Passive Stability And Actuation Of Micro Aerial Vehicles

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Abstract

Micro Aerial Vehicles (MAVs) have increased in popularity in recent years. The most common platform, the quadrotor, has surpassed other MAVs like traditional helicopters and ornithopters in popularity mainly due to their simplicity. Yet the quadrotor design is a century old and was intended to carry people. We set out to design a MAV that is designed specifically to be a MAV, i.e. a vehicle not intended to carry humans as a payload. With this constraint lifted the vehicle can continuously rotate, which would dizzy a human, can sustain larger forces, which would damage a human, or can take advantage of scaling properties, where it may not work at human scale. Furthermore, we aim for simplicity by removing vehicle controllers and reducing the number of actuators, such that the vehicle can be made cost effective, if not disposable.

We begin by studying general equations of motion for hovering MAVs. We search for vehicle configurations that exhibit passive stability, allowing the MAV to operate without a controller or actuators to apply control, ideally a single actuator. The analysis suggests two distinct types of passively stabilized MAVs and we create test vehicles for both.

With simple hovering achieved, we concentrate on controlled motion with an emphasis on doing so without adding actuators. We find we can attain three degree of freedom control using separation of time scales with our actuator via low frequency for control in the vertical direction and high frequency for control in the horizontal plane. We explore techniques for achieving high frequency actuator control, which also allow the compensation of motor defects, specifically cogging torque.

We combine passive stability with the motion control into two vehicles, UNO and Piccolissimo. UNO, the Underactuated-propeller Naturally-stabilized One-motor vehicle, demonstrates the capabilities of simple vehicles by performing maneuvers like conventional quadrotors. Piccolissimo, Italian for "very little", demonstrates the merits of passive stability and single actuator control by being the smallest, self-powered, controllable MAV.

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PASSIVE STABILITY AND ACTUATION OF MICRO AERIAL VEHICLES

Matthew Piccoli

A DISSERTATION

in

Mechanical Engineering and Applied Mechanics

Presented to the Faculties of the University of Pennsylvania

in

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ABSTRACT

PASSIVE STABILITY AND ACTUATION OF MICRO AERIAL VEHICLES

Matthew Piccoli
Mark Yim

Micro Aerial Vehicles (MAVs) have increased in popularity in recent years. The most common platform, the quadrotor, has surpassed other MAVs like traditional helicopters and ornithopters in popularity mainly due to their simplicity. Yet the quadrotor design is a century old and was intended to carry people. We set out to design a MAV that is designed specifically to be a MAV, i.e. a vehicle not intended to carry humans as a payload. With this constraint lifted the vehicle can continuously rotate, which would dizzy a human, can sustain larger forces, which would damage a human, or can take advantage of scaling properties, where it may not work at human scale. Furthermore, we aim for simplicity by removing vehicle controllers and reducing the number of actuators, such that the vehicle can be made cost effective, if not disposable.

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Chapter 1: Introduction

In the last two decades, there has been an increased interest in micro air vehicles (MAVs). With the advent of higher density batteries, more efficient motors, and light-weight, high-strength materials, MAVs have become more feasible. These robotic air vehicles can hover and be arbitrarily positioned in 3D space, which is particularly useful for search and rescue, surveillance and reconnaissance, the exploration of hazardous or unreachable locations, the transportation and delivery of payloads, and toys.

Lowering the cost of MAVs to be a commodity product would allow large numbers to be feasible and enable new classes of applications, particularly uses not otherwise considered. Large scale distributed sensing could be used to aid in applications such as forestry and agriculture health monitoring, airport and shopping mall security, or atmospheric weather observation. MAVs sufficiently low in cost, such that they are disposable, enables sensing in extremely hazardous locations such as smoke detection during firefighting or explosive device detection in the battlefield.

1.1 Motivation

Flying rotorcraft have been around in one form or another for thousands of years. One example is the Chinese Top, which dates back to 400bc [25]. Since then, nearly all rotorcraft fall under just a few categories: conventional (one main rotor and one tail rotor), coaxial (two main rotors aligned axially), tandem (two main rotors with offset yet
parallel axes), intermeshing (two main rotors with non-parallel axes and overlap), and multirotors (multiple main rotors, typically of equal size and parallel axes), where the first three require swashplates or servo tabs.

Historically, the concentration was to decrease power loading of hovering vehicles, create control mechanisms, and increase vehicle stability until the early 1950s when gas turbines became mainstream and sufficient experimentation had taken place [25]. Once the general concepts for helicopter flight were figured out, the focus then shifted to increase the size, payload, and other research and development [17].

Emerging technologies allowed a new research branch to build smaller vehicles. Enabled by new battery chemistries with higher specific power, specifically lithium-ion, the use of electric motors are perhaps the MAVs’ equivalent to helicopters’ gas turbines. Microelectromechanical systems (MEMS) sensors and microcontrollers allow fully fly-by-wire systems, once reserved for military and spacecraft, to be shrunk to less than a square centimeter.

Because of these advances, the United States’ Defense Advanced Research Projects Agency (DARPA) initiated a program for a Micro Air Vehicle (MAV) in 1996. The specifications require a maximum vehicle dimension of 150 mm, can carry a payload of up to 20 g a distance of 10 km at a speed of 10 m s\(^{-1}\) to 20 m s\(^{-1}\), and an endurance of 20 min to 60 min [31]. By the end of the program, AeroVironment’s Black Widow was perhaps the closest to the target with its 150 mm fixed wingspan, 2 km range, and 30 min endurance with a camera as payload [20].

In 2005, DARPA released a second call for a small UAV, this time calling it the Nano Air Vehicle (NAV) program. The new requirements required a maximum dimension of
75 mm, mass under 10 g, payload of 2 g, the ability to fly from hover up to 10 m s\(^{-1}\) at a range of 1 km with an endurance of 20 min to 30 min \[14\]. Again, AeroVironment met the call with their Nano Hummingbird, a flapping wing UAV with a 165 mm wingspan, mass of 19 g, maximum speed of 6.7 m s\(^{-1}\), and a 4 min endurance (with versions up to an 11 min endurance) \[19\]. Even this impressive vehicle was unable to match any of DARPA’s stringent NAV goals, leaving the the door open for future researchers.

Perhaps the key to designing vehicles with these yet unobtainable goals is to create flyers that are specifically MAVs. MAVs are unhindered by human passengers, human control rates, and biological construction. Humans are only capable of sustaining ones of g forces and dizzy from continuous rotation, while machines do not have these limitations. Furthermore, humans have control rates in the tens of Hz, while the simplest of microcontrollers easily operate in the hundreds or thousands of Hz. Can removing these constraints give us higher maneuverability, higher payload, higher efficiency, smaller size, increased robustness, and more?

1.2 Related Work

One cannot simply scale down a full sized UAV to MAV or NAV sizes. Reynolds number, the ratio of inertial forces versus viscous forces in a fluid, and system integration are attributed to vehicle size leveling off to 100 mm within a few years after the MAV project \[41\]. This is most exemplified by AeroVironment’s MAV and NAV contributions having sizes of 150 mm and 165 mm respectively, a 10% increase, despite the roughly 10 year difference between the two projects. Where manned aircraft operate at Reynolds numbers greater than 1,000,000, MAVs are expected to fly below 50,000, and a sharp decrease in
the lift to drag ratio occurs at numbers below 100,000 [31, 41]. Furthermore, attitude
stability is expected to perform as well in MAVs and NAVs as in full size UAVs (if not
better due to their faster dynamics), yet there is a smaller mass, volume, and power
budget for the required sensors and actuators [41].

1.2.1 Low Reynolds Number

Some researchers have tackled the low Reynolds number problem. The Mesicopter, a
NASA sponsored project examined low Reynolds number propeller design and manufac-
turing in hopes of constructing a centimeter-scale quadcopter [23]. A vehicle with low
Reynolds number optimized 25 mm propellers, a maximum dimension of 65.6 mm, and a
weight of 17 g hovered out of ground effect while tethered to a power supply. Another ve-
hicle, meant for constrained testing while tethered, used 15 mm propellers [22]. Although
the maximum dimension of this vehicle is not published, it had a minimum possible max-
imum dimension of 36 mm with no rotor spacing, while a more realistic estimate is 39 mm
when using the 15% rotor separation ratio seen in its larger relative.

Another approach is biologically inspired, mimicking the low Reynolds number flight
of flapping-wing insects. The Harvard Robobee imitates the Diptera fly using piezoelectric
actuators attached to 15 mm to 16 mm wings with a maximum dimension of 35 mm [5].
Currently, the Robobee receives power and control via a tether to a computer.

There is some confusion regarding low Reynolds number flapping, rotary, and trans-
lating flight. It is generally agreed that steady translating low Reynolds number flight
produces less peak lift, while Petricca et al. found contradictory sources claiming either
flapping wings or rotary wings have better performance [41]. Much of the discrepancy
is due to the variation of analytical tools (2D versus 3D) and how they are applied to the wings (unsteady versus quasi-steady). Delayed stall, rotational circulation, and wake capture boost flapping wing lift [6]. Unsteady lift from rotational circulation and wake capture have less effect at larger stroke amplitudes since their contributions occur relatively less often. Delayed stall due to leading edge vortices are quasi-steady, as they do not depend on the change in wing angle, and instead depend on the Rossby number, the ratio of inertial forces versus Coriolis force, a 3D effect not captured in 2D analysis [26]. Since steady rotating wings can mimic the Rossby number of flapping wings, they too can produce stable leading edge vortices, thus delaying stall indefinitely. When comparing rotating wings to flapping wings, both achieve roughly the same amount of quasi-static lift, yet rotating wings do so at down to half of the power, suggesting that rotating wing vehicles should be the vehicle of choice at low Reynolds numbers [26].

1.2.2 Stability

Producing sufficient thrust to get in the air in low Reynolds numbers is one challenge, while staying in the air is an equally daunting task. One approach is to estimate the vehicle’s state and actively orient the thrust in the proper direction and is called active stabilization. Traditional quadcopters use active stabilization by estimating its orientation relative to gravity via a 3 axis accelerometer and 3 axis gyroscope, then dynamically vary the speeds of its four propellers to control its attitude and thus thrust. The smallest commercially available quadrotor, the Cheerson CX-STARS, actively stabilizes using this method, with a maximum dimension of 52 mm, weighs 7.71 g, and has an endurance of 3 min 50 s [27]. On the other hand, passive stabilization occurs when the vehicle’s dynamics and aerodynamics
naturally orient the vehicle’s thrust to maintain hover without any active sensing or control.

Vehicles can achieve passive stability in many ways. Small toys and older helicopters use flybars or paddles to mechanically mix in gyroscopic and aerodynamic forces into blade pitch at a frequency of once per revolution. A variant of this mechanism stabilized the Picoflyer, a coaxial helicopter with a maximum dimension of 72 mm using a 60 mm rotor, mass of 3.3 g, and endurance of less than one minute [46]. The 65.6 mm mesicopter demonstrated passive stability by tilting the propellers inwards at an angle of 15° and locating the center of mass (COM) below the the propellers [23]. A similar effect stabilizes a 75 mm, 9.5 g robotic samara through its coning angle, though this version is only capable of vertical flight [50]. Another method of passive stability used on quadcopters by the authors as well as on the Robobee is placing the aerodynamic center of pressure (COP) over the COM [44 49]. Passive stability without COP over COM can also be achieved by coupling differential lift with gyroscopic precession [43 30].

It is important to note that stability usually only refers to attitude stability. Of the stability methods mentioned only the COP over COM and the differential lift with gyroscopic precession methods exhibit horizontal velocity stability as well as attitude stability. Though one could integrate accelerometer measurements, this not common practice and is only used for short time scales on small vehicles since sensor noise is higher.
1.3 Proposed Solutions

Unfortunately, no one vehicle configuration satisfies every requirement. Although they are frequently weighted for the proposed application, multiple papers have tables examining the pros and cons of each configuration [4, 48]. They tend to agree that axial and coaxial helicopters are more compact and simpler to build, but less maneuverable than conventional helicopters. Quadrotors, on the other hand, are even simpler to build, easier to control, and easier to model, but less efficient in both power consumption and size when compared to conventional helicopters.

An ideal device would combine the strengths of the different types of flyers, while eliminating their existing drawbacks. It follows that new configurations should be explored since the drawbacks mentioned above are known and inherent in those configurations. The rigid hub propeller of the axial and quadrotors are simple to construct and model. The single, large propeller of conventional and axial helicopters is aerodynamically most efficient. Providing anti-torque without using or losing additional power like coaxial helicopters improves efficiency further. If the maximum dimension is the propeller diameter, as with axial or coaxial helicopters, the flyer could not be smaller for a given thrust and efficiency. If pulsing of already existing motors is used for cyclic, like some samara flyers, then the additional mass, complexity, and power consumption from a swashplate and its actuators can be eliminated [53, 54, 40, 55]. If the design is inherently stable as with appropriately designed rotating wing devices, the mass of mechanical stabilizers like flybars or the sensors and computation for an actively stabilized vehicle can be removed.

One device that eliminates many of these drawbacks is the Tracking Device patented
Figure 1.1: Yim’s Tracking Device with passive stability fins (125) and offset mass for steering (126).

by Yim [34]. This proposed vehicle consists of just one motor with two propellers, one attached to the stator and one attached to the rotor, depicted in Figure 1.1. It uses passive stability via fins attached to one of the propellers to ensure its propellers’ spin axis is nominally vertical. A weight is added to partially destabilize the vehicle so that this axis wobbles. As the axis varies direction the controller increases or decreases motor speed, therefore varying thrust, so that the average thrust has a non-zero horizontal component, causing the vehicle to travel horizontally in a helical motion.

We will discuss the equations that govern vehicles like this Tracking Device and their implications. Since this vehicle was only simulated, we will go over some practical restric-
tions of the device. One example is the passive stability fins and the asymmetry between
the two propellers. This is likely to cause an imbalance in angular momentum, which
plays a substantial role in stability. Our passive stability analysis will suggest some de-
sign changes to alleviate this problem. Similarly, pulsing the speed of a rigid propeller at
propeller speeds is not likely to achieve the desired control authority, nor will the vehicle
behave in the expected manner, which our actuation section will explain.

1.4 Organization

To achieve a vehicle like the Tracking Device, we explore the two required categories:
passive stability and actuation, then tie the two categories together at the end as shown
in Figure 1.2. The passive stability will ensure the vehicle will remain in the air, despite
having a single motor and little or no method to control its attitude. First we formally
define and model passive stability in Chapter 2. Then we discuss the implications of the
results from Chapter 2 in Chapter 3 and provide demonstration vehicles and experiments.
We move from passive stability to vehicle actuation in Chapter 4. Some types of actuation
require high frequency motor torque pulsing so we discuss motor control and some other
useful applications of high frequency torque pulsing in Chapter 5. Finally, we describe
two vehicles that combine the concepts from the previous chapters to make a single motor,
passively stabilized, three controlled degree of freedom vehicles in Chapter 6.
Figure 1.2: A high level map of the topics discussed in this thesis. Green is the motivation, partially done by Mark Yim. Blue is theory. Orange is experimental hardware not designed by the author. Yellow is experimental hardware created by the author.
Chapter 2: Passive Stability

We begin by studying passive stability of flying vehicles. Passive stability is widely used in the aviation industry, particularly on light and older aircraft, older helicopters, and even blimps and hot air balloons. Because there are many different types of vehicles that use passive stability, there are also many definitions. Since this thesis concentrates on hovering flyers, we will only discuss those types of stability that are immediately important, namely roll and pitch attitude and horizontal position stability. Roll and pitch attitude stability ensures that the vehicle’s orientation remains in a nominal direction. Horizontal position stability prevents motion perpendicular to the direction of gravity, solving two of the three directions required to hover. For this analysis, we assume the direction parallel to the gravity vector is externally controlled, such as by a pilot, pressure sensor, or distance sensor.

It is important to make the distinction that even among hovering vehicles passive stability’s definition is loosely defined. Some analyses use rate stability, which are solely on the roll and pitch rates and define that stability is when these rates approach zero [50]. This definition is essentially a rate damper with no mention of the vehicle’s orientation. Vehicles with this type of stability could happily remain at an attitude with a large pitch or roll, causing large translational accelerations, velocities, and displacements, which makes this definition inadequate for a hovering vehicle. This type of stability is analogous to a multi-copter in rate mode commanding zero rates.
A more stringent definition of passive stability is attitude stability, which includes the vehicle’s roll and pitch angles. An attitude stable vehicle will return to a nominal orientation after being perturbed. There is still no notion of the vehicle’s velocity, so an attitude stabilized vehicle could have the desired orientation but still translate indefinitely, making even this definition unsuitable for hovering vehicles. This type of stability is similar to a multi-copter in attitude mode with a command that aligns the vehicle’s thrust axis with that of gravity.

To take the definition one step further, we now include the vehicle’s velocity and we’ll call this velocity stability. This form of stability will drive the vehicle’s velocity towards zero. Since hovering can be defined as flying with zero velocity, this form of stability can determine if a vehicle can passively hover, so we will concentrate velocity stability in this stability analysis.

Finally, position stability includes position information and attempts to drive position error to zero. This is useful for vehicles that attempt to hover in a specific location. Because this form of stability is in reference to an absolute location, position sensors like GPS or motion capture, outward facing sensors like cameras or range finders, or other exteroceptive sensors are generally required.

This chapter starts by studying the generalized dynamics of MAVs. We then linearize the dynamics to perform a velocity stability analysis. We narrow in on two designs that promise a good balance of control from electronics and actuators versus natural dynamics. Our goal is to find passive vehicle dynamics that take the larger portion of the burden, so that the power and complexity of the electronics and actuators can be reduced.
2.1 Vehicle Dynamics

To get an initial understanding of the final vehicle's dynamics, we start with a model of a simple vehicle. Figure 2.1 shows a UFO Disc Toy with three frames, the inertial, $I$, the flyer, $F$, and the body, $B$, along with aerodynamic forces, $f_a$, and moments, $\tau_a$. The inertial frame is a world fixed frame. The flyer frame is a fictional frame that is fixed to the vehicle's COM, but neglects yaw. This can also be thought of as the pilot frame. The body frame is fixed to the body and rotates with it. Keeping consistent with various flight dynamics and aerodynamics texts \cite{7,17,38,56}, the $\hat{x}$ axis is positive forwards, the $\hat{y}$ axis is positive right, and the $\hat{z}$ axis is positive down and aligned with gravity. The vehicle's single lifting propeller is mounted such that the thrust vector nominally goes through the COM, lying on the $\hat{z}$ axis in the flyer and body frames. Forces and moments are computed in the flyer frame. We assume the vehicle is rotationally symmetric about the $\hat{z}$ axis where $I_{XX} = I_{YY}$, which is true for standard multi-rotor vehicles, flying discs, and samaras of three or more blades. Furthermore, we assume the $\hat{x}$, $\hat{y}$, and $\hat{z}$ directions are the principle axes of inertia, so:

$$ I = \begin{bmatrix} I_{XX} & 0 & 0 \\ 0 & I_{XX} & 0 \\ 0 & 0 & I_{ZZ} \end{bmatrix} \quad (2.1) $$

We borrow our notation from \cite{56}, where subscripts are points and superscripts are frames. If a value has two superscripts or two subscripts, it is read "(first script) with respect to (second script)". Furthermore, lowercase bold indicates a vector, while uppercase bold indicates a tensor. Non-bold terms are scalars. Bold, capital, non-italicized
terms are frames.

With this notation, Newton’s equation for our example vehicle in the flyer frame is:

$$\dot{\mathbf{v}}^I_F = (\mathbf{f}_a + m\mathbf{g} - m\omega^{FI} \times \mathbf{v}^I_F)/m$$

(2.2)

where $m$ is the vehicle’s mass and $\mathbf{g}$ is gravity. $\mathbf{v}^I_F$ is the velocity of a point at the origin of the flyer frame from the origin of the inertial frame. $\omega^{FI}$ is the angular velocity of the flyer frame with respect to the inertial frame.

Likewise, Euler’s equation for our example vehicle in the flyer frame is:

$$\dot{\omega}^{FI} = I^{-1}(\tau_a - \omega^{FI} \times I\omega^{BI})$$

(2.3)

where $\omega^{BI}$ is the angular velocity of the body frame with respect to the inertial frame. This term is $\omega^{FI}$, but also includes the body’s rotation about the $\hat{z}$ axis.
Throughout this analysis, \( v_F^I = \begin{bmatrix} u & v & w \end{bmatrix}^T \), \( \omega^{BI} = \begin{bmatrix} p & q & r \end{bmatrix}^T \) will denote \( \hat{x}, \hat{y}, \hat{z} \) linear and angular velocities in the flyer frame, \( \phi, \theta, \psi \) are the inertial to body Euler angles, and \( \mathbf{f}_a = \begin{bmatrix} X & Y & Z \end{bmatrix}^T \), \( \tau_a = \begin{bmatrix} L & M & N \end{bmatrix}^T \) are the forces and moments in the flyer frame.

### 2.1.1 Simple Stability Simulation

We can manually integrate this equation for some theoretical vehicle to gain intuition about passive stability. Since our goal is for velocity stability, we want the vehicle to turn away from the direction of motion so its thrust vector aims away from the velocity vector. This will apply a force that opposes the motion and slows down the vehicle. This means that velocities in a given direction need rotations about the perpendicular axis in the horizontal plane.

### 2.1.2 COP Vs. COM

One type of stabilization can occur by introducing moments that directly orient the thrust vector in the desired direction. Intuitively, as a vehicle with high dragplates (i.e. the COP has more negative z value than COM) translates through air, the air pushes on the dragplates causing a moment about the COM which results in the downward thrust turning in the direction of translation and slowing the vehicle down, exhibiting passive velocity stabilization. We will call this phenomenon COP\(>\)COM and was shown by Teoh et al. to stabilize the Robobee platform \[49\].

As an example, assume a theoretical vehicle with this COP\(>\)COM feature is perturbed with a linear velocity \( u \). For stability, we ultimately want a decelerating force in the \(-\hat{x}\) direction, \( X < 0 \). In Newton’s equation, Equation 2.2, \( \mathbf{f}_a \) is likely to have a \( X < 0 \).
component from linear drag. The response in Euler’s equation, Equation 2.3, is trickier. The COP > COM creates a moment, $M$, causing an angular acceleration which double integrates to a $\theta$ and vectors the thrust to suppress the $u$ velocity, giving:

$$\dot{\omega}^F = \begin{bmatrix} 0 & M \\ \frac{M}{I_{xx}} & 0 \end{bmatrix}^T$$

(2.4)

This result is favorable for the Robobee which has no net angular momentum. Angular momentum will cause secondary reactions from gyroscopic effects. In the case of the simple sample vehicle, integrating Equation 2.4 over time will result in a nonzero $q$. The $q$ will yield a precession as seen from Equation 2.3:

$$\dot{\omega}^F = \begin{bmatrix} -qrI_{zz} \\ \frac{M}{I_{xx}} \end{bmatrix}^T$$

(2.5)

Taking another step results in an added nutation about $\hat{y}$.

$$\dot{\omega}^F = \begin{bmatrix} -qrI_{zz} \\ \frac{M + prI_{zz}}{I_{xx}} \end{bmatrix}^T$$

(2.6)

A number of values must be in the proper range for this method to stabilize a vehicle. In this case, with $u > 0$, stability requires $\dot{q} > 0$. First, $M > 0$ must be true. This occurs when the COP is above the COM. Second, $M >> -qrI_{zz}$ to keep precession, and thus nutation, at a minimum.

