Global asymptotic stability of a passive juggler: a parts feeding strategy

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Global Asymptotic Stability of a Passive Juggler: A Parts Feeding Strategy

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1 Introduction

As industry moves toward faster product cycles, smaller production runs, and shorter product development time, the idea of flexible manufacturing as a means of improving the quality, variety, and overhead cost of producing goods has caught on. Programmable mechanisms — robots, NC controlled milling machines, etc. — abound. It is becoming cheaper and easier to use flexible equipment all the time. However, all of this machinery suffers from a common drawback: parts need to be fed, one at a time, and absent sensors [1], each part must be fed in a precise orientation at a precise location. This is the Parts Feeding Problem. "Ultimately, the smartest assembly robot and the best assembly machine in the world are useless without the mechanism that delivers the parts." [10]

*This work was supported in part by the NSF under grant IRI-912366, and in part by Deneb Robotics, Inc.

[1] Two basic vision strategies are used for parts feeding. The first is rejection based: parts already in the correct orientation are removed from the feeder, while those with incorrect orientation are ignored and recirculated. The second is orientation based: the vision system identifies the orientation of each part, and a robot re-orients the part appropriately. Both vision approaches are extremely flexible, but limitations of the technology often prevent vision from implementation in an industrial setting.

1.1 Parts Feeding: The Orientation Problem

The Parts Feeding Problem can be broken into three sub problems: singulation, orientation, and presentation. Singulation is the process of separating the mass of parts into individual parts, and can be very difficult if the parts nest within each other easily (like thimbles) or become entangled (like paper clips). Orientation is the process of reorienting the randomly oriented parts to a small pre-determined subset of the possible orientations (typically only one). Presentation is the action of moving the singulated and oriented part to a known location, where a machine tool or robot can easily perform an operation on it.

The orientation problem lies at the heart of the parts feeding problem. Little is known about how to orient an arbitrary part beyond decades of craftsmanship and experience. Even in textbooks, the orientation problem is presented with a cookbook approach, in the manner of "this approach worked with this type of part." [4, 3, 8, 15]

Current technology in parts feeders relies heavily on rejection techniques. These techniques randomize the orientation of the parts, often by shaking or dropping then, and then reject all those parts which are not in the correct orientation. The rejected parts are then recirculated, and the process repeats indefinitely. Research into the probability distribution of stable random part orientations [25] suggests that for a typical part, a minority [2] of the parts will randomly assume the correct orientation. For this reason, rejection based methods are very inefficient [5].

[2] We note that flat or round parts have a high probability of randomly assuming a useful orientation, and are easily fed with existing technology. We consider here more complex parts, such as irregular polyhedrons.

1.2 Background and Contributions of this Paper

We present simulation results from a 3 DOF simplification of the parts feeding problem, and analysis of a system which is a 1 degree of freedom further simplification. It is hoped that this analysis will lead to more general results which will be useful for industrial parts feeding applications. Section 2 states the general problem and develops the simplified setting. Section 3 presents an analysis of that setting and summarizes our formal results.

Mason and colleagues have pioneered the analysis and potential assembly applications of sensorless manipulation in the robotics field [9]. Canny and Goldberg have enlarged and have begun to formalize this program in the effort to minimize sensing and automation complexity without unduly compromising its usefulness [7]. An interesting and rather different approach to the parts feeding problem considered here has recently been taken by Böhringer, et al. [2], who consider the possibility of re-orienting planar parts through nodal shapes introduced by plate vibrations in the supporting table. Antecedent to this work, Singer and Seering [20] investigated the problem of parts rocking (rather than bouncing) on a vertically vibrating table. Sony's APOS system [19] nests multiple parts on a vibrating tray with indentations shaped to conform to the desired pose. Grossman and Blasgen's tilted vibrating box randomized motion, capturing parts in a limited number of predictable poses which could be distinguished by simple probe measurements [11].

We seek in this paper to enlist properties of dynamical manipulation in the program of reduced sensory and actuator complexity just described. We adapt suggestive work by Atkeson and Schaal on the "Shannon juggler" to the sensorless manipulation paradigm [18]. The robotics literature reports a growing number of experimental successes with dynamical manipulation, mostly involving hopping, walking, or juggling mechanisms [16, 14, 1, 17, 6, 20]. Analysis of these machines has also been reported, albeit with more limited success [26, 13, 23]. There is a large and growing analytical literature surrounding the 1 degree of freedom bouncing ball that we study here, most of it motivated by an interest in chaos[22, 12]. We, of course, are interested in stable motion.

