allocation rules $\left\{C_t^*, \left\{z_{j,t}^*\right\}_{j\in\Omega}, Z_t^*, L_t^*+1, N_t^*+1, D_t^*+1, K_{f,t+1}^*\right\}_{t=0}^\infty$ in a competitive equilibrium.
APPENDIX C: Welfare Costs of Capital Requirement

The compensating variation \( \varpi \), which is the permanent change in consumption that households are willing to accept to move from the economy with a fixed level of capital requirement \( \zeta^0 \) to the world with a capital ratio of \( \zeta^1 \), is defined by:

\[
E_{\zeta^0} \left[ \sum_{t=0}^{\infty} \beta^t \left( \frac{(1 + \varpi) \bar{C}_t}{1 - \gamma} \right)^{1-\gamma} - 1 \right] = E_{\zeta^1} \left[ \sum_{t=0}^{\infty} \beta^t \frac{\bar{C}_t^{1-\gamma} - 1}{1 - \gamma} \right].
\]  

(C.1)

or, equivalently:

\[
\varpi = \left( \frac{E_{\zeta^1} \left[ \sum_{t=0}^{+\infty} \beta^t \bar{C}_t^{1-\gamma} \right]}{E_{\zeta^0} \left[ \sum_{t=0}^{+\infty} \beta^t \bar{C}_t^{1-\gamma} \right]} \right)^{\frac{1}{1-\gamma}} - 1.
\]

To obtain an estimate of expected lifetime utility, I first solve for agents’ decision rules in a competitive equilibrium, varying the capital regulation in place. Next, I perform 1,000 simulations of a Ramsey equilibrium. Within each simulation, at a random point on the time path, a new regime of capital charges is introduced. Then, using agents’ decision rules, I generate 10,000 observations for the households’ consumption and deposits from the period when a new regime has been introduced on. Finally, I evaluate the realized utility under the new regime and average it over the simulations. This approach allows me to take into account the transition dynamics between different policy regimes.

It is crucial to take into the account the transition dynamics to get the correct welfare ranking. In a competitive equilibrium with a lower level of capital requirement, the financial intermediaries accumulate a higher stock of capital. In the long-run, this delivers a higher level of consumption, but it is costly for the households in the short run, since they have to reduce consumption in order to boost capital investment. So, if I were to compare the realized utility based on the simulations where I burn initial observations, I would find that economies with strict capital requirements always generate lower welfare results than economies with low capital ratios. To understand why the compensation variation is affected
by the transition dynamics and changes in the steady state levels, define consumption as 
the product of its cycle component, denoted by \( \tilde{C}_t^\ast \), and level component, denoted by \( g_t \):\(^1\)

\[
\tilde{C}_t = g_t \tilde{C}_t^\ast.
\]

By taking a second-order Taylor expansion of the momentary utility function, I obtain:

\[
E \left[ \left( \tilde{C}_t^\ast \right)^{1-\gamma} \right] \approx 1 - \frac{1}{2} \gamma (1 - \gamma) \sigma_{\tilde{C}_t^\ast}^2.
\]

Substituting the above expression in C.1 delivers:

\[
(1 + \varpi)^{1-\gamma} \left( 1 - \frac{1}{2} \gamma (1 - \gamma) \sigma_{\tilde{C}_t^\ast}^2 \right) \left( \sum_{t=0}^{\infty} \beta^t \left( g_t^{0} \right)^{1-\gamma} \right) = \left( 1 - \frac{1}{2} \gamma (1 - \gamma) \sigma_{\tilde{C}_t^\ast}^2 \right) \left( \sum_{t=0}^{\infty} \beta^t \left( g_t^{1} \right)^{1-\gamma} \right).
\]

By taking logs and using that \( \log (1 + x) \approx x \), I get:

\[
\varpi \approx -\frac{1}{2} \gamma \left( \sigma_{\tilde{C}_t^\ast}^2 \gamma^1 - \sigma_{\tilde{C}_t^\ast}^2 \gamma^0 \right) + \frac{1}{1 - \gamma} \left( \log \left( \sum_{t=0}^{\infty} \beta^t \left( g_t^{1} \right)^{1-\gamma} \right) - \log \left( \sum_{t=0}^{\infty} \beta^t \left( g_t^{0} \right)^{1-\gamma} \right) \right).
\]

