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Essays in Macroeconomics, Finance and Growth

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Abstract
This dissertation studies the growth process from two different approaches. First, the measurement of misallocation of production inputs in China is analyzed within the context of a dynamic investment model that presents adjustment costs, with the purpose of assessing how much of the measured misallocation arises due to the presence of these costs. Given that these are technological constraints, rather than imperfections in markets or distortions arising from sub-optimal institutional features of a country, these are unavoidable. Thus, the potential aggregate productivity gains that would arise in a static model if inputs were perfectly allocated are over estimated, as these would be faced also by a social planner. Second, the product cycle as a feature of economic growth is studied. Using historical data from the United States, a model where growth occurs through innovations where production and entry-exit dynamics is estimated, and we use it to learn about the features of the product cycle: the number of firms operating in a sector peaks at 19 years and the number of firms that they imply varies greatly, yet how much output they imply is much more compact.

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ESSAYS IN MACROECONOMICS, FINANCE AND GROWTH

A Melissa,

mis padres Minerva y Rafael,

mis hermanos Odile y Carlos,

y mis sobrinos, Gustavo, Eduardo y Alejandro.

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Gustavo Camilo

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ABSTRACT

ESSAYS IN MACROECONOMICS, FINANCE AND GROWTH

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This dissertation studies economic growth from two different approaches. First, the measurement of misallocation of production inputs in China is analyzed within the context of a dynamic investment model that presents adjustment costs, with the purpose of assessing how much of measured misallocation arises due to the presence of these costs. Given that these are technological constraints, rather than imperfections in markets or distortions arising from sub-optimal institutional features of a country, these are unavoidable. Thus, the potential aggregate productivity gains that would arise in a static model if inputs were perfectly allocated are over estimated, as these would be faced by a social planner. Second, the product cycle as a feature of economic growth is studied. Using historical data from the United States, a model where growth occurs through exogenous innovations with entry-exit dynamics is estimated, and we use it to learn about the features of the product cycle: the number of firms producing an innovation peaks at 19 years and the number of firms that they are associated with varies greatly, yet how much output they imply is much more homogeneous.

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Introduction

The growth process has been studied through many lenses, and misallocation of inputs has been the focus of a recent series of paper, Restuccia and Rogerson [2008], Hsieh and Klenow [2009], Midrigan and Xu [2014], Moll [2014], Hsieh and Song [2015]. Misallocation refers to having producers of high productivity be relatively small, while having low productivity producers operating at large scales. This is measured by using the firm's average products, when there is a lot of variation in this variable, this is taken as evidence of the presence of misallocation. The typical counterfactual asks the question: if capital were perfectly reallocated such that average products of capital are equalized across firms, how much larger would aggregate productivity be?

The first chapter extends previous research by Asker et al. [2015] that attempts to measure what fraction of these productivity losses are generated by the presence of adjustment costs of investment. These costs make adjusting the capital stock both in response to productivity shocks that change the optimal size of the firm and during the initial growth phase of the firm more difficult, inducing variation in average products.

I build and estimate a dynamic investment model similar to those used by Gomes [2001] and Hennessy and Whited [2007] using Chinese firm level data that matches the data well. I use this model to assess what fraction of the productivity losses relative to the world where capital is perfectly allocated, which amount to 59% of TFP, are generated by the presence of adjustment costs. I find that a third of them are removed once I take out adjustment costs from the model. I also conduct and experiment to highlight an amplification channel that the model features where collateral constraints on the amount of debt a firm can issue interact with adjustment costs. When collateral constraints are tight, high adjustment costs generate much larger productivity losses than when they are loose, as more unproductive firms survive.

The second chapter studies the product cycle and it's relationship to long run growth. In joint work with Cecilia Fieler we develop a multi-sector growth model that features the dynamics of the product cycle as described by Gort and Klepper [1982] and estimate it using historical U.S. data. The structure of the model allows us to use aggregate data to recover deep parameters that determine the dynamics of the product cycle. We learn that the typical cycle peaks in 19 years, a magnitude similar to that measured by Gort and Klepper [1982], and that firms take a long time to exit. The number of firms that enter and exit during the cycle depend on the particulars of the innovation and presents large amounts of variation.

1 Adjustment Costs, Financial Constraints and the Persistence of Misallocation in China

1.1 Introduction

In the economics literature, differences in per-capita income across countries have been historically explained using differences in capital deepening, human capital and technology levels. Recently a literature has analyzed how these resources are allocated across different producers within a country and found large differences of allocative efficiency between developing and developed countries, which implies aggregate productivity levels far below optimal. Misallocation occurs when the most productive firms use a small share of the economy's inputs, and thus operate at a small scale, while less productive firms control a large share of inputs and produce a large share of the output. If inputs were reallocated from the least to the most productive firms, output and measured TFP would rise. Hsieh and Klenow [2009] provide evidence of substantial efficiency losses in the Chinese manufacturing sector due to misallocation of productive inputs, in the order of 86% of total factor productivity (TFP) if it were perfectly allocated, and 40% when compared with the efficiency of the United States.

These efficiency losses are typically mapped to distortions in the economy, either on the input or the product markets that make it difficult for the most productive firms to grow and capture a large share of the market. Hsieh and Song [2015] find that in the input side most of these distortions seem to arise from the capital side, as there is more variation in the average product of capital than in labor in the Chinese economy.

In this paper I will argue that a large proportion of this variation can be explained in a model that faces dynamic investment decisions and adjustment costs, and as such are technological constraints that would be faced by a social planner, implying that a large proportion of the TFP losses measured in the data should not necessarily be considered to be policy failures, but as the natural outcome of an economy where heterogeneous firms that are advancing in their life cycle and responding optimally to productivity shocks.

Using firm level data on all Chinese publicly traded manufacturing firms in the years 2000-2013, this paper documents the presence of misallocation within the companies studied. Given this facts, I build a model of the firm that faces dynamic investment and financing decisions in the presence of financial constraints and adjustment costs, where lenders can potentially treat firms differently according to their ownership. I will use the firm level data to estimate the structural parameters of the model. Most recent models of misallocation from the industrial sector are not able to generate large TFP losses stemming from misallocation [Midrigan and

Xu, 2014, Gopinath et al.], but this model is able to generate dispersion in TFPR that is close to the data. I use this framework to explore counterfactual exercises where I compare the levels of aggregate productivity to the efficient level when the firms face adjustment costs, and when these are removed. I find that 35% of the misallocation generated by the model in baseline economy disappears once I remove convex adjustment costs from the model.

The channel that introduces this dispersion is simple: the level of adjustment costs that are necessary to match the investment moments in the data induce the path of capital accumulation to vary more, being more dependent on the particular history of productivity shocks that a firm faces. When these adjustment costs are removed, more firms grow faster to their optimal level ending up looking more alike, reducing the degree of variation in marginal products, the main measure of misallocation. The key point is that adjustment costs are unavoidable and would be faced by a social planner.

Finally, I explore how do these adjustment costs interact with financial constraints, here modeled as collateral constraints and I establish that adjustment costs amplify the TFP losses generated through a type of selection: when collateral constraints are tight and adjustment costs high, more firms are small due to either exiting when they get a bad shock, or by simply slowing down capital accumulation enough. When these collateral constraints are loosened, increasing adjustment costs does not spread out the distribution as much as firms can use debt to grow more quickly.

Related Literature: This paper is a contribution to the growing literature of misallocation, with early contributions by Restuccia and Rogerson [2008] and Hsieh and Klenow [2009], and the research of misallocation associated with financial frictions. Midrigan and Xu [2014], Hsieh and Song [2015], and **?** agree in finding substantial TFP losses due to misallocation in China. Moll [2014] shows that parametrization choices of models of misallocation can have large results in the measured aggregate TFP losses. This paper is most closely related to Asker et al. [2015], who make a similar point in remarking that much of the variation of average products can be generated by adjustment costs as firms respond optimally to productivity shocks. This paper provides a model with which the actual aggregate productivity losses can be quantified. Buera and Shin [2013] study the effects of financial frictions on the persistence of misallocation after reforms that distort production, and the length of time they take to unwind; suggesting that gradual, rather than one off reforms must have been the case in developing countries as their model grows to the steady state too quickly.

This paper is related to the literature of dynamic models of corporate finance, which Strebulaev and Whited [2012] survey. My model draws from the work of Gomes [2001], Hennessy and Whited [2007] and Li et al. [2016]. Cooper and Haltiwanger [2006] make an important contribution highlighting the importance of capital adjustment costs in being able to match firm investment moments in this literature, and they highlight the importance of including both convex and non-convex costs of adjustment which are not very large, but statistically and economically significant.

The rest of the paper proceeds as follows, Section 1.2 describes the data and presents the main misallocation results. Section 1.3 describes the model and Section 1.4 provides functional forms and gives a brief description of the estimation procedures. Section 1.5 highlights the results and provides intuition for them, and finally Section 1.6 gives concluding remarks.

1.2 Misallocation and Adjustment Cost Facts

I have Balance Sheet, Cash Flow, Income Statements, and shareholder information on all publicly traded firms in China for the period 2000-2013 in the Shanghai and Shenzhen exchanges from the CSMAR database. To determine the ownership of each type of firm I follow a methodology similar to Hsieh and Song [2015], where I code a firm as SOE if: it has more than 50% shares officially listed as state owned, or if the largest shareholder is the state or a state owned entity. I only keep manufacturing firms, which leaves me with a sample with 13,608 annual observations of 1644 firms. As Hsieh and Song [2015] point out, it will be important to also keep track separately of firms which at one point during the sample were SOE, but have since been privatized, as these tend to be fall in between SOE and private firms for most observable characteristics.

To measure misallocation I will use the framework introduced by Hsieh and Klenow [2009], which uses unspecified wedges to capture all distortions that drive the economy away from a perfect allocation of resources. Suppose the economy is populated by N different sectors, and that in each sector s in period t there are M_{ts} monopolistically competitive firms each producing differentiated foods y_{tsi}. These are combined into the aggregate sectoral good Y_{ts} with a Constant Elasticity of Substitution (CES) aggregator with elasticity ν:

$$Y_{ts} = \left[\sum_{i=1}^{M_{ts}} y_{tsi}^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}}$$

Each firm uses a Cobb-Douglas production function with idiosyncratic productivity z_{tsi} and labor share $1 - \alpha_s$ that is sector specific:

$$y_{tsi} = z_{tsi} k_{tsi}^{\alpha_s} l_{tsi}^{1-\alpha_s}.$$

The firm observes demand and chooses prices, capital and labor to maximize profits π_{si}:

$$\pi_{tsi} = \max_{p_{tsi}, k_{tsi}, l_{tsi}} p_{tsi}(y_{tsi}) y_{tsi} - (1 + \tau_{tsi}^{l}) w l_{tsi} - (1 + \tau_{tsi}^{k})(r + \delta) k_{tsi}$$

where $p(y_{si}) = P_s(y_{si}/Y_s)^{\frac{-1}{\nu}}$ is the usual downward sloping demand curve that results from consumer optimization under monopolistic competition, τ_{si}^{l} and τ_{si}^{k} are firm specific wedges that affect the cost of labor and capital respectively. With these I am trying to capture all possible restrictions, subsidies and any other distortions that affect the firm's marginal product of capital and labor. First order conditions are the usual, prescribing that the marginal benefits, here titled as marginal revenue products of labor and capital, equal their marginal costs:

$$MRPL_{tsi} \equiv (1-\alpha_s)\left(\frac{\nu-1}{\nu}\right)\frac{p_{tsi}y_{tsi}}{l_{tsi}} = (1+\tau^l_{tsi})w \tag{1.2.1}$$

$$MRPK_{tsi} \equiv \alpha_s\left(\frac{\nu-1}{\nu}\right)\frac{p_{tsi}y_{tsi}}{k_{tsi}} = (1+\tau^k_{tsi})(r+\delta) \tag{1.2.2}$$

Note that the marginal costs include the wedges that prohibit the firm from reaching its optimal size, as these wedges raise (if there is a distortion), or lower (if there is a subsidy) marginal costs, rendering firms unable to as optimality prescribes equalize costs and benefits. A firm that has difficulties in accessing credit, for example, would have a high τ^k_{tsi}, implying that they have a high marginal revenue product of capital and that it can't borrow to increase their capital stock. Following the literature I define a firm's total factor revenue productivity, $TFPR_{tsi}$, as:

$$TFPR_{tsi} \equiv p_{tsi}z_{tsi}$$

A result of consumer optimization is that if there are no distortions present in the economy, $TFPR_{tsi}$ should be equalized across firms, since every firm faces the same problem they would choose the same allocations after controlling for productivity. If you are a very productive firm, which maps to having a high z_{tsi}, you should capture a large share of the market, which under monopolistic competition leads to a lower price p_{tsi}. When $TFPR_{tsi}$ is high relative to other firms, it implies that the firm is very productive but small, and when it is low, that the firm is large but unproductive. Thus, if this variable presents a large degree of variation, this is evidence of abundant misallocation.

To measure $TFPR_{tsi}$ I will use:

$$TFPR_{tsi} \equiv p_{tsi} z_{tsi} = \frac{p_{tsi} y_{tsi}}{k_{tsi}^{\alpha_s} l_{tsi}^{1-\alpha_s}}, \tag{1.2.3}$$

Where $p_{tsi} y_{tsi}$ will be a measure of value added[1], k_{tsi} will be fixed assets computed with a perpetual inventory method, and l_{tsi} the labor input measured by cash paid to and on behalf of employees. For this analysis only firms with positive value added are kept. The sector specific labor shares $1 - \alpha_s$ are computed from aggregate data and are detailed in Appendix C. In Figure 1.1 I plot the distribution of log deviations of $TFPR$ from industry means[2] by current ownership. Summary statistics are reported in Table 1.1.

First, the presence of misallocation is clear, as there is wide variation in TFPR within industries. Second, on average, SOE tend to be larger than their productivity warrants, with mean deviation of -.048, as they tend to have low TFPR relative to industry average; while privately owned firms tend to be smaller than their

[1] Value added will be defined as the sum of Operating Profits (EBIT), cash paid for and on behalf employees and depreciation.

[2] The statistic is $log(TFPR_{tsi}/\overline{TFPR}_s)$, where $\overline{TFPR}_s$ is an industry average of $TFPR_{tsi}$, defined in Appendix F.

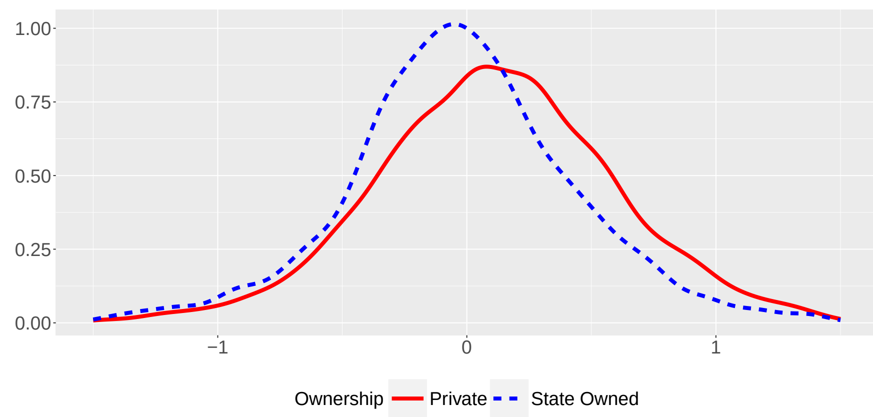


Figure 1.1: Distribution of log($TFPR$) Deviations from industry means

Table 1.1: Statistics of $TFPR$ deviations from industry means

	N	Mean	Median	S.D.	75 - 25	90 - 10
By Ownership						
SOE	6,680	-.048	-.042	.614	.559	1.204
Private	6,052	.096	.105	.624	.643	1.286
By Status						
SOE	5,663	-.053	-.040	.594	.549	1.183
Private	4,972	.136	.138	.578	.623	1.240
Privatized SOE	2,097	-.048	-.049	.749	.658	1.430
All Groups						
All Years	12,732	.015	.025	.623	.613	1.250
H-K Data 2001	108,702			.68	.88	1.71
H-K Data 2005	211,304			.63	.82	1.59

Notes: The table reports summary statistics of $\log(TFPR_{tsi}/\overline{TFPR_s})$ over all firms. S.D is standard deviation, 75-25 is the interquartile range and 90-10 is the difference between the 90th and 10th percentiles. Industries are weighted by share of value added. Only firms with positive value added are kept.

productivity warrants with mean deviation of .095. A Kolmogorov-Smirnoff test of whether the distribution for private firms lies below (and hence has more mass on higher values) than the SOE distribution has a p-value of 2.2×10^{-16}. The same pattern is observed when I consider Privatized SOE separately, with privatized being slightly less large for their productivity than SOE, while presenting more variation of TFPR than both other types of firms.

If I conduct the same counterfactual study as Hsieh and Klenow [2009] of eliminating all distortions in the economy I find smaller TFP gains, 19% compared to 86% in their paper. This result is expected, given that they use a survey that covers all firms above a revenue threshold, and I only observe publicly traded firms, hence my sample presents much less variation in TFPR than theirs does, as can be seen in the bottom panel of Table 1.1. The difference between the 75th and 25th percentiles of TFPR in my data is .613, and for them it is between .82 and .88, and the same happens with the 90th and 10th percentiles. Furthermore, if I don't trim the tails of the distortions and productivity measures, the losses increase to 52%.

To consider the misallocation of capital and labor separately, I plot the standard deviation of log marginal revenue products of capital and labor separately in Figure 1.2, which, according to equations (1.2.1)-(1.2.2) should capture the extent of distortions in these two inputs. If there is significant variation in each one of these, it would be evidence for distortions of that input, modeled as variation in the respective wedge. The data points to there being more distortion of capital than labor, a fact consistent with Hsieh and Song [2015] who document that the labor productivity of SOE had largely converged with that of the private firms by 2007 while capital productivity in SOE remained smaller. They argue that this reflects lower redundant employment in SOE as they shed 3.6 million workers from 1998-2007, while the higher capital productivity of private firms is likely driven by their difficulty in accessing funding for capital investment. It is interesting to note that distortions peak around the crisis era, but have since fallen back to pre-crisis level, potentially an effect of the stimulus policies undertaken during the crisis being targeted sub-optimally, but having since been undone.

In this paper I will focus on the distortions to capital exclusively, and will not have any channel that distorts labor choices. Given that it's typically modeled as a static choice, I will assume it has been optimized away.

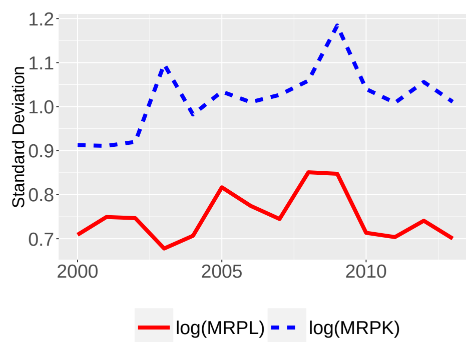


Figure 1.2: Path of dispersion of $\log(MRPK)$ and $\log(MRPL)$

I will now provide some reduced form evidence of the importance of adjustment costs in amplifying the spread of MRPK in the data. If it is the case that they are significant, then when a firm receives a productivity shock, this should amplify the dispersion of marginal products. As output, the numerator of MRPK in equation (1.2.2) will rise, but the denominator, the capital stock will take a long time to grow in order to equalize. This is different from the effect of the distortions, as they have an effect on the levels of MRPK, rather than a dynamic effect.

To test if this is the case, I will estimate innovations to each firm's $TFPR$ process using a dynamic panel model with the methodology of Arellano and Bond [1991]. The model is:

$$\log(TFPR_{t+1si}) = \rho \log(TFPR_{tsi}) + v_i + \sum_{j=2001}^{2013} \gamma_{tj} + \varepsilon_{tsi} \tag{1.2.4}$$

This results in $\hat{\rho} = .286$. With these estimated innovations $\hat{\varepsilon}_{tsi}$ I will estimate the following regression as in Asker et al. [2015]:

$$\log(MRPK_{tsi}) = \beta_0 + \beta_1 \hat{\varepsilon}_{tsi} + \beta_2 \log(k_{tsi}) + \beta_3 \log(l_{tsi}) + \beta_4 \log(TFPR_{t-1si}) + u_{tsi}. \tag{1.2.5}$$

These regressions ask the following question: faced with the same capital and lagged productivity levels, do firms who receive different productivity shocks end up with different marginal products of capital? If adjustment costs were irrelevant, firms would invest/disinvest optimally and $MRPK$ would remain constant across firms, rendering $\beta_1 = 0$. On the other hand, if it takes time to adjust the capital stock following a shock, a high TFPR innovation today would raise a firm's marginal revenue product for a long time, implying $\beta_1 > 0$. I present the results in Table 1.2.

