1-1-2015

Essays on Macroeconomics and Finance

Qiusha Peng
University of Pennsylvania, pengqiusha2010@gmail.com

Essays on Macroeconomics and Finance

Abstract

This dissertation consists of two essays on macroeconomics and finance. Chapter 1 develops a novel theory of "bubble" dynamics in a tractable noisy rational expectations model with endogenous capital flows. I show that the unique linear partially revealing rational expectations equilibrium features a dramatic non-fundamental rise and fall of asset prices driven by speculation. Specifically, two layers of uncertainty---uncertainty about the fundamental value and uncertainty regarding the probability with which the fundamental value is fully revealed in each period, generate the hump shape in prices; gradual capital inflows lead to dramatic price movements and also trading frenzies. Simulation results show that the model equilibrium can produce various realistic bubble episodes. In Chapter 2, I investigate the role of business deregulation and financial reform in both stock and credit market in explaining the rapid growth of China in the past twenty years. To do so, I build a dynamic general equilibrium growth model with heterogeneous consumers and firms, and I show that structural reforms that facilitated business formation and growth lead to a significant increase in the aggregate output. The reason is resource reallocation resulting from stronger market competition, in particular caused by a massive influx of new firms. Quantitative results using firm-level data find a sizable effect of these reforms, especially through the extensive margin, and counterfactual experiments show that different policies aiming to promote entry and post-entry growth have very distinct impacts on the economic performance.

Degree Type

Dissertation

Degree Name

Doctor of Philosophy (PhD)

Graduate Group

Economics

First Advisor

Joao F. Gomes

Keywords

Finance, Macroeconomics, Theory

Subject Categories

Economics | Finance and Financial Management

ESSAYS ON MACROECONOMICS AND FINANCE

Qiusha Peng

A DISSERTATION

in

Economics

Presented to the Faculties of the University of Pennsylvania

in

Partial Fulfillment of the Requirements for the

Degree of Doctor of Philosophy

2015

Supervisor of Dissertation

João F. Gomes
Howard Butcher III Professor of Finance

Graduate Group Chairperson

George Mailath
Professor of Economics, Walter H. Annenberg Professor in the Social Sciences

Dissertation Committee

João F. Gomes, Howard Butcher III Professor of Finance
Guillermo L. Ordoñez, Assistant Professor of Economics
Itay Goldstein, Joel S. Ehrenkranz Family Professor, Professor of Finance
Richard E. Kihlstrom, Ervin Miller-Arthur M. Freedman Professor of Finance and Economics
Andrew Postlewaite, Harry P. Kamen Professor of Economics, Professor of Finance

ESSAYS ON MACROECONOMICS AND FINANCE

Dedicated to my parents

ACKNOWLEDGEMENTS

The best thing during my doctoral study is to have Professor João Gomes as my main advisor. His support has led me through this critical stage of my life. He trained me to do research. More importantly, without him, I would not have the belief, confidence and courage, and this dissertation would be impossible.

I also would like to thank my other advisors, Professors Guillermo Ordoñez (chair), Itay Goldstein, Richard Kihlstrom and Andrew Postlewaite. I have learned a lot from them. Their insightful questions pushed me to think and have a better understanding of my own work, and their valuable comments helped shape the current version of this dissertation.

Last but not least, I greatly appreciate all the generous help along the way. It is never an easy road, but to all my friends,

"Growing old along with me! The best is yet to be."

-Robert Browning

ABSTRACT

ESSAYS ON MACROECONOMICS AND FINANCE

Qiusha Peng

João F. Gomes

This dissertation consists of two essays on macroeconomics and finance. Chapter 1 develops a novel theory of "bubble" dynamics in a tractable noisy rational expectations model with endogenous capital flows. I show that the unique linear partially revealing rational expectations equilibrium features a dramatic non-fundamental rise and fall of asset prices driven by speculation. Specifically, two layers of uncertainty—uncertainty about the fundamental value and uncertainty regarding the probability with which the fundamental value is fully revealed in each period, generate the hump shape in prices; gradual capital inflows lead to dramatic price movements and also trading frenzies. Simulation results show that the model equilibrium can produce various realistic bubble episodes. In Chapter 2, I investigate the role of business deregulation and financial reform in both stock and credit market in explaining the rapid growth of China in the past twenty years. To do so, I build a dynamic general equilibrium growth model with heterogeneous consumers and firms, and I show that structural reforms that facilitated business formation and growth lead to a significant increase in the aggregate output. The reason is resource reallocation resulting from stronger market competition, in particular caused by a massive influx of new firms. Quantitative results using firm-level data find a sizable effect of these reforms, especially through the extensive margin, and counterfactual experiments show that different policies aiming to promote entry and post-entry growth have very distinct impacts on the economic performance.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF ILLUSTRATIONS

Chapter 1

Price Dynamics: Differential Information and Capital Flows

1.1 Introduction

Throughout history, asset prices sometimes appear to deviate spectacularly from their fundamental values. From the historic Dutch tulipmania and the South Sea Bubble to the recent IT and housing booms, asset prices increased dramatically following a rise in investor exuberance and then collapsed as the fad receded. Explaining these phenomena and the associated trading frenzies has long intrigued and challenged economists. Although there is a vast literature on this subject, almost all of it has focused on either the existence of bubbles or their collapse. Very few papers seek to understand the entire dynamic evolution of prices during these events.

Bubble episodes are usually associated with periods of high uncertainty. The supply of rare tulip bulbs was limited and uncertain during the Dutch tulipmania of 1634-1637, and there was great uncertainty about the value of government debt-for-equity swaps during the South Sea Bubble in 1720. During stock market booms accompanying technological revolutions like railroads in Britain (1830-1861) and IT in the U.S. (1995-2002), there was substantial uncertainty about the extent to which these new technologies would change our lives.[1]

In light of those historical facts, in this paper, I develop a novel theory of endogenous asset price dynamics to show how speculation driven by uncertainty can generate plausible bubble-like episodes. Specifically, I show how uncertainty regarding the speed at which

[1] See Garber (2000) pp. 39-42 and 115-120 about the Dutch tulipmania and the South Sea Bubble; see Pástor and Veronesi (2009) for technological revolutions and stock market bubbles.

fundamental values are revealed, together with an endogenous influx of new investors, can lead to a unique linear partially revealing rational expectations equilibrium in which asset prices rise significantly before returning to their intrinsic values.

Uncertainty about fundamental asset values, together with differential private information and short-sale constraints, means that asset prices will reflect the marginal investor's belief about future optimistic investors' beliefs. As a result, the marginal investor is willing to pay more than his perceived fundamental value and speculative "bubbles" arise.

While this type of individual uncertainty is important, my paper focuses on a different, and arguably more fundamental, form: uncertainty about the speed of learning. Specifically, in my model, investors are learning about the resolution probability of fundamentals. Investors do not know whether the uncertainty about fundamentals is going to be resolved quickly, like with the majority of earnings announcement events, or slowly like during the IT boom. As time goes by, if fundamentals are not fully revealed, investors begin to speculate more and more pushing prices higher and higher.

The second novel feature in this paper is the introduction of endogenous capital inflows, that are linked to optimal entry decisions by potential new investors. This increase in asset demand by new investors is driven by the same uncertainty that drives up asset prices and can greatly amplify the observed fluctuations.

In equilibrium, both uncertainty and entry determine asset prices and the magnitude of price movements. Two layers of uncertainty generate the hump shape in prices, and capital inflows can lead to dramatic price movements. At first, learning about how long the speculation can last dominates, there are gradual capital inflows, and asset prices are pushed up continuously—as long as the fundamental value is not fully revealed. Eventually however, as investors' beliefs gradually converge, asset prices fall back to their intrinsic values.

Capital inflows also cause trading frenzies. The influx of new investors bids up asset prices, so assets change hands from existing investors to newcomers and trading volume skyrockets.

After entry stops, since over time, new private information has a weaker and weaker effect on investors' beliefs, investors adjust their holding positions less and less frequently and thus trading volume falls gradually.

The model can produce various realistic bubble episodes. First, it is able to capture salient features of observed bubble events, like the declining economic and policy uncertainty and the prominent rise and fall of housing prices, the price-rent ratio, the number of first-time buyers and existing home sales during the recent U.S. housing bubble. Second, the model can generate bubble episodes of different shapes. By adjusting the relative rate of information flows, the equilibrium price can exhibit a slow build-up before collapse like during the IT boom, or a long-lasting downturn after a surge observed during the Japanese real estate bubble in the late 1980s. Moreover, in the model, in general, the market crashes if the fundamental value is fully revealed during the gradual learning process.

Related Literature. This paper contributes to the literature on bubbles. First, I provide a price formation mechanism for "bubbles". Understanding price formation is important because only after this can we discuss policy implications. Among various explanations for the existence of bubbles, speculative bubbles led by heterogenous beliefs and short-sale constraints are most closely related to this paper.[2] Those papers attribute heterogenous beliefs to differences of opinion in Harrison and Kreps (1978), to heterogenous priors in Morris (1996), or to overconfidence in Scheinkman and Xiong (2003). In this paper, it results from differential privation information and sustained by the presence of noise traders who prevent prices from fully revealing the fundamental value. Besides, some papers like Allen, Morris, and Postlewaite (1993) have shown that how bubbles can be supported in equilibrium but not how those prices are formed. In this paper, I use a noisy rational expectations framework to link the equilibrium price with investors' demand schedules. Allen, Morris, and Shin (2006) also discusses bubbles within this framework, but in their paper, bubbles come from the higher-order uncertainty, they occur only when investors' prior

[2]Examples of other explanations include rational bubbles by Tirole (1982, 1985), Blanchard and Watson (1982), and Santos and Woodford (1997), and churning bubbles by Allen and Gorton (1993).

mean is above the fundamental value and the equilibrium price is monotonically decreasing over time.

My paper also explains the whole price evolution process during bubble episodes. Previous literature focuses on bursts of bubbles or stock market crashes.[3] In Abreu and Brunnermeier (2003), the bubble grows exogenously, and investors are trying to ride the bubble and get out of it before it bursts. In this paper, instead, price processes are endogenous, and investors are active traders, trying to profit from price fluctuations instead of the price trend. Besides, recently, two papers have discussed price dynamics. Pástor and Veronesi (2009) explains the bubble-like stock price behavior. The differences from this paper are, prices there always reflect investors' perceived fundamental value, and it is specific to technological revolutions in which the risk associated with some new technologies gradually change from idiosyncratic to systematic as these technologies get widely adopted and this change depresses stock prices. The other paper, Burnside, Eichenbaum, and Rebelo (2013) considers a different mechanism—social dynamics, that is, agents with tighter prior are more likely to convert others to their beliefs. By contrast, in this paper, it is learning and endogenous capital inflows that drive price dynamics.

In addition, this paper also adds to the literature on trading frenzies. To explain this phenomenon, researchers have discussed various ways to generate strategic complementarities in speculators' information acquisition behavior. Examples include complementarities that result from short trading horizons by Froot, Scharfstein, and Stein (1992), the riskiness of positions by Hirshleifer, Subrahmanyam, and Titman (1994), fixed information acquisition costs by Veldkamp (2006a,b), the extra dimension of supply information by Ganguli and Yang (2009), relative wealth concerns by García and Strobl (2011), and the feedback effect from financial markets to the real investment decision by Goldstein, Ozdenoren, and Yuan (2013). By contrast, this paper describes trading frenzies as being from large cap-

[3]For example, Grossman (1988), Gennotte and Leland (1990), Romer (1993), Caplin and Leahy (1994), Avery and Zemsky (1998), Lee (1998), Zeira (1999), Abreu and Brunnermeier (2003), Barlevy and Veronesi (2003), Hong and Stein (2003) and Veldkamp (2006b).

ital inflows, and without strategic complementarities, it features a unique linear partially revealing equilibrium.

Moreover, this paper has several other contributions. First, to my knowledge, this is the first theoretical paper to emphasize the importance of capital flows to bubble episodes. Empirically, Singleton (2012) documents that investor flows had a significant impact on crude-oil futures prices. Theoretically, Merton (1987) shows that a larger size of the investor base can reduce the risk premium and increase the asset value, but it is not about bubbles or dynamics of capital flows. Second, in my model, the interaction between two layers of uncertainty leads to gradual capital inflows, which provides an alternative explanation for the slow-moving capital (see Duffie (2010)).

Technically, my model fits within noisy rational expectations models. The differential information feature of this model is similar to the tractable dynamic framework by He and Wang (1995).[4] Based on this, I explore another layer of uncertainty and capital flows.

Structure. The rest of the paper is organized as follows. I introduce the setup of the model in Section 2 and characterize equilibrium conditions in Section 3. In Section 4, I discuss price and trading dynamics. Section 5 shows that the model can produce various realistic bubble episodes, and explores several other extensions and implications. Section 6 concludes.

1.2 The Model

The model embeds uncertainty about the resolution probability and endogenous capital flows into the context of a dynamic noisy rational expectations model in which investors have differential private information and face a short-sale constraint, and highlights the importance of these new features to "bubble" dynamics. In the model, investors learn from prices, so different from differences of opinion literature, they do not agree to disagree.

[4] The seminal work on classical noisy rational expectations models is Grossman (1976), which was later extended by Hellwig (1980) and Diamond and Verrecchia (1981).

Next, I introduce the setup, discuss model assumptions and define the equilibrium.

1.2.1 Setup

Consider an economy in discrete time with an infinite horizon, $t = 0, 1, 2, \ldots$. It is populated by a continuum of infinitely-lived risk-neutral investors with a time discount rate r.

Investment Opportunities. There are two assets. One is riskless with infinitely elastic supply, yielding a return r. I take this asset to be the numeraire and normalize its price to one. The other is a risky asset with a stochastic supply due to noise traders to be described later. This risky asset pays out a stochastic dividend in each period,

$$D_t = r\Pi + \tilde{\epsilon}_{D,t}$$
$$\tilde{\epsilon}_{D,t} \sim N(0, r^2/\tilde{\rho}_D)$$

Since investors are risk neutral, they use the riskless interest rate r as the discount rate to price assets. This implies that Π is the fundamental value of the risky asset. If Π is known, asset prices are simply the fundamental value $P_t = \Pi$, and any deviation of prices from Π will be arbitraged away in equilibrium. As a result, risk-neutral investors are indifferent at investing in the riskless or risky asset when there is no uncertainty. In contrast, if Π is unknown, asset prices can deviate from their fundamentals.

For investors trading the risky asset, there are two restrictions on their strategies. One is a short-sale constraint, that is, selling assets that are not currently owned, and subsequently repurchasing them. This assumption creates an asymmetry between buying and selling. As a result, on average, equilibrium prices will be above the market average of investors' beliefs.[5] The other restriction is that, to prevent risk-neutral investors from taking unlimited

[5] All I need is the asymmetry created by costly short sales. Even if the cost is very small, with substantial uncertainty and large capital inflows, we can still have dramatic price movements. Below is some evidence for costly short sales in reality. For the real estate market, houses are rarely sold short. For the stock market, examples of costs include, according to the Federal Reserve Regulation T§220.12, the margin requirement for short sales is at least 100% of the current market value of the security; Securities and Exchange Commission [SEC] Rule 10a-1 imposes the up-tick rule, that is, the constraint is triggered when a security's price decreases by 10% or more from the previous day's closing price and is effective until the close of the next day; the

positions, I assume the maximum position they can hold is 1. Thus, for the risky asset, the position investors can take is within $[0, 1]$.

Information Structure. At the beginning of period 0, some news about the risky asset is released. As a result, there starts to have uncertainty about its fundamental value. Assume the uncertainty is resolved at $t = T$, where T can be large so that by then investors will have learned of the fundamental value almost exactly. Investors are assumed to have a common prior over the fundamental value, defined as

$$\Pi \sim N(\Pi_0, 1/\rho_0^c)$$

Now I focus on those T speculative periods. At the beginning of each period, public information is released and a dividend is paid out. Besides the price and dividend history, there can be some other pieces of public information, like media coverage. For simplicity, I incorporate those information into the dividend, and rewrite it as[6]

$$D_t = r\Pi + \epsilon_{D,t}$$
$$\epsilon_{D,t} \sim N(0, r^2/\rho_{D,t})$$

Assume the signal precision is stochastic. In each period, it can grow at a low rate η with probability $1 - \lambda$, or a high rate that I put to the extreme, ∞, which implies that the fundamental value is fully revealed immediately and the uncertainty is resolved, with

derivatives trading to mimic short sales is also costly as documented by Ofek and Richardson (2003).

[6]I have used the following result: given two noisy signals, $x_1 = \theta + \epsilon_1$, $x_2 = \theta + \epsilon_2$, where $\epsilon_1 \sim N(0, \sigma_1^2)$, $\epsilon_2 \sim N(0, \sigma_2^2)$ and $cov(\epsilon_1, \epsilon_2) = \rho\sigma_1\sigma_2$, they are as informative as a synthesized signal,

$$\begin{aligned} x_3 &= \mu x_1 + (1-\mu)x_2 = \theta + (\mu\epsilon_1 + (1-\mu)\epsilon_2) \\ &\sim N(\theta, \frac{(1-\rho^2)\sigma_1^2\sigma_2^2}{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2}) \end{aligned}$$

where $\mu = \frac{\sigma_2^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2}$.

probability λ. Specifically,

$$\rho_{D,t} = e^{\eta_t t} \rho_D$$

where

$$\eta_t = \begin{cases} \eta & \text{with probability } 1-\lambda \\ +\infty & \text{with probability } \lambda \end{cases}$$

The resolution probability λ measures the average learning speed or how long speculation can last. With a lower λ, it is less likely that the fundamental value will be fully revealed by dividends in each period, and thus investors expect the learning to be slower. The motivation behind this setup is, different events have different learning speed. Some are really fast like the majority of earnings announcement events, and some are very slow like IT boom.

Here, I introduce the second layer of uncertainty, uncertainty about this resolution probability λ or how long speculation can last. Investors start from a common prior over λ following $Beta(\beta, \gamma)$ with the prior mean $\beta/(\beta+\gamma)$. In each period, after observing η_t, they use Bayes' rule to update their belief.[7] Their perceived expected resolution probability λ_t is given by the following proposition:

Proposition 1.2.1. *If the uncertainty is not resolved till period m, then for $0 \leq t \leq m$,*

$$\lambda_t = \frac{\gamma}{\beta + \gamma + t + 1}$$

which is decreasing over time.

Proof. See Appendix A. □

The intuition is, as time goes by, if not much new information arrives, investors will become

[7] Here, I assume investors know η. It can also be, investors know that the uncertainty gets resolved almost sure if the realized dividend is the same as in the last period.

increasingly confident that uncertainty will not get resolved very soon.[8]

In the financial market, while some investors are trading the risky asset, others are not.[9] I call the latter potential investors. In each period, after receiving the public signal, potential investors make entry decisions based on the history of public information $\mathcal{F}_t^e = \{\mathcal{F}_0, P_\tau, D_\tau, \eta_\tau, D_t, \eta_t : 0 \leq \tau \leq t-1\}$. If they decide to enter and start to trade the risky asset, they need to pay an entry cost e. We can think it as a one-time information acquisition cost or the opportunity cost associated with the portfolio adjustment. Denote the set of entrants at period t as $I_{e,t}$. Right after entry, each new investor receives a private signal

$$\begin{aligned} \tilde{S}_t^i &= \Pi + \epsilon_{\tilde{S},t}^i \\ \epsilon_{\tilde{S},t}^i &\sim N(0, 1/\rho_{\tilde{S},t-1}) \end{aligned} \tag{1.1}$$

where $i \in I_{e,t}$ and $\rho_{\tilde{S},t-1}$ is the cumulative precision of private signals received by existing investors before period t. By this specification, the belief distribution across newcomers is the same as existing investors, so there is no need to keep track of cohorts.

Denote the set of rational investors trading the risky asset at period t as I_t. Assume in each period, each existing investor also receives a private signal, for $i \in I_t$,

$$\begin{aligned} S_t^i &= \Pi + \epsilon_{S,t}^i \\ \epsilon_{S,t}^i &\sim N(0, 1/\rho_{S,t}) \end{aligned}$$

After receiving private information, investors make trading decisions based on their own information set. By rational expectations, investors learn from their own net trades, or

[8] This intuition is also reflected in the conjugate prior I use. Two shape parameters β and γ represent the number of historical realizations η and ∞ respectively, so the prior mean of the resolution probability is $\gamma/(\beta+\gamma)$. In the next period, if uncertainty is not resolved, we add 1 to β and the posterior mean of the resolution probability decreases to $\gamma/(\beta+\gamma+1)$.

[9] One justification is that mutual funds have different investment objectives. According to Wiesenberger, Strategic Insight and Lipper Objective codes, mutual funds investing in the domestic equity can be classified by sector like technology or health stocks, by capitalization like large or small cap stocks, or by style like growth or income stocks.

from the current asset price because prices are informative. As a result, each investor submits their demand schedule which depends on the current price. After that, a Walrasian auctioneer like an electronic system sets the equilibrium price to clear the market.

The timeline for each period is summarized in Figure 1 below.

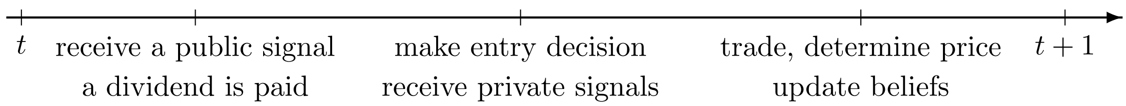

Figure 1.1: Timeline.

For any stochastic process $\{Z_t\}$, define $\underline{Z}_t \equiv \{Z_0, \dots, Z_t\}$ to be the history of Z_t up to and including t. Using this notation, I define the total public information available to all the investors to be

$$\mathcal{F}_t^c = \{\mathcal{F}_0, \underline{P}_t, \underline{D}_t, \underline{\eta}_t\}$$

Similarly, define the total information available to investor $i \in I_t$ to be

$$\mathcal{F}_t^i = \{\mathcal{F}_0, \underline{P}_t, \underline{D}_t, \underline{\eta}_t, \underline{S}_t^i\}$$

where $\underline{S}_t^i = \{\tilde{S}^i_{t_e^i}, S^i_{t_e^i}, \dots, S^i_t\}$ and t_e^i is investor i's entry time.[10]

Assume the structure of the economy is common knowledge. Thus, investors are only asymmetrically informed about the fundamental value of the risky asset.

Investors' Problem. Investors behave competitively. Given the initial wealth and asset prices, investors make consumption, investment, and also entry decisions if they are potential investors, to maximize their discounted lifetime utility from consumption. For potential

[10]For investors trading the risky asset before the announcement of news, by default, their entry time is period 0.

investors, they have the following problem:

$$\max_{c,x,t_e^i} E_0[\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} c_t^i | \mathcal{F}_0] \tag{1.2}$$

s.t.

$$W_t^i = W_{t-1}^i(1+r) - c_t^i - 1_{\{t=t_e^i\}} e \text{ for } 0 \leq t \leq t_e^i$$

$$W_t^i = W_{t-1}^i(1+r) + x_{t-1}^i(P_t + D_t - P_{t-1}(1+r)) - c_t^i \text{ for } t > t_e^i$$

$$x_t^i \in [0,1], \; 0 \leq t_e^i \leq T-1$$

$$W_0^i \text{ is given}, \; \text{non-Ponzi condition}$$

where W_t^i, c_t^i and x_t^i are investor i's wealth, consumption and position on the risky asset at period t respectively, conditional on the history which for simplicity I do not write out. After the entry, investors' return from investment consists of two parts, the riskless interest rate and the excess return from trading the risky asset. In addition, I impose a non-Ponzi condition to preclude the possibility that investors keep rolling over their debt forever.

For the entry decision, since it is based only on the public information, all the potential investors will make the same decision. Thus, from now on, I omit its superscript i. Besides, I restrict the entry decision to be made during those T speculative periods, because trading the risky asset makes a difference only when there is uncertainty about its fundamental value. Moreover, investors can always be inactive if they do not perceive any profitable trading opportunities, so given the entry cost, entry is a one-time decision for them.

Some investors are initially randomly endowed with the risky asset. They have the same problem except that they do not need to make entry decisions. For them, $t_e = 0$.

Capital Flows. Capital flows in this paper are defined as the measure of new rational investors trading the risky asset in each period. Although by this definition it is more accurate to call them investor flows, I choose capital flows because the intuition holds in a general sense.

The total measure of investors trading the risky asset $\tilde{n}_t$, consists of two parts, rational investors with measure n_t and noise traders with measure $\epsilon_{n,t}$. As the standard noise trading story, the asset supply available in the market is $1 - \epsilon_{n,t}$ with $\epsilon_{n,t}$ held by noise traders. $\epsilon_{n,t}$ and thus the asset supply are stochastic and unobservable.

The specification for noise traders is as follows. For tractability, instead of imposing a stochastic structure directly on $\epsilon_{n,t}$, I introduce a new variable $\tilde{q}_t$ and assume

$$\begin{aligned} \tilde{q}_t &= q_t + \epsilon_{q,t} \\ \epsilon_{q,t} &\sim N(0, 1/\rho_q) \end{aligned} \tag{1.3}$$

where

$$\tilde{q}_t = \Phi^{-1}(\frac{\tilde{n}_t - 1}{n_t}),\ q_t = \Phi^{-1}(1 - \frac{1}{n_t})$$

Here, n_t is strictly increasing in q_t, so throughout the paper, I will use n_t and q_t interchangeably. In fact, $\tilde{q}_t$ measures how many units of standard deviation the equilibrium price is above the market average of investors' beliefs. The intuition for this specification is explained in Appendix B.

