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Shareholder-Optimal Design of Cash Balance Pension Plans

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Shareholder-Optimal Design of Cash Balance Pension Plans

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# Shareholder-Optimal Design of Cash Balance Pension Plans

## Abstract

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The Shareholder-Optimal Design of
Cash Balance Pension Plans

Abstract

In 1980 and 1981, Fischer Black and Irwin Tepper showed that shareholders would gain if corporate defined benefit pension assets were invested in taxable fixed income securities instead of equities. This paper extends this analysis into the cash balance plan arena, concluding that additional shareholder gains arise when plan liabilities mimic equities. A numerical example demonstrates that the present value of riskless gains to shareholders can exceed the entire after-tax value of plan assets. Lack of transparency in actuarial methods and assumptions is shown to impede implementation.

I Introduction

From Modigliani-Miller to Black and Tepper, Briefly

The capital structure literature that begins with the indifference propositions of Modigliani and Miller (1958), divides into two major branches. While one considers bankruptcy issues and their challenge to structural indifference, the other focuses on tax concerns. This tax branch includes Miller (1977) which considers "gains from leverage" as a function of the tax rates applicable to corporations ($\tau_c$), to individuals holding bonds ($\tau_{pb}$) and to individuals holding stocks ($\tau_{ps}$) and solves for a leverage-indifferent relationship between these rates:

\[(1-\tau_c)(1-\tau_{ps}) = (1-\tau_{pb})\]

Treynor (Bagehot, 1972) introduces the idea of the "augmented balance sheet" to address the corporate structure role of the defined benefit (DB) pension. This approach depicts the pension plan as a transparent financial subsidiary of the corporation. Treynor asserts a financial integration of the corporation and the plan which deliberately ignores the separation of the entities under law and regulation. Although, the 1974 passage of the Employee Retirement Income Security Act (ERISA) strengthens the legal
separation, the Treynor paradigm prevails in the literature. The present paper, for the most part, continues this tradition.

Sharpe (1976) observes that the establishment, by ERISA, of the Pension Benefit Guaranty Corporation (PBGC) creates a put option for pension plans which may be exercised (with restrictions) to the advantage of the plan and the corporation when plan liabilities exceed plan assets. Subsequent legislation has substantially tightened the restrictions and reduced the importance of the "PBGC put". This put and an "excise tax call" (characterized as such for the first time herein) enacted in a series of steps during the late 1980's are discussed further in Appendix D.

Black (1980) and Tepper (1981) combine the tax branch descended from Modigliani-Miller and the augmented balance sheet approach of Treynor to conclude that the assets of corporately sponsored defined benefit pension plans should be invested entirely in fixed income securities subject to high rates of tax. The plan liabilities are modeled by each author as exogenous and bond-like. The recommended fixed-income investment strategy will allow favorable hedging at the corporate level (Black) or in the hands of shareholders (Tepper); a tax-arbitrage that represents a gain to shareholders in comparison to the common practice that includes substantial equity investments in corporate pension portfolios.

The subsequent literature has been primarily empirical and has generally concluded\(^1\) that corporate plan sponsors have not availed themselves of the tax benefits outlined in the Tepper and Black papers.

**New directions**

This paper extends the work of Tepper and Black into the present arena which includes a new type of pension plan called "Cash Balance". The next subsection details the features of these plans.

Although almost every Cash Balance (CB) plan in existence today has liabilities that closely resemble bank accounts or short-term cash investments, we endogenize the

\(^1\) A recent exception may be found in Myers (1999).
liability investment type and conclude that the tax-arbitrage benefit to shareholders will be maximized by equity returns to participant accounts. Consistent with Tepper and Black, we conclude that plan assets should be invested entirely in fixed income.

We also review the claim by Black and Tepper that their approaches produce arbitrage gains and we conclude that they are correct when the common practice or status quo is the starting point for measure. When we consider plans without surplus assets, we find that there is no opportunity to profit from a DB plan (in comparison to cash compensation or defined contribution (DC) plan substitutes) unless liability returns are equity based. The Tepper and Black approaches merely undo the tax-arbitrage losses inflicted by plan sponsors upon themselves.

Our primary model assumes that the participants are indifferent to the liability measure (in effect, their total compensation is exogenous and unvarying). We go beyond the primary model in a subsection entitled "Employee choice plans – closer to equilibrium" at the end of Section V. Therein, symmetry motivates us to look for tax-arbitrage losses on the part of participants. We find some losses but also observe that various conditions may cause such losses to be much less than the gains to shareholders. From this we conclude that contract-improving arrangements may be made between the shareholders and the employees (within the structure of the corporation) to the detriment of taxpayers in general. A practical contract improvement can be achieved simply by allowing individual employees to choose the liability benchmark that they prefer on a continuum from all-equities to all-fixed-income.

We also consider an entirely original approach (a "CBSOP") wherein the employee benchmark choices would include stock in the sponsoring corporation. This would improve upon an admitted weakness in the Black hedging approach. Regulatory impediments might prove intractable, however.

Because the focus of the entire literature line from Modigliani-Miller through Black and Tepper is on a single firm that is presumed to deal in securities (including its own) in a liquid and deep market that makes each agent a price-taker, none of these analyses (including the present one) describe general equilibria. The endogenization of the liabilities and the contract-improving employee choice parts of this paper move in the
direction of a wider equilibrium. In the subsection entitled "Upsetting the Equilibrium" in Section IV, we ask and briefly reply to the equilibrium-seeking question: what would happen to the equity risk premium if the lessons of Tepper and Black and this paper were widely adopted by corporations? Because we do not endogenize the taxpayers or the government we stop well short of general equilibrium.

This paper also addresses the issue, hinted at by Black, of the diversification effects that arise when corporations invest their pension assets in the equity of other corporations. We show that, absent bankruptcy issues, such diversification is valueless to shareholders and that they will act to unwind its effects and restore their effective portfolios to their original holdings of the underlying corporate assets. This in turn produces an interesting observation about the correct definition of the market portfolio. To the extent, if any, that corporate pension assets are supported by corporate equity capital, the capitalization of the market portfolio is seemingly increased (by the exchange of pieces of paper alone). Because U.S. pension funds hold a sizable fraction of the market portfolio, a multiplier or leveraging effect may be revealed.

We reconcile the Black and Tepper approaches and derive the necessary and sufficient tax conditions for them to achieve identical arbitrage gains. Delightfully, and intuitively, this turns out to be the set of rates that satisfy Miller's leverage-indifference criterion.

Black and Tepper, writing when they did, did not need to respond to a question which today's writer must address: if the arbitrage gains are genuine and substantial, as claimed, why does it appear that so few corporations have adopted the Tepper-Black investment strategies? We hypothesize that opaque and perverse actuarial methods and assumptions provide strong motivation to act in contradiction of the strategy. To this end we demonstrate that (Section VI), under an actuarial regime, the common practice wherein liabilities are fixed and assets include equities allows sponsors to provide a dollar's worth of employee account balances at a substantial discount. This same

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2 The partial equilibrium environment of corporate finance pension papers is in contrast to the literature that addresses the future of the U.S. Social Security system. The pervasiveness of Social Security makes it necessary that its literature deal with general equilibria. Some of the indifference results of that literature (e.g., Smetters, 1997, Proposition 1) bear a superficial resemblance to our pre-tax Result 1 but reasoning by analogy beyond that point is not supportable.
actuarial regime causes the shareholder-optimal strategy (determined under a transparent regime) to result in a cost of more than one sponsor dollar per employee dollar. This issue is explored in a broader and deeper fashion in a subsequent paper.

The Tepper and Black models address one year at a time. Because their strategy for investment as applied to a fixed liability plan constitutes an asset-liability match, the implications over several years may be represented by fixed rate perpetuities (Tepper). Because our strategy, that defines liability benchmarks as equities, mismatches assets and liabilities, we have added a subsection (Some Dynamic Considerations) in Section V that looks at some of the ensuing dynamics.

Because the tax-arbitrage maximizing strategy for cash balance plans is not an asset-liability match, we note that bankruptcy concerns may discourage full implementation for weaker companies. We also note that the excise tax call may rein in the tax gains for overfunded plans when, subsequent to implementation, the equity market declines substantially over time. In a transparent environment, reductions to gains that derive from these sources constitute an agency cost attributable to a need for risk capital.

This paper is timely in two regards. First, the current controversy over cash balance plans may be mitigated by plan sponsors' incorporation of equity-based liabilities and employee choice. Second, the existence of cash balance plans has made the long-standing dissonance between the actuarial process and a transparent examination more obvious. How is it possible for a $1 credit to an employee account (that is nonforfeitable and generally immune to default) to cost the employer substantially less than $1 (a pre-tax to pre-tax comparison) and how is it possible that a contemporaneous change of asset allocation can change the cost of that $1 credit? In the context of a traditional DB plan, these questions could not be so easily framed, despite the fact that the underlying actuarial distortion pervades valuations of both CB and traditional DB plans.

**Cash balance plans**

Much of the recent publicity concerning cash balance pension plans sponsored by U.S. corporations has focused on the controversy that attaches to their implementation as conversions of traditional defined benefit pension plans. This controversy is tangential to
the main thesis of this paper. My thesis, derived as an extension to the work of Fischer Black (1980) and Irwin Tepper (1981), is that the shareholders will be able to achieve gains from tax arbitrage if the plans promise equity-based investment credits and invest the plan assets entirely in taxable fixed income securities.

This strictly contradicts the common practice of cash balance plan sponsors who typically promise investment crediting rates based on fixed income benchmarks and invest the plan assets in a diversified mix that is more than half allocated to equities. This results in tax arbitrage losses to shareholders.

Because a cash balance plan is a special form of defined benefit plan, its primary financial aspects are transmitted to shareholders through an actuarial filter. This filter smoothes the cash flows from the corporation to the plan, smoothes the expenses (income) reported by the plan and anticipates returns on risky investments before the risks have been borne (Gold, 1999). The actuarial use of expected returns on assets with little or no attention paid to the stochastic distribution of returns implies, incorrectly, that the common practice is a profitable strategy. The actuarial firm that invented the cash balance plan, in its introductory publication (Kwasha Lipton, 1985) said “A ‘5% of pay plan’ might require a contribution of only 4% of pay, after a realistic investment differential is taken into account.”

A key element of our analysis is the assumption of transparency with respect to the financial intermediary that is the corporation’s pension plan. This implies that a vested compensation credit equal to 5% of pay cannot be financed at any cost less than 5% of pay. Why such transparency may not regularly prevail is discussed in Section VI and in a subsequent paper.

**Some basic definitions**

Retirement plans sponsored by corporate employers in the United States may be one of two types. These are explained by McGill et al (1996)\(^3\):

\(^3\) Pp. 27-28.
• “One approach is to establish and maintain a pension plan that promises a determinable set of benefits at retirement… The plan sponsor, typically the employer, undertakes to provide the funds, through periodic contributions and investment earnings on the plan assets, that are needed to pay the promised benefits as they become payable… this type of plan is identified in pension literature as a defined benefit [DB] plan. It is characterized by definitely determinable benefits… and by indeterminable future costs.”

• “The other approach is to specify the basis on which contributions will be made to the plan, with no contractual commitment as to the level of benefits that will be provided. Individual accounts are maintained for the participants, the accounts being credited with their allocable share of employer (and employee) contributions and investment earnings… In contrast to the defined benefit approach, the employer’s future cost, as a percentage of covered payroll, is known in advance; but the amount of retirement benefit is not determinable in advance… Thus it may be that the future cost of the plan is predictable but the benefits are unpredictable. This approach to retirement planning is known as a defined contribution [DC] plan, also referred to as an individual account plan.” [bolding and abbreviation added].

A DC plan is a "pass-through" financial entity much like a mutual fund or a mutual fund family; the assets and liabilities are always equal in amount and all of the plan assets may be assigned to individual participant accounts. A DB plan, on the other hand, is a financial intermediary that is more analogous to a bank or an insurance company; the liabilities are promises made by the plan to the participants; the aggregate assets, not generally equal to the aggregate liabilities, serve to collateralize the promises.

A Cash Balance [CB] plan is known as a “hybrid” because it combines some elements of each of the above plan types. The primary feature borrowed from DC plans is the individual account balance which accumulates for each participant. Thus, from a participant perspective, a CB plan looks much like a DC plan.

Almost every other aspect of CB plans, including actuarial methodology, GAAP accounting and statutory qualification under the Employee Retirement Income Security Act (ERISA) and the Internal Revenue Code (IRC) conform to the DB model. From a regulatory perspective, a CB plan is a special type of DB plan. Virtually every cash balance plan in existence today has come about as a conversion of (via plan amendments to) an older defined benefit plan.
Using the mutual fund versus bank or insurance company distinction above, it is fair to say that the CB plan is most like a bank because the participant liabilities are represented by account balances while the traditional DB plan is most like an insurance company because the participant liabilities are primarily annuities. Importantly, the mutual fund analogy applicable to DC plans applies neither to the traditional nor to the CB type of defined benefit plan.

All of the assets of defined contribution plans are allocated to individual employee accounts and the sum of the account balances is equal to the total assets of the plan. In contrast, under a cash balance plan, the account balances are notional and the aggregate of the individual account balances will only equal the total plan assets by rare coincidence. The assets of cash balance plans, like those of other DB plans, are held unallocated on behalf of the entire plan and serve the primary purpose of providing collateral for the benefit promises of the plan.

The traditional defined benefit plan may be understood to hold a long position in the various assets of the plan and to be short the benefits promised to the employees (the accrued liabilities). Since the accrued liabilities of a cash balance plan are the account balances, the CB plan may be understood as a financial entity that is long its various assets and short the account balances of its participants.

Cash balance plans utilize two elements to control the development of plan balances in a fashion that mimics the operation of DC plans:

- Periodic additions to account balances are credited as a function of the employee’s compensation. These additions are called compensation credits; the percentage of compensation is called the compensation crediting rate. Typical compensation crediting rates are single digit percentages that are usually constant but may, in accordance with plan rules, vary with the age and/or service of the employee.

- Account balances also receive investment credits based on an investment crediting rate. The rate may be benchmarked to a marketed security (e.g., the one-year T-bill rate) or set arbitrarily (e.g., 7%). Usually the rate is fixed in advance for a period of no longer than one year.
Subject usually to a vesting schedule, a terminating employee is entitled to receive the current account balance. Various other options including periodic annuity benefits commencing at a later date are usually available.

**Controversial current events**

In the last several years, the pace of major company conversions of traditional DB plans into CB plans has increased. The motivations for the conversions are outlined in Section II. One important motivation is a change in the pattern of accrual of benefits over an employee’s career (Appendix A). In the last year, a substantial controversy has arisen focusing on the conversion process (which almost invariably leads to a reduction in projected retirement benefits for some currently employed plan participants). Some mitigation of these reductions may be effected by a transition treatment (Appendix B). Typically the oldest and longest service employees are protected by “grandfather” rights which allow them to finish their careers under the old plan provisions. Many mid-career participants who are not grandfathered, however, are likely to be disappointed. Their degree of disappointment depends on the transition methodology as outlined in the appendix. Those who are subjected to a “wearaway” transition are likely to be particularly upset.

**Remaining sections**

Section II discusses the history of cash balance plans, the motivations that lead employers to convert existing DB plans to CB plans, and the varying impact that this has on their current employees. Section III presents the assumptions and philosophy of the model (Section IV) that represents the heart of this paper. Section V presents the implications of the model results. Section VI discusses impediments to implementation. Conclusions are summarized in Section VII.

**II Why Corporations are Converting to Cash Balance Plans**

**A brief history**

From the 1940’s through the 1970’s the traditional defined benefit plan provided an efficient method for employers to enable and encourage the retirement of their
superannuated employees. These employees were, at least early in the period, predominantly male heads of nuclear families who were likely to spend the bulk of their careers with one or two employers.

The design of the plans provided incentives for employees to remain with the same employer, particularly from ages 35 through 55 and then, depending on the degree to which the plan subsidized early retirement, encouraged them to retire in the period from age 55 to 65.

The 1970’s were a period that challenged traditional defined benefit plans. ERISA, adopted in 1974, provided federal insurance protection for employee pensions, created standards for minimum funding, enhanced nondiscrimination, vesting, benefit accruals, fiduciary behavior, and disclosure. Each of these features made traditional defined benefit plans more expensive for employers to create and maintain.

The bear market in stocks in 1973 and 1974, accompanied by inflationary surges that recurred several times during the 1970’s and into the early 1980’s, marked down the plan assets and, since actuaries were slow to recognize rising interest rates, raised (actuarially measured) plan liabilities.

Workforce trends towards more mobility and greater female participation accelerated in the 1970’s leading to exposure of the weaknesses of defined benefit plans for those whose careers were not largely spent working for one employer.

In the early 1980’s the pace of pension legislation increased substantially and, with each layer of legislation, defined benefit plans became less attractive to employers. In late 1981, the IRS issued regulations under IRC Section 401(k) which had been added to ERISA by a 1978 act. These regulations provided for pre-tax employee contributions and thus made defined contribution plans an attractive alternative to defined benefit plans. While most of the large, financially strong corporations that are the focus of this paper already had both DB and DC plans, employer-initiated plan improvements thereafter were provided to 401(k) plans while defined benefit plans were cut back when possible and enhanced only when new legislation made such changes necessary.
During 1981, interest rates on long-term Treasury bonds exceeded 14%. Actuarial interest rates were still single digits. This resulted in an overestimate of liabilities and in contribution rates that were arguably much too high. In the four following years, as the stock market began a bullish trend that has not yet ended, the overfunding of many corporate plans became evident. A spate of plan terminations accompanied by the reversion of surplus assets to employers followed. This in turn led to additional legislation, sponsored by Senator Howard Metzenbaum (D. Ohio, 1977-1995), imposing and then sharply increasing excise taxes on these asset recaptures⁴ (Appendix C).

**The invention of the Cash Balance plan**

In 1984, Kwasha Lipton, a leading employee benefit consulting term, unveiled⁵ its newly designed and newly named “Cash Balance” plan. The first major employer to adopt such a plan was Bank of America which did so in 1985. In its explanatory material issued at that time, Kwasha Lipton (1985) identified the following advantages provided by the new design in comparison to traditional DB plans:

- “A basic distinguishing characteristic of the typical Cash Balance plan, as compared to a traditional pension plan, is its relatively more generous treatment of younger employees and employees who terminate before retirement.”

- “Age-independent credits represent equal pay for equal work, without discrimination on the basis of age, sex or marital status.”

- “The company’s contribution is clear-cut and easily understood”⁶

- “A range of [actuarial] funding methods is available. Turnover and other expected experience can be anticipated and discounted in advance, and gains and losses can be amortized.”