For all specifications of the regression $\beta_1 > 0$ at the .01% level, implying that firms don't optimally adjust their capital stock immediately after receiving a productivity shock, even after controlling for industry and individual productivity, and

Table 1.2: Dispersion of $MRPK$ and Productivity Shocks

	Dependent variable:			
	log($MRPK$)			
	(1)	(2)	(3)	(4)
log($TFPR$) Innovation	0.467***	0.469***	0.703***	0.596***
	(0.032)	(0.032)	(0.018)	(0.023)
Capital Stock	−0.600***	−0.606***	−0.775***	−0.933***
	(0.017)	(0.018)	(0.014)	(0.024)
Labor Input	0.623***	0.641***	0.736***	0.791***
	(0.016)	(0.017)	(0.014)	(0.035)
Lagged log($TFPR$)	0.158***	0.164***	0.674***	0.440***
	(0.019)	(0.019)	(0.022)	(0.030)
Constant	−1.928***	−2.074***	−0.186*	1.530***
	(0.170)	(0.170)	(0.108)	(0.391)
Year F.E.	No	Yes	Yes	Yes
Industry F.E.	No	No	Yes	No
Individual F.E.	No	No	No	Yes
Observations	11,122	11,122	11,122	11,122
R^2	0.298	0.307	0.834	0.885

*p<0.1; **p<0.05; ***p<0.01

I report the results of the regression of equation (1.2.5) for different sets of controls, where the log($TFPR$) Innovation is given as the estimated residual of the model of equation (1.2.4), the Capital Stock is given by fixed assets and $TFPR$ is defined as in equation (1.2.3). In parentheses are heretoskedasticity robus standard errors.

their current input levels. The coefficient implies that a 1% innovation in TFPR increases the marginal revenue product of capital about .5%, which imply that adjustment costs are present in economically and statistically significant amounts. In Appendix E I add more lags of $\hat{\varepsilon}_{tsi}$ as a robustness check for timing inconsistencies, and I find that at both one and two lags of TFPR innovations the coefficient is positive and significant, providing further evidence that marginal products take a long time to adjust.

To summarize this section, I have constructed a measure of misallocation in China that agrees with the results of Hsieh and Klenow [2009], where there is present a large degree of variation of marginal products, mostly of capital. I thus conduct a reduced form exercise to test whether adjustment costs could be a plausible explanation of this variation, and I find that they must be present in order to justify the dynamic decisions of firms in the data.

1.3 Model

This section presents the model used to study the investment decisions of firms, and I highlight the sources of variation of marginal products implied by it. The model follows closely a modified version of Li et al. [2016], which is a dynamic investment model with a contracting problem, but I add an exit decision modeled as in Clementi and Palazzo [2015], with monopolistic competition.

There is a continuum of monopolistically competitive producers making differentiated goods y_t, whose objective is to maximize the expected value of the firm. Within the continuum of firms, there will be three types, denoted $i \in \{a, b, c\}$, respectively private, privatized and SOE firms. For clarity any parameters that depend on the type of the firm will have an i index. Each firm enters a period with a productivity shock $z \in \mathcal{Z}$, debt $b \in \mathcal{B}$ and capital $k \in \mathcal{K}$, where $\mathcal{K} \times \mathcal{B} \times \mathcal{Z} \subset \mathbb{R}^3$. The firm faces a demand curve given by $p(y) = y^{-1/\nu}$. Once it enters the period the firm produces using its capital stock according and the technological shock, which follows a Markov process with transition function $Q^i(dz', z)$ which satisfies the Feller property. Firm output is given by the production function F:

$$y(k, z, i) = F(k, z, i)$$

Operating profits are given by

$$\pi(k, z, i) = p\left(y(k, z, i)\right) y(k, z, i)$$

After production, the firm chooses whether to pay a random fixed cost c_f with distribution function C_f^i to continue operating. If it chooses to exit, then it pays off its debts $(1 + r_b)b$ and returns to shareholders the produced output and the depreciated value of capital. If the firm decides to pay the fixed cost, it must choose how much capital to have next period k' and how much to borrow b' at rate $1 + r$ from a deep pockets bank, subject to a collateral constraint:

$$(1+r)b' \leq \theta^i(1-\delta)k'$$

Li et al. [2016] derive this constraint as a reformulation of an enforcement constraint on a long term debt contract, but restrict equity distributions to be positive. The firm discounts with β, pays adjustment costs of capital according to the function $A^i(k,k')$, and if it issues negative distributions, it must pay equity issuance costs which are captured in the function $\Phi^i(\cdot)$. The capital and borrowing decisions imply how large are equity distributions e, to be defined below.

1.3.1 Recursive Problem

The recursive problem for a firm with type i can be written down as follows:

$$V(k,b,z,i) = \int_{c_f} \max_{\text{exit,operate}} \left\{V^E(k,b,z,i), V^O(k,b,z,i) - c_f\right\} dC^i(c_f) \tag{1.3.1}$$

The value of exiting is distributing profits and depreciated capital net of debts:

$$V^E(k,b,z,i) = \pi(k,z,i) + (1-\delta)k - (1+r)b. \tag{1.3.2}$$

The value of operation after paying the fixed cost is given by:

$$V^O(k,b,z,i) = \max_{k',b'} \Phi^i(e(k,k',b,b',z,i)) + \beta \int_{z'} V(k',b',z',i)Q^i(dz',z) \tag{1.3.3}$$

equity distributions are given by:

$$e(k, k', b, b', z, i) = \pi(k, z, i) + (1 - \delta)k - (1 + r)b - k' - A^i(k, k') + b' \quad (1.3.4)$$

subject to

$$(1 + r)b' \leq \theta^i(1 - \delta)k' \quad (1.3.5)$$

The equity issuance costs function Φ^i charges a constant fraction ϕ^i of the raised value, paid only if distributions are negative:

$$\Phi^i(e) = e + \mathbf{1}_{[e<0]}\phi^i e. \quad (1.3.6)$$

Since the fixed cost enters linearly, the exit decision is given by a cutoff rule where there exists a realization of the fixed cost $c_f^*(k, b, z, i)$ that renders the firm indifferent between exiting and staying, and thus the firm will exit with probability $\mathbb{P}[c_f \geq c_f^*(k, b, z, i)] = x^i(k, b, z)$, and stay with complementary probability. By a law of large numbers, at each state the measure of firms that choose to stay will pay an expected fixed cost of

$$\mathbb{E}[c_f | c_f \leq c_f^*(k, b, z, i)] \equiv c_f^e(k, b, z, i)$$

The solution to this problem is given by policies $k'(k, b, z, i)$, $x^i(k, b, z)$, $b'(k, b, z, i)$, $e(k, b, z, i)$ and a value function $V(k, b, z, i)$. We can rewrite the value function as:

$$V(k,b,z,i) = x^i(k,b,z)V^E(k,b,z) + (1 - x^i(k,b,z))[V^O(k,b,z,i) - c_f^e(k,b,z,i)]$$

1.3.2 Aggregation

Firms are born with initial capital endowment k_0^i and zero debt, and a shock drawn from the stationary distribution of $Q^i(dz',z)$, denoted as $\overline{Q}^i(dz)$. Any firm that exits is replaced with a newborn firm. Let $\mu^i(k,b,z)$ denote the mass of firms in the state $(k,b,z) \in \mathcal{K} \times \mathcal{B} \times \mathcal{Z}$, for any for any subset $S = (K,B,Z) \in \Sigma(\mathcal{K}) \times \Sigma(\mathcal{B}) \times \Sigma(\mathcal{Z})$, where $\Sigma(X)$ denotes the minimum sigma algebra for set X. For compactness denote $s = (k,b,z)$. The law of motion for the measure of firms of type i can be characterized in the following way, for the sets that contain the state at which firms are born, if $(k_0^i, 0) \in (K,B)$

$$\mu'^i(S) = \int_{\mathcal{K}} \int_{\mathcal{B}} \int_{\mathcal{Z}} \int_{Z} \chi^i(s,S) \left[\mu^i(ds)(1 - x^i(s))\right] Q^i(dz',z) + \int_{\mathcal{Z}} M^i \overline{Q}^i(dz) \quad (1.3.7)$$

for cases when it is not, $(k_0^i, 0) \notin (K,B)$

$$\mu'^i(S) = \int_{\mathcal{K}} \int_{\mathcal{B}} \int_{\mathcal{Z}} \int_{Z} \chi^i(s,S) \left[\mu^i(ds)(1 - x^i(s))\right] Q^i(dz',z). \quad (1.3.8)$$

Where χ^i is an indicator function of whether the policy functions points to a specific state:

$$\chi^i(s,S) = \begin{cases} 1 & \text{if } (k'(s,i), b'(s,i)) \in (K,B) \\ \\ 0 & \text{otherwise} \end{cases} \tag{1.3.9}$$

and M^i is the mass of exitors, whom are just reborn at initial capital and zero debt:

$$M^i = \int_K \int_Q \int_Z x^i(s)\mu^i(ds) \tag{1.3.10}$$

This equation takes into account the exit, investment and borrowing decisions of firms to keep track of the state of each firm of type i across the whole distribution. To obtain the aggregate measure μ, add the three measures together $\sum_{i,s} \mu^i(s) = \mu(s)$.

Since the equilibrium definition will be a stationary industry equilibrium, it will be the case that $\mu' = \mu = \mu^*$ so that μ defines the mass of firms in a stationary equilibrium.

Using this stationary distribution, I can define aggregate variables in the following way, aggregate debt is given by

$$B = \sum_{i\in\{a,b,c\}} \int_S b\mu^i(ds) \tag{1.3.11}$$

aggregate capital,

$$K = \sum_{i\in\{a,b,c\}} \int_S k\mu^i(ds) \tag{1.3.12}$$

aggregate output,

$$Y = \left[\sum_{i \in \{a,b,c\}} \int_S y(k,z,i)^{\frac{\nu-1}{\nu}} \mu^i(ds) \right]^{\frac{\nu}{\nu-1}}, \tag{1.3.13}$$

aggregate TFP will be measured as

$$TFP \equiv \frac{Y}{K} = \frac{\overline{TFPR}}{P} = \left[\sum_{i=a,b,c} \int_S \left(z \frac{\overline{TFPR}}{TFPR(s,i)} \right)^{\nu-1} \mu^i(s) \right]^{\frac{1}{\nu-1}} \tag{1.3.14}$$

where $\overline{TFPR} \equiv PY/K$ is aggregate revenue productivity and $TFPR(s,i) = p(y(s,i))z$ is defined as in the data section. Finally, given that efficient allocation of resources is achieved when all firms equalize $TFPR(s,i) = \overline{TFPR}$, so that if capital is perfectly allocated, the efficient level of TFP is

$$TFP^e = \left[\sum_{i=a,b,c} \int_S z^{\nu-1} \mu^i(s) \right]^{1/(\nu-1)}. \tag{1.3.15}$$

These equations are derived in Appendix G.

1.3.3 Stationary Industry Equilibrium

Definition 1. : A stationary industry equilibrium is a set of policies for each type of firm $k'(k,b,z,i)$, $b'(k,b,z,i)$, $x^i(k,b,z)$, a value function $V(k,b,z,i)$ and three firm measures $\mu = (\mu^a, \mu^b, \mu^c)$ such that

- Firm decision rules and value function solve each firm's problem (1.3.1)-(1.3.6);

- The measures μ satisfy the law of motion (1.3.7)-(1.3.10) with $\mu' = \mu$.

Proposition 1. *There exists a stationary industry equilibrium.*

Proof. See Appendix B □

1.3.4 Sources of Misallocation

To clarify what are the sources of misallocation in the model, I will describe the optimality condition with respect to capital. Suppose that the Lagrange multiplier of the collateral constraint is $\lambda\beta$, and denote δ_h as the partial derivative with respect to h, and I omit the arguments of the policy functions. The first order condition of next period capital, k', in the case of positive distributions (for simplicity), is given by

$$
\begin{aligned}
1 + \delta_{k'} A^i(k, k') &= \beta \mathbb{E}_{z'}[\delta_{k'} V(k', b', z', i) + \lambda\theta^i(1-\delta)|z] \\
1 + \delta_{k'} A^i(k, k') &= \beta \mathbb{E}_{z'}\big[\alpha^i \left(\frac{\nu - 1}{\nu}\right) p(z' k'^{\alpha^i}) z' k'^{\alpha^i - 1} + (1-\delta) \\
&\qquad + X(k', b', z', i) + \lambda\theta^i(1-\delta)|z\big]
\end{aligned}
$$

Rearranging the discounting and depreciation terms,

$$
\begin{aligned}
\frac{1 + \delta_{k'} A^i(k, k')}{\beta} + \delta - 1 &= \mathbb{E}_{z'}\left[\alpha^i \left(\frac{\nu - 1}{\nu}\right) \frac{p'y'}{k'} + X(k', b', z', i)|z\right] + \lambda\theta^i(1-\delta) \\
\frac{1 + \delta_{k'} A^i(k, k')}{\beta} + \delta - 1 &= \mathbb{E}_{z'}\left[MRPK' + X(k', b', z', i)|z\right] + \lambda\theta^i(1-\delta)
\end{aligned}
$$

The left hand side of this equation is the user cost of capital, and the right hand side is marginal revenue product of capital, plus the shadow value of relaxing the collateral constraint. The term $X(k', b', z', i)$ captures the marginal returns related with the exit decision and adjustment costs:

$$X(k', b', z', i) = \delta_{k'} x^i(\cdot)[k'' + A^i(k', k'') - b' + \mathbf{1}_{[e<0]} \phi^i e' + c_f^e - \beta V'']$$
$$+ x^i(\cdot)[\delta_{k'} A^i(k', k'') + \mathbf{1}_{[e<0]} \phi^i + \delta_{k'} c_f^e]$$

In the model variation in the measured marginal revenue product of capital MRPK, which is defined here identically as for the data analysis in equation (1.2.2), is generated by variation of the adjustment costs firms are facing, equity issuance costs and the exit decision through the X term, risks in investment, and the collateral constraint, which affects the choice of capital at its shadow value λ. Implicit in this equation is also the initial level of capital k_0^i, which can be interpreted as IPO size of the firm, which affects where along distribution of marginal products of capital the firms are located in the stationary distribution.

The two effects that this paper wants to highlight are the contribution of adjustment costs to generating dispersion in MRPK through X and the time-to-build technology that requires that capital purchased today to be productive tomorrow. I will also study how do collateral constraints (θ^i) and adjustment costs interact to make the growth process slower, and the TFP losses from misallocation larger.

1.4 Estimation and Identification

Now that the equilibrium has been defined, this section describes the functional forms chosen for the model, the estimation procedure and the identification of parameters.

1.4.1 Functional Forms

For the firms, I will assume a Cobb-Douglas production function, where the capital share will be the same for al types of firms:

$$F(z,k,l) = zk^{\alpha^i}$$

The stochastic process for the technology shock is given by

$$\log(z') = (1-\rho^i)m_z^i + \rho^i \log(z) + \sigma_z^i \epsilon$$

where $\epsilon \sim N(0,1)$. This process allows for firms of different ownership types to have productivity with different means, variances and persistence levels. The distribution of the random fixed costs C_f^i will be lognormal with parameters $(m_{c_f}^i, \sigma_{c_f}^{i2})$. The functional form of adjustment costs of capital feature fixed and convex costs, as is typical in the investment literature, and is key to fitting the investment moments in the data (see **?**Cooper and Haltiwanger [2006]),

$$A^i(k,k') = \mathbf{1}_{[k' \neq k]}\kappa_0^i k + \frac{\kappa_1^i k}{2}\left(\frac{k' - (1-\delta)k}{k}\right)^2.$$

1.4.2 Method of Simulated Moments & Identification

I will estimate most of the structural parameters of the model using the method of simulated moments (MSM), however some of the parameters will have to be estimated separately or fixed from outside data. The discount factor is set to the average real 1-year lending rate in China for 2000-2013, which was 5.6%, so that $\beta = \frac{1}{1.056}$. As is typical of the investment literature, in order for the firm's problem to be well defined even in the long run, it is required that $\beta < \frac{1}{1+r}$, basically to render the firm less patient than it's lenders. This is typically achieved if taxes are included in the model, but since I don't explicitly model taxes, and this gap is very difficult to identify in the data, I will use the corporate tax rate of 25% to set $r = (1 - .25) \times 0.056 = 0.042$.[3] This rate implies that agents don't have an incentive to pay down their debts once they reach their optimal size, and thus are either newborn with zero debt, or at their constraints at all times.

The depreciation rate is set at $\delta = .09$ to match the ratio of depreciation to fixed assets in the data. The elasticity of substitution is $\nu = 3$ to match the TFPR analysis of Section 1.2. The means of the stochastic processes for the technol-

[3] Given that if the firm borrows one dollar and pays r of interest on it, they get to deduct r dollars from their taxable income, which pays rate tax, this renders the effective interest rate as $(1 - tax) \times r$.

ogy shock are estimated using the method of Wooldridge [2009], which is a GMM refinement of Olley and Pakes [1996]. I average the resulting log productivity measures across firm types, and express them as ratios to SOE log productivity so that $m_z^c = \log(1)$, private firms productivity is $m_z^a = \log(2.2)$ and privatized productivity is $m_z^b = \log(1.2)$. Given that I can't identify in my data if a firm is delisted, closes or is taken private, I will take the exit rates from the aggregate data of Hsieh and Song [2015], where the average exit rate from 1998-2007 was 12% for private firms and 13.2% for SOE. For privatized firms I'll take the average of these two.

The parameters that I will estimate are $p = (\{\rho^i, \sigma_z^i, \theta^i, \omega^i, k_0^i, \kappa_0^i, \kappa_1^i, \alpha^i\}_{i \in \{a,b,c\}})$, where ω^i stands for the ratio of the absolute value of the mean to the standard deviation of the fixed cost. I will express k_0^i as a percentage of the non-stochastic steady state level of capital. Given that the exit rate and equity issuance rates are almost uniquely dependent and monotone on the mean of the fixed cost $m_{c_f}^i$ and the equity issuance cost parameter ϕ^i respectively, I will for every solution of the model at parameters p use a bisection algorithm over these two to match the two moments very closely. The relative mass of each type of firm will be given by the share of aggregate output each produced within the sample, so that aggregate quantities match the relative contributions of each type. This results in SOE being 66%, Private firms 23% and Privatized SOE 11%.

The idea of MSM is to estimate the parameters of an average firm by choosing them so that the average moments that result from a simulation are close to those from the data. In a model where the stationary distribution of firms is obtained, I can avoid simulation error by computing population moments in the model by using the distribution and policy functions of the firms.

I will match the mean and standard deviation of equity issuance, and the fraction of periods with positive issuance[4]. I will also match the mean, standard deviation and AR(1) coefficients of investment, operating profits and debt, where debt is defined as the sum of short term debt, long term debt and bonds outstanding. All moments will be measured relative to fixed assets, as the model does not include any investment in intangibles or working capital. My model counterparts will be $(k' - (1-\delta)k)/k$ for investment, b/k for debt, profits will be π/k, and equity issuance will be $\mathbf{1}_{[e<0]} e/k$.

In MSM identification comes from carefully choosing moments that are sensitive to the structural parameters of the model. A parameter is well identified by the MSM estimator if it has a monotonic relationship with a data moment [Strebulaev and Whited, 2012]. It is also important that the moments are related to the variables that will be affected in the counterfactual. In my model, the standard deviation and autocorrelation of profits are most informative about ρ^i and σ_z^i. A higher collateral constraint parameter θ^i corresponds to larger levels of mean debt while the correlation of debt and investment is useful for placing debt within the pecking order of sources of financing, mostly through k_0^i.

[4]This frequency is normalized by the number of periods a firm is in the sample.

Mean investment and equity issuance depend on the cost parameter ϕ^i and the initial level of capital k_0^i. The standard deviation of investment informs the curvature of the production function α^i as firms won't invest as aggressively as a result of a productivity shock if the firm presents large decreasing returns to scale. The adjustment costs parameters, κ_0^i and κ_1^i are related to the standard deviation of investment and to its autocorrelation. The computational method to solve the model and technical details about how I solve the model and on MSM are given in Appendix A.

