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Multistable Phase Regulation for Robust Steady and Transitional Legged Gaits

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Disciplines

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1 Introduction

Building effective legged machines, rivaling even the simplest of biology’s legged creatures, remains a principal challenge of robotics. While legs offer significant advantages over other locomotive strategies, particularly for sparse and irregular terrain, the design difficulties attending the typically high degree of freedom limbs and their coordinated control have impeded progress. This paper introduces two new concepts to the growing inventory of methods for coordinating a robot’s “gait”. By this term we mean the recurring temporal pattern of leg phasing whose specification and control introduces a key abstraction barrier between the body’s overall progress and the legs’ detailed motions. The abstract set of relative leg phase patterns we term the “gait space”. We first introduce a family of multistable vector fields on the gait space that induce numerous, equally qualified gaits, each ready to be expressed when a complex terrain ejects a leg’s controller from the basin of one to land in the basin of another. Next, we exploit the robustness achieved by these families of task-equivalent basins to command safe transitions between different types of gaits, allowing the operator to choose arbitrarily which types should be switched in or out, even during active locomotion, and in the presence of persistent environmental disturbances. Finally, we demonstrate empirically the value of these formal constructions by reporting measurements taken from a variety of contrasting experiments with the RiSE climbing robot (Spenko et al., 2008).

In contrast with wheeled and tracked vehicles that maintain continual surface contact, legged locomotion requires repetitive intermittent contacts: legs alternate between supporting the body to generate propulsion, and lifting off the ground to recirculate forward. Gaits, and gait space, thus emerge as a natural periodic abstraction of this recurring alternation. We use these abstracted periodic representations as feedforward reference signals that cue the detailed pose variations of the individual legs over their recurring stride, thereby decoupling the details of geometry and infinitesimal kinematics (joint velocities, torques and instantaneous stiffnesses) from their relative timing. Of course, when operating in a real unstructured environment, these notional geometric details must be relaxed by allowing local proprioceptive measurements to transiently but continually alter individual leg pose and compliance properties or else the actual ground reaction forces will go awry and locomotion may become inefficient or fail outright. In turn, such real-time pose and compliance adjustments generally must project back into the abstract gait space as departures from the notionally prescribed state of relative timings (the phase in gait space). Because of these environmentally driven perturbations, the imperatives of overall leg coordination demand the introduction of some dynamical feedback adjustment to the nominal feedforward gait. This problem of gait regulation—essentially, the continuous, real-time adjustment of phase velocity with the aim of maintaining or restoring limb coordination—comprises the focus of this paper.

1.1 Related Work

Control methods associated with the traditional robotics “sense-think-act” paradigm often presume the availability of local world models amenable to planned paths that, when executed, produce successful locomotion. Often applied to footfall planning (Wettergreen et al., 1990; Chestnutt et al., 2005; Shapiro et al., 2005; Bretl, 2006; Hodoshima et al., 2004), these methods require accurate sensor information as well as detailed environmental representations. Notwithstanding their rational design appeal, these methods are difficult to implement on small, fast, and possibly dynamic legged machines. Partly in response to these difficulties, an interest in purely reactive architectures (Beer et al., 1998; Brooks, 1997), based upon proprioceptive, feedback-driven, neuromechanical models of arthropod locomotion (Wilson, 1966; Cruse, 1990; Dürr et al., 2003), has emerged. In these schema, motor commands arise from the interaction of local control reflex networks excited by sensory signals, with the resulting decentralized exchange yielding locomotion. Despite their physiological basis and conceptual elegance, such reactive or “self-exciting” control architectures are potentially amenable to mathematical analysis (Klavins et al., 2002), however they are not straightforward to understand and the behaviors they produce reportedly can occur by accident as much as by design (Brooks, 1989), entailing extensive manual or automated learning of parameters to generate successful locomotion (Lewis et al., 1993).

In recent years, roboticists have returned to explore a complementary bio-inspired approach to gait generation that traces back to the nearly century old tradition of neural oscillators and central pattern generators (Brown, 1912; Grillner, 1975) whose relatively tractable feedforward representation has attracted decades of mathematical analysis (Cohen et al., 1982). Indeed, given the cyclic nature of legged locomotion, one would expect a robot to choose repetitive leg motions when presented with a uniform terrain. This leads to the idea of cyclic gait patterns for encoding locomotion, in which robots store and replay prespecified motion patterns (Altendorfer et al., 2001; Autumn et al., 2005; Cham et al., 2002; Poulakakis et al., 2005). Using little or no sensing, a robot can use one of many possible gaits to achieve impressive feats of locomotion, including true energetic running, and incorporating methods of tuning open-loop gaits for increased performance, (Weingarten et al., 2004b; Chernova and Veloso, 2004; Zykov et al., 2004; Kohl and Stone, 2004).

While many approaches for central pattern generators have been proposed in the robotics literature (Buchli et al., 2006; Brambilla et al., 2006; Frasca et al., 2004) (often derived from studies of neural oscillators (Pearson, 1973; Delcomyn, 1980; Cohen et al., 1988)), we take our point of departure in the work of (Klavins and Koditschek, 2002), in which vector fields over the gait space are constructed through attraction or repulsion between pairs of oscillators. There is active research on the analysis of gradient-like dynamics resulting from such systems (e.g. (Mirollo and Strogatz, 2005) providing stability conditions for networks of all attracting oscillator pairs, (Jadbabaie et al., 2005) giving some conditions for arbitrary interconnections, and (Mallada and Tang, 2010) studying methods of global attraction for smooth, symmetric couplings). We add to this tradition of seeking globally attractive limit cycles by introducing (we believe for the first time) the notion of multistability to the robotics locomotion control literature. While the systems biology literature acknowledges multistability as a central strategy for robustness (Kitano, 2004), with increasingly sophisticated methods for identifying its presence (Angeli et al., 2004), in our application, multistability confers a welcome degree of indifference to a multitude of possible gaits available during locomotion over unknown terrains.

Coordination amongst oscillator pairs in the manner of (Klavins and Koditschek, 2002) has been applied to the hexapod, RHex (Saranli et al., 2001), scrambling over rough terrain (Weingarten et al., 2004a), but focused upon inducing a single global basin associated with a single gait—the alternating tripod. In contrast, the present paper extends these methods of gait regulation to the design of multistable closed loops, generalizing in suitable archival form the specific notions of the “crawl” gait family first introduced in (Haynes and Rizzi, 2006a) that we now expand upon and begin to systematically exploit for a broader range of alternative gaits and gait families. Furthermore, instead of level ground running, the empirical target of the present paper is the climbing robot, RiSE, first announced in (Autumn et al., 2005) and presented archivally in (Spenko et al., 2008), respecting that the perils of locomotion failure are suitably acute as to appropriately motivate our sharp focus on robustness. Finally, it should be noted that this paper is a greatly condensed (but somewhat more analytically developed) version of the material presented in (Haynes, 2008).

1.2 Specific Contributions

This paper introduces algorithms and techniques to regulate and control the gait of a hexapedal robot, allowing the machine to return to and transition amongst various gaits through the careful management of basins of attraction surrounding each limit cycle gait. This specific case arises from a general approach to composing vector fields for stabilizing limit cycles on the $(N - 1)$ -torus, where N is the number of legs. Although the method is developed for an arbitrary number of legs, our experimental results are focused on the case of a robot for which $N = 6$. Our method allows attracting limit cycles to emerge from the interaction between repelling sets we place at any combination of pairwise diagonal subspaces of the torus (any subspace of the torus where two legs have equal phase). In so doing, we introduce a natural cellular decomposition of the gait space into $(N - 1)!$ “top cells” (those with open interiors) corresponding to every possible cyclic ordering of relative phases. As the constituent repelling functions of our constructions are convex over each top cell, we are able to show that when all possible repelling fields are simultaneously activated, each of these $(N - 1)!$ cells induces a “crawl” gait—an asymptotically stable limit cycle

corresponding to periodic recirculation of N legs maximally separated in relative phase—whose basin of attraction is exactly the cell in which it is contained. The stability properties of the resulting family of crawl gaits, represent this paper’s central contribution, introducing techniques of controller multistability to the problem of gait regulation.

The paper makes three further notional contributions to the gait regulation literature whose proper formalization remains the subject of subsequent work in progress:

Higher Codimension Gaits: By removing specific constituent repelling fields selectively as a function of gait space phase, we find that the multistable basins of the crawl gait family can be fused to yield attracting cycles located on higher codimension cells of the gait space decomposition corresponding to useful, familiar gaits. We describe and explore in simulation some particular common instances for the case of quadruped ($N = 4$) and hexapod ($N = 6$) locomotion, such as the trot or the alternating tripod gait. The resulting closed loop controllers have a hybrid nature as we find it convenient to implement them via recursively-defined logical predicates, yielding vector fields that are smooth over connected components of the gait space formed by the unions of neighboring top cells and their no-longer repelling boundaries. Because the constituent fields are all locally convex, we conjecture, and corroborate through numerical simulation, that the resulting closed loop dynamics continue to confer exactly one cycle and basin for each of these connected components: for example, when enough repelling constituents have been removed to yield a single connected component, we observe the resulting gait is (essentially) globally asymptotically stable. We illustrate the empirical utility of these methods through experiments with a hexapedal robot.

Stance Leg Constraints: In the specific case of a climbing robot, we call attention to a new class of gait space obstacle that is based upon quasi-static constraints and is of broad importance for legged locomotion. For a robot to remain statically stable while climbing, some subset of legs must remain in contact with the climbing substrate at all times. This constraint results in a combinatorially-defined obstacle set, introducing a different gait space decomposition that is familiar to topologists as a “moment angle complex” (Bahri et al., 2009). Because our controllers are constructed with constituent gradient fields that repel away from the skeletal “core” of these obstacles in gait space, we show through numerical simulation that the resulting closed loops tend to stay away from the “dangerous” phase timings, those in the moment angle complex (although, with current results, only approximately avoiding the obstacle set).

