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Hierarchical Feedback Controllers for Robotic Assembly

Daniel E. Koditschek

University of Pennsylvania, kod@seas.upenn.edu

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NOTE: At the time of publication, author Daniel Koditschek was affiliated with Yale University. Currently, he is a faculty member in the Department of Electrical and Systems Engineering at the University of Pennsylvania.

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Hierarchical Feedback Controllers for Robotic Assembly

Abstract

How is navigating a unicycle amidst obstacles like manipulating beads on a necklace? This paper addresses that question by introducing a simple control system that appears to offer a common dynamical model for both situations. System (1) consists of a trivially controlled set of “steering” state variables and an additional set of “body” state variables that can be “driven” in a direction determined by the steering variables. The task will be to bring the state of the system to some desired equilibrium using feedback. A defect in the “steered” vector field precludes the possibility of stabilization via a single continuous feedback controller - the essence of what feedback is all about - incurs the necessity of a hierarchical system.

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Comments

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Hierarchical Feedback Controllers for Robotic Assembly

Daniel E. Koditschek *

Center for Systems Science
Yale University, Department of Electrical Engineering

1 Introduction

How is navigating a unicycle amidst obstacles like manipulating beads on a necklace? This paper addresses that question by introducing a simple control system that appears to offer a common dynamical model for both situations. System (1) consists of a trivially controlled set of “steering” state variables and an additional set of “body” state variables that can be “driven” in a direction determined by the steering variables. The task will be to bring the state of the system to some desired equilibrium using feedback. A defect in the “steered” vector field precludes the possibility of stabilization via a single continuous feedback controller. Persisting in the desire for a purely event driven controller — the essence of what feedback is all about — incurs the necessity of a hierarchical system.

The apparently different settings of Figure 1 and Figure 2 appear to exhibit in miniature the essential problems raised by general “assembly” tasks wherein few actuated degrees of freedom are required to manipulate a larger number of unactuated degrees of freedom [3, 4]. Thus, these apparently simple situations appear to be worth considerable study. Both seem to admit the same class of dynamical model, raise similar formal problems, and seem to yield to analogous solution methods. It is important to emphasize that the planning and control problems these examples raise are entirely trivial if an open-loop control strategy — a plan taking the form of a pre-specified curve in configuration space together with a standard classical controller to track it — is admitted. The novelty arises when we require a solution in the form of a feedback controller. Suddenly the problem is not even solvable in traditional terms. A solution requires recourse to the discontinuously switched (hence, “hierar-

chical”) methods introduced here. Unfortunately, these methods, at present, do not suffice to solve more generalized problem versions of the kind depicted in Figure 3 and Figure 4. Possibly, the approach to hierarchical control outlined here can be extended to solve such general problems of intelligent manipulation.

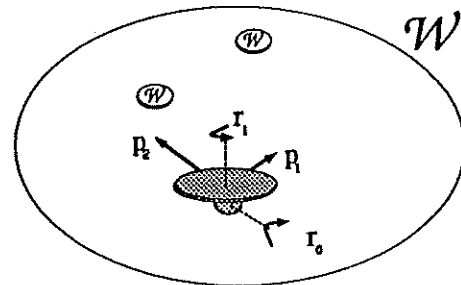


Figure 1: A radially symmetric unicycle must maneuver among fixed obstacles in its workplace, \mathcal{W} , and navigate to a specified destination, $d = (p_1^*, p_2^*)$ at a specified orientation, $n = r_1^*$.

2 The Dynamical Model

Let \mathcal{S}, \mathcal{B} be compact connected (oriented) manifolds (possibly) with boundary. We will call their cross product the *configuration space*, $\mathcal{Q} \triangleq \mathcal{S} \times \mathcal{B}$. A simple dynamical model that is valid for both the scene in Figure 1 and in Figure 2 takes the form

$$\begin{aligned} \dot{s} &= u_1 \\ \dot{b} &= c(s, b) \delta(u_0, u_1), \end{aligned} \quad (1)$$

where $u_1 \in \mathcal{U}_1 \subset \mathcal{TS}$, and $u_0 \in \mathcal{U}_0 \subset \mathbb{R}$, with $\mathcal{U} \triangleq \mathcal{U}_0 \times \mathcal{U}_1$ a compact set, and where $c : \mathcal{Q} \rightarrow \mathcal{TB}$ and

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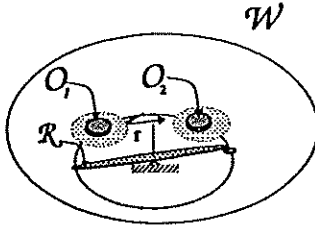


Figure 2: A rotating, arm, \mathcal{R} , must approach and grip two different objects $\mathcal{O}_1, \mathcal{O}_2$, in order to move them into a desired final position, d , before moving to its "nest" configuration, n .

