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Toward a Vocabulary of Legged Leaping

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Toward a Vocabulary of Legged Leaping

Abstract

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Toward a Vocabulary of Legged Leaping

Aaron M. Johnson and D. E. Koditschek

Abstract—As dynamic robot behaviors become more capable and well understood, the need arises for a wide variety of equally capable and systematically applicable transitions between them. We use a hybrid systems framework to characterize the dynamic transitions of a planar “legged” rigid body from rest on level ground to a fully aerial state. The various contact conditions fit together to form a topologically regular structure, the “ground reaction complex”. The body’s actuated dynamics excite multifarious transitions between the cells of this complex, whose regular adjacency relations index naturally the resulting “leaps” (path sequences through the cells from rest to free flight). We exhibit on a RHex robot some of the most interesting “words” formed by these achievable path sequences, documenting unprecedented levels of performance and new application possibilities that illustrate the value of understanding and expressing this vocabulary systematically.

I. INTRODUCTION

Stable steady state dynamic legged locomotion was achieved more than two decades ago in the laboratory [1, 2], and more recently exported to real outdoor operation over rough terrain [3, 4]. A growing body of literature has arisen to explore the stability properties of dynamical steady state template [5] locomotion [6–8], as well as formal design methodologies for increasingly practical, high degree of freedom dynamical running and walking robots [9–12].

In contrast to the maturity of steady state locomotion research, while dynamical transition behaviors have been formulated [13] in terms of appropriately composed steady state constituents that can be generalized and strengthened for locomotion in a computationally tractable manner [14], there seems little *a priori* rationale for requiring that the words of transition be spelled out only in the letters of existing attractor basins¹. In this paper we explore the intrinsic vocabulary of a particularly simple transition: the legged leap on a solid level substrate from a motionless state to some desired aerial apex condition in a high energy regime, such as the leap onto a ledge in Fig. 1.

A. Contributions and Organization of the Paper

The remainder of this section motivates the leap as a necessary antecedent to many interesting subsequent behaviors of obvious utility. Next, in Section II, we review some preliminary formal ideas concerning the central object of study, a two legged sagittal plane hopper, and exhibit the topological space — the “ground reaction complex” (in

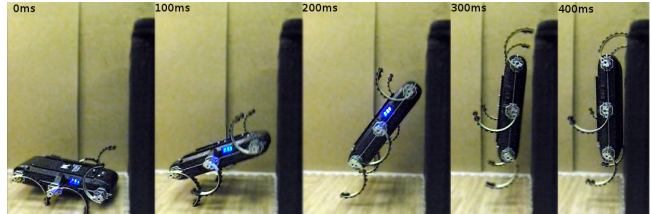


Fig. 1: XRL [16] leaping upward onto a 73cm high table, nearly 1.5 times its bodylength. Frames taken every 100ms from a high speed video.

this case a simplicial tetrathedron) — over whose variously dimensioned cells the Hamiltonian flows of its holonomically constrained body evolve as directed by the ground reaction forces. This cellular construction indexes in a computationally effective (“grammatical”) manner the realizable sequences of continuous dynamics that are physically available, providing crucial intuition for hand-designed behaviors (as suggested by the new capabilities we document) as well as parameterizing the various sequences of constraints that would be required for any automated method of behavior generation (i.e. a learning or optimization based approach).

The value of working out the cell adjacency relations in the ground reaction complex is the resulting catalog it affords of all possible leaps (transitions from the rest state to the fully aerial state). Presented in Section III, this is shown to take the form of variously triggered hybrid dynamic transitions between adjacent cells. These cell-labeled sequences of gradually ascending dimensional flows comprise this hopper’s vocabulary of leaps. In Section IV we document empirically a variety of the very different terminal aerial phase conditions that can result from these various leaps through appropriately coordinated open loop maneuvers implemented on XRL [16], a recent update of RHex [3]. In Section V we show how two different instances of these leaps lead to evidently useful behaviors heretofore unachieved by a general purpose legged robot²: a two hop vault across a gap 20% wider than the robot’s body length; and a high jump onto a ledge almost 50% taller than the robot is long. We conclude with some brief remarks about implications and future work.

B. Motivation

Leaping is a key transition from rest to a variety of high energy behaviors. It allows us to engage in nearly pure form one of the foundational questions of robotics: how can we program the transfer of energy in a robot’s battery or fuel tank to its mechanical state?

²By which we mean a power-autonomous robot without specialized jumping (e.g. [17]) or climbing (e.g. [18]) mechanisms.

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¹Although there is some tantalizing evidence to suggest that rhythmic human transitions are indeed composed of snippets from steady state oscillatory primitives [15].

When jumping onto a ledge or across a gap, sometimes a single leap is all that is needed. However the leap can also be used to setup a second step, as exemplified by the behaviors documented in Section V. In this paper, the second step will be essentially governed by the dynamics of the SLIP template (i.e., the spring-mass hopper literally instantiated by Raibert [1], and empirically used by all running animals [19]), wherein the state of the SLIP system (height, forward velocity, etc) at apex before a hop determines the reachable set after the hop [20, 21]. Naturally the second hop can lead to a third, and thus the leap can be a quick transition into a high kinetic energy running gait from a seated position.

