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Yoshio Yamamoto  
*University of Pennsylvania*

Xiaoping Yun  
*University of Pennsylvania*

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### Abstract

A mobile manipulator in this study is a manipulator mounted on a mobile platform. Assuming the end point of the manipulator is guided, *e.g.*, by a human operator to follow an arbitrary trajectory, it is desirable that the mobile platform is able to move as to position the manipulator in certain preferred configurations. Since the motion of the manipulator is unknown *a priori*, the platform has to use the measured joint position information of the manipulator for motion planning. This paper presents a planning and control algorithm for the platform so that the manipulator is always positioned at the preferred configurations measured by its manipulability. Simulation results are presented to illustrate the efficacy of the algorithm. The use of the resulting algorithm in a number of applications is also discussed.

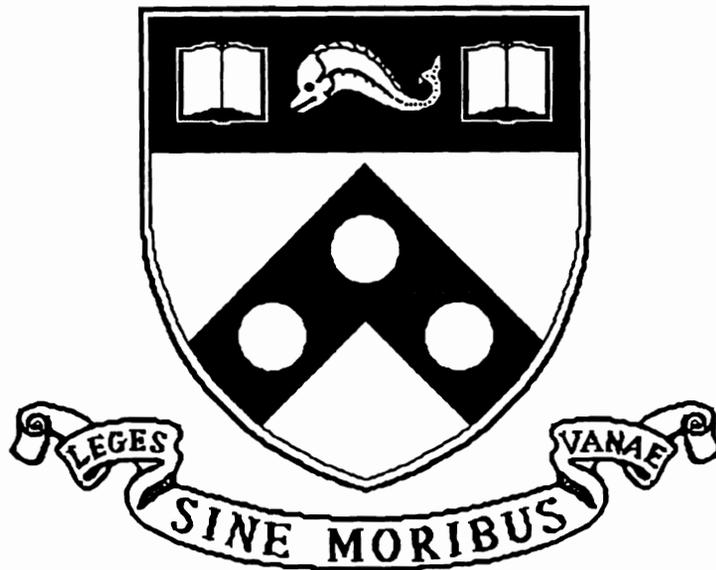
### Comments

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# Coordinating Locomotion and Manipulation Of A Mobile Manipulator

MS-CIS-92-18  
GRASP LAB 304

Yoshio Yamamoto  
Xiaoping Yun



University of Pennsylvania  
School of Engineering and Applied Science  
Computer and Information Science Department  
Philadelphia, PA 19104-6389

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# Coordinating Locomotion and Manipulation of a Mobile Manipulator

Yoshio Yamamoto and Xiaoping Yun  
General Robotics and Active Sensory Perception (GRASP) Laboratory  
University of Pennsylvania  
3401 Walnut Street, Room 301C  
Philadelphia, PA 19104-6228

## ABSTRACT

A mobile manipulator in this study is a manipulator mounted on a mobile platform. Assuming the end point of the manipulator is guided, *e.g.*, by a human operator to follow an arbitrary trajectory, it is desirable that the mobile platform is able to move as to position the manipulator in certain preferred configurations. Since the motion of the manipulator is unknown *a priori*, the platform has to use the measured joint position information of the manipulator for motion planning. This paper presents a planning and control algorithm for the platform so that the manipulator is always positioned at the preferred configurations measured by its manipulability. Simulation results are presented to illustrate the efficacy of the algorithm. The use of the resulting algorithm in a number of applications is also discussed.

## 1 Introduction

When a human writes across a board, he positions his arm in a comfortable writing configuration by moving his body rather than reaching out his arm. Also when humans transport a large and/or heavy object cooperatively, they tend to prefer certain configurations depending on various factors, *e.g.*, the shape and the weight of the object, the transportation velocity, the number of people involved in a task, and so on. Therefore when a mobile manipulator performs a manipulation task, it is desirable to bring the manipulator into certain preferred configurations by appropriately planning the motion of the mobile platform. If the trajectory of the manipulator end point in a fixed coordinate system (world coordinate system) is known *a priori*, then the motion of the mobile platform can be planned accordingly. However, if the motion of the ma-

nipulator end point is unknown *a priori*, *e.g.*, driven by a visual sensor or guided by a human operator, then the path planning has to be made locally and in real time rather than globally and off-line. This paper presents a planning and control algorithm for the platform in the latter case, which takes the measured joint displacement of the manipulator as the input for motion planning, and controls the platform in order to bring the manipulator into a preferred operating region. While this region can be selected based on any meaningful criterion, the manipulability measure [1] is utilized in this study. By using this algorithm, the mobile platform will be able to “understand the intention of its manipulator and respond accordingly.”

This control algorithm has a number of immediate applications. First, a human operator can easily move around the mobile manipulator by “dragging” the end point of the manipulator while the manipulator is in the free mode (compensating the gravity only). Second, if the manipulator is force-controlled, the mobile manipulator will be able to push against and follow an external moving surface. Third, when two mobile manipulators transport a large object with one being the master and the other being slave, this algorithm can be used to control the slave mobile manipulator to support the object and follow the motion of the master, resulting in a cooperative control algorithm for two mobile manipulators.

Although there has been a vast amount of research effort on mobile platforms (commonly referred to as mobile robots) in the literature, the study on mobile manipulators is very limited. Joshi and Desrochers considered a two link manipulator on a moving platform subject to random disturbances in its orientation [2]. Wien studied dynamic coupling between a planar vehicle and a one-link manipulator on the vehicle [3]. Dubowsky, Gu, and Deck derived the dynamic equa-

tions of a fully spatial mobile manipulator with link flexibility [4]. Recently, Hootsmans proposed a mobile manipulator control algorithm (the Mobile Manipulator Jacobian Transpose Algorithm) for a dynamically coupled mobile manipulator [5]. He showed that with the algorithm the manipulator could successfully compensate a trajectory error caused by vehicle's passive suspension with the help of limited sensory information from mobile vehicle.

What makes the coordination problem of locomotion and manipulation a difficult one is twofold. First, a manipulator and a mobile platform, in general, have different dynamic characteristics, namely, a mobile platform has slower dynamic response than a manipulator. Second, a wheeled mobile platform is subject to nonholonomic constraints while a manipulator is usually unconstrained. These two issues must be taken into consideration in developing a planning and control algorithm.

Dynamic modeling of mechanical systems with nonholonomic constraints is richly documented by work ranging from Neimark and Fufaev's comprehensive book [6] to more recent developments (see for example, [7]). However, the literature on control properties of such systems is sparse [8]. The interest in control of nonholonomic systems has been stimulated by the recent research in robotics. The dynamics of a wheeled mobile robot is nonholonomic [9], and so is a multi-arm system manipulating an object through the whole arm manipulation [10].

Bloch and McClamroch [8] first demonstrated that a nonholonomic system cannot be feedback stabilized to a single equilibrium point by a smooth feedback. In a follow-up paper [11], they showed that the system is small-time locally controllable. Campion *et al* [12] showed that the system is controllable regardless of the structure of nonholonomic constraints. Motion planning of mobile robots has been an active topic in robotics in the past several years [13, 14, 9, 15, 16]. Nevertheless, much less is known about the dynamic control of mobile robots with nonholonomic constraints and the developments in this area are very recent [17, 18, 19].