Assume there are unlimited potential investors. This and the assumption that risk-neutral investors have access to the same information before their entry indicate that the measure of rational investors trading the risky asset should satisfy the free-entry condition, that is, in each period investors keep entering till entry is no longer profitable. Denote investors' expected discounted lifetime utility by following an entry rule $\tilde{t}$ as $\tilde{V}(\tilde{t})$, given asset prices $\{P_t\}$ and the measure of rational investors trading the risky asset $\{n_t\}$. Thus, the optimal entry decision is the optimal stopping time maximizing $\tilde{V}(\tilde{t})$, and I denote the set of optimal entry decisions as T_e. Additionally, denote investors' maximized expected discounted lifetime utility from only trading the riskless asset as $\tilde{V}_0$. Then the free-entry condition is given by

$$\tilde{V}(t_e) = \tilde{V}_0,\ \forall t_e \in T_e \tag{1.4}$$

Moreover, in equilibrium, we also need a consistency condition, that is, whenever there are capital inflows, investors must be willing to enter at that point. Mathematically, this requires that, given the probability space $(\Omega, \mathcal{F}^e_{T-1}, P)$ in which given the relevant state variables $(\Pi, \underline{\epsilon}_{D,t}, \underline{\epsilon}_{q,t}, \underline{\eta}_t)$, $\Omega = \mathbb{R}^{2T+1} \times \Lambda^T$ with $\Lambda = \{\eta, +\infty\}$, for $\omega \in \Omega$,

$$\{t : n_t(w) > n_{t-1}(w), 0 \le t \le T-1\} \in T_e(w)\ (a.s.) \tag{1.5}$$

Here it is abuse of notation but for simplicity, $T_e(w) \equiv \{t_e(w)\}$, i.e., $T_e(w)$ is the set of the optimal entry time $t_e(w)$ for $w \in \Omega$.

Discussion. The discussion below is on modeling strategies. First, the fundamental value Π is assumed to be constant, but it is easy to introduce time-varying fundamentals. One such example is Wang (1993) which considers a time-varying growth rate of dividends. Besides, it is also straightforward to extend the model to study independent multi-asset markets.

Second, I synthesize all the public information into dividends. The only change resulting from this simplification is that as fundamentals, dividends are less volatile and realized dividends are affected by other public signals. However, this change does not affect investors' trading decision and the equilibrium asset price. This is because, since investors are risk neutral, when making trading decisions, they only consider expected future dividends.

Third, the specification for the precision of public signals is parsimonious, but rich enough to include some standard cases discussed in the literature. Specifically, $\eta_t = 0$ corresponds to the constant flow of information, $\eta_t = -\infty$ implies the concentrated flow of information, and $\eta_t = \eta > 0$ captures the speedup in information flows, which can be caused by the increasing media coverage over time and also more time to revise previous reports.

Fourth, notice that n_t is not included in the public information set, because as we will see later, it contains no extra information.

Fifth, I assume that the precision of private signals received by new investors equals the cumulative precision of historical private signals received by existing investors. Under this assumption, among newcomers, some are optimistic and some are pessimistic. What I need for capital inflows is at every period there is an inflow of optimistic investors which will put more upward pressure on the asset price. In light of this, there is no loss in intuition from the specification I use for $\rho_{\tilde{S},t}$. Moreover, I choose this specific form for tractability. On one hand, there is no need to keep track of cohorts entering at different periods; on the other hand, as we will see later, this set-up can avoid the extreme complexity caused by investors' differential beliefs about future capital flows.

At last, the precision of the private signal received by each existing investor can be time varying. This includes two standard cases in the literature, a concentrated flow of information, $\rho_{S,0} > 0$ and $\rho_{S,t} = 0$ for $t > 0$, and a constant flow of information, $\rho_{S,t} = \rho_S > 0$ for all $0 \leq t \leq T-1$.

1.2.2 Equilibrium

Notation. Denote the expectation on the fundamentals based on the public information and each existing investor's own information set $\mathcal{F}_t^i$, $i \in I_t$ as

$$E_t^c[\Pi] = E[\Pi|\mathcal{F}_t^c], \ E_t^i[\Pi] = E[\Pi|\mathcal{F}_t^i]$$

Denote each existing investor's own expectation on the next-period price as

$$E_t^i[P_{t+1} + D_{t+1}] = E[P_{t+1} + D_{t+1}|\mathcal{F}_t^i]$$

and the market average of their expectation as

$$\bar{E}_t[P_{t+1} + D_{t+1}] = \int_{i \in I_t} E_t^i[P_{t+1} + D_{t+1}]di$$

Besides, denote the belief precision based on the public information and each existing investor's own information set as

$$\rho_t^c = \frac{1}{var(\Pi|\mathcal{F}_t^c)}, \ \rho_t = \frac{1}{var(\Pi|\mathcal{F}_t^i)}$$

where $i \in I_t$. By symmetry, $var(\Pi|\mathcal{F}_t^i)$ should be the same for all the existing investors, so I omit the superscript i.

In addition, due to differential private information, investors have different forecasts on future prices. Thus, denote the standard deviation of the forecast mean across investors as

$$\sigma_t(P_{t+1} + D_{t+1}) = \sigma(E_t^i[P_{t+1} + D_{t+1}])$$

At last, let I be the set of all the investors.

Equilibrium. State variables in this economy are $(\Pi, \underline{\epsilon}_{D,t}, \underline{\epsilon}_{q,t}, \underline{\eta}_t, (\underline{\epsilon}_{S,t}^i)_{i \in I_t}, (\underline{\epsilon}_{\tilde{S},t}^i)_{i \in I_{e,t}})$. Since the noise in investors' private signals will be smoothed out in the aggregation, prices only depend on $\mathcal{F}_t^P = \{\Pi, \underline{\epsilon}_{D,t}, \underline{\epsilon}_{q,t}, \underline{\eta}_t\}$.[11] For simplicity, assume all the shocks to the economy are independent of each other.

We can define a rational expectations equilibrium as follows:

Definition A rational expectations equilibrium consists of asset prices $\{P_t\}$, the measure of rational investors trading the risky asset $\{n_t\}$, investors' consumption decision $\{c_t^i\}$ and trading strategy $\{x_t^i\}$ adapted to $\{\mathcal{F}_t^i\}$ for $i \in I$, potential investors' entry decision t_e which

[11] Technical problems exist for the validity of the law of large numbers with a continuum of private signals (see Judd (1985)). Several papers have discussed possible remedies (examples include Feldman and Gilles (1985), Bewley (1986), Uhlig (1996) and Al-Najjar (2004)). Here following Feldman and Gilles (1985), we can relax the independence assumption of private signals across investors to get no aggregate uncertainty.

is an optimal stopping time of $\{\mathcal{F}_t^e\}$, and investors' beliefs about the fundamental value and the resolution probability, $\{E_t^c[\Pi], \rho_t^c, \lambda_t, (E_t^i[\Pi], \rho_t)_{i \in I_t}\}$, s.t.

- Given $\{P_t\}$ and $\{n_t\}$, investors' choice $(\{c_t^i, x_t^i\}, t_e)_{i \in I}$ is the solution to their utility maximization problem (2.1).

- Given $\{P_t\}$ and $\{n_t\}$, investors' beliefs $\{E_t^c[\Pi], \rho_t^c, \lambda_t, (E_t^i[\Pi], \rho_t)_{i \in I_t}\}$ are updated according to Bayesian rules.

- The measure of existing rational investors $\{n_t\}$ satisfies the free-entry condition (2.2) and the consistency condition (2.3).

- The asset market clears:[12]

$$\int_{i \in I_t} x_t^i di = 1 - \epsilon_{n,t}$$

1.3 Characterization of the Equilibrium

Since investors are risk neutral, they make consumption and investment decisions separately and their objective function is equivalent to maximizing the present value of their wealth at the end of the speculative period. This implies a simple optimal trading strategy, that is, they are willing to hold the risky asset as long as its expected excess return is positive.

Proposition 1.3.1. *The optimal trading strategy is given by, for $t_e \leq t \leq T-1$,*

$$x_t^i = \begin{cases} 1 & \text{if } E_t^i[P_{t+1} + D_{t+1}] \geq (1+r)P_t \\ 0 & o.t.w. \end{cases}$$

Proof. See Appendix A. □

This optimal trading strategy can be implemented by limit orders. The current price is

[12] Throughout this paper, I always mean almost surely or almost everywhere.

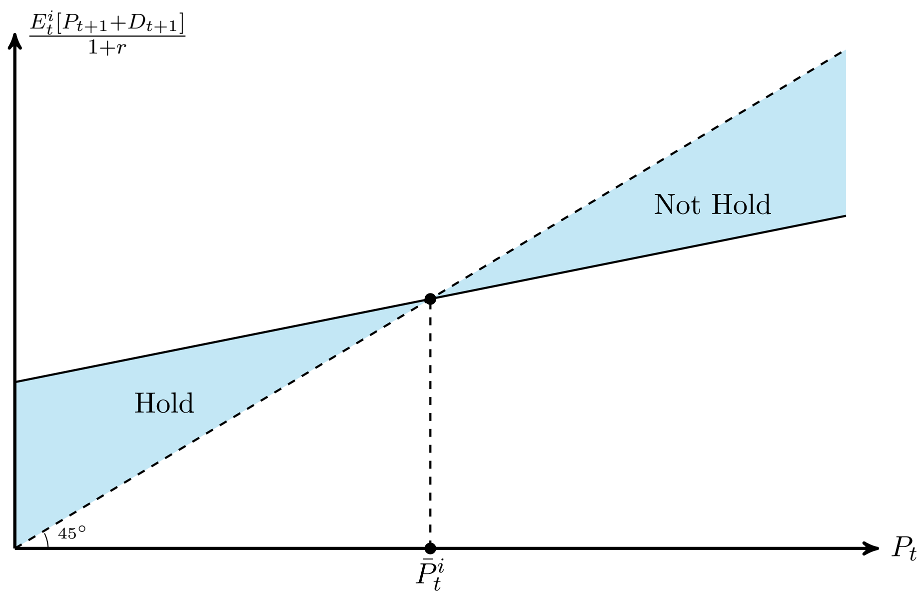

Figure 1.2: Demonstration of the optimal trading strategy. The intersection between the price target $E_t^i[P_{t+1} + D_{t+1}]/(1 + r)$ shown by the solid line and the 45-degree line defines a price threshold $\bar{P}_t^i$. Investors are willing to hold the asset iff $P_t \leq \bar{P}_t^i$, which is a limit order.

informative but also noisy, so conditional on the current price, investors only partially adjust their price target $E_t^i[P_{t+1} + D_{t+1}]/(1 + r)$. This implies the existence of a price threshold. Investors are willing to hold the asset if and only if the current price is below this price threshold. Specifically, if investors hold one unit of asset, they want to sell it whenever the current price is above the price threshold, and vice versa. This is exactly a limit order. The intuition for this result is demonstrated in Figure 2.2.

In this paper, I focus on the linear partially revealing rational expectations equilibrium. In my model, there is a fully revealing equilibrium in which the equilibrium price equals the fundamental value and investors are indifferent between buying and selling. However, it is hard to justify how the information is incorporated into the price when investors' demand schedules contain no information. In light of this, I study the partially revealing equilibrium. Besides, for tractability, I restrict to the linear equilibrium in which prices are linear in state variables and q_t which is equivalent to the measure of rational investors trading the risky asset. In the linear equilibrium, first-order expectations are enough to solve the complexity of the higher-order uncertainty from the beauty-contest metaphor in Keynes (1936).

The equilibrium asset price is determined by supply and demand. It is characterized by the following proposition.

Proposition 1.3.2. *In a linear partially revealing rational expectations equilibrium, at period t, if the uncertainty is resolved, $P_t = \Pi$, otherwise,*

(1) the equilibrium price has the following form:

$$P_t = (1 - p_{\Pi,t})E_t^c[\Pi] + p_{\Pi,t}\Pi + \sum_{m=t}^{T-1} p_{q,tm}q_m + p_{\epsilon,t}\epsilon_{q,t} \tag{1.6}$$

(2) the belief about the fundamental value based on the public information is updated by

$$E_t^c[\Pi] = \frac{\rho_{t-1}^c}{\rho_t^c}E_{t-1}^c[\Pi] + \frac{\rho_{D,t}}{\rho_t^c}\frac{1}{r}D_t + \frac{\rho_q/\mu_t^2}{\rho_t^c}\xi_t$$

where $\xi_t = \Pi + \mu_t\epsilon_{q,t}$ with $\mu_t = p_{\epsilon,t}/p_{\Pi,t}$, and

$$\rho_t^c = \rho_{t-1}^c + \rho_{D,t} + \frac{\rho_q}{\mu_t^2}$$

each existing investor's belief about the fundamental value based on their own information set is updated according to

$$E_t^i[\Pi] = (1 - \alpha_t)E_t^c[\Pi] + \alpha_t\frac{1}{t+1}\sum_{m=0}^{t} S_m^i \tag{1.7}$$

with

$$\alpha_t = \frac{\rho_{\tilde{S},t}}{\rho_t^c + \rho_{\tilde{S},t}}$$

and

$$\rho_t = \rho_{t-1} + \rho_{D,t} + \frac{\rho_q}{\mu_t^2} + \rho_{S,t}$$

Proof. See Appendix A. □

The equilibrium price function consists of three components. The first is the fundamental belief component $(1 - p_{\Pi,t})E_t^c[\Pi] + p_{\Pi,t}\Pi$. It is a weighted average of two parts—the belief based on the public information and the fundamental value reflecting the aggregate private information. The last component is a noise term $p_{\epsilon,t}\epsilon_{q,t}$ led by noise traders and it prevents prices from fully revealing the fundamental value.

The second component $\sum_{m=t}^{T-1} p_{q,tm}q_m$ is driven by the supply and demand force, and it is the key to non-fundamental price movements in this model. With short-sale constraints, asset prices reflect the marginal investor' forecast about future marginal investors' price forecasts. In general, marginal investors are optimistic. Thus, prices are above the market average of investors' beliefs about the fundamental value, and their distance is captured by this second component in the price function.

As will be shown later, in equilibrium, it is common knowledge that investors have the same forecast on rational capital flows. For now, let us take it as given and discuss the price formation mechanism.

Investors hold different beliefs about the fundamental value. This heterogeneity is caused by differential private information. Investors with a history of good private signals are optimistic among all the investors, and vice versa. Moreover, the heterogeneity is sustained by the presence of noise traders which prevents prices from fully revealing the fundamental value and thus preserves asymmetric information across investors. In each period, investors try to extract the information about the fundamental value from all the noisy information they have. For instance, taking the price function (2.4) as given, we have the information contained in the price to be

$$\xi_t = \Pi + \mu_t \epsilon_{q,t}$$

with

$$\mu_t = p_{\epsilon,t}/p_{\Pi,t}$$

If the current price is higher than expected, investors will ascribe this difference partly to the possibility that the fundamental value is higher than their expectation, and partly to the high demand from too much noise trading. If the price is very informative about the fundamental value, investors will give a high credit to the asset value.

With heterogenous beliefs about the fundamental value, investors differ in the price they are willing to pay for the risky asset and they submit different limit orders. Optimistic investors have high price thresholds for their limit orders relative to pessimistic investors. This generates a downward-sloping demand curve, and the equilibrium price is set such that the asset demand equals the supply of that period.

The equilibrium price reflects the marginal investor's price target. The marginal investor is the least optimistic investor holding the asset currently. Since private signals are assumed to follow a normal distribution, from the equilibrium price function (2.4), across investors, their price target $E_t^i[P_{t+1}+D_{t+1}]/(1+r)$ also follows a normal distribution $N(\bar{E}_t(P_{t+1}+D_{t+1})/(1+r), \sigma_t^2(P_{t+1}+D_{t+1})/(1+r)^2)$. By the market clearing condition,

$$n_t(1-\Phi(\frac{P_t-\bar{E}_t[P_{t+1}+D_{t+1}]/(1+r)}{\sigma_t(P_{t+1}+D_{t+1})/(1+r)}))=1-\epsilon_{n,t}$$

using the specification for noise traders (2.5), we have the equilibrium price to be determined by the marginal investor's price target,

$$P_t=\frac{\bar{E}_t[P_{t+1}+D_{t+1}]}{1+r}+\frac{\sigma_t(P_{t+1}+D_{t+1})}{1+r}(q_t+\epsilon_{q,t})$$

which on average is above the market average of investors' price targets $\bar{E}_t[P_{t+1}+D_{t+1}]/(1+r)$. The intuition for the price determination is illustrated in Figure 1.3. In a dynamic context, prices in general reflect current optimistic investors's beliefs about future prices which further reflect future optimistic investors' beliefs on even future prices, and so on... This is where the non-fundamental price components or "bubbles" come from.

The equilibrium price depends on how many rational investors are trading the risky asset,

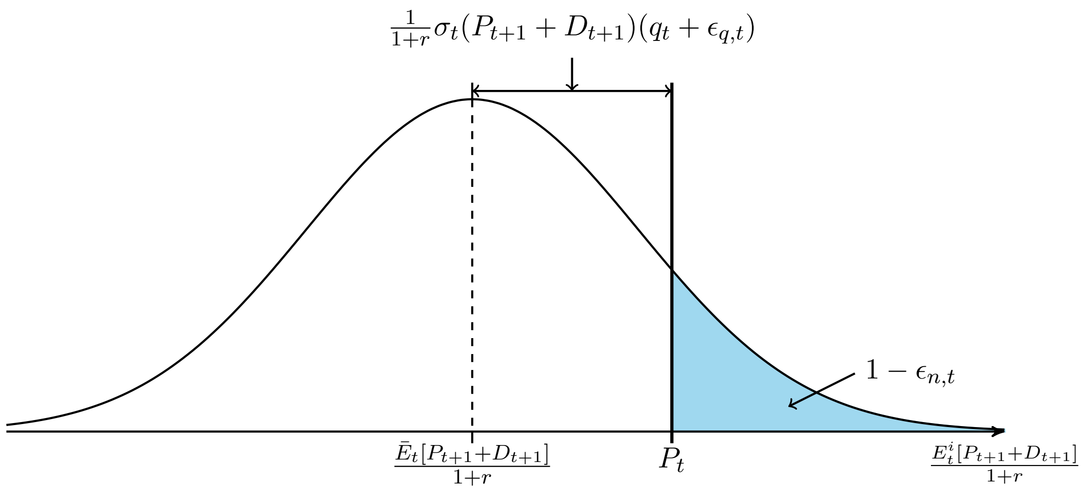

Figure 1.3: Demonstration of the equilibrium price. Given the asset supply $1-\epsilon_{n,t}$, with short-sale constraints, the equilibrium price is above investors' average price target by $q_t+\epsilon_{q,t}$ units of investors' belief dispersion.

which is determined by the free-entry condition and characterized in the proposition below:

Proposition 1.3.3. *At period t, the equilibrium measure of rational investors trading the risky asset n_t or equivalently q_t satisfies:*

(1) It is nondecreasing over time.

(2) If there are capital inflows,

$$V_t \geq e \tag{1.8}$$

where

$$V_t = \sum_{m=t}^{T-1}(\frac{1}{1+r})^{m-t}(\Pi_{j=t}^{m-1}(1-\lambda_j))v_m$$

is the discounted expected excess returns if entering at period t.

(3) If there are capital inflows at both period t and $t+1$,

$$v_t = (1-\frac{1-\lambda_t}{1+r})e \tag{1.9}$$

where v_t is the expected excess return from trading the risky asset at period t if the *uncertainty is not resolved, conditional on* $\mathcal{F}_m^e$, $\forall 0 \leq m \leq t$, *and it is given by*

$$v_t = \sigma_t(P_{t+1} + D_{t+1})(\sqrt{1 + \frac{1}{\rho_q}}\phi(-\frac{q_t}{\sqrt{1 + \frac{1}{\rho_q}}}) - q_t\Phi(-\frac{q_t}{\sqrt{1 + \frac{1}{\rho_q}}}))$$

with $\sigma_t(P_{t+1} + D_{t+1}) = (1 + r)p_{\Pi,t}\sqrt{\frac{1}{\rho_{\tilde{S},t}}}$, *and* $\phi(\cdot)$ *and* $\Phi(\cdot)$ *are the PDF and CDF of the standard normal distribution respectively.*

(4) If $1 - (1 - \lambda_t)/(1 + r)$ *is decreasing faster than* $\sigma_t(P_{t+1} + D_{t+1})$ *over time, we have gradual inflows of investors till* $V_t < e$.

Proof. See Appendix A. □

The measure of rational investors trading the risky asset is nondecreasing over time. This is because, investors can always choose not to hold any risky assets if they do not perceive any profitable opportunities, and given the costly entry, those investors have no incentive to exit. In Section 5, I discuss the possibility of exit and its implications.

It is common knowledge that investors have the same perfect foresight about future capital flows and capital flows in each period are deterministic. Given the equilibrium price function, the expected excess return is given by

$$E_t^i[P_{t+1} + D_{t+1}] - (1 + r)P_t = (1 + r)p_{\Pi,t}\frac{1}{t + 1}\sum_{m=0}^{t}\epsilon_{S,m}^i - \sigma_t(P_{t+1} + D_{t+1})(q_t + \epsilon_{q,t})$$

which depends on the noise in private signals and also the measure of noise traders. Since potential investors have access to only public information before entry, so their ex-ante expected excess returns in each period are: first, identical across potential investors; second, unrelated with the fundamental value, so private information does not help existing investors forecast capital flows, and all the investors, including existing and potential investors, have the same perfect forecast; third, independent with the history of public information till that period, so capital flows in each period are deterministic.

Equilibrium capital flows satisfy condition (1.8) and (1.9). First, in each period, if potential investors are willing to enter, their discounted expected excess returns from trading the risky asset after entry must be enough to cover entry cost, i.e., $V_t \geq e$. Second, potential investors also need to decide when to enter, now or next period. With capital inflows at two consecutive periods, potential investors must be indifferent, that is, the benefit from entering now which is the flow payoff from speculation, should equal the benefit from waiting which is the entry cost that can be saved if uncertainty gets resolved in the next period. Here, the flow payoff from speculation is increasing in investors' belief dispersion. This is because with a larger belief dispersion, asset prices are more volatile and thus expected speculation profits are higher. Besides, this flow payoff is decreasing in the current measure of rational investors trading the risky asset. The reason is that, with more investors, asset prices will be bid up higher, and given future asset prices, this squeezes out speculation profits.

Two layers of uncertainty determine equilibrium capital flows. This is shown by the indifference condition (1.9). Mathematically, if $1 - (1 - \lambda_t)/(1 + r)$ which captures investors' expected resolution probability for the next period, is decreasing faster than investors' belief dispersion $\sigma_t(P_{t+1} + D_{t+1})$, q_t will increase which means capital inflows. Intuitively, equilibrium capital flows depend on the relative speed of learning about two layers of uncertainty—the fundamental value and the resolution probability. On one hand, if the fundamental value is not fully revealed over time, investors will become increasingly confident that uncertainty will not get resolved very soon, speculation will last long, and total expected speculation profits will be higher. This creates an incentive for entry. On the other hand, as learning proceeds, investors' beliefs gradually converge. This reduces speculation profits, so investors have less incentive to enter. These two forces work in opposite ways and equilibrium capital flows depend on which force is stronger. If learning about the resolution probability is faster, there are inflows of investors; otherwise, we have no capital flows.

Without strategic complementarities, given exogenous information flows, we have the following result for the equilibrium existence and uniqueness:

Proposition 1.3.4. *There exists one and only one linear partially revealing rational expectations equilibrium in this economy.*

Proof. See Appendix A. □

The basic idea for this result is as follows. First, exogenous information flows uniquely determine the coefficients in the equilibrium price function. Second, given those price coefficients, since two layers of uncertainty have opposite effects on entry decisions, they uniquely determine equilibrium capital flows. At last, equilibrium capital flows and price coefficients together determine the unique equilibrium price function.

1.4 Price and Trading Dynamics

1.4.1 Capital flows

Proposition 1.4.1. *Under certain regularity conditions (A1)-(A3) stated in Appendix A.6, we have gradual inflows of investors first and they stop till any of the conditions* (1.8) *and* (1.9) *is violated.*

Proof. See Appendix A. □

From now on, all the results are discussed under those regularity conditions. As explained before, capital flows depend on expected speculation profits which are determined by the relative speed of two layers of learning. Those regularity conditions guarantee that learning about the resolution probability is faster than the convergence of investors' beliefs initially and slower later. Thus, at the beginning, period after period, investors expect a longer speculation process and higher expected speculation profits, and this brings gradual capital inflows; however at the same time, investors are learning about the fundamental value which leads to declining profits eventually, so capital inflows stop after some periods.

Table 1 lists the value of baseline parameters, which will be used for all the numerical results unless otherwise specified. Here I consider a case in which public information comes faster and faster as suggested by $\eta > 0$ and private information arrives in constant flows, $\rho_{S,t} = \rho_S$.[13] This is only for numerical illustrations, as all the analytical results hold in general cases. A numerical example for capital flows is shown in Figure 1.4.