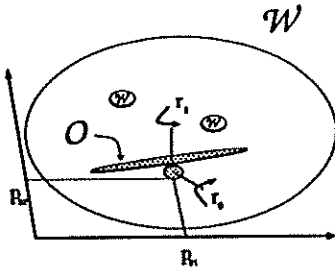


Figure 3: An asymmetric unicycle has a freespace that cannot be expressed as a simple cross product since its heading and translational degrees of freedom both interact with obstacles.

$\delta : \mathcal{U} \rightarrow \mathbb{R}$ are smooth. This will now be shown.

2.1 Navigating a Unicycle

In Figure 1, the robot unicycle rides on a single wheel whose drive axis is denoted r_0 and whose steering axis is denoted r_1 . We suppose that both degrees of freedom are actuated by controlled sources of torque, u_0, u_1 , respectively. The vehicle's translational degrees of freedom are specified by the vector $p = (p_1, p_2)$. Because the realizable translational velocities depend upon the rotational position, this machine is subject to nonholonomic constraints. These velocity constraints may be expressed by the function

$$\alpha(q) = \begin{bmatrix} \dot{p}_1 \\ \dot{p}_2 \end{bmatrix} - \begin{bmatrix} \dot{r}_0 \cos r_1 \\ \dot{r}_0 \sin r_1 \end{bmatrix} = 0 \quad (2)$$

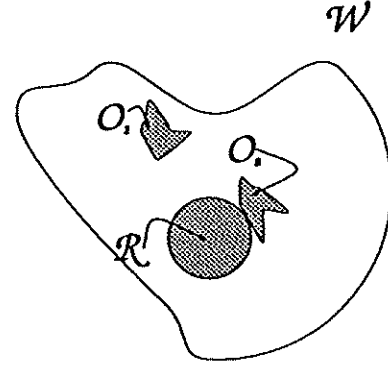


Figure 4: An assembly problem involving a robot, \mathcal{R} , that "inhabits" the same workspace as the objects to be manipulated, $\mathcal{O}_1, \mathcal{O}_2$. The robot must approach the objects, and drag them one at a time toward a specified final assembly. The configuration space of this problem cannot be expressed as a simple cross product since each robot-object mating gives rise to a different effective freespace [1].

For $q = (p, r) = (p_1, p_2, r_0, r_1)$ note from (2) that

$$A(q) = [A_1, A_2](q); \quad A_1 = I_{2 \times 2}; \quad A_2 = \begin{bmatrix} 0 & -\cos r_1 \\ 0 & -\sin r_1 \end{bmatrix}$$

hence the method of undetermined multipliers yields equations that may be written

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \begin{bmatrix} \ddot{p} \\ \ddot{r} \end{bmatrix} = \begin{bmatrix} \lambda \\ A_2^T \lambda + u \end{bmatrix} \quad (3)$$

$$= -A_2 \dot{r}.$$

Differentiating the second equation,

$$\ddot{p} = -A_2 \ddot{r} - \dot{A}_2 \dot{r},$$

and substituting $M_1 \ddot{p}$ for λ in the second component of the first equation yields

$$(M_2 + A_2^T M_1 A_2) \ddot{r} = -A_2^T M_1 \dot{A}_2 \dot{r} + u$$

Assuming that M_1, M_2 are constant and diagonal, since

$$A_2^T \dot{A}_2 \equiv 0 \text{ and } A_2^T A_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix},$$

the equations of motion are now specified by

$$\begin{aligned} \ddot{r} &= \begin{bmatrix} 1/\mu_1 & 0 \\ 0 & 1/(\mu_1 + \mu_2) \end{bmatrix} u \\ \dot{p} &= c(r_1) \dot{r}_0; \quad c(r_1) = \begin{bmatrix} \cos r_1 \\ \sin r_1 \end{bmatrix}. \end{aligned} \quad (4)$$