Beyond its value in reaching across obstacles and setting up other behaviors, there are a number of tasks that may entail a leap as an intrinsic goal. The robot may need to flip over if it is not completely symmetric or if there are payloads only available on one side [22]. It also may need to reach a certain height to gain a better vantage point for its sensors.

II. HYBRID HAMILTONIAN DYNAMICS OVER THE GROUND REACTION COMPLEX

We are concerned with a planar rigid body, $b \in \text{SE}[2]$, possessed of two massless limbs whose revolute joints $\theta_i \in \Theta_i := \mathbb{S}^1, i \in \{1, 2\}$, relative to the body are actuated by the motors. The resulting five degree of freedom kinematic system, $q := (\theta_1, \theta_2, b) \in \mathcal{Q} := \Theta_1 \times \Theta_2 \times \text{SE}[2]$, is further subject to a set of unilateral holonomic constraints, $g_j(q) \geq 0, j \in \mathcal{J}$, specified by smooth maps, $g_j : \mathcal{Q} \rightarrow \mathbb{R}$ (and an index set, \mathcal{J} , that we introduce below), that define the base topological space and thereby comprise in part the “guard” or “boundary” conditions on the dynamical flows over the base cells. We will simplify the body contact by assuming two contact points (“front” and “rear” along the bottom), reducing the possible contact conditions to an enumeration of constraint equations over the powerset of \mathcal{P} ,

$$\mathcal{P} := \{p_{k,l} \in \mathbb{R} : (k,l) \in \mathcal{I} := \{F,R\} \times \{B,L\}\}$$

where $\{F,R\}$ indexes the “front” or “rear” location and $\{B,L\}$ indexes the “body” or “leg” terminal. It now follows that there are $2^{|\mathcal{I}|} = 16$ different logically possible contact conditions yielding 16 different Lagrangian dynamical systems whose physical features we will specify below.

While compliance in the legs almost certainly helps achieve some of the behaviors documented here, for the most part the body will follow the rigid linkage path with the springs acting to force the robot onto that trajectory, and so we will assume rigid legs³. We will assume that the actuators can deliver the greatest amount of work to the body when they are individually doing the most work they can on their individual motor shafts. The infinitesimal kinematics of rigid closed kinematic chains generically accord unequally weighted contribution to the net body wrench (see [24] for one example). However, none of the closed chains relevant to leaps against the simple level substrate encounter sign changes in these weights, so actuators might “waste” energy generating internal forces but will not impart negative work

³Though compliance can easily be added back, as in [23] and others.

to the body when they are asserting their maximum torque in the direction of shaft travel⁴.

We further assume that the actuators are each capable of and are restricted to delivering a constant torque (in either direction) throughout their operation, which is saturated by the motor controller current limit. This, of course, does fly in the face of physical reality [25, 26], and power limitations are well understood to play a critical role in fast moving legged robot limbs [27, 28]. Fortunately, here much of the action takes place at relatively low limb speeds, and so there is relatively little back EMF to substantially reduce the output torque. For similar reasons, we neglect damping in the joints and limbs and ignore any other source of energy loss throughout the paper.

A. The Ground Reaction Complex

In [29] a cell complex [30] was used to index all possible abstract coordination schemes that a legged machine might undertake and in [31] this cell complex was used to organize the possible gait transitions and recovery strategies of a quasistatic vertical climbing robot, treating the varying ground contact conditions experienced along the way as mere “noise” shown to be robustly rejected by a proper feedback implementation of the coordination controller. Here we explore what is in some sense the opposite extreme case: we are only interested in characterizing the possible direction and magnitude of ground reaction forces in consequence of different limb configurations; we are only interested in the high energy dynamical regime; and we wish to factor out all the inessential details of interlimb coordination.

Hence, although the kinematic system just introduced has as many as five degrees of freedom, we now exploit the assumption of massless limbs to introduce a coordination assumption that will cut away the inessential dimensions with no loss of generality regarding the ground reaction force interactions of central focus. Namely, we will assume when either limb is free of ground contact that there is some “mirror law” [9], of the form $\theta_i = m_i(q), i = 1, 2$ that the joint actuators track exactly.

Denote by π_z the projection onto the second coordinate of some world frame representation of the body and leg contact points⁵. Consider the family of constraint equations,

$$\pi_z p_{k,l} = 0 \quad (k,l) \in \mathcal{I}_j \subset 2^{\mathcal{I}}$$

where the subscript, j , on the active-constraint set, \mathcal{I}_j , indexes each subset of \mathcal{I} through a Boolean string denoting membership (or its absence) respecting the lexicographic ordering of \mathcal{I} , (FB, FL, RB, RL) so that, for example $\mathcal{I}_{0111} = \{FL, RB, RL\}$.

With this nomenclature in place we now enumerate all of the 16 possible ground contact conditions that form the base space on which our hybrid system is defined, grouping them

⁴As a motivating extension beyond the scope of the present paper, we do document one instance in Section IV-A.2 where the compliance in the legs allow for a novel trajectory, where this maximal torque assumption fails.

⁵We must cut off the “north pole” of the bodys rotational component by always requiring $\pi_z p_{FB} < \ell$, where ℓ is the robot’s bodylength (to ensure each cell is truly contractible as formally required).