In this paper, we first present the theoretic formulation of a general nonholonomic system. Next we apply the formulation to the specific mobile platform used for the experiments in order to derive the dynamic equations. Then we describe the path planning algorithm and show the simulation results, followed by concluding remark.

## 2 Nonholonomic Systems

### 2.1 Dynamic Equations of Motion

Consider a mechanical system with  $n$  generalized coordinates  $q$  subject to  $m$  bilateral constraints whose equations of motion are described by

$$M(q)\ddot{q} + V(q, \dot{q}) = E(q)\tau - A^T(q)\lambda \quad (1)$$

where  $M(q)$  is the  $n \times n$  inertia matrix,  $V(q, \dot{q})$  is the vector of position and velocity dependent forces,  $E(q)$  is the  $n \times r$  input transformation matrix<sup>1</sup>,  $\tau$  is the  $r$ -dimensional input vector,  $A(q)$  is the  $m \times n$  Jacobian matrix, and  $\lambda$  is the vector of constraint forces. The  $m$  constraint equations of the mechanical system can be written in the form

$$C(q, \dot{q}) = 0 \quad (2)$$

If a constraint equation is in the form  $C_i(q) = 0$ , or can be integrated into this form, it is a holonomic constraint. Otherwise it is a kinematic (not geometric) constraint and is termed nonholonomic.

We assume that we have  $k$  holonomic and  $m-k$  nonholonomic independent constraints, all of which can be written in the form of

$$A(q)\dot{q} = 0 \quad (3)$$

Let  $s_1(q), \dots, s_{n-m}(q)$  be a set of smooth and linearly independent vector fields in the null space of  $A(q)$ , *i.e.*,

$$A(q)s_i(q) = 0 \quad i = 1, \dots, n-m.$$

Let  $S(q)$  be the full rank matrix made up of these vectors

$$S(q) = [s_1(q) \quad \dots \quad s_{n-m}(q)] \quad (4)$$

and let  $\Delta$  be the distribution spanned by these vector fields

$$\Delta = \text{span}\{s_1(q), \dots, s_{n-m}(q)\}$$

It follows that  $\dot{q} \in \Delta$ .  $\Delta$  may or may not be involutive. For that reason, we let  $\Delta^*$  be the smallest involutive distribution containing  $\Delta$ . It is clear that  $\dim(\Delta) \leq \dim(\Delta^*)$ . There are three possible cases (as observed by Campion, *et al.* in [12]). First, if  $k = m$ , that is, all the constraints are holonomic, then  $\Delta$  is involutive itself. Second, if  $k = 0$ , that is, all the constraints are nonholonomic, then  $\Delta^*$  spans the entire space. Finally, if  $0 < k < m$ , the  $k$  constraints are integrable and  $k$  components of the generalized coordinates may be eliminated from the motion equations. In this case,  $\dim(\Delta^*) = n - k$ .

<sup>1</sup> $E(q)$  is an identity matrix in most cases. However, if the generalized coordinates are chosen to be some variables other than the joint variables, or if there are passive joints without actuators, it is not an identity matrix.

## 2.2 State Space Representation

We now consider the mechanical system given by (1). (3) Since the constrained velocity is always in the null space of  $A(q)$ , it is possible to define  $n - m$  velocities  $\nu(t) = [\nu_1 \ \nu_2 \ \cdots \ \nu_{n-m}]$  such that

$$\dot{q} = S(q)\nu(t) \quad (5)$$

These velocities need not be integrable.

Differentiating Equation (5), substituting the expression for  $\ddot{q}$  into (1), and premultiplying by  $S^T$ , we have

$$S^T(MS\dot{\nu}(t) + M\dot{S}\nu(t) + V) = S^T E\tau \quad (6)$$

by noting that

$$A(q)\dot{q} = 0. \quad (7)$$

Using the state space variable  $x = [q^T \ \nu^T]^T$ , we have

$$\dot{x} = \begin{bmatrix} S\nu \\ f_2 \end{bmatrix} + \begin{bmatrix} 0 \\ (S^TMS)^{-1}S^TE \end{bmatrix} \tau \quad (8)$$

where  $f_2 = (S^TMS)^{-1}(-S^TM\dot{S}\nu - S^TV)$ . Assuming that the number of actuator inputs is greater or equal to the number of the degrees of freedom of the mechanical system ( $r \geq n - m$ ), and  $(S^TMS)^{-1}S^TE$  has rank  $n - m$ , we may apply the following nonlinear feedback

$$\tau = ((S^TMS)^{-1}S^TE)^+(u - f_2) \quad (9)$$

where the superscript  $+$  denotes the generalized matrix inverse. The state equation simplifies to the form

$$\dot{x} = f(x) + g(x)u \quad (10)$$

where  $f(x) = \begin{bmatrix} S(q)\nu \\ 0 \end{bmatrix}$ ,  $g(x) = \begin{bmatrix} 0 \\ I \end{bmatrix}$ .

## 2.3 Control Properties

The following two properties of the system (10) have been established in [12, 8, 11] for the special case in which *all* constraints are nonholonomic.

**Theorem 1** *The nonholonomic system (10) is controllable.*

**Theorem 2** *The equilibrium point  $x = 0$  of the nonholonomic system (10) can be made Lagrange stable, but can not be made asymptotically stable by a smooth state feedback.*

In the rest of this section, we discuss the more general case in which Equation (3) consists of both holonomic and nonholonomic constraints.

**Theorem 3** *System (10) is not input-state linearizable by a state feedback if at least one of the constraints is nonholonomic.*

**Proof:** The system has to satisfy two conditions for input-state linearization: the strong accessibility condition and the involutivity condition [20]. It is shown below that the involutivity condition is not satisfied.

Define a sequence of distributions

$$D_j = \text{span}\{L_f^i g \mid i = 0, 1, \dots, j - 1\}, \quad j = 1, 2, \dots$$

Then the involutivity condition requires that the distributions  $D_1, D_2, \dots, D_{2n-m}$  are all involutive. Note that the dimension of the state variable is  $2n - m$ .  $D_1 = \text{span}\{g\}$  is involutive since  $g$  is constant. Next we compute

$$L_f g = [f, g] = \frac{\partial g}{\partial x} f - \frac{\partial f}{\partial x} g = - \begin{bmatrix} S(q) \\ 0 \end{bmatrix}$$

Since the distribution  $\Delta$  spanned by the columns of  $S(q)$  is not involutive, the distribution  $D_2 = \text{span}\{g, L_f g\}$  is not involutive. Therefore, the system is not input-state linearizable.  $\square$

Although a system with nonholonomic constraints is not input-state linearizable, it is input-output linearizable if a proper set of output equations are chosen. Consider the position control of the system, *i.e.*, the output equations are functions of position state variable  $q$  only. Since the number of the degrees of freedom of the system is instantaneously  $n - m$ , we may have at most  $n - m$  independent position outputs equations.

$$y = h(q) = [h_1(q) \ \cdots \ h_{n-m}(q)] \quad (11)$$

The necessary and sufficient condition for input-output linearization is that the decoupling matrix has full rank [20]. With the output equation (11), the decoupling matrix  $\Phi(x)$  for the system is the  $(n - m) \times (n - m)$  matrix

$$\Phi(q) = J_h(q)S(q) \quad (12)$$

where  $J_h = \frac{\partial h}{\partial q}$  is the  $(n - m) \times n$  Jacobian matrix.  $\Phi(x)$  is nonsingular if the rows of  $J_h$  are independent of the rows of  $A(q)$ .

