University of Pennsylvania
ScholarlyCommons

Publicly Accessible Penn Dissertations

2017

Essays On Taxation And Competition Under Firm Heterogeneity And Financial Frictions

Daniel S. Wills
University of Pennsylvania, dwills@sas.upenn.edu

Follow this and additional works at: https://repository.upenn.edu/edissertations

Part of the Economics Commons

Recommended Citation

Wills, Daniel S., "Essays On Taxation And Competition Under Firm Heterogeneity And Financial Frictions" (2017). *Publicly Accessible Penn Dissertations*. 2636.
https://repository.upenn.edu/edissertations/2636

This paper is posted at ScholarlyCommons. https://repository.upenn.edu/edissertations/2636
For more information, please contact repository@pobox.upenn.edu.

Essays On Taxation And Competition Under Firm Heterogeneity And Financial Frictions

Abstract

In this dissertation, I study the implications of taxation -and other regulations- in environments with financial frictions and firm entry.

The first chapter asks if there is a role for the regulation of the market of funds for firms that lack collateral and have a large uncertainty about their ability to generate profits. To answer the question, it characterizes optimal financial contracts in a competitive environment with risk, adverse selection, and limited liability. In this environment, competition among financial intermediaries always forces them to fund projects with negative expected returns both from a private and from a social perspective. Intermediaries use steep payoff schedules to screen entrepreneurs, but limited liability implies this can only be done by giving more to all entrepreneurs. In equilibrium, competition for the profitable entrepreneurs forces intermediaries to offer better terms to all customers. There is cross-subsidization among entrepreneurs and intermediation profits are zero. The three main features of the framework (competition, adverse selection, and limited liability) are necessary in order to get the inefficient laissez-faire outcome and a role for financial regulation. The result remains robust when firms can collateralize some portion of the credit as long as there is an unsecured fraction. These results provide a motive for regulating the market for unsecured financing to business start-ups.

The second chapter quantifies the effect of replacing the corporate income tax by a tax on business owners. This is done by constructing a model with heterogeneous firms, borrowing constraints, costly equity issuance and endogenous entry and exit. Calibrating the model to the U.S. economy, the chapter documents that replacing the corporate income tax with a revenue-neutral common tax on shareholders, the steady-state output would increase by 6.8% and total factor productivity (TFP) by 1.7%.

Due to financial frictions, taxes levied at the corporate income level and at the shareholder level are not perfect substitutes because they distort different margins. In the model, firms are hit by productivity shocks and aim to adjust their capital stock in pursuit of optimal size. Optimal firm behavior often dictates reliance on retained earnings for growth. The corporate income tax reduces retained earnings available for investment, thereby delaying capital accumulation. As the retained earnings are not paid back to shareholders, the friction described does not occur when taxes are levied at the dividend level. The mechanism is amplified by endogenous entry and exit and by general equilibrium feedback.

Degree Type

Dissertation

Degree Name

Doctor of Philosophy (PhD)

Graduate Group

Economics

First Advisor

Dirk Krueger

Second Advisor

Harold Cole

Subject Categories
Economics

This dissertation is available at ScholarlyCommons: https://repository.upenn.edu/edissertations/2636

ESSAYS ON TAXATION AND COMPETITION UNDER FIRM HETEROGENEITY AND FINANCIAL FRICTIONS

Daniel S. Wills

A DISSERTATION

in

Economics

Presented to the Faculties of the University of Pennsylvania

in

Partial Fulfillment of the Requirements for the

Degree of Doctor of Philosophy

2017

Supervisor of Dissertation

Dirk Krueger, Professor of Economics

Co-Supervisor of Dissertation

Harold Cole, Professor of Economics

Graduate Group Chairperson

Jesús Fernández-Villaverde, Professor of Economics

Dissertation Committee

João F. Gomes
Howard Butcher III Professor of Finance, Wharton School of Business

Ali Shourideh
Assistant Professor of Economics, Tepper School of Business, Carnegie Mellon University

ESSAYS ON TAXATION AND COMPETITION UNDER FIRM HETEROGENEITY AND FINANCIAL FRICTIONS

© COPYRIGHT

2017

Daniel Santiago Wills

This work is licensed under the
Creative Commons Attribution
NonCommercial-ShareAlike 3.0
License

To view a copy of this license, visit

http://creativecommons.org/licenses/by-nc-sa/3.0/us/

To my family

ACKNOWLEDGEMENT

I owe to my advisors Harold Cole and Dirk Krueger and to the members of my committee João Gomes and Ali Shourideh my deepest gratitude for their invaluable guidance and advice. I feel lucky to have encountered such an accessible and caring group of professors.

I am also deeply indebted to my co-authors Juan Hernández and Gustavo Camilo for all they have taught me. I continue to learn from them every day.

Many others have provided very helpful feedback and advice in different parts of this thesis. Alberto Ciancio, Michael Chirico, Ilenin Condo, R. Matthew Darst, Alessandro Dovis, Rodolfo Manuelli, Fernando Martin, Gaston Navarro, Guillermo Ordóñez, Víctor Ríos-Rull deserve a special mention.

Completing this dissertation would not have been possible without the continuous love and support of my friends and family.

Daniel Wills

April 8th, 2017

Philadelphia, PA

ABSTRACT

ESSAYS ON TAXATION AND COMPETITION UNDER FIRM HETEROGENEITY AND FINANCIAL FRICTIONS

Daniel S. Wills

Dirk Krueger

Harold Cole

In this dissertation, I study the implications of taxation -and other regulations- in environments with financial frictions and firm entry.

The first chapter asks if there is a role for the regulation of the market of funds for firms that lack collateral and have a large uncertainty about their ability to generate profits. To answer the question, it characterizes optimal financial contracts in a competitive environment with risk, adverse selection, and limited liability. In this environment, competition among financial intermediaries always forces them to fund projects with negative expected returns both from a private and from a social perspective. Intermediaries use steep payoff schedules to screen entrepreneurs, but limited liability implies this can only be done by giving more to all entrepreneurs. In equilibrium, competition for the profitable entrepreneurs forces intermediaries to offer better terms to all customers. There is cross-subsidization among entrepreneurs and intermediation profits are zero. The three main features of the framework (competition, adverse selection, and limited liability) are necessary in order to get the inefficient laissez-faire outcome and a role for financial regulation. The result remains robust when firms can collateralize some portion of the credit as long as there is an unsecured fraction. These results provide a motive for regulating the market for unsecured financing to business start-ups.

The second chapter quantifies the effect of replacing the corporate income tax by a tax on business owners. This is done by constructing a model with heterogeneous firms, borrowing

constraints, costly equity issuance and endogenous entry and exit. Calibrating the model to the U.S. economy, the chapter documents that replacing the corporate income tax with a revenue-neutral common tax on shareholders, the steady-state output would increase by 6.8% and total factor productivity (TFP) by 1.7%. Due to financial frictions, taxes levied at the corporate income level and at the shareholder level are not perfect substitutes because they distort different margins. In the model, firms are hit by productivity shocks and aim to adjust their capital stock in pursuit of optimal size. Optimal firm behavior often dictates reliance on retained earnings for growth. The corporate income tax reduces retained earnings available for investment, thereby delaying capital accumulation. As the retained earnings are not paid back to shareholders, the friction described does not occur when taxes are levied at the dividend level. The mechanism is amplified by endogenous entry and exit and by general equilibrium feedback.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

CHAPTER 1 : Fighting for the Best, Losing with the Rest: On the Desirability of Competition in Financial Markets

This chapter is co-authored with Juan Hernández.

1.1. Introduction

Innovation has being widely recognized as the key source of economic growth, at least going back to the work of Schumpeter (1934). Arrow (1962) argued that potential entrants have stronger incentives to undertake innovation than incumbent monopolists, and indeed, an important fraction of new projects and products come from start-up firms. However, in many situations the firm or entrepreneur having access to a potential project lacks the financial resources needed to undertake it and has to rely on external lenders or investors.

Financial markets for small R&D intensive start-ups feature several frictions that generically lead to inefficient outcomes. First, the profitability of a particular endeavor is not guaranteed, the research project or new product may or may not succeed and this uncertainty can only be resolved after the investment is made. In that sense, projects are *risky.* Moreover, since innovations require specific and sophisticated knowledge, it is likely that the entrepreneur will know more about the project's prospects than investors do. This introduces a second friction, *asymmetric information.* Third, by the nature of the project, its most important assets are the knowledge, time and effort, devoted by its team of workers. If the project fails, the salvage value is close to zero. As a consequence, there is severe *limited liability.*

Recent legislation has intended to deregulate the market for funding for start-ups. Specifically, The Jumpstart Our Business Startups (JOBS) act of 2012 eased securities regulation making it easier for companies to both go public, and raise capital privately. This motivates us to ask if complete deregulation is desirable in a market featuring risk, asymmetric information and limited liability as described above.

To answer the question, we characterize the contracts offered in equilibrium by a competitive financial sector to entrepreneurs facing risk and limited liability, in an environment with adverse selection. We then proceed to describe the welfare implications of the resulting equilibrium. Our main result is that the optimal contract delivers an inefficient outcome. This is in contrast with the screening literature in environments with linear types (such as ours): when utility is linear in the type, optimal contracts achieve first best allocations. In our environment, the presence of limited liability and an outside option for entrepreneurs do not allow to achieve the first best outcome. Interestingly, if financial intermediaries were able to collude, the first best outcome would be achieved. Alternatively, under competition, the inefficiency can be corrected using simple policy tools such as a tax per financial contract.

In our model, there is a continuum of entrepreneurs, each one having access to a risky project. The entrepreneurs are heterogeneous on the probability with which their project will succeed. Also, they typically won't have enough resources to fund their projects and will need to rely on financial intermediaries. Financial intermediaries supply financial contracts in a competitive way, aiming to maximize their profits, but do not observe the ex ante probabilities of success. Beyond the information friction and limited liability, the set of contracts to be offered is completely unrestricted. Naturally, all intermediaries would like to attract the best entrepreneurs, and competition will force them to offer good terms on borrowing for the best types. They also will need to provide incentives in the right way to distinguish good entrepreneurs from bad entrepreneurs which implies a (very specific) gap between the contracts offered to good and bad types. However, limited liability imposes a lower bound on the terms of a contract. Hence, in order to sustain those incentives, bad types must also receive good terms. In equilibrium, intermediaries will break even, but they will fund both projects with positive expected profits (from good types) and projects with negative expected profits (with bad types). The inefficiency arise because entrepreneurs have an outside option (or equivalently a utility cost). We show that if an intermediary is only funding socially efficient projects (i.e: projects with expected profit higher that the cost of capital plus the opportunity cost of the entrepreneur), she is making positive profits. As a consequence, her competitor is willing to improve borrowing terms for good types, even though some low types with inefficient projects will take the contract. The inefficiency can be corrected with simple tax instruments.

In the basic model, entrepreneurs cannot collateralize their assets. We extend the model to the case in which entrepreneurs have some collateral. We show that as long as loans cannot be fully collateralized, the inefficiency is reduced but not removed. We then proceed to study how the inefficiency changes when the parameters of the model change. The effect of the entrepreneur's outside option is non-monotonic: the inefficiency is zero if there is no outside option (all projects are socially efficient), but it is also zero if the outside option is so high that nobody want to undertake risky projects (no project is efficient). Further, the inefficiency (relative to the net economic surplus) increases with the cost of capital faced by financial intermediaries and with the relative density of lower types. When we allow for collateralizable assets, the inefficiency decreases with the mean of the asset distribution.

Starting from Stiglitz and Weiss (1981), and extensive body of literature has studied asymmetric information in financial markets. The main message of Stiglitz and Weiss (1981) is that the interest rate cannot clear the credit market because of a standard "lemons" problem, and as a result, there is credit rationing. Subsequent papers allow the financial intermediaries (banks) to use different tools, other than the interest rate, to screen borrowers' types. A first strand of papers allow intermediaries to use collateral, on top of the interest rate, to screen types. Bester (1985) turns down the credit rationing result, by allowing intermediaries to offer interest rate/collateral contract pairs. By using collateral in addition to the interest rate, banks can screen borrowers: risky borrowers will accept to pay higher interest rates in order to benefit from a lower collateral requirement. However, in Bester (1985)'s economy, there is no limit to the amount of collateral that borrowers can provide. The question of limits to collateral is studied by Besanko and Thakor (1987). The environment is similar to Bester (1985)'s, and safer types will prefer loans with low interest rate and high collateral. Nonetheless, it may be that the borrower has not enough wealth to provide the required collateral. In that case, the collateral/interest rate pair cannot achieve the sufficient spread in payoffs necessary to separate types. Besanko and Thakor (1987) solve this issue by allowing the contracts to additionally depend on the probability of approval. To achieve the necessary spread of utilities, low interest - high collateral credits will be denied with positive probability. An interesting point in Besanko and Thakor (1987) that relates to our result is that the paper compares welfare when the financial intermediation sector is competitive or a monopoly, and finds that monopoly may lead to a higher welfare, depending on parameter values.

Another strand of papers have departed from the Stiglitz and Weiss (1981) result by allowing intermediaries to screen borrowers using the size of the loan. A contract is hence a pair interest rate - loan size. In Milde and Riley (1988), borrowers are entrepreneurs with access to a project with risky returns. The return on the project depends on both the borrower's type and the size of the loan. The interaction between type and loan size in the project's payoff allows to separate types using interest rate - loan size menus. The outcome, however, depends strongly on the specific function mapping the type and loan size to the return of the project. In general good types take bigger loans accompanied by higher interest rates. However, Milde and Riley (1988) provide examples of production functions for which the opposite happens: good types take smaller loans and pay lower interests. A point to keep in mind from Milde and Riley (1988) is that projects won't be funded to its optimal, full-information size.

More recently, Martin (2009) uses a similar framework to study the relation between entrepreneurial wealth and aggregate investment. In his model, intermediaries can use both collateral and the size of the loan to screen types. He shows that when entrepreneurial wealth is high, collateral can be used to separate types. When entrepreneurial wealth is low, screening is mainly done by restricting the level of investment, and becomes more costly. As a result, in the later case, a pooling equilibrium is more likely. However, Martin

(2009) restricts the interest rate to be un-contingent. We show that, when transfers contingent of success of the project are allowed (say by a contingent interest rate or an equity-like contract), the intermediaries never distort the level of investment to screen types. Instead they find optimal to use the contingent transfer.

Although close in terms of topic, all the papers cited above impose ad-hoc restrictions to the space of contracts potentially offered by the financial sector. In contrast, in our model, the set of contracts is only restricted by the features of the environment.

Lester et al. (2015) develop a model where screening contracts are unrestricted. Their environment features adverse selection between informed sellers and uninformed variables. Beyond asymmetric information, they introduce imperfect information coming from search theoretic frictions. The later feature allows them to do comparative statics on the degree of imperfect competition and how it interacts with the severity of adverse selection. As in our environment, they find that increasing competition may reduce welfare when markets are competitive.

In our model, the financial intermediation is competitive. Intermediaries fund projects in which they expect to loose, but are socially efficient because the payoff of the entrepreneur compensates the intermediary's loses. They also fund projects that are socially inefficient in the sense that they generate a dead-weight loss. Our environment is close to Rothschild and Stiglitz (1992), in which adverse selection is introduced to a competitive market. In their environment the equilibrium (when it exists), is separating. The limited liability constraint in our model prevents the separating outcome. In our model, the inefficiency results from the interaction of several forces: first, there is asymmetric information that introduces a “lemons” problem; second, limited liability puts a bound on the screening that can be done by financial intermediaries; third, competition among intermediaries introduces profitable deviations from the efficient outcome (that would be reached by a monopolist lender).

The rest of the paper is organized as follows: In the next section we describe the model which is the core of the paper. Then, in section 1.3 we extend the model to allow for a distribution of assets among entrepreneurs. In section 1.4 we introduce numerical example, and show how the deadweight loss changes with the parameters of the model. Concluding remarks are made in section 1.5.

1.2. Basic Model

1.2.1. Environment

In this section we describe the main mechanism of the paper in a partial equilibrium static economy. The economy is populated by a continuum of agents with mass 1 indexed by their heterogeneous ability $\theta \in [0, 1]$. Each agent can work for a wage w or undertake a project with a risky outcome, that depends both on the ability of the agent and the capital invested. If an agent decides to start his own project, he or she will have to borrow funds from a financial intermediary[1].

The financial intermediaries have access to capital at the (gross) risk free rate R. They cannot observe the entrepreneurs ability and will have to provide incentives in order to get that information. Intermediaries can observe investment in the project, i.e. agents cannot divert funds from their projects without being caught. Both agents and intermediaries are risk neutral. However, if a project fails the intermediaries cannot exert any claims on the entrepreneurs. In that sense, projects in this economy are subject to limited liability.

The ability of each agent will determine the probability that an entrepreneurial venture succeeds. We denote $G(\theta)$ the cumulative distribution of abilities. More specifically, if the agent decides to become an entrepreneur and invests an amount k of capital in the project, the project will succeed with probability $p(\theta, k)$

Assumption: $p(\theta, k)$ is multiplicatively separable: i.e. $p(\theta, K) = g(\theta)f(k)$. Where f is

[1]This w can also be interpreted as the opportunity cost, pecuniary or not, of running the project for the entrepreneur.

continuous and $f' > 0,\ f'' < 0,\ f(0) = 0$ and $\lim_{k\to\infty} f(k) = 1$. Without loss of generality, we can set $g(\theta) = \theta$ since we are just renaming the unobservable types. . Although we are interpreting θ as entrepreneurial ability, notice that it could be anything that is known by the entrepreneur, but not by the intermediary, and that increases the probability that the project succeeds.

The assumption of multiplicative separability is important because it allows us to abstract from Riley-style signaling distortions in investment and highlight our main mechanism where competition in the intermediation sector generates overinvestment. It also helps to keep the model tractable and allows a better characterization of the optimal contract.[2]

In line with the endogenous growth literature, we interpret a successful project as the arrival of a new innovation, which allows the entrepreneur to create or "steal" some market. We denote π the payoff of a successful project which, in turn, can be interpreted as the value of the innovation. As a result, the expected surplus of an entrepreneurial venture is given by,

$$\theta f(k)\pi - Rk - w$$

As we will show later, in equilibrium, the surplus will be shared between the entrepreneur and the financial intermediary who lends the funds.

In principle, a contract is a triple, (k, x_1, x_0) where the k is the size of the loan, x_1 the repayment in case of success and x_0 the repayment if the project fails. However, for convenience we make a linear transformation of the contract that will allow a more straightforward relation with the mechanism design literature. Set $x = -x_0$ and $z = \pi - (x_1 - x_0)$ From the point of view of the agent the contract (k, x, z) prescribes a fixed

[2] Without multiplicative separability the optimal contract is harder to characterize, however the main message of the paper remains: the optimal contract yields an inefficient outcome. When the cross-partial derivatives of $\ln(p(\theta, k))$ are not zero, the entrepreneurs will use k to signal type as in Riley (????). As a result, the project size k will be distorted which will lead to another source of inefficiency. Relative to our results, this will lead to less extensive inefficiency (fewer *ex-ante* suboptimal projects are started) but more intensive efficiency (all projects will be run at a suboptimal scale). This is in contrast with the equilibrium of the linear environment we present, in which k is always the full information optimal level.

pay for the agent x, an additional payment contingent on success z and an investment amount k that determines the probability of the contingent payment happening.

The intermediaries' objective is to offer a profit maximizing contract schedule, taking into account that agents would choose the better option available to them, and also the competition from other entrepreneurs.

1.2.2. Contract menus and entrepreneur choices

The game

In this subsection we formally define the game. For simplicity we assume there are only two financial intermediaries, indexed by $i \in \{1,2\}$, competing *a la* Bertrand for entrepreneurs.[3]

- **Players:** 2 intermediaries, 1 entrepreneur. The intermediaries are identical. The entrepreneur has a private type θ drawn from a distribution $G(\theta)$.

- **Timing:** Intermediaries move simultaneously, posting arbitrary sets of contracts. Then the entrepreneur chooses among the available contracts an her outside option.

- **Strategies:** For intermediaries the strategy space contains any subset of contracts of the form (k, x, z). The strategy space is hence the power set of $\mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}$ denoted $\mathcal{P}(\mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R})$. We denote the strategy of intermediary i (or his contract menu) by $\mathcal{C}_i$. For the entrepreneur a strategy is a probability distribution $s : \Theta \times \mathcal{P}(\mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R})^2 \to \Delta\mathbb{R}^3$ such that $Supp\{s(\theta, \mathcal{C}_1, \mathcal{C}_2)\} \subseteq \mathcal{C}_1 \cup \mathcal{C}_2 \cup \{(0, w, 0)\}$. Above, $\Delta\mathbb{R}^3$ denotes the set of probability measures over $\mathbb{R}^3$, $Supp\{f\}$ denotes the support of f and $(0, w, 0)$ is the outside option. Abusing notation, we let $s(\theta, \mathcal{C}_1, \mathcal{C}_2)[k, x, z]$ be the cumulative density function of s evaluated at (k, x, z).

  Note that we allowed for mixed strategies for entrepreneurs. They will be able to

[3] As long as the entrepreneurs observe all offered contracts, the outcome will be the same with more intermediaries, although the optimal strategies of each one may differ. Hence this is just a notational simplification for free entry in the intermediaries sector.

randomize over any subset of the offered contracts, including the outside option. We focus on the case in which intermediaries play pure strategies.

- **Payoffs:** All players are risk neutral and care only about expected payoff. For an entrepreneur of type θ the expected payoff of signing a contract (k, x, z) is

$$u\big(\theta, (k, x, z)\big) = \theta f(k) z + x.$$

in particular, if the entrepreneur take his outside option, his payoff is $u\big(\theta, (0, w, 0)\big) = w$.

The entrepreneur's expected payoff is,

$$U(\theta, s, \mathcal{C}_1, \mathcal{C}_2) = \int_{(k,x,z)\in\mathbb{R}^3} \big(\theta f(k) z + x\big) ds(\theta, \mathcal{C}_1, \mathcal{C}_2)[k, x, z]$$

Conditional on an entrepreneur of type θ signing a contract (k, x, z) with intermediary i her expected payoff is: $\theta f(k)(\pi - z) - x - Rk$.