Gait Transition: Many gaits differ in their locomotive characteristics—some cautious and slow, others promoting speed over safety. This diversity of capabilities, one of the chief putative attractions of legged locomotion relative to tracks or wheels, should be freely available to the locomotion executive (whether a human operator or automated “higher level” intelligence), selectable upon command throughout a walk or climb. Because the union of the basins of each multistable family of gaits induced by our construction essentially covers the entire gait space, the nominal guard conditions for sequential composition (Burridge et al., 1999) are always satisfied thus we are able to switch between one gait family to another arbitrarily. Furthermore, by utilizing control policies that approximately avoid the obstacle set defined by the moment angle complex, our transition methods allows a robot to switch from potentially distant gaits without any additional likelihood of failure, even in the face of significant terrain-induced external perturbations.

We demonstrate the value of these various control techniques through a series of empirical studies with the RiSE robot (Autumn et al., 2005; Spenko et al., 2008), in the context of legged climbing wherein gait failures generally incur dramatic and easily verified consequences. We show how multistability lends robustness, providing an automated defense against perturbations by switching amongst many similar gaits without explicit command. When proprioceptive feedback terms unpredictably force the system to cross boundaries between basins surrounding different attractors, the robot transparently and safely switches gaits without requiring operator attention. We also explore the implications of stance leg constraints. The simultaneous recirculation of certain sets of legs results in loss of static stability, a potentially fatal mistake for a climbing robot. We show how to use the underlying algorithmic construction to carefully deploy the induced basins of attraction so as to place their boundaries approximately coincident with the leg recirculation obstacles. Finally, we provide examples of online transitions for a class of “simple” source-target gait family pairs mentioned above.

1.3 Organization

Section 2 presents our gait reference dynamical system, establishing the spatio-temporal decomposition of gaited locomotion control that separates relative phase (and stance-flight timing) from the geometry of limb interaction. Within this modular architecture, we discuss proprioceptive feedback loops shown to play an essential role in past running (Weingarten et al., 2004a) and climbing (Spenko et al., 2008) studies, and how this feedback necessarily links perturbative influence of a given terrain back onto both the “geometric” and “coordination” layers of gait timing.

Section 3 introduces the gait regulation techniques and extensions described above, distinguishing these new ideas from a variety of prior methods and exploring their comparative features through a series of simulation studies.

Section 4 presents the results of a series of empirical studies using the RiSE robot, illustrating the application of these techniques to the problem of legged climbing by means of which our gait generators allow the robot to carefully manage its preferred gait timings, and allow its operator to actively transition amongst variously desired gaits while climbing.

2 Parameterization of Robot Gaits

We impose the gait abstraction by factoring limb motion patterns within a stride into geometric and temporal components, translating the latter by angular addition of a phase offset vector. We use the example of the RiSE climbing robot to illustrate the utility of this parameterization. More details are provided in (Spenko et al., 2008) where these ideas are first introduced.

2.1 A Spatio-Temporal Decomposition of Gaits

Gaits can be most simply described as cyclic patterns of coordinated limb and body movement. Due to their periodicity and the inherent difference between stance, the portion of gait phase during which a specific leg is in contact with the ground, and recirculation, when the leg swings forward before a new stance begins, we can parameterize various timings of gaits, separate from the parameterization of their joint-level motions. We describe a complete gait as a mapping from a phase coordinate, $\theta \in S^1$, to the configuration space of a robot’s joints, $\mathbb{Q}$. Phase is the cyclic coordinate of the unit circle S^1 , a differentiable manifold we represent using the quotient space $[0, 1]/\{0, 1\}$ (the unit interval with endpoints identified). We define an individual gait, $g(\theta)$, as a cycle, that is, a smooth embedding of the circle:

$$g(\theta) : S^1 \rightarrow \mathbb{Q} \quad (1)$$

Rather than considering all possible cycles, we restrict our attention to gaits in which stance for each leg occurs precisely once per stride, in support of which we introduce a new set of parameters, the stance phase offsets, $\mathbf{r} = [\rho_1 \ \dots \ \rho_N] \in \mathbb{T}^N$, that prescribe the phase at which each indexed leg should ideally make contact with the ground.

An additional set of temporal parameters relate the amount of phase typically spent in stance for a given leg, the *duty factors* of a gait, defined as $\mathbf{d} = [\delta_1 \ \dots \ \delta_N] \in \mathbb{T}^N$. If ρ_i is the phase at which a leg begins stance, $\rho_i + \delta_i \bmod 1 = \rho_i - (1.0 - \delta_i) \bmod 1 \in S^1$ is the corresponding beginning of recirculation. As such, the stance phase offset and duty factor parameters fully define the overall timing of stance and recirculation in a gait. Following (Haynes and Rizzi, 2006a; Spenko et al., 2008) we now further assume that the gait cycle (1) admits a leg-indexed factorization,

$$g(\theta) = g_1(\theta) \times \dots \times g_N(\theta) \quad (2)$$

$$g_i(\theta) : S^1 \rightarrow \mathbb{Q}_i \quad (3)$$

$$g_i(\theta) = g_{i,s} [g_{i,t}(\theta - \rho_i, \delta_i)] \quad (4)$$

into a “temporal” component, $g_{i,t} : S^1 \rightarrow S^1$, that can effectively capture the relative speeds of legs in stance versus flight (and in which the phase offsets, $\mathbf{r}$ are subtracted as an offset to the coordination regulator) in composition with a “spatial component”, $g_{i,s} : S^1 \rightarrow \mathbb{Q}_i$, that maps the “timed” regions of stance and flight into a desired geometric foot path trajectory.

This paper concerns the control of a gait’s phase offsets, $\mathbf{r}$, in order to achieve transient, feedback-driven behaviors, capable of recovering from environmental perturbations that occur during climbing by a hexapedal robot, as highlighted in Sec. 4. In that example, intermittent but significant disturbances, owing to the complex interaction of the robot’s feet with a climbing surface, affect the relative timing of legs in stance, from which our gait controller must recover. As such, we have developed reactive control laws that attempt to recover from these locomotive failures by modifying the gait timing of legs in stance (Haynes and Rizzi, 2006a; Spenko et al., 2008). During recirculation, however, legs are free from the ground, and we take advantage of their mechanical decoupling from the body and each other to restore their coordination via a controlled process we term “gait regulation”. Implemented as dynamic systems on the phase offset vector, $\mathbf{r} \in \mathbb{T}^N$, these controllers impose basins of attraction around the translated cycle, $\theta - \mathbf{r}(t)$, that constitutes the input to the temporal component of a gait. As perturbative influences force the robot away from a desired gait, gait regulation attempts to bring it back, or to potentially transition to a different desired gait when necessary. The design of these translated cycles and their associated basins is the central focus of this paper.

2.2 Examples of Common Gaits

With these prior (Haynes and Rizzi, 2006a; Spenko et al., 2008) definitions in force, it is now clear that the values of the offset parameters, $\mathbf{r}$, dictate the relative phasing properties and hence the coordination of cyclic gait patterns, and we now list a variety of useful such patterns by way of concrete example, with specific examples of gaits for hexapedal systems ($N = 6$):

Pentapedal Crawl Gaits : These are hexapedal gaits that recirculate a single leg at a time, marked by high duty factors, slow speeds, and great stability. Owing to the fact that there are $(N - 1)!$ unique orderings of N elements¹, there are a total of

¹These gaits correspond to cyclic reorderings of the N^{th} roots of unity, $\rho_i := e^{2\pi i/N}$.

Figure 1: RiSE is shown climbing a crushed stone wall using its compliant feet with embedded microspine structures. Leg numbering conventions are noted. *Inset*: the upper left foot (#1) attached to the surface of the wall.

$(6 - 1)! = 120$ unique pentapedal crawl gaits.

Tetrapedal Gaits : Any hexapedal gait that recirculates two legs at a time, keeping four legs on the ground, is termed a tetrapedal gait. While there exist many combinatorially possible tetrapedal gaits, we focus on only a fraction, those that produce statically stable walking. While faster than pentapedal gaits, they are also slightly less stable.

Alternating Tripod : A widely observed biological legged timing pattern, this gait synchronously recirculates three legs at a time, alternating between two unique tripods of support. While there exist many gaits that recirculate three legs at a time, this gait does so with the largest possible stability margin, resulting in a fast gait that is also statically stable during locomotion.

Our convention for leg numbering is in order from front to back, left to right, on the body of a robot (leg 1 being front left, leg 4 being front right), as noted in Fig. 1. In addition to examples relevant to the hexapedal system of Sec. 4, this paper also discusses examples of familiar quadrupedal gaits, such as walking crawl gaits (recirculating a single leg at a time) and the trot gait (one of three gaits that recirculate precisely two legs at a time).

2.3 Proprioceptive Behavioral Control of Climbing Gaits

Our target empirical platform, RiSE (Fig. 1), is a biologically inspired robot designed for vertical climbing as well as horizontal mobility (Autumn et al., 2005; Saunders et al., 2006; Spenko et al., 2008). Similar to its predecessor, RHex (Saranli et al., 2001), RiSE relies upon underactuation to achieve effective locomotion while reducing the number of required actuators.

With only twelve actuated degrees of freedom for a six-legged robot, RiSE is severely underactuated and relies upon tuned passive mechanics at multiple stages—upper leg, ankle, and foot—to locomote effectively. While interchangeable, the specific feet used in this paper’s experiments allow the robot to climb vertical stucco and brick, and consist of arrays of microspines, each individually embedded into a multi-material structure with tuned compliances to attach to surface asperities (Asbeck et al., 2006; Spenko et al., 2008). The compliances of the system allow the robot to use only two actuators per leg yet adapt to variations in surface. A full system description of RiSE is available in prior work (Autumn et al., 2005; Saunders et al., 2006; Spenko et al., 2008).

To climb vertical building surfaces using compliant feet, the motions of RiSE’s legs are hand-tuned so that feet contact the surface while loading compliance, carry the robot’s weight during stance, and unload compliance to recirculate. Per our spatio-temporal decomposition of gaits, we describe each gait as the composition of functions dictating leg timing with functions describing the overall geometry of the leg trajectories. While open-loop gaits may be successfully used for vertical locomotion, the addition of behavioral feedback greatly increases the achievable climbing distance and robustness (Spenko et al., 2008). By modifying gait parameters, we have built controllers that successfully seek the surface, reattach after slip events, and balance traction forces amongst the robot’s legs (Spenko et al., 2008).

