2-1985

A Comment on Sigma Model Anomalies

Aneesh Manohart

Gregory Moore

Philip C. Nelson
University of Pennsylvania, nelson@physics.upenn.edu

Recommended Citation

Manohart, A., Moore, G., & Nelson, P. C. (1985). A Comment on Sigma Model Anomalies. *Physics Letters B, 152* (1-2), 68-74. http://dx.doi.org/10.1016/0370-2693(85)91141-4

At the time of publication, author Philip C. Nelson was affiliated with Harvard University. Currently, he is a faculty member in the Physics & Astronomy Department at the University of Pennsylvania.

Abstract

Some non-linear sigma models with fermions are known to be ill-defined because of a global obstruction to any consistent quantization. Sigma models relevant to phenomenological theories of dynamical symmetry breaking must satisfy the additional constraint of appropriately realizing the flavor symmetries of the underlying theory at the one-loop level. This is possible if and only if 't Hooft's anomaly condition is satisfied. In particular, we show that there always exists a Wess-Zumino term which correctly reproduces the flavor anomalies, and the global obstruction vanishes, whenever 't Hooft's condition is satisfied.

HUTP-84/A088

A Comment on Sigma Model Anomalies*

Aneesh Manohar†, Gregory Moore‡, and Philip Nelson†

Lyman Laboratory of Physics
Harvard University
Cambridge, MA 02138

Some non-linear sigma models with fermions are known to be ill-defined because of a global obstruction to any consistent quantization. Sigma models relevant to phenomenological theories of dynamical symmetry breaking must satisfy the additional constraint of appropriately realizing the flavor symmetries of the underlying theory at the one-loop level. This is possible if and only if 't Hooft's anomaly condition is satisfied. In particular, we show that there always exists a Wess-Zumino term which correctly reproduces the flavor anomalies, and the global obstruction vanishes, whenever 't Hooft's condition is satisfied.

12/84

* Work supported in part by the National Science Foundation under grant No. PHY-82-15249
† Harvard Society of Fellows
‡ NSF Graduate Fellow

1. Introduction and Summary

Sigma models with non-linear boson fields coupling to massless fermions arise in a number of contexts, for example in string theories [1], supergravity, and in the low-energy approximations to strongly interacting chiral gauge theories. Recently, it has become clear that classical sigma models of this type cannot always be quantised [2][3][4][5]: in certain cases, a global obstruction prevents quantisation of the Fermi fields in a consistent way for all background boson field configurations. Loosely speaking, there is no appropriate choice of basis for the space of fermion paths in the functional integral [3].

The above obstruction arises when we place the mildest restriction possible on an "appropriate" choice of basis, namely that it should give rise to a Fermi effective action $\Gamma_f[\phi]$ which is well-defined for every boson configuration $\phi(x)$. In the case of sigma models which actually arise as low-energy reductions, however, we can and should ask more. In this case some underlying linear theory with a global chiral symmetry under a group G breaks part of G down to H. Any phenomenological theory describing the consequences of this breaking must reproduce the (possibly anomalous) Ward identities of the original linear theory. In the language of the previous paragraph, we must quantise the putative low-energy theory using a choice of bases which is not only well-defined, but is also covariant under the action of G. We can rephrase this demand in more familiar language if we note that [3] changing fermion bases changes the fermion effective action by the addition of a functional of $\phi(x)$ of the Wess-Zumino type [6]. Thus we can start with an arbitrary quantisation of a sigma model which is known to make sense, say the one used in standard perturbation theory, and demand that there exist a *local counterterm* of the Wess-Zumino type which modifies the Ward identities to the form required by the underlying theory. It is not obvious that such a counterterm always exists.

In this letter we establish a condition for the existence of such a counterterm. Since we now require predictable behavior under G transformations, the behavior of the theory on all of G/H should come from its behavior near any point; hence we expect a purely local condition which includes the previous global one. This will indeed be the case, just as in gauge theory.