The total expected payoff of intermediary i can be written as:

$$v_i(s, \mathcal{C}_1, \mathcal{C}_2) = \int_0^1 \int_{(k,x,z)\in\mathbb{R}^3} \Big[\theta f\big(k\big) \cdot \big(\pi - z\big) - x - R \cdot k\Big] ds(\theta, \mathcal{C}_1, \mathcal{C}_2)[k, x, z] dG(\theta),$$

Given s, $\mathcal{C}_1$ and $\mathcal{C}_2$ it is useful to define the set of types *strictly* willing to take a contract from intermediary i,

$$A_i(s, \mathcal{C}_1, \mathcal{C}_2) = \big\{\theta : Supp\,(s(\theta, \mathcal{C}_1, \mathcal{C}_2)) \subseteq \mathcal{C}_i \setminus \mathcal{C}_{-i}\big\},$$

the set of types indifferent between the two intermediaries,

$$B(s, \mathcal{C}_1, \mathcal{C}_2) = \big\{\theta : Supp\,(s(\theta, \mathcal{C}_1, \mathcal{C}_2)) \subseteq \mathcal{C}_i \cap \mathcal{C}_{-i}\big\}$$

and the set of types willing to sign a contract, rather than taking the outside option,

$$A = A_1 \cup A_2 \cup B$$

Notice that we are using the maintained assumption that when indifferent between being an entrepreneur or a worker, agents prefer entrepreneurship.

Equilibrium definition

The equilibrium concept applicable to this framework is the Bayes-Nash Equilibrium.

Definition A strategy profile $(\mathcal{C}_1^*, \mathcal{C}_2^*, s^*)$ is a *Bayes-Nash Equilibrium* if:

1. For all $\theta \in \Theta$

$$\begin{aligned} Supp\big(s^*(\theta, \mathcal{C}_1^*, \mathcal{C}_2^*)\big) \subseteq \qquad & \arg\max_{(k,x,z)} \theta f(k) z + x \\ & \text{s.t: } (k,x,z) \in \mathcal{C}_1^* \cup \mathcal{C}_2^* \cup \{(0,w,0)\} \end{aligned}$$

2. For each intermediary $i \in \{1,2\}$, given the entrepreneur strategy s^* and the competitor's contract menu $\mathcal{C}_{-i}^*$ her own contract menu $\mathcal{C}_i^*$ maximizes her expected utility

$$\begin{gathered} \mathcal{C}_i^* \in \arg\max_{\mathcal{C}_i} v_i(s^*, \mathcal{C}_i, \mathcal{C}_{-i}^*) \\ s.t. \qquad \forall (k,x,z) \in \mathcal{C}_i \subset \mathbb{R}^3 \\ k \geq 0, \quad x \geq 0, \quad x + z \geq 0. \end{gathered}$$

The conditions $x \geq 0$ and $x + z \geq 0$ make sure that limited liability is satisfied: if the project fails, the entrepreneur cannot make any payment to the intermediary, but the intermediary could potentially make a transfer to the entrepreneur.

1.2.3. The Equilibrium Contract

In this section we state a sequence of claims leading to the characterization of the equilibrium contract. As will be shown, intermediaries make zero profits in any equilibrium and the entrepreneur's payoff is linear in their type. Although the equilibrium is by no means unique, all equilibria are payoff equivalent.

The Payoff of Entrepreneurs

Let $U(\theta;\mathcal{C}_1,\mathcal{C}_2)$ be the *potencial* payoff that a θ-type agent could get, conditional on becoming an entrepreneur, when intermediaries play $\mathcal{C}_1$ and $\mathcal{C}_2$. That is,

$$U(\theta;\mathcal{C}_1,\mathcal{C}_2) = \max_{(k,z,x)\in\mathcal{C}_1\cup\mathcal{C}_2} \theta f(k)z + x$$

Let $\big(k(\theta), z(\theta), x(\theta)\big)$ be a representative of the (equivalence class of) maximizers. Under the assumptions for θ and $f(k)$, the Spence-Mirrless conditions (single crossing) hold and local incentive compatibility conditions are equivalent to the global incentive compatibility conditions. Hence, Myerson's lemma can be applied:

Lemma 1. *If $\mathcal{C}_1^*$ and $\mathcal{C}_2^*$ are part of an equilibrium:*

$$U(\theta;\mathcal{C}_1^*,\mathcal{C}_2^*) = U_i(0) + \int_0^\theta f\big(k(s)\big)z(s)ds, \tag{1.1a}$$

$$f\big(k(\theta)\big)z(\theta) \text{ is non-decreasing.} \tag{1.1b}$$

$$x(\theta) = U(0) + \int_0^\theta f\big(k(s)\big)z(s)ds - \theta f\big(k(\theta)\big)z(\theta), \tag{1.1c}$$

We refer to $\big(k(\theta), z(\theta), x(\theta)\big)$ as an *incentive compatible* contract menu.

The equilibrium payoff of a θ-type agent is $\max\{w, U(\theta)\}$.

Project/Loan Size

We now aim to characterize the amount of capital lent to each entrepreneur in equilibrium. First, define $k^*(\theta)$ as the full information optimal investment in a project of type θ.

$$k^*(\theta) = \arg\max_k \{\theta f(k)\pi - Rk\}$$

And let $S(\theta)$ be maximum gross surplus generated by an entrepreneur of type θ,

$$\begin{aligned} S(\theta) &= \max_k \{\theta f(k)\pi - Rk\} \\ &= \theta f\big(k^*(\theta)\big)\pi - R \cdot k^*(\theta). \end{aligned}$$

$S(\theta)$ is a gross surplus because it doesn't include the opportunity cost of forgoing the outside option w. Note that under the assumptions for $f(k)$, the optimal project size $k^*(\theta)$ is a continuous and strictly increasing function of θ.

The payoff of the entrepreneur only depends on k through the product $f(k)z$. Because of the multiplicative separability, given k^0, z^0, the intermediary can offer $k^*(\theta)$ and adjust z to keep $f(k)z$ constant.

Claim 1 formalizes the argument above. All proofs are in the appendix.

Claim 1. *Contracts signed in equilibrium have $k(\theta) = k^*(\theta)$ for (almost) every $\theta \in A$.*

The Payoff of Intermediaries

The market structure resembles Bertrand competition, and it is natural to conjecture that if an intermediary were to make profits, his competitor could offer contracts slightly more generous and steal the market.

Claim 2 (Zero Profit Condition)**.** *In any equilibrium, the profits for intermediaries is zero.*

Fighting for the Best

We just showed that entrepreneurs make zero profits. In some sense, entrepreneurs have all the bargaining power and one could suspect that intermediaries will make zero profits type by type and each entrepreneur would receive all the economic surplus she produces. That suspicion would be correct in a similar framework without limited liability, or without asymmetric information. However, the interaction between the two frictions does not allow for that to happen in our economy. On the contrary, the expected surplus of the project tends to grow much faster than incentives can be provided: whenever expected profits are positive, locally expected revenues increase faster than expected costs. In a nutshell, intermediaries want to invest more in more able types, but cannot increase rewards too fast to keep incentives.

Claim 3. *Suppose $S(\hat{\theta}) \geq U(\hat{\theta})$ for some $\hat{\theta} > 0$. Then, $S(\theta') > U(\theta')$ for almost every $\theta' > \hat{\theta}$.*

Because both intermediaries are making zero profits, and U and S are continuos, there is a θ such that $U(\theta) = S(\theta)$. The claim above implies that such θ is necessarily unique: although intermediaries make zero profits on average, they make strict profits with the best types and strict loses with other types.

The result above is in *sharp contrast* with Rothschild and Stiglitz (1992). In such an environment, if there were profits to be made with a particular type, the market will "cream skim" it, until profits are zero type by type. We will show that the limited liability constraint binds and prevents that from happening. Before, we need to characterize the set A of types choosing to be entrepreneurs.

The next claim states that A takes the form of an interval, and the lowest type willing to take one of the offered contracts rather than the outside option is well defined. We will refer to such a type as θ_L.

Claim 4. *The set A of all types that are willing to accept at least one of the offered* *contracts is an interval of the form* $A = [\theta_L, 1]$ *for some* $\theta_L \in [0,1]$. *Moreover, if* $\theta_L > 0$, $U(\theta_L) = w$.

When entrepreneurs are risk neutral, it is natural to expect that if a project fails, intermediaries won't pay anything to entrepreneurs, and hence $x(\theta) = 0$. If that was not the case, intermediaries could "cream skim" the market. That is, intermediary i could deviate to a contract serving all the profitable types, and leave all the unprofitable types to his competitor.

Claim 5. *Competition in the intermediation market and the limited liability constraint imply that any contract offered and signed in equilibrium has* $x(\theta) = 0$ *for all* $\theta > \theta_L$.

It is useful to note that the result above would not hold, absent limited liability. In that case, the optimal mechanism specifies that entrepreneurs get all the profits from the project and pay $R \cdot k^*(\theta)$ no matter if the project succeeds or fail. Limited liability puts a bound to the separation of types. As a result, high types are more profitable for intermediaries.

Limited Incentive Provision

Given the above results, incentives can only be be provided using z. Claim 5 implies that $U(\theta) = \theta f\big(k^*(\theta)\big)z(\theta)$ for all theta. Hence $U(0) < w$ and then $\theta_L > 0$. Also, the set of maximizers of $\theta f(k)z$ is independent of θ. It follows that $f(k(\theta))z(\theta)$ cannot depend on θ. Since, $\theta_L f(k(\theta_L))z(\theta_L) = w$, we get from the condition above:

$$f(k(\theta))z(\theta) = \frac{w}{\theta_L}. \tag{1.2}$$

This together with claims 1 and 5, fully characterize the equilibrium optimal contract.

Proposition 1. *In equilibrium, the contract signed by almost every* θ *is:*

$$k^*(\theta) = \arg\max_k \{\theta f(k)\pi - Rk\}, \qquad z^*(\theta) = \frac{w}{\theta_L f\big(k^*(\theta)\big)}, \qquad x^*(\theta) = 0.$$

where θ_L, the lowest type who accepts the contract, is such that the financial intermediaries make zero profits.

Some Implications

The expected payoff for the entrepreneur of type θ is $\frac{\theta}{\theta_L}w$, the gross rate of return is $\frac{\theta}{\theta_L k^*(\theta)}w$. Thus the expected return may be increasing or decreasing in θ, depending on f.

The expected payoff for the intermediary on a θ-type project is $\theta f\big(k(\theta)\big)\pi - \frac{w\theta}{\theta_L} - Rk(\theta) = S(\theta) - U(\theta)$. It follows from claim 3 that this payoff can be zero only for one particular θ and will be positive for higher types. Moreover $S(\theta) - U(\theta)$ is a convex function of θ.[4] The good types are very valuable for intermediaries: the profits coming from them will compensate for the losses from bad types.

Interestingly, *firms cannot fight for segments of the market (i.e: try to steal only a subset of* $\Theta = [0, 1]$*).* The expected return for entrepreneurs is independent of k and z. If a firm offers a more attractive contract for some θ, it must be because θ_L is lower, and hence the contract is better for every θ. In other words, in a world where the limited liability constraint is binding, intermediaries are not able to "skim the cream" up to the point of making zero profits type by type. This is in contrast with Rothschild and Stiglitz (1992).

The following picture illustrates the equilibrium described by proposition 1. The figure shows several monetary payments as functions of θ. We plot the entrepreneurs' expected payoff $U(\theta) = \frac{\theta}{\theta_L}w$ and their outside option w (which is assumed to be independent of θ); the expected surplus of the project, net of financial costs, $S(\theta) = \theta f\big(k^*(\theta)\big)\pi - Rk^*(\theta)$.[5] The area shaded in gray represents the profits of an intermediary. From the figure it is clear that changes in θ_L change the slope of the entrepreneurs expected payoff. Thess changes, in turn modify the intermediaries' payoffs and hence θ_L can be adjusted so that

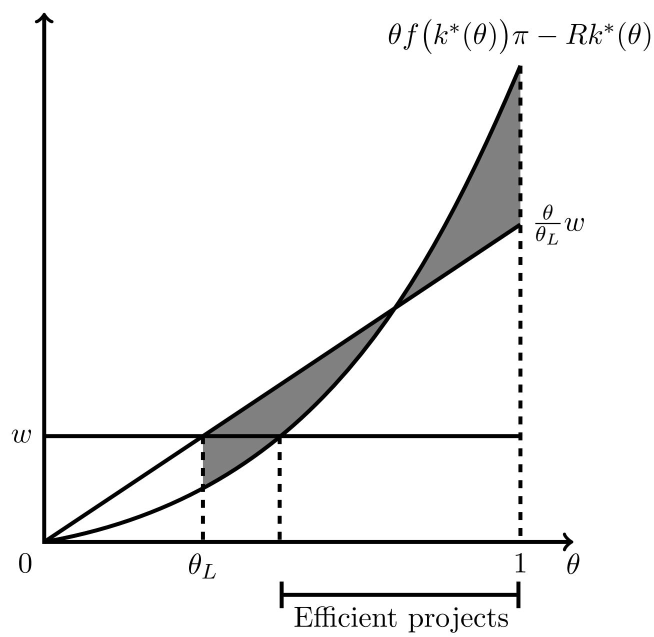
[4] By the envelope theorem, the derivative is $\pi f\big(k^*(\theta)\big) - \frac{w}{\theta_L}$

[5] Note that the expected surplus is similar to the profit function of a competitive firm facing an output price of θ. It is well known that profit functions are convex in prices and hence the surplus is convex in θ, as displayed.

intermediaries make zero profits.

Figure 1 illustrates the key force in this economy: to provide the correct incentives, the payoff to entrepreneurs cannot grow as fast as the total surplus does. That means that intermediaries always make higher profits with the highest types. Since high types are so profitable, intermediaries are willing to lose money with low types to offer more attractive contracts to (profitable) high types and maintain incentives.

Figure 1: Equilibrium Contract and Zero Profits



1.2.4. Welfare and optimal policy

The expected social surplus of a project is $\theta f\big(k^*(\theta)\big)\pi - Rk^*(\theta) - w$. Hence a social planner sets $\theta f'\big(k^*(\theta)\big)\pi = R$ as before, but only for those θ such that the expected social benefit is not negative. As the surplus is increasing in θ by the envelope theorem, there is a lower bound θ_P, for those socially valuable projects. Then it is worth for the society to devote resources to all projects with $\theta \geq \theta_P$ where $\theta f\big(k^*(\theta_P)\big)\pi - Rk^*(\theta_P) = w$.

By contrast, in the decentralized equilibrium, θ_L yields zero profit for intermediaries:

$$\int_{\theta_L}^{1} \left\{\theta f\big(k^*(\theta)\big)\pi - Rk^*(\theta)\right\} dG(\theta) - \int_{\theta_L}^{1} \theta \frac{w}{\theta_L} dG(\theta) = 0$$

Proposition 2. *In the decentralized solution, socially inefficient projects are enacted. That is $\theta_L < \theta_P$. Moreover, efficiency will be restored if any of the following becomes true:*

- *No adverse selection: Types become public information*
- *No limited liability: Intermediaries become able to recover any contracted amount.*
- *No competition: There is only one intermediary or intermediaries collude.*

The intuition behind it is as follows: if only efficient projects were financed, the intermediaries would make profits in all the projects. But then, positive profits attract new intermediaries who can "steal" the market by offering more generous contracts. More generous contracts involve some cross-subsidization, and as a result socially inefficient projects will be active.

If types are public information but all other conditions remain the same, intermediaries will break even on each type. This implies that all signed offer is such that $U(\theta) = S(\theta)$, and only types $\theta > \theta_P$ sign contracts. Contracts will not be completely determined since many combinations of $x(\theta)$ and $z(\theta)$ yield $U(\theta) = S(\theta)$, but $k(\theta) = k^*(\theta)$. The equilibrium payoff of type θ is $U(\theta) = \max\{w, S(\theta)\}$.

If there is no limited liability but all other conditions hold, the only IC contract schedule in equilibrium is $k(\theta) = k^*(\theta)$, $x(\theta) = -R \cdot k^*(\theta)$ and $z(\theta) = \pi$, which implies $U(\theta) = S(\theta)$. This is just the risk free debt contract. As it is well understood, when the entrepreneur is risk neutral, transferring all the risk to her solves the incentive problem.

If there is only one intermediary, but adverse selection and limited liability still hold, the only equilibrium is as follows: the contract that maximizes profits for the intermediary is

$k(\theta) = k^*(\theta)$, $x = w$, $z = 0$ and entrepreneurs with types $\theta \geq \theta_L$ take it, all others reject. The intermediary will take all the surplus and her profits would be $\int_{\theta_P}^1 \big(S(\theta) - w\big) dG(\theta)$.

Optimal policy

In this section we consider two possibilities for taxation. A fixed sum tax per contract to financial intermediaries ϕ and a tax rate τ on entrepreneurs profits[6] . In both cases the full information optimal allocation can be achieved.

Claim 6. *Let ϕ^* be a fixed tax per contract defined by:*

$$\phi^* \equiv \left[\int_{\theta_P}^1 \{\theta f(k^*(\theta))\pi - Rk^*(\theta)\} \, dG(\theta) - \int_{\theta_P}^1 \theta \frac{w}{\theta_P} dG(\theta) \right] \left(\int_{\theta_P}^1 \frac{\theta w}{\theta_P} dG(\theta) \right)^{-1}$$

Then only efficient projects (and all of them) are funded.

This tax can also be seen as fixed subsidy on w, or any instrument that increases the outside option of every entrepreneur. However the revenue implications for the tax authority would be different. Since we do not model the nature of the wage or the outside option, we stick to the contract fee tax interpretation.

Now suppose there is a profit or dividend tax. Intermediaries make zero profit, so they wont be affected by such tax. However entrepreneurs do make profits if the project is successful. Hence a tax rate τ on profits would make any contract $\big(k, x, z\big)$ look to the entrepreneurs like $\big(k, x, (1-\tau)z\big)$

Claim 7. *Let $\tau^* = \frac{\phi^*}{w+\phi^*}$ be the tax rate on profits, where ϕ^* is the fixed tax rate found in claim 6. Then only efficient projects (and all of them) are funded.*

Taxing R would be troublesome since it will distort $k^*(\theta, R)$ reducing the total surplus,

[6] A tax to entrepreneurs could be considered but it would require them to have some external funds that the government can seize even in case of failure. In that case:

$$\phi^* \equiv \theta_P \left[\int_{\theta_P}^1 \theta dG(\theta) \right]^{-1} \left[\int_{\theta_P}^1 \{\theta f(k^*(\theta))\pi - Rk^*(\theta)\} \, dG(\theta) - \int_{\theta_P}^1 \theta \frac{w}{\theta_P} dG(\theta) \right]$$

which is the standard result on capital income taxation. In principle this would be at odds with claim 7, but this is because we assume taxation on capital income is made at the individual level. Hence as long as the intermediaries can aggregate profits on investments before taxes, or claim back taxes on dividends from entrepreneurs firms in which they have a stake, they wont be affected by the taxation, only the entrepreneurs. So far we have avoided to make any interpretation of the legal stance the contracts will have. We can think of the contracts as debt contracts with limited liability clauses or as equity stakes in a firm, those details are irrelevant for our discussion above but become important once we face a complex tax law. Of course the discussion of that topic is far away from the scope of this paper.

1.3. Capital Holdings

As the previous section shows, for θ high enough, the return of the project for the entrepreneur would be higher if he had access to the risk free rate. Hence skilled entrepreneurs who own capital are willing to use it on their project. The same will happen if the asset owned is not liquid but is pledgeable: the entrepreneurs are willing to pledge their assets if doing so gets them better loan terms. We allow for that possibility in the current section.

1.3.1. Observable Capital Holdings

Assume entrepreneurs have assets $a \in \mathcal{A}$, distributed according to $G(\theta, a)$. Two interpretations are possible, both will yield the same results : for a project of size k the entrepreneur provides a as capital and the intermediary finances $k - a$ or the intermediary finances k collateralized by Ra from the entrepreneur. In what follows the latter will be used.

The asset holdings from entrepreneurs serve two purposes, they relax the limited liability constraint for the intermediaries and increase the outside option for the entrepreneurs.

Claim 8. *The outside option $O(\theta, a)$ of an entrepreneur of type θ that holds assets a is* *characterized as follows:*

$$O(\theta, a) = \max\Big\{w + Ra, \theta F\big(\min\{a, k^*(\theta)\}\big)\pi + R \cdot \max\{0, a - k^*(\theta)\}\Big\}$$

The expression for the outside option results from the fact that an entrepreneur of type θ holding a units of capital can always produce using his skills and own capital, or work and lend his capital.

With observable assets, contract schedules are contingent on a. A strategy for an intermediary specifies a subset of $\mathbb{R}^3$ for each $a \in \mathcal{A}$.

To facilitate interpretation, let the equilibrium payoff of a type θ entrepreneur with assets a be $U(\theta, a) + Ra$.

Myerson's lemma and claims A.1.1 to 2 will still hold for each a, since nothing in their proofs depends on the limited liability condition being exactly zero. With minor changes to the proof we can state:

Proposition 3. *In equilibrium with observable assets, the contract offered by all financial intermediaries to almost every* (θ, a) *is:*

$$k^*(\theta, a) = k^*(\theta) = \arg\max_k\{\theta f(k)\pi - Rk\}, \quad z^*(\theta, a) = \frac{O\big(\theta_L(a), a\big)}{\theta_L(a) f\big(k^*(\theta)\big)}, \quad x(\theta) = -Ra.$$

where $\theta_L(a)$*, the lowest* θ *among those entrepreneurs with* a *assets who accepts the contract. For each* a*, it must be the case that:*

$$\int_{\theta_L(a)}^{1} \Big\{\theta f\big(k^*(\theta)\big)\pi - \theta\frac{O(\theta_L(a), a)}{\theta_L} - R\big(k^*(\theta) - a\big)\Big\}\, dG(\theta|a) = 0$$

The following figure illustrates the equilibrium,

As long as entrepreneur's capital holdings are observable several loan markets will be active, one for each asset level a. Those markets will be described by proposition 3. In

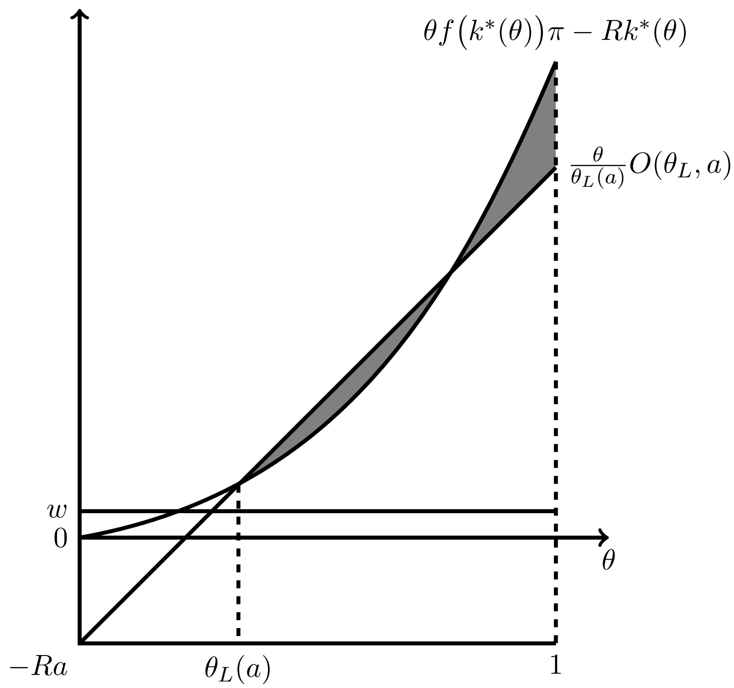


Figure 2: Equilibrium Contract with Observable Asset Level a

the next subsection we deal with the case of unobservable asset holdings.

