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Dynamic Contracting With Unobserved Progress

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Abstract
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In Chapter 3, I consider several modifications to the modeling assumptions and discuss their implications. First, I study the case that the two breakthroughs need not be in a particular order, and the agent can choose which task to work on first. It is shown that it is optimal to induce the agent to work on the more difficult task first. Second, I consider the scenario where there is an ex ante probability that the project is a bad one and breakthroughs never come. The optimal contract is no longer efficient conditional on the termination date. Last, I allow the principal to receive informative signals on whether the intermediate breakthrough has occurred.

In Chapter 4, I extend the baseline model by introducing randomly arriving buyers and apply it to study the financing of startup firms with opportunities to be acquired. I show that the potential acquisition increases the cost of providing incentives. Since an agent with low level of progress is ``bailed out" when an offer is made to acquire firms with both high and low levels of progress, the agent has more incentive to shirk. In response, the principal reduces the likelihood that the firm with high level of progress is sold. Moreover, the total financing provided by the principal is less compared to the environment without buyers.

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ABSTRACT

DYNAMIC CONTRACTING WITH UNOBSERVED PROGRESS

Zehao Hu

George Mailath

This thesis investigates agency problems in projects whereas the principal cannot effectively monitor the progress. In Chapter 2, the baseline model is studied. It is assumed that the success of innovation requires an intermediate breakthrough and a final breakthrough, but the occurrence of the intermediate breakthrough is privately known to the agent. The principal provides incentives to the agent through a termination date and a reward for the final success. Two properties of optimal contracts are identified. First, conditional on the termination date, the optimal contract induces efficient actions from the agent. Second, the reward for success to the agent is in general non-monotone in success time and later success may be rewarded more.

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the termination date. Last, I allow the principal to receive informative signals on whether the intermediate breakthrough has occurred.

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Chapter 1

Introduction

Most projects require continuing effort exerted or investment made before they succeed. Examples include scientists conducting scientific research, researchers working on R&D in a company, and entrepreneurs developing startup firms. Agency problems naturally arise in such contexts, and previous work has provided useful insights on how the agent should be incentivized (e.g., Bergemann and Hege (1998), Bergemann and Hege (2005), Manso (2011), Hörner and Samuelson (2013) and Halac, Liu, and Kartik (2013)). In existing literature, the success of a project is often modeled as a random arrival process as effort are being exerted. This treatment ignores an important feature of such projects: Typically a project consists of multiple steps and the success requires several breakthroughs rather than being one-shot. In other words, before the agent can work toward the final breakthrough that brings success, some intermediate breakthroughs must first be achieved. For example, before a chemist proceeds to synthesize the final product, she may have to first find a way to produce an important intermediate chemical.

The multi-step nature of a project may have significant implications on how contracts should be designed. What complicates the agency problem even more is that the agent often has better knowledge about the progress of the project than the principal. After the final breakthrough, the outcome is observable to all and performance can be tested. But before that, it may be difficult for people outside the team, including the principal, to monitor the level of progress. For example, Keil, Smith, Iacovou, and Thompson (2014b) review a series of 14 studies, including Snow, Keil, and Wallace (2007), Keil, Smith, Iacovou, and Thompson (2014a) and Iacovou, Thompson, and Smith (2009), which demonstrate the difficulties in monitoring the progress of IT projects. They conclude that project staff tend to misreport information on project status. Moreover, it does not necessarily help track the project progress to use an audit team or to have a senior executive involved in the project; in fact, misreporting may increase in some cases.

In such environment, the progress of the project and chance of future success depend (stochastically) on the agent's past actions. Since the agent is better informed of the project progress, information asymmetry endogenously arises along the way, even if ex ante it is a problem of pure moral hazard. This creates difficulties for the principal to design contracts and to provide incentives. The agent's incentive to work depends on the level of the progress (which affects the probability of future success). Ideally, the principal would like to choose different financing decisions and reward schemes based on how much progress has been achieved. But when the principal cannot differentiate agents with different levels of progress, the contract terms have to be the same for agents with different levels of progress;[1] yet their relevant incentives conditions have to be satisfied simultaneously under the contract.

In this thesis, I investigate this agency problem in the context of a model of two-step project, and study how the principal provides incentives to the agent when she cannot monitor the intermediate progress of the project. By contrasting with the case where the progress is observable and the case where the success only requires one breakthrough, I highlight the implications of the unobserved progress on the contract structure. The information asymmetry about the progress can actually lead to more efficient actions induced in an optimal contract. Moreover, it results in possible non-monotonicity in the reward schedule: later success may be rewarded more. Thus, the unobserved progress not only is a more realistic assumption, but also allows us to gain additional insights.

I also extend the model to investigate a more complex contracting environment in which the agent's private knowledge about the progress of innovation is an important concern. The application I consider is the financing of startup companies in the presence of occasionally arriving buyers that make acquisition offers. A large portion of startups end up being acquired or merged by other firms, and whether to "get big" or "get bought" is an important choice. The startup's founder usually knows better than the investor or outside buyers about how much progress has been made on innovation and the future prospects of the startup if not sold. In such an environment, the agent's private infor-

[1]In principle, the contract can induce reports from the agent on the innovation progress, but in this model, inducing and contracting on report do not improve the principal's payoff, as discussed in section A.1.

mation about the progress matters both for incentives to work and for decision-making on acquisition bids. These two aspects interact with each other: the principal would like to provide proper incentives so that the agent can use his information for better decisions about selling the company, but also needs to take into account that the incentives provided upon acquisition offers will in turn affect the moral hazard problem in innovation. I thus analyze the impact of potential acquisitions on agent's incentives to innovate and principal's finance decisions. Moreover, I examine how moral hazard in the innovation project affects company sale decisions.

In the rest of the introduction, I preview the models and results of each chapter in section 1.1, and discuss related literature in section 1.2.

1.1 Preview of the Models and Results

1.1.1 Baseline

Chapter 2 studies the baseline model. A principal finances an agent on an innovation project (e.g., an investor finances an entrepreneur to launch a startup company). The success of innovation requires *two* breakthroughs. In each period, if and only if a costly investment is made, *one* breakthrough occurs with positive probability.[2] Success is publicly observable, but only the agent observes the first breakthrough. In each period, the principal finances the agent with the cost of investment. There is a moral hazard problem:

[2]This feature that an action in a period affects the probability of the arrivial of some event is related to Biais, Mariotti, Rochet, and Villeneuve (2010), who look at an environment in which the agent can exert effort to reduce the probability of a large loss.

the agent can shirk and divert funds for his private consumption instead of truly investing in the project. As the project goes on, the agent develops into two possible types. I call the agent that has made the first breakthrough the *stage 1 agent* and the one that has made no breakthrough the *stage 0 agent*. The principal and agent can commit to a long-term contract, but payments and financing decisions can only depend on the event of success (but not the intermediate breakthrough).

In the first best outcome, the project is financed as long as success has not occurred,[3] but this is not optimal for the principal because it gives the agent too much rent. In an optimal contract, the principal chooses to finance the project until some termination time, and rewards the agent depending on the date of success. The stage 1 agent will exert effort as long as he is financed. In contrast, the stage 0 agent will stop investment and start fund diversion some time before the termination date. This is because it becomes too expensive to induce effort from the agent with no progress when it is close to the termination date. The reward has to be very high because the probability of making two breakthroughs becomes very low as time moves close to the termination time. Moreover, whenever the stage 0 agent is induced to work, his incentive compatibility constraints are binding and he is kept indifferent between working and shirking. In this way, the principal's cost of providing incentives is minimized.

I identify two properties of optimal contracts driven by the multi-step nature of the innovation and the agent's private knowledge of the project progress.

[3] This is because it is assumed that there is no ex ante uncertainty about the quality of the project.

First, given the termination time of the principal's financing, the induced agent's actions maximize the total social surplus. In other words, the stage 1 agent always works and the stage 0 agent is induced to work if and only if it is socially efficient to do so. The reasons are as follows: Since financing cannot terminate only for the stage 0 agent (because the principal does not know whether the agent is at stage 0 or stage 1), the total rent available to the agent is independent of when the stage 0 agent stops working. Given the total investment financed by the principal, the agent's ex ante payoff is fixed. In order to maximize the principal's payoff, the contract needs to induce actions from the agent so that the total social surplus from the project is maximized. I refer to this as *efficiency from ignorance*, because this efficiency result will not hold if the first breakthrough is public and contractible. In that environment, the principal prefers to terminates financing the stage 0 agent earlier than socially optimal to provide extra incentives to work in earlier periods. This comparison needs to be interpreted with caveats, because when the first breakthrough is contractible, the total financing provided by the principal is different, and depends on when the first breakthrough occurs. However, if there is a binding exogenous deadline for innovation, due to either time or budget constraint, then the principal's inability to monitor the agent's progress turns out to be beneficial from a social point of view.[4]

[4]The *efficiency from ignorance* result resonates with the desirability of an arm's-length relationship between principal and agent illustrated by Crémer (1995) and Bergemann and Hege (2005). In those environments, the benefit of loose monitoring comes from the lack of commitment, and arm's-length relationship makes threat of termination more credible. In my model, there is full commitment, so the principal cannot benefit from not observing the intermediate breakthrough, but her inability to monitor the progress may lead to more socially efficient outcome under some circumstances.

Second, the reward scheme that minimizes incentive cost is non-monotonic in the date of success. That is to say, the agent is not necessarily rewarded more for achieving success earlier. If success only requires one breakthrough, the reward is strictly decreasing in the date of success which Hörner and Samuelson (2013) identifies as the dynamic agency cost. In that setting, earlier success needs to be rewarded more throughout the contracting periods, because by working and possibly making success happen early, the agent gives up the opportunity to divert a large amount of funds in the future. Back to the setting of this paper, in the periods when the stage 0 agent is induced to work, the reward for success is decreasing due to the dynamic agency cost. However, in some of the periods after the stage 0 agent stops working, the reward for success is higher than if success happens in earlier dates. The intuition is that as time passes, it becomes increasingly difficult for stage 0 to reach success, so the increase in reward provides the extra incentive needed for him to work when it is socially efficient to do so. Moreover, this is the most cost-effective way to provide incentives because only the stage 1 agent will have a chance to achieve success and get the higher rewards in these periods. The non-monotonicity of reward is consistent with the use of time-vested stocks as part of the compensation for entrepreneurs in startup companies.

1.1.2 Extensions and Discussions

In Chapter 3, I consider several modifications to the modeling assumptions and discuss their implications.

First, I study the case that the two breakthroughs need not be in a particular order, and the agent can choose which task to work on first. It is shown that it is optimal to induce the agent to work on the more difficult task first.

Second, I consider the scenario where there is an ex ante probability that the project is a bad one and breakthroughs never come.The optimal contract is no longer efficient conditional on the termination date.

Next, I allow the principal to receive informative signals on whether the intermediate breakthrough has occurred. The social surplus generated is between the baseline case and the perfectly observable progress case.

1.1.3 Application: Acquisition Offers

In Chapter 4, the model is extended to study the role of potential acquisitions in the financing of startups. In addition to the baseline model discussed, I assume that at the end of each period, a buyer randomly arrives and makes an offer to acquire the company, and the agent decides to accept or reject it. The agent has a tendency to keep the firm independent, since by doing so he will continue to have access to the funds and may get rewarded for success in the future. This tendency is stronger for the stage 1 agent due to a larger probability of success. To induce the agent to accept an offer, the contract has to specify a severance payment no less than the agent's continuation value after the offer is rejected. In an optimal contract, in addition to the termination time of financing and the reward scheme for success, severance payments are specified contingent on sale prices so that certain offers will be accepted.[5] In each period there is a pair of price cutoffs for the stage 0 and 1 agents such that the agent accepts an offer if and only if the offer is higher than the corresponding cutoff. Since the continuation value for the stage 1 agent is higher, his cutoff price for acceptance is also higher.

I show that the possibility of acquisitions incurs additional cost to incentivize innovation. If the agent always shirks from the beginning, not only does he consume funds financed by the principal, but there is also a chance that a high enough acquisition offer arrives such that the company should be sold no matter it is at stage 0 or stage 1. In that case, the stage 0 agent receives a severance pay equal to what the stage 1 agent would receive, which is higher than his continuation value for keeping the company unsold. Therefore, the agent receives a *acquisition rent*, and his value from always shirking is higher than without acquisition offers. To induce effort, the principal needs to provide higher rewards for success.

The increased incentive cost due to potential buyers has an impact on the company sale decisions induced by the optimal contract. In the case without agency problem, an offer should be accepted if and only if it is higher than the continuation surplus to be generated when the company is not sold in that period. However, because of the moral hazard problem, acquisition rent arises due to the possibility that the stage 0 agent may get the stage 1 agent's continuation value when the stage 1 company is sold. In order to reduce

[5] The use of severance pay to induce the agent's optimal use of his private information is related to work on CEO turnover such as Laux (2008) and Inderst and Mueller (2010). They look at problems in which the CEO has private information about the profitability of the firm under his management, and severance pay may be used together with a steep incentive pay to induce low types of CEO to reveal his information and to leave the firm.

the incentive cost and the agent's acquisition rent, the principal would like to reduce the likelihood that the stage 1 company is sold, and the cutoff offer for the stage 1 company is higher than the continuation surplus after rejecting the offer. Selling only the stage 0 company does not affect the agent's payoff, and thus the cutoff offer for stage 0 is equal to the continuation surplus of the project. In other words, the company with more progress is only sold at a premium over its value if kept independent. I call this premium the *moral hazard premium*. This result suggests that the moral hazard and the agent's private information about the innovation progress may together aggravate the lemon problem in the market for startups.

Finally, the possibility of acquisitions will also affect the principal's optimal amount of financing. On the one hand, it is more costly to induce one more period of investment due to the acquisition rent; on the other hand, the benefit of financing investment for one more period is smaller, because it is possible that the company has already been sold off and there is no need for the additional investment. As a result, the total investment will be less than when there are no buyers.

1.2 Literature

The closest paper to this thesis is a contemporaneous work by Green and Taylor (2014). They study a continuous time model with no discounting where (as here) the success of a project takes two breakthroughs. The key difference between their model and mine is that Green and Taylor assume there is an efficiency loss when the agent diverts funds financed by the principal while in my model the agent can divert funds efficiently. In their case, even without a public randomizing device, it is no longer without loss generality to only consider contracts with no reports, and they focus on the role of communication between the principal and the agent. My model allows me to have the optimal contract in a simpler form and to highlight the insights we obtain by introducing unobserved progress: *efficiency from ignorance* and *rewards for later success*. Moreover, without the need to consider communication and randomization, we can extend the model to study more complex applications such as the financing of startups with arriving acquisition offers.

This thesis contributes to the literature on agency problems in innovation and experimentation. Bergemann and Hege (1998), Bergemann and Hege (2005) and Hörner and Samuelson (2013) study experimentation models where there is an ex ante probability that the project is of bad quality. Breakthrough happens in one shot and there is a lack of commitment power. These papers highlight the impact of learning about the quality of the project on the agent's incentives to work. Halac, Liu, and Kartik (2013) look at long term contracts for experimentations and allow for agent's private information about his own ability, but they do not impose limited liability on the agent. Manso (2011) looks at a different innovation problem which consists of two periods. Apart from working and innovating, the agent has a safe option, and he found that to motivate innovation, the incentive scheme needs to be tolerant with early failure. Our key novelty is to model the innovation as a multi-step process, and it enables us to study implications of the information asymmetry about the progress of innovation. The *efficiency from ignorant* result resonate the desirability of an arm's-length relationship between principal and agent illustrated by Crémer (1995) and Bergemann and Hege (2005). In their settings, the benefit of loose monitoring comes from lack of commitment, and arm's-length relationship makes threat of termination more credible. In our environment, no monitoring is socially efficient under some circumstances even under full commitment.

The paper is also broadly related papers on principal-agent problems where the profitability in each period depends on the value of a changing state. For example, Kwon (2014) and DeMarzo and Sannikov (2011) look at problems where the principal and the agent share common initial beliefs and the state evolve exogenously over time. The agent can interfere with the principal's learning by private deviations in effort. Garrett and Pavan (2012) and Garrett and Pavan (2013) consider the case where the productivity of the agent is his own private information, and focus the mechanism design aspect.

The application of our framework on the financing of startups with randomly arrived buyers connects with the literature on takeovers. Grossman and Hart (1980) shows that takeovers can play a disciplinary role for the management, because the company of a manager with poor performance may get taken over. The raider profits from the takeover and incumbent manager loses his job. On contrary, Stein (1988) argues that if stockholders are imperfectly informed, the takeover threat will lead to managerial myopia, but the conclusion is reached with the assumption of no agency problem. This result is often used as a justification for entrenchment. But our result shows that the principal's lack of information on innovation progress and the entrenchment of the agent together cause the takeover rent, and investment level will decrease, which is more inefficient.

Finally, the use of severance pay to induce agent's private information is related to work on CEO turnover such as Laux (2008) and Inderst and Mueller (2010).

Chapter 2

Baseline

2.1 Model

A principal (she) hires an agent (he) to work on an innovation project. Time is discrete $t = 0, 1, 2, ..., \infty$. The principal and the agent share a common discount factor $\delta \in (0, 1]$ and are both risk neutral. There is no ex ante uncertainty about either the type of the agent or the type of the principal. The project requires *two* breakthroughs to succeed. Once successful, the project generates a positive constant flow profit per period; before success, the profit generated is 0. The discounted value of cash flows for a successful project is Y. I say the project or the agent is at stage n if exactly $n \leq 2$ breakthroughs have been made. In each period, if an investment is made to develop the project, then a stage 0 project becomes stage 1 with probability q_1, and a stage 1 project becomes stage 2 (successful) with probability q_2.[1] The cost of investment per period is fixed at $c > 0$. Once the project is successful, no further investment is needed. The agent does not have financial resources and must acquire funding for the investment cost from the principal. If the agent receives the investment cost c in a period t, he can choose to make the investment honestly for a chance of a breakthrough (denoted by $a_t= 1$), or to divert it for his own private consumption, ($a_t = 0$).[2] Alternatively, I refer to making honest investment as *working* or *exerting effort*, and fund diversion as *shirking*.

Without the agency problem, the social planner's problem is straightforward. If the value of the successful project Y is large enough compared to the cost of investment c and the probabilities of breakthroughs q_1 and q_2, the planner would like to keep on investing until success. Otherwise, the planner will not invest at all. I assume that it is socially efficient to invest in the project, i.e., the expected discounted value of profits from the project is greater than the expected cost of investment. See the Appendix for the detailed calculations.

Assumption 1 (Efficiency)**.**

$$Y \geq \frac{[1-\delta+\delta(q_1+q_2)]}{\delta q_1 q_2} c.$$

The agent observes each breakthrough and thus knows exactly the stage of the project,

[1] In the experimentation literature (e.g.,Hörner and Samuelson (2013)), there is an ex ante probability of a poor quality project where investment will never lead to a breakthrough. I will discuss the implication of it in Section 3.2.

[2] It makes no difference if we allow the agent to invest a portion of the investment funds and divert the rest, as long as the breakthrough probability is linear in the portion of funds invested.

but the principal observes the final success. At any time t, there are two kinds of contractible histories: either the success has not occurred by time t, or the success happened in some period $t' < t$. The agent cannot make reports about the progress to the principal. In section A.1, I show that this restriction on the space of contracts is without loss of generality as long as there is no public randomizing device in the sense that contracting on reports cannot improve the principal's payoff in an optimal contract. If a public randomizing device is available and reports are contractible, then the principal can indeed obtain a higher payoff.

Ex ante, the principal and the agent can commit to a long-term contract that specifies in each period the funding decision (whether the principal advances the investment cost c to the agent) and an additional payment, contingent on contractible histories. The agent has limited liability: all payments must be non-negative. The principal's payoff is the (discounted) total profits from the project minus payments to the agent and investment costs transferred, whereas the agent's payoff is the value of total funds diverted plus payments received.

2.2 Optimal Contracts

I begin by defining a class of contracts that I call *cutoff contracts*.

Definition 1. A contract is a cutoff contract if there exists some $T > 0$ such that the agent is financed with the investment cost if and only if $t \leq T$.

In cutoff contracts, the agent is financed until some termination date T, and there is no delay of financing. I first restrict attention to the class of cutoff contracts. Proposition 2 characterizes the optimal cutoff contract given a fixed termination date T, and Proposition 3 characterizes the optimal termination date. Then in Proposition 4, it is shown that the restriction to cutoff contracts is without loss of generality. The proof consists of two parts. First, it can be shown that conditional on a deterministic financing strategy, it is suboptimal to delay financing. Loosely speaking, if the continued financing after suspension is profitable for the principal, then he would rather not delay financing so that he can collect the profits earlier; if the continued financing is not profitable, then he is better off by terminating financing instead of delaying it. Second, we need to show that randomized financing strategy cannot improve the principal's payoff.

The lemma below helps further restrict the set of contracts that need to be considered.

Lemma 1. *In an optimal contract, the agent receives a positive payment (apart from the investment cost) only when success occurs.*

Intuitively, unconditional payments to the agent do not help providing incentives to work. An unconditional payment in some period can be replaced by some payment conditional on success such that the agent's incentives will still be satisfied, but the total expected payment to the agent is smaller.

I now focus on contracts that consist of two components: i) a termination time of investment T; ii) a reward schedule $\mathbf{w} = \{w_t\}_{t=0}^{T}$ that specifies a payment w_t if the success occurs in period t.

Denote the agent working (shirking) at stage n in period t by $a_t^n = 1$ ($a_t^n = 0$). Take any contract $(T, \mathbf{w})$. For any sequence of actions of the agent $\mathbf{a} = \{a_t^n\}$, a probability distribution $\mathbb{P}_{T,\mathbf{a}}$ is induced over the time of the first breakthrough and the time of final success t^*. Let n_t be the stage of the project in period t. If success does occur, i.e., $t^* \leq T$, the ex post payoffs of the principal (u) and the agent (v), and the total surplus (π) are given by

$$u = \delta^{t^*}(Y - w_{t^*}) - \frac{1-\delta^{t^*+1}}{1-\delta}c;$$

$$v = \delta^{t^*} w_{t^*} + \sum_{t=0}^{t^*} \delta^t (1 - a_t^{n_t})c;$$

$$\pi = \delta^{t^*} Y - \sum_{t=0}^{t^*} \delta^t a_t^{n_t} c.$$

If success does not occur, then

$$u = -\frac{1-\delta^T}{1-\delta}c;$$

$$v = \sum_{t=0}^{T} \delta^t (1 - a_t^{n_t})c;$$

$$\pi = -\sum_{t=0}^{t^*} \delta^t a_t^{n_t} c.$$

The corresponding ex ante values U, V, Π are calculated by taking expectations with respect to $\mathbb{P}_{T,\mathbf{a}}$.

In principle, the agent's strategy depends not only on calendar time t and whether the intermediate breakthrough has occurred $n \in \{0, 1\}$, but also on his entire private history including his past actions and when the intermediate breakthrough occurred. He may also choose a mixed strategy. Nevertheless, the probability distribution $\mathbb{P}_{T,\mathbf{a}}$ over the time of breakthroughs is uniquely pinned down by the termination date T and the realized choice of actions of the agent $\mathbf{a} = \{a_t^n\}$. Similarly, the total surplus Π is determined by T and $\mathbf{a} = \{a_t^n\}$, with the additional terms of the contract of the rewards $\mathbf{w}$ determining the agent's payoff V and the principal's payoff U.