3 Gait Regulation

The central contribution of this paper is a family of gait regulation controllers, with specifically designed convergence properties relative to certain useful gaits, that also tend to repel transients away from violating stance leg constraints mentioned in the

Figure 2: Phase spaces of low-dimensional gaits

introduction. We implement these regulators by means of a state-based “switching” function that activates or deactivates the individual controller components in accord with a cellular decomposition of the torus, algorithmically connecting regions of gait space in order to produce flows that converge to desired gaits. We formally establish the asymptotic properties of these regulators by noting their local convexity, utilizing numerical simulations to suggest how they improve the performance of our gait generators relative to methods reported in prior literature.

3.1 Obstacles in Gait Space

A statically stable climbing robot must keep some set of its feet in contact with the ground surface at all times, resulting in stance leg count (SLC) constraints. When considering the high-dimensional torus on which we define the phasing of individual legs in a gait, SLC constraints are obstacles that must be avoided to keep the robot stable. Consider Fig. 2, showing the obstacle sets for two- and three-legged gait spaces. For each axis of the torus, some portion corresponds to stance ($0 < \psi_i < \delta_i$, where $\psi_i = \theta - \rho_i$, the “leg clock”) and for recirculation ($\delta_i < \psi_i < 1.0$). The duty factor parameters affect the size of the obstacle region. The phase offsets of a gait, defining a gait cycle on the torus, affect the intersection of the resulting flows with these obstacles.

For the case of a bipedal robot, instability occurs if both legs attempt to recirculate together, an obstacle noted in the upper right quadrant. For three-legged gaits, in $\mathbb{T}^3$, recirculating all three legs at once is surely forbidden in quasi-static climbing. However, depending upon the fragility of the surface and strength of individual legs, it may be the case that recirculating two legs at a time should also be considered an obstacle. In such a case, the obstacle set has greater complexity, as is depicted in Fig. 2b. For both the two- and three-legged examples, however, a select family of open-loop gaits exist that do not intersect the obstacle.

The combinatorial complexity of these SLC generated obstacles grows with the number of legs in a manner whose precise characterization takes the form of a “moment angle complex” (Bahri et al., 2009). While ongoing work has begun to systematically study how a hybrid gait planner can be used to systematically avoid these SLC generated obstacles (Haynes et al., 2009), the present study focuses upon control methods that roughly avoid these obstacles, suggested through a series of numerical studies and through experimental results demonstrating robustness of legged robot climbing.

3.2 A Non-Smooth, Convex Potential on $\mathbb{T}^2$

Our control primitives are formed from gradient vector fields generated by potential functions over the gait space defined by leg phases, $\mathbb{T}^N$. Following the design goals of a vector field planner (Koditschek and Rimon, 1990), we seek a control system that produces an attracting limit cycle at the target gait while insuring that the transient flow avoids the obstacle set we define on the torus. Fundamental topological obstructions preclude the possibility of any such perfect design, as the homotopy type of a basin must match that of its attractor, the asymptotic flow of a dynamical system defining a deformation retraction (Bhatia and Szegö, 2002). The homotopy type of the gait cycle, S^1 , differs from $\mathbb{T}^N$, the space of gaits, for any $N > 1$. This necessitates that we

Figure 3: The basis function for pairwise leg repulsion, ϕ , defined on the 2-torus. A minima appears along the cycle where legs are maximally separated in phase.

introduce boundaries on the torus that delimit the basin. Motivated by a desire for rapid transient responses to changing phases, we introduce a family of non-differentiable potentials, organized by a cellular decomposition of the torus, that provide a rough approximation to the obstacle-free subspace (the combinatorially-defined “moment angle complex”).

Potential functions are common for robot navigation tasks (Choset et al., 2005) in which obstacles and goals are encoded as maxima and minima of a real-valued function on the configuration space of the robot. In our case, the configuration space of a gait’s phase offsets is the N -torus, while we place global minima at desired gait limit cycles and maxima that are coincident with obstacle sets. Navigation of phase space follows the negative gradient of a potential function, Φ :

$$\Phi : \mathbb{T}^N \rightarrow \mathbb{R} \quad (5)$$

$$\dot{\mathbf{r}} = -\nabla_{\mathbf{r}} \Phi(\mathbf{r}) \quad (6)$$

Our approach follows that of (Klavins and Koditschek, 2002), by summing pairwise potentials between leg pairs. We depart from that approach, however, by making use of only repulsion between leg pairs in the construction of our controllers, rather than both attraction and repulsion. We make use of a scalar valued measure of relative phase error, $\phi : \mathbb{T}^2 \rightarrow \mathbb{R}$, that may be selectively activated for any or all $\binom{N}{2}$ leg pairs². A vector, ϕ , of all $\binom{N}{2}$ possible pairwise repulsions, may be summed together using a Boolean selection vector, $c \in \{0, 1\}^N$, to activate the specific repulsions required by the overall potential function, Φ ³:

$$f(\mathbf{r}) = [\phi(\rho_1, \rho_2) \quad \dots \quad \phi(\rho_{N-1}, \rho_N)]^T \quad (7)$$

$$\Phi(\mathbf{r}) = c^T f(\mathbf{r}) \quad (8)$$

A significant departure from prior work is our choice the constituent scalar phase error, ϕ . In (Klavins and Koditschek, 2002), pairwise repulsion is formed using the simple trigonometric function, $\phi(\rho_i, \rho_j) = \cos[2\pi(\rho_i - \rho_j)]$. This function achieves a smooth maximum value when legs are in-phase, $\rho_i - \rho_j = 0$, and a smooth minimum for out of phase pairs, $\rho_i - \rho_j = 0.5$. By contrast, we have replaced this with an analogous function that produces smooth, differentiable minima, but has “sharper” (continuous but not smooth) maxima at the cycle $\rho_i - \rho_j = 0$:

$$\phi(\rho_i, \rho_j) = 1 - \sin[\pi(\rho_i - \rho_j \bmod 1.0)] \quad (9)$$

The function, ϕ is depicted as a surface over $\mathbb{T}^2$ in Fig. 3. The gradient field associated with the resulting potential (8) is well defined over the complement of the pairwise in-phase subspace (a discontinuity in its gradient occurs along $\rho_i = \rho_j$, motivated by our desire for rapid transient dynamics). We apply this 2-dimensional function to the entire torus by recourse to a hybrid tie-breaking rule that chooses a set of component functions, via c , in order to follow “sensible” directions of convergence. While ϕ suffers from discontinuities in its gradient along the diagonal subspace of $\mathbb{T}^2$, we are amply repaid with a potential function that is convex over the complement of the diagonal subspace, a property that will be shown to be critical to our approach, as we now discuss⁴.

3.3 Navigating Gait Space

The *diagonal* subspaces of $\mathbb{T}^N$, in which some subset of the $\binom{N}{2}$ pairwise phases are equal, $\rho_i = \rho_j$, whose union we will denote as $\Delta[N]$, define an arrangement that, together with its complement, $\Delta^C[N]$ (termed the “configuration space” of N

²While we limit our discussion to pairwise repulsion, it should be noted that pairwise attraction, can be produced by translating pairwise repulsion, $\phi(\rho_i, \rho_j + 0.5)$.

³The vector representation we use is a terser alternative to the connection matrices previously in (Klavins and Koditschek, 2002), however the two are equivalent (Haynes, 2008).

⁴It should be noted that the convexity property of our ϕ function is not unique, and alternative functions, such as the quadratic $\phi = ((\rho_i - \rho_j \bmod 1.0) - 0.5)^2$, have been tested and shown to be similarly effective. Convexity over the interior of the non-diagonal subspace, however, seems to be a requirement, and the alternative function provided in (Klavins and Koditschek, 2002) does not suffice.

distinct points in $\mathbb{T}^N$ (Fadell, 2001)), comprises a cellular decomposition of gait space. Discussed more carefully in (Haynes et al., 2009), we observe here that $\Delta^C[N]$ is comprised of $(N-1)!$ *top cells* corresponding to the $(N-1)!$ possible cyclic orderings of unique leg phases. Dual to this cell structure, we define a graph whose vertices correspond to the top cells, the elements of $\Delta^C[N]$, and whose edges represent the codimension 1 boundaries of these cells (corresponding to the $\binom{N}{2}$ pairwise components of $\Delta[N]$ characterized by the single constraint $\rho_i = \rho_j$) in which some single pair of leg phases are equal. The gradient vector field of each repelling potential (9) is well defined on $\Delta^C[N]$ and generates a flow that repels trajectories away from these various top cells' boundaries. Exploiting this very regular cellular structure, we introduce a variety of spatially determined switching functions for turning on or off various combinations of these repelling fields as a means of generating a desired class of multistable gaits, or a passage from one to another such class, including, possibly gait classes with only one attracting cycle possessed of an essentially global basin.

3.3.1 Convexity of Pairwise Potentials on $\Delta^C[N]$

The imposition of “complete” leg repulsion, turning on all $\binom{N}{2}$ repellor fields (9), generates the multistable class of $(N-1)!$ “crawl” gaits as follows. Each of the isolated $(N-1)!$ open top cells (one corresponding to each possible cyclic ordering of the legs) comprising $\Delta^C[N]$ is rendered positive invariant by the repellor fields. At the “center” of each cell is a crawl gait, an asymptotically stable limit cycle, that recirculates each leg in turn, one after another, in the particular cyclic ordering corresponding to that cell. For example, in Sec. 4, this multistable class of crawls is instantiated for the case of a hexapedal robot ($N=6$), entailing the emergence of 120 distinct stable limit cycles (that we term “pentapedal” gaits for this set of six-legged crawls), the union of whose basins, $\Delta^C[6]$, covers all but a set of measure zero, $\Delta[6]$, in $\mathbb{T}^6$. We will show that the resulting profusion of effective alternative gaits and their accompanying array of basins covering essentially the entirety of gait space plays a decisive role in safe recovery from the inevitable feedback-induced perturbations to coordination during locomotion. We will offer evidence to show that this mechanism for transitioning opportunistically and indifferently amongst the various crawl gaits measurably contributes to the empirical success of the RiSE vertical climber reported in (Haynes and Rizzi, 2006a; Spenko et al., 2008).

