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## Semirigid Supergravity

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## Semirigid Supergravity

### Abstract

We formulate two-dimensional topological gravity starting from *local*  $N = 2$  superconformal geometry. The theory is free from the very beginning. The usual “twisting” of the  $N = 2$  algebra emerges from symmetry breaking when we expand about a nonzero value for one of the ghost fields. The mysterious linear term in the supercurrent emerges automatically, as does a full superfield formalism for the whole system including ghosts. We analyze the moduli space of the “semirigid” super Riemann surfaces associated with this theory, including their allowed degenerations.

### Disciplines

Physical Sciences and Mathematics | Physics

# Semirigid Supergravity

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We formulate two-dimensional topological gravity starting from *local*  $N = 2$  superconformal geometry. The theory is free from the very beginning. The usual “twisting” of the  $N = 2$  algebra emerges from symmetry breaking when we expand about a nonzero value for one of the ghost fields. The mysterious linear term in the supercurrent emerges automatically, as does a full superfield formalism for the whole system including ghosts. We analyze the moduli space of the “semirigid” super Riemann surfaces associated to this theory, including their allowed degenerations.

Two-dimensional topological gravity still has many mysterious features. On one hand it is supposed to be a cohomological quantum field theory, like topological Yang–Mills [1]. From this viewpoint it makes sense to construct it from a double complex with differentials  $Q_S$  and  $Q_V$  corresponding respectively to a scalar supersymmetry and the usual BRST charge of the symmetry algebra of conformal gravity, in this case the Virasoro algebra. This approach was followed in [2][3][4][5][6][7][8] and elsewhere. A related, but distinct, construction obtains the theory from Chern-Simons theory for the super analog of  $SL(2, \mathbb{R})$  [9]. It seems remarkable that after a rather arduous gauge-fixing procedure only a very small set of fields remain in the end, and these are *free*.

On the other hand it is by now quite clear that topological gravity has an intimate association with  $N = 2$  superconformal symmetry. The matter systems which can couple to pure topological gravity invariably seem to come from “twisting” systems with  $N = 2$  susy [10][11][12][13][14]. If this is not an accident then we should see this symmetry entering in some essential way in the geometry underlying this system, and in particular in the gravity sector. Indeed there are hints, but they raise as many questions as they answer.

Verlinde and Verlinde have shown that the same “twisted”  $N = 2$  symmetry algebra just mentioned also acts on the ghost and Liouville systems of topological gravity, which are all that remain of the gravity sector after gauge fixing [9]. But what physically is responsible for this “twisting”? If only a fragment of local  $N = 2$  susy remains, what has broken it? Is there another phase of the theory where this symmetry is not broken? The question is particularly acute because the twisted algebra controls even the pure gravity theory; we cannot appeal to the matter system for help in breaking the  $N = 2$  symmetry. We do not know of another system where gravity breaks its own supersymmetries, leaving *unbroken* the ordinary coordinate group (or its conformal subgroup).

A second question relates to the specific form of the residual symmetry generators  $L_n$ ,  $G_n$ , and  $Q_S$  found in [9]. While  $L_n$  and  $Q_S$  are *bilinear* in fields (like any Noether charge in free field theory), one sees that to make the theory work one needs a *linear* term in  $G_n$ . In refs. [9][15] this term is added by hand, but it should come out of some symmetry principle.<sup>1</sup>

Thirdly, in [14][15] an elegant superspace construction was proposed in which matter fields and allowed vertex operators assemble into superconformal fields. Unfortunately the ghost fields,  $b, \beta, c, \gamma$  do not seem to form superfields, and hence the generators  $L_n, G_n, Q_S$

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<sup>1</sup> A different suggestion can be found in ref. [8].

do not have superfield forms. This seriously limits the utility of superspace; we would like to use super contour-deformation arguments and so on. Moreover, it is hard to see how the ghosts, with the inhomogeneous transformation law implied above, can be tensors on *any* super Riemann surface. These puzzles cast doubt on whether the supermoduli space corresponding to the super Riemann surfaces of [14][15] is the right one to integrate over. The statement that this space was trivially fibered over ordinary moduli space played a role in the analyses of [15][8], so we need to be sure it is right. In particular it is crucial to understand the degeneration of these surfaces.

