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Abstract

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Comments

NOTE: At the time of publication, author Daniel Koditschek was affiliated with Yale University. Currently, he is a faculty member in the Department of Electrical and Systems Engineering at the University of Pennsylvania.

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Natural Motion for Robot Arms

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This paper describes some initial steps toward the development of more natural control strategies for free motion of robot arms. The standard lumped parameter dynamical model of an open kinematic chain is shown to be stabilizable by linear feedback, after nonlinear gravitational terms have been cancelled. A new control algorithm is proposed and is shown to drive robot joint positions and velocities asymptotically toward arbitrary time-varying reference trajectories.

1. Introduction

Typical robotic tasks may be described by appeal to dynamical models of motion rather than explicit geometric means such as curves or knots. The terminology "natural motion" expresses the hope that those models which in some way match the internal dynamics of the robot will afford more accurate performance with less effort. This approach requires (i) specification of a suitable class of reference models; (ii) a control strategy for their use; resulting in (iii) asymptotically stable error equations. This paper addresses the second and third aspects of the problem while suggesting future directions of investigation within the first.

A familiar model for the dynamics of a robot arm with n joints whose positions and velocities are represented by the generalized coordinates, θ, ω, respectively, may be written as

$$\begin{aligned} \dot{\theta} &= \omega \\ \dot{\omega} &= - M^{-1}(\theta)[\, B(\theta, \omega)\omega + k(\theta) - \tau\,], \end{aligned} \tag{1}$$

where M, the positive definite moment of inertia matrix, and k, the gravitational vector field depend upon products of transcendental terms in θ; and B (representing coriolis and centripetal forces) is linear in ω, and involves products of transcendental terms in θ. A commonly proposed paradigm for the control of robot arm motion involves the a priori generation of an algebraic reference trajectory, resolved into joint space, and introduced to a set of actuating motors in the presence of error and state feedback. Let θ_r, ω_r, and $\dot{\omega}_r$ specify some desired motion, velocity, and acceleration profile, and define the errors

$$\begin{aligned} \delta &\triangleq \theta_r - \theta \\ \epsilon &\triangleq \omega_r - \omega. \end{aligned}$$

The control strategy

$$\tau_{PI} \triangleq B\omega + k(\theta) + M[\, \dot{\omega}_r + \gamma_0\delta + \gamma_1\epsilon]; \quad \gamma_1, \gamma_0 > 0 \tag{2}$$

may be shown, theoretically, to drive the position and velocity errors to zero, exponentially in time, for arbitrary initial values. This method has been called the "computed torque" technique [1], or the "inverse problem" [5]. The general scheme proposed independently in [3] amounts to just this strategy in the context of robot arm control. An analogous procedure, termed "resolved acceleration control" [5] can be used when the reference signals are expressed in Cartesian or task space coordinates.

As a practical matter, a number of objections to this strategy come to mind quite readily. The global stability of the scheme is founded upon such assumptions as that

(i) equation (1) provides an accurate model of the true robot arm dynamics;
(ii) the terms in (2) may be computed sufficiently quickly.
(iii) the terms in (2) may be computed with sufficient accuracy;
(iv) the actuators are capable of delivering the torques required by (2).

Work by a number of researchers, most notably in [4], has persuasively demonstrated that the second assumption is, or may soon be justifiable. However, the rest of these assumptions merit much less confidence. The new algorithm reported here may ameliorate some of the problems hidden by these assumptions, although analysis that might conclusively demonstrate its superiority to (2) is not yet available. Since it may lead toward appropriate dynamical reference models, the algorithm should be interesting in its own right.

