November 1989

Exploration of Unknown Mechanical Assemblies Through Manipulation

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University of Pennsylvania Department of Computer and Information Science Technical Report No. MS-CIS-89-74.

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Abstract

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ACKNOWLEDGEMENTS:
The authors wish to thank Mario Campos, University of Pennsylvania, Susan Lederman, Queens University, and Roberta Klatzky for discussions on exploration through manipulation. This work was in part supported by: Airforce grant AFOSR F49620-85-K-0018, Army/DAAG-29-84-K-0061, NSF-CER/DCR82-19196 Ao2, NASA NAG5-1045, ONR SB-35923-0, NIH grant NS-10939-11 as part of Cerebro Vascular Research Center, NIH 1-RO1-NS-23636-01, NSF INT85-14199, NSF DMC85-17315, ARPA N0014-88-K-0632, NATO grant NO. 0224/85, by DEC Corp., IBM Corp., Axis Inc. and LORD Corp.

SPIE's 1989 Symposium on Advances in Intelligent Robotics Systems, Nov. 5-10, 1989, Philadelphia, PA.

1196-09

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ABSTRACT

If robots must function in unstructured environments, they must also possess the ability to acquire information and construct appropriate models of the unknown environment. This paper addresses the automatic generation of kinematic models of unknown objects with moveable parts in the environment. If the relative motion between moving parts must be observed and characterized, vision alone cannot suffice. An approach in which manipulation is used with vision for sensing is better suited to the task of determining kinematic properties. In this paper, algorithms for constructing models of unknown mechanical assemblies and characterizing the relative motion are developed. Results of a simulation are described to demonstrate the role of manipulation in such an endeavor.

1 Introduction

1.1 Robotic Exploration

The ultimate goal of robotics is to build robots that function in completely unstructured environments. This is particularly important, since even if some information about the environment is available, it may be inaccurate or worse still, the environment may be changing. There are several motion planning and control schemes that allow robots to operate in different environments and are quite effective [7, 9]. All these algorithms require a complete and accurate model of the system, which includes that of the robot manipulator, the end effector, the sensor devices, the environment or external object, and the controller itself. However, the model of the environment or object is, in most cases, not known in unstructured environments.

A solution to this problem is to incorporate in the robot system the "learning ability" to acquire knowledge about the properties of the environment. This paper describes work which is part of a research program on exploratory robots that can acquire knowledge about properties of the environment [2]. In particular, the emphasis is on learning the dynamic characteristics of the environment in order to build an accurate model for the purpose of controlling the robot. The key feature is that the robot must *directly interact* with the environment and therefore, manipulation is an integral part of the exploration process

In particular, it is the kinematic properties of objects, which are mechanical assemblies, that are of interest in this paper. The kinematic properties of a mechanical system describe the mobility in the system. For example, the presence of mating parts in assemblies or mechanical systems allows relative motion between different parts of the object. It is extremely difficult if not impossible to detect the presence of joints or fasteners without actually manipulating and changing the state of the object or mechanical system. In addition, it is often important to know the kinematic parameters (for example, the Denavit-Hartenburg parameters in serial linkages) for the joints.

1.2 Scope

In this paper, we investigate the issue of determining the presence of mating parts in unknown mechanical assemblies or linkages. We are further interested in characterizing the nature of the relative motion that is possible within the assembly. The problem is one of exploration - we assume that complete knowledge of the model(s) of the robot(s) is available and the objective is to determine the kinematic parameters that describe the unknown object or assembly. Our approach is similar to the computational approach of Atkeson to learning [1]. We use the rigid body assumption in order to be able to describe the relative motion with a finite number of parameters and to be able to compare. what would otherwise appear to be completely dissimilar motions. We further restrict ourselves to cases in which the object can be modeled by an open chain of serially connected links. If the mechanical assembly is more complicated, it is assumed that the problem can be decomposed into several simpler ones. each of which involves a single serial chain. Algorithms for the two subproblems of determining the mobility of the linkage and the estimation of the kinematic parameters for the linkage are developed here. Results from simulations are also presented to illustrate the use of the proposed algorithms.

1.3 Methodology - Two handed manipulation

The determination of the unknown kinematics of a mechanical assembly requires the formation of a *closed kinematic chain*. This is most conveniently done by two armed manipulation, though attaching one end of the object to a vice at a known reference position while manipulating the other end with one arm also accomplishes the same objective. The basic objective is to design an exploratory motion that is *sufficiently exciting* so that the mobility and the unknown kinematic parameters can be determined, while at the same time, does not damage the robot-object system.