In this letter we will address all the above issues. Our point of departure is extremely simple. We begin with *local*  $N = 2$  superconformal gravity[16]. The corresponding geometry was studied by Cohn [17]. We then find a field in the gravity sector (there is no matter) whose vacuum expectation value breaks the full symmetry to the “twisted” subalgebra. There is only one field available which can do this job without spoiling ordinary conformal invariance. We describe the corresponding reduced geometry, which we have dubbed “semirigid” geometry, because half of the local  $N = 2$  supersymmetries get broken down to rigid susy. This may seem strange, since in  $N = 1$  there is no such thing as rigid susy on an arbitrary Riemann surface. In  $N = 2$  however, we can consistently attribute spin zero to one supercharge, and hence give invariant meaning to its zero mode.

Remarkably, many of the mysterious elements of the complicated cohomological field theory construction emerge automatically in this approach. For example, both ghost and vertex-operator superfields can coexist in this geometry.<sup>2</sup> And due to the vev, all the ghost parts of  $L_n$ ,  $G_n$  and  $Q_S$  including the linear term come directly from the unbroken bits of the full  $N = 2$  stress tensor. We will *not* inquire into the dynamical origin of the symmetry breaking proposed here, however. This question is of course the great mystery of any topological gravity theory. Finally we describe some of the main features of the supermoduli spaces  $\widehat{\mathcal{M}}$  appropriate to semirigid geometry. We will prove that  $\widehat{\mathcal{M}}$  is indeed split (*i.e.*, trivially fibered) over the ordinary  $\mathcal{M}$ .

Our simple construction thus puts  $N = 2$  supersymmetry at center stage. The cohomological interpretation is a byproduct due to the fact that  $Q_S + Q_V$  happens to be an invariant, nilpotent operator, where again  $Q_V$  is the Virasoro BRST operator. In contrast, TYM theory has a superspace version where  $Q_S = \partial/\partial\theta$  is gauge-invariant all by itself [18][6]. The difference is as usual that gravity is not quite a gauge theory; its symmetry

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<sup>2</sup> We are grateful to H. Verlinde for suggesting this.

generators get tangled up in spacetime. This difficulty seems responsible for the complexity of the cohomological approach to topological gravity.

To get started we recall some  $N = 2$  geometry, beginning with a coordinate-invariant definition of  $N = 2$  super Riemann surfaces (SRS).<sup>3</sup> An  $N = 2$  super Riemann surface  $\hat{\Sigma}$  is patched from pieces of  $\mathbf{C}^{1|2}$ . Generalizing the  $N = 1$  discussion of [19] we take  $\hat{\Sigma}$  to be equipped with two odd line bundles  $\mathcal{D}, \tilde{\mathcal{D}} \subseteq T\hat{\Sigma}$ , and require  $\mathcal{D}, \tilde{\mathcal{D}}$  each to be integrable:  $[D, D'] \propto D$  for any sections  $D, D'$  of  $\mathcal{D}$ , *etc.* We also require that  $[D, \tilde{D}]$  be linearly independent of  $D$  and  $\tilde{D}$ . Then there are local coordinates  $\mathbf{z} \equiv (z, \theta, \xi)$  such that  $D_\theta \equiv \partial_\theta + \xi\partial_z$  and  $\tilde{D}_\xi = \partial_\xi + \theta\partial_z$  span  $\mathcal{D}, \tilde{\mathcal{D}}$  respectively. Any other  $\mathbf{z}' = (z', \theta', \xi')$  are related by a transformation preserving  $D$  and  $\tilde{D}$  up to a multiplier[17]:

$$\begin{aligned} z' &= f + \theta a \rho + \xi e \alpha + \theta \xi \partial(\alpha \rho) \\ \theta' &= \alpha + \theta a + \theta \xi \partial \alpha \\ \xi' &= \rho + \xi e - \theta \xi \partial \rho \end{aligned} \tag{1}$$

where  $f, a, e$  are commuting functions of  $z$ ;  $\alpha, \rho$  are anticommuting functions of  $z$ ,  $\partial = \partial/\partial z$ , and we impose

$$ea = \partial f + \alpha \partial \rho + \rho \partial \alpha \quad .$$

Thus there are two independent even and two odd sets of symmetry generators, the usual  $L_n, J_n, G_n, \tilde{G}_n$ .