1.3.2. Unobservable Capital Holdings

In principle, an entrepreneur with wealth a could hide part of his own wealth and take a contract designed for a lower a if it is more profitable. This is akin to an entrepreneur setting up a corporation but only investing a fraction of his wealth. We ruled out this possibility by assuming that a was known (observable) by the intermediary. In this subsection we drop that assumption.

Just as in the previous section, the strategy of intermediary i, $\mathcal{C}_i$ specifies a subsets of $\mathbb{R}^3$ for each $a \in \mathcal{A}$. But now, an entrepreneur with type (θ, a) solves,

$$U(\theta, a; \mathcal{C}_1, \mathcal{C}_2) = \max_{(k,z,x) \in \mathcal{C}(a)} \theta f(k)z + x$$

where $\mathcal{C}(a) \equiv \bigcup_{a' \leq a} \mathcal{C}_1(a') \cup \bigcup_{a' \leq a} \mathcal{C}_2(a')$

Let $\big(k(\theta, a), z(\theta, a), x(\theta, a)\big)$ denote a representative solution of the above problem. Incentive compatibility across assets only requires $U(\theta, a)$ to be nondecreasing in a. This because an entrepreneur cannot pledge more collateral or invest more capital than the amount he owns, which implies he can only lie by hiding some assets. For the incentive compatibility over θ, Myerson lemma holding fixed each asset level is enough. Limited liability in this environment still requires $x(\theta, a) \geq -Ra$.

It can be shown that, as before, $x(\theta, a)$ is non increasing in θ, $k(\theta, a) = k^*(\theta)$ for (almost) all (θ, a), $U(\theta, a)$ will be nondecreasing in θ. If we define the set $A_i(a)$ as those θ such that types (θ, a) are willing to take the contract from intermediary i it is still the case that $\bigcup_i A_i(a) = A(a) = [\theta_L(a), 1]$. Also there would be a zero profit condition but in an aggregate sense across assets. Hence proofs depending on a zero profit condition per asset level need to be updated.

The unobservability of the assets may (will) imply limited liability is not binding for some cases. However we still can state an updated version of claim 5.

Claim 9. *Competition in the intermediation market and the limited liability constraint imply that any equilibrium contract schedule satisfies $x(\theta, a) = -Ra$ for all (θ, a) such that $S(\theta) \geq U(\theta, a) > w$.*

This implies that limited liability will bind for those types for which the intermediaries expect to make some profits. Now, for each a define $\theta_e(a)$ as the solution to $U(\theta, a) = S(\theta)$ which means the intermediary expects to break even with this type. Above those $\theta_e(a)$ claim 9 implies $x = -Ra$ and therefore for all $\theta > \theta_e(a)$:

$$U(\theta, a) = \theta \frac{S\big(\theta_e(a)\big) + Ra}{\theta_e(a)} - Ra \tag{1.3}$$

Now, fix the non decreasing function $\theta_e(a)$. That completely determines the expected earnings for intermediaries among the profitable contracts. For types below $\theta_e(a)$ losses are incurred in expectation so the intermediary wants to offer as little as possible. However, the IC constraint across assets may bind. Equation (1.3) is then a lower bound for the surplus given to those types. If the IC across assets binds for a type (θ, a) is because there exists some $0 \leq a' < a$ such that:

$$\theta \frac{S\big(\theta_e(a')\big) + Ra'}{\theta_e(a')} - Ra' > \theta \frac{S\big(\theta_e(a)\big) + Ra}{\theta_e(a)} - Ra.$$

After fixing $\theta_e(a)$, a profit maximizing intermediary has no way to improve. The contracts she is profiting with are fully determined, and for those types she is expected to lose she has a lower bound on the utility she has to deliver. Hence for $\theta < \theta_e(a)$ a profit maximizing intermediary sets:

$$U_i(\theta, a) = \sup_{0 \leq a' \leq a} \theta \frac{S\big(\theta_e(a')\big) + Ra'}{\theta_e(a')} - Ra' \tag{1.4}$$

Equations (1.3) and (1.4) fully characterize the contracts given $\theta_e(a)$. Figure 3 illustrates this feature. The zero profit condition is not enough to pin down that function, since zero profits must hold on aggregate across types. To pin down that function consider the following:

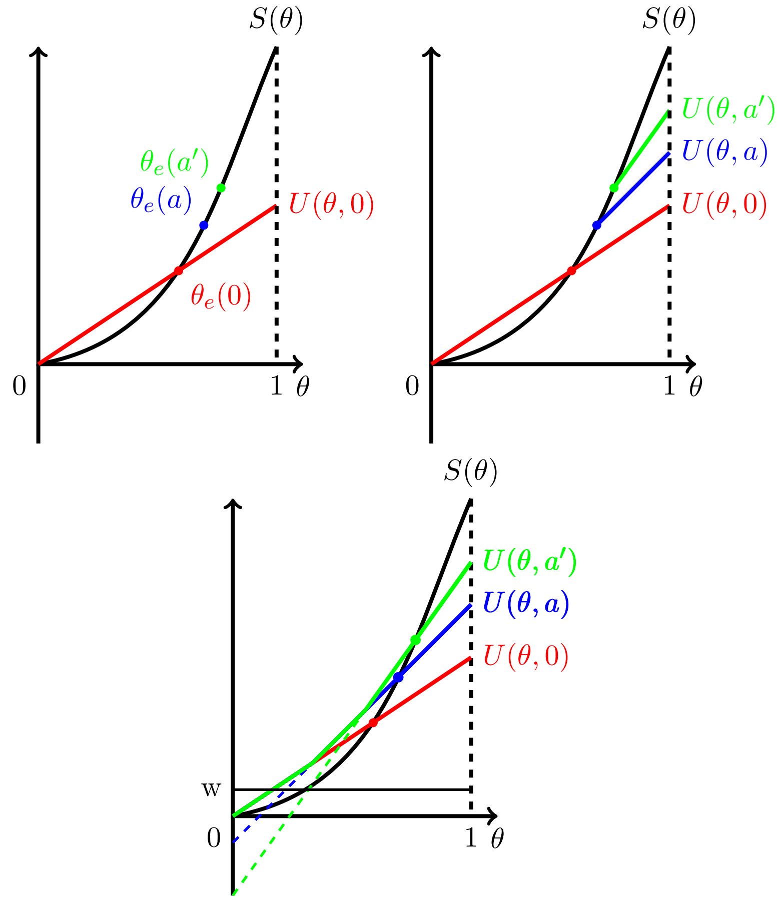
Claim 10. *In any equilibrium, contract schedules must offer the same utility to (almost) all types willing to sign at least one of the offered contracts.*

The intuition rests on the fact that the average of two IC contracts is IC because the expected utility for entrepreneurs is linear in the varying elements of the contract (x, z).

An intermediary can decide not to offer any contract with $x(\theta, a) = R\bar{b}$ for some asset level $\bar{b} > 0$. By doing so she gives up the expected profit of types with $\theta \geq \theta_e(\bar{b})$ and assets $\bar{b}$, she avoids the losses with those types $\theta < \theta_e(\bar{b})$ with assets $\bar{b}$. But also there could have been some types (θ, a) with $\theta \leq \theta_e(a)$ and $a > \bar{b}$ but such that:

$$U(\theta, a) = \theta \frac{S\big(\theta_e(\bar{b})\big) + R\bar{b}}{\theta_e(\bar{b})} - R\bar{b}.$$

Figure 3: Characterization of the profit maximizing contract given $\theta_e(a)$



If that was the case, half of those types were taking the contract with intermediary i before she dropped those contracts. To quantify the effect of that action define

$$A(a|b) = \Big\{\theta : x(\theta, b) = -Ra\Big\}$$

Incentive compatibility implies that if two types are offered the same $x(\theta, a)$ they should be offered the same $f\big(k^*(\theta)\big)z(\theta, a)$. Hence for all $\theta \in A(a|b)$ it must be the case that:

$$U(\theta, b) = \theta \frac{S\big(\theta_e(a)\big) + Ra}{\theta_e(a)} - Ra$$

Definition Let $P(a) = \frac{S\left(\theta_e(a)\right)+Ra}{\theta_e(a)}$ be the slope of all contracts such that $x(\theta, b) = -Ra$.

Claim 11. *The sets $A(a|b)$ are intervals. For all $b \leq c$ $A(a|c) \subset A(a|b)$.*

Given that, we can now write a closed form for intermediary's profits given some non decreasing function $\theta_e(a)$.

$$\Pi = 0.5 \int_{a=0}^{a=\infty} \int_{b=a}^{b=\infty} \int_{\theta \in A(a|b)} \big[S(\theta) - \theta P(a) + Ra\big] d^2 G(\theta, b) da \tag{1.5}$$

Now define $\Pi(c)$ as the profits brought by contracts with $x(\theta, a) \leq -Rc$:

$$\Pi(c) = 0.5 \int_{a=c}^{a=\infty} \int_{b=a}^{b=\infty} \int_{\theta \in A(a|b)} \big[S(\theta) - \theta P(a) + Ra\big] d^2 G(\theta, b) da$$

Claim 12. *For all c profits brought by contracts with $x(\theta, a) \leq -Rc$ are zero.*

Proof. If $\Pi(c)$ is negative, the intermediary can always drop all those contracts and increase profits. If $\Pi(c)$ is positive, analogously to the zero profit condition, the intermediary can give some $\varepsilon > 0$ more to all types with assets equal or higher than c, stealing the half of the market serviced by the other intermediary and getting $2\Pi(c) - \varepsilon$. $\square$

Taking derivative of $\Pi(c)$ we obtain that for all a

$$0 = \int_{b=a}^{b=\infty} \int_{\theta \in A(a|b)} \big[S(\theta) - \theta P(a) + Ra\big] d^2 G(\theta, b) \tag{1.6}$$

which implies that for each a the intermediaries should break even on those contracts such that $\underline{x}(\theta, b) = -Ra$. This result is analogous to Rothschild and Stiglitz (1992), where zero profits should be made contract by contract. In our setting all contracts with $x(\theta, b) = -Ra$ are equivalent for the entrepreneurs because $f\big(k^*(\theta)\big) z(\theta, a)$ has to be constant across all those contracts, but different to the intermediary because of the different expected surplus. Next the following proposition fully characterizes the equilibrium.

Proposition 4. *In equilibrium, the contract offered by all financial intermediaries to* *almost every θ and every a is:*

$$\begin{aligned}
k^*(\theta, a) &= \arg\max_k \theta f(k)\pi - Rk, \\
U(\theta, a) &= \max_{0 \le a' \le a} \theta P(a') - Ra' \\
x(\theta, a) &= -R\Big[\arg \max_{0 \le a' \le a} \theta P(a') - Ra'\Big].
\end{aligned}$$

where $\theta_e(\cdot)$ is such that for all a the financial intermediaries make zero profits over all entrepreneurs taking a contract with $x = -Ra$. That is,

$$\int_{b=a}^{b=\infty} \int_{\theta \in A(a|b)} \big[S(\theta) - \theta P(a) + Ra\big] d^2 G(\theta, b) = 0.$$

1.4. A Numerical Example

We have established that in the environment described above, competition among financial intermediaries yields to an inefficient outcome. The result raises a natural question: in which markets, industries or countries will be the inefficiency more pronounced? In the current section we use a numerical example to illustrate the changes in the deadweight loss when various parameters of the model change.

1.4.1. Parameterization

For the numerical example we set the entrepreneurs' outside option $w = 15$; the output of the project in case of success, $\pi = 100$; and the gross interest rate is $R = 1.02$.

Recall that a project succeeds with probability $p(\theta, k) = \theta f(k)$. We let $f(k) = 1 - \exp(-\beta k^\alpha)$ for $\beta > 0$ and $\alpha \in (0, 1)$. This functional form has several properties. First, it is continuous and strictly increasing and strictly concave on $\mathbb{R}_+$. Second, $f(0) = 0$ and $\lim_{k\to\infty} f(k) = 1$. Third, $\lim_{k\to 0} f'(k) = \infty$. The last condition ensures that for every $\theta > 0$ there is a scale such that the θ-type project is profitable (Inada condition). A way to interpret the above functional form is that the $p(\theta, k)$ is the product of θ and the probability that an exponential random variable is lower than k^{α} [7]. As exponential variables are usually employed for waiting times for a poisson process, it can be interpreted as the waiting time until the arrival of a new innovation (success), an amount k of capital allows the entrepreneur to run the project for k^{α} periods and hence the probability of a good idea arriving would be $f(k)$.

We set $\beta = 0.1$, implying that the latent exponential random variable would have a mean of 10. Under the above interpretation increasing k increases the probability that the random falls below k^{α} at a decreasing rate. We set $\alpha = 0.1$.

Last but not least, the joint distribution of assets and types will be key to compute intermediaries profits. We focus on the conditional distribution of θ given a, and let $g(\theta|a) = \frac{a+1}{\eta}\theta^{\frac{a+1}{\eta}-1}$ be its density. The exponent $\frac{a+1}{\eta} - 1$ controls the participation of high types on the conditional distribution. The higher is the exponent, the higher will be the density of types higher than a fixed value θ (Saffie and Ates (2013)). We let the exponent to be increasing on the asset level a, meaning that assets are positively correlated with the types. In addition, we assume the unconditional distribution of assets has density $\lambda \exp(-\lambda a)$. Hence an average entrepreneur has an asset level of 2. All the parameters and functional forms used in the numerical example are summarized in Table 1.

1.4.2. Benchmark Results

We begin by describing the case where entrepreneurs do not have assets. In order to facilitate comparability with the upcoming section where assets are introduced, we use use the marginal distribution of types that is consistent with the joint distribution of assets and types that will be used next (i.e: intermediaries will face the same population in both cases).

As defined before, an equilibrium is fully described by the triple $\big(k(\theta), x(\theta), z(\theta)\big)$. We

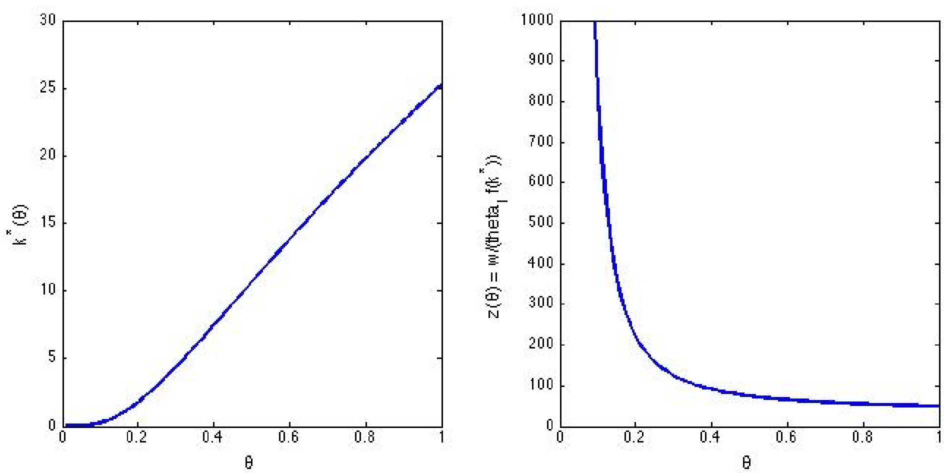
[7]Equivalently, $p(\theta, k)$ is the product of θ and the probability that a Weibul random variable is lower to k. In that case, the waiting time interpretation would be that thelonger it takes for a project to succeed the less likely it wil succeed in the future, Jovanovic and Szentes (2013) use a similar approach hence our functional forms may be regarded as a reduced form of their results

Table 1: Benchmark Parameters

w	15
π	100
R	1.02
$f(k)$	$1 - \exp(-\beta k^{\alpha})$
α	0.7
β	0.1
$g(\theta\|a)$	$\frac{a+1}{\eta}\theta^{\frac{a+1}{\eta}-1}$
η	1
$g(a)$	$\lambda\exp(-\lambda a)$
λ	0.5

established that $x(\theta) = 0,\ \forall\theta$. The functions $k(.)$ and $z(.)$ are plotted in the Figure 4 below. An important equilibrium object is the lowest type taking the contract, θ_L. For

Figure 4: Optimal Contract



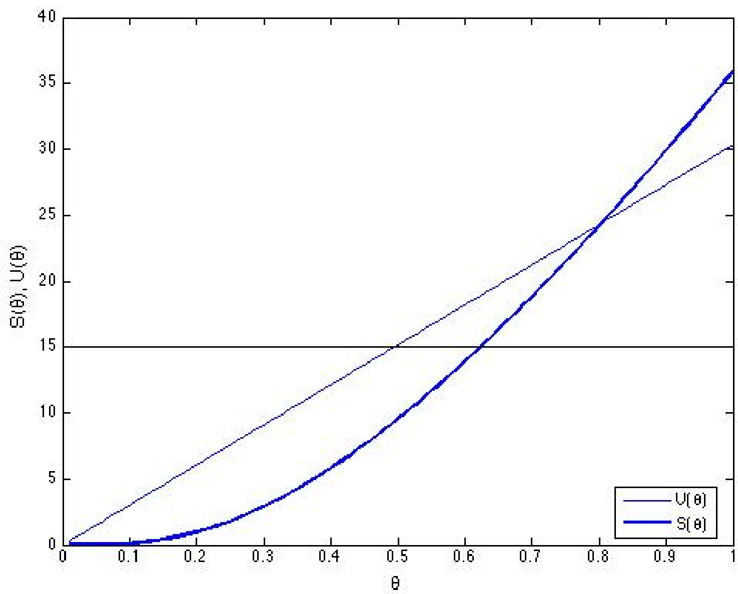
this example, the value of θ_L is 0.49, which implies that 74% of the entrepreneurs take the contract. Compare θ_L with the lowest type that would be funded by a social planner $\theta_P = 0.62$.

In the Figure 5 we plot $S(\theta) = \pi\theta f\big(k^*(\theta)\big) - Rk^*(\theta)$ as well as the expected payoff of entrepreneurs $U(\theta) = w\theta/\theta_L$. In addition we plot the the wage (horizontal line) to allow the reader to picture the equilibrium deadweight loss. In what follows we will use the relative inefficiency, defined as the ratio of the deadweight loss to the net economic surplus,

$$\mathcal{I} = \frac{\int_{\theta_L}^{\theta_P} \left(w - S(\theta)\right) dG(\theta)}{\int_{\theta_P}^{\theta_L} \left(S(\theta) - w\right) dG(\theta)}$$

For our benchmark parameterization, the relative inefficiency is $\mathcal{I} = 5.03\%$. In the next section we study how this inefficiency responds to changes in the parameters of the model.

Figure 5: Surplus, Payoff, Inefficiency

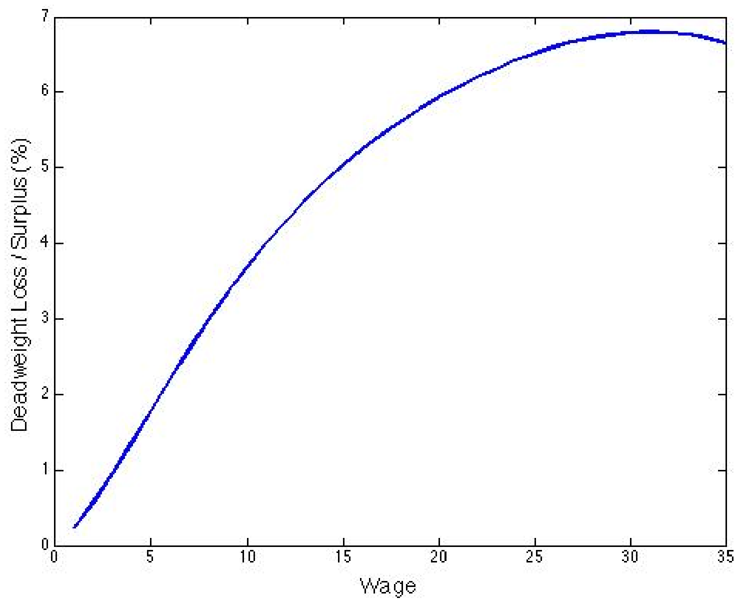


1.4.3. The Relative Inefficiency: Comparative Statics

In the current section we change, one by one, all the parameters of the model, holding the other parameters at their respective benchmark values.

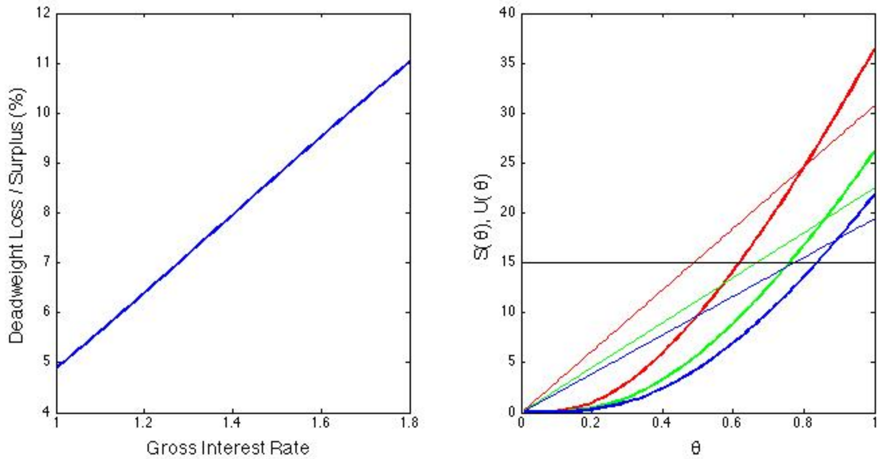
The outside option of entrepreneurs, w, is very important because, absent this cost, the economy would not be inefficient. In fact, when $w = 0$, all projects are socially profitable - by assumption -, and they are all funded at the optimal scale. An increase in w directly increases the deadweight loss and decreases the net economic surplus (everything else equal). It also induces an endogenous adjustment of θ_L. A higher w tends to increase θ_L, but this increment is dampen because the pool of types is increased and competition induces intermediaries to increase the terms of the contract. However, increases in θ_L reduce the deadweight loss. As illustrated in Figure 6, when the wage is very high, the force is strong enough to actually decrease the inefficiency. Note that is the wage is high enough, all the entrepreneurs take the outside option and the deadweight loss disappears. The relative inefficiency is maximized when $w = 31$, attaining 6.8% of the net surplus.

