2001

# Reply to Comment by S. J. Cox and D. Weaire on "Free Drainage of Aqueous Foams: Container Shape Effects on Capillarity and Vertical Gradients"

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## Recommended Citation

Saint-Jalmes, A., & Durian, D. J. (2001). Reply to Comment by S. J. Cox and D. Weaire on "Free Drainage of Aqueous Foams: Container Shape Effects on Capillarity and Vertical Gradients". *Europhysics Letters, 55* (3), 447-448. http://dx.doi.org/10.1209/epl/i2001-00435-1

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# Reply to Comment by S. J. Cox and D. Weaire on "Free Drainage of Aqueous Foams: Container Shape Effects on Capillarity and Vertical Gradients"


## Abstract

Cox and Weaire [1] rightly emphasize that our solution of the drainage equation for the “Eiffel Tower” geometry does not treat the boundary conditions. There should be a no- flow condition at the top, and, after leakage begins, the liquid fraction should be pegged to $\varepsilon_c \approx 0.36$ at the bottom. They then show how approximating the no-flow conditions at the top can improve agreement with numerical solution. But as argued in [2], we maintain that the neglect of capillarity coming from boundary conditions at the bottom dominates, and that this cannot explain our measurements. At short times, capillarity can delay the onset of leakage, and at long times it can counter gravity and retain liquid in the foam indefinitely; in either case, leakage is *slower* than our approximate solution, contrary to experiment. Therefore, we speculated that the discrepancy arose from neglect of coarsening, whereby the average bubble size increases via gas diffusion from smaller to larger bubbles. This is an important puzzle because, while the drainage equation successfully predicts forced-drainage experiments, it fails dramatically for free-drainage experiments



## Disciplines

Physical Sciences and Mathematics | Physics

## Comments

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EUROPHYSICS LETTERS 1 August 2001

*Europhys. Lett.*, **55** (3), pp. 447–448 (2001)

*Reply*

# Reply to the Comment by S. J. Cox and D. Weaire on "Free drainage of aqueous foams: Container shape effects on capillarity and vertical gradients"

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(received 14 February 2001; accepted 25 May 2001)

PACS. 82.70.Rr – Aerosols and foams.
PACS. 47.60.+i – Flows in ducts, channels, nozzles, and conduits.
PACS. 47.55.Mh – Flows through porous media.

Cox and Weaire [1] rightly emphasize that our solution of the drainage equation for the "Eiffel Tower" geometry does not treat the boundary conditions. There should be a no-flow condition at the top, and, after leakage begins, the liquid fraction should be pegged to $\varepsilon_c \approx 0.36$ at the bottom. They then show how approximating the no-flow conditions at the top can improve agreement with numerical solution. But as argued in [2], we maintain that the neglect of capillarity coming from boundary conditions at the bottom dominates, and that this cannot explain our measurements. At short times, capillarity can delay the onset of leakage, and at long times it can counter gravity and retain liquid in the foam indefinitely; in either case, leakage is *slower* than our approximate solution, contrary to experiment. Therefore, we speculated that the discrepancy arose from neglect of coarsening, whereby the average bubble size increases via gas diffusion from smaller to larger bubbles. This is an important puzzle because, while the drainage equation successfully predicts forced-drainage experiments, it fails dramatically for free-drainage experiments [2].

While Cox and Weaire [1] consider conditions at the top of the sample, we clarify here the role of boundary conditions at both top and bottom. Numerical results from the drainage equation are shown in figs. 1(a)-(b) for an initial liquid fraction of $\varepsilon_0 = 0.36$, and in fig. 1(c) for several different $\varepsilon_0$. In all cases the sample height is $H = 70$ cm, the flaring length is $z_0 = 25$ cm, the capillary rise scale is $\xi = 5$ cm, the characteristic flow speed is $u_0 = 0.026$ cm/s, and the dissipation exponent is $m = 1$. We predicted [2] that the liquid fraction profile is $\varepsilon(z,t) = \varepsilon_0/(1 + t/t_0)$, and that the normalized volume of drained liquid is $V(t)/V_f = 1/(1 + t_0/t)$, where $t_0 = z_0/(\varepsilon_0 u_0)$, *i.e.* that drainage is uniform and all the liquid eventually leaks out. By contrast the full numerical solution for $\varepsilon(z,t)$, in fig. 1(a), shows that the sample becomes drier at the top and remains wet at the bottom. Consequently, in fig. 1(b), $V(t)/V_f$ does not approach 1 at long times. Also in figs. 1(a)-(b), we see that the liquid fraction at the top, $\varepsilon(0,t)$, is not zero as assumed in [1].

