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Value of Speed in Public Transit Services

Abstract
High speed is always a desirable feature of public transportation services. Any measure that increases public transport speeds results in benefits to users in terms of saved travel time, and benefits to the operator in reduced operating costs and in eventual reduction of fleet size.

At the same time, it is known that some major efforts for increasing speeds, often involving considerable cost (such as increasing maximum technical speed of vehicles) result in relatively small increases in average passenger travel speed. This problem is common for a number of different modes of transportation. An excellent example can be found in air transportation; increases of aircraft cruising speed are costly and have relatively little impact on the passenger average travel speeds, particularly for short-and medium-haul trips. The same problem is observed with both rapid and surface transit in urban areas. In order to derive more specific results applicable in practice, this research is limited to the latter transportation systems: public transportation in urban areas.

Disciplines
Engineering | Systems Engineering | Transportation Engineering
VALUE OF SPEED IN
PUBLIC TRANSIT SERVICES

by

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A Report on the

VALUE OF SPEED IN
PUBLIC TRANSIT SERVICES

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The work was carried out under the auspices of the Transportation Studies Center of the University of Pennsylvania. The thoughts and findings expressed in this report however do not reflect any official position of the University.
I. INTRODUCTION

High speed is always a desirable feature of public transportation services. Any measure that increases public transport speeds results in benefits to users in terms of saved travel time, and benefits to the operator in reduced operating costs and in eventual reduction of fleet size.

At the same time, it is known that some major efforts for increasing speeds, often involving considerable cost (such as increasing maximum technical speed of vehicles) result in relatively small increases in average passenger travel speed. This problem is common for a number of different modes of transportation. An excellent example can be found in air transportation; increases of aircraft cruising speed are costly and have relatively little impact on the passenger average travel speeds, particularly for short- and medium-haul trips. The same problem is observed with both rapid and surface transit in urban areas. In order to derive more specific results applicable in practice, this research is limited to the latter transportation systems: public transportation in urban areas.

A. THE PROBLEM

Stated in somewhat simplified terms, the cost of providing transit service is a function of its speed. User benefits from that service are also functions of speed. Further, speed is one of the basic elements of a system's level of service and thereby has an impact on mobility in the city. The basic problem studied in this research is to define --quantitatively or qualitatively --
all the relationships between speed and costs and benefits of a transit service. Those relationships would make it possible to develop a systematic method of evaluation of transit speed for any mode of urban public transportation. Such a method can be utilized in the decisions about the improvements of existing as well as planning of new systems.

The effectiveness of various methods for reducing transit travel times on the average travel speed is also examined. Some attention is given to the costs and implementation possibilities of these methods of transit speed increases.

B. WORK BY OTHERS

The fairly extensive literature search conducted for this study has not discovered any published works dealing directly and exclusively with this problem. Relationship of operating costs and operating speeds of various transit lines, based on the data from New York and San Francisco, has been given respectively by Holden\(^1\) and by Homburger\(^2\), who quoted the data from a transit study in San Francisco\(^3\). The relationships these authors developed, however, give the operating cost as a function of operating speed for different lines in a city. Since these lines may differ in characteristics other than operating speed (density of travel, facility used, speed of boarding-alighting, etc.) -- and this is typically the case -- the studied relationship may not always be causal. In other words, the curves developed by these authors cannot be applied to estimate the change in operating cost if the speed on a given line is changed, while most of the other parameters would remain

\(^{*}\)Superscripted numbers refer to Bibliography at the end of the report.
basically the same.

In the recently conducted study by the Organization for Economic Coordination and Development (OECD) in Europe on future directions for research in urban transportation, it has been stated that "there is no urgent need for increase of speed in urban rail transit." This somewhat surprising statement becomes clear when the reader follows the discussion and discovers that this change of speed refers only to the maximum technical speed; the statement reflects the fact that increased technical speed has little bearing on the average operating speed.

A number of studies related to this problem have been utilized in the research. References to these studies are given at the appropriate places in the text.

C. PURPOSE AND SCOPE

This research has several purposes. The major ones are:

- To derive a method for estimation of consequences (benefits and costs) of transit speed changes;

- To derive the effect of changes in individual time elements on the average travel speed;

- To analyze which parties are affected (passengers, operator and others), and in what way, by speed changes;

- To establish the trade-offs between individual benefits and costs due to speed changes;
- To provide a means for evaluation of alternative transit systems involving different speeds; and

- To find the methods for speed increase which will result in the greatest cost-effectiveness.

Consequences of different actions resulting in speed changes are identified and expressed either mathematically or defined qualitatively to the degree of precision which is realistically justified by the accuracy of the assumptions, models, and the best data which can be obtained in practice.

D. STRUCTURE OF RESEARCH

The research is based on a model of a transit service. The model incorporates all relevant time segments involved in service; sensitivity of travel speed to these individual elements is explored.

Emphasis is given on the travel time within the transit system; other travel times, such as access to and departure from the line, are mentioned, but are not analyzed deeper.

A detailed conceptual analysis of the consequences of speed change on the affected groups (passengers, transit operator, community) is made, taking into account realistic assumptions of changes in the service (e.g. that if the speed is increased, the number of vehicles in service can be reduced, assuming that the number of passengers remains constant; or, that the number of vehicles will be increased if the number of passengers increases, etc.).
the analysis includes also feedback effects of increased speed through the increased patronage and increased frequency of service, etc.

Each of the relationships within this conceptual analysis is defined quantitatively or qualitatively.

Based on this analysis an algorithm is developed for evaluation of speed change and it is illustrated by an example. The last part of this research analyzes some of the possible methods of increasing the speed and gives an estimate of their cost effectiveness.

A. THE ASSUMPTIONS

The model of the transit line is based on the assumption that the vehicle stands in a station for an interval of time, the length of which depends on the number of passengers and several other factors. The vehicle then accelerates to its maximum cruising speed which it then maintains to the point where it has to start deceleration for the next station. This process is repeated for each interstation spacing. In addition to this, it is assumed that the vehicles have a certain terminal time interval at each end of the line. With respect to the passengers, their arrival, transfer and departure times are divided into individual segments.
II. TRANSIT SYSTEM MODEL

In analyses of urban travel and transit system operating characteristics, travel time and its inverse, speed, are used interchangeably. Since in most cases travel of persons is studied between two given points, it is more meaningful to use travel time as a travel characteristic. From the point of view of the system, however, it is more convenient to use the speed, since it is independent of the length of individual trips. This research includes transit service as it affects both the passenger and operator, so that both of these variables -- travel time and speed -- will be used at different points, as the analysis requires.