Rewriting the the scheme (2) as

$$\tau_{PI} = (B - \gamma_1 M\,)\omega + k(\theta) - \gamma_0 M\,\theta + M[\,\dot{\omega}_r + \gamma_1\omega_r + \gamma_0\theta_r]$$

makes obvious its intuitive analogy to the technique of pole placement. In a linear time invariant setting, state feedback is used to cancel undesired coefficients of the characteristic polynomial and replace them with coefficients which result in the desired root locations prior to the introduction of the reference command signal. In the present context, the cancellation of all nonlinear terms of the vector field reflects the designer's uncertainty as to which of them should be considered "undesirable". Notice that after the first three state feedback (pole placing) terms of τ_{PI}, above, is a set of three terms involving the reference signal. If M were constant (and assuming ω_r is indeed assigned to be the derivative of θ_r), these terms would constitute the output of a system whose transfer function is the inverse of exactly that linear time invariant system whose poles the plant has been assigned. It will be convenient in the sequel to use this analogy by referring to the first three terms of τ_{PI} (and the corresponding terms of the new algorithm, $\tau_{P\cdot I}$, proposed in Section 4) as the "pole placement" portion, and the last three terms as the "inverse filter" portion of the control strategy.

Intuition might suggest, to the contrary, that terms which do not contribute to instability ought not to be cancelled: if inverse filtering is countenanced, it should be as closely matched to the stable part of the plant as possible. The central result of this paper, presented in Section 3, is the demonstration that system (1) can be stabilized by linear state feedback in the absence of the gravitational term of the vector field, $k(\theta)$. In consequence, a new algorithm which incorporates the moment of inertia and coriolis matrices in the precompensating "inverse filter" rather than "pole placement" portion of the strategy, is shown, in Section 4, to drive θ and ω toward arbitrary reference trajectories. This arrangement begins to suggest the sort of reference models whose output might specify "natural motion."

2. Notation and Preliminary Results

To aid the exposition of the main result, it is worth pausing to review the derivation of (1) and introduce some notation. If $g: \boldsymbol{R}^n \rightarrow \boldsymbol{R}^m$ has continuous first partial derivatives, denote its m $\times$ n jacobian matrix as dg. When we require only a subset of derivatives, e.g. when $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $x_i \in \boldsymbol{R}^{n_i}$, $n_1 + n_2 = n$, and we desire the jacobian of g with respect to the variables x_1, as x_2 is held fixed, we may write

$$d_{x_1} g \triangleq dg \begin{bmatrix} I_{n_1} \\ 0 \end{bmatrix},$$

where I_{n_1} is the $n_1 \times n_1$ identity matrix, and 0 is the $n_2 \times n_1$ zero matrix. If $A \in \boldsymbol{R}^{n \times m}$ then its representation in $\boldsymbol{R}^{nm}$ formed by "stacking" each column below the previous - Bellman's "stack" operator [2] - will be denoted A^s. If A is an n $\times$ m array, and B is a p $\times$ q array then their Kronecker Product is defined as the np $\times$ mq array

$$B \times A \triangleq \begin{bmatrix} Ab_{11} \dots Ab_{1q} \\ \vdots \\ Ab_{p1} \dots Ab_{pq} \end{bmatrix}.$$

If $g(x) \triangleq A(x)b$, where b is a constant vector in $\boldsymbol{R}^m$, then it may be shown that

$$dg = [\, b \otimes I_n \,]^{\mathsf{T}}\, dA^s,$$

where I_n denotes the n $\times$ n identity matrix.

To derive (1) we will use the Lagrange formulation of the dynamical equations. If θ is a vector of generalized coordinates, with derivatives, ω, then it can be shown that the total kinetic energy of the robot arm is given by

$$\kappa \triangleq \tfrac{1}{2}\, \omega^{\mathsf{T}} M\, \omega,$$

where M depends upon transcendental terms in θ, arising from the kinematics of the arm. If ϕ is the gravitational potential, and τ is the vector of generalized forces applied at each joint, then the equations of motion obtain from

$$\tau^{\mathsf{T}} = \frac{d}{dt}\, d_{\omega} L - d_{\theta} L$$

where $L \triangleq \kappa - \phi$. If, as in this case, ϕ does not depend upon ω, then we may write

$$\frac{d}{dt}\, d_{\omega} L = \frac{d}{dt}\, d_{\omega} \kappa$$
$$= [M\, \dot{\omega} + \dot{M} \omega]^{\mathsf{T}}.$$