Recall [7] that a phenomenological theory with matter is specified by a choice of the underlying symmetry group G (a compact Lie group), an unbroken subgroup H imbedded in G, and a representation ρ_H of H for the matter fields.[1] In our case, ρ_H describes the fermions kept massless by unbroken chiral symmetries.[2] If there are no massless composite fermions, as in the reduction of QCD, all the anomalies in G must be reproduced by Goldstone boson poles in the Green functions, and in particular the underlying fermions must be anomaly free under H, since the unbroken currents cannot produce single Goldstone bosons out of the vacuum. More generally, there will be surviving low-energy fermions also contributing to the G-anomalies. Then we no longer demand that H must be anomaly free; instead, the H anomalies of ρ_H must match those of $\rho_G|_H$ [8]. Here ρ_G, a representation of G, is the fermion content of the underlying theory, and $\rho_G|_H$ is its restriction to H. Our result is that the necessary matching of H anomalies is also sufficient, both for a proper realization of the G symmetries and for the vanishing of the global obstruction.

There is an easy heuristic argument for this result. To define the effective theory locally, we can use the prescription of [7] for the fermion effective action $\Gamma_f(\rho_H)$. We can conveniently describe the anomalous Ward identities for $\Gamma_f(\rho_H)$ by coupling the theory to external flavor gauge fields A for G and discussing the anomalous variation of Γ_f under *local* G-transformations. Now consider adding the H-representation $\rho_G|_H$, and its complex conjugate. This does not change the anomalies, so we have

$$\Gamma_f(\rho_H) \doteq \Gamma_f(\rho_H + \bar{\rho}_G|_H) + \Gamma_f(\rho_G|_H) \ ,$$

where the symbol $\doteq$ means "has the same anomalous variation under G". But $\rho_H + \bar{\rho}_G|_H$ has no H-anomaly by hypothesis so the first term on the right hand side has no G-variation (see next section). Also, the fermions in the second term on the right hand side correspond to those in the effective action of the underlying linear theory, $\Gamma_f^{linear}(\rho_G)$. Before we can conclude that $\Gamma_f(\rho_H) \doteq \Gamma_f^{linear}(\rho_G)$, however, we must investigate the transformation from the Fermi fields ψ in the Callan-Coleman-Wess-Zumino (CCWZ) basis to fields Ψ transforming linearly under G. Following [7] this is

$$\rho_G(s(\phi(x)))\,\psi(x) = \Psi(x) \ , \tag{1.1}$$

where s is a map from G/H to G used in defining Γ_f. Since the fields Ψ transform in the same way as the underlying fermions the flavor anomalies are the same. As shown in [9] in the case of the chiral quark model, the change of variables(1.1) contributes an anomalous Jacobian factor to Γ_f. In that case the

---

[1] This representation can be reducible.

[2] All fermions will be considered to be left-handed.

change in phase could be compensated by the addition of a local counterterm, however, and so we expect

$$\Gamma_f(\rho_H) + F \doteq \Gamma_f^{linear}(\rho_G) \ . \tag{1.2}$$

Hence the addition of a local counterterm F in each patch should be sufficient to realise the G symmetry properly.

In general, the map s above will be defined only on a neighborhood $\mathcal{U} \subseteq G/H$, so we need to use a collection of maps $\{s_\alpha\}$ on patches $\{\mathcal{U}_\alpha\}$ covering G/H. These define a set of Γ_f^α, and by equation (1.1) a corresponding set of F^α. Since the right hand side of equation (1.2) is independent of s_α, we have that $e^{-(\Gamma_f^\alpha + F^\alpha)}$ regarded as a section of a line bundle has trivial transition functions. Thus the global anomaly of [2] is absent.

In the sequel we give a derivation which bears out the above expectations. We simply compute the local counterterm $F^\alpha[\phi]$ needed to fix Γ_f^α. We show how the 't Hooft conditions are enough to guarantee the existence of F^α and the vanishing of the global obstruction.

2. Classical and Quantum Sigma Models

For convenience we always work in four Euclidean spacetime dimensions. Consider the projection $\pi : G \to G/H$, where g maps to the coset gH.[3] This defines a principal H-bundle. To couple matter fields to the Goldstone fields $\phi : S^4 \to G/H$ we first cover G/H by a collection of contractible patches $\{\mathcal{U}_\alpha\}$. We will always consider maps which take all of spacetime into some patch $\mathcal{U}_\alpha$.