2.2 Placing Beads on a String

Suppose in Figure 2 that a robot, \mathcal{R} , with moment of inertia, μ_3 is much more massive than its "load" — $\mathcal{O}_1, \mathcal{O}_2$, whose mass is μ_1, μ_2 , respectively. Suppose, moreover, that \mathcal{R} pivots on some nearly frictionless bearing, measured by the angle, r , and actuated by a controlled torque source, u_1 . In contrast, \mathcal{O}_1 and \mathcal{O}_2 , whose positions are denoted by the planar vectors p_1, p_2 respectively, must slide over a rough surface with pronounced Coulomb friction that opposes their motion according to the standard velocity step function, $-g_0 \dot{p}_i / \|\dot{p}_i\|$. Further assume that $g_0 \gg \mu_1, \mu_2$, so that objects $\mathcal{O}_1, \mathcal{O}_2$, when not forced by the actuated bar, \mathcal{R} , effectively experience no motion — the opposing force, $g_0/\mu_i, i = 1, 2$ is simply too great.

Suppose the end of the pivoting robot bar is endowed with a "perfect gripper" that is capable of holding the objects fast (e.g., it contains an electromagnet and the objects are metal) with a force u_0 , when located exactly over them, and that this force falls off rapidly as the relative proximity decreases. To model this situation, it is helpful to introduce a *smooth but not analytic* scalar "interpolation" function, that vanishes identically outside some small interval near zero, $\epsilon > 0$, and takes unit value at zero,

$$\xi(\delta) \triangleq \begin{cases} 0 & : \delta \geq \epsilon \\ 1 & : \delta = 0 \end{cases} \quad (5)$$

Assuming that the objects are initially within the robot's reach, $\|p_i\| = 1$, the robot can engage either one when

$$\beta_i(r, p_i) \triangleq [r - \arctan(p_{i2}/p_{i1})]^2$$

vanishes. The force of the contact between \mathcal{R} and \mathcal{O}_i may now be modeled as $u_0 \cdot c_i(r, p)$ where $c_i = \xi \circ \beta_i; i = 1, 2$.

Since $\mathcal{O}_1, \mathcal{O}_2$ can only be in motion when engaged by \mathcal{R} in this manner, it follows that these two objects must match the velocity of the normal surface of \mathcal{R} at their point of contact (we simplify the kinematics here by treating the objects as point masses)

$$\begin{bmatrix} \dot{p}_1 \\ \dot{p}_2 \end{bmatrix} = u_0 \dot{r} \begin{bmatrix} c_1 \cdot J p_1 \\ c_2 \cdot J p_2 \end{bmatrix}; \quad J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

Notice that the model of Coulomb friction we have employed here results in the imposition of two constraints on motion. The first constraint is holonomic, and, asserts that $\frac{d}{dt} \|p_i\| = 0$. Now defining

$$\theta_i \triangleq \arctan(p_{i2}/p_{i1}),$$

we may take as our configuration space the variables $q = (\theta_1, \theta_2, r) \in \mathbb{R}^3$, coordinatizing T^3 — the three-torus. The second constraint is nonholonomic and takes the form $A(q)\dot{q} = 0$ where

$$A(q) = [A_1, A_2](q); \quad A_1 = I_{2 \times 2}; \quad A_2 = -u_0 c(q) \quad (6)$$

where $c = [c_1, c_2]^T$. This may be verified after recalling the relation $\dot{\theta}(p_j) = p_j^T J^T \dot{p}_j / \|p_j\|$.

We now have (3) with

$$M_1 = \begin{bmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{bmatrix} \text{ and } M_2 = \mu_3.$$

Proceeding to derive the equations of motion as above, we must now compute

$$\begin{aligned} M_2 + A_2^T M_1 A_2 &= \mu_3 (1 + (\mu_1/\mu_3)c_1^2 + (\mu_2/\mu_3)c_2^2) \approx \mu_3 \\ A_2^T M_1 \dot{A}_2 &= c^T (\mu_1 \dot{c}_1 + \mu_2 \dot{c}_2) \end{aligned}$$