The contract maximizes the principal's ex ante payoff U. Formally, it solves the following problem

$$\begin{aligned} &\max_{T,\mathbf{a},\mathbf{w}} U(\mathbf{a},\mathbf{w},T) = \mathbb{E}_{\mathbb{P}_{T,\mathbf{a}}} u \\ \text{s.t. } & V(\mathbf{a},\mathbf{w},T) \geq V(\mathbf{a}',\mathbf{w},T), \ \ \forall \mathbf{a}' \qquad \text{(IC)} \\ & w_t \geq 0, \ \ \forall t. \qquad \text{(LL)} \end{aligned}$$

The design of optimal contracts must address the following questions:

1. What are the proper incentives to provide to the agents with different levels of progress, or in other words, should an agent at stage n in period t be induced to work or not?

2. Is it possible to provide those incentives using only rewards for success and threat of termination without knowing the agent's progress?

3. What is the least costly way to provide those incentives?

The principal's ex ante payoff is a complicated function of T, $\mathbf{a}$ and $\mathbf{w}$. Moreover, the reward schedule and the termination date have to satisfy the IC constraint that consists a set of inequalities. Rather than solving the principal's problem directly, it is more convenient to look at the problem from another angle. Since the principal and the agent share the same discount factor, $U = \Pi - V$. Given termination time T, the induced action profile $\mathbf{a}$ determines the total surplus of the project Π. The reward schedule $\mathbf{w}$ has to make the induced $\mathbf{a}$ incentive compatible for the agent. The agent will receive an expected payoff V, and this measures the principal's cost of providing incentives. In general, a tradeoff would be expected between the social surplus and the cost of providing incentives. A more socially efficient action profile may also be more costly to induce. However, in this model it turns out that given T the socially optimal action profile $\hat{\mathbf{a}}$ is no more costly to induce than others. More specifically, define $\underline{V}(T) = c(1-\delta^{T+1})/(1-\delta)$. This is the ex ante payoff that the agent can receive if he always shirks and diverts funds, and is a lower bound for the incentive cost for all action profiles. It turns out that the efficient $\hat{\mathbf{a}}$ can be induced at a cost of $\underline{V}(T)$.

In the remainder of this section, I characterize the optimal contract through the following steps. To start, I look for the optimal cutoff contract for fixed termination time T. To do that, I first solve for the action profile $\hat{\mathbf{a}}$ that maximizes the total surplus Π. Next, I show that there indeed exists a reward schedule $\mathbf{w}$ that induces $\hat{\mathbf{a}}$ and gives the agent a payoff of $\underline{V}(T)$. Therefore, $(\hat{\mathbf{a}}, \mathbf{w})$ maximizes the principal's payoff U given T because it simultaneously maximizes Π and minimizes V over all possible incentive compatible

$(\mathbf{a}', \mathbf{w}')$. Then, I solve for the optimal termination time T^*. Finally, I show that the optimal cutoff contract is also optimal among all contracts.

2.2.1 Fixed T

2.2.1.1 Efficient Actions

Given Assumption 1, if the project is at stage 1 and has only one breakthrough to be made, then it is socially optimal for the agent to work instead of shirking for all $t \leq T$. So $\hat{a}_t^1 = 1$ for all $t \leq T$. However, if the player is at stage 0, then as time moves closer to the termination time T, working becomes suboptimal compared to shirking. This is because when it is closer to the termination time, there is less and less chance to make two breakthroughs that are necessary for success, and the expected return from investment becomes smaller than the cost of investment. Therefore, it is socially more efficient to let the agent divert the funds for private consumption. In particular, if the stage is 0 in period T, then there is no chance to succeed because only one breakthrough is possible per period. It is therefore more efficient for the agent to consume the fund rather than invest it for no return.

Use Π_t^n to denote the maximum social surplus available if the project is at stage n at the beginning of period t given termination time T. First note that

$$\Pi_t^2 = Y, \forall t; \;\; \Pi_{T+1}^n = 0, \forall n < 2.$$

At stage 1, working is always efficient. Therefore, the social surplus at stage 1 is characterized recursively by

$$\Pi_t^1 = q_2 Y + \delta(1-q_2)\Pi_{t+1}^1 - c,$$

with boundary condition $\Pi_{T+1}^1 = 0$. Solving the recursive equation gives us

$$\Pi_t^1 = \frac{q_2 Y - c}{1-\delta(1-q_2)}(1-[\delta(1-q_2)]^{T-t+1}). \tag{2.1}$$

In period t at stage 0,

$$\Pi_t^0 = \max\{\underbrace{\delta(q_1\Pi_{t+1}^1 + (1-q_1)\Pi_{t+1}^0) - c}_{\text{Working}},\ \underbrace{\delta\Pi_{t+1}^0}_{\text{Shirking}}\}.$$

Working is efficient if and only if

$$\Pi_{t+1}^1 - \Pi_{t+1}^0 \geq \frac{c}{\delta q_1}. \tag{2.2}$$

At stage 0, in period T it is efficient to shirk, because it is impossible to achieve success; or we can also see it by condition (2.2): $\Pi_{T+1}^1 - \Pi_{T+1}^0 = 0 < \frac{c}{\delta q_1}$. So $\Pi_T^0 = 0$. Check condition (2.2) again for period $T-1$: if

$$\Pi_T^1 - \Pi_T^0 = (q_2 Y - c) - 0 > \frac{c}{\delta q_1},$$

then it is efficient to work in period $T-1$, and

$$\Pi^0_{T-1} = \delta(q_1\Pi^1_T + (1-q_1)\Pi^0_T) - c;$$

otherwise, it is still efficient to shirk, and

$$\Pi^0_{T-1} = \delta\Pi^0_T = 0.$$

As we go back in time, as long as it is efficient to shirk at stage 0 in period t, Π^0_t stays at 0. On the other hand, from equation (2.1), we can see that Π^1_t becomes larger as t becomes smaller. Define

$$t_0 = \max\{t : \Pi^1_{t+1} \geq \frac{c}{\delta q_1}\}, \tag{2.3}$$

where Π^1_t is determined by equation (2.1). Then t_0 is the last period in which condition (2.2) holds, i.e., the last period in which it is efficient to work if the stage is 0. It is shown in the proof to proposition 1 that for $t \leq t_0$, it is always efficient to work at stage 0, and the total surplus functions satisfy

$$\Pi^0_t = \delta(q_1\Pi^1_{t+1} + (1-q_1)\Pi^0_{t+1}) - c. \tag{2.4}$$

The following proposition summarizes the results above. See Figure 2.1 for an illustration.

Proposition 1. *For given T, let t_0 be defined by* (2.3) *and* (2.1).

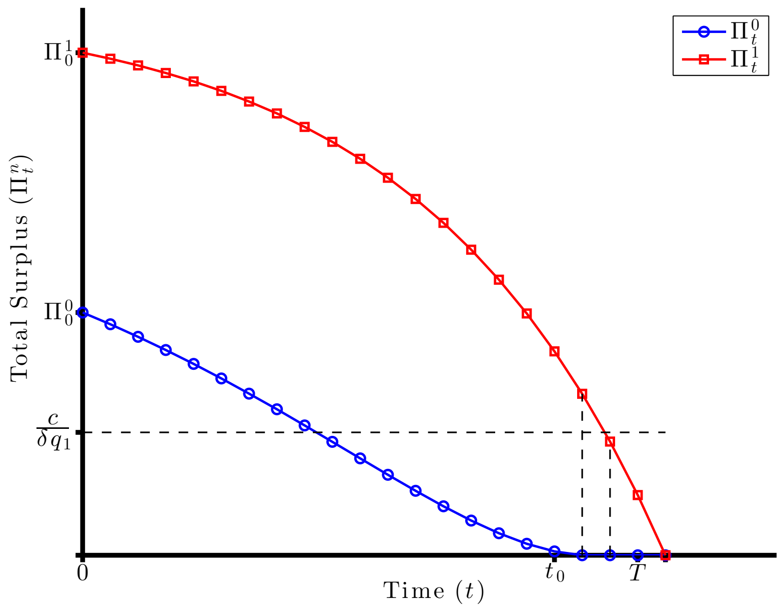

Figure 2.1: Total surplus function Π_t^n
$(Y = 60, q_1 = q_2 = 0.1, c = 1, \delta = 0.99, T = 20)$

1) *It is socially efficient to work at stage 1 for all $t \leq T$; it is socially efficient to work at stage 0 if and only if $t \leq 0$.*

2) *At stage* 1*, the social surplus of the project Π_t^1 is characterized by* (2.1)*. At stage 0, the social surplus Π_t^0 equals* 0 *for $t > t_0$; for $t < t_0$, Π_t^0 is characterized by* (2.4).

2.2.1.2 Values and Rewards

Now that we have determined the socially optimal action choices are $\hat{a}_t^1 = 1$ for all t and $\hat{a}_t^0 = 1$ if and only if $t \leq t_0$, we will show that there exists a reward schedule $\mathbf{w} = \{w_t\}$ that induces $\hat{\mathbf{a}}$ from the agent, while giving the agent a payoff of $\underline{V}$.

Given termination time T, let V_t^n denote the agent's value at stage n in period t under a contract that induces the socially optimal action profile $\hat{\mathbf{a}}$ characterized in proposition 1. The agent's ex ante payoff is $V = V_0^0$. The agent's value in any period only depends on the stage of the project $n \in \{0,1\}$ and his future actions. The agent's IC constraint is equivalent to a set of one-shot IC conditions at both stage 0 and 1 in all periods. At stage 0, the agent's value function follows

$$V_t^0 = \max\{\underbrace{\delta(q_1 V_{t+1}^1 + (1-q_1)V_{t+1}^0)}_{\text{Value from working}}, \quad \underbrace{c + \delta V_{t+1}^0}_{\text{Value from shirking}}\}, \ \forall t = 0, ..., T$$

It is optimal for him to work if and only if

$$V_{t+1}^1 - V_{t+1}^0 \geq \frac{c}{\delta q_1}.$$

So the one-shot IC conditions for the stage 0 agent are

$$V_{t+1}^1 - V_{t+1}^0 \geq \frac{c}{\delta q_1}, \ \forall t \leq t_0;$$
$$V_{t+1}^1 - V_{t+1}^0 \leq \frac{c}{\delta q_1}, \ \forall t > t_0.$$

Stage 0 agent's value function satisfies

$$V_t^0 = \delta(q_1 V_{t+1}^1 + (1-q_1)V_{t+1}^0) \geq c + \delta V_{t+1}^0, \forall t \leq t_0;$$

$$V_t^0 = c + \delta V_{t+1}^0, \forall t > t_0.$$

To minimize the agent's ex ante payoff $V = V_0^0$, we need to make the IC constraint binding for all $t \leq t_0$, so that his payoff is the same no matter whether he works or shirks. For $t > t_0$, his IC constraints are not satisfied, and shirking is optimal. Then his ex ante payoff V is exactly equal to the payoff he can receive if he always shirks:

$$V = V_0^0 = c + \delta V_1^0 = \cdots = c \sum_{t=0}^{T} \delta^t = \underline{V},$$

and stage 0 agent's payoff at any $t \leq T$ is

$$V_t^0 = c \sum_{i=0}^{T-t} \delta^i.$$

Next we solve for the agent's value function at stage 1. First for $t \leq t_0 + 1$, the binding IC constraints at stage 0 for $t \leq t_0$ pins down V_t^1:

$$\delta(q_1 V_t^1 + (1 - q_1) V_t^0) = c + \delta V_t^0,$$

or

$$V_t^1 = V_t^0 + \frac{c}{\delta q_1}.$$

Note that for any $t \leq t_0$, if stage 0 agent's IC constraints are binding, then IC constraints for stage 1 agent are automatically satisfied and stage 1 agent has a strict incentive to work for any $\delta \in (0,1)$:

$$V_t^1 - \delta V_{t+1}^1 = (V_t^0 + \frac{c}{\delta q_1}) - (\delta V_{t+1}^0 + \frac{c}{q_1}) = c + (1-\delta)\frac{c}{\delta q_1} > c.$$

For $t > t_0 + 1$, we need $V_t^1 \leq V_t^0 + \frac{c}{\delta q_1}$ so that stage 0 agent has incentive to shirk in $t > t_0$. Also, it is necessary that $V_{t-1}^1 \geq c + \delta V_t^1$ so that the IC constraints for stage 1 agent are satisfied for $t \geq t_0 + 1$. But optimality does not pin down the value function of stage 1 agent for $t > t_0 + 1$, and value functions V_t^0 and V_t^1 are consistent with optimal contracts as long as the above conditions are satisfied.

Given any $\{V_t^1\}_{t=1}^T$ and $V_{T+1}^1 = 0$, the reward schedule $\{w_t\}$ can be derived from the recursive equations of the agent's value at stage 1:

$$V_t^1 = q_2 w_t + \delta(1-q_2)V_{t+1}^1,$$

or

$$w_t = \delta V_{t+1}^1 + \frac{1}{q_2}(V_t^1 - \delta V_{t+1}^1), \forall t = 1, ..., T.$$

Note that stage 1 agent's IC conditions $V_{t-1}^1 \geq c + \delta V_t^1$ automatically imply the limited liability constraints $w_t \geq 0$.

The following proposition summarizes properties of the optimal contracts given termination time T.

Proposition 2. *Let t_0 be the last period that it socially efficient for stage 0 agent to work.*

An action profile $\mathbf{a}$*, a reward schedule* $\mathbf{w}$ *and the agent's value function* V_t^n *are consistent with an optimal cutoff contract if and only if the following holds:*

1) $a_t^0 = 1$ *if and only if* $t \leq t_0$*;* $a_t^1 = 1$, $\forall t \leq T$.

2) *For all* $t > T$, $V_t^n = 0$, $n = 0, 1$.

   At stage 0,

$$V_t^0 = c \sum_{i=0}^{T-t} \delta^i, \ \ \forall t \leq T.$$

   At stage 1, for all $t \in [1, t_0 + 1]$,

$$V_t^1 = V_t^0 + \frac{c}{\delta q_1} = c \left(\sum_{i=0}^{T-t} \delta^i + \frac{1}{\delta q_1} \right);$$

   for all $t \in (t_0 + 1, T]$, V_t^1 *satisfies*

   (a) $V_t^1 \leq V_t^0 + \frac{c}{\delta q_1}$ *: No incentive to work for stage 0 agent;*

   (b) $V_{t-1}^1 \geq c + \delta V_t^1$*: IC conditions for stage 1 agent;*

3) *For all* $t \in [1, T]$, w_t *satisfies*

$$w_t = \delta V_{t+1}^1 + \frac{1}{q_2}(V_t^1 - \delta V_{t+1}^1).$$

   In particular, $\forall t \in [1, t_0]$,

$$w_t = c \left(\sum_{i=1}^{T-t} \delta^i + \frac{1}{q_1} + \frac{1}{q_2} + \frac{1 - \delta}{\delta q_1 q_2} \right).$$

In an optimal contract, the agent's induced actions **a** maximizes the total surplus and is generically unique. Stage 0 agent's value function V_t^0 is also unique for all t. It is equal to the discounted value of future transfers of investment cost. We can uniquely determined V_t^1 for $t \leq t_0 + 1$ and w_t for $t \leq t_0$, which are also strictly decreasing.

There are multiple values of V_t^1 for $t > t_0 + 1$ and multiple values of w_t for $t > t_0$ that are consistent with optimal contracts. This is because optimality only requires minimizing the *ex ante* value of the agent. Given an optimal contract, suppose we increase the reward in some $t > t_0$ but decrease the reward in another $t' > t_0$. Since the IC conditions for agents at either stage do not bind in general, the incentive to work (or shirk) for stage 1 (or 0) agent will still hold. So as long as the perturbation generates the same value for stage 1 agent in period $t_0 + 1$, then all incentives for $t \leq t_0$ will not be affected and the agent's ex ante payoff remains the same. The perturbed contract is still optimal. Next, we show two examples of optimal contracts with different V_t^1 and w_t for $t \geq t_0$. In one example, stage 1 agent's value V_t^1 is maximized for $t > t_0 + 1$ while in the other V_t^1 is minimized.

Example 1. (Figure 2.2) Set $V_t^1 = V_t^0 + c/\delta q_1$ for $t > t_0 + 1$. Therefore, stage 0 agent is still indifferent between working and shirking for $t > t_0$ but chooses to shirk. Stage 1 agent has a strict incentive to work in every period. Then

$$w_t = c\left(\sum_{i=1}^{T-t}\delta^i + \frac{1}{q_1} + \frac{1}{q_2} + \frac{1-\delta}{\delta q_1 q_2}\right), \forall t \in [1, T-1];\;\; w_T = \frac{c}{q_2}\left(1 + \frac{1}{\delta q_1}\right).$$

w_t is strictly decreasing except possibly in the last period.

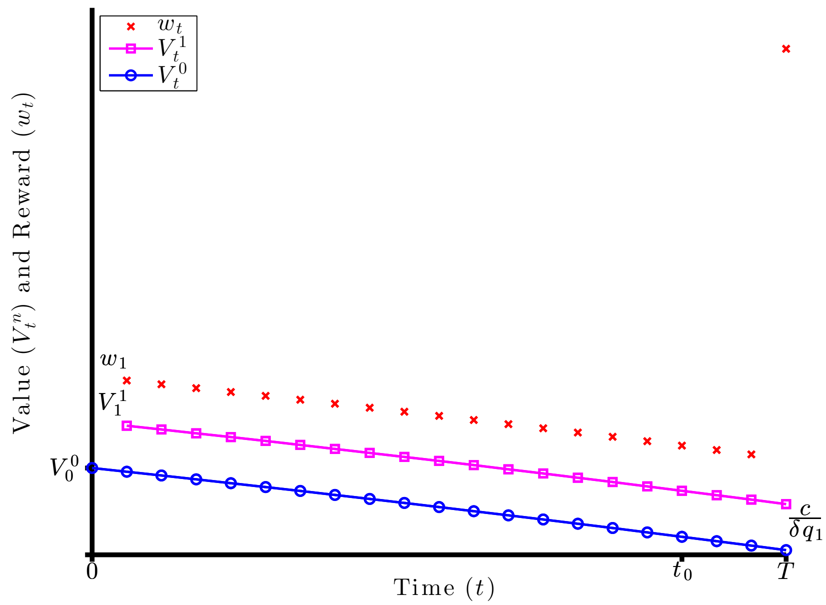

Figure 2.2: E.g.1: Value and reward functions
$(Y = 60, q_1 = q_2 = 0.1, c = 1, \delta = 0.99, T = 20)$

Example 2. (Figure 2.3) Set $V_t^1 = V_t^0$ for $t > t_0 + 1$. In other words, we choose the reward schedule such that stage 1 agent's IC constraints are binding for $t > t_0 + 1$. Stage 0 agent strictly prefers to shirk for $t > t_0$ because the continuation value for working and shirking are the same. The implied reward schedule is

$$w_t = \begin{cases} c\left(\sum_{i=1}^{T-t} \delta^i + \frac{1}{q_1} + \frac{1}{q_2} + \frac{1-\delta}{\delta q_1 q_2}\right) & 1 \le t \le t_0 \\ c\left(\sum_{i=1}^{T-t} \delta^i + \frac{1}{q_2} + \frac{1}{\delta q_1 q_2}\right) & t = t_0 + 1 \\ c\left(\sum_{i=1}^{T-t} \delta^i + \frac{1}{q_2}\right) & t_0 + 1 < t \le T \end{cases}$$

with the summation $\sum_{i=1}^{0}(\cdot)$ defined to be 0.

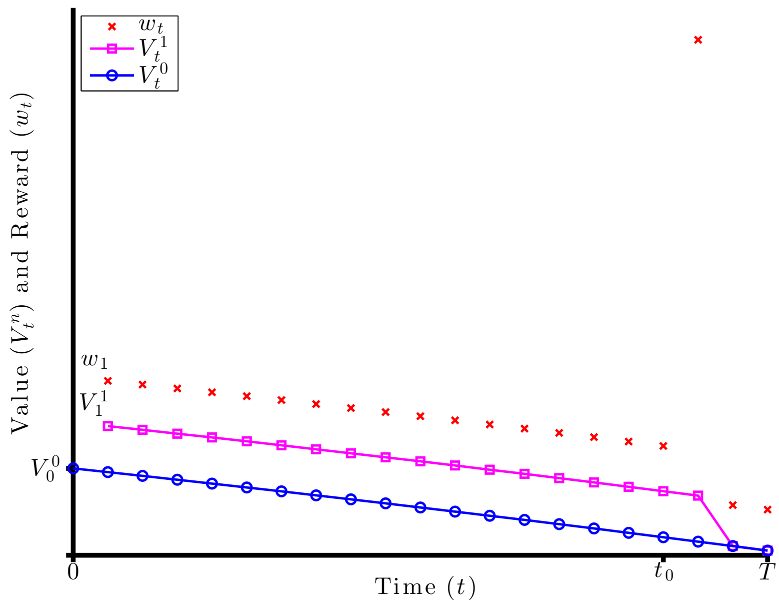

Figure 2.3: E.g.2: Value and reward functions
($Y = 60, q_1 = q_2 = 0.1, c = 1, \delta = 0.99, T = 20$)

As we can see, in the two examples, the reward w_t's are different for $t > t_0$. However, under both reward schedules, the relevant IC conditions are satisfied and the same actions will be induced. Moreover, the agent will receive the same expected payment. Thus both contracts are optimal.

2.2.2 Finding Optimal T

To fully characterize the optimal contracts, it remains to solve for the optimal time of termination T. In first best scenario, T should be infinity, i.e., investment should always be made until success. However, with the moral hazard problem, it is suboptimal to invest infinitely because that gives the agent too many funds to divert. As we have shown in the previous section, for a given T, the agent's ex ante payoff is equal to the discounted value of the T periods' investment costs. Suppose the contract specifies one more period of investment, changing from $T-1$ to T. It leads to an increase of the agent's payoff by $\delta^T c$. This is the marginal cost for the principal to commit to one more period of investment. On the other hand, the marginal benefit is an increase in the probability of success of the project. As we can see, as T increases, the marginal cost remains constant if we ignore discounting. However, the marginal benefit diminishes, because when T goes to infinity, with probability almost 1 success can occur before T. Increasing investment for one more period will hardly increase the overall probability of success. Then the optimal termination time T is the last period that the marginal benefit of investment is larger than the marginal cost.

Define

$$Q_{01}(t) = (1-q_1)^t;$$

$$Q_{02}(t) = (1-q_2)^t;$$

$$Q_1(t) = \sum_{i=0}^{t-1}(1-q_1)^i q_1 (1-q_2)^{t-i}.$$

So $Q_{01}(t)$ is the probability that the first breakthrough does not occur within t periods; $Q_(02)(t)$ is the probability that the second breakthrough does not occur within t periods after the first has occurred. $Q_1(t)$ is the probability that only the first breakthrough occurs within t periods.

