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Keywords

corporate Investment, firm Decisions, indirect inference

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Abstract

We estimate investment policy functions under general assumptions about technology and markets. Policy functions are easy to estimate and summarize the key predictions of any dynamic investment model. Because our method does not rely on Tobin's Q, it does not require information about market values and can be readily applied to study private firms. In addition, unlike Tobin's Q, we show that investment policy functions account for a large fraction of the variation in corporate investment. As such they are much better suited to evaluate and estimate dynamic investment models. Using this superior characterization of firm investment behavior we then use indirect inference methods to estimate deep parameters of a structural model of investment featuring decreasing returns to scale and generalized adjustment cost functions.

Keywords: Corporate Investment, Firm Decisions, Indirect Inference

1 Introduction

Hayashi's (1982) famous elaboration of Brainard and Tobin's Q-theory has influenced the study of corporate and aggregate investment for nearly three decades. The prediction that Tobin's Q is a sufficient statistic to describe investment behavior has been immensely popular among researchers, and the simple investment regressions implied by the linearly homogenous version of the model form the basis for a myriad of empirical studies in economics and finance. Despite a long-standing consensus that Q is poorly measured and that the linearly homogenous model which motivates its use is misspecified, Q-type investment regressions still form the basis for most inferences about corporate behaviors.[1]

In this paper we propose an alternative procedure to describe firm investment under very general assumptions about the nature of markets, production and investment technologies. Our methodology is not only theoretically correct but also straightforward to implement. Importantly, unlike Q theory we do not require information about the market value of the firm and thus our method can be used to study the investment behavior of private firms and to compare it with that of publicly traded corporations.[2]

Like many others, our starting point is a dynamic structural model of corporate investment behavior, but without the stringent, and counterfactual, assumptions about homogeneity and perfect competition.[3] Our procedure exploits the fact that the optimal investment policy can always be directly estimated as function of key state variables of the firm. Unlike marginal q, many of the state variables are either directly observable or can be readily constructed from observables, under fairly general conditions.

We show both in theory and in the data that even a simple low order polynomial

[1] Q-based investment regressions, often augmented by various ad-hoc measures of cash flows, have been used to, among other purposes, test the importance of financial constraints, the effects of corporate governance, and the efficiency of market signals.

[2] Asker, Farre-Mensa and Ljungqvist (2011) offer a recent example on the difficulties of using Q-theory with private firms.

[3] Possible departures from homogeneity due to technological and/or financial frictions include market power or decreasing returns to scale in production (Gomes, 2001; Cooper and Ejarque, 2003; Abel and Eberly, 2010), inhomogeneous costs of investment (Abel and Eberly, 1994, 1997; Cooper and Haltiwanger, 2006) or of external financing (Hennessy and Whited, 2007). Although he relies on homogeneity, Philipon (2009) also offer another alternative to the use of Tobin's Q.

approximation in the key state variables provides a better description of investment than standard Q-type regressions. Investment is simply not closely correlated with Tobin's (or average) Q and moves more in tandem with key state variables like firm size and sales. Alternatively, the covariances between investment and Q, implied by standard regressions, are far less informative about underlying structural parameters, than covariances with the state variables. Specifically, we show that elasticity of regression coefficients to the deep parameters is always significantly higher than those obtained Q regressions. Altogether this evidence suggests policy function estimates should receive considerably more weight in indirect inference studies.

Theoretically, the main novelty of our approach is to explicitly identify firm size and productivity as key state variables for optimal investment behavior under general assumptions about markets and technology. Surprisingly, given its popularity in other empirical applications, firm size is often ignored in the investment literature, and when used, it usually shows up either as a catch-all variable to account for omitted variables in investment regressions or as sorting variable for identification of financially constrained firms.[4] Here we formally establish that firm size naturally is an important determinant of investment, with decreasing returns to scale technologies, even in the absence of financial market frictions. Similarly, our approach also clarifies the role of sales and cash flow variables. Contrary to their once popular use in tests of financing constraints, we show that these variables should matter because they capture underlying movements in the state of productivity and demand or in factor prices.[5]

By avoiding market values we also minimize the serious measurement concerns induced by potential stock market misvaluations (Blanchard, Rhee and Summers, 1993; Erickson and Whited, 2000), and approximations of unavailable market values of debt securities and nonphysical assets such as human capital, intangibles, and goodwill (Er-

[4]A notable recent exception is Gala and Julio (2012). Exploiting variation across industries, they provide direct empirical evidence that firm size captures technological decreasing returns rather than differences in firms' financing frictions.