It follows that the only terms multiplying ω in (1) arise from

$$\dot{M}\, \omega \;-\; \tfrac{1}{2}\, [d_{\theta}\kappa]^{\mathsf{T}}. \tag{3}$$

Now notice that

$$\dot{M} \omega = [\omega \otimes I]^{\mathsf{T}}\, \dot{M}^s$$
$$= [\omega \otimes I]^{\mathsf{T}} \;\; [d_{\theta} M^s] \;\; \omega,$$

while

$$d_{\theta}\kappa = \omega^{\mathsf{T}}\, d_{\theta}(M\, \omega)$$
$$= \omega^{\mathsf{T}}\, [\omega \otimes I]^{\mathsf{T}} \;\; [d_{\theta} M^s]\,.$$

Expression (3) may thus be re-written as

$$\tfrac{1}{2}\;\; \dot{M}\; \omega + \tfrac{1}{2} \left[(\omega \otimes I)^{\mathsf{T}} [\, d_{\theta} M^s \,] - [\, d_{\theta} M^s \,]^{\mathsf{T}} (\, \omega \otimes I \,) \right] \omega$$
$$= \tfrac{1}{2} \;\; [\, \dot{M} - J \,]\, \omega,$$

where J is a skew-symmetric matrix for all ω and θ. In other words, B, the coriolis and centripetal force term in (1) may be written as the sum of the time derivative of the positive definite symmetric moment of inertia matrix,M, and a skew-symmetric matrix, J.

3. A Simpler Stabilizing Feedback Strategy

It is now possible to show that in the absence of gravity, and admitting assumption (i), from Section 1, any kinematic chain possessed of a positive definite moment of inertia matrix is stable in the large. The same Lyapunov function used in the latter demonstration may be again employed to show that the system is stabilizable - i.e., may be made globally asymptotically stable - using linear state feedback. For the sake of brevity, it is preferable to prove a single statement regarding the full robot model presented in the introduction.

Define a simpler pole placement feedback algorithm

$$\tau_{P'} \triangleq k(\theta) - \gamma_1 \omega - \gamma_0 \theta,$$

which gives rise to the closed loop system

$$\begin{aligned} \dot{\theta} &= \omega \\ \dot{\omega} &= -\, M^{-1}(\theta) [\, (B + \gamma_1 I)\omega + \gamma_0 \theta \,]. \end{aligned} \tag{4}$$

Theorem 1: System (4) is globally asymptotically stable.

Proof: Consider the positive definite Lyapunov candidate

$$v \triangleq \tfrac{1}{2}\, (\, \gamma_0 \theta^{\mathsf{T}} \theta + \omega^{\mathsf{T}} M \omega),$$

whose time derivative is given by

$$\dot{v} = \gamma_0 \theta^{\mathsf{T}} \omega \;+\; \tfrac{1}{2} \omega^{\mathsf{T}} \dot{M}\, \omega - \omega^{\mathsf{T}} B \omega - \gamma_1 \omega^{\mathsf{T}} \omega - \gamma_0 \omega^{\mathsf{T}} \theta$$
$$= \tfrac{1}{2} \omega^{\mathsf{T}} J \omega - \gamma_1\, \omega^{\mathsf{T}} \omega.$$

Since J is skew-symmetric, the first term of the last equation is zero, and $\dot{v}$ is negative semi-definite. Thus, the error system is globally stable, and ω converges to zero. To show that the system is globally asymptotically stable - i.e. that θ converges to zero as well - it suffices to show that the vector field in (4) is never directed toward the interior nor lies upon the boundary of the surface described by $\dot{v} \equiv 0$. In the case at hand, this surface is the time-invariant subspace $\omega \equiv 0$, and the vector field restricted to this subspace is given by

$$f|_{\omega \equiv 0} = \gamma_0 \begin{bmatrix} 0 \\ M^{-1}\theta \end{bmatrix}.$$

Since M is positive definite, the condition is satisfied, and the result follows. □

Future research may reveal terms in the gravitational part of the vector field of (1), k, to be harmless from the point of view of stability. This would afford a still simpler version of $\tau_{P'}$.