Table 1.1: Baseline Parameter Values.

Assigned parameters		Value
precision of the prior	ρ_0^c	35
precision of private signals	ρ_S	0.05
precision of noise traders	ρ_q	0.05
precision of dividends	ρ_D	1.6×10^{-4}
growth rate of public information	η	0.2
mean of the prior about the fundamental value	Π_0	0.4
fundamental value	Π	0.4
prior parameter I of the resolution probability	β	1
prior parameter II of the resolution probability	γ	9
entry cost	e	0.01
riskless interest rate	r	0.004
number of speculative periods	T	75

1.4.2 Price dynamics

Denote the average observed price path if the fundamental value is not fully revealed till period T as $\{\bar{P}_t\}_{t=0}^T$. Given the equilibrium price function, price movements in $\{\bar{P}_t\}_{t=0}^T$ are described by the proposition below:

[13] There is one issue with constant flows of private information. Under the common prior assumption, the weight on the private information and thus investors' belief dispersion increase first. As a result, prices go up initially with only uncertainty about the fundamental value. However, this will not happen for other situations like concentrated flows of private information.

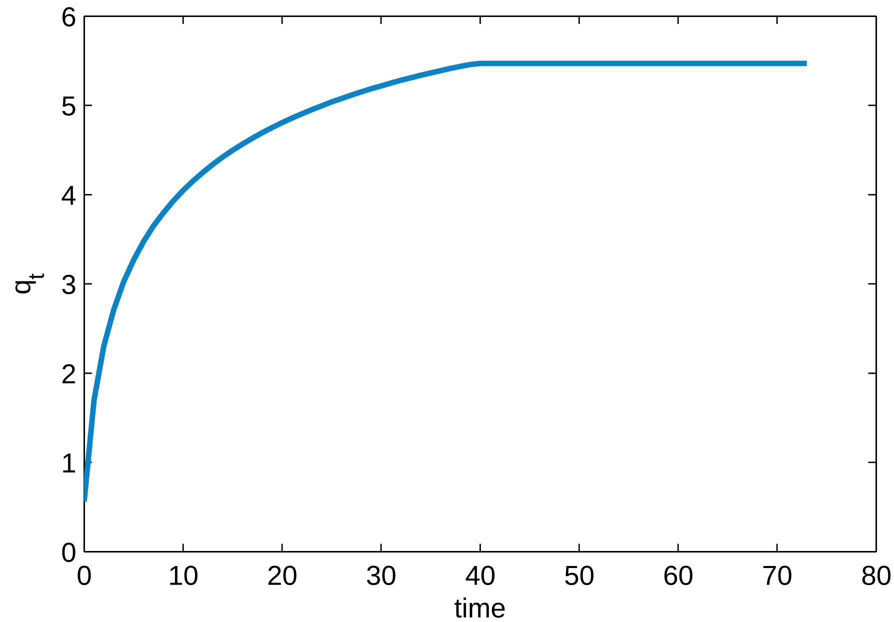

Figure 1.4: The measure of rational investors trading the risky asset indicated by $\{q_t\}$.

Proposition 1.4.2. *If the uncertainty is not resolved at $t+1$, the price movement in the average observed price path is*

$$
\begin{aligned}
\triangle \bar{P}_t &= \bar{P}_{t+1} - \bar{P}_t \\
&= \frac{1}{1+r}(\underbrace{\sum_{m=t+1}^{T-1}(1-\frac{1-\lambda_t}{1+r})(\prod_{j=t+2}^{m}\frac{1-\lambda_j}{1+r})\sigma_m(P_{m+1}+D_{m+1})q_m}_{\triangle n_{1,t}} \underbrace{-\,\sigma_t(P_{t+1}+D_{t+1})q_t)}_{\triangle n_{2,t}}
\end{aligned}
$$

(1) $\triangle n_{1,t} > 0$, is caused by unexpected capital flows; $\triangle n_{2,t} < 0$, is by short-sale constraints.

(2) There exists $\tilde{t}$ s.t. $0 \leq \tilde{t} < T-1$, $\triangle P_t \geq 0$ for $0 \leq t < \tilde{t}$ and $\triangle P_t < 0$ for $\tilde{t} \leq t \leq T-1$.

Proof. See Appendix A. □

The average observed price movement is driven by $\triangle n_{1,t}$ and $\triangle n_{2,t}$. Positive $\triangle n_{1,t}$ is caused by unexpected capital flows. To see this, notice that in each period, with some probability, uncertainty will be resolved and the price will not depend on capital flows any more. Thus, when making trading decisions, investors only take expected future capital flows into account. In the next period, if uncertainty is not resolved, capital flows will be higher than expected. This difference measures unexpected capital flows at period t, given by $\triangle n_{1,t}$, which pushes up the price as long as uncertainty is not resolved. In contrast with this positive $\triangle n_{1,t}$, $\triangle n_{2,t}$ is negative, due to short-sale constraints. In the last section, we have shown that with short-sale constraints and differential private information, the price reflects current optimistic investors' beliefs about future optimistic investors' beliefs, so on average, the market average of investors' price forecasts for the next period is below the current price, and this difference is captured by $\triangle n_{2,t}$.

Two layers of uncertainty and capital inflows drive non-fundamental price movements through these two terms. Uncertainty about the fundamental value is related with the negative term $\triangle n_{2,t}$. As learning proceeds, investors' beliefs gradually converge, and thus eventually, the price goes back to the fundamental value. In contrast, uncertainty about the resolution probability is associated with the positive term $\triangle n_{1,t}$. Initially, expected capital inflows are very small because investors are worried that uncertainty will get resolved very soon. As a result, the asset price is only slightly above the fundamental value. After one period, uncertainty is not resolved, so the price goes up because of large unexpected capital inflows. In this way, as speculation continues period after period, unexpected capital inflows push up the asset price continuously. Meanwhile, since investors become increasingly confident that the resolution probability is low, the size of unexpected capital inflows decreases over time. Thus after some point, this positive force $\triangle n_{1,t}$ is dominated by the negative force $\triangle n_{2,t}$, and the price falls. On top of these two layers of uncertainty, capital inflows amplify price movements, since price movements in the above proposition are proportional to $\{q_t\}_{t=0}^{T-1}$. The intuition is, with capital inflows, more investors are trading the risky asset, and they push the asset price further above the fundamental value.

Figure 1.5 illustrates the result for price dynamics. The red horizontal line is the funda-

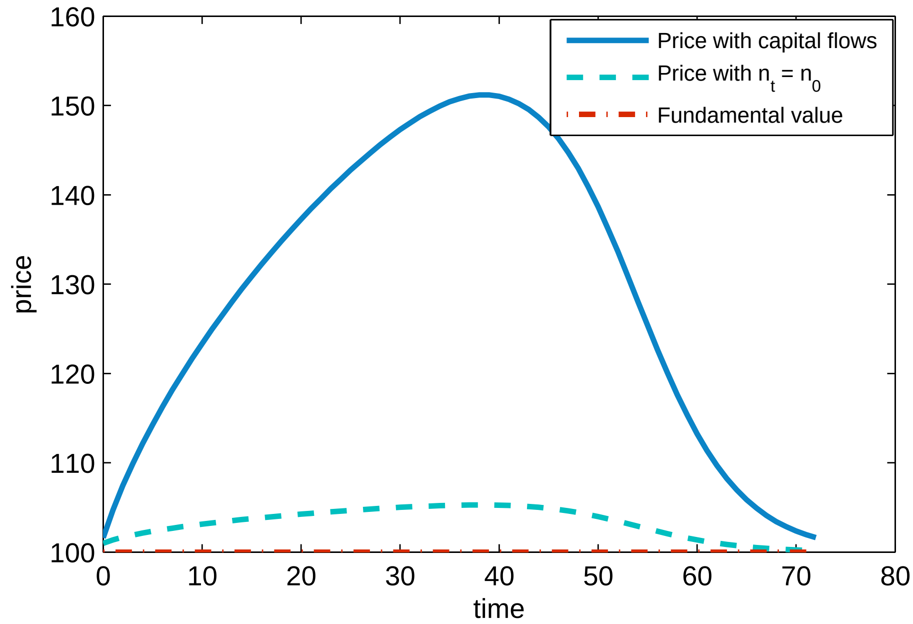

Figure 1.5: The average observed price path. Normalized by the fundamental value.

mental value. Two layers of uncertainty lead to a slightly hump-shaped price pattern, as displayed by the dashed line. On top of this, with capital inflows, price movements are dramatic, which look like a realistic bubble episode.

While the average observed price path is smooth, realized price paths are fluctuating, as shown in the top panel of Figure 1.6. I simulate 1000 price paths and plot the top and bottom one percentile of price realizations in each period. From the figure, we can see that among realized price paths, this rise-and-fall pattern is in general preserved.

Investors are active traders even when the price trend is going down. The average observed price path is given by

$$\bar{P}_t = \Pi_0 + \sum_{m=t}^{T-1} p_{q,tm} q_m$$

This indicates that on average, asset prices are above the fundamental value by $\sum_{m=t}^{T-1} p_{q,tm} q_m$ and it is common knowledge. Thus, all the investors understand this rise-and-fall price pattern. However, investors are still actively trading even when the price trend is going down,

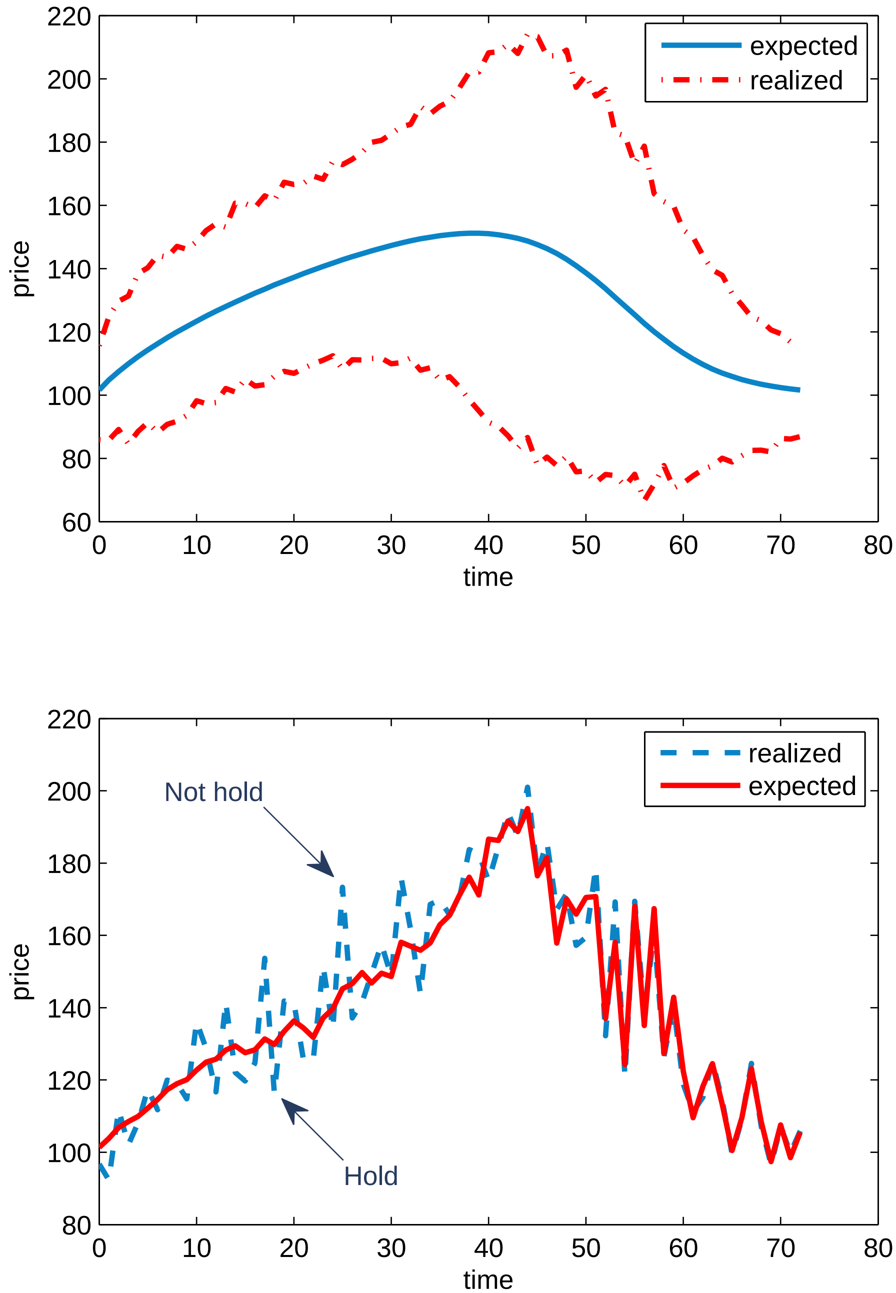

Figure 1.6: Simulated price paths. Normalized by the fundamental value. The top panel shows the average, as well as the top and bottom 1% realizations of simulated 1000 price paths. The bottom panel shows one particular realized price path and the corresponding expected price $E_t^i[P_{t+1} + D_{t+1}]/(1 + r)$ of investors with private signals $\epsilon_{S,t}^i = 0$, $0 \leq t \leq T - 1$.

because instead of the price trend, by making use of their private information, they are trying to profit from price fluctuations caused by noise traders.

One example of the trading process is illustrated in the bottom panel of Figure 1.6, in which I show one realized price path and the corresponding price target $E_t^i[P_{t+1} + D_{t+1}]/(1+r)$ from investors with private signals $\epsilon_{S,t}^i = 0$ for all $0 \leq t \leq T-1$. If the realized price is higher than their price target, investors believe that it is because of the high demand from too many noise traders today, and in the next period, the price will go down. Thus, they choose not to hold the asset, that is, if they already have one share, they sell it, and if not, they do not buy. By the same logic, if the realized price is lower than their price target, they choose to hold. By following this buy-at-low and sell-at-high short-term trading strategy, investors can profit from price fluctuations, and this is why they still trade during the downturn.

Moreover, in Figure 1.6, we can see that speculation profits diminish as investors' beliefs converge over time. Also in this example, the conditional volatility of prices is higher during the downturn, because with an increasing $\rho_{D,t}$, prices become very sensitive to public news.

Comparative Statics. Results for comparative statics are shown in Figure 1.7. Within the parameters that have been tried, I have the following findings:

If the prior precision ρ_0^c increases, price movements are less dramatic due to less uncertainty. Besides, asset prices go up for a longer time, because the convergence rate of investors' beliefs in each period is lower with a higher prior precision. This is also true for concentrated flows of private information.

If the measure of noise traders is less volatile, i.e., ρ_q is higher, we have less striking price movements and also an earlier price reversal. This is because prices are more informative now. As a result, investors' beliefs converge faster, which leads to these two results. Similar results hold for a higher precision of public signals η and ρ_D.

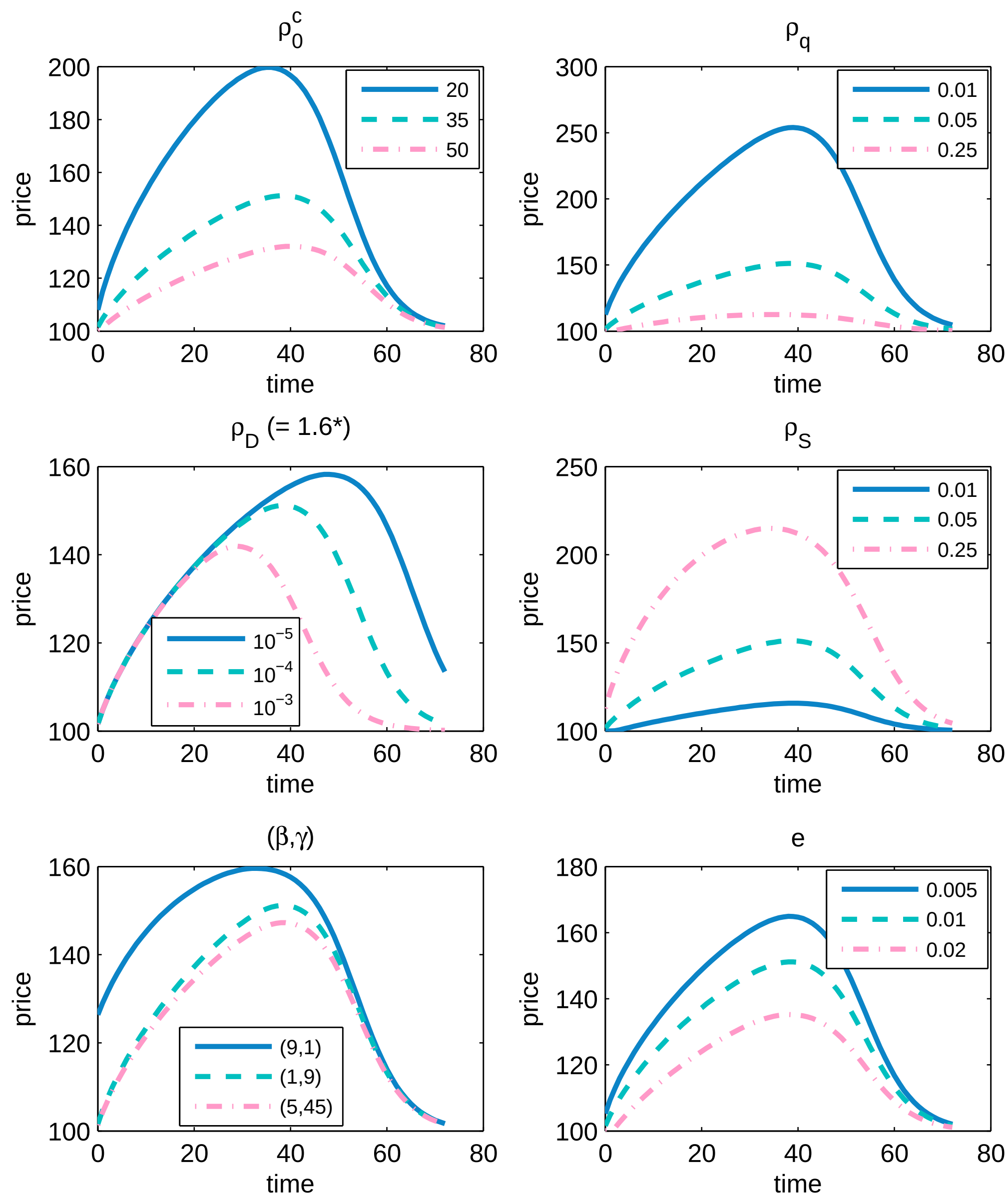

Figure 1.7: Comparative statics.

With a higher precision of private signals ρ_S, investors put more weight on private information and thus their initial forecast dispersion is larger. As a result, we have more capital inflows. This and a higher initial forecast dispersion push up prices to a larger extent. Meanwhile, investors' beliefs converge faster, so the price reversal comes earlier.

If the prior mean of the resolution probability is lower, investors are more aggressive in the beginning, and this brings more capital inflows. As a result, initial prices are higher and prices continue to go up to a larger extent. However, the price reversal is earlier as learning about the resolution probability is slower in each period. This is shown by two cases $(\beta, \gamma) = (1, 9)$ and $(\beta, \gamma) = (9, 1)$. For cases $(\beta, \gamma) = (1, 9)$ and $(\beta, \gamma) = (5, 45)$, the latter one implies a slower learning of the resolution probability. Thus, it generates fewer capital inflows, and price movements are less prominent.

As we raise the entry cost e, we have less capital inflows in each period. This leads to less striking price movements and lower initial prices.

These results on comparative statics have several cross-sectional implications on asset prices. First, results on the initial uncertainty indicate that compared with income stocks, growth stocks experience more dramatic price movements on average. Second, by results on the precision of public signals, it is more likely to observe striking price movements in stocks with low analyst coverage. Third, although not shown in Figure 1.7, by the linearity property of equilibrium prices, price movements of small firms tend to be more prominent. Moreover, if we think of the entry cost as the downpayment for home purchases, the result shows that intuitively, raising downpayment requirements help curb speculation.

1.4.3 Bubbles

Given non-fundamental price movements, can we call those price paths bubbles?

Previous literature has defined bubbles in different ways. Among which, speculative bubbles are the closest to my paper.[14] To quote Allen and Gorton (1993), a bubble is defined to

[14]See, e.g., Harrison and Kreps (1978), Tirole (1982), Allen and Gorton (1993), Allen, Morris, and

be a price path supported by the trading of agents who are "willing to pay more for [the security] than they would pay if obliged to hold it [to horizon]". Another way to state this definition is that every investor knows that the price is above their perceived fundamental value.

In this model, speculators exist in equilibrium, but during each period not all of the investors are speculators. The difference between the price and investors' perceived fundamental value is

$$P_t - E_t^i[\Pi] = \sum_{m=t+1}^{T-1} p_{q,tm} q_m + (P_t - \frac{E_t^i[P_{t+1} + D_{t+1}]}{1+r}) + (1 - \frac{p_{\Pi,t}}{\alpha_t})(E_t^c[\Pi] - E_t^i[\Pi]) \quad (1.10)$$

Among investors who are willing to hold the asset, $P_t - \frac{E_t^i[P_{t+1}+D_{t+1}]}{1+r} \leq 0$. For extremely optimistic investors with very high beliefs on the fundamental value, $P_t < E_t^i[\Pi]$, i.e., the asset price is below their perceived asset value. Thus, investors holding the asset can be divided into two groups, speculators and extremely optimistic investors. The fraction of speculators in each period is depicted in Figure 1.8. The figures shows that during each period except $T-1$, there are some speculators among investors holding the risky asset.

Since the essence of speculative bubbles is speculation, given the existence of speculators, I will treat price paths here as bubbles. Specifically, in this paper, a bubble arises when the marginal investor who is the least optimistic investor holding the risky asset with $E_t^i[P_{t+1} + D_{t+1}]/(1+r) = P_t$, is a speculator, that is, knows that the price is above his perceived fundamental value. The intuition follows from the price formation process we have discussed in the last section. Rational investors are willing to pay more because they expect that future prices, reflecting future optimistic investors' beliefs, will be even higher.[15]

We have several findings in Figure 1.8. First, it shows that most investors are speculators

Postlewaite (1993), Morris (1996), and Scheinkman and Xiong (2003).

[15] The weight on private information is different in the equilibrium price function and investors' beliefs about the fundamental value. This adds some noise to the bubble definition here, as can be seen from the third term in equation (1.10) which is on average negative. However, usually this third term is very small, and apart from it, the results using my definition of bubbles are clear.

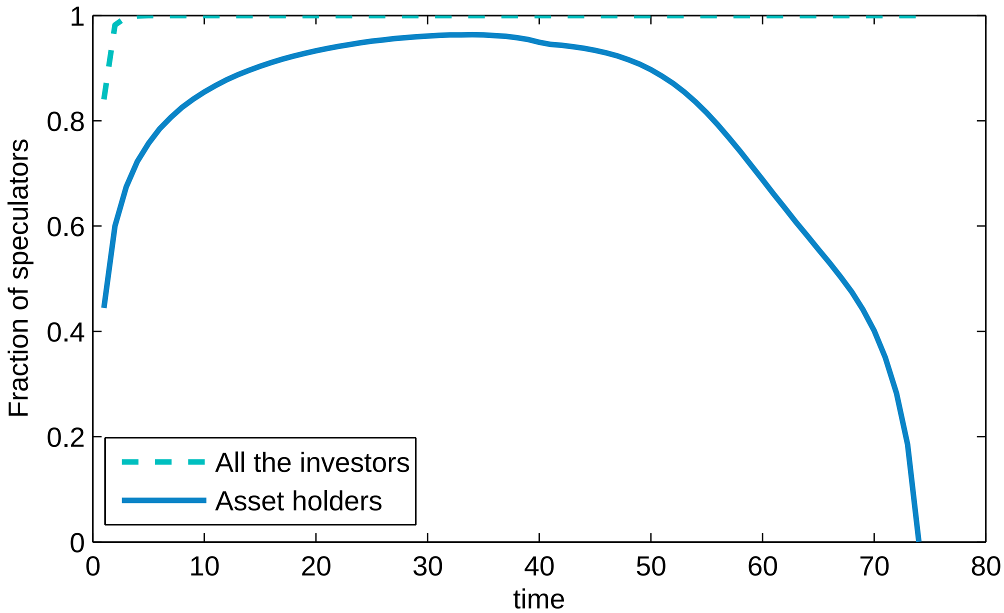

Figure 1.8: The fraction of speculators. This figure shows the fraction of speculators out of all the investors and among asset holders.

during each period. In addition, no speculators hold the risky asset during the last speculative period $T-1$, thus there are no bubbles in that period. This is the result shown in Allen, Morris, and Postlewaite (1993). The reason is, investors know that the asset price in the next period is going to be the fundamental value and thus they are not willing to pay a price higher than their perceived fundamental value. However, bubbles arise when we go backward one more period. In period $T-2$, investors have one trading opportunity to resale the asset to even more optimistic investors at period $T-1$. This creates speculation and thus bubbles.