We begin by observing that the emergence of these gait cycles results from the convexity of our repellors fields over the convex components $\Delta^C[N]$ of $\mathbb{T}^N$. Consider the case of activating all repellors fields, i.e. $c_k = 1$ for all $1 \leq k \leq \binom{N}{2}$. Let $B_{i,j} = e_i - e_j$ be the vector where element i is 1, element j is -1 , and all other elements zero.

$$\Phi(\mathbf{r}) = \sum_{1 \leq i < j \leq N} \phi(\rho_i, \rho_j) \quad (10)$$

$$= \sum_{1 \leq i < j \leq N} 1 - \sin[\pi(\rho_i - \rho_j \bmod 1.0)] \quad (11)$$

$$= \sum_{1 \leq i < j \leq N} 1 - \sin[\pi(B_{i,j}^T \mathbf{r} \bmod 1.0)] \quad (12)$$

The gradient of this function is likewise a summation of terms:

$$\nabla \Phi = \sum_{1 \leq i < j \leq N} \begin{bmatrix} \frac{\partial \phi(\rho_i, \rho_j)}{\partial \rho_1} \\ \vdots \\ \frac{\partial \phi(\rho_i, \rho_j)}{\partial \rho_N} \end{bmatrix} \quad (13)$$

$$= \pi \sum_{1 \leq i < j \leq N} \cos[\pi(B_{i,j}^T \mathbf{r} \bmod 1.0)] B_{i,j} \quad (14)$$

Critical points occur whenever leg phases are evenly spaced, the $(N-1)!$ possible orderings of legs such that the phase difference between sequential leg pairs is $\frac{1}{N}$. The gradient vanishes as the partial derivatives are all odd (with respect to relative phase) causing all non-zero terms of $\nabla \Phi$ to cancel out one another.

The Hessian of this potential function is the sum of a set of outer products, as follows:

$$D^2 \Phi = \pi^2 \sum_{1 \leq i < j \leq N} \sin[\pi(B_{i,j}^T \mathbf{r} \bmod 1.0)] B_{i,j} B_{i,j}^T \quad (15)$$

Each component of the Hessian is positive semi-definite, thus the overall Hessian will be as well⁵. While it is impossible to produce a convex function over the entirety of (the non-convex space) $\mathbb{T}^N$, our repellors in fact partition the torus into a

⁵The rank of the Hessian is $N-1$, as the eigenvector $[1 \dots 1]^T$ has eigenvalue 0. This vector corresponds to the direction of a gait cycle and is the same cycle over which we produce the quotient space of $\mathbb{T}^N$.

Figure 4: Graph representation of the cellular decomposition of the $\mathbb{T}^4$. We show how removing repulsion between individual legs results in a single large region as a collection of four vertices, while two vertices remain separate.

collection of open cells, each with an isolated convex interior⁶.

Thus, upon activation of the $\binom{N}{2}$ repulsive functions, we produce $(N - 1)!$ stable attractors, each a unique crawl gait limit cycle and the basis for our cellular decomposition of $\mathbb{T}^N$. In contrast, if any one of these repulsive functions is deactivated (by zeroing the corresponding element of the vector c), the corresponding codimension 1 face shared between the two specific cells is no longer repulsive, and fewer limit cycles with their basins of attraction are produced. As the number and relative positions of these deactivations is varied, the limiting gaits change and their attracting basins expand or shrink in a complicated manner whose analysis is the focus of ongoing work and which we instead explore here by recourse to numerical study of the various specific choices of switching logic we now introduce.

3.3.2 Regulating Gaits via Deactivation of Pairwise Potentials

The second useful new coordination control idea emerging from our cellular decomposition of $\mathbb{T}^N$ is the possibility of not only regulating to the $(N - 1)!$ crawl basins, but also systematically transitioning to and from gaits with fewer degrees of freedom, for instance the trot, pace, and bound gaits for a quadruped, $N = 4$. Formally speaking, the underlying cellular decomposition of the N -torus arises as a familiar object (the image under the exponential map of the “generic braid arrangement” of the Lie Algebra of $\mathbb{T}^N$) in geometric combinatorics (Stanley, 2004). Notwithstanding our present focus on the top cells (unique orderings of N legs) and codimension 1 faces (two specific legs in-phase), we will find it useful to observe in passing that gaits with fewer degrees of freedom, such as the trot, exist at *higher* codimension boundary components of the underlying cellular decomposition.

Consider the cellular decomposition of $\mathbb{T}^4$, with a collection of $(4 - 1)! = 6$ cells resulting from the control vector:

$$r = \begin{bmatrix} \phi(\rho_1, \rho_2) \\ \phi(\rho_1, \rho_3) \\ \phi(\rho_1, \rho_4) \\ \phi(\rho_2, \rho_3) \\ \phi(\rho_2, \rho_4) \\ \phi(\rho_3, \rho_4) \end{bmatrix} \quad (16)$$

Activating all functions with $c = [1 \ \dots \ 1]^T$, the diagonal subspace of the torus, $\Delta[4]$, is repulsive, and six crawl gaits are locally stabilized, each corresponding to a different cyclic order as listed in Fig. 4a, whose attracting basins corresponds to one of the 6 “top cells”, the 4-dimensional subsets of $\mathbb{T}^4$ comprising $\Delta^C[4]$ cut out by the braid arrangement. The graph of Fig. 4a (formally, the one-skeleton of the dual complex) is defined by six vertices corresponding to these top cells, with edges corresponding to the 3-dimensional subspace boundaries upon which a pair of phases switch order.

Consider the quadrupedal “trot”, a two degree of freedom virtual bipedal gait wherein diagonally opposed legs are locked in phase (there are of course two additional virtual bipedal gaits—the pace and bound—however their construction follows that of the trot in an equivalent manner we will not detail here). To build a gait controller that stabilizes to the trot, we rework the connection graph (8) by deactivating the repulsion functions between leg pairs we would like to be in phase, $\phi(\rho_1, \rho_4)$ and $\phi(\rho_2, \rho_3)$, leaving the four remaining repellers in (16) active. As can be shown from the potential functions analytically, but merely noted in this paper through numerical simulations (Sec. 3.4), two crawl gaits remain as positive invariant basins of attraction, thus preventing global convergence to the trot. Intuitively, these gaits—the circular crawl gaits, as the legs recirculate

⁶In addition to admitting a single attractor, the interiors of each cell are also positive invariant with respect to negative gradient flow, proof of which is forthcoming in a future publication.

Figure 5: Design of the basic alternating tripod gait regulation controller

in either clockwise or counter-clockwise order—are “furthest away” from the trot, as some pair of legs must first cross ordering before the trot gait is within reach. Fig. 4 provides an intuitive description of this effect, when considering the one skeleton of the dual complex. Fig. 4b depicts abstractly the fusing of four of the six top cells of $\Delta^C[4]$ by removing the “non-trot” repellors corresponding to the codimension 1 sets, $\rho_1 = \rho_4$ and $\rho_2 = \rho_3$. The resulting closed loop behavior renders positive invariant two isolated top cells corresponding to the two circular crawls, with one large cell formed by the union of the four remaining top cells. Within each cell lies a single attracting limit cycle, the large region containing the attracting trot gait whose basin includes the fused region. When considering graph distance, as introduced in our algorithmic treatment of this concept, the four fused cells lie at 0 distance from the trot, while the two remaining circular crawls both a distance of 1 away. We emphasize that these ideas are introduced as exemplary heuristics in the present paper, while a more systematic formalization and explanation has been initiated in (Haynes et al., 2009).

As for any regular finite cellular decomposition, the gait complex on the torus gives rise to a partial order on its cells induced by the boundary relation (e.g. any given codimension 1 cell is the boundary of the two top cells it separates), a property that can be effectively described by its associated Hasse diagram (Wachs, 2004). For purposes of the present paper it suffices to leave unspoken this complete model of cell adjacency and merely work with the one skeleton of the dual complex, e.g. the case presented in Fig. 4a, where these cells and boundaries are represented as vertices and edges of a graph⁷.

Specifically, we limit our consideration of gait transitions to the problem of shifting from gaits with some sense of “multistability” (such as a system of $(N - 1)!$ different crawl cells) to a gait characterized by a single limit cycle whose attracting basin exhibits global convergence on the torus. Once again, because the homotopy type of the cycle is distinct from $\mathbb{T}^N$ (for $N > 1$), this can only be accomplished by recourse to a discontinuous controller, implemented via a recursively defined switching function that we detail in the next section.

While these constructions result in a gradient field that is often discontinuous at the selected codimension 1 boundaries between “top cells”, thus conferring the character of a hybrid controller, they provide a relatively simple, yet effective implementation of the gaits we desire. In our systems, we begin with a targeted gait of interest, a cycle within a specified cell of the the gait complex, whose basin is formed by the union of all adjacent top cells (its open “star” (Hatcher, 2002)) that “merge” to form the basin of attraction surrounding the desired gait. In general, such base fields will yield the targeted gait of interest as well as isolated “islands” of undesired gaits whose cycles and basins we must destroy by selectively removing addition repelling terms in a state dependent manner. For cells not initially located within the basin of the target cycle, we determine the shortest graph distance path to the union of top cells comprising the target basin, implementing a version of sequential hybrid control akin to that of (Burridge et al., 1999) to bring the gait timing into the target basin.

3.3.3 A Hybrid Control Approach to Gait Regulation

We now describe the hybrid control methods used to transition to gaits such as the trot or alternating tripod, using our graph representation to describe leg phase adjustments within the cells of the spatial partition it represents. We begin by defining the pairs of legs that should be out of phase with one another in the desired gait, as this affects the activation of the various potential functions via the vector c and is sufficient to realize gaits such as the hexapedal alternating tripod and the quadrupedal trot.

A hexapedal system admits $\binom{6}{2} = 15$ possible pairwise repulsions, of which we remove 6 to attain an alternating tripod gait. The 6 removed correspond to legs that would nominally be in phase with one another in the desired gait, the leg pairs $(1, 3)$, $(1, 5)$, $(3, 5)$, $(2, 4)$, $(2, 6)$, and $(4, 6)$. Fig. 5 shows both the original set of all 15 repulsions, as well as the system including the remaining 9. The associated binary vector is:

$$\text{Alternating Tripod: } c = [1\ 0\ 1\ 0\ 1\ 1\ 0\ 1\ 0\ 1\ 0\ 1\ 1\ 0\ 1]^T \quad (17)$$

⁷A true Hasse diagram, on the other hand, includes all possible boundary relationship, whereas our graph representation only considers codimension 1 boundaries. As such, a graph distance of 1 would correspond to two traversals on the Hasse diagram: one traversal downward to the boundary between two crawl cells (an in-phase locking of two legs), and one traversal upward to a neighboring cell.

