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NOTE: At the time of publication, author Daniel Koditschek was affiliated with the University of Michigan, Ann Arbor. Currently, he is a faculty member in the Department of Electrical and Systems Engineering at the University of Pennsylvania.

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1 Sample Problem Statement: Hand-Eye Calibration

1.1 Ingredients

1.1.1 Open Kinematic Chain

Any single degree of freedom spatial joint can be parametrized by four *link parameters*, $\alpha_{i-1}, a_{i-1}, \theta_i, d_i$, in the standard Denavit-Hartenberg form [2]

$${}^{i-1}T_i = R_X(\alpha_{i-1})D_X(a_{i-1})R_Z(\theta_i)D_Z(d_i),$$

where, for example, $R_X(\alpha)$ is a rotation of magnitude α around the local X -axis, and $D_Z(d)$ is a translation of magnitude d along the local Z -axis. One of the parameters is the *joint variable*, q_i , and the other three comprise the *static link parameter* three-vector, p_i .

An m -d.o.f. kinematic chain can be parametrized using the product of m such frame transformations,

$$w = g_p(q) := \left(\prod_{i=1}^{m+1} {}^{i-1}T_i \right) \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad q := \begin{bmatrix} q_1 \\ \vdots \\ q_m \end{bmatrix}, \quad p := \begin{bmatrix} p_1 \\ \vdots \\ p_m \end{bmatrix},$$

where q, p , denote, respectively, the jointspace and parameter-space points.

1.1.2 Stereo Camera Pair

The stereo projective transformation, $\mathbf{p}_k : \mathbb{R}^3 \rightarrow \mathbb{R}^4$, associates with each spatial point, w , a vector, c , of paired (“left” and “right” camera-) image plane points. Specifically, let Π, π denote the projections from \mathbb{R}^4 that pick out, respectively, the first two, and the third coordinate, of a homogeneous representation of a point. The camera transformation may be written as

$$c = \mathbf{p}_k(w) := \begin{bmatrix} \Pi(w)/l_1 \pi(w) \\ \Pi(H_0 w)/l_2 \pi(H_0 w) \end{bmatrix},$$

where (l_1, l_2) denote the focal lengths, and H_0 relates the frame of one camera to that of the other (chosen, for convenience, as the “world” frame in this problem). Denote this set of parameter values with the symbol $k = (l_1, l_2, H_0)$. This function admits a (family of) pseudo-inverse(s), $\mathbf{p}_k^\dagger : \mathbb{R}^4 \rightarrow \mathbb{R}^3$, with the property that $\mathbf{p}_k^\dagger \circ \mathbf{p}_k$ is the identity transformation of \mathbb{R}^3 .

1.2 Calibration as Hill Climbing

1.2.1 Calibration Problem Statement

One obtains an *input-output pair* of data, (Q, C) by associating the image plane coordinates of a fixed point on the arm, C , with the joint readings sensed, Q , when the picture was taken. The task at hand now is to acquire a *data set*,

$$\mathcal{D} = \{(Q_j, C_j)\}_{j=1}^n$$

of many input-output pairs and use it to identify the parameter vector, (p, k) .

1.2.2 Some (Nonlinear) Least-Squares Problems

A procedure proposed by Hollerbach [1], calls for a Newton-like numerical descent algorithm on the “error” cost function

$$\epsilon(p, k) := \sum_{l=1}^n \|\mathbf{p}_k^l(C_l) - g_p(Q_l)\|^2.$$

When we attempted to implement this procedure for the three degree of freedom Bühgler arm [5] we found it to be extremely sensitive numerically.

Instead, we have had great success with a variant on this idea that substitutes a cost function in the stereo camera image space,

$$\epsilon(p, k) := \sum_{j=1}^n \|C_j - \mathbf{p}_k \circ g_p(Q_j)\|^2,$$

for the previously defined workspace objective.