A. THE ASSUMPTIONS

The model of the transit line is based on the assumption that the vehicle stands in a station for an interval of time, the length of which depends on the number of passengers and several other factors. The vehicle then accelerates to its maximum cruising speed which it then maintains to the point where it has to start deceleration for the next station. This process is repeated for each interstation spacing. In addition to this, it is assumed that the vehicles have a certain terminal time interval at each end of the line. With respect to the passengers, their access, transfer and departure times are divided into individual segments.
This basic model of a transit line is modified to represent two different kinds of transit systems. One system is assumed to operate on a private right-of-way, as is the case with rapid transit. Vehicle movement on this system is assumed to be fully deterministic; also, it is assumed that the vehicles accelerate to the maximum technical speed permitted by the length of the interstation spacing. The second system represents operation of transit vehicles in mixed traffic. The expression for travel time of this system includes an additional interval of time representing delays caused by interference from other traffic. Maximum cruising speed of these vehicles is in most cases lower than the maximum technical speed of the vehicles.

Graphic presentation of the two transit system models is given on the time-distance diagrams in Figures 1 and 2. Each of the diagrams shows graphically the approximate movement of vehicles as well as a sketch of the detailed movement between two stations, including acceleration and deceleration of the vehicles. The diagrams also show the movement of passengers to and from the system. The individual time intervals and speeds, shown on these diagrams, are defined below.

B. DEFINITIONS

Although the emphasis of this research is on travel speed on transit lines, system terminal times and individual passenger travel times of the system are also defined here.

1. Travel Time Intervals

Time which is common to both passengers and the system consists of the following intervals:
FIGURE 2. TIME DISTANCE DIAGRAM OF SURFACE TRANSIT SYSTEM
user waiting time at boarding point;

mean vehicle standing time at stations or stops for passenger boarding

time interval during which vehicle is accelerating from station stop to constant cruising speed;

time interval during which vehicle is moving at constant speed in interstation spacing i;

unscheduled time delay of vehicle due to congestion, breakdowns and other random events in interstation spacing i;

time interval during which vehicle decelerates from constant speed to full stop at a station.

Certain time elements concern only the operator. One such time element is the travel time of vehicles between the yards/garages and the point along the line where they begin revenue service. Since this time interval is constant once the route is located, and it is not affected by most of the speed changes on the line, it will not be considered in this analysis. A second time element which concerns the operator only is:

vehicle terminal time, which takes place at each end of the line. It includes the time required for driver convenience (layover), headway adjustment and transition from one schedule to another (drop-back) and time allowed to recover unforeseen delays (schedule recovery time).
Time spent by the users outside the transit system consists of:

- $t_a$: access time from trip origin to vehicle boarding point (walking, park-and-ride, kiss-and-ride, etc.);
- $t_f$: transfer time between two transit lines;
- $t_e$: exit time from the terminal station or stop to the user's destination.

These passenger times off the system are constant for any given route and station locations. In most cases changes of these times would require major changes in the system. Optimization of services with respect to access times has been studied by several authors, including Holroyd, Vuchic, and Byrne, and this particular problem will therefore not be analyzed in greater depth in this report.

In order to derive several different operating speeds of the system, some of the time intervals will be aggregated into time segments as follows:

$$t_{ri} = t_A + t_{ci} + t_{di} + t_{B'}$$

running time between stations $i$ and $i+1$;

$$T_r = \sum_{i=1}^{n} t_{ri}$$

total running time along the entire line;

$$t_t = \frac{V}{2} \left( \frac{1}{A} + \frac{1}{B} \right) + t_s$$

incremental time lost for stopping at a single station, where $V$ is the constant speed of the vehicle and $A$ and $B$ are acceleration and deceleration rates, respectively, both assumed to be constant.*

*Since the acceleration rate for most vehicles is not constant but decreasing with increased speed, its average value during the acceleration interval must be used. The deceleration rate in most cases is approximately constant.
\[ T_L = n t_L, \]

\[ T_d = \sum_{i=1}^{n} t_{d1}, \]

Total time lost for stopping at all intermediate stations;

\[ T_o = T_r + n t_s, \] (3)

Operating time; and

\[ T \] cycle time, or time interval between two subsequent departures of the same vehicle from the same point along the line in the same direction.

2. **Speeds**

Several different speeds are used in the analyses of transit operations. They are defined as follows:

- \( V \) maximum sustained vehicle speed; with rapid transit systems this is in most cases maximum technical speed of the vehicle. With surface streets operations, this speed refers to maximum vehicle speed achievable under given conditions.

- \( V_r \) running speed; the average speed of operation, not including stopped time at stations. This definition is rarely used since \( V_r \) differs from one interstation spacing to another.

- \( V_o \) operating speed; the average speed of vehicle movement, including stopped time at station; in other words, distance passed divided by total elapsed travel time along the line. Naturally, \( V_o = \frac{L}{T_o} \), where \( L \) is the length of the line, one way.
$V_c$ commercial speed. Average vehicle travel speed on the line, including terminal times. This speed can be computed as

$$V_c = \frac{2L}{T},$$

or as the total vehicle miles driven on the line divided by the total service time.

It is obvious that $V > V_r > V_o > V_c$.

It is appropriate to mention here that two of these speeds are of particular importance. Passengers are particularly interested in high operating speed, $V_o$; the operator is interested in increasing commercial speed, $V_c$.

C. SENSITIVITY ANALYSIS

In order to evaluate the impact of changes in individual time elements on the changes in system's operating and commercial speeds, it is necessary to evaluate sensitivity of speeds to these elements. Sensitivity of travel time to individual time elements will be explored since it yields more convenient expressions and greater clarity than an analysis of the sensitivity of speeds to the time elements. As can be seen from the definitions, it is then very easy to relate individual system speeds to corresponding time intervals.

The most important time intervals for analysis are operating time, $T_o$, and cycle time, $T$, corresponding to the operating speed $V_o$ and commercial speed $V_c$, respectively. For their analysis the following assumptions will be made:
1. The length of any interstation spacing \( i \), \( S_i \), is sufficiently great that the vehicle can reach its maximum speed:

\[
S_i \geq S_c
\]

where \( S_c \) is the shortest distance on which maximum speed can be reached;

2. The acceleration rate \( A \) and deceleration rate \( B \) are constant;

3. Standing time in each station is constant \(- t_s\), while standing time at each terminal exclusive of terminal layover time is \( t_s/2 \);

4. There are no influences of way alignment or grade on vehicle travel; and

5. No coasting is applied.

These assumptions are reasonably realistic and they do not significantly affect the results of this analysis. In those cases in which some of the assumptions are not satisfied (e.g. assumption \#1 for a line with very frequent stops) this fact must be taken into account and the results must be treated with appropriate limitations.

