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NOTE: At the time of publication, author Daniel Koditschek was affiliated with Yale University. Currently, he is a faculty member in the Department of Electrical and Systems Engineering at the University of Pennsylvania.

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Task Encoding for Autonomous Machines: The Assembly Problem

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Abstract

Assembly problems require that a robotic system with fewer actuated degrees of freedom manipulate an environment with a greater number of unactuated degrees of freedom. This paper explores the possibilities of combining a navigation plan for an “animated” version of the environment with a juggling plan that mediates between the conflicting subgoals of that underconstrained world. The hope is to develop a formalism for constructing globally stabilizing feedback controllers for the nonholonomically constrained dynamical systems that represent the underlying problem.

1 Introduction

A general problem of widely acknowledged importance in robotics concerns the coordination of multiple degrees of freedom in the face of an environment possessed of fewer. Whether formulated as the “redundant manipulator” problem, the “cooperating robots” problem, or the “grasping” problem, many researchers have attempted to deploy underconstrained and actuated joints to gain better performance than could have otherwise obtained. Consider, however, how commonly the opposite situation prevails. The paradigm of the monkey, stick, and banana quickly comes to mind. A hungry monkey stares at a ripe banana hanging just over its head out of reach; on the ground at its feet lies a sturdy branch with a forked end. The monkey quickly realizes it can pick up the stick and yank the banana off the limb. The banana and stick have each six degrees of unactuated freedom. The actuated joints that the monkey can bring to bear on the problem are far fewer but it engages its environment in a planned sequence of manipulations that achieves the desired goal state: banana-in-mouth. This is an example of an assembly problem.

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1.1 Assembly: Manipulating Many Using Few Degrees of Freedom

Loosely speaking, say that a robotic task involves an assembly problem if the environment to be manipulated possesses more degrees of (unactuated) freedom than are available to the (actuated) robot system, and the specified goal state is to be achieved starting from arbitrary initial configurations (that is, all unactuated degrees of freedom must be exercised, in general, to complete the task). The unavailability of actuated degrees of freedom might result from limitations inherent in the robot’s design (e.g., the PUMA has only six joints and the widget has twenty parts) or as a function of natural constraints imposed by the environment (e.g., the monkey’s twenty degree of freedom hand has no bearing upon the banana’s six degree of freedom state unless there is contact). A successful assembly plan must develop a sequence of manipulations none of whose single steps can achieve the goal yet each of whose concluding states brings the environment to a more favorable situation than the prior.

Surely, any reasonably interesting task to be carried out in the real (unstructured) world by a solitary robot will have the character of an assembly problem. For example, this paradigm underlies the warehouseman’s problem [10]: in a large hall filled to the ceiling with storage crates lies (in the far corner under a pile of heavy cartons) a box of *Proceedings of the First Yale Workshop on Adaptive and Learning Systems*; the task is to retrieve a copy of the Proceedings. It appears reasonable, moreover, to represent the excavation problem — e.g., “robot bulldozer, go clear out the following dimensioned cavity so that the foundation robot can pour in the footings” — as a (simpler, but still challenging) version of the warehouseman’s problem as well. But even in the most structured factory environment, it is hard to imagine that “design for manufacturability” will obviate the need to assemble widgets with many parts.

1.2 Autonomous Machines: Planning via Stabilizing Controllers

In a recent paper [13], the author reviews the principles underlying a program of robotics research that seeks to develop planning procedures by recourse to dynamical

systems theory. Since physical machines are ultimately force or torque controlled dynamical systems, the specification of input torques must result in certain classes of vector fields. In this light, it makes sense to specify plans in the form of appropriately constructed sensor based feedback controllers whenever possible. As is well known, feedback controllers, unlike open loop plans, are designed to work over large classes of initial configurations (tolerance to state uncertainty) and often succeed even when the underlying dynamics are imperfectly modeled (tolerance to parametric uncertainty). Further, this approach to planning encourages the design of "canonical" procedures for "model" problems which may then be instantiated in particular settings by a change of coordinates [22, 21]. Moreover, the resulting compression of the more usual planning-execution hierarchy tends to force the designer's parametrization of the present environment into the "low level" controller, thus suggesting a means of turning fuzzy questions of learning into clearly posed (albeit, difficult) problems in parameter adaptive control. Finally, such specifications make explicit the resulting (closed loop) dynamical system, and afford the application of well developed mathematical analysis when attempting proofs of correctness.

