2017

Essays In Macroeconomics With Financial Frictions

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Essays In Macroeconomics With Financial Frictions

Abstract
How can governments design policies that alleviate the macroeconomic implications of financial frictions? This dissertation contributes to answer this question focusing on two aspects: international borrowing and crisis prevention at the country’s level, and the impact of taxation and financial regulation on entrepreneurship at the agent’s level. In the first chapter, debt crises arise from the incompleteness of sovereign debt markets: the government cannot credibly commit to repay or default in certain states of the world and this gives way to non-fundamental debt crises. In a strategic default environment, I show that international reserve holdings help to reduce the probability of these market-driven debt crises, advancing the theoretical literature that had struggled to explain why countries hold reserves while indebted. The results are consistent with previous empirical results that had shown countries with greater reserve holdings faced lower spreads in the sovereign debt markets, which is at odds with the previous theories. In the second chapter, a small open economy faces an aggregate borrowing constraint and the agents fail to internalize how their private borrowing decisions push the total debt towards the limit, making the current account adjustment more severe. We model the decentralized and planner’s problem and find the optimal capital control policies, these are very effective to move the economy to the first-best scenario but also very hard to implement, given their state contingent nature. We then address the effectiveness of simpler policy rules, and find that they can bring welfare gains but had to be carefully designed. Finally, in the third chapter, the competition among investors for the most promising entrepreneurs, under adverse selection and limited liability, leads to an excessive entry into entrepreneurship activity and allocates resources to socially inefficient projects. We solve the optimal contracting problem and show that the inefficiency disappears if at least one of the next three is missing: competition in financial intermediation, adverse selection or limited liability. We also show that a small cost or fee per contract, like red-tape requirements, is enough to restore efficiency, making a case for financial regulation.

Degree Type
Dissertation

Degree Name
Doctor of Philosophy (PhD)

Graduate Group
Economics

First Advisor
Enrique G. Mendoza

Keywords
International Reserves, Macroprudential Policy Rules, Self-fulfilling Crises, Sovereign Debt, Startup Financing

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ESSAYS IN MACROECONOMICS WITH FINANCIAL FRICTIONS

Juan M. Hernandez

A DISSERTATION

in

Economics

Presented to the Faculties of the University of Pennsylvania in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy

2017

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ESSAYS IN MACROECONOMICS WITH FINANCIAL FRICTIONS

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To my wife Angela.
Acknowledgments

There are several people without whose contributions significantly improved this dissertation, and to whom I am greatly indebted. First and foremost, I am extremely grateful to my advisor, Enrique G. Mendoza for offering invaluable guidance and support during the entire process and also for giving me the honor of being his coauthor. The third chapter of this dissertation is based on a joint project. Chapter 1 of this dissertation also owes a lot to him.

I also thank the other members of my dissertation committee. I am deeply thankful to professor Harold Cole. My understanding of my own research was greatly improved by his insights. I am very grateful to professor Guillermo Ordoñez for extensive advice and guidance.

I have been fortunate to meet several people who have helped me and inspired me during my graduate studies. I thank Daniel Wills for being an outstanding coauthor and a better friend. Chapter 2 of this dissertation is based on a joint project. I learned a lot from him and I know our collaboration will last throughout my academic career and our friendship beyond that.

My research benefited enormously during these years from the whole faculty at the Department of Economics at the University of Pennsylvania. In particular I would like to thank Dirk Kruger, José Victor Rios Rull and Alessandro Dovis their
very useful advice. I am very grateful to my fellow students, especially to every member of the Macro Club, no place is as challenging and fruitful as my home department.

I was very blessed to have the support of my friends during my years at the University of Pennsylvania. I would like to especially mention Rodrigo Azuero, Mauricio Calani, Gustavo Camilo, Michael Chirico, Alberto Ciancio, Ana Gazzmuri, Cinthia Konichi, Pau Pereira, Rodimiro Rodrigo, Constanza Vergara, Daniel Wills and David Zarruk. My gratitude also goes to two important people: Kelly Quinn and Charmaine Thomas.

Last but not least, I would like to thank my family. To my wife Angela, who has been truly the wind beneath my wings, 10 years ago we were a couple of young bachelors with a shared dream, through ups and downs she made this journey worth every second, I could not have made it this far without her by my side. This is as much her accomplishment as is mine. To my parents Jimmy and Martha for always being there for me even when they are physically distant, and for continuously reminding me of what is important in life. To my brothers Farid, Tomas and Samuel, for all the much needed joyful moments.

Juan M. Hernandez

Philadelphia, PA

April 26, 2017
ABSTRACT

ESSAYS IN MACROECONOMICS WITH FINANCIAL FRICTIONS

Juan M. Hernandez
Enrique G. Mendoza

How can governments design policies that alleviate the macroeconomic implications of financial frictions? This dissertation contributes to answer this question focusing on two aspects: international borrowing and crisis prevention at the country’s level, and the impact of taxation and financial regulation on entrepreneurship at the agent’s level. In the first chapter, debt crises arise from the incompleteness of sovereign debt markets: the government cannot credibly commit to repay or default in certain states of the world and this gives way to non-fundamental debt crises. In a strategic default environment, I show that international reserve holdings help to reduce the probability of these market-driven debt crises, advancing the theoretical literature that had struggled to explain why countries hold reserves while indebted. The results are consistent with previous empirical results that had shown countries with greater reserve holdings faced lower spreads in the sovereign debt markets, which is at odds with the previous theories. In the second chapter, a small open economy faces an aggregate borrowing constraint and the agents fail to internalize how their private borrowing decisions push the total debt towards the limit, making the current account adjustment more severe. We model the decentralized and planner’s problem and find the optimal capital control policies, these are very effective to move the economy to the first-best scenario but also very hard to implement, given their state contingent nature. We then address the effectiveness of simpler policy rules, and find that they can bring welfare gains but had to be carefully designed. Finally,
in the third chapter, the competition among investors for the most promising entrepreneurs, under adverse selection and limited liability, leads to an excessive entry into entrepreneurship activity and allocates resources to socially inefficient projects. We solve the optimal contracting problem and show that the inefficiency disappears if at least one of the next three is missing: competition in financial intermediation, adverse selection or limited liability. We also show that a small cost or fee per contract, like red-tape requirements, is enough to restore efficiency, making a case for financial regulation.
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Introduction

Financial frictions arise from market incompleteness have been found to severely affect macroeconomic outcomes. Three different frictions at different levels are studied here: lack of commitment by a government in the sovereign debt markets, a negative externality private borrowing entails when the economy faces an aggregate borrowing constraint, and adverse selection together with limited liability in the competition among financial intermediaries. In each case a policy response is proposed: international reserve holdings as a mechanism to prevent market-driven debt crises, capital controls to make agents internalize the social cost of their indebtedness and red-tape financial regulation to prevent excessive entry into entrepreneurship.

In the first chapter of this dissertation, “How International Reserves reduce the Probability of Debt Crises” I provide a model to explain why many emerging economies maintain significant positions in both external sovereign debt and foreign reserves, paying spreads of over 250 basis points on average. Arguments advanced in empirical work and policy discussions suggest that governments may do this because international reserves play a role in reducing the likelihood of sovereign debt crises, improving a country’s access to debt markets. I propose a model that justifies that argument.

The government makes optimal choices of debt and reserves in an environment in which self-fulfilling rollover crises a-là Cole-Kehoe and external default a-là Eaton-
Gersovitz coexist. This allows for both fundamental and market-sentiment-driven debt crises. Self-fulfilling crises arise because of a lender’s coordination problem when multiple equilibria are feasible. Conditional on the country’s Net Foreign Asset position, additional reserves make the sovereign more willing to service its debt even when no new borrowing is possible, which enlarges the set of states in which repayment is the dominant strategy and, hence, reduces the set of states that admit a self-fulfilling crisis. From an ex-ante perspective, reserves reduce the probability of crises in the future which lowers current sovereign spreads. The result depends on the existence of roll-over risk and debt not being limited to one period debt. This framework advances existing models by accounting not only for the self-insurance role of reserves against self-fulfilling crises but also for their part in reducing the probability of such events. My findings are in line with the empirical literature on vulnerability measures to sovereign debt crisis that shows the connection between international reserves, the probability of debt crises and sovereign spreads. Quantitatively the model can explain 50% of Mexico’s international reserves holdings, while accounting for key cyclical facts.

The second chapter, joint work with Enrique Mendoza, is titled “Optimal v. Simple Financial Policy Rules in an Equilibrium Model of Credit Booms and Crashes”. There we evaluate the effectiveness of optimal versus simple financial policy rules in a model of a small open economy with production, liability dollarization and “unconventional shocks” in the form of global liquidity shifts and news about future fundamentals. In our model, both tradable and nontradable goods are produced using intermediate goods traded in world markets. Debt is denominated in units of tradables, and a collateral constraint limits debt to a fraction of the market value of total income. The optimal policy has a macroprudential or ex-ante component: a debt tax levied at date \( t \) only when the collateral constraint is not currently binding but may do so at \( t + 1 \), as well as an ex-post component: sectoral pro-
duction taxes/subsidies used when the collateral constraint binds. The optimal policy, although complex, is very effective at reducing the magnitude and severity of financial crises. Simple policies can be effective but need to be constructed carefully otherwise they can be welfare reducing.

The third and final chapter is titled “Fighting for the Best, Losing with the Rest: On the Desirability of Competition in Financial Markets” and was developed with Daniel Wills. Our inquiry was motivated by the Jumpstart Our Business Startups (JOBS) act of 2012, which aimed at increasing funding access for young firms by easing securities regulation. We ask if there is a role for the regulation of the market of funds for firms that lack collateral and have a large uncertainty about their ability to generate profits. To answer that we characterize optimal financial contracts in a competitive environment with risk, adverse selection and limited liability. We find that competition among financial intermediaries always forces them to fund projects with negative expected returns both from a private and from a social perspective. Intermediaries use steep payoff schedules to screen entrepreneurs, but limited liability implies this can only be done by giving more to all entrepreneurs. In equilibrium competition for the profitable entrepreneurs force intermediaries to offer better terms to all customers, there is cross subsidization among entrepreneurs and intermediation profits are nil. The three main features of our framework (competition, adverse selection and limited liability) are necessary in order to get the inefficient laissez-faire outcome and a role for financial regulation. Our result remains robust when firms can collateralize some portion of the credit as long as there is still an unsecured fraction.
Chapter 1

How International Reserves Reduce the Probability of Debt Crises

1.1. Introduction

Several emerging economies have both foreign currency denominated debt outstanding and international reserve holdings. This coexistence is costly for the government, since it pays a higher interest rate on its debt than what it earns on its reserves holdings. This suggests there should be some extra benefit of reserve holdings.

The right panel of Table 1 presents the foreign debt stock and reserves holding as a percentage of GDP in 2015 for a panel of emerging economies. The median country held 16.9% of GDP in reserves while owing 14.9% in foreign currency and paying a spread of 238 basis points over the return on reserves. The numbers from 2015 are not only a product of the recent trend in emerging economies reserves accumulation. The left panel on Table 1 shows the 15 year average, between 2001 and 2015, of debt, reserves and sovereign spreads for the same countries. The data shows that the concurrence of debt and reserves is not a short-term phenomenon. The main reason cited for emerging markets reserve holdings is known as the
Table 1: External Debt, Reserves and Spreads

<table>
<thead>
<tr>
<th>Country</th>
<th>Reserves</th>
<th>Debt</th>
<th>Spread</th>
<th>Reserves</th>
<th>Debt</th>
<th>Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>9.8</td>
<td>31.3</td>
<td>1478</td>
<td>6.0</td>
<td>18.7</td>
<td>458</td>
</tr>
<tr>
<td>Brazil</td>
<td>11.5</td>
<td>7.0</td>
<td>391</td>
<td>23.2</td>
<td>4.4</td>
<td>446</td>
</tr>
<tr>
<td>Bulgaria</td>
<td>31.7</td>
<td>18.1</td>
<td>196</td>
<td>45.5</td>
<td>11.9</td>
<td>68</td>
</tr>
<tr>
<td>Chile</td>
<td>15.2</td>
<td>2.9</td>
<td>147</td>
<td>16.8</td>
<td>2.8</td>
<td>226</td>
</tr>
<tr>
<td>Colombia</td>
<td>10.7</td>
<td>13.1</td>
<td>298</td>
<td>18.3</td>
<td>16.6</td>
<td>332</td>
</tr>
<tr>
<td>Indonesia</td>
<td>12.0</td>
<td>21.0</td>
<td>282</td>
<td>12.1</td>
<td>17.0</td>
<td>317</td>
</tr>
<tr>
<td>Lithuania</td>
<td>15.0</td>
<td>20.9</td>
<td>228</td>
<td>6.5</td>
<td>31.1</td>
<td>122</td>
</tr>
<tr>
<td>Mexico</td>
<td>10.1</td>
<td>12.0</td>
<td>187</td>
<td>16.3</td>
<td>18.7</td>
<td>216</td>
</tr>
<tr>
<td>Peru</td>
<td>24.3</td>
<td>21.2</td>
<td>268</td>
<td>32.9</td>
<td>11.1</td>
<td>235</td>
</tr>
<tr>
<td>Philippines</td>
<td>23.4</td>
<td>24.0</td>
<td>298</td>
<td>28.3</td>
<td>10.8</td>
<td>121</td>
</tr>
<tr>
<td>Poland</td>
<td>16.5</td>
<td>26.6</td>
<td>73</td>
<td>20.1</td>
<td>29.4</td>
<td>51</td>
</tr>
<tr>
<td>South Africa</td>
<td>10.1</td>
<td>9.2</td>
<td>214</td>
<td>16.8</td>
<td>16.7</td>
<td>406</td>
</tr>
<tr>
<td>Turkey</td>
<td>12.0</td>
<td>13.3</td>
<td>356</td>
<td>16.1</td>
<td>13.1</td>
<td>322</td>
</tr>
<tr>
<td>Ukraine</td>
<td>16.0</td>
<td>15.3</td>
<td>712</td>
<td>6.6</td>
<td>30.3</td>
<td>1621</td>
</tr>
</tbody>
</table>

Median: 15.1 14.3 248 16.9 14.9 238

The precautionary motive: reserves are a liquidity buffer that protects the sovereign against adverse developments in financial markets. This motive for holding reserves has been adopted by the mainstream policy institutions, as the following excerpt from the reserves definition in the 6th edition of the IMF’s Balance of Payments Manual shows:

"external assets . . . controlled by monetary authorities for meeting balance of payments financing needs, for intervention in exchange markets . . . , and for other related purposes (such as maintaining confidence in the currency and the economy, and serving as a basis for foreign borrowing)."

Other explanations, like intergenerational wealth transfers and exchange rate management, have been suggested to account for the holding of reserves, but they are still problematic. Although some of those countries with high international reserves have positive Net Foreign Asset (NFA) positions -like China-, the majority of emerging economies are still indebted vis-a-vis the rest of the world, which precludes the wealth transfer motive. Similarly, several of those countries...
have floating exchange rate regimes, weakening the case for an export promotion or currency management motive.

This paper focuses on the precautionary motive for holding reserves, in particular, in their role in preventing debt crises, maintaining confidence in the economy, and enhancing foreign borrowing conditions. Empirical evidence supports this case: Ben-Bassat and Gottlieb (1992), Calvo, Izquierdo, and Loo-Kung (2013) and Tavares (2015) have found that reserves are correlated with smaller spreads and a lower probability of sudden stops, defined as large current account reversals.

Figure 1 hints at the effect of reserves in reducing spreads. The scatter plot shows reserves-to-GDP ratio and sovereign spreads where each point represents a country and quarter in the dataset, described in Appendix A.3. A simple linear relation fitted between the two variables shows a negative correlation.

Ben-Bassat and Gottlieb (1992), Tavares (2015), Gumus (2016), and Levy Yeyati (2008) among others,\(^1\) build country panels and estimate the effect of reserves on

\(^1\)See Petrova, Papaioannou, and Bellas (2010) for a survey of papers assessing the effect of reserves and
the sovereign spreads under different specifications. They find estimates for the coefficient of the reserves-to-GDP ratio on sovereign in the range [-8.0, -2.5] which indicates that for each percentage point increase of the reserves-output ratio the sovereign spread falls between 2.5 and 8 basis points. Table 2 presents estimates of a common panel data regression specification regressing spreads on lagged reserves, debt, current account, output growth, an exchange rate regime dummy and the aggregate EMBI+ spread. The results are in line with previous studies, in particular, the point estimate of the spread reduction, after a 1% increase in the reserves-to-GDP ratio, is between 5.5 and 7.6 basis points.

In a similar setting, Calvo et al. (2013) use a panel of emerging and developed countries to assess the impact of international reserves in the cost and probability of sudden-stops, and find that reserves holdings reduce both of them. Gourinchas and Obstfeld (2012) find that international reserve holdings significantly reduce the probability of future crisis (default, currency and banking) on emerging economies, in particular they report a marginal effect of the reserves-to-GDP ratio on the default probability of -0.59.

On the theoretical side, however, sovereign debt models with strategic default struggle to incorporate reserves. International reserves are protected by the sovereign immunity clause, which means they are not pledgable and cannot be seized by creditors in case of a default. This makes more challenging for these models to sustain both debt and reserve holdings since reserves make default more attractive. Bulow and Rogoff (1989) showed that if the sovereign is allowed to accumulate assets after default, reputation costs are not enough to support lending since

other fundamental variables on sovereign spreads. For a survey of the literature on reserves and the probability of crises see Bussière, Cheng, Chinn, and Lisack (2015).

Calvo et al. (2013) assume that the probability of sudden stops comes from a Probit model including reserves, the output cost of default is linear in reserves, and there is a constant cost of holding reserves. The first order condition for reserves is obtained analytically. Then they proceed to estimate the Probit model for the likelihood of sudden-stops and the linear model for the output cost, and include those estimates in their first order condition. Given the specification uncertainty, they assume the sovereign chooses the most conservative model, which is the one yielding larger reserves. For Mexico, they find the optimal reserve to GDP ratio to be 15% in 2007 and 22% in 2010.
Table 2: Effect of reserves on spreads

<table>
<thead>
<tr>
<th></th>
<th>Pooled OLS</th>
<th>Random Effects</th>
<th>Fixed Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td>L. Reserves/GDP</td>
<td>-7.559***</td>
<td>-5.512***</td>
<td>-5.561**</td>
</tr>
<tr>
<td></td>
<td>(1.141)</td>
<td>(2.032)</td>
<td>(2.260)</td>
</tr>
<tr>
<td>L. Debt/GDP</td>
<td>2.964**</td>
<td>6.911***</td>
<td>7.797***</td>
</tr>
<tr>
<td></td>
<td>(1.226)</td>
<td>(1.930)</td>
<td>(2.059)</td>
</tr>
<tr>
<td>L. Agg. EMBI+</td>
<td>0.369***</td>
<td>0.324***</td>
<td>0.313***</td>
</tr>
<tr>
<td></td>
<td>(0.080)</td>
<td>(0.069)</td>
<td>(0.069)</td>
</tr>
<tr>
<td>L. Current Account/GDP</td>
<td>-3.807*</td>
<td>-10.397***</td>
<td>-10.907***</td>
</tr>
<tr>
<td></td>
<td>(2.095)</td>
<td>(2.045)</td>
<td>(2.072)</td>
</tr>
<tr>
<td>L. Exchange rate regime</td>
<td>205.070***</td>
<td>136.424***</td>
<td>124.703***</td>
</tr>
<tr>
<td></td>
<td>(23.888)</td>
<td>(34.515)</td>
<td>(36.439)</td>
</tr>
<tr>
<td></td>
<td>(7.186)</td>
<td>(5.968)</td>
<td>(5.987)</td>
</tr>
<tr>
<td>Cons.</td>
<td>186.433***</td>
<td>131.684*</td>
<td>152.143**</td>
</tr>
<tr>
<td></td>
<td>(40.297)</td>
<td>(70.990)</td>
<td>(62.884)</td>
</tr>
<tr>
<td>R²</td>
<td>0.313</td>
<td>0.280</td>
<td>0.304</td>
</tr>
<tr>
<td>N</td>
<td>650</td>
<td>650</td>
<td>650</td>
</tr>
</tbody>
</table>

Notes: EMBI spread (basis points) as dependent variable. All ratios are calculated using the current GDP; the Reserves to GDP ratio and Debt to GDP is calculated as the variable divided by four times the quarterly GDP (annualized). Exchange rate regime corresponds to a dummy variable that equals 1 for countries with a fixed exchange rate regime. Additional information about the data used can be found in the appendix. Standard errors in parentheses. The R² statistic of the random effects model (column 2) corresponds to the overall R². * Significance at the 10 percent level. ** Significance at the 5 percent level. *** Significance at the 1 percent level.

default happens when debt is at its highest. With reserves, a front-loaded version of this mechanism applies, since the sovereign can increase its debt quickly and default on it in the next period, keeping the proceeds from debt issuance as reserves.

Recent developments in quantitative models of sovereign debt and international reserves have had some success in explaining the coexistence of debt and reserves by stressing the hedging properties of international reserves in the event of a sudden stop. Jeanne and Rancière (2011) highlight the insurance role of reserves against rollover crises in a simple model with switching exogenous collateral constraints. With those insights in mind, Bianchi, Hatchondo, and Martinez (2012)
added exogenous rollover crises to an Eaton-Gersovitz style model of long-term sovereign debt. Their model can fully explain reserves accumulation in Mexico while keeping track of standard cyclical moments, but the probability of those exogenous crises has to be introduced as a function of reserves. While these papers are able to represent the empirical facts, they model the link between the probability of sudden stops and the reserves as an exogenous object, which does not allow the authors to assess if those holdings are useful for preventing sovereign debt crises. This contrasts sharply with the findings of empirical studies suggesting that international reserves help to prevent debt crises. Those empirical regularities have been used to build several measures of international reserves adequacy, assess optimal reserve holdings, and even prescribe policy, but there is little understanding of the mechanism that drives them.

In the model developed in this paper, self-fulfilling debt crises as developed by Cole and Kehoe (2000) are introduced. Reserves affect the probability of a sudden stop generated from a self-fulfilling debt crisis via an endogenous mechanism. In choosing the share of international reserves in the sovereign’s portfolio, the government takes into account that the frequency of those crises is determined by the extent to which reserves can move the economy in and out of states of nature in which the coordination problem may give way to a self-fulfilling debt crisis. This provides an explanation for reserves accumulation that does not depend on ad-hoc assumptions about their effect on the dynamics of the sovereign’s economy or on other debt market participants.

One fact that is indicative of the potential empirical relevance of self-fulfilling debt crises is the observation of several periods in which countries with different fundamentals all face spread widening and current account reversals. Figure 2 shows the percentage of countries with spread two standard deviations above

\[ \text{In their latest version Bianchi, Hatchondo, and Martinez (2016b) drop the exogenous rollover crises and instead use a risk aversion shift on the lenders’ side, exogenously tied to the sovereign’s income shock. They mention that this link accounts for half of the reserves holdings explained by their numerical exercise.} \]
This figure shows the percentage of countries from the sample whose EMBI spread is more than two times the standard deviation above its corresponding mean. Additional information about the data and sample used can be found in the data appendix.

their corresponding means in each month over the 1994-2015 period. By construction, when multiple countries are facing an unusually high spread, the aggregate spread must be higher as well. Calvo, Izquierdo, and Mejía (2008) identified and named these events Systemic Sudden Stops (3S): periods with high aggregate spreads and where multiple countries face simultaneous current account reversals. These 3S periods suggest a driving external financial factor. Other recent studies have presented evidence in the same line: Aguiar, Chatterjee, Cole, and Stangebye (2016b) found a common exogenous factor driving spreads for a panel of emerging markets. Tavares (2015) includes a time effect in his empirical specifications for the sovereign spread. In Table 2, the coefficient on the EMBI+ spread, which is the same across all countries, captures this effect.

This paper contributes to the existing literature by setting up a model that not only can quantify the impact of international reserves on the probability of self-fulfilling debt crises but also produces an endogenous mechanism linking both.
in the financial markets but only when fundamentals allow for doubts about repayment by the sovereign.

The intuition for the mechanism is that, conditional on a Net Foreign Asset position (NFA), additional reserves make the sovereign more willing to service its debt even if there is an episode of market panic and no new borrowing is possible. This reduces the set of states that admit a self-fulfilling crises. Seen from an ex-ante perspective, the additional holdings of international reserves reduce the probability of a self-fulfilling debt crisis in the future which in turn decrease current sovereign spreads.

The quantitative implications of the model are studied in a numerical exercise calibrated to Mexico. The model can account for 50% of the country’s international reserve holdings, while matching the average outstanding debt, the volatility of the sovereign spread and Mexico’s default frequency. The model also features the negative correlation between spreads and reserves observed in the data.

The model can be used to quantify the optimal portfolio of reserves and external debt for the sovereign. Additionally, this framework can explain empirical regularities regarding reserves holdings and measures of crises vulnerability used in policy circles. The Guidotti-Greenspan rule, which links the probability of sudden stops to the ratio of reserves to short term debt, is the most popular reserve adequacy metric among policymakers but not the only one. This paper proposes a micro-founded explanation for that and other empirical regularities, by endogenously tying the probability of crises to economic fundamentals. Finally the setup is able to assess the effectiveness of contingent lending policies put in place by the IMF and developed countries’ central banks aimed at preventing confidence crises.

4Other measures include ratios of reserves to several variables like: monthly imports, current account deficit, total debt and short term debt. In addition, according to the IMF’s survey of reserve managers, countries approach to assess reserve adequacy usually include a mixture of these ratios.

5IMF’s Flexible Credit Line (FCL) is one of those contingent credit facilities. This was aimed at countries with strong fundamentals for crisis-prevention and crisis-mitigation lending. It allows the sovereign to draw
1.1.1. Related literature

This paper relates to the sudden stops literature which has been able to explain reserve holdings as self-insurance against the risk of sudden reversal in financing. Jeanne and Rancière (2011) highlight the insurance role of reserves against sudden stops in a simple model in which an exogenous collateral constraint suddenly switches preventing new borrowing. Durdu, Mendoza, and Terrones (2009b) present a model in which a planner internalizes how the choice of reserves affects the probability and magnitude of sudden stops driven by the deflation mechanism. However, these papers abstract from modeling sovereign default and the behavior of lenders while this paper models sovereign default and the response of lenders to reserves accumulation, which is key for the role of reserves in preventing crises.

This paper builds on the long strain of strategic sovereign default models pioneered by Eaton and Gersovitz (1981) and further developed by Aguiar and Gopinath (2006) and Arellano (2008) whose work highlighted the quantitative relevance of that setup by replicating key business cycle statistics for emerging market economies. In these models however, the sovereign was not allowed to simultaneously hold assets and debt.

The first attempt to model both debt and reserves in a setup with default was proposed by Alfaro and Kanczuk (2009). They found that reserves are too costly to hold in equilibrium. With only one-period debt, the net position (reserves on the credit line at any time (within a pre-specified period) which turned those funds into contingent reserves without imposing additional conditions for disbursement. Three countries: Colombia, Mexico and Poland used the FCL but none of them withdrew any funds. However it is still believed that at the peak of the global financial crisis in 2009 the mere existence of this arrangement injected confidence in the markets. Given our endogenous self-fulfilling crises framework, our model is in a better position to address the impact of this instrument in precluding the possibility of such events. Bocola and Dovis (2015) use a similar framework to evaluate the ECB’s promise to buy European sovereign bonds (Draghi’s “whatever it takes”) on European spreads.

Durdu et al. (2009b) do not model the portfolio choice of debt and reserves, the latter arise as a higher NFA position. Also their model is not of sovereign default but one of private debt crises driven by Fisherian deflation.

A comprehensive review of the literature that followed can be found in Aguiar et al. (2016b).
minus debt) is the relevant state after the sovereign chooses to repay. But after default, reserves can compensate the direct output costs and allow for consumption smoothing, making exclusion from financial markets more bearable. Hence, conditional on a NFA, more reserves make the sovereign less likely to repay. Lenders react to this by increasing spreads and in equilibrium debt stocks fall. In this paper, reserves still make default more attractive for the sovereign, but allowing for long-term debt and self-fulfilling crises makes the gross portfolio position relevant after repayment, allowing a role for reserves.

More recently, Bianchi et al. (2012) set up a model of endogenous default, long-term debt and exogenous roll-over crises, which can explain Mexico’s reserves holdings. In their model the probability of roll-over crises is assumed to be a decreasing function of reserves. In contrast, the framework developed here endogenously ties the probability of crises to the fundamentals of the economy, and in particular to reserves holdings, which allows for them to play a role in crisis prevention.

This paper also draws from the self-fulfilling debt crises literature which has two main branches: the first one pioneered by Calvo (1988), features multiple current debt prices consistent with future consumption and default decisions. Recent papers on the Calvo tradition are Lorenzoni and Werning (2013) and Nicolini, Teles, Ayres, and Navarro (2015). In a setup similar to bank run, Cole and Kehoe (2000) develop a framework in which both repayment and default can be equilibrium actions depending on the lenders’ decision to roll over debt or not. Recently Chatterjee and Eyigungor (2012) introduce this setup in their Eaton-Gersovitz model of long term debt to make its issuance a superior alternative to short-term bonds.

The self-fulfilling debt crises environment has also been used to assess the role of an International Lender Of Last Resort (ILOR): Roch and Uhlig (2016) introduce an ILOR and determine the minimal intervention needed to rule out self-fulfilling
crises. Bocola and Dovis (2015) measure the role of self-fulfilling crises in recent European debt crises and the impact of the ECB’s Outright Monetary Transactions (OMT) announcement. This paper complements that literature, as reserves emerge as an alternative to the insurance and crisis prevention role of the ILOR. Other attempts have given reserves different roles to explain their coexistence with debt. Tavares (2015) gathers empirical evidence that confirms reserves reduce sovereign spreads, then proposes a model in which reserves are necessary after a default to make a settlement on a fraction of the stock of defaulted debt. Building on the Diamond and Dybvig (1983) bank run framework, Hur and Kondo (2013) set up an optimal contracting problem to find a role for reserves as a collateral that prevents runs on the sovereign debt. In this paper reserves are not given any additional role besides being a non-seizable cash buffer, which makes the task of explaining their coexistence more challenging.

This paper is also related to the credit card puzzle literature of Telyukova (2013) and Telyukova and Wright (2008). In these models, the existence of certain goods that cannot be purchased with credit cards can explain the coexistence of cash balances and credit card debt in households portfolios. This is similar to the sudden stop literature in which the reserves balances of the sovereign can be explained by the existence of certain states where credit is not available. However the legal environments for the two problems differ substantially. Bankruptcy laws prevent households and corporations from holding assets after a default event while international reserves are protected by the Sovereign Immunity clause and cannot be seized by lenders. Hence, cash makes the household credit safer while reserves make the sovereign riskier.

The rest of the paper continues as follows: Section 2 presents the empirical facts. In Section 3 the model is specified and its theoretical properties are discussed. Section 4 presents the quantitative analysis and results. Finally, Section 5 con-
cludes and states directions of further research.

1.2. The Model

The model is based on the classic sovereign default model proposed by Eaton and Gersovitz (1981) with the modifications to introduce long-term debt and self-fulfilling debt crises incorporated by Chatterjee and Eyigungor (2012). The main addition is the introduction of foreign reserves (an asset that the sovereign can hold) and the portfolio choice of debt and reserves.

1.2.1. The Sovereign

Preferences, endowments, choices

The sovereign seeks to maximize the representative agent’s utility:

$$
E \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \right],
$$

where $u(\cdot)$ is a twice-continuously differentiable utility function, $c_t$ is consumption and $\beta < 1$ is the discount factor.

The model can be thought of as a benevolent government maximizing private utility subject to resource constraints, or as the government’s own utility considering its own expenditures and revenues. As is common in the literature, the former is assumed here.

The resource constraint faced by the government varies depending on whether it chooses to repay or not, and on whether there is a self-fulfilling debt crises preventing it from accessing credit markets. Below, the resource constraints under each scenario and the corresponding recursive maximization problem are described.

In all of these scenarios, the government draws a realization of endowment income process denoted $y_t$. The log-income ($\log(y_t)$) follows an AR(1) process.

The sovereign has access to two financial instruments. First, a reserve asset, de-
noted \( a_t \), that earns an exogenous world-determined real rate of return \( r \). Second, long-term bonds specified as in Chatterjee and Eyigungor (2012).

In particular, the government issues long-term bonds described by two parameters: their coupon \( z \) and their maturity \( \frac{1}{\lambda} \). Only one fixed type of \((z, \lambda)\) bond can be issued. Each period a fraction \( \lambda \in (0, 1) \) of the bond matures. The remaining \((1 - \lambda)\) pays the coupon \( z \) and is automatically rolled over. By keeping \((z, \lambda)\) fixed, the model only needs to keep track of current outstanding debt. Remaining bonds look exactly like newly issued bonds. Hence the law of motion of the outstanding debt law of motion is:

\[
b' = (1 - \lambda)b + \text{current issuance}
\]

The government has the option to default on previous obligations and default entails two different costs: first, the government loses access to credit markets, with the standard exogenous probability of re-entry \( \epsilon \in (0, 1) \); second, the government loses a fraction of the income \( \phi(y_t) \) while it remains excluded from credit markets. As Arellano (2008) shows, this is necessary for the model to be able to generate debt stocks consistent with the observed levels and to make default occur at low income realizations. When the government defaults, it is immediately excluded from borrowing in the same period the decision is made. Upon re-entry, all previous obligations are void.

**The Government’s Problem**

The timeline of decisions is as follows: the government enters the period with reserve holdings \( a \) and a stock of outstanding debt \( b \) if it was not in default state the previous period. First the exogenous shocks are realized: income \( y \), the

---

\(^{8}\)Hatchondo and Martinez (2009) and Arellano and Ramanarayanan (2012) also incorporate long term bonds in similar environments but with bonds limited to be a geometrically declining series of coupons, which is the special case of the setup developed by Chatterjee and Eyigungor (2012) when \( z = 1 \). Here \( z \) is set to be equal to the spread plus the risk-free rate, such that on average bonds are issued at par, as is common practice, and nominal and face values are roughly the same which facilitates comparisons.
sunspot variable $\omega$ and the reentry shock if the government was in default. After that, if the government has access to financial markets the state of the economy is $s = (y, a, b, \omega)$ and the value for the government as a function of the state is denoted $W(y, a, b, \omega)$. The government then chooses whether to repay ($\delta = 0$) or default ($\delta = 1$), and the portfolio for next period $(a', b')$. Simultaneously the lenders decide whether to enter the bond auction or not. When lenders enter the auction and the government repays, the value for the government is $V^+(y, a, b, \omega)$; if the government repays but lenders do not enter the auction, the value for the government is $V^-(y, a, b, \omega)$; and when the government defaults, its debt obligations are void and the value is $X(y, a, \omega)$, which is the same value achieved by a government that was in default state the previous period and did not gain access to the market in the current period.

When the sovereign chooses to repay in the current period, it has access to the international capital markets and hence it can choose next period debt $b'$, reserves $a'$ and current consumption $c$ taking as given current income $y$, reserve holdings $a$ and debt outstanding $b$. It faces a pricing schedule for debt $q(y, a', b', \omega)$ that depends on future debt and reserves, and on current income $y$ and a sunspot variable $\omega \in \{0, 1\}$. When $\omega = 1$ the best equilibrium price schedule is offered by the lenders, and when $\omega = 0$ they coordinate on the worst possible pricing schedule and a self-fulfilling crisis may arise (see section 1.2.2). This sunspot variable follows a two-state Markov process to mimic the tranquil and panic regimes in the global financial markets.

Let $V^+(y, a, b, \omega)$ be the value function corresponding to the sovereign problem in case it has access to international capital markets, then:

\[
V^+(y, a, b, \omega) = \max_{a', b', c} \left\{ u(c) + \mathbb{E}_{y, \omega} \left[ \beta W(y', a', b', \omega') \right] \right\},
\]

s.t. \[ c = y + a - \frac{a'}{1 + r} - [\lambda + (1 - \lambda)z]b + q(y, a', b', \omega)[b' - (1 - \lambda)b]. \tag{1.1} \]
Under repayment, consumption equals the income realization plus the net resources generated by reserves and debt, which are equal to the economy’s balance of trade. The resources generated by reserves are given by \( a - \frac{a'}{1+r} \), where \( a \) is the payout on reserves carried over from the previous period and \( \frac{a'}{1+r} \) is the resource cost of buying reserves \( a' \) at the price \( \frac{1}{1+r} \). The term \( [\lambda + (1 - \lambda)z]b \) captures the debt service cost, which includes the fraction of outstanding debt maturing \( \lambda b \) and the coupon payment on the non-maturing part \( z(1 - \lambda)b \). Finally, current issuance of new bonds is \( [b' - (1 - \lambda)b] \) and the revenue collected is \( q(y, a', b', \omega)[b' - (1 - \lambda)b] \).