The operating and cycle times can be expressed as:

\[
T_o = n(t_A + t_B + t_s) + \sum_{i=1}^{n} (t_{d_i} + t_{c_i})
\]

\[
= \frac{nV}{2} \left( \frac{1}{A} + \frac{1}{B} \right) + n t_s + \frac{nS}{V} + T_d
\]

\[
= \frac{L}{V} + T_c + T_d
\]

and

\[
T = 2(T_o + t_t)
\]

with \( S \) being mean interstation spacing and \( n \) the number of spacings.
Sensitivity of $T_o$ and $T$ to individual time elements can be explored by marginal analysis of their expressions. Change of $T_o$ and $T$ due to unit changes in any one of the time elements $-t_s, t_d, t_t$ and parameters $S, n, V, A$ and $B$ (which determine $t_{c_i}^*, t_{A}^*$ and $t_{B}^*$) can be obtained through partial derivatives. If $x$ designates any one of these independent variables, the partial derivative of $T$ can be expressed in the general form:

$$\frac{\partial T}{\partial x} = 2 \left( \frac{\partial T_o}{\partial x} + \frac{\partial t_t}{\partial x} \right)$$  \hspace{1cm} (6)

If $x$ represents any variable except $t_t$, one obtains

$$\frac{\partial T}{\partial x} = 2 \frac{\partial T_o}{\partial x}$$  \hspace{1cm} (7)

Equation (7) indicates that all independent variables except $t_t$ affect both $T_o$ and $T$, which is intuitively clear. Marginal analysis with respect to individual variables will therefore be performed for $T_o$ only. The same expressions, multiplied by 2, will be valid for $T$.

1. Sensitivity of $T_o$ to $A$ and $B$

The relationship between $T_o$ and $A$ (or $B$ in the same way) is given by equation (4) and plotted on the diagram in Figure 3. The diagram shows a family of curves for different values of $n$, i.e. the plotted function is $T_o = f(A; n)$. They are all decreasing with increasing $A$, asymptotically approaching the line:

$$T_o = \frac{L}{V} + \frac{nV}{2B} + T_d$$  \hspace{1cm} (8)

Sensitivity of $T_o$ to changes in $A$ (or $B$) is expressed by its partial derivative:

$$\frac{\partial T_o}{\partial A} = -\frac{nV}{2A^2},$$  \hspace{1cm} (9)
FIGURE 3. OPERATING TIME AS A FUNCTION OF ACCELERATION RATE
which is also plotted in Figure 3.

The curves clearly indicate that with increasing acceleration rates sensitivity of travel time to acceleration decreases, i.e. it is particularly useful to increase acceleration rates when they are low. As one would expect, the diagram also shows that sensitivity is higher when the number of stations is greater.

2. Sensitivity of \( T_o \) to \( V \)

To examine this relationship, \( T_o \) will be expressed and plotted as a function of \( V \), and as a partial derivative with respect to \( V \). In this case \( S \) will be used as an independent parameter rather than \( n \) (the two are equivalent since \( n \cdot S = L = \text{const.} \)) because the possibility of reaching \( V \) on an interstation spacing depends on the magnitude of \( S \). The two expressions plotted in Figure 4 are equation (4) and:

\[
\frac{\partial T_o}{\partial V} = \frac{n}{2} \left( \frac{1}{A} + \frac{1}{B} \right) - \frac{n s}{v^2}
\]  

(10)

It can be seen on the diagram that, for any given value of \( S \), \( T_o \) depends on \( V \) only to a certain point \((S = S_c)\), since beyond that point \( S \) is not sufficiently great for the vehicle to attain speed \( V \) \((S > S_c)\). These points are easily obtained by setting the expression for the partial derivative of \( T_o \) with respect to \( V \) equal to 0 and deriving the expression for \( V^* \):

\[
V^* = \left( \frac{2 A B S}{A + B} \right)^{1/2}
\]

(11)
FIGURE 4. OPERATING TIME AS A FUNCTION OF MAXIMUM SPEED
As expected, the reduction in $T_0$ from 5 unit increases in $V$ is the greatest when the initial value of $V$ is small.

Figure 5 shows the range of values of $V$ for the intersection of Figure 1. This range is commonly found in urban transport operations.

3. Sensitivity of $T_0$ to $t_0$

Equation (4) again gives the relationship of $T_0$ and $t_0$, while the sensitivity of $T_0$ is expressed by:

$$\frac{dT_0}{dt_0} = \eta$$

Plots of these two expressions, (4) and (11), for $t_0$, are given in Figure 6. The diagram shows that operating time is highly sensitive to $t_0$.

**FIGURE 5.** MAXIMUM ATTAINABLE SPEED AS A FUNCTION OF STATION SPACING

**FIGURE 6.** OPERATING TIME AS A FUNCTION OF STATION TIME
As expected, the reduction in $T_0$ from a unit increase in $V$ is the greatest when the initial value of $V$ is small.

Figure 5 shows the range of values of $V^*$ for the parametric values commonly found in urban transport operations.

3. Sensitivity of $T_0$ to $t_s$

Equation (4) again gives the relationship of $T_0$ and $t_s$, while sensitivity of $T_0$ is expressed by:

$$\frac{\partial T_0}{\partial t_s} = n \quad (12)$$

Plots of these two expressions, (4) and (12), for different values of $n$, is given in Figure 6. The diagram shows that operating time is highly sensitive

![Diagram showing sensitivity of $T_0$ to $t_s$](image)

**FIGURE 6. OPERATING TIME AS A FUNCTION OF STATION TIME**
to station standing time: a unit change in the time required for passenger boarding, alighting or fare collection on the vehicle leads to a change of n units in the total operating time or 2n units in cycle time. The change in $T_o$ due to a change in $t_s$ is independent of all variables except the number of interstation spacings on the line and is the same for any initial value of $t_s$.

4. Sensitivity of $T_o$ to n (or S)

Since it is assumed that $n \cdot S = L = \text{const.}$, the sensitivity of $T_o$ to $n$ based on equation (4), is simply:

$$\frac{\partial T_o}{\partial n} = \frac{v}{2} \left( \frac{1}{A} + \frac{1}{B} \right) + t_s = t_l$$

This expression represents marginal time loss due to one additional stopping.

Equations (4) and (13) are plotted in Figure 7.

**FIGURE 7. OPERATING TIME AS A FUNCTION OF NUMBER OF STATIONS**
The diagram shows that the time increase due to additional stations is constant as long as the assumption that $S \geq S_c$ holds. Consequently, reduction of the number of stations is an effective method of reducing travel time. This fact is well understood and yet often underestimated in practice.

5. Sensitivity of Travel Time to $t_t$

Terminal time $t_t$ does not affect operating time $T_o$, but it directly affects cycle time $T$, as (5) indicates. Sensitivity of $T$ to $t_t$ is:

$$\frac{dT}{dt_t} = 2.$$  \hspace{1cm} (14)

The diagram of $T = f(t_t)$, shown in Figure 8, is simple, since other parameters have no influence:

![Diagram](image-url)

**FIGURE 8. SENSITIVITY OF TRAVEL TIME TO TERMINAL TIME**
D. APPLICATION OF FINDINGS

The preceding analysis has examined the relative sensitivity of the transit system model to changes in the various parameters. An example based on ranges of values of parameters typical for urban bus lines will illustrate the significance of changes to each of the parameters in a practical situation.