Recall that the Lucas cost of eliminating business cycle risk in this framework is equal to

\[-\frac{1}{2} \gamma \sigma_{\tilde{C}_t}^2.\]

\(^1\)The level component \( g_t \) captures the transition dynamics (temporary trend) between a steady state of
the economy with a fixed level of capital requirement \( \zeta^0 \) to a steady state of the world with a capital ratio
of \( \zeta^1 \).
## Table D.1: Barclays Index Data

<table>
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<tr>
<th>Index</th>
<th>Start Date</th>
<th>Quality</th>
<th>Minimum Issue Size</th>
<th>Minimum Maturity (or Average Life)</th>
<th>Size ($billion)</th>
<th>Date</th>
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<tr>
<td><strong>Long-Term Debt Components:</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>Dec-2014</td>
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<td>U.S. Corporate IG</td>
<td>Jan-73</td>
<td>IG</td>
<td>$250 m</td>
<td>1 year</td>
<td>$3,892</td>
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<td>U.S. Corporate HY</td>
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<td>HY</td>
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<td>U.S. 144a Ex-Aggregate</td>
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<tr>
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APPENDIX E: Cyclical Properties of Payouts: Robustness

Figure E.1 shows the term structure of asset payout cyclicality based on value-weighted global GDP, value-weighted global GDP excluding the United States, and equal-weighted global GDP. To compute global GDP, we use the OECD quarterly output data for 17 major industrialized countries, such as the United States, Canada, France, Germany, Italy, Japan, the United Kingdom, Australia, Belgium, Denmark, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, and Switzerland.

We perform a variety of checks to assess the robustness of our payout cyclicality results. Specifically, in addition to the benchmark sample from 1975 to 2014, we consider shorter sample which stops in 2006 before the Financial Crisis, as well as the most recent sample from 2007 to 2014. For equity data, we also provide the results for the 1949-2014 and 1949-2006 samples. The benchmark results are based on the changes in annual payouts sampled at annual frequency. To extend the sample size, we consider sampling the data at quarterly frequency. First, we consider changes in annual payouts (that is, the payouts over the past four quarters relative to the payouts over the same four quarters in the previous year), sampled at quarterly frequency. Next, we look at changes in quarterly payouts, which are seasonally adjusted through the band-pass filter or the X-12 ARIMA filter. Finally, we consider year-to-year changes in quarterly payouts (that is, quarterly payouts this year relative to the payout in the same quarter in a previous year), again sampled at quarterly frequency. The results are consistent across all the specifications, and show that cash payouts are generally procyclical, while changes in net issuances and total payouts seem acyclical to mildly counter-cyclical.
Figure E.1: Term structure of Asset Payout Cyclicality: Global Risks

a) Value-Weighted Global GDP

b) Value-Weighted Global GDP, excluding US

Equal-Weighted Global GDP

The Figure shows multi-horizon correlations between changes in total asset payouts, scaled by the global output level, and measures of global output growth. The left panel shows the results for the cash payouts, the middle panel is for the net issuances, and the right panel plots the correlations for the total payouts. Measures of global output include value-weighted GDP, value-weighted GDP excluding the U.S., and equally-weighted GDP. The data are real quarterly observations, sampled on a quarterly frequency and from Q1.1975 to Q4.2014.
Table E.1: Equity Payout Cyclicality