### 2.1.3 Differential Lift

Another type of stabilization with restoring moments can be seen in the Chinese Top that passively orients its attitude using differential lift. This phenomenon applies moments to spinning propellers. As the device moves away from hover, with some linear velocity through the air, one side of the propeller sees a higher relative wind velocity, called the advancing side, and generates excess lift as a result. Conversely, the opposite, retreating
side, generates less lift. This couple results in a moment about the direction of travel. Gyroscopic effects then result in an angular velocity perpendicular to this moment. The Chinese Top’s attitude changes to slow the horizontal translation, passively stabilizing the vehicle.

Returning to our example, assume the simple sample vehicle now has differential lift, as opposed to COP > COM, and again let’s perturb it with a linear velocity \( u \). For stability, we still want a decelerating force in the \(-\hat{x}\) direction, \( X < 0 \). In Newton’s equation, Equation 2.2, \( f_a \) is still likely to have a \( X < 0 \) component from linear drag. Let’s again step through Euler’s equation, Equation 2.3. For differential lift with the UFO toy in Figure 2.1, the vehicle velocity \( u \) yields a negative moment, \( L < 0 \), exclusively about the \( \hat{x} \) axis as a result of differential lift from the left-handed rotation of the vehicle:

\[
\dot{\omega}_{FI} = \begin{bmatrix} \frac{L}{I_{XX}} & 0 & 0 \end{bmatrix}^T
\]

With one integration step we get the precession:

\[
\dot{\omega}_{FI} = \begin{bmatrix} \frac{L}{I_{XX}} & \frac{pr_{IZZ}}{I_{XX}} & 0 \end{bmatrix}^T
\]  

(2.7)

Another step yields the nutation:

\[
\dot{\omega}_{FI} = \begin{bmatrix} \frac{L-pr_{IZZ}}{I_{XX}} & \frac{pr_{IZZ}}{I_{XX}} & 0 \end{bmatrix}^T
\]

(2.8)

For this method to stabilize the sample vehicle with an initial \( u > 0 \), we again need \( \dot{q} > 0 \). This requires the signs of \( p \) and \( r \) to be the same, \( \text{sgn}(p) = \text{sgn}(r) \).

The simplified example suggests two distinct passively stable vehicle categories. The first, COP > COM, case stabilizes itself when the net angular momentum is low and no net differential lift is produced. Larger angular momentum about \( \hat{z} \) increases the destabilizing
effects from the precession and nutation from the COP>COM moments. Vehicles with an even number of symmetric contra-rotating propellers that in general cancel their rotational inertia, such as coaxial and multi-copters, fit this category, as well as propellerless vehicles like ornithopters. A toy that exhibits this behavior is the Air Hogs Skywinder Stunt Rocket [29]. Unfortunately, multiple propellers implies multiple motors or complex transmissions, which adds complexity and cost.

The second category recommends a nonzero $r$ and differential lift. In addition, it is better if the COP and COM are coincident in this case, since a large angular momentum coupled with a COP>COM will result in large destabilizing terms. Vehicles that fall under this category are vehicles with odd number rigid-hub rotors, like traditional helicopters, samaras, vehicles with asymmetric rotors, and the Chinese Top. A toy that stabilizes using this method is the Air Hogs Vectron Wave [30].

So far, we have discussed two categories of passively stabilized vehicles. Related, are their unstable counterparts. If the COP<COM, the flyer will turn into the wind, which is unstable, but could be used for a vehicle that passively rejects wind gusts. On the other hand, the differential lift vehicles could generate differential lift in the opposite direction, also turning into the wind.

2.2 Linearized Dynamics

Though the previous section gave us some intuition into how we can achieve velocity passive stability, we must prove this is the case. Traditionally this is done through a root locus analysis [17, 38]. We first linearize the equations of motion about a desired flight state, $x_0$, in our case hover, giving us a state transition matrix $A$. Subsequent states
are found by integrating the equation $\dot{x} = Ax$, which takes the place of the non-linear Equations 2.2 and 2.3.

Considering only the velocity and attitude terms in the state vector and ignoring rotations and translations in the Z direction, the state vector of the vehicle can be reduced from 18 to 6 terms: $\begin{bmatrix} u & v & p & q & \phi & \theta \end{bmatrix}^T$. To simplify notation, the linearized partial derivatives are rewritten as the force caused by the subscripted velocity normalized by mass or inertia, for example, $\frac{\partial X}{m \partial u} = X_u$ and $\frac{\partial L}{I_{XX} \partial p} = L_p$. The linearized equations of motion become:

$$\begin{bmatrix}
\dot{u} \\
\dot{v} \\
\dot{p} \\
\dot{q} \\
\dot{\phi} \\
\dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
X_u & X_v & X_p & X_q & 0 & -g \\
Y_u & Y_v & Y_p & Y_q & g & 0 \\
L_u & L_v & L_p & L_q & 0 & 0 \\
M_u & M_v & M_p & M_q & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
u \\
v \\
p \\
q \\
\phi \\
\theta
\end{bmatrix}$$

(2.10)

The linear forces perpendicular to the linear motion, $X_v$ and $Y_u$, should all cancel for symmetric vehicles, so we will ignore them. Similarly, linear drag caused by rotating the flyer is sufficiently small, so we can ignore $X_p, X_q, Y_p$, and $Y_q$ as well.

Symmetry around the vertical axis allows the combination of the remaining partial derivatives from the Jacobian. For convenience we rename the derivatives and describe them:

- $a = X_u = Y_v$
  - is the drag force $\parallel$ to $v$ and is always negative
- $b = L_u = M_v$
  - is the differential lift moment $\parallel$ to $v$
\( c = L_v = -M_u \)

is the COP > COM moment \( \perp \) to \( v \)

\( d = L_p = M_q \)

is the drag moment \( \parallel \) to \( \omega \) and is always negative

\( e = L_q = -M_p \)

is the gyroscopic precession \( \perp \) to \( \omega \)

The state transition matrix is now:

\[
\begin{bmatrix}
\dot{u}
\dot{v}
\dot{p}
\dot{q}
\dot{\phi}
\dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
a & 0 & 0 & 0 & 0 & -g \\
0 & a & 0 & 0 & g & 0 \\
b & c & d & e & 0 & 0 \\
-c & b & -e & d & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
u \\
v \\
p \\
q \\
\phi \\
\theta \\
\end{bmatrix}
\]

(2.11)

A standard stability analysis would begin by taking the eigenvalues of the \( A \) matrix.

To do so, we use the characteristic equation \( \det(A - \lambda I_6) = 0 \), where \( I_6 \) is a 6x6 identity matrix, giving:

\[
0 = s^6 - 2(a + d)s^5 + (a^2 + 4ad + d^2 + e^2)s^4 \\
- 2(a^2d + ad^2 + ae^2 + cg)s^3 \\
+ (a^2(d^2 + e^2) + 2g(ca + cd - be))s^2 \\
+ 2ag(be - cd)s + g^2(b^2 + c^2)
\]

(2.12)

If Equation 2.12 has roots with all negative real components the vehicle is stable.

Unfortunately, finding an analytical solution to the polynomial is quite difficult. Furthermore, we can not find a numerical solution since we have not yet settled on a vehicle design. Fortunately, the Routh-Hurwitz Stability Criterion can analytically solve for the boundaries of stability, and for our characteristic polynomial this can yield useful insight.
2.2.1 Routh-Hurwitz Stability Criterion

The Routh-Hurwitz Stability Criterion analytically finds the bounds where the roots of the characteristic polynomial cross over the imaginary axis. The criterion is a necessary and sufficient condition for stability by constraining the polynomial coefficients to remain on a single side of these bounds. Though this criterion is useful for determining if an LTI system is stable, it is not able to accurately determine how stable.

To execute the criterion we group coefficients of the characteristic polynomial such that Equation 2.12 has the form:

\[ 0 = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0 \]  

(2.13)

We apply the coefficients to the bounds determined by the criterion. Though these bounds are easily derivable for polynomials of all orders, the derivation is widely known, so we will only discuss the bounds for the pertinent polynomials. A simple example is of a second order system with a polynomial of the form \( a_2 s^2 + a_1 s + a_0 = 0 \), and states that stability is assured if and only if all \( a_i > 0 \). Higher order polynomials have these constraints as well as others.

Executing the Routh-Hurwitz criteria on our sixth order polynomial in Equation 2.12 gives stability constraints on the partial derivatives. One trivial result from the \( a_5 > 0 \) bound is that \( a \) and \( d \) summed is negative. These terms are simply drag, indicating that the drag forces and moments express themselves in the opposite direction from motion and are always negative.

Another simple result from the \( a_1 > 0 \) constraint is that \( cd > be \), since we already know \( a \) is negative and \( g \) is positive. The first term calls for COP\( > \)COM, since a negative
c is COP>COM and d is always negative. The second term is from differential lift and gyroscopic precession, and suggests they be of opposite sign. This shows the balance of the COP>COM versus differential lift methods.

Another constraint from Routh-Hurwitz commands \( cg > (3ad + a^2 + d^2)(a + d) + de^2 \). The first term on the right side indicates that more drag increases stability. The second term states that if the angular momentum has a large magnitude, COP can be lowered. Both can be thought of as requiring damping. This is of particular interest since it places a second, complementing constraint on \( c \). Further constraints are possible to find; however, their complexity increases greatly from the previous examples.

2.2.2 Aerodynamic With Angular Momentum

We can simplify our equations of motion, state transition matrix, and Routh-Hurwitz analysis if we specifically wish to passively stabilize a vehicle that has angular momentum. In this case, let’s assume we design the COP=COM so that \( c = 0 \). The state transition matrix is now:

\[
\begin{bmatrix}
\dot{u} \\
\dot{v} \\
\dot{p} \\
\dot{q} \\
\dot{\phi} \\
\dot{\theta}
\end{bmatrix}
= 
\begin{bmatrix}
a & 0 & 0 & 0 & -g \\
0 & a & 0 & 0 & g \\
b & 0 & d & e & 0 \\
0 & b & -e & d & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
u \\
v \\
p \\
q \\
\phi \\
\theta
\end{bmatrix}
\]  

(2.14)
and the characteristic polynomial is:

\[ 0 = s^6 - 2(a + d)s^5 + (a^2 + 4ad + d^2 + e^2)s^4 \]  
(2.15)

\[ -2(a^2d + ad^2 + ae^2)s^3 \]  
(2.16)

\[ + (a^2(d^2 + e^2) - 2gbe)s^2 \]  
(2.17)

\[ + 2agbes + g^2b^2 \]  
(2.18)

The Routh-Hurwitz stability criteria now only gives two useful constraints. The first is that \(a_5 > 0\) again requires \(a + d < 0\), and is expected since drag still behaves similarly. The second is that \(a_1 > 0\) now requires \(be < 0\). This is, indeed, a familiar result, and becomes more apparent when Equation 2.3 is written with the linearized partial derivatives. Remembering that \(c = 0\) and \(\tau\) is strictly aerodynamic forces, we have:

\[
\dot{\omega}^{FI} \equiv \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = I^{-1}(\tau - \omega^{FI} \times I\omega^{BI}) = \begin{bmatrix} bu + dp \\ bv + dq \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{I_{ZZ}q}{I_{XX}} \\ -\frac{I_{ZZ}p}{I_{XX}} \\ 0 \end{bmatrix} \quad (2.19)
\]

with \(bu + dp = L\), \(v = 0\) and \(q = 0\), as is the case in our simple simulation example in Section 2.1.3, this equation looks identical to Equation 2.9. Furthermore, a close examination reveals that:

\[
e \equiv L_q = \frac{\partial L}{I_{XX} \partial q} = -\frac{I_{ZZ}r}{I_{XX}} = -\frac{\partial M}{I_{XX} \partial p} = -M_p \quad (2.20)
\]

Knowing \(e \propto -r\) and \(p \propto \int b\), if we must have \(be < 0\) and thus \(\text{sgn}(b) \neq \text{sgn}(e)\), then \(\text{sgn}(p) = \text{sgn}(r)\), which was our conclusion in Section 2.1.3. Continuing with Equation
\[
\begin{bmatrix}
\dot{p} \\
\dot{q} \\
\dot{r}
\end{bmatrix} =
\begin{bmatrix}
bu + dp \\
bv + dq \\
0
\end{bmatrix} -
\begin{bmatrix}
n\frac{I_{ZZ}}{I_{XX}} \\
-n\frac{I_{ZZ}}{I_{XX}} \\
0
\end{bmatrix}
= \begin{bmatrix}
bu + dp + eq \\
0 \\
0
\end{bmatrix}
\]
(2.21)

which expands out to be the two middle rows of Equation 2.14.

In the end, we care about the influence of the differential lift on the stabilizing \( \dot{\omega}^{FI} \). Since the differential lift moment, \( b = \frac{\partial L}{I_{XX} \partial u} \), as well as all other external moments including disturbances, has \( I_{XX} \) in the denominator, which means increasing inertia will decrease \( \dot{\omega}^{FI} \) from external moments. We want the effects from \( bu \) to be significantly larger than any disturbances that the vehicle would experience in flight, thus, \( b \) should be large. However, we do not want the vehicle to actually roll much, so \( I_{ZZ} \) should also be large. On the other hand, if we assume the vehicle is a flat plate, \( I_{ZZ} = 2I_{XX} \), so \( e = -2r \). The only way to directly increase the gyroscopic effects is to spin faster.

Let’s examine how differential lift is made to ensure we have enough to stabilize. The moment caused by a single blade in hover is

\[
\tau = \int_0^R \frac{1}{2} \rho \nabla^2 C_l c d d d d = \int_0^R \frac{1}{2} \rho r^2 d^2 C_l c d d d = \frac{1}{8} \rho C_l c r^2 R^4
\]
(2.22)

where \( R \) is the blade’s radius, \( \rho \) is the air density, \( \nabla \) is the perceived wind velocity on the blade, \( C_l \) is the blade’s coefficient of lift (and is usually a function of \( d \) and angle of attack), \( d \) is the distance from the center of rotation, and \( c \) is the blade’s chord (also usually a function of \( d \)).

When the vehicle is in forward flight at velocity \( u \), the wind velocity must be added.
The perceived wind velocity is \(- \sin(\theta)u\) so the above equation is modified to be

\[
\tau = \int_0^R \frac{1}{2} \rho (rd - \sin(\theta)u)^2 C_l c d d d (2.23)
\]

Now, if we only care about the moments about \(\theta = 0\), and integrate this over \(\theta\) we will get the effective moment across a whole revolution.

\[
\bar{\tau} = \int_0^{2\pi} \sin(\theta) \int_0^R \frac{1}{2} \rho (rd - \sin(\theta)u)^2 C_l c d d d \theta = -\frac{1}{3} \rho C_l c r \pi u R^3 (2.24)
\]

Adding multiple blades to the above equation, we have

\[
\bar{\tau} = -\frac{n}{3} \rho C_l c r \pi u R^3 (2.25)
\]

which states that, if all else is held constant, differential lift is linear with the number of blades, \(n\), the angular rate \(r\), the linear velocity \(u\), and is proportional to blade radius cubed, \(R^3\). Pulling out \(u\) and dividing by \(I_{XX}\) converts this torque to a state transition partial derivative:

\[
b = -\frac{n}{3 I_{XX}} \rho C_l c r \pi R^3 (2.26)
\]

Though the Routh-Hurwitz analysis gives us limits on our Jacobian terms, it is beneficial to know which terms affect which eigenvalues and by how much. To this end, we take the Jacobian terms from the vehicle presented in Section 6.1 and modify each term independently to test that vehicle’s sensitivity to changes.

In Figure 2.2 we vary the linear drag term, \(a\). This only affects the four eigenvalues closest towards the imaginary axis. As suggested from the Routh-Hurwitz analysis, the vehicle goes unstable when \(a = 0\). It appears increasing \(a\) can only help with stability, though practically this would hamper maneuverability which is discussed further in Chapter 4.
Figure 2.2: Varying the linear drag term, $a$, from a simulated vehicle with differential lift and angular momentum. The top plot shows all six eigenvalues, while the bottom plot focuses on the two closest to the imaginary axis with positive imaginary values.
Figure 2.3 adjusts the differential lift sensitivity, $b$. One can see the vehicle goes slightly unstable as $b = 0$, which agrees with the Routh-Hurwitz’s constraint of $be < 0$. The vehicle becomes more unstable as the this constraint more farther from being satisfied. Interestingly, the vehicle also becomes less stable if the magnitude of $b$ is too large and dominates the the angular momentum, indicating there is likely a sweet spot for differential lift.

In this case, the COP versus COM term, $c$, had little effect on stability, as shown in Figure 2.4. This Figure does show that the two closest poles to the imaginary axis do criss cross, indicating that a $c$ of large magnitude in either direction could lead to instability. Interestingly, $c = 0$ is not the most stable condition. This may be due to some COP versus COM moment with gyroscopic precession canceling the direct differential lift, and would be very fortunately if it is true since both are dependent on linear velocity alone.

The angular drag, $d$, is shown in Figure 2.5. Angular drag has a large effect on the eigenvalues that are nominally far from the imaginary axis. As predicted by the Routh-Hurwitz analysis, when $d = 0$ the vehicle becomes marginally stable. Interestingly, reducing the $d$ magnitude improve the stability of the other four eigenvalues. Furthermore, it appears there can be too much angular drag and this constraint may have hidden in the more complicated constraints that we were not able to solve analytically.

Figure 2.6 shows increasing and decreasing angular momentum. As expected more angular momentum is better, both in terms of stability and damping ratio. When the vehicle has no angular momentum, $e = 0$, the vehicle is already unstable, indicating $be < 0$ is not the limiting Routh-Hurwitz constraint for $e$ on this vehicle.

Finally, we want to make sure our assumptions for $L_v = M_u = X_p = X_q = Y_p = Y_q = 0$
Figure 2.3: Varying the differential lift term, $b$, from a simulated vehicle with differential lift and angular momentum. The top plot shows all six eigenvalues, while the bottom plot focuses on the two closest to the imaginary axis with positive imaginary values.
Figure 2.4: Varying the COP vs. COM term, $c$, from a simulated vehicle with differential lift and angular momentum. The top plot shows all six eigenvalues, while the bottom plot focuses on the two closest to the imaginary axis with positive imaginary values.
Figure 2.5: Varying the angular drag term, $d$, from a simulated vehicle with differential lift and angular momentum. The top plot shows all six eigenvalues, while the bottom plot focuses on the two closest to the imaginary axis with positive imaginary values.
Figure 2.6: Varying the angular momentum term, $e$, from a simulated vehicle with differential lift and angular momentum. The top plot shows all six eigenvalues, while the bottom plot focuses on the two closest to the imaginary axis with positive imaginary values.
are justified. Figure 2.7 shows little change for any eigenvalue for any or all assumptions, justifying these assumptions for this vehicle.

For a given configuration the eigenvectors dictate the motion corresponding to each eigenvalue. The UNO V1 vehicle shown in Figures 2.2 to 2.7 has three complex conjugate pairs of eigenvalues. The eigenvectors corresponding to the eigenvalues at about $-8.5 \pm 75i$ are the roll and pitch short period modes, which are damped oscillations in roll or pitch resulting from a roll or pitch respectively. The remaining four eigenvalues correspond to motion dominated by linear translation in one direction and oscillation in the other linear direction. These two complex conjugates have nearly identical forms, but switch axes. Since both have oscillations and are on different axes, the resulting motion from the pair of these complex conjugates are roughly circular trajectories. To a smaller degree these four eigenvectors also contain roll or pitch rates that orient the vehicle away from the direction of motion.

In summary, passively stabilized vehicles with angular momentum require differential lift and a dominating precession. For maximum differential lift, make a vehicle with large, many, and quickly rotating blades. For maximum precession and minimum first order torque effects, make a vehicle with large inertia and quickly rotating blades.

2.2.3 Aerodynamic Without Angular Momentum

We again simplify our equations of motion, linearized state transition matrix, and Routh-Hurwitz analysis, this time for a vehicle without angular momentum. Knowing that $e$ arises from gyroscopic terms, which no longer apply, and $b$ comes from differential lift, let
Figure 2.7: Varying assumptions used in Section 2.2.2 from a simulated vehicle with differential lift and angular momentum. The top plot shows all six eigenvalues, while the bottom plot focuses on the two closest to the imaginary axis with positive imaginary values.
us assume they are zero. The state transition matrix is now:

\[
\begin{bmatrix}
\dot{u} \\
\dot{v} \\
\dot{p} \\
\dot{q} \\
\dot{\phi} \\
\dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
a & 0 & 0 & 0 & -g \\
0 & a & 0 & 0 & g \\
0 & c & d & 0 & 0 \\
-c & 0 & 0 & d & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
u \\
v \\
p \\
q \\
\phi \\
\theta
\end{bmatrix}
\]

(2.27)

We note that now \( u, q, \) and \( \theta \) are dependent on each other, \( v, p, \) and \( \phi \) are dependent on each other, and both sets are independent. We continue by examining linear \( \hat{x} \) and angular \( \hat{y} \) motion, noting that the system behaves identically in the linear \( \hat{y} \) and angular \( \hat{x} \) direction. The resulting linearized state equation becomes:

\[
\begin{bmatrix}
\dot{u} \\
\dot{q} \\
\dot{\phi}
\end{bmatrix} =
\begin{bmatrix}
a & 0 & -g \\
-c & d & 0 \\
0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
u \\
q \\
\phi
\end{bmatrix}
\]

(2.28)

and the determinant is:

\[0 = s^3 - (a + d)s^2 + ads - cg\]

(2.29)

The Ruth-Hurwitz stability criterion gives us three useful constraints again. The first is that \( a_2 > 0 \) and, as before, dictates that drag is in the opposite direction as the motion with \( a + d < 0 \). The \( a_1 > 0 \) constraint reinforces this constraint. The second is that \( a_0 > 0 \), stating we must have \( c < 0 \). This correlates nicely with our simple sample vehicle simulation in Section 2.1.2 and Equation 2.4 when remembering \( c = -M_u \). A \( c < 0 \) gives
us the desired $M_u > 0$. A third constraint, $a_2a_1 > a_3a_0$ says:

$$\frac{a^2d + ad^2}{g} < c$$

and puts a second, complementary constraint on $c$, requiring sufficient damping.
Chapter 3: Passive Stability Experiments

In this chapter we set out to build physical vehicles that achieve passive stability using the methods described in Chapter 2. We identified two classes of vehicles: those that use COP vs. COM for stability, and those that use differential lift and gyroscopic precession. We present them separately, as they contain few similarities.