We can see that $Q_{01}(t)$ and $Q_{02}(t)$ are strictly decreasing in t, and $Q_1(t)$ first increases and then decreases. These values converge to 0 as t goes to infinity.

Given T, let $\Pi^n_{t,T}$ and $V^n_{t,T}$ be the total surplus functions and the agent's value functions under optimal contracts, and let $t_{0,T}$ be stage 0 agent's last working period under optimal contracts. From how $t_{0,T}$ is calculated in proposition 1, we can see that for any termination time T, $T - t_{0,T}$ is constant. Let $\hat{t} = T - t_{0,T}$. This will be the number of periods of financing after stage 0 agent stops working. So one more period of investment will induce both stage 0 and stage 1 agent to work for one more period. Also note that the total surplus $\Pi^1_{t,T}$ at stage 1 in time t when termination time is T only depends on $T - t + 1$, i.e., the number of periods left for investment. Define $\Pi^1(t) = \Pi^1_{T-t+1,T}$ to be the total surplus when the project is at stage 1 when there are t periods of investment left. From equation (2.1) in 2.2.1.1, we know that

$$\Pi^1(t) = \frac{q_2 Y - c}{1 - \delta(1 - q_2)}(1 - [\delta(1 - q_2)]^t).$$

Proposition 3. *Define*

$$T^* = \max_T \{T : \Pi^0_{0,T} - \Pi^0_{0,T-1} \geq \delta^T c\}^3,$$

where

$$\Pi^0_{0,T} - \Pi^0_{0,T-1} = Q_1(t_{0,T})Q_{02}(\hat{t})\delta^T(\delta q_2 Y - c) + Q_{01}(t_{0,T})\delta^{t_{0,T}}(\delta q_1 \Pi^1(\hat{t}) - c).$$

[3] The set is non-empty by Assumption 1.

Then $T^* > 0$ is the termination time in the optimal cutoff contract.

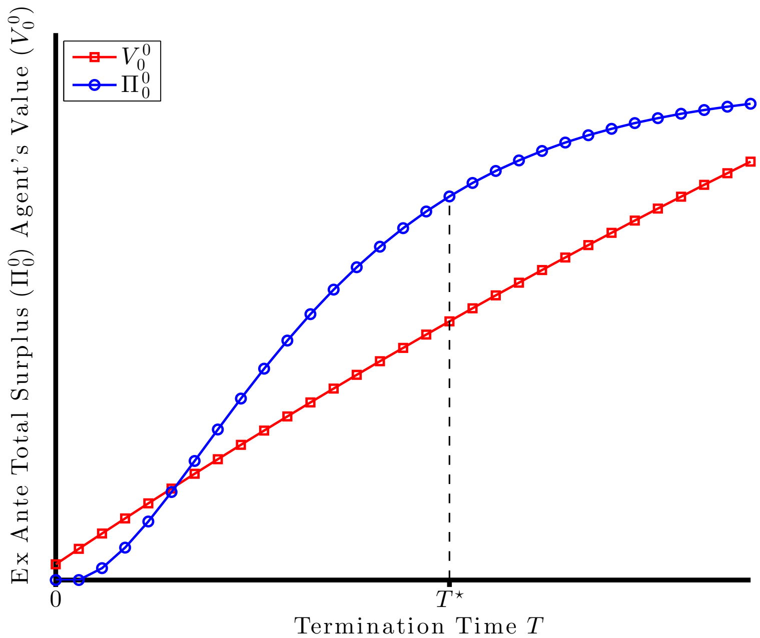

Figure 2.4: Finding opitmal termination T^*
($Y = 50, q_1 = q_2 = 0.15, c = 1, \delta = 0.99, T^* = 17$)

Figure 2.4 illustrates how to determine the optimal termination time. As T increases, the difference between the ex ante total surplus and the agent's payoff $\Pi_{0,T}^0 - V_{0,T}^0$ first increases and then decreases. The optimal T^* is where the gap is the largest if the difference is ever positive. If $\Pi_{0,T}^0 - V_{0,T}^0$ is always negative, then the project is not profitable to finance for the principal.

Proposition 4. *Conditional on the optimal cutoff contract yields non-negative payoff to the principal, the optimal cutoff contract is also optimal in unrestricted class of contracts. Otherwise, the optimal contract specifies no financing at all.*

2.3 Implications of Unobserved Progress

In this section, I discuss in more detail the properties of optimal contracts characterized in section 2.2. Comparing the results to models where the progress is observable and contractible, or where only one breakthrough is required for success (so that the progress of innovation is not modeled), I highlight the implications of progress being unobservable to the principal on incentives and on the contract structure.

2.3.1 Efficiency from Ignorance

From section 2.2, we already know that fixing the termination time T, profit-maximizing contracts for the principal actually induce actions from the agent that maximize the total surplus. However, the optimal termination time T^* is not efficient because first best will require always investing until success. But if there is a binding exogenous deadline $\tilde{T} \leq T^*$ for innovation due to either time or budget constraint such that investment cannot be made after $\bar{T}$, then the optimal contract is socially efficient.

This efficiency result is precisely because progress is unobservable, and we will call it *efficiency from ignorance*. Since the agent with no progress cannot be distinguished from the one with some progress, he will continue collect the rent from diverting the funds when he is supposed to shirk in $t > t_0$. Therefore, it does not help providing incentives to choose a earlier stopping time of working for the stage 0 agent. Given the final investment termination time T, the agent is always guaranteed a payoff of $\underline{V}$ from always shirking. As a result, the best that the principal can do is to induce actions from the agent that maximize the total surplus.

This will not be the case when progress is observable and contractible. In that environment, the agent's value is no longer independent of the stopping time of investment at stage 0 because the principal can observe whether the first breakthrough has occurred or not. If the principal wants stage 0 agent to stop working after some period $\tilde{t}_0$, but wants stage 1 agent to continue working until T, she can do so simply by stopping financing the project if the stage is still 0 after $\tilde{t}_0$. The smaller $\tilde{t}_0$ is, the less money he can divert by always shirking. In other words, earlier termination $\tilde{t}_0$ provides more incentive for stage 0 agent to work. Less reward is needed upon success, and inducing working becomes cheaper. As a result, given the termination time T, the principal may be better off by choosing a $\tilde{t}_0$ that is smaller than the socially efficient t_0, although it reduces the total possible surplus.

Proposition 5. *Suppose the first breakthrough is observable and contractible, and there is an exogenous deadline for innovation $T \leq T^*$.*[4] *The optimal contract the following feature: conditional on the first breakthrough does not occur, the agent is financed if and only if $t \leq \tilde{t}_0$ for some $\tilde{t}_0 < T$.; conditional on the first breakthrough has occurred, the agent is financed until the deadline T.*

Moreover, $\tilde{t}_0 \leq t_0$, where t_0 is the efficient stopping time of investment for stage 0 project characterized in Proposition 1. The inequality is strict under some parameters.

It is worth noting that although, under some conditions, the outcome is less efficient

[4] Recall that T^*, as characterized by Proposition 3, is the optimal termination time of financing when the first breakthrough is not contractible.

when the principal can monitor and contract on the progress, she is still strictly better off than if the progress is not contractible. She can always induce the efficient actions and have stage 0 agent stop in period t_0. Unlike the case where progress is private, she does not need to finance the stage 0 agent after t_0, and the agent's share of the surplus is strictly smaller. However, if the innovation project is funded by a principal that aims to maximize social welfare, such as the government, and there is a binding deadline for innovation, then there is an optimal contract where the principal chooses not to monitor the progress even when monitoring is costless. Another interpretation of the result is that, under some circumstances, the regulator in the economy may choose to impose regulations that prevent tight monitoring of the progress of some projects, although it will hurt the financier.

2.3.2 Non-monotone Rewards

In an optimal contract, the reward function w_t is strictly decreasing in the success time t for $t \leq t_0 + 1$. This is reminiscent of the dynamic agency cost identified by Hörner and Samuelson (2013).[5] Intuitively, when success happens early, the agent loses the opportunity to divert a large amount of fund in the future. So for him to be willing to work for success in early periods, the agent has to be rewarded by more for earlier success. However, as illustrated by example 1 and 2, the reward w_t may jump up in some periods

[5]In their setting, an additional source of the dynamic agency cost is due to the uncertainty of the quality of the project. The agent's private belief is more optimistic than the principal's off the equilibrium path when he deviates to shirking, so shirking is more tempting compared to the static setting because it will lead to larger value in the future.

after t_0, and later success can be rewarded by more.[6] This possible non-monotonicity of w_t on $t \in [t_0, T]$ is again driven by the features that innovation takes more that one step and that the progress is unobservable to the principal. If the success of innovation only requires one breakthrough, and thus there is no information asymmetry about the progress then the optimal reward will be strictly decreasing for all $t \leq T$. As a comparison, we state the result for the case where innovation requires only one breakthrough in the following proposition.

Proposition 6. *When innovation only requires one breakthrough, the optimal contract* $(\mathbf{w}, T)$ *is characterized by*

$$w_t = c\left(\sum_{i=1}^{T-t} \delta^i + \frac{1}{q}\right), \forall t = 0, ..., T;$$

and

$$T = \left\lfloor \log_{1-q} \frac{c}{qY - c} \right\rfloor,$$

where $\lfloor \cdot \rfloor$ *is the floor function:* $\lfloor x \rfloor \equiv \min\{k \in \mathbb{N} : k \leq x\}$.

Mathematically, the reason that w_t may be monotone is the following. As shown in figure 2.2 and 2.3, V_t^0 decreases gradually in t and is equal to 0 when $t = T+1$. For $t \leq t_0 + 1$, V_t^1 is strictly larger than V_t^0 by $c/\delta q$; but at $t = T+1$, $V_t^1 = V_t^0 = 0$. So between $t_0 + 1$ and $T+1$, stage 1 agent's value function V_t^1 shifts from $V_t^0 + c/\delta q$ down to V_t^0, and has to decrease at a higher rate than in periods $t \leq t_0$. In example 1, V_t^1 drops

[6]For all parameters, there exists an optimal contract in which the rewards are non-monotone. Moreover, there exists parameters such that all optimal contracts have non-monotone rewards.

down at $t = T+1$, and in example 2, V_t^1 drops at $t = t_0 + 2$. A faster decrease of V_t^1 in t corresponds to a high w_t. In general, the shift can be more gradual, the the jump in w_t will be less drastic as in the two examples. See figure 2.5 for an example.

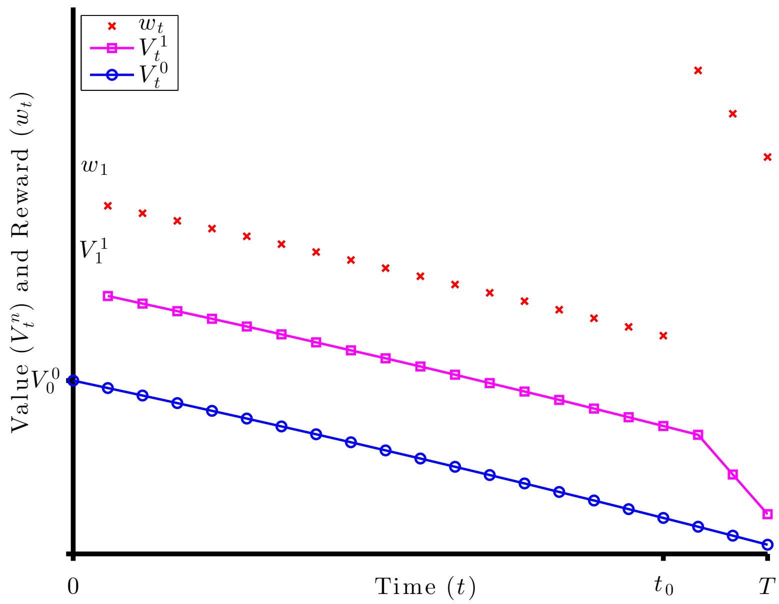

Figure 2.5: E.g.3: Value and reward functions
$(Y = 60, q_1 = q_2 = 0.1, c = 1, \delta = 0.99, T = 20)$

The result of non-monotone rewards can also be interpreted from the perspective of capital structure implementation. In the one-breakthrough case, the decreasing reward function $w_t = c\left(\sum_{i=1}^{T-t} \delta^i + 1/q\right)$ is equivalent to the following arrangement. The principal commits to transfer c to the agent in each period for either investment or consumption until T, even if success has already occurred. When the project is successful, the agent receives a fixed share of the value of success c/q and the principal gets the rest $Y - c/q$; the agent consumes the rest of the transfers of c period. Since the agent can still get the future transfers after success, he has no incentive to delay investment. The share c/q is the minimum reward needed for the agent to be willing to work in a static problem. The optimal contract can be simply implemented by granting a fixed share of stock to the agent.

One might conjecture a similar argument will hold for the two-breakthrough case. Indeed, for $t \leq t_0$,

$$w_t = c\left(\sum_{i=1}^{T-t}\delta^i + \frac{1}{q_1} + \frac{1}{q_2} + \frac{1-\delta}{\delta q_1 q_2}\right). \tag{2.5}$$

The structure is similar to the one-breakthrough case, except now the fixed share granted to the agent becomes $c\left(\sum_{i=1}^{T-t}\delta^i + \frac{1}{q_1} + \frac{1}{q_2} + \frac{1-\delta}{\delta q_1 q_2}\right)$. However, if w_t follows equation (2.5) for all $t \leq T$, some of the incentive conditions for the agent will be violated. Remember that in an optimal contract, stage 0 agent is supposed to work until t_0, where t_0 is the socially efficient time to stop investing in stage 0 project when the value of success is Y. If the agent is given c in each period until T and gets a fixed share

$$FS = c\left(\sum_{i=1}^{T-t}\delta^i + \frac{1}{q_1} + \frac{1}{q_2} + \frac{1-\delta}{\delta q_1 q_2}\right)$$

when the project succeeds, then he will act as if he is a social planner that manages a project that is worth FS upon success. Since $FS < Y$, the agent of stage 0 would like to stop investing earlier than the efficient time t_0. In order to induce stage 0 agent to invest for longer time, the principal could increase the fixed share of the value of success granted to the agent. But this is the suboptimal approach because this gives the agent a larger ex ante payoff. Instead, the optimal contract does not use a fixed share of stock. For $t \leq t_0$, the share given to the agent is $c\left(\sum_{i=1}^{T-t}\delta^i + \frac{1}{q_1} + \frac{1}{q_2} + \frac{1-\delta}{\delta q_1 q_2}\right)$; in one or more periods from t_0+1 to T, the reward function w_t jumps up, corresponding to a larger share of stock given to the agent. This provides stage 0 agent more incentive to work for $t \leq t_0$ without giving him a larger ex ante payoff.

The result is consistent with the wide use of time-vested restricted stock units (RSU). Usually time-vested RSU is understood as a tool to provide incentives for employees to stay on the job. Indeed, this is its main role for workers at middle to low levels in large companies, where individuals' effort have little impact on the overall profitability, and moral hazard may not be the main concern. On the other hand, this model stresses that in contexts such as incentivizing founders of startups, time-vested stocks as part of the compensation scheme not only help retain founders on the project, but also give them more incentives to exert effort before the stocks vest. This extra bit of incentive is especially important if the progress has been slow. The founder will want to work to make more progress on the project so that his vested stocks will be more likely to be valuable.

Chapter 3

Extensions and Discussions

3.1 Which Task First: the Easy or the Difficult?

In this section, suppose the two breakthroughs required for the success of the project do not need to be in a particular order. In other words, there are two tasks that need to be completed, task D(ifficult) and task E(asy). In each period, the agent can choose to work on task D or E. If task i is worked on in a period, the probability that it is completed is q_i, with $q_E > q_D$. The costs for investing in the two tasks are the same, c. For simplicity, assume there is no discounting, i.e., $\delta = 1$. The contractible histories remain the same as in the baseline case. Now a contract specifies at each history a financing decision, a reward for final success and which task to work on. There is an additional set of IC constraints for the agent that he does not deviate to working on the other task. The question is, in the optimal contract, which task is the agent induced to work on first.

Again, it is helpful to consider the planner's problem first. Given a termination time T, if the planner works on task E first, she will exert effort up to some t_0 to complete task E. If task E is completed by t_0, she will work until T on task D; otherwise she stops working. Let $\Pi(ED)$ denote the social surplus when the planner works on task E first and uses the optimal t_0. Let $\Pi(DE)$ denote the social surplus when task D is done first and the planner uses the same t_0 that is optimal for the sequence ED.

Lemma 2.

$$\Pi(DE) > \Pi(ED).$$

The above lemma states that planner obtains a higher payoff by doing task D first even she uses a dropout time t_0 at stage 0 that may not be optimal. Therefore, it is always optimal for the planner to work on the difficult task first. To see the intuition behind the result, think about an extreme case where $q_E = 1$ and $q_D < 1$. If task E is carried out first, it is immediately completed with certainty at $t = 0$. Thus, the choice of t_0 does not matter and it can be set to $T - 1$. Now compare the planner's payoffs under the following two strategies: 1) complete task E at $t = 0$ and use the remaining T periods to work on D; or 2) work on task D first for T periods from $t = 0$ to $T - 1$, and if task D is completed, do task E and succeed; otherwise do not work at $t = T$. Note that the two strategies lead to the same probability of success: under both strategies, success requires task D be completed within T periods. In fact, the two strategies result in the same distribution over success time. Therefore, in the event of success, the payoffs of the planner are the same. However, suppose in the unlucky event that task D is not completed within T

periods (with probability $(1-q_D)^T$), working on D first saves the planner one period's investment cost compared to working on E first. Lemma 2 proves that the intuition from the extreme case holds in general.

Another way to interpret this result is that the continuation value of having completed the difficult task is higher. It is not yet obvious that the difficult task should be worked on first because the probability is lower. The proof shows that even taking into account the lower probability, working on the difficult task still gives higher return.

The next proposition shows that working on the difficult task first is not only efficient, but also generates higher payoff for the principal.

Proposition 7. *The optimal contract induces the agent to work on task D first.*

Proof. Consider best contract that induces the agent to work on task E first. It has a termination date T and a stage 0 dropout time t_0. Let $U(ED)$ be the principal's payoff ($U(DE)$ defined respectively). Then $U(ED) \leq \Pi(ED) - (T+1)c$. Now suppose the agent is restricted to do task D first. Consider the contract describe in Example 2 in Chapter 2 with the same T and t_0, and $q_1 = q_D, q_2 = q_E$. By proposition 2,

$$U(DE) = \Pi(DE) - (T+1)c.$$

Therefore, $U(DE) > U(ED)$.

Lastly, it can be verified that when the wages are defined as in Example 2, the agent indeed does not have an incentive to deviating to working on E first. Recall from Example

2 that the reward schedule that implements DE is

$$w_t = \begin{cases} c\left(\sum_{i=1}^{T-t}\delta^i + \frac{1}{q_D} + \frac{1}{q_E} + \frac{1-\delta}{\delta q_D q_E}\right) & 1 \le t \le t_0 \\ c\left(\sum_{i=1}^{T-t}\delta^i + \frac{1}{q_E} + \frac{1}{\delta q_D q_D}\right) & t = t_0 + 1 \\ c\left(\sum_{i=1}^{T-t}\delta^i + \frac{1}{q_E}\right) & t_0 + 1 < t \le T \end{cases}.$$

Stage 1 agent (the agent that has completed D) is indifferent between working and shirking for $t \in [t_0+1, T)$ and stage 0 agent is indifferent between working and shirking for $t \le t_0$.

Consider another reward schedule

$$w'_t = \begin{cases} c\left(\sum_{i=1}^{T-t}\delta^i + \frac{1}{q_D} + \frac{1}{q_E} + \frac{1-\delta}{\delta q_D q_E}\right) & 1 \le t \le t_0 \\ c\left(\sum_{i=1}^{T-t}\delta^i + \frac{1}{q_D} + \frac{1}{\delta q_D q_D}\right) & t = t_0 + 1 \\ c\left(\sum_{i=1}^{T-t}\delta^i + \frac{1}{q_D}\right) & t_0 + 1 < t \le T \end{cases}.$$

If the agent is restricted to take the sequence ED, then under $\{w'_t\}$, he is exactly indifferent between working and shirking at stage 1 for $t \in [t_0+1, T)$ and at stage 0 for $t \le t_0$. Since $w_t = w'_t$ for $t \le t_0$ and $w_t < w'_t$ for $t > t_0$, if agent deviates to the sequence ED, then after completing task E, his value V_t^1 is strictly smaller than the stage 1 value under $\{w'_t\}$ for $t \le t_0 + 1$. Therefore at stage 0, he does not have an incenitve to work on E under $\{w_t\}$. $\square$

To sum up, there are two reasons to induce the agent to work on the difficult task first. First, the sequence DE yields higher total surplus than ED given any termination time. Second, it is also cheaper to induce the agent to work on the difficult task first. By comparing $\{w_t\}$ and $\{w_t'\}$ in the proof above, we see that w_t is weakly smaller than w_t' for all t and strictly smaller for some t. It is possible to implement the efficient action profile given sequence DE while keeping the agent's ex ante payoff at the low bound $\underline{V} = \sum_{i=1}^{T} \delta^i c$, but for the sequence ED. To induce the agent to work on the easy task first, the incentive cost is higher than $\underline{V}$, because otherwise he would deviate to working on D first.

3.2 Possibility of Bad Project

In this section, assume that with probability $1-\gamma_0$, the project is with bad quality ex ante, and the breakthroughs never arrive. With probability γ_0, the project is good and conditional on that everything is the same as in the baseline model. The principal and agent share the same ex ante belief about the quality of the project.

Same as the baseline model, the only publicly observable event is the final success, so the space of contracts remains the same. Given a contract, the agent faces a decision problem with incomplete information about the quality of the project. His private histories include the events of breakthroughs and past effort choices. The agent updates his belief about the quality of the project based on his private histories.

3.2.1 Characterizing Values and Rewards

A critical variable in this environment is the agent's belief on the project being good, which evolves as the agent works on the project according to the following equation:

$$\gamma_k = \frac{\gamma_{k-1}(1-q_1)}{\gamma_{k-1}(1-q_1)+(1-\gamma_{k-1})}.$$

So γ_k is the agent's belief that the project is good after working but failing to complete the first breakthrough in k periods. Once the first breakthrough occurs, the agent's belief jumps to 1. On the equilibrium path, the principal's belief conditional on no breakthrough is the same as the agent's. However, after deviations, the agent has private beliefs that differ from the principal's.

Again, first consider the constrained planner's problem given a termination time T. At stage 1, it is known that the project is good, so the social surplus function Π_t^1 at stage 1 is the same as in the baseline model:

$$\Pi_t^1 = \frac{q_2 Y - c}{1-\delta(1-q_2)}(1-[\delta(1-q_2)]^{T-t+1}). \tag{2.1}$$

Define

$$t_0^* = \max_t\{\delta\gamma_t q_1 \Pi_{t+1}^1 \geq c\}.$$

It is efficient to work at stage 0 up to t_0^*, and work up to T conditional on being at stage 1. Note that compared with the base line (see equation (2.3)), $T - t_0^*$ is larger. That is to say, given the same T, the efficient time to drop out at stage 0 is earlier when there is uncertainty about the projects quality. The value of completing the first breakthrough at any time remains the same, but the planner is more pessimistic about the chance.

