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Exploiting Passive Stability for Hierarchical Control

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Abstract

The dynamics of a Spring Loaded Inverted Pendulum (SLIP) \template" [1] approximate well the center of mass (COM) of running animals, humans, and of the robot RHex [2]. Running control can therefore be ierarchically structured as a high level SLIP control and the anchoring of SLIP in the complex morphology of the physical system. Analysis of the sagittal plane lossless SLIP model has shown that it includes parameter regions where its gait is passively stabilized, i.e. with the discrete control input | the leg touchdown angle | held constant. We present numerical evidence to suggest that an open loop \clock" excitation of a high degree of freedom hexapedal robot model can lead to asymptotically stable limit cycles that \anchor" [1] the SLIP model in its self stabilizing regime. This motivates the search for completely feedforward SLIP locomotion control strategies, which we now speculate may be successfully used to elicit a self-stabilizing running robot such as RHex.

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Exploiting passive stability for hierarchical control

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ABSTRACT

The dynamics of a Spring Loaded Inverted Pendulum (SLIP) "template" [1] approximate well the center of mass (COM) of running animals, humans, and of the robot RHex [2]. Running control can therefore be hierarchically structured as a high level SLIP control and the anchoring of SLIP in the complex morphology of the physical system. Analysis of the sagittal plane lossless SLIP model has shown that it includes parameter regions where its gait is passively stabilized, i.e. with the discrete control input — the leg touchdown angle — held constant. We present numerical evidence to suggest that an open loop "clock" excitation of a high degree of freedom hexapedal robot model can lead to asymptotically stable limit cycles that "anchor" [1] the SLIP model in its self stabilizing regime. This motivates the search for completely feedforward SLIP locomotion control strategies, which we now speculate may be successfully used to elicit a self-stabilizing running robot such as RHex.

1 INTRODUCTION

It is now well established that the centers of mass of running animals [3] and humans [4] approximately follow the dynamics of the sagittal plane SLIP. It has recently been demonstrated that the COM of the robot RHex [2] also exhibits pronounced SLIP characteristics when its mechanical and control parameters are appropriately tuned [5]. Notably, this stable dynamical regime is presently achieved in RHex by a feedforward controller that drives the machine's few (six) actuators without the benefit of any proprioceptive data other than simple proportional derivative control at the motor shaft. A general framework for "anchoring" the SLIP-"template" mechanics in the far more elaborate morphologies of real animals' bodies has been introduced in [1]. However, both the anchoring as well as the control of SLIP seem to demand sensing, actuation, and computation that may be unrealistic relative to the resources that animals and practical robots might be expected to have on hand. Is it possible to introduce sensorily-cheap, low-bandwidth robotic controllers on the basis of the template-anchor paradigm [1]?

At present, outside the "expensive" active anchoring framework as exemplified in [6, 7], we do not understand enough about RHex's mathematical models to select desired gaits by first principles analysis. Stable steady state SLIP behavior is adjusted at present by systematic but almost purely empirical parameter tuning methods. Mathematical analysis of the (simplified) horizontal plane mechanics of a running cockroach [8] has revealed that self-stabilization can occur in two and three degree of freedom lossless and energetically actuated mechanisms (lateral leg spring (LLS) models, see [9, 10]). These models are mechanically very similar to a horizontal plane version of SLIP, motivating a search for self-stabilized gaits in the sagittal plane. Analogous study of the sagittal plane SLIP model now reveals that it too includes parameter regimes that yield self-stabilizing gaits [11].[1] In Sec. 2 we briefly review the SLIP model. In Sec. 3 we describe feedforward control strategies [13, 14] for SLIP locomotion that are motivated by this discovery of self-stabilized SLIP-gaits. Sec. 4 discusses the passive anchoring of SLIP in RHex and outlines the applicability of the proposed control strategies.

2 REVIEW OF SLIP

The SLIP model is shown in Fig. 1a). It depicts a point mass m with a springy leg attached to the hip H. Also shown are the angle ψ formed between the foothold O to COM vector and the vertical (gravity) axis, and the distance ζ from the foothold to the COM. The leg is endowed with a conservative spring with potential $V(\zeta)$. For most of this paper, a Hooke spring potential with $V(\zeta) = (k/2)\,(\zeta - \zeta_0)^2$ is employed. It is assumed

