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Do Required Minimum Distribution 401(k) Rules Matter, and For Whom? Insights from a Lifecycle Model

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Abstract

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Keywords

life cycle saving, household finance, 401(k) plan, retirement, tax policy

Disciplines

Economics

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ABSTRACT

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Do Required Minimum Distribution 401(k) Rules Matter, and For Whom? Insights from a Lifecycle Model

US tax law has long required older Americans to withdraw a stipulated minimum fraction from their tax-qualified retirement account balances each year, and then pay tax on the income withdrawn. Moreover, if the retiree withdraws too little during the taxable year, she must pay a 50% excise tax on the under-withdrawn amount. Known as the Required Minimum Distribution (RMD) Rule, the starting age of this mandatory withdrawal policy was initially set at age 70.5, with the minimum withdrawal fraction from the account balance rising with age reflecting the account owner's remaining life-expectancy (IRS 2015). The rationale for an RMD policy was that contributions and investment earnings in both tax-qualified employer sponsored 401(k) plans and Individual Retirement accounts (IRA) were tax-exempt until the money was withdrawn.¹ If, however, the retirement saver died prior to drawing down her entire account, the remaining assets would pass to her heirs who could stretch distributions over their own (probably longer) lifetimes and potentially result in much lower tax revenue collected.² Accordingly, the RMD rule was implemented to get retirement account owners to pay more income tax on their pre-tax contributions and investment earnings before their deaths. As Mortenson, Schramm, and Whitten (2019) pointed out, since RMDs happen in the distant future, these seek to limit the cost of tax subsidies designed to encourage retirement saving without adversely impacting their intended

¹ RMD rules apply to Individual Retirement Accounts (IRA) and most employer sponsored defined contribution plans (such as 401(k) and 403(b) plans); the latter may be rolled over to IRAs when the participant retires. We do not consider Roth accounts in this paper; Roth account holders are not subject to RMD rules, though their beneficiaries usually are.

² The 2019 SECURE Act required that inherited qualified retirement accounts for non-eligible designated beneficiaries be paid out over a maximum of 10 years, instead over their life expectancies as under the old law (Gradisher and Tassell-Getman 2020; Hartman 2020). Eligible designated beneficiaries, including a surviving spouse, a minor child, and other individuals not more than 10 years younger than the deceased owner, can still stretch after death distributions over their own lifetimes.

effect, which is to enhance old age security of private households through the accumulation and decumulation of retirement assets.³

The SECURE Act of 2019 recently extended the age for required minimum distributions to age 72, and several advocates seek to raise it even more, to age 75 (Waddell 2019; Kapadia and Hershberg 2020). For instance, in October of 2020, House Ways and Means Committee Chairman Richard Neal (D-MA) and Ranking Member Kevin Brady (R-TX) proposed the “Securing a Strong Retirement Act of 2020,” boosting the RMD age to 75. The previous year, key members of the Senate Finance Committee, Sens Rob Portman (R-Ohio) and Ben Cardin (D-Md.) offered the “Retirement Security and Savings Act of 2019,” which would have eliminated the RMD for retirees having retirement assets worth less than \$100,000 in aggregate,⁴ an approach called a “progressive RMD.” Moreover, a full abandonment of the RMD rules may become a reality, according to some analysts (Berry 2020).

Softening the RMD restrictions is intended to make retirement savings more attractive to private households. Nevertheless, since “RMDs are [intended] to generate taxable income from these distributions, it probably won’t help the federal deficit if they push the age back,” according to financial adviser Malito (2018: np). Indeed, the US Joint Committee on Taxation (JCT 2019) estimated that federal tax revenue would fall by \$8.9 billion over the period 2019-2029, as a result of simply raising the RMD age to 72.

Whether and how Americans might respond to a further RMD deferral, or indeed the rule’s abolition, is the subject of this paper. To investigate potential outcomes on various decision variables of private households, we develop and solve a realistically-calibrated lifecycle model

³ For a historical discussion of RMD rules in the United States, see Warshawsky (1998).

⁴ See S.1431 - Retirement Security and Savings Act of 2019, 116th Congress (2019-2020). <https://www.congress.gov/bill/116th-congress/senate-bill/1431/text#toc-HF935525E46F14165887066E18B94649A>

with taxable and tax-deferred retirement accounts that allows also to evaluate the implications on tax-payments to the government of alternative RMD rules. Specifically, we measure the anticipated impact on work hours/retirement patterns, saving/consumption patterns, and contribution/withdrawal paths in tax-qualified accounts. This allows us to measure both potential direct and indirect effects on optimal withdrawal behaviour resulting from the restrictions established by the proposed changes in RMD rules. Our research, therefore, contributes to the rich literature on financial decisionmaking of private households (see Gomes, Haliassos, and Ramadorai 2020 for an overview) using dynamic consumption and portfolio choice models in discrete time (see Gomes 2020 for an overview), initiated by Cocco et al. (2005) and Gomes and Michaelidis (2005). Specifically, we build on and extend prior work by Gomes, Michaelidis, and Polkovnichenko (2009) on tax-deferred retirement accounts by including endogenous labour supply (as Gomes et al. 2008) and claiming decisions of social security benefits (as in Hubener et al. 2016), while undertaking a detailed consideration of the taxation of withdrawals. We extend the existing empirical literature on the impact of RMD rules (Brown, Poterba, Richardson 2017 and Mortenson et al. 2019) by providing new additional insights from our theoretical life cycle model. In addition, we contribute to the current and ongoing policy debate on the efficient decumulation of the \$20 trillion in assets (ICI 2020) held in tax-qualified defined contribution retirement plans, by analyzing the economic implications of altering the RMD rules.⁵

We show that delaying the RMD age further would have little impact during the worklife, including on workers' savings and asset allocation inside and outside tax-qualified retirement accounts. Additionally, social security claiming behavior is almost unaffected. By contrast, significant changes are observed for distributions from tax-qualified accounts during the retirement

⁵ Early empirical research on IRA withdrawals is available in Sabelhaus (2000).

period, depending on whether older people have a bequest motive or not. For those lacking a bequest motive, the RMD rules are not particularly restrictive, as optimal expected withdrawals from 401(k) plans are substantially higher than the RMD pattern required by the IRS. This conclusion holds for the RMD-70 as well as at ages 72 and 75; we also show that even under a progressive RMD, people's withdrawal behavior and therefore tax-payments would change little. By contrast, when the retiree has a bequest motive, results are rather different. The age-70 RMD rule proves quite restrictive, since many individuals would prefer to make fewer withdrawals than required and use the 401(k) plans as a tax-favored tool to transfer financial wealth to the next generation. We also show that when the RMD age is delayed to 72 or 75, the sum of lifetime tax payments is not much affected, even for persons with a bequest motive. By contrast, if RMD rules were applied only to retirement assets in excess of \$100,000, as under the progressive RMD approach, or were completely suspended, this would result in notably lower lifetime tax payments by high-income individuals having a bequest motive.

In what follows, we develop and calibrate a discrete time lifecycle model using US data for utility-maximizing workers with endogenous work hours, retirement behavior, consumption/saving, and portfolio choice including risky stocks and bonds held in and outside a tax-deferred retirement plan. The model embeds exogenous background risks (labor income, capital market, out-of-pocket medical expenditures), heterogeneity of income profiles and preferences, realistic rules on income taxes, and regulations regarding social security benefit claiming options. Just as importantly, the model also integrates real-world rules characterizing tax-qualified 401(k) accounts including pre-tax contributions, employer matches, and RMD withdrawal amounts.⁶ Our results using calibrated baseline parameters agree closely with observed

⁶ An unpublished working paper closely related to ours by Stuart and Bryant (2020) also builds a structural lifecycle model to investigate the impact of RMD withdrawal penalties on Individual Retirement Account (IRAs). While their

U.S. household saving and social security claiming ages. We then use this realistic calibrated life cycle approach to generate optimal consumption/savings, work/retirement patterns, and portfolio allocations in a baseline case and compare results across different RMD scenarios.