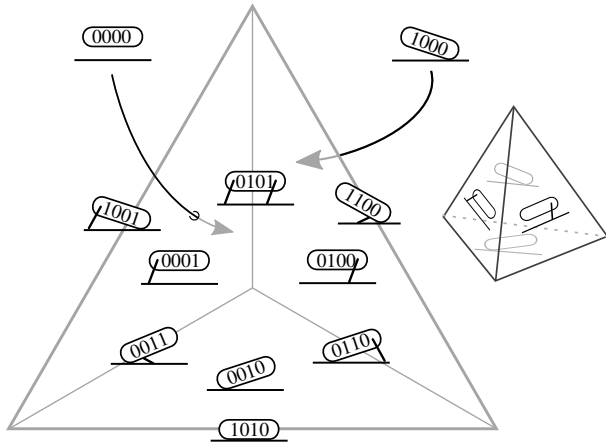


Fig. 2: All possible contact states, represented as a tetrahedron, showing adjacency. The interior volume and bottom face are indicated with arrows.

into the following categories according to their common dynamics as follows:

- One state where the body has three degrees of freedom (3-DOF): the aerial state with no contact $\mathcal{S}_{0000} := \{\}$.
- Two 2-DOF states have one end of the robot on the ground sliding $\mathcal{S}_{1000}, \mathcal{S}_{0010}$.
- Two 2-DOF states have only one leg is down and there is a 2-link open kinematic chain $\mathcal{S}_{0100}, \mathcal{S}_{0001}$.
- Two 1-DOF states have a leg and the opposite side of the body down in a crank-slider configuration $\mathcal{S}_{0110}, \mathcal{S}_{1001}$.
- Two 1-DOF states have a leg and the body on the same side down like a single link chain $\mathcal{S}_{1100}, \mathcal{S}_{0011}$.
- One 1-DOF state has both legs down in a four bar linkage \mathcal{S}_{0101} .
- One 1-DOF state has the body completely on the ground but still able to slide \mathcal{S}_{1010} .
- Four completely constrained states that in general the robot will spend no time in, $\mathcal{S}_{1110}, \mathcal{S}_{1101}, \mathcal{S}_{1011}, \mathcal{S}_{0111}$.
- One degenerate case that is over-constrained with all possible contacts simultaneously on the ground, \mathcal{S}_{1111} .

These states are illustrated in Fig. 2 arranged as a simplicial tetrahedron, with the aerial state in the interior, the 2-DOF states as the faces, and the 1-DOF states as the edges. The 0-DOF states are not illustrated but are the vertex points, and the over-constrained system is not depicted as it represents a degenerate case. Space and time constraints preclude our formal demonstration that the definitions just introduced yield the topological tetrahedron depicted, but it will suffice for the reader to merely keep track of the adjacency relations the figure implies.

B. Hamiltonian Flows

Given present space constraints, we defer to [32] our preferred method of populating (by formal symbolic manipulation) the exact terms in appropriate local coordinates arising in each of the 16 different Lagrangian dynamical systems describing the distinctly different contact mechanics

associated with each GRC cell⁶. We simply exhibit here the formal abstract expression from which each specific instance can be systematically derived. Define the Lagrangian free variable(s) as $y \in \mathcal{Y}$ (related by $h: \mathcal{Y} \rightarrow \mathcal{Q}$ to the state), and thus the kinetic $K: \mathcal{Q} \rightarrow \mathbb{R}^+$ and potential $\Phi: \mathcal{Q} \rightarrow \mathbb{R}^+$ energy⁷ are,

$$H\dot{y} := D_y h \dot{y} = \dot{q} \quad (1)$$

$$K(q, \dot{q}) = \frac{1}{2} \dot{q}^T M(q) \dot{q} \quad (2)$$

$$\tilde{K}(y, \dot{y}) := \frac{1}{2} \dot{y}^T (H^T M(h(y))) H \dot{y} := \frac{1}{2} \dot{y}^T \tilde{M}(y) \dot{y} \quad (3)$$

$$\tilde{\Phi}(y) := \Phi(h(y)) \quad (4)$$

where $M \in \mathbb{R}^{5 \times 5}$ is the mass and inertia matrix. Denote by $f(t): \mathbb{R} \rightarrow \mathcal{Q}$ the flow of the corresponding Hamiltonian dynamics. Conservation of total energy now gives a first integral which, in the most interesting one DOF case, affords a closed form expression for the flow of the system,

$$C = K + \Phi; \quad C = \frac{1}{2} \dot{y}^2 \tilde{M}(y) + \tilde{\Phi}(y) \quad (5)$$

$$\dot{y} = \pm \sqrt{\frac{2C - 2\tilde{\Phi}(y)}{\tilde{M}(y)}} \quad (6)$$

for some constant $C \in \mathbb{R}^+$. For this analysis we will assume that the body of the robot can slide along the ground with minimal friction, while the leg toes have enough friction to act as if it were pinned until it reaches the guard condition⁸.