Figure 6: Abstract representation of hybrid gait regulation control approach

When considering the merged top cells of the trot gait, as presented in Fig. 4, a total of four cells lie within the basin of the trot, thus a graph distance of 0, while two cells are at distance 1. When performing similar analysis upon the graph corresponding to $\mathbb{T}^6$ (using our simplified version of a proper Hasse diagram), our cellular decomposition contains a total of 120 cells, of which we have:

36 cells at graph distance 0
 72 cells at graph distance 1
 12 cells at graph distance 2

Upon activation of the specific set of pairwise repulsions given in (17), we would expect natural convergence to the alternating tripod gait from only 30% ($\frac{36}{120}$) of random initial starting configurations. As we seek to produce global convergence, however, we must necessarily deactivate some portion of pairwise repulsions when the system starts from these initially-disconnected portions of the torus.

Considering distant cells on our graph representation, there are a variety of possible paths between these cells and the set of cells within the basin surrounding our desired gait. In the trot example on $\mathbb{T}^4$, both of the two circular crawl gait cells are graph distance 1 from the trot basin, with four different possible edge traversals. To decide which route is best, we utilize a simple metric that compares individual leg phase differences—evaluating comparisons akin to $\rho_i - \rho_j < \rho_j - \rho_k$ —to choose a path. This has the effect of introducing a geodesic subdivision of distant cells (related to the barycentric subdivision) that partitions each cell based upon the nearest codimension 1 face.

To address these two primary concepts, that of a graph representation of the cell complex of gait cells, as well as of a simple metric subdivision for potentially distant cells, we utilize an algorithmic approach for gait regulation control. If a system starts out within the “natural” basin of attraction of a desired gait, the unmodified c vector may be used, such as that shown in (17). Starting outside of this region would result in local minima, thus we seek to compare various routes of convergence in order to modify c such that convergence to the original basin is attained. If the system starts out in a cell at graph distance k , a total of k pairwise repulsions must be temporarily removed for convergence to occur. Comparisons of metric phase differences ensure that the pairwise repulsion functions removed are the “closest”, the system passing through the nearest codimension 1 face when necessary. This method of control, removing pairwise repulsions when necessary, produces deterministic, global convergence to desired gaits. A full implementation of this algorithm (capable of regulating crawl, alternating tripod, and tetrapod gaits) is provided as MATLAB source code in (Haynes, 2008).

By continuously evaluating this algorithm in real-time as a robot locomotes, the system adapts as it passes into closer-connected regions (in fact, reactivating pairwise repulsions once the system passes through), as well as to unforeseen changes in gait. Fig. 6 provides an abstract representation of the approach.

As we use only repulsion between leg pairs to form our gait regulation controllers, we approximate the obstacle sets defined by the moment angle complex (as two legs out of phase from one another are less likely to recirculate together (Haynes, 2008)).

3.4 Numerical Gait Studies of Heuristic Stance Leg Constraints

To study the transient properties of the gait coordination system defined above, we utilize numerical simulations of oscillator systems, as defined by the phase offsets representing legs in a robot gait. To keep our simulations relevant to gait regulation, we apply control only while legs “recirculate”, assuming that stance legs must be fixed on the ground for some period of a stride. By simulating different control strategies from a random sampling of initial conditions, as well as noting the transient patterns and convergence properties for each system, we suggest how our hybrid control approach tends to avoid recirculation obstacles dramatically better than several prior methods that we introduce.

Figure 7: Example gait regulation simulation. Stance regions are shaded, during which no gait regulation takes place. During recirculation, legs are allowed to speed up or slow down to match the gait timing of a desired gait.

To simulate gaited locomotion, a fixed timestep, first-order integrator runs until a *convergent gait* is reached. In order to compare the convergence properties of various controllers, a total of 5000 random initial conditions are integrated until producing a limit cycle gait, while statistics are collected regarding diagonal subspace crossings (legs changing ordering) as well as incursions into obstacle sets (Sec. 3.1).

As we are interested in control policies that minimize these incursions, we define the following undesired obstacle regions. For a quadrupedal system, the “3-legs” obstacle corresponds to any time three or more legs attempt to recirculate, while “2†-legs” is any recirculation of ipsilateral or contralateral pairs of legs (the recirculating leg pairs of the trot are not considered an obstacle). For six legs, the “4-legs” obstacle similarly defines a recirculation by four or more legs, while “3†-legs” corresponds to any recirculation of three legs not in the specified alternating tripod groupings. For simulation comparisons, we collect statistics on incursions into each of these obstacle sets.

Quadruped simulations are performed with a duty factor⁸ of $\delta = \frac{3}{4}$, while hexapods use $\delta = \frac{5}{6}$. These duty factors are chosen to reflect the nominal and minimum duty factor possible when using a crawl gait for each system. Fig. 7 shows an example of one simulation run, a trot controller. In the example, the system begins at a random set of phase offsets, while gait regulation is performed during leg recirculation to converge to a trot gait. The diagonal shaded regions indicate the portion of phase, at each point of time, dedicated to stance. Legs within stance are constant-valued, with control performed only during recirculation.

3.4.1 Realizing a Trot Gait

We introduce several controllers based upon our methods, using only repulsion amongst leg pairs to produce a trot gait. The first such controller, Φ_{rtrot} (19), applies four pairwise repulsions to achieve a trot gait, but suffers from two local minima (as suggested in Fig. 4b). By using one less repulsion, another controller, Φ_{grtrot} (21), results in global convergence but adds “preference” by explicitly choosing a specific route of convergence. The third controller, Φ_{srtrot} , is our new method that algorithmically chooses which repulsion to deactivate, to evenly partition the torus, constructing the basin of attraction around the trot gait. For each controller, connection diagrams (Klavins and Koditschek, 2002) are shown alongside, depicting pairwise repulsion between legs using -1 . For Φ_{srtrot} , “switch” symbols indicate the algorithmic choice of deactivation.

$$c = [1 \ 1 \ 0 \ 0 \ 1 \ 1] \quad (18)$$

$$\Phi_r(\mathbf{r}) = \phi(\rho_1, \rho_2) + \phi(\rho_1, \rho_3) + \phi(\rho_2, \rho_4) + \phi(\rho_3, \rho_4) \quad (19)$$

$$c = [1 \ 1 \ 0 \ 0 \ 0 \ 1] \quad (20)$$

$$\Phi_{gr}(\mathbf{r}) = \phi(\rho_1, \rho_2) + \phi(\rho_1, \rho_3) + \phi(\rho_3, \rho_4) \quad (21)$$

⁸We assume the duty factor parameter to be identical across all legs.

Table 1: Convergence probabilities for five different trot controllers, using only repulsion between pairwise legs.

Controller	Convergent gait	Convergence probability
Φ_{rtrot}	Trot, $[0 \ 0.5 \ 0.5 \ 0]$	66.3%
	Crawl #2, $[0 \ 0.25 \ 0.75 \ 0.5]$	16.8%
	Crawl #4, $[0 \ 0.75 \ 0.25 \ 0.5]$	16.8%
Φ_{grtrot}	Trot	100%
Φ_{srtrot}	Trot	100%
Φ_{artrot}	Trot, $[0 \ 0.5 \ 0.5 \ 0]$	66.0%
	Degenerate Pace, $[0 \ 0 \ 0.25 \ 0.25]$	16.5%
	Degenerate Pace, $[0 \ 0 \ 0.75 \ 0.75]$	17.6%
$\Phi_{gartrot}$	Trot	100%

$$\Phi_{srtrot}(\mathbf{r}) = c(\mathbf{r})^T f(\mathbf{r}) \quad (22)$$

Compared against these methods, we introduce two related controllers that additionally make use of attraction between leg pairs, in order to converge to a trot: Φ_{artrot} (25), derived from a description of a trot leg coordination scheme in (Raibert, 1986), and $\Phi_{gartrot}$ (28), loosely derived from the successful alternating tripod controller in (Klavins and Koditschek, 2002). In Φ_{artrot} , leg pairs (1, 4) and (2, 3) attract one another, while leg pairs (1, 3) and (2, 4) repulse. This system converges to a trot, but suffers from local minima when using the convex pairwise coordination functions for attraction and repulsion described in the previous section. $\Phi_{gartrot}$ is a similar system, using both attraction and repulsion, but removing the (2, 4) leg repulsion, leaving a single repulsion function in the overall potential. The vector c denotes pairwise repulsion, while a similar vector c_a denotes pairwise attraction (implemented here by simply translating the difference in f through a one-half phase shift, 0.5). As before, connection diagrams are shown alongside each controller equation, with +1 indicating pairwise attraction between legs.

$$c = [0 \ 1 \ 0 \ 0 \ 1 \ 0] \quad (23)$$

$$c_a = [0 \ 0 \ 1 \ 1 \ 0 \ 0] \quad (24)$$

$$\Phi_{artrot}(\mathbf{r}) = \phi(\rho_1, \rho_3) + \phi(\rho_2, \rho_4) + \phi(\rho_1, \rho_4 + 0.5) + \phi(\rho_2, \rho_3 + 0.5) \quad (25)$$

$$c = [0 \ 1 \ 0 \ 0 \ 0 \ 0] \quad (26)$$

$$c_a = [0 \ 0 \ 1 \ 1 \ 0 \ 0] \quad (27)$$

$$\Phi_{gartrot}(\mathbf{r}) = \phi(\rho_1, \rho_3) + \phi(\rho_1, \rho_4 + 0.5) + \phi(\rho_2, \rho_3 + 0.5) \quad (28)$$

Each of the five different controllers introduced, with various arrangements of pairwise attraction and repulsion, produces convergence to the trot gait. The use of “extra” component functions sometimes results in other convergent gaits, however, with Φ_{artrot} and Φ_{rtrot} both converging to local minima roughly $\frac{1}{3}$ of the time⁹. The versions with just one fewer component function all produce global convergence, however, as is shown in Table 1. The slightly modified versions— $\Phi_{gartrot}$ and Φ_{grtrot} , both of which use one fewer component function, as well as Φ_{srtrot} , which uses one fewer when it deems necessary—all produce global convergence to a trot.