Model and Calibration

Time budget, labor income, and retirement benefits. Our lifecycle model assumes a representative worker making annual decisions from age 24 ($t = 0$) until her maximum age of 100 ($T = 76$). This worker can allocate up to $(1 - l_t) = 0.6$ of her available time budget to paid work (assuming 100 waking hours per week and 52 weeks per year). Depending on her work effort $(1 - l_t)$ and the wage rate WR_t , her yearly before-tax labor income during her worklife is:

$$Y_{t+1} = (1 - l_t) \cdot WR_t . \quad (1)$$

The uncertain wage rate $WR_t = w_t \cdot P_t \cdot U_t$ consists of an age-dependent deterministic component (w_t), an uncertain permanent component $P_{t+1} = P_t \cdot N_{t+1}$ with independent lognormal distributed shocks $N_t \sim LN(-0.5\sigma_P^2, \sigma_P^2)$, and a transitory shock $U_t \sim LN(-0.5\sigma_U^2, \sigma_U^2)$ assumed uncorrelated with N_t . We assume heterogeneous individuals and calibrate the deterministic components of the wage rate process and variances of the permanent and transitory wage shocks separately for six groups, namely men and women of three educational levels: less than High School (<HS), High School graduate (HS), and at least some college (Coll+). The estimation procedure draws on the 1975–2015 waves of the Panel Study of Income Dynamics (see Appendix A).

model-simulated outcomes match IRA withdrawal data provided by the IRS, their setup is less rich than ours. Unlike in this paper, they do not include endogenous work hours, social security claiming decisions, portfolio decisions, exogenous shocks from risky stocks, and out-of-pocket medical expenditures. They also eschew heterogeneity analysis, while our results allow six different income profiles (by sex and education), preferences (bequest/no bequest), and three exogenous shocks (stocks, medical expenditures, and labor income).

Between ages $62 \leq K \leq 70$, the individual may retire from work and claim social security benefits which results in a yearly retirement income ($t \geq K$) of:

$$Y_{t+1} = PIA_K \cdot \lambda_K \cdot \varepsilon_{t+1} \quad (2)$$

Old age retirement benefits depend on the retiree's Primary Insurance Amount (PIA) and an adjustment factor λ_K for early or delayed claiming. Thus, the $PIA = \min[0.9AIME; 9,666 + 0.32(AIME - 10,740); 26,954 + 0.15(AIME - 64,764); 36,500]$ in 2018 was a piecewise linear function of (12 times) the worker's average indexed monthly lifetime earnings (AIME). If a worker claims benefits at the system-defined Normal Retirement Age of 66, the PIA replaces 90% of the first \$10,740 of average lifetime earnings, plus 32% of earnings between \$10,740 through \$64,764, plus 15% of earnings over \$64,764 up to the cap (\$128,400).⁷ An adjustment factor permanently decreases (increases) benefits if an individual claims benefits before or after the Normal Retirement Age of 66. More specifically, the factors we use are $\{\lambda_{62} = 0.75; \lambda_{63} = 0.8; \lambda_{64} = 0.867; \lambda_{65} = 0.933; \lambda_{66} = 1.0; \lambda_{67} = 1.081; \lambda_{68} = 1.16; \lambda_{69} = 1.24; \lambda_{70} = 1.32\}$. Finally, the variable ε_t is a independent lognormal distributed transitory shock $\varepsilon_t \sim \text{LN}(-0.5\sigma_\varepsilon^2, \sigma_\varepsilon^2)$, which reflects out-of-pocket medical expenditure shocks in retirement (as in Love 2010). If the individual works beyond age 62, the model stipulates that she must devote at least one hour per week, and our model rules out overtime work in retirement ($0.01 \leq 1 - l_t \leq 0.4$).

Wealth dynamics and budget constraint. The individual can use current cash on hand for consumption C_t , investments in risky stocks $S_t \geq 0$, riskless bonds $B_t \geq 0$, and contributions $A_t \geq 0$ to employer sponsored

⁷ We use 2018 parameter levels; as these increase with inflation each year (US SSA nd_a b), our model focuses on real rather than nominal values. Following Chai et al. (2011), the PIA is approximated using permanent income in the optimization. In the simulation of optimal life cycles, we use the 35 best years of earnings to specify the PIA and adjust the corresponding permanent income state.

tax-deferred qualified 401(k) plans which are taxed according to an EET-regime: that is, workers can deduct contributions to retirement accounts from taxable income up to a yearly limit,⁸ earn pre-tax investment earnings in their accounts, and pay income tax on withdrawals during retirement (when the marginal tax rate is usually lower than during the worklife). After retirement at age K , no further contributions may be made into 401(k) plans $A_t = 0$ ($t \geq K$). Hence, cash on hand X_t in each year is given by:

$$X_t = C_t + S_t + B_t + A_t. \quad (3)$$

One year later, cash on hand is the value of stocks (bonds) having earned an uncertain (riskless) gross return of R_{t+1} (R_f), plus income from work after age-dependent housing costs h_t (as in Love 2010), plus withdrawals (W_t) from the retirement plan, minus taxes (Tax_{t+1}):

$$X_{t+1} = [S_t R_{t+1} + B_t R_f](1 - f) + Y_{t+1}(1 - h_t) + W_t - Tax_{t+1}. \quad (4)$$

Our financial market parameterizations assume a risk-free rate of 1% and lognormal distributed stock return $\ln(R_t) \sim N(0.05; 0.18)$ with a mean of 5% and a return volatility of 18%. For assets outside the tax-qualified retirement account, we assume a yearly management fee of $f = 1\%$ to reflect the (usually) higher cost of retail investment compared to employer-sponsored retirement plans. As in Lusardi, Michaud, and Mitchell (2017), if a worker's cash on hand falls below $X_{t+1} \leq \$5,879$ p.a., the model posits that she receives a minimum welfare benefit of \$5,879 the next year.

Individuals must pay three kinds of taxes: payroll, federal income, and penalty taxes for non-compliant withdrawals from 401(k) accounts. Payroll taxes are proportional to the worker's annual earnings, amounting to 11.65% until retirement (the sum of 1.45% Medicare, 4% city/state

⁸ Our model uses a yearly own contribution limit of \$18,500 until retirement regardless of age; the 2018 IRS constraint permits additional catch-up contributions of \$ 6,000 for those over age 50.

tax⁹, and 6.2% social security contributions up to a cap). After retirement, social security and Medicare contributions are generally no longer paid. In addition, both workers and retirees pay federal income taxes that depend on taxable income and the corresponding progressive marginal tax rates for each of the seven tax brackets (parameters as per 2018). Own contributions into a 401(k) plan reduces the worker's taxable income (see online Appendix B), while withdrawals from 401(k) plans increase it. A penalty tax of 10% is payable on withdrawals from 401(k) accounts prior to age 59½ ($t = 36$), and a penalty of 50% applies if withdrawals are less than required under the RMD rule. Inherited assets are tax-exempt.¹⁰

Tax deferred retirement accounts and RMD rules. The worker's assets in the tax-qualified retirement plan are invested in a portfolio of risky stocks and bonds. Letting $\omega_t^S \geq 0$ be the relative exposure to equity, this portfolio generates a gross portfolio return of $R_{t+1}^{401(k)} = \omega_t^S R_{t+1} + (1 - \omega_t^S) R_f$. In addition to the benefits from deferred taxation, we assume that employers match 100% of employee contributions up to 5% of yearly gross labor income, so that the 401(k) plan is able to avoid complex non-discrimination testing.¹¹ Due to tax regulation (as of 2018), the matching rate is only applied to a maximum compensation of \$275,000, so the overall matching contribution is given by $M_t = \min(A_t, 0.05Y_t, \$13,750)$. After retirement, no additional own contributions are possible ($A_t, M_t = 0$). Prior to the endogenous retirement age $t = K$, the total value (F_{t+1}) of retirement assets at time $t + 1$ is determined by the previous period's value minus any withdrawals ($W_t \leq F_{t+1}$), plus additional own contributions (A_t), plus any employer match (M_t), and returns on stocks and bonds.