In this paper we analytically demonstrate the feasibility of deterministically manipulating the stable dynamic behavior of a one degree of freedom part without the use of feedback. The manipulation strategy of Equation (2) calls for a supporting table that mimics a lossless bouncing ball whose mass is much greater than the part. This is equivalent to juggling without sensing. When the ensuing collisions between the part and the table are governed by a coefficient of restitution that is sufficiently small, it can be guaranteed that every initial condition of the part will be knocked into a unique periodic motion. This result is illustrated in Figure 3 and the precise conditions for global asymptotic stability are summarized in Section 3.

2 Problem Statement

Since rejection techniques are inefficient and remote sensing and orientation techniques are often slow or expensive, we examine a potentially alternative strategy. Namely, we seek to design a table motion which will cause all the "bodies" on the table to asymptotically approach a known state without using feedback. We propose a flat, level, 3 degree of freedom vibratory table as a viable means of orienting pre-singulated parts.

The vibrational strategy should work by bouncing parts gently on the table vertically, while inducing momentary horizontal forces at the contact points which cause a torque to be applied to the center of mass. One would hope that if the vibrations are adequately designed, after a short period of time the parts will all rotate to a stationary pre-determined orientation with a very high probability. The shaking may then be stopped and the parts land in this known orientation, again with very high probability.

The problem now amounts to finding favorable vibration parameters – wave shape, frequency, magnitude – or, indeed, determining whether such parameters exist at all. Despite the intuitively compelling nature of this idea, it turns out that the design of such vibratory strategies seems possible but not obvious, as our preliminary simulation data will suggest.

Figure 1 shows a 3 degree of freedom model of the full 6 degree of freedom problem. We seek to demonstrate global stability of a desired bouncing state for a particular 1 degree of freedom juggling scheme. Sinusoidal motions [22] have an extensive background in the literature, but study of their dynamics has been mostly limited to system parameters leading to chaotic motion. Local stability of the Shannon juggler has been established [18]: we seek a global result motivated by the intended application. Our simulations suggest that by careful parameter selections, we can create globally attracting trajectories even for 3 degree of freedom systems. We hope analytically to prove global re-

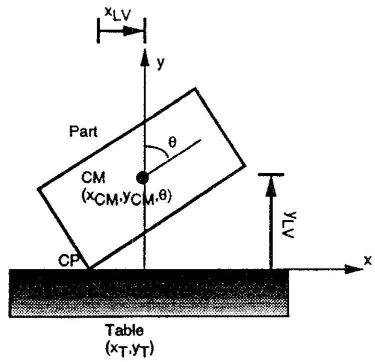

Figure 1: Setup for the 3 degree of freedom part orientation problem. A side view of the part is depicted. The y axis is vertical.

sults in these systems.

For the remainder of the discussion, the terms "robot" and "table" (henceforth "robot-table") will be treated as synonymous, as are the terms "ball" and "part" (henceforth ball-part).

2.1 Simulation Results

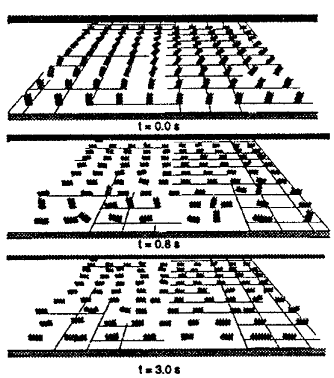

Figure 2: A 3 degree of freedom simulation. (x vibration 30 Hz 1 mm)

Figure 2 depicts a typical simulation of the part-table interaction shown in Figure 1. In this simulation, the parts are constrained to move in vertical planes indicated by the horizontal lines on the table. The simulation uses a standard 3 degree of freedom Newtonian flight model when the part is not in contact with the table, and employs an impact model with friction developed by Wang and Mason [24] when each part contacts the table. Each part is integrated in isolation from all the others: part-part interactions are not modeled. The

x frequency, magnitude	all parts lying down
15 Hz 2 mm	4.0 s
15 Hz 4 mm	Chaotic
30 Hz 1 mm	2.9 s
30 Hz 2 mm	2.1 s
60 Hz 0.1 mm	5.0 s
60 Hz 1 mm	3.1 s