Notice that both expressions vanish outside the ϵ -neighborhood defined in (5). The first expression is dominated by μ_3 on all configurations. The second may not be, depending upon the speed of approach to the contact set, but in the spirit of cleaving to a very simple model, we let $u_1 = \bar{u}_1 - c^T (\mu_1 \dot{c}_1 + \mu_2 \dot{c}_2)$ to write

$$\begin{aligned} \ddot{r} &= \frac{1}{\mu_3} \bar{u}_1 \\ \dot{\theta} &= u_0 \dot{r} c(r, \theta) \end{aligned} \quad (7)$$

2.3 The Associated First Order Dynamics

As matters stand, (1) differs in form from both (4) and (7) in that the latter have a second order component. We will argue in Section 3.1 that the essential planning and control issues inherent in these constrained second order systems may be effectively represented by a first order system. Here, we present the representative first order "planning systems" associated with Figure 1 and Figure 2.

A dynamical model that reduces the second order components of (4) and (7) is readily achieved by appeal to the notion of a "generalized damper" — a physical system where inputs cause a change in position rather than velocity. Such models have been widely accepted in robotic situations where a very tight velocity servo loop may be postulated [8, 12].

If, in (4) we assume that both actuated degrees of freedom, r_0, r_1 are velocity controlled rather than acceleration controlled, that is, $\dot{r}_0 = u_0, \dot{r}_1 = u_1$ then the model may be rewritten

$$\begin{aligned} \dot{r}_1 &= u_1 \\ \dot{p} &= c(r_1) u_0. \end{aligned} \quad (8)$$

This system has the form (1) with $\delta(u_0, u_1) = u_0$. It is not hard to see that (8) is completely controllable [9]. Notice, however, that $c(r_1)u_0$ does not cover a neighborhood of the zero vector in \mathbb{R}^2 on any collection of points (r_1, p, u_0, u_1) and it follows from Brockett's Theorem [2] that the system is not stabilizable.

Similarly, if in (7) we assume that the robot's rotational degree of freedom is velocity controlled rather than acceleration controlled, $\dot{r} = u_1$, then the model may be rewritten

$$\begin{aligned}\dot{r} &= u_1 \\ \dot{\theta} &= u_0 u_1 c(r, \theta).\end{aligned}\quad (9)$$

This system has the form (1) with $\delta(u_0, u_1) = u_0 u_1$. Again, it is not hard to see that (9) is completely controllable [3]. But when $\|p_1 - p_2\| > \epsilon$ the vector c must have a zero in either the first or second component according to (5). It follows that $u_0 u_1 c$ does not cover a neighborhood of the zero vector and this system is again not stabilizable at such configurations according to Brockett's Theorem.

3 Problem Setup

In light of the previous discussion concerning the particular instances (8) and (9), we will suppose that system (1) is completely controllable. However, let there be some region of the configuration space, $\mathcal{A} \subset \mathcal{Q}$ (a "locus of anholonomy,") such that if $(s, b) \in \mathcal{A}$, then the image of c on every sufficiently small neighborhood of that point fails to include a complete neighborhood of $0 \in T_b \mathcal{B}$. According to Brockett's Theorem, it follows that no single continuous feedback law can stabilize (1) around any configuration in \mathcal{A} .

Instead, we will devise a family of three distinct controllers derived from the same "dynamically informed" planning methodology applied to three different subgoals. A higher level automaton will be required to switch discontinuously between these subgoals. In this section we explore the formal machinery and assumptions that seem to be required in order to do this effectively.

3.1 Dynamical Plans for First and Second Order Systems

Let d be an interior point of a compact connected (oriented) manifold (with boundary), \mathcal{M} . A *navigation function for d* , is a smooth Morse function on \mathcal{M} taking values in the interval $[0, 1]$, such that $\varphi^{-1}[1] = \partial\mathcal{M}$,

and $\varphi^{-1}[0] = \{d\}$, and d is the only minimum of φ . Every compact connected oriented manifold with boundary admits a navigation function for any interior point [7].

Given a navigation function on \mathcal{M} with a minimum at some desired destination, $d \in \mathcal{M}$, say that a smooth vector field, f , on \mathcal{M} is a *plan for φ* if the latter is a strict Lyapunov function, that is, $d\varphi f \leq 0$ (with equality only at the critical points of φ) for all $b \in \mathcal{M}$.¹ Note that this implies (i) d is an asymptotically stable equilibrium state of f whose domain of attraction differs from \mathcal{M} by a set of measure zero and (ii) f is complete on \mathcal{M} (there is no finite escape, and, in particular, no trajectory of f escapes through $\partial\mathcal{M}$) [7]. Note as well that every navigation function automatically generates a plan — take $f \triangleq -grad \varphi$ — although there are in general better performing alternative plans that may be readily constructed from a specified navigation function [11].