Under these superconformal transformations we get  $D_\theta = (D_\theta \theta') D_{\theta'}$ , and  $\tilde{D}_\xi = (D_\xi \xi') D_{\xi'}$ , which define the transition functions of the bundles  $\mathcal{D}, \tilde{\mathcal{D}}$ . Henceforth a section of  $\mathcal{D}^{\otimes n}$  will be denoted by a component function with  $n$  raised  $\theta$  indices *etc.* A  $\theta\xi$  index pair will be rewritten as a  $z$  index. Infinitesimal superconformal transformations are generated by

$$V_v \equiv v \partial_z + \frac{1}{2} (Dv) \tilde{D} + \frac{1}{2} (\tilde{D}v) D$$

where  $v \equiv v^z(\mathbf{z})$  is an even tensor field. We find  $[V_{v_1}, V_{v_2}] = V_{[v_1, v_2]}$  where

$$[v_1, v_2] = v_1 \partial v_2 - v_2 \partial v_1 + \frac{1}{2} Dv_1 \tilde{D}v_2 + \frac{1}{2} \tilde{D}v_1 Dv_2 \quad . \tag{2}$$

Finally, computing the Berezin determinant of (1) shows that the volume form  $d\mathbf{z} \equiv [dz|d\theta d\xi]$  is *invariant*. Thus the integral  $\oint d\mathbf{z}$  sets up a Serre duality between  $(p, q)$ -tensors and  $(-p, -q)$ -tensors, unlike  $N = 0$  or 1.

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<sup>3</sup> For reasons to become clear we will not allow the ‘twisting’ considered in [17].

The  $C$  ghost is always determined by the group of allowed coordinate transformations, so we have  $C \equiv C^z(\mathbf{z})$ . The stress tensor and  $B$  ghost are always dual to  $v$ , so  $B \equiv B_z(\mathbf{z})$ ,  $T \equiv T_z(\mathbf{z})$ . We will choose to expand these fields as

$$\begin{aligned} C^z &= c^z + \theta\gamma^\xi + \xi\check{\gamma}^\theta + \theta\xi\check{c} \\ B_z &= \check{b}_z + \theta\check{\beta}_{\theta z} - \xi\beta_{\xi z} + \theta\xi(b_{zz} + \partial_z\check{b}_z) \\ T_z &= J_z + \theta\check{G}_{\theta z} - \xi G_{\xi z} + \theta\xi((T_B)_{zz} + \partial_z J_z) \quad . \end{aligned} \quad (3)$$

These peculiar linear combinations will be useful momentarily. The operator products of the component fields yield

$$B(\mathbf{z}_1)C(\mathbf{z}_2) = \theta_{12}\xi_{12}/z_{12} = C(\mathbf{z}_1)B(\mathbf{z}_2)$$

where

$$\begin{aligned} z_{12} &\equiv z_1 - z_2 - \theta_1\xi_2 - \xi_1\theta_2 \\ \theta_{12} &\equiv \theta_1 - \theta_2 \quad \xi_{12} \equiv \xi_1 - \xi_2 \end{aligned}$$

are elementary translation-invariant functions [17]. The ghost stress tensor can be constructed by the method of [20]: demanding that  $[T[v], \phi] = \mathcal{L}_v\phi$  for  $\phi = B, C$  and  $\mathcal{L}$  the Lie bracket implied by (2) gives

$$T = \partial(CB) - \frac{1}{2}(DB\check{D}C + \check{D}BDC) \quad . \quad (4)$$

