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Credit Ratings With Endogenous Assets

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Abstract

The market prices of securities are heavily dependent on their credit ratings, which can in turn influence the issuers' incentives to invest in asset generation, resulting in inefficiencies. We provide a model with a strategic credit rating agency (CRA) and issuers with endogenous set of assets. Specifically, we consider two ways by which issuers generate assets: (1) bundling assets into securities of varying qualities (securitization) (2) investing in projects funded by issuing corporate bonds (investment). We then analyze the equilibrium of these models and derive the conditions under which ratings can result in over or under investments. Next, we perform comparative statics analysis on the impact of market and macro factors on inefficiency by way of influencing credit ratings. Finally, we show that how ratings models with endogenous assets can explain different ratings performances across different asset classes.

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CREDIT RATINGS WITH ENDOGENOUS ASSETS

Ali Aram

A DISSERTATION

in

Finance

For the Graduate Group in Managerial Science and Applied Economics

Presented to the Faculties of the University of Pennsylvania

in

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ABSTRACT

CREDIT RATINGS WITH ENDOGENOUS ASSETS

Ali Aram

Bilge Yilmaz

The market prices of securities are heavily dependent on their credit ratings, which can in turn influence the issuers' incentives to invest in asset generation, resulting in inefficiencies. We provide a model with a strategic credit rating agency (CRA) and issuers with endogenous set of assets. Specifically, we consider two ways by which issuers generate assets: (1) bundling assets into securities of varying qualities (securitization) (2) investing in projects funded by issuing corporate bonds (investment). We then analyze the equilibrium of these models and derive the conditions under which ratings can result in over or under investments. Next, we perform comparative statics analysis on the impact of market and macro factors on inefficiency by way of influencing credit ratings. Finally, we show that how ratings models with endogenous assets can explain different ratings performances across different asset classes.

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1.1. Introduction

The recent financial crisis drew attention to the role of information intermediaries and their performances, from credit ratings to financial stress tests. Researchers have explored multiple sources of friction in intermediaries' ability to provide information, their incentives to do so truthfully, and their role in improving efficiency and providing risk sharing in the market. Credit rating literature have attributed inflated ratings particularly in asset backed securities, prior to, and their failure after the crash, to a slew of factors including, but not limited to, competition among CRAs, rating shopping, asset complexity, sophisticated versus naive investors, business cycles, reputation concerns, and regulations.

A common feature in the existing theoretical models of credit ratings is that the set of assets for which issuers seek ratings is exogenous. In this paper, we examine credit ratings in a framework in which issuers are able to tailor the composition of their asset portfolios to take advantage of the ratings offered by the CRA, a feature of markets offering securitized instruments, and as we will argue, a key determining factor of rating performance and efficiency.

The market in our model consists of issuers, investors, and a single CRA. Issuers are endowed with assets of differing values, known privately, which they would sell to the uninformed investors at the highest price. Ratings allow issuers to mitigate the information asymmetry by signaling the value of their assets to investors. The rating system consists of an information structure and a fee designed by the CRA with the goal of maximizing the total fee collected.

Our setup builds on the previous works by Lizzeri (1999) and Kartasheva and Yilmaz (2013), by introducing two new features. First we assume that in addition to their endowment assets, issuers have access to a costly *technology* that consists of two bits: 1) A production bit that allows issuers to produce new assets and 2) a securitizaton bit that enables issuers to combine their endowment assets and the newly produced asset in varying proportions to generate new securities pf varying qualities. By utilizing the technology, an issuers is essentially able to select the asset qualities offered in its portfolios of issues.

Second, in our model, issuers and the CRA split the gains from trade of the assets in the market. In comparison, the intermediary in Lizzeri's and the CRA in Kartasheva and Yilmaz's extract all the trade surplus. Whereas the manner in which the surplus is split in models with exogenous pool of assets is in general irrelevant to the their main results, we show that with endogenously determined asset compositions, the degree by which the CRA is able to extract trade surplus critically affects ratings performance, the issuers' composition of assets, and the economy's efficiency.

By combining the above features and solving for the equilibrium, we can examine information disclosure by the CRA, how it is affected by the extent of issuers access to securitization, and what the overall impact on efficiency is. Further, we perform comparative statics on a number of market parameters and their influence on ratings performance and efficiency. Our main results are the following.

Our first result is that in equilibrium all gains from trade are realized and the ratings are informative but noisy and inflated, leading to under-pricing of high quality and over-pricing of low quality assets. We also show that in equilibrium, some issuers would utilize the technology to generate securities that differ in quality from their endowment assets, even if the cost of the technology exceeds the value of the asset it produces. The reason that inefficient investment in the technology is possible is the fact that equilibrium ratings are noisy and induce over pricing of some assets. An issuer can take advantage by constructing securities that replicate the qualities of said assets. We argue that this mechanism can partially explain the over issuance of sub prime mortgages prior to the financial crisis.

Our next result is that credit ratings does not guarantee the efficient outcome, despite the fact that they ensure all gains from trade are realized. The reason is that with endogenous asset composition, ratings some times do induce inefficient investment, which causes the equilibrium total surplus fall short of the first best solution. We derive the most efficient outcome possible in any equilibrium, and show that a reduction in the cost of securitization to issuers can further reduce market efficiency, since it leads to more inefficient asset production and creation of securities. We also show that under certain extreme conditions, the cost of rating induced inefficient investment can fully wipe out the trade surplus, reducing the social benefit of credit ratings to zero.

Next, we perform comparative statics on two otherwise identical markets that differ only in the issuers access to securitization technology. We show that in market with traditional investment assets, ratings outperform those in the market with securitized instruments in the following sense: The probability that a high rated security is of low type is higher in the latter market. The underlying cause for poor performance of ratings of securitized assets is that when equilibrium ratings are inflated, issuers can make a profit by tuning the quality of their issues to those that receive inflated ratings, and securitization technology is the right tool for that. This leads to a decline of the average quality of securitized assets. As a result, all else being equal, we expect ratings of asset-backed securities to under perform their counterparts in corporate bonds market. Importantly, this result holds despite the following properties of the equilibrium in the two markets: (1) the CRA's profit is equal in both markets and (2) the precision of ratings is higher in the market with securitized assets. Put differently, poor performance of ratings of securitized instruments in our model is not the result of the CRA's attempt to draw business by over rating assets, nor is it due to the difficulty of rating complex securities. It is the consequence of the relative ease with which issuers can manipulate the qualities of their issues through securitization, and its interaction with the CRA's choice of ratings.

Next, we find that an increase in the CRA's share of the gains from trade translates into improved efficiency. In other words, market efficiency is directly related to the CRA's market power. A more dominant CRA charges higher fees, which reduces the issuers' profits, and in turn their incentives to undertake inefficient investments. We don't model competition among multiple CRAs. However, as far as the impact of competition on rating fees is concerned, our findings suggest that by increasing competition among rating agencies, ratings performance may suffer.

Finally, we examine the effects on efficiency of business cycles and informed speculation, via their impacts on asset composition. The relative abundance of high quality assets during expansions creates more profit opportunity for the issuers of low quality over priced assets, which results in further inefficient asset production. Informed speculators can therefore improve efficiency by bidding on the under-priced high quality assets.

1.2. Related Literature

The papers closest to ours is Lizzeri (1999) and Kartasheva and Yilmaz (2013). In Lizzeri's, there are a continuum of seller types, risk neutral buyers, and intermediaries who sell certification to sellers. He shows that in equilibrium, a monopolistic intermediary extracts all the surplus without disclosing any information. Also, presence of an intermediary in Lizzeri's framework does not play a role in facilitating trade, but merely transfer of surplus. A key driver of Lizzeri's findings is that a seller's valuation of his asset is 0, for all seller types. Put differently, sellers' reservation values are type independent. In Kartasheva and Yilmaz (2013), issuers have type dependent reservation values. In equilibrium, the CRA discloses information by selling noisy but informative ratings, which facilitates trade and restores market efficiency. Also, they show that winner's curse, caused by investor information heterogeneity, prompts the CRA to increase ratings precision, with the purpose of curbing informed investors' ability to extract surplus.

Our paper extends the above works in two aspects. First, by introducing securitization technology, we allow the composition of assets in our framework to be endogenously determined. We can then examine the impact of ratings on composition of assets and vice versa.

In particular, our model provides a framework to compare ratings performances in markets with traditional investment assets with those offering securitized instruments.

Second, we model market dominance of the CRA by the extent of its ability to extract the surplus. In models with a single CRA and exogenous set of assets, like those in the above works, the way surplus is split among the CRA and the issuers is typically irrelevant. However, we find that with endogenous asset composition, the CRA's market dominance impacts ratings performance and market efficiency: ratings performance and efficiency improve in a market with a more dominant CRA.

Our work is also related to a number of other papers which pursue other factors that influence the performance of ratings. In a model of endogenous reputation, Mathis, McAndrews, and Rochet (2009) show that when a large fraction of a monopoly CRA's income comes from rating complex assets, reputation concerns are not enough incentives to rein in rating inflation. Bolton, Freixas, and Shapiro (2012) examine credit ratings in an environment featuring two CRAs with reputation concerns and the possibility of shopping for ratings. They find that presence of naive investors leads to rating inflation, which is only intensified with competition among CRAs. In Skreta and Veldkamp (2009), CRAs always produce unbiased noisy signals of asset quality. For complex assets, CRAs produce sufficiently different ratings, which together with investor naivete, encourage rating shopping, and in turn, leads to inflation. Opp, Opp, and Harris (2013) show that rating based regulations increase demand for high rated securities, which in turn create incentive for the CRA to inflate ratings.

1.3. The Model

There is an economy with 3 type of risk neutral agents: issuers, a CRA, and investors. Issuer $i \in \{1, 2, 3\}$ is endowed with m_i units[1] of an asset with unit market value $v_i \in V = \{v_1, v_2, v_3\}$, where $0 = v_1 < v_2 < v_3$. We use *type* of an asset to refer its unit value. The asset types are issuers private information. The reservation value of an asset type v for

[1] E.g. dollar face value in fixed income securities.

issuers is δv, with $\delta < 1$.

The difference between assets market and reservation values create an opportunity for issuers to profit by trading them in the market. The total market value of issuers' assets are equal to $\sum_i m_i v_i = m_3 v_3 + m_2 v_2$. Therefore, the total gains from trade available is given by $(1 - \delta)(m_3 v_3 + m_2 v_2)$. Absent the ratings, issuers may fail to realize all or part of the available gains from trade. For instance, consider the condition

$$\frac{\sum_i m_i v_i}{\sum_i m_i} < \delta v_3, \tag{1.1}$$

where the left hand side is the average asset type, as well as its market price if issuers trade all their assets in the absence of ratings. If condition (1.1) holds, issuer 3 would prefer not to sell and there will be a market breakdown. A rating agency can resolve, perhaps partially, the information asymmetry in the market and facilitate trade. We assume the CRA observes asset types and has access to a rating structure which consists of a set of signals and a function which maps each asset type to a probability distribution over the signal set. The CRA charges issuers a flat fee $\phi \geq 0$ per unit of asset rated.

Issuers have access to a *technology* which consists of production and securitization segments. The production's output is an asset type v_p. We assume issuers can produce any amount of type v_p asset. Similar to the endowment assets, the reservation value of a unit of type v_p asset is δv_p. The securitization enables an issuer to combine any ratio of endowment and production assets into a single security that replicates other asset types. The per unit cost of production of v_p asset is denoted by c. Alternatively, we can attribute the cost to the securitization segment of the technology, or any combination of the two. A low c is meant to represent markets with complex securitized asset classes, for instance, mortgage backed securities. Markets with *simple* asset classes, such as corporate bonds, on the other hand, can be modeled with a high c.

We make the following assumption on type v_p and the associated production cost.

$$0 = v_p < c \tag{1.2}$$

Condition (1.2) effectively makes investing in the production of type v_p a negative NPV investment. Absent any information asymmetry, issuers would not invest in this technology. As a result, any potential investment in type v_p asset must be driven by the ratings and the ability of the issuers to combine assets into more complex securities, which is the focus of this paper. And in fact, we will show that in equilibrium, it is sometimes optimal for a profit maximizing issuer to invest in type v_p asset. An example of such investments made viable through access to securitization and ratings market is the sub prime lending prior to financial crisis.

We say a type is *targeted* by the CRA if assets of that type are rated in equilibrium. Consider an arbitrary targeted type v asset held by an issuer. Let $\bar{U}$ denote the average market price of type v, if rated. Aside from purchasing a rating, the issuer can choose between selling the asset unrated or not selling. Let $\bar{u}$ denote the highest average value of the asset to the issuer if not rated. Then the rating creates a surplus of $\bar{U} - \bar{u}$. Clearly, the CRA cannot charge the issuer a fee larger than the surplus or the issuer would refuse to purchase the rating. We place a stronger restriction by assuming that for any rating the CRA successfully sells, the fee ϕ is bounded from above by a fraction μ of the surplusthe rating generates. In the above scenario, for instance, this assumption requires that $\phi \leq \mu(\bar{U} - \bar{u})$. We refer to this upper bound as the issuer's willingness to pay for rating. We can think of μ as the CRA's market power. A CRA with more market power can extract a larger fraction of trade surplus. Our key results require that the issuers receive a non zero fraction of the trade surplus, which is ensured by assuming that $\mu < 1$.[2]

We assume that once decided, the CRA is committed to the rating structure. Finally, the

[2] A setting in which this can happen is where the CRA and the issuers take turns to bid for the rating fee and there is a discount factor between turns of less than 1.

CRA's choice of rating structure and fee is public information. The timing of the game is as follows.

- $t = 0$. The CRA commits to a rating structure.
- $t = 1$. Issuers invest in the technology and decide their security portfolios.
- $t = 2$. The CRA announces the rating fee ϕ.
- $t = 3$. Issuers decide whether or not to solicit a rating for any security in their portfolios.
- $t = 4$. Issuers decide whether or not to sell any of the securities in their portfolios in the market.