⁹ State and local taxes vary widely across the states (e.g. 0% Texas; 13.3% California) and municipalities, so our parameter is an average that we add to the payroll tax for simplicity's sake.

¹⁰ Current U.S. law does not require the filing of an estate tax return for gross assets of less than \$11.58 million for persons dying in 2020. Eligible designated beneficiaries do not have to pay estate tax on inherited IRA assets and can defer payment of income tax.

¹¹ See Willson (2019) for a discussion of 401(k) safe harbor plans. Love (2007) reports a value of 100% matching to 6% in US defined contribution plans.

From a given starting age onwards (RMD_{age}), plan participants must take payouts from the retirement account each year, defined as a certain fraction (m_t) of the account value according to the Required Minimum Distribution rules. This required fraction is increasing with age and is calculated as 1 divided by the distribution period specified in the IRS Uniform Lifetime Table (IRS 2015).¹² For the starting age RMD_{age} , we consider three cases: age 70 (consistent with the rules in place until 2019); age 72 (as implemented in 2020); and the recent proposal to raise the RMD age to 75. Withdrawals below the RMD threshold lead to a penalty tax of 50% on the under-withdrawn amount, taken directly from the 401(k) account. The dynamics for the retirement account evolve as follows:

$$F_t = \begin{cases} (F_t - W_t + A_t + M_t)R_{t+1}^{401(k)} & \text{for } t < K \\ (F_t - W_t)R_{t+1}^{401(k)}, & \text{for } t < RMD_{age} \\ (F_t - W_t)R_{t+1}^{401(k)} - \max(0.5(m_t F_t - W_t), 0), & \text{for } t \geq RMD_{age} \end{cases} \quad (5)$$

Additionally, we also consider the progressive RMD case, where the RMD_{age} rises to 75 but the withdrawal rule applies only to those with tax-qualified retirement accounts worth over \$100,000.

Preferences and numerical solution. The worker derives utility from a composite good consisting of consumption C_t and leisure time l_t (normalized as a fraction of total available time), modelled by the time-separable power utility function $u_t(C_t, l_t) = \frac{(C_t l_t^\alpha)^{1-\rho}}{1-\rho}$. After retirement, the individual enjoys full leisure ($l_t = 1$). The parameter α measures leisure preferences; ρ is the coefficient of relative risk aversion; and β the time preference factor. In addition, she receives utility from bequeathing financial wealth after she dies

¹² If an individual remains employed at the same firm at which she has her retirement plan, she need not take an RMD until she stops working.

to the next generation, from both her tax-qualified and regular saving accounts $Q_t = F_t + S_t + B_t$. The parameter $b \geq 0$ measures the strength of her bequest motive. The recursive definition of the value function of this dynamic stochastic programming problem is given by:

$$J_t = \frac{(C_t l_t^\alpha)^{1-\rho}}{1-\rho} + \beta E_t \left(p_t J_{t+1} + (1-p_t) b \frac{\left(\frac{Q_{t+1}}{b}\right)^{1-\rho}}{1-\rho} \right), \quad (6)$$

with terminal utility $J_T = \frac{(C_T)^{1-\rho}}{1-\rho} + \beta E_T \frac{b \left(\frac{Q_{T+1}}{b}\right)^{1-\rho}}{1-\rho}$. Age-specific annual survival probabilities p_t ($p_0 = 1$ and $p_T = 0$) for males and females are taken from the US population life table in the National Vital Statistics Report (Arias 2010).

We posit that individuals with/without a bequest motive in each of the six subgroups (male/female, with education levels <HS, HS, and Coll+) maximize the value function (6) subject to the constraints and calibrations set out above, by optimally selecting their consumption, work hours, social security claiming ages, contribution/withdrawals from tax-qualified 401(k)-plans, 401(k) equity exposures, and investments in stocks and bonds in non-tax-qualified accounts. The numerical procedure to generate the optimal policy functions in each period assumes a four-dimensional discrete state space grid $35(X) \times 25(F) \times 8(P) \times 9(K)$, with X being cash on hand, F referring to 401(k) assets, P permanent income, and K the claiming age.

Calibration of preference parameters. We believe it is important to calibrate such a rich lifecycle model to both financial and non-financial data. Accordingly, calibration of preference parameters (assumed unique for each of the six sex/education subgroups) follows the procedure outlined in Horneff et al. (2020a). We aim to ensure that the model outcomes simultaneously match empirical claiming rates reported by the US Social Security Administration (US SSA 2015), as well as average assets in 401(k) plans reported for 7.3 million plan participants in EBRI (2017). For each

of the subgroups, we solve the lifecycle model under the tax regime and social security rules in place in 2015 (i.e. using RMD 70), generate 200,000 simulated independent lifecycles using optimal feedback controls, and calculate average claiming rates and 401(k) account balances (with respect to three exogenous random variables: stock returns, permanent, and transitory income shocks). These six subgroups are then aggregated to obtain population mean values using National Center on Education Statistics (NCES 2016) weights.¹³ Repeating this procedure for alternative sets of preference parameters (ρ, β, α) assuming $b = 0$ generates a coefficient of relative risk aversion $\rho = 5$, a time discount rate $\beta = 0.96$, and a leisure parameter $\alpha = 1.3$ which closely match simulated model outcomes as well as empirical evidence on both average assets in tax-qualified retirement accounts and social security claiming ages (see Appendix B). Specifically, the model generates a large peak at the earliest claiming age of 62, along with a second peak at the (system-defined) Full Retirement Age. Our model also matches the current distribution of 401(k) wealth by age rather nicely.

Next, we again use these calibrated preference parameters to solve for the optimal lifecycle policies and generate simulated optimal outcomes for the six subgroups, but this time we implement the 2018 tax and social security rules. The number of simulations for each subgroup depends on NCSE (2016) population weights by sex and education, generating a representative distribution of outcomes for the overall population. For comparison purposes, this procedure is followed for the same heterogeneous households with and without a bequest motive $b = 2$, in line with parameters reported in prior lifecycle studies.¹⁴ The simulated lifecycle outcomes (consumption, income, work-hours, retirement ages, 401(k)-contributions/withdrawals/wealth,

¹³ Specifically, the weights are 50.7% female (61% with Coll+, 28% with HS, and 11% with <HS), and 49.3% male (57% with Coll+, 30% HS, and 13% <HS).

¹⁴ See Gomes and Michaelides (2005) and Love (2010).

stocks/bonds, tax payments) of these two groups are then the basis for our comparisons in the policy analyses below.

What Would Delaying the RMD Age Do?

In what follows, we compare the expected optimal outcomes from the lifecycle model for several different RMD rules. The first set of results adopts the RMD start age of 70 (RMD-70), in effect before 2020.¹⁵ Our second analysis implements the new start age of 72 (RMD-72) effective from 2020; the third assumes that the RMD age is 75 (RMD-75); and last, we posit an age-75 RMD age applied only to accounts with assets over \$100,000 (RMD-75 & W>100K). Table 1 shows average outcomes for the overall population in terms of claiming ages, work hours, 401(k) assets, assets in non-qualified accounts, and consumption over the life cycle. Column A (on the left) represents results for workers without a bequest motive, and Column B (on the right) for those having a bequest motive.