To characterize the zero dynamics and achieve input-output linearization, we introduce a new state space variable  $z$  defined as follows

$$z = T(x) = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} h(q) \\ L_f h(q) \\ \tilde{h}(q) \end{bmatrix} = \begin{bmatrix} h(q) \\ \Phi(q)\nu \\ \tilde{h}(q) \end{bmatrix} \quad (13)$$

where  $\tilde{h}(q)$  is an  $m$ -dimensional function such that  $[J_h^T \ J_{\tilde{h}}^T]$  has full rank. It is easy to verify that  $T(x)$  is

indeed a diffeomorphism and thus a valid state space transformation. The system under the new state variable  $z$  is characterized by

$$\dot{z}_1 = \frac{\partial h}{\partial q} \dot{q} = z_2 \quad (14)$$

$$\dot{z}_2 = \dot{\Phi}(q)\nu + \Phi(q)u \quad (15)$$

$$\dot{z}_3 = J_h^{-1}S\nu = J_h^{-1}S(J_h S)^{-1}z_2 \quad (16)$$

Utilizing the following state feedback

$$u = \Phi^{-1}(q)(v - \dot{\Phi}(q)\nu) \quad (17)$$

we achieve input-output linearization as well as input-output decoupling by noting the observable part of the system

$$\dot{z}_1 = z_2 \quad \dot{z}_2 = v \quad y = z_1$$

The unobservable zero dynamics of the system is (obtained by substituting  $z_1 = 0$  and  $z_2 = 0$ )

$$\dot{z}_3 = 0 \quad (18)$$

which is clearly Lagrange stable but not asymptotically stable.

### 3 Mobile Platform

#### 3.1 Constraint Equations

In this subsection, we derive the constraint equations for a LABMATE<sup>2</sup> mobile platform. The platform has two driving wheels (the center ones) and four passive supporting wheels (the corner ones). The two driving wheels are independently driven by two DC motors, respectively. The following notations will be used in the derivation of the constraint equations and dynamic equations (see Figure 1).

- $P_o$ : the intersection of the axis of symmetry with the driving wheel axis;
- $P_c$ : the center of mass of the platform;
- $P_b$ : the location of the manipulator on the platform;
- $P_r$ : the reference point to be followed by the mobile platform;
- $d$ : the distance from  $P_o$  to  $P_c$ ;
- $b$ : the distance between the driving wheels and the axis of symmetry;
- $r$ : the radius of each driving wheel;
- $m_c$ : the mass of the platform without the driving wheels and the rotors of the

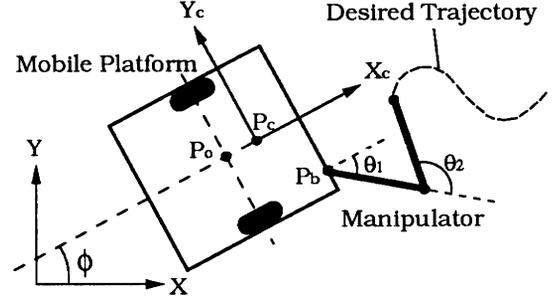


Figure 1: Schematic of the mobile manipulator.

- DC motors;
- $m_w$ : the mass of each driving wheel plus the rotor of its motor;
- $I_c$ : the moment of inertia of the platform without the driving wheels and the rotors of the motors about a vertical axis through  $P_o$ ;
- $I_w$ : the moment of inertia of each wheel and the motor rotor about the wheel axis;
- $I_m$ : the moment of inertia of each wheel and the motor rotor about a wheel diameter.

There are three constraints. The first one is that the platform must move in the direction of the axis of symmetry, *i.e.*,

$$\dot{y}_c \cos \phi - \dot{x}_c \sin \phi - d\dot{\phi} = 0 \quad (19)$$

where  $(x_c, y_c)$  is the coordinates of the center of mass  $P_c$  in the world coordinate system, and the  $\phi$  is the heading angle of the platform measured from the  $X$ -axis of the world coordinates. The other two constraints are the rolling constraints, *i.e.*, the driving wheels do not slip.

$$\dot{x}_c \cos \phi + \dot{y}_c \sin \phi + b\dot{\phi} = r\dot{\theta}_r \quad (20)$$

$$\dot{x}_c \cos \phi + \dot{y}_c \sin \phi - b\dot{\phi} = r\dot{\theta}_l \quad (21)$$

where  $\theta_r$  and  $\theta_l$  are the angular displacement of the right and left wheels, respectively.

Let  $q = (x_c, y_c, \phi, \theta_r, \theta_l)$ , the three constraints can be written in the form of

$$A(q)\dot{q} = 0$$

where

$$A(q) = \begin{bmatrix} -\sin \phi & \cos \phi & -d & 0 & 0 \\ -\cos \phi & -\sin \phi & -b & r & 0 \\ -\cos \phi & -\sin \phi & b & 0 & r \end{bmatrix} \quad (22)$$

<sup>2</sup>LABMATE is a trademark of Transitions Research Corporation.

It is straightforward to verify that the following matrix

$$S(q) = [s_1(q), s_2(q)] = \begin{bmatrix} c(b \cos \phi - d \sin \phi) & c(b \cos \phi + d \sin \phi) \\ c(b \sin \phi + d \cos \phi) & c(b \sin \phi - d \cos \phi) \\ c & -c \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

satisfies  $A(q)S(q) = 0$ , where the constant  $c = \frac{r}{2b}$ . Computing the Lie bracket of  $s_1(q)$  and  $s_2(q)$  we obtain

$$s_3(q) = [s_1(q), s_2(q)] = \begin{bmatrix} -rc \sin \phi \\ rc \cos \phi \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

which is not in the distribution  $\Delta$  spanned by  $s_1(q)$  and  $s_2(q)$ . Therefore, at least one of the constraints is non-holonomic. We continue to compute the Lie bracket of  $s_1(q)$  and  $s_3(q)$

$$s_4(q) = [s_1(q), s_3(q)] = \begin{bmatrix} -rc^2 \cos \phi \\ -rc^2 \sin \phi \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

which is linearly independent of  $s_1(q)$ ,  $s_2(q)$ , and  $s_3(q)$ . However, the distribution spanned by  $s_1(q)$ ,  $s_2(q)$ ,  $s_3(q)$  and  $s_4(q)$  is involutive. Therefore, we have

$$\Delta^* = \text{span}\{s_1(q), s_2(q), s_3(q), s_4(q)\} \quad (23)$$

It follows that, among the three constraints, two of them are nonholonomic and the third one is holonomic. To obtain the holonomic constraint, we subtract Equation (21) from Equation (20).

$$2b\dot{\phi} = r(\dot{\theta}_r - \dot{\theta}_l) \quad (24)$$

Integrating the above equation and properly choosing the initial condition of  $\theta_r$  and  $\theta_l$ , we have

$$\phi = c(\theta_r - \theta_l) \quad (25)$$

which is clearly a holonomic constraint equation.