<table>
<thead>
<tr>
<th></th>
<th>$\frac{\Delta D_t}{C_t}$</th>
<th>$\frac{\Delta NI_t}{C_t}$</th>
<th>$\frac{\Delta ISS_t}{C_t}$</th>
<th>$\frac{\Delta REP_t}{C_t}$</th>
<th>$\frac{\Delta Y_{a,t}}{C_t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sample 1949-2014:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annual change</td>
<td>0.16</td>
<td>0.11</td>
<td>0.18</td>
<td>0.16</td>
<td>-0.10</td>
</tr>
<tr>
<td>Quarterly change, band-passed</td>
<td>0.27</td>
<td>0.21</td>
<td>0.27</td>
<td>0.19</td>
<td>-0.20</td>
</tr>
<tr>
<td>Quarterly change, x12 ARIMA</td>
<td>0.16</td>
<td>0.08</td>
<td>0.12</td>
<td>0.09</td>
<td>-0.09</td>
</tr>
<tr>
<td>Quarterly change, year-to-year</td>
<td>0.25</td>
<td>0.10</td>
<td>0.16</td>
<td>0.15</td>
<td>-0.09</td>
</tr>
<tr>
<td><strong>Sample 1949-2006:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annual change</td>
<td>0.12</td>
<td>0.18</td>
<td>0.21</td>
<td>0.12</td>
<td>-0.17</td>
</tr>
<tr>
<td>Quarterly change, band-passed</td>
<td>0.16</td>
<td>0.26</td>
<td>0.28</td>
<td>0.13</td>
<td>-0.25</td>
</tr>
<tr>
<td>Quarterly change, x12 ARIMA</td>
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<td>0.11</td>
<td>0.13</td>
<td>0.04</td>
<td>-0.12</td>
</tr>
<tr>
<td>Quarterly change, year-to-year</td>
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<td>0.15</td>
<td>0.18</td>
<td>0.09</td>
<td>-0.14</td>
</tr>
<tr>
<td><strong>Sample 1975-2014:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annual change</td>
<td>0.23</td>
<td>-0.12</td>
<td>0.07</td>
<td>0.25</td>
<td>0.14</td>
</tr>
<tr>
<td>Quarterly change, band-passed</td>
<td>0.30</td>
<td>0.02</td>
<td>0.17</td>
<td>0.29</td>
<td>0.00</td>
</tr>
<tr>
<td>Quarterly change, x12 ARIMA</td>
<td>0.19</td>
<td>0.01</td>
<td>0.08</td>
<td>0.16</td>
<td>-0.01</td>
</tr>
<tr>
<td>Quarterly change, year-to-year</td>
<td>0.23</td>
<td>-0.03</td>
<td>0.13</td>
<td>0.22</td>
<td>0.05</td>
</tr>
<tr>
<td><strong>Sample 1975-2006:</strong></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Annual change</td>
<td>0.19</td>
<td>-0.06</td>
<td>0.09</td>
<td>0.21</td>
<td>0.07</td>
</tr>
<tr>
<td>Quarterly change, band-passed</td>
<td>0.02</td>
<td>0.11</td>
<td>0.18</td>
<td>0.18</td>
<td>-0.11</td>
</tr>
<tr>
<td>Quarterly change, x12 ARIMA</td>
<td>0.18</td>
<td>0.05</td>
<td>0.09</td>
<td>0.08</td>
<td>-0.05</td>
</tr>
<tr>
<td>Quarterly change, year-to-year</td>
<td>0.22</td>
<td>0.04</td>
<td>0.14</td>
<td>0.14</td>
<td>-0.03</td>
</tr>
<tr>
<td><strong>Sample 2007-2014:</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Annual change</td>
<td>0.53</td>
<td>-0.34</td>
<td>-0.03</td>
<td>0.28</td>
<td>0.39</td>
</tr>
<tr>
<td>Quarterly change, band-passed</td>
<td>0.51</td>
<td>0.09</td>
<td>0.23</td>
<td>0.16</td>
<td>-0.03</td>
</tr>
<tr>
<td>Quarterly change, x12 ARIMA</td>
<td>0.33</td>
<td>-0.13</td>
<td>0.15</td>
<td>0.37</td>
<td>0.09</td>
</tr>
<tr>
<td>Quarterly change, raw series</td>
<td>0.56</td>
<td>-0.34</td>
<td>0.22</td>
<td>0.58</td>
<td>0.40</td>
</tr>
</tbody>
</table>

The Table reports correlations between changes in the equity payouts, scaled by the consumption level, and consumption growth. The equity payouts include cash payouts, net issuances, issuances, repurchases, and total payouts. Payouts are sampled at quarterly frequency, and are either annual or quarterly, seasonally adjusted through a band-pass filter, X12-ARIMA filter, or by computing year-to-year changes.
Table E.2: Debt Payout Cyclicality