C. Hybrid Dynamics

A unified formalism for the representation of hybrid dynamical systems was worked out roughly two decades ago [34], although the implications for Lagrangian systems of the sort that concern us here remains an active area of inquiry [35, 36]. While the general framework allows for transitions between arbitrary (piecewise) smooth “patches” of state space, our physical setting restricts transitions to occur only between patches that bear a topological “incidence” relationship. For this reason, our major focus of effort concerns mapping out and systematically exploiting these incidence patterns, and the more general, knotty issues associated with hybrid systems recedes to the background.

There is a growing literature on hybrid dynamical systems over stratified sets [37, 38], of the kind we study here that arise from the changing degrees of freedom intrinsic to “regrasped” rigid body manipulation by limbs or fingers of limited physical extent. Although switches across strata can be understood and planned at non-zero velocity [39], in

⁶Of course they can each be derived by classical methods (e.g. [33]) but we prefer the consistent, notationally uniform derivations arising from a “self-manipulation” perspective [24, 32].

⁷Recall that we are able to reduce the effects of the actuators’ torques to the abstraction of a fixed potential field, by virtue of the assumptions introduced at the beginning of this section.

⁸There is one exception: where the legs are fighting against each other — in these cases the large internal force does not necessarily break this friction assumption [24], however in this regime of maximal torque output it will. Therefore when the motors are commanded with opposite directions, the toes will be assumed to be in sliding friction.

this paper we are concerned with the truly dynamical regime wherein the *timing* of actuation is crucial to shepherding effectively a body’s accumulating kinetic energy through the various transitions. As far as we can determine, the recent literature concerned with (self-) regrasping in a high kinetic energy regime has focused on planning, sensing and control of the object capture [40] or self-landing [41, 42] rather than exploring the many routes from rest toward the high energy aerial phase as we do here. Some exceptions include consideration of one or a few most common routes [43–45], and one paper [46] that formulates the space of hybrid system states into a structure, though not a simplicial complex.

In this work it has proven convenient to adopt the specific hybrid systems formalism introduced by Guckenheimer et al. [47]. To complete that specification we must now define the guard conditions, $g_{j,k}$, and reset maps, $r_{j,k}$, that make up the state transitions. In general the robot can transition between any adjacent states. Adjacent states can be found by either adding a contact (resulting in a loss of one degree of freedom) or removing a contact (resulting in the addition of one degree of freedom) from the current state contact set. The full set of all possible transitions can be thought of as the Hasse diagram of \mathcal{S} , with generically $|\mathcal{S}| \cdot 2^{|\mathcal{S}|}$ directed edges, in this case 64 possible transitions.

These transitions can be categorized as: *Control Triggered*, by touching a leg to the ground, as in $\mathcal{S}_{1010} \rightarrow \mathcal{S}_{1101}$ where the guard condition is the zero of $g_{1010,1101} = \theta_1 - \theta_g$ for some θ_g ; *Sometimes Control Triggered*, for example $\mathcal{S}_{0000} \rightarrow \mathcal{S}_{0100}$, where the guard condition is a function of height and pitch and may be positive for all θ (i.e. the set $g_{j,k}^{-1}(0)$ does not include any configurations at the point b); *State Triggered*, but possible based on the dynamics and initial conditions of the system, such as the takeoff condition $\mathcal{S}_{0101} \rightarrow \mathcal{S}_{0001}$ as described below; *Impossible*, the transition where the body lifts off the ground with no action as in $\mathcal{S}_{1010} \rightarrow \mathcal{S}_{1000}$; and *Undesirable*, while the robot is certainly capable of a hard landing $\mathcal{S}_{0000} \rightarrow \mathcal{S}_{1000}$, a behavior designer may wish to avoid it (and furthermore such transitions may not advance the goal of this paper, leaping). Thus the set of transitions which we will consider (i.e. those that are both possible and desirable) is reduced from 64 to only 18, which are shown in Fig. 3. Note that 15 of the 16 contact conditions remain (only \mathcal{S}_{1111} , the overconstrained case, has been eliminated), but the graph of possible transitions is not nearly as dense. Note that the resulting directed graph in Fig. 3.b does indeed specify a formal grammar comprising all paths initiated at the root (rest state which reach the terminus (flight state) — a vocabulary of legged leaps.

The most interesting of these transitions is the “state triggered” takeoff condition. Take for example the case where both legs are on the ground (following [24]). The holonomic constraints $a: \mathcal{Q} \rightarrow \mathbb{R}^4$ that induce forces $\Gamma \in T^*\mathcal{Q}$ that act on the body give rise to the takeoff condition,

$$a(q) \equiv 0; \quad Da := A(q)\dot{q} = 0 \quad (7)$$

$$\Gamma = A^T \lambda; \quad F\lambda \geq 0 \quad (8)$$

$$\lambda = (AM^{-1}A^T)^{-1}(AM^{-1}(-\Phi) + \dot{A}H\dot{y}) \quad (9)$$

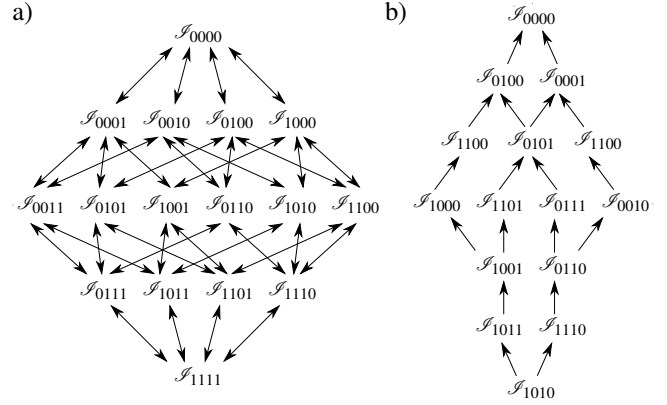


Fig. 3: Logical vs allowable cell transitions over the GRC.