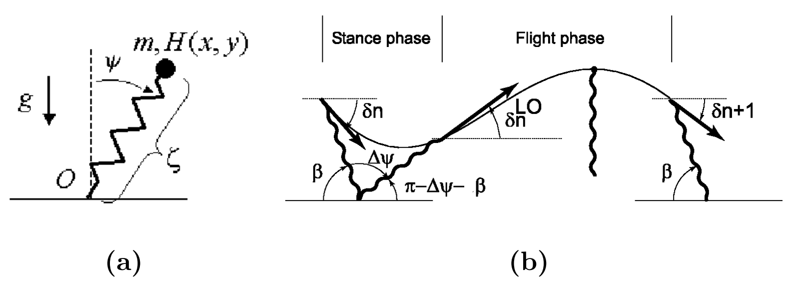


Figure 1. The hopping rigid body a) and the two phases characterising a full stride b).

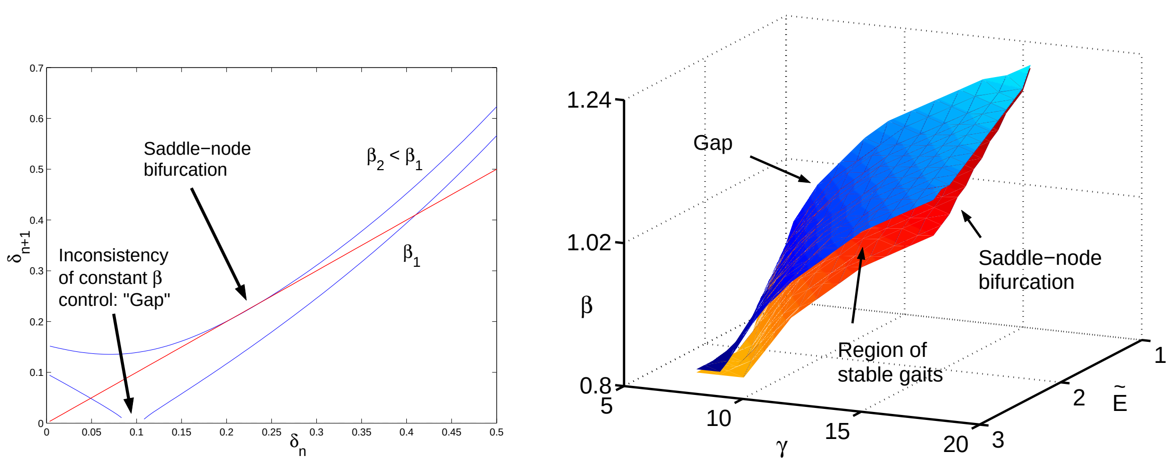
that the spring is at rest length ζ_0 and the leg placed at angle $\beta - \pi/2$ at touchdown; likewise, liftoff is defined as the event when the spring length resumes its rest length. Once the leg has touched the ground, the leg pivots without friction around the foothold O which remains fixed during stance (infinite friction between toe and ground). The body is assumed to remain in the vertical (sagittal) plane and its state, at any point in time, is defined by the position of H, (x, y) referred to a Cartesian inertial frame. Fig. 1b) depicts the SLIP trajectory as a composition of two phases: stance and flight phase. For a sustained forward motion, the leg angle must be changed during flight (e.g. when the COM has reached its apex with apex height y_n^A) from from ψ_{LO} to $\beta - \pi/2$. Since the leg

[1]Self-stabilizing SLIP gaits have already been used in [5] (Fig. 2) to compare SLIP predictions with experimental data. Recently, numerical studies have independently discovered SLIP's passive stability properties [12].

is assumed massless, its angle can be changed with no energy expense during flight, and without influencing the free flight dynamics of the point mass. The simplest feedforward control strategy, namely a constant touchdown angle $\beta_n = \beta \in [\pi/4, \pi/2] \quad \forall n$, with n enumerating subsequent stance phases, is used except when the flight phase apex falls below the COM touchdown height $\zeta_0 \sin\beta$, as we now describe.

3 CONTROL OF VIRTUAL SLIP

As detailed in [11], the dynamics of the flight and stance phase at an asymptotically stable state are determined by three dimensionless parameters: $\tilde{g} = \frac{g\zeta_0}{v^2}$, $\tilde{k} = \frac{k\zeta_0^2}{mv^2}$, and β, where v is the speed of the point mass at touchdown. The control task under consideration is the convergence to a certain apex speed $v_A^* =: \sqrt{g\zeta_0}\hat{v}_A^*$ by changing the leg touchdown angle β. A suitable parameter basis for this control at constant total energy E is $\gamma = \frac{k\zeta_0}{mg}$, $\tilde{E} = \frac{E}{mg\zeta_0}$, and β. The passive stability discussed in [11] assumes constancy of all three parameters during locomotion. However, the parameter regime that yields self-stable gaits is limited; i.e. the Poincaré map as a function of the speed touchdown angle δ has a stable fixed point with a connected domain of attraction only for touchdown angles in an interval $\mathcal{B} = (\beta_1, \beta_2)$ (Fig. 2a). For $\beta > \beta_1$, a gap opens corresponding to COM trajectories that fail to reach touchdown height, and the hopper would stumble.