The strategy of the CRA includes deciding a rating structure and a rating fee. The rating structure is comprised of a non-empty set of signals S and a mapping from the interval $[0, v_3]$, which is the set of all feasible security types, to probability distributions over S. For instance, a viable rating structure is a set of 3 signals (s_1, s_2, s_3) and a uniformly random assignment of ratings to any security type $v \in [0, v_3]$. With this rating structure, the CRA discloses no information to the market. Another example is the following rating structure. This structure divides security types into 3 tiers. Type v_3, types v_2 and above but below

	v_3	v_2	v_1
s_3	1	0	0
s_2	0	1	0
s_1	0	0	1

Table 1.1: Tiered rating with full disclosure

v_3, and finally types below v_2. The mapping is then simply a deterministic assignment of each tier to signals s_1, s_2, s_3. Therefore, the rating fully reveals which tier a security belongs to but does not differentiate between types belonging to the same tier. If the CRA adopts the rating structure of table 1.1, it turns out that in the continuation game, issuers do not invest in the technology. As a result, the only security types present in the continuation game are v_1, v_2, v_3. Then effectively, this rating fully reveals all types. In the process of deriving an equilibrium of the rating game, it will be shown that the structure of table 1.1 cannot be supported in equilibrium. Kartasheva and Yilmaz (2013) demonstrate this fact in detail.

A class of rating structures with 3 signals that is the focus of this paper is demonstrated in table 2.1. Similar to table 1.1, this rating structure divides security types into 3 tires.

	v_3	v_2	v_1
s_3	1	$1-p$	0
s_2	0	p	0
s_1	0	0	1

Table 1.2: Tiered noisy rating with inflation

Unlike table 1.1, however, the mapping from tiers to signals is not 1 to 1 and therefore, ratings does not fully disclose which tier all assets belongs to. The lowest (highest) tier is rated s_1 (s_3) with probability 1, while the middle tier is rated s_2 with probability p and s_3 with probability $1-p$. Notice that for $p = 1$, this rating structure is identical to the one in table 1.1. As such, p measures how accurately the rating reveals the tier to which the security belongs to. Following Kartasheva and Yilmaz, we refer to p as the rating precision. Finally, note that in the rating structure of table 2.1, mid tier securities are assigned the high rating of s_3 with positive probability. On the other hand, no security is ever assigned a rating lower than their corresponding tier. As such, we say that this structure demonstrates rating inflation.

The strategy of an issuer, given the rating structure, is the choice of its security portfolio, probability of soliciting ratings for each security in the portfolio, and the probability of selling a security in the market based on the realized rating. For instance, consider an issuer endowed with 1 unit of type v asset. A feasible security portfolio for this issuer is

$$\left\{\left(\frac{1}{2}, v\right), \left(\frac{1}{2}, \frac{v}{2}\right), \left(1, \frac{v}{4}\right)\right\}. \tag{1.3}$$

Each element in the set 1.3 describe a security, with the first quantity being the number of units and the second quantity the type of the security. For instance, $\left(\frac{1}{2}, \frac{v}{2}\right)$ represents a $\frac{1}{2}$ unit of type $\frac{v}{2}$ asset. The issuer can construct this asset by combining $\frac{1}{4}$ unit of type v asset and $\frac{1}{4}$ unit of v_p asset (the production asset). Constructing this security would cost the issuer $\frac{1}{4}c$. Note that the value of the portfolio is $\frac{1}{2}v + \frac{1}{2}\frac{v}{2} + 1\frac{v}{4} = v$, which is equal to the value of the issuer's endowment asset. The total size of the portfolio is $\frac{1}{2} + \frac{1}{2} + 1 = 2$, which consists of 1 unit of type v and 1 unit of type v_p assets. The total cost of constructing the portfolio is c. A feasible follow-up set of actions for this issuer is 1) to solicit a rating for the first security in 1.3 and sell it in the market if and only if it receives rating s_3, 2) to sell the second security without soliciting a rating, and 3) not to sell the third security.

Finally, there is a market in which the issuers can sell their assets. Since the market is perfectly competitive, the price of a security is set to the expected value of the security. The expectation is taken using the posterior belief of the market over the distribution of types. The posterior belief is formed taking into account the strategy of the CRA and the issuers, as well as the realization of the rating signals. Whenever possible, the posterior must satisfy the Bayes' rule. We can now formally define equilibrium of the rating game.

Definition 1.1. *An equilibrium of the rating game is a set of the CRA's rating structure and rating fee, issuers' security portfolio decisions and probabilities of soliciting a rating for and selling each security in those portfolios, and market prices such that*

1. *The CRA maximizes profit given the strategy of the issuers and market prices.*

2. *Given CRA's strategy and market prices, issuers' maximize their profit.*

3. *Market price of a security offered in equilibrium is the expected value of that security given the CRA and the issuers' strategies and the realization of rating signal, if available. The price of securities not offered can be arbitrarily assigned.*

The strategy space of the CRA accommodates a variety of rating structures and in general multiple equilibria. To narrow down the set of equilibrium candidates, we focus on rating structures of table 2.1. This rating structure has a number of interesting features. The first one is that in an equilibrium utilizing rating structure of table 2.1, the CRA only targets security types in the highest two tiers. A rating in which the CRA targets only a single tier is never supported in equilibrium. Then the rating structure of table 2.1 effectively offers the minimum required rating tiers in any equilibrium.

Another interesting feature of the rating structure of table 2.1 is that it exhibits rating inflation. As will be presented shortly, there always exist an equilibrium in which the rating structure belongs to table 2.1. On the other hand, Kartasheva and Yilmaz show that rating deflation can be supported in equilibrium under certain restrictions. From a practical point of view, issuers typically have the opportunity to observe their prospective ratings before they decide to purchase them. In markets with multiple rating agencies, the issuers also have the opportunity to shop around for an agency that offers the best rating. It is very unlikely for a rating structure with deflation to survive in these environments.

Finally, in rating structure of table 2.1, rating precision is well defined by a single parameter p. Also as important, we can easily quantify rating failure, defined here as the probability that a high (s_3) rated asset is of lower (v_2) type. We use this metric as a measure of ratings performance. In this aspect, by focusing on rating structure of table 2.1, we are able to examine the impact of various parameters in the model on rating precision and ratings performance.

Finally, we should mention that the majority of our results are independent of the choice of equilibrium. For the remainder, one can qualitatively extend the results to other possible equilibria.

1.4. Equilibrium

As we mentioned previously, our candidate for the equilibrium of the rating game has the rating structure of table 2.1. Given the rating structure, the strategy of the CRA is fully characterized by the choice of rating precision $p \in [0, 1]$ and rating fee $\phi \geq 0$. Next, the following lemma allows us to narrow down the strategy space of the issuers in the equilibrium candidate.

Lemma 1.1. *If the rating structure is according to table 2.1, the portfolios of the issuers in the continuation game will include only asset types in the set* $\{v_1, v_2, v_3\}$.

The formal proof is included in the appendix. Intuitively, since any type strictly between two tier cutoffs is rated identical to the type matching the lower cutoff, issuers can profit by decomposing any security type into types matching the cutoffs.

Next, by Lemma 1.1, all s_1 rated securities must be of type v_1 or v_p and is therefore priced 0 by the market. Since the production of type v_p is costly, we can conclude that the portfolio of no issuer would include type v_p securities. In short, the above arguments imply that the portfolios of issuers 1 and 2 only contain type v_1 and v_2 assets, respectively, and that they do not invest in the technology. The portfolio of issuer 3 has the following form.

$$\left\{((1-\alpha)m_3, v_3), \left(\alpha m_3 \frac{v_3}{v_2}, v_2\right)\right\} \tag{1.4}$$

The first entry in portfolio 1.4 is $(1-\alpha)m_3$ units of type v_3 asset, whereas the second entry is $\alpha m_3 \frac{v_3}{v_2}$ units of type v_2 security, made by combining type v_3 and type v_p assets. Notice that the total value of issuer 3's portfolio is $(1-\alpha)m_3v_3 + \alpha m_3 \frac{v_3}{v_2} v_2 = m_3v_3$, as one should expect. The parameter α must be solved for in equilibrium.

Finally, define U_i to be the equilibrium unit price of an asset rated s_i, and U_0 denote the equilibrium unit price of an unrated asset. Also, we partition the set of technology costs c into three subsets according to the following cutoffs.

$$\underline{c} := (1-\mu)(1-\delta)v_2 \tag{1.5}$$

$$\overline{c} := (1-\mu)(1-\delta)\frac{m_3v_3 + m_2v_2}{m_3 + m_2}. \tag{1.6}$$

We refer to intervals $[0, \underline{c}]$, $(\underline{c}, \overline{c})$, $[\overline{c}, \inf)$ as low, intermediate, and high technology costs, respectively.

Proposition 1.1 (Equilibrium of the rating game)**.** *There exists an equilibrium in which the rating structure belongs to table 2.1. Issuers 1, 2, and 3's security portfolios, respectively, are*

$$\{(m_1, v_1\}, \{(m_2, v_2)\}, \left\{((1-\alpha)m_3, v_3), \left(\alpha m_3 \frac{v_3}{v_2}, v_2\right)\right\}. \tag{1.7}$$

Issuer 1 does not solicit a rating and offers the portfolio for sale in the market at unit price $U_0 = U_1 = 0$. Issuers 2 and 3 purchase rating for all the securities in their portfolios. Conditional on rating s_2, both issuers 2 and 3 sell the corresponding securities in the market at unit price $U_2 = v_2$. Conditional on rating s_3, both issuers 2 and 3 sell the corresponding securities in the market at unit price U_3. Equilibrium values of rating precision p, ϕ, α, and U_3 are functions of the production cost c and defined as follows.

$$p = \begin{cases} \frac{\delta(m_3+m_2)}{m_3+\delta m_2} & c \geq \overline{c} \\ \frac{\delta(v_3-v_2)}{\delta v_3 - v_2 + \frac{c}{1-\mu}} & \underline{c} < c < \overline{c} \\ 1 & c \leq \underline{c} \end{cases}$$

$$\phi = \begin{cases} \mu(1-\delta)\frac{m_3 v_3 + m_2 v_2}{m_3+m_2} & c \geq \overline{c} \\ c\frac{\mu}{1-\mu} & \underline{c} < c < \overline{c} \\ \mu(1-\delta)v_2 & c \leq \underline{c} \end{cases}$$

$$\alpha = \begin{cases} 0 & c \geq \overline{c} \\ \frac{(1-\delta)(1-\mu)(m_3 v_3 + m_2 v_2) - c(m_3+m_2)}{m_3 c \frac{v_3 - v_2}{v_2}} & \underline{c} < c < \overline{c} \\ 1 & c \leq \underline{c} \end{cases}$$

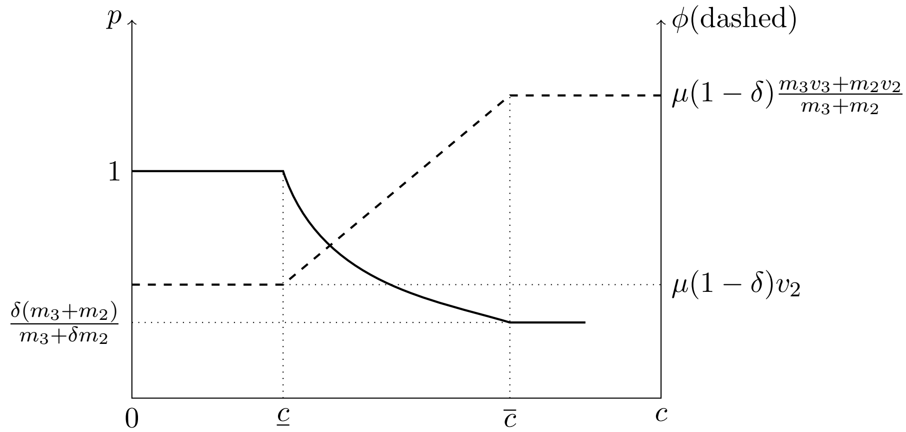

Figure 1.1: Rating precision and rating fee

$$U_3 = \begin{cases} (1-\delta)\frac{m_3v_3+m_2v_2}{m_3+m_2} + \delta v_3 & c \geq \overline{c} \\ \frac{c}{1-\mu} + \delta v_3 & \underline{c} < c < \overline{c} \\ v_2 & c \leq \underline{c} \end{cases}$$

The proof is provided in the appendix. When the technology cost is high ($c \geq \overline{c}$), replicating type v_2 by combining types v_3 and v_p is not profitable for issuer 3. In this scenario, all issuers' portfolios in equilibrium contain only their corresponding endowment assets. The equilibrium outcome in this case is identical to the equilibrium of a modified version of the rating game in which the technology is not available, which we will refer to as the rating game without securitization [3]. in this equilibrium, only issuers 2 and 3 purchase ratings. The rating is informative ($p > 0$), despite being noisy ($p < 1$). The market rationally expects that securities rated s_3 can be either type v_2 or v_3. As a result, on average, type v_3 is under priced, where as type v_2 is over priced. The key feature of the CRA's optimal strategy is to adjust the rating so as to equalize issuers 2's and 3's willingness to pay. That is, the following must hold in equilibrium.

$$U_3 - \delta v_3 = [pv_2 + (1-p)U_3] - \delta v_2. \tag{1.8}$$

[3]This is essentially the game analyzed in detail in Kartasheva and Yilmaz (2013).

The left hand side in equation (1.8) is the expected market price of type v_3 net of its reservation value for issuer 3. The right hand side is the same for type v_2. This allows the CRA to maximize its share of the total gains from trade by setting

$$\phi = \mu(U_3 - \delta v_3). \tag{1.9}$$

Condition (1.8) uniquely determines the rating precision to be $\frac{\delta(m_3+m_2)}{m_3+\delta m_2}$. Notice that p is increasing in $\frac{m_2}{m_3}$, the ratio of the units of type v_2 to type v_3 securities. The reason is that as the relative population size of type v_2 security increases, the market expects a larger fraction of s_3 rated securities to be of type v_2. Consequently, market price of s_3 rated securities decreases. The CRA's response is to increase rating precision to maintain equality of issuers' willingness to pay.

Conditions (1.8) and (1.9) reveal another important feature of the equilibrium. If one considers purchasing ratings an investment by the issuer, the return on type v_3 security, that is $\frac{U_3-\phi}{\delta v_3} - 1$, is smaller than the return on type v_2, which is equal to $\frac{[pv_2+(1-p)U_3]-\phi}{\delta v_2} - 1$. This prompts issuer 3 to replicate type v_2 by utilizing the technology, provided that the cost of the technology is not prohibitively high.