1. An Example

Consider a bus route, length $L = 5$ miles, with the following physical and operating characteristics:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range of Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of interstation spacings - $n$</td>
<td>30</td>
</tr>
<tr>
<td>Stop spacing - $S$ mi</td>
<td>.167</td>
</tr>
<tr>
<td>Standing time per stop - $t_s$ sec</td>
<td>15</td>
</tr>
<tr>
<td>Total delay time - $t_d$ sec</td>
<td>318</td>
</tr>
<tr>
<td>Terminal time at one end - $t_t$ sec</td>
<td>360</td>
</tr>
<tr>
<td>Max. speed - $V$ mi/hr</td>
<td>25</td>
</tr>
<tr>
<td>Acceleration Rate - $A$ m/hr/sec</td>
<td>2</td>
</tr>
<tr>
<td>Deceleration Rate - $B$ m/hr/sec</td>
<td>3</td>
</tr>
</tbody>
</table>

Extreme values of individual parameters which result in lower speed are designated as "low" and v. v. For this analysis "low" values have been adopted as initial on the basis of which all changes of parameters are examined.

For these "low" values one obtains as cycle time $T = 72$ min.; commercial speed $V_c = 8.33$ mi/hr., and operating speed $V_o = 10$ mi/hr.
The diagram in Figure 9 shows the cycle time $T$ on the ordinate. Each parameter is plotted on the abscissa independently at such scale that the range of its value approximately fits the shown diagram. Each curve represents cycle time as it changes due to changes in the respective single parameter; all other parameters remain constant, i.e. retain their "low" values. Therefore, the individual curves in Figure 9 do not represent a vertical combination of the values of parameters plotted along the abscissa. Such a value is shown by the curve marked "composite."

2. **Practical Use of the Diagram**

The diagram in Figure 9 can be very useful in planning improvements for an existing transit service as well as in selecting operational characteristics in planning new systems. If, for example, an operator wants to increase speed of transit services, he can develop a diagram like this one with the existing value of travel time plotted along the ordinate as "low" value. Along the abscissa all parameters which could possibly be changed should be plotted.

With this diagram one can immediately see what reduction in cycle time could be achieved by a given change either in acceleration rate, or speed, standing time, etc., by reading the values on respective curves. For example, in the case diagrammed in Figure 9, a reduction in cycle time $T$ of 2 minutes could be achieved either by reducing terminal time at each end by 1 minute, by increasing acceleration rate from 2.0 to 2.9 mi/hr/sec., or by increasing the maximum speed from 25 to 32 mi/hr.
FIGURE 9. CYCLE TIME AS A FUNCTION OF SYSTEM VARIABLES

L = 5 MILES
V₀ = 10 MPH
Vₖ = 8.33 MPH
Once these alternative improvements are established, costs (and other aspects) of implementing each change must be compared and the lowest cost method for achieving the desired reduction of cycle time can be derived. The following analysis is the bases for such cost estimations.

3. Marginal Cost Analysis

Assume that the total cost in (including both operating and investment amortization) of producing a unit change in one of the above parameters designated as x - can be determined. The cost of a unit change in total cycle time T due to a change in variable x is:

\[
\frac{\delta C_x}{\delta T} = \frac{\delta C}{\delta x} \frac{\delta x}{\delta t} = I_x \left( \frac{\delta T}{\delta x} \right)^{-1}
\]  

(15)

Then the cost of a unit change in T produced by a change in t is:

\[
C_T = I_T/2
\]

(16)

for a change in t,

\[
C_s = I_s/2n
\]

(17)

for a change in V,

\[
C_v = I_v/\left[ n \left( \frac{1}{A} + \frac{1}{B} \right) - \frac{2L}{V^2} \right]
\]

(18)

for a change in A,

\[
C_A = I_A/ \frac{-nV}{A^2}
\]

(19)

and for a change in n,

\[
C_n = I_n/2 t
\]

(20)
In practical applications, the proposals under consideration will rarely produce a unit change in the variables, nor will several proposals produce identical increases in speed (decrease in travel time). If the total cost of providing the change in speed by changing variable \( x \), \( \Delta c_x \), is assumed to be linear for small changes then:

\[
\Delta c_x = \frac{d}{dx} C_x \Delta x,
\]

and an approximate value for \( I_x \) is

\[
I_x = \frac{d}{dx} C_x = \frac{\Delta c_x}{\Delta x}
\]

(21)

If these marginal costs are computed in this way for any existing or planned system, then the costs of each of the considered alternative methods of reducing total travel time by any amount can be immediately estimated and thus the most cost/effective method for the considered improvement may be found.
III. CONSEQUENCES OF SPEED CHANGES

In this chapter a conceptual analysis of the consequences of a speed change is developed. Each of the consequences is then analyzed individually and defined either mathematically or qualitatively.

A. STRUCTURE OF THE PROCESS

If it is supposed that an investment is made to increase transit speed through such action as, for example, reduced stop time at stations, a reduced number of stations, increased acceleration rates, etc., then such an action results in a change of operating cost of the system on one side, and in the increased speed of the system on the other side. Conceptual analysis of the process caused by such an action is presented schematically in Figure 10.

The increased speed permits either reduction of the number of vehicles required for a given service, or increased frequency of service if the number of vehicles is held constant. Also, the increased speed creates a direct benefit to the users on the system by reduction of their travel time. It also attracts additional patronage because of the improved level of service.

Tracing further the consequences of the reduced number of vehicles, one can notice that this can result either in direct savings to the operator, or in reduced fares, if the savings are passed over to the users. Reduced fares therefore represent direct user savings and, on the other hand, they make the service more attractive and thereby result in another reason for increased
FIGURE 10. CONCEPTUAL STRUCTURE OF CONSEQUENCES OF TRANSIT SPEED INCREASES
patronage. This increase in patronage, if substantial, may require increased frequency of service, thereby bringing the whole transit system to a more profitable region of operation.

The increased frequency of service results in users' savings in terms of reduced waiting time and increased convenience; it also results in attraction of additional patronage.

The additional patronage may be either diverted from other systems or induced, i.e., it would represent the travel which would not take place if such an improvement of service was not made.

The increase in patronage caused by these three effects of the speed change results in a number of benefits to the users, transit operator and the community or city as a whole.

The above briefly described process, presented in the chart in Figure 10, is discussed in some detail in the following sections. Each step of this process, called an "event", is described. The benefits and costs, presented in the chart in double frames, are then analyzed in separate sections.

B. EVENTS

1. Speed Increase

The process of increasing transit speed starts, as Figure 10 shows, with an investment (link 1) into an improvement (vehicle, operational, etc). The activity affecting speed usually has a direct impact on service cost (link 2). The change may be an increase or decrease. The increase of speed (link 3), however, causes a chain of operational consequences which further result in
various benefits and costs to the users, system operator and the city or society as a whole.

Before various consequences of speed change are discussed, several different speeds, previously defined in Chapter II, will be discussed.

\( V \) is the technical speed: the maximum sustained speed with which a vehicle runs between stations.

\( V_o \) is the operating or travel speed - average speed of travel on the line. This is the speed which is most significant for the passengers.

\( V_c \) is the commercial or average speed which represents the total distance passed in service divided by the total time in service. This is the speed in which the operator is mostly interested.