Table 1: Sensitivity to horizontal and vertical sinusoidal vibrations. Times indicate approximate onset of steady state where all initial orientations have evolved to the 90° orientation. "Chaotic" denotes a vibration in which both statically stable orientations were destabilized and no simple steady state behavior emerged. The y vibrations used were 60 Hz 0.2 mm, in phase with the x vibrations.

foremost row of parts on the table have initial orientations θ_0 at $t = 0$ from $\theta_0 = 0°$ to $\theta_0 = 9°$, viewed from left to right. Each successive row on the table increases the initial orientation by 10°, up to the lone part in the back row with $\theta_0 = 90°$. All parts are initially in contact with the table in their lowest corners. Letting the parts fall in gravity without vibration, those with orientations $0 - 26°$ are attracted to the 0° orientation, while those with orientations $27 - 90°$ are attracted to the 90° orientation.

Figure 2 now depicts the future evolution of these 91 different initial conditions at times $t = 0s$, $t = 0.8s$, and $t = 3.0s$ in the face of a table vibration. For this particular run, the table vibration was a 60 Hz 0.2 mm sinusoidal vibration in the y direction with an in-phase 30 Hz 1.0 mm sinusoidal vibration in the x direction. Supposing our goal is to end up with a part lying with its long face on the table, this vibration seems to work.

Table 1 illustrates the results of several such simulations. These simulations demonstrate that even for such a simply shaped part, finding an effective vibration strategy is by no means straightforward.

Figure 3 shows the evolution of several initial states using the vastly simpler one degree of freedom model of Figure 4. All initial states rapidly converge to a stable oscillation, for which we have derived analytical stability conditions in [21]. The robot-table oscillation is the bottom set of arcs in the figure.

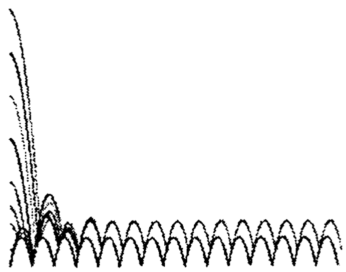

Figure 3: Simulation showing rapid convergence to a stable oscillation from several initial conditions. Figure shows position as a function of time with $\alpha = 0.2$. The bottom set of arcs depict the table's motion – a relaxation oscillator modeled by a lossless bouncing ball.

3 Summary of Analytical Results

In this section, we summarize the analytical results of [21]. Space limitations preclude all but a sketch of this analysis, and we refer the interested reader to our WWW site [3] and the forthcoming paper [21] for a complete account.

3.1 The Effective Environmental Control System

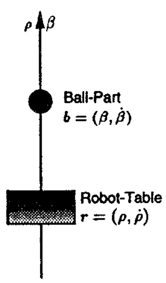

Figure 4: Setup for the 1 degree of freedom ball-part manipulation problem.

The system shown in Figure 4 consists of a one degree of freedom "robot-table,"

$$r = (\rho, \dot\rho)$$

and a lighter "ball-part,"

$$b = (\beta, \dot\beta)$$

[3]Relevant documents by the authors may be found at http://ai.eecs.umich.edu/people/pjswan/pjswan.html

falling in the earth's gravitational field and constrained to move in the direction of gravitational field gradient (vertically). The ball-part falls from some initial position and velocity, b, according to $\ddot b = -\gamma$ and reacts to a collision with the robot-table at some state r according to the coefficient of restitution α.

Now let b denote the state of the ball-part just prior to an impact. Suppose the robot-table impacts with velocity v and allows the ball-part to fall freely for the next τ_c (time to collision) interval of time. Then the state of the ball-part just prior to the next impact is given by

$$f(b, \tau_c, v) := F^{\tau_c} \circ C_v(b) \tag{1}$$

where F indicates the flight law and C represents the impact law. Any effect of the robot-table on the ball-part may be described with regard to this model, which is, in effect, a discrete dynamical control system.

3.2 Control Design and Analysis

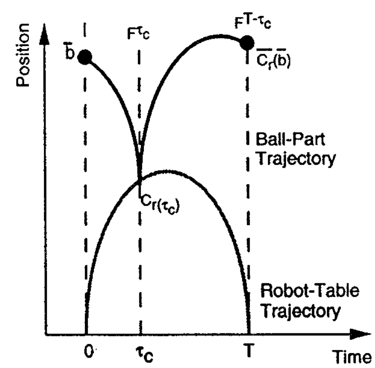

Figure 5: Coupled Oscillators

We choose a relaxation oscillator depicted in Figure 3 as the robot-table trajectory: the robot-table behaves as a lossless bouncing ball in gravity with period T.

$$\begin{gathered} \ddot r = -\gamma \\ r(t^-) = 0 \Rightarrow \dot r(t^+) = -\dot r(t^-) \end{gathered} \tag{2}$$

We then couple the ball-part to the robot-table using Newtonian gravity and restitution, [4] assuming the robot-table mass is so much greater than that

[4]The question of whether Figure 3 is a feasible robot-table trajectory arises; our simulations suggest that the velocity discontinuity at the bottom of the robot-table trajectory may be replaced with a smooth transition without noticeably affecting the stability of the coupled system.

of the part that there is no change in the motion of the robot-table before and after collision.