### 3.2 Dynamic Equations

We now derive the dynamic equation for the mobile platform. The Lagrange equations of motion of the

platform with the Lagrange multipliers  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_3$  are given by

$$m\ddot{x}_c - m_c d(\ddot{\phi} \sin \phi + \dot{\phi}^2 \cos \phi) - \lambda_1 \sin \phi - (\lambda_2 + \lambda_3) \cos \phi = 0 \quad (26)$$

$$m\ddot{y}_c - m_c d(\ddot{\phi} \cos \phi - \dot{\phi}^2 \sin \phi) + \lambda_1 \cos \phi - (\lambda_2 + \lambda_3) \sin \phi = 0 \quad (27)$$

$$-m_c d(\ddot{x}_c \sin \phi - \ddot{y}_c \cos \phi) + I\ddot{\phi} - d\lambda_1 + b(\lambda_3 - \lambda_2) = 0 \quad (28)$$

$$I_w \ddot{\theta}_r + \lambda_2 r = \tau_r \quad (29)$$

$$I_w \ddot{\theta}_l + \lambda_3 r = \tau_l \quad (30)$$

where

$$\begin{aligned} m &= m_c + 2m_w \\ I &= I_c + 2m_w(d^2 + b^2) + 2I_m \end{aligned}$$

and  $\tau_r$  and  $\tau_l$  are the torques acting on the wheel axis generated by the right and left motors respectively. These five equations of motion can easily be written in the form of Equation (1). The matrix  $A(q)$  has been defined in Equation (22). The matrices  $M(q)$ ,  $V(q, \dot{q})$ , and  $E(q)$  are given by

$$M(q) = \begin{bmatrix} m & 0 & -m_c d \sin \phi & 0 & 0 \\ 0 & m & -m_c d \cos \phi & 0 & 0 \\ -m_c d \sin \phi & m_c d \cos \phi & I & 0 & 0 \\ 0 & 0 & 0 & I_w & 0 \\ 0 & 0 & 0 & 0 & I_w \end{bmatrix}$$

$$V(q, \dot{q}) = \begin{bmatrix} -m_c d \dot{\phi}^2 \cos \phi \\ -m_c d \dot{\phi}^2 \sin \phi \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad E(q) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

In this case, owing to the choice of  $S(q)$  matrix, we have

$$\nu = \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} = \begin{bmatrix} \dot{\theta}_r \\ \dot{\theta}_l \end{bmatrix}$$

The state variable is then

$$x = [x_c \ y_c \ \phi \ \theta_r \ \theta_l \ \dot{\theta}_r \ \dot{\theta}_l]$$

Using this state variable, the dynamics of the mobile platform can be represented in the state space form, Equation (8).

### 3.3 Output Equations

While the state equation of a dynamic system is uniquely, modulo its representation, determined by its

dynamic characteristics, the output equation is chosen in such a way that the tasks to be performed by the dynamic system can be *conveniently specified* and that the controller design can be *easily accomplished*. For example, if a 6-DOF robot manipulator is to perform pick-and-place or trajectory tracking tasks, the six-dimensional joint position vector or the 6-dimensional Cartesian position and orientation vector is normally chosen as the output equation. In this section, we present the output equation for the mobile platform and discuss its properties.

It is convenient to define a platform coordinate frame  $X_c$ - $Y_c$  at the center of mass of at the mobile platform, with  $X_c$  in the forward direction of the platform. We may choose an arbitrary point  $P_r$  with respect to the platform coordinate frame  $X_c$ - $Y_c$  as a reference point. The mobile platform is to be controlled so that the reference point follows a desired trajectory. Let the reference point be denoted by  $(x_r^c, y_r^c)$  in the platform frame  $X_c$ - $Y_c$ . Then the world coordinates  $(x_r, y_r)$  of the reference point are given by

$$x_r = x_c + x_r^c \cos \phi - y_r^c \sin \phi \quad (31)$$

$$y_r = y_c + x_r^c \sin \phi + y_r^c \cos \phi \quad (32)$$

The selection of the reference point for the purpose of coordinating locomotion and manipulation is discussed in the following section. Having chosen the reference point,  $x_r^c$  and  $y_r^c$  are constant. By taking the coordinates of the reference point to be the output equation

$$y = h(q) = [x_r \quad y_r]^T \quad (33)$$

we have a trajectory tracking problem studied in [17, 18]. The corresponding decoupling matrix for this output is

$$\Phi(q) = J_h(q)S(q) = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \quad (34)$$

where

$$\Phi_{11} = c((b - y_r^c) \cos \phi - (d + x_r^c) \sin \phi) \quad (35)$$

$$\Phi_{12} = c((b + y_r^c) \cos \phi + (d + x_r^c) \sin \phi) \quad (36)$$

$$\Phi_{21} = c((b - y_r^c) \sin \phi + (d + x_r^c) \cos \phi) \quad (37)$$

$$\Phi_{22} = c((b + y_r^c) \sin \phi - (d + x_r^c) \cos \phi) \quad (38)$$

Since the determinant of the decoupling matrix is  $\det(\Phi(q)) = -\frac{r^2(d+x_r^c)}{2b}$ , it is singular if and only if  $x_r^c = -d$ , that is, the point  $P_r$  is located on the wheel axis. Therefore, trajectory tracking of a point on the wheel axis including  $P_o$  is not possible as pointed out in [18]. This is clearly due to the presence of nonholonomic constraints. Choosing  $x_r^c$  not equal to  $-d$ , we may decouple and linearize the system.

Since  $S^T E = I_{2 \times 2}$ , the nonlinear feedback, Equations (9) and (17), in this case is simplified to

$$\tau = (S^T M S)u + S^T M \dot{S}\nu + S^T V \quad (39)$$

and

$$u = \Phi^{-1}(q)(v - \dot{\Phi}(q)\nu) \quad (40)$$

The linearized and decoupled subsystems are

$$\ddot{y}_1 = v_1 \quad (41)$$

$$\ddot{y}_2 = v_2 \quad (42)$$

## 4 Motion Planning

For simplicity, a two link planar manipulator is considered in this discussion. Let  $\theta_1$  and  $\theta_2$  be the joint angles and  $L_1$  and  $L_2$  be the link length of the manipulator. Also let the coordinates of the manipulator base with respect to the platform frame  $X_c$ - $Y_c$  be denoted by  $(x_b^c, y_b^c)$ . We let the reference point to the end point of the manipulator at a preferred configuration. We choose the configuration that maximizes the manipulability measure of the manipulator. If we specify the position of the end point as the desired trajectory for the reference point, the mobile platform will move in such a way that the manipulator is brought into the preferred configuration. The manipulability measure is defined as [1]

$$w = \sqrt{\det(J(\theta)J^T(\theta))} \quad (43)$$

where  $\theta$  and  $J(\theta)$  denote the joint vector and Jacobian matrix of the manipulator. If we consider non-redundant manipulators, the equation (43) reduces to

$$w = |\det J(\theta)| \quad (44)$$

For the two-link manipulator shown in Figure 1, the manipulability measure  $w$  is

$$w = |\det J| = L_1 L_2 |\sin \theta_2| \quad (45)$$

Note that the manipulability measure is maximized for  $\theta_2 = \pm 90^\circ$  and arbitrary  $\theta_1$ . We choose  $\theta_2 = +90^\circ$  and  $\theta_1 = -45^\circ$  to be the preferred configuration, denoting them by  $\theta_{1r}$  and  $\theta_{2r}$ . Then the coordinates of the reference point with respect to the platform frame  $X_c$ - $Y_c$  is given by

$$x_r^c = x_b^c + L_1 \cos \theta_{1r} + L_2 \cos(\theta_{1r} + \theta_{2r}) \quad (46)$$

$$y_r^c = y_b^c + L_1 \sin \theta_{1r} + L_2 \sin(\theta_{1r} + \theta_{2r}) \quad (47)$$

We emphasize that  $x_r^c$  and  $y_r^c$  are constant and will be used in the representation of the output equation (33).