We have shown elsewhere that plans on a configuration space, \mathcal{M} , can always be "lifted" to produce controllers for Lagrangian (second order) systems defined on the tangent bundle, $T\mathcal{M}$, endowed with any Riemannian metric (that is, a definition of kinetic energy) by the addition of a damping term. These controllers are guaranteed to induce the analogous limit behavior and provide forward completeness of solutions. Gradient plans may be lifted in a straightforward fashion familiar to Lord Kelvin [6]. More general plans require a slightly more involved procedure [5]. Thus, in the present paper, we will consider merely the dynamical planning problem associated with first order versions of (8) and (9). Since our solutions will take the form of dynamical plans, a solution for the original second order problems, (4) and (7), will follow almost directly [3].

3.2 The Contact Set

Beyond the underlying similarity in their governing dynamics, the most important feature common to both Figure 1 and Figure 2 is that the configuration space, \mathcal{Q} , is a simple cross product, $\mathcal{S} \times \mathcal{B}$. Thus, to encode the task it will suffice to put a navigation function, φ , on \mathcal{B} . This favorable situation is decidedly not the case in the examples of Figure 3 and Figure 4.

Given a navigation function, φ , on \mathcal{B} , we may measure the extent to which steering c with s at any given b produces a better or worse plan for φ by taking the

¹The co-vector field generated by a scalar valued function of \mathcal{M} is denoted $d\varphi$. Given a function h with respect to a specified coordinate system, the operation of partial differentiation with respect to the i^{th} slot is denoted $D_i h$, and the jacobian matrix of h is denoted Dh .

projection,

$$\pi(s, b) \triangleq d\varphi(b) c(s, b). \quad (10)$$

Using this measure of likely progress, the *contact set* of system (1) (with respect to the navigation function, φ), may be defined as

$$\begin{aligned} C_\varphi(b) &= \arg \max_{r \in \mathcal{S}} |\pi(r, b)| \\ C_\varphi &\triangleq \bigcup_{b \in \mathcal{B}, s \in C_\varphi(b)} (s, b) \end{aligned} \quad (11)$$

To motivate this definition, note that for Figure 2 $C_\varphi(b)$ will always correspond to the configurations at which the robot is exactly aligned with either one or the other of the bodies. For Figure 1, $C_\varphi(b)$ will correspond to those steering angles that point the disk toward the direction of steepest descent at b . In general, note that by this definition $C_\varphi(b)$ is non-empty for all $b \in \mathcal{B}$.

3.2.1 Unicycle Example

The placements (planar rigid transformations) of the unicycle that do not result in intersections with the workplace, \mathcal{W} , in Figure 1 include all rotations, $\mathcal{S} = SO(2)$ at any legal translation, $\mathcal{B} = \mathbb{R}^2 - \rho\mathcal{W}$ (here, $\rho\mathcal{W}$ denotes the dilation of the set \mathcal{W} by the unicycle's radius, ρ). Thus, \mathcal{B} is a topological sphere world [7]. That is, there exists a smooth and smoothly invertible map, h , between \mathcal{B} and a "model space" whose boundaries are circles (say with centers and radii, $c_i, \rho_i, i = 1, \dots, M$) specified implicitly by the quadratic forms

$$\beta_i(b) = \|b - c_i\|^2 \quad i = 1, \dots, M,$$

where M is the number of distinct configuration space obstacles with which the robot must contend. We have shown how to construct the transformation, h , for a dense subset of the sphere worlds [10] but, for ease of exposition, assume here that \mathcal{B} is itself a model space. That is, after dilating the radii of the objects labeled \mathcal{W} in Figure 1, the resulting obstacles form a set of disjoint disks. It now follows that if γ denotes the squared Euclidean distance from the unicycle's destination, $d \in \mathbb{R}^2$ and β denotes the product of all the obstacle functions, $\prod \beta_i$, then

$$\varphi = \frac{\gamma}{(\gamma^k + \beta)^{1/k}} \quad (12)$$

is a navigation function for d in \mathcal{B} for sufficiently large k .