3.1 COP Vs. COM Vehicle

The generic requirements for this system is a vehicle that can create thrust while hovering with no net angular momentum. In our case, we attach stabilizers, sometimes called dampers or drag sails, to a quadrotor. Typically, one stabilizer is above the COM and a second is below the COM. See Figure 3.1 as a reference. The top stabilizer provides the desired COP > COM moment, which restores the vehicle’s attitude to vertical. The bottom stabilizer is added to increase the effective damping by both increasing linear and angular damping as well as reducing the net COP > COM moment.

If both the top and bottom stabilizer are the same size, shape, and distance from the COM, then the COP = COM, there is no restoring moment (ignoring effects from the vehicle itself), and the stabilizers are purely linear and angular dampers. Net forces from rotating are eliminated when the top and bottom plates are well matched. Although uncommon, a single, well sized top stabilizer can provide both the COP > COM moment and sufficient damping.
3.1.1 Modeling

We build on the linear model from Section 2.2 by modeling the state transition partial derivatives. As described in Section 2.2.3, we only need to model $a$, $c$, and $d$. The below analysis should apply to most vehicles without angular momentum and with dragplates, though some details become specific to vehicles similar to the one in Figure 3.1 as the model is refined after experimental testing.

We define drag as the force felt in the direction of wind and lift in the direction perpendicular to wind. Vehicles operating on pure drag assume that no wind from the thrust producing components of the vehicle blows across the stabilizers [49]. The coefficients generated by this method are significantly smaller than values extracted by test data with our vehicle.
For our vehicle, the inflow from the propellers create wind in the vertical, positive z direction. The vehicle's motion in the world creates the horizontal components of wind. Together, these two sources of wind create angles of attack of less than 10° from the z axis, which falls under both the linear region of the lift slope curve and the small angle approximation. Thus, the lift from this mechanism is linear with u motion and is felt along the x axis. Note that vertical motion also contributes to wind in the vertical direction, where rapid descents can cancel the propeller’s inflow and the aerodynamics fall back to the lower magnitude drag equations. For the remainder of this discussion we assume there is no vehicle motion in the vertical direction. Furthermore, fluid flow through and around rotors is quite complicated and the analysis below is only a guideline.

Momentum theory states that \( T = mg = 2\rho A_p \nu^2 \) where \( T \) is thrust, \( m \) is the vehicle mass, \( g \) is the acceleration from gravity, \( \rho \) is surrounding air density, \( A_p \) is the propeller disc area, and \( \nu \) is the inflow velocity. Solving for \( \nu \) gives us:

\[
\nu = \sqrt{\frac{mg}{2\rho A_p}}
\]  

We assume \( \nu \gg u \) such that \( \nu^2 + u^2 \approx \nu^2 \) and \( \arctan\left(\frac{u}{\nu}\right) \approx \frac{u}{\nu} \). Similarly, we assume \( \nu \gg qd \) where \( d \) is the distance between the center of mass and a stabilizer element. These assumptions hold for a limited flight envelope which is vehicle dependent and is discussed for a test vehicle in Section 3.1.2. We also adjust the inflow according to the distance of the dragplate from the propellers and assume air \( \beta w \) away from the propellers is unaffected by the vehicle, where \( \beta \) was manually tuned to \( \beta = 1.25 \). This gives two
distinct values for inflow:

\[ \nu_{1,2} = \nu(\beta w - |d_2|)/\beta w \]  
\[ \nu_{3,4} = \nu(\beta w - |d_3|)/\beta w \]  

(3.2)  
(3.3)

One should not allow these values to evaluate greater than \( \nu \) or less than 0.

The force generated by the stabilizers is \( F = 1/2 \rho \nu^2 AC_\alpha \) where \( \alpha \) is the angle of attack, \( C_\alpha = 2\pi\alpha \) is the coefficient of lift at a given angle of attack, and \( A \) is the area of the stabilizer element.

With \( \alpha = \frac{\nu}{\beta w} \) and breaking \( A \) into the stabilizer width \( w \) and distance from the COM in the \( \hat{z} \) direction, \( a \) is:

\[ a = \rho w (|d_1 - d_2|\nu_{1,2} + |d_3 - d_4|\nu_{3,4})\pi/m \]  
(3.4)

For flat plate airfoils the aerodynamic center is at the quarter chord, thus \( c \) is:

\[ c = \rho w (|d_1 - d_2| (3/4(d_1 - d_2) + d_2)\nu_{1,2} + |d_3 - d_4| (1/4(d_4 - d_3) + d_3)\nu_{3,4})\pi/I \]  
(3.5)

We use two methods for determining the angular drag. The first also uses the quarter chord as the aerodynamic center of pressure. The key assumption for this method is the variation in wind velocity does not significantly change across the dragplate as the vehicle rotates. This is only reasonable if the dragplate has a small chord relative to the dragplate's distance from the COM. The equation is:

\[ d = -\rho w q (|d_1 - d_2| (3/4(d_1 - d_2) + d_2)^2\nu_{1,2} + |d_3 - d_4| (1/4(d_4 - d_3) + d_3)^2\nu_{3,4})\pi/I \]  
(3.6)

The second method integrates along the chord. The assumption used here is that the pressure along the chord is constant, though this is rarely the case. This method works
better when the chord is large compared to the distance from the COM. The angle of attack is approximated with \( \alpha = \frac{-d_9}{\nu} \), making \( d \):

\[
d = \rho w ((d_1^3 - d_2^3)\nu_{1,2} + (d_3^3 - d_4^3)\nu_{3,4})\pi/(3I)
\]

We use the integral version of computing \( d \) in the next section since we want to design vehicles that are not unnecessarily tall, which means putting the dragplates as close to the COM as possible. Also, for higher fidelity, \( m \) and \( I \) should be a function of \( d_1 \), \( d_2 \), \( d_3 \), and \( d_4 \) as well.

### 3.1.2 Experiments

**Experimental Vehicle Design**

The base of the test vehicle is a standard quadrotor stemming from a low cost design [32]. Although this specific vehicle has an IMU, an Invensense MPU-6050, its information is used for reporting purposes only. Unlike most other quadrotors, there is no active attitude or rate controller running.

Rods are positioned at a distance such that stabilizers strung between them have a 5 mm clearance from the propellers, making the width of the stabilizers 0.135 m shown in Figure 3.1. Despite this, we do not use any configurations that have material near the propellers to ensure that the stabilizers do not collide with the propellers during crashes.
and aggressive maneuvers. The placement also creates a cage, allowing for safe flight in cluttered environments. The stabilizers are 0.0005 in polyester film and cut with a laser cutter or vinyl cutter. We include tabs and slots on the ends of the sheet to make a loop. The design has slots for threading the rods through the stabilizer.

To find the flight envelope that is valid for the assumptions in Section 3.1.1 we set a target mass of 40 g. For that mass and a square duct of side length 0.135 m, \( \nu = 2.90 \text{ m s}^{-1} \). The linear lift coefficient versus angle of attack assumption generally holds until stall, which usually occurs between 10° to 15°. An angle of attack of 10° occurs at a horizontal velocity of 0.52 m s\(^{-1}\), which is 3.9 body lengths per second. The \( \nu^2 + u^2 \approx \nu^2 \) and \( \arctan(\frac{u}{\nu}) \approx \frac{u}{\nu} \) assumptions result in 1.5% and 1.0% error respectively at this velocity, indicating their validity for this vehicle.

**Vehicle Testing**

We test numerous stabilizer configurations to verify that our analysis emulates the real world. Each configuration is placed on the ground in the center of a 3 m long by 3 m wide by 4 m tall room. A Vicon \([51]\) motion capture system tracks the vehicle with a precision of 0.05 mm at up to 375 Hz \([34]\). A position and yaw controller runs off-board on a PC at 100 Hz, which sends motor voltage commands to the vehicle. This differs from a traditional quadrotor where the position controller sends a desired vehicle attitude, and an inner attitude controller on-board the vehicle attempts to achieve it. Instead, we rely on the stabilizers to replace the inner attitude control loop.

Through position control, the quadrotor takes off and climbs to over 1.5 m from the ground. The x, y, and yaw controllers are then switched off, leaving only the z controller,
causing all four motors to receive the same voltage commands. This condition emulates the math derived in Section 3.1. Natural air currents perturb the vehicle.

We build nine configurations picked from the stability analysis to test the model validity at different points in the design space. The design parameters for each vehicle are listed in Table 3.1. The predicted eigenvalues for these variants are shown in Figure 3.3.

Designs (a), (b), and (c) are from the same family of stabilizers, all with a 0.3 m rod, \( d_3 = 0.05 \) m, and \( d_4 = 0.14 \) m. Essentially, we are trading between the sizes of the top stabilizer and the gap between the top stabilizer and the COM. Vehicle (a) has the lowest predicted damping ratio of this family and is exploring the practical limits of low damping ratios. Variant (b) is predicted to have the least stability while still remaining a stable configuration. Configuration (c) should have a COP < COM, causing it to immediately fall over.

Like (a) through (c), vehicles (d) through (g) are also in their own family. These have a rod length of 0.3 m, \( d_3 = 0.00 \) m, and \( d_4 = 0.14 \) m. Configuration (e) is predicted to be the most stable (\( \min(\max(\text{real}(\lambda))) \)) vehicle of those with a rod length of 0.3 m. Variant

<table>
<thead>
<tr>
<th>Quad</th>
<th>( m )</th>
<th>( I_{YY} )</th>
<th>( d_1 )</th>
<th>( d_2 )</th>
<th>( d_3 )</th>
<th>( d_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>39.6 g</td>
<td>1110 g cm²</td>
<td>-0.16</td>
<td>-0.05</td>
<td>0.05</td>
<td>0.14</td>
</tr>
<tr>
<td>(b)</td>
<td>39.2 g</td>
<td>1070 g cm²</td>
<td>-0.16</td>
<td>-0.09</td>
<td>0.05</td>
<td>0.14</td>
</tr>
<tr>
<td>(c)</td>
<td>39.0 g</td>
<td>1050 g cm²</td>
<td>-0.16</td>
<td>-0.11</td>
<td>0.05</td>
<td>0.14</td>
</tr>
<tr>
<td>(d)</td>
<td>40.0 g</td>
<td>1120 g cm²</td>
<td>-0.16</td>
<td>-0.06</td>
<td>0.00</td>
<td>0.14</td>
</tr>
<tr>
<td>(e)</td>
<td>40.1 g</td>
<td>1130 g cm²</td>
<td>-0.16</td>
<td>-0.05</td>
<td>0.00</td>
<td>0.14</td>
</tr>
<tr>
<td>(f)</td>
<td>39.8 g</td>
<td>1100 g cm²</td>
<td>-0.16</td>
<td>-0.85</td>
<td>0.00</td>
<td>0.14</td>
</tr>
<tr>
<td>(g)</td>
<td>39.7 g</td>
<td>1090 g cm²</td>
<td>-0.16</td>
<td>-0.95</td>
<td>0.00</td>
<td>0.14</td>
</tr>
<tr>
<td>(h)</td>
<td>37.4 g</td>
<td>620 g cm²</td>
<td>-0.11</td>
<td>-0.065</td>
<td>0.02</td>
<td>0.09</td>
</tr>
<tr>
<td>(i)</td>
<td>37.2 g</td>
<td>611 g cm²</td>
<td>-0.10</td>
<td>-0.05</td>
<td>0.05</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Table 3.1: Tested vehicle configurations
(d) explores the trade-off between stability and increased damping ratio. Vehicle (f) is predicted to be slightly stable, while vehicle (g) is solidly unstable.

In general, it is desirable to use less material. Configuration (h) is predicted to be the most stable vehicle with a rod length of 0.2 m (using a model that does not include the inflow adjustments), reducing the material used by a third. Variant (i) is a follow-up vehicle discussed in Section 3.1.3.
Experimental Results

There are four easily identifiable cases of stability. The first is that the vehicle is stable and over-damped. This is characterized by a slow response and no oscillations. The eigenvalues of these vehicles are all negative real with no imaginary parts. We will label this case as $\lambda < 0$, $\zeta > 1$. We group critically damped vehicles in this category as cursory examination cannot discern the difference.

In the second case the vehicle is stable and under-damped, having a faster response, but also overshoots and oscillates. Their eigenvalues have negative real components, but also imaginary components. These are labeled $\lambda < 0$, $\zeta < 1$.

In the third case, the vehicle has a COP $<$ COM and the vehicle is unstable. The COP $<$ COM ($M_u < 0$) causes the vehicle to turn toward the direction of motion, and is the result of the bottom stabilizer dominating. These vehicles have positive real eigenvalues with no imaginary parts. Their labels are $\lambda > 0$, $\zeta > 1$, even though damping ratios are not typically used in unstable systems.

Finally, the fourth case is when the vehicle has insufficient damping and is unstable. Here, the COP $>$ COM moment is too strong and the vehicle over-corrects, causing increasing oscillations. The eigenvalues are positive real with imaginary components. They are labeled as $\lambda > 0$, $\zeta < 1$.

Time series of some of the test flights are provided in Figures 3.4 to 3.7. Actual values are those reported by a Vicon motion capture system. Desired values are the positions commanded by the position controller. In the beginning 2 s to 4 s of each time series the vehicle is flown to between 1.5 m to 2.5 m under full position control. When the x, y, and
Figure 3.4: Configuration a: stable but very under-damped.

Figure 3.5: Configuration b: stable and lightly under-damped.
Figure 3.6: Configuration c: unstable and fails the $M_u > 0$.

Figure 3.7: Configuration d: stable, lightly under-damped, and has the desired response.
yaw controllers are switched off, the desired positions and the actual positions are the same.

Configuration (a) in Figure 3.4 chosen for exploring low damping ratios, behaves as expected. Both the x and y directions have 1.5 s oscillations which are very lightly damped, and close to the 1.16 s predicted by the eigenvalues. In gusty conditions, the light amount of damping may not be able to keep the vehicle upright.

Vehicle (b) in Figure 3.5 is less damped than expected. It is predicted to be over-damped, but is actually lightly under-damped with an oscillation period of just under 2 s. This region is extremely sensitive to configuration variations. A vehicle is just a 5 mm difference fits the behavior of Vehicle (b).

Variant (c) in Figure 3.6 shows the x and y velocities growing to the limits of the room with no oscillations. This is characteristic of COP < COM. Interestingly, the position controller is sufficient to stabilize this vehicle during climb, indicating that it is only slightly unstable as predicted.

Configuration (d) in Figure 3.7 chosen for its fast and lightly under-damped response time, behaves as expected. The 2.25 s oscillations are heavily damped and are very close to the predicted period of 1.76 s. The horizontal velocities do not grow beyond 0.35 m s$^{-1}$.

3.1.3 Discussion

Theory Verification

The majority of experiments are consistent with theory. Eight of the nine configurations behave as expected, and the one that did not presents behaves like a vehicle with just 5 mm less top stabilizer. When the vehicles are predicted to be under-damped, their oscillation
periods are similar, yet all are larger than predicted. This is potentially explained by
the lack of an $X_q$ term in the model or inaccuracies in the inertial model. This is an
improvement on the model presented in [44].

Since lift and not drag is believed to be the dominating aerodynamic force, and the
aerodynamic center of lift on flat plates is at the quarter chord (as opposed to half chord
in the case of drag), a vehicle with symmetric stabilizers should have a COP > COM. We
use the follow-up vehicle, (i), shown in Figure 3.2i to test this idea. This configuration is
indeed stable, but has a large bounded linear velocity.

Another observation is that when the controllers turn off, all of the vehicles move in
the negative $x$ and positive $y$ directions. In fact, a close look at Figure 3.6 shows that the
vehicle even changes direction when the controllers turn off. This is consistent with either
an unbalanced vehicle in either thrust or mass distribution, which favors one side of the
vehicle versus another, or the wind in that location of the room is higher than vehicle
(c)’s speed.

The main goal of this analysis is to provide a tool for finding quality configurations
analytically or numerically, not experimentally. Configuration (e) is the result of this
search for our quadrotor. With this set of stabilizers, the vehicle is not only stable without
an attitude controller, but is capable of following trajectories like any other quadrotor.
Furthermore, it is robust to large wind gusts and crashes.

**Vehicle Cost**

One of the main advantages of a passively stabilized MAV is its reduction in cost. In
Table 3.3 we see the cost of the stabilizing components of three actual and one theore-
Table 3.3: Vehicle costs for a run of 1000 in USD

<table>
<thead>
<tr>
<th>What</th>
<th>Crazyflie</th>
<th>Ladybird V1</th>
<th>Passive 1</th>
<th>Passive 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retail $</td>
<td>116.00</td>
<td>89.00</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(\mu c)</td>
<td>32F103CB</td>
<td>XMEGA16D4</td>
<td>32F373CB</td>
<td>ATtiny9</td>
</tr>
<tr>
<td>(\mu c) $</td>
<td>2.82</td>
<td>0.97</td>
<td>2.47</td>
<td>0.39</td>
</tr>
<tr>
<td>Accel</td>
<td>MPU-6050</td>
<td>ITG-3205</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>Accel $</td>
<td>4.62</td>
<td>3.67</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Gyro</td>
<td>MPU-6050</td>
<td>MMA8452Q</td>
<td>None</td>
<td>None</td>
</tr>
<tr>
<td>Gyro $</td>
<td>(4.62)</td>
<td>0.73</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Passive $</td>
<td>-</td>
<td>-</td>
<td>3.54</td>
<td>0.24</td>
</tr>
<tr>
<td>Total</td>
<td>7.44</td>
<td>4.64</td>
<td>6.01</td>
<td>0.63</td>
</tr>
</tbody>
</table>

A physical quadrotor of similar size: the Bitcraze Crazyflie, Walkera Ladybird V1, our passive quadrotor, and a cost optimized passive quadrotor. For a fair comparison, all products were reverse engineered and component costs are listed for production runs of 1000.\(^1\) Not only can we remove the accelerometer and gyroscope from the passive quadrotor, but the microcontroller no longer estimates attitude and controls. A simpler and lower cost microcontroller only reads voltage commands from the radio and outputs them on four PWMs.

The added components are the four rods, assumed to be 0.3 m each, and \(4 \times 0.3 \times 0.135 = 0.162\) m\(^2\) of film. The rods cost 2.62 $/m and film costs 2.34 $/m\(^2\). This leaves the added cost of the passive mechanism to be \(4 \times 0.3 \times 2.62 + 0.162 \times 2.34 = 3.52\), which is on par with the cheapest quadrotors' electronics and without any cost optimization. Replacing the carbon fiber rods with birch wood at 0.23 $/m and the polyester film with polyethylene at 0.09 $/m\(^2\), the cost is merely $0.24. Thus, passive stability can save nearly an order of magnitude on control costs.

\(^1\)Prices from Octopart, McMaster-Carr, and Dragonplate on Feb. 26, 2015


**Efficiency Effects**

To fly, aerial vehicles must support their own weight, so the vehicle’s mass is a critical design constraint. Although we can remove the accelerometer and gyroscope, the mass of the vehicle is not significantly reduced. The base quadrotor weighs 33 g, while the configurations with 0.3 m rods weigh roughly 40 g, which requires 21% more thrust for hover.

Assuming the translational drag on the quadrotor itself remains the same, the added thrust required to move linearly can be derived from \( a = X_u = Y_u \). The horizontal force \( F = mX_u u \). As mentioned in Section 3.1.2, the linear assumptions hold up to 0.52 m/s\(^{-1}\) and perhaps faster depending on the onset of stall. Of the configurations with a rod length of 0.3 m, the average predicted \( \bar{X}_u = -3.98 \), resulting in the added thrust requirement of \( F = 3.98 \times 0.04u = 0.159uN \). For example, the hover thrust of our vehicle is \( mg = 0.04 \times 9.81 = 0.392N \) and the linear drag force moving at 0.5 m/s\(^{-1}\) is \( F = 0.159 \times 0.5 = 0.080N \). So, the required thrust to translate is \( \sqrt{(mg)^2 + F^2} = 0.401N \), a 2.1% increase.

### 3.2 Differential Lift Vehicle

The differential lift based device is minimalistic, composed of just two moving parts attached to a motor. A propeller is attached to the motor’s rotor and a stabilizer is attached to the motor’s stator. There is no swash plate, no anti-torque tail rotor, and only one, instead of four, rotors.

The vehicle is illustrated in Figure 3.8 along with the frame assignments: S attached to the stator and R attached the rotor. The vehicle’s single lifting propeller is mounted
Figure 3.8: Coordinate systems used while computing the equations of motion.

such that the thrust vector nominally goes through the COM, lying on the $\hat{z}$ axis of the collinear rotor and stator frames. Stabilizers, which spin on the same axis, but in opposite direction from the propeller, are used to counter the propeller torque and could provide extra lift. The stabilizers are arranged symmetrically around the propeller axis of rotation so the aerodynamic COP also lies on the $\hat{z}$ axis of the rotor or stator frames. According to the stability analysis, the stabilizers should be located such that the COP is in the same location as the COM.

### 3.2.1 Modeling

It is desirable for the vehicle to be modeled accurately using linear equations of motion. Equations (2.26) and (2.20) which describe the differential lift and gyroscopic precession, are quite accurate under certain conditions. Unfortunately the $a$ and $d$ drag terms for these vehicles are more difficult to model.
Figure 3.9: Blade element velocities, angles, and forces.

To this end, we built a nonlinear simulator to help design vehicles. We can iterate vehicles quickly in the simulator, view their behavior, and determine all of the partial derivatives of the state transition matrix. To do this, we model the propellers and stabilizers using blade element theory, but ignore the bodies. A new frame, $E$ is located along a propeller or stabilizer blade element. The frame $E$ is moved along each blade and the forces, $f_i$, and moments, $\tau_i$, experienced at each position of $E$ are summed to find the aerodynamic forces, $f_a$, and moments, $\tau_a$, felt by the vehicle.

To calculate the amount of lift and drag on the propeller and stabilizers, the relative wind $[\vec{v}_{IE}^E]$ in the propeller frame must first be found. The relative wind on a blade element of a propeller is:

$$
[\vec{v}_{IE}^E] = R_E^F [v_F] + R_E^F [\omega_{SF}] F \times R_E^F [S_{EF}] F + R_E^F [\nu] F
$$

(3.8)

where $R_E^F$ is the rotation matrix from the flyer frame to the propeller element frame. $S_{PF}$ is the displacement vector between the net center of mass to the propeller element. $[\nu]^F = \left[\begin{array}{c} 0 \\ 0 \\ \nu \end{array}\right]^T$ is the induced inflow velocity from Equation 3.1.