(a) (b)

Figure 2. a) The Poincaré map (relating the next angle of the COM velocity vector to the previous) within the useful self-stable regime determined by the leg touchdown angle setting, β. b) Region of stable, periodic forward locomotion.

This gives rise to two surfaces in $(\gamma, \tilde{E}, \beta)$-space, which delimit the region of stable, periodic forward locomotion (Fig. 2b). In the interval $\mathcal{B}$, the variation in apex speed Δv_A is in general so limited ($< 5\%$) that total energy E must be altered as well, if the controlled machine is to offer a usefully wide range of target apex speeds. Therefore we limit the SLIP operating regime to a surface $\mathcal{S}_{\mathcal{O}}(\gamma, \tilde{E}, \beta) = 0$ halfway (in terms of β) between the two surfaces. The dimensionless apex speed $\hat{v}_A$ is given by $\hat{v}_A = \mathcal{V}_{\mathcal{A}}(\gamma, \tilde{E}, \beta)$. Hence the

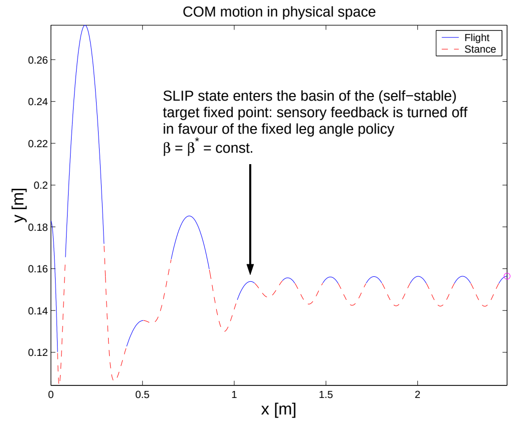


Figure 3. Sample run of a three-stage reduced sensor bandwidth controller.

control is specified by

$$\begin{aligned} \mathcal{S}_{\mathcal{O}}(\gamma, \tilde{E}, \beta) &= 0 \\ \hat{v}_A^* - \mathcal{V}_{\mathcal{A}}(\gamma, \tilde{E}, \beta) &= 0 \end{aligned}$$

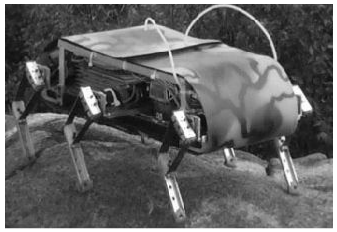
which induces an implicitly defined forward control map $\mathcal{Q}_2 : (\gamma, \hat{v}_A^*) \mapsto (\beta, \tilde{E})$.[2]

Control now proceeds in three stages. First, an energy changing feedback controller with control targets v_A and h_A (apex height) and control inputs (β, ζ_{TD}) moves the SLIP state into the desired energy regime, $\tilde{E}^*$. For $\zeta_{TD} \neq \zeta_0$ energy is injected or removed from the SLIP system. Second, an energy conserving feedback controller with control target v_A and control input β moves the SLIP state in the basin of attraction of the stable fixed point.[3] Finally, the open loop self-stable strategy is engaged, guaranteeing that the desired apex speed, v_A^*, will be achieved and maintained in the face of small perturbations with no need for further sensor involvement. Sensing and control need be reactivated only if the state leaves the domain of attraction (or if a speed (energy) change is desired). This is demonstrated in Fig. 3, where after the first stride the energy-conserving controller kicks in, and is then turned off after the fourth stride, following which the system settles into a “self-stable” periodic gait.