Given that the robot's execution of a task in a specified environment may be represented as a dynamical system on an appropriate space, and that the criterion of success is the achievement of some distinguished goal set in that space, one is in a position to assess the "autonomy" of the resulting behavior with respect to standard ideas from dynamical systems theory. Namely, if autonomy connotes ability to contend with the full spectrum of logically possible circumstances that arise in completion of a task, systems whose goal states are globally attracting represent autonomous behavior.

In summary, control procedures whose resulting vector fields have globally attracting goal states may properly be said to evince autonomous behavior. The question remains as to how abstract user goals may be appropriately translated into the language of stabilizing feedback laws.

1.3 Task Encoding: Translating Goals into Controllers

The investigation of task encoding principles arises from just this effort to render abstract goals into feedback laws. The term "encoding" seems to connote more effectively than "language" our progress to date. A language requires a formal grammar whose symbols need simultaneously have some semantic interpretation (for example, truth) that connects them to the outside world. The techniques we presently employ offer neither a complete symbol system nor any precise notion of how to extract meaning. Yet, several years' experimentation and mathematical analysis begin to suggest a pattern of expression whose inchoate form may be worth exploring in other task domains even at this early juncture.

2 Two Ingredients of Assembly Behavior

This section presents a summary of techniques for encoding two rather different classes of robotic tasks. The motivation for the present paper rests upon the author's (as yet unproven) presumption that assembly tasks may be solved by a suitable combination of techniques drawn from the two distinct classes.

2.1 Navigation

Let a fully actuated mechanism move in a cluttered but perfectly known workplace. There is a particular location of interest and it is desired that the "robot" move to this location from anywhere else in the workplace without colliding with the obstacles present.

2.1.1 Navigation Functions

Initiated by Khatib a decade ago [11], the idea of using artificial potential functions for robot task description and control was adopted or re-introduced independently by a number of researchers [16, 1, 19]. Gradually, there seems to have emerged a common awareness of several fundamental problems with the potential function methodology. Spurious local minima seemed unavoidable, and unrealizable infinite torques were thought to be required at the obstacle boundaries. In fact, an artificial potential function need satisfy a longer list of technical conditions in order to give rise to a bounded torque feedback controller that guarantees convergence to the goal state, from almost every initial configuration. This list comprises comprises the notion of a *navigation function* introduced to the literature two years ago [20].

The question immediately arises whether such desirable features may be achieved in general. In fact, the answer is affirmative: smooth navigation functions exist on any compact connected smooth manifold with boundary [14]. Thus, in any problem involving motion of a mechanical system through a cluttered space (with perfect information and no requirement of physical contact) if the problem may be solved at all, we are guaranteed that it may be solved by a navigation function. There remains the engineering problem of how to construct such functions

2.1.2 Navigation Functions on Sphere Worlds

A "Euclidean sphere world" is a compact connected subset of E^n whose boundary is the disjoint union of a finite number, say $M + 1$, of $(n - 1)$ -spheres. To construct a navigation function on such a space, first form a function which vanishes on the "bad" set of obstacles,

$$\beta \triangleq \prod_{i=0}^M \beta_i,$$

by "OR"-ing the constituent expressions, β_i , that vanish on the boundary of each distinct obstacle. Letting γ be the euclidean distance to the goal state, form the expression "go to $\gamma = 0$ and do not go to $\beta = 0$ " via the quotient

$$\varphi \triangleq \frac{\gamma}{\beta}.$$

Since $\hat{\phi}$ is unbounded, normalize it via the composition with a smooth “squashing” function,

$$\sigma(x) \triangleq \frac{x}{1+x}.$$

Note that the composition

$$\sigma \circ \hat{\phi} = \frac{\gamma}{\gamma + \beta}$$

is a kind of analytic “switch” that vanishes on the goal state, goes to unity exactly on the bad states, and varies smoothly in between. It is shown in [14] that this construction indeed satisfies the conditions of a navigation function.¹