When technology cost enters the intermediate range ($\underline{c} < c < \overline{c}$), replicating type v_2 becomes profitable for issuer 3. In equilibrium, issuer 3's portfolio now contains a mix of type v_2 and v_3 securities. This means that the equilibrium aggregate size of type v_2 securities relative to type v_3 is now larger than $\frac{m_2}{m_3}$. The CRA anticipates this and increases rating precision such that issuer 3 is indifferent between having type v_2 and v_3 securities in its portfolio. At the same time, the CRA's choice of rating precision together with issuer 3's portfolio decision ensures all rated types' willingness to pay for rating are equal. As before, the CRA sets the rating fee equal to the rated types' willingness to pay. Since the relative population size of type v_2 to type v_3 has increased, the equilibrium ϕ is smaller compared to that in high technology cost scenario. However, the addition of type v_p to issuer 3's portfolio also means that the aggregate size of the rated securities has increased. These two effects fully cancel out and consequently, CRA's share of the total gains from trade remains unchanged, despite the change in equilibrium composition of rated securities. As the technology cost decreases, ratio of type v_2 to type v_3 securities in issuer 3's portfolio, and accordingly, rating precision increases (figure 1.1). Meanwhile, ϕ and market price of s_3 rated securities, U_3, decrease.

When technology cost reached the low cutoff $\underline{c}$, issuer 3's optimal portfolio is 100% comprised of type v_2 security. Therefore in equilibrium, all rated securities are of type v_2. Since in this case only a single type is rated, enforcing equal willingness to pay across rated types is trivial. The CRA sets rating fee equal to the willingness to pay of type v_2, that is $\phi = \mu(1-\delta)v_2$. As a result, the CRA's profit remains the fraction μ of the total gains from trade, same as prior scenarios. For any $c < \underline{c}$, issuer 3's portfolio is independent of the technology cost. Naturally, rating precision and fee remain constant (figure 1.1).

Following Proposition 1.1, we can proceed to derive the profit of the CRA and the issuers in equilibrium. The CRA's profit π^{CRA} is defined as the sum of the rating fee collected by the agency. An issuer i's profit π^i is defined as the market price of the issuer's portfolio net of the technology cost and the reservation value.

Proposition 1.2. *The profit of the CRA and issuers 2 and 3 in the equilibrium of Proposition 1.1 is given by*

$$\pi^{CRA} = \mu(1-\delta)(m_3 v_3 + m_2 v_2)$$

$$\pi^3 = \begin{cases} (1-\mu)(1-\delta)m_3 \frac{m_3 v_3 + m_2 v_2}{m_3 + m_2} & c \geq \overline{c} \\ m_3 c & \underline{c} < c < \overline{c} \\ (1-\mu)(1-\delta)m_3 v_3 - \frac{v_3 - v_2}{v_2} m_3 c & c \leq \underline{c} \end{cases}$$

$$\pi^2 = \begin{cases} (1-\mu)(1-\delta)m_2 \frac{m_3 v_3 + m_2 v_2}{m_3 + m_2} & c \geq \overline{c} \\ m_2 c & \underline{c} < c < \overline{c} \\ (1-\mu)(1-\delta)m_2 v_2 & c \leq \underline{c} \end{cases}$$

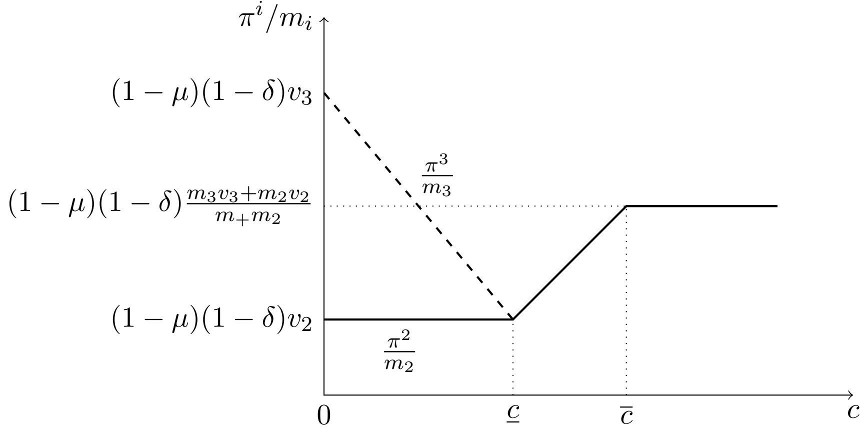

Figure 1.2: Issuer's profit normalized by the endowment size

The total gains from trade available is equal to $(1-\delta)(m_3v_3+m_2v_2)$. Notice that since $v_p=0$, the production of type v_p asset does not add to the gains from trade. Recall that in the equilibrium of Proposition 1.1, the CRA's strategy involves ensuring the equality of all rated types' willingness to pay for rating. Having established that, the CRA then sets rating fee equal to those types willingness to pay. Therefore, the CRA manages to extract fraction μ of the gains from trade, regardless of the technology cost, as presented in Proposition 1.2.

Issuers do not invest in the technology when the cost c is high. The average value of rated securities in this case is $\frac{m_3v_3+m_2v_2}{m_3+m_2}$. Since type v_2 and v_3 have equal willingness to pay for rating in equilibrium, issuers 2 and 3's profits from trading a unit of type v_2 or v_3 securities is then given by the product of the average gain from trade of a unit of type v_2 or v_3, which is equal to $(1-\delta)\frac{m_3v_3+m_2v_2}{m_3+m_2}$, and the fraction of gain from trade not extracted by the CRA, $1-\mu$. Note that absent any resources spent on the technology, issuers and the CRA's profit adds up to the total gains from trade available. That is $\pi^{\text{CRA}}+\pi^3+\pi^2=(1-\delta)(m_3v_3+m_2v_2)$.

For Intermediate technology costs, issuers profits are functions of c. As c decreases, the share of type v_2 security in issuer 3's portfolio, and consequently the total amount of type v_2 in the market increases. Rationally, type v_2 security, on average, is priced lower in the market, and therefore, issuer 2's profit decreases (figure 1.2). The effect of a decrease in c on issuer 3's profit is two fold. On the one hand, the issuer 3's investment in the technology increases. On the other hand, the average market price of issuer 3's portfolio increases. In this case, the former effect always dominates and reduces issuer 3's profit (figure 1.2).

When c falls below $\underline{c}$, issuer 3's investment in the technology is capped. All rated securities are type v_2 and are correctly priced v_2 by the market. Issuer 2's profit is then simply the product of the gains from trade of type v_2 and the fraction $1-\mu$. Issuer 3's share of the gains from trade is similar, which is $1-\mu$ times the gains from trade of type v_3. In addition, issuer 3 bears the cost of investment in the technology. In this case, however, since total units of type v_p in issuer 3's portfolio is constant, a decrease in c results in an increase in the profit of issuer 3, as illustrated by figure 1.2.

An important observation to be made is regarding issuers' profits for intermediate technology costs. In this region, issuer 3's strategy to maximize profit involves investing in the technology with the aim of replicating type v_2. Interestingly, as can be seen in figure 1.2, the profit of issuer 3 in this region is strictly lower than its profit in the high cost region. At the same time, issuer 2's profit also suffers from issuer 3's investment in the technology. In other words, access to the technology has adverse effect on the profits of both issuer 2 and 3. Put differently,

Corollary 1.1. *For $c \in (\underline{c}, \overline{c})$, both issuer 2 and 3 can strictly increase their equilibrium profits if they commit not to invest in the technology.*

1.5. Market Efficiency

We now proceed to evaluate the efficiency of the equilibrium outcome of the rating game. As our fist benchmark for total surplus, we begin by establishing the first best solution. A social planner can maximize surplus by fully disclosing asset types. This would ensure that all issuers trade their assets for their true market values, and in turn, eliminates any incentives to invest in the technology. Therefore, the first best total surplus is equal to

$$W^{\mathrm{FB}} = (1-\delta)(m_3 v_3 + m_2 v_2).$$

The equilibrium total surplus consists of two parts: (1) The trade surplus, or the total gains from trade and (2) The cost of investment in the technology. The CRA's strategy ensures that in equilibrium all available gains from trade are realized, which is equal to $(1-\delta)(m_3 v_3 + m_2 v_2)$, as established by Proposition 1.1. The total cost of investing in the technology is equal to $m_p c$, where m_p denotes the aggregate amount of type v_p asset in the portfolio of the issuers. Then the equilibrium total surplus is given by

$$W^{\mathrm{Eq}} = (1-\delta)(m_3 v_3 + m_2 v_2) - m_p c. \tag{1.10}$$

A quick comparison of W^{Eq} and W^{FB} reveals that when equilibrium outcome involves investment in the technology, market is inefficient. The inefficiency in the equilibrium outcome is equal to the total cost of investing in the technology. We can use Proposition 1.1 to derive the equilibrium total surplus.

Proposition 1.3. *In the equilibrium of of Proposition 1.1, m_p, the amount of type v_p in issuer 3's portfolio, is given by*

$$m_p = \begin{cases} 0 & c \geq \overline{c} \\ \frac{(1-\delta)(1-\mu)(m_3 v_3 + m_2 v_2) - c(m_3+m_2)}{c} & \underline{c} < c < \overline{c} \\ m_3 \frac{v_3 - v_2}{v_2} & c \leq \underline{c} \end{cases} \tag{1.11}$$

The total surplus is calculated as

$$W^{Eq} = \begin{cases} (1-\delta)(m_3 v_3 + m_2 v_2) & c \geq \overline{c} \\ \mu(1-\delta)(m_3 v_3 + m_2 v_2) + c(m_3 + m_2) & \underline{c} < c < \overline{c} \\ (1-\delta)(m_3 v_3 + m_2 v_2) - m_3 \frac{v_3 - v_2}{v_2} c & c \leq \underline{c} \end{cases}$$

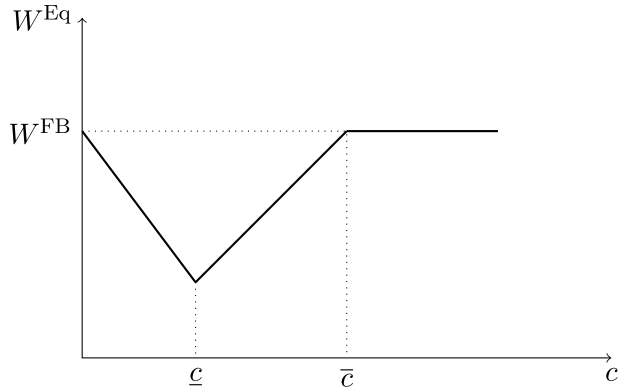

Figure 1.3: Equilibrium total surplus

For high technology costs, issuers do not invest and the equilibrium achieves the first best total surplus. In the intermediate range, investment in the technology decreases the total surplus. As c decreases, total amount of type v_p produced increases. The affect of increase in investment dominates the decrease in c and as a result, the surplus decreases. Finally, when c drops below the threshold $\underline{c}$, total amount of type v_p is capped. Any further decrease in technology cost would then increase the total surplus as illustrated in figure 1.3.

With Proposition 1.1, we established an equilibrium in which all available gains from trade is realized, of which fraction μ is extracted by the CRA. Therefore, in any other equilibrium of the rating game, all available gains from trade must be realized. Then the inefficiency in any equilibrium outcome is equal to the total cost of investment in the technology. It then follows that an equilibrium with smaller m_p generates more surplus, i.e., is more efficient. The rating game in our model allows for multiple equilibria with varying investment levels in the technology. For instance consider the rating structure of table 1.3. Notice that this

	$v_3 - \epsilon$	v_2	v_1
s_3	1	$1 - p'$	0
s_2	0	p'	0
s_1	0	0	1

Table 1.3: An alternative rating structure

is identical to the rating structure in the equilibrium of Proposition 1.1 with the exception that the highest tier cutoff is changed to $v_3 - \epsilon$. Intuitively, this rating structure would induce issuer 3 to have a portfolio of type v_2 and type $v_3 - \epsilon$ securities. Replicating type $v_3 - \epsilon$ requires additional type v_p asset. Then we would expect in an equilibrium in which the rating structure is set to table 1.3, the total amount of type v_p asset to be larger than that in the equilibrium of Proposition 1.1. An important question that follows is whether or not there exists an equilibrium that is more efficient than the equilibrium of Proposition 1.1.

Proposition 1.4. *In any equilibrium of the rating game, the aggregate amount of type v_p in the portfolio of the issuers is bounded from below by the quantity in equation 1.11.*

The proof is in the appendix. The intuition behind Proposition 1.4 is the following. Consider two security types v^l and v^h, where $v^l < v^h$ and both types are rated in equilibrium. Equilibrium requires that replicating type v^l not be profitable, which sets an upper bound on the gain from trade of type v^l. Since a fraction of the gain from trade is extracted by the CRA as rating fee, an upper bound on the gain from trade translates into an upper bound on rating fee. Since the profit of the CRA in any equilibrium is constant, an upper bound on ϕ is equivalent to a lower bound on total type v_p produced in equilibrium. The following is an immediate result of Proposition 1.4.

Corollary 1.2. *The equilibrium of Proposition 1.1 is the most efficient equilibrium of the rating game.*

Our next benchmark for total surplus is the market solution when there is no CRA. Absent ratings, market outcome can vary from complete breakdown to all gains realized. The former can arise if the following conditions are met.

$$\delta v_3 > \frac{m_3 v_3 + m_2 v_2}{m_3 + m_2 + m_1} \tag{1.12}$$

$$\delta v_2 > \frac{m_2 v_2}{m_2 + m_1} \tag{1.13}$$

Inequality 1.12 states that issuer 3's reservation value for type v_3 is strictly higher than the average type of the endowment assets held by issuers. This condition ensures that absent ratings, issuer 3 would not trade its assets in the market, since otherwise the price (the right hand side) would be strictly lower than the reservation value (the right hand side). With issuer 3 out of the market, the average type of the remaining assets is equal to $\frac{m_2 v_2}{m_2+m_1}$, the right hand side of inequality 1.13. Then with condition 1.13 satisfied, issuer 2 would also pull out of the market. Since trade of type v_1 does not generate any gains, whether issuer 1 stays or leaves the market is irrelevant. In this scenario (scenario 1), effectively there is a complete market break down and trade surplus is 0.

Clearly, ratings improve market efficiency in this scenario as they restore trades in the market. On the other hand, inefficient investment can arise in the market with ratings. In comparison, no investment in the technology takes place in the absence of ratings in scenario 1 , since no securities is traded in the market. However, the surplus generated by the ratings is always larger than the total cost of investment in the technology, as is evident in figure 1.3. Therefore, ratings unambiguously improve market surplus in in scenario 1. However, it has to be noted that the scale of this improvement in market efficiency can vary greatly, depending on model parameters. In fact, the following lemma shows that the impact of ratings on efficiency can be made arbitrarily close to 0.

Lemma 1.2. $\lim_{|\mu|+|c-\underline{c}|+|v_2|\to 0} m_3 c = (1-\delta)(m_3 v_3 + m_2 v_2)$.