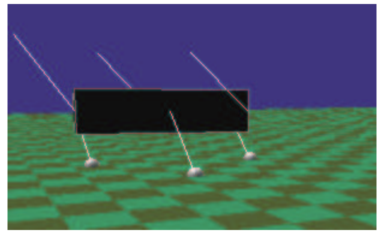
4 SELF-STABILITY OF SLIP IN RHEX

Stable runs with aerial phases as demanded by the SLIP template have been observed at high speeds with the robot RHex [2]. The parameters of these stable runs seem to be in the regime of self-stability of a virtual SLIP embedded in RHex, although, to date, the absence of trajectory data for these runs precludes a rigorous establishment of a SLIP anchoring along the lines of the procedures described in [5]. Firmer evidence for the manifestation of a passively-stabilized SLIP system anchored in RHex's morphology comes from simulations of a RHex-like model with RHex's open-loop controller (simulation environment SimSect [15]). The robot RHex and a snapshot of the GeomView output of a SimSect run is shown in Fig. 4.

[2] This map can be reduced to one strictly monotonic transcendental equation under certain approximations for the spring potential $V(\zeta) = (c/2)\left(1/\zeta^2 - 1/\zeta_0^2\right)$ [14]. For the Hooke spring with gravity in



(a)



(b)

Figure 4. Robot RHex on a rock (a) and a snapshot of a RHex-like model on flat ground in the SimSect simulation evironment (b).

4.1 Passive anchoring of SLIP in SimSect

SimSect simulations were run over a range of initial conditions and clock parameters of RHex's open loop controller (see [2] for a description of those clock parameters). Only those simulations with flight phases (no legs on ground) between consecutive stance phases were considered. Of those, only periodic motions that were stable over at least 10 stance phases were used for further analysis. Here, stability is measured in terms of the maximal deviation of averaged linear and angular velocities for individual stance and flight phases with respect to the respective averaged quantities over 10 stance and flight phases. Fitting a virtual SLIP as in [16] in a range of RHex clock parameters ($t_c \in [0.23, 0.24]s$, $\phi_s \in [0.68, 1.2]rad$, $t_s \in [0.046, 0.12]s$, $\phi_0 \in [-0.04, 0.02]rad$) yielded an average rms error of $(5.3 \pm 3.0)\%$ over 434 stable runs, with an average fitted spring stiffness $\kappa = (5130 \pm 510)N/m$ and an average relaxed spring length $r_0 = (0.172 \pm 0.003)m$. A naive SLIP anchoring in SimSect would give $\kappa = 5400N/m$ and $r_0 = 0.17m$, thus establishing strong evidence for the passive anchoring of SLIP in stable, periodic SimSect simulations with flight phases. However, in order to show the stabilizing influence of the anchored SLIP template in SimSect, we must identify the attractor and its nature that makes our notion of stability in terms of averaged quantities more precise. For this, we selected an operating point with clock parameters $\mathcal{C}_0 = (t_c = 0.24s, \phi_s = 0.92rad, t_s = 0.1056s, \phi_0 = 0.02rad)$.

4.2 Attractors in SimSect

In order to gain insight into the attracting nature of stable SimSect runs as time-trajectories, we record 10 of the 12 dynamical states of the robot's 6 degree-of-freedom rigid body, at 100 consecutive minima z_i $i \in \{1, ..., 100\}$ of the $z(t)$ trajectory. Then the average and the standard deviation for each quantity is computed over all minima and is normalized by the range of the corresponding time trajectory spanning the last 100 minima. This is listed in Table 1, where x, y, z are the COM coordinates, $\theta_p, \theta_y, \theta_r$ are the pitch-, yaw-, and roll-angle, and X stands for any of those quantities. The normalized standard deviations are all smaller than 0.01, strongly suggesting the existence of a period 1 orbit.

We also want to characterize the basins of attraction for the above determined attractor.

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stance, $\mathcal{V}_{\mathcal{A}}$ must be obtained by numerical fitting.

[3]Note that both of these controllers can be realized as modifications of the deadbeat controller discussed in [6].

Table 1. Mean and normalized standard deviation of the coordinates of the robot's Poincaré map, defined by $\dot{z}_i = 0$.

	x	z	$\dot{x}$	$\dot{y}$	θ_p	θ_y	θ_r	$\dot{\theta}_p$	$\dot{\theta}_y$	$\dot{\theta}_r$
$\mathrm{av}(\{X_i\})$	-0.297	0.1422	0.0017	2.078	0.0063	-0.108	-0.0281	1.2765	0.1056	0.2764
$\frac{\mathrm{std}(\{X_i\})}{\mathrm{Range}(X(t))}$	0.009	0.0001	0.0002	0.002	0.0001	0.002	0.0003	0.0002	0.0004	0.0003

Table 2. Initial conditions that flow to an attracting manifold.