The constraint force magnitudes at contact, $\lambda \in \mathbb{R}^4$, and the friction matrix $F \in \mathbb{R}^{4 \times 2}$ will give the guard condition.

The reset maps [47], $r_{j,k}$, taking the state vector in cell j to the state vector in cell k , must ensure that $h_j(y_j) = h_k(y_k)$ (i.e. the position of the body and limbs is the same), and that $K_j(y_j, \dot{y}_j) \geq K_k(y_k, \dot{y}_k)$ (i.e. no kinetic energy is gained in the transition). While more complicated reset maps can be used, for the present study we will assume that the velocity, \dot{q} , after the transition is simply the projection onto the new free twist direction of the velocity vector before the transition, so in the 1-DOF case, $\dot{q}_k = \frac{H_k^T \dot{q}_j}{|H_k|}$.

III. OPEN LOOP CONTROL OF TRANSITIONS ACROSS THE GROUND REACTION COMPLEX

Here we limit the discussion to leaping transitions, namely transitions that take the robot from $\mathcal{S}_{1010} = \{FB, RB\}$ to $\mathcal{S}_{0000} = \{\}$. The transitions directly to the two 2-DOF systems adjacent to the start (\mathcal{S}_{1000} and \mathcal{S}_{0010}) are impossible, so a path through one of the ends of this edge is required, namely the robot must put down either the front or rear leg.

A. Leg Strategy

The saturated torque assumption yields a binary control input for each leg, pushing forwards $+$ or backwards $-$, and the combined leg strategy $S \in \{+, -\} \times \{+, -\}$ on the robot is then specified by an ordered pair such as $(+, +)$.

These four distinct control inputs are each capable of exciting a multitude of pathways through the directed graph of Fig. 3, yielding the large variety of leaps we explore empirically in Section IV. Furthermore, the half circle legs imply that, for the most part, $(+, +)$ produces a forward lunge, while $(-, -)$ produces a flip. The rest of this section will focus on $(+, +)$ as an example of the insight afforded by the grammar of leaps enumerated in (11) – (16), however all four basic strategies (and a representative selection of the varied leaps achievable by suitably coordinating their relative timing) are documented in the experimental section.

B. Coordination Timing

Choose as a reference time the touchdown of the front leg, and consider the relative timings of the other transitions. The

second leg will touch down at t_2 , which is a coordination time, C , that can be chosen arbitrarily. More complicated leg strategies that depart from the assumptions of Section II may have a higher dimension coordination timing, and might well explore a slightly richer subgraph of Fig. 3.a than the more restricted leaping grammar we focus on in this paper. The time of transition to the air for each leg, t_{1a} and t_{2a} , are implicitly defined based on the Hamiltonian flow and the liftoff guard condition on the hybrid dynamics, $g(f(t)) = 0$, which in a deterministic world are fixed by the choice of jumping strategy S and are a smooth function $T : \mathbb{R} \rightarrow \mathbb{R}$ of the coordination timing C ,

$$t_{1a} = T_{1a}^S(C); \quad t_{2a} = T_{2a}^S(C) \quad (10)$$

where in this example $t_{1a} = T_{1a}^{(+,+)}(t_2)$. A closed form for T is not explicitly needed, but even without it some basic properties will trivially be true, such as $0 < T_{1a}$ and $t_2 < T_{2a}$.

C. Transition Paths

Now we can write out all of the possible state transitions for a jump, based on the set of possible cell transitions described above. The transition path, i.e., the “leap-word”, is an ordered list, and the set of words that are possible are thus (with the zero time transition states suppressed, as well as the always present initial \mathcal{I}_{1010} and final \mathcal{I}_{0000} states),

$$(\mathcal{I}_{0110}, \mathcal{I}_{0010}, \mathcal{I}_{0001}) \Leftrightarrow t_{1a}^* < t_2 \quad (11)$$

$$(\mathcal{I}_{0110}, \mathcal{I}_{0101}, \mathcal{I}_{0001}) \Leftrightarrow 0 < t_2 < t_{1a}^*, t_{1a} < t_{2a} \quad (12)$$

$$(\mathcal{I}_{0110}, \mathcal{I}_{0101}, \mathcal{I}_{0100}) \Leftrightarrow 0 < t_2 < t_{1a}^*, t_{2a} < t_{1a} \quad (13)$$

$$(\mathcal{I}_{1001}, \mathcal{I}_{0101}, \mathcal{I}_{0001}) \Leftrightarrow t_{2a}^* < t_2 < 0, t_{1a} < t_{2a} \quad (14)$$

$$(\mathcal{I}_{1001}, \mathcal{I}_{0101}, \mathcal{I}_{0100}) \Leftrightarrow t_{2a}^* < t_2 < 0, t_{2a} < t_{1a} \quad (15)$$

$$(\mathcal{I}_{1001}, \mathcal{I}_{1000}, \mathcal{I}_{0001}) \Leftrightarrow t_2 < t_{2a}^* \quad (16)$$

as shown in Fig. 3.b. Specific physical parameters may well make some words impossible. For RHex the front leg tends to lift off the ground first, and so the (13) word is not realizable.