Table 1 here

For households with no bequest motive, Column A of Table 1 shows that delaying the RMD start age from 70 to 72 or 75, or even applying it only to accounts over \$100,000, has little effect on expected lifecycle patterns, both during the worklife and in retirement. That is, the average social security claiming age remains at age 64.9, work hours average around 34 per week, and average yearly consumption stands at around \$26,000 during the worklife (age 25-61) and \$23,000 in retirement (age 62-100). Moreover, asset accumulation changes only slightly: workers have on average \$1,000 less in their 401(k) accounts with RMD-75 rules, versus the other two other cases. No major changes are identified for the assets held in non-qualified accounts. Overall,

¹⁵ The pre-SECURE Act age was in fact 70.5 but as we solve the model in full years, we consider only changes in full years.

we conclude that, for households lacking a bequest motive, none of the proposed RMD reforms makes retirement savings more attractive. That is, they neither accumulate more wealth during the worklife, nor do they increase consumption during retirement.

More substantive changes are evident when people with bequest motives face the various RMD scenarios. Comparing Columns A and B, we find that individuals with a bequest motive tend to claim old age benefits from social security about 0.4 years later, work three hours more per week, accumulate about \$30,000 more in 401(k) plans, and hold \$5,000 more in non-qualified plans, for all of the RMD settings. The fact that these individuals work more, retire later, and accumulate more assets indicates that they not only wish to bequeath to their heirs, but they also consume more in retirement. The explanation is twofold. First, longer working hours and later claiming lead directly to higher social security benefits. Second, additional accumulated assets to fund the desired inheritances lead to larger investment income that can also be used for more consumption.

Moreover, a comparison of results across the RMD settings in Column B shows that claiming ages, work hours, and consumption are again virtually unchanged across the scenarios, but there are remarkable differences in retiree wealth patterns for those desiring to leave a bequest. By the end of the worklife, there are no appreciable differences in accumulated assets across the four RMD scenarios. Even in the retirement phase, the differences between the age 70, 72, and 75 RMD rules are less than 1% for both assets inside and outside tax-qualified accounts. Yet the progressive RMD rule substantially boosts average assets in retirement accounts late in life (age 92-100), to about \$43,000, or \$10,000 (30%) greater than in the RMD 70 case, while non-qualified accounts are much reduced in later life, by about \$8,000 (50%); see boxed values in Table 1. Accordingly, retirees with a bequest motive retain more assets in their 401(k) accounts when the

progressive RMD rule is in effect, indicating that the tax-qualified retirement accounts are a more effective instrument to finance bequests than are non-qualified accounts. This occurs for two reasons. First, the 401(k) is a tax free transfer of assets to the next generation, and second, assets in retirement account earn higher returns due to lower fees. Simultaneously, retirees use more of the assets in their non-qualified accounts to finance consumption late in life, which for the progressive RMD generates essentially the same consumption as do the other rules.

To sum up: Individuals having a bequest motive retain more assets in their 401(k) plans to finance their bequests. The progressive RMD rule enables this more flexibly than would occur simply by delaying the RMD start age. Yet for individuals with no bequest motive, they do not build up more retirement saving under the progressive RMD rule. Accordingly, an increase in lifetime utility from relaxing the RMD rules can only be expected for households seeking to leave an inheritance.

How Restrictive Are the RMD Rules?

Thus far, we have shown that delaying the RMD age has little impact on households' expected behavior if they lack a bequest motive, whereas people desiring to leave a bequest will seek to withdraw less from their 401(k) accounts than currently. Next, we investigate whether RMDs serve as a binding constraint restricting optimal withdrawal behavior, and for whom. On the one hand, as mentioned by Brown et al. (2017), RMD rules only constrain qualified account holders who would prefer to withdraw less from their account balances than the rules prescribe. A bequest motive or a large growth in retirement assets resulting from very high stock returns could support this argument. On the other hand, retirement accounts are designed to support old-age consumption, and lifetime utility maximizing retirees may wish to distribute more than the RMD

amounts. Mortenson et al. (2019) pointed out that the RMD schedule is modest, since the implied remaining life-expectancies used in the regulations are higher than those from actual mortality tables. For example, the remaining life expectancy or distribution period at age 75 according the IRS uniform table is 22.9 years, which translates into an RMD of $1/22.9 = 4.4\%$ of account balances. Yet the remaining life expectancy according to the US population table (used in our utility function) is much lower, about 13 years for females and 11 years for males, which translates into withdrawal rates of 7.7% and 9.1%, respectively. An additional argument supporting higher than required withdrawals is the low risk-free interest rate of 1% versus the subjective discount factor of 4% used in the utility function.

Expected versus required minimum withdrawals. To investigate this on a quantitative basis, we calculate the difference ($W_{it} - m_t F_{it}$) between the actual 401(k) withdrawals predicted by the lifecycle model minus the withdrawals required by each RMD rule, for all simulated optimal lifecycle paths. Figure 1 illustrates the differences, where individuals lacking a bequest motive are depicted in the first row, and those with a bequest in the second. Column A (on the left) traces expected differences in withdrawals for the RMD-70, -72, and -75 rules, while Column B (on the right) compares differences under the RMD-75 rule and the progressive rule applied to 401(k) accounts holding over \$100,000 (RMD-75 & $W > 100K$).

Figure 1 here

Results in the first row of Column A are clear: specifically, people with no bequest motive optimally withdraw substantially more from their retirement accounts throughout retirement, compared to what each of the RMD rules requires. Additionally, all of the curves decline with age, since as the retiree spends from her account, her 401(k) balance falls with age. We also see that the withdrawal differentials are very similar for all three RMD ages, with the main difference being

the age shift resulting from different RMD starting ages. A similar picture emerges in Column B. The difference between optimal versus required withdrawals is initially much higher for the progressive RMD approach (gray line) versus the RMD 75 rule (black line). This is not due to lower average withdrawals, but instead to the lower RMD required payouts resulting from the \$100,000 exemption limit. Nevertheless, the difference again converges with age, because assets held in the retirement accounts decline. Accordingly, we conclude that the regulatory minimum RMD withdrawal rules are not particularly restrictive for retirees lacking a bequest motive. These individuals intend to spend all of their assets by the time they pass away, so as to generate a constant lifetime utility stream from consumption, and the best way to achieve that goal is to withdraw enough along the way. Any remaining assets transferred to the next generation are random, depending on whether the retiree dies early or late.

Somewhat different conclusions emerge from the second row of Figure 1, where we compare optimal minus RMD withdrawals for retirees having a bequest motive. In the left graph, expected optimal withdrawals again exceed those under the age-based -70, -72- and -75 RMD rules, and again, the difference declines with age. Yet under the RMD -72 and -75 rules, the initial differential is notably higher than under the RMD-70 rule. For example, the retiree's first withdrawal under the RMD-72 rule exceeds the required minimum by \$4,000; for the RMD-70 rule starting two years earlier, the corresponding value at the same age is only \$3,000 (or 8% lower). A similar finding applies to the RMD-75 rule. In other words, under the later RMD ages, retirees withdraw less until they are required to, and then they make larger withdrawals later from the larger remaining assets in their 401(k) plans. Nevertheless, this differential is only temporary as it disappears with age due to rising required withdrawals as remaining life expectancy falls. A much more notable difference arises under the progressive RMD rule, in the second row of Column

B. Specifically, the differential grows with age between the optimal withdrawal for the RMD-75 rule versus the progressive RMD. Even at fairly advanced ages, the retiree retains substantially more assets in her retirement account and pays no penalty tax. In other words, these retirees use their tax-qualified retirement accounts as a means to pass financial assets to the next generation.