1.2.3 Motivation for Sequential Versions

The robot and its environment are constantly changing. Yet accuracy is always important. Calibration must be re-done everytime the physical hardware is shaken or moved. Procedures must be very fast and continually re-started. Since the parametrizations obtain from idealized (approximate) physical models it may prove better in practice to tune calibration parameters using different data sets collected over different operating regimes and to re-tune each time the machine enters or leaves such characteristic regimes

2 Sequential Estimation Techniques

2.1 Sequential Techniques for Linear-in-Parameters Problems

2.1.1 Linear-in-Parameters Problems

An estimation problem,

$$y = f_p(u),$$

involving sensed input-output data pairs (x, y) and unknown parameter vector, p is *linear-in-parameters* if the function

$$F(p, u) := f_p(u)$$

is linear in its first argument, and, thus, may be written,

$$F(p, u) = A(u)p$$

where A is a matrix valued operator.

2.1.2 Sequential Descent

For linear-in-parameters problems with bounded inputs, gradient descent on the “time varying” cost function,

$$\epsilon_j(x) := \frac{1}{2} \|F(x, u_j) - y_j\|^2 = \frac{1}{2} \|A(u_j)[x - p]\|^2 \quad (1)$$

that is to say, an adjustment rule, r_j of the form

$$x_{j+1} = r_j(x_j); \quad r_j(x) := x - \lambda \text{grad}_x \epsilon_j \quad (2)$$

results in the steady decrement of parameter error,

$$\delta(x) := \frac{1}{2} \|x - p\|^2, \quad (3)$$

when the scalar constant λ is sufficiently small.

2.1.3 Descent and Convergence Arguments

The Lyapunov-like function,

$$\begin{aligned}\Delta_j(x) &:= \delta(r_j(x_j)) - \delta(x) \\ &= \frac{1}{2}\lambda^2 \|A^\top A(u_j)[x - p]\|^2 - \lambda [x - p]^\top A^\top A(u_j)[x - p]^2 \\ &= \lambda [x - p]^\top A^\top [A^\top A\lambda/2 - I]A[x - p]^2\end{aligned}$$

is non-positive at every stage j for any x , assuming λ is small enough relative to the bound on $\|A(u_j)\|$ imposed by the bound on $\|u_j\|$. This guarantees that the estimate, x , of p , can only improve. Further arguments may be used to show that the sequence of estimates, $\{x_j\}_{j=1}^\infty$ converges to p in the limit from any initial guess, x_0 , when the data set, \mathcal{D} , is “sufficiently rich.” These arguments are standard in the adaptive systems literature [4].

It is worth noting that the state error, ϵ (1) and parameter error δ (3) use the same Euclidean norms: in general, the one must be “matched” to the other.

2.1.4 Remarks

Although the linear-in-parameters techniques and their convergence arguments have been “standard” for more than a decade, they are worth a great deal of study. The guaranteed success of following such a “data-driven” path along a hill regardless of the particular sequence of data and the initial placement of the estimate on the hill contrasts markedly with the general situation in **nonlinear** regression problems. For example, it is widely known that tuning the parameters in a neural network generally takes a certain amount of artistry.

2.2 Parameter Estimation on Lie-Groups

2.2.1 Log-Linear Problems

If

$$F(p, u) = \exp \{a^T(u)p\}; \quad a^T(u)p = \sum_{i=1}^m a_i(u_i) \cdot p_i; \quad a_i(u_i) \in \mathbb{R}^k.$$

then a very similar procedure to the one outlined above works equally well.

By setting

$$\epsilon_j(x) := \frac{1}{2} (1 - \exp \{F(x, u_j) - y_j\})^2 \quad (4)$$

and following the previous logic one obtains a parameter adjustment rule, r_j , of the form (2) where λ now includes a normalization term of the form

$$\lambda_j(x) := \frac{\lambda_0}{1 + |\text{grad}_x \epsilon_j|} = \frac{\lambda_0}{1 + |(1 - \exp \{F(x, u_j) - y_j\}) \cdot (F(x, u_j) - y_j) a(u_j)|}$$

Again assuming that the sequence of inputs is bounded and that λ_0 is sufficiently small in relation to the bound on $\|a\|$ so induced it follows that Δ is non-positive for all possible parameter estimates, x owing to the sign definiteness of

$$(x - p) \cdot \text{grad}_x \epsilon_j = a^T[x - p] \exp \{a^T[x - p]\} (1 - \exp \{a^T[x - p]\}).$$

2.2.2 Lie Groups

Crudely speaking, a *Lie group* is a manifold (a smooth set that may not be identifiable — that is, put in smooth and smoothly invertible one-to-one correspondence — with any linear vector space) obtained by “exponentiating” a *Lie algebra* (a vector space endowed with a particular kind of bilinear product).