We now have

$$\pi = D\varphi \begin{bmatrix} \cos r_1 \\ \sin r_1 \end{bmatrix}$$

so that the contact set at some b consists of a single direction in S^1 ,

$$C_\varphi(b) = \{\arctan(D\varphi(b))\},$$

when b is not an extremum of φ (here we take the four quadrant arctan function). At the extrema, all directions maximize $|\pi|$ yielding the value $\pi = 0$. Thus C_φ is the graph in \mathcal{Q} of a function from a subset (that includes all but a finite number of isolated points) of \mathcal{B} to \mathcal{S} to which is adjoined a copy of \mathcal{S} in \mathcal{Q} over each extremum of φ .

3.2.2 Beads Example

The placements (triples of rotations about the fixed axis) of the three bodies in Figure 2 that do not result in any of the bodies overlapping include all rotations of the robot, $\mathcal{S} = S^1$ at any legal pairing of the bodies, $\mathcal{B} = S^1 \times S^1 - \mathcal{D}$ (here, \mathcal{D} denotes the "diagonal" around the torus along which the distance between the bodies differs by less than the sum of the two disk radii. Defining a distance function on S^1 (now coordinatized by its covering space, \mathbb{R}) as

$$\text{DIST}(\theta) := \frac{1}{2}(1 - \cos \theta),$$

we may characterize the boundaries of \mathcal{D} by the zero set of

$$\beta = \text{DIST}(\theta_1 - \theta_2) - \rho^2.$$

Given a set of desired bead destinations, (d_1, d_2) , measure the progress to the destination by the function $\gamma = \text{DIST}(\theta_1 - d_1) + \text{DIST}(\theta_2 - d_2)$. It now follows that the construction in (12) yields a navigation function for d on \mathcal{B} for sufficiently large k .

We now have

$$\pi = D\varphi \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} \quad \xi_i \triangleq \xi \circ \text{DIST}(r - \theta_i).$$

Since $\xi_1 \xi_2 = 0$ on \mathcal{B} , $\pi(r, \theta_1, \theta_2) > 0$ only when r is sufficiently close to θ_1 or θ_2 . In these subsets of \mathcal{S} , the local maxima of π are characterized by

$$D_r \pi(r, \theta_1, \theta_2) = 0 \text{ and } [D_r^2 \pi](r, \theta_1, \theta_2) > 0.$$

Note that these conditions hold i) at the two isolated points $r = \theta_1, r = \theta_2$ when neither entry of $D\varphi$ vanishes; ii) at one isolated point, $r = \theta_j$, when only the j^{th} entry is non-zero; iii) nowhere when (θ_1, θ_2) is an extremum of φ . Thus, C_φ is locally the union of the graphs $r = \theta_1, r = \theta_2$, and these join together with \mathcal{S} or vanish in \mathcal{S} at those body placements for which $[D_r^2 \pi](r, \theta_1, \theta_2) = 0$.

3.2.3 Assumptions Concerning the Contact Set

We will assume throughout this paper that $\pi(q) = 0$ for some $(s, b) \in C_\varphi$ implies b is a critical point of φ as is indeed the case in the two examples above.² Thus, $\pi|_{C_\varphi}$ vanishes on a set of measure zero — $S \times \mathcal{E}_\varphi$, where \mathcal{E}_φ denotes the isolated set of extrema of φ . Generically and, in particular, in the two examples above,

$$C_\varphi(b) = \{r \in S : D_s \pi(r, b) = 0; [D_s^2 \pi](r, b) > 0\} \quad b \notin \mathcal{I}_\varphi$$

where \mathcal{I}_φ is not empty (for instance, it contains the extrema, $\mathcal{E}_\varphi \subset \mathcal{I}_\varphi$) but does have measure zero in B . In such cases the implicit function theorem implies that C_φ is the union of smooth co-dimension one submanifolds of Q — the graphs of a finite number of implicit functions, $z_i : B \rightarrow S, i = 1, M$, such that $D_s \pi(z_i(b), b) = 0$ — that may join together at $S \times \mathcal{I}_\varphi$.