We distinguish between the following two situations:

- Exploration with Vision
- Exploration without Vision (blind exploration)

In the first case vision can be used to provide cues or starting points. The object is then subject to forces that cause relative displacements between the mating parts. The changes can be observed by kinesthetic feedback (joint position changes) as well as visual feedback. The feedback is then used for planning the manipulation (forward path). Here, it is noted that detecting changes through visual feedback is a very difficult process and one which cannot easily be accomplished in real-time. Therefore, it is more attractive to use visual feedback at a lower bandwidth and rely primarily on kinesthetic feedback. If the exploratory motions are sufficiently exciting, the exact kinematic parameters are easily obtained by feedback from the joint displacement transducers.

In the second case, surprisingly, the problem is only slightly complicated. The main disadvantage is a description of the possible joint locations in the unknown object and therefore, a description of the manipulation task, is not available. However, the manipulation can still be described and controlled in joint space coordinates with proprioceptive sensing. This case is the focus of the rest of the paper.

2 Kinematics of Closed Chains

2.1 Mobility of linkages

When two arms grasp a common object, they form, together with the object and the ground, a closed kinematic chain. In this section, the basic theory on kinematics of closed loop linkages is reviewed. We assume that all links are rigid to enable a finite dimensional description of the system configuration.

The *mobility* of a closed chain is the minimum number of independent parameters required to uniquely specify the configuration of the chain. It can be computed using the Kutzbach-Grubler criterion [5]:

$$m = d(n - g - 1) + F \tag{1}$$

where m is the mobility of the linkage, n is the number of bodies or links, g is the number of joints, F is the sum of the degrees of freedom in all the joints in the closed chain, and d is equal to 3 for planar mechanisms and 6 for spatial mechanisms. Consider two serial chain robot arm each with r degrees of freedom, and assume that the object has p single degree of freedom joints. Further, assume that each arm grasps the object (mechanical system) rigidly so that no kinematic joint is present at the contact. Then, in the equation above,

$$n = 2r + p, g = 2r + p, F = 2r + p \tag{2}$$

or,

$$m = 2r + p - d \tag{3}$$

In the above example, there are $2r$ actuators (r in each arm) or $2r$ active joints, while only m joint positions can be uniquely specified or controlled. Clearly, if $p < d$, there are $d - p$ surplus actuators in the closed chain resulting in redundancy in actuation. For example, in a situation with two robots grasping a pair of scissors (in three dimensional space), $r = 6$, $d = 6$ and $p = 1$, so that $m = 7$, and there are five $(= d - p)$ surplus actuators. If $p > d$, the object (assembly) cannot be completely constrained (controlled). In this situation, exploration through manipulation alone is not feasible. However, the grasps can be so planned that the number of joints the object "in between" the two grasps is less than d to enable exploration.

2.2 Mathematical modeling of the unknown object kinematics

In robotics, the Denavit-Hartenberg notation is used, almost without exception, to model serial chain linkages [6]. The D-H parameters α_i (the i^{th} twist angle), r_i (the i^{th} link offset), a_i (the i^{th} link length), and θ_i (the i^{th} joint angle) completely specify the position and orientation of the i^{th} reference frame (x_i, y_i, z_i) on link i, relative to the $(i-1)^{th}$ reference frame on link i-1 as shown in Figure 1.

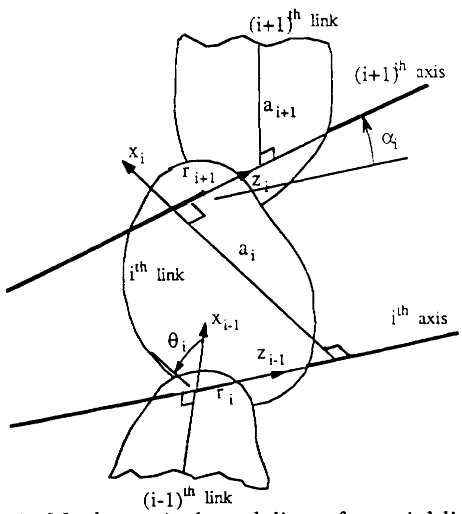

Figure 1: Mathematical modeling of a serial linkage

For each link, one of the parameters α_i, θ_i, r_i or a_i is a variable that can be used to parametrize its motion relative to an adjacent link, while the other three remain constant. The rigid body kinematics of