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⁴ 10% excise tax on asset reversions, IRC Section 4980, added by Pub. L. 99-514, title XI, Sec. 1132(a), for reversions after December 31, 1985. Increased to 15% by Pub. L. 100-647, title VI, Sec. 6069(a), for reversions after December 31, 1988. Increased to 20% by Pub. L. 101-508, title, XII, Sec. 12001 which further provided a rate of 50% unless the employer used at least 20% of the otherwise revertible assets to fund immediate benefit increases or at least 25% to fund a qualified replacement plan, for reversions after September 30, 1990.

⁵ Kwasha Lipton Partner Larry Brennan presented the concept at the Annual Fall Conference of the Council on Employee Benefits in October of 1984.
• “There is flexibility in the amount of funding from year to year…”

• “If the plan is overfunded, no cash outlay is required until the overfunding is eliminated.”

• “The employer can invest for the long term”

• “If the plan outperforms the rates being credited to employees, the cost to the sponsor will be reduced accordingly.”

• “The investment differential can be anticipated.”

• “As a defined benefit plan, a Cash Balance plan is entitled to PBGC\textsuperscript{7} protection which is not available (or meaningful) to defined contribution plans.”

• “Benefit subsidies – over the years, most pension plans have adopted a variety of benefit subsidies, typically geared toward facilitating early retirement or reducing the cost of joint-and-survivor options. Some companies may feel that these subsidies have become cumbersome, expensive or unfair. When that is the case, the introduction of a Cash Balance plan may present an opportunity to eliminate or rethink these subsidies.”

Fourteen years later, these observations still constitute a good summary of the positive features of these plans. The points above may be grouped into fewer categories: i) portability for mobile workforces, ii) age-neutral benefit accrual patterns that better match employee productivity, iii) tangible, comprehensible benefits that mirror defined contribution plans, iv) actuarial flexibility and a potential profit for employers who invest in equities while promising fixed income rates of return.

The first two items amount to something of a two-edged sword. Portability (meaning larger benefits for those who leave a particular employer early in their careers) raises the per-retiree cost of providing benefits to those who remain with one employer until they are old enough to retire. As a result, the employer who wishes to remain cost neutral must reduce the benefits for full service employees to pay for the portable benefits of

\textsuperscript{6} This must refer to the notional contribution added as a compensation credit since the actual employer contributions are based on actuarial funding methodology that is neither clear-cut nor easily understood.

\textsuperscript{7} The Pension Benefit Guaranty Corporation established by ERISA (1974).
those who leave. This introduces some of the transition issues that have aroused much controversy during 1998 and 1999.

The defined benefit accrual pattern (discussed in detail with a graphical example in Appendix A) encourages employees to remain in service (a reward for loyalty in the early history of pension plan design) even after they have become less productive. The pre-cash balance approach has been to provide subsidized early retirement benefits and/or “early retirement windows”. In effect, the high cost for older employees under traditional defined benefit plans made it profitable to “bribe” employees to retire early.

Point iii has, until recently, been an unarguable plus for CB plans (an implied criticism of DB plans). The complexity of the DB plan that is being converted provided an opportunity for employers to communicate the advantages of cash balance plans without necessarily highlighting the disadvantages that fall upon long service employees in the second half of their careers. Weak or even misleading communication, cost neutral or cost-savings conversions and the “wearaway” issue (Appendix B) have given rise to employee protests, press coverage (Schulz, 1999) and Congressional attention.

Point iv is a financial illusion that is the central issue of this paper. In Sections III and IV, we show that the common practice of promising fixed income returns and investing in equities results in losses to shareholders. These losses are masked by actuarial and accounting methodologies that support Kwasha Lipton’s claim, above, that “The investment differential can be anticipated.” The realities of marketed securities and risk averse investors make this statement actuarially true but financially false.

**Why not defined contribution?**

One last question should be addressed as part of *Why Corporations are Converting to Cash Balance Plans*. Why not simply terminate the defined benefit plan and establish or enrich an existing defined contribution plan? While the fourth preceding item provides one possible response, a compelling reason for many employers is found in the Metzenbaum excise taxes. Prior to the enactment of these taxes in 1986 and their sharp

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8 A limited time offer by the pension plans to older employees of higher than usual benefits, typically computed by adding service, age or compensation above the factual values.
upward adjustments through 1990, sponsors of overfunded DB plans might have considered the termination approach. They would have incurred some tax inefficiency since they would have had to pay income taxes when the plan surplus returned to the corporation but subsequent contributions to the DC plan would have offset all but the tax timing differential. The excise taxes were intended to provide serious disincentives for DB plan terminations. They succeeded. It is something of a coincidence that the CB plans were invented just before the excise taxes were enacted. What is not a coincidence, however, is the use of CB plans as a tax-effective exit strategy for sponsors of overfunded DB plans. It is of some note that every CB plan adopted to date has been a conversion of an existing DB plan. It is as if all of the other argued advantages of CB plans in comparison to DC plans (PBGC guarantees, funding flexibility, investment profits) come to naught.

III Model Approach

We employ two models herein. Each relies on plan investments in taxable fixed income securities combined with investment crediting rates on plan balances based on equity returns. Each also relies on offsetting investment and/or financing arrangements engaged in by the corporation and/or its shareholders.

The “Tepper” model traces the risks and returns of the pension plan through to the hands of the shareholders. The shareholders then undertake to neutralize the risks to restore their previously preferred portfolio. They borrow money (or sell fixed income securities) and purchase equities. We measure the after-effects of this riskless arbitrage by looking at the changes in total tax liabilities of the shareholders.

The “Black” model follows the risks and returns of the pension plan to the after-tax corporate balance sheet where they are neutralized by changes in the firm’s capital structure. This means repurchase of the firm’s own shares and an equal value issuance of new corporate debt. For diversified and optimized shareholders this might necessitate the reallocation of their equity holdings between the shares of the plan sponsoring firm and other firms. We measure the effects of the Black model by looking at the tax liabilities of the corporation.


Assumptions

The models are based on the following assumptions about markets (A.1 through A.7), about relative tax rates (A.8), about the operation of the CB plan (A.9 and A.10) and about employee compensation (A.11):

A.1 The shares of the corporation are marketed (i.e., traded in an accessible liquid market).
A.2 Shareholders hold diversified portfolios of assets chosen to reflect their preferred distribution of returns.
A.3 Shareholders also hold some fixed income securities or else can borrow at the market rate of interest.
A.4 Securities may be traded without transaction costs.
A.5 The plan holds, as assets, a portfolio of marketed securities.
A.6 Corporation and pension plan are ongoing; probability of bankruptcy is negligible.
A.7 Transparency: the market values of financial intermediaries accurately reflect the marginal value of any marketed securities held.
A.8 Taxes: total returns on fixed income assets held by individuals are subject to higher effective tax rates than are the total returns on equity assets; corporate pension assets are taxed neither at the plan nor at the corporate level; corporate contributions to pension plans are tax deductible. Tax rates are fixed for all time and companies and shareholders continue to pay taxes in their current bracket.
A.9 The demographic elements of the plan are sufficiently predictable to be modeled without uncertainty.
A.10 The investment crediting rate to be applied to account balances is set periodically in advance and is equal to the total return (whether positive or negative) for the period on a benchmark portfolio comprised of marketed securities.
A.11 Each employee’s compensation, as well as the compensation crediting rate applied thereto, is set without regard to the portfolio used to benchmark the investment crediting rate and without regard to the plan asset amounts and investment returns.
A.6 means that we are focusing on generally well funded plans sponsored by successful companies. We exclude those plans that are so well funded that the plan sponsor cannot, even over time, avoid excise taxes on excess assets. (Appendix D).

A.7 is the most controversial assumption. In the cash balance case, even though the plan assets and liabilities may be readily valued in current dollars, the actuarial methodology that CB plans inherit by virtue of their status as DB plans allows liabilities to be arbitrarily valued and plan costs to bear only the slimmest relationship to the changing current values of assets and liabilities. Appendix E elaborates.

A.8 is discussed in Appendix I.

A.11 treats the employee compensation package as exogenous to the model and is one of several ways in which the model falls short of being a general equilibrium model. At the end of Section V, we examine an alternative assumption which moves towards a wider equilibrium. Near the end of Section IV, we comment on the non-equilibrium result that arises directly from both the Black and Tepper models and the direction that it implies for the equity risk premium in equilibrium. It should be noted that a tax arbitrage model that does not incorporate governmental effects will never produce a general equilibrium result.

IV Transparent Intermediary Model

_with marketed assets and liabilities, without tax considerations_

Consider a transparent intermediary without taxes:

\[ \text{Let } A_p = \text{intermediary’s assets (all marketed)} \]

\[ \text{Let } L_p = \text{intermediary’s liabilities (all marketed)} \]

\[ \text{Let } E_p = A_p - L_p \]

and, since we have assumed transparency, \( E_p \) is the value of the intermediary.
Consider a business entity with all values at market:

Let $A_B =$ business’ assets, valued at market
Let $L_B =$ business’ liabilities, valued at market
Let $E_B = A_B - L_B$

Attach the intermediary to the business entity (in the sense that a pension plan is attached to a corporation). Such an attachment follows the \textit{augmented balance sheet} concept introduced by Jack Treynor (Bagehot, 1972 and Treynor, Regan and Priest, 1976, 1978). The following is based on Figure 1 in the 1978 article:

| Augmented Balance Sheet (at market value) |
|-----------------|-----------------|
| Assets          | Liabilities     |
| $A_p =$ Pension portfolio | $L_p =$ Present value of pension obligations |
| $A_B =$ Corporate assets | $L_B =$ Corporate liabilities |
| $E = E_p + E_B =$ Corporate equity |

Thus the composite corporate entity is valued at (market capitalization):

$$E = A_B - L_B + A_p - L_p$$

We ask the question, how do shareholders of this corporation (presumed to be optimally diversified) react to changes in the allocation of pension assets and liabilities? We conclude:

\textbf{Result 1:} Shareholders act to offset the pension asset allocation. They are able to do so at no cost and thereby restore their preferred distribution of future wealth. Thus they are indifferent to the allocation of pension assets and liabilities among marketed securities.
Consider a shareholder holding his preferred diversified portfolio including his share of interest in the business and the pension plan with its base case asset/liability allocation. The shareholder learns that the pension plan will sell some of its assets and invest the proceeds in other marketed securities. Because the shareholder investment opportunity set is identical to the pension plan’s, the shareholder will restore his preferred investment allocation by purchasing the securities that the plan sells and by selling the securities that the plan purchases. Naturally, these transactions will be scaled to reflect the fractional ownership of the corporation by the shareholder. As a group, the shareholders will sell (and buy) the same securities in the same amounts as the pension plan buys (and sells). In effect, all the transactions could occur between the plan and the shareholder group with no participation by outside parties.

What happens when the plan liabilities are reallocated (i.e., benchmarked to a different set of securities)? As before, the shareholders have chosen their own portfolios in a fashion that recognizes the long and the short positions of the corporate pension plan. Thus each shareholder will adjust his own portfolio by buying the securities that are added to the liability benchmark and will sell those that have been removed. Later we will consider the offsetting transactions that might be effected by plan participants (Section V) but, for now, we assume that the shareholder transactions will involve third party market participants.

These shareholder reactions preserve the current, and restore the future distributions of, wealth for each shareholder. Thus, we conclude that shareholders are indifferent to the asset/liability allocations of the pension plan provided that the plan assets and liability benchmarks are restricted to marketed securities.

When the investment crediting rate is not marketed

In many of the cash balance plans adopted to date, the investment crediting rate is set once a year to a numerical quantity that may be set arbitrarily or may represent a market rate on an instrument of maturity other than one-year (e.g., the current coupon rate on the ten-year Treasury bond) or may be a rounded or adjusted version of some published rate (e.g. prime rate or the one-year T-bill plus 1%). How shall we understand this in
light of the shareholders’ inclination to maintain a preferred investment strategy? We can interpret it as a riskless profit or loss to the shareholders:

Result 2: When the liability allocation changes from a marketed benchmark to a marketed benchmark offset by a measurable amount, the shareholder wealth changes by the amount of the offset.

Because the rate is set one year at a time shortly in advance of the beginning of the year for which it is effective, the proper portfolio adjustment by the shareholders will entail the purchase of the one-year T-bill on the announcement date of the plan’s investment rate.

If, for example, the plan credits a 7% annual effective rate on the prior year’s balance, and the effective annual rate on the one-year bill is 5%, then each shareholder immediately loses approximately 1.9% (2%/1.05) of his share of the aggregate opening account balances of the plan. In effect the plan is offering employees a near riskless return of 7% when the market rate for such an investment is only 5%. There is no riskless investment strategy that the shareholder can use to reduce this loss.

Suppose instead that the plan credits investment returns equal to the S&P index less 1% annually. In this case, of course, the ending plan balances cannot be computed until the S&P index is evaluated at year end. Nonetheless, the riskless portfolio adjustment calls for shareholders to purchase the S&P index which locks in a gain to the shareholders of about .95% (1%/1.05, assuming the same T-bill rate as above) of the opening plan balances.

*With tax considerations*

Next we consider taxes and define:

\[
\begin{align*}
\tau_c &= \text{corporate tax rate} \\
\tau_{pb} &= \text{personal tax rate on bonds} \\
\tau_{ps} &= \text{personal tax rate on stocks}
\end{align*}
\]
Assumption A.8 states that the effective tax on equity returns is less than the effective tax rate on returns from fixed income:

$$\tau_{ps} < \tau_{pb}$$

We assume that there are no future changes to tax laws. Nor do our agents migrate from bracket to bracket over time. Appendix I outlines why Assumption A.8 is realistic under U.S. tax law. For our purposes it is sufficient to note that effective taxes on personal equity holdings can be substantially less than those on fixed income.

We now apply tax rules to the pension plan, the corporation and the shareholder. We note that contributions to the plan by the corporate sponsor are deductible, within limits, when made and investment returns inside the plan are not taxed. We develop Results 3a, 3b and 3c as three properties of these rules.

Result 3a: A dollar inside the pension plan may be equated, at any point in time, to \$$(l - \tau_c)$$ in value on the balance sheet. Equivalently, \$1 on the balance sheet may be equated to a plan asset of \$l/(l - \tau_c).

At the time of the Black (1980) and Tepper (1981) papers, assets returned to the corporate sponsor (“reverted” in the language of pensions) after plan termination and settlement of all of the accrued liabilities of the plan were subject to income tax at the corporate rate, \(\tau_c\). As discussed in Section II, since 1986, the Metzenbaum excise tax means that assets reverting to an employer from its pension plan are taxed at a much higher rate than that which applied when contributions to the plan were made on a tax-deductible basis.

How, then, may we develop the Black-Tepper assertion that a dollar in the plan is worth \$$(l - \tau_c)$$ on the corporate balance sheet? Recall that we have assumed an ongoing corporation and an ongoing DB plan (note that a CB plan conversion is an ongoing DB plan). An ongoing plan will (see Appendix D), at some future date, be required to make

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9 Consistent with Tepper (1981) and Miller (1977)
contributions. We want to track the impact of a marginal dollar of contributions made at time zero that results in reduced contributions at time \( n \).

First we develop the converse, that \( $(1 - \tau_c)$ \) on the balance sheet may be equated to $1 inside the pension plan. This is trivial since the contribution of $1 to the plan\(^{10}\), results in a contemporaneous tax reduction equal to \( $\tau_c \).

Next we note that, since the flow of contributions to the plan continues over time, the existence of a marginal dollar in the plan will drive out a $1 contribution which would have been tax deductible if made, thus adding a net \( $(1 - \tau_c)$ \) to the after-tax balance sheet.

**Result 3b:** A dollar contributed to a plan a time zero and used to reduce future contributions effectively delivers a pre-tax rate of return to the balance sheet after the payment of corporate income tax.

A corporation contributes $1 of its current *earnings before income taxes* [EBIT] to its defined benefit pension plan. Because this dollar may be deducted from the corporation’s income subject to tax, the net after-tax balance sheet effect of the contribution is a reduction in assets of \( $(1 - \tau_c)$ \). Inside the plan, the $1 grows over time to \( $(1+i)^n \), where \( i \) is the annually compounded untaxed rate of return on invested assets. After \( n \) years, the corporation reduces the contribution that it would otherwise have made for the year by \( $(1+i)^n \). This increases the corporate taxes for the year by \( $(1+i)^n \tau_c \) and thus \( $(1+i)^n(1 - \tau_c)$ \) is the net addition to the balance sheet. Since the net investment \( n \) years earlier was \( $(1 - \tau_c)$ \), the after-tax rate of growth may be seen to equal the annual untaxed rate of return, \( i \).

This result relies on the tax-free accumulation of assets within the pension plan and not upon the deductibility of pension contributions. All that is required with respect to deductibility is that the same rules and rates apply as contributions are made at different

\(^{10}\) It is assumed that the $1 is within the annual deduction limits under IRC Section 404(a).
times\(^{11}\). To see that deductibility per se is unimportant, consider the result above in the case where \(\tau_c = 0\). Since 1986, result 3b has also required that assets do not accumulate to such a degree that they may only be realized at the corporate level after the payment of excise taxes (Appendix D).

When we combine Result 3b with the taxes that shareholders must pay on returns they receive for investing in the company shares, we get:

Result 3c: A shareholder’s marginal investment that is contributed to the corporate pension plan earns the market rate of return over time and is taxed at the personal equity tax rate regardless of whether the pension plan invests in fixed income or in equity securities.

We note that the after-tax return to \(S(1 - \tau_c)\) of shareholder investment (which supports a $1 contribution to the pension plan) is \(S(1 + i)^n(1 - \tau_{ps})(1 - \tau_c)\) and that the tax rates are independent of the nature of the asset allocation within the pension plan\(^{12}\).

**Shareholder optimal policy**

Define:

- \(r\) = the riskless return on the one-year T-bill
- \(\bar{q}\) = the one-year stochastic rate of return on equity investment
- \(\bar{q}\) = the one-year expected rate of return on equity investment
- \(\alpha\) = the fraction of assets invested in indexed equities, balance in T-bill
- \(\beta\) = the fraction of liabilities benchmarked to equities, balance to T-bill

\[\bar{e} = \alpha \bar{q} + (1 - \alpha)r\] = one-year stochastic rate of return on an \(\alpha\)-weighted portfolio

\[\bar{e} = a \bar{q} + (1 - \alpha)r\] = one-year expected rate of return on an \(\alpha\)-weighted portfolio

\(^{11}\) Tepper analyzes the case where contributions may be made in excess of IRC deductibility limits and the resulting deductions must be deferred. We do not address this case.
We consider investment/crediting pairs designated as \( \{\alpha, \beta\} \), where each variable is restricted to the range \([0,1]\)\(^{13}\), and ask whether there exists a shareholder optimal pair.

Note that these pairs admit no offsets from the marketed benchmarks and thus, absent tax considerations, shareholders should be indifferent among them. Recall that we have assumed that employee compensation and satisfaction will not vary with the definition of the liability benchmark and that we explore variations to this assumption at the end of this section. Thus all demonstrable differences in shareholder wealth attributable to the cases above must derive from the tax treatment that attaches to each pair.