Figure 6: Relative Inefficiency and Wage



The gross interest rate, R, is similar to the wage, in the sense it represents the outside option, or opportunity cost of the capital in hands of financial intermediaries. A higher gross interest rate not only decreases the payoff of intermediaries, but it also decreases the optimal scale of the projects, and hence their expected return. The condition $\theta\pi f'(k^*) = R$ implies that the function $S(\theta; R) = \pi\theta f(k^*) - RK^*$ is linearly homogenous in (θ, R). Since $\theta \in [0, 1]$, an increase in R can be interpreted graphically as a downward shift of the curve $S(\theta)$ as shown in the right panel of Figure 7. This force increases both the net surplus and the deadweight loss everything else equal. However, θ_L will increase to satisfy the zero profit condition. As shown in the left panel of Figure 7, when we let R vary between 1 and 1.8, the relative inefficiency increases up to 11%, at $R = 1.8$.

Figure 7: Relative Inefficiency and Interest Rate



We move to describing how the inefficiency responds to changes in the shape of function $f(k)$. Figure 5 above suggests that the size of the inefficiency is very related to the concavity of the function $S(\theta)$. This concavity only depends on the shape of f. In fact,

$$S''(\theta) = f\big(k^*(\theta)\big) - \frac{\big(f'(k^*)\big)^2}{f''(k^*)}$$

Not surprisingly, the relative inefficiency is quite sensitive to the parameters α and β, that govern the shape of f in the current example. The results are displayed in Figure 8. The relative inefficiency is decreasing in both α and β. It decreases particularly fast as β increases, getting to 0.05% when $\beta = 1$.

Last, the distribution of types importantly affects the size of the inefficiency. The parameter η governs the shape of the distribution of θ. More precisely, as η increases, the density is shifted toward lower types. Holding the outside option fixed, an increase in η decreases the net surplus, because some density will be shifted from socially profitable projects to unprofitable projects. However, when good types are scarcer, intermediaries will offer less generous contracts, increasing θ_L. The last force tends to decrease the inefficiency. Figure

9 shows that the relative externality actually increases, reaching 7.7% when $\eta = 3$. On the other hand, for values of η close to zero, the relative inefficiency gets close to zero.

1.4.4. Assets

For simplicity, we discretize the space of assets, using the quantiles of the of the marginal distribution described in section 1.4.1, $g(a) = \lambda \exp(-\lambda a)$. We use five asset levels, 0, 0.45, 1.02, 1.83 and 3.22; there will be of 20% of the population holding each of the levels of assets. To put this numbers in perspective, a entrepreneur of type $\theta = 1$ will optimally invests $k^*(1) = 25$, while the for the lowest type taking the contract in the absence of assets, $k^*(\theta_L) = 10.3$ (see Figure 4). Recall that assets and types are correlated, and $g(\theta|a) = \frac{a+1}{\eta}\theta^{\frac{a+1}{\eta}-1}$.

The equilibrium is summarized in Table 2 and Figure 10. It will prove useful to introduce some further notation and let $\theta_F = \inf A(a|a)$ be the lowest type with asset level a, taking the contract with slope $P(a)$. By contrast, $\theta_L(a)$, the lowest type with assets a taking any contract. Hence, $\theta_L(a) = \min_{\hat{a} \leq a} \theta_F(a)$. Then, the relative inefficiency becomes,

$$\mathcal{I} = \frac{\sum_{a=1}^{5} \int_{\theta_L(a)}^{\theta_P} \big(w - S(\theta)\big) dG(\theta) Pr(a)}{\int_{\theta_P}^{\theta_L(a)} \big(S(\theta) - w\big) dG(\theta)}$$

Figure 8: Relative Inefficiency and the Shape of $f(k)$

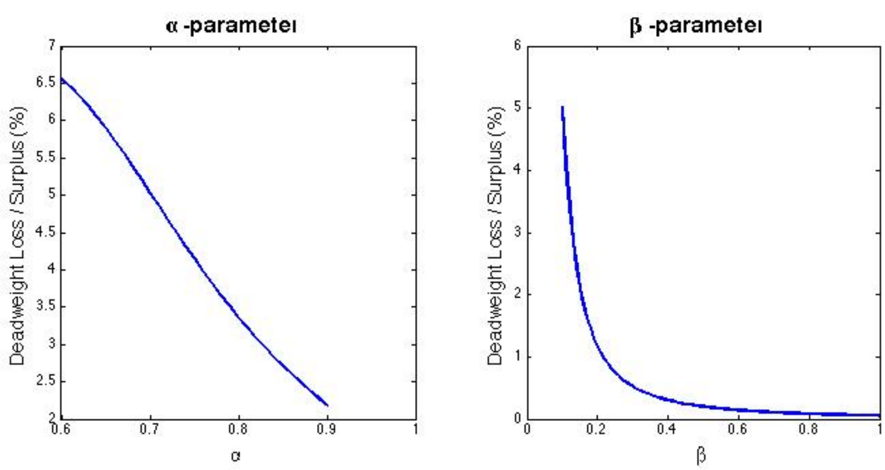
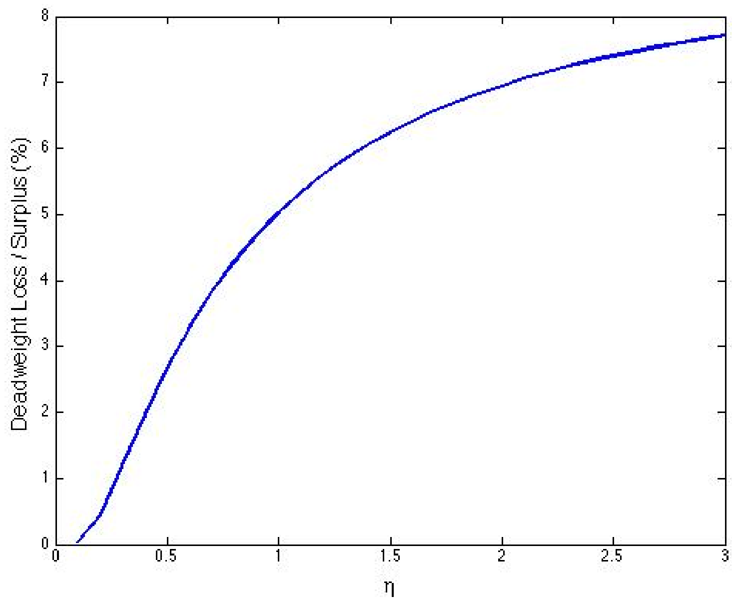


Figure 9: Relative Inefficiency and the Distribution of Types

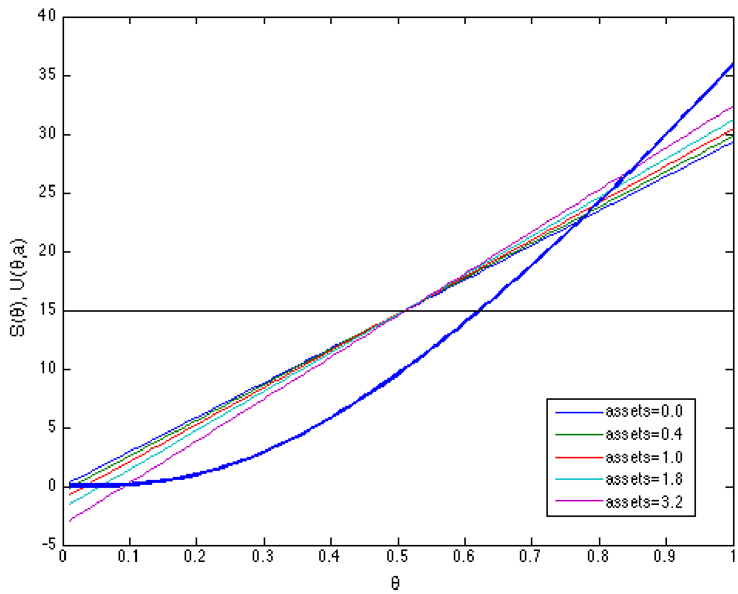


The lowest type taking the 0 asset contract is type $\theta = 0.5123$. Next, the lowest type taking the contract that requires to advance $a = 0.45$ is $\theta_F(0.45) = 0.5108$. This implies that the contract that requires $a = 0.45$ is more generous than the one that doesn't require assets. By contrast, the lowest type taking the contract requiring $a = 3.22$ is 0.5478. When θ is between 0.5136 and 0.5478, an entrepreneur with $a = 3.22$ prefers to take the contract that only requires a collateral of 1.8.

In this setting, the relative inefficiency is 3.94%. Hence, allowing intermediaries to condition their contracts on collateral decreases the inefficiency by more than a percentage point (from 5.03 %) in this example. There are two reasons why the inefficiency is reduced when the contracts depend on assets. First, the introduction of assets relaxes he limited liability constraint. Second, because the collateral is more likely to be held by higher types, conditioning contracts on collateral allows the intermediaries to do some sort of screening.

In order to disentangle the two effects, we compute the equilibrium for an economy with

Figure 10: Optimal Contract Under Asset Holdings



the same marginal distributions of assets and types, but where assets and types are independent. When assets and types are independent, the relative inefficiency is 4.62%. We also consider the case in which intermediaries observe a signal, that is correlated with types in the exact same way that assets were, but (of course) does not affect the limited liability constraint. In that case, the economy has a relative inefficiency of 4.65%.

The results are presented in Table 2. The upper panel displays the optimal contract for entrepreneurs holding assets that are independent of types, and the second panel shows the results when there is an observable signal correlated with the types. In the first case, entrepreneurs with higher asset holdings receive more generous contracts than their peers with lower assets. To understand this, recall that limited liability prevents "cream-skimming" in this environment once $x = 0$. For entrepreneurs with higher assets, the limited liability is not binding until it hits $x = -Ra$. Hence, intermediaries have incentive to skim the cream until they hit the constraint. Under this situation, although the contracts are more generous, they are taken by a lower number of types. In the second case, entrepreneurs with higher signals also get more generous contracts, but simply because they are better on average, and hence intermediaries break even offering higher terms to entrepreneurs. In this case, the lowest type taking the contract will be decreasing on the signal.

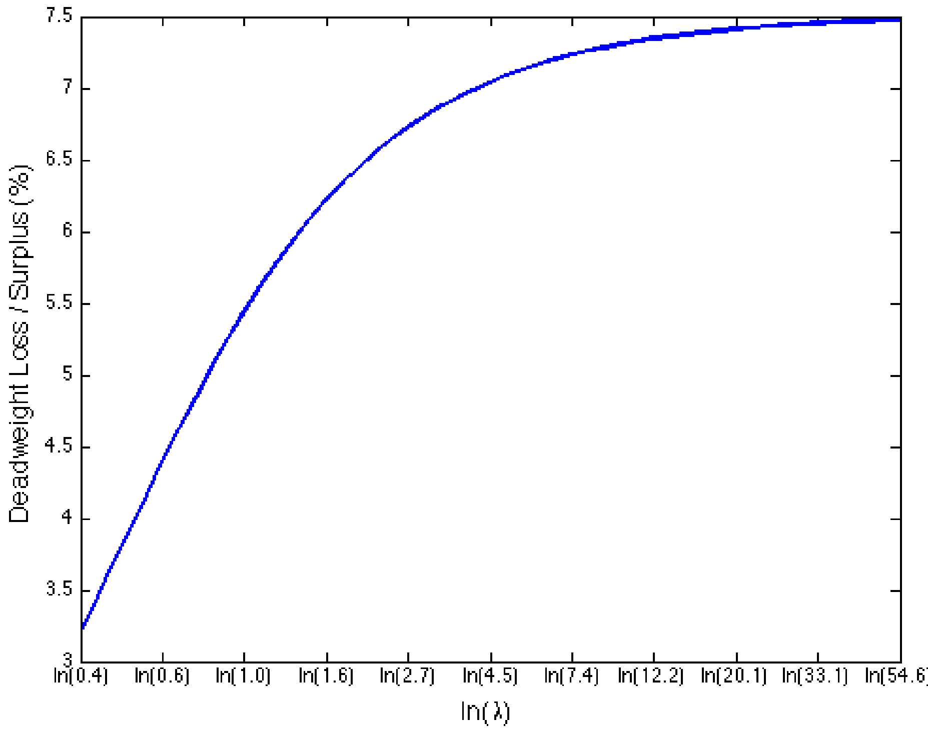
Table 2: Separate Asset Effects

Independent Types					
Asset Level	0	0.45	1.02	1.83	3.22
θ_F	0.5004	0.5248	0.5498	0.5766	0.6378
Expected Payoff	30.8θ	$31.6\theta - 0.5$	$32.5\theta - 1.0$	$33.7\theta - 1.9$	$35.4\theta - 3.3$
Informative Signals					
Asset Level	0	0.45	1.02	1.83	3.22
θ_F	0.5119	0.5065	0.4998	0.4906	0.4769
Expected Payoff	29.3θ	29.6θ	30.0θ	30.6θ	31.5θ

We end this section presenting some comparative statics of the inefficiency to the distribution of assets. In particular, we consider changes in the parameter λ of the exponential distribution of assets. We let λ vary between 0.5 and 50, which implies that the mean of the asset distribution varies approximately between 2 and 1/50 [8]. If the mean of the asset distribution is high enough, the inefficiency can be arbitrarily reduced, because most of the entrepreneurs can fund their own projects. On the other extreme, as $\lambda \to \infty$, the economy converges to one in which all entrepreneurs have zero assets, and the inefficiency converges to 7.5% of the surplus. It is important to note that the limit economy where all entrepreneurs have zero assets is not the same as the benchmark economy described in section 1.4.2. In the benchmark economy, the distribution of types, is the marginal distribution implied by joint distribution of types and assets used throughout the section. I.e, in the benchmark economy $g(\theta) = \int_0^\infty g(\theta, a)da$. When $\lambda \to \infty$, the distribution of types faced intermediaries is $g(\theta|0)$. Because of the positive correlation between assets and types, "lemons" are more abundant in the later economy.

[8] The mean of the exponential distribution is $1/\lambda$. In our case, this is, however, only an approximation because we discretize the distribution.

Figure 11: Inefficiency and Asset Distribution



1.5. Conclusion

We characterized financial contracts in a competitive environment with risk, adverse selection and limited liability. We find that, under the optimal contract the highest types are rewarded below the expected value of their project, and hence financial intermediaries make profits with high types. They also make losses with lower types. Our main result is that the optimal contract generates an inefficient outcome: projects that wouldn't be funded by a social planner are funded in equilibrium. We show that asymmetric information, limited liability and competition are all necessary to generate the result. We also show both analytically and numerically that the ability to use some assets as collateral mitigates but does not eliminate the inefficient outcome.

An implication of our results is that competition among financial intermediaries is not desirable. Note, however, that a monopolist financial intermediary would get all the economic surplus. In fact, the monopolist attains the first best because he has the ability to set loan sizes and interests rates, that is, has the ability to price discriminate. Needless to say, being the monopolist of financial intermediation in the economy would be extremely profitable. Who, if anyone, should get this rents? Even if the government gets the rents, a lot of political economy consideration would arise, that may challenge the efficiency of the result.

The results of this paper suggest at least two directions for future research. On the theoretical front, allowing for dynamics would generate an incentive for firms/entrepreneurs to retain earnings in order to escape from the inefficiency, and can be related to the literature on firm dynamics. On the empirical front, the degree to which working capital can be collateralized varies exogenously across industries. This opens the door to test the implications of the model.

CHAPTER 2 : Taxing Firms Facing Financial Frictions

This chapter is co-authored with Gustavo Camilo.

2.1. Introduction

Many countries, including the United States, tax business income more than once. First, they tax profits at the firm level as corporate income. Next, business owners pay taxes on dividends when distributed, and on capital gains when realized. If the business issues debt, bondholders pay taxes on interest at the personal income rate.

Major tax reforms in the United States and elsewhere have sought to alleviate this "double taxation". For instance, the Jobs and Growth Tax Relief Reconciliation Act of 2003 under the George Bush administration, and the Tax Reform Act of 1986 under the Reagan administration both decreased capital taxes at the shareholder level in order to alleviate double taxation. In this paper we will argue that the allocation of capital in the economy can be improved by alleviating double taxation in the other direction. Namely by decreasing the corporate income tax and replacing the forgone income by a tax on shareholders.

Specifically we ask what is the effect if replacing the corporate income tax by a tax on shareholders, in a revenue neutral way. To answer this question we construct a model with heterogeneous firms, borrowing constraints, costly equity issuance and endogenous entry and exit. After calibrating the model to the U.S. economy, we find that when the corporate income tax is replaced by a tax on shareholders (to be described in more detailed later), steady-state output increases by 6.8% and total factor productivity (TFP) increases by 1.7%.

We compute steady states for all corporate income tax rates between the current statutory rate of 35% and 0, and in each case set the tax on shareholders such that the government revenue is unchanged. As expected, we find that output, total factor productivity and steady state lifetime utility all peak when the corporate income tax rate is zero.

The mechanism behind the efficiency gain is as follows. Firms randomly receive idiosyncratic investment opportunities (modeled as idiosyncratic persistent productivity shocks). When hit by a shock, a firm seeks to adjust it capital stock in pursuit of optimal size. Due to financial frictions, optimal firm behavior often dictates reliance on retained earnings for growth. The corporate income tax reduces retained earnings available for investment, thereby delaying capital accumulation. As the retained earning are not paid back to shareholders, the friction just described is absent when taxes are levied at the dividend level.

The general equilibrium feedback and the endogenous entry and exit plays an important role in the quantification of the mechanism. The revenue neutral tax reform triggers an increase in the demand for capital and labor. In turn, the higher demand increases the wage and as a result less productive firms no longer find it optimal to remain active. Firm turnover increases, and more importantly, in steady state, the average productivity of active firms increases. The higher wage also increases the households labor supply, contributing to the increase in output.

Our paper relates to literature examining taxation and corporate financial policy from a theoretical perspective.[9] This literature has focused on the effect of taxes on firm behavior from a partial equilibrium perspective. The key theoretical result from this literature, proved in Auerbach (1979), Bradford (1981) and King (1974), is that dividend taxes do not distort firm decisions when the firm uses retained earnings as the marginal source of investment. The literature refers to this result as the "new view". The new view is in contrast with an alternative "traditional view" which assumes that equity serves as the marginal source of investment. Dividend taxes reduce the return on equity and hence affect capital accumulation. Thus, the existing literature takes the marginal source of investment as given and examines the implications of a set of taxes on firm behavior.

[9] See Auerbach (2002) for a comprehensive review.

In this paper, we use a model where the marginal source of investment is endogenous. Our model builds on the structural corporate finance models ofCooley and Quadrini (2001), Gomes (2001), Hennessy and Whited (2005) and Hennessy and Whited (2007). In these models, external funds are costly and firms rely on retained earnings unless the benefit of investment is high enough to offset the costs of external financing. That is, endogenously, a fraction of the firms will behave according to the traditional view and another fraction will behave according to the new view. Once calibrated, our model reproduces central features of the data including the investment rate, leverage, the frequency and average of equity issuances, and firm turnover.

Our work also relates to an empirical literature aiming at estimating the elasticity of capital taxes on investment. The estimates (for publicly traded firms) range between -1 and -0.5 (see Hassett and Hubbard (2002)). However, recent studies have used novel data from tax returns finding different results. Zwick et al. (2014) exploits variation in eligibility for bonus depreciation allowances, which are effectively reductions on the corporate income tax base. They find a much larger elasticity: -1.6. Moreover, they find that small firms respond 95% more than big firms. Using the same data,Yagan (2015) exploits the different incidence of a recent tax reform on S-corporations versus C-corporations[10]. He finds no effect of the dividend tax on firm investment. Our model can theoretically rationalize the apparently contradictory results. Firms facing financial frictions will decrease investment in response to an increase in the corporate income tax, as the later decreases their net worth. The effect is particularly important for smaller firms that are more likely to be constrained. On the other hand, as firm endogenously rely on retained earning to finance investment, increases in the dividend tax rate won't decrease investment.

The closest papers to ours use structural corporate finance models to assess the economic impact of the Jobs and Growth Tax Relief Reconciliation Act of 2003, which substantially decreased taxes on dividends and capital gains. Gourio and Miao (2010) study the steady

[10]See Yagan (2015) for a detailed explanation about the differences between S-corporations and C-corporations and the resulting tax treatment.

state effect of the reform, finding that the tax cuts reduce frictions in the reallocation of capital and increase steady-state capital by 4%. In a companion paper, Gourio and Miao (2011) use the same apparatus to predict the transitional dynamics triggered by the same reform. However, the cited papers focus on mature firms and do not consider capital accumulation during the growth process. By contrast, we introduce entry and exit and financial frictions, which allows use to consider the entire firm life-cycle. In particular, in our model the firms go through a non-trivial growth process that is delayed by the corporate income tax. Considering the full life-cycle of firms is crucial to quantifying the mechanism described above.

The rest of the paper is organized as follows. In section 2.2, we present the model and define the equilibrium. Section 2.3 describes our calibration strategy the data used for calculating moments. Section 2.4 presents and discusses the quantitative results. Section 2.5 concludes.

2.2. The Model

2.2.1. Economic environment

We now describe the model, which builds on the industry dynamic literature pioneered by Hopenhayn (1992), but features endogenous investment and financing decisions on the firms side. Time is discrete and the horizon infinite. The model economy consists of representative household, a continuum of ex-post heterogenous firms, and a government who needs to finance an exogenous stream of (constant) expenditures.