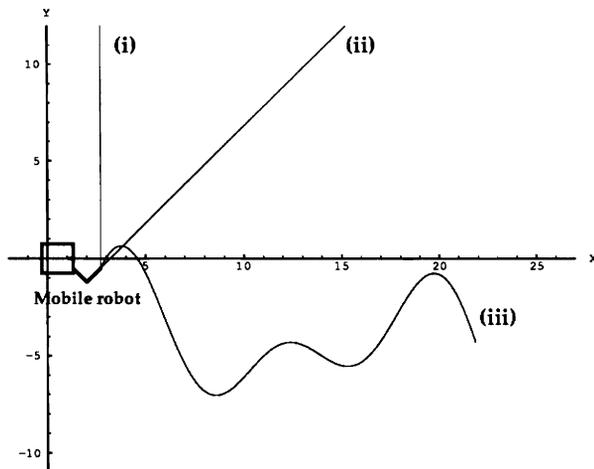


Figure 2: Three desired trajectories.

The manipulator is regarded as a passive device whose dynamics is neglected. It is assumed that a human operator drags the end effector of the manipulator. The position of the end effector is given as the desired trajectory for the reference point  $P_r$ . The manipulator will be kept in the preferred configuration provided that the reference point is able to follow the desired trajectory. Any tracking error of the reference point will leave the manipulator out of the preferred configuration, resulting in a drop in manipulability measure. To count for measurement and communication delay, the current position of the end effector is made available to the mobile platform a fixed number of sampling periods later (five periods in the simulation). Further, before given to mobile platform as the desired trajectory, the position data of the end effector is approximated by piecewise polynomial functions generated in real time by *singular value decomposition*. This approximation is to eliminate high frequency (noise) components and to allow differentiation of discrete data in order to obtain desired velocity for the reference point.

## 5 Simulation

The mobile platform is initially directed toward positive  $X$ -axis at rest and the initial configuration of the manipulator is  $\theta_1 = -45^\circ$  and  $\theta_2 = 90^\circ$ . Three different paths used for the simulation are shown in Figure 2. The velocity along the paths is constant.

1. A straight line perpendicular to the  $X$ -axis or the initial forward direction of the mobile platform,
2. A forward slanting line by 45 degree from  $X$ -axis.
3. A general curved path.

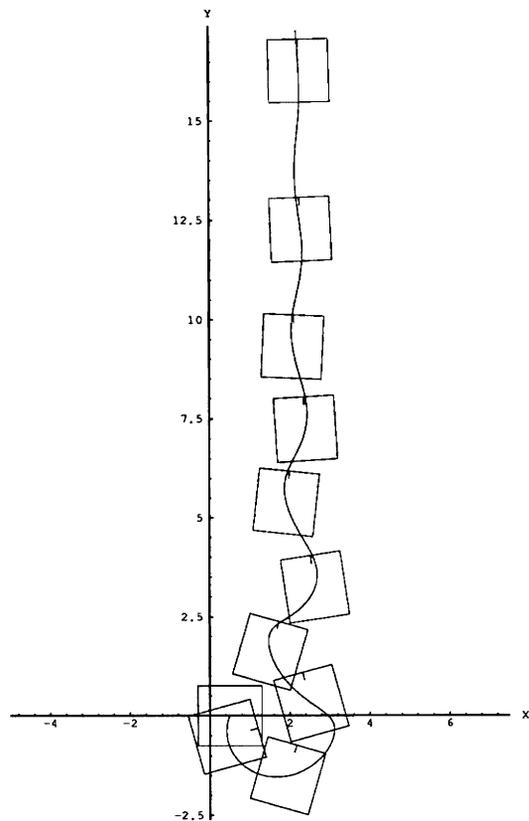


Figure 3: Trajectory of the point  $P_o$  for the desired trajectory (i).

The sampling rate is 0.01 sec. The linear state feedback gains for the two subsystems (41) and (42) are chosen so that the overall system has a natural frequency  $\omega_n = 2.0$  and a damping ratio  $\zeta = 1.2$ . The higher damping ratio is to simulate the slow response of the mobile platform. For each simulation, we plot the trajectory of  $P_o$ , the trajectory of the reference point  $P_r$ , the manipulability measure, the joint angles of the manipulator, the heading angle of the platform, and the velocity of the  $P_o$ .

1. Figure 3 shows the trajectory of point  $P_o$ , in which a box<sup>3</sup> and a notch on one side represent the mobile platform and its forward direction, respectively. Note that the desired trajectory is given for the reference point  $P_r$ .  $P_o$  has no desired trajectory. Figure 4 shows the desired and actual trajectories of the reference point  $P_r$ . The manipulability measure, the joint angles, the heading angle, and the velocity of point  $P_o$  are shown in Figure 5,6,7, and 8, respectively. Figure 5 shows a

<sup>3</sup>These boxes are not equally distributed in time.

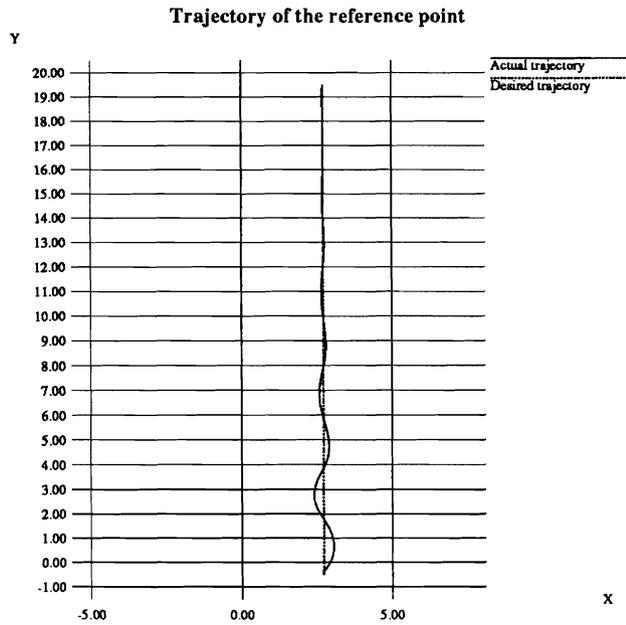


Figure 4: Desired and actual trajectories of the reference point for (i).