Since the patches $\mathcal{U}_\alpha$ are contractible we can choose sections s_α. These are the maps considered above; they must satisfy $s_\alpha(p)\, H = p$. For example, in [7] the section near the identity is $s_0(p) = e^{\xi \cdot X}$ where ξ are the normal coordinates of p and X^r are the "broken generators," i.e. generators orthogonal in the Cartan metric to the generators T^a of H. Define the function $h_\alpha(p; g_0)$ by

$$g_0 s_\alpha(p) = s_\alpha(g_0 \cdot p) h_\alpha(p; g_0) \tag{2.1}$$

where $h_\alpha(p; g_0) \in H$. For the choice in [7] we thus have $h_0(p; g_0) = e^{u' \cdot T}$ where $u'(p; g_0)$ is defined by $g_0 e^{\xi \cdot X} = e^{\xi' \cdot X} e^{u' \cdot T}$. Examining the action of G defined in [7] now shows that the matter fields live on the vector bundle B associated

[3] We will also denote points of G/H by p.

to $G \to G/H$ by the representation ρ_H of H. That is, taking account of its spinor properties, $\psi(x)$ is a section of $S^+ \otimes \phi^* B$, just as in [3].

To define fermion dynamics we first choose coordinates for $B|_{\mathcal{U}_\alpha}$, so that $\psi(x)$ is a vector in the representation space of ρ_H. If we change sections to $s_\beta(p) = s_\alpha(p) h_{\alpha\beta}(p)$, then $\psi(x)$ is rotated by $\rho_H(h_{\alpha\beta}(\phi(x)))$. Next, we note that there is a natural connection on B given on $\mathcal{U}_\alpha$ by $\Theta_\alpha = \rho_H(s_\alpha^{-1} ds_\alpha|_T)$. The notation $|_{T^a}$, $|_{X^r}$, etc. means orthogonal projection of an element of the Lie algebra $\mathcal{L}(G)$ of G onto its T^a, X^r etc. component. The derivative $\not{D}_\Theta \psi$ built from this transforms covariantly under changes of s_α and is the one defined in [7], so we can take the classical action to be $\mathcal{L}_f = \overline{\psi} \not{D}_\Theta \psi$. More generally, when we also want to couple gauge fields A, which we consider as one-forms with values in $\mathcal{L}(G)$. Let

$$Q_0^\alpha[A, \phi] = \rho_H(A^{s_\alpha^{-1}(\phi)}|_T) \ , \tag{2.2}$$

where $A^g = gAg^{-1} + gd(g^{-1})$. Q_0 is a connection on the pullback $\phi^* B$, so we can take

$$\mathcal{L}_f = \overline{\psi} \not{D}_{Q_0} \psi .$$

By (2.1) the G-transformation

$$\begin{aligned} A &\to A^\ell \ , \\ \phi &\to \ell \cdot \phi \ , \end{aligned} \tag{2.3}$$

induces the transformation

$$Q_0^\alpha \to (Q_0^\alpha)^{\rho_H[h_\alpha(\phi;\ell)]} \ . \tag{2.4}$$

In certain cases we can add to Q_0 extra terms. This requires only that the representation $\overline{\rho_H} \otimes \rho_H$ of $\overline{\psi}\psi$ contain in its Clebsch-Gordan decomposition a copy of the representation $D^{(br)}$ defined by $[T^a, X^r] = D_{sr}^{(br)}(T^a) X^s$, that is, the representation of H formed by the Goldstone bosons. Let C_r be a set of Clebsch-Gordan matrices in the representation space of ρ_H which extract $D^{(br)}$, and let

$$Q^\alpha = Q_0^\alpha + g_A C_r (A^{s_\alpha^{-1}(\phi)}|_{X^r}) \tag{2.5}$$

with a sum on r. For example, in the chiral quark model g_A is the axial-vector coupling constant [9][10]. The full Q^α again transforms properly to be a connection on B.