1.5 Results

In Table 1.4 I present the data and simulated moments that resulted from the estimation. The model does fairly well at matching the data. The moments that are the most important for the counterfactuals to be studied, those relating to investment, are well captured, as the mean, standard deviation and autocorrelation of investment moments are very close to the data. The profit moments, which relate to production are also well captured, as also is the debt level. The model has difficulty achieving equity issuance levels that match the data and at the same time reasonable profitability and investment levels, which occurs because the firms would be on average smaller when equity issuance is high, inducing the ratios of investment and profits to assets to be much larger. Given that those moments are better measured in the data (the variance-covariance matrix from which the weights are derived presents a much smaller variance for mean investment and profits), the estimator prefers to match these two over equity issuance.

Table 1.3: Parameter Estimates

Type	θ^i	ρ^i	σ_z^i	ω^i	k_0^i	κ_0^i	κ_1^i	α^i	χ^2
Private	0.637	0.834	0.387	23.786	0.004	0.000	1.811	0.976	5.366
SOE	0.731	0.875	0.465	17.899	0.014	0.033	2.031	0.971	4.314
Privatized	0.838	0.861	0.477	12.129	0.026	0.023	1.264	0.959	5.274

Notes: The table reports the parameter values estimated with Method of Simulated Moments. χ^2 is the statistic for the J test of overidentifying restrictions. The 95th percentile of the distribution is 11.07.

Table 1.3 contains the point estimates of parameter values that result from the estimation. The debt to capital ratios are monotonic in the debt levels that they match, a typical feature of models with collateral constraints. Private firms start smaller than SOE, which in turn start smaller than privatized firms, inducing the patterns in equity issuance observed in the data, but at much smaller scales given the difficulties discussed before.

The model is fit well, as can be evidenced by the results of the J-test of over identifying restrictions, given by the last column of Table 1.3, where the test statistics for all three estimations are well to the left of the 95th quantile, failing to reject the null hypothesis that the model is not identified. Some non-targeted moments that the model also fits well are the time series moments of productivity, ρ^i and σ_z^i, which are very close to those that arise from the previously estimated productivity measures using the same dynamic panel regression as in equation (1.2.4). In the data, the autocorrelation coefficient has a value of .801, with standard error of the regression of .511, which are close to the values found across the firm types in Table 1.3.

Finally, to assess how closely the model captures measured misallocation relative to the data, it is useful to compare the variation in $\log(TFPR)$ from Table 1.1, and the aggregate results from the baseline parametrization in Table 1.5. The standard deviation was .62 while in the model it was .47, therefore the model does not generate the same degree of variation, and it can be seen that it is due to the model not having that many firms in the tails, as the log difference of 90th and 10th quantiles is 1.05, while in the data it was 1.25, yet the gap between the 75th and the 25th quantiles are very close. Given that the model is very parsimonious in how distortions from optimality are generated, this is to be expected.

The model is able to generate TFP losses relative to the efficient allocation of resources of 59%, in the range of those found by Hsieh and Klenow [2009] who determine they are 86%, much larger than those from the data used in this paper, which were only 19%, again, mostly due to a much smaller sample of more homogeneous firms.[5] In the next counterfactual, I explore the contribution of the presence of adjustment costs to this large degree of inefficiencies.

---

[5]This low number is also due to following the same trimming procedure of Hsieh and Klenow [2009], where as if I don't trim the 1% tails of TFPR and the distortions, the TFP losses relative to the efficient are 52%, implying that the tails of this distribution are very important in determining the degree of misallocation generated.

Table 1.4: Method of Simulated Moments Results

Model	Moments	Private Data	Private Model	SOE Data	SOE Model	Privatized Data	Privatized Model
$\frac{k'-(1-\delta)k}{k}$	Mean Investment / Assets	0.341	0.226	0.202	0.118	0.166	0.112
	SD Investment / Assets	0.149	0.180	0.107	0.104	0.110	0.118
	AR(1) Investment / Assets	0.600	0.589	0.692	0.596	0.600	0.542
$\frac{\pi}{k}$	Mean Profits / Assets	0.309	0.267	0.150	0.204	0.102	0.179
	SD Profits / Assets	0.147	0.121	0.126	0.091	0.160	0.083
	AR(1) Profits / Assets	0.501	0.615	0.508	0.671	0.446	0.644
$\frac{b}{k}$	Mean Debt / Assets	0.476	0.490	0.671	0.554	0.774	0.640
	Correlation Debt and Investment	-0.036	-0.358	0.009	-0.178	0.039	-0.002
$\frac{\mathbf{1}_{[e<0]}e}{k}$	Mean Equity Issuance / Assets	0.217	0.022	0.154	0.003	0.245	0.001
	SD Equity Issuance / Assets	0.417	0.081	0.079	0.025	0.062	0.009
	Frequency of Equity Issuance / Assets	0.779	0.226	0.157	0.023	0.133	0.042
	Correlation Equity and Investment	0.149	0.047	0.091	0.017	0.058	0.020

Notes: The table reports simulated and data moments for each firm type. The data comes from the CSMAR database and covers the years 2000-2013, all moments are relative to fixed assets, I trim 3% of the tails of each ratio before taking the statistics.

In order to clarify the effects of capital deepening and selection in the model, the fourth column of Table 1.5 shows the aggregate effects of greatly relaxing financial constraints, by increasing the collateral constraint parameter of all types of firms to $\theta^i = 2$. The main thing to notice is that measured TFP falls by 22%, which is the result of two forces: first, given that the production function of the firms presents decreasing returns to scale, and TFP is defined as Y/K, keeping other things constant, increasing the scale of the economy will lower the average marginal productivity, which is captured as lower TFP. Thus, while output is greatly increased, the quantity of capital required to produce it is more than proportionally larger, and measured TFP falls. Second, the average physical productivity of the surviving firms falls, as looser financial constraints allows more small and low productivity firms to survive. Due to this selection effect, the different distributions give different levels of efficient TFP, TFP^e in equation (1.3.15), so for the rest of the paper I'll present both TFP losses relative to the baseline, and to the counterfactual efficient levels of TFP.

1.5.1 Counterfactuals

In this subsection I present the results of my counterfactual exercises. For all of these only the parameters detailed are changed, so there is no new bisection performed, as can be seen by the changes in exit rates, an important channel through which adjustment costs and financial constraints interact to affect TFP losses.

Table 1.5: Aggregate Results

	Data	Baseline	$\theta^i = 2$
Aggregate TFP		2.41	1.87
TFP Losses	19%	59%	85%
TFP Losses To Baseline		59%	106%
Output (Y)		763	1946
Capital (K)		315	1040
Debt (B)		192	1806
SD log($TFPR$)	.62	.47	.58
75-25 log($TFPR$)	.61	.60	.62
90-10 log($TFPR$)	1.25	1.05	1.41
Exit Rate	12%	13%	6%

The table contains the aggregate variables of the economy, computed as in equations (1.3.11)-(1.3.15). The first column names the aggregate moments, the second column contains the relevant moments from the data, the third column has the baseline parametrization and the fourth column contains a counterfactual there the collateral constraint parameter is set for all firms at $\theta^i = 2$. The TFP losses are measured as $100(TFP^e/TFP - 1)$, where TFP^e can be either the model's, or the one from the baseline economy and represent how much TFP would rise if it were perfectly allocated. 75-25 and 90-10 stand for the difference of the quantiles of the stationary distribution of log($TFPR$).

The counterfactual exercise of Table 1.6 removes the adjustment costs by setting either $\kappa_0^i = 0$, $\kappa_1^i = 0$, or both, and presents the same aggregate values, divided by those of the baseline economy for clarity. From the second column it can be seen that removing fixed adjustment costs is able to both increase the scale of the economy and marginally increase TFP, and from the third column, removing convex adjustment costs increased TFP by 10% and brought the economy much closer to the efficient level, by 20 percentage points. When both are removed TFP increases by only 6%, and this case is the one that minimizes variation of TFPR at .34, approximately half of the variation in the data. However, removing fixed adjustment costs makes it easier for SOE and privatized firms to capture a larger share of the market, and because these are the least productive types of firms, aggregate productivity falls.

This key result implies that for this model to generate sizable TFP losses relative to an efficient TFP level, it requires a technological constraint would also be encountered by a social planner, and has been established to be necessary in order to accurately capture the timing of investment decisions found in the data, and thus is unavoidable. This implies that a large proportion of the measured misallocation by the literature may not be caused by bad policies, but is an inherent part of the firm life cycle as they respond to productivity shocks along their capital accumulation process.

Note that the increase in output is fivefold, and it occurs because without these adjustment costs, the firms optimally choose to grow very quickly, and are at their optimal size in one period by issuing large amounts of equity. Deviations of optimality only occur when the productivity shock changes, or when they exit because they received a large fixed cost shock. Thus, these firms spend most of the time in their optimal size, rather than being distributed along the path to it as in the baseline, and this distribution produces much more output. This fact is robust to lager values of the equity issuance costs.

Table 1.6: Counterfactual: Removing Adjustment Costs (Relative to Baseline)

	Baseline	Remove Fixed AC	Remove Convex AC	Remove Both
Aggregate TFP	1	1.01	1.10	1.06
TFP Losses	59%	55%	38%	37%
TFP Losses to Baseline	59%	57%	44%	50%
Output (Y)	1	1.17	5.29	4.61
Capital (K)	1	1.16	4.78	4.36
Debt (B)	1	1.17	4.87	4.44
SD $\log(TFPR)$	.47	.47	.39	.34
Exit Rate	13%	12%	9%	6%

The table contains results of counterfactual exercises, with the aggregate variables computed as in equations (1.3.11)-(1.3.15). The first column names the aggregate moments, all of which will be expressed a ratio to the baseline economy. The second column contains the baseline economy, the third column the counterfactual of removing fixed adjustment costs, the third of removing convex adjustment costs and the fourth of removing both types of adjustment costs. The TFP losses are measured as $100(TFP^e/TFP - 1)$, where TFP^e can be either the model's, or the one from the baseline economy and represent how much TFP would rise if it were perfectly allocated. 75-25 and 90-10 stand for the difference of the quantiles of the stationary distribution of $\log(TFPR)$.

In order to study the interaction of adjustment costs and financial constraints I first present in Table 1.7 the aggregate results of equalizing collateral parameters θ^i and initial capital levels k_0^i at their value added weighted means. I map k_0^i to IPO size and can be thought of as a financial parameter. In the data SOE firms have a mean log assets in their first year of being listed of 21.01, while private of 20.75, so the estimated differences of k_0^i are broadly in line with the data . As can be seen in this table equalizing these constraints does not increase TFP by a large amount, just about 2%. However, this process improves selection as the TFP losses relative to the baseline efficient TFP are smaller than those of the counterfactual efficient TFP, indicating that the surviving firms are on average more productive when θ^i is equalized. Thus, in the presence of adjustment costs, the misallocation due to different financial treatment across firm types are about 20% of those that arise from adjustment costs.

In Table 1.8 I compare the interaction of the collateral constraints and adjustment costs channels by comparing how much do TFP losses increase as the level of adjustment costs increases for tight financial constraints ($\theta^i = .5$), and loose financial constraints ($\theta^i = 1$). I solve the model after I set the level of adjustment costs low: 10% of the baseline adjustment costs parameters, medium at 50% and high at 200% of the adjustment costs levels.

The effects of relaxing collateral constraints are consistent with the capital deepening and decreasing returns to scale technology, as discussed in the beginning of

Table 1.7: Counterfactual: Equalizing Financial Constraints (Relative to Baseline)

	Baseline	$\theta^i = .72$	$k_0^i = .01$	Both
Aggregate TFP	1	1.02	1.01	1.02
TFP Losses	59%	58%	56%	59%
TFP Losses to Baseline	59%	55%	57%	56%
Output (Y)	1	.74	1.02	.96
Capital (K)	1	.72	1.01	.94
Debt (B)	1	.61	1	.92
SD log($TFPR$)	.47	.49	.44	.47
Exit Rate	13%	15%	12%	15%

The table contains results of counterfactual exercises, with the aggregate variables computed as in equations (1.3.11)-(1.3.15). The first column names the aggregate moments, all of which will be expressed a ratio to the baseline economy. The second, third and fourth columns contain the aggregate quantities that result from equalizing $\theta^i = .72$, $k_0^i = .01$, and from equalizing both simultaneously respectively. The TFP losses are measured as $100(TFP^e/TFP - 1)$, where TFP^e can be either the model's, or the one from the baseline economy and represent how much TFP would rise if it were perfectly allocated. 75-25 and 90-10 stand for the difference of the quantiles of the stationary distribution of log($TFPR$).

this section: for all levels of adjustment costs, relaxing financial constraints reduces aggregate TFP as a result of firms operating at a larger scale with decreasing returns to scale technology. What's important to highlight is how these magnitudes differ at different levels of collateral constraints.

If collateral constraints are tight, increasing the level of adjustment costs increases the TFP losses by 20 percentage points, whereas doing the same for loose financial constraints increases TFP losses by the smaller amount of 13 percentage points. The main channel is the fattening of the right tail: when financial constraints are tight and adjustment costs high, more firms exit relatively early and are born with a small level of capital, which is associated with high TFPR: the 90th quantile is .60 when constraints are tight and .53 when they are loose. In contrast, when financial constraints are loose, more of these firms survive, and bring aggregate TFP down. Thus, adjustment costs and collateral constraints interact to make the losses from misallocation larger: when constraints are tight, higher adjustment costs make the size distribution of firms more spread out as firms are kept small, yet if these are loose, occurs happens much faster.

Table 1.8: Counterfactual: Adjustment Costs Experiment (Relative to Baseline)

	$\theta = .5$			$\theta = 1$		
	Low	Mid	High	Low	Mid	High
Aggregate TFP	1.09	1.04	1.00	1.01	1.01	.97
TFP Losses	43%	55%	63%	52%	61%	65%
TFP Losses to Baseline	46%	52%	59%	58%	60%	64%
Output (Y)	2.55	1.37	.51	2.60	1.77	.48
Capital (K)	2.38	1.31	.51	2.57	1.79	.50
Debt (B)	1.64	.90	.31	3.64	2.50	.64
SD log($TFPR$)	.40	.47	.53	.42	.49	.52
10th Q TFPR	.15	.14	.21	.14	.15	.22
25th Q TFPR	.21	.21	.22	.19	.19	.22
50th Q TFPR	.24	.27	.37	.22	.22	.30
75th Q TFPR	.28	.38	.48	.27	.34	.49
90th Q TFPR	.38	.45	.60	.37	.45	.53
Exit Rate	11%	13%	18%	12%	12%	16%

The table contains results of counterfactual exercises, with the aggregate variables computed as in equations (1.3.11)-(1.3.15). The first column names the aggregate moments, all of which will be expressed a ratio to the baseline economy, as well as the 10th, 25th , 50th 75th and 90th quantiles of the distribution of TFPR. The second to seventh columns display these moments for different values of adjustment costs, which vary from Low (10% of each model's adjustment costs in the baseline estimation), Medium (50%) and High (200%), and the level of financial constraints.

1.6 Concluding Remarks

In this paper I study misallocation within the context of a dynamic investment model in order to measure how important are adjustment costs to misallocation as is measured by the variation in marginal products, the most common measure of the literature. I find that my small sample presents a large degree of variation in these marginal products and that TFP losses from misallocation are large, similarly to studies done with a larger sample.

I build and estimate a dynamic investment model of a firm that faces adjustment costs and financial constraints using Chinese data and the method of simulated moments. This model is able to generate variation in marginal products and TFP losses from misallocation that are in the order of what was measured in the data.

Using the model I conduct several counterfactual exercises to examine if adjustment costs are a big source of the TFP losses found. To do this I remove adjustment costs from the model and conclude that 35% of the losses arise due to the presence of adjustment costs, while a negligible amount arises from financial constraints. This result provides evidence that much of the measured TFP losses may be unavoidable by a social planner, and thus efficient. Finally, I study how financial constraints and adjustment costs interact to worsen the allocation of resources, which occurs through both a selection effect (more unproductive firms survive), and a capital deepening effect.

2 Growth and Product Cycle

2.1 Introduction

Economic growth in the XX century America is marked by the arrival of large innovations—e.g., synthetic fiber and rubber in the early decades, cars and assembly lines in the 1920s, household appliances in the 1940s and 1950s, and personal computers in the later decades.[6] Gort and Klepper [1982] document empirical regularities following large successful innovations. Initially, new firms enter, varieties are created, and production processes and product features improve significantly. In a maturing stage, firms settle on preferred processes and product features, the number of firms stabilizes and innovation slows down. Eventually, the arrival of a new innovation drive entrepreneurs and labor out of the sector.

We develop a quantitative growth model with multiple sectors and overlapping generations of types of products that captures this process of Schumpeterian creative destruction. Consumers have nested CES preferences over goods that are classified into types, that are themselves classified into sectors. A product type corresponds to a nest and each firm produces one variety. The arrival of a new type is followed

[6] Harberger [1998]

by endogenous entry of firms because new types are on average more productive than old ones, and by exogenous within-type productivity growth. The expansion of new types drives down the economy's price index, thereby inducing exit from old types. Because the quality of each type is stochastic, growth is heterogeneous across sectors. If the number of sectors is small, then growth also varies across decades. If it is large, growth is balanced and the model has a closed form solution.

In sum, the tension between disruptive new types and imitators give rise to the entry and exit of firms in and out of sectors that render the model uniquely suited to analyze product cycles in aggregate data. We use data from the *Historical Statistics of the United States* on value added, number of workers, and number of establishment classified into 20 sectors over the period of 1899 through 1999. Growth is highly heterogeneous across sectors. In a typical decade the fastest growing sector grows at an annualized rate 4.6 percentage points faster than the median sector, and the slowest growing sectors experience negative growth. Relative to other sectors, fast-growing sectors grow in number of employees (not surprisingly), number of firms and number of employees per firm.

While these facts suggest the presence of Schumpeterian product cycles, we do not directly observe innovations. For example, the sector "electrical equipment and supplies" grew disproportionately in the 1950s, but we do not observe household appliances within the sector. We estimate the model using the method of simulated moments. The estimation uses moments from the data together with the structure of the model to infer characteristics of the product cycle. For example, the correlation of growth rates within sectors, across decades provide information on the average duration of different stages of the product cycle. If cycles were short lived, then this correlation would be low because new innovations would quickly drive out old ones. Figure 2.13 illustrates a typical product cycle in the model. The cycle peaks after 19 years from the first entry. This estimated duration is broadly consistent with Gort and Klepper. Using data on firm entry and exit for 46 large innovations in the United States, they find that the product cycle appears in predictable stages for all of these products, with different durations. On average, the typical innovation has a period of fast entry and then a shake out where many firms exit, and these stages last 22 years. Comin and Hobijn [2004] study technological adoption across countries and find that innovations disseminate in predictable patterns in this dimension as well.

This result shows that our estimation strategy is reasonably successful in extracting from aggregate data unobservable features of product cycles. Relative to Gort and Klepper, the approach relies on stronger structural assumptions, but it allows us to analyze the universe of manufacturing firms in the United States, and data on value added and number of workers per establishment, not just the number of firms. Our estimates imply a large variation in the number of establishments and value added across cycles. The ratio of the peak number of firms in the median cycle to the 90th percentile largest cycle is four and the 90th percentile the sector has over double the market share of the median.

The features of the data highlighted above are not captured by existing models. Schumpeterian growth models pioneered by Grossman and Helpman [1991], Aghion and Howitt [1992] and more recently extended by Klette and Kortum [2004], Akcigit and Kerr [2010] and Acemoglu et al. [2013] typically feature only one firm per product line. Arkolakis et al. [2014] model rich firm dynamics but do not have a notion of sectors or disruptive innovation. Jovanovic and MacDonald [1994], Gort and Klepper [1982], Klepper [1996] model the dynamics of a single product cycle, but do not have aggregate economic growth or the arrival of new innovations. The paper is also related to vintage models such as Cooley et al. [1997] and Greenwood and Jovanovic [2001]. These papers study movements of investment and capital across sectors, not movements of firms.

Our paper complements this previous work by analyzing new aspects of the data, at the cost of simplifying features of other models. Most critically, new types arrive exogenously and deterministically, one per sector and per decade.

The paper is organized as follows, section 2.2 describes the data set and describes the variables we construct, section 2.3 describes in more detail the stylized facts we uncover from this data, section 2.4 develops our model, section 2.6 describes the estimation procedure, section 2.7 presents our results and section 2.8 provides concluding remarks.