Curves in C_φ project onto curves in B that follow the steepest possible descent with respect to φ that (1) will admit. Thus, an obvious strategy for bringing b to d in B is to steer our system onto a component submanifold of C_φ and remain there until some singularity at \mathcal{I}_φ “confuses” the local direction of descent. We hope that everywhere but \mathcal{E}_φ , where there is no longer any descent information, we will be able to find an alternative branch of C_φ that is locally a good submanifold. This prompts the assumption

Smooth Contact Cover: If $b \notin \mathcal{E}_\varphi$ then there can always be found some $r \in C_\varphi(b)$ such that

$$D_s \pi(r, b) = 0 \text{ and } [D_s^2 \pi](r, b) > 0$$

3.3 Additional Assumptions

Finally, we impose a number of regularizing assumptions on the effect of the input signals in (1) that reflect the situation in the examples discussed above. Evidently, the ability to place the steering direction anywhere in S implies that \mathcal{U}_1 includes a neighborhood of the zero section of TS . To permit “driving forward and backward” we will further suppose that δ includes in its range a neighborhood of $0 \in \mathbb{R}$. Moreover, we shall assume that “steering” and “driving” may be effectively decoupled by letting $u_0 = 0$ imply $\delta \equiv 0$ for all $u_1 \in \mathcal{U}_1$.

The problem at hand may now be stated as follows: Given a desired final robot “nest point”, $n \in S$, a goal, $d \in B$, and a navigation function, φ , for d , on B find a (necessarily discontinuous) feedback controller under

²One suspects that the complete controllability of the overall system (1) might imply this is so for a “reasonable” φ .

whose action the closed loop system (1) takes almost every initial condition to the the configuration, $(n, d) \in Q$. In solving this problem we shall require of system (1) that the following properties hold:

Decoupled Obstacles: $Q = S \times B$

Decoupled Inputs: $u_0 = 0$ implies $\delta \equiv 0$.

Full Steering Range: \mathcal{U}_1 contains a neighborhood of the zero section in TS .

Full Driving Range: $\text{Im } \delta$ contains an interval around zero, $[-\underline{\delta}, \underline{\delta}] \subset \mathbb{R}$.

Complete Contact: There exists a navigation function, φ , for d on B with the property that for all $b \in B$ not in the set of critical points there can be found an $s \in S$ such that $\pi(s, b) \neq 0$.

4 Sketch of the Solution Paradigm

We now propose an obvious but rather general two-level feedback control scheme. A (very simple) “higher level” switching logic dictates the manner in which several “low-level” continuous feedback laws are discontinuously switched in and out of operation.

4.1 Three “Low Level” Dynamical Plans

4.1.1 Subgoal: Done

Encode the nest goal with a navigation function for n on S , γ_n , and let \tilde{g}_n be an associated plan. This yields a “steering-but-no-driving” feedback law of the form $(u_1, u_2) = g_n := (\tilde{g}_n, 0)$.

This feedback law will be switched in by the higher level automaton when as much progress as desired has been made by $b \in B$ toward the destination, d .

4.1.2 Subgoal: Steer

Assume that a finite number of smooth scalar valued functions on regions of Q have been constructed, $\{\gamma_{c,i}\}_{i=1}^M$, that measure the distance to the correspondingly indexed graphs, the component of C_φ in Section 3.2.3, such that for each (s, b) in the domain of $\gamma_{c,i}$, this is a navigation function for some $s_d \in C_\varphi(b)$. Let $\mu : B - \mathcal{E}_\varphi \rightarrow \{1, 2, \dots, M\}$ denote the index valued map that chooses the graph component, z_i in $C_\varphi(b)$ at which

$|\pi(z_i(b), b)|$ is greatest. This is a well defined function according to the **Smooth Contact Cover** assumption.

For each $b \in \mathcal{B} - \mathcal{E}_\varphi$, each navigation function, $\gamma_{c,i}(\cdot, b)$ has an associated (smooth) plan, $\tilde{g}_{c,i}(\cdot, b)$, on \mathcal{S} . Similarly to the nesting plan above, each of these contact seeking plans, $\tilde{g}_{c,i}$, above, gives rise to a feedback law, $(u_1, u_2) = g_{c,i} := (\tilde{g}_{c,i}, 0)$ that steers-with-no-driving toward a contact.