The relative wind angle from the tangent is

$$
\gamma = \arctan\left(\frac{[\vec{v}_{IE}^E] \cdot [\hat{z}]^E}{[\vec{v}_{IE}^P] \cdot [\hat{x}]^E}\right)
$$
with $n = 1, 2$. The angle of attack, $\alpha$, is

$$\alpha = \beta_P + \gamma$$

where $\beta_P$ is the blade pitch at the blade element. The dimensionless coefficients of lift, $C_L = f(\alpha, R_e)$, and drag, $C_D = f(\alpha, R_e)$, for a given airfoil are used to compute the lift, $L$, and drag, $D$, using

$$L = \frac{\rho}{2} \| [V^E]_E \cdot \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \|^2 c_P C_L$$

(3.9)

and

$$D = \frac{\rho}{2} \| [V^E]_E \cdot \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \|^2 c_P C_D$$

(3.10)

where $R_e$ is the Reynold’s number and $c_E$ is the chord length at the blade element. With a simple rotation, this becomes to the normal $N$ and tangential $T$ forces with

$$N = L \cos(\gamma) + D \sin(\gamma)$$

and

$$T = -L \sin(\gamma) + D \cos(\gamma)$$

The forces felt by the blade at a distance $d$ from the $\hat{z}$ axis along the blade is then $f_P = \begin{bmatrix} -T & 0 & -N \end{bmatrix}^T$. Therefore, the total aerodynamic force felt by the blade $i$ is

$$[f_i]^F = \left( \int_0^R f_P \, dd \right) R_F^E$$

(3.11)

and the moment is

$$[\tau_i]^F = \left( \int_0^R R_F^E [S_{PF}]^F \times f_P \, dd \right) R_F^E$$

(3.12)

where $R$ is the radius of the propeller. Finally, $f_a = \sum_{i=1}^B f_i$ and $\tau_a = \sum_{i=1}^B \tau_i$ where $B$ is the number of blades and stabilizers.

53
Using Equations 3.11, 3.12, 2.2, and 2.3, a simulator can predict the motion of the vehicle. Furthermore, the state transition matrix, $A$, is generated by perturbing the vehicle in each direction, running the simulator for a single time step, and noting its corresponding accelerations per perturbation. With these virtually generated $A$ matrices, we take the eigenvalues to determine stability.

### 3.2.2 Experiments

**Experimental Vehicle Design**

The stability analysis guided two designs, a tall variant and a wide variant (Figure 3.10), each with variable elements. Some of the variable elements, which are shown in Figure 3.11, include stabilizers of different sizes, location, and number as well as mounting arms for propellers with varying inertial and lift properties.

In the tall version, the stabilizers have no lift and only provide anti-torque, while having very low angular momentum. This allows the propeller’s angular momentum to outweigh that of the stabilizers. Furthermore, the propeller is the only source of differential lift. The stabilizers were created from a laser cut ABS frame and covered in a polyester film. Nine stabilizers were constructed with varying COP distance in 5mm increments along $Z$.

The wide version features mounting points for up to eight stabilizers. Unlike in the tall version, these stabilizers both provide lift and inertia. Beams of varying stabilizer mounting heights were constructed, again out of laser cut ABS, and increment every 5mm with a total of 14 positions. Three types of airfoils are mounted to these frames. One type is seen mounted on the vehicle in Figure 3.10 and another is visible on the top right
A base housing the electronics and motor was created. A custom motor driver, discussed in detail in Chapter 5, communicates to a computer running MATLAB over an AT86RF radio. The STM32F373 microcontroller commutes the E-Flight Park 400 740Kv brushless motor using an AS5145B encoder, records IMU data from an MPU-6050, and transmits data back to the computer. The vehicle does not use the IMU anything other than reporting data. This base is mounted to both the tall and wide versions, and can be seen in Figure 3.10.

**Vehicle Testing**

Throughout the test, three parameters are varied. The most obvious is the height of COP vs COM by use of the interchangeable stabilizers. Another is by varying the inertia through interchanging propellers and adding mass at the ends of the stabilizers. Inertia is again varied by changing from the tall to wide version. Finally, the number of stabilizer blades is varied to confirm that differential lift is the stabilizing moment and increases
The results of a subset of trials is shown in Table 3.4. Versions marked with a U in the Stable column are unstable. Those marked with U* are unstable, but remained aloft for more than 5 seconds before reaching a critical angle. The lone S is the stable vehicle and is the one shown in Figure 3.10. The $I_{ZZ}$ refers to the rotor or stator $I_{ZZ}$, which ever is dominant (i.e. rotor $I_{ZZ}$ for all tall configurations and stator for all wide configurations).

Despite having more configurations, the tall variant is always unstable. Since the stabilizers were designed with a $\pi/2$ radian angle of attack, they create no lift, and thus no differential lift. Thus, the propeller is both the body with the most differential lift and the most angular momentum.

The wide version is designed to generate lift from its stabilizers. Initial versions utilized large Reynolds number airfoils at various angles of attack. Despite their position, shape,
and size, no set has larger differential lift than the propeller’s, which is in the opposite direction. A version that spans the width of the crossbar is created, flown, and shows promise. The body of the version with four stabilizers spins faster than the onboard IMU’s limit of 34.9 rad/s, and therefore the listed angular momentum is <-0.02. This version precesses despite large changes in COP>COM, which is an indication that the differential lift was insufficient, thus more blades are added. With eight blades, and small tweaking of the COM, the vehicle flies stably.

The COP to COM column in Table 3.4 lists the separation distance along Z, where negative values have COP above COM (the COP>COM condition). COM location is estimated from a 3D model and COP location is estimated as the centroid of the projected area on a vertical plane.

Note that the wide configurations all have positive COP to COM which is destabilizing to both COP>COM and differential lift mechanisms. It is likely that the centroid method of COP estimation likely indicates COP lower than it should be. This is because the thin bar and hoop structures on the wide configuration will have little aerodynamic effect and minimal pressure difference, yet will present a projected area that is significantly far from the estimated COP erroneously increasing their effect. Furthermore, shape and speed alter the drag significantly, and could be why the second to last configuration in Table 3.4 has a more favorable COP>COM than the last, stable configuration. The difference between the two configurations is the vertical location of the outer ring, visible in Figure 3.10. The unstable version has it at the bottom, while the stable configuration is with it as depicted. The ring may generate more drag in the unstable configuration (while in the downwash of the propeller) or the stabilizers may create more drag than estimated.
Table 3.4: Stability results of various vehicle configurations

<table>
<thead>
<tr>
<th>Ver</th>
<th>$I_{XX}$</th>
<th>$I_{ZZ}$</th>
<th>$L$</th>
<th>blades</th>
<th>$t_{COP_{COM}}$</th>
<th>Stable</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$kgm^2$</td>
<td>$kgm^2$</td>
<td>Nms</td>
<td>#</td>
<td>$mm$</td>
<td></td>
</tr>
<tr>
<td>tall</td>
<td>3.3E-4</td>
<td>9E-5</td>
<td>-0.03</td>
<td>2</td>
<td>12</td>
<td>U</td>
</tr>
<tr>
<td>tall</td>
<td>3.3E-4</td>
<td>9E-5</td>
<td>-0.03</td>
<td>2</td>
<td>1.4</td>
<td>U</td>
</tr>
<tr>
<td>tall</td>
<td>3.3E-4</td>
<td>9E-5</td>
<td>-0.03</td>
<td>2</td>
<td>-1.2</td>
<td>U</td>
</tr>
<tr>
<td>tall</td>
<td>3.3E-4</td>
<td>9E-5</td>
<td>-0.03</td>
<td>2</td>
<td>-6.4</td>
<td>U</td>
</tr>
<tr>
<td>tall</td>
<td>3.3E-4</td>
<td>9E-5</td>
<td>-0.03</td>
<td>2</td>
<td>-9.0</td>
<td>U*</td>
</tr>
<tr>
<td>tall</td>
<td>3.3E-4</td>
<td>9E-5</td>
<td>-0.03</td>
<td>2</td>
<td>-15</td>
<td>U</td>
</tr>
<tr>
<td>tall</td>
<td>5.2E-4</td>
<td>2.1E-4</td>
<td>0.13</td>
<td>2</td>
<td>-8.4</td>
<td>U</td>
</tr>
<tr>
<td>tall</td>
<td>5.2E-4</td>
<td>2.1E-4</td>
<td>0.13</td>
<td>2</td>
<td>-3.1</td>
<td>U*</td>
</tr>
<tr>
<td>tall</td>
<td>5.2E-4</td>
<td>2.1E-4</td>
<td>0.13</td>
<td>2</td>
<td>2.0</td>
<td>U*</td>
</tr>
<tr>
<td>tall</td>
<td>5.2E-4</td>
<td>2.1E-4</td>
<td>0.13</td>
<td>2</td>
<td>9.9</td>
<td>U*</td>
</tr>
<tr>
<td>wide</td>
<td>1.2E-3</td>
<td>3.5E-3</td>
<td>&lt;0.02</td>
<td>4</td>
<td>12</td>
<td>U</td>
</tr>
<tr>
<td>wide</td>
<td>2E-3</td>
<td>3.5E-3</td>
<td>-0.04</td>
<td>8</td>
<td>8.2</td>
<td>U*</td>
</tr>
<tr>
<td>wide</td>
<td>2.6E-3</td>
<td>4.8E-3</td>
<td>-0.06</td>
<td>8</td>
<td>8.7</td>
<td>U*</td>
</tr>
<tr>
<td>wide</td>
<td>2.6E-3</td>
<td>4.8E-3</td>
<td>-0.06</td>
<td>8</td>
<td>10.3</td>
<td>S</td>
</tr>
</tbody>
</table>

* These unstable versions flew successfully for short periods (seconds)
Regardless, this measurement indicates that the COP did move, and certain locations are more stable than others.

**Discussion**

Vehicle models are able to guide designs, yet ultimately true stability is not proven until a real vehicle flies. The models claimed the virtual machines would be more stable than their physical counterparts. The real world is complicated, with wind and imperfect vehicle manufacturing that continuously add small perturbations. Furthermore, the interaction of aerodynamic bodies is difficult to model. We estimate center of pressure with three methods: silhouettes (which are inaccurate since 3D geometry and depth are not taken into account), the simulator (which does not include bodies and only limited aerodynamic interaction), and using a 6 degree of freedom force/torque sensor (which requires the entire vehicle to be in flying conditions, including wind being blown across it, and saturated our ATI Nano17 sensor). No method is perfect. Each could use further development.

While no tall variants in Table 3.4 are stable, it is worth noting that a tall variant has hovered stably. Unfortunately no numbers are available for this vehicle since it crashed on descent. Tall versions are inherently less stable since $b$ and $e$ are smaller due to the increased $I_{XX}$, decreased $I_{ZZ}$, and less and smaller blades. These push some of the eigenvalues more towards positive reals. While the stability analysis considers many of these vehicles stable, most cannot withstand the constant perturbations from flying in the real world.

It is much simpler to create a stable wide version. When designed with contra rotating propellers (where the stabilizers are propellers) versus small propellers for counter torque
like [15, 37, 50], care must be taken to ensure the stabilizers, not the main propeller, generates more differential lift. This means use a low differential lift propeller and use a high differential lift stabilizer. A simple way to ensure the propeller generates low differential lift is to shroud it in a duct, as is done in [30].

We apply these lessons to follow up vehicles. In particular, we improved simulation accuracy by using a radial inflow distribution and including ducts on all vehicles constructed after the ones found in Figure 3.10.
Chapter 4: Actuation Methods

In Chapters 2 and 3 we explored the design and implementation of vehicles that would passively hover. With throttle control these vehicles are capable of translation in the \( \hat{z} \) direction, while rejecting motion in the \( \hat{x} \) and \( \hat{y} \) directions. In this chapter we seek methods for making these passive flyers controllably translate in the horizontal directions.

Typically, cyclic control of the propeller’s blade pitch or varying the speed or collective blade pitch of offset propellers creates torques or forces. These torques and forces either directly or indirectly cause the vehicle to translate. First we discuss implementations suitable for the proposed vehicles, which are evolutions of the differential lift with gyroscopic precession vehicle presented in Section 3.2.2. Then we analyze their responses. Though the implementations are specific to the proposed vehicles, the analysis is method agnostic.

The basic idea for a fixed pitch single actuator flyer controllable in three dimensions was introduced in [54]. To vary control directions with only one motor we can utilize separation of time scales. This means that low frequency thrust modulations cause \( \hat{z} \) motion, while high frequency modulations cause \( \hat{x} \) and \( \hat{y} \) motion, where the phase of the high frequency modulations determines the direction in the horizontal plane.

We examined multiple methods for creating horizontal motion. The first is angling the propeller at a slight angle with respect to the vehicle body so that the vehicle’s thrust has a horizontal component. The second is offsetting the propeller from the vehicle’s
center of mass so that the vehicle’s thrust is always creating a roll or pitch moment, which was demonstrated on the monospinner [55]. The motor must pulse its thrust at a rate of once per body revolution to achieve any net moments or forces. A third method was developed by Paulos et al., which mimics cyclic control by pulsing the motor at the propeller frequency, not the body frequency [40].

For these methods of applying force and torque, we assume the pulsing frequency of the motor (the body rate for the first two and the propeller rate for the last one) occurs significantly faster than the stability dynamics. We turn to the fastest eigenvalue from our stability analysis for a reference to the minimum speed that pulsing can occur. This eigenvalue is typically the pitch short period oscillation. In practice we have found that a factor of six is sufficient for this assumption.

We begin the analysis by adding control forces to our linear model. Our updated model now has the form $\dot{x} = Ax + Bu$. The control states are $u^T = [fx \ fy \ \tau_x \ \tau_y]$, which are the control forces and torques about the $\hat{x}$ and $\hat{y}$ directions in the flyer frame. As in the previous linear analysis, we assume the vehicle holds altitude by applying a thrust who’s vertical component opposes the gravitational force on the vehicle at all times. To fill in our $B$ matrix, we introduce two control sensitivities, $f$ and $h$. These sensitivities are simply $f = 1/m$ and $h = 1/I_{XX}$. For convenience, we list all of the sensitivities:

- $a =$ linear drag force sensitivity, $\parallel$ to linear motion
- $b =$ differential lift sensitivity, $\parallel$ to linear motion
- $c =$ COP$\neq$COM moment sensitivity, $\perp$ to linear motion
- $d =$ angular drag moment sensitivity, $\parallel$ to rotation
- $e =$ gyroscopic precession sensitivity, $\perp$ to rotation
- $f =$ control force sensitivity, $\parallel$ to force
• \( g \) = thrust force sensitivity, \( \perp \) to thrust angle, 9.81 m/s\(^{-2}\)
• \( h \) = control moment sensitivity, \( \parallel \) to moment

Our linear model expands to:

\[
\begin{bmatrix}
\dot{u} \\
\dot{v} \\
\dot{p} \\
\dot{q} \\
\dot{\phi} \\
\dot{\theta}
\end{bmatrix} = \begin{bmatrix}
a & 0 & 0 & 0 & 0 & -g \\
0 & a & 0 & 0 & g & 0 \\
b & c & d & e & 0 & 0 \\
-c & b & -e & d & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix} \begin{bmatrix}
u \\
v \\
p \\
q \\
\phi \\
\theta
\end{bmatrix} + \begin{bmatrix}
f & 0 & 0 & 0 \\
0 & f & 0 & 0 \\
0 & 0 & h & 0 \\
0 & 0 & 0 & h \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
f_x \\
f_y \\
f_x \\
f_y \\
\tau_x \\
\tau_y
\end{bmatrix}
\tag{4.1}
\]

### 4.1 Control Using Forces

Let us assume the vehicle is capable of applying horizontal forces only. An example of this method is angling the propeller at a slight angle with respect to the vehicle body so that the vehicle’s thrust has a horizontal component. The thrust must go through the vehicle’s COM to prevent any torques. If this motor is pulsed at the body frequency, the resulting net horizontal force can propel the vehicle.

In general, we expect the vehicle to reach a steady state velocity where wind resistance has an equal and opposite force to our control force. To find the steady state velocity, let’s assume the vehicle is only moving in the \( \hat{x} \) direction, so that \( u = k_u \) and \( v = 0 \), and generating no control torques, so that \( \tau_x = \tau_y = 0 \). Furthermore, by definition, any rate states must be constant in steady state, yielding \( p = k_p, q = k_q, \phi = k_\phi, \theta = k_\theta \).
resulting equation of motion is:

\[
\begin{bmatrix}
0 & a & 0 & 0 & 0 & -g \\
0 & 0 & a & 0 & 0 & g \\
0 & b & c & d & e & 0 \\
0 & -c & b & -e & d & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
k_u \\
k_p \\
k_q \\
k_{\phi} \\
k_{\theta}
\end{bmatrix}
= 
\begin{bmatrix}
f & 0 & 0 & 0 \\
0 & f & 0 & 0 \\
0 & 0 & h & 0 \\
0 & 0 & 0 & h \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
f_x \\
f_y \\
f_z \\
f_w
\end{bmatrix}
\]

(4.2)

The fifth and sixth rows require \( k_p = 0 \) and \( k_q = 0 \). With this, the third and forth rows now force \( k_u = 0 \). The remaining first and second rows become \( gk_{\theta} = ff_x \) and \( -gk_{\phi} = ff_y \).

In summary, if a horizontal thrust is used, the passive stability of the vehicle will orient its attitude such that the formerly vertical component of thrust \( (gk_{\theta}) \) cancels the applied horizontal thrust \( (ff_x) \). The vehicle will return to a hover, though slightly displaced from its initial position, and have an attitude with a non-zero pitch. Thus, the angled motor method or any other method that produces solely forces is insufficient for continuous translation.

### 4.2 Control Using Torques

Now let us assume the vehicle is only capable of applying torques, much like a standard multi-copter. In the case of our example vehicle we can leave the propeller vertical, but offset it from the center of mass. A depiction of this type of vehicle shown in Figure 4.1 along with the coordinate systems used in the non-linear analysis.

To repeat the above analysis for an offset motor, we set \( f_x = f_y = 0 \) and now allow
\( \tau_x \) and \( \tau_y \) to be non-zero. Our equation of motion is now:

\[
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\begin{bmatrix}
a & 0 & 0 & 0 & -g \\
0 & a & 0 & 0 & g \\
b & c & d & e & 0 \\
-1c & b & -e & d & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 \\
\end{bmatrix}
\begin{bmatrix}
k_u \\
k_p \\
k_q \\
k_\phi \\
k_\theta \\
\end{bmatrix}
\begin{bmatrix}
f & 0 & 0 & 0 \\
0 & f & 0 & 0 \\
0 & 0 & h & 0 \\
0 & 0 & 0 & h \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

Again, the fifth and sixth rows require \( k_p = k_q = 0 \). Now, from the third row, we have

\[ k_u = -h\tau_x/b \]

(4.4)

giving the non-zero steady state velocity we were looking for. Similarly, the forth row gives

\[ k_u = h\tau_y/c \]

(4.5)

Furthermore, subbing in these results into the first row yields \( k_\theta = -ah\tau_x/(gb) \) or \( k_\theta = ah\tau_y/(gc) \).

Assuming the vehicle initiates its maneuver from hover, we can approximate the initial roll and pitch rate by setting \( u = v = \phi = \theta = 0 \). The remaining non-trivial equations of motion are:

\[
\begin{bmatrix}
\dot{p} \\
\dot{q}
\end{bmatrix}
=
\begin{bmatrix}
d & e \\
-e & d \\
\end{bmatrix}
\begin{bmatrix}
p \\
q
\end{bmatrix}
+
\begin{bmatrix}
h & 0 \\
0 & h
\end{bmatrix}
\begin{bmatrix}
\tau_x \\
\tau_y
\end{bmatrix}
\]

(4.6)

For Equation 4.6 to estimate the initial roll and pitch rates, we must have \( d >> h \) and/or \( e >> h \) such that \( \dot{p} \) and \( \dot{q} \) are driven to zero quickly.

We can see a clear distinction in control between the two types of passive stability discussed in Chapters 2 and 3. For a properly designed COP Vs. COM vehicle with a
non-zero $c$ and with $b = e = 0$, a torque about the $\hat{y}$ axis, $\tau_y$, propels a vehicle along the $\hat{x}$ axis, as is the case with the passive multi-copter shown in Section 3.1.2. Its initial pitch rate is $q_0 = -h\tau_y/d$. On the other hand, the differential lift with angular momentum vehicles from Section 3.2.2 with $c = 0$ and non-zero $b$ and $e$ terms require a torque about the $\hat{x}$ axis, $\tau_x$, to translate along the $\hat{x}$ axis. For either $e >> d$ or small roll and pitch rates, we can ignore the aerodynamic drag term $d$, to find an initial pitch rate of $q_0 = -h\tau_x/e$. Recalling $e = -I_{ZZ}r/I_{XX}$ and $h = 1/I_{XX}$, the above initial pitch rate simplifies to $\tau_x/(I_{ZZ}r)$.

### 4.3 Actuator Authority

Let us now concentrate on creating the control forces and moments. From the previous section, we learned that our vehicles should create torques about the horizontal axes, as opposed to forces, in order to translate in the horizontal directions. If we want to make the differential lift with gyroscopic precession vehicle maneuverable, Equation 4.4 suggests we increase $h$, increase $\tau_x$, or decrease $b$. Since $h = 1/I_{XX}$, increasing $h$ translates to decreasing $I_{XX}$, but this is deceptive, since $b \propto 1/I_{XX}$ as shown in Equation 2.26, thus making any changes to $I_{XX}$ a wash. Another option is to decrease the differential lift effects in $b$, but this has a negative effect on passive stability. The only remaining option is to increase the control torques, $\tau_x$ and $\tau_y$. We come to a similar result for the COP Vs. COM vehicles using Equation 4.5, where increasing $h$ has no benefit, decreasing $c$ could be destabilizing, so increasing control torques is the only option.

From here on, we will concentrate on the differential lift with angular momentum class of vehicles, which use only a single motor. Creating torque with both the rigid
Figure 4.1: Coordinate systems: $\mathbf{I}$ for inertial, $\mathbf{F}$ for flyer, $\mathbf{B}$ for body. $\mathbf{F}$ and $\mathbf{B}$ are located at COM. The forces and torques generated by the motor and propeller are $\mathbf{f}$ and $\mathbf{\tau}$. The offset between the COM at the body frame and the propeller’s force is $\mathbf{o}$. 
propeller and the swashplateless cyclic propeller presented by Paulos et al. require high frequency pulsing of the motor that drives the propellers. Since much of the analysis of the swashplateless cyclic propeller exists in other publications, we target our pulsing discussion towards the fixed pitch propeller styles of actuation, noting that much of the analysis transfers [40].

We hone in on the design shown in Figure 4.1. The vehicle is similar to the vehicle presented in Section 3.2 except the single motor is mounted with a spin axis parallel to the \( \hat{z} \) axis, while offset by a distance \( o \). Normally other mass is redistributed to ensure the COM remains in the center of the vehicle. Because the thrust is offset from the COM the vehicle always generates some roll torque in the body frame, \( B \). If the thrust of the vehicle does not change throughout a full rotation of the body, then all of the torques cancel, yielding \( \tau_x = \tau_y = 0 \) in the flyer frame, \( F \). So long as the vehicle has sufficient inertia and spins quickly, the torque does not change the vehicle’s attitude a significant amount before the vehicle rotates into an orientation to cancel the original torque.