Note bond prices are forward looking but because of the persistence of the stochastic processes of \( y \) and \( w \), their current realizations provide information that is relevant for forecasting their future realizations, which is in turn relevant for expectations of future utility and prices.

When the government has chosen default and is outside the international markets it takes as given its income, stock of reserves and sunspot variable, \( s = (y, a, w) \). In this case \( \omega \) has no contemporaneous effect on consumption and there is no debt issuance, but it helps to predict \( \omega' \) which matters for the continuation values \( W \) and \( X \) next period. Let the value function in this situation be \( X(y, a, \omega) \):

\[
X(y, a, \omega) = \max_{c,a'} \left\{ u(c) + \beta(1 - \epsilon) E_{y,\omega} \left[ X(y', a', \omega') \right] + \beta \epsilon E_{y,\omega} \left[ W(y', a', 0, \omega') \right] \right\},
\]

\[
s.t. \quad c = y - \phi(y) + a - (1 + r)^{-1}a'.
\]

Under default, consumption equals the income realization plus the net resources generated by reserves. Income in this case is \( y - \phi(y) \) where \( \phi(\cdot) \) is the direct income cost of default. The resources generated by reserves are given by \( a - \frac{a'}{1+r} \), where \( a \) is the payout on reserves carried over from the previous period and \( \frac{a'}{1+r} \) is the resource cost of buying reserves \( a' \) at the price \( \frac{1}{1+r} \).
The value function $V^-$ represents the value the sovereign obtains in states in which it chooses to repay but it cannot issue new debt because lenders refused to enter the auction. It is very similar to the function $V^+$ but with an extra constraint forbidding new bond issuance $b' \leq (1 - \lambda)b$.

$$V^-(y, a, b, \omega) = \max_{a', b'} \left\{ u(c) + \mathbb{E}_{y', \omega} \left[ \beta W(y', a', b', \omega') \right] \right\},$$

s.t. $c = y + a - \frac{a'}{1 + r} - [\lambda + (1 - \lambda)z]b + q(y, a', b', \omega)[b' - (1 - \lambda)b].$  \hspace{1cm} (1.3)

To complete the formulation of the sovereign’s problem, it is needed to define the unconditional value function $W$ which is determined by the default decision. Given the equilibrium in the default rollover game (defined in the next section), the sovereign’s default decision is:

$$\delta^*(y, a, b, \omega) = \begin{cases} 
0 & \text{if } X(y, a, \omega) \leq V^-(y, a, b, \omega) \\
1 & \text{if } X(y, a, \omega) > V^+(y, a, b, \omega) \\
0 & \text{o/w if } V^-(y, a, b, \omega) < X(y, a, \omega) \leq V^+(y, a, b, \omega), \omega = 1 \\
1 & \text{o/w if } \omega = 0,
\end{cases}$$ \hspace{1cm} (1.4)

where $\delta(\cdot) = 1$ indicates default and $\delta(\cdot) = 0$ repayment. The unconditional value ($W(\cdot)$) of the sovereign before the default decision is made, implied by the default
decision above is:

\[
W(y, a, b, \omega) = \begin{cases} 
V^+(y, a, b, \omega) & \text{if } X(y, a, \omega) \leq V^-(y, a, b, \omega) \\
X(y, a, \omega) & \text{if } X(y, a, \omega) > V^+(y, a, b, \omega) \\
V^+(y, a, b, \omega) & \text{o/w if } \omega = 1 \\
X(y, a, \omega) & \text{o/w if } \omega = 0 
\end{cases}
\]  

(1.5)

This function encompasses the default decision \(\delta(\cdot)\). Whenever \(W(\cdot) = X(\cdot)\) the sovereign is optimally choosing to default \(\delta^*(\cdot) = 1\), and it is choosing to repay in the other cases. In the first case \(X \leq V^- \leq V^+\), repayment is better no matter if there is new lending or not. In the second \(X > V^+ \leq V^-\) and default is the dominant strategy. But when \(V^- < X < V^+\) multiplicity arises and the choice depends on \(\omega\). The next section presents the auction game and the reasoning behind this value function and the default-repay decision.

1.2.2. Rollover crises

Self-fulfilling crises are introduced as in Chatterjee and Eyigungor (2012). In every period, the sovereign and the lenders make simultaneous decisions. The sovereign chooses between default and repaying and the lenders choose whether to enter the auction for newly issued bonds or not. Multiplicity will arise when the government finds it better to repay if it can issue new bonds, but rather defaults if there is no new lending.\(^9\)

The Game of Default and Rollover

The government and the lenders will play a default-rollover game every period. The government has two actions, default or repay. The payoffs for the sovereign come from the value functions defined in the previous section. If the government

\(^9\)Recent work by Auclert and Rognlie (2016) established uniqueness of equilibrium in the Eaton-Gersovitz type of models under very mild conditions. However, they acknowledge that multiplicity exists when long-term debt is present (as in Chatterjee and Eyigungor (2012)) and when the sovereign savings are unbounded, both of which are true in this setup.
defaults, it earns $X(y, a, \omega)$ no matter what the lenders do. This assumes that in the case of a default, the sovereign does not receive the funds set aside by the lenders that entered the auction. When the government repays and all lenders enter the auction, the sovereign gets $V^+(y, a, b, \omega)$, but if instead no lender enters the auction, the sovereign obtains $V^-(y, a, b, \omega)$.

There is a continuum of risk-neutral and symmetric lenders each endowed with a finite amount of funds. New lenders are always available each period. Previous bondholders cannot limit the actions of new lenders unless the country is excluded from the financial markets after a default event. Lenders have two possible actions, enter the bond auction and lend, or stay out and not lend. Lenders form beliefs on what other lenders and the government are doing and at equilibrium those beliefs are consistent with the actions of the players. When a lender choose not to lend she will earn zero no matter what other players do. When all lenders choose to lend and the sovereign chooses to repay, the lenders drive the bond price down until they break even, earning zero. If a lender enters the auction and the sovereign defaults, the lender loses an amount $\Delta > 0$ for the opportunity cost.  

All players move simultaneously. The equilibrium concept used here is a pure-strategies Markov equilibrium.  

When the combined funds of the set of lenders that entered the auction is less than the breakeven price for the bond times the new issuance, the bonds are sold at a lower price.

In equilibrium, symmetry implies all lenders choose the same action unless both actions yield the lenders the same utility. But for the lenders to be indifferent, it

---

10 The nature of this cost $\Delta$ is immaterial for the equilibrium as long as it is positive. It can be equal to the total bid the lender made, or just a small fraction representing as the opportunity cost of setting aside funds for an auction.

11 Recently Aguiar et al. (2016b) generalized this timing structure, allowing for a detailed within period auction-settlement separation and microfounding some of the assumptions made here. In the end they showed that under pure strategies the equilibrium is exactly the same as the one coming out from this setup. In that sense this setup is a simplified version of theirs.
must be the case that the sovereign is repaying (because $\Delta > 0$) and that the bond price in the auction is the break-even price (a lot of lenders enter the auction). This implies that either the auction is fully subscribed or no lender enters, and the outcome is equivalent to the one under the assumption that all lenders do the same. Hence, the payoff matrix is given by:

<table>
<thead>
<tr>
<th></th>
<th>Repay</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lend</td>
<td>0, $V^+(s)$-$\Delta$, $X(s)$</td>
<td></td>
</tr>
<tr>
<td>Don’t Lend</td>
<td>0, $V^-(s)$ 0, $X(s)$</td>
<td></td>
</tr>
</tbody>
</table>

Conditional on a state vector $s = (y, a, b, \omega)$ the game reduces to a two player simultaneous-move game described by the payoff matrix above. Its Nash Equilibria depend on the government’s payoffs.

Case 1) If $X(s) < V^-(s) < V^+(s)$, then repay is the strictly dominant strategy for the sovereign and (lend, repay) is the unique Nash equilibrium.

Case 2) When $V^-(s) < V^+(s)$ then default is the strictly dominant strategy and thus (Don’t, Default) is the only Nash equilibrium.

Case 3) If $V^-(s) < X(s) < V^+(s)$ then there is no dominant strategy. In this case both (lend, repay) and (Don’t, Default) are Nash Equilibria. Here the sunspot variable $\omega \in \{0, 1\}$ plays a role: when $\omega = 1$ all agents coordinate to the (lend, repay) equilibrium, otherwise the (Don’t, Default) is played.

Lenders’ optimality condition

Let $\delta^*(y, a, b, \omega)$, $\alpha^*(y, a, b, \omega)$ and $b^*(y, a, b, \omega)$ be the policy functions for default, reserves and debt for the next period. The world risk-free interest rate $r$ is
exogenous. The zero-profits condition of lenders is:

\[ q(y, a', b', \omega) = (1 + r_f)^{-1} \mathbb{E}_{y, \omega} \left[ (1 - \delta^*(y', a', b', \omega')) \right. \times \ldots \]

\[ \left. \left[ \lambda + (1 - \lambda)z + (1 - \lambda)q(y', a'(y', a', b', \omega'), b^*(y', a', b', \omega')) \right] \right] . \tag{1.6} \]

1.2.3. Equilibrium

A Markov Perfect Equilibrium in this model consists of value functions \( W^+ : S \rightarrow \mathbb{R} \), \( V^+ : S \rightarrow \mathbb{R} \), \( V^- : S \rightarrow \mathbb{R} \), \( X : S_b \rightarrow \mathbb{R} \), and functions \( a^* : S \rightarrow \mathbb{R} \), \( b^* : S \rightarrow \mathbb{R} \) and \( \delta^* : S \rightarrow \mathbb{R} \) and a price function \( q : S \rightarrow \mathbb{R} \) defined over the state space \( S = \mathbb{R}_3^3 \times \{0, 1\} \) such that:

1. Given the unconditional value function \( W \), the default value function \( X \) solves the subproblem (1.2).

2. Given the price schedule \( q(\cdot) \), the unconditional value function \( W \) and the default value function \( X \), the value function \( V^+ \) solves subproblem (1.1).

3. Given the price schedule \( q(\cdot) \), the unconditional value function \( W \) and the default value function \( X \), the value function \( V^- \) solves subproblem (1.3).

4. Given the policy functions \( a^*, b^*, \delta^* \) the price function \( q \) satisfies (1.6).

1.3. The Model’s Mechanism

1.3.1. Description of the mechanism

A key aspect of the model is the specification of the rollover crises, which splits the state space into three regions. An upper region where default is never optimal even if there is no new lending, a bottom region where default is always optimal no matter the market conditions, and a middle region where repayment is optimal if and only if there is new lending. In this middle region the sunspot variable acts as a coordinating device that determines if there is a self-fulfilling debt crises
(ω = 1) or not (ω = 0). As a result, the probability of facing a self-fulfilling debt crisis is the product of two probabilities: the probability that the sunspot variable takes the value ω = 1 and the probability that the economy’s fundamentals land in the vulnerability region.

The effect of adding reserves to the standard Eaton-Gersovitz setup with one-period debt is as follows: take as given the country’s net foreign asset position, which is equal to reserves minus debt (i.e. NFA = a − b). Since reserves cannot be confiscated, it is known that an increase in reserves increases the value of default X(y, a, ω) for sure. Assuming the coupon z is such that debt is issued at par, an NFA neutral increase in reserves is matched with an increase in debt of equal size, and causes a decrease in the value of repayment V+(y, b, a, ω) next period. This happens because the sovereign loses the difference between the return on those assets. Hence, an NFA neutral increase in reserves shifts resources from repayment to default states, and increases the default probability next period. The resource shifting is undesirable ex-ante because resources are more scarce on repayment states. In addition, the higher default probability implies a lower revenue on issuance. Putting those two together it follows that an NFA neutral increase in reserves is undesirable, which implies no reserves in equilibrium, consistent with the findings of Alfaro and Kanczuk (2009).

The key channel in this paper is that reserve holdings also increase the value of repaying when there is no new lending V−(y, b, a, ω). This happens because extra reserves can be used to service the additional debt and, with long debt maturity, some will be left for consumption smoothing. This makes repayment the dominant strategy for the government in more states of the world, precluding the bad equilibrium of no-lending and default in those states. Hence, more international reserves shrink the vulnerability region of the state space (the region in which \( V^- < X < V^+ \)). From an ex-ante perspective, more reserves reduce the probabil-
ity of landing in that region, and thus the probability of a self-fulfilling crisis is reduced.

It is possible to shed further light on the mechanism by which reserves reduce the likelihood of self-fulfilling debt crises, and the conditions that allow the model to support a portfolio with debt and reserves, by assuming that the value functions, policy functions and bond pricing function are differentiable, which is done in this section. This is not necessary for the quantitative solution method to work.

1.3.2. Default thresholds

For a given debt, reserves and sunspot realization \((a, b, \omega)\), define the default thresholds \(d_f(a, b, \omega)\) and \(d_s(a, b, \omega)\), depicted on Figure 3, as the income realization that leaves the sovereign indifferent between repaying and defaulting. They are defined implicitly by:

\[
V^+(d_f(a, b, \omega), a, b, \omega) = X(d_f(a, b, \omega), a, \omega), \quad (1.7)
\]

\[
V^-(d_s(a, b, \omega), a, b, \omega) = X(d_s(a, b, \omega), a, \omega). \quad (1.8)
\]

For the case when there are no self-fulfilling debt crises \((\omega = 1)\), figure 3a depicts the value after default \(X(\cdot)\) and after repayment \(V^+(\cdot)\) as a function of the income realization \(y\). As it is standard in the sovereign default literature, the default output cost function is set to be increasing (and convex) in output to match the fact that default tends to happen for low realizations of the income, which implies the graph of \(X(\cdot)\) is above the graph of \(V^+(\cdot)\) for low realizations of output. The crossing point determines the fundamentals default threshold \(d_f\). A fundamentals default occurs if \(y < d_f(a, b, \omega)\). The unconditional value function \(W(\cdot)\) is just the upper envelope of \(X\) and \(V\), also depicted in Figure 3a.

On the other hand, when \(\omega = 0\) the value after repayment but without new lending \(V^-(\cdot)\) plays a role. In Figure 3b the graph of \(V^-(\cdot)\) is depicted along those of \(V^+(\cdot)\) and \(X(\cdot)\). For high income realizations the no-lending constraint does not
bind, since the sovereign finds optimal to buy back debt, hence \( V^+(\cdot) = V^-(\cdot) \). But as income decreases, the constraint starts binding and \( V^- \) decreases faster than \( V^+ \).

Figure 3: Default thresholds for a fixed debt-reserves pair.

![Diagram of default thresholds](image)

The crossing point between \( V^- \) and the default value \( X \) determines the Self-fulfilling crises default threshold \( d^s \). A self-fulfilling default occurs if \( \omega = 0 \) and \( d^f(a, b, \omega) < y < d^s(a, b, \omega) \). Each point \( s = (y, a, b, \omega) \) of the state space is classified to be in one of three regions, also depicted in Figure 3b:

i) Safe region: when \( y > d^s(a, b, \omega) \), repayment is the dominant strategy for the sovereign no matter what the lenders do and thus the state is part of the safe region.

ii) Default region: When \( y < d^f(a, b, \omega) \), default is the best option for the government given the weak fundamentals and thus the state is part of the default region. Defaults occurring in this region are denominated fundamentals defaults.

iii) Multiplicity: When \( d^f(a, b, \omega) < y < d^s(a, b, \omega) \), the state is in the multiplicity region. Here default happens if and only if \( \omega = 0 \), and when it happens it is denominated a self-fulfilling default.
Note that the unconditional value function $W(\cdot)$ when $\omega = 0$ is discontinuous at the self-fulfilling default threshold $y = d^f$. This is because of the outcome of the game of default and rollover described in Section 1.2.2. The value $V^-$ is never realized on the equilibrium path, only $V^+$ or $X$ are realizations of Nash equilibria. Given the thresholds, the probability of default next period can be written as:

$$
\Pr(\text{def}) = \Pr(\delta(y', a', b', \omega') = 1) = \Pr(y' < d^f) + \Pr(\omega' = 0)\Pr(d^f < y' < d^s).
$$

(1.9)

And the equilibrium bond price function can be compactly specified as:

$$
q(1 + r) = (1 - \Pr(\text{def})\left(\lambda + (1 - \lambda)z + (1 - \lambda)\mathbb{E}_{y, \omega}[q'|\text{repay}]\right).
$$

(1.10)

With the features of the model defined, the mechanism can be stated briefly: issuing extra debt to finance reserves accumulation, increases the fundamentals default threshold but reduces the self-fulfilling one. In aggregate, it reduces the next period default probability and thus the sovereign spread. These statements are developed in the next subsection.

1.3.3. Consolidating debt and reserves

Assume that in an equilibrium the economy is at a state $s = (y, a, b, \omega)$ in which both chosen reserves and debt are positive $a^*(s) > 0, b^*(s) > 0$ and there is repayment. After repayment is chosen, define a “consolidation of debt and reserves” as the following portfolio adjustment made at the current equilibrium debt price $q = q(y, a^*(s), b^*(s), \omega)$: the sovereign issues $\varepsilon$ less of face-valued debt and decreases reserves by $\bar{q}(1 + r)\varepsilon$ tomorrow. To keep continuation values accounting easy, this consolidation is fully undone in the next period, that is: for each realization of the exogenous states $(y', \omega')$ in the next period, the asset holdings chosen for two periods ahead are the same before and after the consolidation.

Next, it will be argued that the consolidation operation is always beneficial to the
sovereign in the standard Eaton-Gersowitz setup where no self-fulfilling crises occur, but that it can leave the government worse off once self-fulfilling crises are possible. To determine the overall effect of consolidation, the impact on current consumption and on next period consumption has to be addressed. Consolidating debt and reserves is not neutral on current consumption since bond prices react. How do they react depends in part on how the thresholds in the next period move. This in turn depends on the consolidation effect on next period consumption since continuation values two periods ahead are kept constant. In the next period, the consolidation effect on available resources is as follows. In the states in the default region, reserves change by

$$-\bar{q}(1+r)\epsilon$$

and this is the total effect since debt is wiped away. The effect on resources under repayment has two components: the first one comes from the fall in reserves (equation 1.11), the second one comes from the reduction in debt outstanding

$$\epsilon[\lambda + (1-\lambda)(z + q')]$$,

it includes the lower debt service $\lambda + (1-\lambda)z$ and the extra room for new issuance $(1-\lambda)q'$. Using condition (1.10) for $\bar{q}$, the total effect on resources after repayment is then:

$$\epsilon\bar{q}(1+r)\frac{Pr(def)}{1-Pr(def)} + \epsilon(1-\lambda)(q' - E_{y,\omega}[q'|repay]).$$

The consolidation effect on resources next period is an actuarially fair transfer. The first term of equation (1.12) captures the average transfer into each of the repayment states, which is just the negative of the average transfer out of each of the default states (equation 1.11) times their relative masses.
The consolidation operation keeps fixed the debt outstanding two periods ahead (b‘′), but there is a change in the periods that debt is issued. After the consolidation, some debt is issued in the next period instead of the current one. The second term on equation (1.12) captures the fact that market conditions may change between these two periods and, depending on the price schedule, it may or may not be cheaper to issue in the current period and save as reserves than to issue in the next period. In the current period, lenders anticipate this potential for capital gains or losses and adjust the price accordingly such that on expectation it vanishes away. Hence the second term is just a zero-sum reshuffling of resources among repayment states.

The effect on resources is actuarially fair, but since the sovereign is risk averse, there is room for utility changes. The expected marginal change in utility next period from consolidation can be decomposed in four parts:

1. The expected utility loss in the default states because of less resources available: 
   \[ -\Pr(\text{def}) \cdot (\varepsilon \bar{q}(1 + r) \mathbb{E}[u_c(c')|\text{def}] \]

2. The expected utility gain in repayment states coming from the net resource transfer from the default states:
   \[ (1 - \Pr(\text{def})) \cdot \varepsilon \bar{q}(1 + r) \frac{\Pr(\text{def})}{1 - \Pr(\text{def})} \mathbb{E}[u_c(c')|\text{repay}] \]

3. The expected utility gain or loss coming from shifting the issuance period of the surviving debt: 
   \[ +\varepsilon(1 - \lambda) \text{Cov}(u_c(c'), q') \]

4. The expected utility gain or loss coming from the change in the self-fulfilling default threshold.

\[ -\Pi_\omega(\omega' = 0)[V^+(d^s, a', b', 0) - X(d^s, a', b', 0)] f(d^s|y) \Delta d^s, \quad (1.13) \]
where $c_x$ is the consumption policy in case of default and $\Delta d^x$ is the change in the self-fulfilling default threshold due to the consolidation operation. This fourth term appears because of the discontinuity of the function $W(\cdot)$ when $\omega = 0$, which happens because at the self-fulfilling default threshold the sovereign is not indifferent between repay and default.

Let $z = \varepsilon \bar{q}(1 + r) \Pr(\text{def})$. Abstracting from the effect coming from the change in thresholds, the marginal effect of consolidation on utility next period can be written as:

$$-z \mathbb{E}[u_c(c_x')|\text{def}] + z \mathbb{E}[u_c(c')|\text{repay}] + (1 - \lambda) \text{Cov}(u_c(c'), q').$$  \hfill (1.14)

The case of one-period debt

When ($\lambda = 1$) the resource transfer into repayment states (equation 1.12) reduces to $+\varepsilon \Pr(\text{def})$. Those resources come from the default states, $-\varepsilon (1 - \Pr(\text{def}))$ (equation 1.11).

The fact that resources increase in repayment states and decrease in default directly implies lower default thresholds. Consequently, bond prices move in favor of the sovereign and, since $\Delta d^x < 0$, there are gains from enlarging the safe zone (equation 1.13). However, the benefit of this bond appreciation should be compared against the expected utility cost (or benefit) of that transfer of resources. Given that reserves are a risk-free asset and debt provides some insurance against low realizations of income by the means of default, it is reasonable to think that in equilibrium the expected marginal utility under default ($\delta = 1$) is the same as the expected marginal utility under repayment ($\delta = 0$), that is $\mathbb{E}[u_c(c_x')|\delta = 1] = \mathbb{E}[u_c(c')|\delta = 0]$. However this is not the case in the Eaton-Gersovitz framework because the lack of commitment: the government cannot choose separately the default and repayment states and the amount borrowed, because more borrowing implies more default states. In appendix A.1.3 the sovereign’s problem with
commitment is stated along the lines of Rios-Rull and Mateos-Planas (2016) and the first order condition showing $\mathbb{E} [u_c(c'_x)|\delta = 1] < \mathbb{E} [u_c(c')|\delta = 0]$ is presented. So far it has been established that in the case of one-period debt, the consolidation of debt and reserves decreases default thresholds in the next period, which implies a better price schedule in the current period and some utility gain in the next period coming from those states getting out of the multiplicity zone. It also moves resources from the default states to the repayment states, which is ex-ante desirable because the lack of commitment forces the sovereign to default in more states than what it would have committed to. All that adds up to debt and reserves not coexisting in equilibrium, because the consolidation increases not only current but also expected utility. This is consistent with the results found by Alfaro and Kanczuk (2009).

The case of long-term debt without self-fulfilling crises

In this case, the only threshold that matters is the one for fundamentals default. The change in that threshold depends on the change of the value functions at the old threshold, more precisely the variation in $X(d^f)$ minus the change in $V^+(d^f)$, due to the consolidation operation:

$$\Delta X(d^f) \simeq -\varepsilon u_c(\hat{c}'_x)\hat{q}(1 + r), \quad (1.15)$$

$$\Delta V^+(d^f) \simeq \varepsilon u_c(\hat{c}')\left(\frac{\hat{q}(1 + r) \cdot Pr(def)}{1 - Pr(def)} + (1 - \lambda)(\hat{q}' - \mathbb{E}_{y,\omega}[q'|\text{pay}])\right), \quad (1.16)$$

where a variable with a ˇ indicates it is evaluated at the threshold.

Figure 4 depicts the effect of consolidation on this threshold, on the same plane of income and utility used in Figure 3a. The consolidation effect on the value of default is the same as in the case with one-period debt: less reserves imply a reduction for all future states. It can be seen in Figure 4 as a new graph for the value function $X'$ (in red) drawn below the original curve $X$ (in gray).
As stated before, there is a net transfer of resources into repayment states (first term of equation 1.12). However, the valuation effect on resources \((1 − \lambda)(q' − \mathbb{E}_{y,\omega}[q'|\text{repay}])\) is negative for low income realizations like \(y = d^f\), even though on average it is zero. This implies that \(\Delta V^+\) may be negative. Numerically, this effect is small and the change in \(V^+\) at the threshold is still greater than \(\Delta X\). Hence, the threshold \(d^f\) goes down which makes the bond price in the current period to increase. Figure 4 depicts the new curve \(V'^+\), which is close to the original graph of \(V^+\).

Figure 4: Consolidation effect on \(d^f\) threshold

Regarding the change in utility in the next period, the inequality \(\mathbb{E} [u_c(c'|\text{def})] < \mathbb{E} [u_c(c'|\text{repay})]\) still holds. The same argument from the one-period case holds here: the lack of commitment implies a trade-off between the default threshold next period and the resources brought into the current period. However, in this case the covariance term of the consolidation effect in expected utility (equation 1.14) is negative. This could potentially generate an incentive for reserves holding, although there is an extra cost of long-term debt, which is that lenders now incorporate dilution risk in the price schedule. Numerically, the gains in utility due to the covariance are of second order and less relevant than the losses due to

---

\(^{12}\text{Rios-Rull and Mateos-Planas (2016) show that under commitment long term debt is equivalent to short term debt.}\)
the lack of commitment, which are of first order. Hence the main conclusion still is that the consolidation operation reduces the default threshold, which implies higher bond prices, and shifts resources into repayment states, which is desirable. It also reshuffles resources among repayment states, which may hurt the sovereign. Numerically the first two effects dominate and the consolidation operation increases the value for the government, which implies no debt and reserves coexistence.

The case for reserves with self-fulfilling crises

In this case, the behavior of the fundamentals default threshold is the same described for the case without self-fulfilling crises. The difference now is that the behavior of the self-fulfilling default threshold matters. The consolidation effect on \( d^s \) depends on the change in the value of repayment with no rollover \( V^- \):

\[
\Delta V^- = -\varepsilon q R V_a^- - \varepsilon V_b^- = \varepsilon u_c(c') \left[ -q R + (\lambda + (1-\lambda)(z + q')) \right] - \mu(1-\lambda), \quad (1.17)
\]

where \( \mu \) is the multiplier of the no-rollover constraint \( b' \leq (1-\lambda)b \). As in the previous case, the term in square brackets is still generically positive. However the multiplier term is of first order as long as \( \lambda < 1 \). Directly, less debt outstanding is worse in case of a rollover crisis because it tightens the borrowing constraint. Indirectly, a lower amount reserves implies less room to smooth consumption when additional borrowing is not possible, which increases the value of the multiplier. Hence \( V^- \) can fall significantly with the consolidation operation.

Figure 5 graphs new \( V'^- \) relative to the old one depicted in Figure 3b. The bigger the gap between the old \( V^+ \) and \( V^- \) is before the consolidation operation, the more binding the constraint is and the bigger the fall in \( V'^- \) after the consolidation. Evaluated at the \( d^s \) threshold, the fall in \( V^- \) due to the lower reserves holdings will generally be greater than the reduction in the default value \( X \) due to the same cause. In both cases, more borrowing is not possible, but in default there is no
need for debt servicing and the output loss is small since it happens at low income realizations. Hence, available resources are more tight under repayment and no rollover, which implies the self-fulfilling threshold $d^s$ increases. This increase in the threshold is depicted in Figure 5.

1.3.4. The role of reserves in preventing self-fulfilling debt crises

Given that consolidation increases the self-fulfilling default threshold, it has the potential to be very damaging, since that increase translates into discrete expected utility losses in some states in addition to bond price reductions due to the increased probability of default. In this subsection an operation opposite to consolidation is considered: a debt financed reserve holdings increase.

The discussion in the previous subsection still applies but the effects on the thresholds are the opposite. In particular, debt financed reserves increase the fundamentals default threshold $d^f$ but they reduce the self-fulfilling threshold $d^s$. Figure 6 shows the effect of this operation on both default thresholds, which is the inverse of reserves-debt consolidation. It abstracts from the value function curves and focuses only on the $x$-axis of Figures 4 and 5.

The debt financed accumulation of reserves shrinks the multiplicity zone and enlarges both the default and safe zones. The effect on default and bond prices is
Zones: Default \( d_f \) Multiplicity \( d_f' \) Safe \( d_s \) \( d_s' \) \( y' \)

harder to isolate, but some intuition can be given. In the long run, default rates are low which implies the sovereign avoids falling into default. There is little mass in the default region, most of it is in the safe one. The probability of default next period is:

\[
\Pr(\delta' = 1) = \Pr(y' < d_f) + \Pr(\omega' = 0)\Pr(d_f < y' < d_s)
\]  

(1.18)

Hence, when taking expectations \textit{ex-ante} the effect of the enlarging safe region dominates. The \textit{future} probability of default falls and that causes \textit{current} spreads to go down as well.

\textbf{Taking stock:} the previous discussion outlined the mechanism through which international reserve holdings can reduce sovereign spreads and the probability of self-fulfilling debt crises. However, the model does not yield unambiguous predictions about the strength of the mechanism driving the link between reserves, self-fulfilling crises and spreads, and the overall direction of the effects connecting these variables. For that reason the numerical predictions of the model are studied in the next section.

\section*{1.4. Quantitative analysis}

\subsection*{1.4.1. Solution Method and Calibration}

The model is solved on a discretized state space using the method proposed by Chatterjee and Eyigungor (2012). \footnote{Which is the best algorithm for computing a numerical solution of sovereign default models with long term debt is an open question. As mentioned in Chatterjee and Eyigungor (2012), with long term debt the budget sets for the sovereign are not convex, hence infinitesimal changes in the value function can lead to large changes in policies, causing convergence problems in global methods. They introduced a small purification i.i.d. endowment shock to smooth the value and policy functions. Under monotonicity assumptions, they are able to guarantees the existence of an equilibrium on discretized state spaces.} Gordon and Guerrón-Quintana (2013) de-
velop a variant of that algorithm to deal with multiple assets in an efficient manner, reducing the dimensionality curse of endogenous states.\textsuperscript{14}

Debt and reserves come from equally spaced grids, with 100 points each. Log-output is assumed to follow an AR(1) process:

\[
\ln(y_t) = \rho \ln(y_{t-1}) + \nu_t,
\]

where \(\nu_t \sim \mathcal{N}(0, \sigma_\nu)\). The endowment process is discretized using Rouwenhorst’s method to a grid of 50 points. Kopecky and Suen (2010) found this method to have better properties for highly correlated processes.

The utility function is the standard CRRA:

\[
u(c) = \frac{c^{1-\gamma}}{1-\gamma}.
\]

The function that characterizes the income cost of default is the same as in Chatterjee and Eyigungor (2012):

\[
\phi(y) = \max\{0, d_0 y + d_1 y^2\}
\]

where \(d_0 \in \mathbb{R}\) and \(d_1 \geq 0\). Note that \(d_0\) can take negative values, which implies zero cost for low income realizations.\textsuperscript{15}

The sunspot variable \(\omega \in \{0, 1\}\) is assumed to follow a two-state Markov chain, with transition matrix:

\[
\Gamma = \begin{bmatrix}
g_{11} & 1 - g_{11} \\
1 - g_{22} & g_{22}
\end{bmatrix}.
\]

\textsuperscript{14}For a given state, the method by Chatterjee and Eyigungor (2012) finds the output default threshold and also the thresholds for portfolio allocation by comparing all the feasible choices. Gordon and Guerrón-Quintana (2013) variant proposes a way to efficiently discard big portions of the portfolio state space.

\textsuperscript{15}Arellano (2008) introduced a \textit{kinked} specification for the default cost, and showed it is necessary to prop up the sustainable debt and to make default occur in bad times. Chatterjee and Eyigungor (2012) show that the quadratic form allows this models to better match the spread’s standard deviation.
Two key parameters in this model are the $\Gamma_{11}$ and $\Gamma_{22}$. Very little is known about them because they are not directly observable and very hard to estimate from data. A stance is taken here to pin them down for a baseline calibration, but not without acknowledging that the values found should not be taken as precise estimates.

The model is calibrated to Mexican data, in line with the recent literature on international reserves that focuses on Mexico (Bianchi et al., 2012; Tavares, 2015). Although the sovereign default literature generally uses Argentina, the fact that it had a fixed exchange rate until 2001 implied its reserve holdings had to back the money in circulation, making the precautionary-savings motive for reserves of second order.

Unless stated otherwise, the data covers the period between 1994:Q1 and 2015:Q4. The model has 12 parameter values to select. Seven of these parameter values can be set directly using the data. The corresponding parameter values are shown in Table 3.

The bond duration parameter $\lambda$ is set to $20^{-1} = 0.05$ yielding an average maturity of 20 quarters (5 years). Broner, Lorenzoni, and Schmukler (2013) find an average maturity of 10 years at issuance which implies an average outstanding debt maturity of 5 years. The interest rate spread on debt is taken from J.P. Morgan Emerging Market Bond Index (EMBI+). The average annualized blended spread is 228 basis points.

The quarterly real risk-free rate $r$ is set to 0.38%, which is one quarter of the average nominal yield on 2 year constant maturity treasury bills deflated by the US PCE, using data from the St. Louis FRED database. Two reasons motivate this choice: first, no less than 70% of international reserves were invested at maturities longer than 1 year between 2007 and 2010 (McCauley and Rigaudy, 2011); second, the EMBI spread is calculated as the yield difference against a portfolio of US
bonds of similar duration, which justifies the use of longer maturities. The quarterly coupon $z$ is set to 0.95%, which is the risk-free rate plus one quarter of the annualized average spread (228 bp). This parameter just scales up or down the cash-flow associated with debt but it is useful to make the debt stock in the model easier to compare with that in the data. Sovereign debt in the real world is measured at face value, and interest payments only enter the debt stock statistics when they are due, hence it is useful to have both -the market value and face value of debt- coincide (the latter is 1). This happens when the coupon is exactly the risk-free rate plus the spread. In addition, is common practice among sovereign issuers to set the promised coupons in their bond auctions aiming to sell debt at par.

The parameters $\rho$ and $\sigma_\nu$ that govern the endowment are set to match the cyclical properties of Mexican real GDP times the real exchange rate. This captures not only real output volatility but also changes in the debt burden arising from real exchange rate fluctuations. Hence, the Mexican GDP in US dollars is first deflated with the US PCE index to leave it in global basket consumption terms. Then, an AR(1) process is fitted to the Hodrick-Prescott detrended series. The parameters for the income autocorrelation and the standard deviation of the income shock are $\rho = 0.76$ and $\sigma_\nu = 6.0\%$.

The parameter $\epsilon$ governing the re-entry probability is set at 0.128 which implies an average exclusion time of 8 quarters. This is consistent with the findings of Gelos, Sahay, and Sandleris (2011) for the average exclusion time for default episodes.

16The sovereign default literature has mainly focused on Argentina, which has a very volatile real income process, with some success (Arellano, 2008; Chatterjee and Eyigungor, 2012). Recently, Aguiar et al. (2016b) showed the struggles Eaton-Gersovitz models face when trying to match the quantitative success of those models on Mexico, which has a much less volatile real income process. One overlooked difference among the two economies is the exchange rate arrangement: Mexico has a floating exchange rate while Argentina had a fixed one until its most recent default episode. Floating exchange rates are known to be a buffer to absorb external shocks, which can account for the lower volatility of Mexican real income relative to the Argentinian one. However, floating exchange rates expose the sovereign to the currency mismatch problem, as the exchange rate volatility translates into debt burden volatility relative to income. Eichengreen, Hausmann, and Panizza (2007) discuss how international reserve holdings can lessen the currency mismatch problem.
in the 90’s. Also, as standard in the sovereign debt literature, the relative risk aversion coefficient is set to 2.