Preferences and endowments

The preferences over consumption and labor are given by,

$$\sum_{t=0}^{\infty} \beta^t u(C_t, L_t) \tag{2.1}$$

The households own all the firms in the economy. Denote by $\Phi(s)$ the measure of stocks of type s in the household's portfolio. The firm type s is its state, which will be described in more detail in the next subsection. Since for each type of firm, the total number of shares outstanding is normalized to 1, in equilibrium $\Phi(s)$ is also the total measure of firms of type s in the economy (by market clearing). Let $E(s)$ the cum-dividend price of a firm of type s and $D(s)$ the payout to its shareholders.

Besides dividends, payouts could take the form of equity repurchases. Although modeling share repurchases is beyond the scope of this paper, when firms make share repurchases regularly, the Internal Revenue Service (IRS) treats these as dividends. The same will be assumed in the context of the model. For calibration purposes $D(s)$ will be mapped to the sum of common and preferred dividends and equity repurchases. Throughout the paper, for simplicity, we refer to $D(s)$ as "dividends".

Technology

Firms are heterogenous in their idiosyncratic productivity z_t, which they observe at the beginning of the period, and as in Hopenhayn (1992) the production technology features decreasing returns to scale and a fixed cost of operation. A firm with productivity z_t, k_t units of capital and l_t units of labor produces $F(z_t, k_t, l_t)$ units of final good. We define operating profits as

$$\pi(k, z) = \max_{l} \; F(z_t, k_t, l_t) - wl \tag{2.2}$$

The investment technology is standard: one unit of final good invested at time t increases the capital stock at $t+1$ by one unit, and the capital stock depreciates at rate δ. There are no adjustment costs or irreversibilities in capital accumulation: $k_{t+1} = i_t + (1-\delta)k_t$.

Entry and exit

After productivity is observed and before undertaking production, a firm can exit the market. In such case, the firm liquidates its assets and pays its debts. The remaining funds -always positive because of the collateral constraint- are distributed back to households as dividends. The constraint on borrowing guarantees that default is never a possibility.

If profitable, a positive mass of firms enters in every period. An entrant firms has no capital or debt, and as a result its output is zero on the first period. It is forced to issue equity to pay for the fixed cost of operation. After paying the fixed cost, the entrant observes its productivity shock.

Market structure

Households, firms and the government can trade one period risk free bonds b_t, subject to financial frictions. In this sense, financial markets are incomplete and firms cannot insure against idiosyncratic productivity shocks[11]. Bonds are in zero net supply.

Following the corporate finance literature we constrain the use of debt and of equity in a reduced form way. As first used by Kiyotaki and Moore (1995), debt is subject to a collateral constraint, $b_t \leq \theta(1-\delta)k_t$. Following Hennessy and Whited (2005) and Hennessy and Whited (2007), issuing equity is costly. In particular, for a firm to raise e units of equity it requires an investment of $\lambda_0 + \lambda_1 e$. The parameters $\lambda_0, \lambda_1 > 0$ are meant to capture technological underwriting and floatation costs incurred when issuing equity. They will be calibrated to match the frequency and average of equity issuances.

[11] Although firms are risk neutral, because of the financial frictions, they value cash flows more in certain states of the world.

Government policy

The government needs to finance an exogenous stream of expenditures G. It can do so by using linear taxes at four different levels. First, it can tax firms at the corporate income level, at rate τ_c. The corporate income tax is charged on operating profits and allows for depreciation and interest deductions. Total collections from an individual firm are given by $\tau_c\big(\pi(k,z) - \delta k - rb\big)$.

In addition, the government can tax dividends at rate τ_d and interest payments at rate τ_i. Following the literature, (see Gourio and Miao (2010), Gourio and Miao (2011)) capital gains can be taxed on *accrual* at rate τ_g. In particular, capital losses are fully deductible.[12] This will be important for equity issuances: in the context of our model, when a firm issues equity, its current shareholders are subject to a capital loss on the shares outstanding before the issuance, as their position is diluted. We also allow for a tax on labor income τ_l. Although we will hold the tax on labor income constant in our policy analysis, its presence is important because the wage is endogenous: a better allocation of capital results in higher wages, and tax collections increase. As in Krueger and Ludwig (2016) the labor income tax allows for a deduction Δ. The deduction allows to match both the base of the labor income tax and its the marginal rate in a simple way. Total collections from the labor income tax are therefore given by $\tau_l(w_t L_t) = \tau_l w_t L_t - \Delta$.

We constrain the government to have a balanced budget in steady state.

2.2.2. Competitive Equilibrium

In this subsection we formulate the recursive problem for firms and households and characterize the aggregate variables. The subsection finishes with the definition of a recursive competitive equilibrium.

[12] According to the U.S. tax code capital losses are deductible as long as they are offset by capital gains. This is consistent with our model because in steady state the value of the portfolio is constant.

Timeline

In the interest of clarity, before formulating the agents' maximization problems, we briefly describe the decisions within a period in a timeline.

1. The period begins. Households have assets A, incumbent firms arrive to the period with capital k and debt b.
2. Before the shocks z are observed, a mass M of firms choose to enter.
3. For each form, the shock z is realized. *Incumbents* choose to continue or exit.[13]
4. Production takes places. Wages, taxes and debt are paid.
5. Firms make investment and borrowing decisions by choosing k' and b'.
6. The corporate sector distributes dividends and issues equity.
7. Households collect dividends, interest payment and labor income, and uses them for consumption or savings.

Recursive problem of households

The households are subject to the following budget constraint,

$$\begin{aligned} C + B' + \int E(s)\phi'(s)ds &= B(1 + (1-\tau_i)r) \\ + \int \Big(E(s) + (1-\tau_d)D(s) - \tau_g(E(s) - E^{-}(s))\Big)\phi(s)ds &+ wL - \tau_l(wL) \end{aligned} \tag{2.3}$$

We define assets A as,

$$A = B(1 + (1-\tau_i)r) + \int \Big(E(s) + (1-\tau_d)D(s) - \tau_g(E(s) - E^{-}(s))\Big)\tilde{g}_t(s)ds \tag{2.4}$$

In equilibrium, the household will only hold assets yielding the same return and hence it

[13] Entrants commit to operate during the first period.

is enough to only keep track of the total value of the assets.

The recursive formulation of the household problem is,

$$V(A) = \max_{B', \phi'(s)} u(C, L) + \beta V(A') \tag{2.5a}$$

$$\text{s.t: } C + B' + \int E(s)\phi'(s)ds = A + (1 - \tau_l)wL - \tau_l(wL) \tag{2.5b}$$

$$\text{s.t: } A' = B'(1 + (1 - \tau_i)r) \tag{2.5c}$$

$$+ \int \Big(E'(s) + (1 - \tau_d)D'(s) - \tau_g(E'(s) - E(s))\Big)\tilde{\phi}'(s)ds$$

$$\text{s.t: } B' \geq \underline{B} \tag{2.5d}$$

Here $\underline{B}$ is the natural borrowing limit.

To gain intuition about the way our model works and how taxes affect agents decisions, we summarize here the optimality conditions of the household problem.

First, the Euler equation is

$$u'(C, L) = (1 + (1 - \tau_i)r)\beta u'(C', L') \tag{2.6}$$

We refer to $\frac{\beta u'(C', L')}{u'(C, L)}$ as the household discount factor.

Second, the no-arbitrage condition implies,

$$(1 - \tau_i)r = (1 - \tau_g)\frac{E' - E}{E} + (1 - \tau_d)\frac{D}{E} \tag{2.7}$$

Third, labor supply

$$-u_l(C, L) = (1 - \tau_l)wu_c(C, L) \tag{2.8}$$

The no-arbitrage equation allows to derive the firms' market value. Define the cum-dividend price of the firm $P(s)$ to be the ex-dividend price plus after-tax dividends and net of equity dilutions, $N(s)$: $P(s) = E(s) + \frac{1-\tau_d}{1-\tau_g} D(s) - N(s) - \Lambda(N(s))$.

Iterating on the no-arbitrage equation 2.7, the cum-dividend value of the firm satisfies the following recursion,

$$P(s) = (1 - \tilde{\tau}_d) D(s) - N(s) - \Lambda(N(s)) + \tilde{\beta} P'(s) \tag{2.9}$$

We defined the effective tax on dividends by $1 - \tilde{\tau}_d = \frac{1-\tau_d}{1-\tau_g}$, and the firm's discount factor by $\tilde{\beta} = \left(1 + \frac{1-\tau_i}{1-\tau_g} r\right)^{-1}$. In particular when $\tau_g = 0$, the effective tax on dividends is equal to the dividend tax rate and the firm's discount factor is equal to the household's discount factor.

Recursive problem of firms

In our environment, the incentives between firms' managers and shareholders are aligned, and the objective of the firms is to maximize its market value as stated in equation 2.9 above.

Firms never it find optimal to issue equity and distribute dividends in the same period. For simplicity, we denote by d net distributions, and refer to it as dividend payments when $d > 0$ and equity issuance when $d < 0$. It will prove useful to introduce the indicator function $j = 0$ if $d \geq 0$ and $j = 1$ otherwise.

For each individual firm, the uses and sources of funds need to be equal according to,

$$\pi(z,k) + b' - (1+r)b = d + k' - (1-\delta)k + \tau_c\big(zk^\alpha - f - \delta k - rb\big) \tag{2.10}$$

The equation above states that operating profits and net borrowing are used to pay for (net) distributions, investments and taxes.

Define net worth as, $\omega = \pi(k,z) - \tau_c\big(\pi(k,z-\delta k - rb)\big) + (1-\delta)k - (1+r)b$. This is a sufficient statistic for the firms dynamic decisions.

The problem of a firm be written recursively as,

$$P(\omega,z) = \max_{k',b'} \omega - k' + b' - (1-j)\tilde{\tau}_d(\omega - k' + b') - j\Lambda(k' - \omega - b') \tag{2.11a}$$

$$+ \tilde{\beta}\mathbb{E}_{z'}\Big[\max\big\{(1-\tau_d)\big((1-(1-\tau_c)\delta)k' - (1+(1-\tau_c)r)b'\big), P(\omega',z')\big\}\,|z\Big] \tag{2.11b}$$

$$\text{s.t: } \omega' = \pi(k',z') - \tau_c\big(\pi(k',z') - \delta k' - rb'\big)\big) + (1-\delta)k' - (1+r)b' \tag{2.11c}$$

$$\text{s.t: } \theta(1-\delta)k' \geq b' \tag{2.11d}$$

Once again, optimality conditions allow to built intuition about the way the model works.

We start by discussing exit. Using standard dynamic programing arguments, it can be shown that $P()$ is strictly increasing in z. Hence there is a unique threshold $z^*(k',b')$ such that the firm is indifferent between staying active and exiting,

$$\begin{aligned}&(1-\tilde{\tau}_d)\big((1-(1-\tau_c)\delta)k' - (1+(1-\tau_c)r)b'\big)\\ &= P\big((1-\tau_c)\pi(k',z^*) + (1-(1-\tau_c)\delta)k' - (1+(1-\tau_c)r)b', z^*\big)\end{aligned} \tag{2.12}$$

As described above, when a firm exits it liquidates its capital stock, pays its debts and distributes the differences as dividends. Notice that upon exit, capital depreciation and interest payments are deductible are still deductible.

To the extent that taxes decrease firm value P, they decrease the value of remaining active vis a vis exiting. Everything else equal (including prices and distributions) a higher τ_c is associated with lower exit rates. However when liquidations are taxed, $\tilde{\tau}_d$ decreases the value of exit. As it will be clear later, in an economy where equity issuances aren't allowed, the effective dividend tax does not distort the exit margin. When equity issuances are allowed and the cost of equity capital is not deductible, a higher effective tax is associated with higher exit, because the effective dividend tax is not paid in every state of the world. When capital gains and dividends are taxed at the same rate, equity issuances are tax deductible via capital losses. Under such a tax system, the common dividend - capital gains tax does not distort the exit margin.

Then next step is to consider capital accumulation. We first discuss the case when firms optimally distribute dividends in the current period. Using Leibniz rule, the first order condition with respect to k' (when the objective is differentiable[14]) is,

$$
\begin{aligned}
(1-\tilde{\tau}_d)\tilde{\beta}^{-1} =& (1-\tilde{\tau}_d)(1-(1-\tau_c)\delta)Pr\big(z' < z^*(k',b')\big) \\
&+ \int_{z^*(k',b')}^{\overline{z}} P_\omega\big(\omega', z'\big)\Big(\big((1-\tau_c)\pi_k(k,z)-\delta\big)+1\Big)pr(z'|z)dz' \\
&+ \theta(1-\delta)\mu
\end{aligned} \tag{2.13}
$$

The left hand side of the equation above is the marginal cost of investment, namely not distributing the marginal dollar to shareholders. The right hand side is the marginal benefit. If the firm chooses to exit -with probability $Pr\big(z' < z^*(k',b')\big)$- it gets the after-tax, un-depreciated value of capital. If instead it chooses to stay active, in addition it gets the marginal product of capital, weighted by the value of internal funds on that period: $P_\omega\big(\omega', z'\big)$. This value satisfies,

[14] The value function has kinks at the point where dividends are zero, and at the exit threshold.

$$
\begin{aligned}
P_\omega\big(\omega', z'\big) &= 1 - \tilde{\tau}_d && \text{if } d(w', z') > 0 \\
&= 1 + \lambda_1 && \text{if } d(w', z') < 0 \\
&\in [1 - \tilde{\tau}_d, 1 + \lambda_1] && \text{if } d(w', z') = 0 \qquad (2.14)
\end{aligned}
$$

The marginal benefit also includes the term $\theta(1-\delta)\mu$, where μ is the Lagrange multiplier on the collateral constraint. Since the capital stock relaxes the borrowing constraint. Since debt offers a debt shield, in this model there will be over-investment.

We now provide some intuition of how taxes change capital accumulation decisions. For this discussion, we assume that prices do not respond to changes in tax policy. If a firm never were to issue equity, or alternatively all costs of equity issuance, including the opportunity cost of capital are deducted, the choice of k' is not distorted by $\tilde{\tau}_d$. This is the "new view", first introduced by Auerbach (1979), Bradford (1981) and King (1974). In this model, firms endogenously avoid to issue equity, but do so in some states of the world. A firm that is distributing dividends will invest more than a firm facing the same prices but no effective dividend taxes.

Firms that are hit by a very high productivity shock will find it optimal to issue equity. In such case, the marginal cost of investing one dollar is $\big(1+\lambda_1\big)\beta^{-1}$, and -fixing prices- $\tilde{\tau}_d$ decreases investment.

By contrast, the corporate income tax *decreases* investment in all states of the world: even firms using exclusively retained earning to finance investment invest less than they would in a world with the same prices but no corporate income taxes.

Last, the firm discount factor $\tilde{\beta}$ decreases the marginal cost of investment in every state of the world and regardless of the current liquidity regime.

Besides exit and investment, firms choose the value of debt to be issued. The first order condition with respect to b' is given by,

$$
\begin{aligned}
&(1-\tilde{\tau}_d)+\mu \\
&=\tilde{\beta}(1+(1-\tau_c)r)\Bigg((1-\tilde{\tau}_d)Pr\big(z'<z^*(k',b')\big) \quad +\int_{z^*(k',b')}^{\overline{z}} P_\omega\big(\omega',z'\big)pr(z'|z)dz'\Bigg)
\end{aligned}
$$

Before discussing the equation above, consider a government policy such that equity has a tax benefit over debt, $(1-\tau_i)<(1-\tau_c)(1-\tau_g)$. In such a case, the household like to borrow infinite amounts from firms, and no equilibrium exists. The reason is the following: when the household holds debt, she gets a tax subsidy on her interest payments that is higher than the tax paid by firms on the interests. As a result, the household gets a subsidy proportional to the value of its debt and would demand an infinite amount of it. To rule out such "loophole", we restrict taxes to satisfy $(1-\tau_i)\geq(1-\tau_c)(1-\tau_g)$. The last inequality implies $\tilde{\beta}(1+(1-\tau_c)r)\leq 1$.

Whenever $\tilde{\beta}(1+(1-\tau_c)r)<1$, the tax advantage of debt pushes firms to borrow up to the constraint. if the correlation of shocks is high enough, firms will always be at the collateral constraint. However,when the correlation of shocks if low enough, reducing the stock of debt allows firms to insure against low productivity shocks, and they trade-off the tax advantage with precautionary saving motives.

Entry

At the beginning of every period, an unbounded mass of potential entrants with zero net worth is available. Potential entrants will enter as long as it is profitable to do so. However, entry is costly. Before observing productivity shocks, entrants have to pay an entry cost c_e. Upon entry, firms will draw a productivity shock from its ergodic distribution and pay the fixed cost of operation. In particular, entrants are forced to issue equity. The actual mass of entrant M satisfies,

$$\mathbb{E}P(0,z) = c_e \ , \text{ if } M > 0$$

$$\mathbb{E}P(0,z) < c_e \ , \text{ if } M = 0$$

The entry margin is distorted to the extent that taxes reduce firm value. Both $\tilde{\tau}_d$ and τ_c, but the later has a larger effect: $\left|\frac{\partial P}{\partial \tilde{\tau}_d}\right| < \left|\frac{\partial P}{\partial \tau_c}\right|$. Since $\tilde{\tau}$ is increasing in τ_d, the dividend tax decreases firm value. The firm discount factor decreases firm value as well, meaning that τ_i increases firm value. Notice that if τ_i, τ_g, τ_d are set equal, any change the the common rate does not distort the entry margin.

Having discussed the behavior of each firm in isolation, in the next section we describe the aggregate variables.

Aggregation

Since each firm is characterized by its state $s = (\omega, z)$, in this section we describe the law of motion for the measure Φ over the space of feasible net worth and productivity shocks.

Let $\otimes$ and $\mathcal{Z}$ be measurable sets. The probability of going from ω, z into the set of states $\otimes \times \mathcal{Z}$, is given by,

$$Q(\omega, z, \otimes, \mathcal{Z}) = \int \{\omega'(z', \omega, z) \in \otimes\} x(z', \omega, z) d\Gamma(dz'|z) \tag{2.16a}$$

where, $x(z', \omega, z) = 1$ if $z' > z^*\big(k'(\omega, z), b'(\omega, z)\big)$ and Γ denotes the distribution of productivity shock. That is, a firm with state ω, z goes into $\otimes \times \mathcal{Z}$ if its realized next worth, given its optimal choices of investment and borrowing is in $\otimes$ with probability $\int_{\mathcal{Z}} d\Gamma(dz'|z)$, provided that under the realization of z' the firm doesn't want to exit.

Using the Markov transition function Q, the measure of firms is evolves according to,

$$\begin{aligned}
\Phi'(\otimes, \mathcal{Z}) &= \int Q(\omega, z, \otimes, \mathcal{Z}) d\Phi(\omega, z) && \text{if } 0 \notin \otimes \\
\Phi'(\otimes, \mathcal{Z}) &= \int Q(\omega, z, \otimes, \mathcal{Z}) d\Phi(\omega, z) + M \int d\Gamma^e(z) && \text{otherwise} \qquad (2.16b)
\end{aligned}$$

Where $\Gamma^e(z)$ is the ergodic distribution of productivity shocks. Notice that the distribution Φ is associated with a distribution $\tilde{\Phi}$ over (k, b, z) by $\tilde{\Phi}(k, b, z) = \Phi(\pi(k, z) - \tau_c\big(\pi(k, z - \delta k - rb)\big) + (1-\delta)k - (1+r)b, z)$

Next we use the measure $\tilde{\Phi}$ to define the aggregate quantities in the corporate sector. These are the (net) aggregate supply of final goods, total profits, the aggregate investment, and total financial costs, defined respectively as,

$$\begin{aligned}
Y_t &= \int \left(z_t k_t^{\alpha_k} l_t^{\alpha_l} - f\right) d\tilde{\Phi}_t(k_t, b_t, z_t) \\
\Pi_t &= \int \pi(z_t, k_t) d\tilde{\Phi}_t(k_t, b_t, z_t) \\
I_t &= \int \left(k_t + 1 - (1-\delta)k_t\right) d\tilde{\Phi}_t(k_t, b_t, z_t) \\
\Lambda_t &= \int \left(\lambda_0 + \lambda_1 d_t\right) j_t d\Phi_t(\omega_t, z_t)
\end{aligned}$$

Definition of equilibrium

Given an initial stock of assets in hands of the household A_0, an initial distribution of firm $\Phi_0()$ and a government spending requirement G and tax on labor τ_l, a recursive competitive equilibrium is a sequence of value and policy functions for the household $\{V_t, C_t, B_t', L_t\}_{t=0}^{\infty}$, a sequence of value and policy functions for the firms $\{P_t, k_t', b_t', l_t, x_t\}_{t=0}^{\infty}$, sequences of masses of entrants M_t and measures Φ_t, sequences of prices $\{w_t, r_t\}$ and sequences of government policies $\{\tau_{d,t}, \tau_{c,t}, \tau_{i,t}, \tau_{g,t}\}$ such that,

1. Given prices and government policies $\{V_t, C_t, B'_t, L_t\}_{t=0}^{\infty}$ solves the household problem (2.5).
2. Given prices and government policies $\{P_t, k'_t, b'_t, l_t, x_t\}_{t=0}^{\infty}$ solves the individual firm problem (2.11).
3. M_t is consistent with the free entry condition (2.15).
4. The government policy satisfies the budget constraint

$$\begin{aligned} & G_t + (1+r_t)B_t^G - B_{t+1}^G \\ & = \tau_{i,t} r_t B_t + \tau_{d,t} D_t + \tau_{c,t} \left(\Pi_t - \delta K_t - r_t B_t - f\right) + \tau_{g,t} \left(E_t - E_{t-1}\right) \end{aligned} \tag{2.17}$$

5. Markets clear in every period:

$$L_t = \int l_t(w_t) dG_t(\omega_t, z_t) \tag{2.18}$$

$$Y_t = C_t + I_t + \Lambda_t + G_t \tag{2.19}$$

6. The law of motion of the measure Φ_t is consistent with firms' policy functions according to (2.16)

$B' = \int q'(\omega, z) d\Phi(\omega, z)$ $L = \int l'(\omega, z) d\Phi(\omega, z)$

A stationary equilibrium is a competitive equilibrium in which all functions and aggregate variables are constant over time.