The other extreme case (scenario 2), is when absent ratings, all assets are traded in the market. This happens when condition 1.12 is reversed, in which case assets are priced equal to the average type $\frac{m_3 v_3+m_2 v_2}{m_3+m_2+m_1}$. Between the market without a CRA in scenario 2 and the market with ratings, the one that invests less in the technology is clearly more efficient. Absent ratings, issuers would not invest in the technology if the cost c exceeds the market price, that is

$$\frac{m_3 v_3 + m_2 v_2}{m_3 + m_2 + m_1} < c. \tag{1.14}$$

Then with scenario 2 and condition (1.14) satisfied, the market without CRA achieves first best total surplus. On the other hand if $c < \bar{c}$, the equilibrium of the rating game involves investment in the technology (Proposition 1.1). If all these conditions are jointly satisfied, the total surplus in the market without a CRA is strictly higher than that in the one with a CRA.

Proposition 1.5. *The set of parameters satisfying the following set of constraints is non-empty.*

$$\delta v_3 < \frac{m_3 v_3 + m_2 v_2}{m_3 + m_2 + m_1} < c < \bar{c}. \tag{1.15}$$

For the set of parameters in 1.15, presence of a CRA reduces total surplus.

To summarize our findings, we showed that the introduction of ratings does not always improve market efficiency, and when it does, the magnitude of this improvement can vary greatly based on economic conditions, asset classes, and composition of assets among others.

1.6. Comparative statics and model predictions

In this section we study the predictions of our model. For this section, we make the following assumption on the technology cost.

Assumption 1.1. *Technology cost satisfies $\underline{c} < c < \bar{c}$.*

Assumption 1.1 ensures that a non zero amount of type v_p asset is produced in any equilibrium. In the equilibrium of Proposition 1.1, this assumption implies that issuer 3's portfolio choice problem has an interior solution. In other words, issuer 3's portfolio contains non zero amounts of both type v_3 and v_2 securities. For other technology costs, issuer 3's portfolio is either 100% type v_2 or 100% type v_3 securities. In those cases, issuer 3's portfolio decision does not react to changes in parameters of the model.

Our first prediction pertains to the impact of technology cost c on rating precision and performance. Rating precision is a measure of how truthful the CRA is in assigning a rating to an asset. In the equilibrium of Proposition 1.1, this is captured by parameter p.

By rating performance we refer to how accurately ratings signal the quality of corresponding assets. Here, we use failure rate p_f, which is defined as the probability of an s_3 rated asset being type v_2, as a measure for rating performance. A lower failure rate corresponds to a better performance. For the rating structure of Proposition 1.1, we can use the Bayes' rule to derive p_f.

$$\begin{aligned} p_f =& \Pr(v_2|s_3) \\ =& \frac{(1-p)\Pr(v_2)}{(1-p)\Pr(v_2)+\Pr(v_3)}. \end{aligned}$$

Proposition 1.6. *A decrease in technology cost leads to an increase in rating precision and a decrease in performance.*

$$\frac{\partial p}{\partial c} < 0, \frac{\partial p_f}{\partial c} < 0.$$

A decrease in c prompts issuer 3 to allocate a larger portion of its portfolio to type v_2 security. The CRA's response to the increase in overall type v_2 population size in the market is to increase rating precision, with the purpose of keeping type v_3's and v_2's willingness to pay for rating equal.

The impact of a decrease in technology cost on rating performance is not as straightforward. One the one hand, an increase in rating precision improves performance. On the other hand, as argued above, a decrease in c results in an increase in the overall population size of type v_2 securities. All else being equal, more of type v_2 securities in the market results in higher probability of s_3 rated assets being of type v_2, namely, poorer rating performance. The net effect of the two forces is a decrease in rating performance.

Proposition 1.6 allows us to examine ratings performance variations across different asset types. In particular, we can compare corporate bonds, a high c market, and mortgage backed securities, a low c one. Proposition 1.6 suggests that all else being equal, MBS ratings perform poorly compared to that in corporate bonds market, which is consistent with several empirical findings [4]. In addition, our findings suggest inefficient investment in assets used in the construction of securities to be higher compared to that in traditional corporate investments (figure 1.3)).

Rating agencies faced heavy criticism for their role in the sub prime mortgage crisis, and poor ratings performance were often cited as evidence of failure of the industry. In one common line of criticism, opponents would point to the rating agencies' incentives to purposely inflate their ratings to attract more businesses. This argument, however, fails to explain why performance of ratings varies across asset classes. Our framework provides an alternative mechanism of generating poor ratings performance which departs from the above common argument in two major ways. First, the equilibrium profit of the CRA in our model does not depend on c, or the issuers' investment in the technology. In other words, the CRA is no more inclined to inflate ratings of MBS than corporate bonds. Second, all else being equal, Proposition 1.6 predicts higher precision of ratings of MBS than that for corporate bonds, which is contrary to the critics' argument.

Our next prediction is about the effect of the CRA's market dominance on rating performance. We use μ as a measure of the CRA's dominance in the market. A CRA with more market dominance can extract a larger fraction of trade surplus by charging higher fees, which is equivalent to higher μ in our model.

Proposition 1.7. *Rating performance is increasing in μ, whereas, inefficient investment is decreasing in μ.*

$$\frac{\partial p_f}{\partial \mu} < 0, \frac{\partial}{\partial \mu}(m_p c) < 0.$$

As μ increases, issuers' shares of the trade surplus decrease. This in turn reduces the

[4] Ashcraft et al. (2010) find that ratings quality declined with the increase in MBS issuance between 2005 and 2007. Specifically, subordination levels for subprime and Alt-A MBS were lower during this time, followed by subsequent ratings downgrades. Benmelech and Dlugosz (2009) find that around 70% of the value of securities issued by CLOs is rated AAA backed by collaterals rated B to B+ on average. Griffin and Tang (2012) find that the increase in the size of the AAA tranche of the CDOs from 2003 to 2007 is positively related to future downgrades. Stanton and Wallace (2017) find that subordination levels for CMBSs fell considerably in the years leading to the financial crisis, leaving scant protection for "safe" tranches.

fraction of issuer 3 portfolio dedicated to type v_2 securities. Put differently, issuer 3's investment level in the technology is reduced, hence, lower inefficiency. As a result, the overall population size of type v_2 in the market is smaller, which results in lower failure rate.

Easing the barriers to entry in the credit rating industry and encouraging competition among CRAs has been a major policy response to the poor performance of the three large rating agencies in recent years (White (2010)). Competition among multiple CRAs can affect two fundamental aspects of ratings: rating structure and rating fee. This framework makes no predictions on the impact of competition on ratings structure. However, so far as the role of competition in reducing rating fees is concerned, our model associates poor performance and inefficient investment with competition, particularly in highly securitized asset classes.

Next, we examine how changes to $\frac{m_3}{m_2}$, the ratio of population sizes of the endowment assets of issuers 2 and 3, affect ratings and efficiency. We offer two interpretations for changes to $\frac{m_3}{m_2}$. In one, we use $\frac{m_3}{m_2}$ as an indicator for business cycles. During growth periods, market is bullish in their risk assessment and therefore, high quality assets are in relatively large supply, namely high $\frac{m_3}{m_2}$. Low $\frac{m_3}{m_2}$, in turn, corresponds to recessions.

In another case, $\frac{m_3}{m_2}$ allows us to comment on the role of informed speculation on ratings and economic efficiency. Recall that a consequence of the CRA's strategy to equate rated types' willingness to pay for rating is that high quality assets are on average under priced by the market. This creates an opportunity for informed speculation. For instance, consider a period prior to $t = 0$ in our model, in which informed speculators can bid for issuers assets. Suppose that speculators have perfect information and are able to predict the rating structure the CRA is going to implement in the future. In that case, speculators know that type v_3 assets will be under-priced in equilibrium, and there is an opportunity for profitable trade. On the other hand, since type v_2 assets are on average overpriced in equilibrium, there is no opportunity for profitable trade of this type. In short, informed speculation reduces $\frac{m_3}{m_2}$.

Proposition 1.8. *Rating precision and performance are not functions of* $\frac{m_3}{m_2}$. *Inefficiency is increasing in* $\frac{m_3}{m_2}$.

The first part of Proposition 1.8 is a notable feature of the equilibrium of our rating game, according to which, the CRA's design for the ratings does not depend on the composition of issuers' endowment assets. Notice that, however, according to Proposition 1.1, issuer 3's portfolio decision is in fact a function of $\frac{m_3}{m_2}$. An implication of issuer 3's portfolio choice is that in equilibrium, the ratio of the population of type v_3 to type v_2 in issuers portfolios, which is equal to $\frac{(1-\alpha)m_3}{m_2+\alpha m_3 \frac{v_3}{v_2}}$, remains constant with respect to $\frac{m_3}{m_2}$. In other words, if the sizes of issuers' endowments change, issuer 3 rebalances its portfolio such that the composition of assets in the market remains unchanged. Therefore, Proposition 1.8 is not in disagreement with Kartasheva and Yilmaz' finding that composition of assets is a key decider of the CRA's rating design.

As $\frac{m_3}{m_2}$ increases, to maintain the composition of assets in the market, issuer 3 must increase the size of type v_2 security in its portfolio. This requires issuer 3 to increase investment in the technology, which results in higher inefficiency. Note that this part of Proposition 1.8 does not rely on the choice of the equilibrium and is true in general for any equilibrium of the rating game.

Proposition 1.8 predicts that the level of inefficient investment is higher in growth periods. This is not to be confused by over investment during economic bubbles, brought about by overestimation of the true value of assets or its future prospects. Note that in our model, issuers are perfectly informed of the true value of the assets. Furthermore, the average market price is always equal to the average asset type, contrary to market prices during a bubble. The over investment in our model is due to mis-pricing of assets, which the issuers can take advantage of by issuing securities matching the type of over priced assets. In addition, Proposition 1.8 suggests that not only economic bubbles involve over investment in assets that are over-valued, but also has a positive feedback on inefficient investments in other assets underlying securitized instruments.

Kartasheva and Yilmaz (2013) showed that the winner's curse due to heterogeneity of investors' information would prompt the CRA to increase the precision of ratings. Proposition 1.8 suggests a different role for informed investors in which, rating precision is not affected by informed speculation. Instead, informed speculators help mitigate inefficient investment by bidding for under priced assets and in the process, reducing their population in the market.

1.7. Conclusion

Our model proposes access to securitization technology as an important factor in explaining the differences in ratings of ABSs and corporate bonds. We show that the well documented poor performances of ratings of ABSs prior to the financial crisis can be simulated in a rational setting with a strategic CRA and privately informed issuers who have access to a costly production and securitization technology. We show that as the cost of accessing this technology decreases, the equilibrium performance of ratings declines. In addition, we show that access to securitization can negatively impact efficiency as it gives rise to inefficient investments in poor quality assets. Finally, we show that the inefficient investment induced by securitization is procyclical.

To achieve a closed form solution for our rating game, we made simplifying assumptions that may be worth expanding upon. We assumed that the addition of the production asset to the issuers' portfolios does not change the total available gains from trade. As such, the CRA is not able to increase its profit by further inflating the ratings to prompt more investment in the production by issuers. Relaxing this assumption can bear interesting results.

CHAPTER 2 : Credit Ratings and Investment

2.1. Introduction

The focus of the existing literature in credit ratings has been primarily on the ability of ratings to mitigate information asymmetry in the market and the mechanisms through which ratings performance are affected such as competition among rating agencies, rating shopping, composition of assets being rated, complexity of the asset, and regulations, among others. The other side of the coin, and perhaps as important, is the mechanisms through which ratings impacts go beyond their obvious role as a source of information for investors, and influence decisions that would have significant ramifications for the market and even the economy as a whole. In this work, we explore how investment decisions are influenced directly by ratings, or other market conditions through their impact on ratings.

We consider a model with a continuum of firms and a continuum of investment opportunities, or projects, with decreasing returns. Investing in these projects require an initial capital injection which firms lack. In order to raise capital, the firm have to choose between borrowing from informed investors who demand information rent, or issuing bonds to competitive uninformed investors. Firms that decide to issue bonds can sell them at a higher price by soliciting ratings from a single CRA in exchange for a ratings fee. The CRA is strategic in designing the ratings, and maximizes collected rating fees. This in general implies a ratings system that is informative without fully revealing the type of issuers.

We first solve the first best solution, assuming away the information asymmetry between investors and firms issuing bonds. We then prove the existence of an equilibrium of the game and derive the conditions for which the total amount invested in the projects in the equilibrium outcome is equal to that in the first best solution.

Next, we show that if the projects' expected returns remain unchanged, an increase in the riskiness of the projects leads to more investment in equilibrium. In the special case that the equilibrium outcome coincides with first best, this implies that more risk induces over investment and reduced risk, under investment. Note that this is purely a result of firms financing through issuing bonds with imperfect credit ratings. In particular, the well known phenomenon that a firm's equity holders are prone to excessive risk since they capture the upside while their downside is limited, is not present in our model.

We next consider the effect of a risk shock that equally affects the value of all issue types on firms' equilibrium aggregate investment. We find that under certain conditions, risk shocks leads to under investment. This may partially explain the slow recovery following the recent financial crisis.

We also examine the influence of the CRA's market power on the efficiency of the equilibrium outcome. We show that an increase in the CRA's market power unambiguously reduces equilibrium aggregate investment. However, the effect on efficiency is dependent on the macro conditions. During growth periods that are commonly accompanied by over investment, a more dominant CRA is a positive force towards efficiency. On the other hand, during recessions, this market dominance hampers recovery by reducing firms' incentives to invest.

2.2. Related Literature

Our framework builds on Lizzeri (1999) and Kartasheva and Yilmaz (2013). Lizzeri explores the incentives if an intermediary to disclose information in a model with a continuum of sellers holding assets with varying values to the buyers, but crucially worth 0 to sellers themselves. He then shows that there is an equilibrium in which the intermediary extracts all the trade surplus without disclosing any information to the buyers. In Kartasheva and Yilmaz (2013), issuers of assets have non zero, type dependent reservation values for their assets. They show that in equilibrium, ratings do disclose information. Moreover, ratings improve efficiency by facilitating trade. They also explore how the composition of assets in the market and presence of informed speculators would impact ratings precision.

This work explores the flip side of ratings market and studies the impact of ratings on issuers incentives to make investments. To that end, we endogenize the set of issues by considering issuers as firms facing investment opportunities without the required capital. To raise capital, firms issue bonds whose market price depends on the ratings assigned to them by the CRA. As strategic investors, firms take into account not only the intrinsic NPV of the investment opportunity, but also their effective cost of capital which is influenced by ratings. We then analyze the impact of ratings on inefficient investment and the role of economic environment in reducing this inefficiency, or otherwise.