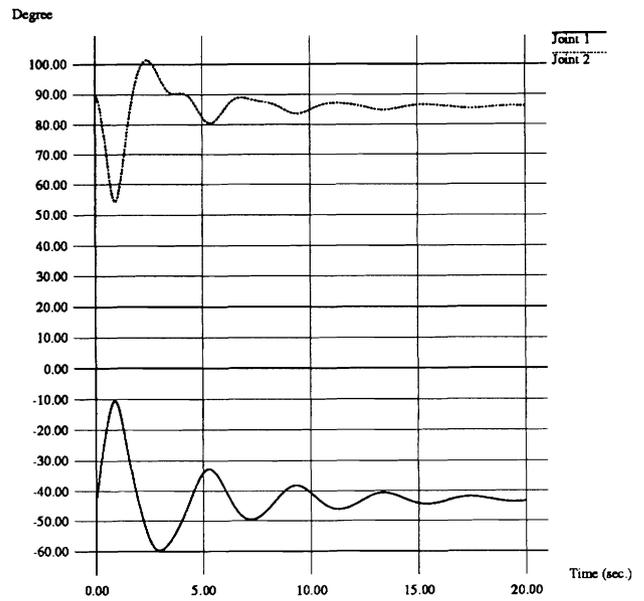


Figure 6: Joint angles for trajectory (i).

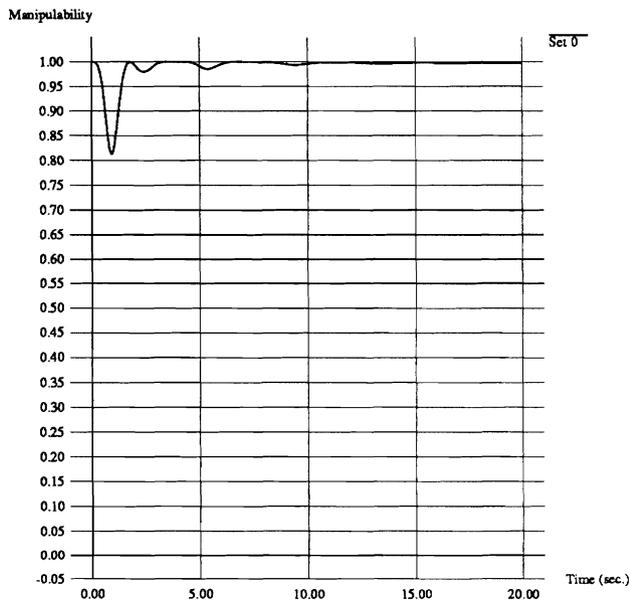


Figure 5: Manipulability measure for trajectory (i).

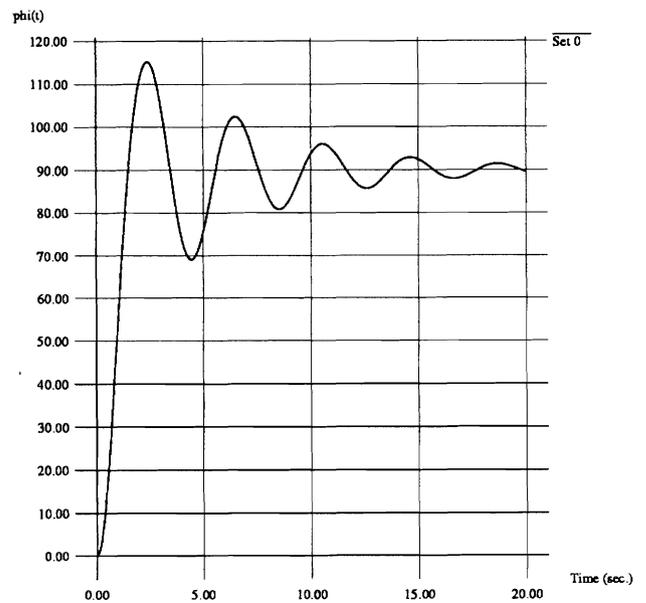


Figure 7: Heading angle for trajectory (i).

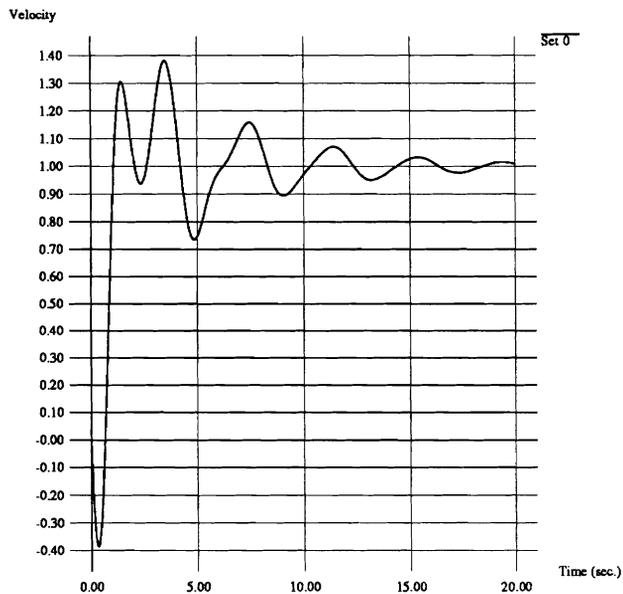


Figure 8: Velocity of the point  $P_o$  for trajectory (i).

little degradation of manipulability measure corresponding to the early maneuver by the mobile platform. Figure 7 shows that the heading angle rapidly increases and exceeds  $90^\circ$  at the beginning, and eventually settles at  $90^\circ$ . The negative value in Figure 8 indicates that the mobile platform moved backwards for a short period of time at the very beginning in order to achieve the needed heading angle. Note that the motion of the platform, or exactly the trajectory of point  $P_o$  is not planned. Therefore, the exhibited backward motion is not explicitly planned and is a consequence of the control algorithm.

2. The results for the slanting trajectory are shown in Figure 9 through 14. Figure 10 shows that the reference point follows the desired trajectory successfully. From Figure 11, the degradation of manipulability measure is smaller than that of the previous case as expected. Figure 14 indicates that that no backward motion occurs.
3. Figure 15 shows the trajectory of point  $P_o$  for the third simulation. The reference point follows the desired trajectory reasonably well (Figure 16). The manipulability measure and the joint angles are shown in Figure 17 and Figure 18, respectively. It is clear that the maintenance of the high manipulability measure is achieved at the cost of platform sharp maneuvers. The current study investigates a planning algorithm which takes into account both manipulability of manipulator and

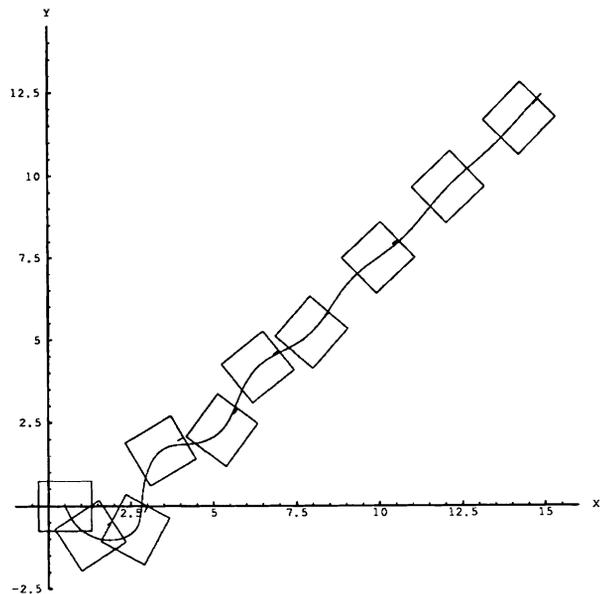


Figure 9: Trajectory of the point  $P_o$  for trajectory (ii).