Since our goal is to maximize net torque production, we examine the equations that describe average \( \tau_x \) and \( \tau_y \):

\[
\tau_x = \int_0^{2\pi} -\cos(\psi_b) f_z o d\psi_b = \int_0^{2\pi} -\cos(\psi_b) k_f r_p^2 o d\psi_b
\]

\[(4.7)\]

and

\[
\tau_y = \int_0^{2\pi} \sin(\psi_b) f_z o d\psi_b = \int_0^{2\pi} \sin(\psi_b) k_f r_p^2 o d\psi_b
\]

\[(4.8)\]

where \( \psi_b \) is the angle between the flyer frame, \( F \), and the body frame, \( B \), about the \( \hat{z} \) direction, and where \( o = \begin{bmatrix} 0 & o & 0 \end{bmatrix}^T \). Propeller thrust is commonly modeled as \( f_z = k_f r_p^2 \), where \( f_z \) is the thrust in N, \( r_p \) is the propeller speed in rad/s, and \( k_f \) is the thrust constant.
with units Ns²/rad². The only way for Equations 4.7 and 4.8 to not evaluate to zero is to set \( r_p = f(\psi_b) \), which reinforces our statement that we could create torques by pulsing the propeller’s speed at the body’s rotational frequency.

To improve actuation authority we have two options. We can increase the sensitivity to changes in propeller thrust or we can increase the variation of propeller thrust. Looking at equations 4.7 and 4.8 we see \( \tau_{x,y} \propto k_f \) and \( \tau_{x,y} \propto o \), thus increasing these will increase the sensitivity to thrust changes. Increasing the moment arm, \( o \), is useful up to a point. One must ensure that the torque generated by the non-pulsing thrust at the offset does not overcome the vehicle’s angular momentum and destabilize the vehicle. An example of this is the monospinner, which is not capable of taking off from standstill since it would flip over, but when launched with some spin it has sufficient angular momentum to remain stable [55]. To boost \( k_f \) we can add more blade pitch to the propeller, increase the number of blades, or lengthen the propeller radius. Unfortunately, lengthening the radius and increasing the number of blades will increase inertia and decrease speed change, so it is recommended to use a high pitch propeller.

Since propeller speed is dictated by \( \dot{r}_p = \tau_m/I_p \) where \( \tau_m = f(\psi_b) \) is the motor torque and \( I_p \) is the propeller and motor’s rotor inertia, we should increase the change in \( \tau_m \) or decrease \( I_p \). Reducing the propeller’s radius will reduce \( I_p \), but is also detrimental to speed change sensitivity.

For the following analysis, we assume the motor is electric. This is not a requirement, but we see in Chapter 5 that electric motors are capable of extremely high frequency pulsing, so we concentrate on them. Increasing motor torque is both straightforward, as well as complicated by multiple nuances. In standard hovering flight, the motor’s
voltage is Pulse Width Modulated (PWM) at a high frequency such that the propeller’s speed remains constant and provides a thrust that is equal and opposite to the force from gravity. We can use this PWM value as a feed-forward in our motor torque pulsing controller, except we change the frequency to the body’s rotational frequency. This creates a bang-bang controller with a 0% to 100% PWM square wave that is simple to compute and generates a large resulting torque.

Unfortunately, not all 0% and 100% PWMs are created equal. If the drive circuit for the motor is an open collector or open drain, 0% PWM means no current flows through the motor. The drag of the propeller and the friction of the motor slow the motor down. We model the effective drive voltage as

\[ V_{\text{app}} = V_{\text{sup}} \delta + (1 - \delta)(r_b - r_p)k_e \]  

(4.9)

where \( V_{\text{app}} \) is the voltage applied to the motor, \( V_{\text{sup}} \) is the supply or battery voltage, \( \delta \) is the PWM duty cycle, and \( k_e \) is the motor’s back electromotive force. Essentially, this type of drive circuit can only apply voltages between \( V_{\text{bemf}} \rightarrow V_{\text{sup}} \). If the drive circuit is synchronous, 0% PWM means the lines of the motor are shorted. Drag, friction, and motor breaking slow the motor down, resulting in larger speed changes. We model this simply as

\[ V_{\text{app}} = V_{\text{sup}} \ast \delta \]  

(4.10)

Since 0% is indeed \( V_{\text{app}} = 0 \), synchronous drive circuits can apply 0 \( \rightarrow V_{\text{sup}} \), which is a larger range, and thus will create more variation in motor speed when pulsing. Synchronous rectification has the added benefit of reduced stress on the supply, since current flow does not stop between switching, so the supply does not have to battle inductance to
restart the flow. This also reduces the torque ripple produced by the motor since torque is proportional to current.

To complicate matters further, $V_{sup}$ is not a fixed value at the supply or battery voltage. Transmission wires and batteries themselves have some resistance, $R_b$, and in many cases this plays an important role. We designate the system’s input voltage $V_{in}$, and the input current $I_{in}$, so that

$$V_{sup} = V_{in} - I_{in}R_b$$  \hfill (4.11)

If there are no losses in the drive circuit, $I_{in} = I_m\delta$, where $I_m$ is the current through the motor. This gives us $V_{sup} = V_{in} - I_m\delta R_b$. Meanwhile, if one ignores inductance, the model for a motor is $V_{app} = (r_b - r_p)k_e + I_mR_m$ where $R_m$ is the motor’s resistance. For a synchronous drive circuit, combining the two above equations with Equation 4.10 yields

$$V_{app} = (V_{in} - I_m\delta R_b)\delta = (r_b - r_p)k_e + I_mR_m$$  \hfill (4.12)

Solving for the motor current, $I_m$ gives

$$I_m = \frac{V_{in}\delta - (r_b - r_p)k_e}{R_m + R_b\delta^2}$$  \hfill (4.13)

Since the motor torque is $\tau_m = I_mk_\tau$, where $k_\tau$ is the motor torque constant and in SI units is equal to $k_e$, we can now see the effect that $R_b$ has on motor performance.
Chapter 5: Motor Control

5.1 Background

Electric motors are the most common motor used in medium to small MAVs. Electronic speed controllers (ESCs) regulate the voltage or current going through the motors. Current flows through coils in the motor to produce an electromagnetic field. This magnetic field pushes against a second magnetic field. The second field can either come from permanent magnets or a current induced in a second set of coils. The largest challenge in motor design and control is how to make the input currents change, or commute, as the motor’s position changes. Furthermore, to implement the control from Section 4.3, the ESCs must be able to vary the magnitudes of these currents at the vehicle body’s rotation rate or the vehicle’s propeller rate, as well as do so phase locked to the body or propeller’s angle. First, we’ll talk about the different types of motors and how they are controlled.

DC brushed motors use a mechanical solution to commute. They have contacts attached to the rotor and spin with the rotor. As the rotor spins, the contacts’ positions rotate relative the conducting brushes attached to the stator. The coils in the rotor change current as the contacts change, maintaining the appropriate current in each coil to match the surrounding magnetic field, typically from permanent magnets. This mechanism effectively changes the input direct current into alternating current through the coils [18]. Figure 5.1 shows some commutation schemes for brushed motors, including the two states
of asynchronous and the two states of synchronous drives discussed in Section 4.3.

Brushless motors are simply motors that do not have brushes. They require external electrical commutation. Asynchronous motors, like AC induction motors, spin at different rates than the commutation rate. This difference in speed is called slip and causes the induction. Synchronous motors require that commutation happens at some multiple of the rotor’s rate [18]. Some motors that fall under this category are permanent magnet synchronous motors (PMSM) which includes brushless DC motors (BLDCM) and brushless AC motors (BLACM), and stepper motors [33].

Robot designers often use DC brushed motors in low cost applications, since their control is simple, or high speed applications. On the other hand, robot designers often use PMSMs when motor torque, precision, performance, or reliability is a concern to robot operation. PMSMs exhibit high torque to weight and inertia ratios. Compared to their AC induction counterparts, they are more efficient and simpler to control. Unlike brushed motors, PMSMs do not require brushes to commute and can be made more reliable and cheaper to manufacture [33]. However, the commutation cost and complexity is now pushed to external controllers. Advancements in computation and miniaturization in power electronics are outpacing advancements in electric motors, so PMSMs are becoming even more attractive from a cost standpoint. This caused a proliferation of PMSMs designed for small to medium sized MAVs, and now a wide selection of these motors are available and at low cost. With a sophisticated controller, these PMSMs can be made to perform well in all aspects of motor control.

The method in which commutation occurs is very important in terms of motor control quality. BLACMs are intended to be driven off of AC mains, yielding a sinusoidal
Figure 5.1: Possible driven states of a brushed motor. Asynchronous configurations are on the left and synchronous configurations are on the right.

Figure 5.2: Possible driven states of a three phase motor with 120° on the left, 180° on the right, and null or motor braking in the center. Arrows represent motor lines.
commutation. BLDCMs are meant to be driven from a constant voltage source with a three phase inverter and a trapezoidal waveform. A simplified version of commuting BLDCMs is using rectangular waveforms, also called $120^\circ$ commutation, and is very common among electronic speed controllers meant for high speed rotation only, like those for driving propellers. On the other hand, a tri-half bridge inverter can mimic sinusoidal or trapezoidal waveforms using $180^\circ$ commutation and pulse width modulation (PWM). Using this method, virtually any waveform within the supply limits can be generated, notably one that cancels all of the various types of torque ripple \[24\]. A depiction of inverter states for $120^\circ$ and $180^\circ$ commutation is in Figure 5.2. Four wire bipolar stepper motors can also be driven from a three phase inverter. Unlike the three coils in a three phase motor, the two coils in a bipolar stepper motors are unconnected. If one ties two ends of each coil together, only three lines need to be driven. The waveforms on each phase for the different types of commutation are depicted in Figures 5.3, 5.4, 5.5, and 5.6.

We want to drive the best possible waveform for a given motor with our inverter. Sinusoidal motors have sinusoidal back electro motive force (bEMF) with respect to electrical angle, which indicates that each coil on those motors benefit the most from sinusoidal driving voltages and currents. Generally, Clark and Park transforms are used to convert from three phase voltages to electric angle and the inverses convert in the other direction. After a Clark transform, sinusoidal commutation become circles, trapezoidal commutation become a hexagon, and rectangular commutation are six discrete locations in the center of the straights on the trapezoid in the $\alpha\beta$ vector space. Figure 5.7 gives a visual representation. For a given shape, we want to be able to draw the largest shape possible when constrained to a set voltage or current. For example, if three sinusoidal voltages
Figure 5.3: Sinusoidal PWM values and the resulting phase voltages with respect to neutral.
Figure 5.4: Trapezoidal PWM values and the resulting phase voltages with respect to neutral.
Figure 5.5: Rectangular PWM values and the resulting normalized phase voltages with respect to neutral.
Figure 5.6: Quadrature PWM values and the resulting normalized phase voltages with respect to the shared line.
offset by 120° are applied to a sinusoidal motor, the maximum voltage differential from a line to neutral (the voltage that causes current to flow) has the same amplitude as the input and gives a circle with a radius of 0.75 in vector space. If, however, the largest possible circle was drawn in vector space, it would have a diameter of $\sqrt{3}/2 = 0.87$.

Two fundamental vectors, those in Figure 5.2 are added to make a three phase voltage that is not one of the fundamental vectors. Algorithms for finding the ideal waveforms for trapezoidal, sinusoidal, and quadrature are below. $\theta_e$ is the electrical angle, starting at the positive x axis in Figure 5.7 and going counter clockwise. $s$ is the sector number, from 0 to 6 or 0 to 4, starting and rotating with $\theta$ and increments when passing a fundamental vector. $t_0$ is the time spent on a fundamental null state. $t_1$ is the time for the nearest fundamental vector at a lower $\theta_e$ than the current $\theta_e$. $t_2$ is the time for the nearest fundamental vector at a higher $\theta_e$ than the current $\theta_e$. $a$ is the per unit, 0 to 1, desired amplitude.

**Algorithm 1 Sinusoidal**

\[
\theta_{es} \leftarrow \theta_e - n\pi/3 \\
t_1 \leftarrow a \sin(\pi/3 - \theta_{es}) \\
t_2 \leftarrow a \sin(\theta_{es}) \\
t_0 \leftarrow 1 - t_1 - t_2
\]

**Algorithm 2 Trapezoidal**

\[
\theta_{es} \leftarrow \theta_e - n\pi/3 \\
t \leftarrow \sin(\theta_{es}) / \sin(\theta_{es} + \pi/3) \\
t_1 \leftarrow a(1 - t) \\
t_2 \leftarrow at \\
t_0 \leftarrow 1 - a
\]
**Algorithm 3 Quadrature**

\[ \theta_{es} \leftarrow \theta_e - n\pi/2 \]
\[ t_1 \leftarrow a \sin(\pi/2 - \theta_{es})/\sqrt{2} \]
\[ t_2 \leftarrow a \sin(\theta_{es})/\sqrt{2} \]
\[ t_0 \leftarrow 1 - t_1 - t_2 \]

Figure 5.7: Space vector patterns for three phase drivers with 180° vectors drawn as black arrows and quadrature vectors drawn in yellow
5.2 Anticogging

Since we are ultimately designing this motor driver for high-speed control with high-speed torque fluctuations, we want to ensure that the output torque is precisely what was commanded. There are many reasons why a torque discrepancy could arise. As alluded to in Section 5.1, one source of torque error is if the wrong waveform is commanded for a motor. This is called mutual torque and mutual torque ripple, and is discussed below.

In general, torque ripple is the periodic fluctuation in the motor torque as the output shaft rotates. We desire a torque ripple for steering our passive vehicle, but usually torque ripple is undesirable. This has been recognized as a problem in a variety of robot applications [28] [36] [52]. In the case of our MAV, it could cause unwanted roll or pitch moments. In haptic rendering it is especially troublesome, where direct-drive, high torque motors are desirable and often essential [11]. Transmissions, such as gear boxes, add non-linear torque variations that are difficult to model and compensate, making direct-drive favorable. PMSM’s high torque capability allow them to be used direct-drive and would be ideal if not for torque ripple.

The recent growth in the electric hobby RC market (in particular flying vehicles) has provided a wide range of high torque density, low cost motors. For example, the Exceed RC Rocket 86MA10 motor is 1/8th the price of a Maxon EC45 261501, is smaller and has higher maximum torque, but has a peak to peak torque ripple of 16 N mm, over 440% that of the Maxon. When a motor spins at high speeds, torque ripple creates high frequency speed fluctuations that generate sound and vibration. In haptic rendering, humans are sensitive to periodic motions especially higher frequencies, 40 Hz to 100 Hz. With good
compensation for the ripple, these unwanted vibrations can be reduced, a multi-rotor’s motors make less noise, robotic arms have smooth motion, and haptic textures are rendered more accurately. At very low speeds, torque ripple can cause relatively large speed fluctuations, even causing the motor to stop or move in discrete increments. In servo control, precise positioning is impossible with a traditional proportional or proportional-integral controller due to the ripple's nonlinearity.

Figure 5.8 shows a graph of a sampling of hobby RC brushless motors along with some high performance ones (e.g. a Maxon EC 45, the right-most data point). A measure of torque ripple is shown in the figure as the torque ripple ratio ($TRR$, detailed later in Equation 5.13), which is the peak torque ripple normalized by the motors maximum torque. From this graph, one can see a correlation between lower priced motors and higher $TRR$.

We present an anticogging method to compensate for cogging torque ripple that yields high performance from motors that are a fraction of the cost of inherently low torque ripple motors. By enabling low cost yet high performance motors, this work has the potential to transform the robotics industry by opening consumer markets for high performance robots that are practical and low cost enough for a wide range of useful tasks in the home.

**Types of Torque Ripple**

There are four main types of torque ripple: mutual, reluctance, cogging, and friction.

*Mutual torque* is caused by the mutual interaction of the rotor’s permanent magnets and the stator’s currents [13] [39]. In a PMSM, this is the primary source of torque
Figure 5.8: Nominal and compensated torque ripple ratio vs. price for a set of motors of nominally same size.
production, having the largest DC component[1]. A mismatch of the rotor’s magnetic field and the stator’s current waveform causes dips in the produced torque and contributes to torque ripple. Some sources of mutual torque ripple are driving a BLAC with a trapezoid or a BLDC with a sine wave, phase shifts or delays in the wave, low PWM resolution, and low PWM frequency. In general, if the motor’s ideal shape in Figure 5.7 does not match the driven shape a ripple arises.

Reluctance torque is a result of variance in the stator’s self-inductance due to the rotor magnet saliency. The magnitude of reluctance torque is a function of current [42]. In an ideal BLACM (perfect sinusoidal back EMF and currents), reluctance torque does not exist or only contains a DC component. BLDCMs and non-ideal BLACMs contain reluctance torque ripple.

Cogging torque, also known as detent torque, comes from the rotor’s permanent magnets’ attraction to the salient portions of the stator [47]. It is not current-dependent and cannot be detected by a current sensor. It also has no DC component, and thus only contributes to torque ripple.

Friction torque is not always axially symmetric, since bearings within the motor may contain eccentricities. These torque ripples are distinguishable from cogging torque by their once per mechanical revolution frequency and change in sign upon a change in direction.

---

[1] When referring to DC components or DC signals, we are referring to the non-oscillating offset components in the frequency domain, rather than current.
Anticogging Background

Torque ripple minimization has been a topic of research for over 25 years. Many researchers have proposed finding an optimal current waveform offline using various methods and using a current controlled inverter to play back waveforms [13, 39, 24, 8, 10]. Some use current feedback [2]. Others use speed feedback at low speeds for online estimation [21]. Yet others use both [12]. In practice, speed control loops and estimation have limited success in minimizing torque ripple at higher speeds due to measurement delays, but it has been show that it can be used at low speeds [12].

Cogging torque cannot be detected from current measurements, all forms of torque ripple are seen via added mechanical sensors [12]. While a few prior works do mention the possibility of adding cogging torque suppression to their current based algorithms, none explore the specifics of finding the necessary waveform [13, 8]. Most reduction methods leave the suppression of cogging torque to the motor designers, typically by skewing the stator slots. In place of a speed loop, some use an external force sensor as feedback to compensate for torque ripple at higher frequencies [47]. This method suppresses all forms of torque ripple, but the required sensor could cost more than the motor itself.

Despite the progress in the above solutions, torque ripple minimization is not yet widely used in robotics. Torque ripple minimization is either incomplete when using current sensing methods or is prohibitively expensive when using an external torque sensor. However, it has been shown on robotic arms behind a gearbox that it is possible to measure torque ripple via position sensing by ramping current until an encoder indicates a position change as well as using acceleration feedback to model torque ripple [8]. Unfortu-
nately, observing accelerations may not work at high operational speeds, but monitoring speed and its ripple at low nominal speeds is comparatively simple and is possible with a position encoder. Data gathered at low speeds can be applied at high speeds open loop with notable results [3, 12]. An alternative method to monitoring speed ripple is to monitor position errors during position control. In the case of an unloaded motor during position control, cogging torque and friction are the primary torque perturbations. Therefore, position error under position control can be used to make a cogging torque map and friction torque map.

We show that cogging torque waveforms can be estimated either by mapping speed fluctuations with respect to position or by mapping position error with respect to commanded position. Neither method requires more than an added position sensor which is already required for servo control, and both methods can capture all forms of torque ripple. The methods work with voltage control or current control with little change. One of the methods can be applied to sub-rotation intervals if the motor is constrained to certain positions, as in servo control of a joint. The results can be added to other algorithms to achieve complete torque ripple suppression [13, 8].

5.2.1 Anticogging Proposed Approach

If the torque ripple for a given state of the motor is known, a controller can suppress the ripple simply by commanding a torque that subtracts the ripple torque from the desired torque. Cogging torque is a function of position, so a map of cogging versus position must first be generated. The large number of torque sources, combined with various non-linearities, make the torque ripple map generation challenging. Generating this waveform
map is the crux of torque ripple suppression and can be estimated from a number of
sources, including commanded position error and accelerations. These values must be
measured or converted to units that are useful to the motor driver, typically voltage or
current versus position.

Many variables are used throughout this section and their details are discussed at
their introduction. $\theta$ indicates an angle and $f$ is a frequency. As mentioned in Section
4.3, $V$ is a voltage, $I$ is a current, $\tau$ is a torque, $\delta$ is a duty cycle. These variables can
have one or more subscripts. $m$ indicates a mechanical value, $e$ is an electrical value, $i$
and $j$ are encoder indices, $clk$ is the microcontroller clock, $sup$ is supplied, $des$ is desired,
$app$ is applied, $cmd$ is commanded, $est$ is estimated, $act$ is actual (measured), $pwm$ is
from the pulse width modulation, $RMS$ is the root mean squared value, $min$ is the
minimum value $max$ is the maximum value, $pp$ is peak to peak, $anti$ is anticogged, $nom$
is nominal (without anticogging), $fw$ is forward, and $bw$ is backward. Torque sources are
also subscripted. $cog$ stands for cogging, $st$ is stiction, $res$ is resolution, $frq$ is frequency,
$enc$ is the encoder, $fr$ is friction, and $mtl$ is mutual.

**Assumptions**

This paper makes the following assumptions which are generally true even for hobby grade
motors and ESCs under normal operating conditions:

1. Each motor winding has equal resistance and inductance.
2. A half-H bridge inverter is used to control each phase.
3. The supply voltage and the inverter’s current rating are high enough that the motor
   inductance does not prevent the creation of the desired waveform.
4. Cogging and friction torque ripple are time-invariant.

We use signed scalar values for current and voltage, as if the motor is brushed and the
supply has positive, negative, and ground rails. Negative values are treated as positive values with 180° added to the electrical position, $\theta_e$. The conversion between electrical position and mechanical position, $\theta_m$, is $\theta_e = p\theta_m \mod 2\pi$ where $p$ is the number of magnetic pole pairs, as visualized in Figure 5.9. Control values need to be converted from the desired input quadrature current to phase currents and all feedback needs to be converted from phase currents back to quadrature currents using Clarke and Park transforms. Using these conversions, the motor model can be represented by Eqn. 5.1

$$V_{app} = \dot{\theta}_m K_e + IR + L \frac{dI}{dt} \tag{5.1}$$

where $V_{app}$ is the voltage applied to the motor, $K_e$ is the electromotive force constant, $I$ is the current, $R$ is the motor resistance, and $L$ is the motor inductance.
5.2.2 Waveform Collection

Two methods of collecting the torque ripple waveform were explored. Both exploit the fact that cogging torque is visible from the mechanical state, i.e. position and speed of the rotor.