The following works study other factors affecting ratings performance. Mathis, McAndrews, and Rochet (2009) consider the role of reputation in reducing a monopolistic CRA's incentive to inflate ratings. They show that reputation is an effective mechanism only if a small fraction if the CRA's profit is based on rating complex securities. In a model with two competing CRAs, Bolton, Freixas, and Shapiro (2012) show that when issuers can shop for ratings, competition can lead to further inflated ratings. Finally, increase demand for high rated securities, created by regulations, is another source of inflated ratings as demonstrated in Opp, Opp, and Harris (2013).

2.3. Model

2.3.1. Market

There is a market with 3 type of risk neutral agents: issuers, a CRA, and investors. Issuers possess debt and they intend to sell them in the market. We assume there are 3 types of debt, v_1, v_2, v_3, where $v_i \in [0, 1]$ denotes the market value of a unit face value of type v_i and types satisfy $0 = v_1 < v_2 < v_3$. We refer to the aggregate face value of type v_i debt by m_i. For simplicity, we assume m_1 and m_3 are exogenous, unlike m_2 which we need to solve for. Debt types are issuers private information. Finally, all market participants are risk neutral and the discount rate is normalized to 0.

There are two potential buyers of issuers' debt: uninformed investors and informed investors. Selling debt to uninformed investors is meant to capture issuing bonds. Uninformed investors are competitive but do not observe types of issues and therefore price them at their expected values given all the publicly available information. Informed investors, on the other hand, observe debt types but extract information rent. Specifically, informed investors pay $\delta \in (0, 1)$ for a debt of value 1. We assume that informed investors do not participate in the bond market, or they are small enough not to impact bond prices.

If an issuer decides to sell its debt to uninformed investors, that is issue bonds, it can enhance issuers information by soliciting ratings. We assume there exist a single CRA in the market, which observes issue types and has access to a rating technology which consists of a set of signals and a function which maps each issue type to a probability distribution over the signal set. The CRA charges issuers a flat fee $\phi \geq 0$ per unit of face value of issue rated. We assume that once decided, the CRA is committed to the rating technology. Finally, the CRA's choice of rating technology and fee is public information.

We say an issue type is *targeted* by the CRA if the type is rated in equilibrium. Consider an arbitrary targeted type v held by an issuer. Let $\bar{U}$ denote the average market price of type v, if rated. Aside from soliciting a rating, the issuer can choose to sell the bonds unrated or sell the issue to the informed investors. Let $\bar{u}$ denote the highest average value of the asset to the issuer if not rated. Then the rating creates a surplus of $\bar{U} - \bar{u}$. Clearly, the CRA cannot charge the issuer a fee larger than the surplus or the issuer would refuse to purchase the rating. We place a stronger restriction by assuming that for any rating the CRA successfully sells, the fee ϕ is bounded from above by a fraction μ of the surplus the rating generates. In the above scenario, for instance, this assumption requires that $\phi \leq \mu(\bar{U} - \bar{u})$. We refer to this upper bound as the issuer's willingness to pay for rating. We can think of μ as the CRA's market power. A CRA with more market power can extract a larger fraction of trade surplus. Our key results require that the issuers receive a non zero fraction of the trade surplus, which is ensured by assuming that $\mu < 1$.[1]

[1] A setting in which this can happen is where the CRA and the issuers take turns to bid for the rating fee and there is a discount factor between turns of less than 1.

2.3.2. Firms

We now discuss in detail the issuers of type v_2 debt. There exists a continuum of firms and investment opportunities (*projects*), both represented by the positive real numbers. The outcome of a project x is binomial which are either a success where the project's return is $R(x)$ or a fail in which case the return is 0. The probability of success is identical for all projects and is denoted by q.

Undertaking a project requires investment of 1 unit such that the aggregate required capital to invest in all the projects between a and b for arbitrary $a < b$ is given by $\int_a^b dy = b - a$. Besides the capital requirement, a firm incurs a cost c for investing in a project. This cost can be firms' opportunity cost of investing in the projects or the effort required to develop the project. We make the following assumption regarding $R(.)$.

Assumption 2.1. *R is a continuous and monotonically decreasing function. Further, $qR(0) > c + \frac{1}{\delta}$ and $\lim_{a \to +\infty} R(a) = 0$.*

Assumption 2.1 ensures that the equilibrium exists, is unique, and is an interior solution. Notice in particular the part $qR(0) > c + \frac{1}{\delta}$. This condition states that if firms' could only borrow from informed investors, the total invested in the projects by all firms would be strictly positive. Since informed investors are the most expensive source of capital for the firms, this condition ensures that in equilibrium, the aggregate investment in the projects is always non zero.

Firms lack the required capital to invest in the projects. We assume the only method of raising capital is issuing debt. A debt issued by firm x is a contract that promises to pay the lenders firm x's cash flows from the project up to a specified amount $D(x)$, which we refer to as the face value of debt. We also assume that firms are committed to invest all the funds they raise by issuing debt in the projects. In particular, this means that (1) the firms are not allowed to extract any amount of the raised funds and (2) cancel investment based on the outcome of the fund raising. The total face value of debt m_2 is equal to

$$m_2 = \int_0^\infty D(x)dx$$

We assume that if firm x decides to invest in a project, x's cash flows are limited to those generated by the project and that lenders are able to verify the outcome of the projects. We make the following assumptions regarding firms' capacity to raise capital and projects' outcomes.

Assumption 2.2. *The face value of firm i's debt D_i is bounded from above by 1.*

The above assumption is intended to make sure that the total investment of any firm is infinitesimal. As such, the upper bound of 1 is arbitrary and has no impact on the aggregate investment in the projects or firms' cost of capital. Under assumption 2.2, when making investment decisions, firm x can ignore the impact of its decision on the market. Then if firm x decides to invest, it will issue as much debt as possible which by assumption 2.2 is equal to 1.

Assumption 2.3. *The outcome of all projects are perfectly correlated.*

Assumption 2.3 ensures that an arbitrary firm i's investment strategy would only impact the firm's expected return and not it's risk profile due to diversification. In this case, regardless of the firm's investment decision, the outcome is binary with probability of success given by q.

We also assume that if a firm decides to invest in the projects, the cash flow generated by the project if it succeeds is sufficient to pay off the debt holders, regardless of how much capital the firm raises by issuing bonds. A sufficient condition that ensures this outcome is a lower bound on cost c.

Assumption 2.4. $c > \frac{1}{\delta}$.

To see how assumption 2.4 ensures the intended outcome, notice that to borrow 1 unit, the most a firm must promise to pay to debt holders is $\frac{1}{\delta q}$, which is the price of borrowing from private investors. Since a firm would not invest in a project if the expected return is not at least as large as the cost c, assumption 2.4 yields

$$
\begin{aligned}
R >& \frac{c}{q} \\
>& \frac{1}{\delta q}
\end{aligned}
$$

Given risk neutrality and 0 discount rate, the market value of a unit of face value of firm x's debt is equal to q provided that if the project outcome is a success, the cash flow is sufficient to pay all the debt holders, which is guaranteed by assumption 2.4. Then the debt of all firms investing in these projects have identical unit value, which is equal to q. In other words, the type of these debts is given by $v_2 = q$.

2.3.3. Incentives and strategies

The timing of the investment game is the following.

1. Firms decide whether or not to invest. Firms which do invest, issue debts of face value 1. Without loss of generality, suppose firms $[0, x]$ have decided to invest.

2. The CRA decides rating precision p and fee ϕ.

3. Issuers decide whether to solicit a rating and whether to sell the issues to uniformed or informed investors.

4. Starting from firm 0, each firm $y \in [0, x]$ invests the raised capital in a project not taken by firms $[0, y)$.

5. Project outcome are determined. If a project succeeds, the firm pays its debt holders from project cash flows, and the rest is pocketed by the firm. If a project fails, the firm and its debt holder get nothing.

In making investment decisions, a firm's objective is to maximize the value of the equity.

However, since in our model firms' debt is sold at their expected market value, debt holders break even, and as such, maximizing equity value is equivalent to maximizing enterprise value. The strategy of a firm x is the probability of investing in the projects, given the strategy of other firms, the strategy of issuers, and the CRA.

An Issuer's objective is to maximize the raised capital per unit face value of debt issued. The strategy of an issuer consists of the probability of soliciting a rating, and the probability of selling the issues to the uninformed investors.

	q_3	q_2	q_1
s_3	1	$1-p$	0
s_2	0	p	0
s_1	0	0	1

Table 2.1: Tiered noisy rating with inflation

The CRA's objective is to maximize the total collected rating fees. To that end, the CRA's strategy is a choice of rating system and rating fee that maximizes its profit given the strategies of firms and issuers. We focus on a specific class of rating technologies summarized in table 2.1. This rating technology would always reveal type v_1 issues, but does not fully reveal type v_2 and v_3 issues. As will be discussed in more detail, with such a rating technology, type v_1 issuers do not solicit ratings. Therefore, this is effectively a two tiered noisy rating technology with which employs rating inflation.

2.4. First Best Solution

Absent any information asymmetry, all issuers are able to sell their issues in the market for their true values. Since we assumed firms which invest in the projects issue debts of unit face value, the capital raised by one such firm is equal to q. Then the expected profit of firm x, provided that all firms $y < x$ have invested, is given by

$$\begin{aligned} qqR(qx) - qc - q &= 0 \\ qR(qx) - c - 1 &= 0 \end{aligned} \tag{2.1}$$

The left hand side of equation (2.1) is decreasing in x and is negative for large enough x by assumption 2.1. Further, assumption 2.1 ensures that the left hand side of equation (2.1) is strictly positive for $x = 0$. Therefore, there exists a unique $x^{\mathrm{FB}} > 0$ that solves equation (2.1). Firm $x^{\mathrm{FB}} > 0$ is the firm which is indifferent between investing and not, provided that all firms $y < x^{\mathrm{FB}}$ invest. Another way to consider the first best solution is to look for the aggregate investment in the projects I^{FB} for which an extra unit of investment yields an expected return of 0. Clearly the first best investment level, I^{FB}, is equal to qx^{FB}, and therefore we can rewrite equation 2.1 as

$$qR(I^{\mathrm{FB}}) = c + 1 \tag{2.2}$$

Condition (2.2) equates the expected return of project I^{FB} (the left hand side), to a firm's cost c of investing in a project plus the cost of capital, which is equal to 1 since investors are competitive by assumption. Note that the first best solution is purely determined by the projects expected return $qR(.)$ and does not react to changes in q and $R(.)$ as long as the expected return remains constant. This is not the case for the equilibrium solution, however, as it will be demonstrated in the next section. We will utilize this property of the first best solution in our comparative statics analysis in later sections.

2.5. Equilibrium

We start by giving the formal definition of an equilibrium of the investment game.

Definition 2.1. *An equilibrium of the investment game is a set of firms' investment decisions, the CRA's and issuers' strategies, and issue prices such that*

- *Given the strategy of the CRA and issuers, and prices, any profitable investment in the projects is undertaken.*
- *Given firms' investment decisions, issuers' strategies, and prices, the CRA's profit is maximized.*

- *Provided the rating technology and fee, issuers maximize the capital raised.*
- *Given the investment decisions and the CRA and issuers' strategies, uninformed investors' prices for issues offered in the market is set to their corresponding expected values. The expectations are calculated over uninformed investors' beliefs, derived by Bayes rule whenever possible.*

We now proceed to derive an equilibrium of the investment game. We start by taking firms' investment decisions as given and solve the equilibrium of the rating sub-game. Note that since the face value of debts issued by all investing firms is equal to 1, then the total face value of debts issued by investing firms, m_2, is equal to the measure of firms which invest in the projects. Then in the rating sub-game, there are 3 issuer types v_1, v_2, v_3 and a total of m_1, m_2, m_3 face value of each type of issue, respectively. This rating sub-game[2] has a unique equilibrium in which the rating system belongs to table 2.1, described by the following lemma.

Lemma 2.1. *There exists an equilibrium of the rating sub-game in which the CRA sets* $p = \frac{\delta(m_3+m_2)}{m_3+\delta m_2}$ *and* $\phi = \mu(1-\delta)\frac{m_3 v_3+m_2 v_2}{m_3+m_2}$. *Type* v_1 *issuers do not solicit ratings. Market prices of unrated issues, as well as* s_1 *rated ones, are set to 0. Type* v_2 *and* v_3 *issuers do solicit ratings. Conditional on rating* s_3*, price of types* v_2 *and* v_3 *is set to* $U_3 = (1-\delta)\frac{m_3 v_3+m_2 v_2}{m_3+m_2} + \delta v_3$. *Conditional on rating* s_2*, price of type* v_2 *is set to* v_2. *All issues are traded at set market prices.*

Ratings enable issuers to profit by selling their assets to uninformed investors who demand lower rates (0 in our model) compared to informed investors. The CRA benefits by extracting a fraction of these profits. Lemma 2.1 states that the optimal strategy of the CRA in the rating sub-game is to choose a noisy rating system. The intuition behind this result is that in order for the CRA to maximize profit, that is, to charge the highest possible ϕ, it needs to choose rating precision p such that the willingness to pay for ratings of all the targeted types are equal. In the equilibrium of Lemma 2.1, only types v_2 and v_3 are targeted. By

[2]This game is similar to the one studied in Kartasheva and Yilmaz (2013)

assigning rating s_3 to type v_2 issues with some non zero probability, the willingness to pay of these two types are made equal.