Probability distribution of withdrawal rates. A more granular look at this result is provided by investigating the probability distribution of optimal withdrawals for the RMD-75 and RMD-75&W>100K rule generated from 200,000 simulated lifecycle patterns. Figure 2 reports the relative frequencies of optimal withdrawals as a fraction (in 0.5% increments) of retirement account balances for retirees at ages 75, 85, and 95. Row 1, which depicts results with no bequest motive, shows that for all scenarios, the probability mass is broadly distributed over the entire spectrum of possible withdrawal ratios. In most of the cases, the retiree wishes to withdraw more than the RMD rule prescribes (reflected by the arrows). For example, the RMD-75 rule requires a 75-year-old retiree to withdraw 4.4% from her retirement account. There is only a moderate peak in the probability mass of 3.2% around this withdrawal rate, while the mode of this distribution is a much higher withdrawal rate of 9.5%. By age 95, the RMD-75 rule requires a withdrawal of 11.6% of the account, while the distribution's modal withdrawal rate is 100%. The relative frequencies for the progressive RMD-75 rule (column B of row 1) are also very broadly distributed over the various withdrawal rates, with no peak around the RMD-age-based fractions. Overall, then, we conclude that the RMD rules are not binding for retirees lacking a bequest motive.

Figure 2 here

A completely different picture emerges for the RMD-75 rule when retirees do have a bequest motive (row 2o, column A). In all cases, the modal payout value corresponds to the required RMD fraction, and the probability mass increases significantly with age. For example, at

age 75, about 20% of retirees withdraw the required minimum fraction of 4.4%. Few retirees take out less than required, and those who take out more, do so at a moderate rate. If retirees survive to age 85 (95), about 40% (75%) follow the RMD payout rule. This underscores the result that people prefer to withdraw less than the required minimum fractions, yet the high penalty taxes prevent them from doing so. Accordingly, our result confirms Mortenson et al.'s (2019) empirical finding that there is a high concentration of probability mass for withdrawals around the RMD levels. In other words, we find theoretically (and those authors conclude empirically) that the RMD turns out to be a binding constraint for many people. The fact that our theoretical results match up with evidence on real-world retirement plan participants' withdrawal patterns can therefore be interpreted as evidence that many retirees take plan payouts as though they have a bequest motive.¹⁶

The situation would change significantly for retirees with a bequest motive if the progressive RMD-75 were to be implemented only for account values above \$100,000 (row 2, column B). Many retirees now elect withdrawal rates well below the RMD rule, and indeed the modal value stands at a withdrawal rate of 0% for all three age groups. For example, about 35% of retirees surviving until age 95 take no withdrawals. Evidently, these individuals seek so far as possible to utilize the RMD exemption limit of \$100,000, to pass on money to heirs that is largely untaxed and which has earned higher returns after costs.

A final look at this result is provided by calculating the probabilities that an individual following optimal lifecycle behavior will withdraw less than the RMD amount conditional on still being alive. In this case, it will be recalled that the retiree would need to pay the 50% penalty tax

¹⁶ Additional factors explaining the high correspondence between empirical and regulatory minimum withdrawal rates might include behavioral (non-expected utility) preferences or optimization constraints, but such factors are beyond the framework of our model.

on any withdrawal shortfalls. Results for the three education levels of interest and the overall population are reported in Table 2. Without a bequest motive (Column A), individuals at all education levels are quite unlikely to withdraw less than required. For example, the likelihood of paying a penalty tax for retirees having a college education and no bequest motive amounts to only 2.7% under the RMD-70 rule, and just 2.5% for the RMD-72 rule. For people with less education ($\leq$ High School), the probabilities still remain low. In other words, households lacking a bequest motive generally wish to avoid the 50% penalty, so they take at least as much as the RMD rules require. If anyone in this group pays penalty taxes at all, it is coincidental: for example, if there are unusually high stock returns, 401(k) asset values can rise sharply along with the RMD amounts that must be withdrawn. At the same time, the retiree will not wish to withdraw her full RMD to avoid depleting her retirement account too quickly, in case the market subsequently drops. In the progressive RMD case, the probability of paying the penalty is essentially zero for members of all education groups.

Table 2 here

For households with a bequest motive (Column B), the chance that a retiree would optimally elect to withdraw less than the RMD starting at ages 70, 72, or 75 rises to 6-12%, depending on the subgroup. In fact, some households would even willingly pay the penalty tax to favor the bequest over own consumption. Yet as we see, this rarely happens, since most retirees withdraw at least the amounts required under the RMD rules. Again, penalty probabilities are essentially zero in the progressive RMD case.

These results thus support the conclusion that households having a bequest motive are those most likely to be constrained by RMD rules. They do their best to use tax-advantaged retirement

accounts not only for old age consumption, but also to fund an inheritance. By contrast, households lacking such a motive are hardly constrained at all by RMD rules such as those examined here.

Implications for Lifetime Tax Payments

We next explore the potential impact of delaying RMD ages for tax payments over the lifecycle. Using the simulated optimal lifecycle profiles for the six subgroups, we calculate for each individual i the income tax payments (including penalty taxes) $IT_{i,t}$ from age 70 ($t = 46$) to the maximum age 100 ($T = 76$). To obtain a representative distribution for the total population, we weight the 200,000 simulation paths for males/females in each of the education groups (<HS, HS, Coll+) by population weights. To reflect mortality risk, we multiply tax payments by the multi-year survival probability that the individual is still alive at this age $p_{0,t} = \prod_{j=0}^t p_j$. Formally, this is defined as:

$$IT_{i,t}^{pop} = p_{0,t} \cdot IT_{i,t}, \quad (7)$$

and it reflects the probability distribution of annual income tax payments per individual at a certain age. We perform this analysis for each of the four RMD approaches considered, only for households having a bequest motive. To evaluate the implications of the different RMD rules for income tax payments, we calculate the (cross-sectional) mean value and the 99% quantile (wealthiest 1 percentile) at each age. Results in Figure 3, Column A (left side), compare results for the RMD-70, the RMD-72, and the proposed higher starting age RMD-75. Column B (right side) compares the RMD-75 (black line) with the progressive RMD-75 rule (gray line).

Figure 3 here

Expected tax payments appear in the top row of the Figure. On the left, compared to the old RMD-70 (blue line) rule, tax payments under the RMD-72 rule (black line) are lower between

age 70 and 72. This is because retirees make fewer withdrawals from their 401(k) accounts in this window. Starting from age 72, tax payments due with the RMD-72 rule are now somewhat higher (as seen in Figure 1). This results from a “catch-up” effect: as more assets remain in the 401(k) plan until age 72, the withdrawals and thus the tax payments are higher when the RMD-72 rule takes effect. Accordingly, tax payments between age 70 and 72 are postponed to the later RMD age. Actual income tax shortfalls occur only if the individual dies in the meantime. Exactly at age 72, the blue and black lines intersect; thereafter, households pay more taxes, on average, under the new RMD-72 than under the old RMD-70 rule, until age 85. In fact, simply looking at the differences between the two lines suggests that the RMD-72 scenario actually generates more tax revenue over the lifecycle compared to the old RMD 70 rule. Nevertheless, this is only true for households with a bequest motive.

This catch-up effect would also apply if the RMD age were further postponed to age 75, as is evident in Figure 3. Now, the black line (RMD-75) intersects the blue line (RMD-70) at age 75 and remains above it until about age 80. In general, households with an inheritance motive pay less taxes until age 75 and more thereafter. Nevertheless, as can be seen in the right panel of Column B, top row, the “catchup” effect disappears under the progressive RMD approach. Retirees with a bequest motive use the \$100,000 exemption limit to leave assets in their 401(k) plans without fear of penalty taxes, resulting in lower tax payments at all ages.

The second row of Figure 3 (left) illustrates the effects for the 1% of the people paying the highest tax (the 99% quantile). Here the catch-up effect becomes even clearer: the dashed line (RMD-72) intersects the blue line (RMD-70) at age 72 and remains above it until about age 80. A similar pattern applies for a RMD age of 75. Again, the catch-up effect disappears under the progressive RMD-75 rule (right panel), as lower taxes are paid at all ages.