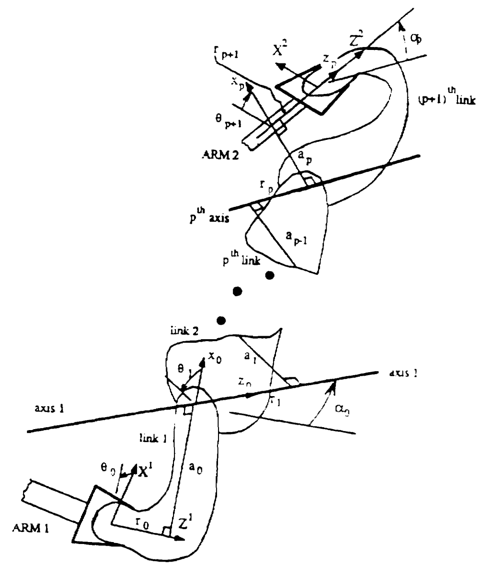

Figure 2: Kinematic representation of manipulation by two arms

the unknown object (assembly) is completely specified, if the three D-H parameters, that are constant, are known. Thus the problem of exploration reduces to the determination of α_i, r_i and a_i if the i^{th} joint (between the $(i-1)^{th}$ member and i^{th} member) is revolute, and the determination of α_i, θ_i, and a_i if the i^{th} joint is a sliding joint.

If the position and orientation of the p^{th} link with respect to the o^{th} link (as shown in Figure 2) is represented by T_p, clearly T_p is a function of the D-H parameters.

$$T_p = T_p(a_i, r_i, \alpha_i, \theta_i) \qquad\qquad i = 1, \ldots, p$$

However to relate $x_0 y_0 z_0$ to $X^1 Y^1 Z^1$, the reference frame fixed to the end-effector of arm 1 we introduce 4 constant parameters, a_0, θ_0, α_0, and r_0 (recall assumption that the grasp permits no relative motion). Similarly to relate $X^2 Y^2 Z^2$ to the reference frame fixed to the end-effector of arm 2 we introduce 2 more constants, r_{p+1} and θ_{p+1}.

Therefore, if T is the transformation between the end-effector fromes of arms 1 and 2, T is a function of $3p+6$ constant parameters and p variables:

$$T = T(a_i, \alpha_i, \theta_j, r_j) \qquad\qquad i = 0, \ldots, p, j = 0, \ldots, p+1$$

For the planar case, α_i and r_i are no longer required, so that we have $p+3$ constants and p variables

$$T = T(a_i, \theta_j) \qquad\qquad i = 0, \ldots, p, j = 0, \ldots, p+1$$

It is convenient to partition the D-H parameters so that x is a vector of the $(3p+6)$ constant parameters while q is the $p \times 1$ vector of joint variables

$$T = T(x, q) \tag{4}$$

These equations are the closure equation for the linkage and are, in general, quite nonlinear.

3 Procedure for Exploration of Kinematic Properties

3.1 General

The exploratory procedure for determining the kinematic parameters for the unknown mechanical system essentially consists of two sequential steps:

1. Determination of Mobility
2. Identification of Kinematic Parameters

Both these steps involve exploratory movements which must satisfy the following basic requirements:

- The interaction forces in the closed chain must be monitored to prevent causing damage to the arms and the unknown system.
- The exploratory motion must be *sufficiently exciting* so that the problem of estimating the kinematic parameters is well-conditioned.

We next discuss each of these two steps in detail.

3.2 Mobility

We describe here a control algorithm which will allow the controlled manipulation with two manipulators without damaging the grasped object or the arms. It is assumed however that both grasps are rigid. We first develop the framework of *partioned actuator sets* in order to explain the strategy.

Since the mobility is only $m(= 2r + p - d)$, any m actuators can be used to control the configuration (position) of the closed chain linkage, while the other $2r - m$ actuators can be made passive (by commanding them to exert zero torques/forces). In such a situation, the m actuators could be grouped into a *primary actuator* set (PAS) while the remainder constitute a *secondary actuator* set (SAS). Then there are ${}^{2r}\mathrm{C}_m$ such primary actuator sets (and secondary actuator sets), since any m actuators can be selected to control the position of the linkage.

We use a control algorithm in which in a preferred PAS is selected and the m primary actuators in the PAS are position controlled. The $2r - m$ are force controlled so that the forces/torques exerted by the secondary actuators are made to equal zero. Thus, the primary actuators are commanded to execute a desired *exploratory trajectory* while the secondary actuators merely comply to the PAS. In some ways, this can be interpreted as a hybrid control scheme in joint space.