Bubbles in this paper are different from the bubbles generated by the higher-order uncertainty in Allen, Morris, and Shin (2006). In their paper, when forecasting prices, investors need to forecast other investors' forecasts, so they put more weight on public information. This result indicates that when the fundamental value is below the prior mean, prices are above the market average of investors' beliefs about the asset value, because investors believe that other investors have even higher beliefs. As a result, this higher-order uncertainty leads to bubbles.

1.4.4 Trading volume

As traditional noisy rational expectations models, in contrast with the no-trade theorem by Milgrom and Stokey (1982), the presence of noise traders brings about trade by itself and also speculative trade because the information in the price does not "swamp" private information. To compute trading volume, we can keep track of the selling side. In this model, the trading volume is given by

$$TV_t = n_{t-1} Pr(E^i_{t-1}[P_t + D_t] \geq (1+r)P_{t-1}, E^i_t[P_{t+1} + D_{t+1}] < (1+r)P_t) + (\epsilon_{n,t-1} - \epsilon_{n,t})^+$$

Although it contains some information about the measure of noise traders, it can be treated as another noisy signal about the fundamental value like the price, and there is no reason to believe that including this signal will change the mechanism. Because of this and also the computation complexity, I assume investors do not observe trading volume in the model as previous literature.

For tractability, I study the case $\rho_{S,t} = \rho_S$. It can be shown that

$$TV_t = n_{t-1} \int_{q_{t-1}+\epsilon_{q,t-1}}^{\infty} \Phi(\sqrt{t+1}(q_t + \epsilon_{q,t}) - \sqrt{t}y)\phi(y)dy + (\epsilon_{n,t-1} - \epsilon_{n,t})^+$$

It consists of two parts, speculative trading and noise trading. This paper focuses on the speculative trading whose volume is measured by the equilibrium trading volume with $\epsilon_{n,t} = 0, \forall 0 \leq t \leq T-1$.[16]

Proposition 1.4.3. *In period t, if the uncertainty is not resolved, the equilibrium volume of speculative trading is*

$$\overline{TV}_t = n_{t-1} \int_{q_{t-1}}^{\infty} \Phi(\sqrt{t+1}q_t - \sqrt{t}y)\phi(y)dy \tag{1.11}$$

[16] To be precise, it is the median path of the volume of speculative trading.

where $\Phi(\cdot)$ and $\phi(\cdot)$ are the CDF and PDF of the standard normal distribution respectively. With sufficient capital inflows, it increases over time, and it falls after capital inflows stop.

Proof. See Appendix A. $\square$

The intuition for the dynamics of trading volume is as follows. In each period, the selling decision is induced by two factors. One is that capital inflows bring more investors, especially optimistic investors, to the market, and they bid up asset prices. The other one is that investors may become pessimistic if they receive a bad private signal. The first factor tends to increase trading volume as assets change hands from existing investors to newcomers. For the second factor, as learning proceeds, investors' beliefs are less and less affected by new private information, and thus trading volume decreases over time. Therefore, with sufficient capital inflows, the first factor dominates and trading volume trends up. But after capital inflows stop, only the second factor works, so trading volume falls gradually.

1.4.5 An example: U.S. housing bubble

After characterizing price and trading dynamics, in this section, I compare model implications with a realistic example, the recent U.S. housing bubble. The sample period I pick is May 2003 when the housing price started to surge, to May 2009 when it reached a low.

Uncertainty. An important factor driving fluctuations in housing prices is the land price (Davis and Heathcote (2007)), which moves together with macroeconomic variables over business cycles (see Liu, Wang, and Zha (2013)). Thus, it is reasonable to believe that uncertainty about the housing value is closely related with the economic uncertainty. In panel (a) of Figure 1.10, I show two related indices for the economic uncertainty. One is the Economic Policy Uncertainty Index constructed by Scott Baker, Nicholas Bloom and Steven J. Davis, and the other is the CBOE Volatility Index (VIX). The panel (a) of Figure 1.9 displays the uncertainty index in the model, defined as $\sigma_t(P_{t+1} + D_{t+1})$.

As is shown in the figure, the uncertainty is in general declining, which is consistent with the

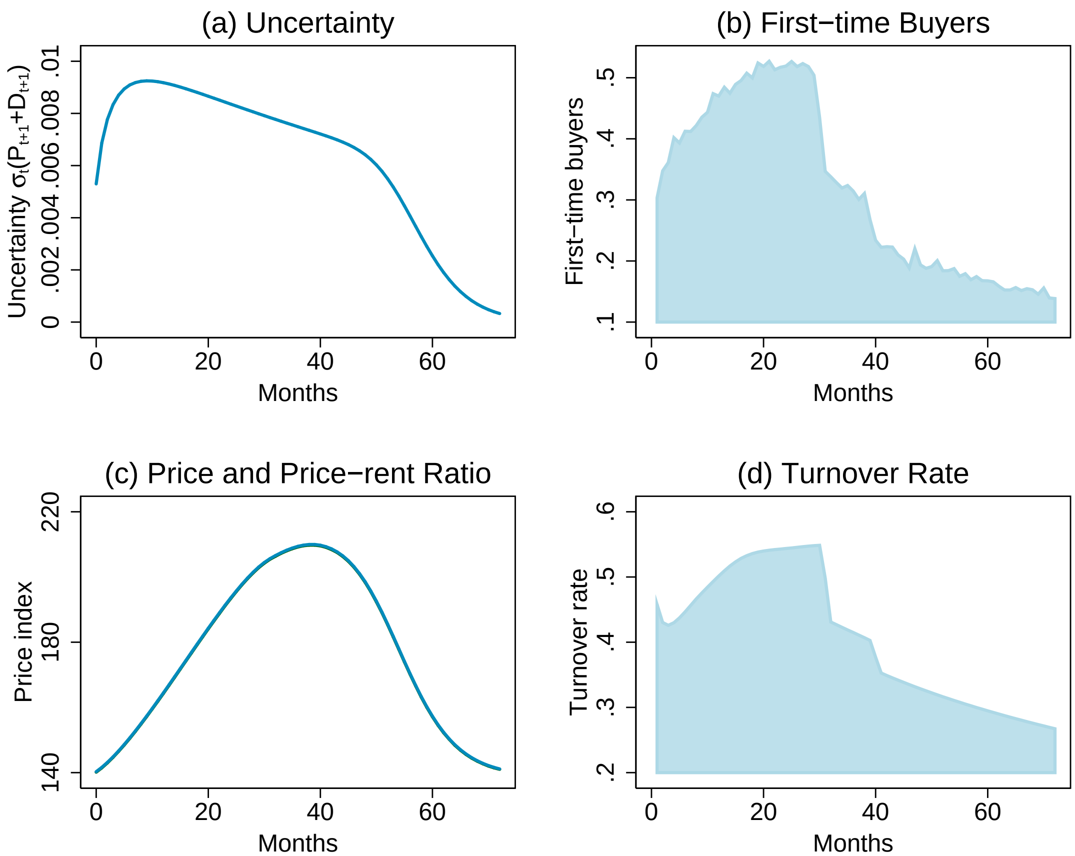

Figure 1.9: The U.S. housing bubble: model. Panel (a) plots the uncertainty measure in the model $\sigma_t(P_{t+1} + D_{t+1})$. Panel (b) shows the measure of first-time buyers (averaged over 8000 times of simulation). Panel (c) displays asset prices. Since the fundamental value is a constant, it also measures the trend of the price-rent ratio. Panel (d) shows the turnover rate. Given that the average asset supply is one, it is also the volume of speculative trading.

Figure 1.10: The U.S. housing bubble: data. Panel (a) displays two uncertainty indicators, the Economic Policy Uncertainty Index constructed by Scott Baker, Nicholas Bloom and Steven J. Davis, and the CBOE Volatility Index (VIX) from WRDS. Panel (b) shows the measure of first-time buyers. The data (with 12-month simple moving average) is calculated using the share of first-time buyers and existing home sales provided by National Association of Realtors. Panel (c) plots housing prices and the price-rent ratio. The housing price is measured by S&P Case-Shiller 20-city home price index released by S&P Dow Jones Indices LLC. The price-rent ratio is constructed using the price index and the rent measured by CPI: owners' equivalent rent of residences from Bureau of Labor Statistics. The initial value of the constructed ratio is normalized to be the same as in the model. Panel (d) shows the turnover rate calculated by existing home sales from National Association of Realtors over owner occupied housing units from U.S. Department of Commerce: Census Bureau, with 12-month simple moving average.

model. Right after the IT bubble burst in 2000, the U.S. economy slipped into a recession in 2001. Together with 9/11 and the 2nd Gulf War, it brought about very high uncertainty about the economic prospect at that time. Later, as the economy gradually recovered, the uncertainty went down over time till it soared again in 2008 because of financial crisis.

Capital Flows. In the model, investors gradually flow into the risky asset sector, and inflows stop at some point. However, by the property of learning, the speed of capital inflows is so fast in the beginning that trading volume shoots up immediately and keeps declining after that. Since in reality, it is reasonable that the information diffusion takes time and the entry cost is decreasing over time due to easier information acquisition and a smaller opportunity cost for portfolio adjustment, to remedy the above issue, I introduce an exogenous cap on capital flows over time, which takes the following parsimonious form:

$$\bar{q}_t = -0.0018t^2 + 0.21t + 0.5$$

The linear term captures constant capital inflows while the quadratic term guarantees an upper limit. Notice that this constraint only changes the speed of capital inflows.

In the data, an indicator for capital flows is the number of first-time buyers. I get a rough estimate using the annual share of first-time buyers provided by National Association of Realtors. Assume each buyer only trades once per period and the monthly share is the same throughout the year. Thus, multiplying the share with the existing home sales gives us the number of first-time buyers. Panel (b) of Figure 1.10 shows the data after 12-month simple moving average. We can see that the number remained high till September 2005 and then fell quickly before it rebounded in 2009. The model prediction shown in panel (b) of Figure 1.9 fits this trend broadly. According to the same mechanism for trading volume, in the model, the measure of first-time buyers trends up and then down.

Prices. By Proposition 4.2, the price soars first and falls later. This is consistent with the data, S&P Case-Shiller 20-city home price index released by S&P Dow Jones Indices LLC..

The housing price in the U.S. grew steadily through 1990s. But after 2001-2002 recession, it surged and peaked in early 2006. After faltering for a few months, it started to decline. Panel (c) of Figure 1.9 and Figure 1.10 show that the model can match the data very well.

Price-rent Ratios. I construct the price-rent ratio from the data in the following way. The price is measured by the S&P Case-Shiller 20-city home price index, and I use the CPI for all urban consumers: owners' equivalent rent of residences from Bureau of Labor Statistics as a proxy for the rent.[17] Although the price-rent ratio obtained in this way cannot give us the right number, it captures the dynamics of the price-rent ratio well, which is what I need. To make a comparison, I normalize the constructed price-rent ratio so it has the same initial value as my model predictions.

Panel (c) of Figure 1.9 and Figure 1.10 show the result. In the data, the rent has remained fairly stable all the time, so the price-rent ratio follows price pattern very closely. It went up by about 40% when the housing price peaked and then dropped to its initial level, which is an indication of speculative bubbles.[18] In the model, given the constant fundamental value, the price-rent ratio has exactly the same dynamics as prices. As can be seen from the figure, the model captures the trend in the data.

Turnover Rate/Trading Volume. In the model, given that the average asset supply is one, the turnover rate of the risky asset equals its trading volume. With the cap on capital inflows, the trading volume or the turnover rate has a rise-and-fall pattern, as displayed in panel (d) of Figure 1.9. Two kinks happen when the cap constraint becomes unbinding and when capital inflows stop.[19] In panel (d) of Figure 1.10, taking the increasing housing supply into consideration, I plot the turnover rate calculated by existing home sales from National Association of Realtors over owner occupied housing units from U.S. Department of Commerce: Census Bureau, after 12-month simple moving average. From the figure, we

[17] Results are very similar using other measures like the CPI for all urban consumers: rent of primary residence.

[18] See Shiller (2005) in which he shows one feature of those episodes of speculative bubbles in the stock market is incredibly high price-earnings ratios.

[19] Kinks happen because the slope of capital flows is not continuous in the discrete time model.

can see that it shows the same pattern of trading frenzies.

1.5 Implications and Extensions

1.5.1 Bubbles of various shapes

Underneath the up-and-down pattern during bubble episodes, prices evolved in their own ways. As shown in Figure 1.11, for some events like the IT bubble, it took a long time for prices to build up before they plummeted, and for some other events like the Japanese real estate bubble in the late 1980s, prices skyrocketed before entering a long-lasting downturn.

In the model, the main force to drive these two different patterns is the flow rate of information. With a lower ρ_D and a higher η, which implies a lower level but faster growth in information flows over time, prices evolve like those during the IT bubble. The reason is that, a lower ρ_D indicates slower convergence of investors' beliefs initially and thus prices can go up for a longer time, and a higher η means that investors' beliefs converge much faster at a later stage which drives down prices quickly. By contrast, price dynamics during the Japanese real estate bubble can occur with a higher ρ_D and a lower η.

In reality, the slow build-up of prices during the IT bubble may be caused by high uncertainty and slow learning about the value of IT, and one possible explanation for the long-term downturn in Japan is the delayed policy response like reducing non-performing loans.

In addition, the model can produce exogenous price crashes. In each period, the fundamental value may get fully revealed with certain probability. If this happens, prices will go back to the fundamental value immediately and this looks like a crash, as shown by Figure 1.12. In fact, the crash mechanism here works in a more general environment. It can also be the possibility of government intervention which ends speculation in each period. This is not rare in real life. For instance, according to the description of the South Sea Bubble in Garber (2000), at the end of August 1720, South Sea share prices collapsed due to a liquidity crisis brought by the enforcement of the Bubble Act which precluded the growth

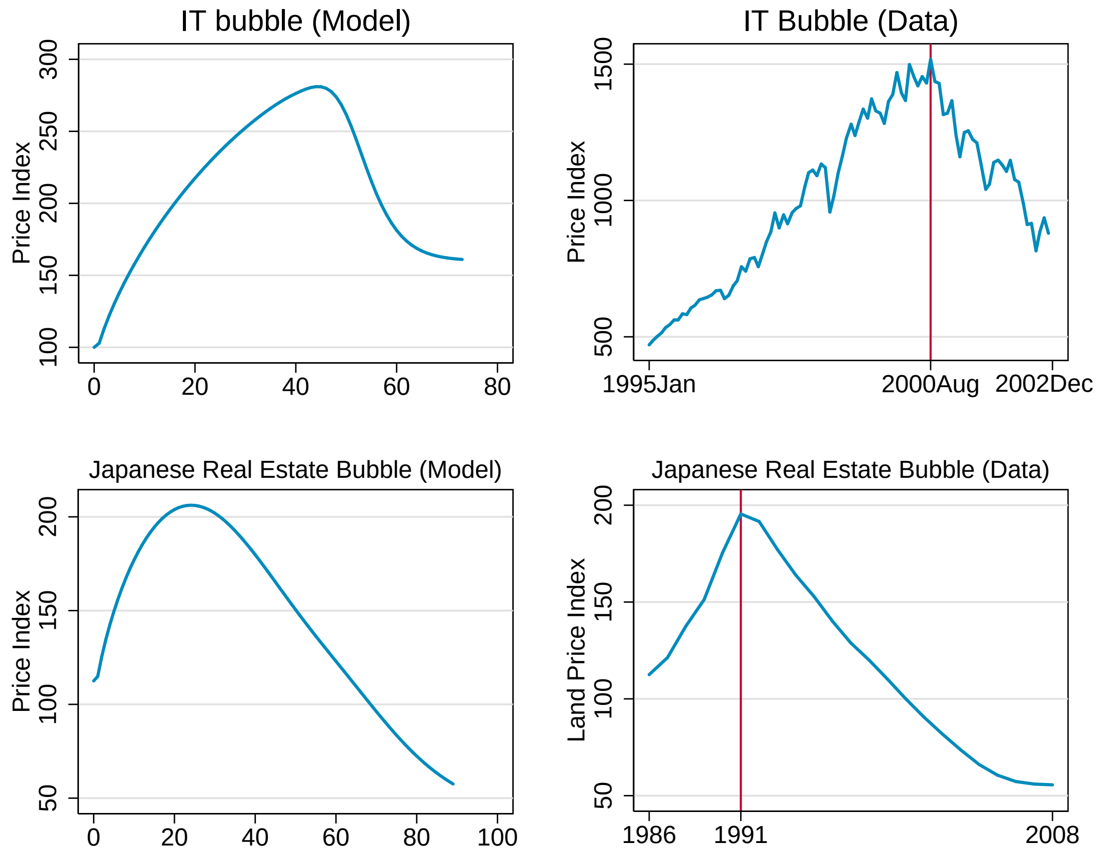

Figure 1.11: Bubbles of different shapes. Parameters that drive these two different shapes are the level parameter ρ_D and the growth rate η of the precision of public signals. I set $\rho_D = 1.6 \times 10^{-5}$ and $\eta = 0.25$ for the IT bubble, and $\rho_D = 25 \times 1.6 \times 10^{-4}$ and $\eta = 0.1$ for the Japanese real estate bubble. Besides, by comparing prices in the initial period and the last period, I set the fundamental value $\Pi = 0.32$ and its prior mean $\Pi_0 = 0.2$ for the IT bubble, and $\Pi = 0.15$ and $\Pi_0 = 0.4$ for the Japanese real estate bubble. In addition, to match the magnitude of price movements, I choose the initial uncertainty $\rho_0 = 20$ for both episodes, the entry cost $e = 0.02$ for the IT bubble, and $e = 0.04$ and the precision of private signals $\rho_S = 0.2$ for the Japanese real estate bubble. In the data, the price index for the IT bubble is S&P 500 Index, and for the Japanese real estate bubble, it is the urban land price of nationwide: commercial, from Japan Statistical Yearbook 2014, Statistics Bureau, Ministry of Internal Affairs and Communications, Japan.

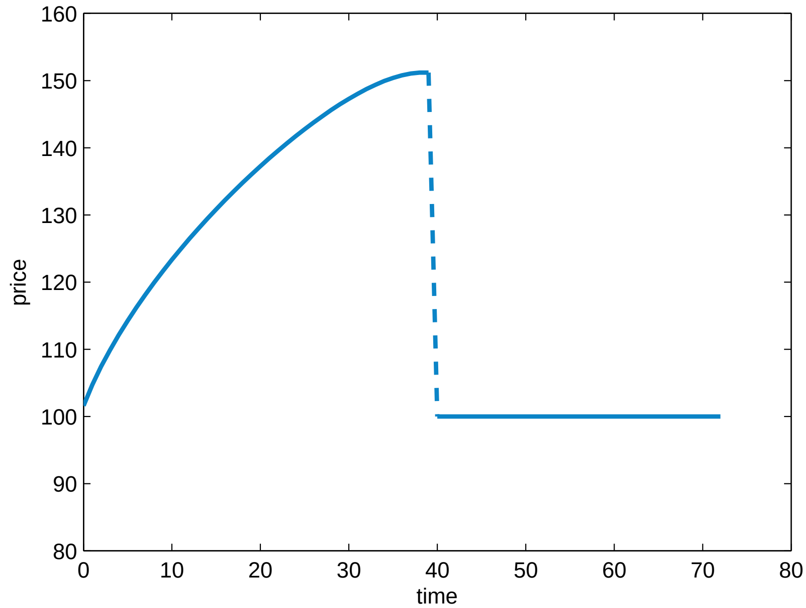

Figure 1.12: Market crashes.

of South Sea company's competitors and thus drove down stock prices of those competitors.

1.5.2 Capital outflows

In the model, there are no capital outflows. In this subsection, I consider an exogenous capital outflows case. In each period, assume 5% of rational investors will exit and stop trading the risky asset. From Figure 1.13 which plots capital flows and the average observed price path, we can see that the main difference is that the price is declining faster now because of capital outflows. This dramatic rise-and-fall pattern is preserved, due to two reasons: first, instead of entry, expected speculation profits lead to net entry; second, investors know the resolution possibilities, so this additional feature, exogenous capital outflows if uncertainty is not resolved in the distant future, will not have a large impact on investors' trading decision and thus prices today.

Another concern with capital outflows is that they reflect pessimistic investors' beliefs and thus contain information about the fundamental value. In general, we can treat capital

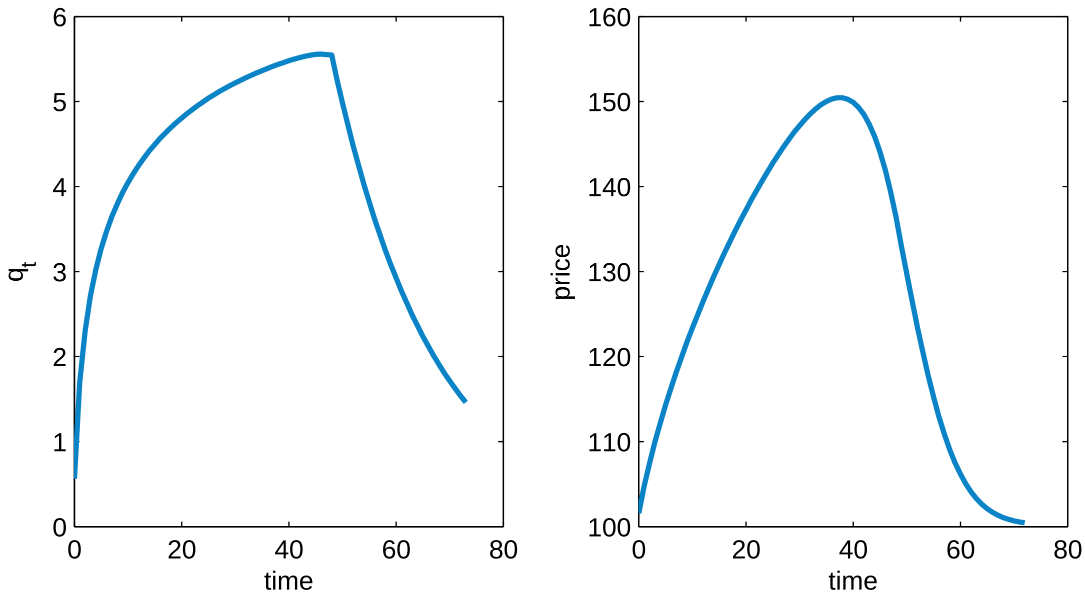

Figure 1.13: Exogenous capital outflows: flows and the average observed price path. Assume 5% exogenous capital outflows in each period. The left panel displays the measure of existing rational investors indicated by q_t. The right panel shows the average observed price path normalized by the fundamental value.

outflows as another noisy signal like prices.[20] There is no reason to believe that it will change model results, but technically it will be much more involved.

1.5.3 Endogenous information acquisition

The model can be justified in an endogenous information acquisition framework. Take the entry cost e as a one-time information acquisition cost and assume an upper bound for the signal precision that can be acquired. It is obvious that in equilibrium investors will choose this upper bound, which is equivalent to exogenous information flows. The rest results carry over into this setup.

Admittedly, multi-equilibria might arise in different endogenous information acquisition settings. Previous literature has a lot of discussion on the equilibrium uniqueness and results varies with the setup.[21] Similarly, it is not robust in this model. Consider one

[20]Noises can be from risky outside production possibilities as in Wang (1990).

[21]Examples include, Grossman and Stiglitz (1980) gets the uniqueness result through information substi-

example in which the information acquisition cost is increasing in the signal precision to be acquired. With endogenous capital flows, we may get two equilibria, a high signal precision with a small amount of rational investors, and a low signal precision with a large amount of rational investors, or even we can get a continuum of equilibria.

1.5.4 Negative shocks: speculation on ST stocks in China

One important implication of this model is that uncertainty drives speculation and thus non-fundamental price movements. For stock market booms during the technological revolutions, they were from good news with uncertainty about the future of those technologies. In this subsection, I provide an example for bad news with uncertainty.

On April 22, 1998, to enhance listed companies' governance practice and protect investors' interests, Shanghai and Shenzhen Stock Exchanges in China introduced a new rule, by which, they would give special treatment to the stocks of listed companies with abnormal financial conditions like shareholders' equity is lower than the registered capital in the last fiscal year. Those stocks are called ST stocks as an abbreviation thereafter, and within one year if their conditions are not improved, they may be suspended for trading and even delisted. For those stocks, surprisingly, investors react enthusiastically, and stock prices go up insanely compared with their negative net profits sometimes.

This phenomenon can be explained by this model, speculation driven by uncertainty. With ST designation, stocks have more uncertainty than before, especially when they start to restructure. One such example is Jiangxi Changjiu Biochemical Industry Company.[22] It received special treatment on April 26, 2011 due to negative net profits in two consecutive fiscal years. After that, from time to time, there were rumors about the possibility that another company with rare earth resources as its main business, Ganzhou Rare Earth Group

tutability, Barlevy and Veronesi (2000) and Chamley (2008) provide a complementarity example in which more information makes prices less informative by introducing the correlation between fundamentals and the noise, and also Veldkamp (2006b) gets strategic complementarities through fixed costs in the information production.

[22]Data can be obtained from CSMAR database. The stock code (STKCD) is 600228.