2.2 Description of the dataset

The dataset we use is the *Historical Statistics of the United States: Millenial Edition*. In particular we use Series Dd-13-231 which include the detailed Manufacturing data for 1899-1999. From this dataset we collect value added, number of establishments and number of employees for 20 manufacturing sectors according the the Standard Industrial Classification (SIC) system. We deflate value added values using the Minneapolis Federal Reserve's Consumer Price Index dataset, which covers 1800-2015.

We study growth over decades to overcome missing data and to focus on long run growth. We average the observations one year before, during and after the start of a decade, and refer to this as the decade's value.[7] To compute growth rates we use percentage change formulas over these.

The results are 154 data points of value added levels, 157 data points of number of establishments and 140 data points of number of employees. Four sectors only begin reporting after the 1940s, and we have only 5 decade observations, but for thirteen of them we are able to assemble the full data of 9 decades from 1900-1980.

[7] For example, to construct the value added by the Food and Kindred Products we average the values for 1899, 1900 and 1901.

2.3 Stylized Facts

In this section we describe the main stylized facts we uncover with this data set, focusing on the relationships between growth and market shares of value added, employment and number of establishments across and within decades. In table 2.1 we present the sectors and their average annualized[8] growth rates of value added, number of establishments and employment. The sectors that grew the fastest in value added, Electrical Equipment and Supplies (electrical), Instruments and Related Products (instruments), Petroleum and Coal Products (petroleum), Rubber and Plastics Products (rubber) and Transportation Equipment (transportation) coincide with the qualitative evidence of having experienced large innovations during the 20th Century.

The table also suggests that sectors that high growth rates of value added also experienced high growth in both the number of establishments and employees. In order to assess this, we construct table 2.2 where we compute the correlations with value added growth of these variables. Growth in value added is highly correlated with growth in number of establishments and employment. The contemporaneous correlation between growth in value added and in number of employees is 0.83, and between growth in value added and number of establishments is 0.61. The relationships, displayed as a scatterplot on figure 2.1[9], are well approximated by

[8] We annualize by raising the decate growth rate to the power of $\frac{1}{10}$ for convenience, and explicitly denote where we use these transformed rates.

[9] In graph (b) we exclude the growth of rubber establishments in the 1950s as it is a large outlier

Table 2.1: Annualized Average Growth Rates

sector	Value Added	Establishments	Employment
apparel	1.025	1.009	1.011
chemicals	1.044	1.005	1.018
electrical	1.063	1.040	1.049
fabricatedmetal	1.051	1.034	1.017
food	1.035	0.988	1.004
furniture	1.032	1.015	1.016
instruments	1.056	1.026	1.032
leather	1.011	0.992	0.996
lumber	1.018	1.003	0.997
machinery	1.055	1.047	1.018
miscellaneous	1.019	1.003	0.996
paper	1.043	1.015	1.019
petroleum	1.053	1.024	1.017
primarymetal	1.035	1.019	1.000
printing	1.033	1.009	1.019
rubber	1.050	1.048	1.036
stone	1.031	1.006	1.010
textile	1.018	1.002	1.001
tobacco	1.023	0.947	0.988
transportation	1.053	1.041	1.012

a line. A similar pattern holds for levels in figure 2.3, where we plot the sectoral shares of value added, employment and establishments. Given that we have some decades with missing observations, the shares are normalized by multiplying by the number of sectors that have data. This is equivalent to dividing by the decade mean value added, rather that the sum of value added. Sectors with high value added have higher employment and number of establishments, and these relationships are statistically significant.

The dynamics of these growth rates are such that sectors that grow fast in

(growth of 88% in the number of establishments) and distorts the graph. We do not exclude it in our estimations.

Table 2.2: Lagged correlations with value added growth

	$t = 0$	$t = 1$	$t = 2$
value added growth	1	0.190	0.044
	-	(0.042)	(0.674)
employment growth	0.826	0.338	0.217
	(0.000)	(0.001)	(0.068)
number of establishments growth	0.608	0.231	-0.078
	(0.000)	(0.013)	(0.453)

one decade, do not necessarily grow fast in the subsequent decade. The lagged correlation of value added growth rates is only 0.19 on table 2.2. Furthermore, the variation in growth rates across sectors and decades is large, as can be seen in figure 2.4. Some decades, as the 1940s experience growth in value added that is much higher for all sectors than others. Over all decades, average value added growth is 3.7% per year with a standard error of 2.8%. Variations across sectors within decades accounts for 64% of the variance in growth rates, and the variation across decades accounts for 38%.[10]

However, the distribution of value added, employment and number of establishments shares across sectors is very stable over time. See figures 2.5-2.7. The coincidence of this fact, with the very large variation of growth rates is surprising, as one sector experiencing very high growth could increase variance of the share

[10]These numbers are the result of a standard variance decomposition. They do not add to one because of the covariance.

distribution, but these present a very predictable pattern.

Finally, there is a tight relationship between the size of sectors from one decade to the other, but over the course of the eight decades in the data, this correlation disappears, as can be seen in figure 2.8. The same holds for employment and number of establishments (not shown).

2.4 Model

2.4.1 Environment

Time is discrete and infinite, $t = -\infty, ..., -1, 0, 1, ..., \infty$. Goods are divided into sectors, types and vintages. The set of sectors S is fixed, finite and has measure 1. In each period a new type arises exogenously in each sector. Denote with $s\tau$ the type of good that arose in time τ and sector $s \in S$. Vintage τ is the set of types that arose in period τ.

In any time period, there is a large mass of entrepreneurs that may enter a type and create a new differentiated variety. We denote firms and varieties with ω and write $\omega \in s\tau$ when variety ω is in type $s\tau$. At any point in time, each of the existing types has a continuum of varieties. Firms observe demand and set prices. Consumers observe prices and choose quantities. In each period, each consumer is endowed with one unit of labor, which he supplies inelastically in a perfectly competitive market. Population L_t evolves exogenously at a constant rate g_L: $L_{t+1} = g_L L_t$.

2.4.2 Demand

A representative consumer maximizes a discounted flow of per capita utility:

$$
\begin{aligned}
U &= \sum_{t=0}^{\infty} \beta^t \ln(C_t/L_t) \\
\text{where } C_t &= \left[\sum_{s \in S} \sum_{\tau=-\infty}^{t} C_{ts\tau}^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)} \\
C_{ts\tau} &= \left[\int_{\omega \in s\tau} c_t(\omega)^{(\sigma-1)/\sigma} d\omega \right]^{\sigma/(\sigma-1)},
\end{aligned}
$$

$\beta \in (0,1)$ is the discount rate and $c_t(\omega)$ is the quantity of variety ω consumed at time t. The intertemporal elasticity of substitution is one for simplicity. The per period utility C_t is nested a CES of the existing types and varieties within a type. The elasticity of substitution between types is $\eta > 1$ and the elasticity of substitution between varieties is $\sigma > \eta$. With $\eta > 1$, an increase in a type's price decreases spending on it. The assumption that the elasticity of substitution between types within and across sectors is the same to save on notation.[11] The consumer's budget constraint is

$$
\sum_{t=0}^{\infty} \sum_{s \in S} \sum_{\tau=-\infty}^{t} \int_{\omega \in s\tau} c_t(\omega) p_t(\omega) \leq W
$$

where $p_t(\omega)$ is the price of variety ω at time t and lifetime earnings is $W = \tilde{W}_0 + \sum_{t=0}^{\infty} L_t w_t$ with $\tilde{W}_0$ is initial wealth and $L_t w_t$ is period t's income. The allocation

[11]This assumption would matter only if the distribution of new technology draws depended on previous draws within the sector. Below, we assume that technologies of new types are independently distributed across sectors and time.

of consumer income is

$$X_t = P_t C_t = \frac{\beta^t}{1-\beta} W \qquad \text{total spending at } t \tag{2.4.1}$$

$$X_{ts\tau} = \left(\frac{P_{ts\tau}}{P_t}\right)^{1-\eta} X_t \qquad \text{spending on type } s\tau \tag{2.4.2}$$

$$x_t(\omega) = \left[\frac{p_t(\omega)}{P_{ts\tau}}\right]^{1-\sigma} X_{ts\tau} \qquad \text{spending on variety } \omega \in s\tau$$

$$\text{where } P_t = \left[\sum_{s\in S} \sum_{\tau=-\infty}^{t} P_{ts\tau}^{1-\eta} ds\right]^{1/(1-\eta)}$$

$$P_{ts\tau} = \left[\int_{\omega \in s\tau} p_t(\omega)^{1-\sigma} d\omega\right]^{1/(1-\sigma)}.$$

2.4.3 Supply

All firms producing type $s\tau$ at time t have the same technology. The labor requirement to produce c units of good ω of type τ is

$$f_{ts\tau} + \frac{c}{z_{ts\tau}}$$

where $f_{ts\tau}$ and $z_{ts\tau}$ are class and time-specific productivity parameters that evolve exogenously according to

$$z_{ts\tau} = \bar{z}_\tau z_{s\tau} (t-\tau)^{\epsilon_z},$$

$$f_{ts\tau} = \bar{f}_\tau f_{s\tau} (t-\tau)^{\epsilon_f}$$

$$\text{where} \quad \bar{z}_{\tau+1} = g_z \bar{z}_\tau$$

$$\bar{f}_{\tau+1} = g_f \bar{f}_\tau$$

$g_z > 1$, $\epsilon_f < 0$, $\epsilon_z > 0$. The expressions for $z_{ts\tau}$ and $f_{ts\tau}$ have three terms. First, $\bar{z}_\tau$ and $\bar{f}_\tau$ govern long-run growth. Relative to the previous vintage, labor productivity of a new vintage is g_z times larger in the variable cost and $1/g_f$ times larger in the fixed cost. Because in the data the number of employees per firm generally increases over time, we allow fixed cost to be potentially larger for newer types, $g_f > 1$, though we assume that $g_z^{\eta-1} > g_f$ in order for the price index to be finite below. Second, the pair $(z_{s\tau}, f_{s\tau})$ is a random variable independently drawn across vintage and sectors from a time-invariant joint cumulative distribution G. This distribution captures idiosyncratic growth in value added and number of establishments across time and sectors. If G is degenerate, then all sectors grow at the same rate. For convenience, we assume that G is bivariate log-normal with mean parameters normalized to zero. Third, $(t-\tau)^{\epsilon_z}$ and $(t-\tau)^{\epsilon_f}$ capture learning within types. Since $(z_{s\tau}, f_{s\tau})$ are independent across vintages, ϵ_z and ϵ_f govern the correlation of growth rates of value added and of number of establishments across time within sectors—sectors with a large technology draw today grow faster in the following periods if ϵ_z and ϵ_f are large.

2.5 Balanced growth path

We study only the balanced growth path. There are two equilibrium conditions. First, assuming goods are perishable, the labor market clears when $X_t = L_t w_t$ for all t. From intertemporal maximization in equation (2.4.1), wages evolve at a constant rate:

$$\frac{w_{t+1}}{w_t} = \frac{\beta}{g_L} \tag{2.5.1}$$

Second, there is free entry. At any period t, there is an infinite mass of entrepreneurs that may exit or enter any of the existing types of goods $s\tau$ with $s \in S$ and $\tau \leq t$. The decision is static since there are no sunk costs. Entry requires only the fixed cost of production $w_t f_{ts\tau}$. Firms that produce type $s\tau$ at time t set prices $p_{ts\tau} = \frac{\mu w_t}{z_{ts\tau}}$ where $\mu = \frac{\sigma}{\sigma-1}$ is the markup. The zero-profit condition is

$$\pi_{ts\tau} = \frac{X_{ts\tau}}{\sigma M_{ts\tau}} - w_t f_{ts\tau} = 0$$

$$\equiv \quad M_{ts\tau} = \frac{X_{ts\tau}}{\sigma w_t f_{ts\tau}} = \left(\frac{p_{ts\tau}}{P_t}\right)^{\frac{(1-\eta)(\sigma-1)}{(\sigma-\eta)}} \left(\frac{L_t}{\sigma f_{ts\tau}}\right)^{\frac{\sigma-1}{\sigma-\eta}} \tag{2.5.2}$$

$$\text{and} \quad P_{ts\tau} = M_{ts\tau}^{1/(1-\sigma)} p_{ts\tau} = \left(p_{ts\tau}^{\sigma-1} P_t^{1-\eta} \frac{\sigma f_{ts\tau}}{L_t}\right)^{\frac{1}{\sigma-\eta}} \tag{2.5.3}$$

where line (2.5.2) uses equation (2.4.2), $P_{ts\tau} = M_{ts\tau}^{1/(1-\sigma)} p_{ts\tau}$, and $X_t/w_t = L_t$. Substituting in the definition of P_t, we have (appendix H):

$$P_t = \left[\sum_{s\in S} \sum_{\tau=-\infty}^{t} P_{ts\tau}^{1-\eta}\right]^{1/(1-\eta)}$$

$$= \left(\frac{L_t}{\sigma \bar{f}_t}\right)^{\frac{1}{1-\sigma}} \frac{\mu w_t}{\bar{z}_t} (\tilde{P}_t)^{\frac{\sigma-\eta}{\sigma-1}} \tag{2.5.4}$$

$$\text{where} \quad \tilde{P}_t = \left\{\sum_{\tau=-\infty}^{t} \left[g_1^{\tau-t}(t-\tau)^{\epsilon_1} \sum_{s\in S} (z_{s\tau}^{1-\sigma} f_{s\tau})^{\frac{(1-\eta)}{(\sigma-\eta)}}\right]\right\}^{\frac{1}{1-\eta}} \tag{2.5.5}$$

$$g_1 = \left[g_z^{(1-\sigma)} g_f\right]^{\frac{1-\eta}{\sigma-\eta}} > 1$$

$$\epsilon_1 = \left(\frac{1-\eta}{\sigma-\eta}\right) [(1-\sigma)\epsilon_z + \epsilon_f]$$

Term $\left[\left(\frac{L_t}{\sigma \bar{f}_t}\right)^{\frac{1}{1-\sigma}} \frac{\mu w_t}{\bar{z}_t}\right]$ is the standard price index in a model without types or productivity dynamics, with $\left(\frac{L_t}{\sigma \bar{f}_t}\right)^{\frac{1}{1-\sigma}}$ capturing the variety effect and $\frac{\mu w_t}{\bar{z}_t}$ capturing the price per variety. Term $\tilde{P}_t$ is the stochastic component of the price index. As the number of sectors approaches infinity, it converges to a constant

$$\tilde{P}_t \to (\Phi_1 \Gamma_1)^{\frac{1}{1-\eta}}$$

$$\text{where} \quad \Phi_1 = \int \left[z^{(1-\sigma)} f\right]^{\frac{1-\eta}{\sigma-\eta}} dG(z, f) \tag{2.5.6}$$

$$\Gamma_1 = \sum_{r=0}^{\infty} g_1^{-r} r^{\epsilon_1} \approx \log(g_1)^{\epsilon_1+1} \Gamma(\epsilon_1 + 1) \tag{2.5.7}$$

and Γ is the gamma function. Heterogeneity across sectors appears in constant $\Phi_1 < \infty$ with G log-normal. All learning is captured in Γ_1, which is a constant because fast learning in new vintages is exactly offset by a slowdown in the rate of learning of old vintages. In the definition of Γ_1, $(g_1^{-r} r^{\epsilon_1})$ is the probability density function of a gamma distribution with parameters $\ln(g_1)$ and $(\epsilon_1 + 1)$.[12] When the number of sectors is large, the price index evolves as

$$\frac{P_{t+1}}{P_t} = \frac{\beta}{g_z} \left(\frac{g_f}{g_L^{\sigma}}\right)^{\frac{1}{\sigma-1}} < 1$$

since $g_z^{\sigma-1} > g_f$. Normalizing $w_t = 1$, equation (2.5.4) gives the price index P_t as a function of exogenous variables. Plugging P_t back in equations (2.5.2), (2.5.3) and (2.4.2), we get the mass of firms, price index and spending on each type. The

[12] The approximation in equation (2.5.7) is only didactic. It is not used in the computation. If the model were in continuous time *and* the birth of new vintages occurred continuously, then equation (2.5.7) would hold with an equal sign. But modeling the birth of new vintages as a continuous is unappealing because it fails to capture the phenomenon we are trying to explain. The invention of mass-produced automobiles, household appliances and personal consumers were all discontinuous breaks from past inventions.

distribution of the mass of firms across types is

$$\frac{M_{ts\tau}}{M_t} = \frac{\left(z_{ts\tau} f_{ts\tau}\right)^{\frac{1-\sigma}{\sigma-\eta}}}{\sum_{s\in S}\sum_{\tau=-\infty}^{t}\left(z_{ts\tau}^{1-\eta} f_{ts\tau}\right)^{\frac{1-\sigma}{\sigma-\eta}}} \tag{2.5.8}$$

$$= \frac{g_2^{\tau-t}(t-\tau)^{\epsilon_2}\left(z_{s\tau}^{1-\eta} f_{s\tau}\right)^{\frac{1-\sigma}{\sigma-\eta}}}{\tilde{M}_t} \tag{2.5.9}$$

$$\text{where} \quad \tilde{M}_t = \sum_{s\in S}\sum_{\tau=-\infty}^{t} g_2^{\tau-t}(t-\tau)^{\epsilon_2}\sum_{s\in S}\left(z_{s\tau}^{1-\eta} f_{s\tau}\right)^{\frac{1-\sigma}{\sigma-\eta}} \tag{2.5.10}$$

$$g_2 = \left[g_z^{(1-\eta)} g_f\right]^{\frac{1-\sigma}{\sigma-\eta}} > 1$$

$$\epsilon_2 = \left(\frac{1-\sigma}{\sigma-\eta}\right)\left[(1-\eta)\epsilon_z + \epsilon_f\right]$$

Analogous to the price index, $\tilde{M}_t$ is a stochastic component. If the number of sectors is sufficiently large, it converges

$$\tilde{M}_t \to (\Phi_2\Gamma_2)$$

$$\text{where} \quad \Phi_2 = \int \left[z^{(1-\eta)} f\right]^{\frac{1-\sigma}{\sigma-\eta}} dG(z,f)$$

$$\Gamma_2 = \sum_{s=0}^{\infty} g_2^{-s} s^{\epsilon_2} \approx \log(g_2)^{\epsilon_2+1}\Gamma(\epsilon_2+1)$$

The total mass of firms at time t is

$$M_t = \left(\frac{L_t}{\sigma\overline{f}_t}\right)\tilde{M}_t\tilde{P}_t^{\eta-1} \tag{2.5.11}$$

The stochastic component $\tilde{M}_t\tilde{P}_t^{\eta-1} \to \frac{\Phi_2\Gamma_2}{\Phi_1\Gamma_1}$ as the number of sectors approaches infinity. The term $\left(\frac{L_t}{\sigma\overline{f}_t}\right)$ is the mass of firms in a monopolistic competition model with no sector and fixed entry cost $\overline{f}_t$. It grows at a constant rate g_L/g_f. Using prices $P_{ts\tau}$ and P_t in equation (2.5.3) and (2.5.4), we get the distribution of revenue across types:

$$\frac{X_{ts\tau}}{X_t} = \left(\frac{P_{ts\tau}}{P_t}\right)^{1-\eta} = \frac{g_1^{\tau-t}(t-\tau)^{\epsilon_1}\left(z_{s\tau}^{1-\sigma}f_{s\tau}\right)^{\frac{1-\eta}{\sigma-\eta}}}{\tilde{P}_t^{1-\eta}} \tag{2.5.12}$$

$$\text{where} \quad \tilde{P}_t = \left\{\sum_{\tau=-\infty}^{t}\left[g_1^{\tau-t}(t-\tau)^{\epsilon_1}\sum_{s\in S}(z_{s\tau}^{1-\sigma}f_{s\tau})^{\frac{(1-\eta)}{(\sigma-\eta)}}\right]\right\}^{\frac{1}{1-\eta}}$$

and $X_t = w_t L_t$. This completes the solution to the model. All aggregate variables, prices P_t, mass of firms M_t and spending X_t and time-type specific prices $P_{ts\tau}$, mass of firms $M_{ts\tau}$ and total spending $X_{ts\tau}$ and spending per firm $x_{ts\tau}$ are functions of exogenous variables.