We can now show that the usual Feynman diagram expansion for $\Gamma_f[\phi, A]$ (fig.1) has the wrong anomalous variation under flavor gauge transformations. In particular, by (2.4) the transformation in (2.3) changes the effective action by the integrated anomaly [6]. Let $\ell_t(x) = L(x,t)$ be a sequence of gauge transformations with $L(x,0) \equiv 1 \in G$ and $L(x,1) = \ell(x)$.[4] This induces a sequence of H-transformations

$$K(x,t) = h[\phi(x); L(x,t)] \tag{2.6}$$

and we find

$$\Gamma_f[\ell\cdot\phi; A^\ell] - \Gamma_f[\phi; A] = \int_0^1 dt \int_{S^4} \omega_4^1[\rho_H(\dot K K^{-1}); Q^{\rho_H(K)}] \tag{2.7}$$

where $\dot K = \frac{\partial}{\partial t} K$ and ω_4^1, the usual anomalous variation, is given by

$$\omega_4^1[v, B] = \frac{1}{24\pi^2} tr\, v((dB)^2 + \frac{1}{2} dB^3)$$

for matrix-valued one-forms B and v in the matrix representation of a Lie algebra[11].

The variation (2.7) is an inappropriate realisation of the (possibly anomalous) G-symmetry of the underlying theory, because the anomalous variation in (2.7) involves the Q fields instead of the A fields. The appropriate realisation demands that the right hand side of (2.7) be replaced by

$$I[\rho_G(A); \rho_G(\ell)] = \int_0^1 dt \int_{S^4} \omega_4^1[\rho_G(\dot L L^{-1}); \rho_G(A^L)] \mod 2\pi i n\,. \tag{2.8}$$

The notation in (2.8) for the integrated anomaly I from A to A^ℓ in the representation ρ_G suggests that I is independent of the path $L(x,t)$ chosen. In fact, by standard arguments one can show that I is a *local* functional of the gauge field A and the map $\ell(x)$ given by

$$\frac{1}{240\pi^2} \int_{I\times S^4} tr\,(\rho_G(L^{-1} dL)^5) + \int_{S^4} \alpha[\rho_G(\ell^{-1} d\ell); \rho_G(A)] \quad .$$

A formula for α can be found in [11]. The integral of the Maurer-Cartan form is a local functional of $\ell(x)$, up to an ambiguity of $2\pi i\times$integer.

---

[4] We have assumed $\pi_4(G) = 0$ which is true for any simple factor of G which has anomalous representations.

3. Local counterterms

We now construct a local counterterm in the fields ϕ and A, $F[\phi, A]$ such that $\Gamma_f[\phi, A] + F[\phi, A]$ has the appropriate anomalous variation given in (2.8). Such a counterterm must satisfy

$$F[\ell\cdot\phi, A^\ell] - F[\phi, A] = I[\rho_G(A); \rho_G(\ell)] - I[Q; \rho_H(k)] \quad , \tag{3.1}$$

where $k(x) \equiv K(x,1) = h(\phi(x); \ell(x))$ is in H. In particular, if we take $\ell(x) = s(\phi(x))^{-1}$ then, observing that

$$h(\phi(x); s(\phi(x))^{-1}) = 1$$

and $s(\phi(x))^{-1}\,\phi(x) = 1 \cdot H \equiv \phi_0(x)$ we find, up to $2\pi i\times$integer, that F must be given by

$$F[\phi; A] = F[\phi_0; A^{s(\phi)^{-1}}] - I[\rho_G(A); \rho_G(s(\phi)^{-1})] \tag{3.2}$$

We must now study the behavior of each of the terms in (3.2) under G transformations. We begin with the integrated anomaly, which is now a local functional of ϕ and A. By considering the composition of paths from 1 to g_1 and from g_1 to $g_2 g_1$ one establishes the composition law

$$I[A; g_2 g_1] = I[A^{g_1}; g_2] + I[A; g_1] \tag{3.3}$$

which holds in any representation of the gauge group. Eq (3.3) states that the Wess-Zumino functional is a 1-cocycle in the sense of [12]. By eq. (2.1), $s(\ell\cdot\phi) = \ell\cdot s(\phi) k^{-1}$, and two applications of eq. (3.3) yield

$$\begin{aligned} &I[\rho_G(A^\ell); \rho_G(s(\ell\cdot\phi)^{-1})] - I[\rho_G(A); \rho_G(s(\phi)^{-1})] \\ &\qquad = I[\rho_G(A^{s(\phi)^{-1}}); \rho_G(k)] - I[\rho_G(A); \rho_G(\ell)] \end{aligned} \tag{3.4}$$