Company, would go public through reverse merger with Changjiu company. In response, on December 28, 2012, Changjiu company made a public announcement to negate the rumor but not decisively. This tone ignited a speculative wave among investors. The stock price went up from 14.95 RMB to almost 40 RMB in June 2013, and later gradually went down to 29.02 RMB on November 4, 2013 right before Weihua company unexpectedly declared that the rare earth resources company would corporate with them. Since that day, the stock price of Changjiu company plummeted, and after dropping consecutively for seven days, it fell to 13.89 RMB. During the period from December 28, 2012 to November 4, 2013, the average annual turnover rate of Changjiu stock was 864.84% while for Shanghai A share market, it was only 8.53%.[23]

1.5.5 Mutual fund flows: "performance chasers"

Interestingly, investors act like return chasers in this model. They flock to assets with high past returns. This is because large inflows of investors and the resulting high returns are two features of speculative fads driven by uncertainty. The superior high past returns of assets indicate that those assets are more likely to have high uncertainty which further implies higher expected capital flows in the future. Thus, those rational investors turn out to look like return chasers. As an illustration, I simulate 5000 stocks for 50 periods, and Table 2 shows regression results using the simulated data. As predicted, the period t return has positive and statistically significant effects on capital flows in period $t+1$, but when controlling for the period t uncertainty, flows become less sensitive to past returns, and when including a fourth-degree polynomial of the period t uncertainty index, the estimate becomes statistically insignificant.

This result can be related to the well-documented flow-performance relationship for mutual funds (see, e.g., Chevalier and Ellison (1997) and Sirri and Tufano (1998)). Mutual funds pick stocks, so their returns are likely to be positively correlated with stock returns.

[23] The turnover rate is computed as, the value of shares traded divided by the market value of tradable shares.

Table 1.2: The Flow-Performance Relationship

The dependent variable is capital flows calculated as the number of new optimistic rational investors who expect to trade immediately. It is zero if uncertainty gets resolved and $1 - n_{t-1}\Phi(-q_t)$ if not. Uncertainty is measured as $0.1\sigma_t(P_{t+1} + D_{t+1})/(1 - (1 - \lambda_t)/(1 + r))$. Simulated data consists of 5000 assets for 50 periods (with the first 50 periods as the "burn-in" period). In each period, 50 different assets are hit by independent uncertainty shocks randomly. The parameters are listed in Table 1 with $\rho_D = 1.6 \times 10^{-3}$, $\eta = 0$, $(\beta, \gamma) = (4, 6)$.

	(1)	(2)	(3)	(4)	(5)
Lag return (0.01%)	10.543***	2.266***	-0.002	10.543***	-0.002
	(0.276)	(0.115)	(0.002)	(1.892)	(0.015)
Uncertainty		4.415***	25.876***		25.876***
		(0.004)	(0.004)		(0.350)
4th poly. of uncertainty	No	No	Yes	No	Yes
Cluster-Robust SE (on asset)	No	No	No	Yes	Yes
Asset fixed effect	Yes	Yes	Yes	Yes	Yes
N	240100	240100	240100	240100	240100
adj. R^2	0.06	0.84	1.00	0.06	1.00

Standard errors in parentheses * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Meanwhile, investors pick funds, so mutual fund flows can be an indicator for capital flows to assets. By the above argument, uncertainty drives fund flows to chase performance. This can be an alternative explanation in addition to the existing theories like investors are learning about managerial skills from the past performance by Berk and Green (2004).

1.6 Conclusion

In this paper, I have developed a novel theory of "bubble" dynamics—the dramatic non-fundamental rise and fall of asset prices and the associated trading frenzies. As in the literature, speculative "bubbles" arise because of heterogeneous beliefs and short-sale constraints. However in this paper, heterogeneous beliefs are led by differential private information and sustained by the presence of noise traders which prevents prices from fully revealing the fundamental value and preserves asymmetric information across investors. For the price formation, uncertainty about assets' fundamental values, together with short-sale constraints, means that asset prices reflect the marginal investor' belief about future optimistic investors' beliefs, and as a result, the marginal investor is willing to pay more than his perceived fundamental value. This is where speculative "bubbles" come from.

For the price evolution, two layers of uncertainty—uncertainty about the fundamental value and uncertainty regarding the probability with which the fundamental value is fully revealed in each period, generate the hump shape in prices, and gradual capital inflows bring dramatic price movements. Moreover, capital inflows also lead to trading frenzies. Numerically, simulation results show that the model equilibrium can produce various realistic bubble episodes. The importance of uncertainty and capital flows demonstrated in this paper sheds light on the future policy analysis on bubble events.

Chapter 2

Financial Frictions, Entry and Growth: A Study of China

2.1 Introduction

During the past twenty years, we have witnessed the incredibly rapid growth of China. The average annual growth rate of real GDP per capita between 1990 and 2010 was 9.5% while within the same period the average global growth was 3.6% and between 1950 and 1990 the Chinese economy grew at only 4.3% per year.[1] Understanding this striking phenomenon is undoubtedly an important growth experiment, and it has attracted a great deal of attention. In particular, Hsieh and Klenow (2009) and Song, Storesletten, and Zilibotti (2011) have focused on the role of the reduced misallocation in China.

While misallocation is measured among incumbent firms, this paper emphasizes the reallocation induced by entry. Specifically, the paper investigates the role of structural reforms—business deregulation and financial reform in both credit and stock market, in explaining this rapid growth, especially through the entry channel. Furthermore, with heterogenous firms, the model is used to evaluate policies targeting particular groups of firms, and the results show that these policies such as tax deduction and favorable loan terms on small firms can have strikingly different impacts on the aggregate TFP and output.

The paper is motivated by the growing experience of China, as shown in Figure 1 panel (i)-(iii). Since the market-oriented reform of China was extended to the whole country in

[1] The global growth is computed using world gross domestic product in constant prices from the IMF World Economic Outlook Database (October 2014). Data for the Chinese economy is the purchasing power parity converted GDP per capita (chain series) for China at 2005 international dollars per person from the Penn World Table 7.1. During the 1980s, the economy experienced a boom and bust, but since 1990, the economy has remained at a high growth stage.

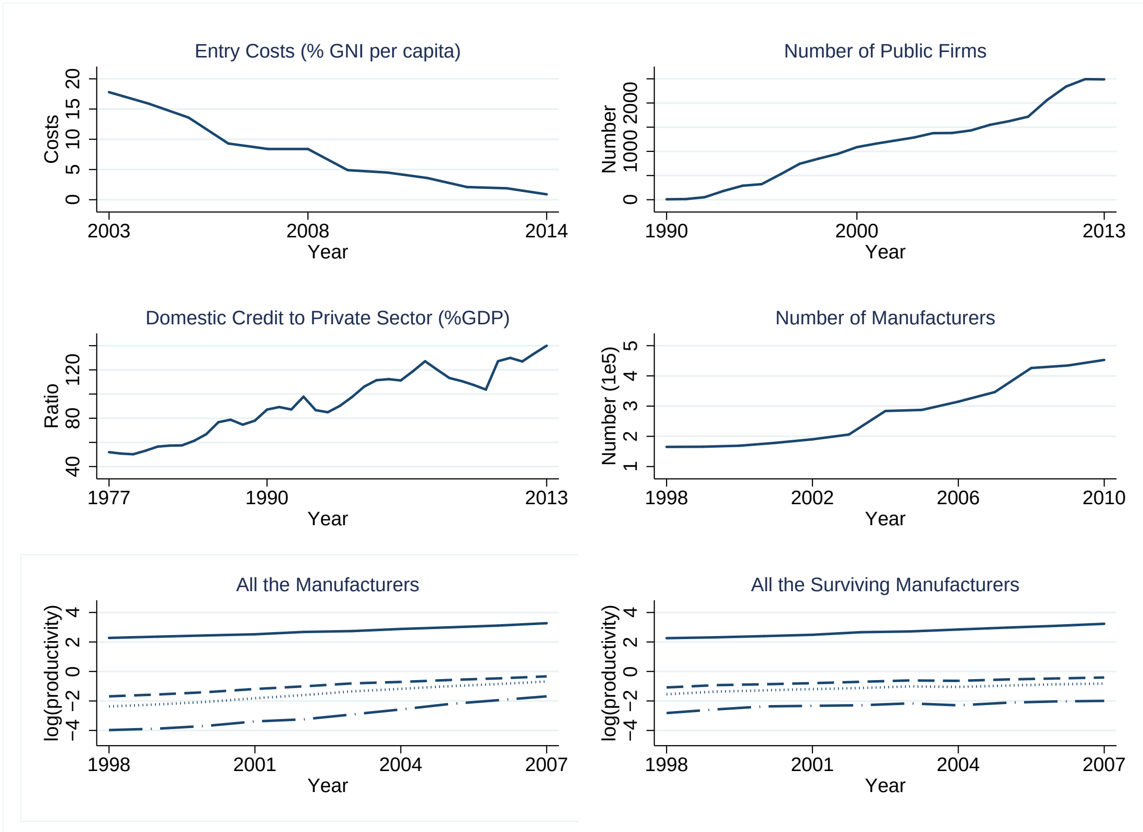

Figure 2.1: Motivation Facts of the Chinese Economy. Data for the costs of business start-up procedures (% of GNI per capita) and domestic credit to private sector (% of GDP) is from the World Bank. Data for the number of public firms and manufacturers is from the China Statistical Yearbook (2014). Firm-level productivity is calculated by $y/(k^{\alpha}l^{\gamma})$ using Annual Surveys of Industrial Production for China 1998-2007.

1992, entry barriers in terms of costs of business start-up procedures have been reduced dramatically. Meanwhile, stock market is growing rapidly after it was established at the end of 1990, as reflected in the climbing number of public firms. Credit market indicated by domestic credit to private sector (%GDP) has also been improved gradually. Based on these facts, I construct a structural model to assess the effects of reforms including business deregulation in the form of declining entry costs, the establishment of stock market, and the improvement in the credit market reflected in the relaxation of credit constraints.

The paper stresses the importance of entry. Structural reforms lead to a massive influx of new firms and thus a higher business density. As a result, the market competition becomes fiercer and more inefficient firms have to exit. This induces resources to be reallocated from inefficient to efficient firms and thus results in a higher aggregate output. The model implication on productivity improvement at the extensive margin is supported by the firm-level data in China. As shown in Figure 1 panel (iv)-(vi), at the post-reform stage, we have observed a sharp increase in the number of manufacturers and in the mean time, a gradual improvement in the bottom percentiles of the productivity distribution for all the manufacturers but not among all the surviving manufacturers.

To quantify the contribution of different aspects of reforms to the economic growth in China through this entry channel, a tractable growth model integrating costly entry, credit frictions, IPO and dynamics of heterogenous firms and consumers is presented. The model is close to Aiyagari (1994) on the consumer side in which individuals can work subject to idiosyncratic labor income shocks or they can set up their own business, while on the firm side, the model is built on Hopenhayn (1992) in which firms differ in their productivity and net worth, and operate with credit constraints and IPO options.

Policy parameters in the model are identified as follows. Entry barriers in the form of entry costs are closely associated with business density. The fixed cost of IPO determines the fraction of firms going public. Lastly, the tightness of credit constraints is linked with the domestic credit to private sector (%GDP).

Quantitative results show a very sizable effect for these reforms. In total the model can explain 46.6% of the increase in real GDP per capita of China from 1990 to 2010. Interestingly, business deregulation alone explains 17.2% while financial reform accounts for 9.3% of the GDP change, suggesting a multiplicative effect of those reforms. Moreover, within the model, 22.3% of the output change is from the intensive margin and 77.7% is through the extensive entry and exit margin, which indicates the importance of the entry channel.

The model is then used to evaluate policies targeting small firms. Specifically, I compare tax deductions with favorable loan terms. Both policies aim to promote entry and post-entry growth. However, since firms grouped by net worth and productivity receive different benefits from two policy tools, it is hard to access which policy is more effective analytically. In light of this, making use of this model, I conduct a numerical policy experiment and find that tax deduction has worked better for the Chinese economy.

This paper contributes to the literature on the Chinese economy. Many researchers have conducted growth accounting exercises for China (examples include Zheng and Hu (2009), Bosworth and Collins (2008), Perkins and Rawski (2008) and Brandt and Zhu (2009)) and found that aggregate TFP and capital stock explain the growth of real GDP per capita half by half. Different from the accounting approach, this paper tries to provide a micro-foundation for the rapid growth. Two papers are closely related to this paper. One is Hsieh and Klenow (2009). They empirically document the decline in the degree of misallocation among incumbent firms over time. The other one is Song, Storesletten, and Zilibotti (2011) which models the gradual reallocation between two sectors. As discussed above, both papers focus on the misallocation within existing enterprises, and this paper emphasizes the reallocation caused by entry. Moreover, by modeling the behavior of individual firms, the framework can be used to evaluate firm-specific policies.

This paper also adds to the literature on entry barriers and the market selection. Jovanovic (1982) explicitly models the selection and associated industry dynamics, and Hopenhayn (1992) discusses the relationship between entry costs and the selection. More recently, some studies make cross country comparisons about entry costs and aggregate economic performance using structural models (e.g., Barseghyan and DiCecio (2009), Boedo and Mukoyama (2012)), and some other papers focus on specific forms of entry costs like the difficulty of imitating incumbents by entrants in Luttmer (2007). Among these papers, Aghion, Fally, and Scarpetta (2007) conducts a cross country empirical study and finds that financial system is important to business creation. This result provides additional empirical support for this paper. Besides, Acemoglu, Akcigit, Bloom, and Kerr (2013) considers a different selection process on innovative capacities and the resulting reallocation of skilled labor. Relative to the literature, this paper focuses on the growth of one specific country and investigates the impact of various types of entry barriers.

In addition, the vast literature on financial development and economic development is closely related to this paper. See Levine (2005), Banerjee and Duflo (2005) and Matsuyama (2007) for excellent surveys. In general, for the quantitatively oriented literature, the mechanism can be divided into extensive margin and intensive margin.[2] This paper focuses on the extensive margin, i.e., entry.

The rest of the paper proceeds as follows. Section 2 introduces the institution background. Section 3 describes the model and characterizes the equilibrium. Section 4 conducts quantitative analysis. Section 5 concludes.

2.2 Institution Background

During the past twenty years, the Chinese government has taken out the market-oriented reform step by step and costs of business start-ups have decreased gradually. Before 1992, state-owned enterprises were the major market participants. However, after realizing the importance of the market, during the 1980s, the government carried out some market reform experiments in a few special areas. Later, the reform was extended to the whole country in 1992, and in 1997, the government officially endorsed the role of private economy and proposed to create equal opportunities for all types of firms to compete in the market. In 2003, the law for small and medium-sized enterprises became effective. The law explicitly states that the government should provide various services and benefits like information to these firms, and also the local government should simplify the register process. In 2004, the government further abolished all the administrative licenses except 500 items related to national security, natural resources and so on. In 2008, the new law for the corporate income tax reduces the tax rate of small-scale enterprises with low profits to 20% while the standard tax rate is 25%. All those steps show the decreasing trend for the entry cost. By

[2] Some recent examples include Giné and Townsend (2004), Jeong and Townsend (2007), Amaral and Quintin (2010), Greenwood, Sanchez, and Wang (2013) and Buera, Kaboski, and Shin (2011).

a direct indicator from the World Bank, in China, the cost of business start-up procedures has decreased from 17.8% of GNI per capita in 2003 to 4.5% of GNI per capita in 2010.

Along with business deregulation, the financial sector has also experienced fundamental changes. For the stock market, nowadays there are two main stock exchanges in mainland China, Shanghai and Shenzhen Stock Exchange. Both of them were founded at the end of 1990, and afterwards grow very quickly, with the total number of listed companies increasing from the initial 10 to 2343 in 2011 and the annual raised capital from the initial 500 million yuan to 581.42 billion yuan in 2011. Moreover, according to the World Federation of Exchanges, in 2012, Shanghai Stock Exchange was the world's 7th largest stock market with the domestic market capitalization at $2.5 trillion. Besides, in the credit market, the bankruptcy efficiency has been improved a lot. According to the World Bank (2001), during the 1980s, when firms went bankruptcy, creditors' rights were always violated for the sake of settling affected workers. Although a Trial Bankruptcy Law for State-Owned Enterprises, which became effective in 1988, separated secured assets from the estate, the Capital Structure Optimization Program Circular No.492 of 1996 made even mortgaged land use rights available for worker claims. Nowadays, the situation has changed with the development of the social security and pension system. In China, till 1990, the employee pension was mainly provided by enterprises. However, the transition towards the social security system started from 1991, and after a 6-year trial process in some provinces from 2001 to 2006, the government started to expand the social security system to the whole country. As a result, now creditors' rights are guaranteed by the new enterprise bankruptcy law which became effective in June 2007. As an indicator for financial development, domestic credit to private sector in China has increased from 78% of GDP in 1989 to 130% in 2010, according to World Development Indicators from the World Bank.

2.3 Model

The model is built on Aiyagari (1994) and Hopenhayn (1992) with financial frictions. In the model, on the one hand, individuals differ in their wealth and they can set up their own businesses; on the other hand, firms have heterogenous productivity and net worth, they produce subject to financial constraints, they launch IPO when they become sufficiently productive, and they exit when they cannot make money any more. Within this environment, I study the effect of structural reforms including business deregulation by lowering entry costs, the improvement in the credit market through relaxing the collateral constraint, and the development of stock market in the form of an IPO option.

Next, I introduce a benchmark model first. After that, I discuss the compatibility between some salient features of the Chinese economy and the model framework.

2.3.1 Consumers

The economy is populated by a continuum of infinitely-lived individuals with measure L.[3] They maximize the expected life-time utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma} - 1}{1-\sigma} \tag{2.1}$$

where c_t is consumption in period t and β is the time discount factor.

In each period, individuals work, save and consume. They can be either workers ($\mu_t = 0$) or entrepreneurs ($\mu_t = 1$). If an individual is a worker in period $t-1$, he can continue to work and earn wage income w_t in period t, or he can set up his own business and become an entrepreneur. In the latter case, he needs to pay an entry cost $c_e w_t$, and in the next period, the firm starts to produce after its initial productivity is drawn from an exogenous distribution. Details about firms are provided in the next section 3.2. The worker's budget constraint is given by

$$c_t + a_{t+1} \leq 1_{\{\mu_t=1\}}(-c_e w_t) + 1_{\{\mu_t=0\}} w_t y_t + (1+r_t)a_t \tag{2.2}$$

Here I impose a borrowing constraint $a_{t+1} \geq 0$. In addition, as in Aiyagari (1994), workers

[3] The phenomenon of migration from rural to urban areas will be considered in section 4.4 Robustness Check.

suffer from a labor income shock y_t which follows

$$\log(y_t) = \rho \log(y_{t-1}) + \epsilon_t$$

where $\rho \in [0, 1)$ and $\epsilon_t \sim N(0, \sigma_y^2)$.

For an entrepreneur, in period t, he can operate the business and collect the profit $pi_t^e(z_t, a_t)$ where z_t measures the firm's productivity. If not, he can close the firm and in the next period switch back to be a worker. More importantly, he has one more option provided by a new element I consider here, the stock market. Assume there is a fixed cost of IPO $c_p w_t$ which captures all the size-irrelevant costs such as expenses on external auditors and registration/filing costs. Under this specification, the entrepreneur will choose to take the firm public if the business is sufficiently profitable. Since through public trading, the ownership and thus the idiosyncratic risk can be perfectly diversified among consumers, the firm's value from the standpoint of investors $v_t^f(z_t)$ is simply the discounted value of all the future profits. Assume going public also incurs a transaction cost which is ϕ_p fraction of the firm's value. This ϕ_p captures the costs netted against proceeds like the underwriter's discount. In summary, by taking the firm public, the entrepreneur receives revenue $(1 - \phi_p)v_t^f(z_t) - c_p w_t$.

Given these options, an entrepreneur's budget constraint is

$$c_t + a_{t+1} \leq 1_{\{\mu_t=1\}} \pi_t^e(a_t, z_t) + 1_{\{\mu_t=0\}} \max((1 - \phi_p)v_t^f(z_t) - c_p w_t, 0) + (1 + r_t)a_t \tag{2.3}$$

2.3.2 Firms

In equilibrium, there is a continuum of firms. They are private firms when operated by entrepreneurs, and become public firms after their IPO. Firms use the following technology

$$y_t = (\bar{Z}_t z_t) k_t^\alpha (l_t - \bar{l})^\gamma$$

where $\bar{Z}_t = \bar{Z}_0(1+g)^t$ is the technology frontier moving forward at an exogenous rate g, and z_t measures the individual firm's relative position with respect to the technology frontier, which takes value within $(0,1]$. A higher z_t indicates that the firm is in a more profitable position relative to other firms. In light of this, to be rigorous, z_t will be called relative productivity from now on. Besides, in the production function, $\bar{l}$ is the overhead labor cost. For private firms, assume $\bar{l}^e = 0$, while for public firms, considering the cost of being IPO like the need to develop external reporting, I assume $\bar{l}^f > 0$. In addition, assume the technology exhibits decreasing returns to scale, i.e., $0 < \alpha, \gamma < 1$ and $\alpha + \gamma < 1$.

In each period, after the realization of individual productivity, firms make production decisions—how much capital to rent and how many workers to employ. As in Buera and Shin (2013), Moll (2014) and Song, Storesletten, and Zilibotti (2011), I assume that the capital that can be rented by private firms is limited by a collateral constraint $k_t \leq \lambda a_t$, where $\lambda \geq 1$ and a_t is the entrepreneur's wealth. This constraint can be derived from a model of limited contract enforcement. Here λ captures the frictions in the credit market. An increase in λ indicates the development of credit market, and $\lambda = \infty$ corresponds to a perfect credit market. Denote $\pi_t^e(z_t)$ as private firms' profit function derived from the static profit maximizing problem,

$$\pi_t^e(a_t, z_t) = \max_{0 \leq k_t \leq \lambda a_t, l_t \geq \bar{l}} (\bar{Z}_t z_t) k_t^{\alpha} (l_t - \bar{l})^{\gamma} - (r_t + \delta) k_t - w_t l_t \tag{2.4}$$

and $k_t^e(a_t, z_t)$, $l_t^e(a_t, z_t)$ are policy functions of capital and labor respectively. For public firms, since they could raise funds through equity issuance, I assume their capital rental is not restricted. Their profit function is thus

$$\pi_t^f(z_t) = \max_{k_t \geq 0, l_t \geq \bar{l}} (\bar{Z}_t z_t) k_t^{\alpha} (l_t - \bar{l})^{\gamma} - (r_t + \delta) k_t - w_t l_t \tag{2.5}$$

and $k_t^f(z_t)$, $l_t^f(z_t)$ are corresponding policy functions. Simple algebra shows that profits are decreasing in factor prices r_t and w_t. This indicates that higher factor prices led by fiercer competition will squeeze firms' profits, and thus more firms will exit.

Assume firms draw their initial relative productivity from an exogenous distribution $G(z)$. Besides, using an exogenous version of productivity evolution process in Pakes and McGuire (1994) and Ericson and Pakes (1995), I assume

$$z_{t+1} = \begin{cases} z_t(1+g)^{n_u} & \text{with prob } p_u(z_t) \\ z_t & \text{with prob } 1 - p_u(z_t) - p_d(z_t) \\ z_t/(1+g)^{n_d} & \text{with prob } p_d(z_t) \end{cases}$$

where in general $p_u(z_t) < p_d(z_t)$ which captures the intuition that deterioration is easier than improvement and also is consistent with the Chinese data. Under this setting, firms' productivity is improving from time to time, but their relative position in the whole productivity ladder is in general falling gradually. As a result, firms exit eventually when they lag too far behind the technology frontier.

While entrepreneurs make exit decisions for their own private firms, public firms exit whenever their value falls below zero. This implies an exit threshold of productivity. I denote it as $\underline{z}_t^f$. Given the stochastic discounting factor $M_{t+1} = 1/(1 + r_{t+1})$, public firms' problem can be written in the following recursive form,

$$v_t^f(z_t) = \pi_t^f(z_t) + M_{t+1} E_t \max\{v_{t+1}^f(z_{t+1}), 0\} \tag{2.6}$$

2.3.3 *Financial Institutions*

In the credit market, assume there are competitive financial intermediaries which take deposits from consumers and rent out capital to firms. In return, firms pay back at a rental rate $r_t + \delta$, and after the deduction of depreciation rate δ, consumers receive an interest rate r_t on their deposits. As described above, the friction in this market is that firms face a credit constraint of which the tightness is measured by λ.

Besides, with a stock market, firms can go public after paying IPO costs consisting of a fixed cost $c_p w_t$ and also a ϕ_p fraction of the firm value. In spite of these costs, IPO is still attractive. This is because it enables risk-averse entrepreneurs to sell the ownership to the public who can bear more risk by diversification and also take back the investment like entry cost which is beneficial given the borrowing constraint. Moreover, another reason is that as an additional funding channel, IPO relaxes the firm's credit constraint and thus increases the firm value.

2.3.4 Equilibrium

In equilibrium, individuals are heterogenous in their wealth a due to different history paths of their career choices, labor income shocks and realizations of firm productivity, and firms differ in their relative productivity z and for private firms/entrepreneurs also in net worth or wealth a. The competitive equilibrium can be defined as follows.