Result 4: Shareholders gain as \( \alpha \) is decreased and as \( \beta \) is increased. With each variable restricted to the range \([0,1]\), the optimal investment/crediting pair is \( \{\alpha = 0, \beta = 1\} \), i.e., the plan invests entirely in T-bills and credits equity returns on employee account balances!

Following Tepper, we assume that shareholders offset pension allocation decisions in their personal portfolios after adjusting for corporate taxes by multiplying by \((1 - \tau_c)\). So, for example, a $1 increase in pension equities which is accompanied by a $1 decrease in pension T-bills will be offset by personal portfolio transactions aggregated for all shareholders: sales of \$(1 - \tau_c)\) of equities and purchases of the same amount of T-bills.

The shareholder personal transactions restore all cash flows to shareholders prior to the payment of taxes on personal portfolio income. Thus we can measure the effectiveness of any pension asset or liability allocation by looking to the taxes paid after the shareholder offset transactions are effected.

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\(^{12}\) We keep as our consistent measure $1 of pension assets or \$\text{\(1 - \tau_c\)}\) of corporate assets. 

Tepper (1981) is inconsistent in this regard as will become clear in our results reconciliation.

\(^{13}\) This range is arbitrary but convenient. We can certainly design crediting and investing strategies that would extend outside these boundaries. The linearity of the arbitrage results makes the implications obvious. At some point, within or without this range, the linearity must fail as we exhaust the opportunity for tax gains or as the asset-liability mismatch raises the probability of cash flow crises and bankruptcy above a negligible level.
In order to normalize our analysis, we begin with all assets and liabilities in T-bills \( \{\alpha = 0, \beta = 0\} \) which we identify as the base pair. Because Tepper and Black assumed that all liabilities could be represented by fixed income securities (i.e., \( \beta = 0 \)) and concluded that all plan assets should be invested in fixed income (\( \alpha = 0 \)), our base pair matches their preferred choice.

In this base case, shareholders will pay taxes at their personal stock tax rate, \( \tau_{ps} \), based on the net income of the pension plan diminished by the corporate tax:

\[
\tau_{ps}(1-\tau_c)rE_p .
\]

(1)

Suppose that the pension plan is now allocated as \( \{\alpha, \beta\} \). Shareholders act to offset this allocation by selling \( S(1-\tau_c)(\alpha A_p - \beta L_p) \) of equities and purchasing the same amount of T-bills. They pay taxes on their income generated by the pension plan:

\[
\tau_{ps}(1-\tau_c)[(\tilde{q}-r)(\alpha A_p - \beta L_p) + rE_p]
\]

and they pay additional taxes on the offsetting personal transactions:

\[
(1-\tau_c)(\alpha A_p - \beta L_p)(r_{tb} - \tilde{q} \tau_{ps})
\]

a total of:

\[
\tau_{ps}(1-\tau_c)rE_p + (\tau_{pb} - \tau_{ps})(1-\tau_c)r(\alpha A_p - \beta L_p)
\]

(2)

where the first term of (2) may be recognized as the base case (1) and so the second term represents the incremental taxes associated with the change in pension allocation. Since we have \( \tau_{pb} - \tau_{ps} > 0 \), taxes increase with \( \alpha \) and decrease with \( \beta \). Without leverage, taxes are minimized with \( \{\alpha = 0, \beta = 1\} \) which means that our shareholder optimal pension investment is 100% in T-bills with the liability crediting rate.
benchmarked 100% to an equity portfolio. The maximum tax case is presented by \( \{\alpha = 1, \beta = 0\} \). The typical corporate plan today may be identified as \( \{\alpha, \beta = 0\} \) which constitutes an inferior strategy for shareholders which becomes progressively worse with increasing \( \alpha \). Note that the base case \( \{\alpha = 0, \beta = 0\} \) is superior to the typical plan, as is shown by both Tepper and Black. A locus of cases equivalent to the base case is traced out by strategies that follow \( \{\alpha = \frac{L_p}{A_p}, \beta \} \). These are also the cases where the pension plan’s effect on corporate earnings is most certain.

**Reconciliation with Tepper**

The Tepper paper assumes \( \beta = 0 \) at all times. Thus we need to reconcile his Table 1 to the after-tax shareholder values that we develop with the same assumption. In addition, we restate the incremental part of (2) in the perpetuity form favored by Tepper and Miller by dividing by the after-tax riskless rate of return, \( r(1-\tau_{pb}) \):

\[
\frac{(\tau_{pb} - \tau_{ps})(1-\tau_c)(\alpha A_p)}{1-\tau_{pb}}
\]

Since we are considering an asset allocation change without any change in plan funding, our treatment may be compared to the difference between the top and bottom rows of Tepper’s Table 1. This amount is the same regardless of financing and is shown in box (3) of Table 1. We recognize that \( \alpha A_p \) when \( \alpha = 1 \) is the same as what Tepper defines as \( F \). When \( \alpha = 0 \), this goes to zero. Because this equation measures incremental taxes, the perpetuity value received by the shareholder when the plan switches from equity assets to debt assets is:

\[
\frac{(\tau_{pb} - \tau_{ps})(1-\tau_c)F}{1-\tau_{pb}}
\]  

which differs from Tepper’s quadrant (3) by a factor of \( (1-\tau_c) \). We have developed our equations based on the dollar amount of pension assets and liabilities and have tracked
them through to the shareholder. Thus, we contend \( \frac{(\tau_{pb} - \tau_{ps})(1 - \tau_c)}{1 - \tau_{pb}} \) represents the shareholder gain from switching one pension dollar from equity to debt. Recall that a pension dollar must be multiplied by \((1 - \tau_c)\) to determine its equivalent balance sheet value.

Tepper defines \( F_S \) and \( F_D \), and implicitly \( F \) as well, as dollars of pension fund investment but he uses them as though they were the balance sheet equivalent. Thus to reconcile to Tepper, we must start with his gain equation and modify it accordingly:

\[
\text{Gain} = \frac{1}{r(1 - \tau_{pb})}\{(rF_D + \rho F_S)(1 - \tau_{ps}) - rL(1 - \tau_c)(1 - \tau_{ps}) - r(F_D - L)(1 - \tau_{pb})\}
\]

where we have replaced an "\( e \)" with the equivalent "\( \rho \)". This allows cancellation and a simpler restatement:

\[
\text{Gain} = \frac{1}{r(1 - \tau_{pb})}\{rF_D(1 - \tau_{ps}) - rL(1 - \tau_c)(1 - \tau_{ps}) - r(F_D - L)(1 - \tau_{pb})\}
\]

Next we multiply \( F_D \) by \((1 - \tau_c)\) as indicated above so that we may properly deem \( F_D \) to be pension assets rather than their balance sheet equivalent:

\[
\text{Gain} = \frac{1}{r(1 - \tau_{pb})}\{rF_D(1 - \tau_{ps})(1 - \tau_c) - rL(1 - \tau_c)(1 - \tau_{ps}) - r(F_D - L)(1 - \tau_{pb})\}
\]

and rearrange terms:

\[
\text{Gain} = \frac{1}{(1 - \tau_{pb})}\{F_D(1 - \tau_{ps})(\tau_{pb} - \tau_{ps}) - L((1 - \tau_c)(1 - \tau_{ps}) - (1 - \tau_{pb}))\}
\]
By letting \( F_D = F \) and \( L = 0 \), we get our (3). Next we consider that the borrowing, \( L \), necessary to support a pension investment of \( F \), need only be equal to \( F(1-t_c) \) and so we set \( L \) to that value and we let \( F_D = F \) again to derive:

\[
\text{Gain} = \frac{(1-t_c)F(1-t_{ps})}{(1-t_{pb})}
\]

which differs from Tepper’s quadrant (1) by the same \((1-t_c)\) that we used to adjust quadrant (3). Lastly, with the same value for \( L \) and letting \( F_D = 0 \):

\[
\text{Gain} = (1-t_c)F(1-\frac{(1-t_c)(1-t_{ps})}{(1-t_{pb})})
\]

we get our modification of Tepper’s quadrant (2). This may be compared to Miller’s equation for “the gain from leverage, \( G_L \), for the shareholders in a firm…”\(^{14}\):

\[
G_L = (1-\frac{(1-t_c)(1-t_{ps})}{(1-t_{pb})})B_L
\]

where \( B_L \) is the amount of corporate borrowing. If \( B_L \) is the amount of borrowing necessary to support \( F \), it must be that \( B_L = (1-t_c)F \).

**The Black variation**

Fischer Black (1980) developed a strategy where the tax benefits of investing pension assets entirely in bonds arise at the corporate balance sheet level. This version would seem to provide greater incentive to corporate managers than does the Tepper approach which relies on shareholder action. As we show below, the basic Black approach (he offers more than one) may rely on shareholder action as well.

Black proposes that a corporation:

\(^{14}\) Miller (1977), pp. 267.
• Sell all stock held by its defined benefit pension plans.
• Invest the plan assets entirely in taxable fixed income securities.
• Borrow, on the corporate balance sheet, \((1-\tau_c)\) times the amount transacted in the pension plan.
• Use the borrowed funds to repurchase the corporation’s own stock.

Black argues from the augmented balance sheet perspective of Treynor (i.e., he assumes transparency and ignores some ERISA technicalities and existing bond covenants) that lenders should be willing to provide the funds after they recognize that these transactions are leverage-neutral. As we do here, he considers well-funded pension plans maintained by corporate sponsors where bankruptcy is deemed to have a very low probability.

Black parses the four transactions as two pairs:

• Sale of pension plan stock holdings and purchase of \((1-\tau_c)\) as much of the corporation’s own stock.
• Borrowing of \(\$(1-\tau_c)\) on the balance sheet to support each \$1 of bond purchases (equal to stock sales) inside the pension plan.

As we do, he equates \$1 in the plan with \(\$(1-\tau_c)\) on the balance sheet and makes the necessary supporting assumptions, some explicitly, others implicitly. The stock transactions above have no tax implications since neither pension plan transactions nor the corporate transactions in company stock are taxable. The entire tax effect is thus derived from borrowing at the after-tax rate \((1-\tau_c)r\) while earning the pre-tax rate \(r\). The after-tax annual gain is \(\tau_c(1-\tau_c)rA_p\) with a perpetuity value of \(\tau_cA_p\). Black comments that, with a 50% combined federal-state value for \(\tau_c\), this is equivalent to borrowing \((1-\tau_c)A_p\) in perpetuity without ever paying interest or principal. This is comparable to the results in our numerical example in Section V.
Black observes that the sale of diversified equities by the plan accompanied by the repurchase of company stock does not constitute a perfect hedge. He says that the company is now more idiosyncratic, which should not be a problem in diversified shareholder portfolios, and he asks “would many investors pay five percent per year for the sake of added diversification within their holdings of a single firm’s stock?”

He suggests an alternative approach which provides a better hedge at a small tax cost. Instead of repurchasing one’s own stock, he proposes that the proceeds of the corporate borrowing be invested in diversified balance-sheet equity implemented via a mutual fund designed to convert capital gains to dividends.

Stock diversification within a single firm

We look at a model of the basic Black proposal in order to examine the stock diversification issue more closely. As we did with the Tepper model, we will look first at the transaction without regard to taxes. For ease of exposition, we will not look directly at the “undiversification” that arises when a company sells diversified stocks and buys its own. Instead, we will assume a starting place where no company owns shares of another and consider what occurs when a company issues its own stock in order to purchase shares of others.

Because we are looking at the narrow issue of stock diversification only, we will assume, without loss of generality, that the assets of corporations are financed exclusively with equity:

\[
E_i = E_{B,i} + E_{P,i} = A_i = A_{B,i} \\
L_i = L_{B,i} + L_{P,i} = 0
\]

---

15 Thus the Sharpe (1976) PBGC pension put is not considered. See Appendix D
16 Computed as $r_c \tau_c$ in an era where the riskless rate might be ten percent and the combined federal-state tax rate might be fifty percent.
17 In a tax regime where 85% of dividends received by a corporation from other corporations was tax exempt. Today this rate is 70% (IRC Section 243). Corporate capital gains are taxed at the $\tau_c$ rate.
where the subscript $i$ added to the earlier notation for the augmented balance sheet, identifies the $i^{th}$ firm of $n$ such firms ($i = 1, n$). $E_i$ is the equity issued by company $i$ which may also be labeled “market capitalization” and $A_i$ are the business assets of firm $i$ which may alternatively be identified as the “value of the firm.” Since we begin with no pension plan assets, we can equate the market capitalization and the ex-pension value of the firm. We normalize by assuming that each company issues one share of $E_i$ at a per share value equal to the value of the firm, $A_i$. The equality of $A_i$ and $E_i$ may seem redundant at the moment but we intend that $A_i$ will not change as new shares of $E_i$ are issued to finance pension investments in the equity shares of other firms, $E_j$.

Before any company buys the stock of another, public investors rationally create preferred portfolios of equities by weighting their investments in each of the companies. An example of such a procedure is embodied in the capital asset pricing model (CAPM) where asset weightings based on a covariance matrix minimize portfolio risk for a given level of portfolio expected return. We rely on less restrictive investor rationality\(^\text{18}\): we do not posit any optimization scheme, merely requiring that, given a single set of portfolio choices, the investor always makes the same selection. The essence of the selection process is that investors compute preferred weights for their exposure to the components of the $n \times 1$ firm value vector:

$$A = \{A_i\}$$

With $m$ shareholders indexed by $k$ ($k = 1, m$), we define a matrix of weights, $G = \{g_{k,i}\}$ where each $g_{k,i}$ is a fraction that is applied to the business assets, $A_i$, of company $i$ to compute $k$’s holding. The aggregate investor wealth equals the aggregate market portfolio:

$$GA = i_n^T A$$

where $i_n$ signifies an $n \times 1$ vector of ones. Naturally each column of $G$ sums to unity.

Next we suppose, still without tax concerns, that the pension plan of each company acquires stock in other companies. The sponsor company finances the purchase by issuing more of its own equity. While this might be done by exchanges among the firms alone, we will assume that additional shares are issued to public investors who finance their purchases by selling some of their own diversified portfolios to the issuer. We look at the case where company \( i \) issues its own shares in an amount sufficient to purchase a fraction, \( f_{ij} \), of the share \( E_j \) of company \( j \). After this exchange, the number of issued and outstanding shares (and the market capitalization) of the diversifying company will have increased:

\[
E_i = (1 + \frac{f_{ij}E_j}{A_i})A_i = A_i + f_{ij}E_j
\]  

(4)

where the parenthetic expression represents the number of outstanding shares for company \( i \). Noting that, for company \( i \), all of its shares are held by the public, we also define:

\[
E_i^* = E_i
\]

as the portion of the market capitalization of company \( i \) held by the public (i.e., not cross-owned by the other corporations). Using this notation for company \( j \) as well, but observing that the public no longer holds all of its outstanding shares, we see:

\[
E_j^* = (1-f_{ij})E_j = (1-f_{ij})A_j
\]

and

\[
E_i^* + E_j^* = A_i + A_j
\]

Letting:

\[
E^* = \{E_i^*\}
\]

\[
E = \{E_i\}
\]

we conclude that:

\[
l_n^TE^* = l_n^TA \text{ and } l_n^TE \geq l_n^TA
\]
apply generally as companies' pension plans buy, but do not sell short, each other's shares. Thus we see that, although the business asset base has not expanded, the total market capitalization seems to have increased through cross-ownership. But the "market portfolio", which consists only of those shares held by public investors, still owns, in the aggregate, exactly all the assets that we have designated as \( A \).

There is no reason to conclude that, after these issuer-initiated transactions, each investor will be left holding the same weightings of \( A \) as he did before, but it must be the case that the preferred portfolio for each investor may still be computed using the original weights on each \( A_i \). It is also the case that the investor will be able to weight (long and short) positions in the shares contained in \( E^* \) in order to replicate his preferred holdings of \( A \) (and that the value of each investor's portfolio will be as before). The details of this replication are shown in Appendix F.

What then has happened? Each of the original companies has now become a diversified mutual fund (and, if they have substantially diversified, the CAPM beta for each company will have moved from its original value to a value closer to one). There is no change in the equilibrium exposure to real assets among shareholders. What this tells us is that, absent tax considerations, companies that wish to diversify across the market may do so by purchasing each others' stock without cost to the system\(^{19} \). What both Black and Tepper tell us is that, when taxes are taken into account, companies stand to gain or lose as they diversify depending on how they do so\(^{20} \). More specifically, they show that holding equity within tax-sheltered defined benefit pension plans is costly.

We have just seen that, with our assumptions, a company should not be concerned that it does not internally diversify since shareholders will be able to achieve their own diversification. This means, however, that the Black variation, like the Tepper, relies on shareholder actions. The distinction between the two, therefore, is not in the need for shareholders to react, but rather in the location of the tax benefits. Tepper captures

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\(^{19}\) Our use of the no taxes, no bankruptcy assumptions is necessary to support this statement.

\(^{20}\) One can hypothesize a comprehensive equilibrium, inclusive of a government which must collect an immutable amount of corporate taxes, in which internal diversification would produce no gains or losses.
them in shareholder personal accounts because the equity/fixed reallocation occurs in both the pension plan and shareholder portfolios. Black captures them at the corporate level because shareholders need only realign their equity holdings while the corporate equity/fixed reallocation occurs both on the balance sheet and within the pension plan.

**Black in a cash balance plan**

As noted above, the sale of diversified stocks by the plan and the repurchase of company stock is not a hedge. Under a cash balance plan, as with any other defined benefit plan, the sponsor who wishes to sell pension equities and buy its own stock will not be hedging (we have seen, however, that shareholders can correct this on their own). But our cash balance proposal goes one step further and calls for the promise of equity returns to participant accounts and the repurchase of additional company stock.

This suggests an approach to plan design that has not yet been tried: a company may choose to credit the total return on its own stock to employee plan balances\(^{21}\). This approach would, with respect to the liability side of the plan, constitute an exact hedge at the company level after allowance for corporate taxes (multiplying plan shares by \((1-\tau_c)\) to compute balance sheet repurchases). In the tradition of new designs in the employee

---

\(^{21}\) This raises a number of ERISA issues. Employee benefits that are dependent on employer stock performance usually are qualified as Employee Stock Ownership Plans (ESOPs). Some non-ESOP defined contribution plans provide that some of the assets will be invested in the sponsor’s stock and thus employee accounts are dependent on the employer stock performance. Defined benefit plans are generally restricted to investing no more than 10% of their assets in the stock of the plan’s sponsor. While even the settled ERISA issues are well beyond the scope of this paper, this approach raises ERISA issues that have never before been addressed.

\(^{22}\) The similarity to the actual issuance of company shares will undoubtedly raise issues under the jurisdiction of the Securities and Exchange Commission (SEC).