<table>
<thead>
<tr>
<th>Corr $(\cdot, \Delta c_t)$</th>
<th>$\frac{\Delta D_t}{C_t}$</th>
<th>$\frac{\Delta N_t}{C_t}$</th>
<th>$\frac{\Delta D_{a,t}}{C_t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sample 1975-2014:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annual change</td>
<td>0.21</td>
<td>0.13</td>
<td>-0.10</td>
</tr>
<tr>
<td>Quarterly change, band-passed</td>
<td>0.33</td>
<td>0.17</td>
<td>-0.14</td>
</tr>
<tr>
<td>Quarterly change, x12 ARIMA</td>
<td>0.19</td>
<td>0.11</td>
<td>-0.09</td>
</tr>
<tr>
<td>Quarterly change, year-to-year</td>
<td>0.30</td>
<td>0.11</td>
<td>-0.08</td>
</tr>
<tr>
<td><strong>Sample 1975-2006:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annual change</td>
<td>0.03</td>
<td>0.08</td>
<td>-0.08</td>
</tr>
<tr>
<td>Quarterly change, band-passed</td>
<td>0.09</td>
<td>-0.06</td>
<td>0.07</td>
</tr>
<tr>
<td>Quarterly change, x12 ARIMA</td>
<td>0.03</td>
<td>-0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>Quarterly change, year-to-year</td>
<td>0.08</td>
<td>-0.01</td>
<td>0.02</td>
</tr>
<tr>
<td><strong>Sample 2007-2014:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Annual change</td>
<td>0.26</td>
<td>0.24</td>
<td>-0.23</td>
</tr>
<tr>
<td>Quarterly change, band-passed</td>
<td>0.39</td>
<td>0.45</td>
<td>-0.44</td>
</tr>
<tr>
<td>Quarterly change, x12 ARIMA</td>
<td>0.34</td>
<td>0.22</td>
<td>-0.20</td>
</tr>
<tr>
<td>Quarterly change, year-to-year</td>
<td>0.63</td>
<td>0.48</td>
<td>-0.44</td>
</tr>
</tbody>
</table>

The Table reports correlations between changes in debt payouts, scaled by the consumption level, and consumption growth. The debt payouts include cash payouts, net issuances, and total payouts. Payouts are sampled at quarterly frequency, and are either annual or quarterly, seasonally adjusted through a band-pass filter, X12-ARIMA filter, or by computing year-to-year changes.
Table E.3: Asset Payout Cyclicality

<table>
<thead>
<tr>
<th>Table entries</th>
<th>ΔD_t/C_t</th>
<th>ΔNI_t/C_t</th>
<th>ΔD_{a,t}/C_t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual change</td>
<td>0.25</td>
<td>0.00</td>
<td>0.05</td>
</tr>
<tr>
<td>Quarterly change, band-passed</td>
<td>0.41</td>
<td>0.16</td>
<td>-0.11</td>
</tr>
<tr>
<td>Quarterly change, x12 ARIMA</td>
<td>0.26</td>
<td>0.05</td>
<td>-0.02</td>
</tr>
<tr>
<td>Quarterly change, year-to-year</td>
<td>0.34</td>
<td>0.06</td>
<td>-0.02</td>
</tr>
</tbody>
</table>

**Sample 1975-2014:**

<table>
<thead>
<tr>
<th>Table entries</th>
<th>ΔD_t/C_t</th>
<th>ΔNI_t/C_t</th>
<th>ΔD_{a,t}/C_t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual change</td>
<td>0.10</td>
<td>-0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>Quarterly change, band-passed</td>
<td>0.08</td>
<td>0.07</td>
<td>-0.06</td>
</tr>
<tr>
<td>Quarterly change, x12 ARIMA</td>
<td>0.12</td>
<td>0.03</td>
<td>-0.01</td>
</tr>
<tr>
<td>Quarterly change, year-to-year</td>
<td>0.17</td>
<td>0.03</td>
<td>-0.01</td>
</tr>
</tbody>
</table>