Definition Given the initial distributions, a competitive equilibrium consists of allocations $\{c_t^w(a,y), a_{t+1}^w(a,y), \mu_t^w(a,y), c_t^e(a,z), a_{t+1}^e(a,z), \mu_t^e(a,z), k_t^e(a,z), l_t^e(a,z), k_t^f(z), l_t^f(z), \underline{z}_t^f\}_{t=0}^{\infty}$, prices $\{w_t, r_t\}_{t=0}^{\infty}$, and distributions $\{F_t^w(a,y), F_t^e(a,z), F_t^f(z)\}_{t=0}^{\infty}$ such that

1. Given $\{w_t, r_t\}_{t=0}^{\infty}$, $\{c_t^w(a,y), a_{t+1}^w(a,y), \mu_t^w(a,y), c_t^e(a,z), a_{t+1}^e(a,z), \mu_t^e(a,z)\}_{t=0}^{\infty}$ solves individuals' problems (2.1)-(2.3).

2. Given $\{w_t, r_t\}_{t=0}^{\infty}$, $\{k_t^e(a,z), l_t^e(a,z), k_t^f(z), l_t^f(z)\}_{t=0}^{\infty}$ solves firms' problems (2.4) and (2.5), and $\{\underline{z}_t^f\}_{t=0}^{\infty}$ is given by $v_t^f(\underline{z}_t^f) = 0$.

3. Markets clear for any $t \geq 0$.

   (a) Labor market clears:

$$L = \int_{a\geq 0}\int_{y\geq 0} \mu_t^w(a,y) c_e dF_t^w(a,y) + \int_{a\geq 0}\int_{(1-\phi_p)v_t^f(z)\geq c_p w_t} (1-\mu_t^e(a,z)) c_p dF_t^e(a,z)$$
$$+ \int_{a\geq 0}\int_{z\in(0,1]} \mu_t^e(a,z)(l_t^e(a,z)+1) dF_t^e(a,z) + \int_{z\in[\underline{z}_t^f,1]} l_t^f(z) dF_t^f(z)$$

(b) Capital market clears:

$$\int_{a\geq 0}\int_{y\geq 0} adF_t^w(a,y) + \int_{a\geq 0}\int_{z\in(0,1]} adF_t^e(a,z)$$
$$= \int_{a\geq 0}\int_{z\in(0,1]} \mu_t^e(a,z)k_t^e(a,z)dF_t^e(a,z) + \int_{z\in[\underline{z}_t^f,1]} k_t^f(z)dF_t^f(z)$$

4. Distributions $\{F_t^w(a), F_t^e(a,z), F_t^f(z)\}_{t=0}^{\infty}$ follow the law of motion:

(a) Distribution of workers:

$$F_{t+1}^w(a,y) = \int_{\tilde{a}\geq 0}\int_{\tilde{y}\geq 0} 1_{\{\mu_t^w(\tilde{a},\tilde{y})=0, a_{t+1}^w(\tilde{a},\tilde{y})\leq a\}} p(y|\tilde{y})dF_t^w(\tilde{a},\tilde{y})$$
$$+ \int_{\tilde{a}\geq 0}\int_{z\in(0,1]} 1_{\{\mu_t^e(\tilde{a},z)=0, a_{t+1}^e(\tilde{a},z)\leq a\}} f(y)dF_t^e(\tilde{a},z)$$

(b) Distribution of entrepreneurs/private firms:

$$F_{t+1}^e(a,z) = \int_{\tilde{a}\geq 0}\int_{y\geq 0}\int_{\tilde{z}\in(0,1]} 1_{\{\mu_t^w(\tilde{a},y)=1, a_{t+1}^w(\tilde{a},y)\leq a, z_{t+1}(\tilde{z})\leq z\}} dG(\tilde{z})dF_t^w(\tilde{a},y)$$
$$+ \int_{\tilde{a}\geq 0}\int_{\tilde{z}\in(0,1]} 1_{\{\mu_t^e(\tilde{a},\tilde{z})=1, a_{t+1}^e(\tilde{a},\tilde{z})\leq a, z_{t+1}(\tilde{z})\leq z\}} dF_t^e(\tilde{a},\tilde{z})$$

(c) Distribution of public firms:

$$F_{t+1}^f(z) = \int_{\tilde{z}\in(0,1]} 1_{\{z_{t+1}(\tilde{z})\leq z\}} dF_t^f(\tilde{z})$$
$$+ \int_{a\geq 0}\int_{(1-\phi_p)v_t^f(\tilde{z})\geq c_p w_t} 1_{\{\mu_t^e(a,\tilde{z})=0, z_{t+1}(\tilde{z})\leq z\}} dF_t^e(a,\tilde{z})$$

Notice that distributions are cumulative functions of frequency not density.

We can further define a balanced growth path (BGP). Along BGP, the interest rate is a constant, denoted as r. Besides, the wage rate, the profits, the value of public firms, the consumption and the wealth are all growing at the same rate $(1+g)^{\frac{1}{1-\alpha}}$. In addition, given the utility function, I can define the effective time discount factor as $\tilde{\beta} = \beta(1+g)^{\frac{1-\sigma}{1-\alpha}}$. The optimization problem for workers now can be written in the following stationary and recursive form (all the variables except r and z are detrended),

$$v^w(a,y) = \max_{\mu\in\{0,1\},c,a'\geq 0}\{\frac{c^{1-\sigma}}{1-\sigma} + \tilde{\beta}(1_{\{\mu=0\}}Ev^w(a',y') + 1_{\{\mu=1\}}Ev^e(a',z'))\}$$

subject to

$$c + a' \leq 1_{\{\mu=0\}}wy + 1_{\{\mu=1\}}(-c_e w) + (1+r)(1+g)^{-\frac{1}{1-\alpha}}a$$

where the initial relative productivity z' is drawn from $G(z')$.

Correspondingly, entrepreneurs' problem is given by

$$v^e(a,z) = \max_{\mu\in\{0,1\},c,a'\geq 0}\{\frac{c^{1-\sigma}}{1-\sigma} + \tilde{\beta}(1_{\{\mu=0\}}Ev^w(a',y') + 1_{\{\mu=1\}}Ev^e(a',z'))\}$$

subject to

$$c + a' \leq 1_{\{\mu=0\}} \max((1-\phi_p)v^f(z) - c_p w, 0) + 1_{\{\mu=1\}}\pi^e(a,z) + (1+r)(1+g)^{-\frac{1}{1-\alpha}}a$$

where by redefining the stochastic discount factor $\tilde{M} = (1+g)^{\frac{1}{1-\alpha}}/(1+r)$, we have the value of public firms to be

$$v^f(z) = \pi^f(z) + \tilde{M}E\max\{v^f(z'), 0\}$$

2.3.5 Discussion

In this paper, I take the year 1992 as a turning point and consider structural reforms in two aspects. One is business deregulation, that is, the Chinese government started to allow and promote the growth of private economy from 1992. This change is reflected in the decline of entry cost c_e in the model. The other aspect is financial reform. Stock market was first established at the end of 1990 in China. Thus at the pre-reform stage, transaction cost of

IPO ϕ_p is set to be 1, and the reform features a drop in ϕ_p. Besides stock market, credit market has also improved, which is characterized by the relaxation of collateral constraint, that is, an increase in λ.

One phenomenon accompanying structural reforms in China is the massive migration from rural to urban areas. I take into account this change in the size of labor force for manufacturing sector in the quantitative analysis by assuming the measure of individuals after the reform is $L' > L$. As will be discussed later, it is a linear amplification effect.

It is well documented that state-owned enterprises (SOEs) are less efficient than private enterprises (PEs).[4] This paper can endogenize the TFP discrepancy in the following way. Before 1992, SOEs were in the majority in China. However private economy has been expanding rapidly since 1992. As Song, Storesletten, and Zilibotti (2011) documents, the employment share of domestic private enterprises (DPEs) out of DPEs and SOEs has increased from 4% in 1998 to 56% in 2007. Given these facts, we can take all the firms before the reform as SOEs and all the entrants as PEs after the reform. Thus, SOEs are less productive because they are older and grew in a less competitive environment.

Moreover, as discussed in Song, Storesletten, and Zilibotti (2011), SOEs have much better access to bank loans than PEs. Although in the model SOEs receive no preferential treatment, the result here is a conservative estimate on the impact of those reforms. The reason is that, before the reform, the economy with a mixture of SOEs and PEs with differential policies is less productive than the economy with only one type of firm, while during the post-reform stage, the difference has no effect since nearly all the firms will be private firms along the new BGP given almost all the entrants are PEs.

[4] For example, Hsieh and Klenow (2009).

2.4 Quantitative Analysis

2.4.1 Parameterization

I parameterize the model to match some salient features of the Chinese economy.

The preference parameter σ is set to be standard 1.5. The time discount rate is chosen so that the equilibrium interest rate is standard 4.5%. Besides, the technology frontier expands at the rate $g = 2\%$ which matches the TFP growth rate of the U.S. economy. The capital depreciation rate is $\delta = 0.1$. In addition, since functionally overhead labor cost incurred for public firms serves as a per-period fixed cost and thus the stationary distribution of public firms is well-defined, I choose it to be $\bar{l}^f = 0.5$. Results under different $\bar{l}^f$ will be shown later as a robustness check. Moreover, the initial technology frontier $\bar{Z}_0$ and the measure of workers L are scale parameters, so I set $\bar{Z}_0$ to be 0.8 and L to be 1,000. As discussed above, due to labor migration, the size of labor force in the industrial sector has increased a lot after the reform. According to the China Statistical Yearbook, the employment size in the industrial sector is 1365 in 2010 if it is 1000 in 1991 right before the reform.[5] Thus, I set $L' = 1365$.

For the production function, the output elasticity with respect to labor, taken from the careful study of Young (2003) on the labor income share, is on average 0.46 from 1978-1995.[6] The span of control, in particular for firms whose credit constraints are non-binding, can be calculated by

$$\alpha + \gamma = 1 - \frac{revenue\, from\, principal\, business - cost\, of\, principal\, business}{revenue\, from\, principal\, business}$$

Using the data from the China Statistical Yearbook on aggregate statistics for large and medium-sized manufacturers which are taken to be not financially constrained, I find that

[5]The employment size in the industrial sector is the employment size in the secondary industry consisting of industry and construction sector minus that in the construction sector.

[6]To be rigorous, γ equals $w(l - \bar{l})/y$ which is not exactly the labor income share wl/y. However, the difference is numerically very small, so I treat them equally.

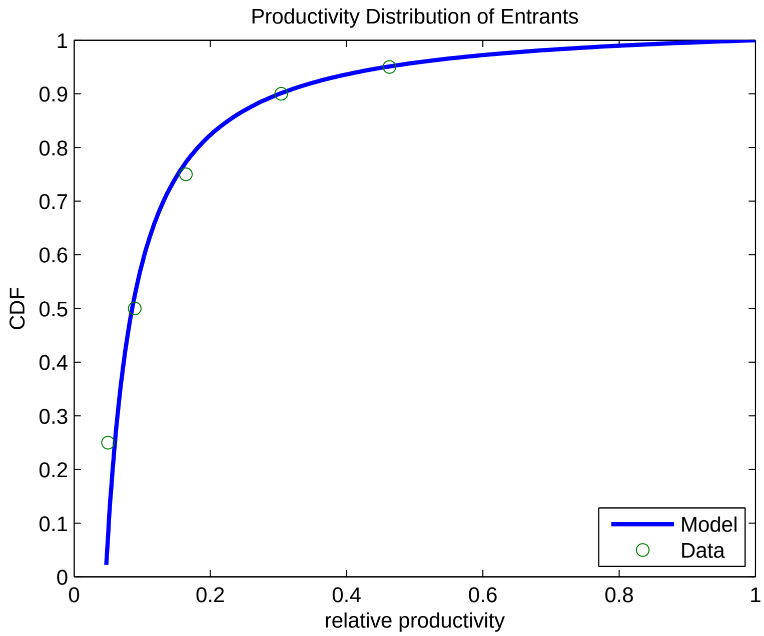

Figure 2.2: CDFs of the Relative Productivity of Entrants from Model and Data.

from 1998 to 2011, this ratio varies within $[0.79, 0.85]$, so I take the median 0.83. Accordingly, the output elasticity with respect to capital is $\alpha = 0.83 - 0.46 = 0.37$.

The productivity distribution of entrants $G(z)$ is computed from Annual Surveys of Manufactures (ASM) from 1999 through 2007 conducted by the National Bureau of Statistics of China.[7] The details are in Appendix B. Since the distribution can be well captured by a Pareto distribution, I assume

$$g(z) = \frac{\tau \underline{z}^{\tau} z^{-\tau-1}}{1 - \underline{z}^{\tau}}$$

for $z \in [\underline{z}, 1]$. I choose the tail index τ and the minimum support $\underline{z}$ to match the 95%, 90%, 75%, 50% and 25% percentiles of entrants' productivity distribution. The CDFs generated by the model and the data are depicted in Figure 2.2, which shows the success of a Pareto distribution specification.

For the transition matrix of productivity, the up and down step-size (n_u, n_d) is the corre-

[7] I am very grateful to Professor Heng Yin for providing me the data.

sponding median step-size computed from ASM. Besides, the transition probability $(p_u(z), p_d(z))$ is computed as the probability of productivity going up by at least $n_u/2$ steps and down by at least $n_d/2$ steps respectively. The ASM data shows that $p_u(z)$ is increasing in z while $p_d(z)$ is decreasing, which reflects the intuition that it is harder for productive firms to improve further but easier to deteriorate. Thus, I assume for any $z \in [\underline{z}, 1]$,

$$p_u(z) = \frac{p_u(1) - p_u(\underline{z})}{1 - \underline{z}}(z - \underline{z}) + p_u(\underline{z}), \ p_d(z) = \frac{p_d(1) - p_d(\underline{z})}{1 - \underline{z}}(z - \underline{z}) + p_d(\underline{z})$$

The process of labor income shocks is imputed from Xu (2010). Using the China Health and Nutrition Survey (CHNS) data, he estimated labor income shocks consisting of an AR(1) permanent component and a MA(1) transient component. By matching two moment conditions on the variance in labor income shocks and the autocovariance of the first-order difference in labor income shocks in two adjacent periods, I compute the corresponding parameters (ρ, σ_y) for the AR(1) specification used in this paper.

Next we determine policy parameters. For the entry cost c_e at the pre-reform stage, since they are highly correlated with firms' entry and exit decision and the economy before the policy change can be treated as along the old BGP, I choose this parameter to match the pre-reform business density. Business density is defined as the number of enterprises over the employment size in the industrial sector. The China Statistical Yearbook has provided such information.[8] I take the median business density from 1980 to 1991, 0.13%, as the pre-reform business density.

As for the entry cost during the post-reform stage, I directly infer the change of entry costs using World Development Indicators from the World Bank. According to this database, the cost of business start-up procedures has decreased from 17.8% of GNI per capita in 2003

[8] The statistical standard was adjusted in 1998, and since then non-state-owned enterprises with annual sales revenue below five million yuan have been excluded. Thus for consistency concern, I substitute the number of collective-owned enterprises in 1998 for the number before 1998. Since before 1992, the major participants in the market are state-owned and collective-owned enterprises, the sum of the number of these two types of firms is the total number of industrial enterprises.

to 4.5% of GNI per capita in 2010. Assuming the entry cost in the model is proportional to the cost of start-up procedures and adjusting the change in the GNI per capita (using real GDP per capita from the Penn World Table 7.1 instead), I find that the entry cost in China has decreased by 64.13% from 2003 to 2010. The Chinese government extended the market-oriented reform to the whole country in 1992, and since then, domestic private economy has grown very fast. We can take this institution change as the starting point of business deregulation. Thus, given the data from 2003 to 2010, I extrapolate the entry cost till 1992, and by my calculation, the entry cost has decreased by 92.84% from 1992 to 2010.

Now three parameters governing financial frictions (ϕ_p, c_p, λ). For ϕ_p, since the stock market was not established till 1990, it is reasonable to choose $\phi_p = 1$ for the old BGP. As described in the institution background part, during the past 20 years, the stock market in China has rapid development. Nowadays, the underwriter's discount, which is the main part of the IPO cost, is about 5-7% of gross proceeds in China according to the Wind financial database. Together with other types of cost like legal-related fees, costs netted against proceeds is roughly 10% of gross proceeds. Thus, I set $\phi_p = 0.1$ along the new BGP. For the fixed cost c_p, it is directly associated with the IPO decision, so I choose it to match the fraction of public firms out of all the firms, which is 0.0045 in 2010.[9] The credit friction λ is calibrated to match the indicator, domestic credit to private sector (% of GDP), from world bank, as in Buera, Kaboski, and Shin (2011). In China, this indicator has changed from 62.6% (average between 1977 and 1989) to 103.3% (average from 1990 to 2007 right before the financial crisis). I choose year 1989 as the breaking point because a Trial Bankruptcy Law for SOEs became effective in 1988 which could be seen as the start of the credit market reform.

Table 2.1 lists the value of assigned parameters I use in the numerical analysis, and calibrated parameters as well as matched moments other than percentiles of entrants' productivity distribution are listed in Table 2.2 below.

[9]The data I use is the number of listed A-share companies and the total number of industrial enterprises from the China Statistical Yearbook (2014).

Table 2.1: Assigned Parameter Values.

Parameter		Value
CRRA coefficient	σ	1.5
Correlation coefficient of labor income shock	ρ	0.54
Standard deviation of labor income shock	σ_y	0.68
Output elasticity with respect to capital	α	0.37
Output elasticity with respect to labor	γ	0.46
Overhead labor cost	$\bar{l}^f$	0.5
Probability of improving relative productivity	p_u	[0,0.39]
Probability of deteriorating relative productivity	p_d	[0.68,0.37]
Productivity change (up)	$(1+g)^{n_u}$	1.46
Productivity change (down)	$(1+g)^{-n_d}$	0.69
Capital depreciation rate	δ	0.1
Initial technology frontier	$\bar{Z}_0$	0.8
Technological growth rate	g	0.02
Tail index of entrants' productivity distribution	τ	1.08
Minimum support of z	$\underline{z}$	0.047
Measure of workers	L	1000
IPO costs netted against proceeds	ϕ_p	[1,0.1]

Table 2.2: Matched Moments.

	Data	Model	Parameter
Interest rate	0.045	0.045	$\beta = 0.78$
Business density	0.0013	0.0014	$c_e = 3$
Credit/GDP (%)	[62.6, 103.3]	[62.3, 101.2]	$\lambda = [2.77, 1000]$
Fraction of public firms	0.0045	0.0037	$c_p = 2485$

2.4.2 Results

The main results are listed in Table 2.3 below. Under these structural reforms, aggregate output has increased by a factor of 2.4. Compared with the data, which is the purchasing power parity converted GDP per capita (chain series) for China at 2005 international dollars per person from the Penn World Table 7.1, it accounts for 46.6% of the change from 1990 to 2010 if the economy in 2010 is close to the new BGP. Moreover, with the labor migration into the manufacturing sector, since the size of labor force increases and meanwhile more individuals start their own business, the labor migration overall has a scale effect. As shown in Table 2.3, it amplifies the output from 341.3 to 465.9 by a factor of 1.365, while business density and prices remain the same. In addition, in the model, the technology frontier is moving forward exogenously over time, so aggregate output is growing at 3.27% per period along the BGP. If we take this exogenous change into account, the model can generate $(3.66 \times (1 + 3.27\%)^{20} - 1)/5.18 = 115\%$ of the growth.

The effect can be further decomposed by different types of reforms. Without financial development, lowering entry costs alone can explain 17.2% of the output increase, and with entry costs unchanged, financial development alone can explain 9.3% of the increase. The sum of these two results is much smaller than the total effect 46.6%, which suggests a multiplicative effect. Intuitively, financial development and lower entry costs are complements, so their effect will be mutually strengthened. Moreover, notice that the effect of credit market reform is slightly larger than the effect of reforms in both credit and stock market. This is because IPO introduces a trade off between risk diversification and profits (relaxed credit constraints but higher overhead labor costs) for entrepreneurs, and in this numerical analysis, risk diversification dominates. Besides, on top of business deregulation and credit market reform, the establishment of stock market only increases the GDP by 1%, which is because credit and stock markets are substitutes.

Table 2.3 also demonstrates the role of these structural reforms—reallocating resources from inefficient to efficient firms through the increased market competition brought by more

Table 2.3: Effects of Structural Reforms.

	Output	Business density	Wage	Interest rate
Benchmark	100	0.0014	100	0.045
Reforms	341.3	0.0043	335.4	0.135
+ Labor migration	465.9	0.0043	335.4	0.135
Data[a]	618	0.0026	663	
Decomposition				
Entry costs	189.3	0.0073	183.9	0.035
Credit market	150.1	0.0039	146.1	0.16
Finance total	148.0	0.0010	144.1	0.155
Entry + Credit	340.4	0.0043	335.4	0.135

[a] Data source: Penn World Table 7.1, China Statistical Yearbook (2009, 2012).

entrants. Specifically, business deregulation encourages entry and financial development promotes post-entry growth. Overall they lead to a larger entry size, which results in a higher business density. As the market becomes more active, the competition is also fiercer. As a result, the wage goes up which forces more inefficient firms to exit. Here, business density defined as the number of enterprises over the size of labor force has increased to 0.0043. This is acceptable compared with the data in 2010, 0.0026. Besides, the wage rate has risen by 235.4 times. It is half of that suggested by the data, 563.[10]

Table 2.4 decomposes the effect at the intensive and extensive margin. Specifically, first, I start from the pre-reform BGP and compute the output with the firms' distribution and policies (c_e, ϕ_p, λ) along the old BGP but using the business density and equilibrium prices (r, w) on the new BGP. This measures the output change caused by a higher business density. Next, I calculate the output under new policies, business density and prices but fixing the

[10]We have two series for the real wage. One is the growth rate of average real wage for all the workers from 1991 to 2008, and the other is for all the urban workers from 1995 to 2010. Since there is a subtle difference between two series, I combine the growth rate for all the workers from 1991 to 1994 with that for all the urban workers from 1995 to 2010 to compute the growth of real wage between 1990 and 2010.

Table 2.4: Decomposition of Effects.

	(a) Business Density	Reallocation (b) Intensive	Reallocation (c) Extensive	Entry (a) + (c)
z distribution*	O	O	N	
Policies	O	N	N	
Measure	N	N	N	
Prices	N	N	N	
Effects (%)**	-24.74	22.32	102.42	77.68

*In this table, O represents old (pre-reform) and N is new (post-reform).

**Effects are calculated by dividing (a)-old, (b)-(a) and (c)-(b) by the total change respectively.

firms' productivity distribution on the old BGP. The further change in the output in this step captures the effect of reallocation at the intensive margin, that is, among incumbent firms. At last, the difference between the post-reform output and the output in the last step characterizes the effect of reallocation at the intensive margin. From Table 2.4, we can see that output decreases through a pure business density channel. This is because the wage rate has been pushed up very high due to the increased market competition caused by a higher business density. Besides, reallocation at the intensive margin accounts for 22.32% of the total GDP change in the model, and entry channel through both a higher business density and reallocation at the extensive margin explains the rest 77.68%, which shows the importance of entry.

2.4.3 Policy Analysis

After realizing the importance of market economy, the Chinese government has carried out various policies to encourage entry and promote post-entry growth. For example, before 2008, corporate income tax rate for small firms is 18% or 27% (depends on the taxable income level) while the standard tax rate is 33%, and under the new corporate tax law carried out in 2008, small firms pay 20% income tax and large firms pay 25%. Besides, the Chinese government has provided favorable loan terms to small firms. Analytically, it is hard to assess which policy is more effective. The reason is as follows. Although inefficient firms with high net worth have no external finance needs and thus do not benefit from loan subsidies, the saved benefits have been allocated to both efficient and inefficient firms with low net worth. Since benefits to efficient firms increases the output and these to inefficient firms are detrimental to the economy, the result depends on which force is stronger.

Since this model characterizes the firm-level behavior and thus is suitable to conduct firm-level policy analysis, in this section, I compare these two policy tools numerically. The experiment has the following steps. First, I impose 25% corporate income tax on all the firms in the pre-reform economy, and use the correspondent equilibrium output as the benchmark. Next, I assume small firms only need to pay 20% tax, denoted as $\tau_\pi = 0.05$. Here small firms are defined as those with taxable income below a certain threshold which is determined by dividing the cutoff income level 300,000 yuan (specified by the new corporate income tax law) by the average corporate income 11,710,000 yuan in 2010 and then multiplying with the average corporate income during the pre-reform stage in the model. At last, consider loan subsidies on small firms, that is, they pay less interest rate $r + \delta - \tau_r$. τ_r is chosen to satisfy the equivalent revenue rule. I also tried $\tau_\pi = \{0.10, 0.25\}$ as a robustness check. The results are listed in Table 2.5. From the table, we can see that according to this model, tax deduction works better than loan subsidies for the Chinese economy, and both policies have large allocation effects.

Table 2.5: Comparison of Policies.

$\tau_\pi =$	0.05	0.10	0.25
Tax deduction	1.38%[a]	3.34%	6.74%
Loan subsidies	0.76%	1.63%	4.94%
Total subsidies	0.006%	0.014%	0.048%

[a] The results in the first two rows are the output change. The unit for all the results in the table is % of benchmark output.