The user of the system is concerned, in addition to the speed on the line, with his total travel speed - or, for a given distance, travel time.

His travel time is defined as:

\[
T_u = t_a + t_w + \frac{d}{V_o} + t_f + t_e
\]

(22)

where \( d \) is the distance he travels on the vehicle, and his total travel speed is \( V_u = D/T_u \),

(23)

\( D \) being the total distance between trip origin and destination.

The above expressions show that an increase of \( V \) results in increased \( V_o, V_c \) and \( V_u^* \), so that both operator and users benefit from it. The case is the

\[ \text{The speed and time variables following a speed increase are designated by a prime (e.g. } V_o', t', \text{ etc.)} \]

31
same with increasing A and B or decreasing n and t\_s. Terminal time reduction benefits only the operator, while the user's travel time can also be reduced by decreasing t\_a, t\_w, t\_f, or t\_e. Access and departure times - t\_a and t\_e, respectively - are functions of the spacing of lines and stations and access mode. Decrease of these elements usually requires higher density of stations and/or lines, which results in lower V\_o and service headways. Optimization of these factors has been studied by Vuchic\textsuperscript{6} with respect to interstation spacings and Byrne\textsuperscript{7} for the optimum route spacing problem.

Consequently, speed increase through change of any element except t\_t results in travel time savings of the users P who are on the system at the time of change (link 4 in the chart).

On the other hand, increased speed will attract new passengers from other systems - ΔP\_1 and induce new trips - ΔN\_1 (link 5). The increase in patronage may slightly reduce the initial speed increase due to longer boarding and alighting time. The final equilibrium must, however, be at a higher level of speed (V\_0' > V\_o) and increased number of passengers (ΔP\_1 + ΔN\_1 > 0) in comparison with the initial situation.

2. CHANGE IN FLEET SIZE

As a direct consequence of increased speed the operator has the choice either to reduce the number of vehicles on the line or to increase the frequency of service, as shown by link 6. Choice of the former action (link 6a on the chart) is the only way for the operator to obtain direct benefits and recover part or all of his investment (link 7a). For some given headway h or frequency
of service $f$, the number of required vehicles is:

$$N = \left[ \frac{2L}{hV_c} \right] = \left[ \frac{T}{h} \right],$$  \hspace{1cm} (24)

where $h = 1/f$; square brackets mean integer value $\geq$ the given number.

Thus the number of vehicles is a step-wise non-increasing function of $V_c$, as shown in Figure 11.

**FIGURE 11. FLEET SIZE AS A FUNCTION OF COMMERCIAL SPEED**

Examining the relation of $N$ to $T$ leads to the following figure.
and to the value $\Delta T = \Phi$ as the reduction in cycle time required for reducing
the fleet size by one vehicle. The increase in commercial speed required
for a reduction in fleet size of $x$ vehicles is:

$$\Delta V_C = \frac{xV_C}{N - x}$$

(25)

3. **INCREASED FREQUENCY OF SERVICE**

As mentioned in the preceding section, the operator may elect to main-
tain the existing fleet size and utilize the increase in commercial speed, $\Delta V_C$, to increase the frequency of service by $\Delta f$ (link 6b):

$$\Delta f = \frac{N \Delta V_C}{2L}$$

(26)

The effect of this decision is to reduce the average passenger waiting time,
$\tilde{t}_w$ (link 8).
As in other events where the level of service is improved, it is necessary to consider the impact of increased service frequency on the number of passengers (link 9). This event has a multiplier effect in that the increased patronage may require a further increase of frequency which in turn further increases patronage. The relationship between patronage and frequency is:

\[ f = \frac{P_{\text{max}}}{\alpha C_v} \]  \hspace{1cm} (27)

where \( P_{\text{max}} \) - Number of passengers at maximum load point,
\( \alpha \) - vehicle utilization factor,
\( C_v \) - vehicle capacity.

The increased patronage may be defined as:

\[ P'_{\text{max}} = P + \Delta P_3 + \Delta N_3 \]  \hspace{1cm} (28)

which may lead to a requirement for either a further increase in frequency, \( f'' \), or an increase in the fleet size since

\[ f = \frac{N' \frac{V_c}{L}}{\frac{P_{\text{max}}}{\alpha C_v}} \]

so that

\[ N' = \frac{2 L P'_{\text{max}}}{\frac{V_c}{\alpha C_v}} \]  \hspace{1cm} (29)

Except for the rare cases of highly inefficient and deficitary service this increase of fleet cannot be considered as additional cost since the cost of additional vehicles would be more than offset by the increased patronage.
Another, possibly important, event resulting from the operator's decision to increase the frequency of service is the increase in line capacity $C_L$ and improvement of the level of service (as expressed in the utilization factor, $\alpha$):

\[
C_L = P_{\text{max}} = \alpha C_v f; \text{ and}
\]

\[
\Delta C_L = \alpha C_v (f' - f), \text{ or}
\]

\[
\Delta \alpha = P_{\text{max}} / C_v (f' - f). \tag{30}
\]

Thus it is possible to increase both the line capacity and to some extent the comfort of users (reduced utilization coefficient-fewer standees) through the increase in speed.

It is significant to point out that the operator has the choice in decision 6 to either take the savings from increased $V_c$ for himself by decreasing the fleet (he can then decide what to do further with the savings) - 6a - or to pass the savings to the users by increasing frequency of service (from which he may still benefit indirectly through increased patronage) - 6b.

If the speed increase is very significant, he may also have the choice to combine the two changes, i.e. reduce the fleet and increase frequency.

4. FARE CHANGE

As a result of the operator's decision to reduce the number of vehicles, certain cost savings will be available either for collection as a benefit to the operator or for distribution to present users in the form of reduced fares.
The operator's decision is indicated by a "trade-off switch," shown as relationships (7a) and (7b) in the chart.

Because fare structures currently used by most urban transport agencies are based on coin denominations, it is conceivable that the savings may not be large enough to allow a reduction of even the smallest coin in common use. In the United States, fares are generally set in five-cent multiples, which means that the minimum cost saving that could be distributed as a fare reduction is:

\[ CS = 0.05 P - \Delta P_2 \left[ F' - C \left/ (P + \Delta P_2) \right. \right] \]  \hspace{1cm} (31)

where

- \( \Delta P_2 \) is increased patronage due to the fare reduction,
- \( F' \) is the reduced fare, and
- \( C \) is the annual cost of operating the line.

The annual cost of operation can be expressed as:

\[ C = K_1 c_1 h_d N \] \hspace{1cm} (32)

where

- \( k_1 \) is the number of workday-equivalents per year,
- \( c_1 \) is the cost of a vehicle hour (including depreciation and proportionate share of fixed costs),
- \( h_d \) is the average hours of service per vehicle per day.
Alternatively to decreasing fares, the operator can transfer some of his gains from reduced costs to system users by maintaining existing fares in an inflationary economy. It is doubtful that this has the same impact on ridership as a reduction in fares.