The Euclidean sphere world, of course, corresponds to a rather simplistic freespace. Fortunately, the navigation properties are invariant diffeomorphism [14]. This suggests that we might consider the Euclidean sphere world as a “model space” and use it to induce navigation functions on more interesting “real spaces” in its analytic diffeomorphism class. The problem of constructing a navigation function on a member of this class reduces to the construction of an analytic diffeomorphism from this space onto its model. Our constructive results to date of this nature encompass almost all of the topological equivalence class of the Euclidean Sphere worlds [21, 22, 23]. Although a proper discussion of these constructions is beyond the scope of the present paper, it is worth noting in passing that a critical component in this work has been the systematic use of the “analytic switch” introduced above that “turns on” when a “bad set” ($\beta = 0$) is encountered and “turns off” when a “good set” ($\gamma = 0$) is encountered.

2.1.3 Navigation Functions on Mating Worlds

The sphere world equivalence class arises naturally in robotics only in situations where the robot is (or can profitably be approximated by) a single spherically symmetric rigid body [12]. More relevant to the assembly problem is a class of freespaces whose obstacles arise from the mutual intersections of many independently actuated rigid bodies. For example, a jigsaw puzzle (whose pieces live either on the plane or in space) has a freespace embedded in the cross product of as many copies, say n , of the Euclidean group (rotations and translations) as there are pieces in the puzzle. If each is a Euclidean ball then the freespace is \mathbb{R}^{3n} with all the diagonal sections, $\|x_i - x_j\|^2 - (\rho_i + \rho_j)^2 < 0$, removed. This simple space may be a topological model for all those puzzles whose pieces are star-shaped sets since the latter are known to be path connected [9]. We do not yet know what a good topological model for more general instance of such “mating spaces” might be. Accordingly, we have no explicit constructions of navigation functions for the finished puzzles — only the assurance that a navigation function does indeed exist for any solvable puzzle [14].²

¹To be precise, we find it necessary for technical reasons to take the k^{th} power of the first quotient followed by the k^{th} root of the composition.

²In point of fact, this statement is not quite accurate if the finished puzzle has its pieces actually touching. In such a case

Clearly, a navigation function on a mating space prescribes a rather general “proto” assembly strategy that we might term a “mating flow”. That is, if each piece could be simultaneously manipulated at will (with no further self-intersections to worry about) then the gradient flow of the navigation function would result in the motion of the parts to the finished assembly. In fact, intuition suggests that such a mating flow might constitute a reasonable first step along the way to an assembly plan. Lest one too quickly reject the very notion as absurd, consider the ubiquity of exploded-parts diagrams in human assembly instructions.

2.2 Juggling

This section introduces a task domain where the robot has indeed fewer degrees of freedom than the environment. Consider a frictionless plane inverted into the earth’s gravitational field. Two pucks are allowed to slide freely on this plane except when batted by a simple “robot” — a revolute motor with a bar attached to it whose axis of rotation is orthogonal to the plane. The robot has one degree of actuated freedom (perhaps one and a half, if one considers the recourse to “whole arm” manipulation [24] as adding freedom) while the environment possesses two for each puck. The robot is given the task of repeatedly batting the two pucks so that each one attains a periodic trajectory whose apex lies at a specified vertical and horizontal locus on the inverted plane.

2.2.1 The Primitive Task

We have argued [6] that the “vertical one-juggle” task — batting a single puck on an inclined plane so that it eventually attains a repeated purely vertical motion at a specified horizontal position — may be encoded as a fixed point of a certain dynamical system. Moreover, we have shown how to construct a sensor based feedback strategy for the robot that accomplishes the task [8] and is provably correct as well [5, 13]. Let b denote the puck’s position and r denote the robot’s joint angle. The effective strategy calls for the robot to track a “mirrored” reflection of the puck’s trajectory, $r = \mu(b, b)$.

2.2.2 Autonomous Scheduling of Conflicting Subgoals

We have reported in [4] a scheme for extending the primitive one-juggle solution to the case of two pucks simultaneously whose goal trajectories are located symmetrically on either side of the robot’s joint axis. Although the procedure is heuristic, extensive experimental study reveals that it works remarkably well in practice. There are two primitives, μ_l, μ_r , that solve the vertical one-juggle for the left and right hand pucks, respectively. Given only one robot (with one degree of freedom), the question arises as to how these two reference trajectories should be “assigned” to the robot, r .

the goal state would be on the boundary of the freespace rather than its interior as the existence result of [14] requires. Assume, then, that the puzzles are built with sufficient tolerance that the mating configuration does not require touching.