\(^{23}\) While plan sponsor’s have often encouraged employee ownership of company stock with and without the use of qualified retirement plans, companies have never “shorted” their own stock in any such program (they have, however, often used shares repurchased or held as treasury shares for that purpose). The idea that shorting the sponsor’s stock inside a defined contribution plan might reduce the risk that employees face because their earnings and their savings were highly correlated has always surrendered to the motivational advantages claimed for increasing that risk. Additionally, the public relations implications of a company or its employees shorting its stock has always resulted in an automatic declination of such ideas. In this instance, of course, the company would be shorting its stock to its employees (thus putting them in the same position as shareholders) who, as before would be long the stock and face the correlated risk thus implied. Lastly, it would seem that the “shares” implicitly “sold to participants” using this approach should result in an accounting dilution.
benefit arena, such a plan might naturally be dubbed a CBSOP, thus highlighting its ESOP-like features.

**Upsetting the equilibrium**

We look again at the Black and Tepper proposals and ask whether, with tax considerations but without governmental reaction, the undiversification can occur without upsetting the market equilibrium. Note the \((1-\tau_c)\) factor in each of the transaction pairs proposed by each writer. Because of this factor, we see that the transactions increase the demand for fixed income securities and reduce demand for equities. Although we have insufficient information to determine the magnitude of the resulting price changes, the direction is clear. If every DB sponsor followed Black or Tepper, equity prices in general would decline and so too would corporate borrowing rates. We can anticipate that corporations would borrow more and issue less equity but that the new equilibrium would still be characterized by lower values for \(r\) and higher values for \(\bar{q}\).

Viewed from an earlier starting point, we may conclude that investment in equities by corporate pension plans has provided, ceteris paribus, equity investors with too large a premium to date and, if Black or Tepper were followed forward, a lower future premium.

**Reconciling Black and Tepper**

Black's after-tax gain on the balance sheet is:

\[
r(1-\tau_c)\tau_c
\]

annually per $1 of pension assets reallocated from stocks to bonds. This is equivalent to a shareholder after-tax gain of:

\[
r(1-\tau_c)(1-\tau_{ps})
\]

Tepper's shareholder after-tax gain:

\[
r(1-\tau_c)(\tau_{pb}-\tau_{ps})
\] (5)
The Tepper gain less the Black gain:

\[ r(l^{-\tau_c})[(\tau_{pb}-\tau_{ps})-\tau_c(l^{-\tau_{ps}})] \]

Which equals zero when:

\[(\tau_{pb}-\tau_{ps}) = \tau_c(l^{-\tau_{ps}})\]
\[(l^{-\tau_{ps}})-l^{-\tau_{pb}} = \tau_c(l^{-\tau_{ps}})\]
\[(l^{-\tau_c})(l^{-\tau_{ps}}) = (l^{-\tau_{pb}})\]

which is Miller’s formulation for leverage-indifferent tax rates. When the LHS, representing the shareholder cost of corporate borrowing is less than the RHS, representing the cost of shareholder personal borrowing, there is an advantage to borrowing at the corporate rather than at the personal level. When there is an advantage to borrowing at the corporate level, the Black gain exceeds the Tepper gain and vice versa. This makes sense because the Black proposal borrows on the corporate balance sheet while the Tepper version relies on personal borrowing (investor sales of fixed income securities).

Result 5a: Absent gains from leverage, the gains from tax arbitrage using the Black and Tepper approaches are identical.

Because not all shareholders actually have the same marginal tax rates, some shareholders may prefer the approach of Black and others prefer that of Tepper. An important special case arises for tax-exempt institutions (including, perhaps misguidedly, pension plans). In this case:

Result 5b: When the shareholder is tax exempt, \( \tau_{ps} = \tau_{pb} = 0 \), and \( \forall \tau_c > 0 \) \( \text{LHS} < \text{RHS} \) and the Black proposal is preferable.
**Black and Tepper gains merely offset losses**

The common strategy for both CB and DB plans consists of a fixed income promise combined with an investment strategy that includes equities. Black and Tepper describe their proposals to invest the pension fund entirely in fixed income securities as a “gain”. Certainly it represents a comparative gain vis-a-vis common practice, but we prefer to describe it as a recovery of losses created by an ill-advised equity investment strategy.

We do not make this characterization arbitrarily. It may be developed by an *ab ovo* look at the exchange of ordinary compensation for a pension benefit of any sort (DC, DB or CB). Suppose a company creates a DC plan and contributes a percentage of each employee’s pay in lieu of an equal amount of compensation. Such a plan would preserve total compensation cost for the company on both pre- and post-tax bases. The employees, however, might benefit from the tax deferral. To the extent that they have alternative tax-advantaged savings (e.g., pre-tax IRAs) available, they should not be expected to accept a reduction in total compensation. To the extent, if any, that the pension plan extends their tax advantages or that they value professional asset management and administrative convenience, total compensation may be reduced. Without denying this possibility, for our analytic purposes, we will assume that total compensation is unaffected by the plan creation.

Thus with a DC or our DC/CB clone arrangement (defined in Appendix A), the tax advantages inure to the benefit of the employees. In the case of a DB plan with contributions equal to annual increases in the value of accrued benefits, and with fixed income investments matching any benefits promised, we are in the same position as in the DC and DC/CB plans. If the corporation subsequently decides to sell the fixed income assets and to buy equities, as so many DB plan sponsors did during the period when trusteed plans replaced insured plans, needless taxes are thereafter inflicted on shareholders with no effect on employees. A shorthand synopsis of this is:
Result 6: Our base case, \( \{0,0\} \), is identical to the Tepper and Black proposals is identical to cash compensation\(^{24}\).

Viewed from this perspective, it is clear that the first opportunity for substantial shareholder gains from pension tax arbitrage arises with the advent of the cash balance plan and the concomitant power to set the value of \( \beta \) to a value greater than zero.

V Implications

The obvious first implication is that companies should invest all defined benefit pension plan assets in taxable fixed income, as observed by both Tepper and Black. Further, with a cash balance plan, shareholders should desire an equity benchmark for the plan’s investment crediting rate. The first subsection below presents a numerical example of the value of such decisions. Then, because the Tepper and Black proposals are essentially one period, and because, with the extension into the CB arena, the choice of plan design affects the evolution of the tax gain opportunity, we look at some plan dynamics. These effects, which derive from mismatches between plan assets and liabilities, are then generalized and represented with a graphical assist. In the last subsection, because the assumption of employee indifference in A.11 is narrow, we look at employee preferences and at a plan structure that includes employee choices with respect to the investment crediting rate.

A numerical example

With the top personal federal tax rate now equal to 39.6% and the corresponding corporate rate equal to 35%, we can use the following assumptions (Appendix I) to develop a numerical example of the value of shareholder optimal design of a CB plan:

\[
\tau_{pb} = .4 \\
\tau_{c} = .35 \\
\tau_{ps} = .15
\]

\(^{24}\) Except for the value, if any, reflected off of employee utility gains and further excepted for the
We add the assumption that $\alpha = .6$, the "classic" 60:40 asset allocation for DB plans. Using the second term of equation (2) we can compute taxes in excess of those on the base case:

$$(.4-.15)(1-.35)r(.6A_p) = 9.75\% \text{ of } rA_p$$

This may be compared to the shareholder optimal allocation $\{\alpha = 0, \beta = 1\}$ and resulting tax where the minus sign indicates a deduction from the base case:

$$(.4-.15)(1-.35)r(-L_p) = -16.25\% \text{ of } rL_p$$

If, by chance, the assets of the plan equal the liabilities (total account balances), the loss of potential value to the shareholders is 26% of the riskless return on the plan. In the perpetuity form of Miller and Tepper, the value of such additional annual returns is:

$$\frac{.26rA_p}{r(1-\tau_{pb})} = .4333A_p$$

So that the shareholders of a corporation with a $1 billion CB plan crediting the T-bill rate will give up $433 million of after-tax present value. Consider the common case where the plan credits 1% over the T-bill rate. This adds:

$$\frac{(1-\tau_c)(1-\tau_{pb})(.01)L_p}{r(1-\tau_{pb})} = .5525(.01)L_p = \left(\frac{.00921}{r}\right)L_p$$

to the shareholder loss. With the riskless rate less than 5%, this exceeds $184$ million.

This total after-tax loss of $617$ million may be compared to a hypothetical case in which the plan liabilities are instantly and miraculously erased and where the plan assets are then returned to the corporation without being subjected to the enormous post-1990 excise tax\textsuperscript{25} on plan reversions without replacement plans. In such a most-

\textsuperscript{25} gains (losses) attributable to the tax shelter applied to positive (negative) pension equity $E_p$, 50% in addition to the ordinary income tax rate of 35%, IRC Section 4980.
extraordinarily-favorable case the corporation would pay $350 million in taxes leaving $650 million. Even if shareholders were taxed at only the 15% rate developed above (for a presumably longer holding period), the net proceeds would then be $552.5 million. In effect, the misdesign and misinvestment of the CB plan is more costly than the instantaneous destruction of the entire $1 billion of plan assets!

**Some dynamic considerations**

With our assumptions, the tax gains from setting \( \{\alpha, \beta\} \) optimally and making the offsetting adjustments are developed one year at a time with certainty at the balance sheet or shareholder level. Using the Tepper approach, the one year certain gain is given by:

\[
(\tau_p - \tau_0)(1 - \tau_c)r(\alpha_0 A_p + (1 - \beta_0)L_p)
\]

adapted from equation (2). \( \{\alpha_0, \beta_0\} \) represent the investment allocation parameters before they are set to \( \{0,1\} \). This gain is increasing in both \( A_p \) and \( L_p \) which simply verifies that, for a suboptimally managed plan, there are gain opportunities on both sides of the pension balance sheet. It also implies that, ceteris paribus, larger gains are possible with larger plans. Observe that \( L_p \) is the primary driver in this since all actuarial processes impel \( A_p \) to pursue \( L_p \) asymptotically.

We have thoroughly explored the tax gain opportunities for a single period where we know the values of \( A_p \) and \( L_p \). In the Tepper paper, the one-year case may be extended to a perpetuity because the liabilities grow with the population (which he implicitly assumes to be stationary) and with the risk free rate. In the CB case, the liabilities may grow more rapidly and thus we ask, what can we say about the progress of \( A_p \) and \( L_p \) over time?

We want to look at five plan events that impact the future values of \( A_p \) and \( L_p \) and sign the shareholder effects for these events for three allocation cases: 1) \( \{0,1\} \), 2) \( \{1,0\} \) and 3) \( \{0,0\} \). We know that the exchange of pension benefits for cash compensation,
ignoring secondary benefits for shareholders reflected off the employees, is profitable to shareholders in case 1, unprofitable in case 2 and a wash in case 3. The five events are: i) new compensation credits (sans funding), ii) funding, iii) retiree and teminee withdrawals, iv) investment growth in assets and v) investment growth in liabilities.

- **New compensation credits** (unaccompanied by funding) – imply an increase in next year’s $L_p$ and lead to a loss in case 2 and 3 and a wash in case 1. In cases 2 and 3, the company has retained the unpaid compensation on its balance sheet in exchange for pension promises (account balance increments) to employees, effectively a borrowing. But it is tax-inefficient to borrow inside the plan where the pre-tax cost of such debt will be delivered to the balance sheet on an after-tax basis. If equity is the liability benchmark, as in case 1, the corporation (Black) or the shareholders (Black and Tepper) will react to effect a no-risk tax wash.

- **Funding** leads to a gain in cases 1 and 3 and to a wash in case 2. In cases 1 and 3, cash is removed from the corporate balance sheet and invested in taxable fixed-income assets inside the plan. The adjustment is on-balance-sheet borrowing (Black) and the result is after-tax borrowing supporting pre-tax rates of return. In case 2, the offset is sales of shares such that the pre-tax return on equity is offset by a matched pre-tax cost of equity capital. This finding is consistent with assertions by both Black and Tepper that funding is gainful only if invested in taxable fixed-income assets.

We find it natural to combine new compensation credits and funding and to go one step further and define **contemporaneous funding** to be the exact amount of new (presumed to be currently and algebraically deductible – i.e., taxable without excise tax if negative) corporate contributions necessary to result in:

$$E_{p,t+1} = (1+r)E_{p,t}$$

which might equivalently be defined as funding equal to the new compensation credits plus (minus) the investment credits in excess of the risk free rate minus (plus) the excess investment returns on assets:
where \( i_A \) and \( i_L \) each take on the value \( q \) or \( r \) depending on the plan strategy. The first line above shows that all excess returns on assets and liabilities are immediately recognized in the funding calculation\(^{26}\). The last line shows that returns on assets fully reduce the amount of funding.

The combination of these first two events is just like the existence of the plan itself. That is, the net result is added future gain in case 1, added future loss in case 2 and a wash in case 3.

- **Retirements and withdrawals** cause equal reductions in \( A_p \) and \( L_p \) and thus reduce future tax gains or losses depending on the case. The impact is the opposite of the effect that arises from new compensation credits that are contemporaneously funded without regard to investment credits and investment returns.

- **Investment growth in assets** has no impact on the tax gain for the concurrent period because we have assumed full hedging. For the same reason, it creates no economic gain or loss to shareholders other than that identified herein as the tax gain. For a given level of \( L_p \), an asset-side investment gain has the same affect as an equivalent amount of contemporaneous funding. But for such a given \( L_p \) it replaces an equivalent amount of funding and, to avoid double counting, we must recognize this entire event as a wash.

\(^{26}\) This follows neither actuarial nor accounting practice, but is consistent with the economic logic applied throughout this paper.
For example, consider case 2, \{0,1\}, and assume that assets earn \( k = (q - r)A_p \) more than the risk free rate. Because of full hedging and because the assets have exceeded the risk free return by \( k \), we must have \( k(1-\tau_c) \) less in resources outside the plan. But this is exactly reversed by contemporaneous funding (other than the concurrent tax effect, which remains external to the plan).

- **Investment growth in liabilities** has an effect that is very similar to the granting of additional compensation credits which is to say that it leads to a loss in cases 2 and 3 and to a wash in case 1. Unlike compensation credits, however, it is not costly to the firm because of the assumed hedge. Further, because it compels funding, the effect of an excess liability return is to increase the subsequent tax gain opportunity.

Looking at the last two events ensemble, in case 3, nothing happens. Although the assets and liabilities are larger or smaller depending on the direction of investment returns, the design strategy forces future gains and losses to be unaffected (except for the risk free return on \( E_p \) which itself is merely compounding at the risk free rate with no adjustment for the swelling or contracting plan size). If, we are operating in case 1 or 2, however, we find that when asset returns exceed liability returns, we lose less (case 2) or gain less (case 1) in subsequent periods. When liability returns exceed asset returns, we lose more (case 2) or gain more (case 1) thereafter.

**Result 7a:** The tax loss strategy \( \{1,0\} \) gets worse when liabilities grow faster than assets but the expected case is that assets will grow faster than liabilities and thus some of the damage is expected to be self-mitigating.

**Result 7b:** The tax gain strategy \( \{0,1\} \) gets better when liabilities grow faster than assets and, since this is the expected case, the gains are likely to be self-enhancing.
A general and graphical look at strategy

Figure 1 provides a graphical view of some of the issues discussed in the previous subsection. The origin represents the base case, \( \{0,0\} \), and the most prominent line represents the locus of points with the same tax and hedging implications as the base case. It is given by \( \{\alpha,\alpha \frac{A_p}{L_p}\} \) with axes defined by \( \{\alpha,\beta\} \). The line will pass through the point \( \{1,1\} \) iff \( A_p = L_p \). A plan with assets greater than 100% of the account balances will cause the line to intersect the top of the chart space, while assets less than 100% imply an intersection with the right hand space border. The example presented has assets equal to 125% of liabilities.

The lines parallel to the base case locus represent equivalence classes for taxes and hedging implications. The “Plus 10%” case contains all points where \( \beta - \alpha \frac{A_p}{L_p} = 10\% \) which is a tax gain strategy. The “Minus” cases represent tax loss strategies. This allows us to observe that all favorable strategies are found to the upper left with the unfavorable strategies to the lower right.

Along the base case line \( E_p \) grows at the riskless rate (assuming new compensation credits are contemporaneously funded) and the tax situation in the following year will be no different from the current year regardless of the performance of equities relative to the riskless rate. All other strategies create uncertainty with respect to the potential tax situation in the following year and the uncertainty is proportional to the displacement relative to the base case. Thus all strategies in an equivalence class (e.g., all those on the “Plus 10%” line) will lead to the same increase or decrease in the tax potential for the following year. When equity provides a positive excess return, “plus” strategies become more “plus” while “minus” strategies become less minus. This extends Result 7a to all minus strategies and Result 7b to all plus strategies. The opposite occurs when the excess return on equities is negative.

This suggests that the sponsor of any CB plan (unless substantially overfunded or underfunded as discussed in Appendix D) that is presently in the lower right should alter the plan to move to the left and upward at least as far as it takes to locate along the base
case line (i.e., Black-Tepper is generally a winning strategy compared to most DB plans). Plans that are on or above the base case will provide greater tax gains to shareholders if they are moved further upwards and leftwards but the sponsors will have to consider the likelihood that they will encounter some of the Appendix D concerns.

**Employee choice plans – contract improvement**

The model presented so far treats employee compensation exogenously, assuming that the equity fraction of the liability crediting rate, $\beta$, is inconsequential to the determination of other compensation. In this subsection we examine the role that $\beta$ might play in the employment contract.

We describe a new $\beta$ as "contract-improving" if shareholders and/or employees gain without loss to the other group. This will be achieved at the expense of taxpayers in general. Because employee preferences may depend on individual access to investment resources, degree of risk aversion, tax bracket and anticipated period of deferral, we consider the determination of $\beta_i$ by employee $i$ in the context of a plan that permits individual choice. Because taxable shareholders gain and employees lose with increasing $\beta_i$, our starting place for potential improvements is the $\beta_i = 0$ plan.

We will generally find a rational opportunity for contract improvement when the shareholder gain exceeds the employee loss for some employees. When employees are heterogeneous, it may be optimal for shareholders to chose a level of incentive that will motivate only a subset of the motivatable employees.

After considering rational opportunities, we will look at the experience in employee-choice 401k plans and suggest some practical ways for employers to use employee choice for the benefit of shareholders.

**Rational opportunities**

There are two possible sources of contract improvement that we will rule out because each would allow arbitrage opportunities unrelated to the selection of $\beta$. The first of these would entail the exchange of taxable wages for qualified plan benefits. It always
seems that tax gains may be generated by salary reduction and benefit increases. But it is clear that this process is limited in at least two ways: i) tax code provisions and ii) utility costs that are unspecified in our model; many employees simply do not want more than a limited exchange of future tax benefits for loss of current income. We will assume, therefore, that no further compensation reductions will be part of any contract improvement that we identify.

A second source is a violation of the Miller indifference condition. If the equation does not apply, then we might seem to generate shareholder benefits by altering the corporate capital structure. We have seen (Appendix I) that the indifference equation does not hold exactly. But, as with the salary reduction case above, we may view this as indicative of costs that are not specified in our model. Therefore, we assume that the Miller condition applies. This further implies that we can proceed using the Tepper approach, knowing that results would be identical under Black.