In the optimal contract, the principal induces the agent to work at stage 0 up to some t_0, and to work at stage 1 up to T. Let $V_t^0(\gamma)$ be the agent's value at stage 0 in period t when his belief is γ; let V_t^1 be the agent's value at stage 1 in period t.

On the equilibrium path, the agent's value function in period t at stage 0 satisfies

$$V_t^0(\gamma_t) = \delta\left[\gamma_t q_1 V_{t+1}^1 + (1-\gamma_t q_1)V_{t+1}^0(\gamma_{t+1})\right], \forall t \le t_0; \tag{3.1}$$

$$V_t^0(\gamma_{t_0+1}) = c + \delta V_{t+1}^0(\gamma_{t_0+1}), \forall t \ge t_0 + 1.$$

The IC constraints at $t \le t_0$ requires

$$V_t^0(\gamma_t) \ge c + \delta V_{t+1}^0(\gamma_t).$$

Note that attention can be restricted to IC constraints on the equilibrium path. Suppose the IC is always satisfied on the path of the play. Then had the agent shirked in one or more periods, his belief would be more optimistic than on the equilibrium path, and he would be more willing to work.

The stage 0 agent is supposed to shirk in $t \ge t_0 + 1$. In fact the contract can be designed such that stage 0 agent prefers to shirk for $t \ge t_0 + 1$ for any belief. For example, the rewards w_t can be chosen such that $V_t^1 = c\sum_{i=0}^{T-t}\delta^i$ for all $t \geq t_0+2$. Therefore,

$$V_t^0(\gamma) = c\sum_{i=0}^{T-t}\delta^i, \forall t \geq t_0+1, \forall \gamma \in [0,1]. \tag{3.2}$$

Before proceeding to solve the agent's value function under the optimal contract, observe that $V_t^0(\gamma)$ is linear affine in γ:

$$V_t^0(\gamma) = \gamma V_t^0(1) + (1-\gamma)\delta^{t_0+1-t}V_{t_0+1}^0 = \delta^{t_0+1-t}V_{t_0+1}^0 + \gamma(V_t^0(1) - \delta^{t_0+1-t}V_{t_0+1}^0).$$

Define $\overline{V}_t^0(\gamma) = V_t^0(\gamma) - \delta^{t_0+1-t}V_{t_0+1}^0$. Then

$$\overline{V}_t^0(\gamma) = \gamma\overline{V}_t^0(1).$$

Optimality requires the IC constraints to be binding for all $t \leq t_0$, i.e.,

$$V_t^0(\gamma_t) = c + \delta V_{t+1}^0(\gamma_t), \forall t \leq t_0.$$

Subtracting $\delta^{t_0+1-t}V_{t_0+1}^0$ from both sides,

$$\overline{V}_t^0(\gamma_t) = c + \delta\overline{V}_{t+1}^0(\gamma_t) = c + \delta\frac{\gamma_t}{\gamma_{t+1}}\overline{V}_{t+1}^0(\gamma_{t+1}).$$

Solving it recursively with boundary condition $\overline{V}_{t_0+1}(\gamma_{t_0+1}) = 0$

$$\overline{V}_t^0(\gamma_t) = c\left(\sum_{i=0}^{t_0-t} \delta^i \frac{\gamma_t}{\gamma_{t+i}}\right).$$

Therefore,

$$V_t^0(\gamma_t) = c\left(\sum_{i=0}^{t_0-t} \delta^i \frac{\gamma_t}{\gamma_{t+i}} + \sum_{i=t_0-t+1}^{T-t_0} \delta^i\right) \tag{3.3}$$

The value function at stage 1 in $t \leq t_0 + 1$ can then be derived from equation (3.1):

$$V_t^1 = V_t^0(\gamma_t) + \frac{V_{t-1}^0(\gamma_{t-1}) - \delta V_t^0(\gamma_t)}{\delta \gamma_{t-1} q_1}. \tag{3.4}$$

Similar to the benchmark case, there are multiple sets of V_t^1 values for $t \geq t_0 + 2$ that are consistent with optimality. A sufficient condition is that

$$V_t^1 \leq V_t^0(1) + \frac{c}{\delta q_1} \text{ and } V_{t-1}^1 \geq c + \delta V_t^1, \forall t \geq t_0 + 2 \tag{3.5}$$

This ensures that in $t \geq t_0 + 1$ the agent has incentive to work at stage 1, and has incentive to shirk at stage 0 for any belief. Finally, the reward schedule w_t is derived from recursive equation of the agent's value at stage 1:

$$V_t^1 = q_2 w_t + \delta(1 - q_2) V_{t+1}^1, \forall t = 1, 2, ..., T. \tag{3.6}$$

The following proposition summarizes the properties of the value and reward functions in optimal contract:

Proposition 8. *In the optimal contract, the principal finances the agent up to some termination time T. The stage 0 agent is induced to work up to some $t_0 < T$; the stage 1 agent is induced to work up to T. The agent's value functions and the reward schedule are characterized by equation* (3.2) – (3.6).

3.2.2 Comparison with the Baseline

Recall the two key properties of the optimal contracts that are identified in the baseline model: *efficiency from ignorance* and *non-monotone rewards*. In the environment with the ex ante possibility of a bad project, it can be seen that the non-monotone rewards property continues to hold. That is to say, the reward for success bumps up in at least one of $t = t_0 + 1, ..., T$.

However, the efficiency from ignorance result no longer holds. The stopping time for working of the stage 0 agent t_0 is no longer conditionally efficient given the termination time T. To see that, observe that the agent's ex ante value is

$$V = V_0^0(\gamma_0) = c\left(\sum_{i=0}^{t_0} \delta^i \frac{\gamma_0}{\gamma_i} + \sum_{i=t_0-t+1}^{T} \delta^i\right). \tag{3.7}$$

V is now strictly larger than the payoff from always shirking. The agent gains additional *rents from experimentation* in the periods that he is supposed to work at stage 0: $t = 0, ..., t_0$. This is because the agent's private beliefs become more optimistic after deviation to shirking. Although on the equilibrium path the agent is always indifferent between working and shirking, after having shirked in previous periods, he would strictly prefers to work. Also note that unlike the baseline model, the agent's ex ante value is no longer independent of the choice of t_0. The longer that the principal would like the stage 0 agent to work, the more rents from experimentation she needs to give up to the agent. The principal now faces a tradeoff: with a t_0 smaller than the efficient t_0^*, less social surplus is generate from the project, but the principal also saves on incentive cost. Thus, in the optimal contract, the principal does not necessarily induces the stage 0 agent to work up to the efficient stopping time t_0^*. In fact, in the continuous time limit, the principal always chooses a t_0 that is strictly smaller than t_0^*. This is because near t_0^*, the change on the social surplus is of second order, while the save on incentive cost is of first order.

Proposition 9. *Given any termination time T, $t_0 \leq t_0^*$, and there exists parameter values such that the inequality is strict.*

Lastly, I compare the optimal termination time T^* when there is uncertainty about the project quality with the optimal termination T^{*BL} in the baseline case.

Proposition 10.

$$T^* \leq T^{*BL}.$$

It is not very surprising that with uncertainty of the project quality, the total financing is less. Loosely speaking, the marginal benefit of increasing T by one more period is that if the project is at stage 1 after T, there is one more period to try to complete it. However, with uncertainty of the project's quality, the probability that only the first breakthrough occurs by time T is smaller. There are two reasons for this. First, there is a chance that breakthrough never come, and second, $t_0 \leq t_0^*$ so that less time is allowed to complete the first breakthrough. At the same time, the marginal cost of increasing T, which is the increased agent's ex ante payoff is at least the same as the baseline case. Therefore, the principal would like to decrease the total level of financing when there is uncertainty about the project's quality.

3.3 Noisy Signals of the Progress

In this section, assume that when the project is at stage 1 and if the agent is financed,[1] then at the beginning of a period, a signal arrives publicly with probability κ, independent over time. At stage 0, no signal is received. Moreover, in this section, it is assume that there is a time constraint $T \leq T^*$, where T^* is the optimal termination time in the baseline model. For simplicity, it is assumed that $\delta = 1$.

Now a public history includes not only the event of success, but also the histories of signals. Once one signal arrives, the project is publicly revealed to be at stage 1. Thus future signals no longer matter. There are four kinds of public histories: a history that success has occurred without any signal received, a history that success has occurred after

[1] The reason to assume signals are only available when the agent is financed is that otherwise, the principal can delay financing for arbitarily long and wait for signals over time. If the principal is patient enough, it is almost as if the principal can perfectly observe the progress. But the behavior of waiting to verify the progress is not very realistic and is not the purpose of this excercise. This additional assumption eliminates the desirability to delay financing and wait for signals.

a signal was received, a history that success has not occurred but the first breakthrough has been revealed, and a history without success and signals. A contract again maps a public history to a financing decision and a payment to the agent.

The agent's private histories include the event of the first breakthrough and his past actions in addition to the public histories. Before the final success, there are now three possible states, publicly known stage 1, private stage 1, and stage 0. Let Π_t^1, $\hat{\Pi}_t^1$ and Π_t^0 denote the social surplus conditional on the contract at the three states respectively, and let V_t^1, $\hat{V}_t^1$ and V_t^1 be the respective value functions.

Given any T, the principal again finances the agent and induces him to work up to T if the project has been confirmed to be stage 1 by some signal. If no signal has arrived, the principal finances the agent up to some $t^G \leq T$; the stage 1 agent is induced to work up to t^G and the stage 0 agent is induced to work up to some $t_0 < t^G$. Also, now the reward schedule not only can be a function of the success time, but also may depend on whether the signal has arrived. Let w_t denote the reward paid to the agent for success at time t if the signal has arrived, and let $\hat{w}_t$ be the reward for success before the signal arrives.

Π_t^1 follows the same recursive equations as in the baseline model:

$$\Pi_t^1 = q_2 Y + (1 - q_2)\Pi_{t+1}^1 - c, \forall t.$$

Thus

$$\Pi_t^1 = \left(Y - \frac{c}{q_2}\right)\left(1 - (1 - q_2)^{T-t+1}\right).$$

Same as the baseline model, the efficient time to stop working on a stage 0 project is defined by

$$t_0^* = \max\{t : \Pi_{t+1}^1 \geq \frac{c}{q_1}\}.$$

When the stage is 1 but not yet observed by the principal, the social surplus function satisfies

$$\hat{\Pi}_t^1 = \kappa \Pi_t^1 + (1-\kappa)\hat{\Pi}_{t+1}^1, \forall t \leq t^G,$$

and

$$\hat{\Pi}_t^1 = 0, \forall t > t^G.$$

At stage 0,

$$\Pi_t^0 = q_1 \hat{\Pi}_{t+1}^1 + (1-q_1)\Pi_{t+1}^0, \forall t \leq t_0,$$

and

$$\Pi_t^0 = 0, \forall t > t_0.$$

The agent's value function at stage 0 satisfies

$$\begin{aligned} V_t^0 = q_1 \hat{V}_{t+1}^1 + (1-q_1)V_{t+1}^0 \geq c + V_{t+1}^0, \forall t \leq t_0; \\ V_t^0 = c + V_{t+1}^0, \forall t \in (t_0, t^G]; \\ V_t^0 = 0, \forall t > t^G. \end{aligned} \tag{3.8}$$

At stage 1 before the signal arrives,

$$\begin{aligned}\hat{V}_t^1 =& \kappa V_t^1 + (1-\kappa)q_2\hat{w}_t + (1-\kappa)(1-q_2)\hat{V}_{t+1}^1 \\ \geq& \kappa V_t^1 + (1-\kappa)(c+\hat{V}_{t+1}^1), \quad \forall t \leq t^G; \\ \hat{V}_t^1 =& 0, \forall t > t^G.\end{aligned} \tag{3.9}$$

At stage 1 after the signal arrives,

$$V_t^1 = q_2 w_t + (1-q_2)V_{t+1}^1, \forall t. \tag{3.10}$$

Similar to the environment with observable progress, given termination time T, the stage 0 agent's IC constraints may not be binding. There are two cases to consider.

Lemma 3. *If $q_2\kappa(T-t^G)c < c$, then $t_0 \leq t^G$, and the IC constraints are binding for $t \leq t_0$ at stage 0. If $q_2\kappa(T-t^G)c \geq c$, then $t_0 = t^G$, and stage 0 agent's IC do not bind for $t < t^G$.*

Case 1

Suppose the IC constraints are binding at stage 0 for $t \leq t_0$,

$$V_t^0 = c(t_G - t + 1), \forall t \leq t^G, \tag{3.11}$$

and

$$\hat{V}_t^1 = c\left(t_G - t + 1 + \frac{1}{q}\right), \forall t \leq t^0 + 1. \tag{3.12}$$

The values of $\hat{V}_t^1$ for $t_0 + 2 \leq t \leq t^G + 1$ are not unique: they are consistent with optimal contracts as long as stage 0 agent has incentives to shirk and unconfirmed stage 1 agent has incentives to work:

$$
\begin{aligned}
(q_1\hat{V}^1_{t+1} + (1-q_1)V^0_{t+1}) &\leq c + V^0_{t+1}, \\
\hat{V}^1_t \geq \kappa V^1_t + (1-\kappa)(c+\hat{V}^1_{t+1}), &\quad \forall t \in [t_0+1, t^G].
\end{aligned} \tag{3.13}
$$

Without loss of generality, the IC constraint at stage 1 before signals arrive can (but not need to) be set to binding, i.e., inequality in (3.9) holding with equality. This would pin down $\hat{w}_t$ and V^1_t for $t \leq t_0+1$:

$$\hat{w}_t = \hat{V}^1_t + \frac{c}{q}; \tag{3.14}$$

$$V^1_t = \hat{V}^1_t + c. \tag{3.15}$$

Again, the values of V^1_t and w_t are not unique.

Proposition 11. *Given T, t^G and t_0, if $q_2\kappa(T-t^G)c < c$, then the value functions and reward functions in the optimal contracts are characterized by* (3.8) – (3.13).

Although the values and rewards in optimal contracts are not unique, but they all lead to the same ex ante payoff for the agent.

Proposition 12. *Given T and t^G, if $q_2\kappa(T-t^G)c < c$, then t_0 is characterized by*

$$t_0 = \max\{t : \hat{\Pi}^1_{t+1} \geq \frac{c}{q_1}\}.$$

Proof. Given t^G, the agent's ex ante value is fixed when the stage 0 IC are always binding. Therefore, t_0 should be chosen to maximize the total surplus. If $\hat{\Pi}^1_{t+1} \geq \frac{c}{q_1}$, but the agent is induced to shirk at stage 0. Then the surplus is not maximized by working in t and shirk afterwards,

$$\Pi^0_t = q_1 \hat{\Pi}^1_{t+1} - c,$$

while by starting to shirk the surplus is 0. Conversely, if $\hat{\Pi}^1_{t+1} < \frac{c}{q_1}$ and the agent works in period t at stage 0, then the surplus is negative, which is suboptimal. □

Case 2

Suppose $q_2 \kappa (T - t^G) \geq c$. Then at t^G, the agent at stage 0 has an incentive to work as long as he is promised to be financed till T if he becomes stage 1 and a signal arrives at $t^G + 1$. Moreover, this will be true not only at t^G but at all $t \leq t^G$. Then to minimize the agent's value, the continuation contract after a signal arrives at time t is the same as the one breakthrough case described in proposition 6, and the agent's continuation value is equal to $c(T-t)$:

$$w_t = c\left(T - t + \frac{1}{q}\right) \tag{3.16}$$

$$V^1_t = c(T-t). \tag{3.17}$$

The value function $\hat{V}^t_1$ still satisfies (3.9). Moreover, to minimize the agent's ex ante value, the inequality should hold with equality:

$$
\begin{aligned}
\hat{V}_t^1 =& \kappa V_t^1 + (1-\kappa) q_2 \hat{w}_t + (1-\kappa)(1-q_2)\hat{V}_{t+1}^1 \\
=& \kappa V_t^1 + (1-\kappa)(c + \hat{V}_{t+1}^1), \quad \forall t \leq t^G; \\
\hat{V}_t^1 =& 0, \forall t > t^G.
\end{aligned}
\tag{3.18}
$$

This pins down $\hat{V}_t^1$ and $\hat{w}_t$.

Finally, V_t^0 is characterized by

$$
\begin{aligned}
V_t^0 = q_1 \hat{V}_{t+1}^1 + (1-q_1) V_{t+1}^0, \forall t \leq t^G \\
V_t^0 = 0, \forall t > t^G.
\end{aligned}
\tag{3.19}
$$

Proposition 13. *Given T, t^G and t_0, if $q_2\kappa(T-t^G)c \geq c$, then the value functions and reward functions in the optimal contracts are characterized by* (3.16) – (3.19).

Now that the optimal contracts have been characterized in both cases, binding or non-binding IC at stage 0, next I show that the optimal contract with imperfect signals is the same as the fully observable progress case when the signal is very informative. The result is true irrespective of whether the IC constraints are binding or not at stage 0. Similarly, when the signal is very uninformative, the optimal contract is the same as the situation with no signals at all, wherein the IC constraints are binding at stage 0.

Given $T(\leq T^*)$, let t_0^{BL} be the stage 0 agent's stopping time in the baseline model, and let $\tilde{t}_0$ be the stage 0 stopping time in the fully observed progress case.

Proposition 14. *There exists $\overline{\kappa}$ and $\underline{\kappa}$ such that: 1) if $\kappa > \overline{\kappa}$, then $t^G = t_0 = \tilde{t}_0$; 2) if $\kappa < \underline{\kappa}$, then $t^G = T$, $t_0 = t_0^{BL}$.*

The above proposition confirms that the original baseline model is robust to small perturbation with respect to the principal's ability to observe the progress of the project.

Chapter 4

Application: Acquisition Offers

In this chapter, I extend the model to study the problem of financing of an innovative startup company whereas buyers randomly arrive and make offers to acquire the company. The founder (the agent) of a startup company aims to develop a new product, and the venture capitalist (VC, the principal) finances the founder the cost of R & D. Startup companies attract acquisition offers from time to time, and whether to "get bought" or "get big" is a critical decision to make. Typically, the founder is better informed about the innovation progress, and thus possesses private information of the company's value if kept independent. The VC would like the founder to use his private information appropriately to make better decisions responding to acquisition offers, but their interests in general do not align. For him to be willing to accept an offer, the founder has to be sufficiently compensated by the contract for losing the opportunity to manage the company and be rewarded for a possible success. However, the terms of contracts regarding acquisitions in turn affects the founder's incentive to work at the first place. In this section, I examine how the potential acquisition interacts with the moral hazard problem in innovation, and study how it affects the principal's financing decisions.

The problem of financing startups with randomly arrived buyers relates to the literature on takeovers. Grossman and Hart (1980) shows that takeovers can play a disciplinary role for the management, because the company of a manager with poor performance may get taken over. The raider profits from the takeover and incumbent manager loses his job. On the contrary, Stein (1988) argues that if stockholders are imperfectly informed, the takeover threat will lead to managerial myopia; but this conclusion is reached with the assumption of no agency problem. This result is often used as a justification for entrenchment. In this section, I will show that the principal's lack of information on innovation progress and the entrenchment of the agent together lead to increase in the cost of incentivizing the agent. Consequently, total financing decreases due to potential acquisitions, which is less efficient.

Consider the same environment as described in Chapter 2. It requires *two* breakthroughs for the product to be successfully developed, and there is a moral hazard problem that the founder may divert the investment for private consumption. Only final success is observable to the investor. For notational convenience, assume $q_1 = q_2 = q$. In addition, assume that at the beginning of each period t, there is a probability λ that there is a buyer that arrives and makes an offer p_t to acquire the startup. I assume that buyers are non-strategic and $p_t = z + \mathbb{1}\{n_t = 2\}Y$, where z is a random variable that follows some distribution $G(\cdot)$ with density $g(\cdot)$ and support $[0,\infty)$.[1] The part z in the offer reflects the part of the buyer's valuation for the startup that does not rely on the success of the project. For example, it could be for the expertise of the research team that the startup has built, or for some existing patent or product that is valuable to the buyer. It could even be for the benefit of eliminating a potential competitor. In addition, if the project has been successful, the acquisition offer will take into account the value of the future cash flows Y. The arrivals and values of acquisition offers are independent across periods. Upon receiving the acquisition offer, the agent (the founder of the startup) chooses whether or not to sell the company. The agent may receive payment from the principal upon selling the company, and again the payment has to be non-negative. Afterward, the agent receives 0 continuation value.

I take as exogenous that the agent has the control right. This is typically the case for startup companies in technology industry nowadays, especially for firms at a younger age. In corporations, various forms of anti-takeover defense are used widely such that it is often very difficult for outsiders to acquire a company without the consent of the incumbent management. While it is interesting to study the optimal allocation of control right, this is a complex problem affected by many factors, many of which are not the focus of this paper. Given founder control and takeover defense are the prevailing practices, my goal is to highlight the impact of the unobserved progress of innovation on the principal-agent problem when there are potential buyers interested in take over the company.

[1]The results are not driven by the offers being non-strategic. In subsection 4.4, I will discuss the implication of strategic offers.

With potential buyers, in addition to the final success, the principal now can also observe offers made by arrived buyers and whether the agent accepts an offer or not. I assume that besides the date of success, the principal and the agent can contract the date and the price of the sale of the company, but (dates and prices of) the rejected offers are not contractible. There are two reasons for this assumption. First, communication of offers is often informal and thus difficult to verify and contract on. Indeed, it is uncommon that terms in a contract are contingent on an offer of some price being rejected at some date. In contrast, payments made contingent on an accepted offer are very common, and can be easily implemented by equity and severance packages. Second, if the contractual terms are contingent on some offer being rejected, then potentially the agent may have an incentive to present a fake offer and reject it in order to get more favorable treatment. Here, another set of incentive constraints would be needed, complicating the model without providing much more insight. So instead, it seems reasonable to assume that besides the success of R & D project, only the sale of the company (including the date and price) is contractible.

In the environment with buyers, it is still the case that in an optimal contract, the principal will finance the agent for the project until some termination time, and there is no benefit of delaying investment. Also similar to lemma 1, in an optimal contract, payments only need to be made either at the time of success as a reward or at the time of sale as a severance pay. Moreover, whenever the project has succeeded, or has been terminated, there is no future financing of the project and the continuation value for the agent is zero. Then no payment is needed to induce the agent to sell the company, and I assume that he makes optimal decisions for the principal.[2] Note the selling problem after success or termination is stationary, and will not interfere with the agent's incentives.

In summary, we can focus on contracts given by $(T^B, \mathbf{w}^B, \mathbf{s}(p), \overline{p}, \underline{p})$, where $T^B \in \mathbb{N}$ is the last period that the agent is financed, $\mathbf{w}^B = \{w_t^B\}$ is the reward to the agent when success occurs at time t, $\mathbf{s}(p) = \{s_t(p)\}_{t<T}$ specifies a severance pay to the agent conditional on the company being sold in period t for price p when success has not happened, $\overline{p}$ is the cutoff price above which the company is sold when the project has succeeded, and $\underline{p}$ is the cutoff price when the project has terminated without success.