Table 3: Parameters related to moments in the data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Data moment</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau$</td>
<td>0.038</td>
<td>Avg. 2yr US Treasury real yield</td>
<td>FRED</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.05</td>
<td>Govt. external debt duration</td>
<td>Broner et al. (2013)</td>
</tr>
<tr>
<td>$z$</td>
<td>0.096</td>
<td>$r$ plus EMBI spread</td>
<td>Datastream</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.76</td>
<td>GDP autocorrelation</td>
<td>WB-GEM</td>
</tr>
<tr>
<td>$\sigma_{\nu}$</td>
<td>0.06</td>
<td>GDP standard dev.</td>
<td>WB-GEM</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2</td>
<td>Standard in literature.</td>
<td>-</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>0.125</td>
<td>Avg. default exclusion time</td>
<td>Gelos et al. (2011)</td>
</tr>
<tr>
<td>$\Gamma_{22}$</td>
<td>0.875</td>
<td>Avg. duration of SS events</td>
<td>Jeanne and Rancière (2011)</td>
</tr>
</tbody>
</table>

The transition probability parameters governing the sunspot Markov process determine the frequency and duration of self-fulfilling debt crises. The former is hard to measure directly in the data since those crises are only observed when the sovereign is in the multiplicity zone. On the other hand, the duration of systemic sudden stops can be observed in the data. Bianchi et al. (2012) report a sudden-stop duration of 1.12 years which implies $1 - \Gamma_{22} = \frac{1}{1.12} = 0.22$ and Jeanne and Rancière (2011) find a duration of 4 years which would imply $1 - \Gamma_{22} = \frac{1}{4} = 0.032$. The baseline calibration is set to the middle point of those estimates $1 - \Gamma_{22} = 0.125$ which implies a duration of 2 years.

The remaining four parameter values: $\beta$, $d_0$, $d_1$ and $\Gamma_{11}$ are set using a Simulated Method of Moments (SMM) algorithm.

Table 4: Parameters set by SMM

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Targeted Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.983</td>
<td>Debt to GDP ratio</td>
</tr>
<tr>
<td>$d_0$</td>
<td>-0.460</td>
<td>Default frequency</td>
</tr>
<tr>
<td>$d_1$</td>
<td>0.595</td>
<td>Spread Std. dev.</td>
</tr>
<tr>
<td>$\Gamma_{11}$</td>
<td>0.949</td>
<td>Spread-GDP corr.</td>
</tr>
</tbody>
</table>

In the sovereign debt literature, the goal is to match the mean debt to GDP ratio,
the average spread and the standard deviation of the spread. As argued by Aguiar et al. (2016b), the first three parameters have been found to be enough to match the debt level and average spread, but they find that either the spread volatility is too low or the correlation between the spread and income predicted by the model is very high. In this framework, the possibility of self-fulfilling crises allows multiple bond price schedules to be consistent with the same fundamentals, weakening the correlation between those fundamentals and sovereign spreads. Building of this, the calibration targets the correlation between spread and output cycle. In addition, Aguiar et al. (2016b) document the role of risk premium as a driver of sovereign debt returns. They point to a gap between the realized returns on the EMBI+ index and the return on US govt indexes of similar maturity. Given that in this framework the lenders are assumed to be risk neutral, spreads are going to be closely related with the default probability. In the SMM calibration, instead of the spread, the target is the default probability which Aguiar et al. (2016b) find at 2% per year for Mexico.

Table 4 presents the values found during the procedure. The discount factor $\beta = 0.983$ corresponds to a yearly discount factor of 0.934 which is in line with the values found in the literature. The default cost parameters $d_0 = -0.46$ and $d_1 = 0.59$ imply a 15.6% proportional default cost when output is at its long run average, 8.1% when it is one standard deviation below average and 0.7% when it is two standard deviations below average.

The parameter governing the frequency of the sunspot switching from the good equilibrium to the bad one (a panic) obtained was $1 - \Gamma_{11} = 0.051$. It implies that those events happen roughly $\frac{1}{0.051} \approx 20$ quarters (5 years). This parameter together with the probability of going back to the good equilibrium $1 - \Gamma_{22} = 0.125$ implies that financial markets are in the high beliefs regime $\frac{0.051}{0.051+0.125} = \approx 28.5\%$ of the time.
Aguiar, Chatterjee, Cole, and Stangebye (2016c) estimated a two-state regime-switching model of Mexico’s EMBI+ and found a transition of 0.12 from the low bond price into the high bond price regime, which is similar to the corresponding value used in the baseline calibration $1 - \Gamma_{22} = 0.125$. They also report a transition probability of 0.028 from the high bond price regime into the low bond price one, which implies a bad regime frequency of once every $\frac{1}{0.028} \approx 36$ quarters (9 years), as they identified two episodes in their 20 year sample when the spread jumped more than what was granted by fundamentals. In the calibration here, the probability of the sunspot switching to the self-fulfilling crises prone regime is higher (0.051), but it is important to stress that in this framework not all periods where the sunspot variable point to the bad regime imply a spike in spreads since the economy’s fundamentals can be deep inside the safe zone.

1.4.2. Results

<table>
<thead>
<tr>
<th>Targeted Moments</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt to GDP ratio</td>
<td>15.8</td>
<td>15.9</td>
</tr>
<tr>
<td>Default probability</td>
<td>2.0</td>
<td>1.9</td>
</tr>
<tr>
<td>Std. dev. of spread</td>
<td>1.1</td>
<td>1.2</td>
</tr>
<tr>
<td>Spread-GDP corr.</td>
<td>-0.67</td>
<td>-0.68</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Non-targeted Moments</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserves to GDP ratio</td>
<td>8.0</td>
<td>4.0</td>
</tr>
<tr>
<td>Average Spread</td>
<td>2.3</td>
<td>1.8</td>
</tr>
</tbody>
</table>

All in percentage points except Spread-GDP correlation which is scalar.

Table 5 presents the model’s performance with respect to data moments. The model is able to match all the targeted moments, particularly the countercyclical and volatile spread together with observed debt holdings. The model can explain half of the average reserves holdings: 4% of GDP in the model vs 8.0% in the data. This result leaves room for other motives for reserve
accumulation. On the spread side, the model is able to generate high mean spreads (177 b.p), but still they are 50 b.p. below the data average. As mentioned before, the lack of a risk premium on sovereign debt can explain this shortfall.

Figure 7: Effect of debt and reserves on spreads

(a) Next period Debt

(b) Next period Reserves

The current debt and reserve holdings are at their mean values. Yavg: mean income. Ylow: mean minus 1.6 standard deviations of income. Panic: $\omega = 0$.

Figure 7 illustrates the effect of debt and reserve choices on sovereign spreads. The left panel depicts the spread schedule for different levels of future debt which has the standard convex shape reflecting higher spreads for higher debt positions. The right panel of figure 7 presents the spread schedule for reserve holdings choices. Consistent with the empirical evidence, reserves holdings are correlated with lower spreads.

Figure 8 presents the optimal debt and reserves policies as a function of current output. The state of the economy is assumed to be at the mean debt and reserve holdings and the sunspot variable pointing to no self-fulfilling crises ($\omega = 1$). The results show that reserves are accumulated in periods of high output while the debt response is somewhat muted, which is consistent with the findings of Aguiar, Amador, Hopenhayn, and Werning (2016a). Aguiar et al. (2016a) argue

\[17\] It its important to acknowledge that there seems to be a recent upward trend in reserves accumulation in the data and a downward trend in the spread. A parameter shift can accommodate such trends, further research could delve into the precise nature of that shift and its quantitative performance.
The current debt and reserve holdings are at their mean values. The sunspot variable value is $\omega = 1$.

debt buybacks are very costly because the bond prices move against the government and they find that adjustments are better performed in the short-term margin, which in this framework corresponds to the reserves margin. Debt and reserve accumulation are thus procyclical, which is consistent with the findings by Broner et al. (2013) on gross capital flows.

Table 6 presents the results of a regression of the sovereign spread on debt, reserves, current account, growth and the sunspot variable using data simulated from the model.

The results are very similar to those found in the data (Table 2). The reserves coefficient is negative and slightly larger in magnitude than the coefficient on debt, indicating that, in the model, higher levels of both debt and reserves are not associated with higher spreads.

The coefficient on the sunspot variable is small, which seems odd but in fact is consistent with the features of the model. First, in the safe and in the default region the sunspot is not very relevant, besides some impact through the expected value of landing in the multiplicity region in subsequent periods, which is small. Second, in the multiplicity region the direct effect of $\omega = 0$ is to cause a self-
Table 6: Spread regression on model simulated data

<table>
<thead>
<tr>
<th>Dep. var.: Spread</th>
<th>Pooled OLS</th>
<th>( \beta / \text{s.e.} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>L.Reserves/GDP</td>
<td>( -9.598^{* * * } )</td>
<td>(0.066)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L.Debt/GDP</td>
<td>( 8.797^{* * * } )</td>
<td>(0.109)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L. Sunspot</td>
<td>( 1.560^{* * } )</td>
<td>(0.749)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L.Current Account/GDP</td>
<td>( -10.033^{* * * } )</td>
<td>(0.188)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L.GDP growth</td>
<td>( -4.005^{* * * } )</td>
<td>(0.049)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>( 83.872^{* * * } )</td>
<td>(2.063)</td>
</tr>
</tbody>
</table>

All variables are simulated from the model using the baseline calibration. "L." indicates one period lagged variable. Standard errors in parentheses. * Significance at the 10 percent level. ** Significance at the 5 percent level. *** Significance at the 1 percent level.

Fulfilling crises where the country defaults, but those periods are dropped in the regression since the spread goes to infinity. Hence the coefficient in the regression shown in Table 2 is capturing just the indirect effect of the sunspot on spreads.

To further assess the impact of the sunspot variable and the proposed mechanism, Table 7 presents the results of alternative specifications in which either the frequency or the duration of the bad equilibrium regime is reduced.

The Low Frequency specification in Table 7 shows the numerical performance of a specification in which the parameter \( \Gamma_{11} \) is set such that the sunspot variable switches to the self-fulfilling crises prone equilibrium once every 8 years (instead of the 5 years in the baseline calibration). This specification can still match the average debt level, the spread volatility and spread correlation with income, but it is not able to match the default probability which decreases from 2.0% to 1.6%.
Table 7: Alternative specifications of the sunspot process

<table>
<thead>
<tr>
<th>Targeted Moments</th>
<th>Data</th>
<th>Baseline</th>
<th>Low frequency</th>
<th>Low duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debt to GDP ratio</td>
<td>15.8</td>
<td>15.9</td>
<td>15.7</td>
<td>15.5</td>
</tr>
<tr>
<td>Default probability</td>
<td>2.0</td>
<td>1.9</td>
<td>1.6</td>
<td>2.0</td>
</tr>
<tr>
<td>Std. dev. of spread</td>
<td>1.1</td>
<td>1.2</td>
<td>1.1</td>
<td>1.4</td>
</tr>
<tr>
<td>Spread-GDP corr.</td>
<td>-0.61</td>
<td>-0.60</td>
<td>-0.60</td>
<td>-0.59</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Non-targeted Moments</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Reserves to GDP ratio</td>
<td>8.0</td>
</tr>
<tr>
<td>Average Spread</td>
<td>2.3</td>
</tr>
</tbody>
</table>

1 All in percentage points except Spread-GDP correlation which is scalar.
2 Baseline specification described in Tables 3 and 4. On average, the self-fulfilling regime happens once every 5 years and lasts 2 years.
3 Low frequency specification: the self-fulfilling regime happens on average once every 8 year. Expected duration still 2 years.
4 Low duration specification: the self-fulfilling regime is expected to last 1 year. On average occurs once every 5 years.

On the non-targeted moments, the spread is still 1.8% but reserves holdings fall to 3.0% (compared to the baseline at 4.0%). Making the coordination problems less frequent reduces the perceived risk of the multiplicity zone which in turn lowers the incentive to accumulate reserves, and also reduces the frequency of self-fulfilling defaults.

The last column in Table 7 shows the quantitative results of the Low Duration specification, in which the parameter $\Gamma_{22}$ is set such that the sunspot stays in the self-fulfilling crises prone regime for 1 year on average (compared to 2 years on the baseline calibration). This specification matches the average debt level, the default probability and the correlation between spread and income, but yields a higher spread volatility (1.4%). The spread level attained is higher that the one in the baseline (2.0% vs 1.8%) but the reserve holdings fall significantly to 2.6% (baseline: 4.0%). Reducing the duration of the self-fulfilling regime means that in the case of repayment without rollover the government expects to quickly regain access to borrowing, which implies that a lower amount of reserves is required to smooth consumption and service debt if that is the case. In addition, the
lower duration of this regime reduces its expected impact, meaning that the value functions \( V^+ \) and \( V^- \) are closer, and the multiplicity region is then smaller.

To sum up, reducing either the frequency or the duration of the self-fulfilling crises prone regime significantly reduces the average reserve holdings the sovereign finds optimal. This shows the relevance of the mechanism in generating a role for reserves.

1.5. Concluding Remarks
This paper developed a quantitative model of sovereign default and international reserves to address the optimal portfolio choice of the government. The model embedded Cole-Kehoe style self-fulfilling crises in a standard Eaton-Gersovitz setup with long-term debt. In this framework lenders were risk-neutral, but the government faced a rollover risk arising from coordination failures on the lenders’ side.

The mechanism described here explained the role of reserves in reducing spreads and the probability of debt crises. It was shown that for a given Net Foreign Asset position, additional reserves reduce the set of states that allow multiple equilibria regarding the repayment and rollover decisions. Higher reserve holdings preclude some of the risk of a self-fulfilling debt crises in the future and consequently reduce current sovereign spreads.

The model was calibrated to replicate the average external public debt, the default frequency and the volatility and countercyclicality of sovereign spreads. In the simulations, the model generates mean reserve holdings that are half as big as the observed one. In addition, the mean spread level in the model is 78% of the one observed in an environment with risk-neutral lenders.

This paper caught up with the empirical literature on vulnerability measures to sovereign debt crisis that has established the connection between higher reserve holdings and lower crises probability.
Further research can use the developed framework to evaluate policies aiming at preventing crises in sovereign debt markets. Those include the standard policy prescription rules, like the reserves adequacy ratios to imports, debt, output, short-term debt or debt service, or the more elaborated and widely used Guidotti-Greenspan rule.

In addition, this model is in position to evaluate the different contingent lending arrangements put in place by the IMF and other developed countries’s central banks. Those are motivated as means to generate confidence in sovereign debt markets and prevent self-fulfilling crises, and are not supposed to bail-out or subsidize lending to the recipient countries.
Chapter 2

Fighting For The Best, Losing With The Rest: A case for regulation in the entrepreneurship financing markets

2.1. Introduction

Innovation has being widely recognized as the key source of economic growth, at least going back to the work of Schumpeter (1934). Arrow (1962) argued that potential entrants have stronger incentives to undertake innovation than incumbent monopolists, and indeed, an important fraction of new projects and products come from start-up firms. However, in many situations the firm or entrepreneur having access to a potential project lacks the financial resources needed to undertake it and has to rely on external lenders or investors. Financial markets for case of small R&D intensive start-ups present are plagued with frictions that generically lead to inefficient outcomes. First, the profitability of a particular endeavor is not guaranteed, the research project or new product may or may not succeed and this uncertainty can only be resolved after the in-

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1 Coauthored with Daniel Wills
vestment is made. In that sense, projects are risky. Moreover, since innovations require specific and sophisticated knowledge, it is likely that the entrepreneur will know more about the project’s prospects than investors do. This introduces a second friction, asymmetric information. Third, by the nature of the project, its most important assets are the knowledge, time and effort, devoted by its team of workers. If the project fails, the salvage value is close to zero. As a consequence, there is severe limited liability.

Recent legislation has intended to deregulate the market for funding for start-ups. Specifically, The Jumpstart Our Business Startups (JOBS) act of 2012 eased securities regulation making it easier for companies to both go public, and raise capital privately. This motivates us to ask if complete deregulation is desirable in a market featuring risk, asymmetric information and limited liability as described above.

To answer the question, we characterize the contracts offered in equilibrium by a competitive financial sector to entrepreneurs facing risk and limited liability, in an environment with adverse selection. We then proceed to describe the welfare implications of the resulting equilibrium. Our main result is that the optimal contract delivers an inefficient outcome. This is in contrast with the screening literature in environments with linear types (such as ours): when utility is linear in the type, optimal contracts achieve first best allocations. In our environment, the presence of limited liability and an outside option for entrepreneurs do not allow to achieve the first best outcome. Interestingly, if financial intermediaries were able to collude, the first best outcome would be achieved. Alternatively, under competition, the inefficiency can be corrected using simple policy tools such as a tax per financial contract.

In our model, there is a continuum of entrepreneurs, each one having access to a risky project. The entrepreneurs are heterogeneous on the probability with which
their project will succeed. Also, they typically won’t have enough resources to fund their projects and will need to rely on financial intermediaries. Financial intermediaries supply financial contracts in a competitive way, aiming to maximize their profits, but do not observe the ex ante probabilities of success. Beyond the information friction and limited liability, the set of contracts to be offered is completely unrestricted. Naturally, all intermediaries would like to attract the best entrepreneurs, and competition will force them to offer good terms on borrowing for the best types. They also will need to provide incentives in the right way to distinguish good entrepreneurs from bad entrepreneurs which implies a (very specific) gap between the contracts offered to good and bad types. However, limited liability imposes a lower bound on the terms of a contract. Hence, in order to sustain those incentives, bad types must also receive good terms. In equilibrium, intermediaries will break even, but they will fund both projects with positive expected profits (from good types) and projects with negative expected profits (with bad types). The inefficiency arise because entrepreneurs have an outside option (or equivalently a utility cost). We show that if an intermediary is only funding socially efficient projects (i.e: projects with expected profit higher that the cost of capital plus the opportunity cost of the entrepreneur), she is making positive profits. As a consequence, her competitor is willing to improve borrowing terms for good types, even though some low types with inefficient projects will take the contract. The inefficiency can be corrected with simple tax instruments.

In the basic model, entrepreneurs cannot collateralize their assets. We extend the model to the case in which entrepreneurs have some collateral. We show that as long as loans cannot be fully collateralized, the inefficiency is reduced but not removed. We then proceed to study how the inefficiency changes when the parameters of the model change. The effect of the entrepreneur’s outside option is non-monotonic: the inefficiency is zero if there is no outside option (all projects
are socially efficient), but it is also zero if the outside option is so high that nobody want to undertake risky projects (no project is efficient). Further, the inefficiency (relative to the net economic surplus) increases with the cost of capital faced by financial intermediaries and with the relative density of lower types. When we allow for collateralizable assets, the inefficiency decreases with the mean of the asset distribution.

Starting from Stiglitz and Weiss (1981), and extensive body of literature has studied asymmetric information in financial markets. The main message of Stiglitz and Weiss (1981) is that the interest rate cannot clear the credit market because of a standard “lemons” problem, and as a result, there is credit rationing. Subsequent papers allow the financial intermediaries (banks) to use different tools, other than the interest rate, to screen borrowers’ types. A first strand of papers allow intermediaries to use collateral, on top of the interest rate, to screen types. Bester (1985) turns down the credit rationing result, by allowing intermediaries to offer interest rate/collateral contract pairs. By using collateral in addition to the interest rate, banks can screen borrowers: risky borrowers will accept to pay higher interest rates in order to benefit from a lower collateral requirement. However, in Bester (1985)’s economy, there is no limit to the amount of collateral that borrowers can provide. The question of limits to collateral is studied by Besanko and Thakor (1987). The environment is similar to Bester (1985)’s, and safer types will prefer loans with low interest rate and high collateral. Nonetheless, it may be that the borrower has not enough wealth to provide the required collateral. In that case, the collateral/interest rate pair cannot achieve the sufficient spread in payoffs necessary to separate types. Besanko and Thakor (1987) solve this issue by allowing the contracts to additionally depend on the probability of approval. To achieve the necessary spread of utilities, low interest - high collateral credits will be denied with positive probability. An interesting point in Besanko and
Thakor (1987) that relates to our result is that the paper compares welfare when
the financial intermediation sector is competitive or a monopoly, and finds that
monopoly may lead to a higher welfare, depending on parameter values.
Another strand of papers have departed from the Stiglitz and Weiss (1981) result
by allowing intermediaries to screen borrowers using the size of the loan. A con-
tract is hence a pair interest rate - loan size. In Milde and Riley (1988), borrowers
are entrepreneurs with access to a project with risky returns. The return on the
project depends on both the borrower’s type and the size of the loan. The inter-
action between type and loan size in the project’s payoff allows to separate types
using interest rate - loan size menus. The outcome, however, depends strongly on
the specific function mapping the type and loan size to the return of the project.
In general good types take bigger loans accompanied by higher interest rates.
However, Milde and Riley (1988) provide examples of production functions for
which the opposite happens: good types take smaller loans and pay lower inter-
ests. A point to keep in mind from Milde and Riley (1988) is that projects wont
be be funded to its optimal, full-information size.
More recently, Martin (2009) uses a similar framework to study the relation be-
tween entrepreneurial wealth and aggregate investment. In his model, interme-
adiaries can use both collateral and the size of the loan to screen types. He shows
that when entrepreneurial wealth is high, collateral can be used to separate types.
When entrepreneurial wealth is low, screening is mainly done by restricting the
level of investment, and becomes more costly. As a result, in the later case, a pool-
ing equilibrium is more likely. However, Martin (2009) restricts the interest rate
to be un-contingent. We show that, when transfers contingent of success of the
project are allowed (say by a contingent interest rate or an equity-like contract),
the intermediaries never distort the level of investment to screen types. Instead
they find optimal to use the contingent transfer.
Although close in terms of topic, all the papers cited above impose ad-hoc restrictions to the space of contracts potentially offered by the financial sector. In contrast, in our model, the set of contracts is only restricted by the features of the environment.

Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2015) develop a model where screening contracts are unrestricted. Their environment features adverse selection between informed sellers and uninformed variables. Beyond asymmetric information, they introduce imperfect information coming from search theoretic frictions. The later feature allows them to do comparative statics on the degree of imperfect competition and how it interacts with the severity of adverse selection. As in our environment, they find that increasing competition may reduce welfare when markets are competitive.

In our model, the financial intermediation is competitive. Intermediaries fund projects in which they expect to lose, but are socially efficient because the payoff of the entrepreneur compensates the intermediary’s loses. They also fund projects that are socially inefficient in the sense that they generate a dead-weight loss. Our environment is close to Rothschild and Stiglitz (1992), in which adverse selection is introduced to a competitive market. In their environment the equilibrium (when it exists), is separating. The limited liability constraint in our model prevents the separating outcome. In our model, the inefficiency results from the interaction of several forces: first, there is asymmetric information that introduces a “lemons” problem; second, limited liability puts a bound on the screening that can be done by financial intermediaries; third, competition among intermediaries introduces profitable deviations from the efficient outcome (that would be reached by a monopolist lender).

The rest of the paper is organized as follows: In the next section we describe the model which is the core of the paper. Then, in section 2.3 we extend the
model to allow for a distribution of assets among entrepreneurs. In section 2.4 we introduce numerical example, and show how the deadweight loss changes with the parameters of the model. Concluding remarks are made in section 2.5.

2.2. Basic Model

2.2.1. Environment

In this section we describe the main mechanism of the paper in a partial equilibrium static economy. The economy is populated by a continuum of agents with mass 1 indexed by their heterogeneous ability \( \theta \in [0, 1] \). Each agent can work for a wage \( w \) or undertake a project with a risky outcome, that depends both on the ability of the agent and the capital invested. If an agent decides to start his own project, he or she will have to borrow funds from a financial intermediary\(^2\).

The financial intermediaries have access to capital at the (gross) risk free rate \( R \). They cannot observe the entrepreneurs ability and will have to provide incentives in order to get that information. Intermediaries can observe investment in the project, i.e. agents cannot divert funds from their projects without being caught. Both agents and intermediaries are risk neutral. However, if a project fails the intermediaries cannot exert any claims on the entrepreneurs. In that sense, projects in this economy are subject to limited liability.

The ability of each agent will determine the probability that an entrepreneurial venture succeeds. We denote \( G(\theta) \) the cumulative distribution of abilities. More specifically, if the agent decides to become an entrepreneur and invests an amount \( k \) of capital in the project, the project will succeed with probability \( p(\theta, k) \)

Assumption: \( p(\theta, k) \) is multiplicatively separable: i.e. \( p(\theta, K) = g(\theta)f(k) \). Where \( f \) is continuous and \( f' > 0, f'' < 0, f(0) = 0 \) and \( \lim_{k \to \infty} f(k) = 1 \). Without loss of generality, we can set \( g(\theta) = \theta \) since we are just renaming the unobservable types.

\(^2\)This \( w \) can also be interpreted as the opportunity cost, pecuniary or not, of running the project for the entrepreneur.
anything that is known by the entrepreneur, but not by the intermediary, and that increases the probability that the project succeeds.

The assumption of multiplicative separability is important because it allows us to abstract from Riley-style signaling distortions in investment and highlight our main mechanism where competition in the intermediation sector generates over-investment. It also helps to keep the model tractable and allows a better characterization of the optimal contract.\(^3\)

In line with the endogenous growth literature, we interpret a successful project as the arrival of a new innovation, which allows the entrepreneur to create or “steal” some market. We denote \(\pi\) the payoff of a successful project which, in turn, can be interpreted as the value of the innovation. As a result, the expected surplus of an entrepreneurial venture is given by,

\[
\theta f(k)\pi - Rk - w
\]

As we will show later, in equilibrium, the surplus will be shared between the entrepreneur and the financial intermediary who lends the funds.

In principle, a contract is a triple, \((k, x_1, x_0)\) where the \(k\) is the size of the loan, \(x_1\) the repayment in case of success and \(x_0\) the repayment if the project fails.

However, for convenience we make a linear transformation of the contract that will allow a more straightforward relation with the mechanism design literature. Set \(x = -x_0\) and \(z = \pi - (x_1 - x_0)\) From the point of view of the agent the contract \((k, x, z)\) prescribes a fixed pay for the agent \(x\), an additional payment contingent on success \(z\) and an investment amount \(k\) that determines the probability of the

---

\(^3\) Without multiplicative separability the optimal contract is harder to characterize, however the main message of the paper remains: the optimal contract yields an inefficient outcome. When the cross-partial derivatives of \(\ln p(\theta, k)\) are not zero, the entrepreneurs will use \(k\) to signal type, leading to the standard Riley distortion. As a result, the project size \(k\) will be distorted which will lead to another source of inefficiency. Relative to our results, this will lead to less extensive inefficiency (fewer \(ex-ante\) suboptimal projects are started) but more intensive efficiency (all projects will be run at a suboptimal scale). This is in contrast with the equilibrium of the linear environment we present, in which \(k\) is always the full information optimal level.
contingent payment happening.
The intermediaries’ objective is to offer a profit maximizing contract schedule, taking into account that agents would choose the better option available to them, and also the competition from other entrepreneurs.

2.2.2. Contract menus and entrepreneur choices

The game

In this subsection we formally define the game. For simplicity we assume there are only two financial intermediaries, indexed by $i \in \{1, 2\}$, that compete à-la Bertrand for entrepreneurs.\footnote{As long as the entrepreneurs observe all offered contracts, the outcome will be the same with more intermediaries, although the optimal strategies of each one may differ. Hence this is just a notational simplification for free entry in the intermediaries sector.}

- **Players:** 2 intermediaries, 1 entrepreneur. The intermediaries are identical. The entrepreneur has a private type $\theta$ drawn from a distribution $G(\theta)$.

- **Timing:** Intermediaries move simultaneously, posting arbitrary sets of contracts. Then the entrepreneur chooses among the available contracts an her outside option.

- **Strategies:** For intermediaries the strategy space contains any subset of contracts of the form $(k, x, z)$. The strategy space is hence the power set of $\mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}$ denoted $\mathcal{P}(\mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R})$. We denote the strategy of intermediary $i$ (or his contract menu) by $\mathcal{C}_i$. For the entrepreneur a strategy is a probability distribution $s : \Theta \times \mathcal{P}(\mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R})^2 \to \Delta \mathbb{R}^3$ such that $\text{Supp}(s(\theta, \mathcal{C}_1, \mathcal{C}_2)) \subseteq \mathcal{C}_1 \cup \mathcal{C}_2 \cup \{(0, w, 0)\}$. Above, $\Delta \mathbb{R}^3$ denotes the set of probability measures over $\mathbb{R}^3$, $\text{Supp}(f)$ denotes the support of $f$ and $(0, w, 0)$ is the outside option. Abusing notation, we let $s(\theta, \mathcal{C}_1, \mathcal{C}_2)[k, x, z]$ be the cumulative density function of $s$ evaluated at $(k, x, z)$.

Note that we allowed for mixed strategies for entrepreneurs. They will be
able to randomize over any subset of the offered contracts, including the outside option. We focus on the case in which intermediaries play pure strategies.

- **Payoffs:** All players are risk neutral and care only about expected payoff. For an entrepreneur of type $\theta$ the expected payoff of signing a contract $(k, x, z)$ is

$$u(\theta, (k, x, z)) = \theta f(k)z + x.$$  

in particular, if the entrepreneur take his outside option, his payoff is

$$u(\theta, (0, w, 0)) = w.$$  

The entrepreneur’s expected payoff is,

$$U(\theta, s, C_1, C_2) = \int_{(k, x, z) \in \mathbb{R}^3} (\theta f(k)z + x) ds(\theta, C_1, C_2)[k, x, z]$$

Conditional on an entrepreneur of type $\theta$ signing a contract $(k, x, z)$ with intermediary $i$ her expected payoff is: $\theta f(k)(\pi - z) - x - Rk$.

The total expected payoff of intermediary $i$ can be written as:

$$v_i(s, C_1, C_2) = \int_0^1 \int_{(k, x, z) \in \mathbb{R}^3} \left[ \theta f(k) \cdot (\pi - z) - x - R \cdot k \right] ds(\theta, C_1, C_2)[k, x, z] dG(\theta),$$

Given $s$, $C_1$ and $C_2$ it is useful to define the set of types *strictly* willing to take a contract from intermediary $i$,

$$A_i(s, C_1, C_2) = \{ \theta : \text{Supp} \ s(\theta, C_1, C_2) \subseteq C_i \setminus C_{-i} \},$$
the set of types indifferent between the two intermediaries,

\[ B(s, C_1, C_2) = \{ \theta : \text{Supp}(s(\theta, C_1, C_2)) \subseteq C_i \cap C_{-i} \} \]

and the set of types willing to sign a contract, rather than taking the outside option,

\[ A = A_1 \cup A_2 \cup B \]

Notice that we are using the maintained assumption that when indifferent between being an entrepreneur or a worker, agents prefer entrepreneurship.

**Equilibrium definition**

The equilibrium concept applicable to this framework is the Bayes-Nash Equilibrium.

**Definition 2.1** A strategy profile \((C^*_1, C^*_2, s^*)\) is a Bayes-Nash Equilibrium if:

1. For all \(\theta \in \Theta\)
   \[
   \text{Supp}(s^*(\theta, C^*_1, C^*_2)) \subseteq \arg \max_{(k,x,z)} \theta f(k)z + x \\
   \text{s.t.} \quad (k,x,z) \in C^*_1 \cup C^*_2 \cup \{(0,w,0)\}
   \]

2. For each intermediary \(i \in \{1, 2\}\), given the entrepreneur strategy \(s^*\) and the competitor’s contract menu \(C^*_i\), her own contract menu \(C^*_i\) maximizes her expected utility
   \[
   C^*_i \in \arg \max_{C_i} v_i(s^*, C_i, C^*_{-i}) \\
   \text{s.t.} \quad \forall (k,x,z) \in C_i \subseteq \mathbb{R}^3 \\
   k \geq 0, \quad x \geq 0, \quad x + z \geq 0.
   \]

The conditions \(x \geq 0\) and \(x + z \geq 0\) make sure that limited liability is satisfied: if
the project fails, the entrepreneur cannot make any payment to the intermediary, but the intermediary could potentially make a transfer to the entrepreneur.

2.2.3. The Equilibrium Contract

In this section we state a sequence of claims leading to the characterization of the equilibrium contract. As will be shown, intermediaries make zero profits in any equilibrium and the entrepreneur’s payoff is linear in their type. Although the equilibrium is by no means unique, all equilibria are payoff equivalent.

The Payoff of Entrepreneurs

Let \( U(\theta; C_1, C_2) \) be the potential payoff that a \( \theta \)-type agent could get, conditional on becoming an entrepreneur, when intermediaries play \( C_1 \) and \( C_2 \). That is,

\[
U(\theta; C_1, C_2) = \max_{(k, z, x) \in C_1 \cup C_2} \theta f(k)z + x
\]

Let \( (k(\theta), z(\theta), x(\theta)) \) be a representative of the (equivalence class of) maximizers. Under the assumptions for \( \theta \) and \( f(k) \), the Spence-Mirrless conditions (single crossing) hold and local incentive compatibility conditions are equivalent to the global incentive compatibility conditions. Hence, Myerson’s lemma can be applied:

**Lemma 2.2** If \( C_1^* \) and \( C_2^* \) are part of an equilibrium:

\[
U(\theta; C_1^*, C_2^*) = U_1(\theta) + \int_0^\theta f(k(s))z(s)ds,
\]

(2.1a)

\[ f(k(\theta))z(\theta) \text{ is non-decreasing.} \]  

(2.1b)

\[
x(\theta) = U(0) + \int_0^\theta f(k(s))z(s)ds - \theta f(k(\theta))z(\theta),
\]

(2.1c)

We refer to \( (k(\theta), z(\theta), x(\theta)) \) as an incentive compatible contract menu.

The equilibrium payoff of a \( \theta \)-type agent is \( \max\{w, U(\theta)\} \).
**Project/Loan Size**

We now aim to characterize the amount of capital lent to each entrepreneur in equilibrium. First, define \( k^*(\theta) \) as the full information optimal investment in a project of type \( \theta \).

\[
\begin{align*}
  k^*(\theta) &= \arg \max_k \{ \theta f(k) \pi - Rk \} \\
  S(\theta) &= \max_k \{ \theta f(k) \pi - Rk \} \\
  &= \theta f(k^*(\theta)) \pi - R \cdot k^*(\theta).
\end{align*}
\]

\( S(\theta) \) is a gross surplus because it doesn’t include the opportunity cost of forgoing the outside option \( w \). Note that under the assumptions for \( f(k) \), the optimal project size \( k^*(\theta) \) is a continuous and strictly increasing function of \( \theta \).

The payoff of the entrepreneur only depends on \( k \) through the product \( f(k)z \). Because of the multiplicative separability, given \( k^0, z^0 \), the intermediary can offer \( k^*(\theta) \) and adjust \( z \) to keep \( f(k)z \) constant.

Claim 2.3 formalizes the argument above. All proofs are in the appendix.

**Claim 2.3** Contracts signed in equilibrium have \( k(\theta) = k^*(\theta) \) for (almost) every \( \theta \in A \).

**The Payoff of Intermediaries**

The market structure resembles Bertrand competition, and it is natural to conjecture that if an intermediary were to make profits, his competitor could offer contracts slightly more generous and steal the market.

**Claim 2.4 (Zero Profit Condition)** In any equilibrium, the profits for intermediaries is zero.
Fighting for the Best

We just showed that intermediaries make zero profits. In this case, entrepreneurs have all the bargaining power and one could suspect that intermediaries will make zero profits type by type and each entrepreneur would receive all the economic surplus she produces. That suspicion would be correct in a similar framework without limited liability, or without asymmetric information. However, the interaction between the two frictions does not allow for that to happen in our economy. On the contrary, the expected surplus of the project tends to grow much faster than incentives can be provided: whenever expected profits are positive, locally expected revenues increase faster than expected costs. In a nutshell, intermediaries want to invest more in more able types, but cannot increase rewards too fast to keep incentives.

Claim 2.5 Suppose $S(\hat{\theta}) \geq U(\hat{\theta})$ for some $\hat{\theta} > 0$. Then, $S(\theta') > U(\theta')$ for almost every $\theta' > \hat{\theta}$.