5. INCREASED PATRONAGE

The precise determination of the demand for transportation, either total or for any particular mode, presents the greatest challenge to the transportation planner or operator. In this research emphasis is placed on the demand for public transportation, involving principally users diverted from other modes, although it is recognized that improvements may reduce the total time and money cost of tripmaking to the point where latent demand becomes manifest in the form of new transport users. This latter case is represented by $\Delta N_1$, $\Delta N_2$, and $\Delta N_3$ in Figure 10.

Users who are diverted from other modes are designated as $\Delta P_1$, $\Delta P_2$, and $\Delta P_3$, the subscripts indicating the reason for diversion.

a. Demand Elasticity. Classical demand theory defines point elasticity as:

$$E = \frac{d Q}{d P} \cdot \frac{P}{Q}$$

where $P$ and $Q$ represent respectively the price and quantity demanded at some instant; $dQ$ and $dP$ are the small changes in $Q$ and $P$ at point $(Q, P)$. A more useful concept for this work is that of arc elasticity:

$$E_a = -\frac{\frac{Q_1}{P_1} - \frac{Q_o}{P_o}}{\frac{P_1}{Q_1} - \frac{P_o}{Q_o}} \cdot \frac{P_1 + P_o}{Q_1 + Q_o} = \frac{\Delta Q}{\Delta P} \frac{P_1 + P_o}{Q_1 + Q_o}.$$
which is the "average" elasticity over the range of Q and P values through which the changes occur. It is of course possible to determine the elasticity of demand with respect to other variables than price, particularly time.

Numerous attempts have been made to estimate the elasticity of demand for public transportation and to estimate the modal split within the total transportation demand. The factors considered significant are relative travel time, relative travel cost, relative level of service, economic status of the trip-maker and trip purpose. These five factors were used by Hill and Von Cube in their forecasting models for Washington, D.C. and various combinations of these variables have formed the basis for most modal split analyses. For purposes of this study the relevant variables are travel time, cost (fares) and relative level of service (as measured by frequency of service).

b. Patronage Change. The sources of increased patronage are identified, with subscripts indicating the reason for their attraction to the system, as follows:

\[ \Delta P_1, \quad \Delta N_1 \] - passengers attracted because of increased speed of the system

\[ \Delta P_2, \quad \Delta N_2 \] - passengers attracted because of a fare reduction

\[ \Delta P_3, \quad \Delta N_3 \] - passengers attracted because of increased frequency of service.
The rationale for disaggregating $\Delta P_3$ and $\Delta N_3$ from $\Delta P_1$ and $\Delta N_1$ is because of the difference in the imputed values of walking and waiting time vs. riding time and because of the observed reluctance of people to walk, wait or make transfers. Lisco found in analyzing Skokie Swift data that "commuters appear to be willing to spend three times as much to save walking time (at least at the margin), as they are to save time in their cars or in mass transit....."

Relatively little data are available indicating the responsiveness of demand to decreases in travel time on public transit systems, perhaps because of the rarity of that event during the last several decades. This is particularly true of surface transit systems which have been adversely affected by the general long term increase in congestion and consequent reduction in speed. Curtin includes diversion curves which imply a highly elastic relation between ridership and time saved for the modal split of transit patrons between rapid and surface transit. For example, with no fare differential, a reduction in travel time (saving) of 2 minutes for a 10 minute trip results in a diversion corresponding to $E_a = 4.1$. The range of arc elasticities was estimated to be between 1.0 and 10.0.

New facilities, such as the Lindenwold Line, Philadelphia, Jersey Arrow, Metroliner, Skokie Swift and BARTD may build impressive ridership by offering higher speed (Lindenwold Line has over 35,000 daily riders after 2 years operation in a corridor previously accessible only by bus and automobile).
Their impact, however, is a result of a complex combination of fares, speed, amenities, image, and system promotion so that the responsiveness of demand to the increased speed is difficult to ascertain. It seems reasonable to conservatively assume that a well advertised speed increase can lead to $\Delta P_1 + \Delta N_1$ corresponding to an $E_a = 1.0$ to $5.0$. The higher values would pertain to speed increases which reduce transit travel times below existing automobile travel times.

The problem of evaluating the responsiveness of demand to fare decreases, link 10, may be attacked from an analysis of the response to fare increases. Curtin$^{10}$ analyzed the "shrinkage" in ridership due to 77 fare increases and found it to be relatively inelastic with respect to fares (0.3 to 0.8). As would be expected for $E_a$ values less than 1.0, Curtin found that for fares below $0.35$ the increases in fares produced increased net revenues for the operators.

Lassow$^{11}$ analyzed the 1966 fare increase in New York City and found that diversion from rapid transit (overall composite) was much more inelastic (0.08) than for surface transit (0.30). Of particular note was the fact that shrinkage in low income areas was greater than in high income areas where peak-hour ridership actually increased.

There is some question as to whether the above relations are reversible. The only reference to a fare decrease is reported by Curtin$^{10}$ for Iowa City where fares were halved and ridership doubled, a case of unit elasticity.
One of the difficulties in analyzing diversion due to fare increases is the fact that the fare represents only a portion of the user's total cost. Therefore a cash fare increase of 50% may represent a total cost increase of only 10%, in which case the shrinkage may indicate a unit elasticity or even higher.

Considering the questions of elasticity of demand with respect to decreased fares, a reasonable estimate of values appears to be that $E_a = 0.3$ to 0.9.

Estimating the increased patronage resulting from an increased frequency of service, link 9 is even more difficult than for the two previous cases. One problem to be considered is the perceptibility of a change in headways, particularly on surface lines where the variance in headways is large.

C. BENEFITS

The impact of changes in transit service examined in the preceding section on the users, the operator and the community is broadly referred to as benefits. These benefits are designated by double line boxes in Figure 10.

The following discussion will attempt to identify the various benefits, (links 4, 7a, 8, 11 and 12), either savings in time or money (i.e. decreased costs) or increases in accessibility, mobility and economic activity. The benefits are classified by the different affected groups.
Associated with each of the benefits to be discussed in some "cost", representing a negative effect of speed increase, such as increased access time or air pollution, reduced net revenues, etc. A complete analysis of the consequences of speed increases on the line will therefore determine the direct and indirect costs to the users, the operator and the community during the process of calculating benefits.

1. **User Savings**

The analysis of user savings is based on the assumption that transportation users are willing to pay certain total cost for travel including fares, time, irritation (lack of comfort and convenience) and the risk of accidents. Further it will be assumed for purposes of this specific analysis that:

a. all costs and benefits may be expressed in monetary terms;

b. there is no difference in savings due to differences in user income levels; and

c. demand for transport is a monotonic inverse function of price.

The assumption that money and time saved have the same value for all persons is necessary to preclude judgment as to the manner in which each person spends them; it is not to say that the amount of time or money each person is willing to devote to a particular trip is the same for all people.

These concepts can be diagrammed as a classical demand curve shown in Figure 13, where $\gamma$ is the average value of user time.
This section will cover benefits accruing only to patrons of the line prior to a change. Users attracted to the line following the change will be treated in the following section.