This "hybrid" control algorithm requires knowledge of the mobility m $(= 2r+p-d)$, which in turn depends on number of joints in the unknown system. We start with a conservative assumption that the mechanical system has no moveable parts ($p = 0$). This would mean that the mobility of the linkage is completely determined by the geometry of the two robots, and therefore $m_0 = 2r - d$ is the assumed mobility. If the actual mobility m is greater than m_0 (i.e. $p > 0$), and the control algorithm described above is employed to command a trajectory with small displacements, since the mechanism is only partially constrained (m_0 forces/torques applied to a mechanism with m independent degees of freedom), large (finite) displacements will occur at one or more joints. This can be detected through joint position transducers. Such trajectories can be executed by assuming mobilities of $m_1 = 2r - d + 1$ (assuming $p = 1$), $m_2 = 2r - d + 2$ (assuming $p = 2$), and so on until the mechanism no longer appears to be only partially constrained. If the assumed value of m_i is higher than m, the use of this algorithm will result in high interaction forces which can be detected by joint force/torque transducers once again. Thus, if dc motors are used, for example, the currents would be monitored and high currents on the PAS motors would be indicative of an overconstrained system and of the assumed value of mobility being higher than the actual value.

Thus we can start out with an initial value of zero for p and gradually increase p until we arrive at the correct value. This is an iterative but systematic procedure for identifying the kinematic mobility in the object. Since there are ${}^{2r}\mathbf{C}_m$ PAS, the iterative procedure can be carried out with every PAS if it is necessary. It should be noted this method yields the number of independent relative degrees of freedom in the object or mechanical assembly between the two ends that are grasped. If the object itself contains a mechanical linkage that is a closed chain, or if there are moveable parts that are not excited into motion because of the inadequacy of the actual two-handed grasp, the value of p would not reflect the number of moveable parts, which could now be more than p.

3.3 Kinematic Parameters

3.3.1 Trajectory Generation

Once the number of serially connected parts (between the two grasps) in the object assembly is known, it is often of interest to be able to describe the relationship between. As discussed earlier, the most tractable way is to assume a rigid body model (structure) for the system which can be parametrized by the Denavit-Hartenberg (DH) parameters. We now describe a systematic exploratory procedure to determine the DH parameters for the parts in the object.

The exploratory procedure requires the execution of an *exploratory trajectory*. The performance of the algorithms for determining the mobility and the kinematic parameters is directly effected by the exploratory trajectory. Hence this trajectory should be designed carefully. It is probably easier to synthesize this trajectory by formulating this problem in joint space, since in the absence of *a priori* knowledge about the environment, the task space or the constraint space is unknown. However, it turns out that simple cartesian trajectories often provide adequate information for the estimation algorithms. Nevertheless a better rate of convergence and better accuracy can be obtained with well designed exploratory trajectories. The problem of designing such trajectories is not addressed in this paper.

3.3.2 Identification of parameters

The next step is the identification of the parameters. Let T_1 and T_2 be the transformations relating the position and orientation of the end-effector of arms 1 and 2, respectively, with respect to some world coordinate system as shown in Figure 2. If both arms are assumed to grasp the object rigidly, the position and orientation of the arm 2 relative to arm 1 is given by

$$T = T_1^{-1}T_2 \tag{5}$$

If the two robots are equipped with joint transducers and the kinematics of the arms are known, a direct kinematics algorithm can be used to obtain T_1 and T_2, and from (5), T, in any configuration. From the l measurements obtained by exciting the unknown system, we have equations of the type (4) which can now be used to determine the vector x. In the rest of this section, we describe two methods based on this general approach, which are then demonstrated through two examples in the next section.

Method I Let us rewrite equation (4) in the form

$$z = z(x, q) \tag{6}$$

where z is now a 6×1 vector (3×1 in the planar case) containing the relative position and orientation information which is contained in T. Since q is a variable we eliminate q from equation (6) to obtain a set of $(6 - p)$ equations ($3 - p$ in the planar case)

$$y = y(x) \tag{7}$$

If an estimate for the unknown parameters, $\hat{x}$, is available, we can calculate $\hat{y}$ from (7) which will be different from the measured y unless the estimate $\hat{x}$ is correct. Therefore, if the difference $x - \hat{x}$ is small, we get

$$y - \hat{y} = \frac{\partial y}{\partial x}(x - \hat{x}) \tag{8}$$

Since l measurements are available $y^1, y^2, \ldots, y^l$, we can write (8) l times to get $(6 - p)l$ equations