2.3. Data and Calibration

In this section we calibrate the model. A first subset of parameters is chosen to match steady state moments to their data counterparts. Those include the parameters ruling the financial frictions, the productivity process and the amount of turnover in the economy. We will argue that those are the most important parameters to quantify the mechanism. A second set of parameters is fixed following estimates typically found in the literature.

2.3.1. Functional forms and fixed parameters

We use the same production technology as Gomes (2001), namely $F(z,k,l) = zk_k^{\alpha} l_l^{\alpha} - f$. We set the labor share of output $\alpha_l = 0.64$ as in Prescott (1986). Using plant-level data, Lee (2007) finds that returns to scale in manufacturing vary from 0.83 to 0.91, depending on the estimator. We fix $\alpha_k = 0.23$ so that total returns to scale fall in the midpoint of the reported range. The fixed cost of operation f will be included among the estimated parameters. We assume free entry and set the cost of entry, c_e to zero. Firm level idiosyncratic productivity follows $\log z_{t+1} = \rho_z \log z_t + \sigma_z \varepsilon_t$, where ε is a standard normal innovation.

As first introduced by Greenwood et al. (1988), in our economy household preferences are given by

$$u(C,L) = \frac{1}{1-\sigma}\left(C - \frac{H}{1+\frac{1}{\gamma}} L^{1+\frac{1}{\gamma}}\right)^{1-\sigma}$$

Underlying this preference specification is the assumption of no wealth effects on the labor-leisure trade-off. This is done for computational simplicity, but it may overestimate the efficiency gains of the reforms. We assume log utility and choose an elasticity of labor supply $\gamma = 0.5$ as in Chetty et al. (2011). The discount rate β is fixed at 0.972 to match a (before tax) interest rate of 4% in the benchmark steady state.

Tax system

We argue that flat tax rates are a good approximation to the actual tax system. Although the tax rate are actually progressive, the rate structure produces a flat 34% tax rate on incomes from $335,000 to $10,000,000, gradually increasing to a flat rate of 35% on incomes above $18,333,333. Moreover the marginal rate already hits 34% for income above

75,000. In our sample, 99% of firms report a pretax income above 128,000 (conditional on reporting profits). We set the corporate income tax to the top statutory rate $\tau_c = 0.35$.

Since the Jobs and Growth Tax Relief Reconciliation Act of 2003 was enacted, dividend and capital gains are tax at 15% for all incomes falling on or above the third tax bracket[15]. Accordingly we set $\tau_g = \tau_d = 15\%$.

For personal income taxes, the rate structure is more progressive and a linear tax is less of a good approximation. As described in section 2.2.1 we model this in a parsimonious way by allowing for a constant marginal rate and a deduction. We follow Krueger and Ludwig (2016) and calibrate the deduction to match 35% of the household income. This matches the sum of standard deductions and exemptions from the tax code. Krueger and Ludwig (2016) choose the marginal rate to balance the government budget and find 27.5%. This is consistent with the 28% rate reported by Mendoza et al. (1994). We set $\tau_l = \tau_i = 0.28$.

2.3.2. Targeted Parameters

Our environment is in between two benchmarks. On the one hand, in an economy where λ_0 or λ_1 and θ are infinite, households cannot finance firms. In such economy firms only use internal funds to finance investment and dividends can be thought as an endowment to households. The taxation of such endowment is not distortive. On the other extreme is an economy in which $\lambda_0 = \lambda_1 = \theta = 0$ and the economy is similar to the neoclassical benchmark, where household finance investment and are indifferent to do so using equity or debt. Because of their importance we include those parameter in the vector to be calibrated. In addition we include in the vector of calibrated parameters, the fixed cost of operation f, the parameters ruling the process of productivity ρ_z, σ_z, the depreciation rate of physical capital δ and the disutility of effort H[16].

[15]For instance in 2013, the 15% rate applied for incomes above \$36,250 for single taxpayers and above \$72,500 for those filing jointly.

[16]The disutility of effort is sensitive to changes in the wage, and the equilibrium wage turns out to be very responsive to the value of the fixed cost of operation f. Hence we include H in the calibrated parameters.

Selection of moments

We choose moment that are a priori informative about the parameters we seek to calibrate. First, in steady state, aggregate investment is given by δK. In order to pin down δ we include the average investment rate $\int i(\omega,z)/kd\tilde{\Phi}(k,b,z)$. In order to pin down λ_0 and λ_1 we include the frequency of equity issuance, $\int j(\omega,z)d\Phi(\omega,z)$, and the average of equity issuance as a fraction of total capital, $\int d(\omega,z)j(\omega,z)d\Phi(\omega,z)/\int kd\tilde{\Phi}(k,b,z)$. In order to identify ρ_z and σ_z we include the serial autocorrelation and the standard deviation of the profits to capital ratio π/k[17]. The turnover rate defined as the mass of entrants over the mass of incumbents, $M/\int d\Phi$, should be informative about the cost of entry c_e. Average leverage $\int b/k \; d\tilde{\Phi}(k,b,z)$ is associated with the value of θ. The fixed cost of operation is related to the ratio of dividends to profits. Last, the scale parameter H is chosen to match one third of time spent at market work. All the moments were computed using the dataset described in the next subsection except for two. The first is time spent at market work, which we set at 0.33 as is standard in the literature. The second is turnover. Firms appear and disappear from Compustat for several reasons other than entry and exit. Consequently, we target the turnover value of 6% reported by Lee and Mukoyama (2015) using the Annual Survey of Manufactures from the US Census Bureau.

Data Description

We use data from the Compustat Monthly Updates - Fundamentals Annual File, North America from WRDS. Following the literature, we discard all regulated utilities (SIC 4900-4999), financial firms (SIC 6000-6999) and quasi-governmental and non-profit firms (SIC 9000-9999) because our model is not well suited for the analysis of such firms. Next we drop Canadian and foreign ADRs, as the American tax system does not apply for those

[17] Following the literature, for the computation of second moments, we remove fixed effects from the data. Accordingly, the variance of profits to assets is computed after substracting the average for each firm. For the autocorrelation, we fit a panel autoregression using the method by Arellano and Bond (1991). Assuming that the variance of profits is constant over time, the slope coefficient corresponds to the autocorrelation of profits.

firms.

We define capital as Compustat variable total assets (AT); investment as the difference between capital expenditures and sales of property, plant and equipment (CAPX-SPPE); dividends (total payouts) include common and preferred dividends, and equity repurchases (DVC + DVP + PRSTKC); debt as long term debt plus short term debt (DLTT + DLC), equity issuances as sales of common and preferred equity (SSTK)[18]; and operating profits as earnings before interest, taxes, depreciation and amortization (EBITDA). All variables are winsorized at the top and bottom 5%.

Our model abstracts from unobserved heterogeneity in firms' characteristics. To be consistent, we use firm (and time) fixed effects when computing second moments.

The dataset covers fiscal years between 2003 and 2015. This correspond to the period since the last major tax reform, for which the statutory rates used are relevant. After dropping observation with missing or inconsistent information for any of the variables used, we end up with 42,546 observations. The number of firms ranges between a minimum of 2,711 in 2015 and a maximum of 3,955 in 2003.

The following table presents the data estimated parameters and the model counterparts.

[18]We use the filter proprosed by McKeon (2015) to clean the Compustat reported data from employee's exercise of options.

	Data	Model
Average investment rate	0.05	0.05
Standard deviation of profits	0.11	0.09
Average leverage	0.17	0.21
Average equity issuances	0.09	0.11
Frequency of equity issuances	0.19	0.23
Autocovariance of profits	0.39	0.61
Turnover (Lee, Mukoyama 2015)	0.06	0.05
Time at work	0.33	0.33
Average dividends to profits	0.48	0.43

Parameter values

We find a collateral constraint parameter value $\theta = 0.23$. This is lower that the 0.36 estimate found by Li et al. (2016) in an environment without equity issuances. The fix cost of equity issuances $\lambda_0 = 0.025$ and the fixed cost of operations is $f = 1.44$. For the marginal cost of equity issuances we find $\lambda_1 = 0.24$. This value is in between 0.028 reported by Gomes (2001) and 0.059 found by Hennessy and Whited (2005). The deprecation rate is $\delta = 0.08$.

We end this section by summarization all parameter values and their respective targets in the following table,

Parameter	Interpretation	Value
	Calibrated Parameters	
θ	collateral constraint	0.23
λ_0	fixed cost of equity issuance	0.025
λ_1	linear cost of equity issuance	0.24
ρ_z	autocorrelation of productivity shocks	0.75
σ_z	std. deviation of productivity shocks	0.1
δ	depreciation rate	0.08
H	disutility of labor	3.4
f	fixed cost of operation	1.44
c_e	cost of entry	0.04
	Fixed Parameters	
β	discount rate	0.972
γ	labor supply elasticity	0.5
σ	intertemporal elasticity of substitution	1
α_k	capital share	0.21
α_l	capital share	0.64
	Taxes	
τ_c	corporate income tax	0.35
τ_d	dividend tax	0.15
τ_g	tax on capital gains	0.15
τ_i	interest income tax	0.28
τ_l	labor income tax	0.28

2.4. Thought Experiment

In this section we use the model to explore computationally the effects of different tax regimes on economic outcomes, focusing particularly of TFP.

Our first exercise consists on decreasing the corporate tax rate from benchmark rate of 35% to 0%, in revenue neutral way. In the baseline model, the value of G is 13.3% of GDP. By comparison, between 2003 and 2015, the average revenue from corporate and personal income taxes was 9.05% of GDP in the U.S. In order to hold G constant, we replace the revenue from the corporate income tax by increasing a common tax on shareholders $\tau_d = \tau_g = \tau_i = \hat{\tau}$.

We do so in two steps. The first step consists on finding the common rate $\hat{\tau}$ achieving budget balance when the corporate income tax is at its benchmark level of 35%. Such reform increases TFP and output by decreasing over-accumulation of capital: since debt provides a tax-shield, and firms are borrowing constrained, they over-accumulate capital as long as the benefit from relaxing the borrowing constraint to exploit the tax shield is higher than the cost, a lower marginal product of capital.

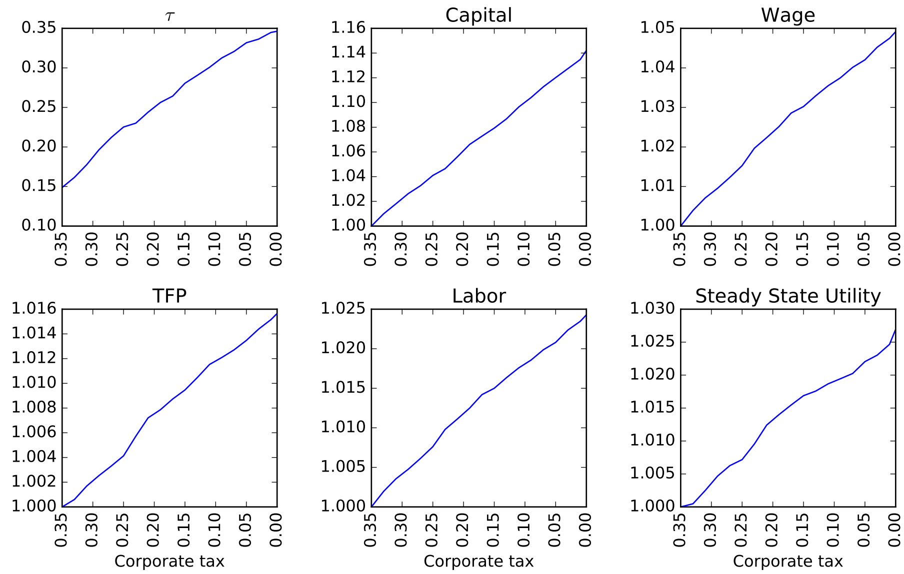
The effect of the first step of the reform is quantitatively very small. Steady-state TFP increases by 0.07%, output by 0.6% and the wage by 0.35%.

The second step consists on decreasing the corporate income tax gradually from 35% to 0%, and adjusting $\hat{\tau}$ such that,

$$G = \hat{\tau} r B + \hat{\tau} D + \tau_c \left(\Pi - \delta K - rB - f\right) + \hat{\tau}\left(E' - E\right) + \tau_l(wL)wL$$

Figure 12 depicts the effect of the described change in government policy on several variables of interest. As shown by the first panel, the corporate income tax can be replaced by a common tax of shareholders of 34.6%. The rate is comparatively low and suggest substantial efficiency gains. In fact, the second panel shows that TFP increases from 0.568 to 0.577, that is 1.7%. Both output and consumption increase substantially: 6% and 6.5% respectively. The wage increases by 4.9%. This increase in the wage combined with the 2.4% in labor displayed in the last panel are important because they substantially increase labor income tax collections, explaining why they 35% corporate income tax can

Figure 12: Effects of Tax Reform



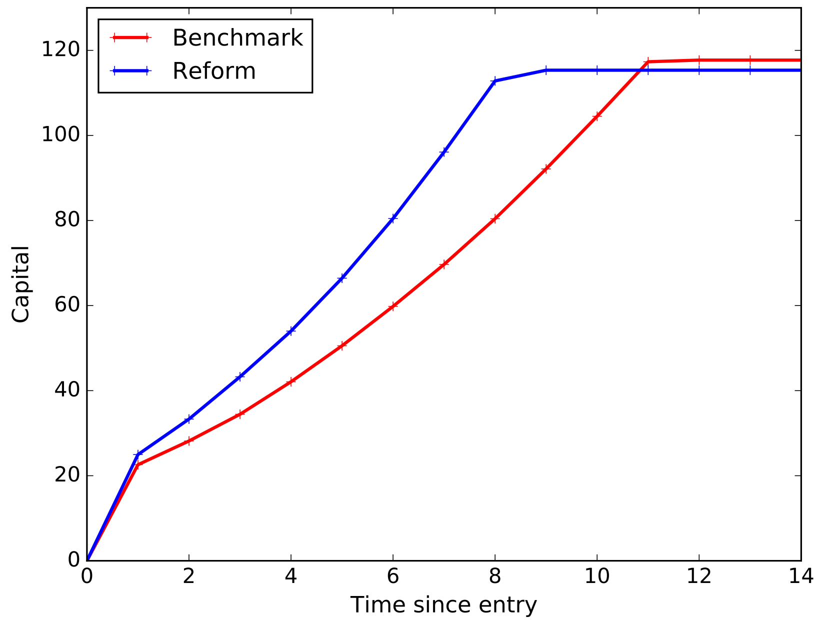
be replaced increasing dividend and corporate income rates by 20 percentage points.

Behind the increase in TFP is an improvement in the capital allocation: the elimination of the corporate income tax allows growing firms to accumulate capital more rapidly. The acceleration in the capital accumulation process is depicted in figure 13. The figure shows the sequence of capital, starting at entry, for a firm hit every period by the same productivity shock. In the benchmark economy, it takes 11 periods for such a firm to grow up to its unconstrained optimal level. When the corporate income tax is eliminated, the same firm reaches its optimal level of capital in 9 periods. As a result, for each level of productivity, the distribution gets more compressed closer to the optimal size, which reduces capital misallocation in the economy.

2.5. Conclusion

In this paper we use a model of heterogenous firms subject to borrowing constraints, costly equity issuances and endogenous entry and exit to study the effects of corporate income taxation and compare them with the effects of taxation at the shareholder level. We

Figure 13: Capital Accumulation Before and After Reform



argued that total factor productivity is 1.7% higher in a steady state where the corporate income tax is replaced by a higher common tax on shareholders in a revenue neutral way.

For our tax experiments we assumed that capital gains are taxed on accrual. Under that assumption, when a firm issues equity, the value of the current shareholders stock decreases and the capital loss is tax deductible. In reality, in the U.S. capital gains are taxed upon realization, not on accrual. Relaxing the former assumption requires a richer model that is out of the scope of this paper. We leave that for future reserch.

CHAPTER A : Appendix

A.1. Proofs

A.1.1. *Zero assets proofs*

The following is a useful consequence of lemma 1.

Lemma A.1.1. *In any IC contract schedule, $x(\theta)$ is non-increasing.*

Proof of lemma A.1.1

Proof. Let $\theta' > \theta$ from equation (1.1c) it follows that:

$$x(\theta) = U(0) + \int_0^{\theta} \Big[f\big(k(s)\big) z(s) - f\big(k(\theta)\big) z(\theta) \Big] ds \quad \text{then,}$$

$$x_i(\theta') - x(\theta) =$$

$$(\theta' - \theta) \Big[f\big(k(\theta)\big) z(\theta) - f\big(k(\theta')\big) z(\theta') \Big] + \int_{\theta}^{\theta'} \Big[f\big(k(s)\big) z(s) - f\big(k(\theta')\big) z(\theta') \Big] ds$$

And both terms in the last equation are negative or zero because of equation (1.1b) hence the claim holds. □

Proof of claim 1

Proof. Start with and equilibrium in which an entrepreneur of type θ receives payoff $U^0(\theta)$. Let the equilibrium incentive compatible contract schedule be $\big(k^0(\theta), x^0(\theta), z^0(\theta)\big)$. Finally let A^0 denote the set of types taking one of the contracts rather than the outside option. Suppose there is a set $C \subseteq A^0$ such that $k^*(\theta) \neq k^0(\theta)$ for all $\theta \in C$. We will show that contract schedule is profit maximizer for intermediary 1 only if C has measure zero.

Construct $\mathcal{C}'$ as the set $\big\{(k'(\theta), x'(\theta), z'(\theta)) | \theta \in A^0\big\}$, where,

$$k'(\theta) = k^*(\theta) \qquad z'(\theta) = \frac{f\big(k^0(\theta)\big) z^0(\theta) + \delta\theta}{f\big(k^*(\theta)\big)} \qquad x'(\theta) = x^0(\theta) + \frac{\delta}{2}(1 - \theta^2) \tag{A.1}$$

If an entrepreneur of type θ takes the contract $\big(k'(\hat{\theta}), z'(\hat{\theta}), x'(\hat{\theta})\big) \in \mathcal{C}'$, her payoff will be,

$$\theta f(k^0(\hat{\theta}))z^0(\hat{\theta}) + x^0(\hat{\theta}) + \delta\hat{\theta}\theta + \frac{\delta}{2} - \frac{\delta\hat{\theta}^2}{2}$$

The above payoff is maximized at $\hat{\theta} = \theta$ and the term $\delta\hat{\theta}\theta - \delta\hat{\theta}^2/2$ ensures the maximizer is *unique*. The resulting payoff is $U'(\theta) \equiv U^0(\theta) + \frac{\delta}{2}\big(1 + \theta^2\big) > U^0(\theta)$. Thus every entrepreneur $\theta \in A^0$ signs a contract with $k^*(\theta)$.

Next, we will show that offering $\mathcal{C}'$ constitutes a profitable deviation for intermediary 1. Let v_i^0, $i = 1, 2$ be the intermediaries payoff in the original equilibrium. Note that, because intermediaries have always the option of offering empty sets of contracts, $v_i^0 \geq 0$.

The following equation follows from the definition of payoffs,

$$\int_{A^0} \Big\{\pi f\big(k^0(\theta)\big) - Rk^0(\theta)\Big\} dG(\theta) = v_1^0 + v_2^0 + \int_{A^0} U^0(\theta) dG(\theta) \tag{A.2}$$

Let,

$$M \equiv \int_{A^0} \Big\{\pi f\big(k^*(\theta)\big) - Rk^*(\theta)\Big\} dG(\theta) - \int_{A^0} \Big\{\pi f\big(k^0(\theta)\big) - Rk^0(\theta)\Big\} dG(\theta)$$

Since C has positive measure, the definition of $k^*(\theta)$ implies, $M > 0$.

Now, let v_1' be the payoff of intermediary 1 when she deviates to $\mathcal{C}'$ and A' the set of types signing a contract after the deviation.

$$
\begin{aligned}
v_1' &= \int_{A'} \Big\{\pi f\big(k^*(\theta)\big) - Rk^*(\theta) - U'(\theta)\Big\} dG(\theta) \\
&= \int_{A'\setminus A^0} \Big\{\pi f\big(k^*(\theta)\big) - Rk^*(\theta) - U'(\theta)\Big\} dG(\theta) + \int_{A^0} \Big\{\pi f\big(k^*(\theta)\big) \\
&\quad - Rk^*(\theta) - U'(\theta)\Big\} dG(\theta) \\
&= \int_{A'\setminus A^0} \Big\{\pi f\big(k^*(\theta)\big) - Rk^*(\theta) - U'(\theta)\Big\} dG(\theta) + M + v_1^0 + v_2^0 \\
&\quad - \int_{A^0} \{U(\theta) - U^0(\theta)\} dG(\theta)
\end{aligned}
$$

The last equality follows from equation A.2.

The measure of A^0 is obviously bounded by one and $U'(\theta) - U^0(\theta) \leq \frac{\delta}{2}\big(1+\theta^2\big) \leq \delta$. Hence,

$$
\int_{A^0} \{U(\theta) - U^0(\theta)\} dG(\theta) \leq \delta
$$

Moreover, $\pi f\big(k^*(\theta)\big) - Rk^*(\theta) - U'(\theta)$ is bounded and it follows that, as the measure of $A \setminus A^0$ goes to zero,

$$
\int_{A\setminus A^0} \Big\{\pi f\big(k^*(\theta)\big) - Rk^*(\theta) - U'(\theta)\Big\} dG(\theta) \to 0
$$

Now,

$$
A \setminus A^0 = \{\theta : U^0(\theta) < w \leq U'(\theta)\} \subset \{\theta : w - \delta < U^0(\theta) < w\}
$$

And it is clear that the measure of $A \setminus A^0$ goes to zero as δ goes to zero.

As a result of the previous observations, there exists a value for δ, low enough, such that,

$$
M > -\int_{A\setminus A^0} \Big\{\pi f\big(k^*(\theta)\big) - Rk^*(\theta) - U'(\theta)\Big\} dG(\theta) + \int_{A^0} \{U(\theta) - U^0(\theta)\} dG(\theta)
$$

For such a δ,

$$v_1' - v_1^0 > 0$$

Since the intermediary payoff is strictly higher, if the measure of C is strictly positive the alternative schedule $\mathcal{C}'$ will achieve strictly higher profits for intermediary 1. $\square$

Proof of claim 4

Proof. We begin by showing that $U(\theta)$ is non-decreasing in θ. Once that is established, it will be straightforward to see that A must be an interval.