Given a contract, with some abuse of notations, I again use V_t^n, U_t^n and Π_t^n to denote the agent's value, the principal's value and the total surplus between them at stage n in period t. Then the agent at stage n is willing to sell the company in period t at price p if and only if

$$s_t(p) \geq \delta V_{t+1}^n, n = 0, 1.$$

In other words, if the principal would like the stage n company to be sold in period t, then she has pay stage n agent a severance pay at least equal to his continuation value. Also, it is obvious that $V_t^1 \geq V_t^0$ for all t. So if stage 1 agent is induced to sell the company at some price p, stage 0 agent is also willing to accept the offer. By choosing $s_t(p)$, the principal is choosing at price p, whether the company will be sold at neither stage, or only at stage 0, or at both stage 0 and stage 1.

[2]Equivalently, I can assume that after success or termination, the agent leaves the company, and the principal makes selling decisions by herself.

Lemma 4. *If an optimal contract exists, then there is an optimal contract where for all $t < T$, $s_t(p)$ satisfies*

$$s_t(p) = \begin{cases} 0 & p < p_t^0 \\ \delta V_t^0 & p \in [p_t^0, p_t^1) \\ \delta V_t^1 & p \in [p_t^1, \infty) \end{cases} \tag{4.1}$$

for some $p_t^0, p_t^1 > 0$ such that $p_t^1 > p_t^0$.

Lemma 4 states that we can focus on contracts where the principal does not pay more than stage n agent's continuation value to induce the company at that stage to be sold. Moreover, if it is optimal to have the company of stage n sold at price p, then it is also optimal to do so for any $p' > p$. Choosing $\mathbf{s}(p) = \{s_t(p)\}_{t<T}$ is equivalent to choosing a sequence of cutoff pairs $(\mathbf{p}^0, \mathbf{p}^1) = \{(p_t^0, p_t^1)\}_{t<T}$ such that the company of stage $n = 0, 1$ will be sold in period $t < T$ if and only if the offer p is no less than p_t^n. Define

$$\mu_t^n = 1 - G(p_t^n), \;\; n = 0, 1, \;\; t < T.$$

So μ_t^n is the probability that the company at stage n in period t will be sold. Choosing $(\mathbf{p}^0, \mathbf{p}^1)$ is equivalent to choosing $(\mu_t^0, \mu_t^1) = \{(\mu_t^0, \mu_t^1)\}_{t<T}$, and later on I use them interchangeably when referring to a contract.

It is a standard search problem to characterize the optimal $\overline{p}$ and $\underline{p}$.

Lemma 5. *Let $\overline{\Pi}$ be the value of the principal when success has happened and $\underline{\Pi}$ be her* *value when the project has terminated without success.* $\underline{\Pi}$ *is the unique solution to*

$$(1-\delta)\underline{\Pi} = \lambda \int_{z \geq \delta\underline{\Pi}} (z - \delta\underline{\Pi})g(z)dz,$$

and $\overline{\Pi} = \underline{\Pi} + Y$*. The optimal* $\overline{p}$ *and* $\underline{p}$ *are* $\overline{p} = \delta\overline{\Pi}$ *and* $\underline{p} = \delta\underline{\Pi}$*.*

A contract $(T^B, \mathbf{w}^B, \mathbf{p}^0, \mathbf{p}^1, \overline{p}, \underline{p})$ will determine the value functions of the agent V_t^n and total surplus functions Π_t^n. The agent will be induced to work or shirk at each stage in each period. Similar to the no buyer case, it can be shown that in optimal contract stage 1 agent is always induced to work for $t \leq T^B$, and there exists some $t_0^B < T^B$ such that stage 0 agent is induced to work if and only if $t \leq t_0^B$. The problem can now be written as

$$\begin{aligned}
&\max_{T^B, t_0^B, \mathbf{w}, \mathbf{p}^0, \mathbf{p}^1} \Pi_0^0 - V_0^0 \\
\text{s.t.}\quad & V_t^0 \geq c + \delta V_{t+1}^0, \forall t \leq t_0^B, \\
& V_t^0 = c + \delta V_{t+1}^0, \forall t \in (t_0^B, T^B]; && \text{(IC: Stage 0)} \qquad \text{(P)} \\
& V_t^0 \geq c + \delta V_{t+1}^0, \forall t \leq T_0^B; && \text{(IC: Stage 1)} \\
& w_t \geq 0, \ \ \forall t \leq T_0^B. && \text{(LL)}
\end{aligned}$$

The following proposition establishes the existence of the optimal contract.

Proposition 15. *There exists a solution to problem* (P')*. The solution characterizes the optimal contract if the maximized value is positive; otherwise no investment is optimal for the principal.*

From now on I assume that the value to problem (P') is positive.

In the rest of the section, I will characterize the properties of the optimal contracts. Again, the key tension in the model is the endogenous information asymmetry of the progress of innovation. As in the setting without buyers, one main problem regarding this asymmetric information is how to use rewards for success and threat of termination to simultaneously provide agents at different levels of progress with proper incentives to work or shirk. In addition to that, with potential acquisitions offers, the other important concern is what selling decisions to induce from agents with different levels of progress. The two problems interact closely with each other. On the one hand, the selling decisions in period t will affect the agent's incentives to work in previous periods and the cost of providing incentives; on the other hand, the moral hazard problem of the innovation project will affect the optimal prices to sell a company.

4.1 Incentive Cost Minimization Given $(T^B, t_0^B, \mathbf{p}^0, \mathbf{p}^1)$: Acquisition Rent

In this subsection, I take as given the termination time T^B, the stopping time of working for stage 0 agent t_0^B and the cutoff selling prices $(\mathbf{p}_t^0, \mathbf{p}_t^1)$ (or selling probabilities (μ_t^0, μ_t^1)) . Then the total surplus between the principal and the agent is fixed, and I study how to minimize the cost of providing incentives to the agent. I will characterize the agent's value functions V_t^n and the reward function w_t^n in terms of $(T^B, t_0^B, \mathbf{p}_t^0, \mathbf{p}_t^1)$ (or $(T^B, t_0^B, \mu_t^0, \mu_t^1)$).

As in section 2.2, there are a set of recursive equations that V_t^1 and V_t^1 satisfy, but this time the possibility of acquisitions needs to be taken into account. For $t \leq t_0^B$, stage 0 agent is supposed to work, and

$$V_t^0 = \delta(q + (1-q)\lambda\mu_t^1)V_{t+1}^1 + \delta(1-q)(1-\lambda\mu_t^1)V_{t+1}^0.$$

So if the stage 0 agent works, then there are two cases that he will have stage 1 agent's continuation value δV_{t+1}. It is either when he makes the first breakthrough with probability q, or when he accepts a acquisition offer greater than p_t^1 and receives a severance pay δV_{t+1}^1. Otherwise, his value in the next period is V_{t+1}^0. To prevent him from shirking, the agent's IC condition is for all $t \leq t_0^B$,

$$\begin{aligned} V_t^0 &= \delta(q + (1-q)\lambda\mu_t^1)V_{t+1}^1 + \delta(1-q)(1-\lambda\mu_t^1)V_{t+1}^0 \\ &\geq c + \delta\lambda\mu_t^1 V_{t+1}^1 + \delta(1-\lambda\mu_t^1)V_{t+1}^0 \end{aligned} \tag{4.2}$$

When $t > t_0^B$, the agent is supposed to shirk, so

$$V_t^0 = c + \delta\lambda\mu_t^1 V_{t+1}^1 + \delta(1-\lambda\mu_t^1)V_{t+1}^0, \tag{4.3}$$

and in particular $V_{T^B}^0 = c$.

Again the agent has the option to always shirk, and that provides a lower bound for the agent's ex ante payoff V_0^0. But unlike the case without buyers, by always shirking, the agent is getting more than the discounted value of total funds for investment. This is because when a acquisition offer $p \geq p_t^1$ arrives, by selling the company the agent can receive the stage 1 agent's continuation payoff δV_t^1, which is larger than his own continuation payoff. The next proposition solves for the minimum agent's ex ante payoff V_0^0, and the associated V_t^n and w_t for $t = 1, ..., T^B, n = 0, 1$.

Proposition 16. *In an optimal contract with $(T^B, t_0^B, \mathbf{p}_t^0, \mathbf{p}_t^1)$ (or $(T^B, t_0^B, \mu_t^0, \mu_t^1)$), the agent's ex ante payoff is*

$$V_0^0 = c\sum_{i=0}^{T^B} \delta^i + c\sum_{i=0}^{t_0^B} \delta^i \frac{\lambda \mu_i^1}{q(1-\lambda \mu_i^1)}.$$

Stage 0 agent's IC conditions are binding for $t \leq t_0^B$ and stage 1 agent's IC conditions are binding for $t > t_0^B + 1$. The agent's value functions V_t^n are

$$V_t^0 = c\sum_{i=0}^{T^B - t} \delta^i + c\sum_{i=t}^{t_0^B} \delta^{i-t} \frac{\lambda \mu_i^1}{q(1-\lambda \mu_i^1)}, \ \ \forall t = 0, ..., t_0^B;$$

$$V_t^0 = c\sum_{i=0}^{T^B - t} \delta^i, \ \ \forall t = t_0^B + 1, ..., T^B.$$

$$V_t^1 = V_t^0 + \frac{c}{\delta q(1-\lambda \mu_{t-1}^1)}, \ \ \forall t = 1, ..., t_0^B + 1;$$

$$V_t^1 = V_t^0 = c\sum_{i=0}^{T^B - t} \delta^i, \ \ \forall t = t_0^B + 2, ..., T^B.$$

The reward function w_t is

$$w_t = \delta V^1_{t+1} + \frac{V^1_t - \delta V^1_{t+1}}{q}, \ \ \forall t = 1, ..., T^B.$$

Proposition 16 shows that as in the case without buyers, the cost-minimizing way to provide incentives is to make stage 0 agent always indifferent between working and shirking when he is supposed to work. The agent's ex ante payoff is equal to the payoff he can get by always shirking.

However, unlike in the previous section, now incentive cost minimization also requires stage 1 agent's IC to be binding for $t > t^B_0 + 1$. Recall that in the environment without buyers, there are multiple $\{V^1_t\}$ for $t > t^B_0 + 1$ that are consistent with optimal contracts. The reason there is that for $t > t^B_0$, stage 0 agent is supposed to shirk, and he will not reach stage 1. Therefore the agent's ex ante payoff, which is the payoff from always shirking, will not be affected by V^1_t for $t > t^B_0 + 1$. So V^1_t can be chosen arbitrarily for $t > t^B_0 + 1$ as long as the relevant IC conditions are satisfied, and V^1_t is not necessarily minimized for $t > t^B_0 + 1$. With arrivals of acquisitions offers and possibility of sale, this is no longer the case. Even when stage 0 agent is shirking in period t, he may still receive stage 1 agent's continuation payoff δV^1_{t+1}, because there may be a acquisition offer high enough such that the principal wants induce stage 1 agent to accept the offer. Therefore, to minimize the agent's ex ante payoff, the contract must also make V^1_t as small as possible in each period, and the function V^1_t is uniquely determined for all $t = 1, ..., T^B$.

From Proposition 16, we can see that μ_t^0 does not enter the agent's value functions. That is to say, inducing only stage 0 agent to sell does not incur extra incentive cost. This is because the principal only needs to pay δV_t^0 to stage 0 agent for him to accept an offer, which is the same as what he will get by not selling. In contrast, one key result is that the agent's ex ante payoff is strictly increasing in the probabilities that the stage 1 firm is sold μ_t^1 for $t \leq t_0^B$:

$$V_0^0 = c\sum_{i=0}^{T^B} \delta^i + c\sum_{i=0}^{t_0^B} \delta^i \frac{\lambda \mu_i^1}{q(1-\lambda \mu_i^1)}.$$

The first part in the agent's ex ante payoff is the discounted value of total investments to divert; the second part is the agent's *acquisition rent*. If the principal wants to use lower cutoff p_t^1 and have lower offers accepted for stage 1 company in period t, then not only stage 1 agent, but also stage 0 agent will more likely receive the severance pay δV_{t+1}^1. In periods prior to t, the agent of stage 0 understands that even he shirks, there is a larger chance that the company will be sold as if the stage is 1. Therefore he has more incentive to shirk in previous periods and inducing him to work becomes more costly. Note the acquisition rent is only caused by potential buyers arriving before $t_0^B + 1$, because after that stage 0 and stage 1 agents have the same values and stage 0 agent will not get compensated by more than his continuation value for selling the company.

I have shown that potential acquisitions increase the incentive cost gives the agent a acquisition rent. In the next two subsections, I will show how the acquisition rent affects the selling prices of the company and the principal's financing problem.

4.2 Optimality on $(\mathbf{p}^0, \mathbf{p}^1)$: Moral Hazard Premiums

Suppose $(T^B, t_0^B, \mathbf{w}, \mathbf{p}^0, \mathbf{p}^1)$ is an optimal contract. In this subsection, I characterize optimality conditions regarding $(\mathbf{p}^0, \mathbf{p}^1)$. Choosing what offers to accept will affect the principal's payoff $U = \Pi - V$ through both the total surplus available from the project $\Pi = \Pi_0^0$ and the agent's ex ante payoff $V = V_0^0$. Subsection 4.1 shows the impact of $(\mathbf{p}^0, \mathbf{p}^1)$ on V_0^0. Next I study its impact on the total surplus.

First note that if the project is successful, the total surplus is $\overline{\Pi}$, and if the project is terminated without success, the total surplus is $\underline{\Pi}$ which comes solely from sale of the company, where $\overline{\Pi}$ and $\underline{\Pi}$ are defined as in lemma 5. If the project is at stage 1 in period $t \leq T^B$, then the total surplus Π_t^1 follows

$$\Pi_t^1 = q\overline{\Pi} + (1-q)\left[\lambda \int_{p_t^1}^{\infty} zg(z)dz + (1-\lambda(1-G(p_t^1)))\delta\Pi_{t+1}^1\right] - c. \tag{4.4}$$

With probability q the project becomes successful and the total surplus increases to $\overline{\Pi}$; without success, if the buyer arrives and offers a price higher than the cutoff p_t^1, then the company will be sold; otherwise, the project moves to the next period staying at stage 1.

If the project is at stage 0, for $t \leq t_0^B$,

$$\begin{aligned}\Pi_t^0 =& q\left[\lambda \int_{p_t^1}^{\infty} zg(z)dz + (1-\lambda(1-G(p_t^1)))\delta\Pi_{t+1}^1\right] \\ &+ (1-q)\left[\lambda \int_{p_t^0}^{\infty} zg(z)dz + (1-\lambda(1-G(p_t^0)))\delta\Pi_{t+1}^0\right] - c;\end{aligned} \tag{4.5}$$

for $t = t_0^B, \ldots, T^B$,

$$\Pi_t^0 = \lambda \int_{p_t^0}^{\infty} zg(z)dz + (1-\lambda(1-G(p_t^0)))\delta\Pi_{t+1}^0. \tag{4.6}$$

Let $\mathbb{P}$ be the probability measure induced over the set of all outcomes. Define

$$\rho_t^n = \mathbb{P}(n_t = n \cap \text{no sale}), \;\; n = 0,1,2;$$

So ρ_t^n is the probability that the project is at stage n and the company has not been sold.

Proposition 17. *In an optimal contract, the cutoff prices p_t^0, p_t^1 satisfy*

$$p_t^0 = \delta\Pi_{t+1}^0, \;\; \forall t \leq T^B;$$

$$p_t^1 = \begin{cases} \delta\Pi_{t+1}^1 + \frac{c}{(\rho_t^0 q + \rho_t^1(1-q))q[1-\lambda(1-G(p_t^1))]^2}, & t \leq t_0^B \\ \delta\Pi_{t+1}^1, & t_0^B < t \leq T^B \end{cases}.$$

As discussed in section 4.1, for $t \leq T^B$, inducing stage 0 agent to sell does not affect the agent's payoff, because the severance pay needed is exactly equal to his continuation value δV_t^0. Therefore, the optimal cutoffs should equal to the total surplus after rejecting the offer. In other words, in these cases the principal should make a severance pay to induce the agent to accept the offer if and only if the price is higher than the total surplus that can be generated from the project after rejecting the offer. Given the investment choices, the selling decisions of the stage 0 project are made as if there is no agency problem. The same is true for selling stage 1 project in periods $t > t_0^B$ because $V_{t+1}^1 = V_{t+1}^0$ for $t > t_0^B$.

However, if the principal wants to induce agents at stage 1 to accept an offer in period $t \leq t_0^B$, she has to pay δV_{t+1}^1, which is larger than stage 0 agent's continuation value. The agent therefore receives extra rent because of the potential acquisition, and inducing incentives in earlier periods becomes more costly. As a result, the principal would like the stage 1 project to be sold less often to reduce the agent's acquisition rent. She may not want to induce the agent to accept an offer even if the offer is higher than the surplus from continuing the project. More specifically, at stage 1, an offer p will be accepted in period $t \leq t_0^B$ if and only if

$$p - \delta \Pi_{t+1}^1 \geq \frac{c}{(\rho_t^0 q + \rho_t^1 (1-q)) q [1 - \lambda(1 - G(p_t^1))]^2} = MHP.$$

I call the right hand side of the above inequality the *moral hazard premium* (MHP). This is extra amount of money the buyer needs to pay in order for the offer to be accepted by a stage 1 agent in addition to the continuation value of keeping the firm. MPH is decreasing in $\rho_t^0 q + \rho_t^1 (1-q))$, which is the probability that the project is unsold and is at stage 1 after the investment in period t. In general, this probability is first increasing and then decreasing, and thus MPH is first decreasing then increasing. The cutoffs p_t^1 is non-monotone in t. Also, MPH is increasing in per period cost c. When the agent can divert more investment in a period, the agency problem is worse, and the moral hazard premium is higher.

In this model, there is one startup company with an innovation project. Imagine a world with many such innovating companies with heterogeneous values. Buyers that cannot observe the progress of innovations face a lemon problem: agents on projects with less progress are more likely to accept an offer. This is true even without the moral hazard problem because projects with less progress have lower value to continue. The result on moral hazard premium suggests that the agency problem in innovation aggravates the lemon problem. To reduce the acquisition rent received by agents, the projects with more progress are sold even less likely at even higher prices. Conditional on an offer being accepted, the probability that the project is with slow progress is larger than the case without the moral hazard problem.

4.3 Investment Choices (T^B, t_0^B)

In this subsection, I analyze the impact of acquisition rent on the total amount of investment the principal chooses to finance, and the total amount of investment made by stage 0 agent.

In the environment without buyers, recall that T is the last period in which the agent is financed by the principal and t_0 is the last period that stage 0 agent is induced to work. Let T^B and t_0^B be the corresponding values in the case with buyers.

Proposition 18.

$$T^B \leq T.$$

Moreover, there exist parameter values such that the inequality is strict.

Proposition 18 states that if the buyers arrive often enough and their valuations are high enough, then the investment on innovation with potential acquisition offers is less than the case without acquisitions offers in two aspects: First, the total investment that the principal commits to finance the agent is less; second, the agent with slow progress gives up on the project earlier.

Intuitively, the presence of potential buyers gives the agent additional acquisition rent and makes inducing honest investment more costly. To balance the increased incentive cost, in an optimal contract, the principal would like to commit to a smaller amount of total investment, and ask the agent with little progress to stop working earlier.

This result have empirical implications on the relationship between investment lengths and success probabilities of innovation projects. During economic bubbles such as the "dot-com" bubble at the end of 20th century, hot money flowed in and buyers' valuations over tech firms surged due to frenzy speculations. Acquisitions became much more probable. However, since the operation of startups are more or less opaque and it is hard for the initial investors and outside buyers to monitor the progress of the innovation, the possibility of acquisition offers created huge inventive problems for entrepreneurs. They had strong incentives to shirk because they knew that even little progress was made, it was likely that they would be bailed out by selling the companies to buyers making wild offers. In response to this increased incentive costs, the initial investors became more impatient. They invested in projects hoping for some quick outcome or sell-off, and were less willing to commit to longer investment periods. Partly because of this, although more projects were financed due to the capital inflow, the overall quality of startups in terms of probability of success and survival time became worse, which might have in turn contributed to the burst of the bubble.

Saffie and Ates (2013), using Chilean data of 1998, found that firms born during economic downturn tended to grow better than in other times. Their main theory is that due to credit shortage, financial institutions were more careful at screening and selecting projects, and therefore projects that actually got financed were intrinsically of better quality. My model suggests an alternative and complementary explanation to the phenomenon. During credit shortage, acquisitions were less a concern, and the agents' incentive problems were alleviated. Investors were willing to commit to longer investment and the probabilities of success were higher.

4.4 Discussion: Strategic Buyers

In this chapter, I have been assuming that buyers are non-strategic and offers are random variables drawn from some distribution. This is a reasonable assumption in situations where buyers' valuation for the startup are observable to the agent, and the agent has all the bargaining power. In this section, I discuss the other extreme, namely, buyers have all the bargaining power and make take-it-or-leave-it (TIOLI) offers upon arrivals. Here

I argue that the qualitative results are not driven by that buyers are non-strategic. Details on the results are available on request.

First, the acquisition rent still arises and the agent's ex ante payoff is larger than if he always diverts funds financed by the principal. Its driving force is that when the company is sold off, the principal cannot distinguish agents with different levels of progress. Therefore, if the principal wants to sell when the project is stage 1, she has to pay stage 1 agent's continuation value δV_{t+1}^{1} to both stage 1 and stage 0 agent. Therefore, even if the agent has always been shirking, there is a chance that in some period he gets a payoff equal to as if he has made some progress. So the acquisition rent exists no matter whether offers by buyers are strategic or not, as long as the stage 1 project is sold with positive probability. Moreover, the agent's acquisition rent is larger when the stage 1 project is sold more likely.

Second, since the agent's acquisition rent is increasing in the probability that stage 1 project is sold, in an optimal contract, the stage 1 project is induced to be sold less likely, and the cutoff offers for acceptance are higher than if there is no agency problem. When buyers are non-strategic, the cutoff offers in no-agency-problem benchmark are the continuation surplus of the project after the current offer is rejected. When buyers are strategic, another consideration kicks in. They will no longer simply bid their valuations; instead they will infer from the contract that whether an offer will be accepted or rejected by the agent with different stages. Depending on their valuations, they will either not make an offer, or bid the cutoff price for stage 0 agent and only buys stage 0 companies, or bid the cutoff price for stage 1 agent and buy companies of both types. In other words, by specifying different cutoff prices for acceptance, the contract can affect the offers made by the buyers. Thus while it is the buyer that makes the TIOLI offer, by committing to a contract, the startup actually has the the bargaining power; the situation is equivalent to a monopoly posting a pair of prices for the company at each stage. So without the agency problem, the contract would specify the cutoff prices that maximize the monopoly profit given the buyers value distribution with the cost being the continuation surpluses of not selling. Unlike the non-strategic buyer case, the cutoff prices will be higher than the continuation surpluses of not selling even without the agency problem. Essentially, by contracting with an agent, the principal can change the agent's payoff in the bargaining game between the agent and the buyer, and obtain more commitment power. This idea similar to Fershtman and Judd (1987), Fershtman, Judd, and Kalai (1991) and Cai and Cont (2004).