Employees may elect $\beta_i > 0$ because they do not have the resources to acquire equity investments outside the plan. We will explore this further below, but in a temporary effort to narrow the opportunities for contract improvement, we will assume that employees can borrow and lend at the risk-free rate in amounts sufficient to acquire all the equity they desire outside of the plan.

We have now narrowed the opportunities for contract improvement to those that derive from the differential tax positions of employees and shareholders. We define the following tax rates as they apply to employees:

$$
\tau_{eb,i} = \text{employee i tax rate on bonds}
$$

$$
\tau_{es,i} = \text{employee i tax rate on stocks}
$$

$$
\tau_{ep,i} = \text{employee i tax rate on pension income}
$$

The first two rates are analogous to the corresponding definitions for shareholders and might, in fact, be identical to the shareholder rates. The third rate is a decreasing function of the remaining time that pension earnings are to be deferred. If pension
earnings this year will be paid out \( n \) years hence, then, assuming no bracket migration by \( i \) and a constant riskless return \( r \):

\[
\tau_{ep,i} = 1 - \frac{(1+r)^n(1-\tau_{eb,i})}{(1+r(1-\tau_{eb,i}))^n}
\]

If pension earnings are to be paid out to the employee within the current tax year, then \( n = 0 \) and \( \tau_{ep,i} = \tau_{eb,i} \). As \( n \) increases, \( \tau_{ep,i} \) decreases and may become negative. A negative value implies that $1 of earnings on pension assets in the current period is worth more than $1 of after-tax funds currently in the hands of the employee. Selected values of \( \tau_{ep,i} \) as a function of \( \tau_{eb,i} \) and \( n \) are shown with \( r = 5\% \) and \( r = 10\% \):

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<th>( r = 5% )</th>
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<td>( \tau_{ep,i} )</td>
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<td>40%</td>
<td>28%</td>
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<tr>
<td>30</td>
<td>-7</td>
<td>-8</td>
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We have already determined that the shareholder annual gain from the shift of $1 of pension liability from fixed income to equity is given by equation (5):

\[
r(l-\tau_c)(\tau_{pb}-\tau_{ps})
\]

The corresponding loss to the employee is given by:
Shareholders will be able to provide incentives for employee elections of $\beta_i > 0$ when:

\[
(1 - \tau_c)(\tau_{pb} - \tau_{ps}) > \frac{(1 - \tau_{eb})(\tau_{eb} - \tau_{es})}{(1 - \tau_{es})}
\]

If shareholders are generally in high brackets (e.g., $\tau_{pb} = 40\%, \tau_{ps} = 15\%$), the above condition will be met for all employees who are in the lowest bracket (i.e., $\tau_{eb} = 15\%, \tau_{es} = 10\%$). For those in the middle (e.g., $\tau_{eb} = 28\%, \tau_{es} = 13\%$), because $(1 - \tau_{ep})/(1 - \tau_{es})$ is greater than $(1 - \tau_c)$ for all employees with sufficiently long time horizons\(^{27}\), the opportunity will only exist for pension income that will be deferred for a relatively short time. For employees with the same tax status as the shareholders, no opportunity for rational tax-based contract improvements will apply. Therefore, only employees with lower salaries and smaller liability account balances may be susceptible to incentives. Shareholders would prefer to see higher-balance employees elect $\beta_i > 0$.

We return now to the case where employees may not have the financial resources to adjust their nonpension investments to prefer $\beta_i = 0$ in their liability accounts. Since we are looking for rational contract improvement opportunities, we expect higher paid, more wealthy employees to prefer small or zero values for $\beta_i$. Again, this should be a disappointment to shareholders.

The experience of employee-choice 401k plans, however, contradicts these findings. Employees at all levels of compensation and wealth select $\beta_i$ values substantially greater than zero\(^{28}\).

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\(^{27}\) Employees can continue to defer even after termination of employment and generally well past the time that they retire.

\(^{28}\) Bodie and Crane (1997).
Practical opportunities

Pensions and Investments (2000) reports that the largest corporate defined contribution plans averaged 31.8% of assets in company stock and 36.7% in other equity in 1999. The company stock allocation may not reflect employee choice, but the other 36.7% (54% of the amount not in company stock) does. In public sector plans, P&I reports 57.5% is invested in equities. Bodie and Crane (1997) find that slightly more than half of retirement accounts of a 1996 TIAA-CREF sample are invested in equities with these same individuals allocating just under half of their nonretirement accounts to equities. These percentages vary very little by wealth quartile.

Bodie and Crane observe that "generally accepted investment principles" include the advice that fully taxable investments should be held primarily inside tax shelters and that less taxable investments should be held in individuals' taxable personal accounts. Shoven and Sialm (1999) adopt a more rigorous approach and conclude "that coupon bonds and stocks with high annual distributions have a preferred location in the tax-deferred environment and that tax-exempt bonds and stocks with low annual distributions have a preferred location in conventional savings accounts."

In light of the experience of employee-choice DC plans, it is likely that firms may conclude that offering employees a choice of liability benchmarks is an attractive part of implementing the tax based design strategy proposed by this paper. Such choices might well include indexed equity (managed equity makes no sense since the asset side of these plans should be invested entirely in fixed income), company stock and a short-term Treasury rate. For those firms that do not offer employee-choice DC plans (a minority), no positive offsets (e.g., T-bills plus 1%) should be considered by the firm and no negative offsets are likely to be well received by employees. When firms offer employee choice in both a DC plan and a CB plan, shareholders may benefit by adding a small sweetener to the DC plan equity returns (e.g., if the DC plan offers an index fund, the CB plan might offer the index plus, say, 10 basis points). The sweetener

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29 A set of principles advocated by practitioners that might also be called "common wisdom of investment professionals".
30 James F. Moore suggests that options (e.g., equity exposure with a positive guarantee less than Treasury rates) would be feasible, attractive to employees and consistent with the shareholder objective of adding equity to the liability crediting rate.
should encourage employees (who might otherwise be indifferent) to concentrate equity investments in the CB plan and fixed income in the DC plan\textsuperscript{31}.

Although this design is not shareholder-optimal in the sense in which we have used that expression earlier, it should provide shareholders with several benefits in comparison to the usual design:

- Investments by the CB plan in fixed income can assure that shareholders will not be losers when the plan is compared to the base case or to a cash compensation alternative. This will only hold strictly if the plan is at least fully funded as measured by $E_p \geq 0$.

- Employee elections of company stock will allow the Black version of the plan to be implemented thus benefiting nontaxable as well as taxable shareholders without adding idiosyncratic leveraging which can increase bankruptcy risk and threaten managerial interests.

- Employee elections of index stock allow tax gains to be achieved using the Black (with idiosyncratic leverage) or Tepper approaches.

- Employee utility enhancements derived directly from their choices and from their opportunities for non-corner allocations should inure to shareholders in various ways including reduced total compensation.

Interestingly, at least two firms have recognized this last advantage and have offered such plans to their employees\textsuperscript{32}. Industry sources, however, report that each of these firms continues to invest the plan assets such that:

$$\alpha A_p - \beta L_p > 0$$

\textsuperscript{31} A popular fixed income choice in DC plans is stable value (formerly GICs). These investments take advantage of employee persistency to offer "up-the-yield-curve" returns on money market terms. These should not be offered on the liability side of the CB plans.

\textsuperscript{32} Anand (1999) identifies several firms offering employees "investment options" including BankAmerica and its sister NationsBank, and PricewaterhouseCoopers LLP
believing that this inequality measures the firm’s advantage. As employees alter $\beta$, the sponsors may adjust $\alpha$ to perpetuate the relationship.

VI Impediments to Implementation

The Tepper-Black tax arbitrage was articulated by its authors in 1981 and 1980 respectively. Until 1999, empirical researchers failed to find evidence in support of corporate implementation of the theory. In a working paper, Myers (1999) cites Bodie et al (1987), Friedman (1983), Landsman et al (1992) and Peterson (1996) as empirical studies that did not find support for popular acceptance of the Tepper-Black prescription. Myers has, for the first time, found a positive empirical response to the tax-arbitrage theory, reporting a significant relationship between corporate tax benefits from leverage and the percentage allocation to bonds in defined benefit pension plans. She estimates that approximately one-third of the potential benefit from the tax arbitrage opportunity is utilized with the other two-thirds representing the “aggregate costs of other factors.”

The arbitrage theory stands upon two cornerstones: transparency and the augmented balance sheet. In this section we review three major impediments to implementation of the arbitrage. Mainstream actuarial practice defies transparency and leads to the first two impediments by encouraging: i) the anticipation of returns to risky investment prior to the acquittal of the risk (Actuarial Standard of Practice 27, Appendix F); ii) the smoothing of volatile results from all sources (including both equity and interest rate risks) by amortization. Statutory separation of the pension plan and its sponsor creates the third impediment by challenging the applicability of the augmented balance sheet.

*Decision makers contemplating a $\{0,1\}$ cash balance plan face pension expenses and cash flows in excess of the value received by the employees; with an $\{\alpha,0\}$ strategy the employees receive more value than the employer must contribute or recognize as expense.*

This paper has used the transparent intermediary argument to assert that the economic liabilities of a cash balance plan at any time must equal the sum of the individual account balances$^{33}$. We have also stated that the cost of the plan is equal to the compensation

$^{33}$ Assuming full vesting.
credits granted. If the financing of the plan is taken into account, we expect risk-adjusted income (cost if negative) equal to $rE_p$ to offset (increase) that cost. In contrast, Kwasha Lipton has been quoted in the Introduction: “A ‘5% of pay plan’ might require a contribution of only 4% of pay, after a realistic investment differential is taken into account.” This statement describes an actuarial process which violates the transparency assumption, A.7.

In order to show why the Kwasha Lipton claim violates A.7 and presents an illusion of bargain cost for the employer, we will first show how the Kwasha Lipton claim may be made. We will do so circa 1999 although the claim was articulated in the accounting, actuarial and legislative context of 1985. Even though changes have occurred since 1985 that reduce the generality of the claim, it is still the case that actuarial and accounting processes will very often support the Kwasha Lipton position.

The view of Kwasha Lipton and most pension actuaries and actuarial consulting firms derives from another statement made by Kwasha Lipton and quoted in Section II: “The investment differential can be anticipated.” The author has addressed this directly (Gold, 1999, 2000) elsewhere. Here we will show why many actuaries and their plan sponsor clients believe that \{\alpha, \theta\} plans are “profitable” in the sense that an employer can provide $1 to an employee at a cost well below $1. We will also show that, by the same measure, \{0,1\} plans are unprofitable.

There are three overlapping ways to demonstrate the \{\alpha, \theta\} “bargain”: i) a long view of pension plans implying that employers can profit by accepting risks that their employees will not bear; ii) accounting support for the same conclusion under FAS 87; and iii) support derived from the cash contribution calculations prescribed by ERISA.

Employer profits by accepting risks

This view may be articulated as a fixed-income-like promise to employees combined with an \alpha\-portfolio that provides an equity premium commensurate with \alpha. We can decompose this into: i) a compensation credit \((k\%C_{x,t})\) which costs the employer as much as an equal amount of compensation plus ii) a borrowing \((L_{p,t+1} + E_{p,t+1})\) used to
finance the $\alpha$-portfolio purchase. This borrowing at a fixed marketed rate to invest in the equity market creates the long run employer profits. From a financial perspective, however, this transaction adds no value; it merely realigns available risks and rewards.

Accounting gains under FAS 87

Almost every large employer will see accounting results for an $\{\alpha,0\}$ plan that reinforces the idea that this is a profitable arrangement. Using the algebra from Appendix E, we investigate the relationship between the plan’s service cost and its compensation credits. We will determine a condition under which these are the same. Then we will repeat the analysis for the relation between a plan’s projected benefit obligation and its opening account balances. Finally we look at a typical conversion plan which commences with assets in excess of the PBO. The service cost at end of year $[t,t+1]$ is given by:

$$SC_{t+1} = \frac{k\% (1+i)^{x+t+\delta}}{65-x} \sum_{w=0}^{64-x} C_x(l+s)^w(l+i_L)^{64-x-w}$$

We know that FAS 87 requires that the settlement rate be related to high quality fixed income investments and, in an $\{\alpha,0\}$ plan, $i_L$ is likely to be similarly defined. It is also reasonable to expect salary increases that are approximately equal to these interest rates. Although it is unlikely that any particular plan would set these rates identically, it is useful to see what results from the assumption that $i = i_L = s$:

$$SC_{t+1} = k\% (1+i)^t C_x = k\% C_{x+t}$$

which is identical to the end-of-year compensation credit. Many of the realistic variations on the assumption of equal rates (e.g., a higher settlement rate) will tend to reduce the service cost below the compensation credit. Although it is not difficult to develop a case where the service cost might exceed the compensation credit, it is fair to observe that an employer that wishes to recognize a service cost no greater than the compensation credit should be able to work with its actuary to achieve that result.
Using the same reasoning as we did for the service cost, we can see that when \( i = i_L = s \), the PBO at the beginning of year \([t, t + 1]\):

\[
PBO_t = \frac{tk\%(1+i)^{t-65}}{65-x} \sum_{w=0}^{64-x} c_x (1+s)^w (1+i_L)^{64-x-w}
\]

becomes:

\[
PBO_t = tk\% C_{x+t-1}
\]

which, under the same assumptions, has been shown (Appendix A) to be the beginning-of-year plan balance. As with the service cost, when the employer desires that the PBO not exceed the plan balances, the actuary should be able to develop suitable assumptions. Since the assets of most conversion plans exceed the account balances, we can conclude that it is highly likely that:

\[
PBO \leq MRV
\]

Since FAS 87 prescribes that \( i < j \) for an \( \{\alpha, 0\} \) plan, we will almost always be able to conclude that the accounting cost will be less than the compensation credits and that any \( E_p \), as we have defined this term, will be credited with an accounting return in excess of the riskless rate.

There are many combinations of assumptions that may be employed by the plan’s actuary that will result in the bargain cost that Kwasha Lipton identified as a CB plan advantage. As long as no adjustments are made for risk or for tax effects, the bargain is likely to persist as long as expected returns on invested assets exceed investment crediting rates.

Note that the gain from comparing the service cost to the compensation credits or the PBO to the plan balances is likely to be small when compared to the substantial bargain that derives from the excess of rate \( j \) over rate \( i \) when the MRV and the PBO are of similar magnitude. This would be wiped out if the plan were designed as a \( \{0,1\} \) plan with the result that the expected return on assets \( (j) \) would be a fixed income rate.
approximately equal to the settlement rate \((i)\). Further, the use of an investment crediting rate \((i_t)\) that included an equity premium would almost always raise the service cost above the compensation credit. Thus a \(\{0,1\}\) plan would either sharply reduce or reverse the bargain cost inherent in an \(\{\alpha,0\}\) plan under current accounting rules.

An FAS 87 Example

Buck Consultants, another leading actuarial consulting firm, has issued a study of assumptions used by 552 of the Fortune 1000 companies for their 1998 FAS 87 computations.\(^3^4\) The average values are:

\[
i = 6.77\% \\
j = 9.11\% \\
s = 4.54\%
\]

Consider the accounting cost for one employee hired at age 25, retiring at age 65, using these assumptions with no early exits, a 6% investment crediting rate, contemporaneous funding and with the MRV defined to equal the market value of the assets. Note that if all assumptions are met, the MRV will equal the PBO at all times:

\[
PBO_0 = MRV_0 = 0 \\
PBO_t = (1 + i)PBO_{t-1} + SC_t \\
\text{Expense} = \text{Funding} = iPBO_{t-1} + SC_t - jMRV_{t-1} \\
MRV_t = \text{Funding} + (1 + j)MRV_{t-1} = iPBO_{t-1} + SC_t + MRV_{t-1} \\
= iPBO_{t-1} + SC_t + PBO_{t-1} = PBO_t
\]

The following Table I presents the resulting compensation credits and plan expenses assuming initial year’s compensation of $10,000 and a 10% compensation credit at the end of each year of employment (the ratio is independent of these last assumptions).

---

\(^3^4\) Buck Consultants (1999). Many of these plans are traditional DB rather than CB.
<table>
<thead>
<tr>
<th>Age x+t</th>
<th>Compensation Credit</th>
<th>Expense</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>26</td>
<td>1000</td>
<td>583</td>
<td>58%</td>
</tr>
<tr>
<td>30</td>
<td>1194</td>
<td>691</td>
<td>58%</td>
</tr>
<tr>
<td>35</td>
<td>1491</td>
<td>844</td>
<td>57%</td>
</tr>
<tr>
<td>40</td>
<td>1862</td>
<td>1011</td>
<td>54%</td>
</tr>
<tr>
<td>45</td>
<td>2325</td>
<td>1181</td>
<td>51%</td>
</tr>
<tr>
<td>50</td>
<td>2903</td>
<td>1331</td>
<td>46%</td>
</tr>
<tr>
<td>55</td>
<td>3624</td>
<td>1420</td>
<td>39%</td>
</tr>
<tr>
<td>60</td>
<td>4525</td>
<td>1377</td>
<td>30%</td>
</tr>
<tr>
<td>65</td>
<td>5650</td>
<td>1089</td>
<td>19%</td>
</tr>
</tbody>
</table>

Cash gains under ERISA using Projected Unit Credit

The apparent bargain in the FAS 87 accounting costs derives substantially from the difference between the expected return on assets and the liability discount rate. Under the projected unit credit (PUC) actuarial cost method, the rates are equal to each other and each incorporates an equity risk premium consistent with the $\alpha$-portfolio.

Under ERISA, using the PUC method, a bargain is derived from the understatement of both the normal cost\(^{35}\) and the accrued liability\(^{36}\). It is clear that the PUC understates the normal cost and the accrued liability in comparison to the FAS 87 service cost and projected benefit obligation because the formulas are identical and the PUC discount rate includes an equity premium. Further, for an \{\alpha,0\} plan with a significantly nonzero $\alpha$, while FAS 87 produces values that are ambiguous in comparison to the compensation credit and the account balances, the PUC is highly likely to develop a normal cost well below the compensation credit and an accrued liability well below the PBO. Although it is possible to construct cases where this might not be true, in almost every practical situation these comparisons will hold.

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\(^{35}\) PUC counter part of SC, see Appendix E.

\(^{36}\) PUC counter part of PBO, see Appendix E.
In a well-funded plan, with $AL < L_{P,t}$, the interest credit, $i \cdot (AAV - AL)^{37}$, will exceed the economic value, $rE_p$. Combined with a normal cost less than the compensation credit, once again we find a bargain cost.