$$\begin{bmatrix} y^1 - \hat{y} \\ y^2 - \hat{y} \\ \vdots \\ y^l - \hat{y} \end{bmatrix} = \begin{bmatrix} \left[\frac{\partial y}{\partial x}\right]^1 \\ \left[\frac{\partial y}{\partial x}\right]^2 \\ \vdots \\ \left[\frac{\partial y}{\partial x}\right]^l \end{bmatrix} (x - \hat{x})$$

which can be written in the form

$$Y = \Gamma X \tag{9}$$

Γ is a $(6 - p)l \times (3p + 6)$ matrix ($(3 - p)l \times (p + 3)$ matrix in the planar case) in which, when l is large, the number of rows is much greater than that of columns. A least square solution for $x - \hat{x}$ can be obtained from (9) which may be then used to get a better estimate for $\hat{x}$.

$$\hat{x}_{new} = \hat{x}_{old} + \Gamma^+ Y \tag{10}$$

where $\Gamma^+ = (\Gamma^T\Gamma)^{-1}\Gamma^T$. This algorithm converges rapidly to the correct solution, x.

Method II If the closure equations are complicated it may not be possible to eliminate q from equation (6) to get the set of $(6 - p)$ equations in equation (7). If this is the case, method I cannot be applied. It is however possible to retain the unknown variables q and follow a similar procedure.

Let q^1, q^2, ... q^l denote the values of q at each of the l configurations. Each of the q^i $(i = 1, \ldots, l)$ may be treated as constant parameters similar to x. Thus equation (6) can be rewritten as for the i^{th} measurement as

$$z^i = z^i(x, q^i) \tag{11}$$

and in general, for any of the l data sets,

$$z = z(x, q^1, q^2, \ldots, q^l) \tag{12}$$

Taking partial derivatives of the above equation we get

$$z - \hat{z} = \frac{\partial z}{\partial x}(x - \hat{x}) + \frac{\partial y}{\partial q^1}(q^1 - \hat{q}^1) + \frac{\partial y}{\partial q^2}(q^2 - \hat{q}^2) + \ldots + \frac{\partial y}{\partial q^l}(q^1 - \hat{q}^l) \tag{13}$$

Since we can write (13) l times we get the same matrix equation as in (9)

$$Z = \Gamma X \tag{14}$$

However the vector X now consists of the parameters x as well as $q^1, q^2, \ldots, q^l$, the pl values for the (p) unknown joint variables. Therefore, Γ is a $(6l) \times (3p + 6 + pl)$ matrix for the general three dimensional case, and a $3l \times (p + 3 + pl)$ matrix for the two-dimensional case. Clearly, l must be sufficiently large so that $(6l)$ is much greater than $(3p + 6 + pl)$. Now the least squares solution for X from equation (14) can be used in the algorithm of equation (10) in the same way to iteratively obtain the correct solution for x. As a byproduct, we also obtain the joint variables $q^1, q^2, \ldots, q^l$ at the l configurations, though this information is not usually of any interest.

3.3.3 Remarks

The general scheme is similar to that used for robot calibration [3, 8]. In one case (robot calibration), we are interested in estimating the kinematic parameters of the robot, while in the other (exploration) we are interested in determining the kinematic parameters of the unknown object. There are three major differences however. Firstly dual arm manipulation is used here. Secondly the initial estimates in robot calibration problems are typically quite good. The calibration procedure only serves to obtain a more accurate reading of the D-H parameters. In this case, an initial estimate can only be obtained from vision. Even if vision is available, it is extremely difficult to identify the presence of joints and estimate their location. Therefore the problem of estimating the unknown parameters is much more difficult. Thirdly, and probably most importantly, since the unknown "system" is the robot, the joints are instrumented and the joint variables are known except for the inaccuracy and noise in the transducers. In this problem however, the joint variables are completely unknown. Either they must be eliminated from the equations (as in equations (7, 8), Method I) or they must be tracked ((12, 13), Method II). This feature is unique to the exploration problem. Finally, we once more note that Method II is completely general and widely applicable in robotic exploration problems since the elimination of joint variables from the closure equations is not necessary.

4 Examples

In this section, the methods described in the previous section are applied to two examples. In both cases, the unknown system is taken to be planar, while the two robots are both two link planar manipulators. However this does not take anything away from the generality of this scheme - the formulation is for a perfectly general geometry with the only restriction being, $p < d$. As mentioned earlier (Section 2.1), the two grasps can be planned so that this condition is always met. In both examples it is assumed that phase 1 of the exploration involving the determination of mobility has already been carried out. In other words, the value of p is assumed to be known. Also, the control algorithms are simulated in these examples. Instead, an arbitrarily chosen exploratory trajectory is used. In Example 1, arm 1 is commanded to follow a circular path while arm 2 merely complies to arm 1 and in example 2, arm 1 executes a straight line trajectory while the proximal link of arm 2 is held still while the distal link complies to arm 1. It is shown that even with such arbitrarily chosen trajectories, the algorithm produces good results.