Position Based

The position based collection of the current or voltage waveform maps the current or voltage required to maintain a given rotor position. This is done according to Algorithm 4 and is outlined below. An ideal waveform is initially assumed, i.e. trapezoidal for a BLDCM or sinusoidal for a BLACM. A proportional position controller with a high gain commands the rotor to positions according to Eqn. 5.2 with encoder positions, $i$, in monotonically increasing order.

\[
\theta_{m,\text{cmd},i} \quad \forall i \in \mathbb{N} \mid \theta_{m,\text{min}} \leq \theta_{m,\text{cmd},i} \leq \theta_{m,\text{max}} \quad (5.2)
\]

For a motor with continuous rotation the encoder position wraps so the minimum encoder position, $\theta_{m,\text{min}}$, equals $\theta_{m,\text{max}}$, the maximum encoder position, and $i$ spans the full encoder count range. At each commanded position $i$, measurements are recorded including: the actual position, $\theta_{m,\text{act},i}$, applied PWM duty cycle in Per Unit (PU or %/100), $\delta_i$, supply voltage, $V_{\text{sup},i}$, and current, $I_i$.

Upon each new command, the motor must come to a complete stop and $dI/dt = 0$ before sampling data so that Equation 5.1 can be simplified to $V_{\text{app}} = IR$. Since the motors do not always go to commanded positions, inconsistencies can occur where

\[
\theta_{m,\text{act},i} = \theta_{m,\text{act},j} \quad \forall i \neq j
\]

90
Figure 5.10: Position method collected data showing duty cycle required to hold position from motor M4 in Table 5.1. This process is described in Section 5.2.6. (a) A full 360° dataset with forward and backward trials and (b) a magnified section showing difference between forward and reverse.

In these cases, the lower magnitude values are discarded.

The above process is repeated commanding $\theta_{m,cmd,i}$ with $i$ monotonically decreasing to find the waveform map in the reverse direction. Figure 5.10 displays these waveforms taken from the experiments outlined in Section 5.2.6. Note that rotating in the reverse direction results in significantly different mapping.

**Algorithm 4** Position Based Waveform Collection

```plaintext
for all $i$ such that $\theta_{m,min} \leq \theta_{m,cmd,i} \leq \theta_{m,max}$ do
    Command $\theta_{m,cmd,i}$
    while $\dot{\theta}_m \neq 0$ do
        Wait
    end while
    $\theta_{m,act,i} \leftarrow \theta_{m,act}$
    $\delta_i \leftarrow \delta$
    $V_{sup,i} \leftarrow V_{sup}$
    $I_i \leftarrow I$
end for
```
**Acceleration Based**

Algorithm 5 is used to map the rotor velocities versus rotor positions under a constant duty cycle. We can then determine the current or voltage waveform based on the rotor accelerations by differentiating the velocities. As in the position based method, an ideal waveform is initially assumed. The motor begins at rest. The PWM duty cycle is incremented for each time step that the motor is stationary. The lowest duty cycle that starts the motor and allows continuous rotation is \( \delta_{\text{max}} \) (not to be confused with the maximum possible \( \delta = 1 \)), and is the lowest duty cycle that overcomes the largest cog, stiction, and deadtime (the period of time in switching when no current flows, detailed in Section 5.2.3). The duty cycle is decremented until the motor stops, then incremented once to find the duty cycle, \( \delta_{\text{min}} \), that runs the motor at the minimum open loop speed. The motor is restarted by commanding \( \delta_{\text{max}} \) until it reaches a steady-state average speed, then \( \delta_{\text{min}} \) is commanded. The test period is long enough to capture the majority of encoder locations \( m \), storing position, \( \theta_m \), and its time derivative, \( \dot{\theta}_m \). \( \dot{\theta}_m \) is sampled by counting encoder counts in a set time period or counting the time period to see a set number of encoder counts. Repeating this process in the opposite direction yields cogging waveforms similar to the original direction (unlike the Position Based method).

**5.2.3 Waveform Analysis**

For cogging compensation, the data collected in Algorithms 4 and 5 must be converted to voltage or current waveforms, \( I_{\text{cog},i} \) or \( V_{\text{cog},i} \). It is not guaranteed that a \( V_{\text{sup},i} \) exists for all \( i \) from the position method, nor a \( \dot{\theta}_{m,j} \) for all \( j \) in the acceleration method. Fast Fourier Transforms (FFTs) and bi-cubic splines have been used for fitting similar voltage,
Algorithm 5 Acceleration Based Waveform Collection

\[ \delta_{\text{max}} \leftarrow 0 \]

\[ \text{while } \dot{\theta}_m \neq 0 \ \forall \theta_{m,i} \ \text{do} \]
  \[ \text{if } \dot{\theta}_m = 0 \ \text{then} \]
  \[ \delta_{\text{max}} \leftarrow \delta_{\text{max}} + \min \Delta \delta \]
  \[ \text{end if} \]
  \[ \text{Command } \delta_{\text{max}} \]
\[ \text{end while} \]

\[ \delta_{\text{min}} \leftarrow \delta_{\text{max}} - \min \Delta \delta \]

\[ \text{while } \dot{\theta}_m \neq 0 \ \text{do} \]
  \[ \text{Wait one revolution} \]
  \[ \delta_{\text{min}} \leftarrow \delta_{\text{min}} - \min \Delta \delta \]
  \[ \text{Command } \delta_{\text{min}} \]
\[ \text{end while} \]

\[ \delta_{\text{min}} \leftarrow \delta_{\text{min}} + \min \Delta \delta \]

\[ \text{Command } \delta_{\text{max}} \]

\[ \text{Wait } \bar{\theta} = \text{steady state} \]

\[ \text{Command } \delta_{\text{min}} \]

\[ \text{Wait } \bar{\theta} = \text{steady state} \]

\[ j \leftarrow 0 \]

\[ \text{while } \text{Rotations} < n \ \text{do} \]
  \[ \theta_{m,j} \leftarrow \theta_m \]
  \[ \dot{\theta}_{m,j} \leftarrow \dot{\theta}_m \]
  \[ t_j \leftarrow t \]
  \[ j = j + 1 \]
\[ \text{end while} \]
current, or velocity waveforms in order to fill gaps in collected data and make the data differentiable \cite{35}. FFTs are of particular interest since most ripple sources are periodic with respect to the mechanical angle \cite{24}. Unfortunately, the raw data cannot be directly fit. Two values, deadtime (explained below) and static friction (also called stiction), complicate matters.

Inverters used to generate waveforms can take one of four states at any given time: high-side transistor conducting, low-side transistor conducting, both conducting, and neither conducting. It is undesirable for both to be conducting, as the inverter will have shoot-through current damaging the circuit. Supply level voltages are produced when only high or only low are conducting, and utilizing PWM between the two an intermediate voltage can be approximated. When neither conduct, the voltage floats or current is sent through flyback diodes. This state is used in 120° commutation on one phase at all times. Deadtime, $\delta_{dt}$, is known as the short period when neither conduct while switching between low and high and vice versa so that it can be guaranteed that both transistors never conduct at the same time\textsuperscript{2}. For accurate open-loop voltage control (via PWM) the controller must account for this deadtime so that the transistors have the desired on-time pulse ratio. This can be accomplished by adding $d_{dt}$ (in PU) to the commanded on-time PWM pulse, $\delta$ (in PU). The effective applied voltage due to deadtime is:

$$V_{\text{app}} = \begin{cases} 
V_{\text{sup}}(\delta - \delta_{dt}) & \text{if } \delta - \delta_{dt} \geq 0, \\
0 & \text{if } \delta - \delta_{dt} < 0.
\end{cases}$$

(5.3)

where $V_{\text{sup}}$ is the DC supply voltage.

\textsuperscript{2}Deadtime refers to only the time that neither transistor is conducting, and not deadzone, the range of mechanical position slop.
If the deadtime is not already known and compensated for by the driver, the data collected using the position based method, Algorithm 4, is sufficient to determine $\delta_{dt}$ using Algorithm 6. All measured duplicates of $\theta_{m,act}$ are consolidated by storing the maximum and minimum commanded duties and currents in $\delta_{max,i}$, $\delta_{min,i}$, $I_{max,i}$, and $I_{min,i}$ respectively. The averages of these are the cogging waveforms, $\delta_{cog,i}$ and $I_{cog,i}$. Half of the maximum difference of the duty cycle across the motor’s position range is the duty cycle required to overcome the maximum deadtime and stiction, denoted $\delta_{dt,st,max}$. All commanded duty cycles with magnitudes below $\delta_{dt,st,max}$ correspond to overcoming both stiction and deadtime and are averaged to get $\delta_{dt,st}$. All commanded duty cycles with magnitudes above $\delta_{dt,st}$ correspond to overcoming stiction only. The mean of these duty cycles, $\bar{\delta}_{st,k}$, is subtracted from $\delta_{dt,st}$ to find the deadtime duty cycle, $\delta_{dt}$. Likewise, the stiction current, denoted $I_{st}$, is the mean of half of the current range at each position.

Stiction manifests as a torque. In the open loop case it can be compensated for with a voltage, $V_{st}$, since at steady currents and no velocity voltage is linear with current, $I_{st}$, and thus is linear with torque. However, because deadtime is a time, it is compensated by modifying the PWM duty cycle on-time by $\delta_{dt}$, in both current and voltage control.

The effects of deadtime and stiction are shown in Figure 5.10b. The average $\pm \delta_{dt,st}$ is shown as horizontal lines. Note that the duty cycles between those lines do not produce motion.

Once deadtime and stiction have been identified, the voltage or current waveforms can be extracted. When using the position method, $I_{cog,i}$ falls out from Algorithm 6 and $V_{cog,i}$ can be found using $\delta_{cog,i}$ as $\delta$ in Equation 5.3

When using the acceleration method, the accelerations are found by taking the time
derivative of the FFT fitted speeds, \( \dot{\theta}_{m,i} = \frac{dF\dot{\theta}_{m,i}}{dt} \). Noting that the rotor inertia, \( J \), is constant, the cogging torque is:

\[ \tau_{cog,i} = J\ddot{\theta}_{m,i} \]  

(5.4)

The motor parameters can be used to find the mapping between \( \tau_{cog,i} \), \( I_{cog,i} \), and \( V_{cog,i} \). If \( J \) is not given, \( \delta_{\text{min}} \) with Equation 5.3 can be used to scale the acceleration waveform to find \( V_{cog,i} \), then to \( \tau_{cog,i} \) and \( I_{cog,i} \).

5.2.4 Waveform Suppression

For either current or voltage control, FFTs are fitted to the data with respect to mechanical position as mentioned in Section 5.2.3. The fits can be evaluated on the controller in runtime for low orders. Alternatively, a lookup table indexed by encoder position \( i \), similar to Equation 5.2, stores precomputed fitted values for \( V_{cog,i} \) or \( I_{cog,i} \). Stiction values could also be position dependent, but require more analysis to compute than in Algorithm 6. These values are added to the desired voltage or current, \( V_{\text{des}} \) or \( I_{\text{des}} \) as indicated in the following:

\[ V_{\text{out}} = V_{\text{des}} + \text{sgn}(V_{\text{des}})V_{st,i} + V_{cog,i} \]  

(5.5)

\[ \delta = \frac{V_{\text{out}}}{V_{\text{sup}}} + \text{sgn}(V_{\text{out}})\delta_{dt} \]  

(5.6)

or

\[ I = I_{\text{des}} + \text{sgn}(I_{\text{des}})I_{st,i} + I_{cog,i} \]  

(5.7)

The suppression of cogging torque involves varying current, \( I \), which adds additional mutual and reluctance torque ripples. With the assumption that mutual and reluctance torques are linear with current, and noting that the feedback throughout this process, \( \theta_m \),
Algorithm 6 Position Based Waveform Analysis

for all $i$ such that $\theta_{m,\text{min}} \leq \theta_{m,\text{cmd},i} \leq \theta_{m,\text{max}}$ do
  for all $j$ in range of $\theta_{m,\text{act},j}$ do
    if $\theta_{m,\text{cmd},i} = \theta_{m,\text{act},j}$ then
      if $\delta_j > \delta_{\text{max},i}$ then
        $\delta_{\text{max},i} \leftarrow \delta_j$
        $I_{\text{max},i} \leftarrow I_j$
      end if
      if $\delta_j < \delta_{\text{min},i}$ then
        $\delta_{\text{min},i} \leftarrow \delta_j$
        $I_{\text{min},i} \leftarrow I_j$
      end if
    end if
  end for
  $\delta_{\text{dt},\text{st},i} \leftarrow \frac{\delta_{\text{max},i} - \delta_{\text{min},i}}{2}$
  $\delta_{\text{cog},i} \leftarrow \frac{\delta_{\text{max},i} + \delta_{\text{min},i}}{2}$
  $I_{\text{st},i} \leftarrow \frac{I_{\text{max},i} - I_{\text{min},i}}{2}$
  $I_{\text{cog},i} \leftarrow \frac{I_{\text{max},i} + I_{\text{min},i}}{2}$
end for

$\delta_{\text{dt},\text{st,max}} = \max_i \delta_{\text{dt},\text{st},i}$

$k \leftarrow 0$

for all $i$ such that $\delta_{\text{dt},\text{st,max}} \geq \delta_{\text{max},i}$ or $\delta_{\text{dt},\text{st,max}} < \delta_{\text{min},i}$ do
  $\delta_{\text{dt},\text{st},k_{\text{temp}}} \leftarrow \delta_{\text{dt},\text{st},i}$
  $k \leftarrow k + 1$
end for

$\delta_{\text{dt},\text{st}} \leftarrow \delta_{\text{dt},\text{st},k_{\text{temp}}}$

$k \leftarrow 0$

for all $i$ such that $\delta_{\text{dt},\text{st,max}} < \delta_{\text{min},i}$ or $\delta_{\text{dt},\text{st,max}} > \delta_{\text{max},i}$ do
  $\delta_{\text{st},k} \leftarrow \delta_{\text{dt},\text{st},i}$
  $V_{\text{st},k} \leftarrow \delta_{\text{dt},\text{st},i} V_{\text{sup},i}$
  $k \leftarrow k + 1$
end for

$V_{\text{st}} \leftarrow \delta_{\text{st},i} V_{\text{sup},i}$
$I_{\text{st}} \leftarrow I_{\text{st},i}$
$\delta_{\text{dt}} \leftarrow \delta_{\text{dt},\text{st}} - \delta_{\text{st},i}$

Algorithm 7 Acceleration Based Waveform Analysis

$\ddot{\theta}_{m,i} = \frac{d^2 \theta_{m,i}}{dt^2}$

for all $i$ such that $0 \leq \theta_{m,i} \leq 2\pi$ do
  $\delta_{\text{cog},i} \leftarrow \delta_{\text{min}} \theta_{m,i} / \max_i \theta_{m,i}$
end for
is a mechanical value and thus captures all torque ripple sources, these additional torques are already compensated for within the algorithm.

5.2.5 Ripple Modeling

In our previous work [45], we found that the anticogging performance changed with the PWM resolution. To understand this, we model the sources of torque ripple to determine the goodness of the anticogging wave fit as well as evaluate design parameter tradeoffs, chiefly PWM resolution. Outlined below are six identified sources that combine to form our model: PWM resolution ($\tau_{\text{res}}$), PWM frequency ($\tau_{\text{frq}}$), deadtime ($\tau_{\text{dt}}$), encoder phase shifting ($\tau_{\text{enc}}$), cogging torque ($\tau_{\text{cog}}$), friction torque ($\tau_{\text{fr}}$), and mutual torque ripple ($\tau_{\text{mtl}}$). All torque ripple sources are modeled as RMS and assumed to have no covariance so that the total RMS torque ripple can be calculated with:

$$\tau_{\text{RMS}} = \sqrt{\tau_{\text{res}}^2 + \tau_{\text{frq}}^2 + \tau_{\text{dt}}^2 + \tau_{\text{enc}}^2 + \tau_{\text{cog}}^2 + \tau_{\text{fr}}^2 + \tau_{\text{mtl}}^2}$$  \hspace{1cm} (5.8)

**PWM resolution** torque ripple, $\tau_{\text{res}}$, stems from the discretization of the desired waveform, where the desired waveform is both the standard sine or trapezoidal signal as well as its change in amplitude according the anticogging waveform, so both nominal motor control as well as anticogged motor control are affected. The error is approximately a sawtooth wave, thus the RMS ripple is the amplitude over $\sqrt{3}$, which is:

$$\tau_{\text{res}} = V_{\text{sup}} f_{\text{pwm}} K_{\tau} / (R f_{\text{clk}} \sqrt{3})$$  \hspace{1cm} (5.9)

where $f_{\text{pwm}}$ is the PWM frequency, $K_{\tau} = 60/(2\pi K_v)$ is the torque constant, and $f_{\text{clk}}$ is the clock frequency. In practice, this is a lower bound since the error is not an exact sawtooth.
PWM frequency is a design parameter so we should model its effect on torque ripple. During PWM, when the pulse is on, the driver drives current through the coils in the motor to cause a torque on the rotor, and not when the pulse is off. It is best that this is faster than the motor’s time constant, $\tau_{pwm} = L/R$, since a notable torque ripple is produced at the PWM frequency if it is slower. The effective frequency is that of the duty cycle’s on time, $\omega_{pwm} = 2\pi f_{pwm}/\delta$. Since high rates are not guaranteed, the AC RMS of a low passed signal, like $\tau_{frq}$, is:

$$
\tau_{frq} = \frac{V_{sup}K_{r}\sqrt{\delta}\sqrt{1-\delta}}{R\sqrt{1+\tau_{pwm}^{2}\omega_{pwm}^{2}}} \tag{5.10}
$$

**Deadtime torque ripple** is the ripple caused by the cessation of current flowing through the motor during the transistor switching time. As the frequency of the PWM increases, the switching deadtime, $\delta_{dt}$, becomes a larger portion of the PWM period. We assume the PWM frequency is above $1/(2\pi\tau)$ so that the PWM voltage is low pass filtered to be $V_{sup}\delta$. If $\delta_{dt}$ is large at frequencies below $1/(2\pi\tau)$, the inverter used is too slow for the motor and is likely too large for the motor. With this assumption, the deadtime torque ripple, $\tau_{dt}$, is:

$$
\tau_{dt} = \frac{V_{sup}\delta K_{r}\sqrt{\delta_{dt}}\sqrt{1-\delta_{dt}}}{R} \tag{5.11}
$$

**Encoder phase shift** is another source of torque ripple that comes from delays in sensing and calculation. If the controller makes PWM updates at the PWM frequency and the PWM frequency is lower than the encoder’s change in position rate, then the controller misses position steps. In reality, this always happens to some extent, even with high PWM frequencies, unless updates are interrupt driven. This encoder phase shift affects both anticogging and the nominal commutation, so both waveforms must
be known. This value is velocity dependent and should be calculated for the motor’s intended velocity. A Monte Carlo simulator simulates calculation start times and various sample times between calculations to find the distribution of encoder phase shifts. Because the encoder is discrete, the simulator returns a vector, $\vec{t}_{enc}$, of length $1 + \lceil \frac{f_{enc}}{f_{pwm}} \rceil$ containing relative times spent with phase shifts of 0 to $\lceil \frac{f_{enc}}{f_{pwm}} \rceil$ encoder counts. The RMS of each phase shift is calculated and stored in $\vec{V}_{\Delta i}$. These are combined by taking the square root of the sum of the squares of the element wise product or Hadamard product, $\odot$, of the times and voltages, $V_{enc} = \sqrt{\sum (\vec{V}_{\Delta i} \odot \vec{t}_{\Delta i}) \odot (\vec{V}_{\Delta i} \odot \vec{t}_{\Delta i})}$, then converted to a torque with $\tau_{enc} = V_{enc} K_\tau / R$.

**Cogging torque** is the primary concern of this paper. We set $\tau_{cog} = \text{RMS}(\tau_{cog,i})$, which is the RMS of the data sampled with algorithm 4 and analyzed with algorithm 6.

**Friction torque** ripple is from position dependent friction, perhaps from eccentricities in the motor’s bearings. To find friction’s effect, we perform algorithm 4 in both the forward and backward direction, giving us $\tau_{cog,i,fw}$ and $\tau_{cog,i,bw}$. The friction torque ripple is then found with $\tau_{fr} = \text{RMS}((\tau_{cog,i,fw} - \tau_{cog,i,bw})/2)$.

**Mutual torque ripple** is from a mismatching of inverter to back EMF waveforms. While it is possible to apply any one of the many mutual torque correction algorithms outlined in section 5.2 for simplicity we assume this is not done. This ripple is typical of driving an ideal waveform (trapezoidal or sinusoidal) on a motor with a non-ideal back EMF shape. If the back EMF shape does not match the driven voltage shape, the error between the waves grows linearly with voltage amplitude and is zero at no voltage amplitude. This ripple could be modeled by sampling the back EMF while the motor is generating, then comparing the sampled wave to the driven wave. Since in our tests we
do not put the motor under any load, which is proportional to current, other than cogging and friction, any mutual torque ripple will be minimal and we ignore it in our model.

5.2.6 Design and Experimental Results

To demonstrate the applicability of the proposed technique for robot arms, a two degree of freedom (DOF) planar robot arm was created that displays smooth motion suitable for simple tool-mediated haptic rendering. A model for specifications of this arm is the popular commercial haptic device, the PHANTOM Omni, now called the Geomagic Touch [9]. The arm specifications includes a planar 2 DOF subset of the Geomatic Touch workspace. This workspace is advertised as rectangular area (160 × 120) mm. However, it is a polar device with workspace measurements 100 mm < radius < 270 mm and 90° in angular range. The maximum continuous force output is 880 mN.
Figure 5.11 shows the two link serial chain used for final validation of the presented method. The length of the first link between the motors is 220 mm and the second link between the second motor and the end effector is 163 mm. This has an effective envelope of $57 \text{mm} < \text{radius} < 383 \text{mm}$, and $360^\circ$ angular range which encompasses the required $(160 \times 120) \text{mm}$ workspace. An onboard wireless radio combined with a battery powering the second joint allows control of the second joint without wires crossing the first joint, reducing external friction sources. Encoders with 12 bit resolution (4096 count) yield a translation positional resolution of 0.087 mm when the links are parallel and 0.59 mm when the links are perpendicular. This is large compared to the 0.06 mm resolution of the Geomagic Touch, but is one sacrifice for obtaining a low cost yet larger workspace.

With the arm lengths chosen, the motor torque required to generate desired max force can be determined. The translation forces applicable by the end effector depend on the joint angles. The nominal position is defined to be identical to the Geomagic Touch with the second joint at $90^\circ$. The maximum applicable force occurs with the shorter lever arm creating the largest static force. This gives a target max motor torque of $0.88 \text{N} \cdot 163 \text{mm} = 143 \text{Nmm}$.

**Experimental Setup**

To determine the most suitable motor, various motors of the appropriate size were evaluated before and after anticogging was applied, but without robot arm links attached to ensure the only sensed torque was from cogging torque. Experiments used a custom motor controller and driver. A Texas Instruments TMS320F28035 provides indirect field-oriented control at 100 kHz. A 600 W, 3 phase inverter, pulse-width modulated at 50 kHz.
symmetrically (up/down), enables updates at 100 kHz with a 300 count PWM. A diametrically aligned magnet affixed to the rotor of each motor and an Austria Microsystems AS145B 12-bit (4096 count) magnetic rotary encoder attached to the stator measure position. The cost of this encoder and magnet pair is $6.69 USD at quantity of 1000 with similar solutions as low as $1.91 USD using the AS5601. The encoder magnet pair is the only required addition to standard hobby ESCs.