With a \{0,1\} plan, the discount rate would be set based on purely fixed income investments while the liability crediting rate would be the expected return on an all-equity portfolio. This would almost certainly result in a normal cost in excess of the current compensation credit and an accrued liability in excess of plan balances. The overstated liability would decrease the income based on a positive $E_p$ and might turn it negative. Thus a \{0,1\} plan is highly likely to require contributions in excess of the economic cost of the plan.

A PUC Example

With the same assumptions as in the previous subsection, the PUC cost will consist of normal cost alone computed with a discount rate of 9.11%. Table II shows the costs at various ages for the same employee shown in Table I. Because the contribution requirements start out lower than the FAS 87 expenses, the assets of Table II are everywhere lower than those of Table I except at age 65 when they are equal. Since a real plan would have the same market value of assets (but not necessarily identical MRV and AAV) for FAS 87 and for ERISA, the tables are not directly comparable.

---

37 AAV is PUC counter part of MRV, see Appendix E.
Table II – Compensation Credits and PUC Contributions

<table>
<thead>
<tr>
<th>Age x+t</th>
<th>Compensation Credit</th>
<th>Contribution</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>26</td>
<td>1000</td>
<td>250</td>
<td>25%</td>
</tr>
<tr>
<td>30</td>
<td>1194</td>
<td>355</td>
<td>30%</td>
</tr>
<tr>
<td>35</td>
<td>1491</td>
<td>548</td>
<td>37%</td>
</tr>
<tr>
<td>40</td>
<td>1862</td>
<td>848</td>
<td>46%</td>
</tr>
<tr>
<td>45</td>
<td>2325</td>
<td>1311</td>
<td>56%</td>
</tr>
<tr>
<td>50</td>
<td>2903</td>
<td>2028</td>
<td>70%</td>
</tr>
<tr>
<td>55</td>
<td>3624</td>
<td>3136</td>
<td>87%</td>
</tr>
<tr>
<td>60</td>
<td>4525</td>
<td>4850</td>
<td>107%</td>
</tr>
<tr>
<td>65</td>
<td>5650</td>
<td>7499</td>
<td>133%</td>
</tr>
</tbody>
</table>

Although the cost incidence is quite different in Tables I and II, they each represent a bargain for a population average. The crossover point between the two tables is age 43 with a 52% ratio and this suggests that, for a typical corporate population, the bargain might be roughly one half of the compensation credits.

A \{0,1\} Plan Example

With a \{0,1\} plan, the assumed rates might become:

\[ j = i = 6.77\% \]
\[ i_L = 9.11\% \]
\[ s = 4.54\% \]

Now the FAS 87 and PUC approaches are the same (because \( j = i \)) and we get the following Table III:
Table III – \{0,1\} Plan Example

<table>
<thead>
<tr>
<th>Age (x+t)</th>
<th>Compensation Credit</th>
<th>Cost</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>26</td>
<td>1000</td>
<td>1139</td>
<td>114%</td>
</tr>
<tr>
<td>30</td>
<td>1194</td>
<td>1480</td>
<td>124%</td>
</tr>
<tr>
<td>35</td>
<td>1491</td>
<td>2054</td>
<td>138%</td>
</tr>
<tr>
<td>40</td>
<td>1862</td>
<td>2850</td>
<td>153%</td>
</tr>
<tr>
<td>45</td>
<td>2325</td>
<td>3955</td>
<td>170%</td>
</tr>
<tr>
<td>50</td>
<td>2903</td>
<td>5487</td>
<td>189%</td>
</tr>
<tr>
<td>55</td>
<td>3624</td>
<td>7614</td>
<td>210%</td>
</tr>
<tr>
<td>60</td>
<td>4525</td>
<td>10565</td>
<td>233%</td>
</tr>
<tr>
<td>65</td>
<td>5650</td>
<td>14659</td>
<td>259%</td>
</tr>
</tbody>
</table>

In sum, the argument that an \(\{\alpha,0\}\) plan is an actuarial and accounting bargain is reinforced by methodologies that contradict the implications of transparency. Many corporate managers will find the accounting outcome sufficient to lead them to conclude that the \(\{\alpha,0\}\) plan is a bargain and the \(\{0,1\}\) plan is costly. Those managers who are able to “see through” the accounting cannot count on their financial constituents to join them and thus they will often conclude that the \(\{\alpha,0\}\) plan represents the wiser course.

**Actuarial cost methods smooth out equity investment volatility allowing the corporation to earn expected risk premiums without apparent risk. The Tepper-Black arbitrage is not viable under this regime.**

Actuarial cost methods (ACM’s) were designed to facilitate an orderly budgeting process for cash contributions to defined benefit plans. In order to achieve orderliness, each ACM incorporates a self-correcting process for coping with year-to-year deviations from long-term expected progress of the plan.

ACM’s always assume a convergence of assets and liabilities at some distant horizon (e.g., 15 or 30 years hence). Each year, an actuarial valuation is performed which develops a vector of expected contributions that, if all assumptions are met, will assure such convergence. The expected contribution vector is inherently smooth; it may, for example, represent a constant percentage of each future year’s expected compensation. Thus, on an expected basis, volatility is not part of the scheme.
Each year’s valuation is also designed to align the expected contribution for this year (the second element in last year’s expected contribution vector) with the leading element in this year’s vector. The procedure must be able to account for differences between actual and expected population data, asset values and plan cash flows between the previous and current valuations. Additionally, there must be provision for changes in plan benefits and actuarial assumptions about future rates of mortality and retirement and interest, if any. The aggregate impact of all these variations in the present value of plan liabilities less plan assets is called “actuarial gain and loss.”

If the total actuarial gain and loss were attached to the leading value in the expected cost vector, there could be substantial year-to-year changes in required contributions. Instead, each ACM has an amortization scheme which effectively identifies the entire gain and loss and then spreads it out over the whole vector. Part of the rationale for this procedure is the desire to create an orderly process but an equally important element is the actuarial tenet that fluctuations over time tend to cancel out. In the case of non-market sources of gain and loss, this is an application of the law of large numbers working much like the averaging of daily takes by a gambling casino. In the case of market based fluctuations, however, it represents a subscription by an entire profession to a mean reverting market model.

Because the pension actuarial process is arcane (somewhere between translucent and opaque), the major impact that pension plans transmit to their sponsor’s financial status flows through the annual pension expense into corporate P&L. Prior to the adoption of FAS 87, plan expenses were identical to plan cash contributions except for occasional, generally minor, timing differences.

FAS 87 removed some of the smoothing tools from the actuarial toolbox and standardized the application of others so as to make company-to-company comparisons easier. A series of legislative acts, notably the Omnibus Budget Reconciliation Act of

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38 For details of gain and loss analysis see, e.g., Berin (1976).
39 “Any market downswing experienced in one generation will be offset by an upswing in later generations.” (Burrows, 1999)
40 Prior to the adoption of FAS 87, Accounting Principals Board Opinion 8 (APB 8) ruled.
1987 (OBRA ‘87) and the legislation enabling the General Agreement on Tariffs and Trade of 1994\(^41\) (GATT), reduced actuarial flexibility with respect to cash contributions.

Nonetheless, the amortization of actuarial gains and losses over many years survives powerfully today. Under FAS 87 (Appendix E), the expected return \(\text{(j)}\) on smoothed plan assets \(\text{(MRV)}\) flows directly into the pension expense. The difference between the actual return on the market value of plan assets and \(j\text{MRV}\) goes into a suspense account. Differences between the newly computed \text{PBO}\ and its expected value based on the prior valuation is lumped into the same bucket which is rolled forward from year to year in order to accommodate any offsets that occur. As long as the entire bucket value remains in a “corridor” (a range of error equal to plus or minus 10% of the greater of the \text{PBO} and the \text{MRV}), the accumulated gains and losses do not affect the current year’s pension expense. Once the corridor is exceeded, a fraction (e.g. 1/15) of the overflow is added or subtracted in the current year.

To the extent that financial analysts deal with pension plan effects primarily through pension expenses, the Tepper-Black arbitrage is not viable. If the shareholder does not experience the financial impact implied by the transparent intermediary model, then the arbitrage is without merit.

Smoothing pension plan assets, liabilities and expenses not only defeats the arbitrage, but it also introduces systematic bias into the valuation of securities by financial analysts. This is an issue addressed in Gold (2000).

*The “augmented balance sheet” is a theoretical nicety but it ignores the strong legal separation between the pension plan and the corporation. The legal reality implies that the corporate leveraging recommended by Black may constitute an unacceptable risk to lenders.*

Treynor’s augmented balance sheet is designed to emphasize the financial integration of a corporation and its pension plan. As such it suggests integrated financial management of the kind employed by Black and Tepper. It does not, however, describe an entity with all the freedom that we usually associate with a single balance sheet. We cannot freely

\(^{41}\) GATT legislation of 1994 incorporated provisions of bills sometimes identified as The
move assets across the horizontal border between the corporation and the plan. We have already discussed the tax considerations that govern this border crossing and know that these can work for or against shareholders depending on plan design.

There is also a strong statutory barrier. Although many of the financial implications of a corporation’s contribution strategy and a plan’s asset allocation affect shareholders (particularly in a well-funded plan), Sharpe (1976) and Treynor (1977) discuss employee interests before and after the passage of ERISA and the PBGC’s interests since then.42

One issue sometimes raised and often dismissed is the ERISA admonition: “…a fiduciary shall discharge his duties with respect to a plan solely in the interest of the participants and beneficiaries….”43 Some misunderstand this to mean that some effort to maximize returns is required. Because the primary societal role of plan assets is to collateralize benefit promises, a better interpretation is that the probability that all promises are met should be kept high. In the context of amending ERISA to strengthen the PBGC, Congress declared: “is desirable to increase the likelihood that full benefits will be paid to participants and beneficiaries of such plans.”44 Under this interpretation, plan strategies along the $\{\alpha, \frac{A_p}{L_p}\}$ line of Section V might be deemed especially prudent.

In the context of the Black version of the arbitrage proposed in Section IV, the most important impediment to implementation derives from the fact that plan assets may not be easily reached by corporate creditors. This means that any proposal to increase corporate leverage by borrowing to buy back company stock will face an uphill battle. Not only can the assets not be pledged or assigned, but the lenders cannot have any

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42 Retirement Protection Act of 1994 (RPA ’94).

43 Pre-ERISA, promises received by employees were only as good as the assets of the plan allowed. If plan assets fell below plan liabilities, the corporation had the right to put the assets to the plan participants without incurring a residual liability. This put option represented an asset of the corporation that Treynor records in the augmented balance sheet. With the passage of ERISA, the PBGC was inserted as a guarantor of a substantial portion of benefits accrued by participants and the PBGC had the right to pursue the sponsor to recoup the shortfall. ERISA also tightened fiduciary standards for plan managers in the name of protecting employee interests. Treynor observes that the major beneficiary of these standards was the PBGC.

44 29 USC 1104(a)1.
assurance that the plan will continue to invest in bonds rather than in equities. Even after all parties thoroughly understand the financial issues, lenders (banks and bond holders) will be placed at some disadvantage. Covenants in current borrowings may make it difficult or impossible to add leverage even if new borrowing is available.

How much of a disadvantage depends of course on the numbers but we know that any borrowing done to support the transaction will have to be done at a higher cost than if the augmented balance sheet representation were statutorily correct. In effect, the benefits of the arbitrage plan are certain to be reduced by this and other friction costs. This cost should be modest when compared to the value of the arbitrage but the cost will be greater for those corporations whose borrowing costs are already relatively high.

Black argues that the value of the arbitrage derives from the reallocation of plan assets and that the rest of the arbitrage is offered by him primarily to defend or prove his case. His point is that, after tax considerations, the corporation’s risk adjusted returns will have been increased whether or not the corporate leveraging is executed.

VII Conclusions

This paper extends the Tepper-Black pension tax arbitrage into a cash balance plan environment. Black and Tepper each assumed that defined benefit pension liabilities were well represented by fixed income instruments. Here we extend the liability model to include arbitrary marketed securities. This model tells us that shareholders benefit when the liabilities are defined in terms of equities.

Traditional defined benefit plans were designed to retain long-service older employees. The enabling mechanism is a sharply convex cost curve, poorly matched to average productivity by age, that offers little value to those who leave early in their careers. Young, mobile, presumably productive, employees are not retained by their employers, nor do they accumulate sufficient retirement resources as they change jobs. Older, presumably less productive, employees are encouraged by traditional DB plans to remain in service. This has led to expensive employer bribes (e.g., subsidized early retirement benefits and open window plans) to motivate their departures.

44 29 USC 1001(b) as modified by SEPPAA (1986).
Employers and younger employees both see that defined contribution plans better serve their needs. Startup employers offer defined contribution plans exclusively. Some sponsors have terminated their DB arrangements and offered DC substitutes. More mature employers with substantial DB plan surpluses find that excise taxes present a barrier to such exit. CB plans offer an attractive alternative to this latter group. The CB liabilities are designed to mimic DC plans while the assets (and surpluses) may be retained for the benefit of shareholders without excise tax penalties.

The process of converting DB plans to CB plans has created a new set of difficulties. Often older employees find that the steep accrual curve that they anticipated has been flattened leading to significantly lower projected pension benefits. Although younger employees generally like CB plans, they are not always happy with the fixed income rates of return on plan balances, especially when they compare these returns to their DC plan equity investments. Dissatisfied employees of various ages have joined together to create a political voice which is now being heard in Washington.

At least part of the problems now in view may be ameliorated by the design opportunities presented in this paper. The idea that the liability equity allocation $\beta$ can be raised above zero producing benefits for shareholders and for some employees offers potential for a whole class of innovative CB plan designs. Certainly the employee choice plan (individual $\beta$'s) combined with fixed income investments ($\alpha = 0$) can work to the benefit of all direct constituents (arguably, taxpayers are the losers).

The major impediments to the implementation of designs in this new class include actuarial and accounting systems that obfuscate financial realities. The old admonition: "eschew actuarial obfuscation" has new justification. The very nature of the cash balance plan (where liabilities are first stated as present values rather than as contingent future cash flow streams) may make the intellectual hurdle lower than it was when the Black and Tepper papers were published. The question: "how can it be possible to give an employee $1 at a cost of $.50?" may gain some traction among financial officers, shareholders, accountants and actuaries.
The impediments related to the statutory separation of the plan and the sponsor may persist but for a great number of solid companies with well-funded plans, this impediment is more a process that requires management than it is a barrier to action.
Appendices

A Patterns of Accrual

Employer economic costs for defined contribution and cash balance plans

Under a defined contribution plan, the employer cost each year is equal to the contribution made and, in the generic case, this is a constant percentage of compensation. The contribution is tax deductible to the corporation and, from the employer/shareholder point of view, is the same as an equal amount of compensation. From the employee perspective, a tax-sheltered contribution may be superior to taxable ordinary compensation. We discuss the employee perspective further in Section ?

Under a cash balance plan, if i) compensation credits are funded contemporaneously, ii) the liability benchmark is a marketed security and iii) assets are invested in the liability benchmark, the economic cost to the employer is equal to the compensation credits added to employee accounts, as it must be since a plan so designed and implemented virtually clones a DC plan. Although no CB plans are designed and managed as clones, it is analytically useful to identify separately the opportunities for economic gains and losses that derive from deviations from the three provisos above. Collectively, we identify the choices available in these three areas as the “strategic” opportunities associated with a CB plan and we identify the resulting economic gains or losses as “strategic” as well:

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45 ERISA provides that plans must may be designed so that employer contributions are not vested until some time after they have been made. To a small degree, this reduces the cost borne by the employer. This relatively small matter is ignored throughout this paper. Further, although rarely exercised, many DC plans and all CB plans offer an annuity alternative to the account balance; we assume that this is merely a choice (not a valuable option) and that any annuity is priced at its expected value.

46 Since an employee separating from service is entitled to his entire vested plan balance.

47 We ignore small differences in administrative costs and PBGC premiums.
• To the extent that cash contributions exceed compensation credits, the employer is accelerating funding; Tepper shows\textsuperscript{48} that this is economically gainful. Conversely, funding less than the compensation credits (as very frequently occurs in practice\textsuperscript{49}) is costly.

• As we show in Section IV, Result 2, when returns on non-marketed securities are used to benchmark investment credits, the economic cost must be adjusted to reflect the difference between the liability benchmark rate and an appropriate marketed rate.

• When taxes are taken into consideration, as shown in Section IV, Result 4, the economic cost is increased when asset equity exposure exceeds liability equity exposure and is decreased in the opposing case.

For the remainder of this appendix we work with DC plans and only those CB plans that are clones. Thus, when comparing to DB plans, we treat DC and CB plans as identical and refer to them generically as either “accumulation” plans or as “DC/CB” plans. In the same vein, we may refer to compensation credits or contributions without distinction.

**Employer costs for traditional defined benefit plans**

Under a traditional defined benefit plan, benefit promises usually provide for a fixed income benefit stream and this has been modeled by Black, Tepper and many others as equivalent to a marketed fixed income instrument, most often without credit risk. Although the benefit streams are uncertain with respect to individual participants due to demographic uncertainty, this uncertainty is deemed to be orthogonal to market risk and fully diversifiable for large groups of employees. We adhere to this model herein.

The value associated with a DB plan that has the same economic significance as the compensation credit may be expressed as the algebraic excess of i) the present value of

\textsuperscript{48} In order to be most gainful, such accelerated contributions must be currently deductible by the employer under IRC Section 404 and, as discussed in Appendix D, must not significantly add to the probability that the plan will fall into an “overfunding trap.”

\textsuperscript{49} As discussed in Appendix E, actuarial methodology enables and encourages optimistic underfunding.
future benefit payments assuming the employee’s service is terminated at yearend (for reasons other than death or disability) over ii) the corresponding value as of the beginning of the year increased by the accrual of interest, mortality and disability discounts thereon.50

One feature that distinguishes accumulation plans from DB plans is that the value of the accumulation plan is fixed even when alternative payout schedules may be selected. Under a DB plan, various payout choices may be offered with substantially different values. For example, an employee whose service has terminated may, if eligible, choose between a benefit commencing when he attains age 65 or a reduced benefit commencing at an earlier date. Often the earlier benefit will be more valuable (i.e., costly) from the plan’s point of view even though not all employees will elect the earlier commencement. It is also common for DB plans to permit the form of annuity to be adjusted from a life annuity to another form (spousal joint and survivor annuities must be available per ERISA) or to a single lump sum payment and these conversions are often effected at rates that incorporate some subsidization.

Thus, when we seek to measure “the present value of future benefit payments assuming the employee’s service is terminated at yearend (for reasons other than death or disability)” we must do so on an expected basis that assigns probabilities to each of the value-altering choices offered under the plan. While this does not make the comparison of economic costs across plans inaccurate, it does mean that the value of the DC/CB account balance is much more readily assessable by, and tangible to, employees. This is deemed to be a very attractive aspect of accumulation plans.