To see the role of boundary conditions, we disable them one at a time by taking $\partial\varepsilon/\partial z = 0$. When only the bottom conditions are correct, we find the dotted curve of fig. 1(b), in which the volume of drained liquid is correct at short and long times. The final approach is too

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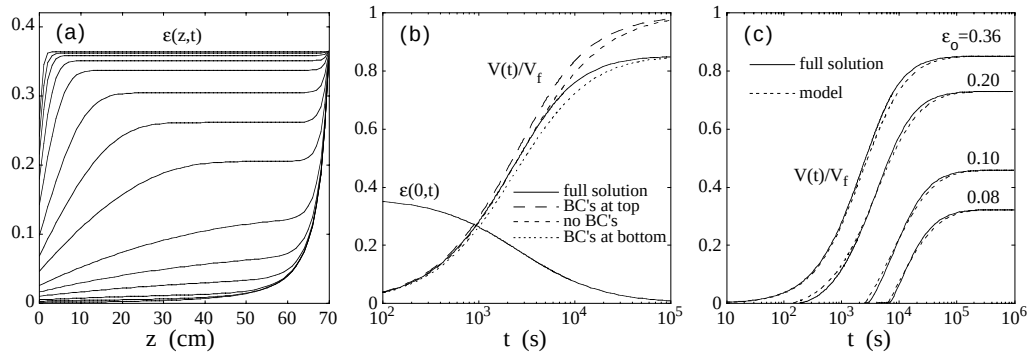


Fig. 1 – (a) Liquid fraction *vs.* depth; from top to bottom the times shown follow a 1-2-5-10 sequence from 10 s to $10^5$ s. The bottom-most curve is the equilibrium profile. (b) Volume of drained liquid *vs.* time for various combinations of boundary conditions, and (c) for different initial liquid fractions.

slow because liquid enters via "flow" conditions at the top. When only the top conditions are correct, we find the long-dashed curve of fig. 1(b), in which the volume of drained liquid is correct only at short times. For later times the drainage is too fast, and all the liquid eventually leaves the sample, because capillary forces do not act at the bottom boundary. When neither boundary condition is correct, we recover our original approximation, the short-dashed curve of fig. 1(b). The errors introduced at top and bottom act in opposite directions, extending the agreement with the full solution to later times. However, the error due to neglect of the bottom boundary conditions eventually dominates and all the liquid leaves the sample.

From fig. 1(b) we conclude that the bottom boundary conditions are most crucial. This is emphasized in fig. 1(c), where numerical solutions show that, as the initial liquid fraction deceases, the onset of leakage grows later and a smaller fraction of liquid eventually leaves. Inspired by fig. 1(a), we model the liquid fraction profile by our original approximation except near the bottom, where it is taken according to the equilibrium profile $\varepsilon_c/[1 + (H - z)/2\xi]^2$. These two forms intersect at a point that travels upwards until equilibrium is reached. The resulting volume of drained liquid, shown in fig. 1(c) by the dashed curves, agrees very well with the crucial features of the full numerical solution. This gives significant improvement over not only our original approximation, but over that of [1] as well. Note that our modeling approach cannot be used in a rectangular column, where the liquid fraction in the central portion remains constant until altered by boundary effects that propagate inwards. This shows the advantage of the "Eiffel Tower" geometry introduced in [2].

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This work was supported by NASA through the microgravity fluid physics program.

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