4. A New Algorithm

The strategy of τ_{PI} - pole placement preceded by inverse filtering of the pole-placed system - may be imitated in light of Theorem 1 by the following feedback algorithm:

$$\tau_{P'I} \triangleq k + M\dot{\omega}_r + B\omega_r + \gamma_1\epsilon + \gamma_0\delta. \tag{5}$$

The reader will notice that a portion of this expression exactly matches $\tau_{P'}$ while the remaining terms involving the reference signals constitute the "inverse filter" of system (4). The resulting error equations may be written as

$$\begin{aligned} \dot{\delta} &= \epsilon \\ \dot{\epsilon} &= - M^{-1} [B \epsilon + \gamma_1\epsilon + \gamma_0\delta], \end{aligned} \tag{6}$$

and are seen to be globally asymptotically stable by appeal to the methods of Theorem 1.

Corollary 2: System (6) is globally asymptotically stable .

Proof: Define the Lyapunov candidate

$$v \triangleq \tfrac{1}{2} (\gamma_0\delta^T\delta + \epsilon^T M\epsilon).$$

and follow the same argument as in the proof of Theorem 1. □

Of course, it is no more possible to match coefficients exactly in an "inverse" pre-filter than to cancel them via state feedback: $\tau_{P'I}$ has, as yet, no better theoretical claim to having addressed the problems of assumption (iii) in the introduction than has τ_{PI}. However, assuming that $k(\theta)$, the gravitaional torques, may be computed with sufficient accuracy (they are computationally less complex), there is some reason to believe that $\tau_{P'I}$ is preferable in this regard. In a linear setting, the stability of the closed loop may be compromised by trying to cancel parameters whose values are not known with sufficient accuracy in the feedback pathway: an innaccurate precompensating filter followed by a stable plant (stabilized without prescribed poles) may cause innaccurate tracking but will never produce instability. Independent theoretical arguments are needed to establish the analogous result for $\tau_{P'I}$, since the computation of M and B still depends upon plant variables: this precompensating filter is not external to the nonlinear feedback loop.

A fundamental goal of this research is the means of command by dynamical reference models. The algorithm presented in (5) suggests regarding θ_r, ω_r as the state variables of the exponentially stable linear time varying system[1]

$$\begin{aligned} \dot{\xi} &= \varsigma \\ \dot{\varsigma} &= - M^{-1}[(B + \gamma_1 I)\varsigma + \gamma_0 \xi - \sigma]. \end{aligned} \tag{7}$$

That is to say, setting the command input to be

$$\tau_{P'I'} \triangleq k(\theta) + \sigma - \gamma_1\omega + \gamma_0[\xi - \theta], \tag{8}$$

it follows that ω and θ must converge to ς and ξ, respectively, according to Theorem 1.

The implication here is that suitable dynamical models of desired motion should include (7). Since the correct estimates of M and B will ultimately depend upon particular features of the task - for instance the mass and moments of a workpiece - as well as the current state of the plant, it makes some sense to isolate them from the plant error feedback loop. There would still remain the question of a suitable earlier subsystem of the reference model which must estimate a "proper" value of σ. Since (7) is linear and exponentially stable, there is some reason to hope that specifying a natural class of σ commands (i.e. those which produce the desired velocity and position reference signals) will be more tractable than the original problem of determining "natural motions" of (1), or even of the globally (exponentially) asymptotically stable, but nonlinear system, (4).

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[1]Note that the stability of the unforced system is again established using the scalar Lyapunov function.