With the monopoly pricing as the no-agency-problem benchmark, it still holds that due to the moral hazard problem, the cutoffs for stage 1 agent are higher than the optimal monopoly prices without agency problem. The intuition is the same: Raising the cutoff prices for stage 1 agent will decrease the agent's acquisition rent; although it decreases the expected profit from the sold-off, the principal is still better off.

Finally, with strategic buyers, the optimal amount of financing is still less than without potential acquisitions. The driving forces in the case of non-strategic buyers are the increased cost of financing due to the acquisition rent and decreased benefit of financing because of the possibility of a sold-off before success. These forces still exist when the buyers are strategic.

I have argued that similar results hold when assuming strategic buyers. There is one additional complication that is worth noting. With non-strategic buyers, future offers will not depend on whether a offer was rejected in the past, and since the contract does not depend on rejected offers, the agent's valuation only depends on the stage of the project. This is no longer true when buyers are strategic and can observe past offers. Buyers will make inferences about the agent's progress based on these past offers. On the one hand, rejects of past offers will affect the buyers' valuations if they care about partial progress. On the other hand, since agents with different levels of progress have different acceptance cutoffs, buyers will have different beliefs about whether an offer will be accepted depending on past offers. Therefore, buyers' strategies depend on past offers, and so do the agent's value functions. More specifically, there will be three relevant value functions for the agent: the value when he is at stage 1, the value when he is stage 0 but buyers think that he is at stage 1 because he rejected an offer that stage 0 is supposed to accept, and the value when he is stage 0 and has deviated.

Appendix A

Appendices to Chapter 2

Throughout the proofs, define $\sum_i^j(\cdot) = 0$ for all $j < i$.

A.1 Characterization of Optimal Contracts

Calculation of Assumption 1. We calculate the planner's payoff by always investing until success Π^*. The probability that success occurs in period $t^* \geq 1$ is

$$\sum_{t'=0}^{t^*-1} q_1 q_2 (1-q_1)^{t'} (1-q_2)^{t^*-t'-1} = \frac{q_1 q_2}{q_1 - q_2}\left[(1-q_2)^{t^*} - (1-q_1)^{t^*}\right]$$

So

$$
\begin{aligned}
\Pi^* = &\sum_{t^*=1}^{\infty} \frac{q_1 q_2}{q_1 - q_2} \left[(1-q_2)^{t^*} - (1-q_1)^{t^*} \right] \left(\delta^{t^*} Y - \sum_{t=0}^{t^*} \delta^t c \right) \\
= &\frac{q_1 q_2}{q_1 - q_2} \sum_{t^*=1}^{\infty} \left[(1-q_2)^{t^*} - (1-q_1)^{t^*} \right] \left(\delta^{t^*} Y - c \frac{1-\delta^{t^*+1}}{1-\delta} \right) \\
= &\frac{q_1 q_2}{q_1 - q_2} Y \sum_{t^*=1}^{\infty} \left[\delta^{t^*} (1-q_2)^{t^*} - \delta^{t^*} (1-q_1)^{t^*} \right] \\
&- \frac{q_1 q_2}{q_1 - q_2} \cdot \frac{c}{1-\delta} \sum_{t^*=1}^{\infty} (1-\delta^{t^*+1}) \left[(1-q_2)^{t^*} - (1-q_1)^{t^*} \right] \\
= &\frac{q_1 q_2}{q_1 - q_2} Y \left[\frac{\delta(1-q_2)}{1-\delta(1-q_2)} - \frac{\delta(1-q_1)}{1-\delta(1-q_1)} \right] \\
&- \frac{q_1 q_2}{q_1 - q_2} \cdot \frac{c}{1-\delta} \left[\left(\frac{1-q_2}{q_2} - \frac{1-q_1}{q_1} \right) - \delta \left(\frac{\delta(1-q_2)}{1-\delta(1-q_2)} - \frac{\delta(1-q_1)}{1-\delta(1-q_1)} \right) \right] \\
= &\frac{\delta q_1 q_2 Y}{[1-\delta(1-q_1)][1-\delta(1-q_2)]} - \frac{c}{1-\delta} \left[1 - \frac{\delta^2 q_1 q_2}{[1-\delta(1-q_1)][1-\delta(1-q_2)]} \right] \\
= &\frac{1}{[1-\delta(1-q_1)][1-\delta(1-q_2)]} \left[\delta q_1 q_2 Y - (1-\delta+\delta(q_1+q_2))c \right]
\end{aligned}
$$

So $\Pi^* \geq 0$ implies

$$
Y \geq \frac{[1-\delta+\delta(q_1+q_2)]}{\delta q_1 q_2} c.
$$

□

Proof of Lemma 1. Suppose the optimal contract Γ specifies a payment scheme $\{w_t\}$ conditional on success in period t and a unconditional payment $\beta > 0$ in some period τ. Now consider an alternative contract $\hat{\Gamma}$ that specifies the same termination time T and induces the same actions as Γ. The only difference is that $\hat{\Gamma}$ specifies 0 unconditional payment in period t, and $\hat{w}_\tau = w_\tau + \beta/q$. We will show that under $\hat{\Gamma}$ the IC conditions are still satisfied and the agent's ex ante value is strictly smaller.

First note that the value functions for both stage 0 and stage 1 agents are not affected for $t > \tau$. We have $\hat{V}_t^0 = V_t^0$ for $t > \tau$. So the IC conditions are not affected for $t \geq \tau$. Moreover, stage 1 agent's value function remain the same even for $t \leq \tau$, i.e., $\hat{V}_t^1 = V_t^1$ for all $t \leq T$, and stage 1 agent's IC conditions are still satisfied for all $t \leq T$. For the stage 0 agent, his value at τ is strictly smaller because of the loss of the unconditional payment β, i.e., , $\hat{V}_\tau^0 < V_\tau^0$. Hence, stage 0 agent's value for $t < \tau$ also becomes smaller, including the ex ante payoff. Since the gap between the stage 0 and stage 1 agents' values are larger, it is easier for stage 0 agent's IC conditions to satisfy for $t < \tau$. $\square$

Proof of Proposition 1. It remains to prove that for all $t < t_0$, condition (2.2) holds, i.e.,

$$\Pi_{t+1}^1 - \Pi_{t+1}^0 \geq \frac{c}{\delta q}. \tag{2.2}$$

We prove it by induction.

1. In period t_0, condition (2.2) holds by construction.

2. Suppose condition (2.2) holds for some $t \leq t_0$. Then

$$\Pi_t^0 = \delta(q\Pi_{t+1}^1 + (1-q)\Pi_{t+1}^0) - c.$$

Thus

$$\begin{aligned}
\Pi_t^1 - \Pi_t^0 &= \Pi_t^1 - \delta(q\Pi_{t+1}^1 + (1-q)\Pi_{t+1}^0) + c \\
&> \Pi_{t+1}^1 - \delta(q\Pi_{t+1}^1 + (1-q)\Pi_{t+1}^0) + c \\
&= (1-\delta q)\Pi_{t+1}^1 - (1-\delta q)\Pi_{t+1}^0 + (1-\delta)\Pi_{t+1}^0 + c \\
&= (1-\delta q)(\Pi_{t+1}^1 - \Pi_{t+1}^0) + (1-\delta)\Pi_{t+1}^0 + c \\
&\geq (1-\delta q)\frac{c}{\delta q} + (1-\delta)\Pi_{t+1}^0 + c \\
&\geq \frac{c}{\delta q},
\end{aligned}$$

where the first inequality holds because Π_t is strictly decreasing in t.

So condition (2.2) holds for all $t < t_0$ with strict inequality. □

Proof of Proposition 3. As discussed, the marginal cost of investment in period T is the increase in the agent's ex ante payoff, $\delta^T c$. Next we calculate its marginal benefit.

Compare the total surplus generated by the project between contract with termination time $T-1$ and one with T. Stage 0 agent stops working after $t_{0,T-1}$ and $t_{0,T}$ respectively, with $t_{0,T} = t_{0,T-1} + 1$. There are three cases:

1. Conditional on the event that two breakthroughs occur by period $t_{0,T-1}$, the two contracts generate the same total surplus.

2. Conditional on the event that exactly one breakthrough occurs by period $t_{0,T-1}$, the contract with T generates more surplus because stage 1 agent has one more period

to make investment. The increase in surplus is

$$Q_1(t_{0,T})Q_0(\hat{t})\delta^T(\delta qY - c).$$

3. Conditional on the event that no breakthrough occurs by $t_{0,T-1}$, the contract with T generates more surplus because it gives one more chance to reach stage 1 in period $t_{0,T}$. The increase in surplus is

$$Q_0(t_{0,T})\delta^{t_{0,T}}(\delta q\Pi^1(\hat{t}) - c)$$

Summing up, the marginal benefit to invest in period T is

$$\Pi^0_{0,T} - \Pi^0_{0,T-1} = Q_1(t_{0,T})Q_0(\hat{t})\delta^T(\delta qY - c) + Q_0(t_{0,T})\delta^{t_{0,T-1}}(\delta q\Pi^1(\hat{t}) - c), \ \ \forall T \geq \hat{t}.$$

Note that $(\Pi^0_{0,T} - \Pi^0_{0,T-1})/\delta^T$ first increases and then decreases in T, and converges to 0 as T goes to infinity. So either $\Pi^0_{0,T} - V^0_{0,T}$ is maximized at

$$T^* = \max_T\{T : \Pi^0_{0,T} - \Pi^0_{0,T-1} \geq \delta^T c\},$$

or $\Pi^0_{0,T} - V^0_{0,T}$ is always negative for all T. □

Proof of Proposition 2. The proof is broken down to two parts:

First, I prove that within the class of deterministic contracts without reports, the optimal contract does not delay financing.

Take any contract Γ where financing is provided in $k \in \{2,3,...,\infty\}$ phases for $t \in \cup_{i=1}^{k}[\tau_i,\tau_i')$ where $\tau_{i+1} > \tau_i'$ for all $i \in \{1,...,k-1\}$. I will show there is a contract consisting of only one financing phase that gives the principal at least the same payoff as Γ does. Define $\ell_i = \tau_i' - \tau_i$ for $i = 1,...,k$ and $\ell_0 = 0$; define $\sigma_i = \tau_{i+1} - \tau_i'$ for $i = 1,...,k-1$ and $\sigma_0 = \tau_1$. So ℓ_i is the length of the ith financing phase and σ_i's are the lengths of intervals in which financing is suspended.

Recall that Π_t^n is the total surplus of the project at stage n in period t. Let $\tilde{\Pi}(i)$ be the expected total surplus in period τ_i conditional on success did not occur before the τ_i, the ith financing phase, i.e.,

$$\tilde{\Pi}(i) = \mathbb{P}(n_{\tau_i} = 0|n_{\tau_i} \neq 2)\Pi_{\tau_i}^0 + \mathbb{P}(n_{\tau_i} = 1|n_{\tau_i} \neq 2)\Pi_{\tau_i}^1,\ i = 1,...,k;\ \tilde{\Pi}(k+1) = 0.$$

Let $\Pi(i) = \tilde{\Pi}(i) - \delta^{\ell_i+\sigma_i}\tilde{\Pi}(i+1)$. Then $\Pi(i)$ is the total surplus at the beginning of the ith phase conditional on success occurs in the ith phase.

Define

$$V(i) = c\sum_{t=0}^{\ell_i-1}\delta^t,$$

and define

$$\tilde{V}(i) = V(i) + \sum_{m=i+1}^{k}\delta^{\ell_m+\sigma_m}V(m).$$

Since the agent has the option to always shirk, his value at τ_i given by contract Γ is bounded from below by $\tilde{V}(i)$. In particular, his ex ante payoff is bounded from below by

$\sigma_0\tilde{V}(1)$.

Define

$$j = \min\left\{i \in \{1,...,k\} : \tilde{\Pi}(i) - \tilde{V}(i) < 0\right\},$$

and let $j = k+1$ when the set is empty. Note that if $k = \infty$, then the set will never be empty, and it is dominated by some contract with finite periods of financing.

If $j \geq 2$, let contract Γ' be an optimal cutoff contract with termination date $T = \sum_{i=1}^{j-1} \ell_i - 1$ as characterized in Proposition 2. If $j = 1$ then the project is not financed at all in contract Γ'. I will argue contract Γ' yields at least the same payoff to the principal as Γ.

If $j = 1$, $\tilde{\Pi}(1) - \tilde{V}(1) < 0$. Under Γ, the ex ante total surplus is $\sigma_0\tilde{\Pi}(1)$, and the agent's ex ante payoff is bounded from below by $\sigma_0\tilde{V}(1)$. Therefore the principal's ex ante payoff is negative, and she is better off by not financing the project at all.

If $j \geq 2$, under Γ, the ex ante total surplus generated from the project can be expressed as

$$\Pi = \sum_{i=1}^{j-1} \delta^{\tau_i}\Pi(i) + \delta^{\tau_j}\tilde{\Pi}(j);$$

the agent's ex ante payoff is bounded from below by

$$\sum_{i=1}^{j-1} \delta^{\tau_i}V(i) + \delta^{\tau_j}\tilde{V}(j).$$

Thus, the principal's ex ante payoff U under Γ satisfies

$$U \leq \sum_{i=1}^{j-1} \delta^{\tau_i} \left[\Pi(i) - V(i)\right] + \delta^{\tau_j} \left[\tilde{\Pi}(j) - \tilde{V}(j)\right].$$

Under Γ', the project is financed for the same length as the total length of the first $j-1$ phases under Γ. Suppose the same sequence of actions were induced as in the first $j-1$ phases in Γ, but delays are eliminated, then the total surplus would be

$$\sum_{i=1}^{j-1} \delta^{\sum_{m=0}^{i-1} \ell_m} \Pi(i).$$

By Proposition 2, we know that Γ' induces socially efficient actions given termination time $\sum_{i=1}^{j-1} \ell_i - 1$. So the total surplus Π' under Γ' is bounded from below by the above expression. Also by Proposition 2, under Γ', the agent's ex ante payoff V' is exactly equal to the payoff that he can get from always shirking, i.e.,

$$V' = \sum_{i=1}^{j-1} \delta^{\sum_{m=0}^{i-1} \ell_m} V(i).$$

Thus, the principal's ex ante payoff U' under Γ' satisfies

$$U' \geq \sum_{i=1}^{j-1} \delta^{\sum_{m=0}^{i-1} \ell_m} \left[\Pi(i) - V(i)\right].$$

Comparing U' and U,

$$\begin{aligned} U' - U &\geq \sum_{i=1}^{j-1} \left(\delta^{\sum_{m=0}^{i-1} \ell_m} - \delta^{\tau_i}\right) [\Pi(i) - V(i)] - \delta^{\tau_j} \left[\tilde{\Pi}(j) - \tilde{V}(j)\right] \\ &\geq \sum_{i=1}^{j-1} \left(\delta^{\sum_{m=0}^{i-1} \ell_m} - \delta^{\tau_i}\right) [\Pi(i) - V(i)], \end{aligned}$$

because $\tilde{\Pi}(j) - \tilde{V}(j) < 0$.

To complete the proof that $U' - U \geq 0$, I show by induction that for all $i' \leq j-1$,

$$\sum_{i=i'}^{j-1} \left(\delta^{\tau_{i'-1} + \sum_{m=i'-1}^{i-1} \ell_m} - \delta^{\tau_i}\right) [\Pi(i) - V(i)] \geq 0. \tag{A.1}$$

By the definition of j, it is true that for all $h \leq j-1$,

$$\sum_{i=h}^{j-1} \delta^{\tau_i} [\Pi(i) - V(i)] \geq 0; \tag{A.2}$$

otherwise, $\tilde{\Pi}(h) - \tilde{V}(h) < 0$, and it violates the definition of j.

1. Let $h = j-1$ in (A.2), then $\Pi(j-1) - V(j-1) \geq 0$. So the inequality (A.1) holds for $i' = j-1$.

2. Suppose inequality (A.1) holds for $i' = i'' \in (1, j-1]$, i.e.,

$$\sum_{i=i''}^{j-1} \left(\delta^{\tau_{i''-1} + \sum_{m=i''-1}^{i-1} \ell_m} - \delta^{\tau_i}\right) [\Pi(i) - V(i)] \geq 0. \tag{A.3}$$

   It remains to show that (A.1) holds for $i' = i'' - 1$.

Equation (A.3) implies

$$\sum_{i=i''}^{j-1} \delta^{\tau_i} [\Pi(i) - V(i)] \leq \sum_{i=i''}^{j-1} \delta^{\tau_{i''-1} + \sum_{m=i''-1}^{i-1} \ell_m} [\Pi(i) - V(i)]. \tag{A.4}$$

Let $h = i'' - 1$ in (A.2):

$$\sum_{i=i''-1}^{j-1} \delta^{\tau_i} [\Pi(i) - V(i)] \geq 0; \tag{A.5}$$

Equations (A.4) and (A.5) imply

$$\begin{aligned}
0 &\leq \sum_{i=i''-1}^{j-1} \delta^{\tau_i} [\Pi(i) - V(i)] \\
&\leq \delta^{\tau_{i''-1}} \left[\Pi(i'-1) - V(i'-1)\right] + \sum_{i=i''}^{j-1} \delta^{\tau_{i''-1} + \sum_{m=i''-1}^{i-1} \ell_m} [\Pi(i) - V(i)] \\
&\leq \delta^{\tau_{i''-2}} \left[\Pi(i'-1) - V(i'-1)\right] + \sum_{i=i''}^{j-1} \delta^{\tau_{i''-2} + \sum_{m=i''-1}^{i-1} \ell_m} [\Pi(i) - V(i)] \geq 0 \\
&\leq \sum_{i=i''-1}^{j-1} \delta^{\tau_{i''-2} + \sum_{m=i''-2}^{i-1} \ell_m} [\Pi(i) - V(i)],
\end{aligned}$$

which implies inequality (A.1) holds for $i' = i'' - 1$.

Next I prove that random contracts are dominated by deterministic contract.

Take any contract with random financing strategies. Ex ante, it induces a probability distribution $\mathbf{P}$ over the random termination time $\tilde{T}$. The agent's payoff is bounded from below by

$$\mathbb{E}_{\mathbf{P}} \left(c \sum_{i=0}^{\tilde{T}} \delta^i \right).$$

Conditional on each realized $\tilde{T}$, the total surplus generated is bounded above by the social planner's surplus under the realization $\Pi_{\tilde{T}}$. So the principle's payoff is no larger than

$$\mathbb{E}_{\mathbf{P}}\left(\Pi_{\tilde{T}} - c\sum_{i=0}^{\tilde{T}}\delta^i\right).$$

However, under the optimal deterministic contract, the principal's payoff is

$$\max_T\left(\Pi_T - c\sum_{i=0}^{T}\delta^i\right).$$

So no random contract yields a higher payoff to the principal than the optimal deterministic contract. □

In the main body of the paper, contracting on reports of the project's progress is not allowed. This assumption may be justified by practical reasons. In reality, the progress of a project in reality may be difficult to understand for outsiders and it may be impossible for the agent to communicate effectively with the principal. Even they can communicate and understand, it is often difficult to contract on such reports and enforce it. In the following, I show that contracting on report not only may be infeasible in practice, but also does not make a difference when the principal and agent can only write deterministic contract.

Proposition 19. *If only deterministic contracts are allowed, then contracting on progress reports cannot improve the principal's payoff.*

Proof. Suppose now the agent can report the progress to the principal and the contractible histories include all past reports. Consider any contract Γ, it can be shown that there exists another contract that does not depend on the reports that gives the principal at least the same payoff. Let T^M be the largest termination time for all possible reports under Γ. The agent's payoff under Γ is at least $\sum_{i=0}^{T^M} \delta^i c$, since he has the option to always shirk and make reports such that he is financed until T^M. Let $\Pi(T^M)$ be the planner's payoff in the planner's problem with a termination time T^M. Then the principal's payoff under Γ is at most $\Pi(T^M) - \sum_{i=0}^{T^M} \delta^i c$. Now consider the optimal contract without reports with a termination time T^M, Γ'. In proposition 2, it has been proved that the principal's payoff under Γ' is $\Pi(T^M) - \sum_{i=0}^{T^M} \delta^i c$. So contracting on reports cannot improve the principal's payoff.

$\square$

The key reason that contracting on report does not help in this case is that to induce truthful report, the principal needs to compensate the agent for the forgone financing. But since the optimal contract without reports is efficient given the length of financing anyway, the principal would rather just give the agent the same amount of financing regardless of the reports.

It is worth noting that if the parties can use randomized contracts that are contingent on report, the principal will be strictly better off. Consider a three-period example without discounting, $t = 0, 1, 2$ and the following financing strategy. The agent is financed at $t = 0$ and is asked to report whether the first breakthrough has occurred. If reporting yes, then the agent is financed for only one more period for sure; if no, then the agent is financed for zero or two more periods with 1/2 probability each. The rewards for final success are designed such that the agent is indifferent between working and shirking at stage 0. Under this contract, the agent is weakly willing to report the truth, and receives an ex ante payoff of $2c$. But the social surplus is higher than a two-period planner's problem. Conditional on a breakthrough in period 0, the social surplus is the same. Conditional on no breakthrough in period 0, with 1/2 probability the project is terminated, which is what happens in a two-period planner's problem; but with 1/2 probability the project is financed for two more periods, which yields higher social surplus. So randomized contracts that depend on report can strictly improve efficiency without increasing the incentive cost.

A.2 Implications of Unobserved Innovation Progress

Proof of Proposition 5. The proof is broken down to four steps.

Step 1 Claim that if the stage 0 agent is financed if and only if $t \leq \tilde{t}$ for some termination time $\tilde{t}_0$, then conditional on the first breakthrough has occurred by $\tilde{t}_0$, the agent is always financed until $\tilde{T}$, regardless of when the time of the first breakthrough. To show this claim is true, consider two cases, $c\sum_{i=1}^{\tilde{T}-\tilde{t}_0}\delta^i \geq c/(\delta q)$ and $c\sum_{i=1}^{\tilde{T}-\tilde{t}_0}\delta^i < c/(\delta q)$.

Case 1. Suppose $c\sum_{i=1}^{\tilde{T}-\tilde{t}_0}\delta^i \geq c/(\delta q)$. Consider the following kind of continuation contracts after the first breakthrough: some non-negative rewards (0 at $\tilde{t}_0$)

at are made for the first breakthrough; the agent is financed up to period $\tilde{T}$, and the rewards for success are as specified in Proposition 6. Then in the continuation contracts, the agent is indifferent between working and shirking. Conditional on that the first breakthrough occurs in period $t \in \{0, 1, ..., \tilde{t}_0\}$, his continuation value (after rewards for the first breakthrough) is exactly $c \sum_{i=1}^{\tilde{T}-t} \delta^i$. In period $\tilde{t}_0$, since $c \sum_{i=1}^{\tilde{T}-\tilde{t}_0} \delta^i \geq c/(\delta q)$, the stage 0 agent has a strict incentive to work under the specified continuation contract without additional rewards. For $t \leq \tilde{t}_0$, the stage 0 agent's incentive to work becomes weaker and weaker as t decreases. At some point, additional rewards for the first breakthrough are needed to make the agent just indifferent. Therefore, the specified contract is incentive compatible. By Proposition 1 and 6, since $\tilde{T} \leq T^*$, it can be verified that the optimal termination time of financing T when the innovation requires only one breakthrough is larger than $\tilde{T}$. Then the specified contract maximizes the principal's payoff conditional on any date of the first breakthrough, and thus maximizes the principal's ex ante payoff.