2.5.1 Characteristics of equilibrium

Aggregate growth Assuming the number of sectors is sufficiently large so that $\tilde{P}$ and $\tilde{M}$ are constants, we have the following growth rates:

$$\frac{X_{t+1}/P_{t+1}}{X_t/P_t} = \left(\frac{g_L^{\sigma}}{g_f}\right)^{\frac{1}{\sigma-1}} g_z \qquad \text{aggregate real value added} \tag{2.5.13}$$

$$\frac{w_{t+1}/P_{t+1}}{w_t/P_t} = \left(\frac{g_L}{g_f}\right)^{\frac{1}{\sigma-1}} g_z \qquad \text{per capita real value added}$$

$$\frac{M_{t+1}}{M_t} = \frac{g_L}{g_f} \qquad \text{total mass of firms} \tag{2.5.14}$$

Vintages Summing over sectors, we get the dynamics of a vintage:

$$\frac{X_{t\tau}}{X_t} = \frac{g_1^{\tau-t}(t-\tau)^{\epsilon_1}}{\Gamma_1} \qquad \text{share of value added} \tag{2.5.15}$$

$$\frac{M_{t\tau}}{M_t} = \frac{g_2^{\tau-t}(t-\tau)^{\epsilon_2}}{\Gamma_2} \qquad \text{share of mass of firms}$$

The evolution of market shares of a vintage has the shape of the probability density function of a gamma distribution. As in Gort and Klepper [1982], when a new type of goods arises, the mass of firms initially increases, reaches a point of zero net entry and exit, and eventually declines with a shake out. The same dynamics holds for value added.

Types The distribution of sales and plants and across types within a vintage is independent of the vintage and of its age. When the number of sectors is large and G is a bivariate log-normal, this distribution is also bivariate log-normal:

$$\begin{aligned}
&\text{Prob}\left(\frac{X_{ts\tau}}{X_{t\tau}} \leq x \text{ and } \frac{M_{ts\tau}}{M_{t\tau}} \leq m\right) = \left(\left[\frac{P_{ts\tau}}{P_{t\tau}}\right]^{1-\eta} \leq x \text{ and } \frac{M_{ts\tau}}{M_{t\tau}} \leq m\right) \\
&= \text{Prob}\left(\left[z^{(1-\sigma)} f\right]^{\frac{1-\eta}{\sigma-\eta}} \leq \Phi_1 x \text{ and } \left[z^{(1-\eta)} f\right]^{\frac{1-\sigma}{\sigma-\eta}} \leq \Phi_2 m\right) \\
&\Rightarrow \left(\frac{M_{ts\tau}}{M_{t\tau}}, \frac{X_{ts\tau}}{X_{t\tau}}\right) \sim \log \mathcal{N}(0, \Sigma_{xm})
\end{aligned}$$

where Σ_{xm} appears in appendix I.

Sectors The observable distribution of market shares across sectors is an affine transformation of the bivariate normal the sum of log-normal distributions $\left(\frac{X_{t\tau}}{X_t}, \frac{M_{t\tau}}{M_t}\right)$:

$$\begin{aligned}
\frac{X_{ts}}{X_t} &= \sum_{\tau=-\infty}^{t} \frac{g_1^{\tau-t}(t-\tau)^{\epsilon_1}}{\Gamma_1} \frac{X_{t\tau}}{X_t} \\
\frac{M_{ts}}{M_t} &= \sum_{\tau=-\infty}^{t} \frac{g_2^{\tau-t}(t-\tau)^{\epsilon_2}}{\Gamma_2} \frac{M_{t\tau}}{M_t}
\end{aligned}$$

which can only be solved for numerically. In the limit, when g is large or ϵ is small, only the output of the most recent vintage matters, and the distribution of markets shares across sectors is equal to the distribution across types. If older types are slow to disappear, the distribution of market shares across sectors is degenerate—

all sectors have the same market shares because all past draws $(z_{ts\tau}, f_{ts\tau})$ have the same weight as current draws.[13]

2.6 Estimation

Parameters We estimate the model using the simulated method of moments. There are 5 parameters and 17 moments. The data are described in section 2 above. We fix the elasticity of substitution across sectors $\eta = 3$ and $\sigma = 7.5$ from Broda and Weinstein [2006].[14] We take the growth rates $g_z = 1.18$, $g_L = 1.21$, $g_f = 1.07$ to exactly match aggregate growth rate in real value added, employment and number of establishments over the period of 1900 to 1980 in the data.[15] The five estimated parameters are $(\epsilon_z, \epsilon_f, \Sigma)$. Parameters ϵ_z, ϵ_f determine the evolution of exogenous firm productivity and fixed costs within a vintage. Type-specific productivity and fixed costs $(z_{\tau s}, f_{\tau s})$ are jointly distributed according to a log-bivariate log-normal with mean zero and variance-covariance matrix Σ, itself with variance parameters

[13] These cases are the limits as the variance of the gamma distributions $g_1^{\tau-t}(t-\tau)^{\epsilon_1}$ and $g_2^{\tau-t}(t-\tau)^{\epsilon_2}$ goes from zero to infinity. With continuous time, this variance is $(1+\epsilon)/\log(g)^2$.

[14] At this stage we estimated the σ parameter even though we believe we need more data for the identification.

[15] The value of g_L is the geometric average of the growth in employment per decade in the data. g_f is then imputed by solving for it in equation (2.5.14), where the left hand side is the geometric average of the aggregate growth of establishments per decade. Finally g_z is solved in the same way in equation (2.5.13) where the left hand side is the geometric average of aggregate real value added growth per decade, using the current estimate of σ.

σ_z, σ_f and correlation ρ_{zf}.

Moments Selected moments describe the stylized facts of section 2, on value added, number of establishments, and employment by sector and decade between 1900 and 1980. A period in the model is one decade in the data. We match the first two autocorrelations of value added and establishment growth and the contemporaneous correlation between value added and establishment growth. We match the 10th, 25th, 50th, 75th and 90th quantiles and maximum of the distribution of sectoral growth rate of value added within a decade, normalized by the mean growth rate of the decade.[16] We repeat these moments for the distribution of growth rates of number of establishments.

Identification The parameters to estimate are well identified. The learning parameters, ϵ_z and ϵ_f are associated with autocorrelations of growth rates. As ϵ_z increases, most productivity growth occurs within a vintage. As a result, successful shocks (high-z types) take longer to be overtaken, and the correlation of value added growth rates across decades within sectors decreases. Similarly, ϵ_f governs the autocorrelation in growth rates of establishments across decades within sectors. The variance parameters σ_z and σ_f govern the distribution of growth rates. The variance of productivity σ_z governs the variance of value added growth rates across sectors, and the variance of fixed costs σ_f govern variance of number of establish-

[16]These latter moments are calculated for each decade and then normalized by the decade mean growth. The average of each moment across decades is the reported value.

ments across sectors. The correlation of these shocks ρ_{zf} governs cross-sectional correlation between the growth of value added and number of establishments.

Simulation We simulate the model for $S = 20$ sectors and $T = 50$ periods. We fix two matrices with dimensions $2T \times N$ with random draws from a bivariate log-normal distribution, truncated at the top 5% for $z_{s\tau}$, and the bottom 5% for $f_{s\tau}$. For each parameter guess, we use Σ to transform these distributions to get draws $(z_{s\tau}, f_{s\tau})$. Details of this procedure is given in Appendix J. There are $2T \times N$ draws because we simulate the model for periods $t = T+1, T+2, ..., 2T$ and use the first T draws to construct the history of shocks for the initial periods. For each period $t = T+1, ..., 2T$ and each sector $s = 1, ..., S$, we calculate the market shares from equations (2.5.9) and (2.5.12):

$$\frac{X_{ts\tau}}{X_t} = \frac{g_1^{\tau-t}(t-\tau)^{\epsilon_1}\left(z_{s\tau}^{1-\sigma} f_{s\tau}\right)^{\frac{1-\eta}{\sigma-\eta}}}{\tilde{P}_t^{1-\eta}}$$

$$\frac{M_{ts\tau}}{M_t} = \frac{g_2^{\tau-t}(t-\tau)^{\epsilon_2}\left(z_{s\tau}^{1-\eta} f_{s\tau}\right)^{\frac{1-\sigma}{\sigma-\eta}}}{\tilde{M}_t}$$

$$\text{where} \quad \tilde{P}_t = \left[\sum_{\tau=t-T}^{t} g_1^{\tau-t}(t-\tau)^{\epsilon_1} \sum_{s=1}^{S} \left(z_{s\tau}^{1-\sigma} f_{s\tau}\right)^{\frac{1-\eta}{\sigma-\eta}}\right]^{\frac{1}{1-\eta}}$$

$$\tilde{M}_t = \sum_{\tau=t-T}^{t} g_2^{\tau-t}(t-\tau)^{\epsilon_2} \sum_{s=1}^{S} \left(z_{s\tau}^{1-\eta} f_{s\tau}\right)^{\frac{1-\sigma}{\sigma-\eta}}$$

$\tilde{M}_t$ and $\tilde{P}_t$ vary across periods only because of randomness in the draws $(z_{s\tau}, f_{s\tau})$. They differ slightly from their theoretical counterparts because they are calculated over a finite number of periods. Sectoral shares in value added and number of establishments are

$$\frac{X_{ts}}{X_t} = \sum_{\tau=t-T}^{t} \frac{X_{ts\tau}}{X_t} \qquad \text{for } t = T+1, ..., 2T \text{ and } s = 1, ..., S$$

$$\frac{M_{ts}}{M_t} = \sum_{\tau=t-T}^{t} \frac{M_{ts\tau}}{M_t} \qquad \text{for } t = T+1, ..., 2T \text{ and } s = 1, ..., S$$

Denote these shares with $x_{ts} = \frac{X_{ts}}{X_t}$ and $m_{ts} = \frac{M_{ts}}{M_t}$. Using equations (2.5.4) and (2.5.11), we get sectoral growth rates in real value added and number of establishments:

$$\frac{X_{st+1}}{X_{st}} \frac{P_t}{P_{t+1}} = \frac{x_{st+1}}{x_{st}} \left(\frac{g_L^\sigma}{g_f}\right)^{\frac{1}{\sigma-1}} g_z \left(\frac{\tilde{P}_{t+1}}{\tilde{P}_t}\right)^{\frac{\sigma-\eta}{1-\sigma}}$$

$$\frac{M_{st+1}}{M_{st}} = \frac{m_{st+1}}{m_{st}} \frac{\tilde{M}_{t+1}}{\tilde{M}_t} \left(\frac{\tilde{P}_{t+1}}{\tilde{P}_t}\right)^{\eta-1} \frac{g_L}{g_f}$$

Given our assumptions of frictionless labor markets and constant markups, the distribution of labor across sectors is the same as value added, $l_{ts} = \frac{L_{ts}}{L_t} = \frac{X_{ts}}{X_t}$, and the sectoral growth in employment is $\frac{L_{t+1s}}{L_{ts}} = l_{ts} g_L$. These assumptions are well supported by the linear relationships between value added and employment in figures 2.1(a) and 2.3(a).

2.7 Results

The values in the data and of the simulations for the targeted moments are displayed in table 2.3 and the resulting parameters are in table 2.4. We find that the model does a good job of capturing both the dynamics and distribution of growth rates, given the simple structure and small number of parameters estimated. The dynamics of growth are well captured, in particular the first autocorrelations of value added and establishment growth, which are just under predicted by the model but within the same order of magnitude. Given that the autocorrelations with two lags are non-significant in the data, they are not matched very closely as they have a very small weight in the estimation.

The distribution of value added growth rates has a very similar degree of spread as that of the data, however the lower tail is not well matched. Across larger quantiles the model does better, and captures the top of the tail very closely. Similarly, growth in the number of establishments is well matched, but the model also under predicts the 10th quantile. In table 2.5 we display the quantiles of the distribution of shares of value added and number of establishments from the data and those produced by the model and we see that the model is able to capture the general properties of these two distributions, but fails at capturing both tails of these, as these sectors are generally more homogeneous for these distributions.

To further assess the estimation we display the same plots relating shares and growth rates of value added and number of establishments as in the stylized facts in figures 2.9-2.12. The simulated data maintains the same positive relationship between value added and number of establishments growth, with the slope of the regression line being a bit smaller (.83 vs .61). The sectoral shares of these two variables are also positively correlated, with a smaller slope as well (1.16 vs .72).

Finally, the dynamic relationships of value added shares are closely matched, both in the data and the simulation a sector that has a high market share today has a high market share tomorrow, but this does not significantly predict the share eight decades in the future.

We study the dynamics product cycle by displaying the path of the mass of firms for several quantiles of shocks in figure 2.13, where we use the estimated parameters and equation (2.5.9) while the time since the the vintage was introduced $t - \tau$ increases, assuming that $\tilde{M}_t$ is constant in figure 2.13. The graph can be interpreted as an impulse response as we assume no further shocks occur as age increases.

The product cycle peaks in 19 years and the shakeout period is long lived, even 80 years after the introduction, there are firms producing vintage. This length to the peak number of firms is comparable to that found across products by Gort and Klepper [1982], where on average for their products it takes 24 years. There is much variation in the number of firms that these innovations introduce, which depends on the value of the shocks $(z_{s\tau}^{1-\eta} f_{s\tau})^{\frac{1-\sigma}{\sigma-\eta}}$ that the firm experiences. The 90th quantile shock implies that at the peak it has 4 times more firms than the median shock, and 19 times more firms than the 10th quantile shock. where some have in their peak eight firms, and others have 50.

Value added responds with less variation in response to an innovation for the same values of $z_{s\tau}$ and $f_{s\tau}$ as before, as can be seen in figure 2.14. While the 90th percentile shock is still relatively very large, 9 times larger than the median and 12 times larger than the 10th percentile shock, amongst shocks below the 75th percentile they are much more concentrated as the paths mostly overlap. Note that the sorting is not necessarily preserved, as the shock that produced the 25th percentile number of firms generates more value added than that for the 75th percentile. This occurs as the value added and number of firms shares evaluate the contribution of the $f_{s\tau}$ shock differently: for value added the shock component in equation (2.5.12) is $(z_{s\tau}^{1-\sigma} f_{s\tau})^{\frac{1-\eta}{\sigma-\eta}}$, thus $f_{s\tau}$ is raised to the power $\frac{1-\eta}{\sigma-\eta}$ instead of $\frac{1-\sigma}{\sigma-\eta}$. Given that $\eta < \sigma$, good (small), draws of $f_{s\tau}$ imply relatively lower value added shares than number of establishments shares.

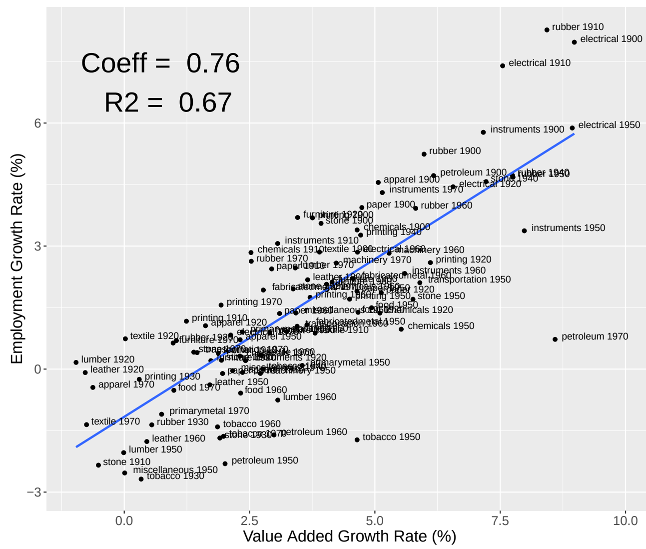
If we were to sort shocks by their contribution to the value added shares, using $(z_{s\tau}^{1-\sigma} f_{s\tau})^{\frac{1-\eta}{\sigma-\eta}}$ as in figure 2.15 , we can see that the 90th percentile is actually much smaller than that induced by the sorting by number of establishments, which is 45% larger. These shocks do vary more amongst themselves than before, yet are still more concentrated than the number of establishments, the 90th percentile shock has 3 times more value added produced than the median and 10 times the 10th percentile shock. Thus, while we observe much variation in the number of establishments that different product cycles feature, they do not imply as large variation in how much output they produce.

2.8 Concluding Remarks

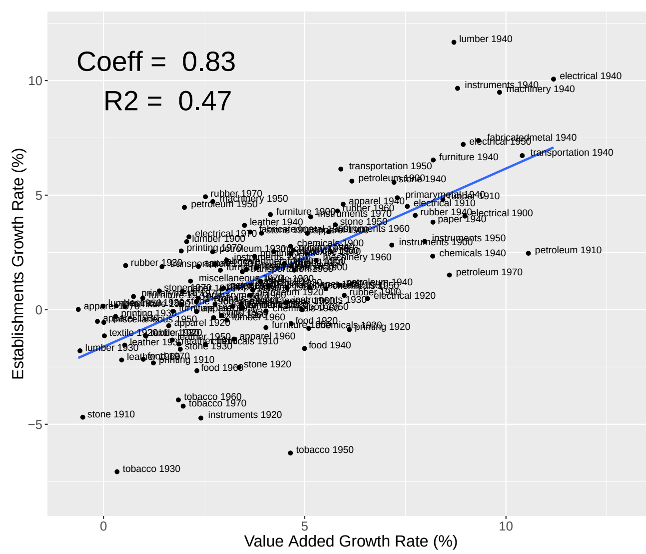
We study the product cycle using a quantitative growth model of exogenous growth. First, we describe historical data on value added, number of employees and establishments for the United States and present several stylized facts that show that growth in value added is very volatile across sectors, and that value added, employment and number of establishments covary very strongly contemporaneously.

Second, we develop and estimate a model that allows us to measure parameters that determine the typical product cycle and also matches the aggregate data. The deep parameters uncover a product cycle that lasts 19 years and is long lived, with firms in operation 80 years after the innovation. These product cycles generate much variation in the number of firms, depending on the characteristics of the particular innovation. They imply much less variation in value added, where these cycles are more similar cross the shock distribution.