The final version of the arm uses this setup with identical hardware, except an updated motor driver using an STMicroelectronics STM32F373 controls phase voltages at 10 kHz with a 1000 count PWM resolution. Haptic feedback is difficult to show in a visual form, so trajectory following was chosen instead to demonstrate smoothness. An example 2.035 m trajectory consisting of 36 line segments, as seen in Figure 5.12, represents a simplified example of a path. Commands are sent and feedback is received synchronously at 150 Hz. Encoder positions are used to calculate the end effector position. We further explore the trade-offs between high PWM frequency versus high PWM resolution (low PWM frequency) with this motor driver, where frequency and resolution are related by

\[ V_{res} = \left\lfloor \frac{72000000}{f_{pwm}} \right\rfloor V_{sup}. \]

For validating the proposed acceleration and position waveform generation methods and measuring frequency-resolution trade-offs, a third method is used to provide ground truth, experimentally determining torque ripple. It uses an external torque sensor, an ATI Industrial Automation Nano17 six-axis force and torque transducer with 1/64 N mm resolution, sampled while performing the acceleration method for five seconds at 20 kHz in MATLAB. We apply a notch filter at the motor and torque sensor’s natural frequency to eliminate noise generated by the test apparatus. These frequencies range from 300 Hz
Figure 5.12: Trajectory of robotic arm with and without anticogging. Cmd is the commanded trajectory, Cog is the actual trajectory without compensation, and Anti is the actual trajectory with anticogging enabled.

The PWM frequency, or encoder shift and too high to be from cogging or mutual torques when spinning at the low test speeds of roughly 1 Hz. The motor’s datasheet provides motor constants to translate torque to voltage and current for this third method. Values of $\theta$, $\dot{\theta}$, $V_{sup}$, and $\delta$ are read at 1 kHz from the controller.

The original controller and driver are tested with eleven motors, demonstrating anticogging’s efficacy across a wide range of motors. Six motors are used throughout this paper as examples and are indicated in Table 5.1. We perform additional tests on motors M1, M2, M3, and M4 in search for an optimal frequency versus resolution trade-off. We use the measured RMS torques from the torque sensor to validate the estimated torque ripple sources from section 5.2.3 for each tested PWM frequency.
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Table 5.1: Motors and Results of Anti-cogging with 300 count PWM at 5 volt.
M1 is a Maxon EC 45 251601.
M2 is an E-flite Park 400 EFLM1300.
M3 is an E-flite Park 300 EFLM1150.
M4 is an Exceed RC Rocket 86MA10.
M5 is a Turnigy Sk3542.
M6 is an ElectriFly Rimfire GPMG4555.
Results

A common metric of torque ripple is the torque ripple factor (TRF) \[\text{[24] [47]}\]. The equation for TRF is:

\[
TRF = \frac{\tau_{pp}}{\tau}
\]

where \(\tau_{pp}\) is the peak to peak torque variation and \(\tau\) is the average applied torque. For mutual and reluctance torque ripple this measurement is constant over different commanded torques, as both torque ripple and desired torque are linear with current and thus \(\tau\). Since cogging torque is independent of current and thus \(\tau\), TRF is not constant and is less useful. TRF is infinite for all motors at zero applied torque because there is still torque ripple from cogging. In place of \(\tau\), a divisor that remains constant for each motor is proposed as Torque Ripple Ratio or TRR, defined as follows:

\[
TRR = \frac{\tau_{pp}}{\tau_{max}}
\]

where \(\tau_{max}\) is the maximum continuous torque that the motor can apply, which can be derived from the motor’s datasheet by multiplying the maximum continuous current and the torque constant. Using this metric, Figure 5.8 shows the relationship between torque ripple and price of 11 arbitrary BLDCMs; a notable inverse correlation before anticogging is evident, while after anticogging the metric is relatively constant.

Figure 5.13 shows a plot of the before and after results of applying anticogging to the 11 tested motors with a 300 count PWM resolution. A line fit shows a 69% average reduction in torque ripple. Table 5.1 shows the details for a subset of the motors from Figure 5.13. \(\tau_{pp\:nom}\) is the nominal peak to peak cogging torque of the motor. \(\tau_{pp\:pos}\) and \(\tau_{pp\:acc}\) are the peak to peak cogging torques after applying the position method and
the acceleration method, respectively.

Data from the PWM frequency versus resolution tests are also in table |5.1| . \( \tau_{RMS \, nom} \) is the RMS torque ripple without anticogging, while \( \tau_{RMS \, anti} \) is the RMS torque ripple with anticogging. Reduction shows the percent torque ripple reduction of the motors. All motors have an RMS reduction of greater than 70% and peak at 88% when using the appropriate PWM frequency and is shown by comparing \( \tau_{nom,act} \) in Figures |5.14| and \( \tau_{anti,act} \) in |5.15| . In contrast to the fixed resolution reductions, picking the proper PWM resolution removes up to \( 3\frac{1}{3} \) times the peak-to-peak ripple.

The values \( f_{pwm \, meas} \) and \( f_{pwm \, est} \) compare the measured and estimated ideal frequencies. Frequency tests are done at the frequencies of \( 1100 \times 1.33^x \) where \( x = 0–17 \) inclusive, giving eighteen frequencies spanning 1100 hz–140 khz. We calculate the estimated RMS torque ripples at the same frequencies and take one with the minimum RMS torque as the best. The estimated best frequencies are all within two calculated frequencies of their best measured, and three out of four were within one. Figure |5.14| and Figure |5.15| show the components of the RMS torques for motor M4 before anticogging and after anticogging respectively. Figure |5.16| shows the four motors’ RMS torques versus PWM frequencies. The data plateaus near the minima, particularly in motors M1 and M3, explaining the small discrepancies in frequency.

A metric that describes the value of a motor is \( \tau_{pp} \times \text{cost} \). From the results in Table |5.1| , motor M2 has the best value before compensation, but motor M4, a motor that fills the same niche in terms of size, torque, and power, wins out after compensation. Conveniently, M4 is also the least expensive of the tested motors.

Since it has the best value, we use motor M4 on the haptic arm, noting it has the
Figure 5.13: Torque ripple after anticogging versus torque ripple before anticogging for eleven tested motors. Fit line is $y = 0.3139x$ with an $R^2 = 0.8922$.

The effect of cogging on the end point position is clearly evident in Figure 5.12. The cartesian root mean squared position error (RMSE) with cogging compensation turned off is 7.38 mm, while the RMSE with acceleration type anticogging on is 3.52 mm.

5.2.7 Discussion

The data presented in Figure 5.8 shows that anticogging gives a low cost motor a TRR lower than that of a motor that is nearly an order of magnitude more expensive. Even with a low resolution of 300 PWM counts across 5 volt, there is an average $\tau_{pp}$ reduction of 69%. At higher resolutions as much as an 88.2% reduction has been seen. Using anticogging, the cartesian RMS position error of a direct drive arm’s end effector can be reduced to less than half.
Figure 5.14: Motor M4 RMS torque versus PWM frequency with anticogging disabled. (+) is $\tau_{res}$, (▪) is $\tau_{freq}$, (*) is $\tau_{cog}$, (.) is $\tau_{fr}$, (x) is $\tau_{dt}$, (□) is $\tau_{cog}$, (⋄) is $\tau_{nom,est}$, (△) is $\tau_{nom,act}$.
Figure 5.15: Motor M4 RMS torque versus PWM frequency with anticogging enabled. (+) is $\tau_{res}$, (o) is $\tau_{frq}$, (*) is $\tau_{cog}$, (.) is $\tau_{fr}$, (x) is $\tau_{dt}$, (□) is $\tau_{cog}$, (φ) is $\tau_{anti, est}$, (△) is $\tau_{anti, act}$
Figure 5.16: Motors M1, M2, M3, and M4 anticogged RMS torque versus PWM frequency predicted and measured. Solid lines are measured and dashed lines are predicted. (∗) is M1, (□) is M2, (⋄) is M3, and (⋆) is M4.
Comparison of Methods

Verifying that both methods of cogging characterization map the torque ripple accurately is crucial. Figure 5.17 displays both methods as well as the ground truth from the external torque sensor detailed in Section 5.2.6. From the plot, the reader can see that all three methods are in agreement in shape, while the position method differs slightly. This is not to say that the position is more or less accurate. Because the speed method and external torque sensor did readings at the same time, they both detect the added mutual torque from bearing friction, while the position method does not. All successful characterizations have a RMS torque error of $<1$ N mm.

It is mentioned in Section 5.2 that speed control loops have limited success suppressing torque ripple, yet the acceleration method, which uses speed feedback, maps cogging torque well. One reason is that the cog mapping is done offline at the lowest possible open loop speed, and thus sensor delay has less impact with respect to position. Furthermore, in a control loop, there must be error to correct and the reactions cause further delays. Another factor may be that all motors tested were smaller hobby or robotics motors in the 18 W to 670 W range. Small size yields smaller inertia as indicated by Equation 5.4, which gives larger, and thus more measurable accelerations for the same torque. The results may not be as favorable for higher inertia motors, motors with higher minimum speeds, or lower frequency speed sensing.

The position method also tracked cogging torque well, despite being based on a different principle. Unlike the acceleration method, which loses DC signal values when taking

\footnote{When referring to DC components or DC signals, the authors are referring to the non-oscillating offset components in the frequency domain, rather than current.}
the derivative, the position method overcomes both the oscillating cogging torque and DC signal friction. Although constant values are easily characterized and compensated, the characterization does introduce a failure mode. The extracted values for deadtime generally agree across motors, as seen in Table 5.1. A supplementary test using a current sensor and the torque sensor on motor M4 found that, while current production starts at \( \delta = 0.071 \), external torque is not felt until \( \delta = 0.083 \). This indicates that the deadtime \( \delta_{dt} = 0.071 \) is the deadtime duty cycle for this motor driver and the stiction is \( \delta_{st} = 0.012 \) or \( V_{st} = 60 \text{ mV} \) at the tested location. The discrepancy between these values and those in Table 5.1 could be because stiction is not consistent across the full range of motion, but the calculations for the compensation assume stiction is consistent. The expensive M1 motor has no detectable stiction, perhaps contributing to its more accurate estimation of \( \delta_{dt} = 0.072 \).

In the process of testing, it was found that with low gains on the position controller, deadtime was not visible in the data. As always, proportional gains that are too high cause the controller to go unstable; thus, gains must be chosen wisely. Excessive gains occasionally prevented more than one iteration of anticogging using the position method.

With a sufficient quality cog map loaded into the driver’s onboard memory, the fidelity of the output waveform is dependent on the controller speed and resolution. At the maximum tested motor speeds (roughly 100 RPM), the encoder incremented around 7 kHz but the controller’s loop speed was significantly higher at 100 kHz, indicating that the loop speed was not a factor. The PWM resolution during these tests were 300 counts across a voltage of 5 V, resulting in 0.017 V increments. Converting this voltage increment into torque increments for each motor using datasheet parameters, \( \tau = K_T I \), and \( V = IR \),
Figure 5.17: Fitted cogging torque ripple data sampled via the position method, acceleration method, and torque sensor versus position on an Exceed RC 86MA10 motor. Voltages are converted to torques using motor datasheet parameters where required.

gives the values indicated by $\tau_{res}$ in Table 5.1. It can be seen that the resolutions are on the same order of magnitude as the anticogged $\tau_{pp}$, between 1 and 5 counts across the full range of motors. This indicates that PWM resolution is the limiting factor of torque ripple reduction in this dataset.

This prompted the PWM resolution versus frequency modeling and tests on motors M1, M2, M3, and M4. Figure 5.14 shows that at high frequencies, where the deadtime is a significant portion of the period, $\tau_{dt}$ is the driving torque ripple source with the exception of $\tau_{cog}$. Figure 5.15 supports that at low PWM resolutions (high PWM frequencies) the RMS torque from resolution error, $\tau_{res}$, also follows the total anticogged RMS torque, $\tau_{RMS}$. This, however, is not the whole story. From Figure 5.15, it can be seen that friction torque is the leading contributor of torque ripple for all of the motors at most
frequencies. Thus, friction torque is justly mentioned in [12] as a torque ripple source. Torque ripple from low PWM frequencies show a sharp increase in RMS ripple at the lowest of frequencies. Particularly on the motors with less inertia, motor vibrations make cog map generation difficult for the position method since extremely low proportional gains must be used to keep the motor stable. At the low speeds of these tests, torque from encoder delay has a negligible effect, but its value should be calculated at the maximum desired motor speed in real application.

With the frequency search calculations verified with experiments, we can compute optimal frequencies for new motors. We calculate $\tau_{pwm}$ and $\tau_{frq}$ straight from datasheet values. The $\tau_{dt}$ is can be calculated from datasheet values and motor driver knowledge, which can be gathered from the position control method. The portion of $\tau_{enc}$ from the nominal sinusoidal or trapezoidal line voltages can easily be calculated knowing the desired motor speed, the motor driver’s clock, and the encoder resolution. Anticogging’s effect on $\tau_{enc}$ is only known after a cog map is generated. Mutual torque’s contribution requires a high quality simulator and model or for the motor to be in hand. While it’s effect can be measured with a torque sensor or current sensor, it is best to apply one of the many mutual torque ripple compensation methods outlined in section 5.2. Since $\tau_{cog}$ and $\tau_{fr}$ are assumed to be constant, they do not contribute to the PWM frequency decision.

Of the four tested motors, all of the minimum RMS torque frequencies lie between 4khz and 14khz as seen in table 5.1. The differences lie in which ripple sources dominate at each frequency for each motor. For a wide range of typical PWM frequencies, the model tracks the actual RMS well. The model tends to underestimate at the frequency extremes. There may be an unmodeled torque ripple source that is either frequency, PWM
duty cycle, or speed dependent, since these all vary proportionally throughout the tests. Perhaps Coulomb friction plays a larger role than expected, as suggested by [12]. Despite these errors, the model not only allow a robot designer to choose the appropriate motor driver frequency, but also predicts the expected amount of RMS torque ripple across a range of motors and frequencies.

Now that we have shown that the anticogging process can suppress torque ripple to a predicted amount and have found the appropriate PWM frequency, we can compare the potential robotic arm motors. If torque ripple is the primary concern, motor M3 is reduced to the lowest $\tau_{RMS}$ thanks to its small torque resolution step size, while the next lowest, M1, is 4.58 times as massive, 3.47 times as expensive, while having only 1.61 times as much continuous torque. If value is the primary concern, motor M4 wins since motor M1 is 1.83 times as massive, 1.60 times larger, 9.43 times more expensive, 0.58 times as much continuous torque than M4, and has 1.2 times the TRR when M4 is anticogged. This is why we chose motor M4 for the robotic arm.

**Arm Test Results**

The results in the previous section guided the design of the updated motor driver used in the robotic arm and is described at the end of Section 5.2.6. Despite the arm having significantly larger inertial loads, which raises the required output torque and lowers the TRF when compared with bare motor cog testing, RMSE decreased by 52% using anticogging. The results are visualized in Figure 5.12.

Comparing the resulting motor capabilities to the desired robot arm requirements, the maximum continuous force is close to the Geomagic Touch. The M4 has 134 N mm
which compares to our target 143 N mm. While most commercial haptic devices do not list torque ripple, they often specify a back-drive friction, which is an error from the desired force output. The Geomagic Touch lists a back drive friction of 0.26 N. Solving $V_{st}$ of motor M4, the stiction force at the end of the second joint is 0.016 N. For a second comparison, we can normalize the back drive friction with the max force, which gives an effective $T_{RR} = 0.30$ for the Geomagic Touch. This is quite large compared to the $T_{RR}$ of the proposed device at 0.04, however the $T_{RR}$ is cyclical and back drive is not. Human touch sensitivity is noticeably stronger when frequencies $>5$ Hz [16]. Nominal human motions move the arm at 120° in 1 second, that would correspond to approximately 5 Hz as the dominant frequency in Figure 5.17 over 120°. Faster motions would result in higher frequencies to which humans are much more sensitive.
We now have all of the tools we need to make a three degree of freedom controllable plus one simulated, single motor flying vehicle. We start with constructing a frame that is passively stable, using the techniques from Chapters 2 and 3. Then we add one of the torque producing methods from Chapter 4, all of which require pulsing of motors, which was discussed in Chapter 5. Finally we combine these principles into two different vehicles: UNO and Piccolissimo. UNO is the Under-actuated, Naturally stabilized, One motor vehicle. Piccolissimo, which is Italian for "very little", demonstrates some of the benefits of having a passively stabilized flying vehicle of this type, specifically size, complexity, and cost.

6.1 UNO

UNO is an iteration on the vehicle presented in Section 3.2.2. It is meant to be a technology demonstrator for both passive stability using the differential lift with angular momentum method and the under-actuated propeller, while showing that our vehicle model is accurate. Utilizing the knowledge learned from the previous vehicles, we design UNO to explore the limits of these types of vehicles. To this end, UNO is not modular like its predecessor, instead it is lightweight and optimized for stability and maneuverability.
6.1.1 Design and Manufacturing

Design Process

Since UNO is not modular, making guess and check processes infeasible, we relied heavily on the theory from Section 2.2.2. The simulator performs a blade element analysis at 5 mm intervals for each blade, both on the propeller and the body. The simulator applies the forces and moments, $f_a$ and $\tau_a$, generated by the blade elements to the dynamical equations from Section 2.1.

The simulator calculates the inflow using the Rankine-Froude method for each blade element radius, which assumes the propeller and stabilizers have a small vertical separation and the inflow velocity does not change between blades. Because the stabilizers are nominally above the propeller, rather than along side like in multi-copters, the inflow from one influences the other. Furthermore, inflow is dependent on the size and shape of the propeller, as shown in Figure 6.1. The combination of these two means different propellers can influence the stabilizers through inflow. In fact, inflow depends on a large number of variables. To name a few, we have: vehicle mass, propeller speed, propeller chord, propeller pitch, propeller radius, body speed, stabilizer chord, and stabilizer pitch. Therefore, we must either optimize the body and stabilizers for a specific configuration or make it robust to many variables. We use the optimized method for UNO.

Once we choose a propeller, we run it through an optimizer in the simulator. The simulator begins with a base body design where the stabilizers have constant chord and pitch along its span. This base vehicle is reproduced in the Computer Aided Design (CAD) software Solidworks with as much detail as possible to have good estimates for
Figure 6.1: Simulated hover inflow versus span for a fixed body and various propellers. Labels are manufacturer, radius (in) x pitch (in/rev) x number blades, style. Style is: MR = Multi Rotor, 3D is reversible, and SF is Slow Flyer.
mass, COM, and inertia. A vehicle design algorithm varies stabilizer blade pitch and
cord at each blade element in the simulator. We impose some limits on the design,
specifically the blade cannot be taller than the material thickness, 40 mm, an upper limit
on disc solidity, 66%, and a minimum chord length of 30 mm for sufficient structure. The
optimizer modifies the chord and stabilizer pitch to reach target angles of attack through
iteration, since varying both chord and pitch vary inflow, which varies angle of attack.
We found ten iterations per configuration is sufficient as long as the configurations are
similar. An example of angle of attack versus stabilizer configuration is in Figure 6.2
In this run we start with a base configuration with a stabilizer blade pitch of 20 deg and
a constant chord of 80 mm. We set targets of −1 deg to 4 deg across the span, though
we also explored variable angle of attack targets across the span. Figure 6.3 shows the
resulting blade pitches that yield the angles of attack from Figure 6.2. Likewise, Figure
6.4 shows the corresponding chord lengths. Note that the 4 deg angle of attack trial
requires more than 4 deg of blade pitch. When the angle of attack increases, so does the
drag for a given speed. To match the torque generated by the propeller the body slows
down significantly. This reduces the horizontal component of the relative wind on the
blade elements, requiring even more blade pitch to hit the desired angle of attack. It
is recursive dependencies like this that demand iterative solutions. Furthermore, as the
blade pitch increases the chord must shorten to remain in the material thickness limits,
like between 0.05 m to 0.125 m on the 3 deg and 4 deg angle of attack trials.

Each iteration begins with the simulation using an altitude controller to find the hover
trim state. Once reached, we sample various pieces of information about the configuration
with an emphasis on the angle of attack at each blade element. It then performs a two-
Figure 6.2: The stabilizer angle of attack, $\alpha$, for various stabilizer angle of attack targets.

Figure 6.3: The stabilizer pitch, $\beta$, for various body and stabilizer configurations. The handedness in the simulator mimics that of the vehicle in Figure 3.10. Thus beta values are $180 - \beta$ from their normal value.
Figure 6.4: The stabilizer chord, $c_E$, for various body and stabilizer configurations.

sided perturbation with small values in each direction to estimate the sensitivity slopes in the Jacobian. The algorithm stores the eigenvalues of the Jacobian of each design, and picks the vehicle with the most negative maximum real eigenvalue. Figure 6.5 shows all six eigenvalues for various configurations. One pair has a high dependence on the body's angular momentum, exhibited by the translation toward the origin as the angle of attacks increased, which decreases body rotational rate. The remaining four eigenvalues concentrate near the origin. Figure 6.6 concentrates on the two with positive imaginary components. We remind the reader that the remaining two eigenvalues have the same real components and negative imaginary components. The most apparent result is that negative angles of attack yield unstable vehicles. This is because the differential lift now works in the opposite direction, and with the angular momentum will turn the vehicle towards direction of travel. Increasing angle of attack improves stability and groups these
Figure 6.5: Eigenvalues of various stabilizer configurations.

two eigenvalues until 2 deg angle of attack. Then the vehicle body slows down too much, causing the differential lift to dominate the gyroscopic precession and decrease stability. Thus, for this set of trials, the stabilizer pitches and chords that cause 2 deg angle of attack are the best choice for our vehicle. Let it be known that this is merely an example and many more trials are performed in actual design and 2 deg angle of attack is not universally the best angle of attack.

**Manufacturing and Construction**

The first step in manufacturing is to reproduce the values for stabilizer blade pitch and chord in the Solidworks CAD model. We process the CAD model in the Computer Aided Machining (CAM) software Solidcam. The result from the CAM is executed on a Southwestern Industries TRAK DPM2 Computer Numeric Control (CNC) mill, which
Figure 6.6: A zoomed in plot of Figure 6.5 focusing on the two eigenvalues closest to the origin and having positive imaginary components.
cuts the body from 50.8 mm Owens Corning Foamular 250 Extruded Polystyrene. We chose Extruded Polystyrene for its high strength-to-weight ratio since, at the time of initial construction, the under-actuated propeller was limited to about 2 N of force, which made the maximum target mass of the vehicle about 200 g \[40\]. We embed plastic M3 standoffs in the foam using UHU POR glue, which allow the attachment of accessories using M3 plastic screws. Accessories include the motor, motion tracking markers, a supplementary rim/duct, and landing gear. The motor controller is the same as the one used in Sections 5.2.6 and 3.2.2 and attached to a DYS Quanum 2206 2000 Kv motor. The onboard gyroscope integrates the measured body yaw rates at up to $35 \, \text{rad s}^{-1}$, $r_b$, giving a body angle about the $\hat{z}$ axis, $\phi_b$. An Light Emitting Diode (LED) illuminates when $\phi_b = 0$, which appears continuously illuminated through persistence of vision, giving the pilot a virtual yaw angle.