Case 2. Suppose $c \sum_{i=1}^{\tilde{T}-\tilde{t}_0} \delta^i < c/(\delta q)$. Then under the continuation contracts specified above, the stage 0 agent does not have a incentive to work. Instead of not rewarding for the first breakthrough, now specify a reward schedule for the first breakthrough $\tilde{\mathbf{w}}_t$ such that the stage 0 agent is indifferent between working and shirking at any time $t \leq \tilde{t}_0$. The agent is still financed until $\tilde{T}$ conditional on the first breakthrough, and the rewards for success remain the same. Under this contract (given $\tilde{t}_0$), the agent's ex ante payoff is minimized (equal to always shirking), and the total surplus of the project is maximized. The principal's ex ante payoff is thus maximized.

Step 2 Claim that there exists some $\tilde{t}_0 < T$ such that the stage 0 agent is financed if and only if $t \leq \tilde{t}_0$. Suppose conversely that the stage 0 is financed for a total of $\tilde{t}_0$ periods, but there is some period in which financing is temporarily suspended.

If $c\sum_{i=1}^{\tilde{T}-\tilde{t}_0} \delta^i \geq c/(\delta q)$, then the contract is dominated by the contract specified in Step 1, Case 1 with termination date of financing $\tilde{t}_0$ for stage 0, because conditional on the number of periods of investment needed for the first breakthrough, the principal's payoff is smaller.

If $c\sum_{i=1}^{\tilde{T}-\tilde{t}_0} \delta^i \geq c/(\delta q)$, then the contract is dominated by the contract specified in Step 1, Case 2, by the same argument as in the proof of Proposition 4.

Step 3 Claim that $\tilde{t}_0 \leq t_0$. Suppose not. Then consider the optimal contract given termination time of financing $\tilde{t}_0 - 1$ for the stage 0 agent. First observe that, under the new contract, the agent's ex ante payoff decreases at least by $(1-q)^{\tilde{t}_0}\delta^{\tilde{t}_0}c$. In Case 2, the decrease is equal to $\delta^{\tilde{t}_0}c$, since the agent's ex ante payoff is equal to as if the value from always shirking. In Case 1, the agent's payoff stays the same conditional on that the first breakthrough occurs by $\tilde{t}_0 - 1$; otherwise, his payoff decreases by at least $\delta^{\tilde{t}_0}c$, because he loses the option to shirk and receives c in period $\tilde{t}_0$. At the same time, since t_0 is the socially efficient time to terminate the stage 0 project, the new contract yields higher total surplus. Hence, the principal's payoff is strictly improved.

Step 4 Let the parameters be parameterized by rates in continuous time and the length of a period Δ, i.e., $q = q_r\Delta$, $c = c_r\Delta$ and $\delta = e^{-r\Delta}$ for some q_r, c_r, and r. The value of success is still denoted by Y. Let the exogenous deadline $\tilde{T}$ be equal to T^*. Claim that there exists $\underline{\Delta}$ such that if $\Delta < \underline{\Delta}$, then $\tilde{t}_0 < t_0$.

First, as Δ goes to 0, by proposition 3, $T^*\Delta$ converges to some S^*, and by proposition 1, $t_0\Delta$ converges to some s_0. Suppose $\tilde{t}_0 = t_0$ for all Δ. Then for Δ small enough, by the argument of Step 3, choosing another contract with termination time $\tilde{t}_0 = t_0 - 1$ for stage 0 decreases the agent's payoff by at least $\left(e^{-(r+q)s_0} - \varepsilon\right) c\Delta$ for arbitrarily small ε. It remains to show that the decrease in total surplus is $o(\Delta)$.

Recall that the total surplus of the stage 1 project in period t given termination time $\tilde{T}$ is

$$\Pi^1_{t,\tilde{T}} = \frac{qY - c}{1 - \delta(1-q)}(1 - [\delta(1-q)]^{\tilde{T}-t+1}).$$

The efficient time to terminate stage 0 project t_0 satisfies

$$\Pi^1_{t_0+1,\tilde{T}} = \frac{qY - c}{1 - \delta(1-q)}(1 - [\delta(1-q)]^{\tilde{T}-t_0}) \geq \frac{c}{\delta q}$$

and

$$\Pi^1_{t_0+2,\tilde{T}} = \frac{qY - c}{1 - \delta(1-q)}(1 - [\delta(1-q)]^{\tilde{T}-t_0-1}) < \frac{c}{\delta q}.$$

So

$$\begin{aligned}
\Pi^1_{t_0+1,\tilde{T}} - \Pi^1_{t_0+2,\tilde{T}} &= [\delta(1-q)]^{\tilde{T}-t_0-1}(qY-c) \\
&= [\delta(1-q)]^{\tilde{T}-t_0-1}(q_rY-c_r)\Delta \\
&= O(\Delta).
\end{aligned}$$

Thus,

$$\Pi^1_{t_0+1,\tilde{T}} = \frac{c}{\delta q} + O(\Delta).$$

In the contract with stage 0 termination time $\tilde{t} = t_0$, the total surplus for stage 0 project at time t_0 is

$$\Pi^0_{\tilde{t}_0,\tilde{T}} = \delta q \Pi^1_{t_0+1,\tilde{T}} - c = \delta q O(\Delta) = e^{-r\Delta} q \Delta O(\Delta) = o(\Delta).$$

In the contract with $\tilde{t} = t_0 - 1$, the total surplus for stage 0 project at time t_0 is 0. The different in total surplus of the stage 0 project at t_0 is $o(\Delta)$, and hence so is the difference in ex ante total surplus.

$\square$

Proof of Proposition 6. When innovation only requires one breakthrough, given a termination time of financing T, the agent is always induced to work. Given a contract $(\mathbf{w}, T)$, his value function V_t follows

$$V_t = \max\{\underbrace{qw_t + \delta(1-q)V_{t+1}}_{\text{Value from working}}, \quad \underbrace{c + \delta V_{t+1}}_{\text{Value from shirking}} \quad\}, \ \ \forall t = 0, ..., T.$$

The IC condition in period t is

$$V_t \geq c + \delta V_{t+1},$$

or

$$w_t \geq \delta V_{t+1} + \frac{c}{q}.$$

To minimize the agent's value, the IC conditions need to be binding in every period, and for all $t \leq T$,

$$V_t = c\sum_{i=0}^{T-t}\delta^i; \ \ w_t = c\left(\sum_{i=1}^{T-t}\delta^i + \frac{1}{q}\right).$$

By increasing the termination time from $T-1$ to T, the increase in total surplus is

$$(1-q)^T\delta^T(qY-c);$$

the increase in the agent's value is $\delta^T c$. So the optimal T should satisfy

$$T = \max\{t : (1-q)^t\delta^t(qY-c) \geq \delta^t c\},$$

or

$$T = \left\lfloor \log_{1-q}\frac{c}{qY-c} \right\rfloor.$$

Note that under a non-generic set of parameters, the agent may be indifferent between termination time T and $T+1$, and so there could be two optimal termination time. Here we choose the larger one whenever there are indifferences. $\square$

Appendix B

Appendices to Chapter 3

B.1 Difficult Task Go First

Proof of Lemma 2. The surplus by doing task E first is

$$\Pi(ED) = \sum_{i=0}^{t_0} q_E(1-q_E)^i \left\{ \sum_{j=0}^{T-1-i} q_D(1-q_D)^j \left[Y-(i+j+2)c\right] - (1-q_D)^{T-i}(T+1)c \right\}$$
$$-(1-q_E)^{t_0+1}(t_0+1)c$$

Using

$$\sum_{i=0}^{t} q(1-q)^i = 1-(1-q)^{t+1}$$

and

$$\sum_{i=0}^{t} (i+1)q(1-q)^i = \frac{1}{q} - \left(\frac{1}{q}+t+1\right)(1-q)^{t+1},$$

we have

$$
\begin{aligned}
&\sum_{j=0}^{T-1-i} q_D(1-q_D)^j\left[Y-(i+j+2)c\right]-(1-q_D)^{T-i}(T+1)c \\
=&\sum_{j=0}^{T-1-i} q_D(1-q_D)^j\left[Y-(i+1)c-(j+1)c\right]-(1-q_D)^{T-i}(T+1)c \\
=&\left[Y-(i+1)c\right]\sum_{j=0}^{T-1-i} q_D(1-q_D)^j-c\sum_{j=0}^{T-1-i}(j+1)q_D(1-q_D)^j-(1-q_D)^{T-i}(T+1)c \\
=&\left[Y-(i+1)c\right]\left[1-(1-q_D)^{T-i}\right]-c\left[\frac{1}{q_D}-\left(\frac{1}{q_D}+T-i\right)(1-q_D)^{T-i}\right] \\
&-(1-q_D)^{T-i}(T+1)c \\
=&\left[Y-(i+1)c\right]\left[1-(1-q_D)^{T-i}\right]-c\left[\frac{1}{q_D}-\left(\frac{1}{q_D}+T-i\right)(1-q_D)^{T-i}\right] \\
&-(1-q_D)^{T-i}(T+1)c \\
=&\left(Y-\frac{c}{q_D}\right)\left[1-(1-q_D)^{T-i}\right]-(i+1)c.
\end{aligned}
$$

So ,

$$
\begin{aligned}
\Pi(ED) =& \sum_{i=0}^{t_0} q_E(1-q_E)^i \left\{ \left(Y - \frac{c}{q_D}\right)\left[1-(1-q_D)^{T-i}\right] - (i+1)c \right\} \\
& - (1-q_E)^{t_0+1}(t_0+1)c \\
=& \left(Y - \frac{c}{q_D}\right)\left[\sum_{i=0}^{t_0} q_E(1-q_E)^i - \sum_{i=0}^{t_0} q_E(1-q_E)^i(1-q_D)^{T-i}\right] \\
& - \sum_{i=0}^{t_0} q_E(1-q_E)^i(i+1)c - (1-q_E)^{t_0+1}(t_0+1)c \\
=& \left(Y - \frac{c}{q_D}\right)\left[1-(1-q_E)^{t_0+1} - \sum_{i=0}^{t_0} q_E(1-q_E)^i(1-q_D)^{T-i}\right] \\
& - c\left[\frac{1}{q_E} - \left(\frac{1}{q_E} + t_0 + 1\right)(1-q_E)^{t_0+1}\right] - (1-q_E)^{t_0+1}(t_0+1)c \\
=& \left(Y - \frac{c}{q_E} - \frac{c}{q_D}\right)\left[1-(1-q_E)^{t_0+1}\right] - \left(Y - \frac{c}{q_D}\right)\sum_{i=0}^{t_0} q_E(1-q_E)^i(1-q_D)^{T-i} \\
=& \left(Y - \frac{c}{q_E} - \frac{c}{q_D}\right)\left[1-(1-q_E)^{t_0+1}\right] \\
& - q_E\left(Y - \frac{c}{q_D}\right)(1-q_D)^{T-t_0}\frac{(1-q_D)^{t_0+1} - (1-q_E)^{t_0+1}}{q_E - q_D}
\end{aligned}
$$

Analogously,

$$
\begin{aligned}
\Pi(DE) =& \left(Y - \frac{c}{q_E} - \frac{c}{q_D}\right)\left[1-(1-q_D)^{t_0+1}\right] \\
& - q_D\left(Y - \frac{c}{q_E}\right)(1-q_E)^{T-t_0}\frac{(1-q_D)^{t_0+1} - (1-q_E)^{t_0+1}}{q_E - q_D}.
\end{aligned}
$$

Taking differences:

$$
\begin{aligned}
&\Pi(ED)-\Pi(DE)\\
=&\left(Y-\frac{c}{q_E}-\frac{c}{q_D}\right)\left[1-(1-q_E)^{t_0+1}\right]\\
&-q_E\left(Y-\frac{c}{q_D}\right)(1-q_D)^{T-t_0}\frac{(1-q_D)^{t_0+1}-(1-q_E)^{t_0+1}}{q_E-q_D}\\
&-\left(Y-\frac{c}{q_E}-\frac{c}{q_D}\right)\left[1-(1-q_D)^{t_0+1}\right]\\
&+q_D\left(Y-\frac{c}{q_E}\right)(1-q_E)^{T-t_0}\frac{(1-q_D)^{t_0+1}-(1-q_E)^{t_0+1}}{q_E-q_D}\\
=&\left(Y-\frac{c}{q_E}-\frac{c}{q_D}\right)\left[(1-q_D)^{t_0+1}-(1-q_E)^{t_0+1}\right]\\
&-\left[q_E\left(Y-\frac{c}{q_D}\right)(1-q_D)^{T-t_0}-q_D\left(Y-\frac{c}{q_E}\right)(1-q_E)^{T-t_0}\right]\cdot\\
&\frac{(1-q_D)^{t_0+1}-(1-q_E)^{t_0+1}}{q_E-q_D}\\
=&\left\{q_E\left(Y-\frac{c}{q_D}\right)\left[1-(1-q_D)^{T-t_0}\right]-q_D\left(Y-\frac{c}{q_E}\right)\left[1-(1-q_E)^{T-t_0}\right]\right\}\cdot\\
&\frac{(1-q_D)^{t_0+1}-(1-q_E)^{t_0+1}}{q_E-q_D}
\end{aligned}
$$

Let $\Pi_t^1(ED)$ be the social surplus at time t when task E has been completed (at stage 1). $\Pi_t^1(DE)$ is defined analogously. Similar to equation (2.1),

$$\Pi_t^1(ED)=\left(Y-\frac{c}{q_D}\right)\left[1-(1-q_D)^{T-t+1}\right]$$

and

$$\Pi_t^1(DE)=\left(Y-\frac{c}{q_E}\right)\left[1-(1-q_E)^{T-t+1}\right].$$

So

$$\Pi(ED) - \Pi(DE) = \left[q_E \Pi^1_{t_0+1}(ED) - q_D \Pi^1_{t_0+1}(DE)\right] \frac{(1-q_D)^{t_0+1} - (1-q_E)^{t_0+1}}{q_E - q_D}.$$

Since

$$\frac{(1-q_D)^{t_0+1} - (1-q_E)^{t_0+1}}{q_E - q_D} > 0,$$

$\Pi(ED) < \Pi(DE)$ is equivalent to $q_E \Pi^1_{t_0+1}(ED) < q_D \Pi^1_{t_0+1}(DE)$, which will be shown by induction:

Step 1 Show that $q_E \Pi^1_T(ED) < q_D \Pi^1_T(DE)$:

$$q_E \Pi^1_T(ED) - q_D \Pi^1_T(DE) = q_E(q_D Y - c) = q_D(q_E Y - c) = c(q_D - q_E) < 0.$$

Step 2 Show that if $q_E \Pi^1_t(ED) < q_D \Pi^1_t(DE)$ and $q_E \Pi^1_t(ED) \leq c$, then $q_E \Pi^1_{t-1}(ED) < q_D \Pi^1_{t-1}(DE)$:

$$\begin{aligned}
& q_E \Pi^1_{t-1}(ED) - q_D \Pi^1_{t-1}(DE) \\
=& q_E \left[q_D Y - c + (1-q_D)\Pi^1_t(ED)\right] - q_D \left[q_E Y - c + (1-q_E)\Pi^1_t(DE)\right] \\
=& c(q_D - q_E) + q_E(1-q_D)\Pi^1_t(ED) - q_D(1-q_E)\Pi^1_t(DE) \\
<& c(q_D - q_E) + c(1-q_D) - c(1-q_E) \\
=& 0.
\end{aligned}$$

Step 3 Since t_0 is the optimal dropout time when task E is done first,

$$q_E \Pi_t^1(ED) \leq c, \forall t \geq t_0 + 2.$$

Then by Step 1 and 2, $q_E \Pi_{t_0+1}^1(ED) < q_D \Pi_{t_0+1}^1(DE)$.

$\square$

B.2 Ex ante Possibility of Bad Project

Proof of Proposition 9. Recall equation (3.7) that characterizes the agent's ex ante value:

$$V = V_0^0(\gamma_0) = c\left(\sum_{i=0}^{t_0} \delta^i \frac{\gamma_0}{\gamma_i} + \sum_{i=t_0-t+1}^{T} \delta^i\right). \tag{3.7}$$

We can see that V is increasing in t_0. Let $V(k)$ denote the agent's ex ante value given $t_0 = k$, and define $dV(k) = V(k) - V(k-1)$. Then

$$dV(k) = \delta^k \left(\frac{\gamma_0}{\gamma_k} - 1\right).$$

Suppose $t_0 = k > t_0^*$. Then consider the alternative contract with $t_0 = k - 1$. The social surplus increases because t_0 is closer to the conditional efficient t_0^*. Moreover, the agent's ex ante value decreases by $dV(k)$. Therefore, the principal is better off. So $t_0 \leq t_0^*$.

For the same reason as in proposition 5, he weak inequality is due to the discreteness of the model. Following the same argument in the proof of Proposition 5, we parametrize the model by rates in continuous time and the length of period Δ. Suppose $t_0 = t_0^*$ for all Δ, then for Δ small enough, if we instead choose $t_0 = t_0^*$, then the decrease in the agent's value is $O(\Delta)$ while the decrease in the social surplus is $o(\Delta)$. The principal will be strictly better off.

$\square$

Proof of Proposition 10. Recall that $Q_1(t) = \sum_{i=0}^{t-1}(1-q_1)^i q_1 (1-q_2)^{t-i}$ is the probability that only the first breakthrough occurred within t periods when the project is good for sure. Define

$$Q_1'(t) = \gamma \sum_{i=0}^{t-1}(1-q_1)^i q_1 (1-q_2)^{t-i}.$$

So Q_1' is the probability that only the first breakthrough occurred within t periods when the project is bad with ex ante probability Γ. Suppose $T^* > T^{*BL}$.

In the baseline case, the principal is worse off by setting the termination time to $T^{*BL} + 1$. Therefore

$$\begin{aligned} &Q_1(t_{0,T^{*BL}})(1-q_2)^{T^{*BL} - t_{0,T^{*B}L}} \delta^{T^{*BL}}(\delta q_2 Y - c) \\ &+ (1-q_1)^{t_{0,T^{*B}L}}(\delta q_1 \Pi^1 (T^{*BL} - t_{0,T^{*B}L}) - c) < \delta^T c. \end{aligned}$$

When $T^* = T^{*BL}$, $t^* > t_0$. So

$$\gamma Q_1(t_{0,T^*}^*)(1-q_2)^{T^{*BL} - t_{0,T^{*B}L}^*} \delta^{T^{*BL}}(\delta q_2 Y - c) < \delta^T c.$$

Since $T^* > T^{*BL}$, and the marginal benefit is decreasing in T,

$$\gamma Q_1(t^*_{0,T^*})(1-q_2)^{T^*-t^*_{0,T^*}}\delta^{T^*}(\delta q_2 Y - c) < \delta^T c.$$

As a result, the principal's payoff will increase if setting the termination time to $T^* - 1$. $\square$

B.3 Impefect Monitoring of Progress

Proof of Lemma 3, Proposition 11 and 13 . Consider the stage 0 agent's incentives at $t = t^G$. By working, his value is

$$q_1 \hat{V}^1_{t^G+1} + (1-q)V^0_{t^G+1} = q_1 \kappa V^1_{t^G+1};$$

by shirking, his value is c. Suppose after the signal reveals stage 1, the principal uses the one breakthrough optimal contract in proposition 6 as the continuation contract, then $V^1_{t^G+1} = c(T-t)$.

If $q_2\kappa(T-t)c < c$, the agent at stage 0 does not have incentive to work without additional payment, and it is possible to find another set of $\{\hat{w}_t,\ w_t\}$ characterized by proposition 11 such that the stage 0 agent's IC constraints are just satisfied for all $t \leq t_0$. Together with the optimally chosen t_0 given T and t^G, the social surplus is maximized while the agent's ex ante value is minimized. The resulting contract is thus optimal.

If $q_2\kappa c(T-t)\geq c$, then the continuation contract after the signal arrives is enough to incentivize stage 0 agent to work. It is impossible to hold the agent's ex ante value to the lower bound $c(t^G+1)$. The principal's payoff can be calculated by taking expectation over the events that the signal revealing stage 1 arrives at time i before success (probability of which denoted as ψ_i) and the events that success occurs at time i before signals arrive (probability of which denoted as ψ_i'):

$$\begin{aligned}U=&\sum_{i=0}^{t^G+1}\psi_i\left[(\Pi_i^1-V_i^1)-c(i+1)\right]+\sum_{i=1}^{t^G}\psi_i'\left[(Y-\hat{w}_t)-c(i+1)\right]\\&-\left(1-\sum_{i=0}^{t^G+1}\psi_i-\sum_{i=1}^{t^G}\psi_i'\right)c(t_0+1).\end{aligned}$$

From the above expression, it can be seen that to maximize U given T, t^G and t_0, we would like to maximize $\Pi_i^1-V_i^1$ and minimize $\hat{w}_t$ for each i. Remember that we assume $T\leq T^*$. Moreover, T^* is smaller than the optimal termination time for project with only one breakthrough $\left\lfloor\log_{1-q}\frac{c}{qY-c}\right\rfloor$. Therefore, conditional on the signal arrives at i, financing the agent till T and using the rewards in the one-breakthrough optimal contract maximize $\Pi_i^1-V_i^1$. Minimizing $\hat{w}_t$ requires the IC constraints in (3.9) are binding.

It remains to show that under the specified contract the stage 0 agent's IC conditions are satisfied not only at t^G, but at all $t\leq t^G$. We prove it by induction, and show that if stage 0 IC holds at $t+1$, it also holds at t.