Additionally there can be degenerate “double” transitions that are quite interesting, such as the basic jump when $t_2 = 0$. The restriction that T_{1a} is strictly greater than zero, and T_{2a} is strictly greater than t_2 , along with the fact that for RHex $T_{1a}^S(0) \neq T_{2a}^S(0) \forall S$, eliminates all higher order degeneracies.

IV. EXPERIMENTS

In order to explore various regions of the space of jumping controllers, (S, C) , and to test the claim that the underlying topological construction predicts interesting behavioral consequences, we have run over 100 trials sampling the space⁹. Each of the four leg strategies was tested with a sampling of coordination timing parameter values. As RHex actually has 3 legs in the plane, in these experiments the “leading” leg was disabled, i.e. the front leg for $(+, +)$, middle leg for $(+, -)$ and $(-, +)$, and rear leg for $(-, -)$, but we will relax this requirement later.

⁹In order to minimize the effect of battery charge level and other time varying effects, the trial order was randomized and the batteries were never allowed to fall below 75% of full.

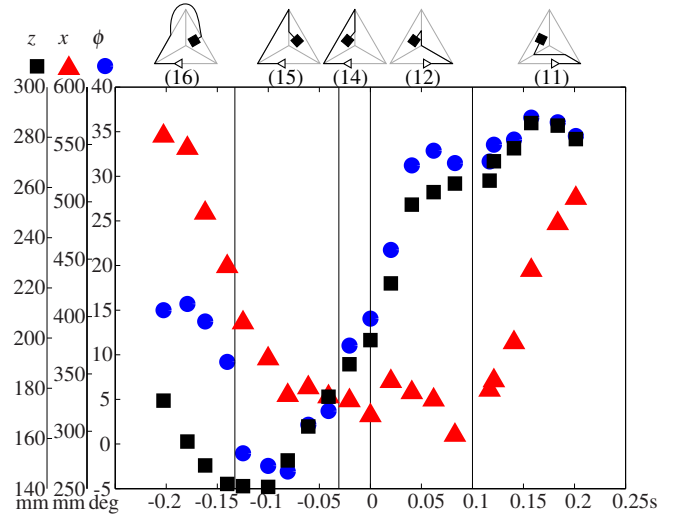


Fig. 4: Apex height (black square), displacement (red triangle), and pitch (blue circle) for $(+, +)$ jumping at various relative leg timings.

Here we report on the height, displacement, and pitch at apex¹⁰, with the $(+, +)$ case highlighted in Fig. 4, and the remaining cases shown in Fig. 5. Full data tables, including additional measurements and higher resolution plots, are available in [48], and the video attachment shows a selection of behaviors. The top of Fig. 4 lists the coordination “word”, (11) – (16), and depicts in a graphical cartoon the different paths through the cell complex, with vertical lines marking approximate transition points¹¹. It is clear that depending on what combinations of these metrics the task requires, several different regions in this space could be useful.

The repeatability can be quantified by comparing the results of 20 additional $(+, +)$ jumps (listed in [48]) to a linearly interpolated estimate based on Fig. 4. This shows an RMS error of 4.3 mm in z , 12.4 mm in x , and 1.4° in ϕ .

But beyond demonstrating which control strategies result in what kinds of jumps, this data clearly shows notable changes near the boundaries between transition paths through the cell complex. For example, the height achieved by the $(+, +)$ strategy has a nearly discontinuous jump just after $t_2 = 0$ — there is a noteworthy advantage in height to letting the front leg start before the rear leg. A similar jump is also present in the pitch however, which may or may not be a good thing depending on the task.

Qualitatively, the leaping strategies are quite different. The $(+, +)$ strategy yields mostly a forward leap, while the $(-, -)$ strategy yields largely a flipping behavior, though for $t_2 > 0$ the robot does not quite complete the flip and instead lands on its nose. The $(+, -)$ strategy causes the robot to jump more or less vertically into the air. The $(-, +)$ strategy

¹⁰Recorded with a Vicon Motion System, <http://www.vicon.com/>

¹¹Paths start at the triangle, end at the square where they transition to the interior (aerial state), and paths outside the triangle represent the bottom face. This data was coded by hand from high speed video of each trial. Takeoff ambiguity is the main reason these transitions are approximate.