The first successful UNO, depicted in Figure 6.7, weighs 184 g and has a diameter of 392 mm. The body is versatile with its ability to accept a wide range of batteries. The center is designed to accommodate a 460 mAh 3S Lithium Polymer battery as well as an 850 mAh 3S Lithium Polymer battery. Four pockets in the tips of the stabilizer blades accept 300 mAh 2S Lithium Polymer batteries, all wired in parallel. Thus, the mass and inertia of the vehicle can be adjusted without redesigning and machining a new body.

Later versions of UNO attempt to improve on the handling of the vehicle. As shown in Section 2.2.2 stability is increased with increased angular momentum. One method of adding angular momentum is to spin faster, so we add a faster gyroscope, capable of 70 rad s$^{-1}$. We also include three 1200 mAh 1S Lithium Polymer batteries arranged in parallel along the rim of the vehicle, increasing the vehicle’s inertia and angular momentum.
These accompany a larger motor, an AX-4005D 650 Kv, and a new set of under-actuated propellers with a flap hinge and more thrust capability. The heaviest of these vehicles tip the scales at 274 g.

6.1.2 Experimental Results and Analysis

We created two versions of UNO. To design the first version we used a uniform inflow model to determine the stabilizer’s 0 deg angle of attack blade pitch, then added an extra 5 deg. This vehicle ultimately performed well, achieving both passive stability and controlled horizontal motion. The vehicle body did spin faster than expected, which lead to saturation of the gyroscope for heavier configurations. The discrepancy between the simulator and experiments lead to the development of the radial inflow model.

The second version features a number of improvements. We design its stabilizer profile
Table 6.1: Speeds of UNO’s body and propeller. All units are rad/sec.

<table>
<thead>
<tr>
<th>Propeller Type</th>
<th>$r_b$ Act.</th>
<th>$r_b$ Sim.</th>
<th>$r_p$ Act.</th>
<th>$r_p$ Sim.</th>
</tr>
</thead>
<tbody>
<tr>
<td>HQ 5x4x3</td>
<td>34.64</td>
<td>40.06</td>
<td>-1273</td>
<td>-1217.06</td>
</tr>
<tr>
<td>APC 9x4.7x2</td>
<td>44.39</td>
<td>48.75</td>
<td>-536.2</td>
<td>-483.25</td>
</tr>
<tr>
<td>Gemfan 10x4.5x2</td>
<td>47.79</td>
<td>50.5</td>
<td>-437.8</td>
<td>-413.8</td>
</tr>
<tr>
<td>Gemfan 11x4.7x2</td>
<td>49.15</td>
<td>51.42</td>
<td>-390</td>
<td>-361.92</td>
</tr>
<tr>
<td>Gemfan 12x4.5x2</td>
<td>57.51</td>
<td>55.68</td>
<td>-384.1</td>
<td>-310.38</td>
</tr>
</tbody>
</table>

with the radial inflow model and eigenvalue analysis from Section 6.1.1. This leads to accurate predictions in body and propeller rotational rates, shown in Table 6.1 with all estimates within 20% error and most within 10% error.

To test stability and disturbance rejection, we apply forces and moments to the vehicle. We use three methods of introducing disturbances to ensure the vehicle can recover from a broad range of states. For all methods, we first launch the vehicle and allow it to achieve a stable hover. Once hovering, the first method is to take a sheet of plastic with a low coefficient of friction and apply a contact force to the landing gear on one side of the vehicle. Since the force is vertical and the linear analysis only models horizontal motion, this disturbance presents itself as a pure torque as far as the linear analysis is concerned. After the disturbance the vehicle nominally reaches some non-zero body angles and rates, but has non-zero linear velocities. If the vehicle returns to a hover we consider it stable.

For the second method, once the vehicle hovers we perturb it with a gust of wind created by waving a large plastic sheet. Unlike the first method, the wind gust method presents itself like a non-linear velocity since aerodynamic Jacobian partial derivatives like $a$ and $b$ are sensitive to wind and not physical motion. Again, if the vehicle returns to hover after the disturbance, we consider the vehicle stable. Finally, the third method is to apply the vehicle with a control torque. We used both the underactuated propeller and
the offset rigid propeller methods, both with successful results. We are able to tune the pulsing phase of the underactuated propeller to provide consistent linear velocities, which indicates that either the control torque or the gyroscopic precession dominates. Since the vehicle returns to hover after we apply a control torque, we can determine that gyroscopic precession dominates. UNO proved stable using these three methods while also showing it is able to translate in a controlled manner.

Though successful, the updated version of UNO faced reliability issues. A number of problems presented themselves. We identify two that affect stability, two that cause vehicle damage, and one that stifles data collection.

The angle of attacks required to allow the body to spin faster are very close to 0, such that small miscalculations in inflow, whether it’s due to body speed error, mass error, or calculation error, cause the angle of attack be negative. When the angle of attacks are negative, the advancing side of the propeller or stabilizer still sees more positive angle of attack, which suggests differential lift may still work. Unfortunately, the advancing side also has a higher relative wind velocity, which means it generates more lift in the direction it is generating lift. Furthermore, lift, $L$, relates to the angle of attack, $\alpha$, as $L \propto \alpha$, but relates to relative wind, $\nabla$, as $L \propto \nabla^2$. Thus, a blade with a negative angle of attack can be stabilizing, but it also can be destabilizing depending on the magnitudes of the changes in $\alpha$ and $\nabla$. We recommend using higher angles of attack to ensure the angle of attack never goes negative. To increase vehicle speed, instead, decrease the chord and disc solidity, as opposed to exclusively reducing angle of attack.

Another source of unreliability is that stability partially relies on the choice of motor. All versions showed instability when mounted with a motor that protruded from the mount
by 51 mm, but were stable with motors that protrude by 33 mm and 37 mm. Either this motor moved the COM in the positive $\hat{z}$ direction enough to destabilize through too much $c$ term or the propeller being close to the bottom of the vehicle rim created unmodeled aerodynamic effects. We settled on the AX-4005D motor due to its short yet wide construction, giving it the necessary power while keeping a low profile.

To complicate matters further, the new propeller with the flap hinge allows the tip path plane to change, which lead to collisions with the vehicle body, essentially destroying the vehicle. This happened both when the vehicle body pitched past some critical angle and when motor pulsing caused sufficient blade flap. Since the earlier underactuated propeller did not have a flap hinge, the only blade flap came from blade compliance, which proved insufficient to collide with the vehicle body. We modified the propeller with hard stops to prevent vehicle damaging flap angles. It is important to remember that the propeller blades also flex through compliance, so adjust the flap stops accordingly.

The compliance of the vehicle’s foam body caused additional issues. The hinged propellers are unbalanced at low rotational rates due to the nature of their angled hinges. This imbalance causes the body to flex, which further exacerbates the imbalance. If the propeller rotates at a vibration mode of the vehicle the imbalance can excite this mode to the point of the foam’s structural failure. A vehicle body that is stiffer can push this frequency above that of the propeller, relieving this issue.

Finally, once all of the vehicle issues are dealt with, our motion capture system also proves too unreliable to collect data. Though the position tracking is satisfactory, the attitude tracking is not, as sometimes it would not update yaw angles for nearly 2 s. With no ground truth for the body’s yaw angle, torque pulsing can occur in unknown
directions. Relying on the onboard gyroscope works for human pilots, which can adjust
the yaw angle for gyro drift, but does not work for replacing the motion capture yaw since
there is no mechanism to adjust for drift.

UNO demonstrated all of the desired traits, including passive stability and vehicle
control with a single motor. Each trait is proven through video footage. The ultimate
goal of the UNO project is to verify the vehicle design model and that it is able to
accurately predict vehicle behavior. Though UNO itself is a successful vehicle we are
unable to gather convincing data that this is due to an accurate model, rather than
chance. Should the vehicle reliability improve and the motion tracking gain the ability
to track high rotational speed objects then we will likely be able to gather the necessary
data.

6.2 Piccolissimo

Now that we have demonstrated that a one motor flying vehicle is capable of passive
stability and steering, we now seek to take advantage of this vehicle’s features. Specifi-
cally, a single motor vehicle has one quarter of the number of moving parts compared to
most simple flying vehicles. With this, we can make extremely small, simple, low cost
MAVs, which would allow flying swarms with numbers in the hundreds, disposable flying
sensors, and low cost toys. To this end, we created the smallest self-powered MAV named
Piccolissimo.

6.2.1 Design and Manufacturing

Piccolissimo is designed to be small, robust, and low cost. Its single moving part, the
motor rotor with an attached propeller, is taken from small commercial multi-rotors.
Power comes from commercially available Lithium Polymer batteries. Many 3D printers are capable of printing the vehicle bodies and 3D printing services have successfully printed bodies for $7.37 USD \[1\].

All versions of Piccolissimo nominally have the motor and propeller in the center of the vehicle, though offset, as discussed in Section \[4.3\]. The batteries are located along the body’s outermost rim in order to increase the body’s inertia. Because no single, curved battery of appropriate dimensions to line the rim with sufficient power exists at the time of writing, we surround the rim with at least three discrete batteries in order to distribute the inertia in the $\hat{x}$ and $\hat{y}$ directions. The three batteries are wired in parallel (1S3P) to keep both the onboard electronics and the motor at a safe voltage. A thin solid rim attached to the body surrounds the vehicle to increase inertia, protect the vehicle in case of a collision, and reduce differential lift on the main propeller (a destabilizing
Figure 6.9: Maneuverable Piccolissimo compared to a US quarter dollar. The motor visualization LED extends out from the vehicle at the top of the photo.

effect). The body’s airfoils are large with a high disc solidity to increase the differential lift effect for passive stabilization. The motor, batteries, and rim are roughly in the \( \hat{x}\hat{y} \) plane to maximize inertia in the \( \hat{z} \) direction, while keeping the other directions close to their ideal minimum, \( I_{XX} = I_{YY} = I_{ZZ}/2 \). Nearly every feature on the body is 0.4 mm thick.

We designed two versions of Piccolissimo: one targeting size, and another targeting mobility. Figure 6.8 displays the size focused Piccolissimo and is called Mini Piccolissimo. Figure 6.9 shows the mobile Piccolissimo, which is called the Maneuverable Piccolissimo.

Mini Piccolissimo

Most vehicles require at least four actuators, abstractly one for each direction of attitude control plus one more for throttle. Piccolissimo’s passive stability allows it to do away with the actuators for attitude control, leaving just a single motor for throttle. With
Table 6.2: Mass distribution of both versions of Piccolissimo in grams.

<table>
<thead>
<tr>
<th>Value</th>
<th>Mini</th>
<th>Maneuverable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frame</td>
<td>0.53</td>
<td>0.80</td>
</tr>
<tr>
<td>Motor</td>
<td>0.68</td>
<td>1.23</td>
</tr>
<tr>
<td>Battery (3x)</td>
<td>1.04</td>
<td>1.84</td>
</tr>
<tr>
<td>Propeller</td>
<td>0.04</td>
<td>0.08</td>
</tr>
<tr>
<td>Electronics</td>
<td>0.19</td>
<td>0.52</td>
</tr>
<tr>
<td>Total</td>
<td>2.48</td>
<td>4.47</td>
</tr>
</tbody>
</table>

this, we can use spare parts from the current smallest, commercial quadcopter to produce a vehicle with a maximum dimension that is roughly half the size and one quarter the planform area. The motor and propeller come from a Cheerson CX-STARS quadcopter which is shown with a Mini Piccolissimo in Figure [6.10] [27].

The motor is driven by a DMN3065LW-13 N-Channel MOSFET whose gate is pulled up by a TEMT7100X01 infrared phototransistor and pulled down by a resistor. An infrared flashlight is pulse width modulated (PWM) above the vehicle, which in turn modulates the on time of the motor driving MOSFET. A CVS-01TB miniature slide switch is in series with the phototransistor. The three batteries are Fullriver 201013HS10C, which give Mini Piccolissimo a demonstrated battery life of 98 s. Mini Piccolissimo’s all up weight is 2.52 g, with a mass distribution shown in table 6.2. Its maximum dimension, the perimeter of the rim, is 28 mm.

**Maneuverable Piccolissimo**

While Mini Piccolissimo is optimized for size and stability, the Maneuverable Piccolissimo is intentionally partially imbalanced. We offset the motor’s location so that the thrust no longer goes through the vehicle’s center. The batteries are also skewed in the opposite direction to further increase the disparity between the COM and the thrust vector. The
Figure 6.10: Mini Piccolissimo next to a Cheerson CX-STARS, which is the source of Mini Piccolissimo’s motor and propeller. Note Piccolissimo’s dimensions are roughly half that of its parts source’s quadcopter.
The electronics on board Maneuverable Piccolissimo are more sophisticated than on Mini Piccolissimo. An Atmega ATTiny20 microcontroller PWMs the motor synchronously using a SiA519EDJ N- and P-Channel MOSFET. It receives commands from the pilot via an upwards facing TSOP57436 remote control infrared receiver. A user controls a transmitter to project an infrared control signal onto the ceiling, which reflects back down onto the remote control receiver. A TEMT7100X01 infrared phototransistor faces the vehicle body’s $\hat{x}$ direction and is pulled down by a 200 kΩ resistor.

As Piccolissimo’s body spins, so does the infrared phototransistor. The software on the microcontroller reads the voltage from the infrared phototransistor and resistor pair, searches for peaks, and runs a simple phase-locked loop. If there is a single infrared source within view, the peak detector will see it and consider that direction North. The phase-locked loop estimates the vehicle’s angle within one revolution. To ensure the peak detector only detects a single peak per revolution, we created an infrared beacon, which is 20 cm tall and has twenty eight wide angle infrared LEDs, and has an effective range of 1.5 m. The Maneuverable Piccolissimo indicates it has lock on the beacon by turning off it’s green LED on the PCB and turning on its red LED on the PCB. To visualize and debug its current behavior, another red LED is located at the rim and on the $\hat{y}$ axis. This LED is on the same side of Piccolissimo as the motor (which is offset from the COM), and is connected to the motor’s leads. Thus, using persistence of vision, we can see when the controller is applying voltage to the motor during pulsing. For this reason, we call this LED the motor visualization LED. Figure 6.11 shows a screenshot of
Figure 6.11: Overhead camera view of Maneuverable Piccolissimo. The motor visualization LED draws a perimeter around Piccolissimo in the top-right and bottom-left of the vehicle, while aliasing hides the LED in the top left, and motor pulsing turns off the LED in the bottom right indicating the direction of thrust.

Achieving sufficient control authority in the Maneuverable Piccolissimo proved difficult. The moment arm of the center of thrust to the COM, $O$, is over 20% of the vehicle’s largest dimension. If this value were any larger, the Maneuverable Piccolissimo would have issues taking off, since the thrust necessary to take off would create a moment large enough to flip the vehicle over. Thus, the only way to increase total torque is by increas-
ing the thrust differential on opposite sides of the vehicle, so we implemented all of the methods described in section 4.3. We found the batteries had an internal resistance comparable to that of the motor, both around 2 Ω. Batteries with a lower internal resistance would appreciably increase the actuation authority and have less losses, leading to greater flight time.

6.2.2 Experiments and Analysis

Power Consumption

We measure Piccolissimo’s power consumption to ensure there are no inherent inefficiencies in the Piccolissimo design. To do so, we first fully charge Mini Piccolissimo’s batteries. Then we fly Mini Piccolissimo. We then recharge Mini Piccolissimo using the same setup and charge cutoff parameters as the first charge. While charging the second time we measure the voltage at the battery and the current entering the battery via a current sense resistor. The voltage and current are sampled at 1 kHz, then multiplied to measure input power, then integrated to measure input energy.

Over the course of 44 s of flying Mini Piccolissimo consumed 64.6 J, resulting in a power draw of 1.47 W. At 2.48 g, Mini Piccolissimo draws 1.68 g W$^{-1}$. The rated combined energy of Mini Piccolissimo’s batteries is 400 J, which gives an estimated flight time of 272 s. Actual battery life of both versions of Piccolissimo and other small multirotors is shown in Table 6.3. Despite the g W$^{-1}$ of Mini Piccolissimo being quite good for a vehicle of its size, its battery life is notably shorter than its larger and commercial counterparts. The discrepancy between the estimated flight time and actual flight time arises from the drop in voltage as the batteries run low. Both versions of Piccolissimo
<table>
<thead>
<tr>
<th>Value</th>
<th>Mini</th>
<th>Maneuver</th>
<th>FY805</th>
<th>FY804</th>
<th>STARS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (g)</td>
<td>2.48</td>
<td>4.47</td>
<td>12.9</td>
<td>7.70</td>
<td>7.71</td>
</tr>
<tr>
<td>Max Dim (mm)</td>
<td>28</td>
<td>39</td>
<td>65</td>
<td>52</td>
<td>52</td>
</tr>
<tr>
<td>Num Motors</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Flight Time (s)</td>
<td>98</td>
<td>161</td>
<td>284</td>
<td>309</td>
<td>230</td>
</tr>
</tbody>
</table>

Table 6.3: Comparison of Mini and Maneuverable Piccolissimo with commercial multicopters. The FY805 and FY804 are made by Fayee, while the CX-STARS is made by Cheerson, and is the source of Mini Piccolissimo’s motor and propeller. The FY804 and the CX-STARS are the current smallest commercial self powered MAVs.

require about 3.6 V to 3.7 V to be able to achieve sufficient thrust to hover, but an empty lithium polymer battery is 3.0 V. This means Mini Piccolissimo is unable to hover when the batteries reach about half capacity, which is consistent with the near factor of two difference between estimated battery life and actual. Reducing the back EMF of the motor can alleviate this problem and can be done simply by using a lower Kv motor or a higher pitch propeller.

Control

Since Mini Piccolissimo can only control vertical motion, which we have shown to work in the past, we use Maneuverable Piccolissimo to measure control in the horizontal plane. To do so, we mount a video camera 2.44 m above the flying area. Its frame rate is 59.94 Hz, which is notably faster than Maneuverable Piccolissimo’s nominal rotation rate of 40 Hz. This causes aliasing of the motor visualization LED, since Maneuverable Piccolissimo only completes about 2/3 of a revolution per frame, yet it does allow us to estimate the body’s rotation rate by viewing the ratio of the perimeter where the LED is visible to where the LED is not visible. The vehicle’s altitude is determined by the radius of the circle drawn by the motor visualization LED as the vehicle spins. The center of this circle determines the $\hat{x}$ and $\hat{y}$ location of the vehicle. We visually identify when the vehicle
begins pulsing the motor voltage by examining the motor visualization LED, shown in
Figure 6.11. Figure 6.12 shows a frame from tracking, the vehicle’s horizontal location at
1/10 s intervals, and LED aliasing.

In the nonlinear simulator with a blade element step of 0.5 mm, we match the condi-
tions at the beginning of pulsing, and mimic the pulsing time and direction from the
experiment. Figure 6.13 shows the forward simulation of position along with the gathered
data. Likewise, Figure 6.14 shows the velocity data. The model tracks the actual data
well, though over-estimates the response from both the applied torque and the restoring
natural stability. Both inertia and aerodynamics play an important role in determining torque sensitivity. Since the frequency of the response is well matched between the simulator and actual data, an inaccurate model of inertia is not likely to be the issue. Instead, the aerodynamic model is likely the culprit. While the radial inflow model used for finding hover conditions and state transition matrix values from very small perturbations worked quite well, it does not capture asymmetric inflow effects which are expected in forward flight and when pulsing an offset motor.

In this sample, the vehicle reached a maximum speed of $0.40 \text{ m s}^{-1}$, which is greater than 10 body-lengths per second. Notably, this was achieved with less than one second of pulsing. Under the same conditions, the simulator estimates a maximum steady state speed of $0.88 \text{ m s}^{-1}$ when pulsing for at least ten seconds, which is 22.5 body-lengths per second. Although the simulator over estimates the response from pulsing and differential lift, in steady state these two torque sources cancel, thus, the source of error may also
Figure 6.14: Both tracked and simulated velocity data superimposed. cancel. This indicates that the Maneuverable Piccolissimo is likely able to be controllable up to a steady wind 0.88 m s$^{-1}$. 
Chapter 7: Conclusion

7.1 Contributions

This dissertation presents design tools for creating novel micro aerial vehicles. We begin by reviewing passive hover stability in order to enable reduced cost, component count, and size of vehicles in Chapter 2. Two discrete mechanisms for passive hover stability emerge and we discuss the specific stability criteria for each. Chapter 3 describes the design, construction, and experimentation of three vehicles that demonstrate the passive stability mechanisms. A quadrotor with stabilizer plates and without an active controller exhibits stability using the COP versus COM method. Two vehicles display the differential lift with angular momentum stability method: one in a tall configuration and one in a wide configuration. We shift to control of passively stabilized vehicles in Chapter 4. We find that pure forces on a passively stabilized vehicle results in no translation in the steady state. Pure torques on a passively stabilized vehicle do result in translation in the steady state. A differential lift with angular momentum stabilized vehicle with an offset vertical axis propeller presents itself. Chapter 5 reviews motor control techniques to achieve the greatest change in motor speed, and thus the largest change in thrust from the offset propeller. We use the same control techniques to create Anticogging, the compensation for cogging torque ripple in brushless motors. Taking reducing torque ripple one step farther, we analyze the effects of varying the controller’s PWM frequency and resolution.
on the total motor torque ripple. Chapter 6 presents combining passive hover stability and high frequency motor torque modulation into two vehicles. UNO demonstrates improved passive stability as well as two methods for steering with a single actuator. Two versions Piccolissimo expose the merits of simple, single motor, passively stabilized micro aerial vehicles. Mini Piccolissimo is the smallest flying robot at a mere 28 mm in its largest dimension. Maneuverable Piccolissimo is the smallest self-powered maneuverable flying vehicle, and has demonstrated the ability to steer with its single motor.

7.2 Future Work

Though the work discussed in this dissertation results in the smallest flying vehicle, there is still much to do. Maneuverable Piccolissimo would benefit from a larger vocabulary since it currently accepts only seven messages: four steering directions, throttle up and down, and shut off. Variable steering amplitudes would allow for translation at different velocities. More variations in the pulsing phase would allow Piccolissimo to steer in more than four cardinal directions. Finally, support for multiple vehicles in the communication architecture would enable swarm behavior.

With Piccolissimo able to swarm, we could have multi-robot systems in the hundreds or even thousands in constrained spaces. To achieve these numbers Piccolissimo should be mass produced. Slight modifications to the design would allow for injection molding. At high enough numbers, one could have custom batteries fabricated that conform to the shape of Piccolissimo’s body. As the manufacturing quantity increases the price decreases, which could allow the price per Piccolissimo to drop below $1.
Bibliography