The homotopy mismatch between the torus and the cycle necessitates the introduction of boundary cuts. Each gait regulation controller, with its unique set of component functions, will produce slightly different paths of convergence. Table 2 summarizes these results by noting both the crossing of diagonal subspaces of the torus (the codimension 1 sets where two legs are in-phase with one another), as well any intersection each convergent path has with the obstacle sets. The use of different component functions affects the crossing of diagonal subspaces; i.e. if a strategy utilizes $\phi(\rho_i, \rho_j)$, that system avoids the set $\rho_i = \rho_j$ ¹⁰. Studying specifically the controllers that utilize repulsion only, Φ_{rtrot} never crosses any subspace corresponding to its four different pairwise repulsion functions. Φ_{grtrot} , for roughly $\frac{1}{3}$ of the simulations (those that produce local minima for Φ_{rtrot}),

⁹The pace gaits produced by Φ_{artrot} are considered degenerate, as they differ from a nominal pace, where $\mathbf{r} = [0 \ 0 \ 0.5 \ 0.5]$.

¹⁰Not noted in the table, but likewise true, is that if a system utilizes $f_a(\rho_i, \rho_j)$, that system will avoid the set $\rho_i = \rho_j + \pi$.

Table 2: Cut crossings and obstacle set analysis

Controller	$\rho_1 = \rho_2$	$\rho_1 = \rho_3$	$\rho_2 = \rho_4$	$\rho_3 = \rho_4$	2†-leg obstacle	3-leg obstacle	Global Convergence
Φ_{rtrot}	0.0%	0.0%	0.0%	0.0%	2.0%	0.12%	No
Φ_{grtrot}	0.0%	0.0%	34%	0.0%	3.0%	0.15%	Yes
Φ_{srtrot}	8.8%	8.3%	8.4%	8.1%	1.2%	0.12%	Yes
Φ_{artrot}	11%	0.0%	0.0%	11%	16%	0.66%	No
$\Phi_{gartrot}$	25%	0.0%	34%	26%	3.9%	1.7%	Yes

will always cross $\rho_2 = \rho_4$. Φ_{srtrot} algorithmically produces even partitions of the torus, and thus crosses the four subspaces with equal probability. Furthermore, systems utilizing pairwise repulsions tend to avoid obstacle sets better, with the best controller, Φ_{srtrot} , offering almost an order of magnitude improvement over those utilizing pairwise attraction.

Toward the goal of producing a rapidly converging, obstacle avoiding trot gait regulation controller, we argue that Φ_{srtrot} is the best available approach among the controllers presented. Its use of only repulsion between leg pairs produces convergence that best avoids obstacle regions. The application of even choice in our algorithm produces decision surfaces that result in non-preferential and fair convergence patterns, while providing global convergence as required.

3.4.2 Realizing the Alternating Tripod Gait

The hexapedal alternating tripod gait, previously introduced in Sec. 2.2 is roughly analogous to the quadrupedal trot. We now demonstrate the application of our hybrid gait coordination method to this ubiquitous pattern. Our switched control methods, operating on a cellular decomposition of 120 cells, plan paths of convergence while safely avoiding obstacle regions, as is noted in simulation comparisons with a prior approach.

For this hexapedal simulation, we once again compare our hybrid switched controller, $\Phi_{srtripod}$ (29), against an alternative method found in (Klavins and Koditschek, 2002). This alternative, $\Phi_{gartripod}$ (30), produces global convergence to the alternating tripod gait by attracting amongst legs within a tripod, while repulsing two legs of opposite tripods. Our switched approach, in comparison, uses only pairwise repulsion to produce a single basin of attraction surrounding the alternating tripod gait, minimizing the number of leg crossings before reaching the final limit cycle. Once again, the introduction of decision surfaces, through metrics induced by our algorithm, produce fair and even convergence patterns from potentially distance starting gaits on the torus.

$$\Phi_{srtripod}(\mathbf{r}) = c(\mathbf{r})^T f(\mathbf{r}) \quad (29)$$

$$\Phi_{gartripod}(\mathbf{r}) = \phi(\rho_2, \rho_5) + \phi(\rho_1, \rho_5 + 0.5) + \phi(\rho_3, \rho_5 + 0.5) + \phi(\rho_2, \rho_4 + 0.5) + \phi(\rho_2, \rho_6 + 0.5) \quad (30)$$

$$(31)$$

In simulation both strategies produce global convergence to the alternating tripod gait. Of note, $\Phi_{srtripod}$, with its hybrid control algorithm carefully managing convergence from all 120 cells, avoids local minima and achieves global convergence. While the number of component repulsion functions is combinatorially greater than Φ_{srtrot} , the system still imposes a pleasingly symmetric pattern of transients toward global convergence by fairly partitioning potential local minima.

Table 3 shows obstacle avoidance for the two systems, and again we note an order of magnitude performance improvement when not using attraction between leg pairs. $\Phi_{srtripod}$ best captures the constraints of obstacles in phase space while always converging to the desired gait.

Rather than listing all diagonal subspace crossings, we count the total number of times a crossing occurs for a given simulation, as shown in Table 4. $\Phi_{srtripod}$, by structuring its convergence to minimize the number of cut crossings, crosses at most 2 diagonal subspaces of the torus. Note also that the crossing probabilities match the overall connectedness of the phase

Table 3: Obstacle values for tripod controllers

Controller	3†-legs	4-legs
$\Phi_{srtripod}$	0.17%	0.017%
$\Phi_{gartripod}$	2.0%	0.75%

Table 4: The number of diagonal crossings per controller over 5000 simulations. $\Phi_{srtripod}$, with its lower valued distribution, performs less cut crossings on average.

Controller	Number of diagonal crossings							Total
	0	1	2	3	4	5	6+	
$\Phi_{srtripod}$	29%	60%	11%					100%
$\Phi_{gartripod}$	22%	19%	22%	14%	10%	7%	6%	100%

space graph for the tripod gait, where there are (36,72,12) cells at graph distances of (0,1,2), respectively. $\Phi_{gartripod}$ performs a greater number of crossings, suggesting greater incursion into the phase space obstacles.

The gait regulation control strategies presented here can be used for a class of different gaits. (Haynes, 2008) provides additional details for building crawl (only a single leg recirculating at a time) and tetrapod (two legs recirculating at a time) gaits. In both of these cases, our control strategies are used to manage multiple basins of attraction that are evenly distributed over the phase space, achieving multistability through the simultaneous creation of multiple, equally valid attracting limit cycles resulting from this single control strategy.

4 Experimental Results

We demonstrate empirically the value of this approach to gait regulation as a critical component of robust climbing behaviors for a robotic hexapod, avoiding dangerous multiple leg recirculations while allowing the application of feedback control to a variety of gaits. Whereas prior work in (Spenko et al., 2008) documents the use of gait feedback to the family of crawl gaits, the work of this paper extends this feedback approach to the generalized family of controllers of Sec. 3.3, not only utilizing pentapedal crawl gaits, but also successively faster tetrapod and tripod gaits.

We reintroduce, in Sec. 4.1, the same individual leg posture and timing feedback controllers of (Spenko et al., 2008), showing that the gait regulation controllers of Sec. 3 play an essential role in stabilizing the leg loads across the family of gaits, with increasing importance from pentapod to tetrapod to tripod gaits. Using our gait regulation controllers as a foundation, we introduce in Sec. 4.2 a new method of transitioning between these various climbing gaits. In contrast to our original specification of pairwise connecting trajectories in gait space (Haynes and Rizzi, 2006b), our covering of multistable gaits permits the implementation of a variant of sequential composition (Burridge et al., 1999). This allows the system to transition freely in a reactive manner during active locomotion to gaits that are capable of traveling close to twice as fast as the single gait control strategy presented in (Spenko et al., 2008), while affording a ready means to transition back down to the slower, more reliable gaits with higher stance leg counts.

4.1 Effect of Leg Feedback upon Climbing with Varied Gaits

We study the salient statistics of the RiSE robot’s wall attachment and ground reaction force patterns during climbing, in order to show how the feedback behaviors arising from the gait regulation framework introduced in this paper increase the reliability of locomotion when compared to the simpler controllers of (Spenko et al., 2008) across a variety of settings including exploration of tetrapedal and alternating tripod gaits in addition to the pentapedal “crawl” gaits that formerly comprised the entire safe climbing repertoire for RiSE.

The purpose of the gait regulation system is to drive the system closer to desired gait timings. On the RiSE robot, this is performed in the presence of real-time sensory feedback that seeks to adjust the quality of the robot’s attachment during climbing. We summarize these constituent proprioceptive feedback components from (Haynes and Rizzi, 2006a; Spenko et al., 2008), noted in Sec. 2.3, as follows:

- An open-loop gait, tuned for the climbing task at hand, is used to approximate the leg motions necessary for climbing.
- A traction force controller, introduced in Sec. 2.3 speeds or retards legs (via phase offset modification) based upon experiencing too little or too large of traction forces.

- A normal force controller helps guarantee foot contact and assists to prevent pitch motions during climbing (Spenko et al., 2008). This controller operates “tangentially” to phase and thus has no effect upon gait timing.
- A pawing controller (Durr, 2001) provides a means of reattaching, by resetting leg phase and quickly recirculating a leg to retry attachment whenever a leg unexpectedly loses grasp.

As these empirically necessary feedback control strategies maintain traction and secure balance by perturbing the gait, we apply gait regulation control. For each gait type, we compare two control arrangements. The first utilizes only an open-loop gait (abbreviated as *FF*) while the second activates all feedback and gait regulation control (abbreviated as *FB*). Prior work (Spenko et al., 2008) studies the relative merits of each component and suggests the merit of all portions of the feedback controller. Robot data is collected climbing a wall panel consisting of crushed quartz gravel¹¹, installed on-site at Boston Dynamics, Inc.¹² The RiSE robot is specifically tuned to climb vertical surfaces using a pentapedal gait, with two force-related issues preventing vertical climbing using other gaits. Specifically, RiSE’s traction force sensors saturate around 15–20 N, while its microspine toes feature an auto-disengage when larger per toe forces are encountered (Asbeck et al., 2006). For these reasons, the climbing wall was angled to 65 degrees (empirically determined to prevent force saturation on legs) to fairly compare all robot gaits and behaviors. Roughly six body lengths of climbing, 3 meters, was performed for each behavior.¹³ Furthermore, to allow fair comparison of speed, the driving frequency of each gait was set such that typical recirculation speeds were identical.