Since we made the assumption that firms have no capital of their own, rating fees must naturally be paid by the proceedings of the issues. Consequently, the funds raised by all issuers who solicit a rating must be at least as large as the rating fee, regardless of the ratings issued. Among all the rated issue types, the smallest sale proceedings is equal to v_2, which is the outcome when a type v_2 is rated s_2. Therefore, imposing the constraint $v_2 > \phi$ ensures that all rated types are able to pay the rating fee. A sufficient condition that ensures $v_2 > \phi$ is

Lemma 2.2. $\mu(1-\delta) < \frac{q}{v_3} \Rightarrow v_2 > \phi$

In making their investment decisions, firms take into account the impact of total investment on the CRA's choice of ratings. Suppose at first that no firm is investing in the projects, that is $m_2 = 0$, and consider the incentives of firm 0. Assumption 2.1 ensures that investing in project 0 is profitable. To see this notice that if firm 0 decides to raise funds by selling bonds to uninformed investors, the amount raised net of the rating fee is at least as large as the amount offered by informed investors which is equal to δq. We can then conclude that the measure of firms investing in the projects in equilibrium must be positive. Now suppose all firms $x < m_2$ have decided to invest while firms $x > m_2$ have decided not to invest and consider the incentives of firm m_2. If firm m_2 invests, the expected return per unit of capital invested is equal to $qR(I)$, where the aggregate investment of all firms in the projects, I, is equal to

$$\begin{aligned} I =& m_2[pv_2 + (1-p)U_3 - \phi] \\ =& m_2 \left[(1-\mu)(1-\delta)\frac{m_3 v_3 + m_2 q}{m_3 + m_2} + \delta q \right], \end{aligned} \tag{2.3}$$

where the last equality follows directly from Lemma 2.1. The cost of investing a unit by firm m_2, on the other hand, is the sum of c and the expected cost of capital, which is equal to

$$
\begin{aligned}
& c+\frac{v_2}{pv_2+(1-p)U_3-\phi} \\
=& c+\frac{m_2 q}{I} \\
=& c+\frac{q}{(1-\mu)(1-\delta)\frac{m_3 v_3+m_2 q}{m_3+m_2}+\delta q}
\end{aligned}
$$

For the measure of firms investing in the equilibrium outcome to be equal to m_2, firm m_2 must be indifferent between investing and not investing, that is

$$
qR(I) = c+\frac{q}{(1-\mu)(1-\delta)\frac{m_3 v_3+m_2 q}{m_3+m_2}+\delta q}. \tag{2.4}
$$

As m_2 increases, the total amount invested in the projects, I, also increases which results in a decrease in the expected return of firm m_2's investment. At the same time, the right hand side of the indifference condition 2.4 is increasing in m_2 since $q = v_2 < v_3$ by assumption. As a result, there exist a unique m_2 for which the indifference condition (2.4) is satisfied.

Proposition 2.1. *The investment game has a unique equilibrium in which the CRA's rating technology belongs to table 2.1. The equilibrium measure of firms investing in the projects* m_2^{Eq} *and the aggregate investment* I^{Eq} *are the unique solutions to the following system of equations.*

$$
\begin{cases}
I^{Eq} & = \quad m_2^{Eq}\left[(1-\mu)(1-\delta)\frac{m_3 v_3+m_2 q}{m_3+m_2}+\delta q\right] \\
qR(I^{Eq}) & = \quad c+\frac{m_2^{Eq} q}{I^{Eq}}
\end{cases}
$$

A comparison of the first best and equilibrium indifference conditions (2.2) and (2.4) reveals how ratings can influence firms investment decisions through their impact on firms' effective cost of capital. In general, the cost of capital in equilibrium, $\frac{m_2^{\mathrm{Eq}} q}{I^{\mathrm{Eq}}}$ will not be equal to 1. Consequently, aggregate investment in equilibrium diverges from first best solution. For a given project expected returns, however, we can find a q for which the two solutions coincide.

Proposition 2.2. *Keep project expected returns, $qR(.)$ constant. Then for $\mu < \frac{1}{1+\frac{I^{FB}}{m_3 v_3}}$, there exists a unique q^* for which the equilibrium outcome and the first best coincide.*

$$q^* = \frac{(1-\mu)m_3 v_3 - \mu I^{FB}}{m_3}.$$

2.6. Comparative statics

In this section, we explore the impact of changes in model parameters on the equilibrium of the investment game. We start by analyzing the reaction of the equilibrium outcome to changes in the riskiness of the projects, captured by parameter q.

Proposition 2.3. *Keep the expected project returns constant. Then an increase in project risk leads to larger total investment in the projects in equilibrium. In other words, subject to $qR(.)$ being constant, $\frac{\partial}{\partial q} I^{Eq} < 0$.*

A decrease in q affects the equilibrium through different mechanisms. On the one hand, when projects are more risky, the firms need to issue more bonds in order to keep the aggregate investment constant. This is because the value of a bond of unit face value is decreasing in its risk. By increasing the volume of their issues, firms will be paying more in rating fees, which has a negative effect on the firms' cost of capital. On the other hand, since the strategy of the CRA involves equating the willingness to pay for ratings of all rated types, a riskier bond is traded, on average, at a higher premium. This effect effectively reduced firms' cost of capital. By Proposition 2.3, the second force is dominant. Then firms' effective cost of capital is decreasing in q, which in turn induces higher aggregate investment by the firms.

A different perspective into Proposition 2.3 can be gained by considering a set of model parameters for which the equilibrium aggregate investment is equal to first best, that is $I^{\mathrm{Eq}} = I^{\mathrm{FB}}$. Since we are keeping the projects' expected returns constant, changing q does not change the first best solution. The equilibrium aggregate investment, on the other hand, is a function of q which translates to over or under investments, depending on the direction of change in q.

Corollary 2.1. *Suppose that the equilibrium outcome coincides with first best, $I^{Eq} = I^{FB}$. Then subject to the expected return being constant, an increase in project risk (decrease in q) leads to over investment. Inversely, a decrease in risk is followed by under investment.*

From another point of view, Proposition 2.3 suggests that all else being equal, riskier projects are more attractive to the firms. In other words, credit ratings can induce excessive risk taking by firms.

Next, we fix the return function $R(.)$ and look at the effect of changes in risk q. Obviously, a decrease in q translates into a decrease in expected return which in turn leads to smaller equilibrium aggregate investment. However, since the aggregate investment in first best also drops as q decreases, it is not immediately clear how a change in risk impacts efficiency of the equilibrium outcome. For that reason, we consider the case in which equilibrium solution and first best coincide. We then apply a risk shock that reduces the market value of all issue types, while maintaining their relative values. The following proposition establishes the effect of such a risk shock.

Proposition 2.4. *Suppose that the equilibrium solution and first best coincide and fix the ratio $\frac{v_3}{q}$. Then provided that $R(x) > \frac{I^{FB}}{x} R(I^{FB})$, a shock to q results in the new equilibrium aggregate investment to fall below the new first best.*

Note that we only require condition $R(x) > \frac{I^{\mathrm{FB}}}{x} R(I^{\mathrm{FB}})$ to hold locally, around the first best solution. Proposition 2.4 states that for such a return curve, a risk shock causes under investment in equilibrium. This suggests that among other factors, ratings can contribute to inefficient fall in aggregate investment, following a financial crisis.

Next we examine the role of the CRA's market power, captured by parameter μ on equilibrium aggregate investment.

Proposition 2.5. $\frac{\partial}{\partial \mu} I^{Eq} < 0.$

As the CRA's market power, that is μ increases, the CRA would charge higher fee, which in effect increases firm's cost of capital, which in turn reduces aggregate investment in the projects. This result is interesting in that it has different implications regarding efficiency, depending on the macro environment. During expansions in which over investment is the likely market inefficiency, a more dominant CRA is a correcting force in that it curbs firms' incentives to invest and therefore improves efficiency. On the other hand, in the after math of a financial crisis where firms are prone to under invest, a dominant CRA is an impediment to recovery, since it further disincentivizes already under investing firms.

Unlike μ, increasing δ unambiguously improves efficiency. Recall that δ is effectively the factor by which informed investors discount the issuers' assets. As δ increases, informed investors' price for the issues tends to their true values. Given the fact that firms always have the option to borrow from informed investors, the CRA's reaction to an increase in δ is to improve rating precision and reduce rating fee, as evident from Lemma 2.1. In short, higher δ leads to firms' effective cost of capital to be closer to their fair value of 1, which translates into higher efficiency. In the limit where $\delta = 1$, a firm's raised capital is equal to the expected value of the bonds it issues, which means the equilibrium solution is the first best.

Proposition 2.6. *Efficiency is increasing in δ.*

In our model, we made the assumption that investors in the bond market are uninformed. We can now discuss qualitatively the effect of relaxing this assumption on equilibrium and efficiency. Kartasheva and Yilmaz (2013) show that when informed speculators are present together with uninformed investors, the winner's curse phenomenon prompts the CRA to increase rating precision in order to limit the informed speculators' information rent. In our model, an increase in rating precision would effectively increase firms' cost of capital, and in turn, reduce their incentives to invest. In other words, informed speculation would reduce aggregate investment in our model. During expansion periods, this mechanism has the potential to reduce firms' over investment and improve efficiency. However, our model suggests that informed speculators can hurt recovery during recessions due to their negative influence on firms' incentives to invest.

Proposition 2.7. *Informed speculation in the bond market reduces aggregate investment.*

2.7. Conclusion

In this work, we presented a framework to study whether and how ratings can affect firms investment decisions. We then established an equilibrium of the game and used the first best solution to measure the efficiency of the equilibrium solution. We derived the conditions under which the equilibrium solution is efficient and analyzed the impact of changes in parameters of the model on efficiency. Among others, our results suggest that ratings can incentivize firms to choose riskier investment when faced with ones with identical expected returns. We also showed that when there is a shock to the economy that effects investment risk, for instance a financial crisis, ratings can be responsible for under investment by the firms.

APPENDIX Chapter 1

Proof of lemma 1.1. First note that in the equilibrium of the continuation game, the market prices of s_2 and s_3 rated securities must be strictly higher than the price of s_1 rated securities, since s_1 rating reveals that the security belongs to the lowest tier with probability 1. The same logic requires s_3 rated securities to be prices at least as high as s_2 rated ones.

Now consider a security type v that does not belong to the set $\mathcal{A} = \{v_p, v_1, v_2, v_3\}$. Then the security must contain at least two asset types in the set $\mathcal{A}$, where at least one type is either v_2 or v_3. Suppose v is made up of b units of type v_p and a units of type v_2. Clearly, v receives rating s_1 by the CRA. Now consider the following decomposition of v into two new securities:1) type v' security made of b units of type v_p and $a - \epsilon$ units of type v_2, and 2) ϵ units of type v_2. Clearly, v' is also rated s_1, while type v_2 is rated either s_2 or s_3. As a result, market price of type v is strictly lower than the aggregate price of the decomposed version. Since the decomposition is costless, it is dominating.

A similar argument can be used for a security that is part v_p type and part v_3 type. It then follows that a dominating strategy by the issuers is to include in their portfolios only security types that match the rating tier cutoffs. $\square$

Proof of Proposition 1.1. First notice that since the total gain from trade is not a function of the CRA or issuer's actions, and in all cases the CRA extracts the maximum possible fraction of the total gain from trade, the CRA's strategy is clearly dominating. It remains to show that the strategies of the issuers constitute best responses.

Suppose $c \geq (1-\mu)(1-\delta)\frac{m_3 v_3 + m_2 v_2}{m_3 + m_2}$. This implies that given $\phi = \mu(1-\delta)\frac{m_3 v_3 + m_2 v_2}{m_3 + m_2}$ and $p = \frac{\delta(m_3 + m_2)}{m_3 + \delta m_2}$, issuer 3 prefers to issue its endowment of type 3 asset without bundling it with the production asset. To see this notice that

$$\begin{aligned}
U_3 - \phi \geq& (1+\lambda)[p_{22}v_2 + (1 - p_{22}U_3)] - \lambda c - (1+\lambda)\phi \\
\Leftrightarrow \lambda(c+\phi) \geq& (1+\lambda)[p_{22}v_2 + (1-p_{22})U_3] - U_3 \\
\geq& (1+\lambda)(\frac{\phi}{\mu} + \delta v_2) - \frac{\phi}{\mu} - \delta v_3 \\
\geq& \lambda\frac{\phi}{\mu} + (1+\lambda)\delta v_2 - \delta v_3 \\
\Leftrightarrow c \geq& \frac{\phi}{\mu} - \phi \\
=& (1-\mu)(1-\delta)\frac{m_3 v_3 + m_2 v_2}{m_3 + m_2}.
\end{aligned}$$

Next, suppose $\frac{(1-\delta)(1-\mu)(m_3v_3+m_2v_2)}{m_3+m_2+\lambda_3 m_3} < c < (1-\mu)(1-\delta)\frac{m_3v_3+m_2v_2}{m_3+m_2}$. Following the above logic, not bundling is not an equilibrium. Then type v_3 bundles fraction α of his asset with $\lambda_3 m_3$ units of the production asset. For this to constitute an equilibrium, issuer v_3 must be indifferent between 1) soliciting a rating and issuing a unit of vanilla type v_3 and 2) packaging that with the production asset, soliciting a rating for the package, and issuing the package in the market. This is the case iff

$$\begin{aligned}
U_3 - \phi =& (1+\lambda_3)[pv_2 + (1-p)U_3] - (1+\lambda_3)\phi - \lambda_3 c \\
\Leftrightarrow \lambda_3(\phi + c) =& (1+\lambda_3)[pv_2 + (1-p)U_3] - U_3.
\end{aligned}$$

Also, for the CRA to extract the maximum possible fraction of the trade surplus, type v_2 and v_3 assets must have equal wilingness to pay for the rating.

$$(U_3 - \delta v_3) = ([pv_2 + (1-p)U_3] - \delta v_2).$$

Finally, $\phi = \mu(U_3 - \delta v_3)$. The solution to the above 3 linear equations in 3 unknowns follows.

$$
\begin{aligned}
\lambda(\phi + C) =& (1+\lambda)(\frac{\phi}{\mu} + \delta v_2) - (\frac{\phi}{\mu} + \delta v_3) \\
=& \lambda\frac{\phi}{\mu} + (1+\lambda)\delta v_2 - \delta v_3 \\
=& \lambda\frac{\phi}{\mu} \\
\Rightarrow \phi + C =& \frac{\phi}{\mu} \\
\phi =& c\frac{\mu}{1-\mu},
\end{aligned}
$$

$$
\begin{aligned}
[pv_2 + (1-p)U_3] - \delta v_2 =& \frac{\phi}{\mu} \\
[pv_2 + (1-p)(\frac{\phi}{\mu} + \delta v_3)] - \delta v_2 =& \frac{\phi}{\mu} \\
p[-v_2 + \frac{\phi}{\mu} + \delta v_3] =& \delta(v_3 - v_2) \\
p =& \frac{\delta(v_3 - v_2)}{\delta v_3 - v_2 + \frac{\phi}{\mu}} \\
=& \frac{\delta(v_3 - v_2)}{\delta v_3 - v_2 + \frac{c}{1-\mu}},
\end{aligned}
$$

$$
\begin{aligned}
U_3 =& \frac{\phi}{\mu} + \delta v_3 \\
=& \frac{c}{1-\mu} + \delta v_3.
\end{aligned}
$$

Now we can use the above to solve for α. We can write U_3 as a function of α according to the following.

$$\begin{aligned}U_3 =&p(v_3|s_3)v_3 + p(v_2|s_3)v_2\\=&\frac{p(s_3|v_3)p(v_3)}{p(s_3)}v_3 + \frac{p(s_3|v_2)p(v_2)}{p(s_2)}v_2\\=&\frac{m_3(1-\alpha)v_3 + (1-p)(m_2+\alpha(1+\lambda_3)m_3)v_2}{m_3(1-\alpha)+(1-p)(m_2+\alpha(1+\lambda_3)m_3)}.\end{aligned}$$