2.9 Figures

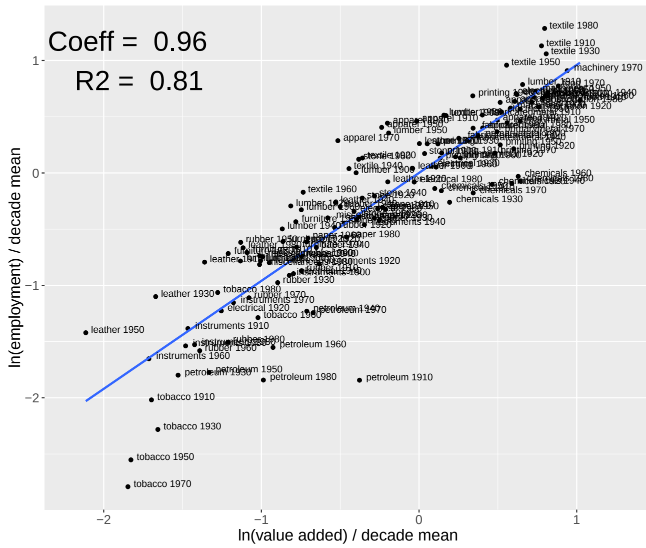


(a) Employment

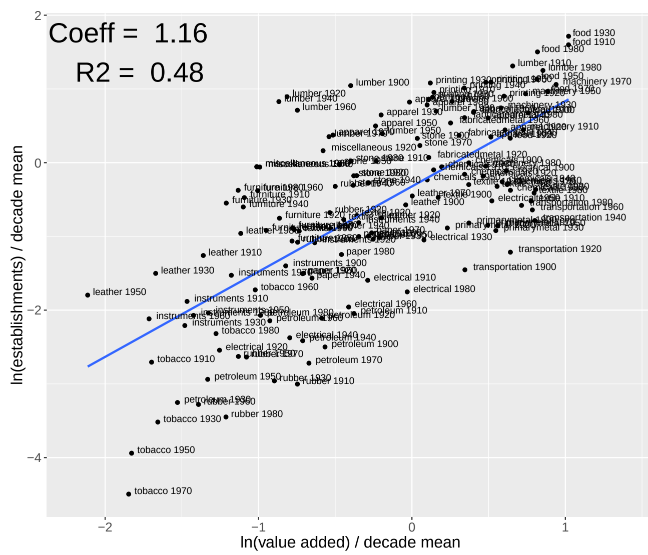


(b) Establishments

Figure 2.1: Annualized growth rate of value added vs annualized growth rates of employment and number of establishments

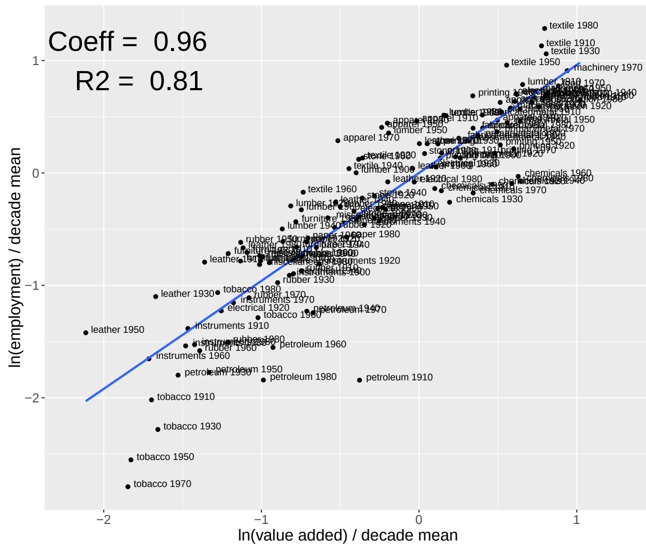


(a) Employment

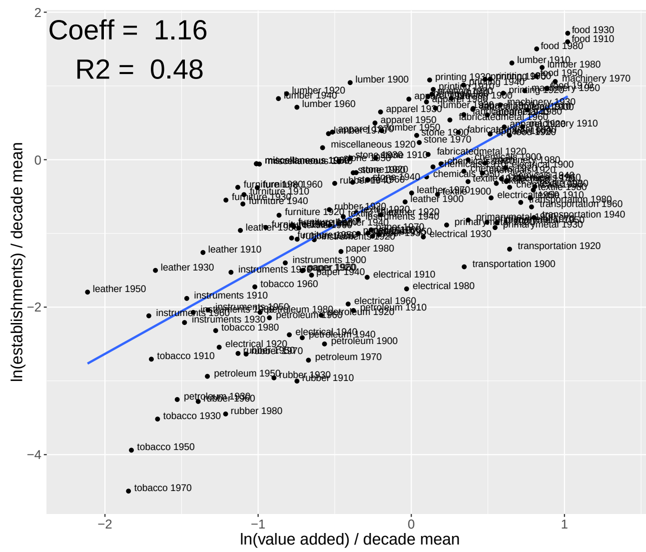


(b) Establishments

Figure 2.2: Normalized sectoral shares of value added vs normalized sectoral shares of employment and number of establishments



(a) Employment



(b) Establishments

Figure 2.3: Normalized sectoral shares of value added vs normalized sectoral shares of employment and number of establishments

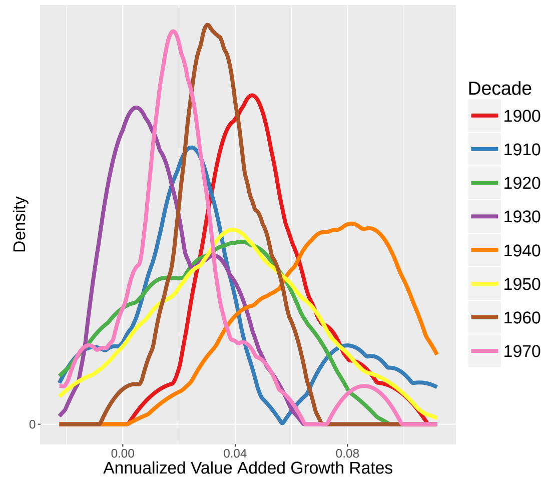


Figure 2.4: Kernel densities of annualized sectoral growth by decade

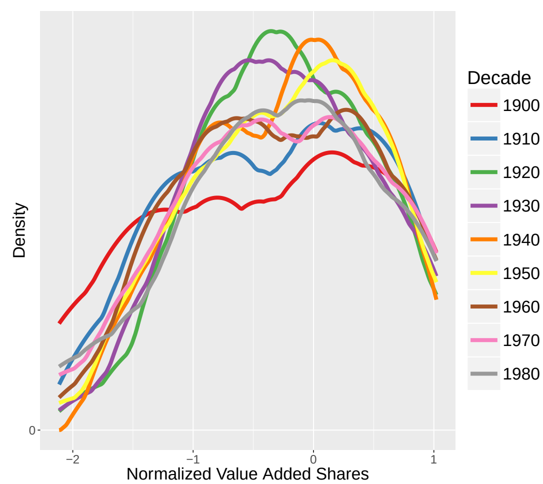


Figure 2.5: Kernel density of sectoral variations in the log of value added by decade.

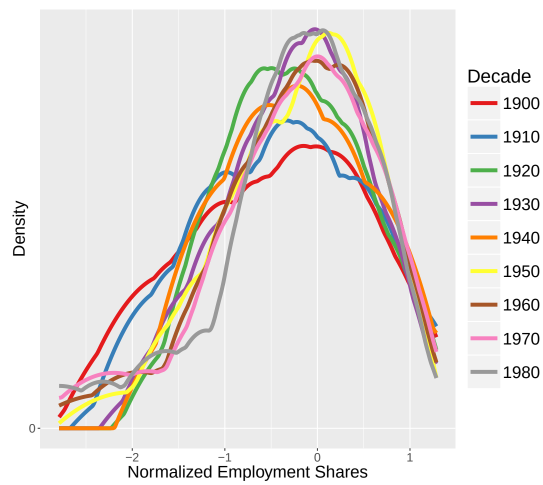


Figure 2.6: Kernel density of sectoral variations in the log of value employment by decade.

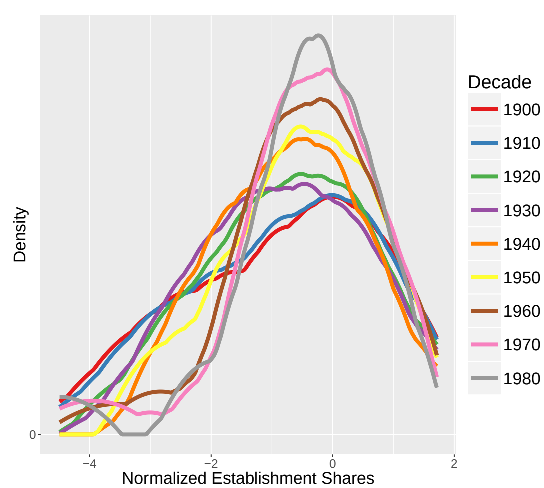
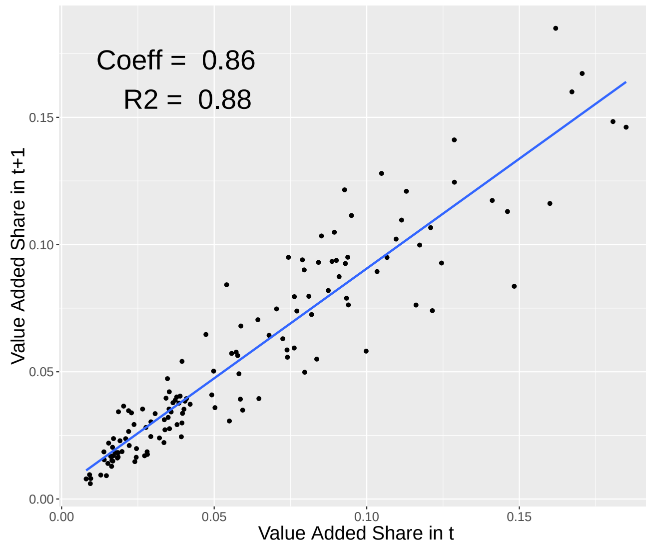
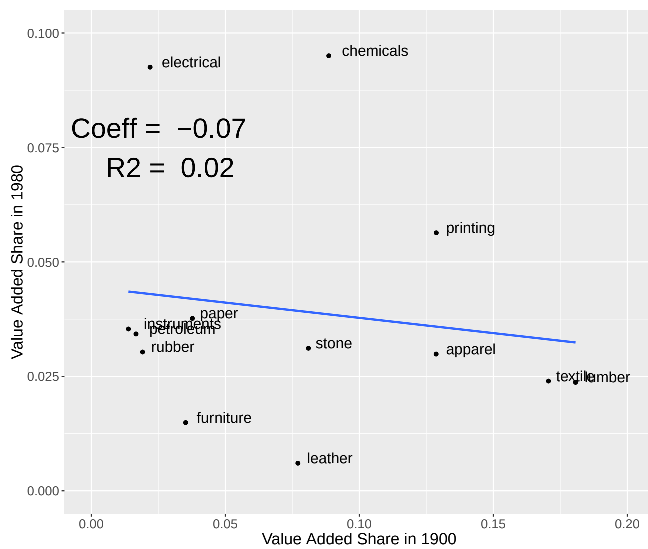


Figure 2.7: Kernel density of sectoral variations in the log of number of establishments by decade.



(a) Value added Shares of decade t and $t+1$



(b) Value added shares of decade t and $t+8$

Figure 2.8: Persistence in value added shares across decades

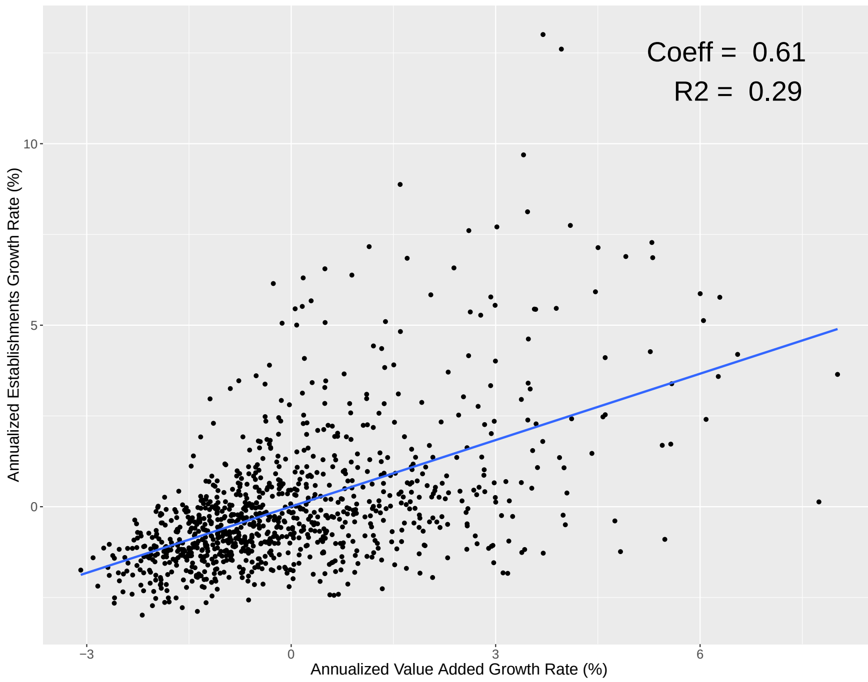


Figure 2.9: Annualized growth rates of value added vs establishments from the simulations

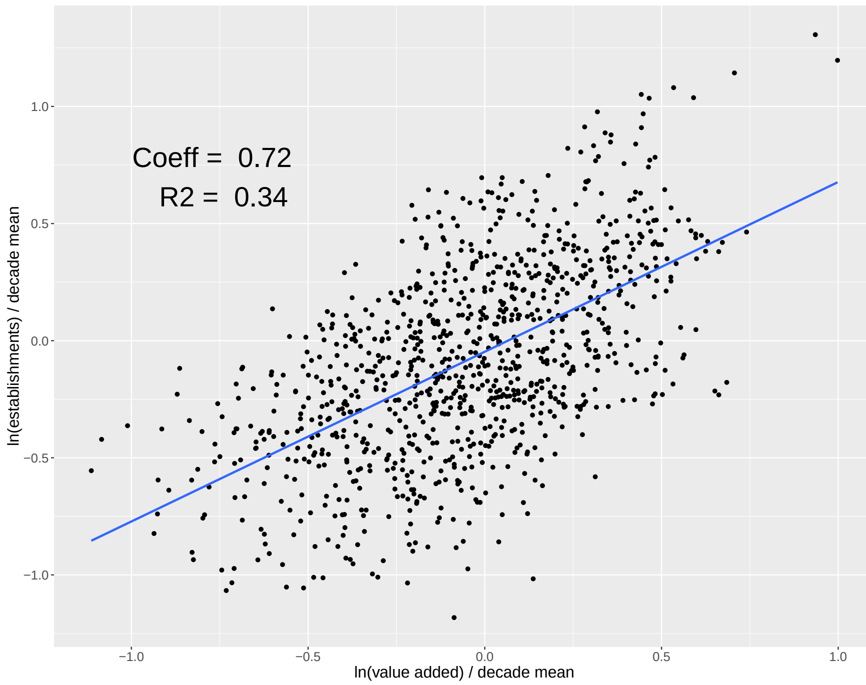


Figure 2.10: Normalized sectoral shares of value added vs normalized sectoral shares of number of establishments from the simulations

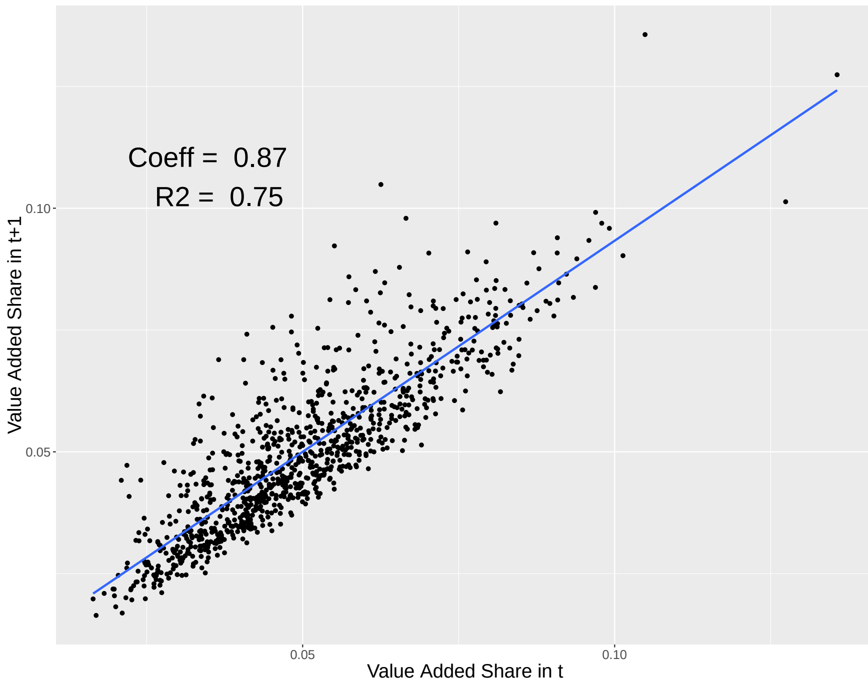


Figure 2.11: Value added shares of decade t and $t + 1$ from the simulations

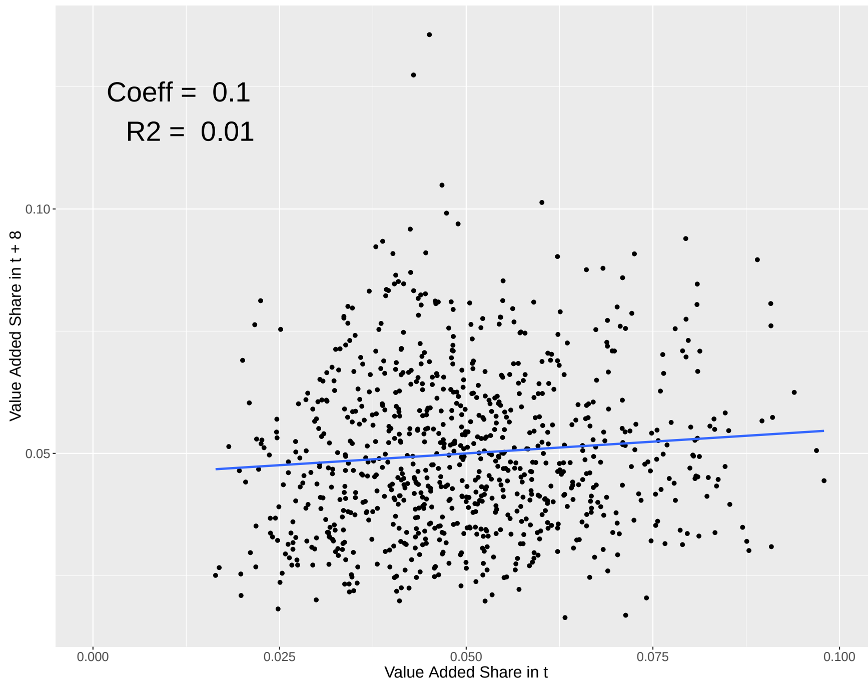


Figure 2.12: Value added shares of decade t and $t + 8$ from the simulations

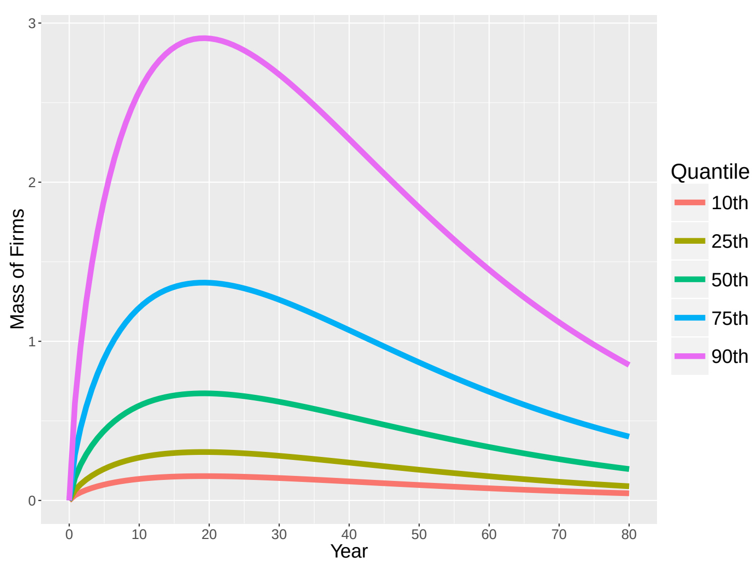


Figure 2.13: Path of number of establishments after an innovation

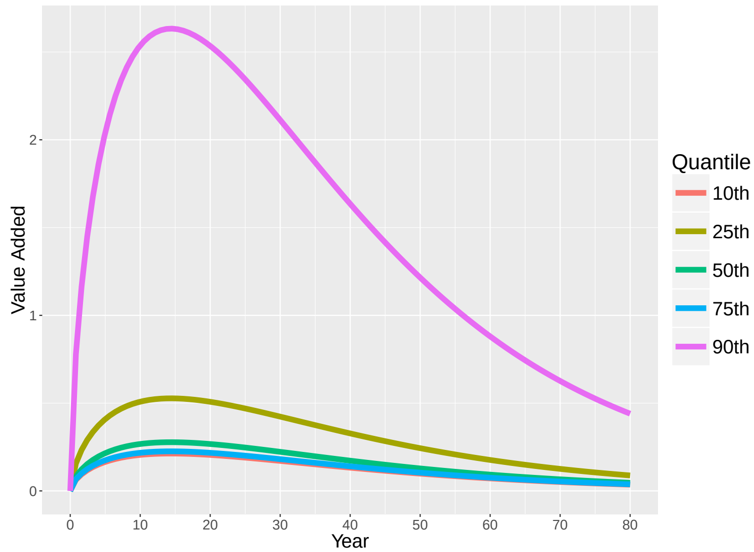


Figure 2.14: Path of number of establishments after an innovation

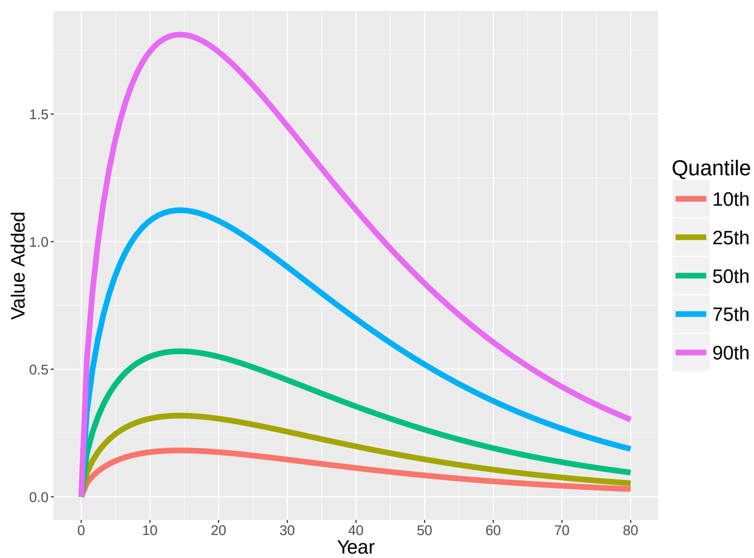


Figure 2.15: Path of number of establishments after an innovation, sorted by value added

2.10 Tables

Table 2.3: Targeted moments

Correlations		
Moment	Data	Model
Autocorrelation VA, $t+1$	0.190	0.108
Autocorrelation VA, $t+2$	0.044	-0.069
Autocorrelation M, $t+1$	0.230	0.129
Autocorrelation M, $t+2$	0.048	-0.024
Cross Correlation VA, M	0.610	0.510
Distribution of value added growth rates		
Moment	Data	Model
VA Growth, 10th Quantile	0.750	0.822
VA Growth, 25th Quantile	0.850	0.864
VA Growth, 50th Quantile	0.960	0.940
VA Growth, 75th Quantile	1.090	1.055
VA Growth, 90th Quantile	1.310	1.212
VA Growth, Maximum	1.510	1.522
Distribution of number of establishments growth rates		
Moment	Data	Model
M Growth, 10th Quantile	0.700	0.831
M Growth, 25th Quantile	0.830	0.865
M Growth, 50th Quantile	0.970	0.927
M Growth, 75th Quantile	1.110	1.019
M Growth, 90th Quantile	1.300	1.199
M Growth, Maximum	1.760	1.683

Results from the estimation of the model, displayed in the first panel are the targeted moments of the autocorrelations of sectoral value added growth with itself, of number of establishments growth with itself, and the cross correlation of contemporary value added growth and number of establishments growth rates. The second and third panels contains the quantiles from the sectoral distribution of value added and number of establishments growth rates, normalized by the mean growth rate of the decade, and afterwards averaged across sectors.