Consider an emergency situation, when both pucks are falling toward the bar nearly at the same time. It is imperative to service the nearest first. Moreover, it is well worth sacrificing any nominally desirable one-juggle performance to the work of keeping both aloft and restoring phase separation between them. This intuition can be readily implemented by use of an “analytic switch” that triggers on the good and bad events interpreted in each puck’s phase space. Specifically, consider the “weighting function”,

$$s = \frac{\sigma(w_l)}{\sigma(w_l) + \sigma(w_r)}, \quad \sigma(w) = \sigma_1(w)\sigma_2(w),$$

where $w = (b, \dot{b})$ denotes a puck’s state in phase space. The function σ_1 evaluates to one when the puck approaches the robot’s “home position” and decreases to zero further away. The function σ_2 scales the previous one in a smooth fashion such that weight is only assigned during the the descending part of the puck trajectory. The function s then simply normalizes the contributions of σ_l and σ_r .

The mirror law extension for the two-juggle may now be written as

$$\mu_{2jug} = s \mu(w_l) + (1 - s) \mu(w_r).$$

In the case of good phase angle separation, each puck gets “full attention” from the robot when close to impact. If the phase angle separation is not good, both pucks can be close to impact simultaneously, and thus both $\sigma(w_l)$ and $\sigma(w_r)$ can approach one. In this case both mirror algorithms $\mu(w_l)$ and $\mu(w_r)$ will contribute to the robot reference trajectory in such a fashion that the puck closer to impact receives more weight.

3 A Missing Ingredient of Assembly Behavior

In some sense the juggling law, μ_{2jug} , solves a restricted kind of assembly problem. It schedules autonomously the conflicting claims for attention demanded by two unactuated parts according to a switching law that is sensitive to “good” and “bad” events as represented in the phase space geometry. On the other hand, the goal is achieved in cooperation with another external force — gravity — that acts upon the environment. In particular, the external force imposes a solution to the coupling problem which we now observe to play a central role in assembly.

3.1 The One Degree of Freedom Unit Assembly Problem

To reduce the problem to simplest terms, consider the “line juggler” scenario [7] — a puck restricted to a single axis of motion engaged by a robot whose linear motion along that axis is controlled by a force we are free to command. Suppose, moreover that the axis is oriented horizontally so that gravity does not come into play. The robot’s surface has a “glue” that immediately holds the puck as soon as contact is made so that pulling as well

as pushing is possible.³ Assume that the puck’s mass is negligible so that its velocity immediately matches that of the robot when engaged by the gripper.

It is desired to relocate the puck at a specified goal location. The problem at hand is to devise an autonomous strategy for the robot that will enable it to move toward the puck, “grab it”, and place it in the arbitrarily designated new location. Recall from the introduction that “autonomous” is used to connote a sensory feedback based strategy whose resulting closed loop has global stability properties.

3.1.1 Nonholonomy

One of the most interesting aspects of both the juggling tasks and unit assembly task described above is that their dynamics include nonholonomic constraints. For the assembly problem, define the variables $q = [r, b]^T$, where r denotes the robot’s position and b that of the puck. The overall system may be written in the form

$$\begin{aligned} M(q)\ddot{q} &= Bu \\ J(q)\dot{q} &= 0 \end{aligned}$$

where M is the constant (diagonal) mass matrix, $B = [1, 0]^T$, and the nonholonomic constraints are expressed as

$$J(q) = \begin{cases} [1, -1] & q = r \\ [0, 1] & q > r \end{cases}$$

i.e., the puck can be moved by the robot if and only if they are touching each other.

A growing literature in robotics concerns open loop planning in the face of nonholonomic constraints (for example, consult the nice review by Murray and Sastry [17]). Indeed, the problem at hand is entirely trivial if open loop strategies are permitted: simply let the robot approach and join the puck, and then let the joint puck-robot mass be moved to the desired location. Even were standard feedback strategies used to make these motions, at least two distinct stages would be required. There is no standard automatic procedure whose output will yield these two controllers in sequence. Formally, there could not be as the matter presently stands, since each motion ends only after an infinite amount of time has passed.⁴

3.1.2 A Feedback Controller for the Analytic Case

Until recently there has been almost no investigation of closed loop planning for nonholonomically constrained control problems. In the last year, however, Bloch and McClamroch have presented a very interesting treatment of the feedback stabilization problem for mechanical systems with classical nonholonomic constraints [2]. They observe that this class of systems generally admits an

³Of course, when more pucks are introduced, the glue will have to be replaced by another “gripper” decision variable.