Now consider the behavior of the other term in (3.2). Under G-transformations we have $A^{s(\phi)^{-1}} \to (A^{s(\phi)^{-1}})^k$. Using (3.2) and (3.4) we find that (3.1) can be satisfied if and only if

$$F[\phi_0; (A^{s(\phi)^{-1}})^k] - F[\phi_0; A^{s(\phi)^{-1}}] = I\left[\rho_G(A^{s(\phi)^{-1}}); \rho_G(k)\right] - I[Q; \rho_H(k)] \tag{3.5}$$

Eq. (3.5) has reduced the problem of cancelling anomalous G-variation in a gauged sigma model to cancelling anomalous H-*variation* in a *pure* gauge theory with gauge fields in $\mathcal{L}(G)$. To see this more clearly, let B be an arbitrary $\mathcal{L}(G)$-valued gauge field and define $Z(B) \equiv F(\phi_0; B)$. Z will be a local functional if and only if F is. Eq. (3.5) can be rewritten as

$$Z(B^h) - Z(B) = I\left[\rho_G(B); \rho_G(h)\right] - I\left[\rho_H(B|_T) + g_A C \cdot (B|_X); \rho_H(h)\right] \quad (3.6)$$

Here we have replaced k by an arbitrary H gauge transformation h, and used the definition of Q from equations (2.2) and (2.5).

Eq. (3.6) can be considerably simplified by introducing the generalisation of Bardeen's counterterm [13]. It is shown in [13] that if A_0, A_1 are two matrix-valued 1-forms with identical gauge transformation properties under δ_v, $v \in \mathcal{L}(H)$:

$$\delta_v A_{0,1} = -(dv + [A_{0,1}, v])$$

then using Cartan's homotopy formula one can construct a local counterterm $R(A_1; A_0)$ such that

$$\delta_v \int R(A_1; A_0) = \int [\omega_4^1(v, A_1) - \omega_4^1(v, A_0)].$$

Explicitly, one has

$$R(A_1; A_0) = \frac{1}{48\pi^2} tr[(A_1^3 A_0 - A_0^3 A_1) + 2(A_0 A_1 F_0 - A_1 A_0 F_1) + (A_0^2 F_1 - A_1^2 F_0) - \frac{1}{2} A_1 A_0 A_1 A_0]$$

where $F_{0,1} = dA_{0,1} + A_{0,1}^2$.

Since $\rho_G(B)$, $\rho_G(B|_T)$ and $\rho_H(B|_T) + g_A C \cdot (B|_X)$, $\rho_H(B|_T)$ are pairs of matrix-valued forms transforming the same way under H-transformations we can define the functional

$$\tilde{Z}(B) = Z(B) + \int R[\rho_G(B|_T); \rho_G(B)] - \int R[\rho_H(B|_T); \rho_H(B|_T) + g_A C \cdot (B|_X)]$$

so that (3.6) will be satisfied if we can find a local functional $\tilde{Z}$ transforming as

$$\tilde{Z}(B^h) - \tilde{Z}(B) = I[\rho_G(B|_T); \rho_G(h)] - I[\rho_H(B|_T); \rho_H(h)] \quad . \qquad (3.7)$$

This is the anomalous variation for an H-gauge theory with gauge field $B|_T$ and fermion representation $\rho_G|_H \oplus \bar{\rho}_H$. It is well known that there is no local functional $\tilde{Z}$ of $B|_T$ which satisfies eq. (3.7) if the right-hand side is nonvanishing. (Otherwise there would be no anomaly in chiral gauge theories!) The vanishing of the rhs of eq. (3.7) is just 't Hooft's anomaly matching condition. Therefore the *local* counterterm F satisfying (3.1) exists if and only if the H anomalies of $\rho_G|_H$ match those of ρ_H. Explicitly, F is given by equation (4.1) below.

Had we formulated the sigma model in terms of maps to G and eliminated the extra degrees of freedom with unphysical H-gauge fields, we might have concluded that ρ_H must have no anomalies at all [4]. The moral of the present derivation is that we must demand locality only in the *physical* pion fields.