[5]Gomes (2001), Cooper and Ejarque (2003) and Abel and Eberly (2010) all argue that cash flow might capture differences between marginal and average Q. Instead, we show that flow variables like sales and/or cash flow, and not Q, should always be the primary determinant of investment, even in the absence of capital market imperfections.

ickson and Whited, 2006). No doubt many of our proposed variables are also subject to some measurement error, but this is likely to be much smaller than the errors in measuring Tobin's Q (Erickson and Whited, 2006, 2011).[6]

We believe our paper contributes to the literature in two significant ways. First, and foremost, it provides a very robust empirical methodology to characterize firm level investment behavior, that can be applied in many settings, including the study of private firms' investment. Second, direct approximation of investment policy functions delivers many more informative empirical moments for the identification and inference of the underlying structural parameters of the model. These are especially more informative than Q-type investment regressions, which are much less general and likely subject to serious measurement error.

With respect to measurement error, our paper delivers perhaps the most logical conclusion to the influential arguments in Erickson and Whited (2000, 2006 and 2011) that "Tobin's Q contains a great deal of measurement error because of a conceptual gap between true investment opportunities and observable measures". Ultimately, our approach offers a simple way to circumvent the problem by avoiding the use of Q entirely, or, at least, limiting its use.

As with any structural method, specification error can be a concern. Here, this manifests itself in the possibility that the model is specified with the wrong state variables. For example, in many models leverage or liquid assets play an important role in the investment decisions of firms. However, our approach offers a very effective way to deal with this issue. By projecting the empirical investment policies on a set of candidate state variables we can let the data inform us as to which ones are appropriate to include in the model. In this way model specification is entirely guided by the data.

The rest of our paper is organized as follows. The next section describes the general

[6]Other authors have also developed methodologies that do not rely on the use of market values. For example, Shapiro (1986), Whited (1992), and Bond and Meghir (1994) propose estimating the Euler equation for investment. Abel and Blanchard (1986) and Gilchrist and Himmelberg (1995, 1998) use VAR forecasts of profitability to estimate marginal q. Our direct estimation of the investment policy function however imposes fewer restrictions on functional forms and allows for the use of much simpler estimation methods.

model and the implied optimal investment policies. In Section 3 we discuss the empirical estimation of the investment policy functions. Section 4 contains the main theoretical and empirical findings. Section 5 generalizes the basic approach to models with leverage and other shocks. We then conclude with a brief discussion of the role of asset prices in estimating investment.

2 Modeling Investment

This section describes a general structural model of investment suitable for empirical work on firm level investment. It provides guidance and imposes discipline on the identification and measurement of relevant state variables. Our model is a generalized version of Abel and Eberly (1994, 1997) and Caballero and Engel (1999). We allow for a weakly concave production technology and an investment technology featuring both non-convex and convex capital adjustment costs which are potentially asymmetric and discontinuous. This environment is flexible enough to include the majority of investment models in the literature as special cases. For exposition purposes, we delay the discussion of additional features such as financial market imperfections in Section 5.

2.1 The Benchmark Model

We examine the optimal investment decision of a firm seeking to maximize current shareholder value in the absence of any financing frictions, V. For simplicity, we assume that the firm is financed entirely by equity and denote by D the value of periodic distributions net of any securities issuance.