The rationale for this is as follows. The control strategy developed below will attempt to steer s toward C_φ in order to drive b down the steepest descent possible with respect to the navigation function φ . In general, it is to be expected that this will lead to a stall situation where, locally maximized though it may be, π becomes smaller and smaller along this curve. Thus, we will need to be able to switch between the various branches of C_φ in order to be guaranteed of eventually coming to the destination it encodes.

4.1.3 Subgoal: Drive

According to the **Full Driving Range** assumption,

$$\delta^{-1}\left[\frac{-\pi(q)}{1 + \underline{\delta}|\pi(q)|}\right] \neq \emptyset, \text{ for all } q \in \mathcal{Q},$$

so there exists at least one function, $p : \mathcal{Q} \rightarrow \mathcal{U}$, such that

$$\delta \circ p(q) = -\frac{-\pi(q)}{1 + \underline{\delta}|\pi(q)|}. \quad (13)$$

This has the effect of sending the body states toward the destination as quickly as the vector field will locally allow.

4.2 A "High Level" Switching Algorithm

A (very simple) Discrete Event System may now be defined with respect to a three state automaton

$$e \in \{\text{DRIVE, STEER, DONE}\}.$$

whose output action is to choose the corresponding low level controller specified above. Roughly speaking, this automaton makes its state transitions as follows. It can enter state DONE only when the body state is sufficiently close to the desired destination. Otherwise, it remains in DRIVE until the contact strength, $|\pi|$ becomes unacceptably small at which point it switches into STEER. Once in STEER, a transition to DRIVE occurs only when the steering variables are sufficiently close to the most immediately promising graph component $z_{\mu(b)}(b)$.

This hierarchical controller succeeds in bringing the system arbitrarily close to the desired goal state, (n, d) , from almost every initial state, $(s, b) \in \mathcal{Q}$. A proof of convergence for the specific case of Figure 2 has been given in [3]. A proof of convergence for the general case (1) is in progress.

References

- [1] Rachid Alami, Thierry Simeon, and Jean-Paul Laumond. A geometrical approach to planning manipulation tasks. the case of discrete placements and grasps. In H. Miura and S. Arimoto, editors, *Robotics Research*, pages 453-463. MIT Press, 1990.
- [2] Roger W. Brockett. Asymptotic stability and feedback stabilization. In Roger W. Brockett, Richard S. Millman, and Hector J. Sussman, editors, *Differential Geometric Control Theory*, chapter 3, pages 181-191. Birkhäuser, 1983.
- [3] D. E. Koditschek. An approach to autonomous robot assembly. *Robotica*, (to appear), 1992.
- [4] D. E. Koditschek. Task encoding: Toward a scientific paradigm for robot planning and control. *Journal of Robotics and Autonomous Systems*, (to appear), 1992.
- [5] Daniel E. Koditschek. Adaptive techniques for mechanical systems. In *Fifth Yale Workshop on Applications of Adaptive Systems Theory*, pages 259-265, New Haven, CT, May 1987. Center for Systems Science, Yale University.
- [6] Daniel E. Koditschek. The control of natural motion in mechanical systems. *ASME Journal of Dynamic Systems, Measurement, and Control*, 113(4):547-551, Dec 1991.
- [7] Daniel E. Koditschek and Elon Rimon. Robot navigation functions on manifolds with boundary. *Advances in Applied Mathematics*, 11:412-442, 1990.
- [8] Tomás Lozano-Pérez, Matthew T. Mason, and Russell H. Taylor. Automatic synthesis of fine-motion strategies for robots. *The International Journal of Robotics Research*, 3(1):3-23, 1984.
- [9] R. Murray and S. Sastry. Grasping and manipulation using multifingered robot hands. In *Robotics, Proceedings of Symposia in Applied Mathematics Vol. 41*, Providence, RI, 1990. American Mathematical Society.
- [10] E. Rimon and D. E. Koditschek. The construction of analytic diffeomorphisms for exact robot navigation on star worlds. *Transactions of the American Mathematical Society*, 327(1):71-115, Sep 1991.
- [11] Louis L. Whitcomb and Daniel E. Koditschek. Automatic assembly planning and control via potential functions. In *Proc. IEEE International Workshop on Intelligent Robots and Systems*, Osaka, Japan, Nov. 1991. IEEE.
- [12] D. E. Whitney. Force feedback control of manipulator fine motions. *ASME J. Dyn. Syst.*, 98:91-97, June 1977.