**Comparative accrual patterns**

In this subsection we look at the way in which the patterns of cost for DB and accumulation plans compare over the working lifetimes of employees. We restrict the DB plan benefit to a life annuity payable at the normal age of 65. The accumulation plan benefit is assumed to be paid in a life annuity form beginning at age 65 as well and we

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50 In general, participant mortality in a DB plan results in a loss of benefit value while disability implications are ambiguous. As with DC plans, typical CB plans provide the full account value in the event of death or disability.
further assume that the cost of a life annuity paying $1/year for a 65 year-old is a fixed
certain number, $a_{65}$.

Our comparison is made for one employee hired at age 25 with an annual rate of
compensation equal to $C_{25}$. We assume that this compensation grows at a compound
annual rate of $s$. The rate of interest used to discount DB plan benefits ($i$) equals to the
liability crediting rate under the accumulation plan $i_L$. The employee survives to age 65
with certainty and without disability.

The CB plan credits $k\%$ of trailing annual compensation to the participant account at
each yearend. Thus $k\%$ of $C_{25}$ is credited just when the participant turns 26 and
$k\%C_{25}(1+s)$ is credited when he turns 27. An investment credit at rate $i_L$ is added to
the prior year’s balance at the same time. Thus the plan balance at age 27 is equal to
$k\%C_{25}[(1+i_L)+(1+s)]$. The plan balance at the beginning of year $[t,t+1]$, when the
employee is exact age $25+t$, is then given by:

$$k\%C_{25+t-1} \left\{ \frac{(1+i_L)^{t-1}}{1+s} \right\} \left[ \frac{(1+i_L)^{-1}}{(1+s)^{-1}} \right]$$

where $C_{25+t-1} = C_{25}(1+s)^{t-1}$

A special case of this occurs when $i_L = s$:

$$tk\%C_{25+t-1}$$

The balance at age 65 will buy a life annuity equal to:

$$\frac{40k\%C_{64}}{a_{65}}$$

Note that the balance at any age $25+t$ will, without further compensation credits, buy a
life annuity at age 65 equal to:
Also note that, as long as the compensation at age \( x + t \) is given by
\[
C_{x+t} = C_{25}(1 + s)^{x+t-25},
\]
the account balance of an employee hired at age \( x \) will, at age \( x + t \), without further compensation credits, also buy a life annuity at age 65 equal to:
\[
\frac{kt\%C_{64}}{a_{65}}
\]

In other words, for an employee on the specified compensation path, the life annuity at age 65 depends on service \( (t) \) but is independent of hire age \( (x) \).

Because both \( k \) and \( a_{65} \) are constants, we can replace \( k/a_{65} \) by \( M \). Thus the life annuity purchased by the plan balance is given by:
\[
M_{t\%C_{64}}
\]

which may be recognized to be equal to the same annuity offered by a DB plan that provides \( M\% \) of final compensation for each year of service (Gold, 1983). Unlike with the DC/CB plan, where the annuity is the same regardless of which \( t \) years are worked, the DB plan benefit will be as shown only if the \( t \) years end at age 65.

If, for example, the \( t \)-year period ended at age 55, the annuity beginning at age 65 would be given by:
\[
M_{t\%C_{54}}
\]

which is smaller than the annuity above.
A graphical example

When we set $x = 25$, $C_{25} = 10,000$, $k = a_{x+5} = 10$, $i_L = s = .06$, we get the results depicted in Figure 2. Compensation, the black line measured by the left hand axis, starts at age 25 at $10,000/year and grows to $97,035 for the year from age 64 to 65. The DC/CB benefits that will be paid as a life annuity commencing at age 65 are shown along the yellow line that is measured by the right hand axis. If the employee works from age 25 to age 65, the yellow line shows that his retirement annuity will be equal to $38,814/year which is 40% of $97,035. If, alternatively, the employee works from age 25 to 45 or for any other twenty years, the age 65 life annuity will amount to $19,407/year or 20% of $97,305.

The DB plan benefits that will be paid as a life annuity commencing at age 65 are shown along the pink curve that is measured by the right hand axis. If the employee works from age 25 to age 65, the pink curve shows that his retirement annuity will be equal to $38,814/year which is 40% of $97,035 just as with the accumulation plan. In the DB case, if the employee works from age 45 to 65, the age 65 life annuity will amount to $19,407/year or 20% of $97,305 (not illustrated). If, however, the employee under the DB plan works from age 25 to 45, the age 65 life annuity will amount to only $6,051/year or less than 7% of $97,305.

Figure 3 shows the annual economic costs for the DB plan (pink curve measured in $/year on the left hand axis, blue curve measured as a percentage of annual compensation on the right hand axis) and for the DC/CB plan (yellow curve measured in $/year on the left hand axis, dark blue horizontal line measured as a constant 10% of annual compensation on the right hand axis). It is clear that the annual economic cost of the DB plan is less than that of the accumulation plan at ages younger than 50 and is greater at older ages.

The DB plan cost is computed as described in the previous subsection as the value of the benefit accrued by age $x + t$:

$$\frac{k\%t C_{x+t-1}}{(1+i)^{55-(x+t)}}$$
less the value of the benefit accrued one year earlier incremented for one year’s interest discount (recall, we have assumed no mortality or disability prior to age 65):

\[
\frac{k\% (t-1) C_{x+t-2}}{(1+i)^{65-(x+t-1)}} (1+i)
\]

which equals:

\[
\frac{k\% C_{x+t-1}(st+1)}{(1+i)^{65-(x+t)}}
\]

and because the DC/CB plan cost is:

\[
k\% C_{x+t-1}
\]

the crossover occurs at the smallest integer value of \( t \) satisfying:

\[
\frac{(st+1)}{(1+i)^{65-(x+t)}} > 1
\]

which, in this example, occurs at age 50 when \( t = 25 \).

**An accounting “solution”**

The accrual pattern shown in Figure 3 is undesirable from several points of view (one of which is the poor mismatch between employee productivity and compensation cost including pension accruals) and this has been a known difficulty with DB plans for at least fifty years. Although the smallest plans circa 1950 (those funded in an “Individual Policy Pension Trust” or IPPT) generally had to follow this accrual pattern (which corresponds to the traditional Unit Credit ACM), larger plans used ACM’s that leveled costs more evenly. For the very special case used to prepare Figure 3, the Projected Unit Credit method which underlies FAS 87 would follow the DC plan lines even though the plan promises follow the DB lines.
In effect, the accounting followed a long-term theory of the employment “contract” under which early excess productivity of employees was deemed to be set aside as a shadow account for an unaccrued pension benefit which was then drawn down in later years. This fiction, arguably not terribly far-fetched in an earlier era, seems to have been strained by today’s mobile workforce.

**A mobile career**

Despite the much higher economic costs for the DB plan in later years, the total economic cost of the DB plan must be less than that for the DC/CB plan because the value carried away by employees who exit from service is strictly less for the DB plan than for the DC/CB plan for all ages less than age 65 and is exactly equal at that age. Thus, a given normal retirement benefit may be more cheaply provided by a DB than by a DC/CB plan. How much less the DB plan costs cannot be determined unless we have some idea of the fraction of employees who separate from service at each attained age.

We can, however, perform an analysis for one employee who works for ten years at each of four separate employers who maintain identical DB plans. The benefit that he accumulates from all four employers by the time he attains age 65 will be of the same value as one employer will pay to four different employees each of whom works for ten years sequentially at progressive ages over the same forty years. This situation is illustrated in Figure 4.

In Figure 4, the pink curve is identical to the pink curve of Figure 2 although this time it is measured on the left hand axis. It represents the accrual of cost for one employee who works for the same employer for all forty years. The lower blue curves represent accruals that an employee generates when he is employed under a DB plan for four consecutive ten-year periods by four different employers. Each plan provides a benefit at age 65 equal to 1% for each of the ten years of service multiplied by the final year’s pay with that employer. Thus at each separation, the employee earns a deferred age 65 annuity equal to 10% of his last years compensation. The dark blue curve sums these four separate accruals across the four employers over time. The employee’s age 65 annuity is now given by:

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Trowbridge, 1952.
instead of the more valuable benefit given by the pink curve as:

\[ 40\%C_{64} \]

A DC/CB plan provides higher benefits for employees who divide their careers among more than one employer. In this example, job switches have no effect upon the ultimate level of benefit earned by the normal retirement age. The preservation of benefit levels despite job changes is usually termed “portability”. In practice, although there may be some gains or losses attributable to changing jobs, portability is an intrinsic property of DC/CB plans. Lack of portability, meaning diminution of career benefits for mobile employees, is an intrinsic property of single employer corporate plans. Social Security provides an intrinsically portable DB plan. Some defined benefit systems maintained by public employers provide portability as long as public employees change jobs within the same system (such as one state or one large city or county or the federal government) and some Taft-Hartley multi-employer plans provide portable benefits for union employees who continue to work in the same industry.

Portability is a plus for the mobile employees of today. This is one of the elements contributing to the rise of DC plans and the conversion of DB plans into CB plans in the last two decades. The lack of portability inherent in the DB plan design was deemed a positive feature by more traditional employers who found it to be a retention incentive.

B  Conversion Designs

Paying for portability

Because a DC/CB plan that matches the normal retirement benefit provided to a long-service employee by a DB plan can be much more expensive than the DB plan, employers who switch to accumulation plans may reduce their contribution level below the \( k\% \) necessary to match the DB plan. In the case of 401k plans, the design will
incorporate, and provide incentives for, tax-effective employee contributions to make up the shortfall. In the case of DB to CB conversions, employers will simply provide compensation credits lower than \( k \% \). Part of their reasoning may be that they are spending more on younger employees and that the level percentage of pay throughout a career is a better match to employee productivity over time than is the rising cost pattern associated with DB plans. This may also, to some degree, represent a reaction to the now more than twenty year old Age Discrimination in Employment Act (ADEA) that makes it more difficult for employers to treat younger workers preferentially\(^{52}\). Figure 5 illustrates a 8% CB plan and a 6% CB accumulation plan in addition to the 10% plan that matches the final pay DB plan at age 65. Based on the actual plan data it is possible to target a contribution level that will match or reduce costs relative to the DB plan. Certainly this contribution level will be substantially less than 10%.

**DB to CB Transitions**

Figure 6 takes the DB plan (pink curve) and the 6% CB plan (blue line) from Figure 5 and illustrates possible transition schemes as they affect an employee aged 58 at the time of conversion. It can be seen that the 58 year-old has accrued a higher annual pension benefit (\$21.3k) under the DB plan than he would have (\$19.2k) under the new 6% CB plan\(^{53}\). ERISA requires that the accrued benefit not be reduced but does not require that any benefits be added for service after the plan converts.

A controversial and ungenerous approach, dubbed “wearaway” grants the employee the larger of his old-plan accrued benefit and what he would have had had he been covered by the new plan throughout his career\(^{54}\). For the employee shown (along the orange angled line), this means that the benefit remains at (\$21.3k) through age 61 and increases along the 6% CB path thereafter. At age 65, this employee will receive a pension equal to \$23.3k which may be compared to the DB pension of \$38.8k.

\(^{52}\) An unanswered question: how does ADEA apply to various cash balance plan designs? VanDerhei (1999).

\(^{53}\) The are many ways for the conversion design to create a minimum plan balance that exceeds the accruing plan balance. The example shown is merely one such instance.

\(^{54}\) In practice, wearaway is more likely to affect somewhat younger employees (e.g., age 50) and to be caused by the application of statutory minimum values attributable to a combination
The grandfather approach allows the employee to continue to be covered by the DB plan formula for the remainder of his career. This results in his earning a pension of $38.8k. Some employers have chosen to offer grandfathering as an option to be elected by employees at the time of conversion. An offer of grandfathering for all employees implies that the employer cannot begin to reduce costs for some time. A common variation allows employees to choose grandfathering only if they meet some age and/or service requirements (e.g., age 50 with 15 years of service as of the transition date). Such an approach tries to balance the employer’s desire to move to the new plan with the needs and expectations of older employees. Inevitably, under such an approach, some employees just miss the cutoff.

An approach which is generally intermediate to the wearaway and grandfathering approaches may be designated as “additive”. Under this approach, shown along the light blue line, the starting CB balance is set equal to the old-plan accrued level of $21.3k but, unlike under the wearaway, the employee immediately begins to add additional benefits equal to the incremental benefits under the new 6% CB plan.

Practical transition schemes may be designed anywhere between the grandfather and the wearaway. One approach, not illustrated, is a variation of the additive scheme wherein eligible employees are credited with a higher “transition” compensation crediting rate. Thus, for example, the additive line might be tilted up to follow a path parallel to the 8% or 10% CB plan instead of, as shown, a line parallel to the 6% CB plan.

C Excise Taxes on Assets Reverted to Plan Sponsors

Since 1986, an excise tax has been imposed on such reversions in addition to (and not deductible against, per IRC Section 275(a)6 applied to Chapter 43) the income tax. This tax has, since 1990, been at the rate of 20% of reverted assets if a qualified replacement plan (typically a DC plan would replace a DB plan) is funded with not less than 25% of subsidized benefits and discount rates prescribed by the PBGC. An illustration of this more practical situation goes beyond the confines of our deliberately streamlined example. 10% excise tax on asset reversions, IRC Section 4980, added by Pub. L. 99-514, title XI, Sec. 1132(a), for reversions after December 31, 1985. Increased to 15% by Pub. L. 100-647, title VI, Sec. 6069(a), for reversions after December 31, 1988. Increased to 20% by Pub. L. 101-508, title, XII, Sec. 12001 which further provided a rate of 50% unless the employer used...
the surplus (the residual after settling plan liabilities), and at a 50% rate in the absence of such replacement.

The impetus for the inclusion of the excise tax was Senator Howard Metzenbaum (D-Ohio, 1977 - 1995) reacting to the reversion of excess plan assets by more than a few large and visible corporations in the early to mid 1980's. The result has been that substantial plan reversions by taxable corporations are now rare.

To see why, consider a surplus of assets over the cost of settling liabilities equal to $1 for a corporation in the 35% ($\tau_c$) federal income tax bracket. If the $1 is returned to the corporation directly, a 50% excise tax is applied in addition to the 35% marginal income tax rate leaving $.15 after tax. If the employer elects to divert $.25 towards a qualified replacement plan, then the revertible amount is reduced to $.75 against which a 55% tax is imposed (20% excise plus 35% income) so that the after-tax return to the corporation is $.3375. Clearly the law intends to induce the latter choice if any reversion is to occur.

The recovery of $.3375 may be compared to a recovery of $.65 ($$(1-\tau_c)$$) before the addition of IRC Section 4980. In Section II, we hypothesize that one of the reasons for the adoption of CB plans by plan sponsors relates to this excise tax provision.

D Funding Considerations

The passage of ERISA in 1974 established the PBGC which introduced federal guarantees for DB pension plans that were unable to meet the promises made to plan participants. Sharpe (1976) characterized the existence of the PBGC guarantee as a put option. If the premium for the PBGC coverage is not properly determined, Sharpe concluded that corporations could maximize shareholder interests by a combination of higher pension liabilities (offered to employees in exchange for reductions in other compensation), lower pension assets (caused by deliberately smaller employer contributions) and investment in risky assets.

at least 20% of the otherwise revertible assets to fund immediate benefit increases or at least 25% to fund a qualified replacement plan, for reversions after September 30, 1990.
Since that time the PBGC has defended itself by successfully pursuing legislative action to raise premiums, to adjust premiums for risk (albeit inadequately), to require faster funding for poorly funded plans, to increase the amount and status of claims that the PBGC can make against the sponsor of a terminated plan, to restrict voluntary termination (put exercise) to plans that are sufficiently funded (thus the put must be out-of-the-money when exercised by the sponsor) or to insufficient plans sponsored by “distressed” employers.

Nonetheless, for a distressed company with an underfunded DB plan, the Sharpe PBGC put may provide an incentive for deliberately risky investment. In a CB plan this can encourage a mismatching of assets and liabilities.

Prior to the enactment of the first of the Metzenbaum reversion excise taxes (Appendix C), overfunding of a plan, regardless of cause, was potentially beneficial to shareholders. As long as the recapture of such excess funding was subject only to income taxes (i.e., at rate $\tau_c$) at the corporate level, the existence of excess assets added to potential shareholder gains. As Tepper shows, if such an $E_p$ were invested in fixed income, shareholders gain; invested in equity, shareholders face a wash. With the advent of the excise tax, however, there exists a tax trap for highly overfunded plans. If the amount of excess is so large that it cannot be absorbed by the regular development of newly earned benefit accruals, it must ultimately be subjected to the excise tax upon termination at some later date. In effect, each plan is short an "excise tax call" option if its assets reach levels sufficiently in excess of liabilities.

Combining the long position in the Sharpe put and the short position in the excise tax call, there is a sinusoidal curve to the value of $E_p$ (picture a sine function from $-\pi/2$ to $+\pi/2$). As the assets fall well below the level of the liabilities, the put increases in value and the PBGC shares in the marginal downside more than it does in the marginal upside resulting in the convex portion of the sinusoid. As the assets rise well above the liability

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57 This is not to say that this is inescapable. A company in such a position may get better value for its surplus by using it to pay for post-retirement health care costs using IRC Section 420 or it may acquire or be acquired by a company with an underfunded plan, etc.
level, the present value of future excise taxes grows in a fashion that creates the concave portion of the sinusoid.

Our assumption A.6 is intended to restrict us to the relatively linear center of the curve and in general we value neither the put nor the call. We do not think that this is a very serious restriction because we are focusing on the large majority of major corporate plans that are generally in the linear section of the curve. Additionally, the accelerated contributions now required for poorly funded plans (together with other PBGC instigated rules) has substantially diminished the significance of the put. Similarly, the IRS has promoted legislation tightening the limits on tax deductible annual contributions (most recently there has been a moderate loosening) making significant overfunding somewhat less likely. The option to transfer excess assets to health care plans under IRC Section 420 may serve to reduce the value of the excise tax call. Further, many plan sponsors are aware of the overfunding trap and their efforts to avoid it also reduce the call value.

E  Actuarial and Accounting Costs

Appendix G provides a verbal summary of Statement No. 87 of Financial Accounting Standards (FAS 87). Appendix J defines notation used throughout this paper. In this appendix we develop the algebra for pension accounting cost (expense) under FAS 87 in the narrow case where actuarial gains and losses and other amortizable components of periodic expense are treated without detail. We also develop the algebra for the employer’s cash contributions to a pension under the Employee Retirement Income Security Act (ERISA). Again this is done narrowly using one of the acceptable actuarial methods, Projected Unit Credit (PUC), and again without detail with respect to amortization computations and various minimums and maximums contained in ERISA.

**FAS 87 expenses**

The general equation for accounting expense under FAS 87 is given by:

\[
\text{Expense} = iPBO_t + SC_{t+1} + AMT-jMRV_t
\]

where \( i \) is the “settlement” rate which is used to compute the Projected Benefit Obligation (PBO) and the Service Cost (SC). This rate is presently interpreted to be no
greater than the Aa bond rate for bonds of similar duration to the liabilities. $j$ is the expected return on assets. AMT represents annual amortization of the difference between PBO and MRV.