4. Global Obstructions

If 't Hooft's anomaly constraints are satisfied, we can construct the local counterterm

$$F^\alpha[\phi; A] = \int R\left[\rho_G\left(A^{s_\alpha^{-1}(\phi)}|_T\right); \rho_G\left(A^{s_\alpha^{-1}(\phi)}\right)\right] - \int R\left[\rho_H\left(A^{s_\alpha^{-1}(\phi)}|_T\right); Q\right] - I\left[\rho_G(A); \rho_G\left(s_\alpha^{-1}(\phi)\right)\right] \; . \qquad (4.1)$$

Note that F^α depends on the choice of section s_α. If ϕ maps spacetime into the intersection of two patches $\mathcal{U}_\alpha \cap \mathcal{U}_\beta$ then there will be a disagreement between the counterterms constructed from s_α and s_β. Thus, F might not be globally defined. However, Γ_f^α also depends on the choice of section, and as we now show, $\Gamma_f^\alpha(\phi, A) + F^\alpha[\phi, A]$ *is* section-independent. Thus if the local obstruction to the appropriate realization of G-symmetry vanishes, then there is no further global obstruction.

Therefore, consider a change of section by a right H transformation γ:

$$s(p) \to \tilde{s}(p) = s(p) \cdot \gamma(p) \; ,$$

where $\gamma : \mathcal{U} \to H$. The connection Q of the CCWZ prescription changes by

$$Q \to \tilde{Q} = (Q)^{\rho_H\left(\gamma^{-1}(\phi)\right)} \; ,$$

so that the effective action changes by

$$\tilde{\Gamma}_f[A; \phi] - \Gamma_f[A; \phi] = I\left[Q; \rho_H\left(\gamma^{-1}(\phi)\right)\right] \; . \qquad (4.2)$$

On the other hand, by equation (3.5),

$$
\begin{aligned}
F[\phi_0; A^{\tilde{s}^{-1}(\phi)}] - F[\phi_0; A^{s^{-1}(\phi)}] &= F[\phi_0; (A^{s^{-1}(\phi)})^{\gamma^{-1}(\phi)}] - F[\phi_0; A^{s^{-1}(\phi)}] \\
&= I\left[\rho_G\left(A^{s^{-1}(\phi)}\right); \rho_G\left(\gamma^{-1}(\phi)\right)\right] \\
&\quad - I\left[Q; \rho_H\left(\gamma^{-1}(\phi)\right)\right] \qquad (4.3)
\end{aligned}
$$

and the composition law (3.3) implies

$$
\begin{aligned}
I\left[\rho_G(A); \rho_G\left(\tilde{s}^{-1}(\phi)\right)\right] &= I\left[\rho_G(A); \rho_G\left(\gamma^{-1}(\phi)\right)\rho_G\left(s^{-1}(\phi)\right)\right] \\
&= I\left[\rho_G(A); \rho_G\left(s^{-1}(\phi)\right)\right] \\
&\quad + I\left[\rho_G\left(A^{s^{-1}(\phi)}\right); \rho_G\left(\gamma^{-1}(\phi)\right)\right] \qquad (4.4)
\end{aligned}
$$

By equations (4.3) and (4.4) we find that the counterterm, equation (3.2) changes by $-I\left[Q; \rho_H\left(\gamma^{-1}(\phi)\right)\right]$ so that $\Gamma^a_f[A;\phi] + F^a[A;\phi]$ is section independent.

We can also see more directly that if 't Hooft's anomaly conditions are satisified, then the obstruction considered in [2][3] vanishes. If B_{ρ_H} denotes the bundle associated to the principal H bundle $G \to G/H$ by the representation ρ_H of H, then 't Hooft's anomaly conditions imply that

$$
ch_3\left(B_{\rho_H}\right) = ch_3\left(B_{\rho_G|_H}\right) \ ,
$$

since the two representations have the same d-symbols. However, for $B_{\rho_G|_H}$ we can choose the Clebsch-Gordan coefficients $C^r_{mn} = \rho_G(X^r)_{mn}$ and the connection on $\mathcal{U}_k$ is just

$$
\rho_G|_H\left(s_a^{-1} ds_a|_H\right) + \rho_G(X^r)\left(\left(s_a^{-1} ds_a\right)|_{X^r}\right) = \rho_G\left(s_a^{-1} ds_a\right) \ .
$$

By the Maurer-Cartan equations, the curvature, and therefore the Chern characters vanish for $B_{\rho_G|_H}$. In particular, $ch_3\left(B_{\rho_G|_H}\right) = ch_3\left(B_{\rho_H}\right) = 0$, so the global obstruction which depends on the non-triviality of $ch_3(B_{\rho_H})$ vanishes.