Because both intermediaries are making zero profits, and $U$ and $S$ are continuous, there is a $\theta$ such that $U(\theta) = S(\theta)$. The claim above implies that such $\theta$ is necessarily unique: although intermediaries make zero profits on average, they make strict profits with the best types and strict loses with other types.

The result above is in sharp contrast with Rothschild and Stiglitz (1992). In such an environment, if there were profits to be made with a particular type, the market will “cream skim” it, until profits are zero type by type. We will show that the limited liability constraint binds and prevents that from happening. Before, we need to characterize the set $A$ of types choosing to be entrepreneurs.

The next claim states that $A$ takes the form of an interval, and the lowest type willing to take one of the offered contracts rather than the outside option is well defined. We will refer to such a type as $\theta_L$. 
Claim 2.6  The set \( A \) of all types that are willing to accept at least one of the offered contracts is an interval of the form \( A = [\theta_L, 1] \) for some \( \theta_L \in [0,1] \). Moreover, if \( \theta_L > 0 \), \( U(\theta_L) = w \).

When entrepreneurs are risk neutral, it is natural to expect that if a project fails, intermediaries won’t pay anything to entrepreneurs, and hence \( x(\theta) = 0 \). If that was not the case, intermediaries could “cream skim” the market. That is, intermediary \( i \) could deviate to a contract serving all the profitable types, and leave all the unprofitable types to his competitor.

Claim 2.7  Competition in the intermediation market and the limited liability constraint imply that any contract offered and signed in equilibrium has \( x(\theta) = 0 \) for all \( \theta > \theta_L \).

It is useful to note that the result above would not hold, absent limited liability. In that case, the optimal mechanism specifies that entrepreneurs get all the profits from the project and pay \( R \cdot k^*(\theta) \) no matter if the project succeeds or fail. Limited liability puts a bound to the separation of types. As a result, high types are more profitable for intermediaries.

Limited Incentive Provision

Given the above results, incentives can only be be provided using \( z \). Claim 2.7 implies that \( U(\theta) = \theta f(k^*(\theta))z(\theta) \) for all theta. Hence \( U(0) < w \) and then \( \theta_L > 0 \). Also, the set of maximizers of \( \theta f(k)z \) is independent of \( \theta \). It follows that \( f(k(\theta))z(\theta) \) cannot depend on \( \theta \). Since, \( \theta_L f(k(\theta_L))z(\theta_L) = w \), we get from the condition above:

\[
f(k(\theta))z(\theta) = \frac{w}{\theta_L}. \tag{2.2}
\]

This together with claims 2.3 and 2.7, fully characterize the equilibrium optimal contract.
Proposition 2.8 In equilibrium, the contract signed by almost every $\theta$ is:

$$k^*(\theta) = \arg \max_k \{\theta f(k)\pi - R_k\}, \quad z^*(\theta) = \frac{w}{\theta_L f(k^*(\theta))}, \quad x^*(\theta) = 0.$$ 

where $\theta_L$, the lowest type who accepts the contract, is such that the financial intermediaries make zero profits.

Some Implications

The expected payoff for the entrepreneur of type $\theta$ is $\frac{\theta}{\theta_L} w$, the gross rate of return is $\frac{\theta}{\theta_L k^*(\theta)} w$. Thus the expected return may be increasing or decreasing in $\theta$, depending on $f$.

The expected payoff for the intermediary on a $\theta$-type project is $\theta f(k(\theta))\pi - \frac{w\theta}{\theta_L} - R_k(\theta) = S(\theta) - U(\theta)$. It follows from claim 2.5 that this payoff can be zero only for one particular $\theta$ and will be positive for higher types. Moreover $S(\theta) - U(\theta)$ is a convex function of $\theta$. The good types are very valuable for intermediaries: the profits coming from them will compensate for the losses from bad types.

Interestingly, firms cannot fight for segments of the market (i.e: try to steal only a subset of $\Theta = [0,1]$). The expected return for entrepreneurs is independent of $k$ and $z$. If a firm offers a more attractive contract for some $\theta$, it must be because $\theta_L$ is lower, and hence the contract is better for every $\theta$. In other words, in a world where the limited liability constraint is binding, intermediaries are not able to “skim the cream” up to the point of making zero profits type by type. This is in contrast with Rothschild and Stiglitz (1992).

The following picture illustrates the equilibrium described by proposition 2.8. The figure shows several monetary payments as functions of $\theta$. We plot the entrepreneurs’ expected payoff $U(\theta) = \frac{\theta}{\theta_L} w$ and their outside option $w$ (which is assumed to be independent of $\theta$); the expected surplus of the project, net of fi-

\footnote{By the envelope theorem, the derivative is $\pi f(k^*(\theta)) - \frac{w}{\theta_L}$}
Financial costs, $S(\theta) = \theta f(k^*(\theta))\pi - Rk^*(\theta)$. The area shaded in gray represents the profits of an intermediary. From the figure it is clear that changes in $\theta_L$ change the slope of the entrepreneurs expected payoff. These changes, in turn modify the intermediaries’ payoffs and hence $\theta_L$ can be adjusted so that intermediaries make zero profits.

Figure 9 illustrates the key force in this economy: to provide the correct incentives, the payoff to entrepreneurs cannot grow as fast as the total surplus does. That means that intermediaries always make higher profits with the highest types. Since high types are so profitable, intermediaries are willing to lose money with low types to offer more attractive contracts to (profitable) high types and maintain incentives.

Figure 9: Equilibrium Contract and Zero Profits

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\[ \theta f(k^*(\theta))\pi - Rk^*(\theta) \]

Note that the expected surplus is similar to the profit function of a competitive firm facing an output price of $\theta$. It is well known that profit functions are convex in prices and hence the surplus is convex in $\theta$, as displayed.
2.2.4. Welfare and optimal policy

The expected social surplus of a project is $\theta f(k^*(\theta))\pi - Rk^*(\theta) - w$. Hence a social planner sets $\theta f'(k^*(\theta))\pi = R$ as before, but only for those $\theta$ such that the expected social benefit is not negative. As the surplus is increasing in $\theta$ by the envelope theorem, there is a lower bound $\theta_P$, for those socially valuable projects. Then it is worth for the society to devote resources to all projects with $\theta \geq \theta_P$ where $\theta f(k^*(\theta_P))\pi - Rk^*(\theta_P) = w$.

By contrast, in the decentralized equilibrium, $\theta_L$ yields zero profit for intermediaries:

$$\int_{\theta_L}^{1} \{ \theta f(k^*(\theta))\pi - Rk^*(\theta) \} \, dG(\theta) - \int_{\theta_L}^{1} \frac{\theta w}{\theta_L} \, dG(\theta) = 0$$

**Proposition 2.9** In the decentralized solution, socially inefficient projects are enacted. That is $\theta_L < \theta_P$. Moreover, efficiency will be restored if any of the following becomes true:

- No adverse selection: Types become public information
- No limited liability: Intermediaries become able to recover any contracted amount.
- No competition: There is only one intermediary or intermediaries collude.

The intuition behind it is as follows: if only efficient projects were financed, the intermediaries would make profits in all the projects. But then, positive profits attract new intermediaries who can “steal” the market by offering more generous contracts. More generous contracts involve some cross-subsidization, and as a result socially inefficient projects will be active.

If types are public information but all other conditions remain the same, intermediaries will break even on each type. This implies that all signed offer is such that $U(\theta) = S(\theta)$, and only types $\theta > \theta_P$ sign contracts. Contracts will not be completely determined since many combinations of $x(\theta)$ and $z(\theta)$ yield $U(\theta) = S(\theta)$, but $k(\theta) = k^*(\theta)$. The equilibrium payoff of type $\theta$ is $U(\theta) = \max\{w, S(\theta)\}$.
If there is no limited liability but all other conditions hold, the only IC contract schedule in equilibrium is \( k(\theta) = k^*(\theta) \), \( x(\theta) = -R \cdot k^*(\theta) \) and \( z(\theta) = \pi \), which implies \( U(\theta) = S(\theta) \). This is just the risk free debt contract. As it is well understood, when the entrepreneur is risk neutral, transferring all the risk to her solves the incentive problem.

If there is only one intermediary, but adverse selection and limited liability still hold, the only equilibrium is as follows: the contract that maximizes profits for the intermediary is \( k(\theta) = k^*(\theta) \), \( x = w \), \( z = 0 \) and entrepreneurs with types \( \theta \geq \theta_L \) take it, all others reject. The intermediary will take all the surplus and her profits would be \( \int_{\theta_p}^{1} (S(\theta) - w) \, dG(\theta) \).

**Optimal policy**

In this section we consider two possibilities for taxation. A fixed sum tax per contract to financial intermediaries \( \phi \) and a tax rate \( \tau \) on entrepreneurs profits\(^7\). In both cases the full information optimal allocation can be achieved.

**Claim 2.10** Let \( \phi^* \) be a fixed tax per contract defined by:

\[
\phi^* \equiv \left[ \int_{\theta_p}^{1} \{ \theta f(k^*(\theta)) \pi - R k^*(\theta) \} \, dG(\theta) - \int_{\theta_p}^{1} \frac{\partial w}{\partial \theta} \, dG(\theta) \right]^{-1} \left( \int_{\theta_p}^{1} \frac{\partial w}{\partial \theta} \, dG(\theta) \right)^{-1}
\]

Then only efficient projects (and all of them) are funded.

This tax can also be seen as fixed subsidy on \( w \), or any instrument that increases the outside option of every entrepreneur. However the revenue implications for the tax authority would be different. Since we do not model the nature of the wage or the outside option, we stick to the contract fee tax interpretation.

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\(^7\)A tax to entrepreneurs could be considered but it would require them to have some external funds that the government can seize even in case of failure. In that case:

\[
\phi^* \equiv \theta_p \left[ \int_{\theta_p}^{1} \theta dG(\theta) \right]^{-1} \left[ \int_{\theta_p}^{1} \{ \theta f(k^*(\theta)) \pi - R k^*(\theta) \} \, dG(\theta) - \int_{\theta_p}^{1} \frac{\partial w}{\partial \theta} \, dG(\theta) \right]
\]
Now suppose there is a profit or dividend tax. Intermediaries make zero profit, so they won't be affected by such tax. However entrepreneurs do make profits if the project is successful. Hence a tax rate $\tau$ on profits would make any contract $(k, x, z)$ look to the entrepreneurs like $(k, x, (1 - \tau)z)$

**Claim 2.11** Let $\tau^* = \frac{\phi^*}{w + \phi^*}$ be the tax rate on profits, where $\phi^*$ is the fixed tax rate found in claim 2.10. Then only efficient projects (and all of them) are funded.

Taxing $R$ would be troublesome since it will distort $k^*(\theta, R)$ reducing the total surplus, which is the standard result on capital income taxation. In principle this would be at odds with claim 2.11, but this is because we assume taxation on capital income is made at the individual level. Hence as long as the intermediaries can aggregate profits on investments before taxes, or claim back taxes on dividends from entrepreneurs firms in which they have a stake, they won't be affected by the taxation, only the entrepreneurs. So far we have avoided to make any interpretation of the legal stance the contracts will have. We can think of the contracts as debt contracts with limited liability clauses or as equity stakes in a firm, those details are irrelevant for our discussion above but become important once we face a complex tax law. Of course the discussion of that topic is far away from the scope of this paper.

### 2.3. Capital Holdings

As the previous section shows, for $\theta$ high enough, the return of the project for the entrepreneur would be higher if he had access to the risk free rate. Hence skilled entrepreneurs who own capital are willing to use it on their project. The same will happen if the asset owned is not liquid but is pledgeable: the entrepreneurs are willing to pledge their assets if doing so gets them better loan terms. We allow for that possibility in the current section.
2.3.1. Observable Capital Holdings

Assume entrepreneurs have assets $a \in A$, distributed according to $G(\theta, a)$. Two interpretations are possible, both will yield the same results: for a project of size $k$ the entrepreneur provides $a$ as capital and the intermediary finances $k - a$ or the intermediary finances $k$ collateralized by $Ra$ from the entrepreneur. In what follows the latter will be used.

The asset holdings from entrepreneurs serve two purposes, they relax the limited liability constraint for the intermediaries and increase the outside option for the entrepreneurs.

Claim 2.12 The outside option $O(\theta, a)$ of an entrepreneur of type $\theta$ that holds assets $a$ is characterized as follows:

$$O(\theta, a) = \max \left\{ w + Ra, \theta F(\min\{a, k^*(\theta)\})\pi + R \cdot \max\{0, a - k^*(\theta)\} \right\}$$

The expression for the outside option results from the fact that an entrepreneur of type $\theta$ holding $a$ units of capital can always produce using his skills and own capital, or work and lend his capital.

With observable assets, contract schedules are contingent on $a$. A strategy for an intermediary specifies a subset of $\mathbb{R}^3$ for each $a \in A$.

To facilitate interpretation, let the equilibrium payoff of a type $\theta$ entrepreneur with assets $a$ be $U(\theta, a) + Ra$.

Myerson’s lemma and claims B.1 to 2.4 will still hold for each $a$, since nothing in their proofs depends on the limited liability condition being exactly zero. With minor changes to the proof we can state:

Proposition 2.13 In equilibrium with observable assets, the contract offered by all finan-
cial intermediaries to almost every \((\theta, a)\) is:

\[
k^*(\theta, a) = k^*(\theta) = \arg \max_k (\theta f(k) \pi - Rk), \quad z^*(\theta, a) = \frac{O(\theta_L(a), a)}{\theta_L(a) f(k^*(\theta))}, \quad x(\theta) = -Ra.
\]

where \(\theta_L(a)\), the lowest \(\theta\) among those entrepreneurs with \(a\) assets who accepts the contract. For each \(a\), it must be the case that:

\[
\int_{\theta_L(a)}^{1} \left\{ \theta f(k^*(\theta)) \pi - \theta \frac{O(\theta_L(a), a)}{\theta_L} - R(k^*(\theta) - a) \right\} dG(\theta|a) = 0
\]

The following figure illustrates the equilibrium,

![Figure 10: Equilibrium Contract with Observable Asset Level \(a\)](image)

As long as entrepreneur’s capital holdings are observable several loan markets will be active, one for each asset level \(a\). Those markets will be described by proposition 2.13. In the next subsection we deal with the case of unobservable asset holdings.
2.3.2. Unobservable Capital Holdings

In principle, an entrepreneur with wealth $a$ could hide part of his own wealth and take a contract designed for a lower $a$ if it is more profitable. This is akin to an entrepreneur setting up a corporation but only investing a fraction of his wealth. We ruled out this possibility by assuming that $a$ was known (observable) by the intermediary. In this subsection we drop that assumption.

Just as in the previous section, the strategy of intermediary $i$, $\mathcal{C}_i$ specifies a subsets of $\mathbb{R}^3$ for each $a \in \mathcal{A}$. But now, an entrepreneur with type $(\theta, a)$ solves,

$$U(\theta, a; \mathcal{C}_1, \mathcal{C}_2) = \max_{(k, z, x) \in \mathcal{C}(a)} \theta f(k) z + x$$

where $\mathcal{C}(a) \equiv \bigcup_{a' \leq a} \mathcal{C}_1(a') \cup \bigcup_{a' \leq a} \mathcal{C}_2(a')$

Let $(k(\theta, a), z(\theta, a), x(\theta, a))$ denote a representative solution of the above problem. Incentive compatibility across assets only requires $U(\theta, a)$ to be nondecreasing in $a$. This because an entrepreneur cannot pledge more collateral or invest more capital than the amount he owns, which implies he can only lie by hiding some assets. For the incentive compatibility over $\theta$, Myerson lemma holding fixed each asset level is enough. Limited liability in this environment still requires $x(\theta, a) \geq -Ra$.

It can be shown that, as before, $x(\theta, a)$ is non increasing in $\theta$, $k(\theta, a) = k^*(\theta)$ for (almost) all $(\theta, a)$, $U(\theta, a)$ will be nondecreasing in $\theta$. If we define the set $\mathcal{A}_i(a)$ as those $\theta$ such that types $(\theta, a)$ are willing to take the contract from intermediary $i$ it is still the case that $\bigcup_i \mathcal{A}_i(a) = \mathcal{A}(a) = [\theta_L(a), 1]$. Also there would be a zero profit condition but in an aggregate sense across assets. Hence proofs depending on a zero profit condition per asset level need to be updated.

The unobservability of the assets may (will) imply limited liability is not binding for some cases. However we still can state an updated version of claim 2.7.
Claim 2.14 Competition in the intermediation market and the limited liability constraint imply that any equilibrium contract schedule satisfies $x(\theta, a) = -Ra$ for all $(\theta, a)$ such that $S(\theta) \geq U(\theta, a) > w$.

This implies that limited liability will bind for those types for which the intermediaries expect to make some profits. Now, for each $a$ define $\theta_e(a)$ as the solution to $U(\theta, a) = S(\theta)$ which means the intermediary expects to break even with this type. Above those $\theta_e(a)$ claim 2.14 implies $x = -Ra$ and therefore for all $\theta > \theta_e(a)$:

$$U(\theta, a) = \theta \frac{S(\theta_e(a)) + Ra}{\theta_e(a)} - Ra$$

(2.3)

Now, fix the non decreasing function $\theta_e(a)$. That completely determines the expected earnings for intermediaries among the profitable contracts. For types below $\theta_e(a)$ losses are incurred in expectation so the intermediary wants to offer as little as possible. However, the IC constraint across assets may bind. Equation (2.3) is then a lower bound for the surplus given to those types. If the IC across assets binds for a type $(\theta, a)$ is because there exists some $0 \leq a' < a$ such that:

$$\theta \frac{S(\theta_e(a')) + Ra'}{\theta_e(a')} - Ra' > \theta \frac{S(\theta_e(a)) + Ra}{\theta_e(a)} - Ra.$$

After fixing $\theta_e(a)$, a profit maximizing intermediary has no way to improve. The contracts she is profiting with are fully determined, and for those types she is expected to lose she has a lower bound on the utility she has to deliver. Hence for $\theta < \theta_e(a)$ a profit maximizing intermediary sets:

$$U_i(\theta, a) = \sup_{0 \leq a' \leq a} \theta \frac{S(\theta_e(a')) + Ra'}{\theta_e(a')} - Ra'$$

(2.4)

Equations (2.3) and (2.4) fully characterize the contracts given $\theta_e(a)$. Figure 11 illustrates this feature. The zero profit condition is not enough to pin down that function, since zero profits must hold on aggregate across types. To pin down
Claim 2.15 In any equilibrium, contract schedules must offer the same utility to (almost) all types willing to sign at least one of the offered contracts.

The intuition rests on the fact that the average of two IC contracts is IC because the expected utility for entrepreneurs is linear in the varying elements of the contract \((x, z)\).

An intermediary can decide not to offer any contract with \(x(\theta, a) = R\bar{b}\) for some asset level \(\bar{b} > 0\). By doing so she gives up the expected profit of types with \(\theta \geq \theta_e(\bar{b})\) and assets \(\bar{b}\), she avoids the losses with those types \(\theta < \theta_e(\bar{b})\) with assets \(\bar{b}\). But also there could have been some types \((\theta, a)\) with \(\theta \leq \theta_e(a)\) and
$a > \tilde{a}$ but such that:

$$U(\theta, a) = \theta \frac{S(\theta_\epsilon(\tilde{a})) + R\tilde{a}}{\theta_\epsilon(\tilde{b})} - R\tilde{b}.$$ 

If that was the case, half of those types were taking the contract with intermediary $i$ before she dropped those contracts. To quantify the effect of that action define

$$A(a|b) = \left\{ \theta : x(\theta, b) = -Ra \right\}$$

Incentive compatibility implies that if two types are offered the same $x(\theta, a)$ they should be offered the same $f(k^*(\theta))z(\theta, a)$. Hence for all $\theta \in A(a|b)$ it must be the case that:

$$U(\theta, b) = \theta \frac{S(\theta_\epsilon(a)) + Ra}{\theta_\epsilon(a)} - Ra$$

**Definition 2.16** Let $P(a) = \frac{S(\theta_\epsilon(a)) + Ra}{\theta_\epsilon(a)}$ be the slope of all contracts such that $x(\theta, b) = -Ra$.

**Claim 2.17** The sets $A(a|b)$ are intervals. For all $b \leq c$ $A(a|c) \subset A(a|b)$.

Given that, we can now write a closed form for intermediary’s profits given some non decreasing function $\theta_\epsilon(a)$.

$$\Pi = 0.5 \int_{a=0}^{a=\infty} \int_{b=a}^{b=\infty} \int_{\theta \in A(a|b)} [S(\theta) - \theta P(a) + Ra] d^2G(\theta, b) da$$  \hspace{1cm} (2.5)

Now define $\Pi(c)$ as the profits brought by contracts with $x(\theta, a) \leq -Rc$:

$$\Pi(c) = 0.5 \int_{a=c}^{a=\infty} \int_{b=a}^{b=\infty} \int_{\theta \in A(a|b)} [S(\theta) - \theta P(a) + Ra] d^2G(\theta, b) da$$

**Claim 2.18** For all $c$ profits brought by contracts with $x(\theta, a) \leq -Rc$ are zero.
Proof. If $\Pi(c)$ is negative, the intermediary can always drop all those contracts and increase profits. If $\Pi(c)$ is positive, analogously to the zero profit condition, the intermediary can give some $\varepsilon > 0$ more to all types with assets equal or higher than $c$, stealing the half of the market serviced by the other intermediary and getting $2\Pi(c) - \varepsilon$. ■ Taking derivative of $\Pi(c)$ we obtain that for all $a$

$$0 = \int_{b=a}^{b=\infty} \int_{\theta \in A(a|b)} \left[ S(\theta) - \theta P(a) + Ra \right] d^2 G(\theta, b)$$ (2.6)

which implies that for each $a$ the intermediaries should break even on those contracts such that $x(\theta, b) = -Ra$. This result is analogous to Rothschild and Stiglitz (1992), where zero profits should be made contract by contract. In our setting all contracts with $x(\theta, b) = -Ra$ are equivalent for the entrepreneurs because $f(k^*(\theta))z(\theta, a)$ has to be constant across all those contracts, but different to the intermediary because of the different expected surplus. Next the following proposition fully characterizes the equilibrium.

Proposition 2.19 In equilibrium, the contract offered by all financial intermediaries to almost every $\theta$ and every $a$ is:

$$k^*(\theta, a) = \arg\max_k \theta f(k) \pi - Rk,$$

$$U(\theta, a) = \max_{0 \leq a' \leq a} \theta P(a') - Ra'$$

$$x(\theta, a) = -R \left[ \arg\max_{0 \leq a' \leq a} \theta P(a') - Ra' \right].$$

where $\theta_e(\cdot)$ is such that for all $a$ the financial intermediaries make zero profits over all entrepreneurs taking a contract with $x = -Ra$. That is,

$$\int_{b=a}^{b=\infty} \int_{\theta \in A(a|b)} \left[ S(\theta) - \theta P(a) + Ra \right] d^2 G(\theta, b) = 0.$$
2.4. A Numerical Example

We have established that in the environment described above, competition among financial intermediaries yields to an inefficient outcome. The result raises a natural question: in which markets, industries or countries will be the inefficiency more pronounced? In the current section we use a numerical example to illustrate the changes in the deadweight loss when various parameters of the model change.

2.4.1. Parameterization

For the numerical example we set the entrepreneurs’ outside option \( w = 15 \); the output of the project in case of success, \( \pi = 100 \); and the gross interest rate is \( R = 1.02 \).

Recall that a project succeeds with probability \( p(\theta, k) = \theta f(k) \). We let \( f(k) = 1 - \exp(-\beta k^\alpha) \) for \( \beta > 0 \) and \( \alpha \in (0, 1) \). This functional form has several properties. First, it is continuous and strictly increasing and strictly concave on \( \mathbb{R}_+ \). Second, \( f(0) = 0 \) and \( \lim_{k \to \infty} f(k) = 1 \). Third, \( \lim_{k \to 0} f'(k) = \infty \). The last condition ensures that for every \( \theta > 0 \) there is a scale such that the \( \theta \)-type project is profitable (Inada condition). A way to interpret the above functional form is that the \( p(\theta, k) \) is the product of \( \theta \) and the probability that an exponential random variable is lower than \( k^\alpha \). As exponential variables are usually employed for waiting times for a poisson process, it can be interpreted as the waiting time until the arrival of a new innovation (success), an amount \( k \) of capital allows the entrepreneur to run the project for \( k^\alpha \) periods and hence the probability of a good idea arriving would be \( f(k) \).

We set \( \beta = 0.1 \), implying that the latent exponential random variable would have a mean of 10. Under the above interpretation increasing \( k \) increases the probability

---

\(^8\text{Equivalently, } p(\theta, k) \text{ is the product of } \theta \text{ and the probability that a Weibul random variable is lower to } k. \text{ In that case, the waiting time interpretation would be that the longer it takes for a project to succeed the less likely it will succeed in the future. Jovanovic and Szentes (2013) use a similar approach hence our functional forms may be regarded as a reduced form of their results.} \)
that the random falls below $k^\alpha$ at a decreasing rate. We set $\alpha = 0.1$.

Last but not least, the joint distribution of assets and types will be key to compute intermediaries profits. We focus on the conditional distribution of $\theta$ given $a$, and let $g(\theta|a) = \frac{a+1}{\eta} \theta^\frac{a+1}{\eta} - 1$ be its density. The exponent $\frac{a+1}{\eta} - 1$ controls the participation of high types on the conditional distribution. The higher is the exponent, the higher will be the density of types higher than a fixed value $\theta$ (Saffie and Ates (2013)). We let the exponent to be increasing on the asset level $a$, meaning that assets are positively correlated with the types. In addition, we assume the unconditional distribution of assets has density $\lambda \exp(-\lambda a)$. Hence an average entrepreneur has an asset level of 2. All the parameters and functional forms used in the numerical example are summarized in Table 8.

<table>
<thead>
<tr>
<th>Table 8: Benchmark Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w$</td>
</tr>
<tr>
<td>$\pi$</td>
</tr>
<tr>
<td>$R$</td>
</tr>
<tr>
<td>$f(k)$</td>
</tr>
<tr>
<td>$\alpha$</td>
</tr>
<tr>
<td>$\beta$</td>
</tr>
<tr>
<td>$g(\theta</td>
</tr>
<tr>
<td>$\eta$</td>
</tr>
<tr>
<td>$g(a)$</td>
</tr>
<tr>
<td>$\lambda$</td>
</tr>
</tbody>
</table>

2.4.2. Benchmark Results

We begin by describing the case where entrepreneurs do not have assets. In order to facilitate comparability with the upcoming section where assets are introduced, we use use the marginal distribution of types that is consistent with the joint distribution of assets and types that will be used next (i.e: intermediaries will face the same population in both cases).

As defined before, an equilibrium is fully described by the triple $(k(\theta), x(\theta), z(\theta))$. We established that $x(\theta) = 0$, $\forall \theta$. The functions $k(.)$ and $z(.)$ are plotted in the
Figure 12 below. An important equilibrium object is the lowest type taking the contract, $\theta_L$. For this example, the value of $\theta_L$ is 0.49, which implies that 74% of the entrepreneurs take the contract. Compare $\theta_L$ with the lowest type that would be funded by a social planner $\theta_P = 0.62$.

In the Figure 13 we plot $S(\theta) = \pi \theta f(k^*(\theta)) - Rk^*(\theta)$ as well as the expected payoff of entrepreneurs $\mathbb{E}(\theta) = w\theta/\theta_L$. In addition we plot the the wage (horizontal line) to allow the reader to picture the equilibrium deadweight loss. In what follows we will use the relative inefficiency, defined as the ratio of the deadweight loss to the net economic surplus,

$$J = \frac{\int_{\theta_L}^{\theta_P} (w - S(\theta))dG(\theta)}{\int_{\theta_L}^{\theta_P} (S(\theta) - w)dG(\theta)}$$

For our benchmark parameterization, the relative inefficiency is $J = 5.03\%$. In the next section we study how this inefficiency responds to changes in the parameters of the model.
2.4.3. The Relative Inefficiency: Comparative Statics

In the current section we change, one by one, all the parameters of the model, holding the other parameters at their respective benchmark values. The outside option of entrepreneurs, \( w \), is very important because, absent this cost, the economy would not be inefficient. In fact, when \( w = 0 \), all projects are socially profitable - by assumption - , and they are all funded at the optimal scale. An increase in \( w \) directly increases the deadweight loss and decreases the net economic surplus (everything else equal). It also induces an endogenous adjustment of \( \theta_L \). A higher \( w \) tends to increase \( \theta_L \), but this increment is dampen because the pool of types is increased and competition induces intermediaries to increase the terms of the contract. However, increases in \( \theta_L \) reduce the deadweight loss. As illustrated in Figure 14, when the wage is very high, the force is strong enough to actually decrease the inefficiency. Note that is the wage is high enough, all the entrepreneurs take the outside option and the deadweight loss disappears. The relative inefficiency is maximized when \( w = 31 \), attaining 6.8% of the net
surplus.

Figure 14: Relative Inefficiency and Wage

![Figure 14](image)

The gross interest rate, \( R \), is similar to the wage, in the sense it represents the outside option, or opportunity cost of the capital in hands of financial intermediaries. A higher gross interest rate not only decreases the payoff of intermediaries, but it also decreases the optimal scale of the projects, and hence their expected return. The condition \( \theta \pi f'(k^*) = R \) implies that the function \( S(\theta; R) = \pi \theta f(k^*) - R k^* \) is linearly homogenous in \((\theta, R)\). Since \( \theta \in [0, 1] \), an increase in \( R \) can be interpreted graphically as a downward shift of the curve \( S(\theta) \) as shown in the right panel of Figure 15. This force increases both the net surplus and the deadweight loss everything else equal. However, \( \theta_L \) will increase to satisfy the zero profit condition. As shown in the left panel of Figure 15, when we let \( R \) vary between 1 and 1.8, the relative inefficiency increases up to 11%, at \( R = 1.8 \).

We move to describing how the inefficiency responds to changes in the shape of function \( f(k) \). Figure 13 above suggests that the size of the inefficiency is very related to the concavity of the function \( S(\theta) \). This concavity only depends on the
shape of \( f \). In fact,
\[
S''(\theta) = f(k^*(\theta)) - \frac{(f'(k^*))^2}{f''(k^*)}
\]
Not surprisingly, the relative inefficiency is quite sensitive to the parameters \( \alpha \) and \( \beta \), that govern the shape of \( f \) in the current example. The results are displayed in Figure 16. The relative inefficiency is decreasing in both \( \alpha \) and \( \beta \). It decreases particularly fast as \( \beta \) increases, getting to 0.05% when \( \beta = 1 \).

Last, the distribution of types importantly affects the size of the inefficiency. The parameter \( \eta \) governs the shape of the distribution of \( \theta \). More precisely, as \( \eta \) increases, the density is shifted toward lower types. Holding the outside option fixed, an increase in \( \eta \) decreases the net surplus, because some density will be shifted from socially profitable projects to unprofitable projects. However, when good types are scarcer, intermediaries will offer less generous contracts, increasing \( \theta_L \). The last force tends to decrease the inefficiency. Figure 17 shows that the relative externality actually increases, reaching 7.7% when \( \eta = 3 \). On the other hand, for values of \( \eta \) close to zero, the relative inefficiency gets close to zero.
2.4.4. Assets

For simplicity, we discretize the space of assets, using the quantiles of the marginal distribution described in section 2.4.1, \( g(a) = \lambda \exp(-\lambda a) \). We use five asset levels, 0, 0.45, 1.02, 1.83 and 3.22; there will be of 20% of the population holding each of the levels of assets. To put this numbers in perspective, an entrepreneur of type \( \theta = 1 \) will optimally invests \( k^*(1) = 25 \), while the for the lowest type taking the contract in the absence of assets, \( k^*(\theta_L) = 10.3 \) (see Figure 12). Recall that assets and types are correlated, and \( g(\theta|a) = \frac{a+1}{\eta} \theta^{a+1} - 1 \).

The equilibrium is summarized in Table 9 and Figure 18. It will prove useful to introduce some further notation and let \( \theta_F = \inf A(a|a) \) be the lowest type with asset level \( a \), taking the contract with slope \( P(a) \). By contrast, \( \theta_L(a) \), the lowest type with assets \( a \) taking any contract. Hence, \( \theta_L(a) = \min_{\hat{a} \leq a} \theta_F(a) \). Then, the relative inefficiency becomes,

\[
J = \frac{\sum_{a=1}^{5} \int_{\theta_L(a)}^{\theta_F(a)} (w - S(\theta))dG(\theta)Pr(\theta)}{\int_{\theta_L(a)}^{\theta_F(a)} (S(\theta) - w)dG(\theta)}
\]

Figure 16: Relative Inefficiency and the Shape of \( f(k) \)
The lowest type taking the 0 asset contract is type $\theta = 0.5123$. Next, the lowest type taking the contract that requires to advance $a = 0.45$ is $\theta_F(0.45) = 0.5108$. This implies that the contract that requires $a = 0.45$ is more generous than the one that doesn’t require assets. By contrast, the lowest type taking the contract requiring $a = 3.22$ is 0.5478. When $\theta$ is between 0.5136 and 0.5478, an entrepreneur with $a = 3.22$ prefers to take the contract that only requires a collateral of 1.8.

In this setting, the relative inefficiency is 3.94%. Hence, allowing intermediaries to condition their contracts on collateral decreases the inefficiency by more than a percentage point (from 5.03 %) in this example. There are two reasons why the inefficiency is reduced when the contracts depend on assets. First, the introduction of assets relaxes the limited liability constraint. Second, because the collateral is more likely to be held by higher types, conditioning contracts on collateral allows the intermediaries to do some sort of screening.

In order to disentangle the two effects, we compute the equilibrium for an economy with the same marginal distributions of assets and types, but where assets
and types are independent. When assets and types are independent, the relative inefficiency is 4.62%. We also consider the case in which intermediaries observe a signal, that is correlated with types in the exact same way that assets were, but (of course) does not affect the limited liability constraint. In that case, the economy has a relative inefficiency of 4.65%.

The results are presented in Table 9. The upper panel displays the optimal contract for entrepreneurs holding assets that are independent of types, and the second panel shows the results when there is an observable signal correlated with the types. In the first case, entrepreneurs with higher asset holdings receive more generous contracts than their peers with lower assets. To understand this, recall that limited liability prevents “cream-skimming” in this environment once $x = 0$. For entrepreneurs with higher assets, the limited liability is not binding until it hits $x = -Ra$. Hence, intermediaries have incentive to skim the cream until they hit the constraint. Under this situation, although the contracts are more generous, they are taken by a lower number of types. In the second case, entrepreneurs
with higher signals also get more generous contracts, but simply because they are better on average, and hence intermediaries break even offering higher terms to entrepreneurs. In this case, the lowest type taking the contract will be decreasing on the signal.