We now explore how to break this large symmetry. Suppose that instead of asking  $D_{\theta'} \propto D_\theta$  we require  $D_{\theta'} = D_\theta$ . In other words we require that  $\mathcal{D}$  be *trivial*, and a global section  $D$  be given. (Note that this precludes “twisting” in the sense of [17].) In  $N = 1$  such a constraint would break us all the way down to rigid Poincaré susy. Now however we find merely that in (1) we need  $\alpha \equiv \text{const.}$ ,  $a \equiv 1$ . Infinitesimally this says that  $\check{D}v \equiv \text{const.}$  Examining (1) we see that it now makes sense to attribute spin 0 to  $\theta$  and spin 1 to  $\xi$ . Thus in (3) the only Bose field whose vev can break  $N = 2$  in this way is  $\check{\gamma}^\theta$ . More invariantly we break  $N = 2$  by the constraint

$$\check{D}C \equiv q \quad , \quad \text{or} \quad \check{\gamma} = q, \quad \check{c} = \partial c \quad . \quad (5)$$

Since  $\check{D}_\xi C^{\theta\xi}$  is a section of  $\mathcal{D}$  we have again supplied a trivialization.  $q$  is some constant.

One readily finds the unbroken generators to be the modes  $L_n, G_n, \tilde{G}_0$  of  $T_B, G, \tilde{G}$  in (3), and these obey the required “twisted  $N = 2$ ” algebra by virtue of (2). Substituting (5) into (4) we find

$$\begin{aligned} T_B &= -2b\partial c - (\partial b)c - 2\beta\partial\gamma - (\partial\beta)\gamma \\ G &= -2\beta\partial c - (\partial\beta)c + \frac{q}{2}b \\ Q_S &\equiv 2\tilde{G}_0 = \oint \gamma b \quad . \end{aligned}$$

Choosing  $q = -2$  yields the formulas of ref. [15] after some trivial changes of notation. Note that we did not *constrain* the  $B$  field at all. The unfamiliar new components  $\check{b}, \check{\beta}$  simply dropped out of the expressions for the unbroken charges; they remain in the other charges, which however are not symmetries. Nevertheless we cannot represent the  $BC$  system by ordinary superfields on any 1|1 superspace.

Because  $Q_S$  has nontrivial commutator with  $G_0$  we cannot regard it as a global charge, any more than  $L_0$  is global on an ordinary Riemann surface.<sup>4</sup> However, the combination  $Q_T = Q_S + Q_V$  *does* commute with the unbroken charges, and so defines the global nilpotent charge needed to get a topological field theory [1].

Let us turn to the moduli of semirigid super Riemann surfaces, or SSRS. We find the moduli by examining the Čech cohomology of the allowed coordinate transformations (see *e.g.* [21]). Since we want the dimension of moduli space to be  $3g - 3|3g - 3$  we further restrict (1) to  $\alpha = 0, a = 1$ :

$$\begin{aligned} z' &= f(z) + \theta\rho(z) \\ \theta' &= \theta \\ \xi' &= \rho(z) + \xi\partial f(z) - \theta\xi\partial\rho(z) \quad . \end{aligned} \tag{6}$$

Infinitesimally this means  $\tilde{D}v = 0$  or  $v^z = v_0^z + \theta v^z + \theta\xi\partial_z v_0^z$  and the desired dimension follows. We next note that since the bundle  $\tilde{\mathcal{D}} \subseteq T\hat{\Sigma}$  is integrable, we can reduce to a manifold with coordinates  $z|\theta$  by modding out the flow of  $\tilde{D}$ . The transition functions then reduce to

$$\begin{aligned} z' &= f(z) + \theta\rho(z) \\ \theta' &= \theta \quad . \end{aligned} \tag{7}$$

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<sup>4</sup>  $Q_S$  is scalar under ordinary  $L_n$  transformations, but this is not good enough.