Table 2.4: Parameter Estimates

ϵ_z	ϵ_f	σ_z	σ_f	ρ_{zf}
0.21	-.07	.17	1.23	.64

Notes: The table reports the parameter values estimated with Method of Simulated Moments.

Table 2.5: Nontargeted moments

Distribution of normalized shares of value added		
Moment	Data	Model
VA Share, 10th Quantile	0.270	0.822
VA Share, 25th Quantile	0.410	0.864
VA Share, 50th Quantile	0.780	0.940
VA Share, 75th Quantile	1.490	1.055
VA Share, 90th Quantile	1.960	1.212
VA Share, Maximum	2.420	1.522
Distribution of normalized shares of number of establishments		
Moment	Data	Model
M Share, 10th Quantile	0.080	0.509
M Share, 25th Quantile	0.240	0.682
M Share, 50th Quantile	0.580	0.871
M Share, 75th Quantile	1.490	1.187
M Share, 90th Quantile	2.510	1.494
M Share, Maximum	3.720	2.121

Results from the estimation of the model, displayed are quantiles of the distribution of shares of value added and the number of establishments.

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Appendices

A Details of the Computational Method

The model is solved globally for the policy and value functions using Value Function Iteration. I discretize the state space (k, b, z) by generating a grid for capital that ranges from 5×10^{-4} to 2 times the non-stochastic steady state value of capital. I generate log-spaced grids of size 60 for these two states. The stochastic process is discretized using Tauchen and Hussey's method, with 5 grid points. As mentioned in Section 1.4.2, for every solution with parameters $p_i = (\rho^i, \sigma_z^i, \theta^i, \omega^i, k_0^i, \kappa_0^i, \kappa_1^i, \alpha^i)$, I will perform a bisection of the mean of the fixed cost parameter $m_{c_f}^i$ and the equity issuance cost parameter ϕ^i to get the exit rate and the average ratio of equity issuance to fixed assets to match the data for every firm type i.

The estimator is from Lee and Ingram [1991]. The goal is to estimate the parameter vector p_i using data vector $\{d_i\}$, where this contains for every firm i all observations T_i of the relevant data series. These vectors are assumed to be assumed to be $i.i.d$ across i but there may be dependence within i. Let h map from the data to the moments, so this function takes the appropriate means and standard deviations, correlations and autocorrelation and frequency of positive equity issuance from the series $h(d_i)$. I estimate each firm's parameters independently. For the simulated data, given that I have the stationary distribution of the policy functions of the model it is not necessary for me to simulate the firms, as I can obtain any moment produced by the model from it. Denote the moments generated by parameter vector p_i by $h^m(p_i)$, using the stationary distributions μ^i, and define the sample moment vector as

$$g_i(x,p) = \frac{1}{N}\sum_{j=1}^{N}\left[h(d_j) - h^m\left(p_i\right)\right]$$

Then, for some random weight matrix $\hat{W}$ which converges in probability to a deterministic positive definite matrix W the Method of Simulated Moments estimator is given by

$$\hat{p}_i = \arg\min_p g_i(x,p)'\hat{W}g_i(x,p)$$

The weight matrix $\hat{W}$ is chosen to be for each firm type i the inverse of the covariance matrix of the moments. I first use a global optimization algorithm (DIRECT-L from the NLopt library) and a local optimizer afterwards for more precise estimation within this minima (Sbplx). The programming language used are Julia and C for the modeling, and R for the data related sections.

B Proofs

Proof of Proposition 1. Similarly to Li et al. [2016], I use Theorem 9.6 in Stokey et al. [1989] to establish the existence of the industry equilibrium defined in Section 1.3.3. Theorem 9.8 establishes uniqueness of the policy functions. The proposition can be restated by defining an operator T in the space of bounded function as

$$T(V)(k,b,z) = x(k,b,z)V^E(k,b,z) + (1 - x(k,b,z))[V^O(k,b,z) - c_f^e(k,b,z)]$$

where

$$V^E(k,b,z) = \pi(k,z) + (1-\delta)k - (1+r)b,$$

$$V^O(k,b,z) = \max_{k',b'} \Phi(e(k,k',b,b',z)) + \beta \int_{z'} V(k',b',z')Q(dz',z),$$

and

$$e(k,k',b,b',z) = \pi(k,z) + (1-\delta)k - (1+r)b - k' - A^i(k,k') + b'$$

subject to

$$(1 + r)b' \leq \theta(1 - \delta)k'$$

Lemma 1. *Let $C(S)$ be the space of all bounded continuous functions, where $S = \mathcal{K} \times \mathcal{B} \times \mathcal{Z}$. The operator T defined above has an unique fixed point $V^* \in C(S)$ for all $v_0 \in C(S)$ initial guesses. (Theorem 9.6 of Stokey et al. [1989])*

Proof. This is established in Stokey et al. [1989], we only have to show that Assumptions 9.4-9.7 hold:

- Assumption 9.4: $\mathcal{K} \times \mathcal{B}$ are convex Borel sets in $\mathbb{R}^2$ with Borel subsets $\Sigma(\mathcal{K}) \times \Sigma(\mathcal{B})$. These assumptions are easily satisfied if for example $\mathcal{K} = [0, \overline{k}]$ and $\mathcal{B} = [\underline{b}, \overline{b}]$. The firm would never choose a capital stock larger than $\overline{k} = \overline{z}\overline{k} + \overline{k}(1 - \delta)$ because otherwise profits are negative. The upper bound for the debt can then be derived from the collateral constraint $\overline{b} = \theta(1 - \delta)\overline{k}/(1 + r)$. The lower bound must also be well defined because given that all firms are born with zero debt and the firms have no incentive to save after they reach their steady state value of capital given that they discount at a higher rate than the debt returns: if it sacrifices one dollar in dividends today, it returns $1 + r$ tomorrow, but they discount this at rate β. Given the assumed value of r as detailed in Section 1.4.2, $\beta \times (1 + r) < 1$, so the firm prefers to set $b = 0$ to $b < 0$, so that $\underline{b} = 0$.

- Assumption 9.5: $\mathcal{Z}$ is a compact (Borel) set in $\mathbb{R}$ with its Borel subsets $\Sigma(\mathcal{Z})$, and the transition function Q on $(\mathcal{Z}, \Sigma(\mathcal{Z}))$ has the Feller property. This is trivially satisfied by the Log-Normal AR(1) process as it is non-negative, and its transition maps the any bounded and continuous function back into a bounded and continuous function.

- Assumption 9.6: The constraint correspondence $\Gamma(k, b, z) : \mathcal{K}\times\mathcal{B}\times\mathcal{Z} \rightarrow \mathcal{K}\times\mathcal{B}$ given by:

$$\Gamma(k, b, z) = \{(k', b')|\theta k'(1 - \delta) \geq b'\}$$

is non-empty, compact-valued and continuous.

1. Non-empty: $k' = 0$ and $b' = 0$ belong to $\Gamma(k, b, z)$, thus non-empty.

2. Compact-valued: Pick any sequence $\{k'_n, b'_n\} \in \Gamma(k, b, z)$ for any z such that it converges, $\{k'_n, b'_n\} \rightarrow (k'_c, b'_c)$. Given that the constraint is linear, it must be the case that $\theta k'_c(1 - \delta) \geq b'_c$, independent of the z. $\Gamma(k, b, z)$ is closed and compact valued (contains all of its limit points).

3. To show that $\Gamma(k, b, z)$ is continuous I will show that it is both upper hemi-continuous and lower hemi-continuous. Since I have shown it is nonempty and bounded (given the definitions in Assumption 9.4), Theorem 3.4 gives that it is upper hemi-continuous. Pick any sequence $\{k'_n, b'_n, z'_n\}$, such that it converges, $\{k'_n, b'_n, z'_n\} \rightarrow (k'_c, b'_c, z'_c)$. To prove lower hemi-continuity I must show that for every $(f'_c, g'_c) \in \Gamma(k'_c, b'_c, z'_c)$ there exists $N \geq 1$ and a sequence $(f'_n, g'_n) \in \Gamma(k'_n, b'_n, z'_n)$ that is convergent $(f'_n, g'_n) \rightarrow (f'_c, g'_c)$ $\forall n \geq N$. Given that $(k'_c, b'_c) \in \Gamma(k'_c, b'_c, z'_c)$. Con-

vergence of the initial sequence means that for all $\{\delta_i > 0 | i \in \{k, b, z\}\}$ there exists some $\{N_i \in \mathbb{Z} | i \in \{k, b, z\}\}$ where for all $n > N_i$, $|k'_n - k'_c| < \delta_k$, $|b'_n - b'_c| < \delta_b$ and $|z'_n - z'_c| < \delta_z$. Let $N = \max\{N_k, N_b, N_z\}$ and $(f'_n, g'_n) = (a_k k'_n, a_b b'_n)$, where $a_k = f'_c/k'_c$ and $a_b = g'_c/b'_c$ then it is the case that $(f'_n, g'_n) \to (a_k k'_c, a_b b'_c) = (f'_c, g'_c)$ for all $n \geq N$.

- Assumption 9.7: $\beta \in (0, 1)$ and the function $F(k, b, z, k', b')$ is bounded and continuous,

$$F(k, b, z, k', b') = \pi(k, z) + (1 - \delta)k - (1 + r)b + (1 - x(k, b, z)) \times [k' - A(k, k') + b' - c^e_f(k, b, z) + \mathbf{1}_{[e<0]}\phi e.]$$

given that $\mathcal{K} \times \mathcal{B}$ is bounded, and all the component functions are continuous, this is satisfied.

□

Lemma 2. *Suppose $S = \mathcal{K} \times \mathcal{B} \times \mathcal{Z}$, Γ, Q, F and β satisfy Assumptions 9.4-9.7. If they additionally satisfy 9.10 and 9.11 then the value function is strictly concave and the optimal policies are unique and continuous. (Theorem 9.8 of Stokey et al. [1989]).*

Proof. • Assumption 9.10: Given z, the F function satisfies, for all $t \in (0, 1)$

$$F\left[tk_0 + (1-t)k_1, tb_0 + (1-t)b_1, z, tk_0' + (1-t)k_1', tb_0' + (1-t)b_1'\right]$$

$$\geq tF\left[k_0, b_0, z_0, k_0', b_0'\right] + (1-t)F\left[k_1, b_1, z_1, k_1', b_1'\right]$$

And strictly if any of the variables are different. This holds given that the production function has decreasing returns to scale, even when we take into account the pricing function. The only concerns are the equity issuance cost, but given that it is linear, it is weakly concave; and the exit decision which is a value along a log-normal CDF, and changes monotonically and continuously in the same direction of the elements contained in the brackets it multiplies

- Assumption 9.11: For all $z \in \mathcal{Z}$ and all $(k_n', b_n') \in \mathcal{K} \times \mathcal{B}$ for $n = 1, 2$, if $y_n \in \Gamma(k_n', b_n', z)$, then $ty_n + (1-t)y_n \in \Gamma(tk_1' + (1-t)k_2', tb_1' + (1-t)b_2', z)$ for all $t \in [0, 1]$. Given that the constraint is linear this holds trivially.

□

C Industry Labor Share

To measure the labor share of each industry for the $TFPR$ analysis I will use a similar definition as Hsieh and Song [2015], where it is defined as

$$\text{labor share} = \frac{\text{labor income}}{\text{labor income} + \text{total profit} + \text{depreciation} + \text{value added tax}}.$$

Ideally, this should be equal to aggregate labor share of 50% [?] if you had a full sample of firms, but due to accounting and statistical discrepancies from reported data this is not the case in the full sample of firms that they use, and is not the case in my restricted sample either. The discrepancy arises from firms reporting only wages as part of payroll and a discrepancy between reported income and reported value added.

I obtain industrial sector Labor Income data from University of Michigan's China Data Center; Depreciation, Value Added Tax and Total Profits from the relevant for each year's China Statistical Yearbook. I obtain this data for all the manufacturing sectors for 2005-2008. As expected, the average labor share for my sample is 32%, less than the aggregate share of 50%, so I will inflate labor income by a constant factor across all years and sectors, increasing by .0001 until the average labor share is exactly 50%.

D Data Comments

I use data on all manufacturing firms from 2000-2013 in the CSMAR database. Manufacturing firms are those that begin with the letter "C" in the 2012 CRSC Industry Code. The precise variable items from the CSMAR database are:

- Fixed Assets: A001212000
- Short Term Debt: A002101000
- Long Term Debt: A002201000
- Bonds: A002203000
- Total Assets: A001000000
- Profits (Operating Profits): B001300000
- Investment: C002006000
- Proceeds from issuing shares: C003001000
- Payroll: C001020000
- Depreciation: D000103000

- Monthly Market Capitalization: MSMVTTL
- The proceeds from issuing shares variable has missing data, but is coded erroneously as ocurring in the beginning of the following year as confirmed by comparing with a secondary source for the data, the ChinaScope database. This is corrected for the periods for which there is data in that entry. Some firms erroneously code their payroll/labor income data and fixed assets as negative, and this is also corrected.

E Different Lags of TFPR Innovations

In this appendix I present the results of adding one and two more lags of TFPR innovations to the regression equation 1.2.5, which shows that the result is robust to longer delays between the period of when the innovation is first observed, and when the capital is successfully adjusted.

Table E.1: Dispersion of $MRPK$ and Productivity Shocks with One Lag of Innovations

	Dependent variable:			
	$\log(MRPK)$			
	(1)	(2)	(3)	(4)
$\log(TFPR)$ Innovation	0.641***	0.641***	0.845***	0.788***
	(0.032)	(0.032)	(0.015)	(0.023)
$\log(TFPR)$ Innovation in $t-1$	0.380***	0.377***	0.253***	0.309***
	(0.031)	(0.031)	(0.010)	(0.014)
Capital Stock	−0.496***	−0.507***	−0.680***	−0.708***
	(0.013)	(0.013)	(0.010)	(0.028)
Labor Input	0.583***	0.604***	0.698***	0.706***
	(0.014)	(0.014)	(0.013)	(0.043)
Lagged $\log(TFPR)$	0.197***	0.200***	0.755***	0.553***
	(0.019)	(0.019)	(0.024)	(0.044)
Constant	−3.361***	−3.409***	−1.328***	−1.068***
	(0.167)	(0.168)	(0.084)	(0.407)
Year F.E.	No	Yes	Yes	Yes
Industry F.E.	No	No	Yes	No
Individual F.E.	No	No	No	Yes
Observations	9,635	9,635	9,635	9,635
R^2	0.295	0.302	0.874	0.907

Note: *p<0.1; **p<0.05; ***p<0.01

I report the results of the regression of equation (1.2.5) for different sets of controls, where the $\log(TFPR)$ Innovations are given as the estimated residual of the model of equation (1.2.4), the Capital Stock is given by fixed assets and $TFPR$ is defined as in equation (1.2.3). In parentheses are heretoskedasticity robus standard errors.

Table E.2: Dispersion of $MRPK$ and Productivity Shocks with Two Lags of Innovations

	Dependent variable:			
	log($MRPK$)			
	(1)	(2)	(3)	(4)
log($TFPR$) Innovation	0.683***	0.682***	0.861***	0.798***
	(0.033)	(0.033)	(0.016)	(0.024)
log($TFPR$) Innovation in $t-1$	0.529***	0.525***	0.283***	0.404***
	(0.038)	(0.039)	(0.015)	(0.024)
log($TFPR$) Innovation in $t-2$	0.274***	0.270***	0.061***	0.160***
	(0.030)	(0.030)	(0.015)	(0.022)
Capital Stock	−0.474***	−0.485***	−0.676***	−0.710***
	(0.014)	(0.014)	(0.012)	(0.035)
Labor Input	0.584***	0.605***	0.691***	0.712***
	(0.015)	(0.016)	(0.014)	(0.051)
Lagged log($TFPR$)	0.161***	0.165***	0.761***	0.490***
	(0.023)	(0.023)	(0.030)	(0.055)
Constant	−3.846***	−3.909***	−1.294***	−1.235***
	(0.181)	(0.182)	(0.089)	(0.456)
Year F.E.	No	Yes	Yes	Yes
Industry F.E.	No	No	Yes	No
Individual F.E.	No	No	No	Yes
Observations	8,171	8,171	8,171	8,171
R^2	0.310	0.317	0.872	0.905

Note: *p<0.1; **p<0.05; ***p<0.01

I report the results of the regression of equation (1.2.5) for different sets of controls, where the log($TFPR$) Innovations are given as the estimated residual of the model of equation (1.2.4), the Capital Stock is given by fixed assets and $TFPR$ is defined as in equation (1.2.3). In parentheses are heretoskedasticity robus standard errors.