By sampling the ball-part's state at the instant of the robot-table's minimum position (as shown in Figure 5), we are able to create a discrete-time dynamical system

$$\begin{aligned} \bar{b}_{k+1} &= \bar{f}_1(\bar{b}_k) \\ &:= F^{T-\tau_c} \circ C_{\dot{\rho}(\tau_c)} \circ F^{\tau_c}(\bar{b}) \end{aligned} \tag{3}$$

that applies when the ball-part is close enough to the robot-table to impact immediately. [5] Otherwise, the ball-part is in free fall for the entire oscillation period and the discrete-time function

$$\bar{f}_2 := F^T \tag{4}$$

applies so that

$$\bar{f} := \begin{cases} \bar{f}_1 & : \tau_c < T \\ \bar{f}_2 & : \tau_c \geq T \end{cases} \tag{5}$$

holds for the entire physically relevant region of the state space, $\bar{\beta} > 0$. [21]

We find that the number of fixed points of the $\bar{f}$ map is determined solely by the choice of restitution coefficient, α. By using an energy-absorbing table ($\alpha < \frac{1}{3}$), we are able to limit the number of fixed points to a single, stable period 1 fixed point, e_1: [21]

$$e_1 = \begin{bmatrix} \frac{4\alpha\dot{\rho}_0^2}{(1+\alpha)^2\gamma} \\ \frac{-(1-\alpha)\dot{\rho}_0}{1+\alpha} \end{bmatrix}$$

We define an energy function, $E(\bar{b})$, as follows,

$$E(\bar{b}) := \gamma\bar{\beta} + \frac{1}{2}\dot{\bar{\beta}}^2.$$

and its first difference function, ΔE,

$$\Delta E(\bar{b}) := E \circ \bar{f}(\bar{b}) - E(\bar{b}).$$

Using these, we find an invariant region $\mathcal{R}$. Defining $\bar{b}_0$ to be any given finite initial state, the maximum time to enter $\mathcal{R}$ is given by $t_{\mathcal{R}}$:

$$\begin{aligned} t_{\mathcal{R}} &:= \frac{2}{\gamma}\sqrt{2E(\bar{b}_0)} \left\lceil \frac{E(\bar{b}_0) - E_{max} - \Delta E_{max}}{-\Delta E_{sup}} \right\rceil \\ E_{max} &:= \frac{1}{2}\left(\frac{1+\alpha}{1-\alpha}\dot{\rho}_0\right)^2 \\ \Delta E_{max} &:= \frac{1}{2}\frac{1+\alpha}{1-\alpha}\dot{\rho}_0^2 \\ \Delta E_{sup} &:= \frac{1}{2}[-\alpha\dot{\bar{\beta}} + (1+\alpha)\dot{\rho}_0]^2 - \frac{1}{2}\dot{\bar{\beta}}^2 \end{aligned} \tag{6}$$

[5] That is, the ball-part impacts the robot-table before the end of the current table oscillation: $\tau_c < T$.

Because of the worst-case assumptions used in this expression, states are typically attracted to $\mathcal{R}$ in a much shorter time than $t_{\mathcal{R}}$. $\mathcal{R}$ is thus globally attracting.

Once inside the attracting invariant set $\mathcal{R}$, the kinetic energy of all trajectories decays to the kinetic energy of the fixed point, e_1. [21] After being attracted to the fixed point's kinetic energy, all trajectories converge to the fixed point using a Lyapunov stability argument. [21] In this manner, for a proper choice of α, the fixed point e_1 is proved to be globally stable.

Acknowledgements

We wish to thank Deneb Robotics for their assistance in the form of their Igrip robotic simulation software used for the 3 degree of freedom table simulation.

We also wish to thank an anonymous reviewer for ICRA '95, whose extensive careful and constructive comments proved immensely helpful in polishing this article.

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