2.4.4 Robustness Check

Table 2.6 below shows the results for various overhead labor costs, and the results have little changes.

Table 2.6: **Robustness Check on Overhead Labor Costs.**

$\bar{l}^f =$	0.25	0.5	1	2	5
Output	341.31	341.31	341.32	341.32	341.32
Business density ($1/10^3$)	4.282	4.276	4.277	4.272	4.272

[a] Data source: Penn World Table 7.1, China Statistical Yearbook (2009, 2012).

2.5 Conclusion

Along with the sustained rapid growth of China in the past twenty years, on the one hand, the business density indicated by the number of firms becomes much higher and the market selection effect is stronger as the wage has gone up and the dispersion of firms' productivity distribution has shrunk, and on the other hand, the cost of business start-up procedures has decreased gradually and the financial sector has experienced significant development. Based on these observations, this paper investigates the effect of structural reforms—business deregulation and financial development in both credit and stock market. A dynamic general equilibrium growth model integrating costly entry, credit frictions, IPO and dynamics of heterogenous firms and consumers is presented. Structural reforms lead to an influx of new firms and thus a higher business density. As a result, the market competition becomes fiercer and more inefficient firms exit. This induces resources to be reallocated from inefficient to efficient firms and thus results in a higher aggregate output. The quantitative analysis using the Chinese firm-level data shows that these structural reforms play an important role in explaining the rapid growth of China. The effect of business deregulation and that of financial development are mutually strengthened, and in total they can explain 46.6% of the growth in the real GDP per capita of China from 1990 to 2010 while the entry margin accounts for 77.68% of the GDP change in the model. Moreover, the model is used to evaluate relevant firm-level policies. By comparing tax deduction and loan subsidies on small firms, the paper shows that both policies are very effective and tax deduction has worked better for the Chinese economy.

Appendix to Chapter 1

A.1 Proofs

A.1.1 Proof of Proposition 2.1

Proof. If uncertainty is not resolved at period m, by Bayes' rule, at period t for $0 \leq t \leq m$, the posterior distribution is $Beta(\beta + t + 1, \gamma)$. Thus, the posterior mean is

$$\lambda_t = \frac{\gamma}{\beta + \gamma + t + 1}$$

which is decreasing over time. □

A.1.2 Proof of Proposition 3.1

Proof. Since investors are risk neutral and $\beta = 1/(1+r)$, their consumption decision and portfolio choice can be studied separately. Here, I focus on the portfolio choice. Since when there is no uncertainty, the excess return from trading the risky asset is zero, investors' objective function can be rewritten as maximizing the present value of their expected wealth at the end of speculative period, and potential investors' problem is

$$\max_{x,t_e} E_0[\frac{1}{(1+r)^{T+1}} W_T^i | \mathcal{F}_0]$$

s.t.

$$W_t^i = W_{t-1}^i(1+r) - 1_{\{t=t_e\}} e \text{ for } 0 \leq t \leq t_e$$

$$W_t^i = W_{t-1}^i(1+r) + x_{t-1}^i(P_t + D_t - P_{t-1}(1+r)) \text{ for } t > t_e$$

$$x_t^i \in [0,1], \; 0 \leq t_e \leq T-1$$

$$W_0^i \text{ is given}$$

Define Lagrange function, take F.O.C., we get the optimal trading strategy, for $t_e \leq t \leq T-1$,

$$x_t^i = \begin{cases} 1 & \text{if } E_t^i[P_{t+1} + D_{t+1}] \geq (1+r)P_t \\ 0 & \text{o.t.w.} \end{cases}$$

□

A.1.3 Proof of Proposition 3.2

Proof. Here, we take the measure of rational investors trading the risky asset $\{n_t\}$ and equivalently $\{q_t\}$ as given. As will be shown in Proposition 3.3, $\{q_t\}$ is common knowledge.

First, we show the result about belief updating. Take the price function as given, we have the information contained in the price to be

$$\xi_t = \Pi + \mu_t \epsilon_{q,t}$$

where $\mu_t = p_{\epsilon,t}/p_{\Pi,t}$.

Notice that $\epsilon_{q,t}$ is i.i.d., which implies signals $\{\xi_t\}$ and $\{D_t\}$ are independent and i.i.d.. Thus, instead of the standard Kalman filter, we can directly apply the Bayesian rule, and get the posterior belief on Π based on $\mathcal{F}_t^c$ to follow $N(E_t^c[\Pi], 1/\rho_t^c)$ with

$$E_t^c[\Pi] = \frac{\rho_{t-1}^c}{\rho_t^c} E_{t-1}^c[\Pi] + \frac{\rho_q/\mu_t^2}{\rho_t^c}\xi_t + \frac{\rho_{D,t}}{\rho_t^c}\frac{1}{r}D_t$$

and

$$\rho_t^c = \rho_{t-1}^c + \rho_{D,t} + \rho_q/\mu_t^2$$

For the belief based on the total information available to investor i, $i \in I_t$, since private signals are i.i.d. and also independent with other noise terms, we have the following Bayesian updating rule:

$$E_t^i[\Pi] = (1-\alpha_t)E_t^c[\Pi] + \alpha_t \frac{1}{t+1}\sum_{m=0}^{t} S_m^i$$

where

$$\alpha_t = \frac{\rho_{\tilde{S},t}}{\rho_t^c + \rho_{\tilde{S},t}}, \ \ \rho_t = \rho_{t-1} + \rho_{D,t} + \frac{\rho_q}{\mu_t^2} + \rho_{S,t}$$

Given belief updating rules, now we show that the price function has the linear form as given in the Proposition. The method we use here is guess-and-verify and backward induction.

At $t = T$, uncertainty is resolved and thus $P_T = \Pi$. Given this, at $t = T-1$, if uncertainty is not resolved, we have

$$\begin{aligned} E_{T-1}^i[P_T + D_T] &= E_{T-1}^i[\Pi + r\Pi + \epsilon_{D,T}] = (1+r)E_{T-1}^i[\Pi] \\ &= (1+r)((1-\alpha_{T-1})E_{T-1}^c[\Pi] + \alpha_{T-1}\frac{1}{T}\sum_{m=0}^{T-1} S_m^i) \end{aligned}$$

Across investors, this generates a distribution following $N(\bar{E}_{T-1}[P_T+D_T], \sigma_{T-1}^2(P_T+D_T))$ with

$$\begin{aligned} &\bar{E}_{T-1}[P_T + D_T] = (1+r)((1-\alpha_{T-1})E_{T-1}^c[\Pi] + \alpha_{T-1}\Pi) \\ &\sigma_{T-1}(P_T + D_T) = (1+r)\alpha_{T-1}\sqrt{1/\rho_{\tilde{S},T-1}} \end{aligned}$$

By Proposition 3.1, investors hold one unit of risky asset iff $E_{T-1}^i[P_T + D_T] \geq (1+r)P_{T-1}$. This and the belief distribution we get above imply the following market clearing condition:

$$n_{T-1}(1 - \Phi(\frac{(1+r)P_{T-1} - \bar{E}_{T-1}[P_T + D_T]}{\sigma_{T-1}(P_T + D_T)})) = 1 - \epsilon_{n,T-1}$$

From which, we have

$$P_{T-1} = \frac{1}{1+r}(\bar{E}_{T-1}[P_T + D_T] + \sigma_{T-1}(P_T + D_T)\Phi^{-1}(1 - \frac{1-\epsilon_{n,T-1}}{n_{T-1}}))$$

Plugging in the specification for noise traders (2.5), we get

$$P_{T-1} = \frac{1}{1+r}(\bar{E}_{T-1}[P_T + D_T] + \sigma_{T-1}(P_T + D_T)(q_{T-1} + \epsilon_{q,T-1}))$$

Using the result for the belief distribution, we have

$$P_{T-1} = (1-\alpha_{T-1})E^c_{T-1}[\Pi] + \alpha_{T-1}\Pi + \alpha_{T-1}\sqrt{\frac{1}{\rho_{\tilde{S},T-1}}}(q_{T-1} + \epsilon_{q,T-1})$$

Matching the coefficients gives us

$$\begin{aligned} p_{\Pi,T-1} &= \alpha_{T-1} \\ p_{q,(T-1)(T-1)} &= p_{\epsilon,T-1} = \alpha_{T-1}\sqrt{\frac{1}{\rho_{\tilde{S},T-1}}} \end{aligned}$$

Thus, the equilibrium price at period $T-1$ has the functional form given in the proposition.

Now assume the equilibrium price at period $t+1$ has the linear form given in the proposition, we want to show that it also holds for period t.

By Proposition 2.1, if uncertainty is not resolved at t, investors expect the resolution probability in the next period to be λ_t. Thus

$$\begin{aligned} E^i_t[P_{t+1} + D_{t+1}] \quad &= \quad (1-\lambda_t)E^i_t[(1-p_{\Pi,t+1})E^c_{t+1}[\Pi] + p_{\Pi,t+1}\Pi + \sum_{m=t+1}^{T-1} p_{n,(t+1)m}q_m \\ &\quad + p_{\epsilon,t+1}\epsilon_{q,t+1} + r\Pi + \epsilon_{D,t+1}] + \lambda_t E^i_t[\Pi + r\Pi + \epsilon_{D,t+1}] \end{aligned}$$

Since by the belief updating rule for $E^c_{t+1}[\Pi]$,

$$\begin{aligned} E^i_t[E^c_{t+1}[\Pi]] \quad &= \quad \frac{1}{\rho^c_{t+1}} E^i_t[\rho^c_t E^c_t[\Pi] + \frac{\rho_q}{\mu^2_{t+1}}\xi_{t+1} + \rho_{D,t+1}\frac{1}{r}D_{t+1}] \\ &= \quad \frac{\rho^c_t}{\rho^c_{t+1}} E^c_t[\Pi] + \frac{\rho_q/\mu^2_{t+1} + \rho_{D,t+1}}{\rho^c_{t+1}} E^i_t[\Pi] \end{aligned}$$

we have

$$E_t^i[P_{t+1}+D_{t+1}] = (1+r-A_t)E_t^c[\Pi] + A_t E_t^i[\Pi] + (1-\lambda_t)\sum_{m=t+1}^{T-1} p_{q,(t+1)m} q_m$$

where

$$A_t = \lambda_t(1+r) + (1-\lambda_t)((1-p_{\Pi,t+1})(1-\rho_t^c/\rho_{t+1}^c) + p_{\Pi,t+1} + r)$$

Using the belief updating rule for $E_t^i[\Pi]$, we further have

$$E_t^i[P_{t+1}+D_{t+1}] = (1+r-A_t\alpha_t)E_t^c[\Pi] + A_t\alpha_t \frac{1}{t+1}\sum_{m=0}^{t} S_m^i + (1-\lambda_t)\sum_{m=t+1}^{T-1} p_{q,(t+1)m} q_m \quad \text{(A.1)}$$

Across investors, it follows $N(\bar{E}_t[P_{t+1}+D_{t+1}], \sigma_t^2(P_{t+1}+D_{t+1}))$ with

$$\bar{E}_t[P_{t+1}+D_{t+1}] = (1+r-A_t\alpha_t)E_t^c[\Pi] + A_t\alpha_t\Pi + (1-\lambda_t)\sum_{m=t+1}^{T-1} p_{q,(t+1)m} q_m \quad \text{(A.2)}$$

$$\sigma_t(P_{t+1}+D_{t+1}) = A_t\alpha_t\sqrt{1/\rho_{\tilde{S},t}}$$

This implies the market clearing condition

$$n_t(1-\Phi(\frac{(1+r)P_t - \bar{E}_t[P_{t+1}+D_{t+1}]}{\sigma_t(P_{t+1}+D_{t+1})})) = 1-\epsilon_{n,t}$$

From which, we have

$$\begin{aligned} P_t &= \frac{1}{1+r}(\bar{E}_t[P_{t+1}+D_{t+1}] + \sigma_t(P_{t+1}+D_{t+1})\Phi^{-1}(1-\frac{1-\epsilon_{n,t}}{n_t})) \\ &= (1-\frac{A_t\alpha_t}{1+r})E_t^c[\Pi] + \frac{A_t\alpha_t}{1+r}\Pi + \frac{1-\lambda_t}{1+r}\sum_{m=t+1}^{T-1} p_{q,(t+1)m} q_m + \frac{A_t\alpha_t}{1+r}\sqrt{\frac{1}{\rho_{\tilde{S},t}}}(q_t+\epsilon_{q,t}) \end{aligned}$$

This confirms our conjecture about the price function, and by matching the coefficients, we have

$$p_{\Pi,t} = \frac{A_t\alpha_t}{1+r}$$

$$p_{q,tm} = \frac{1-\lambda_t}{1+r} p_{q,(t+1)m} \text{ for } t+1 \leq m \leq T-1$$
$$p_{q,tt} = p_{\epsilon,t} = \frac{A_t \alpha_t}{1+r}\sqrt{\frac{1}{\rho_{\tilde{S},t}}}$$

This completes the proof. □

A.1.4 Proof of Proposition 3.3

Proof. We first show that the expected excess return from trading the risky asset at period t if uncertainty is not resolved is the same conditional on any $\mathcal{F}^e_m$ for $0 \leq m \leq t$.

Notice that by the proof of Proposition 3.2,

$$P_t = \frac{1}{1+r}(\bar{E}_t[P_{t+1} + D_{t+1}] + \sigma_t(P_{t+1} + D_{t+1})(q_t + \epsilon_{q,t}))$$

we have

$$\begin{aligned} & E^i_t[P_{t+1} + D_{t+1}] - (1+r)P_t \\ = \; & E^i_t[P_{t+1} + D_{t+1}] - \bar{E}_t[P_{t+1} + D_{t+1}] - \sigma_t(P_{t+1} + D_{t+1})(q_t + \epsilon_{q,t}) \\ = \; & (1+r)p_{\Pi,t}(\frac{1}{t+1}\sum_{m=0}^{t} S^i_m - \Pi) - \sigma_t(P_{t+1} + D_{t+1})(q_t + \epsilon_{q,t}) \end{aligned}$$

where the second equals sign uses (A.1) and (A.2).

Since $\{\epsilon^i_{S,m}\}^t_{m=1}$ and $\epsilon_{q,t}$ are independent with any $\mathcal{F}^e_m$ for $0 \leq m \leq t$, we know that

$$(E^i_t[P_{t+1} + D_{t+1}] - (1+r)P_t)|\mathcal{F}^e_m \sim N(-\sigma_t(P_{t+1} + D_{t+1})q_t, \sigma^2_t(P_{t+1} + D_{t+1})(1 + \frac{1}{\rho_q}))$$

so the expected excess return from trading the risky asset at period t is also the same for any $\mathcal{F}^e_m$, $0 \leq m \leq t$, and can be denoted as v_t. It is given by

$$v_t = E[\max\{E^i_t[P_{t+1} + D_{t+1}] - (1+r)P_t, 0\}|\mathcal{F}^e_t]$$

Using the distribution we have derived above,

$$v_t = \int_0^\infty x\phi(\frac{x + \sigma_t(P_{t+1} + D_{t+1})q_t}{\sigma_t(P_{t+1} + D_{t+1})\sqrt{1 + \frac{1}{\rho_q}}})dx$$

which gives

$$v_t = \sigma_t(P_{t+1} + D_{t+1})(\sqrt{1 + \frac{1}{\rho_q}}\phi(-\frac{q_t}{\sqrt{1 + \frac{1}{\rho_q}}}) - q_t\Phi(-\frac{q_t}{\sqrt{1 + \frac{1}{\rho_q}}}))$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are PDF and CDF of the standard normal distribution respectively.

As a result, total expected excess returns discounted to period t from entering at t is

$$V_t = \sum_{m=t}^{T-1}(\frac{1}{1+r})^{m-t}(\Pi_{j=t}^{m-1}(1 - \lambda_j))v_m$$

By Proposition 3.1, investors want to maximize their discounted expected wealth at period T. Thus, potential investors will enter only if

$$V_t \geq e$$

Moreover, they also need to decide when to enter. Since potential investors make the same entry decision, if there are new entrants at both period t and $t+1$, the following indifference condition must hold:

$$V_t - c = \frac{1}{1+r}E_t[\max\{V_{t+1} - c, 0\}|\mathcal{F}_t^e]$$

which implies

$$v_t = (1 - \frac{1 - \lambda_t}{1+r})e$$

where $\lambda_t = \gamma/(\beta + \gamma + t + 1)$ is investors' expectation on the probability that uncertainty will be resolved in the next period, in which case they will not enter.

In addition, in this model, investors can always choose not to hold the risky asset, so it is obvious that there are always positive expected gains for investors to trade the risky asset before uncertainty gets resolved. Thus, n_t is nondecreasing during those T periods.

At last, we show result (4). Since

$$\frac{\partial v_t}{\partial q_t} = -\Phi(-\frac{q_t}{\sqrt{1+\frac{1}{\rho_q}}}) < 0$$

v_t is decreasing in q_t and thus in n_t. This and the indifference condition indicate that as long as $(1-(1-\lambda_t)/(1+r))/\sigma_t(P_{t+1}+D_{t+1})$ is decreasing over time, we have gradual inflows of investors. In addition, as investors' beliefs gradually converge, v_t decreases over time, so potential investors will keep entering till $V_t < e$. □

A.1.5 Proof of Proposition 3.4

Proof. The proof consists of two parts. We first show the existence and uniqueness for the equilibrium price function, then for the measure of rational investors trading the risky asset.

Recall the proof to Proposition 3.2, we have

$$\mu_t = p_{\epsilon,t}/p_{\Pi,t} = \sqrt{1/\rho_{\tilde{S},t}}$$

thus,

$$\rho_t^c = \rho_{t-1}^c + \rho_{D,t} + \rho_q/\mu_t^2 = \rho_{t-1}^c + \rho_{D,t} + \rho_{\tilde{S},t}\rho_q$$

this gives

$$\rho_t^c = \rho_0^c + \sum_{m=0}^{t}(\rho_{D,m} + \rho_{\tilde{S},m}\rho_q)$$

By the proof of Proposition 3.2, ρ_t^c uniquely determines $\{p_{\Pi,t}, \{p_{q,tm}\}_{m=t}^{T-1}, p_{\epsilon,t}\}$. This suggests the existence and uniqueness of parameters in the price function.

The existence of the measure of rational investors trading the risky asset $\{n_t\}$ is obvious.

Now we show its uniqueness. Suppose not, then there exist two equilibria $N_t^1 \equiv \{n_t^1\}$ and $N_t^2 \equiv \{n_t^2\}$ s.t. $N_t^1 \neq N_t^2$. There must exist $0 \leq \tilde{t} \leq T-1$, s.t. $n_{\tilde{t}}^1 \neq n_{\tilde{t}}^2$ and $n_t^1 = n_t^2$ for $\tilde{t}+1 \leq t \leq T-1$. Without loss of generality, assume $n_{\tilde{t}}^1 > n_{\tilde{t}}^2$. Since $n_t^1 = n_t^2$ for $\tilde{t}+1 \leq t \leq T-1$ and the flow payoff v_t is strictly decreasing in n_t, we have

$$V_{\tilde{t}}(n_{\tilde{t}}^1, n_{\tilde{t}+1}^1, \ldots, n_{T-1}^1) < V_{\tilde{t}}(n_{\tilde{t}}^2, n_{\tilde{t}+1}^2, \ldots, n_{T-1}^2) \leq e$$

This and for any $0 \leq t \leq T-1$, $V_t(n_t, \ldots, n_{T-1}) \leq e$ with "$<$" only if $n_t = n_{t-1}$ imply

$$n_{\tilde{t}}^1 = n_{\tilde{t}-1}^1$$

Thus

$$n_{\tilde{t}-1}^1 = n_{\tilde{t}}^1 > n_{\tilde{t}}^2 \geq n_{\tilde{t}-1}^2$$

Continuing this process, we will get

$$n_0^1 > n_0^2 \geq n_{-1},\ V_0(N_t^1) < V_0(N_t^2) \leq e$$

From $n_0^1 > n_0^2 \geq n_{-1}$, we know that for $\{n_t^1\}$, there are capital inflows at period 0. This implies $V_0(N_t^1) = e$, contradicting with $V_0(N_t^1) < V_0(N_t^2) \leq e$.

Therefore, $\{n_t\}$ must be unique. □

A.1.6 Proof of Proposition 4.1

Proof. First, we introduce some notations:

$$\begin{aligned}
O_t &= \frac{\sigma_t(P_{t+1} + D_{t+1})}{1 - \frac{1-\lambda_t}{1+r}} \\
B_t &= \frac{\frac{1}{1-\frac{1-\lambda_{t-1}}{1+r}}(1 + r - (1-\lambda_{t-1})\frac{\rho_{t-1}^c}{\rho_t^c})\alpha_{t-1}\sqrt{\frac{1}{\rho_{\tilde{S},t-1}}}}{\frac{1}{1-\frac{1-\lambda_t}{1+r}}(1+r)\sqrt{\frac{1}{\rho_{\tilde{S},t}}} - \frac{1}{1-\frac{1-\lambda_{t-1}}{1+r}}(1-\lambda_{t-1})\frac{\rho_{t-1}^c}{\rho_t^c}\alpha_{t-1}\sqrt{\frac{1}{\rho_{\tilde{S},t-1}}}}
\end{aligned}$$

$$
\begin{aligned}
C_t &= \frac{1}{\frac{1-\lambda_{t-1}}{1+r}\frac{\rho^c_{t-1}}{\rho^c_t}\alpha_{t-1}}(B_{t-1} - (1 - \frac{1-\lambda_{t-1}}{1+r}\frac{\rho^c_{t-1}}{\rho^c_t})\alpha_{t-1}) \\
D_t &= B_t \frac{1}{\sqrt{\rho_{\tilde{S},t}}}
\end{aligned}
$$

Assume the following regularity conditions:

(A1) $B_t \geq 0$ for any $0 \leq t \leq T-2$. There exists $\tilde{t}$ s.t. $0 < \tilde{t} \leq T-2$, $D_{t-1} < D_t$ for $0 \leq t < \tilde{t}$, and $D_{t-1} > D_t$ for $\tilde{t} \leq t \leq T-2$.

(A2) $p_{\Pi,T-2} = (1 - \frac{1-\lambda_{T-2}}{1+r}(1-\alpha_{T-1})\frac{\rho^c_{T-2}}{\rho^c_{T-1}})\alpha_{T-2} < B_{T-2}$.

(A3) $c \leq v_0(q_{n,-1})/(1 - \frac{\lambda_0}{1+r})$

Now we show that under those regularity conditions, we have gradual inflows of investors for some periods before they stop. To show this, by (1.8), we only need to show that O_t increases first then decreases.

Since

$$
O_{t+1} - O_t = \frac{\sigma_{t+1}(P_{t+2} + D_{t+2})}{1 - \frac{1-\lambda_{t+1}}{1+r}} - \frac{\sigma_t(P_{t+1} + D_{t+1})}{1 - \frac{1-\lambda_t}{1+r}}
$$

Using the results for $\sigma_t(P_{t+1} + D_{t+1})$ and $p_{\Pi,t}$, we have

$$
O_{t+1} - O_t = \frac{(1+r)p_{\Pi,t+1}\sqrt{\frac{1}{\rho_{\tilde{S},t+1}}}}{1 - \frac{1-\lambda_{t+1}}{1+r}} - \frac{(1+r)(1 - \frac{1-\lambda_t}{1+r}(1-p_{\Pi,t+1})\frac{\rho^c_t}{\rho^c_{t+1}})\alpha_t\sqrt{\frac{1}{\rho_{\tilde{S},t}}}}{1 - \frac{1-\lambda_t}{1+r}}
$$

Using Assumption (A1), we get

$$
sgn(O_{t+1} - O_t) = sgn(p_{\Pi,t+1} - B_{t+1}) \tag{A.3}
$$

Similarly,

$$
sgn(O_t - O_{t-1}) = sgn(p_{\Pi,t} - B_t)
$$

plugging in

$$p_{\Pi,t} = (1 - \frac{\lambda_t}{1+r}(1 - p_{\Pi,t+1})\frac{\rho^c_t}{\rho^c_{t+1}})\alpha_t$$

we have

$$sgn(O_t - O_{t-1}) = sgn(p_{\Pi,t+1} - C_{t+1}) \tag{A.4}$$

Besides,

$$B_{t+1} - C_{t+1} = \frac{1}{\frac{1-\lambda_t}{1+r}\frac{\rho^c_t}{\rho^c_{t+1}}\alpha_t}((\frac{1-\lambda_t}{1+r}\frac{\rho^c_t}{\rho^c_{t+1}}\alpha_t B_{t+1} + (1 - \frac{1-\lambda_t}{1+r}\frac{\rho^c_t}{\rho^c_{t+1}})\alpha_t) - B_t)$$

which gives

$$sgn(B_{t+1} - C_{t+1}) = sgn(D_{t+1} - D_t) \tag{A.5}$$

Assumption (A2) and (A.3) imply $O_{T-2} < O_{T-3}$. By Assumption (A1), $D_{T-3} > D_{T-2}$, and thus $B_{T-2} < C_{T-2}$. This and Assumption (A2) indicate $p_{\Pi,T-2} < C_{T-2}$, and with (A.4), this gives $O_{T-3} < O_{T-4}$. By (A.3), we have $p_{\Pi,T-3} < B_{T-3}$. We can continue this process till period $\tilde{t}-1$, and we have $O_{\tilde{t}-2} > O_{\tilde{t}-1} > \ldots > O_{T-2}$.