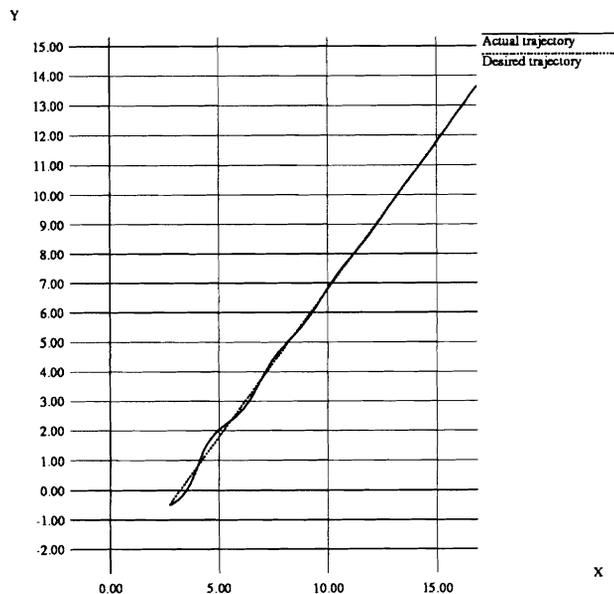


Figure 10: Desired and actual trajectories of the reference point for (ii).

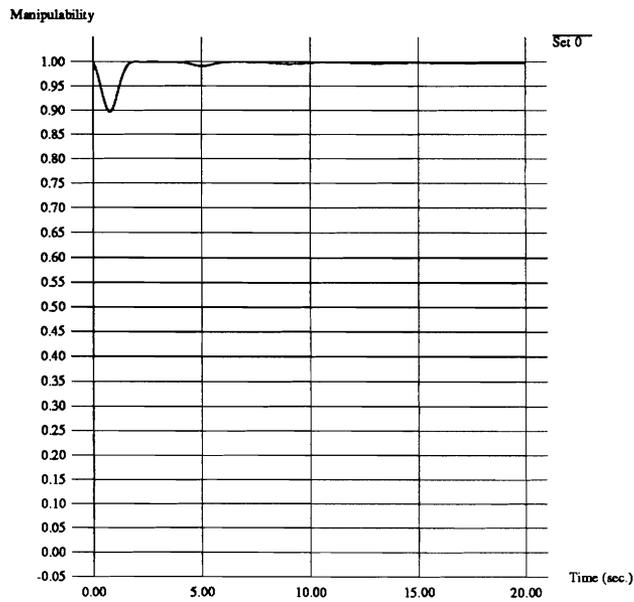


Figure 11: Manipulability measure for trajectory (ii).

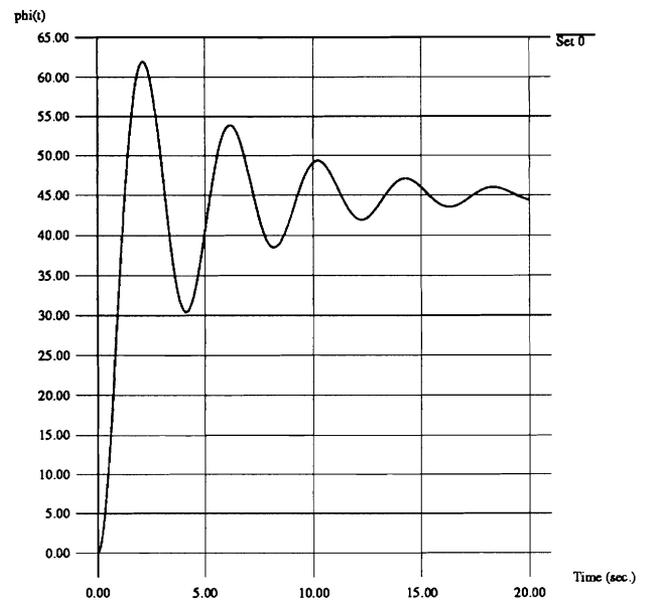


Figure 13: Heading angle for trajectory (ii).

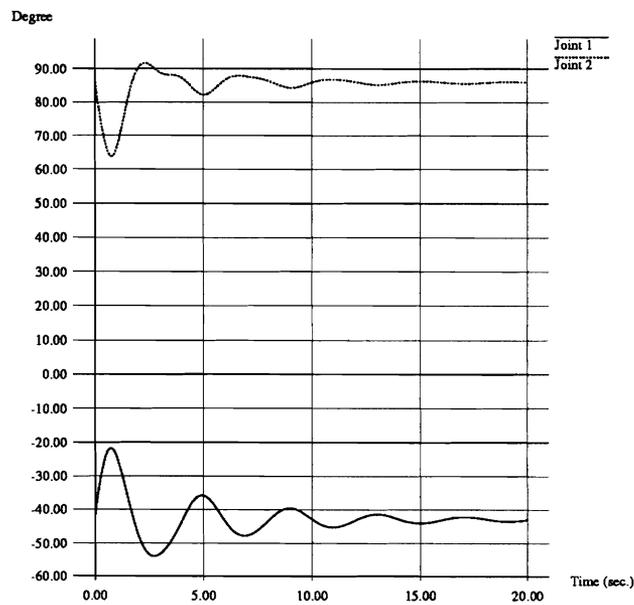


Figure 12: Joint angles for trajectory (ii).

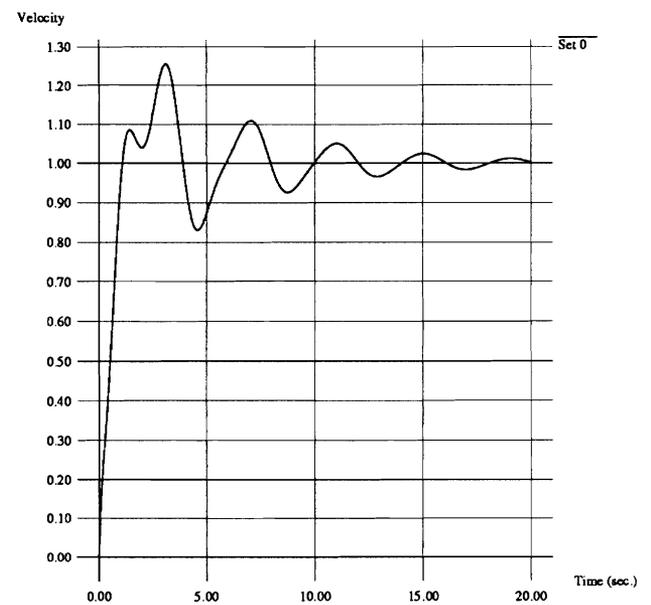


Figure 14: Velocity of the point  $P_o$  for trajectory (ii).