**Sample 1975-2006:**

<table>
<thead>
<tr>
<th>Table entries</th>
<th>ΔD_t/C_t</th>
<th>ΔNI_t/C_t</th>
<th>ΔD_{a,t}/C_t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual change</td>
<td>0.39</td>
<td>0.09</td>
<td>-0.00</td>
</tr>
<tr>
<td>Quarterly change, band-passed</td>
<td>0.59</td>
<td>0.38</td>
<td>-0.35</td>
</tr>
<tr>
<td>Quarterly change, x12 ARIMA</td>
<td>0.43</td>
<td>0.06</td>
<td>-0.02</td>
</tr>
<tr>
<td>Quarterly change, year-to-year</td>
<td>0.70</td>
<td>0.30</td>
<td>-0.20</td>
</tr>
</tbody>
</table>

The Table reports correlations between changes in asset payouts, scaled by the consumption level, and consumption growth. The asset payouts include cash payouts, net issuances, and total payouts. Payouts are sampled at quarterly frequency, and are either annual or quarterly, seasonally adjusted through a band-pass filter, X12-ARIMA filter, or by computing year-to-year changes.
Figure E.2: Term structure of Asset Cash Payout Cyclicality: Normalized Changes versus Log Growth Rate

The Figure shows multi-horizon correlations of consumption growth with changes in total asset cash payouts scaled by the consumption level (solid line) or the log growth rates in asset cash payouts (dashed line). The data are real quarterly observations, sampled on a quarterly frequency and from Q1.1975 to Q4.2014.
APPENDIX F: Wavelet Analysis

The wavelet correlation between two stochastic processes \( x \) and \( y \) for scale \( \lambda_j = 2^{j-1} \) equals to

\[
\rho_{xy}(\lambda_j) = \frac{\text{Cov}\left(\vec{W}_{j,t}^{(x)}, \vec{W}_{j,t}^{(y)}\right)}{\left\{ \text{Var}\left(\vec{W}_{j,t}^{(x)}\right) \text{Var}\left(\vec{W}_{j,t}^{(y)}\right) \right\}^{1/2}},
\]

where \( \vec{W}_{j,t}^{(x)} \) and \( \vec{W}_{j,t}^{(y)} \) are the scale \( \lambda_j \) maximal overlap discrete wavelet transform (MODWT) coefficients for \( x \) and \( y \), respectively. Since this is just a correlation coefficient between two random variables on a scale by scale basis, \(-1 \geq \rho_{xy}(\lambda_j) \leq 1\) for all \( j \). The MODWT coefficient for a stochastic process \( u \) is defined as

\[
\vec{W}_{j,t}^{(u)} = \sum_{l=0}^{L_j-1} \tilde{h}_{j,l} u_{t-l},
\]

where \( \{\tilde{h}_{j,0}, ..., \tilde{h}_{j,L_j-1}\} \) are the wavelet filter coefficients from a Daubechies compactly supported wavelet family, with \( L_j = (2^j - 1)(L - 1) + 1 \).

We estimate the sample wavelet correlation by simply using the estimators of wavelet covariance and wavelet variance, respectively,

\[
\hat{\gamma}_{xy}(\lambda_j) = \frac{1}{T_j} \sum_{t=L_j-1}^{T-1} \vec{W}_{j,t}^{(x)} \vec{W}_{j,t}^{(y)} \quad \& \quad \hat{\nu}^2_x(\lambda_j) = \frac{1}{T_j} \sum_{t=L_j-1}^{T-1} \left( \vec{W}_{j,t}^{(x)} \right)^2,
\]

with \( T_j = T - L_j + 1 \).