For period $\tilde{t}-1$, if $p_{\Pi,\tilde{t}-1} > C_{\tilde{t}-1}$, we have $O_{\tilde{t}-3} < O_{\tilde{t}-2}$ and thus by (A.3), $p_{\Pi,\tilde{t}-2} > B_{\tilde{t}-2}$. Since D_t is increasing before $\tilde{t}$ which implies $B_{\tilde{t}-2} > C_{\tilde{t}-2}$, $p_{\Pi,\tilde{t}-2} > C_{\tilde{t}-2}$ and thus by (A.4), $O_{\tilde{t}-4} < O_{\tilde{t}-3}$. Continuing this process till period 0, we have $O_0 < \ldots < O_{\tilde{t}-2}$. Therefore, O_t is increasing from period 0 to $\tilde{t}-2$ and decreasing afterwards. This means that there are gradual inflows of investors till period $\tilde{t}-2$ or earlier when $V_t < c$. Furthermore, Assumption (A3) guarantees positive inflows at the first period.

If $p_{\Pi,\tilde{t}-1} < C_{\tilde{t}-1}$, by (A.4), $O_{\tilde{t}-3} > O_{\tilde{t}-2}$. There are two possibilities here. One is $p_{\Pi,t} < C_t$ for all $0 \le t \le \tilde{t}-1$. This implies that O_t is decreasing over time. By Assumption (A3), we have inflows of investors at the first period and none afterwards. The other possibility is that there exists a $\tilde{t}'$ s.t. $0 < \tilde{t}' < \tilde{t}-1$ and $p_{\Pi,\tilde{t}'} > C_{\tilde{t}'}$. Following the same procedure as above, we can show that O_t is increasing from period 0 to $\tilde{t}'-1$ and decreasing afterwards, and thus we have gradual inflows of investors first and they stop at period $\tilde{t}' - 1$ or earlier when $V_t < e$. $\square$

A.1.7 Proof of Proposition 4.2

Proof. Using the results for $(\{p_{q,tm}\}_{m=t}^{T-1}, p_{q,tt})$ in Proposition 3.2, we can rewrite price movements as

$$\begin{aligned}
\triangle \bar{P}_t &= (1 - \frac{1-\lambda_t}{1+r})(\sum_{m=t}^{T-2} (\Pi_{j=t+1}^{m} \frac{1-\lambda_j}{1+r})(1+r)(O_{m+1}q_{m+1} - O_m q_m) \\
&\quad - (\Pi_{m=t+1}^{T-1} \frac{1-\lambda_m}{1+r})(1+r) O_{T-1} q_{T-1})
\end{aligned}$$

where $O_t = \frac{\sigma_t(P_{t+1}+D_{t+1})}{1-\frac{1-\lambda_t}{1+r}}$, as defined in the proof of Proposition 4.1.

Under regularity conditions (A1)-(A3), from the proof of Proposition 4.1, we know that O_t is increasing first then decreasing. Thus for period t,

(1) If O_m is decreasing at $t \leq m \leq T-1$, since q_m is nondecreasing, we have $\triangle P_t < 0$.

(2) If $O_{t+1} > O_t$, assume $\triangle P_t < 0$, with nondecreasing q_m, we have

$$\begin{aligned}
\triangle \bar{P}_{t+1} &= \triangle n_{1,t+1} + \triangle n_{2,t+1} \\
&= \frac{1 - \frac{1-\lambda_{t+1}}{1+r}}{\frac{1-\lambda_{t+1}}{1+r}} (\frac{\triangle n_{1,t} + \triangle n_{2,t}}{1 - \frac{1-\lambda_t}{1+r}} - (1+r)(O_{t+1}q_{t+1} - O_t q_t)) \\
&< 0
\end{aligned}$$

This implies that if $\triangle P_t < 0$, $\triangle P_m < 0$ for any $t \leq m \leq T-1$.

Therefore, there exists $\tilde{t}$ s.t. $0 \leq \tilde{t} < T-1$, $\triangle P_t \geq 0$ for $0 \leq t < \tilde{t}$ and $\triangle P_t < 0$ for $\tilde{t} < t \leq T-1$. $\square$

A.1.8 Proof of Proposition 4.3

Proof. From the proof of Proposition 3.3, we have

$$E^i_{t-1}[P_t + D_t] - (1+r)P_{t-1} = (1+r)p_{\Pi,t-1}(\frac{1}{t}\sum_{m=0}^{t-1} S^i_m - \Pi) - \sigma_{t-1}(P_t + D_t)(q_{t-1} + \epsilon_{q,t-1})$$

Thus, $E^i_{t-1}[P_t + D_t] \geq (1+r)P_{t-1}$ implies

$$\sqrt{t\rho_S}(\frac{1}{t}\sum_{m=0}^{t-1} S^i_m - \Pi) \geq \sqrt{t\rho_S}\frac{\sigma_{t-1}(P_t + D_t)}{(1+r)p_{\Pi,t-1}}(q_{t-1} + \epsilon_{q,t-1}) = q_{t-1} + \epsilon_{q,t-1}$$

Here, $\sqrt{t\rho_S}(\frac{1}{t}\sum_{m=0}^{t-1} S^i_m - \Pi) \sim N(0,1)$.

Similarly, $E^i_t[P_{t+1} + D_{t+1}] < (1+r)P_t$ implies

$$\sqrt{\rho_S}(S^i_t - \Pi) \leq \sqrt{t+1}(q_t + \epsilon_{q,t}) - \sqrt{t}(\sqrt{t\rho_S}(\frac{1}{t}\sum_{m=0}^{t-1} S^i_m - \Pi))$$

where $\sqrt{\rho_S}(S^i_t - \Pi) \sim N(0,1)$.

Let $\epsilon_{q,t} = 0$, $\forall 0 \leq t \leq T$. We have at time t, the volume of speculative trading is

$$\begin{aligned} \overline{TV}_t &= n_{t-1}Pr(E^i_{t-1}[P_t + D_t] \geq (1+r)P_{t-1}, E^i_t[P_{t+1} + D_{t+1}] < (1+r)P_t) \\ &= n_{t-1}\int_{q_{t-1}}^{\infty} \Phi(\sqrt{t+1}q_t - \sqrt{t}y)\phi(y)dy \end{aligned}$$

Here, after capital inflows stop, n_t and q_t will stay constant. Denoting them as n and q, given $y \geq q$, we have

$$\frac{\partial(\sqrt{t+1}q - \sqrt{t}y)}{\partial t} < \frac{1}{2}(\frac{1}{\sqrt{t+1}} - \frac{1}{\sqrt{t}})q < 0$$

and thus, $\overline{TV}_t$ is decreasing over time, which also implies it should peak before or when capital inflows stop.

Notice that the volume of speculative trading only depends on capital flows in recent two periods, so we can denote it as $\overline{TV}_t(q_{t-1}, q_t)$ and it is easy to check that it is increasing in q_t. Given q_{t-1} and q_t, define $\triangle\overline{TV}_t(q_{t+1}) = \overline{TV}_{t+1}(q_t, q_{t+1}) - \overline{TV}_t(q_{t-1}, q_t)$. It is continuous and increasing in q_{t+1}, $\triangle\overline{TV}_t(q_t) < 0$, and $\lim_{q_{t+1}\to+\infty} \triangle\overline{TV}_t(q_{t+1}) = 1 - \overline{TV}_t(q_{t-1}, q_t) > 0$. These properties indicate that there exists $\tilde{q}_{t+1} > q_t$, s.t. $\triangle\overline{TV}_t > 0$ for $q_{t+1} > \tilde{q}_{t+1}$. Thus with sufficient capital inflows, the volume of speculative trading increases over time. □

A.2 Discussion on noise traders

The intuition for the specification of noise traders is as follows. In each period, due to differential private information, rational investors differ in the highest price they are willing to pay for the risky asset, which I call price target. Under the assumption of normal distributions for the prior and signals, given all the available information including the current price, the distribution of price targets across rational investors trading the risky asset also follows a normal distribution. This and the short-sale constraint indicate that the current asset price should equal the price target of the investor at the highest $\frac{1-\epsilon_{n,t}}{n_t}$-quantile of this distribution. In most cases, prices are above those investors' average price target. Thus, I define $\tilde{q}_t$ to be how many units of standard deviation the price is above those investors' average price target. In equilibrium, the asset demand equals the asset supply, i.e.,

$$n_t(1 - \Phi(\tilde{q}_t)) = 1 - \epsilon_{n,t}$$

this implies

$$\tilde{q}_t = \Phi^{-1}(\frac{\tilde{n}_t - 1}{n_t})$$

where $\Phi(\cdot)$ is the CDF of the standard normal distribution. Since the total measure of investors trading the risky asset is noisy, this distance measure $\tilde{q}_t$ is also noisy and thus

I assume it equals q_t plus a normal noise term. Under this specification, n_t is strictly increasing in q_t. The intuition is as follows. Given $\epsilon_{n,t} = 0$, with capital inflows, that is, n_t increases, there are more investors, especially optimistic investors, trading the risky asset. Thus the equilibrium price is higher and so is the distance between the price and investors' average price target q_t.

A.3 Computation

The computation procedure for solving the equilibrium is:

(1) Compute $\{\lambda_t\}$ using Proposition 2.1.

(2) By Proposition 3.2, compute $\{\rho_t^c\}$ first, then $\{p_{\Pi,t}, p_{\epsilon,t}, (p_{q,tm})_{m\geq t}\}$.

(3) By Proposition 3.4, the linear partially revealing equilibrium is unique, so we can solve equilibrium capital flows by construction. By Proposition 4.1, we have gradual inflows of rational investors and they stop after some periods. Thus, first, using (1.9), solve $\{n_t\}$ and find the period t_1 after which n_t starts to decrease (with cap restrictions on capital flows $\{\bar{n}_t\}$, denoting the above solution as $\{\tilde{n}_t\}$, we have $n_t = \min\{\tilde{n}_t, \bar{n}_t\}$); next, using (1.8), compute V_t if capital inflows stop at period t, find the first period t_2 when $V_t < e$; $\tilde{t} = \min\{t_1, t_2\}$ is the period when capital inflows stop. Re-solve $n_{\tilde{t}}$ s.t. $V_{\tilde{t}} = e$. Thus, the equilibrium measure of rational investors trading the risky asset is $\{n_1, \ldots, n_{\tilde{t}}, \ldots, n_{\tilde{t}}\}$.

In addition, trading volume is computed using Gauss-Legendre quadrature, and the measure of first-time buyers is by simulation.

Appendix to Chapter 2

B.1 Data: Productivity

The data for Chinese firms is from Annual Surveys of Industrial Production from 1998 through 2008 conducted by the National Bureau of Statistics of China. It contains all non-state owned enterprises with the annual revenue from principal business over 5 million yuan (about 0.6 million dollars) and all state-owned enterprises.

First, I drop mining and utilities industry, which take up 8% of the whole sample, and focus on manufacturing firms. Then I drop observations with no response or non-positive value in either value-added, capital stock or labor, which account for 20.7% of the rest sample. Since firms do not report their value added in year 2008, I use data from 1998 through 2007 to compute individual firms' productivity below.

Next, I use the producer price index for manufactured goods as the deflator for the value-added, and the price index for investment in fixed assets for the capital stock. The capital stock is calculated by using the original value of capital minus the cumulative depreciation. Ideally, we should use the producer price index for each industry as the deflator, but we cannot because the data before 2002 is not available.

Given the production function, I calculate individual firms' productivity by

$$\bar{Z}_t z_{it} = y_{it}/(k_{it}^{\alpha} l_{it}^{\gamma})$$

where y_{it} is the deflated value-added of firm i at year t, k_{it} is the deflated capital stock, and l_{it} is the annual average number of employed workers. Here, to reduce the bias caused by the extreme value, I define the technology frontier $\bar{Z}_t$ as the average TFP of the top 5% firms in year t. I calculate z_{it} for each year and then pool to get the distribution $G(z)$.

BIBLIOGRAPHY

Abreu, D., and M. K. Brunnermeier (2003): "Bubbles and Crashes," *Econometrica*, 71(1), 173–204.

Acemoglu, D., U. Akcigit, N. Bloom, and W. R. Kerr (2013): "Innovation, Reallocation and Growth," NBER Working Paper 18993.

Aghion, P., T. Fally, and S. Scarpetta (2007): "Credit Constraints as a barrier to the entry and post-entry growth of firms," *Economic Policy*, 22, 731–779.

Aiyagari, S. R. (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics*, 109(3), 659–684.

Al-Najjar, N. I. (2004): "Aggregation and the Law of Large Numbers in Large Economies," *Games and Economic Behavior*, 47(1), 1–35.

Allen, F., and G. Gorton (1993): "Churning Bubbles," *Review of Economic Studies*, 60(4), 813–836.

Allen, F., S. Morris, and A. Postlewaite (1993): "Finite Bubbles with Short Sale Constraints and Asymmetric Information," *Journal of Economic Theory*, 61(2), 206–229.

Allen, F., S. Morris, and H. S. Shin (2006): "Beauty Contests and Iterated Expectations in Asset Markets," *Review of Financial Studies*, 19(3), 719–752.

Amaral, P. S., and E. Quintin (2010): "Limited Enforcement, Financial Intermediation, and Economic Development: a Quantitative Assessment," *International Economic Review*, 51(3), 785–811.

Avery, C., and P. Zemsky (1998): "Multidimensional Uncertainty and Herd Behavior in Financial Markets," *American Economic Review*, 88(4), 724–748.

Banerjee, A. V., and E. Duflo (2005): "Growth Theory Through the Lens of Development Economics," *Handbook of Economic Growth*, 1, 473–552.

Barlevy, G., and P. Veronesi (2000): "Information Acquisition in Financial Markets," *Review of Economic Studies*, 67(1), 79–90.

——— (2003): "Rational Panics and Stock Market Crashes," *Journal of Economic Theory*, 110(2), 234–263.

Barseghyan, L., and R. DiCecio (2009): "Entry Costs, Industry Structure, and Cross-Country Income and TFP Differences," Federal Reserve Bank of St. Louis Working Paper.

Berk, J. B., and R. C. Green (2004): "Mutual Fund Flows and Performance in Rational Markets," *Journal of Political Economy*, 112(6), 1269–1295.

Bewley, T. (1986): "Stationary Monetary Equilibrium with a Continuum of Independently Fluctuating Consumers.," in *Contributions to mathematical economics: in honor of Gerard Debreu.*, ed. by M.-C. A. Hildenbrand, W. North-Holland, New York.

Blanchard, O., and M. Watson (1982): "Bubbles, Rational Expectations and Financial Markets,," in *Crises in the Economic and Financial Structure.*, ed. by P. Wachtel. Lexington Books, Lexington, Mass.

Boedo, H. J. M., and T. Mukoyama (2012): "Evaluating the Effects of Entry Regulations and Firing Costs on International Income Differences," *Journal of Economic Growth*, 17, 143–170.

Bosworth, B., and S. M. Collins (2008): "Accounting for Growth: Comparing China and India," *Journal of Economic Perspectives*, 22(1), 45–66.

Brandt, L., and X. Zhu (2009): "Accounting for China's Growth," University of Toronto Working Paper.

Buera, F. J., J. P. Kaboski, and Y. Shin (2011): "Finance and Development: A Tale of Two Sectors," *American Economic Review*, 101(5), 1964–2002.

Buera, F. J., and Y. Shin (2013): "Financial Frictions and the Persistence of History: A Quantitative Exploration," *Journal of Political Economy*, 121(2), 221–272.

Burnside, C., M. Eichenbaum, and S. Rebelo (2013): "Understanding Booms and Busts in Housing Markets," Working Paper.

Caplin, A., and J. Leahy (1994): "Business as Usual, Market Crashes, and Wisdom After the Fact," *American Economic Review*, 84(3), 548–565.

Chamley, C. (2008): "On "Acquisition of Information in Financial Markets"," *Review of Economic Studies*, 75(4), 1081–1084.

Chevalier, J., and G. Ellison (1997): "Risk Taking by Mutual Funds as a Response to Incentives," *Journal of Political Economy*, 105(6), 1167–1200.

Davis, M. A., and J. Heathcote (2007): "The Price and Quantity of Residential Land in the United States," *Journal of Monetary Economics*, 54(8), 2595–2620.

Diamond, D. W., and R. E. Verrecchia (1981): "Information Aggregation in a Noisy Rational Expectations Economy," *Journal of Financial Economics*, 9(3), 221–235.

Duffie, D. (2010): "Presidential Address: Asset Price Dynamics with Slow-Moving Capital," *Journal of Finance*, 65(4), 1237–1267.

Ericson, R., and A. Pakes (1995): "Markov-Perfect Industry Dynamics: A Framework for Empirical Work," *Review of Economic Studies*, 62(1), 53–82.

Feldman, M., and C. Gilles (1985): "An Expository Note on Individual Risk without Aggregate Uncertainty," *Journal of Economic Theory*, 35(1), 26–32.

Froot, K. A., D. S. Scharfstein, and J. C. Stein (1992): "Herd on the Street: Informational Inefficiencies in a Market with Short-Term Speculation," *Journal of Finance*, 47(4), 1461–1484.

Ganguli, J. V., and L. Yang (2009): "Complementarities, Multiplicity, and Supply Information," *Journal of the European Economic Association*, 7(1), 90–115.

Garber, P. (2000): *Famous First Bubbles: the Fundamentals of Early manias*. The MIT Press.

García, D., and G. Strobl (2011): "Relative Wealth Concerns and Complementarities in Information Acquisition," *Review of Financial Studies*, 24(1), 169–207.

Gennotte, G., and H. Leland (1990): "Market Liquidity, Hedging, and Crashes," *American Economic Review*, 80(5), 999–1021.

Giné, X., and R. M. Townsend (2004): "Evaluation of Financial Liberalization: a General Equilibrium Model with Constrained Occupation Choice," *Journal of Development Economics*, 74(2), 269–307.

Goldstein, I., E. Ozdenoren, and K. Yuan (2013): "Trading Frenzies and Their Impact on Real Investment," *Journal of Financial Economics*, 109(2), 566–582.

Greenwood, J., J. M. Sanchez, and C. Wang (2013): "Quantifying the Impact of Financial Development on Economic Development," *Review of Economic Dynamics*, 16(1), 194–215.

Grossman, S. (1976): "On the Efficiency of Competitive Stock Markets Where Trades Have Diverse Information," *Journal of Finance*, 31(2), 573–585.

——— (1988): "An Analysis of the Implications for Stock and Futures Price Volatility of Program Trading and Dynamic Hedging Strategies," *Journal of Business*, 61(3), 275–298.

Grossman, S. J., and J. E. Stiglitz (1980): "On the Impossibility of Informationally Efficient Markets," *American Economic Review*, 70(3), 393–408.

Harrison, J. M., and D. M. Kreps (1978): "Speculative Investor Behavior in a Stock Market with Heterogeneous Expectations," *Quarterly Journal of Economics*, 92(2), 323–336.

He, H., and J. Wang (1995): "Differential Informational and Dynamic Behavior of Stock Trading Volume," *Review of Financial Studies*, 8(4), 919–972.

Hellwig, M. F. (1980): "On the Aggregation of Information in Competitive Markets," *Journal of Economic Theory*, 22(3), 477–498.

Hirshleifer, D., A. Subrahmanyam, and S. Titman (1994): "Security Analysis and Trading Patterns When Some Investors Receive Information Before Others," *Journal of Finance*, 49(5), 1665–1698.

Hong, H., and J. C. Stein (2003): "Differences of Opinion, Short-Sales Constraints, and Market Crashes," *Review of Financial Studies*, 16(2), 487–525.

Hopenhayn, H. A. (1992): "Entry, Exit, and firm Dynamics in Long Run Equilibrium," *Econometrica*, 60(5), 1127–1150.

Hsieh, C.-T., and P. J. Klenow (2009): "Misallocation and Manufacturing TFP in China and India," *Quarterly Journal of Economics*, 124(4), 1403–1448.

Jeong, H., and R. Townsend (2007): "Sources of TFP growth: occupational choice and financial deepening," *Economic Theory*, 32(1), 179–221.

Jovanovic, B. (1982): "Selection and the Evolution of Industry," *Econometrica*, 50(3), 649–670.

Judd, K. L. (1985): "The Law of Large Numbers with a Continuum of IID Random Variables," *Journal of Economic Theory*, 35(1), 19–25.

Keynes, J. (1936): *The General Theory of Employment, Interest and Money*. Macmillan, London.

Lee, I. H. (1998): "Market Crashes and Informational Avalanches," *Review of Economic Studies*, 65(4), 741–759.

Levine, R. (2005): "Finance and Growth: Theory and Evidence," *Handbook of Economic Growth*, 1, 865–934.

Liu, Z., P. Wang, and T. Zha (2013): "Land-Price Dynamics and Macroeconomic Fluctuations," *Econometrica*, 81(3), 1147–1184.

Luttmer, E. G. J. (2007): "Selection, Growth, and the Size Distribution of Firms," *Quarterly Journal of Economics*, 122(3), 1103–1144.

Matsuyama, K. (2007): "Aggregate Implications of Credit Market Imperfections," in *NBER Macroeconomics Annual, vol. 22*, ed. by K. R. Daron Acemoglu, and M. Woodford. MIT press, Cambridge, MA.

Merton, R. C. (1987): "A Simple Model of Capital Market Equilibrium with Incomplete Information," *Journal of Finance*, 42(3), 483–510.

Milgrom, P., and N. Stokey (1982): "Information, Trade and Common Knowledge," *Journal of Economic Theory*, 26, 17–27.

Moll, B. (2014): "Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation?," *American Economic Review*, 104(10), 3186–3221.

Morris, S. (1996): "Speculative Investor Behavior and Learning," *Quarterly Journal of Economics*, 111(4), 1111–1133.

Ofek, E., and M. Richardson (2003): "DotCom Mania: The Rise and Fall of Internet Stock Prices," *Journal of Finance*, 58(3), 1113–1138.

Pakes, A., and P. McGuire (1994): "Computing Markov-Perfect Nash Equilibria: Numerical Implications of a Dynamic Differentiated Product Model," *RAND Journal of Economics*, 25(4), 555–589.

Pástor, Ĺ., and P. Veronesi (2009): "Technological Revolutions and Stock Prices," *American Economic Review*, 99(4), 1451–83.

Perkins, D., and T. G. Rawski (2008): "Forecasting China's Growth to 2025," in *China's Great Economic Transformation*, ed. by L. Brandt, and T. G. Rawski. Cambridge University Press, New York.

Romer, D. (1993): "Rational Asset-Price Movements without News," *American Economic Review*, 83(5), 1112–1130.

Santos, M. S., and M. Woodford (1997): "Rational Asset Pricing Bubbles," *Econometrica*, 65(1), 19–57.

Scheinkman, J. A., and W. Xiong (2003): "Overconfidence and Speculative Bubbles," *Journal of Political Economy*, 111(6), 1183–1220.

Shiller, R. (2005): *Irrational Exuberance*. Princeton University Press.

Singleton, K. J. (2012): "Investor Flows and the 2008 Boom/Bust in Oil Prices," Working Paper.

Sirri, E. R., and P. Tufano (1998): "Costly Search and Mutual Fund Flows," *Journal of Finance*, 53(5), 1589–1622.

Song, Z., K. Storesletten, and F. Zilibotti (2011): "Growing Like China," *American Economic Review*, 101(1), 196–233.

THE WORLD BANK (2001): "Bankruptcy of State Enterprises in China," Working Paper.

TIROLE, J. (1982): "On the Possibility of Speculation under Rational Expectations," *Econometrica*, 50(5), 1163–1181.

——— (1985): "Asset Bubbles and Overlapping Generations," *Econometrica*, 53(6), 1499–1528.

UHLIG, H. (1996): "A Law of Large Numbers for Large Economies," *Economic Theory*, 8(1), 41–50.

VELDKAMP, L. L. (2006a): "Information Markets and the Comovement of Asset Prices," *Review of Economic Studies*, 73(3), 823–845.

——— (2006b): "Media Frenzies in Markets for Financial Information," *American Economic Review*, 96(3), 577–601.

WANG, J. (1990): "Intertemporal Asset Prices under Asymmetric Information," Ph.D. thesis, University of Pennsylvania.

——— (1993): "A Model of Intertemporal Asset Prices under Asymmetric Information," *Review of Economic Studies*, 60(2), 249–282.

XU, S. (2010): "The Decomposition and Estimation of household labor income shocks with irregular data," *Management World*, (9), 67–75.

YOUNG, A. (2003): "Gold into Base Metals: Productivity Growth in the People's Republic of China during the Reform Period," *Journal of Political Economy*, 111(6), 1220–1261.

ZEIRA, J. (1999): "Informational Overshooting, Booms, and Crashes," *Journal of Monetary Economics*, 43(1), 237–257.

ZHENG, JINGHAI, A. B., AND A. HU (2009): "Can China's Growth be Sustained? A Productivity Perspective," *World Development*, 37, 874–888.