Rearranging the terms and we have

$$\begin{aligned}\frac{U_3-v_2}{v_3-U_3} =&\frac{m_3(1-\alpha)}{(1-p)(m_2+\alpha(1+\lambda_3)m_3)}\\(1-p)\frac{U_3-v_2}{v_3-U_3} =&\frac{m_3(1-\alpha)}{(m_2+\alpha(1+\lambda_3)m_3)}\\\frac{-(1-\delta)(1-\mu)v_2+c}{(1-\mu)(1-\delta)v_3-c} =&\frac{m_3(1-\alpha)}{(m_2+\alpha(1+\lambda_3)m_3)}\end{aligned}$$

it immediately follows that

$$\begin{aligned}&m_3[(1-\mu)(1-\delta)v_3-c]+m_2[(1-\mu)(1-\delta)v_2-c]\\=&\alpha m_3[(1-\mu)(1-\delta)v_3-c-(1-\mu)(1-\delta)(1+\lambda_3)v_2+(1+\lambda_3)c],\end{aligned}$$

which, noting that $(1+\lambda_3)v_2 = v_3$, simplifies to

$$\begin{aligned}\alpha\lambda_3 m_3 c =&(1-\delta)(1-\mu)(m_3v_3+m_2v_2)-c(m_3+m_2)\\\alpha =&\frac{(1-\delta)(1-\mu)(m_3v_3+m_2v_2)-c(m_3+m_2)}{\lambda_3 m_3 c}\end{aligned}$$

Finally, if $c \leq \frac{(1-\delta)(1-\mu)(m_3v_3+m_2v_2)}{m_3+m_2+\lambda_3 m_3}$, we need to show that $p = 0$ and $\alpha = 1$ constitute an equilibrium strategy. If $p = 0$ and $\alpha = 1$, then $U_3 = v_2$ and $\phi = \mu(1-\delta)v_2$. We need to show that type v_3 prefers bundling a unit of type v_3 asset with the production asset over offering it as a vanilla type v_3 asset. This is true if

$$
\begin{aligned}
(1+\lambda_3)v_2 - (1+\lambda_3)\phi - \lambda_3 c \geq & v_2 - \phi \\
v_2(1-\mu(1-\delta)) \geq & c
\end{aligned}
$$

which holds if

$$
\begin{aligned}
\frac{(1-\delta)(1-\mu)(m_3v_3+m_2v_2)}{m_3+m_2+\lambda_3 m_3} \leq & v_2(1-\mu(1-\delta)) \\
(1-\delta)(1-\mu)(m_3v_3+m_2v_2) \leq & (1-\mu(1-\delta))(m_3v_3+m_2v_2) \\
(1-\delta)(1-\mu) \leq & (1-\mu(1-\delta)) \\
0 \leq & \delta.
\end{aligned}
$$

Also, we need to show that type v_3 issuer prefers soliciting a rating over its assets reservation value. This is the case if

$$
\begin{aligned}
(1+\lambda_3)v_2 - (1+\lambda_3)\phi - \lambda_3 c \geq & \delta v_3 \\
v_3(1-\mu(1-\delta)) - \lambda_3 c \geq & \delta v_3 \\
(1-\mu)(1-\delta)v_3 \geq & \lambda_3 c,
\end{aligned}
$$

which is true if

$$
\begin{aligned}
\frac{(1-\mu)(1-\delta)v_3}{\lambda_3} \geq & \frac{(1-\delta)(1-\mu)(m_3v_3+m_2v_2)}{m_3+m_2+\lambda_3 m_3} \\
\frac{v_3}{\lambda_3} \geq & \frac{(m_3v_3+m_2v_2)}{m_3+m_2+\lambda_3 m_3} \\
m_3v_3+m_2v_3+\lambda_3 m_3 v_3 \geq & \lambda_3 m_3 v_3 + m_2(v_3-v_2) \\
m_3v_3+m_2v_2 \geq & 0.
\end{aligned}
$$

□

Proof of Proposition 1.2. Proposition 1.1 established that the CRA extracts fraction μ of the trade surplus in equilibrium which immediately yields π^{CRA}. Next,

$$
\begin{aligned}
\frac{\pi^2}{m_2} =& [pv_2 + (1-p)U_3] - \phi - \delta v_2 \\
=& \left(\frac{\phi}{\mu} + \delta v_2\right) - \delta v_2 - \phi \\
=& \frac{1-\mu}{\mu}\phi.
\end{aligned}
$$

The rest immediately follows from Proposition 1.1 by substituting for ϕ. Finally, for $c > \underline{c}$,

$$
\begin{aligned}
\frac{\pi^3}{m_3} =& U_3 - \phi - \delta v_3 \\
=& \left(\frac{\phi}{\mu} + \delta v_3\right) - \delta v_3 - \phi \\
=& \frac{1-\mu}{\mu}\phi.
\end{aligned}
$$

The rest follows immediately by substituting for ϕ from Proposition 1.1. For $c \leq \underline{c}$,

$$
\begin{aligned}
\frac{\pi^3}{m_3} =& v_3 - \delta v_3 - c\frac{v_3 - v_2}{v_2} - \phi\frac{v_3}{v_2} \\
=& (1-\delta)v_3 - \mu(1-\delta)v_3 - c\frac{v_3 - v_2}{v_2} \\
=& (1-\mu)(1-\delta)v_3 - c\frac{v_3 - v_2}{v_2}.
\end{aligned}
$$

□

Lemma A.1. *In any equilibrium of the production game the following must hold.*

1. *Issuers 2 and 3 sell all their assets in the market. All bundles containing type v_2 or v_3 assets are rated.*

2. *The CRA extracts fraction μ of the gain from trade of any rated bundle.*

3. *The profit of the CRA is $\mu(1-\delta)(m_3 v_3 + m_2 v_2)$.*

Proof of lemma A.1. First observe that since the gain from trade of the production asset is 0, regardless of the amount of production asset present in the market, total gain from trade to be made is given by

$$(1-\delta)(m_3 v_3 + m_2 v_2). \tag{A.1}$$

Since by assumption, the CRA can extract at most a fraction μ of the gain from any trade, the profit of the CRA in any equilibrium of the production game is bounded from above by the product of μ and (A.1). Finally, as established by Proposition 1.1, there exist's an equilibrium in which the profit of the CRA is equal to the upper bound $\mu(1-\delta)(m_3v_3+m_2v_2)$ established here. This concludes the argument. □

Proof of Proposition 1.3. We have

$$m_p = \alpha \frac{v_3 - v_2}{v_2} m_3.$$

For $c \geq \bar{c}$ and $c \leq \underline{c}$, α is equal to 0 and 1, respectively, for which m_p is immediately derived. For $\underline{c} < c < \bar{c}$, we substitute for α from Proposition 1.1, and we have

$$m_p = \frac{(1-\delta)(1-\mu)(m_3 v_3 + m_2 v_2) - c(m_3 + m_2)}{c}.$$

□

Lemma A.2. *Suppose a bundle with type v is rated and traded in an equilibrium of the production game. Then the average market price per unit of the bundle is*

$$\overline{U} = \frac{\phi}{\mu} + \delta v.$$

Proof of lemma A.2. The average gain from trade of a unit of the bundle is $\overline{U} - \delta v$. Then by lemma A.1 we have

$$\begin{aligned}\phi =& \mu(\overline{U} - \delta v) \\ \Rightarrow \overline{U} =& \frac{\phi}{\mu} + \delta v.\end{aligned}$$

□

Proof of Proposition 1.4. First note that since the production asset generates no gain from trade, the total gain from trade in any equilibrium is upper bounded by $(1-\delta)(m_3v_3+m_2v_2)$. Next, since the CRA can extract at most fraction μ of the gain from any trade, the CRA's profit cannot exceed the product of the maximum total gain from trade and fraction μ, or $\mu(1-\delta)(m_3v_3+m_2v_2)$. We already established an equilibrium in which the CRA's profit reaches the above upper bound. Consequently, in any equilibrium of the game, the profit of the CRA must be equal to

$$\mu(1-\delta)(m_3v_3+m_2v_2). \tag{A.2}$$

Now consider any equilibrium of the game in which m_p units of the production asset is produced. For the CRA's profit to be equal to (A.2), all type v_2 and v_3 assets must be traded. Further, since the production technology has negative NPV, any units of the production asset is also traded. As a result, rating fee ϕ has to be equal to

$$\phi = \frac{\mu(1-\delta)(m_3v_3+m_2v_2)}{m_3+m_2+m_p}.$$

Now consider the equilibrium strategy of issuer 3. Observe that the average type of issuer 3's equilibrium portfolio must be larger than v_2, otherwise, $m_p \geq m_3\lambda_3$ and we are done. Then there must exist a bundle b_1 in that portfolio whose unit value, $\hat{v}_3$, is strictly above v_2. Then for each unit of type 3 asset in b_1, there are $\hat{\lambda}_1 = \frac{v_3-\hat{v}_3}{\hat{v}_3}$ units of production asset. Now take a look at a unit of type v_3 asset in b. Issuer 3's profit for a unit of type v_3 asset by creating bundle b, soliciting a rating and trading in the market is given by

$$\begin{aligned}
&(1+\hat{\lambda}_1)[\frac{\phi}{\mu}+\delta\frac{v_3}{1+\hat{\lambda}_1}]-\hat{\lambda}_1 c-(1+\hat{\lambda}_1)\phi\\
=&\delta v_3+(1+\hat{\lambda}_1)\phi\frac{1-\mu}{\mu}-\hat{\lambda}_1 c.
\end{aligned}$$

Now consider an arbitrary type $\hat{v}_2 \leq v_2$ bundle in the portfolio of issuer 2. Issuer 3 can deviate from equilibrium action by replacing bundle b_1 with a type $\hat{v}_2$ bundle b_2, soliciting a rating for b_2 and trading b_2 in the market. To form bundle b_2, issuer 3 can simply group a unit of type v_3 with $\hat{\lambda}_2 = \frac{v_3-\hat{v}_2}{\hat{v}_2}$ of the production asset. Issuer 3's profit from deviation is then calculated as

$$\begin{aligned}
&(1+\hat{\lambda}_2)[\frac{\phi}{\mu}+\delta\frac{v_3}{1+\hat{\lambda}_2}]-\hat{\lambda}_2 c-(1+\hat{\lambda}_2)\phi\\
=&\delta v_3+(1+\hat{\lambda}_2)\phi\frac{1-\mu}{\mu}-\hat{\lambda}_2 c.
\end{aligned}$$

Equilibrium requires that the above deviation not be profitable. Thus the following must be true.

$$\delta v_3+(1+\hat{\lambda}_2)\phi\frac{1-\mu}{\mu}-\hat{\lambda}_2 c \leq \delta v_3+(1+\hat{\lambda}_1)\phi\frac{1-\mu}{\mu}-\hat{\lambda}_1 c,$$

which is equivalent to $\phi\frac{1-\mu}{\mu} \leq c$, since $\hat{\lambda}_1 < \hat{\lambda}_2$. Thus equilibrium requires that

$$\begin{aligned}
\phi\frac{1-\mu}{\mu} &\leq c\\
\frac{\mu(1-\delta)(m_3v_3+m_2v_2)}{m_3+m_2+m_p}\frac{1-\mu}{\mu} &\leq c\\
\frac{(1-\mu)(1-\delta)(m_3v_3+m_2v_2)}{c} &\leq m_3+m_2+m_p\\
\frac{(1-\mu)(1-\delta)(m_3v_3+m_2v_2)}{c}-(m_3+m_2) &\leq m_p.
\end{aligned}$$

$\square$

Proof of Proposition 1.5. By definition, $\overline{c} = (1-\mu)(1-\delta)\frac{m_3 v_3 + m_2 v_2}{m_3+m_2}$ which is decreasing in δ. Then there exist a sufficiently small δ such that $\delta v_3 < \overline{c}$. Next, since $\overline{c} < \frac{m_3 v_3 + m_2 v_2}{m_3+m_2}$, there exists m_1 such that the following is true.

$$\delta v_3 < \frac{m_3 v_3 + m_2 v_2}{m_3 + m_2 + m_1} < \overline{c}.$$

Since the above set of constraints does not involve c, all that remains to select an appropriate c that satisfies (1.15). □

Lemma A.3. *The following is true for U_3 and p_f.*

$$U_3 = (1-p_f)v_3 + p_f v_2$$

Proof of lemma A.3. By definition

$$\begin{aligned} U_3 =& \Pr(v_3|s_3)v_3 + \Pr(v_2|s_3) \\ =& (1-p_f)v_3 + p_f v_2, \end{aligned}$$

where that last line follows directly from the definition of p_f. □

Proof of Proposition 1.6. By Proposition 1.1, rating precision is equal to $p = \frac{\delta(v_3 - v_2)}{\delta v_3 - v_2 + \frac{c}{1-\mu}}$, which is clearly decreasing in c.

Next, lemma A.3 and the assumption $v_3 > v_2$ imply that p_f is decreasing in U_3. From Proposition 1.1, $U_3 = \frac{c}{1-\mu} + \delta v_3$. Then since U_3 is increasing in c, then p_f must be decreasing in the same. □

Proof of Proposition 1.7. Since lemma A.3 and the fact that $v_3 > v_2$ imply that p_f is decreasing in U_3, it suffices to show that U_3 is increasing in μ. By Proposition 1.1, $U_3 = \frac{c}{1-\mu} + \delta v_3$, which is increasing in μ and therefore the first part of the Proposition is proved.

Next, by Proposition 1.3, $m_p c = (1-\delta)(1-\mu)(m_3 v_3 + m_2 v_2) - c(m_3 + m_2)$, which is decreasing in μ. $\square$

Proof of Proposition 1.8. The first claim follows from lemma A.3 and the fact that $U_3 = \frac{c}{1-\mu} + \delta v_3$ for intermediate costs and is clearly not a function of $\frac{m_3}{m_2}$.