By lemma 1 it suffices to show $f\big(k(\theta)\big)z(\theta) \geq 0$ for all $\theta > 0$. Suppose there is an equilibrium with IC contract schedule $\big(k(\theta), x(\theta), z(\theta)\big)$, and there exists $\theta_o > 0$ such that $f\big(k(\theta_o)\big)z(\theta_o) < 0$. By claim 1, there exists a sequence $\{\theta^n\} \in [0, \theta_o]$ converging to zero such that $k(\theta^n) = k^*(\theta^n)$ and this implies $\big\{f\big(k(\theta^n)\big)\big\}$ also converges to zero.

Equation 1.1b guarantees that $f\big(k(\theta^n)\big)z(\theta^n) \leq f\big(k(\theta_o)\big)z(\theta_o) < 0$ for all n. Taking limit on both sides implies $\lim_{n\to\infty} z(\theta^n) = -\infty$. But limited liability implies $x(\theta) \geq 0$ and $x(\theta) + z(\theta) \geq 0$. As $x(\theta)$ is non-increasing by claim A.1.1, $x(0)$ is an upper bound for all $x(\theta)$ and thus $z(\theta)$ has to be bounded below, contradicting $\lim_{n\to\infty} z(\theta^n) = -\infty$.

Hence, any contract offered in equilibrium satisfies $U(\theta)$ is nondecreasing.

Now, because $U(\theta)$ is nondecreasing $\theta \in A$ implies $\theta' \in A$ for all $\theta' > \theta$ (when indifferent between being a worker and an entrepreneur, agents choose the later by assumption). The continuity of U implies that if A is nonempty then $A = [\theta_L, 1]$ for some $\theta_L \in [0, 1]$. Also, if $\theta_L > 0$ then $U(\theta_L) = w$. $\square$

Proof of zero profit condition (Claim 2)

Proof. By contradiction suppose intermediary 1 is making profit. We will show intermediary 2 is not choosing an optimal contract schedule.

Let the contract schedules be $\big(k(\theta), x(\theta), z(\theta)\big)$ for $i \in \{1, 2\}$ and $U(\theta)$ the corresponding expected utilities for entrepreneurs. Suppose the profit for intermediary 1 is $M > 0$. Consider the alternative schedule for intermediary 2 in which increases all entrepreneurs' utility by a small amount ϵ. More precisely, this alternative schedule is defined by $\mathcal{C}_2' = \big\{(k'(\theta), x'(\theta), z'(\theta))|\theta \in A\big\}$, where,

$$k'(\theta) = k(\theta) \qquad z'(\theta) = \frac{f\big(k(\theta)\big)z(\theta) + \delta\theta}{f\big(k(\theta)\big)} \qquad x'(\theta) = x(\theta) + \frac{\delta}{2}(1 - \theta^2) \tag{A.3}$$

As shown in the proof of claim 1, the payoff $U'(\theta)$ resulting from optimally choosing a contract in $\mathcal{C}_1 \cup \mathcal{C}_2'$ is strictly greater than the original payoff: $U'(\theta) > U(\theta)$. Hence by deviating to $\mathcal{C}_2$, intermediary 2 will steal half the market intermediary 1 was servicing alone (A_1). The revenue of intermediary 2 increases by at least M. Naturally the costs also increase because entrepreneurs get more generous contracts, but also because the more generous contracts induce an entry of new entrepreneurs. However, as shown in the proof of claim 1 δ can be chosen so that the extra cost are smaller than M. For such a δ, $\mathcal{C}_2'$ is a profitable deviation for intermediary 2. $\square$

Proof of claim 3

Proof. Suppose $U(\theta') \geq S(\theta')$ for some $\theta' > \hat{\theta}$ such that $k(\theta') = k^*(\theta')$ are not positive. By the envelope theorem $S'(\theta) = f\big(k^*(\theta)\big)\pi$, which is increasing in θ because f is increasing and concave. Hence $S(\theta)$ is convex. Remembering that $S(0) = 0 \leq x(\theta')$, by incentive compatibility:

$$U(\hat{\theta}) \geq \hat{\theta} f\big(k^*(\theta')\big)z(\theta') + x(\theta') = \frac{\hat{\theta}}{\theta'}U(\theta') + \frac{\theta' - \hat{\theta}}{\theta'}x(\theta') \geq \frac{\hat{\theta}}{\theta'}S(\theta') + \frac{\theta' - \hat{\theta}}{\theta'}S(0) > S(\hat{\theta})$$

contradicting $S(\hat{\theta}) > U(\hat{\theta})$. As $k(\theta') = k^*(\theta')$ for almost all θ', the lemma follows. □

Proof of claim 5

Proof. Let $\mathcal{C}_1^0, \mathcal{C}_2^0, s^0$ be an equilibrium with corresponding payoffs $v_1^0, v_2^0, U^0(\theta)$. Denote the incentive compatible schedule by $k^0(\theta), z^0(\theta), x^0(\theta)$ and the set of types taking the contract by A^0.

Suppose there there is $\tilde{\theta} \in A^0$ such that $x(\tilde{\theta}) > 0$. Without loss of generality, the associated contract is offered by intermediary 1.

Among the contracts in $\mathcal{C}_1$, there could be some that are dominated by another contract in $\mathcal{C}_1$, for every entrepreneurial type. We focus on the contracts such that this is not the case:

$$\big\{k_1(\theta), x_1(\theta), z_1(\theta) : \theta \in [0,1]\big\}$$

such that $\big(k_1(\theta), x_1(\theta), z_1(\theta)\big)$ is a maximizer of,

$$U_1(\theta) \equiv \max_{(k,x,z \in \mathcal{C}_1)} \theta f(k)z + x \tag{A.4}$$

We will construct a strategy for intermediary 2 allowing him to "cream skim" the market. That is, intermediary 2 will serve all the profitable types, leaving the unprofitable types to intermediary 1.

Because no intermediary would make loses in equilibrium, if a positive measure among the contracts in $\mathcal{C}_1$ are actually signed by entrepreneurs, there must be an positive measure subset over which C_1 yields non-negative profits. Hence there is a $\hat{\theta} > 0$ (in that set) such that $k_1(\hat{\theta}) = k^*(\hat{\theta})$ and with whom the intermediary makes non-negative profits. By claim 3, $S(\theta) > U(\theta)$ for all $\theta > \hat{\theta}$.

If $x_1^0(\hat{\theta}) > 0$, construct

$\mathcal{C}_2' = \{(k_2'(\theta), x_2'(\theta), z_2'(\theta))|\theta \in A^0\}$, where,

$$k_2'(\theta) = k_1^0(\theta) \qquad z_2'(\theta) = \frac{f\big(k_1^0(\hat{\theta})\big)z_1^0(\hat{\theta}) + x_1^0(\hat{\theta})/\hat{\theta} + \delta(\theta - \hat{\theta})}{f\big(k_1^0(\theta)\big)} \qquad x'(\theta) = \frac{\delta}{2}(1 - \theta^2)$$

The payoff of entrepreneur θ among contracts $(k_2'(\theta'), x_2'(\theta'), z_2'(\theta'))$ in $\mathcal{C}_2'$ is

$$U_2'(\theta) = \max_{\theta' \in [0,1]} \theta\big(f\big(k_1^0(\hat{\theta})\big)z_1^0(\hat{\theta}) + x_1^0(\hat{\theta})/\hat{\theta} + \delta(\theta' - \hat{\theta})\big) + \frac{\delta}{2}(1 - \theta'^2)$$

which is uniquely maximized at $\theta' = \theta$. That is,

$$U_2'(\theta) = \theta\bigg(f\big(k_1^0(\hat{\theta})\big)z_1^0(\hat{\theta}) + x_1^0(\hat{\theta})/\hat{\theta}\bigg) + \frac{\delta}{2}(1 - \theta^2)$$

Next, we compare the payoffs of signing the contract with intermediary 1 or 2. We show that for $\theta > \hat{\theta}$, the later is better, while $\theta < \hat{\theta}$ prefers the former.

First consider $\theta < \hat{\theta}$.

By the envelope theorem applied to A.4

$$\begin{aligned} U_1^0(\theta) &= U_1^0(\hat{\theta}) + \int_{\hat{\theta}}^{\theta} f\big(k_1^0(s)\big)z_1^0(s)ds \\ &= f\big(k_1^0(\hat{\theta})\big)z_1^0(\hat{\theta}) + x_1^0(\hat{\theta}) + \int_{\hat{\theta}}^{\theta} f\big(k_1^0(s)\big)z_1^0(s)ds \end{aligned}$$

and $f\big(k_1(s)\big)z_1(s)$ is non-decreasing in s.

Notice that,

$$
\begin{aligned}
U_2'(\theta) &= U_2'(\hat{\theta}) + (\theta - \hat{\theta})\Big(f\big(k_1^0(\hat{\theta})\big)z_1^0(\hat{\theta}) + x_1^0(\hat{\theta})/\hat{\theta}\Big) + \frac{\delta}{2}\left(\hat{\theta}^2 - \theta^2\right) \\
&= U_2'(\hat{\theta}) + \int_{\hat{\theta}}^{\theta}\Big(f\big(k_1^0(\hat{\theta})\big)z_1^0(\hat{\theta}) + x_1^0(\hat{\theta})/\hat{\theta}\Big)ds + \frac{\delta}{2}\left(\hat{\theta}^2 - \theta^2\right)
\end{aligned}
$$

Hence

$$
U_2'(\theta) - U_1^0(\theta) = \int_{\hat{\theta}}^{\theta}\Big\{f\big(k_1^0(\hat{\theta})z_1^0(\hat{\theta})\big) - f\big(k_1^0(s)\big)z_1^0(s)\Big\}\,ds + \frac{\theta - \hat{\theta}}{\hat{\theta}}x_1^0(\hat{\theta}) + \frac{\delta}{2}\left(\hat{\theta}^2 - \theta^2\right)
$$

Since $f\big(k(.)\big)z(.)$ is non-decreasing, for $\theta < \hat{\theta}$, the first term is non-positive. The second term is strictly positive as $x_1^0(\hat{\theta}) > 0$. We chose δ such that $\frac{\theta-\hat{\theta}}{\hat{\theta}}x_1^0(\hat{\theta}) + \frac{\delta}{2}\left(\hat{\theta}^2 - \theta^2\right)$ is still positive. We conclude that for $\theta < \hat{\theta}$

$$
U_2'(\theta) - U_1^0(\theta) > 0
$$

Next, consider entrepreneurs with $\theta > \hat{\theta}$

$$
\begin{aligned}
U_1^0(\hat{\theta}) &= \hat{\theta}f\big(k_1^0(\hat{\theta})\big)z_1^0(\hat{\theta}) + x_1^0(\hat{\theta}) \\
&\geq \hat{\theta}\left(f\big(k_1^0(\theta)z_1^0(\theta) + \frac{x_1^0(\theta)}{\theta}\right) \\
&= \frac{\hat{\theta}}{\theta}U_1^0(\theta)
\end{aligned}
$$

where the week inequality in the second line comes from the envelope theorem and the monotonicity of $f\big(k(.)\big)z(.)$: an argument similar to the one describe in more detail for $\theta < \hat{\theta}$.

Since $\hat{\theta} > 0$,

$$\frac{\theta}{\hat{\theta}} U_1^0(\hat{\theta}) \geq U_1^0(\theta)$$

By construction, $U_2'(\hat{\theta}) = U_1^0(\hat{\theta})$. Moreover,

$$\begin{aligned}
\frac{\hat{\theta}}{\theta} U_2'(\theta) &= U_2'(\hat{\theta}) + \frac{\delta}{2}\left(\frac{\theta}{\hat{\theta}}(1-\hat{\theta}^2) - (1-\theta^2)\right) \\
&> U_2'(\hat{\theta}) + \frac{\delta}{2}\left(\theta^2 - \hat{\theta}^2\right) \\
&> U_2'(\hat{\theta})
\end{aligned}$$

We conclude that,

$$U_2'(\theta) \geq \frac{\theta}{\hat{\theta}} U_2'(\hat{\theta}) = \frac{\theta}{\hat{\theta}} U_1^0(\hat{\theta}) \geq U_1^0(\theta)$$

It follows that entrepreneurs with type $\theta > \hat{\theta}$ are better of contract $\mathcal{C}_2'$ than with contract $\mathcal{C}_1^0$. Hence intermediary 2 makes at least half of the profits over the interval $[\hat{\theta}, 1]$, which are strictly positive by claim 3.

So far we have assumed that $x_1^0(\hat{\theta}) > 0$. If $x_1(\hat{\theta}) = 0$, then it must be the case that $\tilde{\theta} < \hat{\theta}$ (by lemma A.1.1).

In this case, intermediary 2 can deviate to the strategy, $\mathcal{C}_2'' = \left\{(k_2''(\theta), x_2''(\theta), z_2''(\theta)) | \theta \in A^0\right\}$, where,

$$k_2''(\theta) = k_1^0(\theta) \qquad z_2''(\theta) = \frac{f\big(k_1^0(\tilde{\theta})\big) z_1^0(\tilde{\theta}) + x_1^0(\tilde{\theta})/\tilde{\theta} + \delta(\theta - \tilde{\theta})}{f\big(k_1^0(\theta)\big)} \qquad x'(\theta) = \frac{\delta}{2}(1-\theta^2)$$

By the same argument as before, when intermediary 2 deviates to $\mathcal{C}_2''$, Every entrepreneur with type $\theta < \tilde{\theta}$ takes a contract from $\mathcal{C}_1^0$, and every type $\theta > \tilde{\theta}$ takes a contract from $\mathcal{C}_2''$.

It is left to show that intermediary 1 was making loses over $[\theta_L, \tilde{\theta}]$ - and because she wouldn't make loses, intermediary 2 steals a strictly profitable fraction of the market-. If this this was not the case, he must be making profits over a positive measure set and we can redefine $\hat{\theta}$ as a point in that interval. $\square$

Proof of proposition 2

Proof. Suppose $\theta_L \geq \theta_P$, then $S(\theta_L) \geq S(\theta_P) = w = U(\theta_L)$ by the definition of θ_L and θ_P. But then the intermediary expects not to loose with the type θ_L and, by claim 3, expects strictly positive profits with all $\theta' \in (\theta_L, 1]$. That implies the intermediary is making profits strictly positive aggregate profits contradicting the zero profit condition.

If types are public, intermediaries must break even with each type, which implies $U(\theta) = S(\theta)$. For those $\theta < \theta_P$, we have $S(\theta) < w$ hence none of them will take any contract. All the rest will accept the contract offered for their type, hence $\theta_L = \theta_P$ and the inefficiency vanishes.

If limited liability is removed, but all other features remain the same, we will show the equilibrium incentive compatible contract schedule has to be $k(\theta) = k^*(\theta)$, $x(\theta) = -R \cdot k^*(\theta)$ and $z(\theta) = \pi$ for (almost) all $\theta \geq \theta_P$.

We start showing that the following strategy profile is indeed an equilibrium:

Each intermediary offers, $\mathcal{C}_i = \big\{(k_i(\theta), x_i(\theta), z_i(\theta)) | \theta \in [0,1]\big\}$, where,

$$k_i(\theta) = k^*(\theta) \qquad\qquad z_i(\theta) = \pi \qquad\qquad x_i(\theta) = -R \cdot k^*(\theta) \tag{A.5}$$

and entrepreneur θ flips a coin before selecting between $(k_1(\theta), x_1(\theta), z_1(\theta))$ and $(k_2(\theta), x_2(\theta), z_2(\theta))$, - but strictly prefers any of the two compared to any $(k_i(\theta'), x_i(\theta'), z_i(\theta'))$ for $\theta' \neq \theta$ -. As $z_i(\theta)$ is constant, $f\big(k^*(\theta)\big) z_i(\theta)$ is (strictly) increasing, hence the contract satisfies the conditions of lemma **??** and $(k_i(\theta), x_i(\theta), z_i(\theta))$ maximizes entrepreneur θ's utility among the available options. Also $U_i(\theta) = S(\theta)$, and by definition on θ_P, type θ takes the contract if and only if $\theta \geq \theta_P$. If intermediary i is offering the above contract, intermediary j's best response cannot yield her any profit, since she would get only those types such that $U_j(\theta) \geq U_i(\theta) = S(\theta)$, and hence offering the same contract is a best response.

To see all equilibrium are payoff equivalent, notice that claims 1 and 2 still must hold. Define $B_2 = A_2 \cup B$, and suppose that in an equilibrium, intermediary 2 offers a contract schedule such that on a positive measure subset of $B_2 \subset [\theta_P, 1]$, $S(\theta) \neq U_2(\theta)$ for all $\theta \in B_2$. Denote the equilibrium strategies by $\mathcal{C}_1, \mathcal{C}_2$ and s. Define the associated contract schedules $\big(k_1(\theta), x_1(\theta), z_1(\theta)\big)$ and $\big(k_2(\theta), x_2(\theta), z_2(\theta)\big)$ as in equation **??** above.

We will show that intermediary 1 can post a contract schedule that strictly increases her profits.

Define the deviation by, $\mathcal{C}_1' = \big\{(k_1'(\theta), x_1'(\theta), z_1'(\theta))|\theta \in \big\}$, where,

$$k_1'(\theta) = 0.5k^*(\theta) + 0.5k_2(\theta) \quad z_1'(\theta) = 0.5\pi + 0.5z_2(\theta) \quad x_1'(\theta) = -0.5R \cdot k^*(\theta) + 0.5x_2(\theta) \tag{A.6}$$

The new contract schedule to be offered by intermediary 1 is just the average of intermediary 2's and the prescribed equilibrium contracts. Note that the new contract skims the cream: there is a threshold level $\overline{\theta}$, such that every $\theta > \overline{\theta}$ takes the contract offers by intermediary 2 (if any), and lower θs take the contract by intermediary 1.

Define $B_2^+ = \{\theta \in B_2 : U_2(\theta) > S(\theta)\}$ and analogously $B_2^- = B_2 \setminus B_2^+$. If B_2^- has positive measure, this average contract makes profits with all $\theta \in B_2^-$ because $S(\theta) > U_1'(\theta) > U_2(\theta)$; it cannot make loses in B_2^+ since there $U_2(\theta) > U_1'(\theta)$; and, it is irrelevant outside B_2. Hence if B_2^- has positive measure, this contract yields positive profits to intermediary 2 contradicting the zero profit condition. If B_2^- has measure zero and B_2^+ has positive measure, this implies that intermediary 2 is not making profit with any type, since B_2^- has measure zero, but then for her to avoid losses it must be the case that no positive measure of her contracts in B_2^+ is taken in equilibrium, which implies that for all θ in a positive measure set $B_1^+ \subset B_2^+$ we must have $U_1(\theta) > U_2(\theta) > S(\theta)$ which implies intermediary 1 is making loses there. Since she cannot make profits with any positive measure of types, because 2 offers at least $S(\theta)$ to everybody but those in B_2^-, she must be making negative profits over all types.

Hence for all $\theta \in [\theta_P, 1]$ it must be the case that $U_1(\theta) = U_2(\theta) = S(\theta)$. The envelope theorem for $S(\theta)$ yields $S'(\theta) = f\big(k^*(\theta)\big)\pi$ which implies $z_1(\theta) = z_1(\theta) = \pi$ for (almost) all those θ. That in turn implies $x_i(\theta) = -R \cdot k^*(\theta)$.

Last, if there is a unique intermediary facing limited liability and adverse selection, the unique equilibrium is $k(\theta) = k^*(\theta)$, $z(\theta) = 0$ and $x(\theta) = w$ and (almost) all entrepreneurs with $\theta \geq \theta_P$ take the contract. In this case an equilibrium should be a contract schedule and a decision rule for entrepreneurs such that the schedule maximizes profit for the intermediary and the decision rule maximizes return to the entrepreneur. In the proposed equilibrium the intermediary extracts all the surplus, hence it is profit maximizing, and entrepreneurs are always indifferent between accepting or rejecting the contract, hence they are also maximizing. Note that in any equilibrium a type θ entrepreneur must take any contract such that her payoff is higher than w and reject anyone otherwise. So potentially the acceptance rule for some θ' could be to accept the contract offering $k^*(\theta)$ with $\theta \neq \theta'$.

To see that can only happen in measure zero sets, suppose there exists an equilibrium where the acceptance rule differs from the one prescribed above in a set $\Theta \in [0, 1]$ with positive measure. For all $\varepsilon > 0$ the contract with $k(\theta) = k^*(\theta)$, $z(\theta) = \frac{\varepsilon}{\theta_P}$ and $x(\theta) = w - \varepsilon$ is such that $\theta_L = \theta_P$ and the profits for the intermediary are the total surplus less $\varepsilon \int_{\theta_P}^1 \big(1 - G(\theta)\big) d\theta$, this implies that in any equilibrium profits should be equal to the total surplus, otherwise the intermediary could do better with the contract above for some ε.

But to have profits equal to the total aggregate surplus, almost all $\theta \in [\theta_P.1]$ must accept the contract and almost all $\theta < \theta_P$ must reject it. Hence the result follows. $\square$

Proof of claim 6

Proof. A lump sum tax does not change the intermediaries optimal decisions. Then the contract offered in equilibrium is still characterized by proposition 1.