Table 9: Separate Asset Effects

<table>
<thead>
<tr>
<th>Asset Level</th>
<th>0</th>
<th>0.45</th>
<th>1.02</th>
<th>1.83</th>
<th>3.22</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_F$</td>
<td>0.5004</td>
<td>0.5248</td>
<td>0.5498</td>
<td>0.5766</td>
<td>0.6378</td>
</tr>
<tr>
<td>Expected Payoff</td>
<td>30.80</td>
<td>31.60</td>
<td>−0.532.5</td>
<td>−1.033.70</td>
<td>−1.935.40</td>
</tr>
</tbody>
</table>

Informative Signals

<table>
<thead>
<tr>
<th>Asset Level</th>
<th>0</th>
<th>0.45</th>
<th>1.02</th>
<th>1.83</th>
<th>3.22</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_F$</td>
<td>0.5119</td>
<td>0.5065</td>
<td>0.4998</td>
<td>0.4906</td>
<td>0.4769</td>
</tr>
<tr>
<td>Expected Payoff</td>
<td>29.30</td>
<td>29.60</td>
<td>30.00</td>
<td>30.60</td>
<td>31.50</td>
</tr>
</tbody>
</table>

We end this section presenting some comparative statics of the inefficiency to the distribution of assets. In particular, we consider changes in the parameter $\lambda$ of the exponential distribution of assets. We let $\lambda$ vary between 0.5 and 50, which implies that the mean of the asset distribution varies approximately between 2 and $1/50$ $^9$. If the mean of the asset distribution is high enough, the inefficiency can be arbitrarily reduced, because most of the entrepreneurs can fund their own projects. On the other extreme, as $\lambda \to \infty$, the economy converges to one in which all entrepreneurs have zero assets, and the inefficiency converges to 7.5% of the surplus. It is important to note that the limit economy where all entrepreneurs have zero assets is not the same as the benchmark economy described in section 2.4.2. In the benchmark economy, the distribution of types, is the marginal distribution implied by joint distribution of types and assets used throughout the section. I.e, in the benchmark economy $g(\theta) = \int_0^\infty g(\theta, a)da$. When $\lambda \to \infty$, the distribution of types faced intermediaries is $g(\theta|0)$. Because of the positive correlation between assets and types, “lemons” are more abundant in the later

$^9$The mean of the exponential distribution is $1/\lambda$. In our case, this is, however, only an approximation because we discretize the distribution.
2.5. Conclusion

We characterized financial contracts in a competitive environment with risk, adverse selection and limited liability. We find that, under the optimal contract the highest types are rewarded below the expected value of their project, and hence financial intermediaries make profits with high types. They also make losses with lower types. Our main result is that the optimal contract generates an inefficient outcome: projects that wouldn’t be funded by a social planner are funded in equilibrium. We show that asymmetric information, limited liability and competition are all necessary to generate the result. We also show both analytically and numerically that the ability to use some assets as collateral mitigates but does not eliminate the inefficient outcome.
An implication of our results is that competition among financial intermediaries is not desirable. Note, however, that a monopolist financial intermediary would get all the economic surplus. In fact, the monopolist attains the first best because he has the ability to set loan sizes and interests rates, that is, has the ability to price discriminate. Needless to say, being the monopolist of financial intermediation in the economy would be extremely profitable. Who, if anyone, should get this rents? Even if the government gets the rents, a lot of political economy consideration would arise, that may challenge the efficiency of the result.

The results of this paper suggest at least two directions for future research. On the theoretical front, allowing for dynamics would generate an incentive for firms (or entrepreneurs) to retain earnings in order to escape from the inefficiency, and can be related to the literature on firm dynamics. On the empirical front, the degree to which working capital can be collateralized varies exogenously across industries. This opens the door to test the implications of the model.
3.1. Introduction
Recent quantitative studies show that optimal financial policy, defined as policy that implements the allocations that solve a constrained-efficient planner’s problem facing a credit constraint, can be very effective at reducing the magnitude and frequency of financial crises. On the other hand, these studies also show that the optimal policy has a complex state-contingent structure, which raises doubts about its feasibility and suggests that simpler policy rules should be favored. A conjecture implicit in this reasoning is that simple rules are at worse harmless and at best a good approximation to the optimal policy. The findings of recent

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1 Coauthored with Enrique Mendoza. This paper was prepared for the 2016 conference on Policy Lessons and Challenges for Emerging Economies in a Context of Global Uncertainty organized by the Central Bank of Colombia, the Bilateral Assistance and Capacity Building for Central Banks Program of the Graduate Institute Geneva, and the Swiss Secretariat for Economic Affairs. We are grateful to conference participants for their comments and to Eugenio Rojas for excellent research assistance.

2 See, for example, Bianchi (2011), Benigno, Chen, Otrok, Rebucci, and Young (2013, 2014), Bianchi and Mendoza (2010, 2017).
studies suggest, however, that this conjecture is incorrect. Bianchi and Mendoza (2017) showed that simple policies can be welfare-reducing, because they may not match well the prudential characteristics of the optimal policy, which tightens credit-market access in periods of expansion, with a magnitude that varies with the likelihood and severity of future credit crises, and eases credit conditions in the opposite situations. Hence, there is a delicate tradeoff in financial policy design: The optimal policy is too complex to be feasible operationally, but arbitrarily chosen policy rules can be harmful.

This issue is of major policy relevance, because it highlights the importance of the specific rules setting the conditions that trigger the use of financial stability policy instruments and their evolution over time, and yet there is little guidance about the quantitative features that these rules should have. For instance, the Basel III Countercyclical Capital Buffer (CCyB) has the same macroprudential aim of the optimal financial policy described above (i.e. tightening credit in periods of expansion), but it did not define specific rules for when the CCyB is triggered and for how it moves over time. Instead, it left these key features of the CCyB to be determined by BIS member countries using their own judgment. In particular, Basel III indicates that: “each jurisdiction will be required to monitor credit growth and make assessments of whether such growth is excessive and is leading to the build up of system-wide risk. Based on this assessment they will need to use their judgment, following the guidance set out in this document, to determine whether a countercyclical buffer requirement should be imposed. They will also need to apply judgment to determine whether the buffer should increase or decrease over time (within the range of zero to 2.5% of risk weighted assets) depending on whether they see system-wide risks increasing or decreasing. Finally they should be prepared to remove the requirement on a timely basis if the system-wide risk crystallizes.”

In this paper, we compare optimal and simple financial policy rules using a quantitative Fisherian model of financial crises similar to a model widely used in the literature, in which a small open economy faces an endogenously-binding collateral constraint and displays “liability dollarization.”\textsuperscript{4} In particular, debt is denominated in units of tradable goods and cannot exceed a fraction of the income from tradables and nontradables. Models in this Fisherian class feature an endogenous financial amplification mechanism that produce infrequent financial crises with realistic features.

As the literature has shown, the market failure that justifies policy intervention in these models is a pecuniary externality that exists because goods used as collateral are valued at market prices. The social marginal cost of borrowing exceeds the private marginal cost because private agents do not internalize the negative effects of their individual borrowing decisions made in normal times on collateral prices in crisis times. When a crisis hits, the collateral constraint binds inducing agents to fire-sale goods, which in turn causes a collapse in the relative price of nontradable goods, which tightens the constraint further and induces larger declines in relative prices triggering Fisher’s classic debt-deflation mechanism.

The model we propose is based on the one developed by Bianchi, Liu and Mendoza (2016), who introduced noisy news about future economic fundamentals and regime shifts in global liquidity to the workhorse model of macroprudential policy with liability dollarization. We modify this setup by introducing production of tradable and nontradable goods using intermediate goods. This has two important implications. First, it introduces a mechanism by which the collateral constraint causes a drop in output in crisis episodes, because the collapse of the relative price of nontradables causes a collapse in demand for inputs in produc-

\textsuperscript{4}The literature usually refers to this constraint as occasionally binding, but the key feature of the constraint is that whether it binds or not is an equilibrium outcome that depends on endogenous individual choices and aggregate variables. An exogenous credit constraint can be designed to be occasionally binding but it would have very different implications than those obtained with Fisherian models.
tion of nontradables. Second, as a result of the supply-side effects of the collateral constraint, it provides additional vehicles for policy intervention by introducing inefficiencies in sectoral production and factor allocations during crises.

We also characterize optimal policy in the model, showing how the constrained-efficient social planner has incentives to implement both macroprudential and ex-post financial policies. The former reflects the standard pecuniary externality from the existing literature: the optimal policy seeks to increase the cost of borrowing when the collateral constraint is not binding in the current period but can bind with positive probability next period, so as to induce private agents to face the social marginal cost of borrowing in periods of expansion. The ex-post financial policies result from the fact that the effects of the crisis can be mitigated by reallocating resources from production of nontradables to production of tradables in order to prop up the value of collateral and enhance borrowing capacity. Both macroprudential and ex-post financial policies are decentralized as optimal taxes. The macroprudential policy takes the form of a debt tax, and the crisis-management policies take the form of sectoral production taxes and subsidies.5

Following Bianchi, Liu, and Mendoza (2016a), we also characterize the effects of news about fundamentals and global liquidity shifts on the optimal policies. The model is calibrated to data from Colombia and solved to illustrate the model’s crisis dynamics in the absence of policy intervention, the effectiveness of the optimal policy, and the comparison with simple policy rules. The optimal policy reduces the magnitude and severity of financial crises significantly. In contrast, simple policies are much less effective and can be welfare reducing. In particular, time-invariant taxes set to the average values under the optimal policy yield an outcome with lower social welfare than the competitive equilibrium without policy intervention. The time-invariant taxes that yield the largest welfare gain can

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5Modeling macroprudential policy with a debt tax is done only for simplicity. Identical outcomes can be obtained with ceilings on debt-to-income ratios, which are used in practice as a regulatory instrument (for example by the central banks of England, Hungary and Korea).
only produce a gain about half as large as under the optimal policy, have crises in which consumption and the real exchange rate drop nearly four times more, and have crises with a frequency of 1.1 percent (v. nearly zero with the optimal policy).

The rest of the paper is organized as follows. Section 2 describes the decentralized equilibrium of the model without policy intervention. Section 3 examines the problem solved by the financial regulator, characterizes the optimal policy, and shows how the allocations of the social planner can be decentralized using taxes on debt and producers’ input purchases. Section 4 examines the quantitative predictions of the model. Section 5 provides conclusions.

3.2. Model

Our analysis is based on a two-sector model with liability dollarization that has been widely studied in the literature on emerging markets sudden stops and on optimal macroprudential policy (e.g. Mendoza (2002), Bianchi (2011)). In particular, we extend the variant of this model proposed by Bianchi et al. (2016a) that features noisy news about future fundamentals and global liquidity regime-switches by introducing production of tradable and nontradable goods. Durdu, Mendoza and Terrones (2009) proposed a similar setup but with production only in the nontradables sector, keeping tradable goods as an endowment, and they abstracted from studying the normative implications of the model.

3.2.1. Households and Firms

The model represents a small open economy in which a representative household consumes tradable and nontradable goods, denoted $c^T$ and $c^N$ respectively, and representative firms produce tradable and nontradable goods using intermediate goods, denoted $m^T$ and $m^N$ in each industry respectively. The household collects the profits of the firms ($\pi^T$ and $\pi^N$) and has access to a world credit market of non-state-contingent bonds ($b$) denominated in units of tradable goods. Goods and
factor markets are competitive and the prices of traded goods (including both consumption and intermediate goods) and bonds are taken as given from world markets. The credit market is imperfect, because borrowing is limited to fraction of the agent’s income in units of tradable goods.

The preferences of the representative agent are given by a standard intertemporal utility function with constant relative risk aversion (CRRA) defined over a composite good $c_t$:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t), \quad u(c_t) = \frac{c^{1-\gamma}}{1-\gamma}.$$ (3.1)

$\mathbb{E}(\cdot)$ is the expectation operator, $\beta$ is the discount factor, and $\gamma$ is the coefficient of relative risk aversion. The composite good is modeled as a CES aggregator:

$$c_t = \left[ \omega \left( c_{t}^{T} \right)^{-\eta} + (1 - \omega) \left( c_{t}^{N} \right)^{-\eta} \right]^{-\frac{1}{\eta}}, \eta > 1, \omega \in (0, 1).$$ (3.2)

The elasticity of substitution between $c_{t}^{T}$ and $c_{t}^{N}$ is given by which $1/(\eta + 1)$. This is an important parameter because, as we show later, it is one of the main determinants of the collapse of the real exchange rate in periods of financial crises, which in turn is the main driver of crisis dynamics in the model.

Choosing the world-determined price of tradable goods as the numeraire, the agent’s budget constraint is:

$$q_t b_{t+1} + c_{t}^{T} + p_{t}^{N} c_{t}^{N} + A^{T} + p_{t}^{N} A^{N} = b_t + \pi_t^{T} + \pi_t^{N}$$ (3.3)

The left-hand-side of this expression shows the uses of the agent’s income: purchases or sales of bonds $b_{t+1}$ at the world-price $q_t$ (the inverse of which is the world real interest rate $R_t$), plus total expenditures in consumption of tradable and nontradable goods in units of tradables, denoting the relative price of nontradables as $p_{t}^{N}$. $A^{T}$ and $A^{N}$ are constant levels of autonomous expenditures that
represent investment and government expenditures, which are introduced so that
the model can be calibrated to actual consumption-GDP ratios. The right-hand-
side of the budget constraint shows the sources of the agent’s income: Income
from maturing bond holdings \( b_t \) (or repayment of debt if \( b_t < 0 \)), and profits
from production of tradables and nontradables.

Borrowing requires collateral and only a fraction of the agent’s income is pledge-
able as collateral. As a result, the representative agent cannot borrow more than
a fraction \( \kappa \) of total income in units of tradables (i.e. a fraction of total profits):

\[
q_t b_{t+1} \geq -\kappa (\pi^T_t + \pi^N_t)
\]  

(3.4)

This constraint can be interpreted as the result of enforcement or institutional
frictions by which lenders are only able to harness a fraction \( \kappa \) of a defaulting
borrower’s income, or borrowers can only pledge a fraction \( \kappa \) of their income as
collateral. It can also be viewed as resulting from conventional practices in credit
markets, such as the loan-to-income ratios used to limit household credit or in the
construction of credit scores.

As noted earlier, the model’s approach to model two sectors, tradables and non-
tradables, with debt denominated in units of tradables, aims to capture the so-
called liability dollarization phenomenon typical of emerging economies: For-
eign liabilities denominated in hard currencies, which represent tradable goods,
backed up by the income generated in both tradables and nontradables sectors.

The representative agent chooses optimally the sequences \( \{c^T_t, c^N_t, b_{t+1}\} \) to max-
imize (3.1) subject to (3.3) and (3.4), taking \( b_0 \) and \( \{p^N_t, \pi^T_t, \pi^N_t\} \) as given. This
maximization problem yields the following first-order conditions:

\[
\lambda_t = u_T(t)
\]  

(3.5)
\[ p_t^N = \left( \frac{1 - \omega}{\omega} \right) \left( \frac{c_T^T}{c_T^N} \right)^{\eta + 1} \quad (3.6) \]

\[ \lambda_t = \frac{\beta}{q_t} E_t [\lambda_{t+1}] + \mu_t \quad (3.7) \]

\[ q_t b_{t+1} + \kappa \left( \pi_t^T + \pi_t^N \right) \geq 0, \quad \text{with equality if } \mu_t > 0, \quad (3.8) \]

where \( \lambda_t \) and \( \mu_t \) are the Lagrange multipliers on the budget constraint and credit constraint respectively, and \( u_T \) is the marginal utility of consumption of tradables.

The representative firms in both industries use inputs, purchased at a world-price \( p^m \), to produce. They operate Neoclassical production functions \( z_i^i m_i^i \alpha_i \), for \( i = T, N \), facing sectoral TFP shocks \( (z_i^i \text{ for } i = T, N) \) that follow a Markov process to be specified later. We abstract from modeling capital and labor for simplicity. They can be assumed to enter the production technologies in fixed unit supply. Profits are given by:

\[ \pi_T^T = z_T^T m_T^T \alpha_T^T - p^m m_T^T \quad (3.9) \]

\[ \pi_N^N = p_N^N z_N^N m_N^N \alpha_N^N - p^m m_N^N \quad (3.10) \]

The demand for inputs is chosen so as to maximize profits in each sector. This yields standard optimality conditions equating the value of the marginal product of inputs with the relative price in each industry:

\[ \alpha_T z_T^T m_T^T \alpha_T^{-1} = p^m \quad (3.11) \]

\[ \alpha_N p_T^N z_T^N m_T^N \alpha_N^{-1} = p^m \quad (3.12) \]

It is critical that the relative price of nontradables determines the value of the marginal product of inputs in the production of nontradables, because financial crises in the model produce an endogenous drop in output of nontradables, which is induced by a drop in demand for inputs due to a collapse in the relative price.
of nontradables. Note also that fluctuations in production of tradable goods and in the demand for inputs from that sector, are solely driven by the industry’s TFP shock, and hence are unaffected by a financial crisis.

3.2.2. Competitive Equilibrium

The competitive equilibrium is given by sequences of allocations

\[ \{c^T_t, c^N_t, m^T_t, m^N_t, b_{t+1}\}_{t \geq 0}, \]

profits \( \{\pi^T_t, \pi^N_t\}_{t \geq 0} \) and prices \( \{p^N_t\}_{t \geq 0} \) such that:

(a) the representative agent maximizes utility subject to the budget and collateral constraints taking prices and profits as given,

(b) the representative firms maximize profits taking prices as given, and

(c) the market-clearing condition for nontradable goods

\[ (c^N_t + A^N = z^N_t m^N_t^{\alpha N}) \]

and the resource constraint of tradables

\[ (c^T_t + A^T = z^T_t m^T_t^{\alpha T} - p^m(m^T_t + m^N_t) - q_t b_{t+1} + b_t) \]

hold.

Notice that the cost of inputs used in both industries enters in the resource constraint of tradables. Algebraically, this follows from noticing that equilibrium profits are

\[ \pi^T_t = (1 - \alpha T)z^T_t m^T_t^{\alpha T} \]

and \( \pi^N_t = (1 - \alpha N)p^N_t z^N_t m^N_t^{\alpha N} \) in the tradables and nontradables sector respectively, and then using these results to replace profits in the household’s budget constraint, and applying the nontradables market-clearing condition. Intuitively, this makes sense because inputs are assumed to be tradable goods, regardless of whether they are used to produce tradables or nontradables. Hence, the economy’s balance of trade is given by

\[ y^T_t - p^m(m^T_t + m^N_t) - c^T_t - A^T. \]

Note also that gross production and GDP in units of tradable consumer goods are given by

\[ y^G_t = y^T_t + p^N_t y^N_t \]

and \( \text{GDP}_t = y^G_t - p^m(m^T_t + m^N_t) \) respectively.

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3.2.3. News and Global Liquidity Regimes

We use the same formulation of news about fundamentals and global liquidity regime switches as in Bianchi et al. (2016a). They followed the work of Durdu, Nunes and Sapriza (2013) to model news as noisy signals received at date t about the value that $z^T_{t+1}$ may take. Notice that in this setup news about future TFP of the tradables sector or about future world-determined relative prices of exportable goods in terms of a world basket of tradables (i.e. the small open economy’s terms of trade) are equivalent. This is useful because it implies that we can think of the noisy news as related to future real commodity prices, which are a key source of volatility for many emerging economies.

The probability of a news signal conditional on a date-$t+1$ tradables TFP realization is given by the following condition:

$$p(s_t = i | z^T_{t+1} = l) = \begin{cases} \theta & \text{if } i = l \\ \frac{1-\theta}{Z-1} & \text{if } i \neq l \end{cases}$$  \hfill (3.13)

where $s_t$ is the signal at date $t$, $Z$ is the number of possible realizations of $z^T_{t+1}$, and $\theta$ is the signal precision parameter. When $\theta = \frac{1}{Z}$, the signals are completely uninformative, because $p(s_t = i | z^T_{t+1} = l)$ simply assigns a uniform probability of $1/Z$ to all values the signal can take at $t$, regardless of the value of $z^T_{t+1}$. In this case, news do not add any information useful to alter the expectations about $z^T_{t+1}$ that are formed using the probabilistic process of $z^T$ alone. Conversely, $\theta = 1$ implies that the signals have perfect precision. The agent can perfectly anticipate the value of $z^T_{t+1}$ (e.g. $z^T_{t+1} = l$ is expected to occur for sure when the signal $s_t = l$ is observed). Perfect precision does not, however, remove tradables TFP uncertainty completely, because future signals themselves are stochastic, so uncertainty about TFP for dates $t+2$ and beyond remains, although now expectations of future TFP are based only on expectations of future signals.
Following Durdu, Nunes, and Sapriza (2013), we can use Bayes’s Theorem to derive the conditional forecast probability of tradables TFP at date t+1 conditional on a particular date-t pair \((z^T_t, s_t)\), and then form Markov transition probabilities for the joint evolution of \(z^T\) and \(s\):

\[
\Pi(z^T_{t+1}, s_{t+1}, z^T_t, s_t) \equiv \Pr(s_{t+1} = k, z^T_{t+1} = l | s_t = i, z^T_t = j) \\
= \Pr(z^T_{t+1} = l | s_t = i, z^T_t = j) \ldots \\
\times \sum_m \left[ \Pr(z^T_{t+2} = m | z^T_{t+1} = l) \Pr(s_{t+1} = k | z^T_{t+2} = m) \right]
\]

These probabilities are used by the representative agent to form expectations when solving the expected utility maximization problem. Notice that \(\Pi(\cdot)\) combines the information provided by the date-t signal and TFP realization about the likelihood of a particular date-t+1 TFP realization being associated with a particular new signal. The representative agents knows that signals themselves are stochastic, and hence forms rational expectations about their future evolution.

Fluctuations in global liquidity are modeled as a standard two-point, regime-switching Markov process that can drive either fluctuations in the world real interest rate or in the fraction of income that can be pledged in credit markets. The regime realizations are \(x^h\) (low liquidity) and \(x^l\) (high liquidity) with \(x^h > x^l\) for \(x = R, \kappa\). Continuation transition probabilities are denoted \(F_{hh} \equiv \Pr(x_{t+1} = x^h | x_t = x^h)\) and \(F_{ll} \equiv \Pr(x_{t+1} = x^l | x_t = x^l)\), and switching probabilities are \(F_{hl} = 1 - F_{hh}\) and \(F_{lh} = 1 - F_{ll}\). The long-run probabilities of each regime are \(\Pi^h = F_{lh} / (F_{lh} + F_{hl})\) and \(\Pi^l = F_{hl} / (F_{lh} + F_{hl})\) respectively, and the mean durations are \(1/F_{hl}\) and \(1/F_{lh}\).

### 3.3. Optimal Financial Policies

Following Bianchi (2011), we study optimal financial policy by first characterizing the solution to a constrained planner’s (or financial regulator’s) problem, in which
the regulator chooses directly the economy’s bond holdings (i.e. debt) facing the same credit constraint as private agents, but lets all other markets operate competitively.

The regulator’s optimal policy problem can be formulated as a recursive dynamic programming problem, following the standard convention of denoting with a prime variables dated \( t + 1 \):

\[
V(b, e) = \max_{p^T, c^T, c^N, m^T, m^N, b'} \left[ u \left( \left[ \omega \left( c^T \right)^{-\eta} + (1 - \omega) \left( c^N \right)^{-\eta} \right]^{-\frac{1}{\eta}} \right) + \beta \mathbb{E} V(b', e') \right]
\]

subject to

\[
c^T + A^T + p^T (m^T + m^N) + q b' = b + z^{\alpha} m^T
\]

\[
c^N + A^N = z^{\alpha} m^N
\]

\[
q b' \geq -\kappa [ (1 - \alpha^T) z^{\alpha} m_t^T + (1 - \alpha^N) p^N z^N m_t^{\alpha^N} ]
\]

\[
p^N = \left( \frac{1 - \omega}{\omega} \right) \left( \frac{c^T}{c^N} \right)^{\eta + 1}
\]

This Bellman equation has a single endogenous state variable, current bond holdings, \( b \), and four exogenous shocks that are included in the vector of exogenous states: \( e = (z^T, z^N, s, q) \).

The constraints in the regulator’s optimal policy problem include: the resource constraint for tradables (3.16), the market-clearing condition for nontradables (3.17), the credit constraint with profits as determined in competitive markets (3.18), plus an implementability constraint that requires that the equilibrium price of nontradables matches the representative agent’s marginal rate of substitution in consumption of tradables and nontradables (3.19)

Deriving the first-order conditions of the regulator’s problem, simplifying them,
and expressing them in sequential form yields:

$$\lambda_t = u_T(t) + \mu_t \kappa \psi_t$$  \hspace{1cm} (3.20)

$$\lambda_t = \frac{\beta}{q_t} \mathbb{E}_t [\lambda_{t+1}] + \mu_t$$  \hspace{1cm} (3.21)

$$p_t^N \alpha N \z_t^N \m_t^{N\alpha N-1} = p_m \left[ \frac{\lambda_t}{\lambda_t + \mu_t \kappa (1 - \alpha N)} \left[1 - \frac{(p_t^N c_t^N + c_t^T)}{c_t^N} \right] \frac{\lambda_t}{\lambda_t + \mu_t \kappa (1 - \alpha T)} \right]$$  \hspace{1cm} (3.22)

$$\alpha T \z_t^T \m_t^{T\alpha T-1} = p_m \left[ \frac{\lambda_t}{\lambda_t + \mu_t \kappa (1 - \alpha T)} \right]$$  \hspace{1cm} (3.23)

In these expressions, $\lambda_t$ and $\mu_t$ are the Lagrange multipliers on the resource constraint and credit constraint respectively, and $\psi_t \equiv \left( \frac{\lambda_t}{\lambda_t + \mu_t \kappa (1 - \alpha N)} \right) \left[1 - \frac{(p_t^N c_t^N + c_t^T)}{c_t^N} \right]$. The term $\psi_t$ in the first optimality condition measures how the regulator’s choice of $b_{t+1}$ affects borrowing capacity via its effect on tradables consumption and the equilibrium price of nontradables (i.e. by affecting the value of collateral). It follows then that the term $\mu_t \kappa \psi_t$ shows that, when the credit constraint binds, the social marginal benefit from consumption of tradable includes not only the marginal utility of tradables consumption, but also the gains resulting from how changes in tradables consumption help relax the credit constraint. Note also that the magnitude of $\psi_t$ falls with the elasticity of substitution in consumption of tradables and nontradables, because the price of nontradables falls less during a crisis the higher this elasticity, and rises with the ratio of profits from the nontradables sector to consumption of tradables.

Optimal financial policy in this setup includes macroprudential or ex-ante policy, defined as policy used when the credit constraint is not binding at date $t$ but can bind with some probability at $t+1$, and ex-post financial policy, defined as policy that is active when the constraint binds at date $t$. In particular, the aim of the macroprudential policy is to affect credit allocations in normal times because of what those allocations can cause during crisis times. Hence, this type of policy
is active when \( \mu_t = 0 \) but \( E_t[\mu_{t+1}] > 0 \). In this scenario, the regulator’s Euler equation (eq. (21)) takes this form:

\[
\nu_T(t) = \frac{\beta}{q_t} E_t [\nu_T(t+1) + \mu_{t+1} \kappa \psi_{t+1}]
\] (3.24)

Comparing this condition with the household’s Euler equation for bonds shows that this model features the usual wedge between the private and social marginal cost of borrowing from the literature on macroprudential policy, which is given by the term \( \mu_{t+1} \kappa \psi_{t+1} \). In particular, when the credit constraint is expected to bind, the regulator faces a strictly higher marginal cost of borrowing than the representative agent. This is a pecuniary externality, because it results from the fact that the regulator evaluates borrowing choices at \( t \) taking into account that the credit constraint could bind at \( t + 1 \), and if it does the Fisherian debt-deflation mechanism will cause a collapse of the relative price of nontradables (i.e. a collapse of the real exchange rate) that will shrink borrowing capacity. The representative agent takes prices as given, and thus does not internalize these effects.

The terms in square brackets in the right-hand-side of the third and fourth optimality conditions (equations (22) and (23)) capture the regulator’s incentives to use financial policy ex-post, when the credit constraint binds. In particular, when \( \mu_t > 0 \), the regulator finds it optimal to introduce wedges between the value of the marginal product of inputs and their marginal cost in each sector, which indicates that the sectoral social marginal costs of inputs differ from the private marginal cost \( p^m \).

Consider first the wedge in the regulator’s optimality condition for production of nontradables. This wedge captures the effects of changes in inputs allocated to production of nontradables on borrowing capacity induced via changes in the
output and price of nontradables. Borrowing capacity is affected by three effects: First, an increase in demand for inputs, which are tradable goods, reduces resources available for tradables consumption, contributing to lower the nontradables price. Second, the increased production of nontrables obtained with the increase in inputs also lowers the price of nontradables because of the higher supply of these goods. Third, the additional profits generated by the increase in production increase pledgeable resources. The first two effects reduce borrowing capacity while the third increases it. The first two effects are also pecuniary externalities, because they capture price effects of production decisions that are not internalized by private nontradables producers, and the third effect is a non-pecuniary externality that captures the effects of these producers’ decisions on the representative agent’s access to world debt markets, which are not internalized by firms.

The wedge in the regulator’s optimality condition for production of tradables has a similar intuition: It captures the effects of changes in inputs allocated to production of tradables on borrowing capacity induced via changes in the output and price of nontradables. Effects analogous to the first and third effects referred to above are again present (i.e. again higher demand for inputs reduces tradables consumption and makes the price of nontradables and borrowing capacity drop, and again higher profits enhance borrowing capacity). The second effect, however, does not operate because production of tradables does not alter directly the supply of nontradables.

\[6\] The wedge in the regulator’s first-order-condition for \( m_t \) has the form

\[
\begin{bmatrix}
1 + \frac{\mu_t \psi_t}{\mu_t \psi_t} \\
\mu_t \psi_t \frac{\partial p_N^t}{\partial c_T^t} \\
\mu_t \psi_t \frac{\partial p_N^t}{\partial c_N^t} \\
1 + \frac{\mu_t \psi_t}{\mu_t \psi_t} - \frac{\mu_t (1-\alpha)}{u_N(t)}
\end{bmatrix},
\]

where \( u_T \) and \( u_N \) are the marginal utilities of consumption of tradables and nontradables respectively, and \( \partial p_N^t / \partial c_T^t \) and \( \partial p_N^t / \partial c_N^t \) are the derivatives of the equilibrium price of nontradables with respect to tradables and nontradables consumption respectively. The wedge as shown in equation (22) follows from simplifying this expression algebraically using the equilibrium price of nontradables, the definition of \( \psi \) and the nontradables market-clearing condition.
It is critical to note that the social marginal cost of inputs in the production of nontradables is higher than the private marginal cost (i.e. the wedge in equation (22) is less than 1), while the opposite is true in the tradables sector (i.e. the wedge in eq. (23) is higher than 1). This is because the second term in the denominator of (22) is unambiguously negative while the second term in the denominator of (23) is unambiguously positive. Hence, although effects of the regulator’s optimal plans affecting borrowing capacity in opposite directions are at work in both sectors, as explained above, the isoelastic production and utility functions we are using imply that the effects reducing borrowing capacity dominate in the nontradables sector, and the effects increasing borrowing capacity dominate in the tradables sector.

News about future TFP in the tradables sector (e.g. about future terms of trade or commodity prices) and regime-switches in global liquidity have important effects on the externalities driving both macroprudential and ex-post policies. As Bianchi et al. (2016a) explained, “good news” at t about productivity in the tradables sector at t + 1 lead to higher consumption, and since the resulting gain in income has not been realized yet, this leads to an increase in borrowing which makes the economy more vulnerable to hitting the credit constraint. On the other hand, by increasing expected future income, good news also increase on expectation the future borrowing capacity and at the same time reduce future borrowing needs. Similarly, a shift into a regime with high global liquidity leads the economy to take on more debt (e.g. a switch to a lower interest rate makes borrowing cheaper). A sudden shift into a low global liquidity regime can lead to a decline in consumption, which in turn makes the credit constraint tighter and leads to a sharp reduction in both production and capital flows, and a drop in the real exchange rate (i.e. the relative price of nontradables).

The constrained-efficient allocations and prices that solve the planner’s problem
can be decentralized as a competitive equilibrium using various policy instruments, including taxes on debt, debt-to-income ratios, capital requirements or reserve requirements (see Bianchi (2011), Stein (2012)). Since the market failures are in the form of externalities, the natural instruments to consider are standard taxes on the cost of the good associated with each externality. In particular, the regulator can implement the optimal allocations by taxing debt, taxing input purchases in the nontradables sector, and subsidizing input purchases in the tradables sector.

With a debt tax, the cost of purchasing bonds in the budget constraint becomes \( q_t/(1 + \tau_t) b_{t+1} \). We assume also that the revenue of the tax is rebated to the household as a lump-sum transfer to neutralize income effects from this tax. The optimal macroprudential tax is then the value of \( \tau_t \) that equates the Euler equations of bonds of the regulator and the decentralized equilibrium with the tax. Hence, the tax induces private agents to face the social marginal cost of borrowing in the states in which this cost differs from the private cost in the absence of macroprudential policy. When \( \mu_t = 0 \), the optimal macro-prudential tax is:

\[
\tau_t = \frac{E_t [\mu_{t+1} \kappa \psi_{t+1}]}{E_t [u_T(t + 1)]}
\]  

(3.25)

Notice that the numerator of this tax is equal to the expected value of the pecuniary externality.

With a tax \( \tau_t^N \) on input costs in the nontradables industry, the total cost of inputs in that sector becomes \( p_m(1 + \tau_t^N)m_t^N \). Similarly, with a subsidy \( s^T_t \) on input costs in the tradables industry, the total cost of inputs in that sector becomes \( p_m(1 - s^T_t)m_t^T \). The optimal gross tax and subsidy are those such that \((1 + \tau_t^N) \) and \((1 - s^T_t) \) match the wedges in the square-bracket terms in the right-hand-sides of equations.
(22) and (23) respectively. Hence the optimal tax and subsidy are:

\[
\tau^N_t = \left[ \frac{\lambda_t}{\lambda_t + \mu_t \kappa (1 - \alpha N)} \left[ 1 - (\frac{p^N_t c^N_t + c^T_t}{c^T_t}) (1 + \frac{A^T_t}{c^N_t}) \right] \right] - 1
\]

and

\[
s^T_t = 1 - \left[ \frac{\lambda_t}{\lambda_t + \mu_t \kappa (1 - \alpha T)} \right]
\]

Since the social marginal costs of inputs differ from the private marginal cost only when the constraint binds at date \(t\), both the tax and the subsidy are zero if the constraint is not binding. When the constraint binds, the government taxes production of nontradables and stimulates the production of tradables (both the tax and subsidy rates are strictly positive when \(\mu_t > 0\)). Notice also that profits are still given by the same expressions as before, because the optimality conditions for the demand for inputs with the tax and subsidy still imply that profits are

\[
\pi^N_t = (1 - \alpha N) z^N_t m^N_t \alpha N \quad (\pi^T_t = (1 - \alpha T) z^T_t m^T_T \alpha T).
\]

Hence, in order to neutralize the budgetary effects of these production taxes and subsidies at equilibrium, we can assume that households are levied lump-sum taxes to pay for subsidies and lump-sum transfers to rebate tax revenues.

It is worth noting that, according to the above two conditions setting the optimal production taxes and subsidies, we should expect taxes on nontradables to be much larger in absolute value than subsidies on tradables. The expressions for the two are similar, and in addition in our calibration \(\alpha T > \alpha N\), which tends to make the subsidy on tradables larger than the tax on nontradables. But the key difference in the two expressions is the term in square brackets in the denominator of the nontradables tax, which in turn captures the effect missing from the wedge in the optimal allocation of inputs for tradables production relative to that pertaining to nontradables production mentioned earlier: Production of tradables does not alter directly the supply of nontradables, while production of nontrad-
ables does, and this in turn affects the value of collateral \((p^N)\) and thus borrowing capacity. This effect is larger the larger is nontradables consumption relative to tradables consumption.

With the three policy instruments and lump-sum taxes and transfers in place, the budget constraint of the household becomes:

\[
\frac{q_t}{1+\tau_t}b_{t+1} + c_t^T + p_t^Nc_t^N + A^T + p_t^NA^N = b_t + \pi_t^T + \pi_t^N + Tr_t. \tag{3.28}
\]

The government sets \(Tr_t = -q_t b_{t+1} \frac{\tau_t}{1+\tau_t} + \tau_t^N m_t^N - s_t m_t^T\), which is a lump-sum transfer if positive or tax if negative. Substituting profits from the producers’ optimal plans, using the nontradables market-clearing condition, and the government’s total transfers, we recover again the resource constraint for tradables of both the decentralized equilibrium and the planner’s problem.

It is worth noting that this setup also preserves the property that the debt tax is inessential when the collateral constraint binds, as in the setup of Bianchi et al. (2016a). Any value of \(\tau_t\) consistent with the collateral constraint being binding in the competitive equilibrium with taxes (i.e. any \(\tau_t\) such that \(U_T(t) > \frac{\beta}{q_t}(1+\tau_t)E_t[U_T(t+1)]\)) can support the planner’s allocations and prices. With the tax and subsidy on input purchases by producers of nontradables and tradables set to their corresponding optimal values when the constraint binds, the consumption, input and debt allocations are determined without the consumption Euler equation, and therefore without the debt tax. For simplicity, we set the debt tax to zero in these situations, after verifying that it is in the range of inessential debt taxes.
3.4. Quantitative Analysis

3.4.1. Calibration

The parameterization follows the one constructed by Bianchi et al. (2016a), but here we use data for Colombia and use a quarterly frequency. The main difference is in that, since we have added production with intermediate goods into the model, we need to calibrate the sectoral shares of intermediate goods in the production functions of tradables and nontradables and the sectoral TFP shocks. The parameter values used to calibrate the model are shown in Table 10.