4.1 Example 1

In the example, we consider an object (serial linkage) with one movable rotary joint being grasped by two manipulators so that the two end-effectors are on either side of the joint. We assume for the moment that the unknown joint is rotary. A pair of scissors is a typical example of such a single jointed linkage. As mentioned earlier, the exploratory trajectory is not synthesized in joint space. The first manipulator is commanded to trace a circular trajectory while the second manipulator follows the first in a master-slave mode.

Since the mechanism is planar, the unknown parameters are a_0, a_1, θ_0, and θ_2 ($\alpha_i = r_i = 0$ in Figure 2), while the joint variable is θ_1. At any configuration, three closure equations can be written:

$$z_1 = a_0 C\theta_0 + a_1 C(\theta_0 + \theta_1) \qquad z_2 = a_0 S\theta_0 + a_1 S(\theta_0 + \theta_1) \qquad z_3 = \theta_0 + \theta_1 + \theta_2$$

where z_1 and z_2 are the coordinates of the origin of frame 2 and z_3 is the orientation of frame 2 with respect to the reference frame $X_0 Y_0 Z_0$. Therefore in equation (6),

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}, \qquad x = \begin{bmatrix} a_0 \\ a_1 \\ \theta_0 \\ \theta_2 \end{bmatrix}, \qquad q = [\theta_1]$$

$q = [\theta_1]$ can be eliminated from the above equations to get two equations for (4):

$$z_1 = a_0 C\theta_0 + a_1 C(z_3 - \theta_2) \qquad z_2 = a_0 S\theta_0 + a_1 S(z_3 - \theta_2)$$

which can be written (as in (10)) in the form

$$y = y(x)$$

where $y = [z_1, z_2]^T$. $\frac{\partial y}{\partial x}$ is obtained by differentiating these equations. At the i^{th} measurement,

$$\left(\frac{\partial y}{\partial x}\right)^i = \begin{bmatrix} C\hat{\theta}_0 & C(z_3^i - \hat{\theta}_2) & -\hat{a}_0 S\hat{\theta}_0 & \hat{a}_1 S(z_3^i - \hat{\theta}_2) \\ S\hat{\theta}_0 & S(z_3^i - \hat{\theta}_2) & \hat{a}_0 C\hat{\theta}_0 & -\hat{a}_1 C(z_3^i - \hat{\theta}_2) \end{bmatrix}$$

In this example, Method I was adopted except for one slight improvement. Since $\frac{\partial y}{\partial x}$ is a matrix of dimension 2×4, only two measurements are required to formulate a 4×4 system of equations for (9). Thus the inverse of Γ as opposed to a generalized inverse can be used. Further, the algorithm can actually be implemented in an on-line manner by which the estimates for the parameters are continually refined as the exploratory trajectory is executed.

In this example, the mobility, m $(= 2r + p - d)$, can be calculated to be 2 $(= 2 \times 2 + 1 - 3)$. In other words, the PAS must consist of 2 actuators while the SAS consists of the other two actuators. If the two primary actuators both belong to one arm and the other two to the other arm, the framework is identical to that in the leader-follower control strategy for dual arm coordination.

Thus, in the example in Table I, manipulator 1 (the leader) follows a circular trajectory while arm 2 (the follower) merely complies to arm 1 (analogous to a master and a slave). At each position, the kinematic parameters are estimated from the motion data for the prior two configurations. The iterative procedure yields excellent results and an accurate estimate is obtained in five iterations.

4.2 Example 2

In this example, the object is a two jointed serial linkage. A pair of reading glasses, for instance, has three links which are connected by two hinges. The unknown parameters are a_0, a_1, a_2, θ_0 and θ_3 ($\alpha_i = r_i = 0$ in Figure 2), while the joint variables are θ_1 and θ_2. Once more it is assumed that phase 1 of the exploration has already been carried out and that it is known that the number of joints in the linkage, p, is equal to 2.