The stage 0 IC constraint at t is equivalent to $\hat{V}_{t+1}^1-V_{t+1}^0\geq c/q$, which is proved in the following:

$$\begin{aligned}
\hat{V}^1_{t+1} - V^0_{t+1} =& \kappa V^1_{t+1} + (1-\kappa)(c + \hat{V}^1_{t+2}) - V^0_{t+1} \\
=& \kappa V^1_{t+1} + (1-\kappa)(c + \hat{V}^1_{t+2}) - q\hat{V}_{t+2} - (1-q)V^0_{t+2} \\
=& \kappa(c + V^1_{t+2}) + (1-\kappa)(c + \hat{V}^1_{t+2}) - q\hat{V}^1_{t+2} - (1-q)V^0_{t+2} \\
=& c + \kappa(V^1_{t+2} - \hat{V}^1_{t+2}) + (1-q)(\hat{V}^1_{t+2} - V^0_{t+2}) \\
\geq& c + (1-q)\frac{c}{q} \\
=& \frac{c}{q}.
\end{aligned}$$

The first equality is because stage 1 agent before signal arrives is indifferent between working and shirking; the second equality is by the induction assumption that stage 0 agent has an incentive to work at $t+1$; the third equality is because the continuation contract after the signal arrives at time i gives the agent a value of $(T-i)c$. The inequality is again using the induction assumption. $\square$

Proof of Proposition 14. Let $U^F(t)$ be the principal's payoff in the fully observed progress case with stage 0 dropout time t, and $\tilde{U}^F = U^F(t_0)$ is the payoff from choosing the optimal $\tilde{t}_0$. By assumption, $\tilde{U}^F$ is strictly higher than the payoff from any other $U^F(t)$. Suppose $\kappa = 1-\varepsilon$. Since both the total surplus and the agent's ex ante payoff are continuous in κ, there exists a function $f(\kappa)$ such that the principal's payoff $U$from choosing $t^G = t_0 = \tilde{t}_0$ satisfies $U \geq \tilde{U}^F - f(\kappa)$, and $f(\kappa)$ converges to 0 as κ converges to 0. Again by continuity, choosing any other $t^G = t_0'$ gives the principal a payoff no larger than $U^F(t_0)' + f'(\varepsilon)$

for some f' that converges to 0 as ε converges. Therefore, for sufficiently small ε, the principal is strictly better off by choosing $t^G = t_0 = \tilde{t}_0$.

The same continuity argument holds for the case where $\kappa = \varepsilon$ with ε small enough. Since $T \leq T^*$, in the no signal case, the principal would like to finance the agent until T. And she would like to do the same with κ close enough to 0.

$\square$

Appendix C

Appendices to Chapter 4

Proof of Lemma 4. First, we show that $s_t(p) \in \{0, \delta V_t^0, \delta V_t^1\}$.

1. If $s_t(p) \in (0, \delta_t^0)$ for some p, then it is equivalent to $s_t(p) = 0$ because neither type of agent will accept the offer.

2. Suppose for a set of offers B_1 with positive measure, $s_t(p) > \delta V_t^1$ for all $p \in B_1$. Let $\beta_1 = \mathbb{E}\left[s_t(p) - \delta V_t^1 \middle| p \in B_1\right]$. Then it is equivalent to using $s_t(p) = \delta V_t^1$ for all $p \in B_1$ and making an unconditional payment $\mathbb{P}(B_1)\beta_1$ to both types of agents. In lemma 1 we have shown that unconditional payments are suboptimal when there are no buyers. The same argument holds here. We can replace the unconditional payment $\mathbb{P}(B)\beta$ with an increase in reward for success $\mathbb{P}(B)\beta/q$. All incentives conditions will still hold and the total surplus remains the same, but the agent's ex ante payoff is lower.

3. Suppose for a set of offers B_0 with positive measure, $s_t(p) \in (\delta V_t^0, \delta V_t^1)$ for all

$p \in B_0$. Then it is equivalent to using $s_t(p) = \delta V_t^1$ for all $p \in B_0$ and making an unconditional payment only to the stage 0 agent. The principal is better off by setting $s_t(p) = \delta V_t^1$ and not making the unconditional payment. Again incentives conditions will still hold and the total surplus remains the same, but the agent's ex ante payoff is lower.

It remains to show that $s_t(p)$ is weakly increasing in p. Suppose there exist two sets of prices B_0 and B_1 such that

$$s_(p) = \delta V_t^0, \ \forall p \in B_0,$$

$$s_(p) = \delta V_t^1, \ \forall p \in B_1,$$

$$\mathbb{P}(B_0) = \mathbb{P}(B_1),$$

and

$$\mathbb{E}\left[p \middle| B_0\right] > \mathbb{E}\left[p \middle| B_1\right].$$

Then the principal is better off by setting

$$s_(p) = \delta V_t^0, \ \forall p \in B_1,$$

$$s_(p) = \delta V_t^1, \ \forall p \in B_0.$$

□

Proof of Lemma 5. After success or termination, the environment is stationary. Remember that the offer $p_t = z + \mathbb{1}\{n_t = 2\}\delta Y$, with $z \sim G(\cdot)$. So without success, p_t follows distribution $G(\cdot)$, and after success, $p_t - \delta Y$ follows $G(\cdot)$.

When the project has been terminated without success,

$$\underline{\Pi} = \max_{\underline{p}} \left\{ \lambda \int_{z \geq \underline{p}} z g(z) dz + (\lambda G(\underline{p}) + 1 - \lambda) \delta \underline{\Pi} \right\}.$$

The first order condition is necessary and sufficient for optimality, and the value is maximized at $\underline{p} = \delta \underline{\Pi}$. In other words, the acquisition offer should be accepted if and only if it is no less than the continuation value after rejecting it. So

$$\underline{\Pi} = \lambda \int_{z \geq \delta \underline{\Pi}} z g(z) dz + (\lambda G(\delta \underline{\Pi}) + 1 - \lambda) \delta \underline{\Pi},$$

or

$$(1 - \delta) \underline{\Pi} = \lambda \int_{z \geq \delta \underline{\Pi}} (z - \delta \underline{\Pi}) g(z) dz. \tag{C.1}$$

The left hand side (*LHS*) is strictly increasing in $\underline{\Pi}$ and the right hand side (*RHS*) is strictly decreasing in $\underline{\Pi}$. When $\underline{\Pi} = 0$, $LHS < RHS$ and when $\underline{\Pi} \to \infty$, $LHS > RHS$. So equation (C.1) has a unique solution.

Similarly, the principal's value after success follows

$$\overline{\Pi} = \max_{\overline{p}} \left\{ y + \lambda \int_{z + \delta Y \geq \overline{p}} (z + \delta Y) g(z) dz + (\lambda G(\overline{p} - \delta Y) + 1 - \lambda) \delta \overline{\Pi} \right\},$$

and optimal cutoff is $\overline{p} = \delta\overline{\Pi}$. The equation that solves $\overline{\Pi}$ is

$$\overline{\Pi} = y + \lambda \int_{z+\delta Y \geq \delta\overline{\Pi}} (z + \delta Y) g(z) dz + (\lambda G(\delta\overline{\Pi} - \delta Y) + 1 - \lambda)\delta\overline{\Pi}.$$

Arranging terms, we get

$$(1-\delta)(\overline{\Pi} - Y) = \int_{z+\delta Y \geq \delta\overline{\Pi}} (z + \delta Y - \delta\overline{\Pi}) g(z) dz, \tag{C.2}$$

which characterizes $\overline{\Pi}$. Moreover, equations (C.1) and (C.2) imply that

$$\overline{\Pi} = \underline{\Pi} + Y.$$

$\square$

Proof of Proposition 15. The problem (P) is equivalent to

$$\begin{aligned}
&\max_{T^B, t_0^B, \mathbf{w}, \mu^0, \mu^1} \Pi_0^0 - V_0^0 \\
\text{s.t.}\quad & V_t^0 \geq c + \delta V_{t+1}^0, \forall t \leq t_0^B, \\
& V_t^0 = c + \delta V_{t+1}^0, \forall t \in (t_0^B, T^B]; \qquad \text{(IC: Stage 0)} \\
& V_t^0 \geq c + \delta V_{t+1}^0, \forall t \leq T_0^B; \qquad \text{(IC: Stage 1)} \\
& w_t \geq 0, \ \forall t \leq T_0^B. \qquad \text{(LL)}
\end{aligned} \tag{P'}$$

Since it is not optimal to choose $T^B = \infty$, and there exists a $\overline{w}$ such that it is not optimal for any w_t to exceed $\overline{w}$, the maximization problem is to maximize a continuous function over a compact set. Therefore the solution exists. $\square$

Proof of Proposition 16 . In any optimal contract, the agent ex ante payoff V_0^0 must be minimized given $(T^B, t_0^B, \mathbf{p}_t^0, \mathbf{p}_t^1)$. Otherwise, we can implement the same $(T^B, t_0^B, \mathbf{p}_t^0, \mathbf{p}_t^1)$ yielding the same total surplus Π_0^0 while giving the agent a smaller ex ante payoff, and the principal will be strictly better off.

We will show that to minimize V_0^0, the inequalities (4.2) must hold with equality for all $t \leq t_0^B$. That is to say, the stage 0 agent's IC conditions must be always binding whenever he is induced to work. Moreover, the stage 1 agent's IC conditions must also be binding for $t > t_0^B + 1$.

Rearranging stage 0 agent's IC conditions (4.2), we get

$$\delta q(1-\lambda \mu_t^1)(V_{t+1}^1 - V_{t+1}^0) \geq c, \ \ \forall t \leq t_0^B,$$

or

$$V_{t+1}^1 \geq V_{t+1}^0 + \frac{c}{\delta q(1-\lambda \mu_t^1)}, \ \ \forall t \leq t_0^B. \tag{C.3}$$

Substituting (C.3) back into (4.2), we get

$$V_t^0 \geq c + \delta V_{t+1}^0 + \frac{\lambda \mu_t^1}{q(1-\lambda \mu_t^1)} c, \ \ \forall t \leq t_0^B. \tag{C.4}$$

Stage 1 agent is supposed to work for all $t \leq T^B$, and his IC conditions are

$$\begin{aligned} V_t^1 &= qw_t + \delta(1-q)V_{t+1}^1 \\ &\geq c + \delta V_{t+1}^1, \ \ \forall t = 1, ..., T^B. \end{aligned} \tag{C.5}$$

Note that if success does not happen, then stage 1 agent's continuation value is always δV_{t+1}^1, no matter whether the acquisition offer arrives or not, because by selling the company stage 1 agent gets at most δV_{t+1}^1. Recursively using inequalities (C.5) for $t > t_0^B + 1$, we get

$$V_t^1 \geq c \sum_{i=0}^{T^B - t} \delta^i, \ \ \forall t > t_0^B + 1. \tag{C.6}$$

Substituting inequality (C.6) into equation (4.3), for all $t = t_0^B + 1, ..., T^B - 1$,

$$V_t^0 \geq c + \delta\lambda\mu_t^1 c \sum_{i=0}^{T^B - t - 1} \delta^i + \delta(1 - \lambda\mu_t^1)V_{t+1}^0,$$

or

$$V_t^0 - c\sum_{i=0}^{T^B - t} \delta^i \geq \delta(1 - \lambda\mu_t^1)\left(V_{t+1}^0 - c\sum_{i=0}^{T^B - (t+1)} \delta^i\right)$$

Substituting $V_t^0 - c\sum_{i=0}^{T^B-(t+1)} \delta^i$ iteratively and using $V_{T^B}^0 = c$, we get

$$V_t^0 \geq c \sum_{i=0}^{T^B - t} \delta^i, \ \ \forall t = t_0^B + 1, ..., T^B. \tag{C.7}$$

Recursively using inequalities (C.4) for $t \le t_0^B$ and equations (C.7) for $t > t_0^B$, we get

$$V_t^0 \ge c \sum_{i=0}^{T^B-t} \delta^i + c \sum_{i=t}^{t_0^B} \delta^{i-t} \frac{\lambda \mu_i^1}{q(1-\lambda \mu_i^1)}, \forall t \le t_0^B. \tag{C.8}$$

In particular,

$$V_0^0 \ge c \sum_{i=0}^{T^B} \delta^i + c \sum_{i=0}^{t_0^B} \delta^i \frac{\lambda \mu_i^1}{q(1-\lambda \mu_i^1)}. \tag{C.9}$$

If and only if inequalities (4.2) for all $t \le t_0^B$ and (C.5) for all $t > t_0^B + 1$ hold with equalities, inequalities (C.4) for all $t \le t_0^B$ and (C.5) for all $t > t_0^B$ hold with equalities, which is equivalent to (C.8), and in particular (C.9) hold with equalities. So the agent's ex ante value V_0^0 is minimized at the lower bound if and only if stage 0 agent's IC conditions are binding for $t \le t_0^B$ and stage 1 agent's IC conditions are binding for $t > t_0^B + 1$.

When V_0^0 is minimized, the agent's value functions V_t^n are

$$V_t^0 = c \sum_{i=0}^{T^B-t} \delta^i + c \sum_{i=t}^{t_0^B} \delta^{i-t} \frac{\lambda \mu_i^1}{q(1-\lambda \mu_i^1)}, \ \ \forall t = 0, ..., t_0^B;$$

$$V_t^0 = c \sum_{i=0}^{T^B-t} \delta^i, \ \ \forall t = t_0^B + 1, ..., T^B.$$

$$V_t^1 = V_t^0 + \frac{c}{\delta q(1-\lambda \mu_{t-1}^1)}, \ \ \forall t = 1, ..., t_0^B + 1;$$

$$V_t^1 = V_t^0 = c \sum_{i=0}^{T^B-t} \delta^i, \ \ \forall t = t_0^B + 2, ..., T^B.$$

The reward function w_t can be derived from (C.5):

$$w_t = \delta V_{t+1}^1 + \frac{V_t^1 - \delta V_{t+1}^1}{q}, \ \ \forall t = 1, ..., T^B.$$

We have already set stage 0 agent's IC constraints binding for $t \leq t_0^B$ and stage 1 agent's IC constraints binding for $t \geq t_0^B + 2$. It remains to verify that stage 1 agent's IC for $t \leq t_0^B + 1$:

$$\begin{aligned}
\forall t \leq t_0^B, \ \ V_t^1 - \delta V_{t+1}^1 &= V_t^0 - \delta V_{t+1}^0 + \frac{c}{\delta q(1-\lambda \mu_{t-1}^1)} - \frac{c}{q(1-\lambda \mu_t^1)} \\
&= c + \frac{\lambda \mu_t^1 c}{q(1-\lambda \mu_t^1)} + \frac{c}{\delta q(1-\lambda \mu_{t-1}^1)} - \frac{c}{q(1-\lambda \mu_t^1)} \\
&= c + \frac{c}{q}\left(\frac{1}{\delta(1-\lambda \mu_{t-1}^1)} - 1\right) \\
&> c; \\
V_{t_0^B+1}^1 - \delta V_{t_0^B+2}^1 &= V_{t_0^B+1}^0 + \frac{c}{\delta q(1-\lambda \mu_{t_0^B}^1)} - \delta V_{t_0^B+2}^0 \\
&= c + \frac{c}{\delta q(1-\lambda \mu_{t_0^B}^1)} \\
&> c.
\end{aligned}$$

When stage 1 agent's IC constraints are satisfied, automatically w_t is non-negative for all $t = 1, ..., T^B$. So the specified w_t indeed induces the stage 0 agent to work for $t \leq t_0^B$ and stage 1 agent to work for $t \leq T^B$ while minimizing the agent's ex ante payoff. □

Proof of Proposition 17. Fix $t \leq t_0^B$. Iteratively substituting $\Pi_{t'}^0, \Pi_{t'}^1$ using equation (4.4) and (4.5) for $t' \le t$ and get

$$\Pi_0^0 = \rho_t^0 \delta^t \Pi_t^0 + \rho_t^1 \delta^t \Pi_t^1 + K(\{p_{t'}^0, p_{t'}^1\}_{t'<t}), \tag{C.10}$$

where $K(\{p_{t'}^0, p_{t'}^1\}_{t'<t})$ is a function of $p_{t'}^0$ and $p_{t'}^1$ for $t' < t$, and

$$\rho_t^0 = (1-q)^t \prod_{i=0}^{t-1} (1-\lambda(1-G(p_t^0)))$$

is the probability that the project is at stage 0 and no acquisition offer has been accepted by period t, and

$$\rho_t^1 = q(1-q)^{t-1} \sum_{j=0}^{t-1} \left(\prod_{i=0}^{j-1} (1-\lambda(1-G(p_i^0))) \prod_{i=j}^{t-1} (1-\lambda(1-G(p_j^1))) \right)$$

is the probability that the project is at stage 1 and no acquisition offer has been accepted by period t.

First note that for all t', $\Pi_{t'}^n$ only depends on $\{p_t^0, p_t^1\}$ for $t \ge t'$. So from equation (4.4) and (4.5), we have

$$\frac{\partial \Pi_t^0}{\partial p_t^0} = (1-q)\lambda g(p_t^0)(\delta \Pi_{t+1}^0 - p_t^0);$$

$$\frac{\partial \Pi_t^1}{\partial p_t^0} = 0;$$

and

$$\frac{\partial \Pi_t^0}{\partial p_t^1} = q\lambda g(p_t^1)(\delta \Pi_{t+1}^1 - p_t^1);$$

$$\frac{\partial \Pi_t^1}{\partial p_t^1} = (1-q)\lambda g(p_t^1)(\delta \Pi_{t+1}^1 - p_t^1).$$

So

$$\frac{\partial \Pi_0^0}{\partial p_t^0} = \delta^t \rho_t^0 (1-q)\lambda g(p_t^0)(\delta \Pi_{t+1}^0 - p_t^0);$$

$$\frac{\partial \Pi_0^0}{\partial p_t^1} = \delta^t (\rho_t^0 q + \rho_t^1 (1-q))\lambda g(p_t^1)(\delta \Pi_{t+1}^1 - p_t^1).$$

Remember from proposition 16 that

$$V_0^0 = c\sum_{i=0}^{T^B} \delta^i + c\sum_{i=0}^{t_0^B} \delta^i \frac{\lambda \mu_i^1}{q(1-\lambda \mu_i^1)} = c\sum_{i=0}^{T^B} \delta^i + c\sum_{i=0}^{t_0^B} \delta^i \frac{\lambda(1-G(p_i^1))}{q(1-\lambda(1-G(p_i^1)))}.$$

So

$$\frac{\partial V_0^0}{\partial p_t^0} = 0$$

and

$$\frac{\partial V_0^0}{\partial p_t^1} = -\delta^t \frac{\lambda g(p_t^1)}{q[1-\lambda(1-G(p_t^1))]^2} c.$$

So

$$\frac{\partial U}{\partial p_t^0} = \delta^t \rho_t^0 (1-q)\lambda g(p_t^0)(\delta \Pi_{t+1}^0 - p_t^0),$$

and

$$\frac{\partial U}{\partial p_t^1} = \delta^t(\rho_t^0 q + \rho_t^1(1-q))\lambda g(p_t^1)(\delta\Pi_{t+1}^1 - p_t^1) + \delta^t \frac{\lambda g(p_t^1)}{q[1-\lambda(1-G(p_t^1))]^2}c$$
$$= \delta^t \lambda g(p_t^1) \underbrace{\left((\rho_t^0 q + \rho_t^1(1-q))(\delta\Pi_{t+1}^1 - p_t^1) + \frac{c}{q[1-\lambda(1-G(p_t^1))]^2}\right)}_{(*)}.$$

For p_t^0, clearly optimality requires

$$p_t^0 = \delta\Pi_{t+1}^0.$$

For p_t^1, the term (*) is strictly positive when $p_t^1 < \delta\Pi_{t+1}^1$; for $p_t^1 < \delta\Pi_{t+1}$, (*) is strictly decreasing to negative and concave. So optimality requires

$$q[1-\lambda(1-G(p_t^1))]^2(p_t^1 - \delta\Pi_{t+1}^1) = \frac{c}{\rho_t^0 q + \rho_t^1(1-q)},$$

or

$$p_t^1 = \delta\Pi_{t+1}^1 + \frac{c}{(\rho_t^0 q + \rho_t^1(1-q))q[1-\lambda(1-G(p_t^1))]^2},$$

and the above equation has a unique solution and $p_t^1 - \delta\Pi_{t+1}^1$ is increasing in c and decreasing in $\rho_t^0 q + \rho_t^1(1-q)$.

We have characterized optimality conditions regarding p_t^0 for $t \leq t_0$. For $t > 0$, since neither μ_t^0 or μ_t^1 will affect V_0^0, optimality requires p_t^0, p_t^1 maximize Π_0^0, and therefore

$$p_t^0 = \delta\Pi_{t+1}^0; \;\; p_t^1 = \delta\Pi_{t+1}^1.$$

Moreover, since $\Pi_{T^B+1}^0 = \underline{\Pi}$, by recursion we have $\Pi_t = \underline{\Pi}$ for $t > t_0$ and $p_t = \delta\underline{\Pi}$ for $t \geq t_0$. □

Proof of Proposition 18. First, I show that the inequalities hold weakly for any set of parameters.

In the no-buyer case, recall from Proposition 3 that

$$T = \max\{T' : \Pi_{0,T'}^0 - \Pi_{0,T'-1}^0 \geq \delta^{T'} c\},$$

where

$$\Pi_{0,T}^0 - \Pi_{0,T-1}^0 = Q_1(t_{0,T})Q_0(\hat{t})\delta^T(\delta qY - c) + Q_0(t_{0,T})\delta^{t_{0,T}}(\delta q\Pi^1(\hat{t}) - c).$$

Suppose T^B is the optimal termination time in the case with buyers and $T^B > T$. Remember μ_t^1 (μ_t^0) is the probability that the stage 1 (stage 0) company is sold in period t. Note that given $G(\cdot)$, the distribution function of offers, μ_t^1 and μ_t^0 are bounded away from 0.

So

$$\Pi^{0,B}_{0,T^B} - \Pi^{0,B}_{0,T^B-1} = f_1(\mu^t_0) Q_1(t_{0,T^B}) Q_0(\hat{t}^B) \delta^{T^B} (\delta q Y - c)$$
$$+ f_2(\mu^t_0, \mu^t_1) Q_0(t_{0,T^B}) \delta^{t^B_{0,T^B}} (\delta q \Pi^1(\hat{t}) - c).$$

where f_1 and f_2 are positive, smaller than 1 and strictly decreasing in t. So

$$\Pi^{0,B}_{0,T^B} - \Pi^{0,B}_{0,T^B-1} < \Pi^0_{0,T^B} - \Pi^0_{0,T^B-1} < \delta^T c < \delta^{T^B} \frac{\lambda \mu^1_{T^B}}{q(1-\lambda\mu^1_{T^B})} c.$$

Therefore, the principal is better off by choosing $T^B - 1$ instead of T^B as the termination time of financing.

Next I show under some parameters the inequality is strict. Let Δ denote the length of a period, and let the probability of a breakthrough, the cost of investment and the discount factor be functions of the period length, $q = q_r\Delta$, $c = c_r\Delta$, $\lambda = \lambda_r\Delta$ and $\delta = e^{-r\Delta}$. Note that given $G(\cdot)$, μ^0_t and μ^1_t are bounded away from 0 for any Δ, because whenever the offer is larger than $\overline{\Pi}$, the company is sold. Moreover, $\overline{\Pi}$ is bounded as $\Delta \to 0$. As a result, $f_1(\mu^t_0)$ and $f_2(\mu^t_0, \mu^t_1)$ are bounded away from 1, and there exists some ε such that as $\Delta \to 0$,

$$\Pi^{0,B}_{0,T} - \Pi^{0,B}_{0,T-1} < \Pi^0_{0,T} - \Pi^0_{0,T-1} - \varepsilon.$$

At the same time, there exists some ε' such that for all Δ,

$$\Pi^0_{0,T} - \Pi^0_{0,T-1} > \delta^T c - \varepsilon',$$

because by the theorem of maximum, all values are continuous in T for any Δ. $\square$

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