Table 10: Baseline Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$\alpha_T$</td>
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</tr>
<tr>
<td>$\alpha_N$</td>
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</tr>
<tr>
<td>$z^N$</td>
<td>1.000</td>
</tr>
<tr>
<td>$Z$</td>
<td>3</td>
</tr>
<tr>
<td>$\theta$</td>
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</tr>
<tr>
<td>$E[z^T]$</td>
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</tr>
<tr>
<td>$\rho_{z^T}$</td>
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</tr>
<tr>
<td>$\sigma_{z^T}$</td>
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</tr>
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<td>$\gamma$</td>
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<tr>
<td>$\eta$</td>
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</tr>
<tr>
<td>$\kappa$</td>
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<tr>
<td>$\omega$</td>
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</tr>
<tr>
<td>$R^i$</td>
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</tr>
<tr>
<td>$F_{ih}$</td>
<td>0.983</td>
</tr>
<tr>
<td>$F_{ii}$</td>
<td>0.900</td>
</tr>
</tbody>
</table>

As in Bianchi et al. (2016a), we set the coefficient of relative risk aversion to $\gamma = 2$, which is a standard value. We also follow their calibration in setting $\eta = 0.205$. The value of this parameter is important because, as we explained earlier, the elasticity of substitution in consumption of tradables and nontradables $(1/(1 + \eta))$ is a key determinant of the response of the price of nontradables to changes in sectoral consumption allocations, and hence of the size of the pecuniary externality.
and the collapse in the nontradables price when a crisis hits.

The factor shares of intermediate goods in production of tradables and nontradables are set according to information from the Colombian input-output matrix. Since we are abstracting from nontradable inputs, we re-define gross production in each sector as the sum of value added plus tradable inputs, with the breakdown between tradable and nontradable sectors constructed by defining the former as including those sectors for which total trade (exports plus imports) exceeds 10 percent of gross production. The value of $\alpha_T$ is then set equal to the ratio of tradable inputs used in all tradable sectors to the combined gross output of all tradable sectors, and similarly, the value of $\alpha_N$ is set equal to the ratio of tradable inputs used in the nontradables sector to the combined value added of all nontradable sectors. This results in factor shares of 49 and 21 percent in the tradables and nontradables sector respectively. Notice from expressions (26) and (27) that both the tax on nontradables and the subsidy on tradables are higher when their corresponding factor share is lower, with a factor of proportionality that depends on $\mu_t \kappa$.

The joint Markov process of the tradables productivity and news signals is set as follows. First, we set $\rho_{zT} = 0.860$ and $\sigma_{zT} = 0.015$ so as to match the first-order autocorrelation and standard deviation of the HP-filtered cyclical component of GDP from Colombian data. Second, we use the Tauchen-Hussey quadrature algorithm to construct a Markov process with three realizations of tradables TFP shocks ($Z = 3$). Third, to set the value of $\theta$, recall that we are assuming that the signals also have three realizations. Hence, $\theta = \frac{1}{3}$ makes news completely uninformative and $\theta = 1$ makes news a perfect predictor of $y_{t+1}^T$ as of date $t$. Thus, following again Bianchi et al. (2016a), we set $\theta$ to the mid point between these two extremes so that $\theta = \frac{2}{3}$. For simplicity, we also assume that the signal realizations and the vector of realizations of $z^T$ are identical, and we abstract from TFP shocks.
in the nontradables sector.

Global liquidity phases
(ex post real return on 90-day U.S. Tbills)

Figure 20: Global Liquidity Regimes

The regime-switching process of the world interest rate also matches the one used by Bianchi et al. They constructed it so as to capture the global liquidity phases identified in the studies by Calvo, Leiderman, and Reinhart (1996) and Shin (2013), using data on the ex-post net real interest rate on 90-day U.S. treasury bills from the first quarter of 1955 to the third quarter of 2014. Calvo et al. identified in data for the 1988-1994 period a surge in capital inflows to emerging markets that coincided with a trough of $-1$ percent in the net U.S. real interest rate in the second half of 1993. Shin found two global liquidity phases, one in the first half of the 2000s with a real interest rate trough of around $-5.5$ percent in early 2004, and another one in the aftermath of the 2008 global financial crisis, with
the net real interest rate hovering around -3 percent since 2009. Taking the average over the troughs in the Calvo et al. sample and in the first of Shin’s global liquidity phases, we set a −0.82 percent real interest rate for the high liquidity regime, which in gross terms implies \( R_l = 0.9918 \). Given this, and the transition probabilities across regimes calibrated below, we set \( R_h = 1.013 \) so that the mean interest rate of the regime-switching process matches the full-sample average in our data, which was 1 percent.

Constructing estimates of the duration of the global liquidity phases is difficult, because the era of financial globalization, and hence global liquidity shifts, started in the 1980s, and of the three global liquidity phases observed since then, the third is heavily influenced by the unconventional policies used after the 2008 crisis. Using data from the first two phases, it follows that the duration of \( R_l \) was 10 quarters, which thus leaves a duration of 60 quarters for \( R_h \), starting the sample in 1980. This yields \( F_{hh} = 0.983 \) and \( F_{ll} = 0.9 \) at quarterly frequency.

The discount factor \( \beta \) is set to match an average net foreign asset position-GDP ratio of −0.79, which is the quarterly equivalent of the annual average for Colombia in the data of Lane and Milesi-Ferretti (2001). We set \( \omega = 0.460 \) to obtain a share of tradables output over total output of 0.30 for Colombia in a deterministic version of the model with constant \( b \). Given the calibrated value of \( b \), \( \omega \) is obtained from

\[
1 - \omega \left( \frac{y^T}{y_N} \right)^{\frac{\eta + 1}{\eta + 1}} y^N + y^T = 0.30 \quad 7.
\]

Finally, we calibrate the value of \( \kappa \) so that, conditional on all the other calibrated parameter values, the model yields a frequency of crises of 3 percent, in line with estimates of the annual frequency of financial crises and sudden stops (see Mendoza (2010)). This yields a value of \( \kappa = 0.87 \). We target the annual frequency of crises because in the quarterly model simulation we count successive quarters

\[\text{The mean values of tradables and nontradables output are set to one. This is an innocuous assumption. Since we calibrate the model to match the observed share of tradables output in total output, 0.3, a different value for } y_N \text{ would lead to a different calibrated value of } \omega, \text{ which in turn would keep the total income unchanged. Thus a different value of } y_N \text{ would not change the borrowing decisions.} \]
of a financial crises as a single event.

3.4.2. Long-run and Financial Crisis Moments

Table 11 shows a set of the moments that characterize the decentralized equilibrium without policy intervention (DE) and the social planner’s equilibrium (SP) with the optimal financial policy. The top panel shows the mean net foreign asset position-output ratio, the standard deviation of the current account-output ratio, the welfare gain of adopting the optimal policy, the probability of a financial crisis, the probability of observing a binding collateral constraint, and the probability of being in the macroprudential tax region (i.e. the probability that $\mu_t = 0$ and $E_t[\mu_{t+1}] > 0$).\footnote{See the notes to Table 2 for the definitions of crisis and welfare used in these exercises.}

The mean debt ratios are similar in the two scenarios. In the DE baseline calibration, we set $\beta = 0.989$ to match the average quarterly NFA-GDP ratio in the data for Colombia. The mean debt ratio of the planner is slightly smaller (76.7 percent), because of the reduced incentive to borrow once the pecuniary externality is removed. In contrast, the variability of the current account in DE is roughly twice as large as in SP. Thus, the two economies support similar long-run debt positions, but the optimal financial policy halves the volatility of capital flows. This finding is in line with Bianchi (2011), who showed that optimal macroprudential policy achieves a reduction in volatility despite small changes in average debt ratios.

The optimal policy also reduces the probability of crises from 2.95 percent in DE to 0 in SP, and the probability that the constraint binds falls from 16.8 percent to 3.2 percent. Note that in the DE the frequency with which the constraint binds (16.8 percent) is 5.7 times the frequency with which financial crises occur, hence there are several periods in which the constraint binds but there is no crisis (i.e. the correction in the debt position is not enough to trigger a sufficiently large current account reversal). The frequency with which the macroprudential tax is active in the SP scenario is about 10 percent, while the frequency with which the
production taxes and subsidies are used is only 3.2 percent since these taxes are used only when the collateral constraint binds for the planner. These frequency results, however, are not informative about the size of the taxes and subsidies, they only measure the likelihood of observing them.

The optimal policy increases social welfare by a sizable amount, equivalent to a permanent increase of roughly 1.3 percent in consumption. This is interesting because typically welfare gains of smoothing fluctuations are negligible in standard business cycle models with CRRA preferences. In this model, the gains are larger because of both the significant reduction in long-run volatility and the removal of infrequent but dramatic crisis events, in which quarterly consumption drops significantly, as we show next.

<table>
<thead>
<tr>
<th>Table 11: Baseline Model Moments</th>
</tr>
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<tbody>
<tr>
<td>Long-run Moments</td>
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<tr>
<td></td>
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<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>E[B/Y] %</td>
</tr>
<tr>
<td>σ(CA/Y) %</td>
</tr>
<tr>
<td>Welfare Gain 1 %</td>
</tr>
<tr>
<td>Prob. of Crisis 2 %</td>
</tr>
<tr>
<td>Pr(µt &gt; 0) %</td>
</tr>
<tr>
<td>Prob. of MP tax region %</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Financial Crisis Moments</th>
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</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>ΔC%</td>
</tr>
<tr>
<td>ΔRER%</td>
</tr>
<tr>
<td>ΔCA/Y%</td>
</tr>
<tr>
<td>E[τ] pre-crisis 3 %</td>
</tr>
<tr>
<td>E[sT] at-crisis 4 %</td>
</tr>
<tr>
<td>E[τN] at-crisis 5 %</td>
</tr>
</tbody>
</table>

1 Welfare gains are computed as compensating variations in consumption constant across dates and states that equate welfare in the DE and SP. The welfare gain W at state (b,z) is given by \((1 + W(b,z))^{1-a}V^{DE}(b,z) = V^{SP}(b,z)\). The long-run average is computed using the ergodic distribution of the DE.

2 A financial crisis is defined as a period in which the constraint binds and the current account (CA/Y) raises by more than two standard deviations in the ergodic distribution of the decentralized economy, which implies a reversal in (CA/Y) larger than 4.2 percent.

3 Average τ in the period before financial crises.

4 Average sT in starting period of financial crises.

5 Average τN in the starting period of financial crises.
The bottom panel of Table 11 shows moments that summarize the main features of financial crises in both the DE and SP solutions. First we report three statistics about the average magnitude of crises: the drops in aggregate consumption ($\Delta C$) and the real exchange rate ($\Delta RER$), and the reversal in the current account-output ratio ($\Delta CA/Y$). For the DE, these statistics are averages of the impact effects that occur when a financial crisis hits, computed with the corresponding economy’s long-run distribution of the state variables ($b, z$) conditional on the economy being in a financial crisis state. For the SP, we report averages of the responses of the variables under identical sequences of exogenous shocks as in the DE using the SP’s long run distribution. The Table also shows the average macroprudential tax before a crisis occurs ($E[\tau]$ pre-crisis), the subsidy on input costs of the tradables sector ($E[s^T]$), and the tax on input costs of the nontradables sector ($E[\tau^N]$).

The results in the DE column show that financial crises in this model result in large declines in consumption and the real exchange rate, and large current-account reversals. The much smaller fluctuations in the SP column show that the optimal financial policy reduces significantly the severity of crises.

In terms of the policy instruments, the size of the taxes and subsidies is small: On average over the three years before a crisis, the macroprudential debt tax is 0.05 percent, while the averages across crises periods for the subsidy on tradables producers and the tax on nontradables producers are 0.07 and 0.61 percent respectively (with the latter nearly 9 times bigger than the former). Hence, in this model the optimal financial policy is quite effective at reducing the frequency and severity of financial crises with relatively small taxes and subsidies.

The effects of the pecuniary externality on borrowing choices, particularly the incentive to over-borrow in the DE, and the effectiveness of the macroprudential policy at containing these effects are both illustrated in the long-run distributions of bond holdings shown in Figure 21. Note also that the twin-peaked nature
of these distributions results from the twin-peaked distribution of interest-rate shocks characteristic of their two-point, regime-switching specification.

3.4.3. Crisis Dynamics

We study next macroeconomic dynamics around crisis events. Figure 22 plots event-analysis windows for the price of nontradables, the debt choice, aggregate consumption and the current account-GDP ratio that highlight these dynamics. The windows show deviations from long-run averages spanning seven quarters, centered on the quarter a crisis occurs, with the DE shown in continuous, red curves and the SP in dashed, blue curves. The movements observed when financial crises hit in the DE emerge as sharp, non-linear drops in the nontradables prices, debt and consumption, and a sharp current account reversal (i.e. a Sudden Stop), relative to the much smoother pre-crisis patterns. Recoveries after crisis are relatively slow in the DE, with prices, debt and consumption still sharply below their long-run means three quarters after the crisis hits. The effectiveness of the optimal financial policy at reducing the severity of crises is evident in the much smoother dynamics of the SP economy.

Figure 23 shows the composition of the shocks at work in the model in the seven

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9Details of the event-analysis construction are documented in an Appendix available on request.

10In the top-left graph with the plots for $b^t$, a decline indicates a reduction in debt, because both the values of $b_{t+1}$ and the long-run averages are negative.
Figure 22: Baseline DE v. SP around Crises

periods covered in the event windows, by plotting the fraction of realizations of each shock that were observed each quarter. The top panel is for news signals, the mid panel for interest rate regimes, and the bottom panel for tradables TFP shocks. As one would expect, financial crises are periods that largely coincide with high real interest rates and low productivity/income realizations. On the other hand, less than 55 percent of financial crises coincide with bad news (i.e. bad news at $t = 0$ in the top panel of the Figure). The pre-crisis phase is characterized also by mainly high interest rates, and by average or bad TFP shocks. In contrast, and in line with the intuition for the mechanism relating news to financial crises described earlier, in the pre-crisis phase there is a non-trivial fraction of observations of good and average news. Hence, several crisis episodes are
preceded by good and average TFP news in the periods leading up to the crisis, followed by actual low TFP realizations when the crisis hits.

![Exogenous States around Crisis](image)

Figure 23: Exogenous Shocks around Crises Events

3.4.4. Complexity of the Optimal Policy

The results discussed up to this point show that the optimal financial policy is very effective, in terms of reducing the frequency and severity of financial crises and increasing social welfare. We show next that, despite these positive results, the optimal policy is also very complex. Implementing the optimal debt and production taxes is challenging because they require precise knowledge of the state of the economy at each point in time. In particular, the optimal macroprudential debt tax requires knowing the probability that a crisis may hit one period ahead conditional on financial and macroeconomic conditions in the current period. But
even if precise information is available, the results we report below show that the optimal policy has significant variation over time and across states of nature. Figure 24 shows the evolution of the optimal policy instruments in the regulated decentralized economy around financial crises events. The top-left plot shows the probability of the leverage constraint being binding. The other three plots show the evolution of the optimal macroprudential debt tax, the nontradables input tax and the tradables input subsidy. For these three, the plots show unconditional averages for each period of the event windows, and in addition for the input tax and subsides we show averages conditional on the leverage constraint being binding (since when the constraint does not bind both are zero), and for the debt tax we show the average conditional on the constraint not being binding (since when the constraint binds the debt tax is zero).

The plot of the macroprudential debt tax shows the pro-cyclical nature of this policy. Conditional on credit not being constrained, the macroprudential debt tax rises from about 0.03 to a just above 0.08 percent in the quarters before the crisis, and falls to near 0.04 percent in the post-crisis phase. Thus, while debt taxes are low on average, they still display significant variability over time. In addition, debt taxes are very active around crisis times. Recall from Table 2 that the debt tax is active 10 percent of the time in the long run, but the fact that the unconditional and conditional-on-unconstrained-credit averages in the event window are very similar suggests that this policy is active most of the time in the periods covered by the window. The probability with which the debt tax is being used each period cannot be inferred unambiguously from the gap between the two averages, because the constraint not being binding at t is necessary but not sufficient for the debt tax to be used. This requires in addition that the constraint is expected to bind at t + 1 at least in some states of nature. The evolution of input taxes and subsidies around crises is countercyclical, and
the averages conditional on the constraint being binding are much higher than the unconditional averages. This occurs because these two instruments are zero when the constraint does not bind, and the probability of the constraint being binding for the planner is generally low. Focusing on averages when the constraint binds (i.e. when these instruments are used), the tax on inputs for the nontradables sector falls from about 2.8 percent to 2 percent in the pre-crisis phase, and then rises gradually to about 2.5 percent in the post-crisis phase. The subsidy on inputs for the tradables sector falls from 0.33 percent to 0.24 percent, and then rises back to about 0.3 percent. Thus, while less volatile in terms of variance than the debt tax, these two instruments also fluctuate markedly around crisis times. Moreover, in terms of rates at which the three instruments are set, the tax on nontradables has
a much higher average hovering around 2.5 percent (conditional on the constraint being binding). This is because of the direct effect of the nontradables tax on the supply of nontradables, and hence on its relative price, discussed earlier.

The production taxes and subsidies are used much less frequently than the debt tax in the long run, since the probability of the constraint being binding is 3.2 percent v. 10 percent probability of using the debt tax (see Table 2). Around crisis events, the top-right plot of Figure 24 shows that the probability of the constraint being binding for the SP (which is the probability which which the production taxes and subsidies are used) rises monotonically in the pre-crisis phase, from near zero to 20 percent, and post-crisis it falls slowly to about 10 percent. From the period just before the crisis hits to the last post-crisis period of the event windows, the production taxes and subsidies are used with frequencies of 10 to 20 percent.

The complexity of the optimal financial policy is also reflected in significant, non-linear variation of the state-contingent schedules of the three policy instruments. We study this issue by examining how these schedules vary across states of nature, particularly across news signals and liquidity regimes.

We examine first how the macroprudential debt tax varies with the three values that the news signal can take. Figure 25 shows the schedule of debt taxes for bad, average and good news as a function of the value of b organized in four plots: (a) for low $z^T$ and $R_l$, (b) for medium $z^T$ and $R_l$, (c) for low $z^T$ and $R_l$, and (d) for medium $z^T$ and $R_l$.\footnote{In the baseline simulations, for high $z^T$ the macro-prudential tax is always zero.} In all four plots, there is always a threshold value of b above which the debt tax is zero, because the debt is too low for the constraint to bind both contemporaneously and in any state of nature in which the economy can land the following period. Below this threshold, the constraint can bind in some states in the following period, so the debt tax is positive, until b is low enough to reach a second threshold in which the constraint binds contemporaneously, at which point the debt tax becomes zero by construction as explained in the
Consider first states with low global liquidity (high real interest rate) at some date-\(t\). If TFP is low (top, left panel of Figure 25), the debt tax is only used in a narrow range of \(b\) around -0.84, and the tax is higher for good news. With average TFP, however, the top, right panel of the Figure shows that the tax is used in a wider range for \(b < -0.85\), at much higher tax rates, and with higher taxes for bad news. Moreover, the tax rises monotonically as debt rises (\(b\) falls). On the other hand, the two plots in the bottom segment of the Figure show that, if global liquidity is high (low \(R\)), the debt tax is higher with bad news for both bad and average TFP, and has a non-monotonic bell-shape in debt.

Figure 25: Optimal Debt Tax Schedules: Effect of News

These non-linearities of the optimal debt tax are in line with previous findings.
reported by Bianchi et al. (2016a), and reflect the opposing forces affecting borrowing decisions in the presence of noisy news, and their interaction with the actual productivity realization. Since the productivity process is persistent, when the news about next period coincide with the current state, the news add little to the information agents have, but when the news point towards a different productivity level than the current one, agents update their expectations with the news. The planner sees some extra risk in overborrowing, and hence acts as if it were more pessimistic about the news.

Figure 26: Optimal Input Taxes for low TFP shock: Effect of News

Figure 26 shows similar plots for the nontradables sector input tax and the tradables sector input subsidy as those for the debt tax in Figure 25, but for the low TFP shock only, because the production policy instruments are only active when
the collateral constraint binds, and this only happens when TFP is low (and b is sufficiently low) in our baseline calibration. The plots in the left side of the Figure are for the subsidy on tradables sector inputs, and those in the right side are for the nontradables input tax. Nontradable sector input taxes are much larger in magnitude than the tradable sector input subsidies, which as we explained earlier is due to the direct effect of the former on the supply of nontradables, and hence on the value of collateral and borrowing capacity.

These plots show that the production policy instruments are set at higher rates as debt becomes more constrained, which occurs as b falls further below the threshold at which the collateral constraint begins to bind. The production tax and subsidies do vary widely, however, as global liquidity and news change. When global liquidity is low (high R), the rates of the optimal production tax and subsidies are higher than when liquidity is high (low R). With low liquidity the tax and subsidy rates are invariant to the news received, whereas with high liquidity the tax and subsidy rates are higher with bad news than with average or good news.

The optimal policy is also likely to display wide variations depending on the stochastic structure of the underlying shocks driving the economy. For instance, solving the model assuming a constant interest rate (i.e. without global liquidity shocks), the frequency with which the optimal macroprudential debt tax is used increases from 10 percent in the baseline to 23.2 percent, and the frequency with which the optimal production taxes and subsidies are used rises from 3.2 to 11.2 percent. The average debt tax in pre-crisis periods rises from 0.05 to 0.12 percent.

3.4.5. Simple Financial Policies

Given that the optimal financial policy is very effective but also very complex, it is important to consider the possibility that the policymaker may only have access to policy rules that are much simpler than the optimal policy. This raises the
question of whether these simpler policies can still be effective. To shed light on this question, we compare the effects of the optimal policy with those produced by simple policies that restrict the policy instruments to be time- and state-invariant (i.e. constant taxes).

We consider two alternative simple policy rules. First, a rule that sets the constant policy rates equal to the long-run average rates under the optimal policy (denoted CT@SPavg). Hence, under this simple rule the policy rates are set to \((\tau^N, s^T, \tau) = (0.20, 0.02, 0.02)\) in percentages. The second simple rule corresponds to the triple \((\tau^N, s^T, \tau)\) of constant policy rates that attains highest social welfare, in terms of expected lifetime utility (denoted CT@optim). To find this triple, we use a derivative-free routine starting from the unregulated DE, in which \((\tau^N, s^T, \tau) = (0, 0, 0)\). The resulting welfare-maximizing triple of constant policy rates is \((\tau^N, s^T, \tau) = (1.87, 0.05, 0.047)\). The ergodic distributions produced by these simple rules are shown in Figure 27, and the performance of these policies is compared with the optimal policy in Table 12.

Figure 27 shows that the ergodic distributions of bonds widen under the simple policy rules. As can be seen in Table 12, both of the simple rules reduce the average debt of the economy (i.e. increase NFA). However, agents are now less afraid of hitting the constraint, as shown by the position of the peaks in Figure 27.
Table 12 shows that the simple rules we considered are significantly less effective than the optimal policy, and can even be welfare reducing. Relative to the DE, CT@optim still yields a sizable drop in the probability of crises (from 3 to 1 percent), and it also reduces the severity of crises markedly (with smaller drops in consumption and the real exchange rate and a smaller current account reversal). Still, the optimal policy is significantly more effective in terms of reducing the frequency and magnitude of crises, and it yields a welfare gain that is about 60 basis points larger.

Constant policy rates set at the average of the optimal policy with CT@SPavg rule are significantly inferior to both the optimal policy and the CT@optim rule. Under CT@SPavg, the probability of crisis is only slightly less than in the unregulated DE, the magnitude of crises is nearly unchanged, and in fact there is a small welfare loss of -0.02 percent. Hence, agents are worse off with this “poorly designed” financial policy than in the economy without any financial policy.

These results are in line with results obtained by Bianchi and Mendoza (2017). They found that simple rules for macroprudential debt taxes, in a model in which assets serve as collateral, are less effective than the optimal policy even when optimized to maximize welfare gains, and even allowing for time-varying, log-linear policy rules.

It is worth noting that the CT@optim rule sets a constant debt tax and a constant subsidy for inputs used in the tradables sector that are similar to the values set under the optimal policy for the debt tax before crises and for the tradables subsidy during crises. In contrast, it sets a constant tax on inputs for the nontradables sector that is three times bigger than the crises average under the optimal policy. By doing this, the regulator with the CT@Optim rule aims to prop up the relative price of nontradables more than under the optimal policy, and to do so permanently and not just when a crisis hits. This reduces the severity of crises when
they happen, but also makes it less likely that the credit constraint can bind, and thus that crises can happen. On the other hand, the policy is inferior to the optimal policy because it induces misallocation of inputs across sectors at all times, instead of just during crisis events.

Table 12: Comparison of Optimal v. Simple Policies

<table>
<thead>
<tr>
<th>Long-run Moments</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
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</thead>
<tbody>
<tr>
<td>E[B/Y]%</td>
<td>-78.91</td>
<td>-76.65</td>
<td>-70.70</td>
<td>-77.46</td>
</tr>
<tr>
<td>σ(CA/Y)%</td>
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<td>0.99</td>
<td>1.13</td>
<td>2.07</td>
</tr>
<tr>
<td>Welfare Gain%</td>
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<td>1.27</td>
<td>0.69</td>
<td>-0.02</td>
</tr>
<tr>
<td>Prob of Crisis%</td>
<td>2.95</td>
<td>0.00</td>
<td>1.07</td>
<td>2.78</td>
</tr>
</tbody>
</table>

Financial Crisis Moments

| ∆C %            | -5.38    | -1.33   | -3.98  | -5.11 |
| ∆RER%           | -5.58    | -1.33   | -4.74  | -5.59 |
| ∆CA/Y%          | 7.04     | -0.04   | 2.15   | 6.76  |
| E[τ] pre-crisis | n.a.     | 0.05    | 0.047  | 0.02  |
| E[s] at-crisis  | n.a.     | 0.07    | 0.05   | 0.02  |
| E[τN] at-crisis | n.a.     | 0.61    | 1.87   | 0.20  |

1 See notes on Table 11.
2 For the two simple rules with constant policy rates, the averages pre-crisis and at-crisis are the same as the unconditional averages by construction.

Figure 28 shows event analysis windows that compare the macroeconomic dynamics of financial crises under the two simple policy rules and the unregulated DE. These plots illustrate again the result that the CT@SPavg rule has negligible effects in terms of reducing the severity of crises. They also illustrate the mechanism by which the CT@optim rule manages to do much better than CT@SPavg mentioned earlier: When a crisis hits, it implies less severe declines in the relative price of nontradables and debt (recall that higher numbers in the graph for b’ indicate higher debt levels relative to the long-run average), and smaller current account reversals and consumption drops. Moreover, it also sustains higher debt levels at all times, since it reduces the likelihood of crises by propping up collateral values permanently.
3.5. Conclusions

This paper studied optimal financial policy in a liability dollarization model of financial crises driven by an occasionally binding collateral constraint. Agents in a small open economy have access to debt denominated in units of tradable goods, but face a constraint limiting their debt not to exceed a fraction of the market value of their total income also valued in units of tradables, which includes income from the tradables sector and income from the nontradables sector valued at the equilibrium relative price of nontradables (i.e. the real exchange rate). Similar models have been widely used to study macroprudential policy, because they embody a pecuniary externality that justifies policy intervention. In particular, in normal times agents do not internalize the effect of their borrowing decisions on the size of the collapse in the price of nontradables, and hence on the collapse in collateral values and borrowing capacity, in crisis times.

The model we studied is based on the model proposed by Bianchi et al. (2016a).
They examined a liability dollarization model driven by conventional and unconventional shocks, with the latter including fluctuations across regimes of global liquidity (e.g. a regime-switching specification of world interest rate shocks) and noisy news about fundamentals (e.g. news about future tradables income). We modified the Bianchi et al. model by introducing production of tradable and nontradable goods using intermediate goods.

Introducing production has important implications for the optimal design of financial policy. In particular, the optimal policy has both a macroprudential (ex ante) component, which has the standard property of being active only when the collateral constraint does not bind contemporaneously but may bind in the following period with some probability, and ex-post components that are active only when the collateral constraint binds. The macroprudential component is modeled in the familiar form used in the literature, as a debt tax that increases the private marginal cost of borrowing to match the social cost in normal times. The ex-post components take the form of a tax on input purchases levied on producers of nontradables, and a subsidy on input purchases provided to producers of tradables. These two instruments reallocate production across sectors so as to prop up collateral values and hence borrowing capacity in crisis times.

The model was calibrated using Colombian data, and a set of quantitative experiments showed that, while optimal financial policy is a very effective tool for reducing the frequency and severity of financial crises, it is also a very complex policy that entails significant, non-linear variation in policy instruments over time and across states of nature. In addition, the paper shows that if only simple policy rules in the form of time- and state-invariant policy instruments are feasible, these simple rules are at best much less effective than the optimal policy and at worst they can result in lower welfare than in an economy in which financial crises occur without policy intervention.
The findings of this paper indicate that the implementation and design of policies aimed at tackling financial instability should proceed with caution. In particular, specific policy rules need to be the subject of intensive quantitative assessment with macroeconomic models that capture the relevant transmission mechanisms that drive financial crisis and the transitions from normal to crisis times, because otherwise seemingly harmless simple rules can actually be welfare-reducing.
Appendices
Appendix A

Appendix to How International Reserves Reduce the Probability of Debt Crises

A.1. Continuous State Space Heuristics

In this section it is assumed that the state space is continuous and that the value functions $X$, $V^+$ and $V^-$ and their corresponding policy functions are continuously differentiable. The default decision follows from the thresholds defined in equation (1.7) and (1.8) and those are also assumed to be differentiable.

Given the default thresholds, the problem for the government after repayment
can be written as:

\[ V^+(y, a, b, \omega) = \max_{a', b', c} u(c) + \beta \Pi_\omega(\omega' = 1) \left[ \int_{-\infty}^{d_f(a', b', 1)} X(y', a', 1) dF(y'|y) \right. \\
+ \left. \int_{d_f(a', b', 1)}^{\infty} V^+(y', a', b', 1) dF(y'|y) \right] \\
+ \beta \Pi_\omega(\omega' = 0) \left[ \int_{-\infty}^{d_s(a', b', 0)} X(y', a', 1) dF(y'|y) \right. \\
+ \left. \int_{d_s(a', b', 0)}^{\infty} V^+(y', a', b', 0) dF(y'|y) \right] \]

s.t. \quad c = y + a - \frac{a'}{1 + r} + q(y, a', b', \omega)(b' - (1 - \lambda)b) - (\lambda + (1 - \lambda)z)b. \quad (A.1) \]

The equilibrium bond pricing equation (1.6) is then:

\[ q(y, a', b', w) = \frac{\Pi_\omega(\omega' = 1)}{1 + r} \left[ (1 - F(d_f))(\lambda + (1 - \lambda)z) \\
+ (1 - \lambda) \int_{d_f}^{\infty} q(y', a^*(s'), b^*(s'), 1) dF(y') \right] \\
+ \frac{\Pi_\omega(\omega' = 0)}{1 + r} \left[ (1 - F(d_s))(\lambda + (1 - \lambda)z) \\
+ (1 - \lambda) \int_{d_s}^{\infty} q(y', a^*(s'), b^*(s'), 0) dF(y') \right] \quad (A.2) \]

A.1.1. The change in thresholds with respect to debt and reserves

Using the implicit function theorem the impact of changes in reserves and debt on those thresholds can be written as:

\[ \frac{\partial d_f(a, b, \omega)}{\partial i} = d_i^f(a, b, \omega) = -\frac{V_i^+ - X_i}{V_y^+ - X_y} \bigg|_{y = d(a, b, \omega)}, \quad (A.3) \]

\[ \frac{\partial d_s(a, b, \omega)}{\partial i} = d_i^s(a, b, \omega) = -\frac{V_i^- - X_i}{V_y^- - X_y}, \quad (A.4) \]

where \( i \in \{a, b\} \) and \( V_i^+ \) is the partial derivative of the value function with respect to \( i \) and the functions are evaluated at \( y = d(a, b, \omega) \). Notice that the existence of
the default threshold implies that at \( y = d(a, b, \omega), \ V^+_y > X_y \)

The envelope conditions yield:

\[
X_a(y, a, \omega) = \frac{\partial X^y(y, a, \omega)}{\partial a} = u_c(c_d) \quad (A.5)
\]

\[
V^+_a(y, a, b, \omega) = \frac{\partial V^+_y(y, a, b, \omega)}{\partial a} = u_c(c) \quad (A.6)
\]

\[
V^+_b(y, a, b, \omega) = \frac{\partial V^+_y(y, a, b, \omega)}{\partial b} = -u_c(c)(\lambda + (1 - \lambda)(z + \tilde{q})) \quad (A.7)
\]

Next, the effect over the current default threshold of an infinitesimal change in both current debt and reserves is computed. If the ratio of the change in reserves to debt is \( \alpha_1 : \alpha_2 \) then the change in the fundamentals default threshold \( \Delta (d^f) = \alpha_1 d^f_a + \alpha_2 d^f_b \) is:

\[
\Delta (d^f) = (V^+_y - X_y)^{-1}\left(\alpha_1 u_c(\tilde{c}_x) + u_c(\tilde{c})(-\alpha_1 + \alpha_2 \lambda + \alpha_2 (1 - \lambda)(z + \tilde{q}))\right), \quad (A.8)
\]

where \( \tilde{z} \) denotes the value or policy function \( z \) evaluated at the state when output is equal to the threshold: \( s = (d^f(a, b, \omega), a, b, \omega) \), and \( c_x \) is the consumption policy in case of default.

The Self-Fulfilling default threshold change is

\[
\Delta d^s = d^s_a + d^s_b = \frac{-V^-_b - V^-_a + X_a}{V^-_y - X_y} \quad (A.9)
\]

The main difference is that the envelope condition for \( V^-_b \) now includes a multiplier term.

\[
V^-_b = -u(c)[\lambda + (1 - \lambda)z + (1 - \lambda)\tilde{Q}^f] + \mu(1 - \lambda)
\]

NFA-neutral change in holdings

Increase both current debt and reserves by 1$. The fundamentals default threshold change is

\[ \Delta d^f = d^f_a + d^f_b = \frac{-V^+_{b} - V^+_{a} + X_a}{V^+_y - X_y}. \]

Using the envelope conditions it reduces to:

\[ \Delta d^f[u_c(\bar{c}) - u_c(\bar{c}_\delta)] = -u_c(\bar{c})(1 - \lambda)[1 - z - Q(d, a', b', \omega)] + u_c(\bar{c}_\delta) \]

With one period debt \( (\lambda = 1) \), threshold moves up for sure. In the long term debt case \( (\lambda < 1) \), the term \( [1 - z - Q(d, a', b', \omega)] \) is small since debt is issued on average at par.

A.1.2. The Generalized Euler Equation (GEE)

This subsection follows Rios-Rull and Mateos-Planas (2016) and derives the first order conditions for debt and reserves under repayment.