The vectors z, x and q are defined as follows:

$$z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}, \qquad x = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \theta_0 \\ \theta_3 \end{bmatrix}, \qquad q = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

$$\begin{aligned} z_1 &= a_0 C\theta_0 + a_1 C(\theta_0 + \theta_1) + a_2 C(\theta_0 + \theta_1 + \theta_2) \\ z_2 &= a_0 S\theta_0 + a_1 S(\theta_0 + \theta_1) + a_2 S(\theta_0 + \theta_1 + \theta_2) \\ z_3 &= \theta_0 + \theta_1 + \theta_2 + \theta_3 \end{aligned}$$

If q is eliminated from the closure equations above, we get only one remaining equation and $\frac{\partial y}{\partial x}$ is a 1×5 matrix. Therefore, the problem of determining the unknown parameters is not well conditioned. Method II is better suited to the problem and it turns out to be more attractive inspite of the larger computational load.

The mobility m is calculated quite easily:

$$m = 2r + p - d = 2 \times 2 + 2 - 3 = 3$$

The PAS therefore consists of 3 actuators. If we choose, for example, the two actuators on one arm and the proximal actuator on the other, the SAS would consist of the distal actuator on the latter arm. We arbitrarily chose a straight line trajectory for arm 1 while the proximal actuator on arm 2 was held constant (thereby uniquely determining the trajectory of the 3 primary actuators and therefore that of the entire system).

Γ is constructed by arranging l matrices of partial derivatives one below another. In the simulation, ten measurements were made ($l = 10$) to yield a total of 30 equations in 25 unknowns, a_0, a_1, a_2, θ_0, θ_3, θ_1^1, θ_2^1, θ_1^2, θ_2^2, ..., θ_1^{10}, θ_2^{10}. The algorithm converges in about 15 iterations as seen in Table II.

5 Discussion

The examples analysed in the previous section demonstrate the applicability of the general method of determining the mobility and kinematic parameters in unknown objects with two armed manipulation. Though the examples considered were planar, it should be pointed out that most mechanical jointed assemblies are planar. Therefore, planar systems do constitute a special but important case. Further, the formulation of the problem is quite general and its applicability to three dimensional problems cannot be doubted.

An iterative least-squares procedure was found to be adequate and quite tractable from a computational point of view, However more sophisticated numerical methods, such as the Levenburg, Marquadt and Morrison algorithm [4], do exist and might lead to a better results for more complicated cases.

It was assumed that no sliding (prismatic) joints are present in the object. The problem of determining the presence of sliding joints and the associated kinematic parameters is infact simpler because of the linear nature of the equations. As in the case of robot arm kinematics, if the i^{th} joint is sliding, the joint variable is r_i while θ_i is taken to be a constant.

In order to make the exploration technique more powerful, it is necessary to search the joint space efficiently and base the search on the existing knowledge and current (and constantly improving) estimates of the parameters in the model. The development of such exploratory trajectories is an important research issue which will be investigated in the future.

The control algorithm for the determination of the mobility of the linkage was not simulated, since a very detailed dynamic model is required to simulate the uncertainty in the environment, There is a strong experimental component to this project which is currently being pursued. However this work is in a very preliminary stage. Further progress in this area will be reported in future publications.

6 Conclusion

In this paper, we addressed the automatic generation of kinematic models of unknown objects with moveable parts in the environment through robotic exploration. We presented algorithms for estimating the number of joints and the kinematic parameters that characterize the joint motion in a mechanical assembly. In all the algorithms, manipulation was the key to the exploratory procedure. Results of a simulation are described to demonstrate the efficacy of these algorithms.

7 Acknowledgements

The authors wish to thank Mario Campos, University of Pennsylvania, Susan Lederman, Queens University, and Roberta Klatzky for discussions on exploration through manipulation. This work was in part supported by: Airforce grant AFOSR F49620-85-K-0018, Army/DAAG-29-84-K-0061, NSF-CER/DCR82-19196 Ao2, NASA NAG5-1045, ONR SB-35923-0, NIH grant NS-10939 -11 as part of Cerebro Vascular Research Center, NIH 1-RO1-NS-23636-01,NSF INT85-14199,NSF DMC85-17315, ARPA N0014-88-K-0632,NATO grant No.0224/85, by DEC Corp., IBM Corp., Axis Inc. and LORD Corp.