z	$\dot{x}$	$\dot{y}$	$\dot{z}$	θ_p
0.175,0.177	-0.01,0.01	1.59,1.61	-0.01,0.01	-0.01,0.01
θ_y	θ_r	$\dot{\theta}_p$	$\dot{\theta}_y$	$\dot{\theta}_r$
-0.01,0.01	-0.01,0.01	-0.2,0.2	-0.2,0.2	-0.2,0.2

To this end, a neighbourhood around an initial condition $\mathcal{I}_0$ at the operating point $\mathcal{C}_0$ that resulted in stable motion is explored. For vertices on the boundary of the open region defined in Table 2, SimSect runs converged to period 1 orbits in the neighbourhood of the one characterized in Table 1. The fact that period 1 orbits for different initial conditions with different initial total energy are not identical is in accord with the neutral stability of a virtual SLIP with $\beta = const$ policy with respect to energy perturbations (see [11], but note that we have not as yet characterized the dimensionality of the attracting set in this far more complex setting).

To have confidence that the observed phenomenon is not in some sense "exceptional" we wish to ascertain that the qualitative behavior of interest persists within an open ball in parameter space. To that end, we have confirmed that period 1 orbits persist over variations in the clock parameter space around $\mathcal{C}_0$ with initial condition $\mathcal{I}_0$ as characterized in Table 3.

It must be emphasized that while this period one behavior is not "exceptional" it is by no means clear that the behavior is "typical". Namely, we have not yet found such a regime in the clock space that affords significant variation over the physical states of the anchored SLIP. Often - i.e., from many clock parameter settings, the high dimensional system seems to exhibit a period two attractor. However, the physical variations (i.e., forward speed, apex height, etc.) between the two components of the attractor are small enough as to be satisfactorily "constant" in applications settings. A more rigorous investigation of those attractors is currently in progress.

4.3 Passive stability of anchored SLIP

For each stable SimSect run, the dimensionless SLIP quantities $(\gamma, \tilde{E}, \beta)$ and the touch-down velocity angle δ_n can be computed by fitting a virtual SLIP as in Sec. 4.1. Then by numerically computing the SLIP return map (in terms of the COM velocity vector angle δ_n) with these parameters, self-stabilizing behaviour and the location of the fitted virtual SLIP in terms of δ_n in the return map can be determined. Only one out of the 434

Table 3. A neighbourhood in clock parameter space with similar orbits.

t_c $[s]$	ϕ_s $[rad]$	t_s $[s]$	ϕ_0 $[rad]$
0.235...0.24	0.911...0.938	0.101...0.108	0.0196...0.0204

runs resulted in a fixed point free return map, and all other runs had touchdown velocity angles δ_n to the left of the unstable fixed point (see Fig. 2a), indicating that the virtual SLIPs operate inside the basin of attraction of the self-stabilizing fixed point. However, in only five of those 433 runs was the return map without gap — the interval wherein the literal SLIP model fails to return to the next apex (see Fig. 2a). These five cases included the operating point $\mathcal{C}_0$. In most other cases the gap extended from its right boundary to $\delta_n = 0$, "swallowing" the attracting fixed point. From a theoretical point of view, the constant leg touchdown angle control is then inconsistent and the return map does not even have a locally attracting fixed point.

However, in contrast to the literal SLIP, RHex as well as SimSect do not stumble when the gap is encountered by the virtual SLIP because their physical legs recirculate around. Hence the touchdown angles of RHex's six legs are determined by the open loop controller and the interaction with the ground. It seems that the gap is an artefact of the SLIP abstraction which is avoided by recirculating the massless leg driven by a clock. Moreover, as is discussed in [14], return maps with gaps can be made globally consistent, if the $\beta = \beta^*$ control policy is modified to $\beta = \min(\beta : \beta \geq \beta^*)$ where the minimum is taken over the physically allowed touchdown angles β.[4] If this modified control is used, new attractors arise which include period 2 orbits and a "degenerate" SLIP gait with convergence to $\delta_n = 0$ and no flight phase. This might explain the frequent appearance of stable period 2 gaits amongst the 434 stable runs. In summary, taking into account the modified constant touchdown angle control, the specific parameter regimes that characterize anchored virtual SLIP models do indeed operate in the "self-stable" regime of interest.[5]