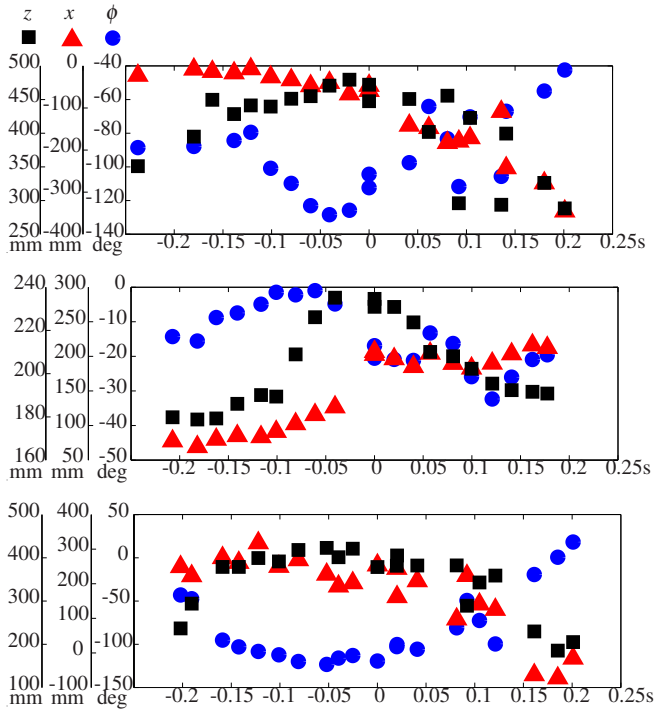


Fig. 5: Apex height (black square), displacement (red triangle), and pitch (blue circle) for $(-, -)$, top, $(+, -)$, middle, and $(-, +)$, bottom, jumping strategies at various relative leg timings.

is mostly a flip, but had trouble for positive values of t_2 . In those trials (as well as a couple for $(-, -)$) the front leg, which is pushing backwards, stretched back along the ground until it hit the middle leg support. Since the motor was at full torque, the leg stuck to the corner of the frame for a short time. Therefore the front leg leading jumps in this strategy would benefit from a more subtle controller to avoid this.

A. Extensions

Here we present some anecdotal results that lie outside the scope of this paper, however can still be informed by the methods presented here.

1) *Three Legged Jumps*: RHex actually has six legs, and not two. While it is easy to anchor the dynamics to the sagittal plane by keeping pairs of legs together, that still leaves three legs. Formally, the third leg will increase the number of hybrid states though not the dimension of the ground reaction complex (which is fixed by body dynamics). However in rigid, non-singular cases only two legs can actually maintain contact on the ground at a time.

However with compliance, and when operating near a singularity (such as $t_2 = 0$, a common occurrence on RHex) it is possible for the three legs to be used, but it may or may not be useful. Initial tests have shown that in the $(+, +)$ case the third leg can only add about 1cm to the final height. In contrast, for the $(+, -)$ case including the middle leg (in the $-$ direction) added 7cm to the apex height, or about a 30% gain in potential energy.

2) *Reversing Strategies*: In a rigid system, reversing the direction of force applied by a motor will simply bleed off some of the energy that is already in the system. However for the compliant half circle legs of RHex, when the leg is moving forward and therefore on the round half of the leg, reversing the torque¹² will sometimes cause the leg to jam and unfold, producing a novel motion. The reverse is not true — if the leg is pushing backwards it will be on the point of the toe, and reversing the direction will usually just lift the leg off of the ground early, or if it did jam simply curl the leg up and slow down the robot. A less extreme reversal has been used in the past [49] to correct the pitch instability of pronk, though the role of the compliant legs was not fully understood. Note that this strategy is taking advantage of the shape change that the compliance allows, but does not recover any energy stored in the unfolded spring.

Since the principle motivation for leg reversal is pitch stabilization, we have tested a hand tuned reversing strategy on the $(+, +)$ jump with $t_2 = 0$, as this may be the most used jump on RHex but does have about 15° of unwanted pitch at apex (more by the time the robot lands). In these initial tests, we have found that in fact stubbing the toe at the end of stroke causes about 20° of pitch correction, albeit at the cost of forward velocity which dropped by 18%. Surprisingly though the stubbed toe experiments did show a slight (2cm) gain in maximum height, which we attribute in part the compliant leg being stretched by this behavior, pushing the robot upwards. Overall the reversing jump had less total energy, but the change in pitch and slight height benefit make it a useful strategy in certain situations.

V. BEHAVIORS

This section applies the preceding catalogue of open loop controllers to the generation of several useful behaviors.

A. Leaping Behaviors

There are many cases where the apex state after a jumping transition is inherently useful. In order to cross a small gap, RHex has previously been shown (but not published) to be capable of crossing a 40cm gap (using the middle and rear legs only). This has been extended to 50.5cm (1 body length) using the $(+, +)$ strategy and $t_2 = 0.02$, a 26% increase. The backflip has been better studied as a way to recover a preferred orientation [22], but has always been completed by rolling on the nose (i.e. never leaving the $\mathcal{S}_{1000} = \{FB\}$ state), implying an apex height of 27cm (though this work did not explicitly optimize for apex height). Fig. 5 documents the $(-, -)$ leap with $t_2 = -.02$ achieving a 48cm apex height (300% of standing height). Because both of these behaviors entail leap-words virtually identical to that past work, we attribute most of the gains to the substantially improved hardware of the current generation robot [16].

While the backflips achieve the highest apex, they are pitched nearly vertical at that state. Fig. 5 reveals a new leap excited by the $(+, -)$ strategy achieving a 23cm apex (143%

¹²A rapid reversal of motor torque requires well hardened electronics with adequate flyback protection, however the electronics in RHex were designed with this in mind [16].