Conversely given patching functions of the form (7) we uniquely get a SSRS. Then *e.g.* any scalar function  $f = f + \theta\phi$  lifts uniquely to the chiral superfield  $F(\mathbf{z}) = f + 2\theta\phi + \theta\xi\partial f$ . Similarly we may instead mod out by the flow of  $D$ , reducing (6) to the form given in [14]:

$$\begin{aligned} z' &= f(z) \\ \xi' &= \xi\partial f(z) + \rho(z) \quad . \end{aligned} \tag{8}$$

Again any set of transition functions of this form induce a unique SSRS by substituting  $f, \rho$  into (6). The relation between (6), (7), and (8) is preserved under composition of maps. Thus (6), (7), (8) all define exactly the *same* moduli space.

Writing any of (6), (7), (8) as  $\mathbf{z}_i = F_{ij}(\mathbf{z}_j)$ , we see that the condition  $F_{ki} = F_{kj} \circ F_{ji}$  implies

$$\begin{aligned} f_{ki} &= f_{kj} \circ f_{ji} \\ \rho_{ki} &= (\partial f_{kj} \circ f_{ji}) \cdot \rho_{ji} + \rho_{kj} \circ f_{ji} \quad . \end{aligned}$$

These say that  $\{f_{ij}\}$  define an ordinary Riemann surface while  $\{\rho_{ij}\}$  define a cocycle in its tangent space. Hence it makes sense to choose  $f_{ij}$  to depend only on the commuting moduli, and  $\rho_{ki}$  to be linear in the anticommuting moduli, in contrast to the situation in  $N = 1$ . This in turn implies that the moduli space  $\widehat{\mathcal{M}}$  is split over  $\mathcal{M}$ , at least away from the boundary; details will appear elsewhere.

The correct notion of puncture on any sort of SRS is [21] that of a divisor of codimension 1|0 obtained from  $z$  by some allowed coordinate transformation of the disk. In the language of (7) we find divisors of the form  $(z - z_0 - \theta\theta_0)$ , yielding 1|1 new moduli for a total of  $3g - 3 + N|3g - 3 + N$  if there are  $N$  punctures. Counting then shows that the correct plumbing fixture for degeneration of SSRS must have 1|1 pinching parameters, similarly to spin pinches in  $N = 1$  (but unlike super pinches) [22]. The correct choice turns out to be to join the  $z|\theta$  plane to the  $u|\zeta$  plane via

$$\begin{aligned} z &= u^{-1}(q + \zeta\delta) \\ \theta &= \zeta \quad . \end{aligned} \tag{9}$$

For these assignments to make sense we need to know that the 3-punctured sphere is rigid<sup>5</sup>. This follows from the observation of a 3|3 parameter group of sphere automorphisms:

$$z' = \frac{(a + \theta\alpha)z + (b + \theta\beta)}{(c + \theta\gamma)z + (d + \theta\delta)} \quad , \quad \theta' = \theta$$

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<sup>5</sup> This is analogous to the sphere in  $N = 1$  geometry with two spin punctures and one super puncture (but unlike the case with three super punctures).

where we may take  $d = 1$ ,  $\delta = 0$ . Note that (9) is of this form.

Now that we know the moduli space one can elaborate the full machinery of constructing the super operator formalism and the string measure for the insertion of weakly physical states [23][24]. Alternately one may try to covariantize the vertex operators by inventing a Liouville sector as in [9].

We have provided a concrete framework for  $d = 2$  topological gravity by starting with *local*  $N = 2$  superconformal gravity. This system is not conformally invariant by itself, but it becomes so once we drop the modes  $\check{c}$ ,  $\check{\gamma}$  fixed by a symmetry-breaking constraint. Thus the broken theory requires no Liouville sector. Indeed it is born free, rather than appearing interacting at first and mysteriously becoming free after gauge-fixing. The system has a global nilpotent charge and hence is topological. It explains why the appropriate matter systems always seem to have “twisted”  $N = 2$  susy, and other mysteries as well. It also urges us to ask about the bigger mystery of the dynamical origin of such a symmetry breaking.

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*Notes added:* Recently we have found that the passage from eqn. (6) to (8) is a special case of a construction given by Dolgikh, Rosly, and Schwarz [25]. The geometrical framework given in this paper can be used to derive recursion formulas such as the dilaton equation, again without recourse to any Liouville sector [26].

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