To compare the overall stability and speed of gaits, we compute the following statistics on data collected from the robot, comprising gait parameters, joint angles, and foot forces. Table 5 displays the overall results for each of the three gaits (contrasting performance with and without feedback) by row, using the following metrics reported in the various columns, along with calculated variances:

Load Count (LC) : the average number of legs carrying load, strictly less than 6. A simple heuristic—empirically developed as positive adhesion force and traction force greater than 2 N—dictates whether a leg is under load.¹⁴ The LC value is dependent upon gait type (as duty factor is related to desired number of legs in stance), and decreases from pentapod to tetrapod to tripod. For the first two, the addition of feedback increases this value, indicating better contact quality. In the case of the tripod, where feedback decreases this value, we presume that the greater speed, as well as the effect of the various proprioceptive controllers, remains a challenge for the gait regulation methods.

Average Leg Stance Force (SF) : the traction force experienced by a single leg, measured during stance and averaged over all six legs. Large variances indicate irregular stance forces, with lower variances desired. These values show that feedback diminishes variances by 54% in the pentapodal gait, 42% for tetrapedal, and 29% in the tripod gait. The decreasing return for gaits with fewer simultaneous legs in stance is presumably due to decreased force regularity overall.

Body Force (BF) : the sum of traction force over all legs. This provides an overall idea of regularity of force acting upon the body during locomotion. Higher variances indicate greater irregularities in overall traction. The data shown in the table shows that feedback reduced the variances by 22%, 25%, and 21% for the pentapod, tetrapod, and tripod gaits, respectively.

Velocity (V) : an average ground speed, distance climbed over time, in cm/s. Across gait types, there is a large increase in speed from the pentapod to tetrapod to tripod gaits. Within each gait type, however, feedback incurs a slight decrease in overall speed (−2%, −5%, and −3% respectively) due to slight retarding perturbative signals induced by each controller.

This table illustrates the general principle that gaits with greater speed—and fewer simultaneous stance legs—have less regularity during locomotion. While the feedforward alternating tripod gait travels the fastest, it does so with less stability, as evidenced in the higher force variances presented in Table 5. Similarly, across all three gait types tested, the feedback versions of each gait produce lower variances in force, thus suggesting greater regularity of locomotion due to the more uniform and consistent forces experienced. Our use of force variances to indicate overall stability assumes that strong fluctuations in ground reaction forces are indicative of unstable climbing, a claim that is supported in our next study, analyzing slip events encountered during climbing. This analysis is further presented qualitatively in Fig. 8.

We note dramatic slip events in the force data as marked drops in ground reaction forces by individual feet or by the whole body. This analysis supersedes that previously performed in (Spenko et al., 2008), by, in addition to collecting statistics on the robot’s ground reaction forces, quantitatively measuring events that could often cause climbing failures. The slip events we study are defined as:

¹¹Stoneflex Stone Aggregate Panels—CEP Panels, Inc., Naperville, IL

¹²Boston Dynamics, Inc., Waltham, MA

¹³The reader is referred to (Spenko et al., 2008) for baseline data of similar experiments performed with pentapedal gaits. The current set of pentapedal experiments are performed on a wall angled at 65 degrees, rather than vertical in previous experiments. Furthermore, while the previous work described the effect each control component has upon the overall behavior, we only compare between the open-loop gait and the full feedback behavior in this work.

¹⁴As described in (Asbeck et al., 2006), the feet on RiSE are designed to be dragged along a surface (to generate traction force) while pressing the foot against the wall (positive normal force). Once engaged, the foot may be pulled on in order to produce adhesion (negative normal force). As such, the heuristic chosen signifies that a foot has successfully completed this attachment sequence.

Table 5: Force Analysis of Regulation Controllers

	LC	σ^2	SF	σ^2	BF	σ^2	V
Units	$n \leq 6$	n^2	N	N^2	N	N^2	cm/s
Pentapod FF	4.1	0.7	8.8	29	46	28	1.2
Pentapod FB	4.4	0.6	9.0	13	49	22	1.2
Tetrapod FF	4.2	0.5	9.7	26	47	33	1.9
Tetrapod FB	4.3	0.8	9.7	15	49	25	1.8
Tripod FF	3.8	1.1	10	23	44	56	2.4
Tripod FB	3.6	1.5	9.5	16	45	44	2.3

Table 6: Occurrence of slipping for various gait types

Controller	Individual Leg Slips							Body Slips	
	1	2	3	4	5	6	Total	Per Stride	Total
Pentapod FF	5	9	5	3	7	3	32	0.67	7
Pentapod FB	7	5	2	1	2	1	18	0.36	6
Tetrapod FF	17	15	5	7	15	3	62	1.3	15
Tetrapod FB	7	6	3	1	5	5	27	0.55	10
Tripod FF	10	34	5	14	19	4	86	1.5	29
Tripod FB	6	30	2	5	14	1	58	1.2	10

Foot slip : In the middle of stance, if a foot loses traction force of 6 N or greater in less than 0.1 seconds, we consider this to be a slip. 6 N was chosen as it is a large fraction (50 – 60%) of the typical traction carried by an individual foot during stance. Feedback behaviors reduced slipping by individual legs by 44% for pentapod, 56% for tetrapod, and 33% for tripod gaits.

Body slip : If the total traction force carried by all six legs drops by roughly 20 N in a similar amount of time (0.1 s), this is a body slip. 20 N is roughly half of the force normally carried by the robot. The addition of feedback reduces total body slipping by 14% for pentapod, 33% for tetrapod, and 66% for tripod. The monotonic relationship is presumably due to the fact that when more simultaneous legs attempt to attach with one another (such as with the tetrapod and tripod gaits), body slips are more likely. By incorporating feedback, overall slips are reduced, thus triggering fewer body slips.

As shown in Table. 6, the incorporation of feedback reduces slippage, both in terms of individual leg slips and overall body slips. The tripod gait behaviors, however, still incur a large number of slips, in particular with middle legs (legs 2 and 5), suggesting the overall challenge of climbing with this gait type. Additionally, and similar to previous force analysis, the occurrence of slipping follows a general trend that faster gaits, per the velocities shown in Table 5, produce more slipping.

The overall stability of locomotion may also be studied by analyzing the traction forces that occur for each stride in the various behaviors. Following on from similar analysis performed for the pentapedal gait controllers in prior work (Spenko et al., 2008), we provide additional results with the tetrapod and tripod gaits, further validating the effect of feedback upon these additional gait types. Fig. 8 shows force profile analysis of the four different controller arrangements, in which aggregate measurements of force data are plotted based upon the stride for each leg. Average values, quartile distances, and upper and lower bounds are indicated for the locomotion, with noticeable improvement, as seen in tighter variances around the nominal values, shown for the feedback behaviors.

4.2 Gait Transitions via Gait Regulation Control

Having demonstrated the contrasting utility of the three gait types, as well as the central importance of leg specific feedback for stabilizing climbs, we now demonstrate, in turn, the crucial role played by feedback regulation of the gait generators proposed in Sec. 3. We focus on two distinct types of transition behaviors afforded by these gait regulation controllers. First, we demonstrate the value of non-volitional, automatic “recovery” transitions enabled by the multistability property of the completion repulsion controller (depicted in Fig. 4a). Our ability to (essentially) cover the gait space with multiple basins corresponding to every possible crawl allows the robot to transparently switch gaits when locomotive perturbations require. Second, equipped with the many different regulation controllers arising from the various switching policies explored in Sec. 3.4 (afforded by appropriately deactivating specific terms of the vector c of (8)), we demonstrate the ability to volitionally switch gaits while maintaining

Figure 8: Traction force profiles for tetrapod and tripod controller arrangements. Each profile begins and ends with recirculation. Thick center lines are force averages throughout stance, while surrounding lines indicate quartile values. Upper and lower bounds are marked every 0.1 phase, excluding statistical outliers. Each profile is computed from six meters of climbing. Similar to previous analysis presented for pentapod gait climbing in (Spenko et al., 2008), the feedback behaviors have smoother and tighter appearances, as compared to the wide variances and jagged effects of the feedforward gaits.

Figure 9: Distribution of the robot’s usage of different gait basins throughout sustained climbing runs. As the robot locomotes faster and incurs greater slippage, it switches basins at a higher rate, with a more even gait usage distribution. An entropy computation summarizing the basin usage frequency in the two cases indicates nearly twice the “spread” of different crawls required to complete the more challenging speed run than required for the safer endurance run.

feedback regulation, allowing the operator to freely exercise the speed-reliability trade-off demonstrated, while continuing to locomote up a surface.

4.2.1 Multistability: The Value of Multiple Basins of Attraction

As detailed in Section 3, our gait regulation controllers induce multistability, with multiple basins of attraction each associated with stable limit cycles, such as the 120 possible limit cycles of the pentapod controller. These basins serve as an automated defense against perturbations during locomotion, allowing the robot to transparently self-transition. We first study the effects of perturbation-induced gait switching in this context. By comparing two different climbing runs, both collected while climbing a multistory building at the Southwest Research Institute¹⁵ (SwRI), we assess the value of multistability by measuring the effect of climbing difficulty upon frequency of behavior perturbation and gait recovery.

Two separate experiments were conducted, both using the pentapedal gait regulation system. In the first, the *endurance run*, the robot climbed 9.35 meters in 17:22, averaging $0.90 \frac{cm}{s}$. Another climb, the *speed run*, climbed at a faster rate, covering 11.05 meters in 15:01, $1.23 \frac{cm}{s}$, using the same behavior but with the gaits driving frequency increased by 50%.