Whitcher et al. (2000) establish a central limit theorem for the estimator of wavelet correlation,

\[
\hat{\rho}_{xy}(\lambda_j) = \frac{\hat{\gamma}_{xy}(\lambda_j)}{\hat{\nu}_x(\lambda_j) \hat{\nu}_y(\lambda_j)},
\]

and construct an approximate confidence interval (CI). An approximate 100 \((1 - 2p)\) % CI
for $\rho_{xy}(\lambda_j)$ is given by

$$tanh \left\{ tanh^{-1}(\rho_{xy}(\lambda_j)) \pm \frac{\Phi^{-1}(1-p)}{\sqrt{T_j - L'_j - 3}} \right\},$$

with $L'_j = (L - 2)(1 - 2^{-j})$. 
APPENDIX G: Model Solution

The equilibrium consumption claim loadings are given by

\[ A_{0,c} = \frac{1}{1 - \kappa_{1,c}} \left( \log(\delta) + \left(1 - \frac{1}{\psi}\right) \mu_c + \kappa_{0,c} + \kappa_{1,c} A_{2,c} (1 - \nu) \sigma_0^2 + \frac{\theta}{2} (\kappa_{1,c} A_{2,c} \sigma_\omega^2) \right), \]

\[ A_{1,c} = \frac{1 - 1}{1 - \kappa_{1,c} \rho_x}, \]

\[ A_{2,c} = \frac{(1 - 1) (\gamma - 1)}{2 (1 - \kappa_{1,c} \nu)} \left(1 + \left(\frac{\kappa_{1,c} \varphi_x}{1 - \kappa_{1,c} \rho_x}\right)^2\right). \]

The market prices of risks are,

\[ \lambda_\eta = \gamma, \]

\[ \lambda_e = - (\theta - 1) \kappa_{1,c} A_{1,c} \varphi_x, \]

\[ \lambda_\omega = - (\theta - 1) \kappa_{1,c} A_{2,c}. \]

The log-linearization coefficients for the corporate asset satisfy,

\[ \kappa_{0,d} = \log \left(1 + \exp \{\varphi_d\} + \exp \{\varphi_c\} - \exp \{\varphi_c\}\right) - \kappa_{1,d} \varphi_d - \kappa_{2,d} \varphi_c - \kappa_{3,d} \varphi_c, \]

\[ \kappa_{1,d} = \frac{\exp \{\varphi_d\}}{1 + \exp \{\varphi_c\} + \exp \{\varphi_c\} - \exp \{\varphi_c\}}, \]

\[ \kappa_{2,d} = \frac{\exp \{\varphi_c\}}{1 + \exp \{\varphi_c\} + \exp \{\varphi_c\} - \exp \{\varphi_c\}}, \]

\[ \kappa_{3,d} = - \frac{\exp \{\varphi_c\}}{1 + \exp \{\varphi_c\} + \exp \{\varphi_c\} - \exp \{\varphi_c\}}. \]
The loadings for the corporate claim are given by,

\[ A_{0,d} = \frac{1}{1 - \kappa_{1,d}} \left( m_0 + \mu_c + \kappa_{0,d} + \kappa_{1,d} A_{2,d} (1 - \nu) \sigma_0^2 + (\kappa_{1,d} A_{3,d} + \kappa_{2,d}) \mu_s (1 - \rho_s) + \ldots ight. \]
\[ + \kappa_{3,d} \mu_h + 0.5 (\kappa_{1,d} A_{2,d} - \lambda \omega)^2 \sigma_0^2 \right), \]

\[ A_{1,d} = \frac{1}{1 - \kappa_{1,d} \psi} \left( 1 - \frac{1}{\psi} \right) + \frac{\kappa_{2,d} \phi_s}{1 - \kappa_{1,d} \rho_s}, \]

\[ A_{2,d} = \frac{1}{1 - \kappa_{1,d} \nu} \left( m_2 + 0.5 \left( (1 - \lambda y)^2 + (\kappa_{1,d} A_{1,d} \varphi_x - \lambda \epsilon)^2 \right) + \right. \]
\[ + \left. (\kappa_{1,d} A_{3,d} + \kappa_{2,d})^2 \varphi_s^2 + 2 \alpha (1 - \lambda y) (\kappa_{1,d} A_{3,d} + \kappa_{2,d}) \varphi_s + \kappa_{3,d} \varphi_h^2 \right), \]

\[ A_{3,d} = \frac{\kappa_{2,d} \rho_s}{1 - \kappa_{1,d} \psi}. \]
BIBLIOGRAPHY


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