The shaded area indicates consumer surplus or the difference between that cost which a user would be willing to pay and the actual total cost of the trip. An increase in consumer surplus is regarded as a benefit.

The major benefit to users would be the time saved by higher operating speed shown as link 4 in the chart. The time saved per year is

$$TS_1 = P \bar{d} \left( \frac{1}{V_o} - \frac{1}{V'_o} \right)$$

(33)

where $P$ is patronage per year prior to speed change

$\bar{d}$ is average trip length on the transit system

$V_o$ and $V'_o$ are, as given above, operating speed before and after the speed change, respectively
This time saving may be translated into monetary units using appropriate dollar values of user riding time, $\gamma_r$. Thus the annual benefit from increased speed is

$$B_{u1} \ [\$/\text{year}] = \gamma_r \ P \bar{d} \left( \frac{1}{V_o} - \frac{1}{V_o'} \right). \quad (34)$$

Preliminary numerical examples using $\gamma_r = \$1.55$ person/hr indicate that potential savings due to relatively modest speed increases may be quite significant.

Further user savings may be realized through reductions in waiting time due to increased frequency of service (link 8). These savings can be expressed as

$$TS_2 \ [\text{per-hrs/year}] = P \left( \frac{h - h'}{2} \right). \quad (35)$$

This expression is valid for an assumption of either uniform or Poisson distributions of passenger arrivals and typical transit headways. For the case where vehicles (trains) operate on a published schedule with sufficiently large headways to encourage passenger arrivals for particular trains, little if any time saving will accrue to the user. He will rearrange his total trip to the new schedule; however, his convenience of selecting trip time increases, which is not conducive to quantification.

The total annual user benefits from decreased waiting time are:

$$B_{u2} \ [\$/\text{year}] = \gamma_w \ P \left( \frac{h - h'}{2} \right), \quad (36)$$
where $\gamma_w$ is the unit value of user waiting time in dollars, which may be different from the value of user riding time. It should be noted that the above analysis assumes a stable system with only a small variance in waiting time. In systems experiencing large random delays, $\gamma_w$ will be a function of the user's perceived waiting time, not a constant.

Some changes in the line's operation, e.g. changing station or stop locations, will affect several time elements and the benefit to users will be the net change resulting from possible trade-offs between access time and riding time:

$$B_{u3} \ [\$/\text{year}] = B_{u1} + B_{u2} + P_a \gamma_a \left[\bar{t}_a + \bar{t}_e\right] - \left(\bar{t}_a + \bar{t}_e\right), \quad (37)$$

where $\gamma_a$ is the unit value of access time, $\bar{t}_a$, $\bar{t}_e$ etc. represent the average values for access and exit times, and $P_a$ is the number of passengers/year affected by the change in access and exit times. In some cases one or more of the terms in equation (37) may be negative.

User benefits from fare reductions (link 11) will vary with the fare structure used. For a flat fare system user benefits will be

$$B_{u4} \ [\$/\text{year}] = P (F_1 - F'_1) \quad (38)$$

for zone fare systems with $k$ zones,

$$B_{u5} \ [\$/\text{year}] = \sum_{i=1}^{k} \sum_{j=1}^{k} P_{ij} (F_{ij} - F'_{ij}) \quad (39)$$
and for systems with fares based on mileage,

\[ B_u 6 \left[ \text{$/year} \right] = P \left( F_2 \bar{d} - F'_2 \bar{d}' \right) \]  

(40)

where \( F'_2 \) and \( F'_2 \) represent the average fare per mile traveled and \( \bar{d} \) and \( \bar{d}' \) represent the average user trip length on the system.

2. New Users

New trips on the system, i.e. person trips which have been attracted to it due to the speed increase, can be divided in two categories: diverted trips and generated trips.

For the diverted trips it can be assumed that the persons have been attracted to the service because the total cost of travel on it (as defined in IIIC-1 above), has become lower than the total cost on the system they had been using. With the assumption that each new user will be attracted to the system when TC equals the total "price" he is willing to pay, the shaded area, A, in Figure 14 (in this case due to a fare reduction, reduction in time is similar) represents the consumer surplus for the new users, diverted from other modes, \( \Delta P_j \). This can be approximated by

\[ B_{ni} \approx \frac{1}{2} \Delta P_j \left( Tc - Tc' \right), \]  

(41)

which leads to a set of equations analogous to (34) and (36)through (40), by substituting \( \Delta P_j /2 \) for \( P \) in those expressions. The subscript \( j \) refers to the appropriate reason for passenger attraction defined in section IIIB-5, above.
Figure 14 also indicates the previously considered benefit to present users $P$, area $B$, which may be calculated by equation (38), (39), or (40) according to the fare structure used.

For the second group - generated trips by users designated $\Delta N_j$ - it may be assumed they had not made trips before the change in speed because the total "price" was higher than they were willing to pay. For these new users the trip value equals the time and money invested plus increased mobility and opportunity. Since benefits have been defined as the increase in consumer surplus, this group of users have negligible or no benefits from the change in speed; they pay for the full value of the trip. Their increased mobility may, however, be considered as an indirect and not necessarily negligible benefit to them as well as to the community, to be treated in section IIIC-4.
3. **Operator Benefits**

The following discussion will treat only those benefits perceived by the operator.

The most direct benefit to the operator comes from a reduction in fleet size; the annual saving for a single transit line may be expressed as:

\[ B_{01} \text{ \$/year} = h_e K_1 (Nc_1 - Nc'_1) \]

\[ = 2 L f \left( \frac{c_1}{V_c} - \frac{c'_1}{V'_c} \right) h_e K_1. \]

The cost per vehicle-hour, \( c_1 \), includes both the short-run costs of driver wages, fuel, maintenance and a portion of fixed operating costs as well as a share of long-run investment costs (depreciation). \( B_{01} \) therefore represents both long and short run benefit. It does not include however the benefit to the operator from being able to maintain the same level of service without purchasing new equipment or to increase the level of service at a lower investment level.

For surface transit lines, operating in mixed traffic there may be some savings in fuel and maintenance costs as running speed approaches a more efficient vehicle operating range, and the number of stops due to street congestion is reduced. Available data aggregates these savings with the larger savings due to the reduction in fleet size, expressed in terms of operating cost/mile, \( V_c^{-1} \), as a function of slowness, \( V_c^{-1} \).

The Transit Research Foundation has reported a number of these curves (see section III D), which are linear in the form
\[ v_o \quad [\$ \text{/mile}] = K_2 \left( 1 / v_c \right) + K_3, \]  
(43)

where \( K_2 \quad [\$ \text{/min}] \) and \( K_3 \quad [\$ \text{/mile}] \) are regression analysis coefficients.

This expression indicates a decrease in bus-mile costs with increasing speed, but because of the \( K_3 \) term, an equivalent expression for costs/bus-hour shows an increase with respect to increasing speed,

\[ \hat{c}_1 \quad [\$ \text{/hour}] = v_c \quad v_o = 60 \quad K_2 + K_3 \quad v_c \]  
(44)

Further research should be directed towards developing more accurate non-linear expressions for vehicle-mile or vehicle-hour costs.