5 DISCUSSION

These preliminary simulation results together with the established SLIP anchoring at lower speeds of RHex in [5] suggest: a) For running gaits, SLIP is anchored in RHex by a not (yet) understood coupling of RHex's morphology, the open-loop controller and the environment; and b) the virtual SLIP anchored in RHex can operate in its passively stabilized regime. These observations provide the begining of theoretical framework to explain the "self-stable" gaits of RHex at high speeds with aerial phases. To complete this explanation, we must establish exactly how the open loop "CPG" signal succeeds in driving RHex into a dynamical regime from which the SLIP template emerges, as may be done in the far simpler one degree of freedom setting [17]. The control application is clear from the previous section: we want to drive RHex *automatically* into the (SLIP-) basin of attraction of the desired (self-stable) apex speed. Although the required state feedback for a deadbeat controller runs contrary our goal of developing sensory cheap low-bandwidth controllers, the deadbeat controller can be turned off after the basin of attraction has been reached, thus lowering the *average* feedback bandwidth and, potentially, eliminating it entirely if subsequent perturbations do not throw the state outside the basin.

[4] This gives rise to a 2-dimensional return map.

[5] A similar observation in the case of human runners was made in [12].

References

[1] R. Full and D.E. Koditschek. Templates and anchors: neuromechanical hypothesis of legged locomotion on land. *Journal of Experimental Biology*, 83:3325–3332, 1999.

[2] U. Saranli, M. Buehler, and D.E. Koditschek. Rhex: A simple and highly mobile hexapod robot. *The International Journal of Robotics Research*, 20(7):616–631, 2001.

[3] R. Blickhan and R. Full. Similarity in multilegged locomotion: Bouncing like a monopode. *J. Comp. Physiol.*, A(173):509–517, 1993.

[4] W.J. Schwind. *Spring Loaded Inverted Pendulum Running: A Plant Model*. PhD thesis, University of Michigan at Ann Arbor, 1998.

[5] R. Altendorfer, N. Moore, H. Komsuoḡlu, M. Buehler, H.B. Brown Jr., D. McMordie, U. Saranli, R. Full, and D.E. Koditschek. Rhex: A biologically inspired hexapod runner. *Autonomous Robots*, 11:207–213, 2001.

[6] U. Saranli, W.J. Schwind, and D.E. Koditschek. Toward the control of a multi-jointed, monoped runner. In *Proc. IEEE Int. Conf. on Rob. and Aut.*, pages 2676–2682, Leuven, Belgium, 1998.

[7] J. Nakanishi, T. Fukuda, and D.E. Koditschek. A brachiating robot controller. *IEEE Trans. Rob. Aut.*, 16(2):109–123, 2000.

[8] T. Kubow and R. Full. The role of the mechanical system in control: a hypothesis of self-stabilization in hexapedal runners. *Philosophical Transactions of the Royal Society of London Series B - Biological Sciences*, 354(1385):849–861, 1999.

[9] J. Schmitt and P. Holmes. Mechanical models for insect locomotion: dynamics and stability in the horizontal plane I. Theory. *Biological Cybernetics*, 83:501–515, 2000.

[10] J. Schmitt and P. Holmes. Mechanical models for insect locomotion: Active muscles and energy losses. *Biological Cybernetics (in review)*, 2002.

[11] R. M. Ghigliazza, R. Altendorfer, P. Holmes, and D. E. Koditschek. Passively stable conservative locomotion. submitted to *SIAM Journal of Applied Dynamical Systems*, 2002.

[12] A. Seyfarth, H. Geyer, M. Günther, and R. Blickhan. A movement criterion for running. *Journal of Biomechanics*, 35:649–655, 2002.

[13] D. E. Koditschek, R. Altendorfer, R. M. Ghigliazza, and P. Holmes. Self-stability mechanisms for sensor-cheap legged locomotion. In *IV. World Congress of Biomechanics (WBC)*, Calgary, Canada, 2002.

[14] R. Altendorfer, R. M. Ghigliazza, P. Holmes, and D. E. Koditschek. Hopping on a springy leg: feedforward control. *In preparation*, 2002.

[15] U. Saranli. Simsect hybrid dynamical simulation environment. Technical Report CSE-TR-437-00, University of Michigan, 2000.

[16] R. Altendorfer, U. Saranli, H. Komsuoḡlu, D.E. Koditschek, H.B. Brown Jr., M. Buehler, N. Moore, D. McMordie, and R. Full. Evidence for spring loaded inverted pendulum running in a hexapod robot. In *Experimental Robotics VII*, pages 291–302. Springer Verlag, 2001.

[17] H. Komsuoḡlu and D.E. Koditschek. Preliminary analysis of a biologically inspired 1-dof clock stabilized hopper. In *4th World Multiconference on Systemics, Cybernetics and Informatics*, Orlando, FL, 2000.