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Table I Identification of kinematic parameters : Example 1

n	x_1	y_1	x_2	y_2	z_1	z_2	z_3	$\hat{a}_0$	$\hat{a}_1$	$\hat{\theta}_0$	$\hat{\theta}_2$
0	1.50	0.50	1.68	0.46	0.07	-0.17	43.60	1.00	1.00	45.00	120.00
1	1.49	0.63	1.68	0.60	0.07	-0.18	45.43	0.18	0.13	34.87	132.40
2	1.43	0.87	1.63	0.86	0.08	-0.18	47.10	0.21	0.17	-11.46	217.02
3	1.31	1.09	1.51	1.10	0.08	-0.18	46.58	0.25	0.20	2.41	165.86
4	1.14	1.27	1.32	1.31	0.07	-0.17	44.17	0.25	0.24	-0.05	183.39
5	0.93	1.40	1.09	1.46	0.06	-0.16	40.15	0.25	0.25	0.00	179.90
6	0.69	1.48	0.82	1.55	0.04	-0.14	34.90	0.25	0.25	0.00	180.00
7	0.44	1.50	0.54	1.57	0.03	-0.12	28.89	0.25	0.25	0.00	180.00
8	0.19	1.45	0.26	1.52	0.02	-0.10	22.64	0.25	0.25	0.00	180.00
9	-0.04	1.34	0.00	1.41	0.01	-0.07	16.60	0.25	0.25	0.00	180.00
10	-0.23	1.18	-0.21	1.23	0.00	-0.05	11.06	0.25	0.25	0.00	180.00

Note: Arm 1 is commanded to move in a circular trajectory whose radius is 1.0 with center at (0.5, 0.5). (x_1, y_1) is the position of the end-effector of arm 1 and (x_2, y_2) that of arm2. (z_1, z_2, z_3) are estimated from the measurements directly, while $(a_0, a_1, \theta_0, \theta_2)$ are the parameters that are estimated during the motion. The actual model has the parameters $x = (a_0, a_1, \theta_0, \theta_2)^T = (0.25, 0.25, 0, 180)^T$ while the initial guess was $\hat{x} = (1, 1, 45, 120)^T$

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Table II Identification of kinematic parameters : Example 2

A. Straight line trajectory (for arm 1)

n	x_1	y_1	x_2	y_2	z_1	z_2	z_3
0	0.50	0.00	0.95	0.09	0.03	0.46	-80.34
1	0.50	0.10	0.90	0.25	0.04	0.42	-101.43
2	0.50	0.20	0.88	0.36	0.10	0.40	-119.56
3	0.50	0.30	0.87	0.47	0.16	0.37	-136.26
4	0.50	0.40	0.87	0.58	0.22	0.35	-151.73
5	0.50	0.50	0.88	0.68	0.27	0.32	-166.16
6	0.50	0.60	0.91	0.78	0.34	0.29	-179.70
7	0.50	0.70	0.94	0.88	0.40	0.26	-192.42
8	0.50	0.80	0.98	0.97	0.47	0.20	-204.29
9	0.50	0.90	1.02	1.05	0.54	0.08	-213.65
10	0.50	1.00	1.07	1.11	0.58	0.04	-225.05

B. Identification of kinematic parameters

n	a_0	a_1	a_2	θ_0	θ_3	n	a_0	a_1	a_2	t_0	t_3
1	0.12	0.37	0.23	9.27	169.08	11	0.20	0.30	0.20	-0.07	177.82
2	0.15	0.33	0.22	3.54	170.14	12	0.20	0.30	0.20	-0.03	179.66
3	0.21	0.28	0.20	-0.91	175.87	13	0.20	0.30	0.20	-0.01	179.87
4	0.19	0.30	0.21	0.71	179.54	14	0.20	0.30	0.20	-0.00	179.99
5	0.21	0.28	0.20	-0.55	174.52	15	0.20	0.30	0.20	0.00	179.99
6	0.19	0.30	0.21	0.32	-179.75	16	0.20	0.30	0.20	0.00	180.00
7	0.21	0.28	0.20	-0.10	174.52	17	0.20	0.30	0.20	0.00	180.00
8	0.20	0.30	0.21	0.04	-179.75	18	0.20	0.30	0.20	0.00	180.00
9	0.20	0.28	0.21	0.10	174.52	19	0.20	0.30	0.20	0.00	180.00
10	0.20	0.30	0.20	-0.10	-179.75	20	0.20	0.30	0.20	0.00	180.00

Note: (x_1, y_1) is the position of the end-effector of arm 1 and (x_2, y_2) that of arm2. (z_1, z_2, z_3) are estimated from the measurements directly, while $(a_0, a_1, a_2, \theta_0, \theta_3)$ are the parameters that are estimated during the motion. The actual model has the parameters $x = (a_0, a_1, a_2, \theta_0, \theta_3)^T = (0.2, 0.3, 0.2, 0, 180)^T$ while the initial guess was $\hat{x} = (0.1, 0.4, 0.25, 10, 160)^T$