Using the envelope conditions. The first order conditions with respect to next period reserves \( (a') \) is

\[
\frac{\partial V}{\partial a'} = -u_c(c)
\left[(1 + r)^{-1} - q_{a'}(b' - (1 - \lambda)b)\right]
+ \beta \Pi_\omega(\omega' = 1)\int_{-\infty}^{d_f(a', b', 1)} u_c(c'_x)dF(y'|y) + \int_{d_f(a', b', 1)}^{\infty} u_c(c')dF(y'|y)
+ \beta \Pi_\omega(\omega' = 0)\int_{-\infty}^{d_s(a', b', 0)} u_c(c'_x)dF(y'|y) + \int_{d_s(a', b', 0)}^{\infty} u_c(c')dF(y'|y)
+ \left[V^+(d^s, a', b', 0) - X(d^s, a', b', 0)\right]f(d^s|y)d_{a'}^s
\]

(A.10)
The FOC with respect to next period debt $b'$ is:

$$
\frac{\partial V}{\partial b'} = + u_c(c) \left[ q(y, a', b', \omega) + q_{b'}(b' - (1 - \lambda)b) \right] 
- \beta \Pi_\omega (\omega' = 1) \int_{d^f(a',b',1)}^{\infty} u_c(c') \left[ \lambda + (1 - \lambda)(z + q') \right] dF(y'|y) 
- \beta \Pi_\omega (\omega' = 0) \left[ \int_{d^s(a',b',0)}^{\infty} u_c(c') \left[ \lambda + (1 - \lambda)(z + q') \right] dF(y'|y) + \left[ V^+(d^s, a', b', 0) - X(d^s, a', b', 0) \right] f(d^s|y)d^s_{b'} \right] 
$$

(A.11)

The debt-reserves consolidation operation

The marginal effect of the consolidation operation at the price $\bar{q} = q(y, a', b', \omega)$ is $-\bar{q}R \frac{\partial V}{\partial a'} - \frac{\partial V}{\partial b'}$, where $R = (1 + r)$ and this is:

$$
\begin{align*}
u_c(c) & \left[ - (\bar{q}R a' + q_{b'}) (b' - (1 - \lambda)b) \right] 
+ \beta \Pi_\omega (\omega' = 1) \left[ - \bar{q}R \int_{d^f(a',b',1)}^{\infty} u_c(c') dF(y'|y) \right] 
+ \left[ \int_{d^f(a',b',1)}^{\infty} u_c(c') \left[ \lambda + (1 - \lambda)(z + q') - \bar{q}R \right] dF(y'|y) \right] 
+ \beta \Pi_\omega (\omega' = 0) \left[ - \bar{q}R \int_{d^s(a',b',0)}^{\infty} u_c(c') dF(y'|y) \right] 
+ \left[ \int_{d^s(a',b',0)}^{\infty} u_c(c') \left[ \lambda + (1 - \lambda)(z + q') - \bar{q}R \right] dF(y'|y) \right] 
+ \left[ V^+(d^s, a', b', 0) - X(d^s, a', b', 0) \right] f(d^s|y) (-\bar{q}R d^s_{a'} - d^s_{b'}) 
\end{align*}
$$

(A.12)

The first term captures the effect on current consumption due to the change in the bond price. The last term captures the discrete gains from states coming out of the multiplicity region. The other terms capture the effect on future consumption depending on the repayment or default decision. From the equilibrium price
equation (1.6):

\[
\lambda + (1 - \lambda)(z + q') - q_R = \frac{\Pr(\text{def})}{1 - \Pr(\text{def})} q_R + (1 - \lambda)(q' - \mathbb{E}[q'|\text{repay}]),
\]

where \(\Pr(\text{def})\) is the probability of default next period. The consolidation effect on utility can be compactly written as:

\[
\begin{align*}
    u_c(c) & \left[ - (q_R a' + q_{b'}) (b' - (1 - \lambda) b) \right] \\
    - \beta \Pr(\text{def}) q_R & \mathbb{E} \left[ u_c(c'|\text{def}) \right] + \beta (1 - \Pr(\text{def})) \mathbb{E} \left[ \frac{\Pr(\text{def})}{1 - \Pr(\text{def})} q_R u_c(c'|\text{repay}) \right] \\
    + (1 - \Pr(\text{def}))(1 - \lambda) & \mathbb{E} \left[ u_c(c')(q' - \mathbb{E}[q']) \right] \\
    + \beta & \Pi_\omega(\omega' = 0) \left[ V^+(d^s, a', b', 0) - X(d^s, a', b', 0) \right] f(d^s|y)(-q_R d^s_a - d^s_b)
\end{align*}
\]

(A.13)

Define the change in price \(\Delta q = -q_R a' - q_{b'}\) and the change in the self-fulfilling default threshold \(\Delta d^s = -q_R d^s_a - d^s_b\), then the consolidation marginal effect is:

\[
\begin{align*}
    u_c(c) & \Delta q \left[ b' - (1 - \lambda) b \right] \\
    - \beta & \Pi_\omega(\omega' = 0) \left[ V^+(d^s, a', b', 0) - X(d^s, a', b', 0) \right] f(d^s|y) \Delta d^s \\
    + \beta & \Pr(\text{def}) q_R \left[ - \mathbb{E} \left[ u_c(c'|\text{def}) \right] + \mathbb{E} \left[ u_c(c'|\text{repay}) \right] \right] \\
    + \beta (1 - \Pr(\text{def}))(1 - \lambda) & \text{Cov} \left[ u_c(c')q'|\text{repay} \right]
\end{align*}
\]

(A.14)

This operation is not revenue neutral in the first period because the bond price reacts to it, and the first term in equation (A.14) captures this effect on current utility. The second term captures the direct effect of the change in the self-fulfilling default threshold \(d^s\) in expected utility, the fundamentals default threshold change has no first order effect since the unconditional value function \(W\) is continuous at that point, but when \(\omega = 0\) there is a jump in \(W(\cdot, 0)\) at \(y = d^s(a, b, 0)\) which captures the fact that crossing the self-fulfilling default threshold causes a discrete loss for the sovereign.
The third and fourth term in equation (A.14) reflect the resource transfer from default to repayment states. If the sovereign had the opportunity to credible commit to default and repay in selected states while keeping the expected repayment constant, the term \(-\mathbb{E}[u_c(c'_{x})|\text{def}] + \mathbb{E}[u_c(c')|\text{repay}]\) would be zero. However, in the Eaton-Gersovitz environment, the expected payment choice and the default threshold choice are made together, at the commitment optimal default threshold, the expected payment is low, which means current revenues are low. The sovereign faces a trade-off between issuing more debt and bringing more resources to the current period and issuing less debt but keep the default threshold low and close to the commitment optimal level.

The last term captures the utility gains from shifting the issuance date of debt outstanding two periods ahead. Before the consolidation operation some of that debt was issued on the current period, after the consolidation it is issued in the next period. As lenders are risk neutral, the current price is consistent with the average prices of that debt in the next period, but for the sovereign this matters because of risk aversion. Since default is more likely at low realizations of output, consumption and bond prices will be positively correlated, hence this covariance term is generally negative.

A.1.3. The sovereign’s problem with commitment

Here, the sovereign problem with commitment is briefly discussed. This section follows closely Rios-Rull and Mateos-Planas (2016).

The setup is the following: at the beginning of the period, before the income shock is realized, the government enters with reserves \(a\) a promise to repay \(k\) in expected value and chooses a default threshold \(d^c\), contingent consumption \(c(y)\) and contingent portfolio positions \(k'(y), a'(y)\).

The previous realization of output \(y_{-1}\) matters only for the expectations. Given
that, the problem for the government can be written in recursive form as:

\[
V^c(y_{-1}, a, k) = \max_{a'(y), k'(y), c(y), d^c} \int_{-\infty}^{\infty} u(c(y)) \, dF(y|y_{-1}) \\
+ \beta \int_{-\infty}^{d^c} X^c(y, a'(y)) \, dF(y|y_{-1}) + \beta \int_{d^c}^{\infty} V^c(y, a'(y), k'(y)) \, dF(y|y_{-1})
\]  

(A.15)

s.t. \( c(y) = y + a - \frac{a'}{1 + r} + 1(y > d^c) \left[ \frac{k'(y)}{1 + r} - \frac{k}{1 - F(d^c)} \right] \).

where \( V^c \) is the value function contingent on current reserve assets \( a \) and a promised expected repayment \( k \). \( X^c \) is the value after default and \( 1 \) is the indicator function.

The first order condition with respect to the default threshold \( d^c \) is:

\[
0 = \frac{\partial V^c(y_{-1}, a, k)}{\partial d^c} = \beta X^c(d^c, a'(d^c)) - \beta V^c(d^c, a'(d^c), k'(d^c)) \\
- \frac{\text{kf}(d^c)}{(1 - F(d^c))^2} \int_{(d^c)}^{\infty} u_c(c(y')) \, dF(y'|y),
\]  

(A.16)

from this first order condition it follows that the commitment default threshold \( d^c \) is determined by:

\[
\beta X^c(d^c, a'(d^c)) = \beta V^c(d^c, a'(d^c), k'(d^c)) + \frac{\text{kf}(d^c)}{(1 - F(d^c))^2} \int_{(d^c)}^{\infty} u_c(c(y')) \, dF(y'|y).
\]  

(A.17)

The difference between this equation and the fundamental’s default equation (1.7) is the third term, which is positive, indicating the sovereign commits to repay in states where ex-post it would prefer to default.

Next, the first order conditions with respect to reserves and promised payments
are derived. First, the envelope conditions yield:

\[
X^c_{a}(y_{-1}, a) = \frac{\partial X^c_{a}(y_{-1}, a, k)}{\partial a} = \int_{-\infty}^{\infty} u_c(c_{x}(y)) dF(y|y_{-1}) \tag{A.18}
\]

\[
V^c_{a}(y_{-1}, a, k) = \frac{\partial V^c_{a}(y_{-1}, a, k)}{\partial a} = \int_{-\infty}^{\infty} u_c(c(y)) dF(y|y_{-1}) \tag{A.19}
\]

\[
V^c_{k}(y_{-1}, a, k) = \frac{\partial V^c_{k}(y_{-1}, a, k)}{\partial k} = \frac{-1}{1 - F(d^c)} \int_{d^c}^{\infty} u_c(c(y')) dF(y'|y) \tag{A.20}
\]

The first order condition with respect to the promised payment \(k'(y)\) is then:

\[
0 = \frac{\partial V^c_{k}(y_{-1}, a, k)}{\partial k'} = \frac{u_c(c(y))}{1 + r} \beta \int_{-\infty}^{\infty} u_c(c_{x}(y')) dF(y'|y) - \frac{\beta F(d^c)}{1 - F(d^c)} \int_{d^c}^{\infty} u_c(c(y')) dF(y'|y). \tag{A.21}
\]

This equation can be compared with the first order condition w.r.t. \(b'\) in the Eaton-Gersovitz setup (equation A.11. First, it is important to note that there are no self-fulfilling crises with commitment. Second, since the borrowing decision is independent of the default decision, there is no price feedback on the amount borrowed (i.e. the term with \(q_b\) in equation A.11). With commitment the expected value of debt service is higher.

The first order condition with respect to reserves \(a'(y)\) is:

\[
0 = \frac{\partial V^c_{a}(y_{-1}, a, k)}{\partial a'} = -\frac{u_c(c(y))}{1 + r} \beta \int_{-\infty}^{\infty} u_c(c_{x}(y')) dF(y'|y) \ldots + \beta \int_{(d^c)}^{\infty} u_c(c(y')) dF(y'|y) \tag{A.22}
\]

Adding both first order conditions, it follows that:

\[
0 = \beta \int_{-\infty}^{(d^c)} u_c(c_{x}(y')) dF(y'|y) - \frac{\beta F((d^c))}{1 - F((d^c))} \int_{(d^c)}^{\infty} u_c(c(y')) dF(y'|y), \tag{A.23}
\]

and this can be rewritten as:

\[
0 = \frac{\beta}{F((d^c))} \left( \mathbb{E} \left[ u_c(c_{x})|\text{def} \right] - \mathbb{E} \left[ u_c(c)|\text{repay} \right] \right) \tag{A.24}
\]
Hence, with commitment, expected marginal utilities under default and repayment are equalized. In the standard Eaton-Gersovitz framework, the expected marginal utility under default is lower than the expected marginal utility under repayment because the sovereign is defaulting on states with higher income. This implies that on the margin, shifting resources from default states to repayment states is efficient and thus the term in the third line of equation (A.14), which is part of the consolidation effect on utility, is positive.

A.2. Computation method

As mentioned in Chatterjee and Eyigungor (2012), with long term debt the budget sets for the sovereign are not convex. This is consequence of the debt dilution problem: with one-period debt, if the government deviates from equilibrium into slightly lower debt levels, bond prices react immediately to indicate a lower default probability. On the other hand, with long-term debt, bond prices almost do not react to the low debt levels since it is expected that the normal level will be chosen in the next period and current bondholders will be diluted.

This same argument explains the convergence problems of global methods solving this problem, the bond price function may not react for infinitesimal changes of debt holdings until a discrete change at a default state happens. In addition, as seen in figure 3, the possibility of self-fulfilling crises make the unconditional value function $W$ discontinuous which exacerbates the convergence problem of global iterative methods.

Chatterjee and Eyigungor (2012) introduced a purification i.i.d. endowment shock to overcome this difficulties. They showed that introducing this shock smooths the value functions and, under monotonicity assumptions, it guarantees the existence of an equilibrium on discretized state spaces.
A.3. Data

This section describes the data sources and the procedures used to aggregate and combine data from different sources. The sample includes data for 18 emerging markets: Argentina, Brazil, Bulgaria, Chile, Colombia, India, Indonesia, Lithuania, Malaysia, Mexico, Peru, Philippines, Poland, Romania, Russia, South Africa, Turkey, and Ukraine \(^1\). It spans the period 1994-Q1 through 2016-Q1 with some significant missing periods for some countries.

**GDP:** Quarterly data on current and constant GDP at market prices, seasonally adjusted, from the Global Economic Monitor by the World Bank. The nominal data was used to calculate all the ratios (Current Account, Debt and Reserves ratio) while the real data was used to calculate the growth rate.

**Spreads:** Data for the monthly Emerging Market Bond Index (EMBI) of each country (blended spread), as well as the composite blended spread or EMBI+, was downloaded from Datastream. The values of March, June, September and December were used for the quarterly data.

**International Reserves:** Monthly Total International Reserves data from the GEM (Global Economic Monitor) was used. The values of March, June, September and December were used for the quarterly data.

**Current Account:** The current account data corresponds to the total net US dollars current account from the World Economic Outlook dataset available from the International Monetary Fund.

**Debt:** Quarterly Public Sector External Debt data comes from different sources. The data for Argentina, Brazil, Chile, Colombia, India, Indonesia, Malaysia, Philippines, and South Africa was downloaded from Haver EMERGE database. All other countries except Ukraine were downloaded from the Quarterly Public Sector Debt database of the World Bank. Neither of these two previous sources pro-

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\(^1\)The same countries were used in Aguiar et al. (2016b). They also include Latvia and Hungary, but due to data availability those could not be included in the regressions on table 2.
vided enough data for Ukraine. To complement this country panel, annual data from the International Debt Statistics database from the World Bank was used. Linear interpolation was used to calculate the quarterly values.

Exchange rate regime: The exchange rate regime classification is taken from the monthly fine classification by Ilzetzki, Reinhart, and Rogoff (2011). This scale is then grouped into a dummy variable identifying those countries with a managed rate regime which includes: No separate legal tender, Pre-announced peg or currency board arrangement, Pre-announced horizontal band that is narrower than or equal to ±2%, De facto peg, Pre-announced crawling peg, Pre-announced crawling band that is narrower than or equal to ±2%, De facto crawling peg, De facto crawling band that is narrower than or equal to ±2%. Pre-announced crawling band that is wider than or equal to ±2%, De-facto crawling band that is narrower than or equal to ±5%, and Moving band that is narrower than or equal to ±2%.
Appendix B

Appendix to Fighting for the Best, Losing with the Rest

B.1. Proofs

B.1.1. Zero assets proofs

The following is a useful consequence of lemma 2.2.

**Lemma B.1** In any IC contract schedule, $x(\theta)$ is non-increasing.

**Proof.** Let $\theta' > \theta$ from equation (2.1c) it follows that:

$$x(\theta) = U(0) + \int_0^\theta \left[ f(k(s))z(s) - f(k(\theta))z(\theta) \right] ds$$

then,

$$x_i(\theta') - x(\theta) = (\theta' - \theta) \left[ f(k(\theta))z(\theta) - f(k(\theta'))z(\theta') \right] \ldots$$

$$+ \int_\theta^{\theta'} \left[ f(k(s))z(s) - f(k(\theta'))z(\theta') \right] ds$$

And both terms in the last equation are negative or zero because of equation (2.1b) hence the claim holds. ■
Proof of claim 2.3

Proof. Start with equilibrium in which an entrepreneur of type \( \theta \) receives payoff \( U^0(\theta) \). Let\( (k^0(\theta), x^0(\theta), z^0(\theta)) \) be the equilibrium incentive compatible contract schedule. Finally let \( A^0 \) denote the set of types taking one of the contracts rather than the outside option. Suppose there is a set \( C \subseteq A^0 \) such that \( k^*(\theta) \neq k^0(\theta) \) for all \( \theta \in C \). We will show that contract schedule is profit maximizer for intermediary 1 only if \( C \) has measure zero.

Construct \( C' \) as the set \( \{(k'(\theta), x'(\theta), z'(\theta))| \theta \in A^0\} \), where,

\[
k'(\theta) = k^*(\theta) \quad z'(\theta) = \frac{f(k^0(\theta)) z^0(\theta) + \delta \theta}{f(k^*(\theta))} \quad x'(\theta) = x^0(\theta) + \frac{\delta}{2}(1 - \theta^2)
\] (B.1)

If an entrepreneur of type \( \theta \) takes the contract \( (k'(\hat{\theta}), z'(\hat{\theta}), x'(\hat{\theta})) \in C' \), her payoff will be,

\[
\theta f(k^0(\hat{\theta})) z^0(\hat{\theta}) + x^0(\hat{\theta}) + \delta \hat{\theta} + \frac{\delta \hat{\theta}^2}{2} - \frac{\delta}{2}
\]

The above payoff is maximized at \( \hat{\theta} = \theta \) and the term \( \delta \hat{\theta} - \frac{\delta \hat{\theta}^2}{2} \) ensures the maximizer is unique. The resulting payoff is \( U'(\theta) \equiv U^0(\theta) + \frac{\delta}{2}(1 + \theta^2) > U^0(\theta) \).

Thus every entrepreneur \( \theta \in A^0 \) signs a contract with \( k^*(\theta) \).

Next, we will show that offering \( C' \) constitutes a profitable deviation for intermediary 1. Let \( v_i^0 \), \( i = 1, 2 \) be the intermediaries payoff in the original equilibrium. Note that, because intermediaries have always the option of offering empty sets of contracts, \( v_i^0 \geq 0 \).

The following equation follows from the definition of payoffs,

\[
\int_{A^0} \left\{ \pi f(k^0(\theta)) - R k^0(\theta) \right\} dG(\theta) = v_1^0 + v_2^0 + \int_{A^0} U^0(\theta) dG(\theta) \] (B.2)

Let,
\[ M \equiv \int_{A^0} \left\{ \pi f(k^*(\theta)) - Rk^*(\theta) \right\} dG(\theta) - \int_{A^0} \left\{ \pi f(k^0(\theta)) - Rk^0(\theta) \right\} dG(\theta) \]

Since C has positive measure, the definition of \( k^*(\theta) \) implies, \( M > 0 \).

Now, let \( v'_1 \) be the payoff of intermediary 1 when she deviates to \( C' \) and \( A' \) the set of types signing a contract after the deviation.

\[
v'_1 = \int_{A'} \left\{ \pi f(k^*(\theta)) - Rk^*(\theta) - \U'(\theta) \right\} dG(\theta)
\]

\[
v'_1 = \int_{A' \cap A^0} \left\{ \pi f(k^*(\theta)) - Rk^*(\theta) \cdots - \U'(\theta) \right\} dG(\theta) + \int_{A^0} \left\{ \pi f(k^*(\theta)) - Rk^*(\theta) - \U'(\theta) \right\} dG(\theta)
\]

\[
v'_1 = \int_{A' \setminus A^0} \left\{ \pi f(k^*(\theta)) \cdots - Rk^*(\theta) - \U'(\theta) \right\} dG(\theta) + M + v_1^0 + v_2^0 - \int_{A^0} \{ \U(\theta) - \U^0(\theta) \} dG(\theta)
\]

The last equality follows from equation B.2.

The measure of \( A^0 \) is obviously bounded by one and \( \U'(\theta) - \U^0(\theta) \leq \frac{\delta}{2}(1 + \theta^2) \leq \delta \). Hence,

\[
\int_{A^0} \{ \U(\theta) - \U^0(\theta) \} dG(\theta) \leq \delta
\]

Moreover, \( \pi f(k^*(\theta)) - Rk^*(\theta) - \U'(\theta) \) is bounded and it follows that, as the measure of \( A \setminus A^0 \) goes to zero,

\[
\int_{A \setminus A^0} \left\{ \pi f(k^*(\theta)) - Rk^*(\theta) - \U'(\theta) \right\} dG(\theta) \to 0
\]

Now,

\[
A \setminus A^0 = \{ \theta : \U^0(\theta) < w \leq \U'(\theta) \} \subset \{ \theta : w - \delta < \U^0(\theta) < w \}
\]

And it is clear that the measure of \( A \setminus A^0 \) goes to zero as \( \delta \) goes to zero.

As a result of the previous observations, there exists a value for \( \delta \), low enough,
\[ M > - \int_{A \setminus A^0} \left\{ \pi f(k^*(\theta)) - Rk^*(\theta) - U'(\theta) \right\} dG(\theta) + \int_{A^0} \left\{ U(\theta) - U^0(\theta) \right\} dG(\theta) \]

For such a \( \delta \),

\[ \nu'_1 - \nu^0_1 > 0 \]

Since the intermediary payoff is strictly higher, if the measure of \( C \) is strictly positive the alternative schedule \( C' \) will achieve strictly higher profits for intermediary 1.

**Proof of claim 2.6**

**Proof.** We begin by showing that \( U(\theta) \) is non-decreasing in \( \theta \). Once that is established, it will be straightforward to see that \( A \) must be an interval.

By lemma 2.2 it suffices to show \( f(k(\theta))z(\theta) \geq 0 \) for all \( \theta > 0 \). Suppose there is an equilibrium with IC contract schedule \( (k(\theta), x(\theta), z(\theta)) \), and there exists \( \theta_0 > 0 \) such that \( f(k(\theta_0))z(\theta_0) < 0 \). By claim 2.3, there exists a sequence \( \{\theta^n\} \in [0, \theta_0] \) converging to zero such that \( k(\theta^n) = k^*(\theta^n) \) and this implies \( \{f(k(\theta^n))\} \) also converges to zero.

Equation 2.1b guarantees that \( f(k(\theta^n))z(\theta^n) \leq f(k(\theta_0))z(\theta_0) < 0 \) for all \( n \). Taking limit on both sides implies \( \lim_{n \to \infty} z(\theta^n) = -\infty \). But limited liability implies \( x(\theta) \geq 0 \) and \( x(\theta) + z(\theta) \geq 0 \). As \( x(\theta) \) is non-increasing by claim B.1, \( x(\theta) \) is an upper bound for all \( x(\theta) \) and thus \( z(\theta) \) has to be bounded below, contradicting \( \lim_{n \to \infty} z(\theta^n) = -\infty \).

Hence, any contract offered in equilibrium satisfies \( U(\theta) \) is non-decreasing.

Now, because \( U(\theta) \) is non-decreasing \( \theta \in A \) implies \( \theta' \in A \) for all \( \theta' > \theta \) (when indifferent between being a worker and an entrepreneur, agents choose the later by assumption). The continuity of \( U \) implies that if \( A \) is nonempty then \( A = [\theta_L, 1] \).
for some $\theta_L \in [0,1]$. Also, if $\theta_L > 0$ then $U(\theta_L) = w$. ■

**Proof of zero profit condition (Claim 2.4)**

**Proof.** By contradiction suppose intermediary 1 is making profit. We will show intermediary 2 is not choosing an optimal contract schedule.

Let the contract schedules be $(k(\theta), x(\theta), z(\theta))$ for $i \in \{1,2\}$ and $U(\theta)$ the corresponding expected utilities for entrepreneurs. Suppose the profit for intermediary 1 is $M > 0$. Consider the alternative schedule for intermediary 2 in which increases all entrepreneurs’ utility by a small amount $\epsilon$. More precisely, this alternative schedule is defined by $C'_2 = \{(k'(\theta), x'(\theta), z'(\theta))|\theta \in \Lambda\}$, where,

$$
k'(\theta) = k(\theta) \quad z'(\theta) = \frac{f(k(\theta))z(\theta) + \delta\theta}{f(k(\theta))} \quad x'(\theta) = x(\theta) + \frac{\delta}{2} (1 - \theta^2) \quad (B.3)
$$

As shown in the proof of claim 2.3, the payoff $U'(\theta)$ resulting from optimally choosing a contract in $C_1 \cup C'_2$ is strictly greater than the original payoff: $U'(\theta) > U(\theta)$. Hence by deviating to $C'_2$, intermediary 2 will steal half the market intermediary 1 was servicing alone ($A_1$). The revenue of intermediary 2 increases by at least $M$. Naturally the costs also increase because entrepreneurs get more generous contracts, but also because the more generous contracts induce an entry of new entrepreneurs. However, as shown in the proof of claim 2.3 $\delta$ can be chosen so that the extra cost are smaller than $M$. For such a $\delta$, $C'_2$ is a profitable deviation for intermediary 2. ■

**Proof of claim 2.5**

**Proof.** Suppose $U(\theta') \geq S(\theta')$ for some $\theta' > \hat{\theta}$ such that $k(\theta') = k^*(\theta')$ are not positive. By the envelope theorem $S'(\theta) = f(k^*(\theta))\pi$, which is increasing in $\theta$ because $f$ is increasing and concave. Hence $S(\theta)$ is convex. Remembering that
\[ S(0) = 0 \leq x(\theta'), \] by incentive compatibility:
\[
U(\hat{\theta}) \geq \hat{\theta} f(k^*(\theta')) z(\theta') + x(\theta') = \frac{\hat{\theta}}{\theta'} U(\theta') + \frac{\theta' - \hat{\theta}}{\theta'} x(\theta') \\
\geq \frac{\hat{\theta}}{\theta'} S(\theta') + \frac{\theta' - \hat{\theta}}{\theta'} S(0) > S(\hat{\theta})
\]
contradicting \( S(\hat{\theta}) > U(\hat{\theta}) \). As \( k(\theta') = k^*(\theta') \) for almost all \( \theta' \), the lemma follows.

\[ \text{Proof of claim 2.7} \]

**Proof.** Let \( \mathcal{C}_1, \mathcal{C}_2, s^0 \) be an equilibrium with corresponding payoffs \( v_1^0, v_2^0, U^0(\theta) \).
Denote the incentive compatible schedule by \( k^0(\theta), z^0(\theta), x^0(\theta) \) and the set of types taking the contract by \( A^0 \).
Suppose there there is \( \tilde{\theta} \in A^0 \) such that \( x(\tilde{\theta}) > 0 \). Without loss of generality, the associated contract is offered by intermediary 1.
Among the contracts in \( \mathcal{C}_1 \), there could be some that are dominated by another contract in \( \mathcal{C}_1 \), for every entrepreneurial type. We focus on the contracts such that this is not the case:
\[
\{ (k_1(x), x_1(z), z_1(t)) : \theta \in [0, 1] \}
\]
such that \( (k_1(x), x_1(z), z_1(t)) \) is a maximizer of,
\[
U_1(\theta) = \max_{(k,z \in \mathcal{C}_1)} \theta f(k) z + x \tag{B.4}
\]
We will construct a strategy for intermediary 2 allowing him to "cream skim" the market. That is, intermediary 2 will serve all the profitable types, leaving the unprofitable types to intermediary 1.
Because no intermediary would make loses in equilibrium, if a positive measure among the contracts in \( \mathcal{C}_1 \) are actually signed by entrepreneurs, there must be an positive measure subset over which \( C_1 \) yields non-negative profits. Hence there
is a $\hat{\theta} > 0$ (in that set) such that $k_1(\hat{\theta}) = k^*(\hat{\theta})$ and with whom the intermediary makes non-negative profits. By claim 2.5, $S(\theta) > U(\theta)$ for all $\theta > \hat{\theta}$.

If $x_1^0(\hat{\theta}) > 0$, construct

$E_2 = \{(k_2^0(\theta), x_2^0(\theta), z_2^0(\theta)) : \theta \in A^0\}$, where,

$$k_2'(\theta) = k_1^0(\theta), \quad z_2'(\theta) = \frac{f(k_1^0(\hat{\theta})) z_1^0(\hat{\theta}) + x_1^0(\hat{\theta})/\hat{\theta} + \delta(\theta - \hat{\theta})}{f(k_1^0(\theta))} \quad x'(\theta) = \frac{\delta}{2}(1 - \theta)$$

The payoff of entrepreneur $\theta$ among contracts $(k_2'(\theta'), x_2'(\theta'), z_2'(\theta'))$ in $E_2$ is

$$U_2'(\theta) = \max_{\theta' \in [0, 1]} \theta \left(f(k_1^0(\hat{\theta})) z_1^0(\hat{\theta}) + x_1^0(\hat{\theta})/\hat{\theta} + \delta(\theta' - \hat{\theta})\right) + \frac{\delta}{2}(1 - \theta)^2$$

which is uniquely maximized at $\theta' = 0$. That is,

$$U_2'(\theta) = \theta \left(f(k_1^0(\hat{\theta})) z_1^0(\hat{\theta}) + x_1^0(\hat{\theta})/\hat{\theta}\right) + \frac{\delta}{2}(1 - \theta)^2$$

Next, we compare the payoffs of signing the contract with intermediary 1 or 2. We show that for $\theta > \hat{\theta}$, the later is better, while $\theta < \hat{\theta}$ prefers the former.

First consider $\theta < \hat{\theta}$.

By the envelope theorem applied to B.4

$$U_1^0(\theta) = U_1^0(\hat{\theta}) + \int_{\hat{\theta}}^{\theta} f(k_1^0(s)) z_1^0(s) ds$$

$$= f(k_1^0(\hat{\theta})) z_1^0(\hat{\theta}) + x_1^0(\hat{\theta}) + \int_{\hat{\theta}}^{\theta} f(k_1^0(s)) z_1^0(s) ds$$

and $f(k_1^0(s)) z_1^0(s)$ is non-decreasing in $s$.

Notice that,

$$U_2'(\theta) = U_2'(\hat{\theta}) + (\theta - \hat{\theta}) \left(f(k_1^0(\hat{\theta})) z_1^0(\hat{\theta}) + x_1^0(\hat{\theta})/\hat{\theta}\right) + \frac{\delta}{2} \left(\theta^2 - \hat{\theta}^2\right)$$

$$= U_2'(\hat{\theta}) + \int_{\hat{\theta}}^{\theta} \left(f(k_1^0(\hat{\theta})) z_1^0(\hat{\theta}) + x_1^0(\hat{\theta})/\hat{\theta}\right) ds + \frac{\delta}{2} \left(\theta^2 - \hat{\theta}^2\right)$$

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Hence

\[
U_2'(\theta) - U_1'(\theta) = \int_\theta^\hat{\theta} \left\{ f(k_1^0(\hat{\theta})z_1^0(\hat{\theta})) - f(k_1^0(s))z_1^0(s) \right\} \, ds + \frac{\theta - \hat{\theta}}{\theta} x_1^0(\hat{\theta}) + \frac{\delta}{2} (\hat{\theta}^2 - \theta^2)
\]

Since \( f(\cdot)z(\cdot) \) is non-decreasing, for \( \theta < \hat{\theta} \), the first term is non-positive. The second term is strictly positive as \( x_1^0(\hat{\theta}) > 0 \). We chose \( \delta \) such that \( \frac{\theta - \hat{\theta}}{\theta} x_1^0(\hat{\theta}) + \frac{\delta}{2} (\hat{\theta}^2 - \theta^2) \) is still positive. We conclude that for \( \theta < \hat{\theta} \)

\[
U_2'(\theta) - U_1'(\theta) > 0
\]

Next, consider entrepreneurs with \( \theta > \hat{\theta} \)

\[
U_1^0(\hat{\theta}) = \hat{\theta} f(k_1^0(\hat{\theta}))z_1^0(\hat{\theta}) + x_1^0(\hat{\theta}) \\
\geq \hat{\theta} \left( f(k_1^0(\theta))z_1^0(\theta) + \frac{x_1^0(\theta)}{\theta} \right) \\
= \frac{\hat{\theta}}{\theta} U_1^0(\theta)
\]

where the weak inequality in the second line comes from the envelope theorem and the monotonicity of \( f(k(\cdot))z(\cdot) \): an argument similar to the one describe in more detail for \( \theta < \hat{\theta} \).

Since \( \hat{\theta} > 0 \),

\[
\frac{\theta}{\hat{\theta}} U_1^0(\hat{\theta}) \geq U_1^0(\theta)
\]
By construction, $U'_2(\hat{\theta}) = U'_0(\hat{\theta})$. Moreover,
\[
\frac{\hat{\theta}}{\theta} U'_2(\theta) = U'_2(\hat{\theta}) + \frac{\delta}{2} \left( \frac{\theta}{\hat{\theta}} (1 - \hat{\theta}^2) - (1 - \theta^2) \right)
\]
\[
> U'_2(\hat{\theta}) + \frac{\delta}{2} \left( \theta^2 - \hat{\theta}^2 \right)
\]
\[
> U'_2(\hat{\theta})
\]

We conclude that,
\[
U'_2(\theta) \geq \frac{\theta}{\hat{\theta}} U'_2(\hat{\theta}) = \frac{\theta}{\hat{\theta}} U'_0(\hat{\theta}) \geq U'_0(\theta)
\]

It follows that entrepreneurs with type $\theta > \hat{\theta}$ are better of contract $C'_2$ than with contract $C'_0$. Hence intermediary 2 makes at least half of the profits over the interval $[\hat{\theta}, 1]$, which are strictly positive by claim 2.5.

So far we have assumed that $x'_0(\hat{\theta}) > 0$. If $x'_1(\hat{\theta}) = 0$, then it must be the case that $\tilde{\theta} < \hat{\theta}$ (by lemma B.1).

In this case, intermediary 2 can deviate to the strategy,
\[
C''_2 = \left\{ (k''_2(\theta), x''_2(\theta), z''_2(\theta)) | \theta \in A^0 \right\}, \text{ where,}
\]
\[
k''_2(\theta) = k'_1(\theta) \quad z''_2(\theta) = \frac{f(k'_1(\hat{\theta})) z'_0(\hat{\theta}) + x'_0(\hat{\theta})}{f(k'_1(\hat{\theta}))} \quad x'(\theta) = \frac{\delta}{2} (1 - \theta^2)
\]

By the same argument as before, when intermediary 2 deviates to $C''_2$, Every entrepreneur with type $\theta < \tilde{\theta}$ takes a contract from $C'_1$, and every type $\theta > \hat{\theta}$ takes a contract from $C''_2$.

It is left to show that intermediary 1 was making loses over $[\theta_L, \tilde{\theta}]$ - and because she wouldn’t make loses, intermediary 2 steals a strictly profitable fraction of the market-. If this this was not the case, he must be making profits over a positive measure set and we can redefine $\hat{\theta}$ as a point in that interval. □
Proof of proposition 2.9

Proof. Suppose $\theta_L \geq \theta_P$, then $S(\theta_L) \geq S(\theta_P) = w = U(\theta_L)$ by the definition of $\theta_L$ and $\theta_P$. But then the intermediary expects not to loose with the type $\theta_L$ and, by claim 2.5, expects strictly positive profits with all $\theta' \in (\theta_L, 1]$. That implies the intermediary is making profits strictly positive aggregate profits contradicting the zero profit condition.

If types are public, intermediaries must break even with each type, which implies $U(\theta) = S(\theta)$. For those $\theta < \theta_P$, we have $S(\theta) < w$ hence none of them will take any contract. All the rest will accept the contract offered for their type, hence $\theta_L = \theta_P$ and the inefficiency vanishes.

If limited liability is removed, but all other features remain the same, we will show the equilibrium incentive compatible contract schedule has to be $k(\theta) = k^*(\theta)$, $x(\theta) = -R \cdot k^*(\theta)$ and $z(\theta) = \pi$ for (almost) all $\theta \geq \theta_P$.

We start showing that the following strategy profile is indeed an equilibrium:

Each intermediary offers, $C_i = \{(k_i(\theta), x_i(\theta), z_i(\theta))|\theta \in [0, 1]\}$, where,

$$k_i(\theta) = k^*(\theta) \quad \quad z_i(\theta) = \pi \quad \quad x_i(\theta) = -R \cdot k^*(\theta)$$

(B.5)

and entrepreneur $\theta$ flips a coin before selecting between $(k_1(\theta), x_1(\theta), z_1(\theta))$ and $(k_2(\theta), x_2(\theta), z_2(\theta))$, but strictly prefers any of these two instead when compared to any other $(k_i(\theta'), x_i(\theta'), z_i(\theta'))$ for $\theta' \neq \theta$. As $z_i(\theta)$ is constant, $f(k^*(\theta))z_i(\theta)$ is (strictly) increasing, hence the contract satisfies the conditions of lemma 2.2 and $(k_i(\theta), x_i(\theta), z_i(\theta))$ maximizes entrepreneur $\theta$’s utility among the available options. Also $U_i(\theta) = S(\theta)$, and by definition on $\theta_P$, type $\theta$ takes the contract if and only if $\theta \geq \theta_P$. If intermediary $i$ is offering the above contract, intermediary $j$’s best response cannot yield her any profit, since she would get only those types such that $U_i(\theta) \geq U_i(\theta) = S(\theta)$, and hence offering the same contract is a best
response.