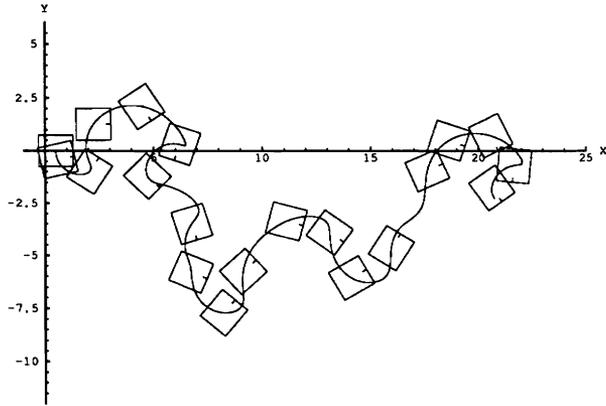


Figure 15: Trajectory of the point  $P_o$  for trajectory (iii).

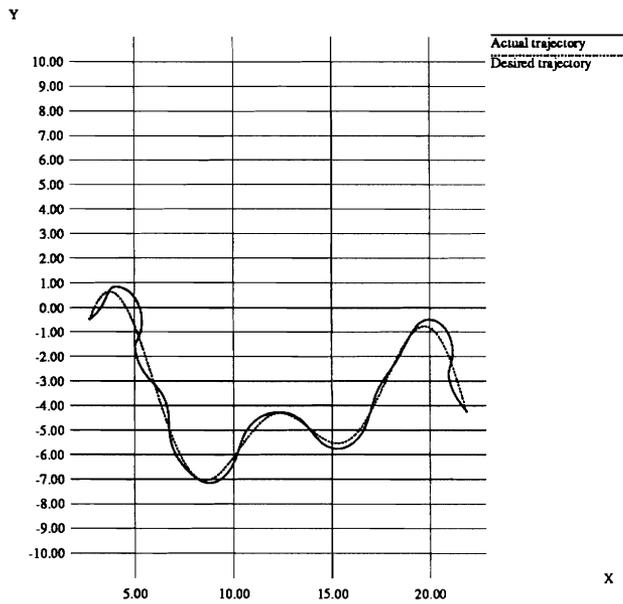


Figure 16: Desired and actual trajectories of the reference point  $P_r$  for (iii).

maneuverability of mobile platform. This consideration will become even more important when a mobile manipulator exerts forces to the environment or is required to cooperate with the other agents.

## 6 Concluding Remarks

We presented a planning and control algorithm for coordinating motion of a mobile manipulator. The design criterion was to control the mobile platform so that the manipulator is maintained at a configuration which maximizes the manipulability measure. We verified the effectiveness of our method by simulations on

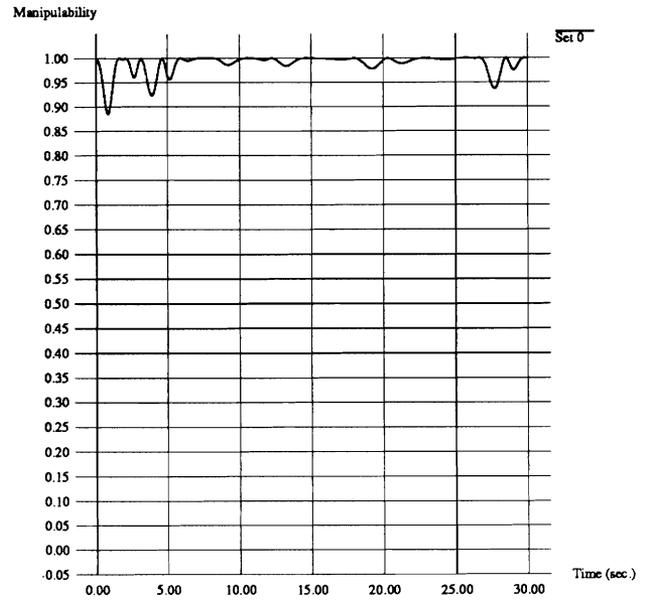


Figure 17: Manipulability measure for trajectory (iii).

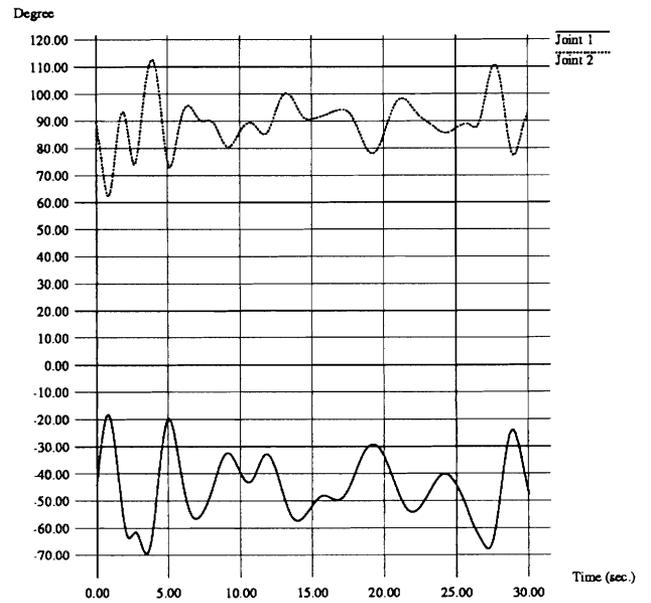


Figure 18: Joint angles for trajectory (iii).

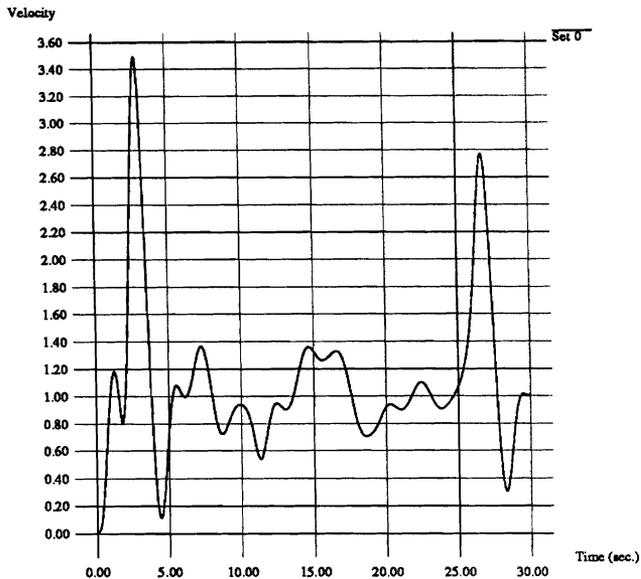


Figure 19: Velocity of the point  $P_o$  for trajectory (iii).

three representative trajectories. For future work, we will investigate the integration of the proposed method and force control. An alternative path planning approach will be explored as mentioned in the previous section such that the maneuverability of mobile platform is taken into consideration as well. The control algorithm is currently being implemented on a mobile manipulator which consists of a Labmate mobile platform and PUMA 250 manipulator.

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