We next investigate whether, over the remaining lifetime from age 70 to 100, the catch-up effect fully compensates for the initial tax losses resulting from postponing the RMD starting age. To this end, we calculate the sum of expected tax payments between ages 70 and 100, $\sum_{t=70}^{100} E(p_{0,t} \cdot IT_{i,t})$, as well as various quantile measures $\sum_{t=70}^{100} Q_a(p_{0,t} \cdot IT_{i,t})$.¹⁷ Using these metrics, we also evaluate the resulting effects on tax payments if the progressive RMD rule were adopted. Finally, we examine how total tax revenues would change if the RMD were to be fully abolished (w/o RMD). Results appear in Table 3, where Panel A contains households without and Panel B with a bequest motive.

Table 3 here

A first finding is that, for households lacking a bequest motive (Panel A), the catch-up effect is insufficient to compensate for initial tax losses resulting from postponing the RMD start age of 70 to later ages. Both the mean and quantile tax loss values for the later RMD ages are generally equal to or below those under the RMD-70 rule. Moreover, tax losses are even larger if the RMD rule were to be abandoned entirely. Comparing the columns labeled w/o RMD and RMD-70, tax losses are the highest though they are still relatively moderate: the mean is 3% lower ($=4.48/4.62-1$), and the value is 4.5% lower for the 99% quantile highest income taxpayers ($=107.47/112.08-1$).

Our conclusions change for households having a bequest motive. Specifically, Panel B of Table 3 shows that the catch-up effect under the RMD-72 approach is so large that it actually results in somewhat higher tax payments than under the RMD-70 rule: that is, average taxes paid

¹⁷ We also calculate the remaining tax-payments over simulated trajectories $TaxSum_{70,i} = \sum_{t=70}^{100} p_{0,t} \cdot IT_{i,t}$. The summary statistics from these probability distribution are not substantially different from those reported in the text.

increase from \$5,080 to \$5,130 per person. This is mostly attributable to more taxes paid by the highest-income retirees, as can be seen from the 95% and 99% quantiles. Nevertheless, the catch-up is too small under the RMD-75 rule to compensate for the initial lower tax payments, though the differences are modest. Under a progressive RMD plan, or if RMDs were eliminated altogether, tax payments would fall more substantially. In particular, high-income retirees (Q99%) use the 401(k) account to bequeath money to their heirs, so as a consequence, tax payments decline by about 7% relative to taxes collected under the current RMD-72 rule ($=118.93/125.95-1$).

Conclusions

We use a calibrated lifecycle consumption and portfolio choice framework embodying realistic institutional considerations to explore how Required Minimum Distribution rules shape individuals' saving patterns, social security claiming ages, and withdrawals from their tax-qualified retirement accounts. We compare results under the historical RMD age 70.5 rule, as well as RMDs at ages 72 and 75, and we also examine the impact of eliminating RMDs for retirees having account values below \$100,000, known as the progressive RMD approach, as well as the complete elimination of the RMD.

We show that delaying the RMD age would have little impact on peoples' financial behavior during their worklives, including for expected savings patterns both inside and outside tax-qualified retirement accounts. Additionally, social security claiming behavior is almost unaffected. Nevertheless, more notable changes are evident during the retirement phase, depending on whether retirees have a bequest motive or not. For those lacking a bequest motive, even eliminating RMD rules would change very little. But for households having a bequest motive, the former RMD age 70.5 rule was quite restrictive, since such a household would prefer to make fewer withdrawals than required and use the 401(k) plans as a tool to transfer financial wealth to

the next generation. Raising the RMD age to 72 permits such a household to postpone account withdrawals and defer taxes for two years. We also show that delaying the RMD from 70 to 72 or even age 75 changes the timing of tax payments, but it hardly affects overall tax payments over the remaining lifecycle. This is due to a “catch up” effect, where lower tax payments early on in retirement are offset by higher tax payments later, due to having higher 401(k) values and larger taxable withdrawals. Under a progressive RMD, households intending to leave a bequest end up paying less tax over their lifetimes. Instead, they use the \$100,000 exemption limit to transfer wealth to the next generation without fear of penalty taxes. This leads to lower tax payments, especially for the wealthiest 1% of taxpayers.

In sum, peoples’ behavior under alternative RMD rules will depend on the extent to which they desire to leave money to their heirs. This implies that financial institutions such as insurance companies and mutual funds offering retirement plans and investment advice would benefit from ascertaining their clients’ bequest intentions, before advising them about RMD strategies. Our conclusions will also interest professional financial planners guiding clients as they make retirement payout choices.¹⁸ Moreover, our results can inform policymakers considering legislation to raise and/or eliminate the RMD policy for 401(k) plan payouts.

We note that our quantitative results regarding tax revenues should be interpreted with caution, since our microeconomic single-generation lifecycle model does not take into account potential macroeconomic effects that may arise with overlapping generations. Moreover, our model does not endogenize the impact of changes in RMD rules on the labor, financial, and goods markets. Nevertheless, since individual behaviors transfer to the macroeconomic level, our results *mutatis mutandis* indicate the direction of how changing RMD rules could affect the federal

¹⁸ Our previous work (Horneff 2020b) has also identified key ways in which deferred annuities can further lower retirees’ RMD payout obligations.

budget. Additionally, we are aware that our model posits that households make rational economic decisions, even though behavioral economists show that they sometimes do not. We leave this extension to future research, as there is no consensus regarding which behavioral aspects should be implemented in normative models and how. Nevertheless, to simultaneously demonstrate the impact of RMD policy changes on key household behaviors including consumption, work hours, saving, labor input, and benefits claiming, is a valuable contribution.

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Figure 1: 401(k) Optimal Withdrawals minus RMD Rule Withdrawals

Notes: The figures report average differences by age between optimal withdrawals predicted by the lifecycle model minus minimum withdrawals required by the corresponding RMD rule. In Column A, retirees lack a bequest motive and Column B, retirees have a bequest motive. Population outcomes are generated using 200,000 weighted simulation paths based on optimal feedback controls for the six subgroups (see weights in Table 1). For additional information on parameters and calibrations see Table 1. Source: Authors' calculations.

Figure 2: Probability Distribution of Optimal Withdrawals Percentages at Different Ages

Column A: RMD-75 Rule

Column B: RMD-75 & W>100K Rule

Notes: The figures depict the predicted probability distribution of withdrawals as fraction of retirement assets (withdrawal ratio) taken by retirees at age 75 (orange bars), 85 (blue bars), and 95 (yellow bars). Relative frequencies by withdrawal ratios are reported in 0.5% steps. The arrows reflect the RMD for those in the corresponding age group, as per the IRS uniform table. In Column A, retirees lack a bequest motive and Column B, retirees have a bequest motive. Population outcomes are generated using 200,000 weighted simulation paths based on optimal feedback controls from the lifecycle models for the six subgroups (see weights in Table 1). Results in the first row are based on the RMD-75 rule, and in the second row, the progressive RMD rule (i.e., RMD-75 & W>100K). For additional information on parameters and calibrations see Table 1. Source: Authors' calculations.

Figure 3: Expected Lifecycle Tax Payments under Various RMD Rules: Retirees Having a Bequest Motive

Column A: Deferring RMD Ages

Column B: Impact of Progressive RMD

Notes: The figures report summary statistics (upper rows = means, lower rows = 99% quantiles) of tax payments (including penalty taxes) for retirees with a bequest motive from age 70 until 100, under various RMD rules. Column A illustrates the effect of changing only the RMD age (RMD-70, RMD-72, RMD-75), while Column B illustrates the impact of the progressive RMD approach (RMD-75 versus RMD-75 for account values above \$100,000). Population outcomes are generated using 200,000 weighted simulation paths based on optimal feedback controls for the six subgroups (see text and weights in Table 1). Authors' calculations.