Both the PBO and the SC represent discounted values based on benefits payable upon several possible events (retirement, termination, disability, death) and at various future dates. For a generic cash balance plan, the benefit payable at each such future date is the account balance at that time. Thus the values may be computed as the sum (over dates) of the discounted projected account balances multiplied by the event probabilities. For ease of exposition, we will assume that the event “retirement at age 65” has a probability of one and all other events have probability zero.

We have defined $L_{p,t}$ as the sum of the account balances at the beginning of a year. Since we have also used $t$ to represent the number of years, as of the beginning of the year, since an employee was hired at age $x$, it is consistent to define $L_p$ for one employee so that $L_{p,65-x} = L_{p,65-x}$ represents the age 65 account balance.

Using notation consistent with Appendix A:

$$L_{p,65-x} = k\% \sum_{w=0}^{64-x} C_x w (1 + i)^{65-x - w} = k\% \sum_{w=0}^{64-x} C_x w (1 + i)^{64-x - w}$$

The PBO is the present value of a linear proration of this future balance as of the beginning of the year $[t, t+1]$:

$$\text{PBO}_t = \frac{tk\%(1 + i)^{x-t+65} \sum_{w=0}^{64-x} C_x (1 + s)^w (1 + i)^{64-x-w}}{65-x}$$

---

58 Consistent with Actuarial Standard of Practice (ASOP) 27, this rate is based on the anticipated asset allocation of the plan.
where the fraction \( \frac{t}{65-x} \) represents years employed divided by total career years.

Similarly, the \( SC_{t+1} \) is present value of one year’s fraction of the future balance as of the end of the year:

\[
SC_{t+1} = \frac{k \%}{65-x} \sum_{w=0}^{64} \frac{C_s(t)w(1+i)^{64-w}}{(1+s)^w(1+i)^{64-w}}
\]

The amortizations can be quite idiosyncratic and path dependent but they generally spread the algebraic difference between the PBO and the MRV over a limited period of future years:

\[
\text{AMT} = \frac{\text{PBO-MRV}}{N}
\]

where \( N \), which changes from year to year, need not be an integer and is no greater than fifteen. When the plan’s actuarial assets (MRV) and liabilities (PBO) are equal, amortizations equal zero. When this is true and all assumptions are met during the year:

\[
PBO_t + iPBO_t + SC_{t+1} = PBO_{t+1}
\]
\[
MRV_t + jMRV_t = MRV_{t+1}
\]

so that the expense rationally links the beginning and ending plan assets and liabilities.

**ERISA contributions using PUC**

The Projected Unit Credit actuarial cost method predates FAS 87 and formed much of its basis. As a result, it is the actuarial cost method that most closely resembles FAS 87. One major difference is that the PUC uses the same interest rate for both assets and liabilities. Although we will use the symbol \( i \) for this rate, under ASOP 27 the rate is based on the expected asset returns based on the plan’s anticipated asset allocation and thus the rate is more like the \( j \) rate of FAS 87.
Another difference is that the PUC incorporates interest on the difference between plan assets and liabilities into the amortization factor. Under the PUC the equation that computes the plan’s annual contribution \((\text{CONT})\) is:

\[
\text{CONT} = \frac{\text{AL}_t - \text{AAV}_t}{A} + \text{NC}_{t+1}
\]

where \(\text{AL}\) is the accrued liability and \(\text{NC}\) is the normal cost under the PUC method. These are defined by the same algebra as are the PBO and the SC under FAS 87 except for the definition of the discount rate \(i\). The actuarial asset value \((\text{AAV})\) is defined similarly to the FAS 87 MRV. The amortization factor \((A)\) is an annuity certain.

Unlike the amortization factor for FAS 87, the PUC factor includes an interest charge that is applied to the unfunded accrued liability. We can make the contribution equation look more like the accounting equation by substituting:

\[
\frac{1}{A} = i + \frac{1}{N'}
\]

then:

\[
\text{CONT} = i\text{AL}_t + \text{NC}_{t+1} + \frac{\text{AL}_t - \text{AAV}_t}{N'} - i\text{AAV}_t
\]

which highlights the application of the discount rate \(i\) to both the assets and the liabilities. \(N'\) is not equal to \(N\) but each serves the purpose of spreading the difference between actuarial assets and liabilities over future periods.

**Cash balance plan economic cost**

In Appendix A we defined the economic cost of a DC/CB clone plan to be equal to the compensation credits provided by the employer. The clone plan matched assets and liabilities. If we allow for a mismatch and allow as well for the returns on assets and liabilities to enter into the “cost” computation without adjustment for risk and taxes, we can create an accounting cost with economic elements:
$\text{Cost} = \text{i}_t L_{P,t} + k \% C_x (1 + s)^t i_A MVA_t$

Under this formulation, with MVA defined as the market value of assets, the plan liabilities equal the account balances and the borrowing and investing (financing) costs may be separated from compensation cost.

**F Actuarial Standard of Practice No. 27, Selection of Economic Assumptions for Measuring Pension Obligations**

Adopted by the ASB in December, 1996, ASOP 27 supercedes ASOP 4 with respect to economic assumptions. Generally, economic assumptions include discount rates, investment returns, inflation, compensation scales and related factors. They may be distinguished from noneconomic (demographic) assumptions relating to rates of death, disability, termination of employment, retirement, marriage, etc.

For the purposes of this paper, Section 3.6 *Selecting an Investment Return Assumption and a Discount Rate* is most relevant. Sections 3.5 *Selecting an Inflation Assumption and Section 3.7 Selecting a Compensation Scale* are pertinent to Section V Counter Arguments.

Section 3.6 states in part that “The discount rate is used to determine the present value of expected future plan payments. Generally, the appropriate discount rate is the same as the investment return assumption. But for some purposes, such as SFAS 87 [herein FAS 87] or unfunded plan valuations, the discount rate may be selected independently of the plan’s investment return assumption, if any.”

Section 3.62 *Constructing the Investment Return Range* details two example methods:

a. Building-Block Method – after identifying asset classes and, for each such class, estimating real returns and inflation, “compute an average, weighted real-return range reflecting the plan’s expected asset class mix; and ... combine [this] ... with the expected inflation range.”
b. Cash Flow Matching Method – “Under the cash flow matching method, the expected future investment return range is viewed as the combination of (i) the internal rate of return on a bond portfolio [of noncallable, high-quality corporate or U.S. government bonds] with interest and principal payments approximately matching the plan’s expected disbursements, and (ii) a risk adjustment range.” The risk adjustment range should reflect “expected future plan investments in equities or other asset classes besides high-quality bonds.”

Clearly, the prescription given by ASOP 27 is that the equity premium (or other risk premia) is to be included in the discount rate when the plan is anticipated to invest in equities and other risky assets. In the present environment, the equity risk premium amounts to several percent per annum and equity allocations frequently represent a majority or near majority of plan assets. Thus, for such plans today, ASOP 27 holds that use of the risk-free rate is outside of the actuarial standard of practice.

G Statement No. 87 of the Financial Accounting Standards Board (FAS 87), Accounting for Pensions by Employers

FAS 87 provides the standards of accounting for private employers who sponsor defined benefit pension plans. The priority of pension accounting is to recognize pension cost over the service period of each employee covered by the plan. This objective leads, in part, to the choice of an accrued benefit actuarial cost method since such methods do not spread costs over aggregations of employees. The chosen method is a modification of an earlier actuarial method often identified as Projected Unit Credit (PUC).

The Projected Unit Credit method assigns to each year of service for each employee a pro rata share of the benefit that will have been earned upon separation from service, reflecting an estimate of any past or future wages that may be taken into account by the defined benefit formula. If, for example, an actuary assumes that an employee, hired at age 35 will retire at age 65 and will have a pay history at age 65 that entitles him to an annual retirement benefit of $30,000, the method will dictate that $1000 of that benefit be allocated to each of the thirty years of service.
If that employee is now 50 years of age, the method will recognize $15,000 of the projected $30,000 benefit as having been accrued to date and will recognize an additional $1000 to be earned in the current accounting year. The $15,000 benefit, multiplied by an appropriate annuity factor and discounted for the period prior to the commencement of the benefit, becomes the Projected Benefit Obligation (PBO) attributable to this employee. The PBO for the plan will be the sum for all current participants of the plan including the remaining annuity value for all those no longer actively employed. The $1000 of benefit to be allocated to the current period will also be multiplied and discounted by the same factors. The resulting value is called the Service Cost (SC) for this employee and will be aggregated across employees to yield the plan’s Service Cost for this period. Using the language of traditional actuarial writings, the PBO would be called the Accrued Liability (AL) under the PUC method. The SC would be called the Normal Cost (NC) under the PUC method.

Under the traditional PUC method, the annual cost for the plan would then be determined as: NC + AMT, where the amortizations are periodic payments with a present value equal to the Unfunded Accrued Liability which is the difference between AL and the Actuarial Value of Assets (AAV): PBO-AAV. The AAV is called the Market Related Value of Assets (MRV) under FAS 87. The amortization amount for the current period would depend upon the history of events that created the divergence between the (AAV) and the (PBO). Such items as an existing difference at the commencement of the plan, or a difference created by changes to the benefit formula of the plan or to changes in actuarial assumptions, or to differences over time between actual and assumed experience (actuarial gain or loss) would be amortized over various fixed periods.

FAS 87 modifies this method in several ways:

- The amortizations in the traditional method include interest on the unfunded accrued liability in each amortization element. FAS 87 separates the interest component and applies it to the PBO and the MRV separately. The unfunded accrued liability arising from the initial application of FAS 87 and any changes therein attributable to
changes in the plan formula or actuarial assumptions are spread (amortized) over time without interest. The gain or loss is accumulated (the asset component of gain and loss is included herein only after it has entered into the MRV) and when it exceeds an optional buffer zone known as a “corridor” it is spread over time without interest.

- The interest component applied to the PBO is computed using a discount rate identified as the “settlement rate”. This same rate, designated i below is also used to compute the PBO itself and the SC. It is determined at each valuation date based on the returns available on high-quality fixed income securities or annuities. In the sense that it contains no equity risk premium and does not take the plan’s asset allocation into account, it is conceptually similar to the risk-free rate used elsewhere in this paper. ASOP 27 identifies this as a “prescribed rate” and exempts it from the general ASOP 27 rules pertaining to the selection of an investment return range.

- The interest component applied to the MRV is computed using an expected long-term rate of return on plan assets that is consistent with the ASOP 27 rules. This rate designated j below is generally left unchanged for several years.

The PUC formula may then be reconstituted as: iPBO + SC + AMT-jMRV. Herein, amortizations are without interest. Amortization of accumulated gains or losses only considers such gains or losses outside of the corridor. The asset component of gains and losses only recognizes the difference in actual versus expected return to the extent that the difference has been filtered through the averaging process used to develop the MRV. As in most ACM’s, the special treatment of the divergence between asset assumptions and experience amounts to an extra degree of smoothing.

H Corporate Diversification

We generalize the development of the diversification that arises when previously non-diversified companies purchase each other’s shares in their own pension plans. We do so absent tax considerations. In what follows, upper case, e.g., G identify matrices and column vectors; singly subscripted lower case, e.g., g_k, designate row vectors embedded in matrices; subscripted upper case, e.g., E_i, and doubly-subscripted lower case, e.g., g_k,i, are scalars. We start before any cross-ownership exists and assume
that each of \( n \) companies \((i = 1,n)\) has issued one share of stock equal in value to the firm asset:

\[
E_i = A_i
\]

\[
E = \{E_i\} = \{A_i\} = A
\]

Further, each of \( m \) investors \((k = 1,m)\) has a preferred portfolio which is expressed as fractions of each company’s one share.

\[
g_{k,i} = \text{investor } k\text{'s fractional interest in company } i
\]

\[
g_k = \{g_{k,i}\} = k\text{'s interest in all } n \text{ companies}
\]

\[
g_k A = W_k = \text{wealth of investor } k
\]

\[
G = \{g_{k,i}\} \text{ is an } m \times n \text{ matrix of } m \text{ investors’ interests in } n \text{ companies, each column sums to 1.}
\]

\[
GA = W, \text{ an } m \times 1 \text{ vector of the wealth of } m \text{ investors}
\]

\[
l_m^T W = \text{total wealth = value of the market portfolio =}
\]

\[
l_n^T E = \text{total market capitalization prior to co-ownership}
\]

As in Section IV, each company, \( i \), purchases fractional interests, \( f_{ij} \), in each other company, \( j \), and issues new shares, each equal in value to \( E_i = A_i \), exactly sufficient to accomplish the purchase. The new market capitalization of company \( i \) is:

\[
E_i = A_i + f_i^T E_i
\]

where \( f_i = \{f_{ij}\} \) is a row vector of company \( i \)’s fractional holdings of each other company’s market capitalization.

Although it is not necessary that \( f_{ij} \geq 0 \), we suppose this will generally be so. The various ownership interests may be summarized in a cross-ownership matrix:

\[
F = \{f_i\} = \{f_{ij}\}
\]

The generalization of equation (4) of Section IV then becomes:
\[ E = A + F E \]  

where the market capitalization \( i_n^T E \) now exceeds the market portfolio \( i_m^T W \). Because any investor transactions that may have occurred during the diversification process did not require any net cash (such a transaction might entail the exchange of newly issued shares of company \( i \) for an equal value of shares held in company \( j \)), neither the market portfolio nor aggregate investor wealth has changed. But investors no longer necessarily hold their desired fractions of each firm’s value, represented by \( G A \). This is true even for investors who participated in none of the transactions. From the relationship in (5), defining \( I_n \) as the \( n \times n \) identity matrix, we derive:

\[ (I_n - F)E = A \]

\[ G(I_n - F)E = GA \]

Define:

\[ H = G(I_n - F) \]

\[ HE = GA \]

This means that investors can achieve their preferred portfolios by acquiring fractions of the market capitalization vector \( E \) in accordance with \( H \). The \( k^{th} \) investor selects the \( k^{th} \) row vector of \( H \), \( h_k \) and owns:

\[ h_k E = g_k A \]

Like \( G \), \( H \) is an \( m \times n \) matrix of weights, but unlike \( G \), its columns each sum to less than one. This is because \( H \) is applied to market capitalizations that are inflated by cross-ownership. As does \( GA \), \( HE \) represents the market portfolio, that portion of the total market capitalization that is held by public investors. Each of the \( n \) columns of \( H \) represents the fractional public ownership of one company’s stock. If we divide the \( i^{th} \) column by \( A_i \), the share price of company \( i \), the investor allocation is now denominated in shares. Dividing every column of \( H \) by the corresponding value from \( A \), we get an
An \( m \times n \) matrix of share ownership that we designate \( S \). Because we have forced the share prices to be given by \( A \) (by initially issuing exactly one share of each), we get the coincidental\(^{59}\) equality:

\[
SA = GA
\]

despite the fact that no value for any \( s_{k,i} \) need equal the value for \( g_{k,i} \). We note that the market portfolio equals the aggregate publicly held shares equals the total investor wealth equals the aggregate firm wealth:

\[
I_n^T E^* = GA = SA \equiv HE = I_m^T W = I_n^T A
\]

where the central identity indicates that \( SA \) and \( HE \) are not only the same in the aggregate but are, in fact, identical.

By choosing his row vector \( h_k \) in \( H \), the investor is able to reproduce his preferred portfolio as a weighted average of exposures to firm values given by \( A \). Every portfolio that the investor might construct must also represent a weighted average of firm values (to see this merely multiply \( H \) by \( (I_n - F)^{-1} \) on the right to retrieve \( G \)). Since \( g_k \) represents the investors preferred choice among all such pre-diversification allocations, \( h_k \) must represent his preferred choice among all post-diversification allocations. What this means is that the actions of the firms in selecting equity portfolios for their pension plans neither help nor hurt the investors in the absence of tax and bankruptcy considerations. With respect to taxes, this supports Black’s contention that his imperfect hedge is perfectly adequate to enable the tax arbitrage that he seeks without additional consequences for shareholders.

\[^{59}\] The only coincidence is that \( A \) represents both share prices and firm values. The aggregate equality would prevail in any case but \( A \) would not appear on both sides of the equation.
I Why $\tau_{ps} < \tau_{pb}$

Why is the effective rate less for equities than for fixed income? The Internal Revenue Code (IRC) distinguishes between ordinary income and capital gains in two ways:

- **Deferral of taxes on unrealized capital gains** - Although equity dividends are taxed when paid, capital gains are not taxed until the asset is sold.

- **Differential rate on realized capital gains** – Even when capital gains are realized and become taxable, the IRC has frequently applied lower rates on such gains than on ordinary income\(^{60}\).

Although it is possible to trade fixed income securities and thus produce capital gains, most of the income on fixed income securities is interest income that is taxed as earned. In the case of bonds bought at par and held to maturity, all of the income earned is taxable interest income. Expected equity income consists of two components: dividends\(^{61}\) and appreciation. Appreciation may be expected to represent the majority of total returns.

The deferral of taxes on appreciated stock delivers an annual effective rate of taxation that is a declining convex function of the holding period. This rate approaches zero as the holding period extends without limit and, for those who cannot wait forever, the entire appreciation is forgiven upon the death of the shareholder. Thus, the $\tau_{ps}$ rate might be represented as $\tau_{ps}(h)$ where $h$ represents the holding period.

In Section V, we use a 39.6% rate for $\tau_{pb}$ which also applies to the dividend portion of the total return on equities. Dividends represent a small (and recently declining) fraction

---

\(^{60}\) Ordinary income marginal rates are presently 15%, 28%, 31%, 36% and 39.6%. Realized long term capital gains are taxed at a 10% rate for those in the lowest bracket and 20% for all others.

\(^{61}\) Company repurchases of outstanding stock have come to some favor in recent years along with lower dividend payout ratios (dividends/free cash flow). This means that stockholders who wish more income can sell their stock and benefit from lower rates on realized capital gains.
of the total expected return. As of 8/27/99, the dividend yield on the S&P is 1.25\% representing perhaps 1/8 of the total expected return based on historic equity premia and the current T-bill rate. The tax on these dividends amounts to no more than .50\% of the market value of the index and about 5\% of the expected return. Capital gains taxes on securities held for more than one year are limited to 20\% as of this writing. The asymptotic rate on equities approaches the full tax rate applied to the dividends only. Thus we can expect to find \( \tau_{ps} \) in the range between .05 and .225 (1/8 \times .4 + 7/8 \times .2).

These assumptions bear a relationship to the Miller equation for neutrality between shareholder and corporate leverage. Using a 15\% rate for \( \tau_{ps} \) in the Miller equation:

\[
(l-\tau_{pb}) = (l-\tau_c)(l-\tau_{ps})
\]

we have the left side equal to .6040 and the right side equal to .5525 implying relatively small value to corporate leverage as an alternative to personal borrowing.

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* Web site citations are vulnerable to updates and address changes.
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