Fig. 9 shows the distribution of the robot’s usage of the 120 different basins of attraction of the pentapedal gait controller, in terms of percentage of climbing time, for the two different experiments. While the second behavior was commanded to climb 50% faster, in actuality an increase of only 37% was observed, indicating greater occurrences of slipping during climbing. While the two histograms may initially appear to be similarly disordered, the entropy of the endurance run, 5.46 bits, is significantly less than that of the speed run, 6.44 bits, indicating a much more widely distributed selection among the 120 variant pentapedal gaits in the latter case. Viewed alternatively, feedback perturbations during the endurance run were significant enough to cause a switch in basins (i.e. a change in crawl gait from one to another cyclic ordering of legs) every 0.99 strides on average. In contrast, during the speed run, feedback perturbations induced a basin switch more than twice as often, every 0.47 strides. We conclude that the availability of a multitude of basins, their union (essentially) covering the space of gaits, allows the robot to switch between acceptable limit cycles transparently, and is ever more valuable in the face of increasingly challenging climbing settings.

4.2.2 Purposeful Switching between Gait Regulation Controllers

Next, the ability to transition between different gaits affords the operator a great variety of choices across a broad range of locomotion properties and speed/stability tradeoffs. In contrast to the perturbation-induced switching across a fixed set of multiple basins studied above, we now allow the operator to purposefully switch between slower, more reliable (higher stance leg count) and faster, riskier (lower stance leg count) gaits at will during active climbing. While we formerly reported on feedforward methods that allow gait switching through the use of pre-specified, open-loop gait transition paths on the torus, such as in (Haynes and Rizzi, 2006b), that approach results in an ad hoc approach limited to transits between specific, a priori chosen gait pairs that ignores the underlying algebraic structure of a combinatorially exploding cell structure in a high dimensional search space. Worse, by using only open-loop gait transitions, the incorporation of feedback control laws, such as our force balancing controller, is precluded. In comparison, the present method of gait transitioning is to simply swap gait regulation controllers,

¹⁵Southwest Research Institute, San Antonio, TX, <http://www.swri.org>

Figure 10: Example of RiSE switching gait regulation controllers while climbing a stucco surface. At $t = 47.5$ sec, the robot switches from pentapedal gait regulation to tripod, taking approximately 5 sec to exhibit tripod-like locomotion. The top plot shows individual leg phase offsets, solid lines, with current desired phase offsets (dashed lines). When the lines are within the lightly shaded green regions, task controllers modify the leg phases to perform force control (with an example of pawing occurring on leg 3 around $t = 42.5$ sec, marked by a sharp discontinuity in phase as the robot reattempts attachment). In unshaded regions, legs recirculate and undergo gait regulation. The 2nd and 3rd plots show commanded duty factor and geodesic distance to the desired gait’s phase offsets, while the bottom plot shows the patterns of stance (shaded) and recirculation for all six legs. An additional control law, described in (Haynes, 2008), lowers gait duty factor and increases stride frequency as the system approaches the alternating tripod gait, thus affecting the shape and length of stance regions.

allowing the natural dynamics of the controller to find a flow path between gaits. This amounts roughly to an instantiation of the (Burridge et al., 1999) sequential composition method, however without the precise need for a “prepares graph” style of guard conditions, as our multistable basins provide essentially global coverage of the entire gait space. Given potentially distant starting conditions, our use of gait regulation controllers that tend to avoid obstacles in phase space—as described in Sec. 3.1 and evidenced using numerical simulations in Sec. 3.4—results in controllers well-suited to the task of gait switching.

We now report on a series of climbing experiments that test the efficacy and quality of this gait transition planner/controller. Five experiments were conducted for each of the six possible transitions between the three gait types described, a total of 30 experiments. Individual examples of transitions are shown in Figs. 10-12. Because perturbations from the environment are continually injected into these hybrid closed loop controllers through the various feedback terms (discussed in Sec. 4.1) the planner’s designated “destination” gait is never exactly achieved in any of these physical trials, and a rougher measure of convergence to a neighborhood of the designated steady state (attracting limit cycle) is required. Thus, we measure success of each gait switch by the robot reaching a certain distance (determined, as usual in dynamical closed loop settings, roughly by the ratio of disturbance magnitude to feedback gain magnitude) from the desired gait, corresponding to the “mean” radius, measured in the geodesic distance, of the attracting neighborhood for that gait, empirically determined by prior observation of repeated steady state climbing on this particular climbing wall with these particular feedback gain settings. Average times and strides for transitions are shown in Tables 7 and 8.

In 30 tests, the robot never failed to execute a successful transition. The longest transitions, in terms of time and strides to completion, but most likely indicative of total distance traveled on the torus, occur from pentapod gaits to other gaits. Switching in reverse, however, took very little time for the system to settle at a nearby pentapod gait. This asymmetry is to be expected. With 120 possible starting locations, the mean distance from a given pentapod to a single gait such as the alternating tripod may be rather large in terms of geodesic distance (a lower bound on the distance each gait regulation controller must travel). In reverse, the system needs only to converge to the nearest pentapod gait of a controller whose resulting 120 multiple, distinct stable basins cover the gait space with rather fine granularity, thus the minimum geodesic (of 120 possible) will be a much lower distance than the mean. This property is reflected in our experimental results in Tables 7 and 8.

Transitions between gaits with differing numbers of stance legs introduce variations in climbing speed, one major motivating factor for transitions. A principal reason for such differences is due to the possible range of duty factors of each gait. Discussed in depth in (Haynes, 2008), duty factors correspond to the portion of phase spent in stance versus flight, with lower duty factors corresponding to faster locomotion. As such, the switching controller described commands both phase modifications as well as

Figure 11: Example of RiSE switching gait regulation controllers while climbing a stucco surface. Switching from a tetrapod gait to a pentapod gait, the system easily transitions to a nearby pentapedal crawl gait, by repelling all recirculating legs away from one another. The new gait is near to the initial tetrapod gait, as noted by the fact that no legs cross one another in phase during recirculation.

Figure 12: Example of RiSE switching gait regulation controllers while climbing a stucco surface. The robot executes a switch of gait regulation strategy from a tripod gait back to the pentapedal crawl gait. The system once again settles to a nearby crawl. While legs may cross ordering during stance, due to the actions of the force balancing controller, legs in flight repel one another and attempt to maintain a pentapod gait after the transition occurs.

Table 7: Average switching time (and *standard deviation*), in seconds, between various gait types

		Desired Gait		
		Pentapod	Tetrapod	Tripod
Previous Gait	Pentapod		9.2 (6.0)	9.4 (4.1)
	Tetrapod	3.1 (0.63)		3.9 (2.7)
	Tripod	2.7 (2.1)	3.6 (2.8)	

Table 8: Average switching time (and *standard deviation*), in terms of gait strides, per gait type

Previous Gait		Desired Gait		
		Pentapod	Tetrapod	Tripod
Previous Gait	Pentapod		1.1 (0.75)	0.95 (0.41)
	Tetrapod	0.37 (0.075)		0.53 (0.34)
	Tripod	0.26 (0.20)	0.50 (0.35)	

changes to the duty factor of a gait as the system moves to the desired gait, the effects of which are seen in Figs. 10-12.

5 Conclusions and Future Work

We introduce methods of gait regulation that, when appropriately paired with previously reported proprioceptive behavioral feedback, are critical to the success of an underactuated climbing robot. We detail the importance of certain convergence and transient properties, in particular the structure of a specific family of repellors on a high-dimensional torus. We introduce a cellular decomposition of the torus organizing a family of hybrid controllers that guarantee convergence to a systematically indexed selection of multistable gaits through transients that tend to avoid dangerous simultaneous leg recirculations. We show empirically how pairing this style of gait regulation with proprioceptive feedback allows a robot to: (i) better sustain desired gaits during challenging locomotion tasks, thereby increasing the run-time performance compared to open-loop climbing; and (ii) transition amongst the selection of gait timings to exercise trade-offs between speed and stability, adding significant functionality to the task of robotic climbing.

A number of theoretical and practical questions arise immediately from the ideas and results of this paper. In the former vein, we are presently engaged in more formal characterization of basins arising from higher codimension cycles, and believe that recent topological insights (Bahri et al., 2009) may provide supplementary control methods that guarantee avoidance of our combinatorial obstacles as well as convergence. In more practical directions, we are convinced that certain common sense exteroceptive and proprioceptive controller extensions, along the lines of (Johnson et al., 2011), could significantly increase robot autonomy, by adding capabilities such as steering around obvious blockages, adjusting overall gait speed, and, of particular note, selecting the use of the various gaits and gait transitions automatically to exploit the tradeoffs between speed and safety on complex terrains.

A different, equally promising direction of future research arises from the introduction of *preference* amongst the various gaits and leg pairs of a robot. Preference is already hinted at when designing gait regulation controllers, as we only consider certain tetrapod and tripod gaits, but could be expanded by considering, in full, the specific roles various legs play during locomotion, for instance by a legged robot climbing over obstacles or transitioning from level ground to climbing. As such, while we currently provide equal likelihood of convergence to the 120 different pentapedal gaits, there are likely individual gaits with greater utility, such as the wave gait that is greatly preferred by biological systems. Providing means of specifying individual desired gaits amongst families that we currently consider equally represents future work in progress. Ideas toward implementing this style of preference have already been implemented in follow-up work to this paper (Haynes et al., 2009), but continue to be expanded with formal studies. Similarly, formal studies in the specific topology of the gait-space obstacles will provide additional insight into control methods. While our proposed approach tends to avoid these dangerous gait timings better than prior methods, we lack a precise understanding of the specific geometry and topology of this obstacle given task space constraints on locomotion.

At a broader level, this paper pursues the paradigm of layering an abstract, adaptive dynamic system over a projection of the parameter space. In this case, this is the gait regulation control (acting upon the space of phase offsets, $\mathbf{r}$, to represent the “physical layer” of a robot climbing a wall. In the end, furthermore, these controllers are reduced to physically grounded symbolic representations (such as “crawl” or “trot”) that are used to control the underlying signal spaces. We expect an extension of this symbol-from-signal abstraction applied to larger projections of the physical layer control parameter space will be even more useful when attempting to provide physically grounded symbolic commands to “higher level” locomotion drivers. For example, future work could further consider constraints upon the duty factor parameters, or even the tangent space of the phase offset parameters—relationships amongst leg phase velocities in a legged system—when designing controllers for dynamic gaits. We expect this to be a rewarding study as we move to dynamic legged robots, for which open-loop gaits have been extremely useful, but challenging to (consequently still rare, e.g. (Weingarten et al., 2004a)) efforts at incorporating feedback strategies.

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