F Industry TFPR

Industry level $TFPR$ appears is used in order to compare which firms are too small relative to their productivity and which ones are too large in the data. Define industry average marginal revenue products as:

$$\overline{MRPK}_s = \left[\sum_{i=1}^{M_s} \frac{1}{MRPK_{si}} \frac{p_{si}y_{si}}{P_s Y_s} \right]^{-1} = \left[\sum_{i=1}^{M_s} \frac{1}{(r+\delta)(1+\tau_{si}^k)} \frac{p_{si}y_{si}}{P_s Y_s} \right]^{-1}$$
$$\overline{MRPL}_s = \left[\sum_{i=1}^{M_s} \frac{1}{MRPL_{si}} \frac{p_{si}y_{si}}{P_s Y_s} \right]^{-1} = \left[\sum_{i=1}^{M_s} \frac{1}{w(1+\tau_{si}^l)} \frac{p_{si}y_{si}}{P_s Y_s} \right]^{-1}$$

where Y_s is as defined in the main text and $P_s = \left[\sum_{i=1}^{M_s} p_{si}^{1-\nu} \right]^{\frac{1}{1-\nu}}$ is the industry level price index. In the static problem, labor demand is given by:

$$l_{si} = (1-\alpha_s) \left(\frac{\nu - 1}{\nu} \right) \frac{p_{si}y_{si}}{w(1+\tau_{si}^l)}$$

Industry s total labor demand is then given by

$$
\begin{aligned}
L_s = \sum_{i=1}^{M_s} l_{si} &= (1-\alpha_s)\left(\frac{\nu-1}{\nu}\right)\sum_{i=1}^{M_s}\frac{p_{si}y_{si}}{w(1+\tau^l_{si})} \\
&= (1-\alpha_s)\left(\frac{\nu-1}{\nu}\right)\sum_{i=1}^{M_s}\frac{1}{w(1+\tau^l_{si})}\frac{p_{si}y_{si}}{P_sY_s}P_sY_s \\
&= (1-\alpha_s)\left(\frac{\nu-1}{\nu}\right)\frac{P_sY_s}{\overline{MRPL}_s}
\end{aligned}
$$

Doing the same for capital, I can rewrite the industry average marginal revenue products as:

$$
\begin{aligned}
\overline{MRPK}_s &= \alpha_s\left(\frac{\nu-1}{\nu}\right)\frac{P_sY_s}{K_s} \\
\overline{MRPL}_s &= (1-\alpha_s)\left(\frac{\nu-1}{\nu}\right)\frac{P_sY_s}{L_s}
\end{aligned}
$$

If we were to rewrite Y_s as a function of industry level capital and labor $Y_s = TFP_sK_s^{\alpha_s}L_s^{1-\alpha_s}$,

$$
\begin{aligned}
TFP_s &= \frac{P_sY_s}{K_s^{\alpha_s}L_s^{1-\alpha_s}}\frac{1}{P_s} \\
&= \left(\frac{P_sY_s}{K_s}\right)^{\alpha_s}\left(\frac{P_sY_s}{L_s}\right)^{1-\alpha_s} \\
&= \left(\frac{\nu}{\nu-1}\right)\left(\frac{\overline{MRPK}_s}{\alpha_s}\right)^{\alpha_s}\left(\frac{\overline{MRPL}_s}{1-\alpha_s}\right)^{1-\alpha_s} \\
&= \overline{TFPR}_s\frac{1}{P_s}
\end{aligned}
$$

To measure this variable I will follow Hsieh and Klenow [2009]: the elasticity of substitution used is $\nu = 3$, and to measure the wedges they use output distortions and capital relative to labor distortions, a setup which has a one to one mapping to this, but allows a more straightforward interpretation to infer them from the data. The optimization problem is:

$$\pi_{si} = \max_{p_{si},k_{si},l_{si}} (1-\tau_{si}^{*y})p_{si}(y_{si})y_{si} - wl_{si} - (1+\tau_{si}^{*k})(r+\delta)k_{si}$$

Distortions are measured as:

$$1+\tau_{si}^{*k} = \frac{\alpha_s}{1-\alpha_s}\frac{wl_{si}}{(r+\delta)k_{si}}$$

$$1-\tau_{si}^{*y} = \frac{\nu}{\nu-1}\frac{wl_{si}}{(1-\alpha_s)p_{si}y_{si}}$$

I infer large capital distortions if the firm has a ratio of labor to capital that is too high, compared to what you would expect given the industry's labor shares. The model infers high output distortions if the share of labor is low when compared from what one would expect given the industry elasticity of output with respect to labor. To remain as close as possible to Hsieh and Klenow [2009], I trim by 1% the tails of the $TFPR$ and $TFPQ = z$ deviations from industry means measures and the capital and output distortions and then recompute the industry average $TFPR$.

The implied $MRPL$ and $MRPK$ with these wedges are

$$MRPL^*_{si} \equiv (1-\alpha_s)\left(\frac{\nu-1}{\nu}\right)\frac{p_{si}y_{si}}{l_{si}} = \frac{w}{1-\tau^{*y}_{si}}$$

$$MRPK^*_{si} \equiv \alpha_s\left(\frac{\nu-1}{\nu}\right)\frac{p_{si}y_{si}}{k_{si}} = \frac{1+\tau^{*k}_{si}}{1-\tau^{*y}_{si}}(r+\delta)$$

These wedges can be made to coincide with the definitions of $MRPL$ and $MRPK$ in the main text (1.2.1)-(1.2.2) with the redefinitions:

$$1+\tau^l_{si} = \frac{1}{1-\tau^{*y}_{si}} \qquad 1+\tau^k_{si} = \frac{1+\tau^{*k}_{si}}{1-\tau^{*y}_{si}}$$

G Aggregate TFP Derivation

In order to compare aggregate TFP for different model calibrations, I will need to do an analogue of the Hsieh and Klenow [2009] derivation of what is the efficient level of TFP for the case of no labor. We can express aggregate TFP in terms of aggregate $\overline{TFPR} \equiv PY/K$

$$TFP = \frac{Y}{K} = \frac{\overline{TFPR}}{P}$$

P being the usual price index that results from monopolistic competition within the model, and defining $s \in S = \mathcal{K} \times \mathcal{B} \times \mathcal{Z}$

$$P = \left(\sum_{i=a,b,c} \int_S p(y(s,i))^{1-\nu} \mu^i(s) \right)^{\frac{1}{1-\nu}}$$

Using the definition of firm $TFPR \equiv pz$, rewrite this same price index as

$$P = \left(\sum_{i=a,b,c} \int_S \left[\frac{TFPR(s,i)}{z} \right]^{1-\nu} \mu^i(s) \right)^{\frac{1}{1-\nu}}$$

Thus, we can rewrite aggregate TFP as,

$$TFP = \frac{\overline{TFPR}}{P} = \left[\sum_{i=a,b,c} \int_S \left(z \frac{\overline{TFPR}}{TFPR(s,i)} \right)^{\nu-1} \mu^i(s) \right]^{\frac{1}{\nu-1}}.$$

Consumer optimization tells us that firms should equalize $TFPR$, we know that the efficient level of TFP is given when all firms are able to set $TFPR = \overline{TFPR}$, so

$$TFP^e = \left[\sum_{i=a,b,c} \int_S z^{\nu-1} \mu^i(s) \right]^{\frac{1}{\nu-1}}.$$

And define the ratio of actual to efficient TFP as

$$\frac{TFP}{TFP^e} = \left[\sum_{i=a,b,c} \int_S \left(\frac{z}{TFP^e} \frac{\overline{TFPR}}{TFPR(s,i)} \right)^{\nu-1} \mu^i(s) \right]^{\frac{1}{\nu-1}}.$$

Thus, for each parametrization we can compute how close this stationary distribution if close to the efficient one using this formula, and the definitions of aggregate output Y and capital K of equations (1.3.12)-(1.3.13).

H Derivation of the Aggregate Price Index

This appendix derives the aggregate price index in section 2.5:

$$
\begin{aligned}
P_t &= \left[\sum_{s\in S}\sum_{\tau=-\infty}^{t} P_{s\tau t}^{1-\eta}\right]^{1/(1-\eta)} \\
&= P_t^{\frac{1-\eta}{\sigma-\eta}} \left(\frac{\sigma}{L_t}\right)^{\frac{1}{\sigma-\eta}} (\mu w_t)^{\frac{\sigma-1}{\sigma-\eta}} \left[\sum_{s\in S}\sum_{\tau=-\infty}^{t} z_{s\tau t}^{\frac{(1-\sigma)(1-\eta)}{\sigma-\eta}} f_{s\tau t}^{\frac{1-\eta}{\sigma-\eta}}\right]^{1/(1-\eta)} \\
&= P_t^{\frac{1-\eta}{\sigma-\eta}} \left(\frac{\sigma}{L_t}\right)^{\frac{1}{\sigma-\eta}} (\mu w_t)^{\frac{\sigma-1}{\sigma-\eta}} \left\{\sum_{s\in S}\sum_{\tau=-\infty}^{t} \left[\bar{z}_\tau z_{s\tau}(t-\tau)^{\epsilon_z}\right]^{\frac{(1-\sigma)(1-\eta)}{\sigma-\eta}} \left[\bar{f}_\tau f_{s\tau}(t-\tau)^{\epsilon_f}\right]^{\frac{1-\eta}{\sigma-\eta}}\right\}^{1/(1-\eta)} \\
&= P_t^{\frac{1-\eta}{\sigma-\eta}} \left(\frac{\sigma}{L_t}\right)^{\frac{1}{\sigma-\eta}} (\mu w_t)^{\frac{\sigma-1}{\sigma-\eta}} \bar{z}_t^{\frac{1-\sigma}{\sigma-\eta}} \bar{f}_t^{\frac{1}{\sigma-\eta}} \\
&\quad \left\{\sum_{s\in S}\sum_{r=0}^{\infty} \left[g_z^{-\frac{(1-\sigma)(1-\eta)}{\sigma-\eta}} g_f^{-\frac{1-\eta}{\sigma-\eta}}\right]^r r^{\epsilon_z \frac{(1-\sigma)(1-\eta)}{\sigma-\eta} + \epsilon_f \frac{1-\eta}{\sigma-\eta}} \left[z_{s\tau}^{\frac{(1-\sigma)(1-\eta)}{\sigma-\eta}} f_{s\tau}^{\frac{1-\eta}{\sigma-\eta}}\right]\right\}^{1/(1-\eta)} \\
&= \left(\frac{\sigma}{L_t}\right)^{\frac{1}{\sigma-1}} \left(\frac{\mu w_t}{\bar{z}_t}\right) \bar{f}_t^{\frac{1}{\sigma-1}} \tilde{P}_t^{\frac{\sigma-\eta}{\sigma-1}} \\
&\approx (\Phi\Gamma_1)^{\frac{\sigma-\eta}{(\sigma-1)(1-\eta)}} \left(\frac{L_t}{\sigma \bar{f}_t}\right)^{\frac{1}{1-\sigma}} \frac{\mu w_t}{\bar{z}_t} \qquad \text{(H.0.1)}
\end{aligned}
$$

where Φ and Γ_1 are constants:

$$\Phi = E_G\left[z^{\frac{(1-\sigma)(1-\eta)}{\sigma-\eta}} f^{\frac{1-\eta}{\sigma-\eta}}\right]$$

$$\Gamma_1 = \sum_{r=0}^{\infty} g_1^{-r} r^{\epsilon_1} \approx \log(g_1)^{\epsilon_1+1}\Gamma(\epsilon_1+1)$$

$$\text{where} \quad g_1 = \left[g_z^{(1-\sigma)} g_f\right]^{\frac{1-\eta}{\sigma-\eta}} > 1$$

$$\epsilon_1 = \left(\frac{1-\eta}{\sigma-\eta}\right)\left[(1-\sigma)\epsilon_z + \epsilon_f\right]$$

and Γ is the gamma function. We assume $\Phi < \infty$. If the number of sectors is sufficiently large, the approximation in line (H.0.1) is precise.

Vintage-Sector Mass is given by:

$$\begin{aligned}
M_{s\tau t} &= \frac{X_{s\tau t}}{\sigma w_t f_{s\tau t}} \\
&= \left(\frac{p_{s\tau t}}{P_t}\right)^{\frac{(1-\eta)(\sigma-1)}{(\sigma-\eta)}} \left(\frac{L_t}{\sigma f_{s\tau t}}\right)^{\frac{\sigma-1}{\sigma-\eta}} \\
&= \left(\frac{L_t}{\sigma}\right)^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{\mu w_t}{P_t}\right)^{\frac{(\eta-1)(1-\sigma)}{\sigma-\eta}} \left(z_{s\tau t}^{1-\eta} f_{s\tau t}\right)^{\frac{1-\sigma}{\sigma-\eta}} \\
&= \left(\frac{L_t}{\sigma}\right)^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{\mu w_t}{P_t}\right)^{\frac{(\eta-1)(1-\sigma)}{\sigma-\eta}} \left(\bar{z}_t^{1-\eta} \bar{f}_t\right)^{\frac{1-\sigma}{\sigma-\eta}} \left[g_2^{\tau-t}(t-\tau)^{\epsilon_2}\right] \left(z_{s\tau}^{1-\eta} f_{s\tau}\right)^{\frac{1-\sigma}{\sigma-\eta}} \\
&= \left(\frac{L_t}{\sigma}\right)^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{\mu w_t}{P_t}\right)^{\frac{(\eta-1)(1-\sigma)}{\sigma-\eta}} \left(\bar{z}_t^{1-\eta} \bar{f}_t\right)^{\frac{1-\sigma}{\sigma-\eta}} \tilde{M}_{s\tau t} \\
&= \left(\frac{L_t}{\sigma \bar{f}_t}\right) \tilde{P}_t^{\eta-1} \tilde{M}_{s\tau t}
\end{aligned}$$

So that the share of mass is given by

$$\frac{M_{s\tau t}}{M_t} = \frac{[g_2^{\tau-t}(t-\tau)^{\epsilon_2}](z_{s\tau t}^{1-\eta} f_{s\tau t})^{\frac{1-\sigma}{\sigma-\eta}}}{\sum_{s\in S}\sum_{\tau=-\infty}^{t}[g_2^{\tau-t}(t-\tau)^{\epsilon_2}](z_{s\tau t}^{1-\eta} f_{s\tau t})^{\frac{1-\sigma}{\sigma-\eta}}} = \frac{[g_2^{\tau-t}(t-\tau)^{\epsilon_2}](z_{s\tau t}^{1-\eta} f_{s\tau t})^{\frac{1-\sigma}{\sigma-\eta}}}{\tilde{M}_t}$$

And growth of a vintage:

$$
\begin{aligned}
\frac{M_{s\tau t+1}}{M_{s\tau t}} &= \frac{m_{s\tau t+1}M_{t+1}}{m_{s\tau t}M_t} \\
&= \frac{m_{s\tau t+1}\tilde{M}_{t+1}}{m_{s\tau t}\tilde{M}_t}\left(\frac{L_{t+1}}{L_t}\right)^{\frac{\sigma-1}{\sigma-\eta}}\left(\frac{P_t w_{t+1}}{P_{t+1}w_t}\right)^{\frac{(1-\sigma)(\eta-1)}{\sigma-\eta}}\left(g_z^{1-\eta}g_f\right)^{\frac{1-\sigma}{\sigma-\eta}} \\
&= \frac{m_{s\tau t+1}\tilde{M}_{t+1}}{m_{s\tau t}\tilde{M}_t}g_L^{\frac{\sigma-1}{\sigma-\eta}}\left(\left(\frac{g_L}{g_f}\right)^{\frac{1}{\sigma-1}}g_z\left[\frac{\tilde{P}_t}{\tilde{P}_{t+1}}\right]^{\frac{\sigma-\eta}{\sigma-1}}\right)^{\frac{(1-\sigma)(\eta-1)}{\sigma-\eta}}\left(g_z^{1-\eta}g_f\right)^{\frac{1-\sigma}{\sigma-\eta}} \\
&= \frac{m_{s\tau t+1}\tilde{M}_{t+1}}{m_{s\tau t}\tilde{M}_t}g_L^{\frac{\sigma-1}{\sigma-\eta}}g_L^{\frac{1-\eta}{\sigma-\eta}}\left(g_f g_z^{1-\sigma}\right)^{\frac{\eta-1}{\sigma-\eta}}\left[\frac{\tilde{P}_t}{\tilde{P}_{t+1}}\right]^{1-\eta}\left(g_z^{1-\eta}g_f\right)^{\frac{1-\sigma}{\sigma-\eta}} \\
&= \frac{m_{s\tau t+1}\tilde{M}_{t+1}}{m_{s\tau t}\tilde{M}_t}g_L\left(g_f g_z^{1-\sigma}\right)^{\frac{\eta-1}{\sigma-\eta}}\left[\frac{\tilde{P}_t}{\tilde{P}_{t+1}}\right]^{1-\eta}\left(g_z^{1-\eta}g_f\right)^{\frac{1-\sigma}{\sigma-\eta}} \\
&= \frac{m_{s\tau t+1}\tilde{M}_{t+1}}{m_{s\tau t}\tilde{M}_t}\frac{g_L}{g_f}\left[\frac{\tilde{P}_t}{\tilde{P}_{t+1}}\right]^{1-\eta}
\end{aligned}
$$

I Distribution of sales and firms across classes

The distribution of sales and plants and across classes within a vintage is:

$$\text{Prob}\left(\frac{X_{s\tau t}}{X_{\tau t}} \leq x \text{ and } \frac{M_{s\tau t}}{M_{\tau t}} \leq m\right) = \text{Prob}\left(\left[z^{(1-\sigma)}f\right]^{\frac{1-\eta}{\sigma-\eta}} \leq \Phi_1 x \text{ and } \left[z^{(1-\eta)}f\right]^{\frac{1-\sigma}{\sigma-\eta}} \leq \Phi_2 m\right)$$

So, the vector of market shares $\mathbf{y} = \log(x_{s\tau t} \quad m_{s\tau t})'$ is an affine transformation of the vector of technology draws $\mathbf{w} = \log(z_{s\tau t} \quad f_{s\tau t})'$: $\mathbf{y} = A\mathbf{w}$, where the elements of matrix A are:

$$a_{11} = \frac{(1-\eta)(1-\sigma)}{(\sigma-\eta)\log(\Phi_1)}, \qquad a_{12} = \frac{1-\eta}{(\sigma-\eta)\log(\Phi_1)},$$
$$a_{21} = \frac{(1-\eta)(1-\sigma)}{(\sigma-\eta)\log(\Phi_2)}, \qquad a_{22} = \frac{1-\sigma}{(\sigma-\eta)\log(\Phi_2)}.$$

Since $\mathbf{w}$ is distributed normally with mean parameter 0 and variance parameter Σ, market shares $\exp(\mathbf{y})$ are distributed according to a log normal with mean parameter 0 and variance parameter $A\Sigma A'$.

J Truncation of the Log-Normal Distribution

The standard way of generating a truncated bivariate normal is to reject observations outside the confidence interval. This alternative does not work for us because we would like to keep the same random draws for all parameter values. These notes show the computation procedure.

We start with a Cholesky decomposition to generate a bivariate normal. Given a vector of two standard normal draws n, $(Z, F)' = An$ is a bivariate normal with the desired variance and correlation parameters if A is the Cholesky decomposition of the variance-covariance matrix. Matrix A has elements

$$a_{11} = (varZ)^{.5}$$

$$a_{12} = 0$$

$$a_{21} = covZF/a_{11} = corrZF \times (varF)^{.5}$$

$$a_{22} = \left[varF + a_{21}^2\right]^{.5} = (varF)^{.5} \times (1 - corrZF)$$

Truncation Denote with Φ the cdf of the normal distribution and with Φ^{-1} its inverse. Given the distribution of $(Z, F)'$, we would like to reject values of Z in the upper *truncZ* share of the unconditional distribution, and values of F in the lower *truncF* share of its unconditional distribution. Parameters *truncZ* and *truncF* typically take values around 0.05. We transform the draws from the uniform distribution originally generating n to deliver values of Z and F in the desired interval. Using $(Z,F)' = An$, $Z = (varZ)^{.5} n_1$. Then,

$$u_z = (1 - truncZ)U \tag{J.0.1}$$

and $n_1 = \Phi^{-1}(u_z)$ has the desired truncation. Again expanding $(Z,F)' = An$, yields $F = a_{21}n_1 + a_{22}n_2$. Let u_f be the uniform distribution generating that unconditional distribution of F.

$$\begin{aligned}
u_f &= \Phi\left(\frac{F}{(VarF)^{.5}}\right) \\
u_f \geq truncF &\Leftrightarrow \left(\frac{F}{(VarF)^{.5}}\right) > \Phi^{-1}(truncF)
\end{aligned}$$

$$\Rightarrow \quad a_{21}n_1 + a_{22}n_2 \geq (VarF)^{.5}\Phi^{-1}(truncF)$$

$$\Rightarrow \quad n_2 \geq \frac{(VarF)^{.5}\Phi^{-1}(truncF)}{a_{22}} - \frac{a_{21}}{a_{22}} n_1$$

$$u_2 = \Phi(n_2) \geq \Phi\left[\frac{(VarF)^{.5}\Phi^{-1}(truncF)}{a_{22}} - \frac{a_{21}}{a_{22}} n_1\right] \equiv \underline{u}_f \tag{J.0.2}$$

So, u_2 truncated at $\underline{u}_f$ for every value of n_1 yields the desired truncation of F.

Computation procedure Let u_1 and u_2 be draws from $U[0,1]$. From equation (J.0.1), get $u_z = (1 - truncZ)u_1$ and $n_1 = \Phi^{-1}(u_z)$ and $Z = varZ^{.5} n_1$. Variable Z has a normal distribution with its upper tail truncated at *truncZ*. For each value of n_1, let $\underline{u}_f \in (0,1)$ be the expression defined in equation (J.0.2). And let

$$u_f = \underline{u}_f + u_2(1 - \underline{u}_f)$$

Then, u_f is a uniform distribution on $[\underline{u}_f, 1]$ and $n_2 == \Phi^{-1}(u_f)$ be the corresponding truncated normal. Then, variable $F == a_{21}n_1 + a_{22}n_2$ has a normal distribution with mean parameter zero and variance parameter $VarF$, and its lower tail is truncated at $truncF$. The correlation parameter between Z and F is $corrZF$.