Now the decision rule for entrepreneurs is as follows: they must accept the best contract offered to their type if $U(\theta) - \phi > w$ and reject if the inequality is reversed. In this sense the tax can also be seen as a subsidy on w. Hence, at θ_L we must have $U(\theta_L) = w + \phi$. Then, the zero profit condition for θ_L is:

$$\int_{\theta_L}^{1} \left\{\theta f\big(k^*(\theta)\big)\pi - Rk^*(\theta)\right\} dG(\theta) - \int_{\theta_L}^{1} \theta \frac{(w+\phi)}{\theta_L} dG(\theta) = 0$$

Which has a unique solution θ_L since its derivative with respect to θ_L is strictly positive. By definition of ϕ, $\theta_L = \theta_P$ solves the equation and is thus the only solution. $\square$

Proof of claim 7

Proof. Suppose intermediaries compete with contracts of the form $\big(k, x, (1-\tau)z\big)$. Then the contract offered in equilibrium is still characterized by proposition 1 but the θ_L now has to solve:

$$\int_{\theta_L}^{1} \left\{\theta f\big(k^*(\theta)\big)\pi - Rk^*(\theta)\right\} dG(\theta) - \int_{\theta_L}^{1} \theta \frac{w}{(1-\tau)\theta_L} dG(\theta) = 0,$$

because intermediaries have to pay $(1-\tau)^{-1}U(\theta)$ if she is supposed to deliver $U(\theta)$ net of taxes to the entrepreneur. Plugging the value for τ^* we obtain:

$$\int_{\theta_L}^{1} \left\{\theta f\big(k^*(\theta)\big)\pi - Rk^*(\theta)\right\} dG(\theta) - \int_{\theta_L}^{1} \theta \frac{(w+\phi)}{\theta_L} dG(\theta) = 0$$

Which by claim 6 has a unique solution $\theta_L = \theta_P$. □

A.1.2. Asset Holdings Proofs

The proofs for claims A.1.1 to 2 are analogous to those for the zero assets case. Fixing the asset level a, the IC constraint across types θ is the same, but now the limited liability restriction is $\underline{x}_i(\theta, a) \geq -Ra$. Recall the expected utility of a type θ entrepreneur with assets a under contract i be $U_i(\theta, a) + Ra$. Then $\underline{x}_i(\theta, a)$ is decreasing in θ and in a competitive equilibrium $k_i(\theta, a) = k^*(\theta)$ for (almost) all (θ, a) and $U_i(\theta, a)$ is nondecreasing in θ

Proof of claim 8

Proof. If $k^*(\theta) \leq a$, the entrepreneur can self-finance the project up to the optimal scale and save the rest, with expected profit $\theta F\big(k^*(\theta)\big)\pi + R\big(a - k^*(\theta)\big)$, which is the best possible outcome for the entrepreneur outside the credit market. If $k^*(\theta) > a$ the project can still be started but at a scale lower than the optimal, concavity of F implies that the best option, conditional on starting the project, is to invest all the assets in it, which yields $\theta F(a)\pi$. In any case that has to be compared with the option of not doing the project and getting the return on the assets. □

Proof of proposition 3

Claim 3 carries over. The proof has to change a little.

Proof. Note that because of the outside option, $\hat{\theta} f(a)\pi < U_i(\hat{\theta}, a) + Ra$ must hold. Also for all θ we have $S(\theta) \geq \theta f(a)\pi - Ra$ with equality only for some θ_a such that $k^*(\theta_a) = a$ Note that no profits can be made with types $\theta < \theta_a$ as those can fully self finance, hence $\hat{\theta} > \theta_a$. Suppose the expected profits for some $\theta' > \hat{\theta} > \theta_a$ such that $k_i(\theta') = k^*(\theta')$ are not positive, that is $U_i(\theta') \geq S(\theta')$. $S(\theta)$ is still convex. Remembering that $S(\theta_a) =$

$\theta_a f(a)\pi - Ra \leq U_i(\theta_a, a)$, by incentive compatibility:

$$
\begin{aligned}
U_i(\hat{\theta}, a) &\geq \hat{\theta} f\big(k^*(\theta')\big) z_i(\theta', a) + \underline{x}_i(\theta', a) = \frac{\hat{\theta}}{\theta'} U_i(\theta', a) + \frac{\theta' - \hat{\theta}}{\theta'} \underline{x}_i(\theta', a) \\
U_i(\hat{\theta}, a) &\geq \frac{\hat{\theta} - \theta_a}{\theta' - \theta_a} U_i(\theta', a) + \frac{\theta' - \hat{\theta}}{\theta' - \theta_a} \left[\frac{\theta_a}{\theta'} U_i(\theta', a) + \frac{\theta' - \theta_a}{\theta'} \underline{x}_i(\theta', a) \right] \\
U_i(\hat{\theta}, a) &\geq \frac{\hat{\theta} - \theta_a}{\theta' - \theta_a} U_i(\theta', a) + \frac{\theta' - \hat{\theta}}{\theta' - \theta_a} \left[\frac{\theta_a}{\theta'} (\theta' f(a)\pi - Ra) - \frac{\theta' - \theta_a}{\theta'} Ra \right] \\
U_i(\hat{\theta}, a) &\geq \frac{\hat{\theta} - \theta_a}{\theta' - \theta_a} U_i(\theta', a) + \frac{\theta' - \hat{\theta}}{\theta' - \theta_a} \left[\theta_a f(a)\pi - \frac{\theta_a}{\theta'} Ra - \frac{\theta' - \theta_a}{\theta'} Ra \right] \\
U_i(\hat{\theta}, a) &\geq \frac{\hat{\theta} - \theta_a}{\theta' - \theta_a} S(\theta') + \frac{\theta' - \hat{\theta}}{\theta' - \theta_a} \left[\theta_a f(a)\pi - Ra \right] \\
U_i(\hat{\theta}, a) &\geq \frac{\hat{\theta} - \theta_a}{\theta' - \theta_a} S(\theta') + \frac{\theta' - \hat{\theta}}{\theta' - \theta_a} S(\theta_a) > S(\hat{\theta})
\end{aligned}
$$

contradicting the expected profits for $\hat{\theta}$. As $k_i(\theta') = k^*(\theta')$ for almost all θ' the lemma follows. $\square$

Using similar arguments as in the proof of claim 5:

Lemma A.1.2. *Let $\mathcal{C}(\theta) = \big(k(\theta, a), x(\theta, a), z(\theta, a)\big)$ be an incentive compatible contract schedule with assets a. For any $\hat{\theta} \in [0, 1]$, define the contract schedule $\mathcal{C}_{\hat{\theta}}$ by,*

$$
\mathcal{C}_{\hat{\theta}}(\theta) = \left(k(\theta, a), -Ra, \frac{f\big(k(\hat{\theta}, a)\big) z(\hat{\theta}, a) + x(\hat{\theta}, a)/\hat{\theta}}{f\big(k(\theta)\big)} \right)
$$

Then,

- *$\mathcal{C}_{\hat{\theta}}$ is incentive compatible.*
- *For all $\theta < \hat{\theta}$, $U(\theta, \mathcal{C}) \geq U(\theta, \mathcal{C}_{\hat{\theta}})$*
- *$U(\theta, \mathcal{C}) > U(\theta, \mathcal{C}_{\hat{\theta}})$ if and only if $x(\hat{\theta}) > -Ra$ or $f\big(k(\hat{\theta}, a)\big) z(\hat{\theta}, a) > f\big(k(\theta, a)\big) z(\theta, a)$ for some $\theta < \hat{\theta}$.*

The proof of this lemma is analogous to the old one. Moreover, once the two lemmas have been established, the proof of claim 5 carries over, completing the proof of proposition 3.

Then the IC constraint across assets is $U_i(\theta, a)$ is nondecreasing in a (recall an entrepreneur cannot lie and give more collateral than what he has).

Linear loss minimization

We define a procedure to minimize losses for a given set of profitable contracts. An (expected) profitable contract is one where $U_i(\theta, a) < S(\theta)$ where $S(\cdot)$ is the surplus function.

For a fixed asset level a, the promised extra utility $U_i(\theta, a)$ curve of an IC contract with limited liability can only cross the curve $S(\theta)$ once[19]. Define $\theta_e(a)$ as the solution of $U_i(\theta, a) = S(\theta)$. Also define $\widehat{fz}(a) = \inf\Big\{f\big(k^*(\theta)\big)z(\theta, a) : \theta > \theta_e(a)\Big\}$, this is the maximum slope the contract can have at $\theta_e(a)$.

For all $\theta < \theta_e(a)$ the intermediary is making losses. Hence it is in his best interest to reduce $U_i(\theta, a)$ for all such theta. However the IC constraint over θ implies that $U_i(\theta, a) \geq S\big(\theta_e(a)\big) - \big(\theta_e(a) - \theta\big)\widehat{fz}(a)$ and the intermediary will try to set that. Unfortunately, there is also the IC constraint over a, that requires $U_i(\theta, a)$ to be nondecreasing in a. Therefore the loss minimization given some contract $\theta_e(a)$ and $\widehat{fz}(a)$ is achieved by setting:

$$U_i(\theta, a) = \sup\Big\{S\big(\theta_e(\hat{a})\big) - \big(\theta_e(\hat{a}) - \theta\big)\widehat{fz}(\hat{a}) : 0 \leq \hat{a} \leq a\Big\}$$

Zero Profits

Zero profits will still happen but the proof needs to be modified. Below are the steps.

1. As before if one intermediary is making profits π, the other can always set a new contract as the max of the two current offered contracts. After that she need to

[19]There may be another cut, but that has to be below or at the outside option curve $O(\theta, a)$

increase her offers by an ε small enough such that she would take over all the market and profits fall just slightly.

2. Because of continuity, the issue is going to be the new entrants. First we need that for every δ there exists some ε such that the change in $\theta_L(a)$ is less than δ for a lot of a (meaning we can make the mass of those not bounded as small as wanted) when we give ε more to everybody.

   (a) The new entrants are determined by the slope of the contract on $\theta_L(a)$. We need a bound for that slope from below that works for every a and is strictly positive. Unfortunately that is not generally possible. However we can find a bound for a lot of values, such that the measure of those not bounded is very small relative to the profits. In what follows we assume the contract, after the max process has been optimized with the linear loss minimization described above.

      i. There is no need to worry about those a such that $\theta_L(a) = 0$. In fact we can disregard all asset levels such that $\theta_L(a) < \delta$. Note that for the remaining a's $\widehat{fz}(a) > 0$.

      ii. First we deal with those a with positive marginal mass. There can be only countable many of those. Let the combined mass of all those a_i be $M_1 \leq 1$. Then there exits finite number of those N_1 such that $\sum_{i=1}^{N_1} m(a_i) > M_1 - \frac{\pi}{5w}$. From now on we will forget about all other a_i with positive mass and take a loss no greater than 0.2π with all those types with capital a_i for $i > N_1$. Now take $0 < f_1 = \min\{\widehat{fz}(a_i) : 0 \leq i \leq N_1\}$

      iii. Take any $\bar{a}$ such that $S\big(\theta_e(\bar{a})\big) > w$ Look at the slope of the contract at $\theta_L(\bar{a})$ after the linear minimization process. We claim that for all $a' > \bar{a}$ the slope of the contract at $\theta_L(a')$ is greater or equal than the minimum between the slope of the contract at $\theta_L(\bar{a})$ and:

$$\frac{S\big(\theta_e(\bar{a})\big) - w}{\theta_e(\bar{a}) - \delta}.$$

Call that minimum $f_2(\bar{a})$. If $\theta_L(a') = \theta_L(\bar{a})$ then, by IC over the assets, the slope of the contract at $\theta_L(a')$ has to be greater or equal than the one at $\theta_L(\bar{a})$. If $\theta_L(a') < \theta_L(\bar{a})$ then, by the envelope theorem, the slope at $\theta_L(a')$ will be $\widehat{fz}(\hat{a})$ for some $\bar{a} < \hat{a} \leq a'$, but all those are bounded below by the slope of line that goes through the points (δ, w) and $\Big(\theta_e(\bar{a}), S\big(\theta_e(\bar{a})\big)\Big)$.

Now, if $S\big(\theta_e(0)\big) > w$ set $f_2 = f_2(0)$ and then the slopes are all (but a small measure) bounded by $\min\{f_1, f_2\}$. If not, since $\theta_e(a)$ is nondecreasing it is measurable, so we can pick an asset level $\bar{a}$ such that the mass of assets $\hat{a}$ such that $S\big(\theta_e(\hat{a})\big) \in \Big(w, S\big(\theta_e(\bar{a})\big)\Big)$ is less than any positive constant we want.

We are going to allow a bigger loss on those $\hat{a}$ types. For all of them the contract offered will be the same as the one for $\bar{a}$, before the ϵ increase. This would generate a loss less than $S\big(\theta_e(\bar{a})\big)$ per entrepreneur, as the contract for $\bar{a}$ makes expected losses only on those θ types that get offered an utility level less than that. Hence we pick $\bar{a}$ such that the mass of $\hat{a}$ such that $S\big(\theta_e(\hat{a})\big) \in \Big(w, S\big(\theta_e(\bar{a})\big)\Big)$ is less than:

$$\frac{\pi}{5S\big(\theta_e(\bar{a})\big)},$$

and set $f_2 = f_2(\bar{a})$.

iv. Now, we have to deal with those $\hat{a}$ such that $S\big(\theta_e(\hat{a})\big) = w$, let $\theta_P = S^{-1}(w)$ then we are talking about those asset levels such that $\theta_e(a) = \theta_P$ they form an interval (a_0, a_1) which may be closed or open at both ends. Notice for all those the slopes $\widehat{fz}(\hat{a})$ are increasing in $\hat{a}$ because of incentive compatibility w.r.t. $\hat{a}$. Now, if $\inf\{\widehat{fz}(\hat{a})\} > 0$ then set f_3 equal to that infimum. Otherwise, as before, for each $\bar{a}_2$ such that $S\big(\theta_e(\bar{a}_2)\big) = w$ we can raise the contract of all those $a' \in [a_0, \bar{a}_2)$ to $U_i(\theta, \bar{a}_2)$. The loss for that change is the maximum difference between the original contracts and this new one, which is bounded by the difference between the line $u = w$ and the line with slope $\frac{Ra+w}{\theta_P}$. As the marginal mass may be distributed in any way, the bound will be the difference between the value of that linear function at $\theta = 1$ and w. So we pick $\bar{a}_2$ such that the mass in $[a_0, \bar{a}_2)$ is less than $\frac{\pi}{5(U_i(1,\bar{a}_2)-w)}$ and let $f_4 = \widehat{fz}(\bar{a}_2)$.

v. Last, we deal with those asset levels (if any) for which $U_i(\theta_w, a) < w$. Again, there is a $\bar{a}_3$ such that the mass of those $a' > \bar{a}_3$ such that $U_i(\theta_w, a') < w$ is less than $\frac{\pi}{5(U_i(1,\bar{a}_2)-w)}$. Again we will lift all those $\bar{a}_3 < \hat{a} < \bar{a}_2$ to $U_i(\theta, \bar{a}_2)$. The loss for doing that is less than 0.2π and let $f_5 = \widehat{fz}(\bar{a}_3)$

vi. To finish take the minimum of the f_i's. That is the lower bound for the slope and hence define $\varepsilon(\delta) = \delta * \min\{f_i : i \in \{1,2,3,4,5\}\}$. So far we lost 0.4π with the unbounded types.

(b) After that we need that for each μ there exist a δ such that the mass of new entrepreneurs is less than μ given a reduction no larger than δ in $\theta_L(a)$ for all a. Measurability of the contracts imply $U_i(\theta, a)$ and $\theta_L(a)$ are measurable. The new entrants are a subset of $U_i^{-1}\big([w-\varepsilon, w)\big) \subset \big[\theta_L(a)-\delta, \theta_l(a)\big)$. Now we want to bound the mass of the latter intervals. For each a there exists some $\delta(a) > 0$ such that

$$G^-\big(\theta_L(a)|a\big) - G^-\big(\theta_l(a) - \delta(a)|a\big) < 0.5\mu$$

If the lower bound of those $\delta(a)$ is positive that is our δ and we are done. Otherwise we find a bound for all but a measure no bigger than $\frac{\pi}{5w}$, on which we take losses of w with all the potential entrants, so total losses wont exceed 0.2π, for the rest take the lower bound which must be positive.

Proof of claim 10

Proof. Suppose that is not the case. Then there exist some positive measure set $B = B^1 \cup B^2$ such that all types in B^i strictly prefer the contract offered by intermediary i than the one offered by j. Notice that the point wise average of two IC contract schedules with $k_i(\theta, a) = k^*(\theta)$ is an IC contract schedule. This is because utility is linear in $z_i(\theta, a)$ and $\underline{x}_i(\theta, a)$ so the IC constraint will be inherited from the IC of the two original schedules. We will show this average IC schedule has to increase profits for one of the intermediaries. For intermediary i, the newly proposed contract reduces the utility offered to those in B^i because that is averaged with the one offered by j to those types, which was assumed to be strictly lower. However types in B^i still prefer intermediary i contract. Outside B nothing changes since offered utilities were equal there, in B^j types still prefer intermediary j. Hence intermediary i by offering the average schedule reduces the surplus given away in the set B^i keeping all other sources of income fixed. If B^i has a positive measure then her profits strictly increase. Hence the measure of B^I must be zero which implies the measure of B has to be zero.

□

Proof of claim 11

Proof. Sup $\theta < \theta' < \theta''$ are such that θ and θ'' are in $A(a|b)$. Let $x(\theta', b) = -Rc$ for some $c \leq b$, notice that $P(a)$ is strictly increasing in a as it is the slope of the line passing through $(0, -Ra)$ and $\Big(\theta_e(a), S\big(\theta_e(a)\big)\Big)$. Now ,as $\theta' \notin A(a|b)$ then $\theta' P(c) - Rc \geq \theta' P(a) - Ra$. If $c > a$, then $P(c) > P(a)$ and as $\theta'' > \theta'$ it follows that $\theta'' P(c) - Rc > \theta'' P(a) - Ra$ contradicting $\theta'' \in A(a|b)$, analogously if $c < a$ implies $\theta \notin A(a|b)$ hence $c = a$ and then $\theta' \in A(a|b)$.

If $\theta \in A(a, c)$ for $c > b \geq a$ then,

$$U_i(\theta, c) = \theta P(a) - Ra = \sup_{0 \leq a' \leq c} \theta P(a') - Ra' \geq \sup_{0 \leq a' \leq b} \theta P(a') - Ra' \geq P(a) - Ra,$$

which implies $\theta \in A(a|b)$

$\square$

BIBLIOGRAPHY

M. Arellano and S. Bond. Some tests of specification for panel data: Monte carlo evidence and an application to employment equations. *The review of economic studies*, 58(2): 277–297, 1991.

K. Arrow. Economic welfare and the allocation of resources for invention. In *The rate and direction of inventive activity: Economic and social factors*, pages 609–626. Princeton University Press, 1962.

A. J. Auerbach. Share valuation and corporate equity policy. *Journal of Public Economics*, 11(3):291–305, 1979.

A. J. Auerbach. Taxation and corporate financial policy. *Handbook of public economics*, 3:1251–1292, 2002.

D. Besanko and A. V. Thakor. Collateral and rationing: sorting equilibria in monopolistic and competitive credit markets. *International economic review*, pages 671–689, 1987.

H. Bester. Screening vs. rationing in credit markets with imperfect information. *The American Economic Review*, 75(4):850–855, 1985.

D. F. Bradford. The incidence and allocation effects of a tax on corporate distributions. *Journal of Public Economics*, 15(1):1–22, 1981.

R. Chetty, A. Guren, D. Manoli, and A. Weber. Are micro and macro labor supply elasticities consistent? a review of evidence on the intensive and extensive margins. *The American Economic Review*, 101(3):471–475, 2011.

T. F. Cooley and V. Quadrini. Financial markets and firm dynamics. *American Economic Review*, pages 1286–1310, 2001.

J. F. Gomes. Financing investment. *American Economic Review*, pages 1263–1285, 2001.

F. Gourio and J. Miao. Firm heterogeneity and the long-run effects of dividend tax reform. *American Economic Journal: Macroeconomics*, pages 131–168, 2010.

F. Gourio and J. Miao. Transitional dynamics of dividend and capital gains tax cuts. *Review of Economic Dynamics*, 14(2):368–383, 2011.

J. Greenwood, Z. Hercowitz, and G. W. Huffman. Investment, capacity utilization, and the real business cycle. *The American Economic Review*, pages 402–417, 1988.

K. A. Hassett and R. G. Hubbard. Tax policy and business investment. *Handbook of public economics*, 3:1293–1343, 2002.

C. A. Hennessy and T. M. Whited. Debt dynamics. *The Journal of Finance*, 60(3): 1129–1165, 2005.

C. A. Hennessy and T. M. Whited. How costly is external financing? evidence from a structural estimation. *The Journal of Finance*, 62(4):1705–1745, 2007.

H. A. Hopenhayn. Entry, exit, and firm dynamics in long run equilibrium. *Econometrica: Journal of the Econometric Society*, pages 1127–1150, 1992.

B. Jovanovic and B. Szentes. On the market for venture capital. *Journal of Political Economy*, 121(3):493–527, 2013.

M. A. King. Taxation and the cost of capital. *The Review of Economic Studies*, pages 21–35, 1974.

N. Kiyotaki and J. Moore. Credit cycles. Technical report, National Bureau of Economic Research, 1995.

D. Krueger and A. Ludwig. On the optimal provision of social insurance: Progressive taxation versus education subsidies in general equilibrium. *Journal of Monetary Economics*, 77:72–98, 2016.

Y. Lee. The importance of reallocations in cyclical productivity and returns to scale: evidence from plant-level data. *US Census Bureau Center for Economic Studies Paper No. CES-WP-07-05*, pages 05–09, 2007.

Y. Lee and T. Mukoyama. Productivity and employment dynamics of us manufacturing plants. *Economics Letters*, 136:190–193, 2015.

B. Lester, A. Shourideh, V. Venkateswaran, and A. Zetlin-Jones. Screening and adverse selection in frictional markets. Technical report, National Bureau of Economic Research, 2015.

S. Li, T. M. Whited, and Y. Wu. Collateral, taxes, and leverage. *Review of Financial Studies*, page hhw008, 2016.

A. Martin. A model of collateral, investment, and adverse selection. *Journal of Economic Theory*, 144(4):1572–1588, 2009.

S. B. McKeon. Employee option exercise and equity issuance motives. *Available at SSRN 1920985*, 2015.

E. G. Mendoza, A. Razin, and L. L. Tesar. Effective tax rates in macroeconomics: Cross-country estimates of tax rates on factor incomes and consumption. *Journal of Monetary Economics*, 34(3):297–323, 1994.

H. Milde and J. G. Riley. Signaling in credit markets. *The Quarterly Journal of Economics*, pages 101–129, 1988.

E. C. Prescott. Theory ahead of business-cycle measurement. In *Carnegie-Rochester conference series on public policy*, volume 25, pages 11–44. Elsevier, 1986.

M. Rothschild and J. Stiglitz. *Equilibrium in competitive insurance markets: An essay on the economics of imperfect information*. Springer, 1992.

F. E. Saffie and S. T. Ates. Fewer but better: Sudden stops, firm entry, and financial selection. 2013.

J. A. Schumpeter. *The theory of economic development: An inquiry into profits, capital, credit, interest, and the business cycle*, volume 55. Transaction publishers, 1934.

J. E. Stiglitz and A. Weiss. Credit rationing in markets with imperfect information. *The American economic review*, 71(3):393–410, 1981.

D. Yagan. Capital tax reform and the real economy: The effects of the 2003 dividend tax cut. *The American Economic Review*, 105(12):3531–3563, 2015.

E. Zwick, J. Mahon, et al. Do financial frictions amplify fiscal policy? evidence from business investment stimulus. *Harvard University*, 2014.