In this paper we have only considered "small" field configurations which lie in some contractible patch on G/H. Since Γ^a_f is a non-local functional, and not the integral of a differential form, our problem does not quite fit into the general framework of [14]. Nevertheless, O. Alvares has given general arguments to show that the existence of Wess-Zumino terms is a problem in cohomology, not homotopy. In our case, these arguments suggest that there

will be no further obstructions to the proper realisation of G symmetry arising from a nonvanishing $\pi_4(G/H)$.

E. Witten, and also P. Ginsparg and L. Alvares-Gaumé have arrived at similar results using different methods. We thank them for discussions. We also thank S. Coleman, H. Georgi, and R. Rohm for helpful conversations.

References

[1] M. Henneaux and L. Mezincescu, "A Sigma Model Interpretation of the Green-Schwartz Covariant Superstring Action," Austin preprint, 1984; D. Friedan, E. Witten, private communication.

[2] G. Moore and P. Nelson, Phys. Rev. Lett. 53 (1984) 1519.

[3] G. Moore and P. Nelson, "The Ætiology of Sigma Model Anomalies," Harvard preprint, HUTP-84/A076, 1984.

[4] P. diVecchia, S. Ferrara, and L. Giradello, "Anomalies of Hidden Local Chiral Symmetries in Sigma Models and Extended Supergravities," CERN TH.4026/84.

[5] E. Cohen and C. Gómes, "Dynamical Implications of Anomalies in Non-linear Sigma Models," CERN TH.4043/84.

[6] J. Wess and B. Zumino, Phys. Lett. 37B (1971) 95.

[7] S. Coleman, J. Wess, and B. Zumino, Phys. Rev. 177 (1969) 2239; C. Callan, S. Coleman, J. Wess, and B. Zumino, Phys. Rev. 177 (1969) 2247.

[8] G. 't Hooft, in *Recent developments in gauge theories,* ed. G. 't Hooft et. al. (Plenum, New York, 1980); Y. Frishman, A. Schwimmer, T. Banks and S. Yankielowicz, Nucl. Phys. B177 (1981) 157; A. D. Dolgov and V. I. Zakharov, Nucl. Phys. B27 (1971) 525; S. Coleman and B. Grossman, Nucl. Phys. B203 (1982) 205.

[9] A. Manohar and G. Moore, Nucl. Phys. B243 (1984) 55.

[10] H. Georgi, *Weak Interactions and Modern Particle Theory,* (Menlo Park, Benjamin/Cummings, 1984).

[11] B. Zumino, "Chiral Anomalies and Differential Geometry," in *Relativity, Groups and Topology II,* (North-Holland, Amsterdam, 1984).

[12] L. Faddeev, Phys. Lett. 145B (1984) 81; B. Zumino, "Cohomology of Gauge Groups: Cocycles and Schwinger Terms," Santa Barbara preprint, NSF-ITP-84-150; Y.-S. Wu, "Local Cohomology of Gauge Fields and Explicit Construction of Wess-Zumino Lagrangians in Nonlinear Sigma Models over G/H," Seattle preprint 40048-32 P4.

[13] L. Alvarez-Gaumé and P. Ginsparg, "The Structure of Gauge and Gravitational Anomalies," Harvard preprint HUTP-84/A016, and references therein.

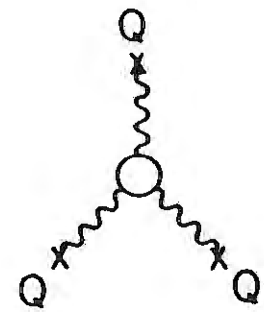
[14] O. Alvarez, "Topological Quantization and Cohomology," Berkeley preprint, UCB-PTH-84/29 and LBL-18658.

Figure Caption

Figure 1: **The effective action according to CCWZ.**

$\Gamma_f\left[\phi, A\right] =$

$+$



$+ \cdots$