Of course, as indicated in link 7b in the chart, the operator may distribute all or some share of his benefit from reduced fleet size to the users in the form of reduced fares. In this case, as in the case of increased speed, the operator looks for increased revenue resulting from increased patronage as a means of wholly or partially offsetting the costs of speed increase. For a fare reduction, the operator's annual benefit (net revenue increase) would be

\[ B_{o2} \quad [\$ \text{/year}] = (P + \Delta P_2 + \Delta N_2) \quad F' - PP; \]  
(45)

It should be noted that \( B_{o2} \) may be negative if the fare reduction does not attract a sufficient number of new users. For changes in other parameters, the operator's expected net revenue increase or benefit will be

\[ B_{o3} \quad [\$ \text{/year}] = (\Delta P_i + \Delta N_i) F + C - C' \]

\[ = (\Delta P_i + \Delta N_i) \quad F + B_{o1}. \]  
(46)
In conclusion, the operator generally benefits from speed increase. There may, however, be cases when his benefits will be negative, which will typically happen if he passes the savings over to the users. This will be the case particularly where changes are made on the basis of a broad-based benefits analysis which puts special emphasis on user and community goals.

4. Community Benefits

Several groups of citizen's make up the beneficiaries identified generally as the community. These groups may be classified as those who benefit directly and those who benefit indirectly.

The direct beneficiaries are users of other transportation systems and commercial vehicle traffic. The indirect beneficiaries are the city's retail firms and those citizens and institutions who share the urban environment and support its activities as taxpayers; basically, all those who are affected by the overall urban mobility and/or suffer from negative side effects of transportation systems.

In order to identify certain groups who benefit from improved transit service, the total number of travelers in the corridor served by the considered transit line before the increase in speed will be designated \((P+Q)\); \(Q\) represents the users of all other systems. Following an increase in speed the patronage of the improved line will be \(P + \Delta P_i\), patronage of the other modes will be \(Q - \Delta P_i\), neglecting for this discussion the generated traffic \(\Delta N_i\). Two cases must be considered.
In the first case the investment in changes to increase the transit speed results in an increase in the operating speed $V_Q'$, of other modes in the same corridor. This case of course applies to surface transit operating in mixed traffic and leads to

$$B_{c1} \ [\$/\text{year}] = \gamma_Q \left( Q - \Delta P_1 \right) d_Q \left( \frac{1}{V_Q} - \frac{1}{V_Q'} \right),$$  \hspace{1cm} (47)$$

where $\gamma_Q$ is the unit value of time for travelers $Q$, and $d_Q$ is the average distance traveled on the affected route. In some instances (e.g. provision of separate bus lanes) $B_{c1}$ may be negative and must be considered as it will reduce the value of the user benefits.

The second case is that in which the change of speed on the transit line (e.g. rapid transit) has no direct effect on the speed or total cost of travel by other modes in the corridor. There may be some long run improvement due to diversion from surface traffic to the improved transit line. If it is assumed that Wardrop's First Principle is applicable here, the users of alternate modes paralleling the improved transit line will divert to transit until the total cost of travel (time and money) again becomes equal for all modes. Thus the benefit to those who remain on the alternate modes is

$$B_{c2} \ [\$/\text{year}] = (Q - \Delta P_1) (TC_1 - TC'_1),$$ \hspace{1cm} (48)$$

where $TC_1$ and $TC'_1$ are the weighted average total trip costs for the alternate modes before and after the speed change, respectively. Reduced street congestion is included in ($TC_1 - TC'_1$).
Commercial vehicles comprise a second group of non-transit users which may benefit from increased speed of surface traffic. The benefit to this group is:

$$B_{c3} [\$/year] = N_c \gamma_c u \bar{d}_c \frac{1}{V_Q} - \frac{1}{V_Q},$$

(49)

where: $\gamma_c$ - the value "added" per hour of time saved;

$N_c$ - number of commercial vehicles per year using the affected route;

$u$ - utilization factor;

$\bar{d}_c$ - average length of trip on affected route.

Haning\textsuperscript{12} analyzed the operations of four U.S. motor carrier groups: general freight common carriers; commodity common carriers; contract carriers and private carriers. He defined $\gamma_c$ as the net revenue plus certain selected expenses for a "composite vehicle", combining the weighted average values for the four classes. The utilization factor, $u$, reflects the trucking firm's inability to collect the total benefit from a time reduction because (1) it may be too small to allow fleet size reduction, (2) the fleet may be currently underutilized or (3) operators' wages may be paid on a mileage rate basis rather than time rate. Haning's research implies that the range of $u$ should be 0.67 - 0.86. The value of $\gamma_c$ in 1961 was estimated at $4.74/hour.

Indirect benefits are difficult to define quantitatively, but continued effort should be made to include as many items as possible in the evaluation of projects. One such benefit is the increased economic activity in retail businesses provided with increased accessibility; another is the benefit
to employers from availability of more potential employees caused by increased mobility in the area. Those persons who because of age, handicap, or economic condition are restricted to public transit riding will benefit from increased opportunities for commerce and employment.

All citizens who inhabit the urban area benefit from an improvement in the environment such as a reduction in air pollution and noise which may result from diversion of auto drivers to transit. Because surface transit vehicles more efficiently utilize city streets including parking requirements than do private automobiles, diversion to transit reduces the required street and parking area and, indirectly, city taxes.

Summary and Conclusions

Speed has been recognized as the dominant characteristic of transit services - or passenger transportation systems in general, for that matter. Efforts are often made to increase the speed of existing and provide high-speed new systems; on the other side, decrease of speed (usually due to street congestion) is known to increase costs of transit service. Quantitative, or at least systematic estimates of the costs and benefits as functions of speed have been seldom made. The exceptions include London Transport which has regularly made estimates of the effect of congestion on its services and some metropolitan area transportation studies which developed and utilized some relationships between costs and speed.
This study has attempted to conceptually clarify this relationship of costs and speed by tracing the consequences of an incremental speed change. The major result of this analysis, presented in Figure 10, is the conceptual organization of all the consequences through different "events", terminating the chain in a set of benefits.

Traditionally, decisions about changes in public transport service have been evaluated on the basis of perceived operator benefits, which include savings in operating and investment costs and increases in net revenue. The operator has generally not been able to evaluate those benefits which he has been unable to recover directly such as those accruing to existing users or patrons of other modes. Classification of benefits and costs by the affected groups consisting of users, operator and community has been performed here, resulting in some new insights.

In addition to the general conceptual structuring of the consequences of speed changes this study has produced the basic computational equations for quantitative estimates of individual events, benefits and costs. Although data have been collected from Philadelphia for application of this model to an actual situation, this task has not been included in the scope of this research. It represents, however, one of its logical continuations.

Another extension of this work is an analysis of possible methods for speed increases which heavily draws on the results of research reported here. This study of methods is currently under way as a new research project.
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