For the second claim to be meaningful, we normalized the inefficiency by W^{FB}. Then by Proposition 1.3,

$$\begin{aligned}
\frac{m_p c}{W^{\mathrm{FB}}} =& \frac{(1-\delta)(1-\mu)(m_3 v_3 + m_2 v_2) - c(m_3 + m_2)}{(1-\delta)(m_3 v_3 + m_2 v_2)} \\
=& 1 - \mu - \frac{c}{1-\delta} \frac{m_3 + m_2}{m_3 v_3 + m_2 v_2} \\
=& 1 - \mu - \frac{c}{1-\delta} \frac{1}{\frac{m_3 v_3 + m_2 v_2}{m_3 + m_2}}.
\end{aligned}$$

The term $\frac{m_3 v_3 + m_2 v_2}{m_3 + m_2}$ represent the average quality of type v_3 and v_2 assets, which is increasing in $\frac{m_3}{m_2}$. Therefore, the normalized inefficiency is also increasing in $\frac{m_3}{m_2}$. $\square$

APPENDIX Chapter 2

Proof of Lemma 2.1. We start by examining the incentives of the issuers. The profit of type v_1 issuers in the equilibrium candidate is equal to 0. If type v_1 issuers deviate by soliciting ratings, their types are revealed and their asset is priced 0. In addition they must pay the rating fee. The other deviation is to sell the issues to informed investors in which case their profit is still 0. Therefore, there are no profitable deviations for type v_1.

For type v_2, the profit by following equilibrium candidate is equal to

$$\begin{aligned}
pv_2 + (1-p)U_3 - \phi =& (1-\delta)\frac{m_3v_3 + m_2v_2}{m_3 + m_2} + \delta v_2 - \phi \\
=& (1-\mu)(1-\delta)\frac{m_3v_3 + m_2v_2}{m_3 + m_2} + \delta v_2 \\
\geq& \delta v_2.
\end{aligned}$$

A deviation by type v_2 either involves selling to informed investors for δv_2, or selling to uninformed investors without soliciting a rating for 0. Clearly none is profitable.

Finally, type v_3's equilibrium candidate profit is given by

$$\begin{aligned}
U_3 - \phi =& (1-\mu)(1-\delta)\frac{m_3v_3 + m_2v_2}{m_3 + m_2} + \delta v_3 \\
\geq& \delta v_3.
\end{aligned}$$

A deviation by type v_3 is either selling to informed investors for δv_3 or selling the issues unrated for 0, neither profitable.

Next, consider the incentives of the CRA. The total value of issuers' assets is equal to $m_3v_3 + m_2v_2$, which is also what they are collectively traded for if they are rated and offered to uninformed investors. Alternatively, ff issuers sell to informed investors, their assets are sold for $\delta(m_3v_3 + m_2v_2)$. Then the ratings generate a trade surplus equal to $(1-\delta)(m_3v_3 + m_2v_2)$. By assumption, the CRA can charge at most fraction μ of surplus generated by ratings in any trade. Therefore, the profit of the CRA cannot exceed $\mu(1-\delta)(m_3 v_3 + m_2 v_2)$. The CRA's profit in the equilibrium candidate is equal to $(m_3+m_2)\phi = \mu(1-\delta)(m_3 v_3 + m_2 v_2)$. Therefore there are no profitable deviations for the CRA.

Finally, we need to confirm that ϕ does not exceed any rated type's willingness to pay.

$$\begin{aligned} \mu(U_3 - \delta v_3) =& \mu(1-\delta)\frac{m_3 v_3 + m_2 v_2}{m_3 + m_2} \\ =& \mu[p v_2 + (1-p)U_3 - \delta v_2] \\ =& \phi. \end{aligned}$$

$\square$

Proof of Lemma 2.2 . Substitute for ϕ from Lemma 2.1

$$\begin{aligned} \mu(1-\delta)\frac{m_3 v_3 + m_2 q}{m_3 + m_2} <& q \\ \mu(1-\delta)(m_3 v_3 + m_2 q) <& (m_3 + m_2) q \end{aligned}$$

Since $q < v_3$, A sufficient condition that guarantees the above inequality is that

$$\mu(1-\delta) v_3 < q.$$

$\square$

Proof of Proposition 2.2.

$$\begin{aligned} m_2 q = I^{\mathrm{FB}} & \\ =& m_2 \left[(1-\mu)(1-\delta)\frac{m_3 v_3 + m_2 q}{m_3 + m_2} + \delta q \right] \end{aligned}$$

$$
\begin{aligned}
I^{\mathrm{FB}} =& m_2 \left[(1-\mu)(1-\delta)\frac{m_3 v_3 + I^{\mathrm{FB}}}{m_3+m_2} + \delta \frac{I^{\mathrm{FB}}}{m_2}\right] \\
=& (1-\mu)(1-\delta)\frac{m_2}{m_3+m_2}(m_3 v_3 + I^{\mathrm{FB}}) + \delta I^{\mathrm{FB}} \\
\Rightarrow (1-\delta) I^{\mathrm{FB}} =& (1-\mu)(1-\delta)\frac{m_2}{m_3+m_2}(m_3 v_3 + I^{\mathrm{FB}}) \\
I^{\mathrm{FB}} =& (1-\mu)\frac{m_2}{m_3+m_2}(m_3 v_3 + I^{\mathrm{FB}}) \\
\Rightarrow 1 + \frac{m_3}{m_2} =& (1-\mu)\left(1 + \frac{m_3 v_3}{I^{\mathrm{FB}}}\right) \\
\frac{m_3}{m_2} =& (1-\mu)\left(1 + \frac{m_3 v_3}{I^{\mathrm{FB}}}\right) - 1 \\
\Rightarrow m_2 =& \frac{m_3}{(1-\mu)\left(1 + \frac{m_3 v_3}{I^{\mathrm{FB}}}\right) - 1} \\
=& \frac{m_3 I^{\mathrm{FB}}}{(1-\mu) m_3 v_3 - \mu I^{\mathrm{FB}}}
\end{aligned}
$$

The solution exists if

$$
\begin{aligned}
(1-\mu)\left(1 + \frac{m_3 v_3}{I^{\mathrm{FB}}}\right) >& 1 \\
1 + \frac{m_3 v_3}{I^{\mathrm{FB}}} >& \frac{1}{1-\mu} \\
\frac{m_3 v_3}{I^{\mathrm{FB}}} >& \frac{\mu}{1-\mu} \\
\Leftrightarrow m_3 v_3 \left(\frac{1}{\mu} - 1\right) >& I^{\mathrm{FB}} \\
\Leftrightarrow \frac{1}{1 + \frac{I^{\mathrm{FB}}}{m_3 v_3}} >& \mu
\end{aligned}
$$

The second condition

$$
\begin{aligned}
q <& (1-\mu)v_3 \\
\frac{I^{\text{FB}}}{m_2} <& (1-\mu)v_3 \\
I^{\text{FB}} <& (1-\mu)m_2 v_3 \\
=& (1-\mu)\frac{m_3 v_3}{(1-\mu)\left(1+\frac{m_3 v_3}{I^{\text{FB}}}\right)-1} \\
(1-\mu)\left(I^{\text{FB}}+m_3 v_3\right)-I^{\text{FB}} <& (1-\mu)m_3 v_3 \\
-\mu I^{\text{FB}} <& 0,
\end{aligned}
$$

which is always true. □

Proof of Proposition 2.3. Recall that the indifference condition for the equilibrium solution is given by the equation

$$
\begin{aligned}
qR(I) =& c + \frac{q}{(1-\mu)(1-\delta)\frac{m_3 v_3 + m_2 q}{m_3+m_2}+\delta q} q \\
=& c + \frac{m_2 q}{I}.
\end{aligned}
$$

Keep I constant. Then by Lemma B.2, an increase in q leads to an increase in the right hand side of the indifference condition. To reestablish the indifference condition then the aggregate investment I must decrease since the left hand side is decreasing in I. □

Lemma B.1. *The solution to the equation* $I = m_2\left[(1-\mu)(1-\delta)\frac{m_3 v_3+m_2 q}{m_3+m_2}+\delta q\right]$ *is unique and equal to*

$$
\begin{aligned}
m_2 =& \frac{1}{2q((1-\mu)(1-\delta)+\delta)}\Big\{I - [(1-\mu)(1-\delta)m_3 v_3 + \delta m_3 q] \\
&+ \sqrt{[I-((1-\mu)(1-\delta)m_3 v_3+\delta m_3 q)]^2 + 4[(1-\mu)(1-\delta)+\delta] I m_3 q}\Big\}
\end{aligned}
$$

Proof.

$$I = m_2\left[(1-\mu)(1-\delta)\frac{m_3v_3+m_2q}{m_3+m_2}+\delta q\right].$$

Define $\alpha = (1-\mu)(1-\delta)$ and we solve for m_2 as a function of I and q.

$$I(m_3+m_2) = m_2[\alpha(m_3v_3+m_2q)+\delta(m_3+m_2)q]$$

Rearranging the terms

$$m_2^2[(\alpha+\delta)q]+m_2[\alpha m_3v_3+\delta m_3q-I]-m_3I = 0$$

The above quadratic equation has a positive and a negative solution. The positive solution is

$$m_2 = \frac{I-(\alpha m_3v_3+\delta m_3q)+\sqrt{[I-(\alpha m_3v_3+\delta m_3q)]^2+4(\alpha+\delta)Im_3q}}{2q(\alpha+\delta)}$$

□

Lemma B.2. *Subject to I being constant, $\frac{\partial}{\partial q}m_2q>0$.*

Proof. Define $\alpha = (1-\mu)(1-\delta)$ and $x = m_3q$. Then by Lemma B.1, we have

$$\begin{aligned} m_2q &= \frac{I-(\alpha m_3v_3+\delta m_3q)+\sqrt{[I-(\alpha m_3v_3+\delta m_3q)]^2+4(\alpha+\delta)Im_3q}}{2(\alpha+\delta)} \\ &= \frac{I-(\alpha m_3v_3+\delta x)+\sqrt{[I-(\alpha m_3v_3+\delta x)]^2+4(\alpha+\delta)Ix}}{2(\alpha+\delta)} \end{aligned}$$

Since the denominator is constant and positive, the derivative of m_2q with respect to q has the same sign as the derivative of $f(x) = 2(\alpha+\delta)m_2 q$.

$$\begin{aligned}
f'(x) >& 0 \\
\Leftrightarrow \frac{[I-(\alpha m_3 v_3+\delta x)](-\delta)+2(\alpha+\delta)I}{\sqrt{[I-(\alpha m_3 v_3+\delta x)]^2+4(\alpha+\delta)Ix}} >& \delta \\
\Leftrightarrow [I-(\alpha m_3 v_3+\delta x)](-\delta)+2(\alpha+\delta)I >& \delta\sqrt{[I-(\alpha m_3 v_3+\delta x)]^2+4(\alpha+\delta)Ix} \\
\Leftrightarrow \{[I-(\alpha m_3 v_3+\delta x)](-\delta)+2(\alpha+\delta)I\}^2 >& \delta^2[I-(\alpha m_3 v_3+\delta x)]^2+4\delta^2(\alpha+\delta)Ix
\end{aligned}$$

where the last line follows since both sides of the inequality are positive.

$$\begin{aligned}
& f'(x) > 0 \\
\Leftrightarrow & \delta^2[I-(\alpha m_3 v_3+\delta x)]^2+4(\alpha^2+\delta^2+2\alpha\delta)I^2+4[I-(\alpha m_3 v_3+\delta x)](-\delta)(\alpha+\delta)I \\
& > \delta^2[I-(\alpha m_3 v_3+\delta x)]^2+4\delta^2(\alpha+\delta)Ix \\
\Leftrightarrow & 4(\alpha^2+\delta^2+2\alpha\delta)I^2+4[I-(\alpha m_3 v_3+\delta x)](-\delta)(\alpha+\delta)I \\
& > 4\delta^2(\alpha+\delta)Ix \\
\Leftrightarrow & (\alpha^2+\delta^2+2\alpha\delta)I^2+[I-(\alpha m_3 v_3+\delta x)](-\delta)(\alpha+\delta)I \\
& > \delta^2(\alpha+\delta)Ix \\
\Leftrightarrow & (\alpha^2+\delta^2+2\alpha\delta)I^2+(-\delta^2-\alpha\delta)I^2+\delta(\alpha+\delta)(\alpha m_3 v_3+\delta x)I \\
& > \delta^2(\alpha+\delta)Ix \\
\Leftrightarrow & (\alpha^2+\alpha\delta)I^2+\alpha\delta(\alpha+\delta)m_3 v_3 I \\
& > 0.
\end{aligned}$$

□

Proof of proposition 2.4. Let q^* be the success probability for which the equilibrium solution is identical to the first best solution and denote the solution by m_2^*, I^{FB}. The indifference conditions for the first best and the equilibrium require that

$$
\begin{aligned}
q^* R(I^{\text{FB}}) =& c + \frac{m_2^* q^*}{I^{\text{FB}}} \\
=& c + 1.
\end{aligned}
$$

Now consider a shock to the value of issues in the form of $v_3' = av_3$, and $q' = aq$. The new first best and equilibrium solutions must satisfy

$$
\begin{aligned}
q' R(I^{\text{FB}}) =& c + 1 \\
q' R(I^{\text{Eq}}) =& c + \frac{m_2^{\text{Eq}} q'}{I^{\text{Eq}}}.
\end{aligned}
$$

We want to show that $I^{\text{Eq}} < I^{\text{FB}}$. It suffices to show that $\frac{m_2^{\text{Eq}} q'}{I^{\text{Eq}}} > 1$. Consider the quantity $\frac{m_2' q'}{I'}$ where $m_2' = m_2^*$ and $I' = aI^{\text{FB}}$. Then

$$
\begin{aligned}
\frac{m_2' q'}{I'} =& \frac{m_2^* aq}{aI^{\text{FB}}} \\
=& 1.
\end{aligned}
$$

We can also confirm that I' is consistent as

$$
\begin{aligned}
I' =& m_2' \left[\frac{m_3 v_3' + m_2' q'}{m_3 + m_2'} + \delta q' \right] \\
=& m_2^* \left[\frac{m_3 a v_3 + m_2^* a q^*}{m_3 + m_2'} + \delta a q^* \right] \\
=& a I^{\text{FB}}
\end{aligned}
$$

We can now examine the quantity $q'R(I')$.

$$\begin{aligned} q'R(I') =& aq^*R(aI^{\text{FB}}) \\ >& q^*R(I^{\text{FB}}) \\ =& c+1 \\ =& c+\frac{m_2'q'}{I'} \end{aligned}$$

where we used the assumption $R(I) > \frac{I^{\text{FB}}}{I}R(I^{\text{FB}})$ from Proposition 2.4. Since $q'R(I') > c + \frac{m_2'q'}{I'}$, then it must be the case that $I^{\text{Eq}} > I'$, which implies that

$$\begin{aligned} \frac{m_2^{\text{Eq}}q'}{I^{\text{Eq}}} >& \frac{m_2'q'}{I'} \\ =& 1. \end{aligned}$$

$\square$

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