To see all equilibrium are payoff equivalent, notice that claims 2.3 and 2.4 still must hold. Define \( B_2 = A_2 \cup B \), and suppose that in an equilibrium, intermediary 2 offers a contract schedule such that on a positive measure subset of \( B_2 \subset [\theta_p, 1] \), \( S(\theta) \neq U_2(\theta) \) for all \( \theta \in B_2 \). Denote the equilibrium strategies by \( c_1, c_2 \) and \( s \). Define the associated contract schedules \((k_1(\theta), x_1(\theta), z_1(\theta))\) and \((k_2(\theta), x_2(\theta), z_2(\theta))\) as in equation (B.4) above.

We will show that intermediary 1 can post a contract schedule that strictly increases her profits.

Define the deviation by, \( c_1' = \{ (k_1'(\theta), x_1'(\theta), z_1'(\theta)) | \theta \in \} \), where,

\[
\begin{align*}
{k_1'(\theta)} &= 0.5k^*(\theta) + 0.5k_2(\theta) \\
{z_1'(\theta)} &= 0.5\pi + 0.5z_2(\theta) \\
{x_1'(\theta)} &= -0.5R \cdot k^*(\theta) + 0.5x_2(\theta)
\end{align*}
\]  \hspace{1cm} (B.6)

The new contract schedule to be offered by intermediary 1 is just the average of intermediary 2’s and the prescribed equilibrium contracts. Note that the new contract skims the cream: there is a threshold level \( \overline{\theta} \), such that every \( \theta > \overline{\theta} \) takes the contract offers by intermediary 2 (if any), and lower \( \theta \)s take the contract by intermediary 1.

Define \( B_2^+ = \{ \theta \in B_2 : U_2(\theta) > S(\theta) \} \) and analogously \( B_2^- = B_2 \setminus B_2^+ \). If \( B_2^- \) has positive measure, this average contract makes profits with all \( \theta \in B_2^- \) because \( S(\theta) > U_2'(\theta) > U_2(\theta) \); it cannot make loses in \( B_2^+ \) since there \( U_2(\theta) > U_2'(\theta) \); and, it is irrelevant outside \( B_2 \). Hence if \( B_2^- \) has positive measure, this contract yields positive profits to intermediary 2 contradicting the zero profit condition. If \( B_2^- \) has measure zero and \( B_2^+ \) has positive measure, this implies that intermediary 2 is not making profit with any type, since \( B_2^- \) has measure zero, but then for her to avoid losses it must be the case that no positive measure of her contracts in \( B_2^+ \) is taken in equilibrium, which implies that for all \( \theta \) in a positive measure set
\( B_1^+ \subset B_2^+ \) we must have \( U_1(\theta) > U_2(\theta) > S(\theta) \) which implies intermediary 1 is making loses there. Since she cannot make profits with any positive measure of types, because 2 offers at least \( S(\theta) \) to everybody but those in \( B_2^- \), she must be making negative profits over all types.

Hence for all \( \theta \in [\theta_P, 1] \) it must be the case that \( U_1(\theta) = U_2(\theta) = S(\theta) \). The envelope theorem for \( S(\theta) \) yields \( S'(\theta) = f(k^*(\theta))\pi \) which implies \( z_1(\theta) = z_1(\theta) = \pi \) for (almost) all those \( \theta \). That in turn implies \( x_i(\theta) = -R \cdot k^*(\theta) \).

**Last, if there is a unique intermediary** facing limited liability and adverse selection, the unique equilibrium is \( k(\theta) = k^*(\theta) \), \( z(\theta) = 0 \) and \( x(\theta) = w \) and (almost) all entrepreneurs with \( \theta \geq \theta_P \) take the contract. In this case an equilibrium should be a contract schedule and a decision rule for entrepreneurs such that the schedule maximizes profit for the intermediary and the decision rule maximizes return to the entrepreneur. In the proposed equilibrium the intermediary extracts all the surplus, hence it is profit maximizing, and entrepreneurs are always indifferent between accepting or rejecting the contract, hence they are also maximizing.

Note that in any equilibrium a type \( \theta \) entrepreneur must take any contract such that her payoff is higher than \( w \) and reject anyone otherwise. So potentially the acceptance rule for some \( \theta' \) could be to accept the contract offering \( k^*(\theta) \) with \( \theta \neq \theta' \).

To see that can only happen in measure zero sets, suppose there exists an equilibrium where the acceptance rule differs from the one prescribed above in a set \( \Theta \in [0,1] \) with positive measure. For all \( \epsilon > 0 \) the contract with \( k(\theta) = k^*(\theta) \), \( z(\theta) = \frac{\epsilon}{\theta_P} \) and \( x(\theta) = w - \epsilon \) is such that \( \theta_L = \theta_P \) and the profits for the intermediary are the total surplus less \( \epsilon \int_{\theta_P}^{1} (1 - G(\theta)) d\theta \), this implies that in any equilibrium profits should be equal to the total surplus, otherwise the intermediary could do better with the contract above for some \( \epsilon \). But to have profits equal to the total aggregate surplus, almost all \( \theta \in [\theta_P,1] \) must accept the contract and
almost all θ < θ_p must reject it. Hence the result follows. ■

**Proof of claim 2.10**

**Proof.** A lump sum tax does not change the intermediaries optimal decisions. Then the contract offered in equilibrium is still characterized by proposition 2.8. Now the decision rule for entrepreneurs is as follows: they must accept the best contract offered to their type if \( U(\theta) - \phi > w \) and reject if the inequality is reversed. In this sense the tax can also be seen as a subsidy on \( w \). Hence, at \( \theta_L \) we must have \( U(\theta_L) = w + \phi \). Then, the zero profit condition for \( \theta_L \) is:

\[
\int_{\theta_L}^{1} \left\{ \theta f(\ell^*(\theta)) \left( \pi - R\ell^*(\theta) \right) \right\} dG(\theta) - \int_{\theta_L}^{1} \theta \frac{(w + \phi)}{\theta_L} dG(\theta) = 0
\]

Which has a unique solution \( \theta_L \) since its derivative with respect to \( \theta_L \) is strictly positive. By definition of \( \phi \), \( \theta_L = \theta_P \) solves the equation and is thus the only solution. ■

**Proof of claim 2.11**

**Proof.**

Suppose intermediaries compete with contracts of the form \((k, x, (1 - \tau)z)\). Then the contract offered in equilibrium is still characterized by proposition 2.8 but the \( \theta_L \) now has to solve:

\[
\int_{\theta_L}^{1} \left\{ \theta f(\ell^*(\theta)) \left( \pi - R\ell^*(\theta) \right) \right\} dG(\theta) - \int_{\theta_L}^{1} \theta \frac{w}{(1 - \tau)\theta_L} dG(\theta) = 0,
\]

because intermediaries have to pay \( (1 - \tau)^{-1}U(\theta) \) if she is supposed to deliver \( U(\theta) \) net of taxes to the entrepreneur. Plugging the value for \( \tau^* \) we obtain:

\[
\int_{\theta_L}^{1} \left\{ \theta f(\ell^*(\theta)) \left( \pi - R\ell^*(\theta) \right) \right\} dG(\theta) - \int_{\theta_L}^{1} \theta \frac{(w + \phi)}{\theta_L} dG(\theta) = 0
\]

Which by claim 2.10 has a unique solution \( \theta_L = \theta_P \). ■
B.1.2. Asset Holdings Proofs

The proofs for claims 2.3, 2.4, and lemma B.1 are analogous to those for the zero assets case. Fixing the asset level \( a \), the IC constraint across types \( \theta \) is the same, but now the limited liability restriction is \( x_i(\theta, a) \geq -Ra \). Recall the expected utility of a type \( \theta \) entrepreneur with assets \( a \) under contract \( i \) be \( U_i(\theta, a) + Ra \). Then \( x_i(\theta, a) \) is decreasing in \( \theta \) and in a competitive equilibrium \( k_i(\theta, a) = k^*(\theta) \) for (almost) all \((\theta, a)\) and \( U_i(\theta, a) \) is nondecreasing in \( \theta \).

**Proof of claim 2.12**

**Proof.** If \( k^*(\theta) \leq a \), the entrepreneur can self-finance the project up to the optimal scale and save the rest, with expected profit \( \theta F(k^*(\theta)) \pi + R(a - k^*(\theta)) \), which is the best possible outcome for the entrepreneur outside the credit market. If \( k^*(\theta) > a \) the project can still be started but at a scale lower than the optimal, concavity of \( F \) implies that the best option, conditional on starting the project, is to invest all the assets in it, which yields \( \theta F(a) \pi \). In any case that has to be compared with the option of not doing the project and getting the return on the assets. ■

**Proof of proposition 2.13**

Claim 2.5 carries over. The proof has to change a little.

**Proof.** Note that because of the outside option, \( \hat{\theta} f(a) \pi < U_i(\hat{\theta}, a) + Ra \) must hold. Also for all \( \theta \) we have \( S(\theta) \geq \theta f(a) \pi - Ra \) with equality only for some \( \theta_a \) such that \( k^*(\theta_a) = a \). Note that no profits can be made with types \( \theta < \theta_a \) as those can fully self-finance, hence \( \hat{\theta} > \theta_a \). Suppose the expected profits for some \( \theta' > \hat{\theta} > \theta_a \) such that \( k_i(\theta') = k^*(\theta') \) are not positive, that is \( U_i(\theta') \geq S(\theta') \). \( S(\theta) \) is still convex.
Remembering that $S(\theta_a) = \theta_a f(a) \pi - Ra \leq U_i(\theta_a, a)$, by incentive compatibility:

\[
U_i(\hat{\theta}, a) \geq \hat{\theta} f(k^*(\theta')) z_i(\theta', a) + x_i(\theta', a) = \frac{\hat{\theta}}{\theta'} U_i(\theta', a) + \frac{\theta' - \hat{\theta}}{\theta'} x_i(\theta', a)
\]

\[
U_i(\hat{\theta}, a) \geq \frac{\hat{\theta} - \theta_a}{\theta' - \theta_a} U_i(\theta', a) + \frac{\theta' - \hat{\theta}}{\theta' - \theta_a} \left[ \frac{\theta_a}{\theta'} U_i(\theta', a) + \frac{\theta' - \theta_a}{\theta'} x_i(\theta', a) \right]
\]

\[
U_i(\hat{\theta}, a) \geq \frac{\hat{\theta} - \theta_a}{\theta' - \theta_a} U_i(\theta', a) + \frac{\theta' - \hat{\theta}}{\theta' - \theta_a} \left[ \theta_a f(a) \pi - \frac{\theta_a}{\theta'} Ra - \frac{\theta' - \theta_a}{\theta'} Ra \right]
\]

\[
U_i(\hat{\theta}, a) \geq \frac{\hat{\theta} - \theta_a}{\theta' - \theta_a} S(\theta') + \frac{\theta' - \hat{\theta}}{\theta' - \theta_a} [\theta_a f(a) \pi - Ra]
\]

\[
U_i(\hat{\theta}, a) \geq \frac{\hat{\theta} - \theta_a}{\theta' - \theta_a} S(\theta') + \frac{\theta' - \hat{\theta}}{\theta' - \theta_a} S(\theta_a) > S(\hat{\theta})
\]

contradicting the expected profits for $\hat{\theta}$. As $k_i(\theta') = k^*(\theta')$ for almost all $\theta'$ the lemma follows. ■

The construction of the alternative schedule in the proof of claim 2.7 will be precisely defined here in a more general setting.

**Lemma B.2** Let $\mathcal{C}(\theta) = (k(\theta, a), x(\theta, a), z(\theta, a))$ be an incentive compatible contract schedule with assets $a$. For any $\hat{\theta} \in [0, 1]$, define the contract schedule $\mathcal{C}_{\hat{\theta}}$ by,

\[
\mathcal{C}_{\hat{\theta}}(\theta) = \left( k(\theta, a), -Ra, \frac{f(k(\hat{\theta}, a)) z(\hat{\theta}, a) + x(\hat{\theta}, a)/\hat{\theta}}{f(k(\theta))} \right)
\]

Then,

- $\mathcal{C}_{\hat{\theta}}$ is incentive compatible.

- For all $\theta < \hat{\theta}$, $U(\theta, \mathcal{C}) \geq U(\theta, \mathcal{C}_{\hat{\theta}})$

- $U(\theta, \mathcal{C}) > U(\theta, \mathcal{C}_{\hat{\theta}})$ if and only if for some $\theta < \hat{\theta}$ either

\[
x(\hat{\theta}) > -Ra, \quad \text{orf} (k(\hat{\theta}, a)) z(\hat{\theta}, a) > f(k(\theta, a)) z(\theta, a).
\]
The proof of this lemma is analogous to the first steps in the proof of claim 2.7. Moreover, once the two lemmas have been established, the rest of the proof of claim 2.7 carries over, completing the proof of proposition 2.13.

B.1.3. Proofs with Unobservable Assets

Then the IC constraint across assets is \( U_i(\theta, a) \) is nondecreasing in \( a \) (recall an entrepreneur cannot lie and give more collateral than what he has).

**Linear loss minimization**

We define a procedure to minimize losses for a given set of profitable contracts. An (expected) profitable contract is one where \( U_i(\theta, a) < S(\theta) \) where \( S(\cdot) \) is the surplus function.

For a fixed asset level \( a \), the promised extra utility \( U_i(\theta, a) \) curve of an IC contract with limited liability can only cross the curve \( S(\theta) \) once\(^1\). Define \( \theta_e(a) \) as the solution of \( U_i(\theta, a) = S(\theta) \). Also define \( \hat{f}_Z(a) = \inf \left\{ f(k^*(\theta)) z(\theta, a) : \theta > \theta_e(a) \right\} \), this is the maximum slope the contract can have at \( \theta_e(a) \).

For all \( \theta < \theta_e(a) \) the intermediary is making losses. Hence it is in his best interest to reduce \( U_i(\theta, a) \) for all such theta. However the IC constraint over \( \theta \) implies that \( U_i(\theta, a) \geq S(\theta_e(a)) - (\theta_e(a) - \theta) \hat{f}_Z(a) \) and the intermediary will try to set that. Unfortunately, there is also the IC constraint over \( a \), that requires \( U_i(\theta, a) \) to be nondecreasing in \( a \). Therefore the loss minimization given some contract \( \theta_e(a) \) and \( \hat{f}_Z(a) \) is achieved by setting:

\[
U_i(\theta, a) = \sup \left\{ S(\theta_e(\hat{a})) - (\theta_e(\hat{a}) - \theta) \hat{f}_Z(\hat{a}) : 0 \leq \hat{a} \leq a \right\}
\]

**Zero Profits**

Zero profits will still happen but the proof needs to be modified. Below are the steps.

\(^1\)There may be another cut, but that has to be below or at the outside option curve \( O(\theta, a) \)
1. As before if one intermediary is making profits $\pi$, the other can always set a new contract as the max of the two current offered contracts. After that she need to increase her offers by an $\varepsilon$ small enough such that she would take over all the market and profits fall just slightly.

2. Because of continuity, the issue is going to be the new entrants. First we need that for every $\delta$ there exists some $\varepsilon$ such that the change in $\theta_L(a)$ is less than $\delta$ for a lot of $a$ (meaning we can make the mass of those not bounded as small as wanted) when we give $\varepsilon$ more to everybody.

(a) The new entrants are determined by the slope of the contract on $\theta_L(a)$.

We need a bound for that slope from below that works for every $a$ and is strictly positive. Unfortunately that is not generally possible. However we can find a bound for a lot of values, such that the measure of those not bounded is very small relative to the profits. In what follows we assume the contract, after the max process has been optimized with the linear loss minimization described above.

i. There is no need to worry about those $a$ such that $\theta_L(a) = 0$. In fact we can disregard all asset levels such that $\theta_L(a) < \delta$. Note that for the remaining $a$’s $\tilde{f}(a) > 0$.

ii. First we deal with those $a$ with positive marginal mass. There can be only countable many of those. Let the combined mass of all those $a_i$ be $M_1 \leq 1$. Then there exits finite number of those $N_1$ such that $\sum_{i=1}^{N_1} m(a_i) > M_1 - \frac{\pi}{5w}$. From now on we will forget about all other $a_i$ with positive mass and take a loss no greater than $0.2\pi$ with all those types with capital $a_i$ for $i > N_1$. Now take $0 < f_1 = \min\{f(a_i) : 0 \leq i \leq N_1\}$

iii. Take any $\bar{a}$ such that $S(\theta_e(\bar{a})) > \bar{w}$ Look at the slope of the contract at $\theta_L(\bar{a})$ after the linear minimization process. We claim that for all
\(a' > \bar{a}\) the slope of the contract at \(\theta_L(a')\) is greater or equal than the minimum between the slope at \(\theta_L(\bar{a})\) and:

\[
\frac{S(\theta_e(\bar{a})) - w}{\theta_e(\bar{a}) - \delta}.
\]

Call that minimum \(f_2(\bar{a})\). If \(\theta_L(a') = \theta_L(\bar{a})\) then, by IC over the assets, the slope of the contract at \(\theta_L(a')\) has to be greater or equal than the one at \(\theta_L(\bar{a})\). If \(\theta_L(a') < \theta_L(\bar{a})\) then, by the envelope theorem, the slope at \(\theta_L(a')\) will be \(\hat{f}z(\bar{a})\) for some \(\bar{a} < \hat{a} \leq a'\), but all those are bounded below by the slope of line that goes through the points \((\delta, w)\) and \((\theta_e(\bar{a}), S(\theta_e(\bar{a}))\).

Now, if \(S(\theta_e(0)) > w\) set \(f_2 = f_2(0)\) and then the slopes are all (but a small measure) bounded by \(\min\{f_1, f_2\}\). If not, since \(\theta_e(a)\) is nondecreasing it is measurable, so we can pick an asset level \(\bar{a}\) such that the mass of assets \(\hat{a}\) such that \(S(\theta_e(\hat{a})) \in (w, S(\theta_e(\bar{a})))\) is less than any positive constant we want.

We are going to allow a bigger loss on those \(\hat{a}\) types. For all of them the contract offered will be the same as the one for \(\bar{a}\), before the \(\epsilon\) increase. This would generate a loss less than \(S(\theta_e(\bar{a}))\) per entrepreneur, as the contract for \(\bar{a}\) makes expected losses only on those \(\theta\) types that get offered an utility level less than that. Hence we pick \(\bar{a}\) such that the mass of \(\hat{a}\) such that \(S(\theta_e(\hat{a})) \in (w, S(\theta_e(\bar{a})))\) is less than:

\[
\frac{\pi}{5S(\theta_e(\bar{a}))},
\]

and set \(f_2 = f_2(\bar{a})\).

iv. Now, we have to deal with those \(\hat{a}\) such that \(S(\theta_e(\hat{a})) = w\), let \(\theta_p = S^{-1}(w)\) then we are talking about those asset levels such that
\( \theta_e(a) = \theta_p \) they form an interval \((a_0, a_1)\) which may be closed or open at both ends. Notice for all those the slopes \( \hat{f}_z(\hat{a}) \) are increasing in \( \hat{a} \) because of incentive compatibility w.r.t. \( \hat{a} \). Now, if \( \inf\{\hat{f}_z(\hat{a})\} > 0 \) then set \( \hat{f}_3 \) equal to that infimum. Otherwise, as before, for each \( \bar{a}_2 \) such that \( S(\theta_e(\bar{a}_2)) = w \) we can raise the contract of all those \( a' \in [a_0, \bar{a}_2) \) to \( U_i(\theta, \bar{a}_2) \). The loss for that change is the maximum difference between the original contracts and this new one, which is bounded by the difference between the line \( u = w \) and the line with slope \( \frac{Ra+w}{\theta_p} \). As the marginal mass may be distributed in any way, the bound will be the difference between the value of that linear function at \( \theta = 1 \) and \( w \). So we pick \( \bar{a}_2 \) such that the mass in \([a_0, \bar{a}_2)\) is less than \( \pi \frac{\pi}{5(U_i(1, \bar{a}_2) - w)} \) and let \( f_4 = \hat{f}_z(\bar{a}_2) \).

v. Last, we deal with those asset levels (if any) for which \( U_i(\theta, w, a) < w \). Again, there is a \( \bar{a}_3 \) such that the mass of those \( a' > \bar{a}_3 \) such that \( U_i(\theta, w, a') < w \) is less than \( \pi \frac{\pi}{5(U_i(1, \bar{a}_2) - w)} \). Again we will lift all those \( \bar{a}_3 < \hat{a} < \bar{a}_2 \) to \( U_i(\theta, \bar{a}_2) \). The loss for doing that is less than \( 0.2\pi \) and let \( f_5 = \hat{f}_z(\bar{a}_3) \).

vi. To finish take the minimum of the \( f_i \)'s. That is the lower bound for the slope and hence define \( \epsilon(\delta) = \delta \times \min\{f_i : i \in \{1, 2, 3, 4, 5\} \} \). So far we lost \( 0.4\pi \) with the unbounded types.

(b) After that we need that for each \( \mu \) there exist a \( \delta \) such that the mass of new entrepreneurs is less than \( \mu \) given a reduction no larger than \( \delta \) in \( \theta_L(a) \) for all \( a \). Measurability of the contracts imply \( U_i(\theta, a) \) and \( \theta_L(a) \) are measurable. The new entrants are a subset of \( U_i^{-1}([w - \epsilon, w]) \subset [\theta_L(a) - \delta, \theta_L(a)] \). Now we want to bound the mass of the latter inter-
vals. For each $a$ there exists some $\delta(a) > 0$ such that

$$G^-(\theta_L(a)|a) - G^-(\theta_L(a) - \delta(a)|a) < 0.5\mu$$

If the lower bound of those $\delta(a)$ is positive that is our $\delta$ and we are done. Otherwise we find a bound for all but a measure no bigger than $\frac{\pi}{5w}$, on which we take losses of $w$ with all the potential entrants, so total losses wont exceed $0.2\pi$, for the rest take the lower bound which must be positive.

**Proof of claim 2.15**

**Proof.** Suppose that is not the case. Then there exist some positive measure set $B = B^1 \cup B^2$ such that all types in $B^i$ strictly prefer the contract offered by intermediary $i$ than the one offered by $j$. Notice that the point wise average of two IC contract schedules with $k_i(\theta, a) = k^*(\theta)$ is an IC contract schedule. This is because utility is linear in $z_i(\theta, a)$ and $x_i(\theta, a)$ so the IC constraint will be inherited from the IC of the two original schedules. We will show this average IC schedule has to increase profits for one of the intermediaries. For intermediary $i$, the newly proposed contract reduces the utility offered to those in $B^i$ because that is averaged with the one offered by $j$ to those types, which was assumed to be strictly lower. However types in $B^1$ still prefer intermediary $i$ contract. Outside $B$ nothing changes since offered utilities were equal there, in $B^j$ types still prefer intermediary $j$. Hence intermediary $i$ by offering the average schedule reduces the surplus given away in the set $B^i$ keeping all other sources of income fixed. If $B^i$ has a positive measure then her profits strictly increase. Hence the measure of $B^i$ must be zero which implies the measure of $B$ has to be zero.

$\blacksquare$

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Proof of claim 2.17

Proof. Sup \( \theta < \theta' < \theta'' \) are such that \( \theta \) and \( \theta'' \) are in \( A(a|b) \). Let \( x(\theta', b) = -Rc \) for some \( c \leq b \), notice that \( P(a) \) is strictly increasing in \( a \) as it is the slope of the line passing through \((0, -Ra)\) and \( \left( \theta_c(a), S(\theta_c(a)) \right) \). Now, as \( \theta' \notin A(a|b) \) then \( \theta'P(c) - Rc \geq \theta'P(a) - Ra \). If \( c > a \), then \( P(c) > P(a) \) and as \( \theta'' > \theta' \) it follows that \( \theta''P(c) - Rc > \theta''P(a) - Ra \) contradicting \( \theta'' \in A(a|b) \), analogously if \( c < a \) implies \( \theta \notin A(a|b) \) hence \( c = a \) and then \( \theta' \in A(a|b) \).

If \( \theta \in A(a, c) \) for \( c > b \geq a \) then,

\[
U_i(\theta, c) = \theta P(a) - Ra = \sup_{0 \leq a' \leq c} \theta P(a') - Ra' \geq \sup_{0 \leq a' \leq b} \theta P(a') - Ra' \geq P(a) - Ra,
\]

which implies \( \theta \in A(a|b) \)

■
Appendix C

Appendix to Optimal v. Simple Financial Policy Rules

C.1. Solution Method

The Matlab code named \texttt{MPPsolve.m} provides the algorithm for solving the model. The code is divided in seven sections (cells).

**Cell 1.** Parameter Values sets the parameters values shown in table 10. We use 100 points in the grid for bonds and three states for $z^T$ shocks, three states for news shocks and two states for interest rates shocks. The convergence tolerance level for the solution of decision rules, as defined in Section 3 below, is set to $\epsilon = 1e^{-5}$.

**Cell 2.** Construction of Markov Chain: This cell builds the Markov Chain for the exogenous shocks $z^T$, $R$ and news, as well as the debt grid.

i) First, discretize $z^T$ shocks using Tauchen and Hussey (1991) method, implemented in function \texttt{tauchenhussey.m}. The Matlab routine and other similar ones are available at \texttt{http://www2.hhs.se/personal/floden/}. The time-series properties of the $z^T$ process that the method targets
are estimates obtained by Bianchi (2011) using data for Argentina, and the corresponding moments are reported in table 10.

ii) Next, incorporate news shocks according to the formulas in the Section 2.3 of the paper. Recall by Bayes rule:

\[
p(z_{t+1}^T = l | s_t = i, z_t^T = j) = \frac{p(s_t = i | z_{t+1}^T = l)p(z_{t+1}^T = l | z_t^T = j)}{\sum_n p(s_t = i | z_{t+1}^T = n)p(z_{t+1}^T = n | z_t^T = j)}
\]  

(C.1)

Hence we can write:

\[
\Pi(z_{t+1}^T, s_{t+1}, z_t^T, s_t) \equiv p(s_{t+1} = k, z_{t+1}^T = l | s_t = i, z_t^T = j) = p(z_{t+1}^T = l | s_t = i, z_t^T = j)p(s_{t+1} = k | z_{t+1}^T = l) = p(z_{t+1}^T = l | s_t = i, z_t^T = j) \times \ldots \sum_m \left[ p(z_{t+2}^T = m | z_{t+1}^T = l)p(s_{t+1} = k | z_{t+2}^T = m) \right],
\]  

(C.2)

iii) Finally we add global liquidity shocks to construct the entire transition matrix, assuming \(z^T\) shocks and global liquidity shocks are independent.

**Cell 3.** Decentralized Equilibrium solves the decentralized equilibrium using the time-iteration method. Intuitively, this algorithm solves the model by a backward recursive-substitution of the model’s optimality conditions written in recursive form. In particular, the algorithm solves for the recursive functions \(m^N(b, e), c^T(b, e), P^N(b, e)\) and \(B(b, e)\) that satisfy equations 3.5, 3.6, 3.7 3.8, 3.3 and 3.12 where \(e\) is a triple \((z^T, q, s)\) that includes the realizations of the exogenous shocks to \(z^T\), the news signal \(s\), and \(q\) (recall that \(q = \frac{1}{R}\)). Note that in the decentralized equilibrium \(m^T(b, e)\) can be solved separately from (3.11), and it depends only on \(z^T\).
Start the algorithm at an initial point defined by setting $K = 1$ and define conjectures for the equilibrium functions at this point, denoted $c^T_K(b, e)$, $p^N_K(b, e)$ and $B_K(b, e)$. Then proceed with the following steps:

**Step 1.** Assume the constraint binds. Rewrite the borrowing constraint as an function of $m^N(b, e)$, $m^T(b, e)$, $b$, $z^t$, $q$ and $s$. For a given $m^N$, consumption of nontradables follows from $c^N_t + A^N = z^N_t m^N_t \alpha^N$, the price of nontradables comes from (3.12). Given $c^N$ and $p^N$ the consumption of tradables can be solved from (3.6). Plugging $m^T$, $m^N$, $c^T(m^N)$ and $p^N(m^N)$ into the leverage constraint, we are left with an inequality on parameters, states, $m^t$ (which depends only on $z^T$) and powers of $m^N$. We then use the fsolve root-finding routine to find the $m^N$ such that the leverage constraint holds with equality and set $m_{K+1}(b, e)$ to that value. The same process gives us a candidate allocation $K + 1$ for all other endogenous variables.

**Step 2.** Compute

$$U \equiv u_T(c^T_{K+1}(b, e), c^N_{K+1}(b, e)) \ldots - \beta \mathbb{E}_e \left[ u_T(c^T_K(B_K(b, e), e'), c^N_K(B_K(b, e), e')) \right] \quad \text{(C.3)}$$

If $U > 0$, the collateral constraint binds and the allocation found in step 1 is the new guess.

**Step 3.** If $U \leq 0$, the collateral constraint does not bind. Discard the values of $m^N_{K+1}(b, e)$ and $C^T_{K+1}(b, e)$ set in Step 1. As in step 1, write $c^T$ and $c^N$ as a function of $m^N$ but in this case solve the Euler equation C.3 with equality using the fsolve root-finding routine. With that $m^N_{K+1}(b, e)$ in hand, compute $p^N_{K+1}(b, e)$, $c^N_{K+1}(b, e)$, $c^T_{K+1}(b, e)$ as before and solve for $B_{K+1}(b, e)$ from the budget constraint (equation 3.3).
Step 4. The above steps will in general produce a new set of functions \(m^N_{K+1}(b,e), c^N_{K+1}(b,e), c^T_{K+1}(b,e), p^N_{K+1}(b,e)\) and \(B_{K+1}(b,e)\) that will differ from the conjectures \(m^N_K(b,e), c^N_K(b,e), c^T_K(b,e), p^N_K(b,e)\) and \(B_K(b,e)\). We thus check the convergence criterion \(\sup |x_{K+1} - x_K| \leq \varepsilon\) for \(x \in \{B, m^N, c^T, p^N\}\). If the criterion fails, the conjectures are replaced with the solutions \(m^N_{K+1}(b,e), c^N_{K+1}(b,e), c^T_{K+1}(b,e), p^N_{K+1}(b,e)\) and \(B_{K+1}(b,e)\) and the procedure returns to step 1 using these new conjectures. If the convergence criterion \(\sup |x_{K+1} - \alpha x_K| \leq \varepsilon\) holds, the recursive functions are a solution to the decentralized competitive equilibrium in recursive form.

**Cell 4.** Social Planner solves the social planner’s problem. The algorithm is also a time-iteration code similar to that of decentralized equilibrium. In this case, to get the planner’s Euler equation, the term \(u_T\) has to be replaced by

\[
\text{u}^\text{SP}_T = u_T + \mu^\text{SP} \psi
\]  

(C.4)

in (C.3), where \(\mu^\text{SP} \geq 0\), with strict inequality if the collateral constraint (equation 3.18) binds, and \(\psi\) is the externality term given by

\[
\psi = \left[(1 + \eta) \frac{p^N_t [(1 - \alpha N) z^N_t m^N_{\alpha N}]}{c^T_t}\right].
\]  

(C.5)

Given that the following steps solve the Social Planner’s (SP) problem.

Step 1. Assume the constraint does not bind. In this case \(\mu^\text{SP} = 0\) and the new guessed allocation can be found as in step 3 of the Decentralized Equilibrium (DE). That is, solve for the \(p^N, c^N\) and \(c^T\) as functions of \(m^N\) that solve the updated planner’s Euler equation and find the debt position from the borrowing constraint. Note
that in this case $m^T$ only depends on $z^T$.

Step 2. Check the borrowing constraint. If it is satisfied, then the previous allocation is the new $K + 1$ guess. If not discard and solve the constrained problem.

Step 3. The main issue with the constrained problem is that it cannot be reduced to one equation in one unknown, since the size of the multiplier $\mu^{SP}$ matters for the input tax subsidies. In this case $m^T$ depends on the endogenous variables. Hence the algorithm consists on reducing the equilibrium to two equations in $m^N$ and $c^T$ to be solved numerically and then use those to generate the new prices and allocations guess.

i) Take as given the pair $(m^N, c^T)$.

ii) Solve $c^N$ from $c^N_t + A^N = z^N_t m^N_t \alpha^N$.

iii) $p^N$ follows from the marginal rate of substitution constraint for the planner, (3.19).

iv) With $p^N$, $c^N$, $c^T$, the value of $\psi$ can be solved.

v) Using the nontradables production optimality condition for the planner, (3.22), solve for the value of $\mu^{SP}$. 

vi) Given $\mu^{SP}$, the tradables input $m^T$ is solved from (3.23), and the budget constraint multiplier $\lambda^{SP}$ is solved from equation (3.21) which is the planner’s Euler equation.

vii) At this point all variables are solved. The two equations left to use are the leverage constraint and the one setting the marginal value of tradables consumption for the social planner, equation (3.20). The numerical routine `fsolve.m` finds the pair $(m^N, c^T)$ that satisfies this two equations.

Step 4. As before, check convergence by comparing the new guess with
the old one.

**Cell 5.** Welfare Calculation takes the optimal policy functions we derived from cells 3 and 4 of the Matlab code, and iterates until convergence to get value functions of the private agent and social planner. We then calculate the welfare gain as in Bianchi (2011).

**Cell 6.** Optimal Tax calculates optimal macro-prudential tax. With taxes on debt \( \tau_t \) the first order condition in a Decentralized Equilibrium is:

\[
u_T(t) - \mu_t^{DE} = (1 + \tau_t) \frac{\beta}{q_t} E_t [u_T(t + 1)] \tag{C.6}
\]

The optimal tax policy is such that the allocation obtained from the planner’s problem solves (C.6). But that allocation solves the planner’s Euler Equation which is:

\[
u_T(t) + \mu_t^{SP}(\psi_t + 1) = \frac{\beta}{q_t} E_t \left[ u_T(t + 1) + \mu_{t+1}^{SP}\psi_{t+1} \right]. \tag{C.7}
\]

Recall we are interested in the optimal macro-prudential tax, the optimal tax when times are good, i.e. constraint not binding \( (\mu_t^{DE} = 0) \).\(^1\) Hence the macro-prudential tax is:

\[1 + \tau_t = \frac{q_t}{\beta} \frac{u_T(t)}{E_t [u_T(t + 1)]} = 1 + \frac{E_t [\mu_{t+1}\psi_{t+1}]}{E_t [u_T(t + 1)]} \tag{C.8}\]

where the first equality is obtained by solving from (C.6) and the second replaces \( u_T(t) \) by (C.7). This is (3.25) in the paper.

The Matlab code named ”MPPsimulation.m” simulates the model. The code is divided in five cells.

\(^1\)The multipliers for the two problems \( \mu_t^{DE} \) and \( \mu_t^{SP} \) do not need to be equal. However if for some allocation the constraint is not binding in the Decentralized Equilibrium, then it will not bind in the planners problem either.
**Cell 1:** Loads solver results. Allocates Grids and initial values for simulation. Randomly draws exogenous variables.

**Cell 2:** Simulates our model for 201,000 periods. The initial bond position is set as mid point of the bond grid for both DE and SP economies. The first 1,000 periods are discarded to eliminate initial condition dependence.

**Cell 3:** Calculates long term moments and displays the top panel of table 11 in the command window.

**Cell 4:** Identifies sudden stop events. Crisis is defined as current account goes beyond two standard deviation and collateral constraint binds in the decentralized economy. The crisis moments are obtained by taking average across all crisis episodes. Displays the bottom panel of table 11 in the command window.

**Cell 5:** Perform event window analysis. Finds the surrounding three periods before and after the sudden stop event. Calculates moments for endogenous variables in the periods before and after the crises.

The Matlab code "MPPfigures.m" generates figures 21, 22, 23, 24, 25 and 26 presented in the paper.
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