Table 1: Model-generated Outcomes for Claiming Ages, Hours Worked, 401(k) and Other Assets, Consumption Flows

	Column A: w/o bequest				Column B: with bequest			
	RMD-70	RMD-72	RMD-75	RMD-75 & W>100K	RMD-70	RMD-72	RMD-75	RMD-75 & W>100K
1. Average Claiming Age								
Age 62-70	64.9	64.9	64.9	64.9	65.3	65.3	65.4	65.4
2. Average Work Hours per Week								
Age 25-61	33.9	33.9	33.9	34.0	36.8	36.8	36.9	36.8
3. Average 401(k) Assets in \$000								
Age 25-61	101.3	101.3	100.3	100.2	131.7	132.3	131.1	131.8
Age 62-90	72.9	71.9	73.1	72.9	107.9	109.0	108.6	109.2
Age 91-100	6.1	5.9	6.1	6.1	33.1	33.3	33.4	42.9
4. Average Non-Qualified Assets in \$000								
Age 25-61	7.2	7.2	7.9	7.8	7.5	7.3	7.8	7.6
Age 62-90	11.9	11.9	11.6	11.6	14.8	14.5	14.6	13.9
Age 91-100	4.6	4.6	4.6	4.6	16.8	16.7	16.7	8.2
5. Average Consumption in \$000								
Age 25-61	26.2	26.2	26.1	26.1	26.7	26.7	26.7	26.7
Age 62-100	23.0	23.0	23.0	23.0	24.7	24.7	24.7	24.7

Notes: We report average outcomes for the age 70 rule (RMD-70); the age 72 rule (RMD-72); the age 75 rule (RMD-75); and the progressive RMD rule (RMD-75 & W>100K). Results in Column A (B) are for individuals without (with) a bequest motive. Averages are derived from 200,000 simulated lifecycles based on optimal feedback controls from the life cycle model using income profiles for six sex/education subgroups. Population averages use education weights for females (males): 61% +Coll; 28% HS; 11% <HS (57% +Coll; 30% HS; 13%<HS); weights for females (males) 49.28% (50.72%) of entire population. The baseline calibration uses the following parameters: risk aversion $\rho = 5$; time preference $\beta = 0.96$; leisure preference $\alpha = 1.3$; results with a bequest motive assume $b = 2$. Endogenous retirement ages 62-70 and social security benefits are based on average permanent income and the bend points in place in 2018. The risk premium for stocks returns is 5% and return volatility 18%; the risk free rate in the baseline case is 1%. Source: Authors' calculations

Table 2: Probability (%) of 401(k) Withdrawals Falling Below the RMD

	Column A: w/o bequest				Column B: with bequest			
	RMD -70	RMD -72	RMD -75	RMD-75 & W>100K	RMD -70	RMD -72	RMD -75	RMD-75 & W>100K
<HS	2.8	2.5	2.1	0.0	12.5	11.8	10.8	0.0
HS	4.2	3.9	3.5	0.0	10.8	10.2	9.2	0.1
Coll+	2.7	2.5	2.1	0.1	6.8	6.5	6.1	0.5
Population	3.2	2.9	2.5	0.0	8.6	8.2	7.6	0.1

Notes: This table reports the probability (%) that an individual with below high school (<HS), high school (HS) or at least college (Coll+) education will pay a penalty tax due to taking 401(k) withdrawals below those required by the RMD rule (conditional on survival). Results in Column A (B) are for individuals without (with) a bequest motive. Outcomes are generated using 200,000 weighted simulation paths based on optimal feedback controls for the six subgroups- (see weights in Table 1). For additional information on parameters and calibrations see Table 1. Source: Authors' calculations

Table 3: Lifetime Tax Payments from Age 70 to 100 under Various RMD Rules (in \$000)

	Panel A: w/o bequest				
	RMD-70	RMD-72	RMD-75	RMD-75 & W>100K	w/o RMD
Mean	4.62	4.51	4.62	4.54	4.48
Median	0.00	0.00	0.00	0.00	0.00
75% Quantile	0.30	0.25	0.17	0.00	0.00
95% Quantile	19.21	18.39	19.57	19.09	18.65
99% Quantile	112.08	110.80	109.95	110.66	107.47
	Panel B: with bequest				
	RMD-70	RMD-72	RMD-75	RMD-75 & W>100K	w/o RMD
Mean	5.08	5.13	5.04	4.93	4.94
Median	0.00	0.00	0.00	0.00	0.00
75% Quantile	0.64	0.59	0.46	0.02	0.01
95% Quantile	21.00	21.15	20.69	20.78	20.92
99% Quantile	125.95	126.91	123.66	118.56	118.93

Notes: This table reports summary statistics for the sum of tax payments between ages 70 and 100 (weighted by survival probabilities) for different RMD rules. Results are based on 200,000 simulated optimal lifecycles for six sex/education groups for households without (Panel A) and with a bequest motive (Panel B). For other parameters and calibrations, see Table 1. Source: Authors' calculations.

Online Appendix A: Wage rate estimation

We calibrated the wage rate process using the Panel Study of Income Dynamics (PSID) 1975-2015 from age 25 to 69. Extreme observations below \$5 per hour and above the 99th percentile are dropped. The wage rate values are expressed in \$2015. During the worklife, each individual's labor income profile has deterministic, permanent, and transitory components with uncorrelated and normally distributed shocks according to $\ln(N_t) \sim N(-0.5\sigma_n^2, \sigma_n^2)$ and $\ln(U_t) \sim N(-0.5\sigma_u^2, \sigma_u^2)$. These are estimated separately by sex and three educational levels: less than High School (<HS), High School graduate (HS), and with at least some college (Coll+). We use a second order polynomial in age and dummies for employment status to estimate the deterministic component using the regression function:

$$\ln(w_{i,y}) = \beta_1 * age_{i,y} + \beta_2 * age_{i,y}^2 + \beta_5 * ES_{i,y} + \beta_{waves} * wave \text{ dummies}, \quad (A1)$$

where $\log(w_{i,y})$ is the natural log of wage at time y for individual i , age is the age of the individual divided by 100, ES is the individual's employment status, and wave dummies control for year-specific shocks. For employment status, we include three groups depending on work hours per week as follows: part-time worker (≤ 20 hours), full-time worker (< 20 & ≤ 40 hours) and overtime worker (< 40 hours). OLS regression results for the wage rate process equations are provided in Table A1.

To estimate the variances of the permanent and transitory components, we follow Hubener et al. (2016). We calculate the difference of the observed log wage and the regression result, and we take the difference of these differences across different lengths of time d . For individual i , the residual is:

$$r_{i,d} = \sum_{s=0}^{d-1} (N_{t+s}) + U_{i,t+d} - U_{i,t}. \quad (A2)$$

We then regress the $v_{id} = \overline{r_{i,d}^2}$ on the lengths of time d between waves and a constant:

$$v_{id} = \beta_1 \cdot d + \beta_2 \cdot 2 + e_{id}, \quad (A3)$$

where the variance of the permanent factor $\sigma_N^2 = \beta_1$ and the $\sigma_U^2 = \beta_2$ represents the transitory shocks.