⁴Note that the resort to a concatenation of feedback controllers produced via finite time optimal control techniques would not satisfy the criterion of autonomy for the same reason.

equivalent representation of the form

$$\begin{aligned} \dot{r}_1 &= r_2 \\ \dot{r}_2 &= u \\ \dot{b} &= c(r_1, b)r_2 \end{aligned}$$

In the present case, note that c , the *coupling function* takes the form

$$c(r_1, b) \triangleq J_2^{-1} J_1(g) = \begin{cases} -1 & b = r_1 \\ 0 & b > r_1 \end{cases}$$

For purposes of exposition, it now proves useful to ignore the fact that c is discontinuous: assume, contrarily, that it is analytic as required by the “classical” nonholonomic constraint formulation [18]. Bloch and McClamroch observe, following a fundamental result of Brockett [3], that there can be no smooth feedback controller whose closed loop gives rise to an isolated asymptotically stable equilibrium state. Fortunately, this negative result will not prove too damaging in the present context, since there is no reason to demand that the robot finish its task in a particular location: we merely require that the puck be at rest in the specified goal state hereinafter designated by the scalar, g .

The task at hand is readily encoded by the zero set of the scalar valued function,

$$v \triangleq \frac{1}{2}r_2^2 + \frac{1}{2}(b-g)^2. \quad (1)$$

To motivate the solution suggested by Bloch and McClamroch, consider the time derivative of v along the motion of the closed loop system when $u \triangleq -r_2 - c(b-g)$,

$$\dot{v} = r_2 [u + c(b-g)] = -r_2^2.$$

According to LaSalle’s invariance principle [15], the limit set of the resulting closed loop is contained within the largest positive invariant set lying in $\dot{v} \equiv 0$ — that is, the $b = r_1$ plane. But the closed loop vector field evaluates to $[0, -c(b-g), 0]^T$ when $r_2 = 0$. Thus, the limit set is contained in the union of the set $b \equiv g$ and $c \equiv 0$. If c is analytic then $c \equiv 0$ describes a set of at most measure zero (unless there is no coupling at all), and the desired conclusion obtains in a straightforward manner.

3.2 Missing: A Stabilization Theory for Nonholonomic Systems

Unfortunately, to the contrary, in our problem c is zero *except* on a set of measure zero. Moreover, it serves no practicable purpose to approximate c in the model with some “very similar” analytic function. For this would result in small motions of the puck even when the robot is very far away, eviscerating the problem entirely. The essential ingredient of the problem is that the puck cannot “feel” the robot’s influence until contact is made. To the best of this author’s knowledge, there is no presently known smooth feedback controller that stabilizes the set $v \equiv 0$ — that is, solves the one degree of freedom unit assembly problem

4 Conclusion: Ex Uno Plura?

Intuitively, there seems good reason to believe that one robot might be able to “juggle” the conflicting motion requirements of several objects incapable of independent simultaneous motion. While mating spaces have a more complicated topology than the sphere worlds for which we presently possess navigation functions, there is suggestive evidence that they are not as bad as the freespaces that arise when single or multiple rigid bodies move in workplaces with fixed clutter [9]. A navigation function on the relevant mating space furnishes a “mating flow.” This last image of a purposefully animated environment suggests, in turn, the possibility of a planning procedure that “projects” the motion of a completely actuated world into an appropriately devised space representing the more constrained possibilities for actuation that are truly available.

Is it possible to obtain an “assembly plan” — a sequence of robot-environment interactions — from a “mating flow” — a dynamical system on a high dimensional space representing the animated environment whose every object is a fully actuated robot? A number of workable projection schemes come readily to mind, and might furnish the basis of an open loop plan. However, a “juggling scheme” that mediates the motions and impacts of a robotic system according to the projected vector field seems to require a better understanding of stabilizing controllers for certain nonholonomically constrained mechanical systems.

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