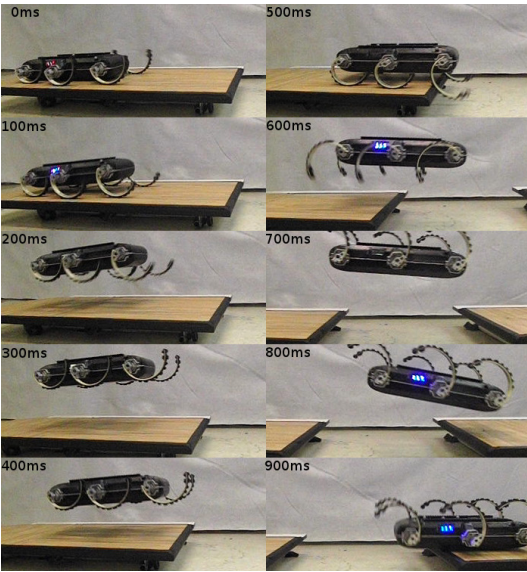


Fig. 6: XRL crossing a 60cm gap. Frames taken every 100ms from a high speed video.

of standing height) at less than 5° pitch. Adding the third pair of legs yields a 30cm apex (nearly 200% of standing height) at 17° pitch. To the best of our knowledge such a near-level vertical leap has never before been elicited from RHex and represents an immediately beneficial consequence of enumerating the entire space of dynamic transitions.

B. Gap Crossing

A variety of compound jumping behaviors benefit significantly from the ability to select a specific initial leap. For example, several high kinetic energy RHex gaits have relatively small basins which can be very effectively “prepared” [50] by selecting the apex state from rest via a leap [32]. However, here, we focus on compound jumps across bigger obstacles than any single leap can afford. Specifically, a leap-step behavior initiated by a 3 legged (+, +) leap with $t_2 = 0$, achieves a high, near-zero pitch apex with significant forward velocity when a reversing strategy is used. Followed by a simple spring-mass stride (with the SLIP parameters adjusted by hand) [20], this leap-step crosses a gap of 60cm (almost 120% of body length), as shown in Fig. 6, representing to the best of our knowledge a 20% gain over the farthest gap jump previously achieved by any general purpose legged robot [4].

C. Jumping on to a Ledge

Another useful application of jumping is to gain access to a high step or ledge. Past quasi-static work on a similar robot has allowed the robot to access an incredible 53% of the body length¹³ [51], the equivalent of a 27cm step up for XRL. By inspecting the results in Fig. 4, it appears that a (+, +) leaping strategy with a large t_2 may be advantageous (i.e. push with the front legs well before the rear legs), as

¹³This work used leg length as the scale, however we feel that under these strategies the robot is gaining much more of an advantage from body length than from leg length.

it reaches a significant height with some forward velocity and a moderate pitch. A timing parameter of about $t_2 = 0.18$ was found to be the best, and was capable of lifting the robot onto a 27cm ledge with either a two or three legged strategy, about the same as the best quasi-static behavior.

For a compound jump onto a ledge, a leap-step similar to the gap crossing behavior reached a ledge of 29cm, a slight improvement. However the previous section reveals far higher leaps are possible, though with significant pitching. This suggests a different compound jump whose initial leap terminates at a vertically pitched apex that vault the legs above a far higher ledge, with the hope of grabbing and pulling the robot up onto it during the second stride. A (-, -) leap with $t_2 = 0.06$ achieves such a (nearly vertical) high apex with some net horizontal displacement. This leap-grab, with no modification, is indeed capable of hooking the robots legs onto a 73cm high table, or 145% of the body length (450% of leg length), as shown in Fig. 1.

The second stride in this compound jump, intended to pull the robot up onto the ledge, is not easy to achieve in the present open loop setting. Absent specialized climbing feet [18], the robot will typically slip off even a coarse-sandpaper-surfaced ledge, as it tries to gain purchase. Extensive tuning (requiring well over 400 attempts) finally achieved a successful stride whose properties lie beyond the scope of the present paper (requiring leg compliance in extension — the rear legs are nearly completely uncurled — and subtle sliding interaction), yet likely is encompassed within the more general self-manipulation framework presently under development [32]. To the best of our knowledge, this compound jump enables to robot to climb onto a ledge higher than that achieved by any previous general purpose legged robot, nearly doubling the best reported prior effort (53% of body length, or 230% of leg length [51]).

VI. CONCLUSION

We have presented the space of legged transitions from complete rest to full flight as generated by combinatorial mixtures of various hybrid dynamical systems indexed by the cells of a “ground reaction complex”. The very regular adjacency relations implied by this topological space organize these sequential mixtures in a sufficiently simple manner as to allow the systematic (“grammatical”) generation of all possible leaps. This enumeration affords a number of new behaviors that significantly extend the range of terrains that the RHex robot can negotiate. Near term future extensions will focus on formal methods of design that exploit this analysis more systematically and effectively than the “hand-crafted” behaviors reported here. Moreover, we are interested in a broader range of dynamical transitions, particularly ones exploiting compliance, including the entirely novel prospect of using the leg springs in extension introduced here.

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