For example, the planar rotations result from exponentiating the one dimensional linear vector space of skew symmetric matrices on \mathbb{R}^2 :

$$SO(2) = \left\{ \exp \left\{ \begin{bmatrix} 0 & \theta \\ -\theta & 0 \end{bmatrix} \right\} : \theta \in \mathbb{R} \right\}.$$

Note that the planar rotations may be identified with the circle on the plane but *not* with the line. Similarly, the spatial rigid transformations result from exponentiating a six dimensional linear vector space and results in a non-Euclidean manifold [3].

2.2.3 Linearly Parametrized Lie Groups

A *linearly parametrized Lie Group* results when the functions a_i defined in the previous section take their range in a set that may be identified with some linear vector space and when the dot product with the parameter component p_i defines a bilinear function into a Lie algebra.

2.3 Commutativity Affords the Log-Linear Technique

When the Lie Group is *commutative* then

$$\exp \left\{ \sum_{i=1}^n a_i(u_i) \cdot p_i \right\} = \prod_{i=1}^n \exp \{ a_i(u_i) \cdot p_i \}$$

and the logic of the Log-linear parametrization may be extended in a straightforward manner by suitable choice of ϵ_j .

2.4 Central Un-solved Problem

When commutativity fails then it is not clear at present how to go about choosing a suitable candidate for ϵ_j (??) and its match, δ , (3).

3 Applications

3.1 Solved

3.1.1 Planar Revolute Machine Tool Calibration

If a planar machine tool possesses only revolute joints then these techniques go through in a straightforward manner. There seems to be some additional utility in considering deviations from the standard Denavit-Hartenberg convention [6]. For example, for a two degree of freedom machine one might consider a parametrization of the form

$$g_p(q) = \begin{bmatrix} \cos(p_{10} + p_{11}q_1 + p_{12}q_2 + p_{13}q_1^2 + p_{14}q_1q_2 + p_{15}q_2^2) \\ \sin(p_{20} + p_{21}q_1 + p_{22}q_2 + p_{23}q_1^2 + p_{24}q_1q_2 + p_{25}q_2^2) \end{bmatrix}.$$

More generally, any argument of the transcendental functions in which the appearance of the parameter vector, p is linear will go through.

3.1.2 Robot Hand Coordination

If, in Section 1, the camera model is omitted, and one postulates the availability of independent Cartesian measurements of the gripper then a poorly behaved variant of these ideas will go through. Namely, if one treats the link parameter matrices as elements in the vector space \mathbb{R}^{12} then the problem is linear-in-the-parameters. Of course, this procedure badly over-parametrizes and one might well expect numerical problems in implementation.

Instead, it would be preferable to identify the four minimal link twist, length and height parameters. Unfortunately, since the spatial rigid transformations are not commutative, the presently available techniques for sequential parameter estimation will not work.

3.2 Un-Solved

3.2.1 General Planar Machine Tool

Since the group of rigid transformations is not commutative (even on the plane), the presently available algorithms for sequential parameter estimation will not work.

3.2.2 Robot Hand-Eye Calibration

As soon as the stereo cameras are added into the spatial robot model, e.g., as in Section 1, the techniques discussed here cannot work. The stereo projective transformation does not lose information (suitable pseudo-inverses p_k^\dagger may be defined) but it does not preserve the structure of a Lie Group.

3.2.3 Artificial Neural Networks

They depart dramatically from the Lie Group structure: they do not even give rise to invertible maps in general. Could a sequential descent technique nevertheless be developed with some guarantees of uniform descent?

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