Table A1: Regression results for wage rates

Coefficient	Male <HS	Male HS	Male +Coll	Female <HS	Female HS	Female +Coll
Deterministic						
Age/100	3.161*** (0.108)	5.972*** (0.049)	9.092*** (0.070)	1.256*** (0.110)	2.767*** (0.046)	4.731*** (0.072)
Age ² /10000	-3.329*** (0.130)	-6.416*** (0.062)	-9.351*** (0.089)	-1.339*** (0.131)	-2.915*** (0.059)	-4.960*** (0.094)
Part-time work	-0.109*** (0.020)	-0.153*** (0.009)	-0.0826*** (0.011)	-0.0858*** (0.006)	-0.129*** (0.003)	-0.0847*** (0.004)
Over-time work	0.00412 (0.004)	0.0506*** (0.002)	0.0949*** (0.002)	0.0158*** (0.006)	0.0748*** (0.002)	0.106*** (0.003)
Constant	1.807*** (0.042)	1.435*** (0.012)	1.151*** (0.015)	2.051*** (0.037)	2.015*** (0.011)	1.938*** (0.017)
Observations	48,762	327,305	293,386	31,788	290,597	225,211
R-squared	0.069	0.102	0.147	0.032	0.044	0.092
Permanent	0.009*** (0.001)	0.013*** (0.0002)	0.019*** (0.0003)	0.008*** (0.0001)	0.013*** (0.0002)	0.019*** (0.001)
Transitory	0.028*** (0.001)	0.031** (0.001)	0.041*** (0.001)	0.023*** (0.002)	0.028*** (0.001)	0.038*** (0.001)
Observations	28,359	175,247	140,984	20,863	176,304	123,145
R-squared	0.214	0.283	0.307	0.146	0.255	0.264

Notes: Regression results for the natural logarithm of wage rates (in \$2015) are based in on information in the Panel Study of Income Dynamics (PSID) for persons age 25-69 in waves 1975-2015. Independent variables include age and age-squared, and dummies for part time work (≤ 20 hours per week) and overtime work (≥ 40 hours per week). Robust standard errors in parentheses. ***: $p > .01$. Source: Horneff, Maurer, and Mitchell (2020).

Online Appendix B: Modeling taxes and retirement accounts

We embed a US-type tax system for workers having access to a qualified tax-deferred retirement account (TDA). All values are in \$2018 and relevant amounts are inflation adjusted yearly. The worker pays federal income taxes on taxable income which is a complex function on labor income (minus housing costs), social security benefits and returns from investments in bonds and stocks.¹⁹ Contributions A_t (up to $D_t = \$18,500$) to the TDA reduce and withdrawals W_t from the TDA increase taxable income. For taxation of social security (Y_{t+1}) benefits after retirement, we use the following rules: when the retiree's *combined income*²⁰ is between \$25,000 and \$34,000 (over \$34,000), 50% (85%) of benefits are part of taxable income. Finally, a general standardized deduction $GD = \$12,000$ reduces the worker's taxable income, which is given by:

$$Y_{t+1}^{tax} = \max[\max(S_t \cdot (R_{t+1} - 1) + B_t \cdot (R_f - 1); 0) + Y_{t+1}(1 - h_t) + W_t - \min(A_t; D_t) - GD; 0]. \quad (B1)$$

In line with US federal income tax, our progressive tax system has $i = 1, \dots, 7$ brackets (IRS 2015) defined by a lower and an upper bound of taxable income $Y_{t+1}^{tax} \in [lb_i, ub_i]$ and determine a marginal tax rate r_i^{tax} .

In 2018, the marginal taxes rates for a single household were 10% from \$0 to \$9,525, 12% from \$9,525 to \$38,700, 22% from \$38,701 to \$82,500, 24% from \$82,501 to \$157,500, 32% from \$157,500 to \$200,000 35% from \$200,001 to \$500,000 and 37% above \$500,000 (see IRS 2018). Based on these tax brackets (deflated to the year 2015), the dollar amount of income taxes payable is given by:

$$\begin{aligned} IT_{t+1}^{tax} = & (Y_{t+1}^{tax} - lb_7) \cdot 1_{\{Y_{t+1}^{tax} \geq lb_7\}} \cdot r_7^{tax} \\ & + \left((Y_{t+1}^{tax} - lb_6) \cdot 1_{\{lb_7 > Y_{t+1}^{tax} \geq lb_7\}} + (ub_6 - lb_6) \cdot 1_{\{Y_{t+1}^{tax} \geq lb_7\}} \right) \cdot r_6^{tax} \\ & + \left((Y_{t+1}^{tax} - lb_5) \cdot 1_{\{lb_6 > Y_{t+1}^{tax} \geq lb_5\}} + (ub_5 - lb_5) \cdot 1_{\{Y_{t+1}^{tax} \geq lb_6\}} \right) \cdot r_5^{tax} \\ & + \left((Y_{t+1}^{tax} - lb_4) \cdot 1_{\{lb_5 > Y_{t+1}^{tax} \geq lb_4\}} + (ub_4 - lb_4) \cdot 1_{\{Y_{t+1}^{tax} \geq lb_5\}} \right) \cdot r_4^{tax} \\ & + \left((Y_{t+1}^{tax} - lb_3) \cdot 1_{\{lb_4 > Y_{t+1}^{tax} \geq lb_3\}} + (ub_3 - lb_3) \cdot 1_{\{Y_{t+1}^{tax} \geq lb_4\}} \right) \cdot r_3^{tax} \\ & + \left((Y_{t+1}^{tax} - lb_2) \cdot 1_{\{lb_3 > Y_{t+1}^{tax} \geq lb_2\}} + (ub_2 - lb_2) \cdot 1_{\{Y_{t+1}^{tax} \geq lb_3\}} \right) \cdot r_2^{tax} \\ & + \left((Y_{t+1}^{tax} - lb_1) \cdot 1_{\{lb_2 > Y_{t+1}^{tax} \geq lb_1\}} + (ub_1 - lb_1) \cdot 1_{\{Y_{t+1}^{tax} \geq lb_2\}} \right) \cdot r_1^{tax} \end{aligned} \quad (B2)$$

where, for $A \subseteq X$, the indicator function $1_A \rightarrow \{0, 1\}$ is defined as:

$$1_A(x) = \begin{cases} 1 & | x \in A \\ 0 & | x \notin A. \end{cases} \quad (B3)$$

Additionally, before retirement ($t = K$) the worker pays payroll taxes proportional to labor income: tax rate social security 6.2% (up to a limit of 128,400); Medicare tax rate 1.45% and city/state tax rate 4% (without limit). Overall payroll taxes are modelled as $PT_t^{tax} = 6.2\% \cdot \max(Y_t, 128,400) + 5.45\% \cdot Y_t$. and $PT_t^{tax} = 5.45\% \cdot Y_t$ after retirement $t \geq K$. Finally, penalty taxes of 10% on early TDA-withdrawals prior to age 59 ½ ($t = 36$), and 50% on non-compliant RMD withdrawals are charged.

¹⁹ For simplicity, we do not distinguish between the different taxation of dividends and capital gains on stocks. Instead, we assume that all investment income (if positive) is taxed at the same rate as labor income at the source.

²⁰ Combined income is sum of adjusted gross income, nontaxable interest, and half of social security benefits (US SSA nd_c).

Online Appendix C: Social Security Claiming Patterns for Males and Females, and 401(k) Asset Values (*model vs data*)

Notes: The top panel compares expected claiming rates generated by our life cycle models and empirical claiming rates reported by the US Social Security Administration for the year 2015 (without disability). The lower panel compares expected account balances generated by the life cycle model versus empirical 401(k) account balances across the US population. Empirical account balance data are from the Employee Benefit Research Institute (2017); age groups referred to as 20s, 30s, 40s, 50s, and 60s denote average values for persons age 20-29, 30-39, 40-49, 50-59, and 60-69. Expected values are calculated from 100,000 simulated lifecycles based on optimal feedback controls for each of six subgroups. Results for the entire female (male) population are computed using income profile three education levels: 61% +Coll; 28% HS; 11% <HS (57% +Coll; 30% HS; 13% <HS); weights for females (males) 49.28% (50.72%) of entire population. Parameters used for the baseline calibration are as follows: risk aversion $\rho = 5$; time preference $\beta = 0.96$; leisure preference $\alpha = 1.2$; endogenous retirement age 62-70. Social security benefits are based on average permanent income and the bend points in place in 2015; minimum required withdrawals from 401(k) plans are based on life expectancy using the IRS-Uniform Lifetime Table in 2015; tax rules for 401(k) plans are as of 2015 as described above. The risk premium for stocks returns is 5% and return volatility 18%; the risk free rate in the baseline case is 1%. Source: Horneff, Maurer, and Mitchell (2020).

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