The operating cash flows or profits of this (representative) firm are summarized by the function Π defined as sales revenues net of operating costs. We formalize this relation as follows:

$$\Pi\left(K_t, A_t, W_t\right) = \max_{N_t} \left\{F\left(A_t, K_t, N_t\right) - W_t N_t\right\}. \tag{1}$$

The function $Y_t = F(A_t, K_t, N_t)$ denotes the value of sales revenues in period t, net of the cost of any materials. Revenues depend on a firm's capital stock and labor input, denoted by K_t and N_t, respectively. The variable A_t captures the exogenous state of demand and/or productivity in which the firm operates. W_t denotes unit labor costs, including wages, taxes and other employee benefits. Both A_t and W_t can vary stochastically over time, thus accommodating any variations to the state of the economy or industry in which a firm operates. We now summarize our main assumptions about revenues and profits.

Assumption 1. Sales. The function $F : A \times K \times N \to R_+$, (i) is increasing in A, and increasing and concave in both K and N; (ii) is twice continuously differentiable; (iii) satisfies $F(hA, hK, hN) \leq hF(A, K, N)$ for all (A, K, N); and (iv) obeys the standard Inada boundary conditions.

Item (iii) is a departure from the standard linear homogeneous model and explicitly allows for decreasing returns to scale. It is straightforward to show that the function $\Pi(K, A, W)$ is also increasing and weakly concave in K.

Installed capital depreciates at a rate $\delta \geq 0$, and capital accumulation requires investment, I_t. We assume that current investment does not affect the current level of installed capacity and becomes productive only at the beginning of the next period:

$$K_{t+1} = (1 - \delta) K_t + I_t. \tag{2}$$

Moreover, there exist costs to adjusting the stock of capital, $\Phi(\cdot)$, which reduce operating profits. Capital adjustment costs depend on the amount of investment and the current stock of capital. Our assumptions about the adjustment cost function are described below.

Assumption 2. Adjustment Cost. The adjustment cost function $\Phi(\cdot) : I \times K \to R_+$ obeys the following conditions: (i) it is twice continuously differentiable for all I, except potentially $I = I^*(K)$; (ii) $\Phi(I^*(K), K) = 0$; (iii) $\Phi_I(\cdot) \times (I - I^*(K)) \geq 0$; (iv) $\Phi_K(\cdot) \leq 0$; and (v) $\Phi_{II}(\cdot) \geq 0$.

Items (ii) and (iii) together imply that adjustment costs are non negative and minimized at the natural rate of investment $I^*(K)$. In most cases this is assumed to be either 0 or δK depending on whether adjustment costs apply to gross or net capital formation. Item (i) allows for general non-convex and potentially discontinuous adjustment costs.

2.2 The Investment Decision

We now define the sequence of optimal investment decisions by the firm as the solution to the following dynamic problem:

$$
\begin{aligned}
V(K_t, A_t, W_t, \Omega_t) &= \max_{\{I_{t+s}, K_{t+s+1}\}_{s=0}^{\infty}} \mathrm{E}_t \left[\sum_{s=0}^{\infty} M_{t,t+s} D_{t+s} \right] && (3) \\
\text{s.t.} \qquad & D_{t+s} = \Pi(K_{t+s}, A_{t+s}, W_{t+s}) - \Phi(I_{t+s}, K_{t+s}) && (4)
\end{aligned}
$$

together with the capital accumulation equation (2). $M_{t,t+s}$ is the stochastic discount factor between periods t and $t+s$, and Ω_t denotes the set of *aggregate* state variables summarizing the state of the economy. Aggregate state variables may include shocks to productivity, wages, capital adjustment costs, relative price of investment goods, and representative household preferences.

2.2.1 Optimal Policies

When item (i) of Assumption 2 holds for any level of investment including $I^*(K)$ - i.e. $\Phi(\cdot)$ is twice continuously differentiable for all I - standard first-order conditions are sufficient to characterize the solution to (3). The optimal investment policy equates marginal benefit and cost of investment:

$$q_t = \Phi_I(I_t, K_t) \tag{5}$$

where q_t is the marginal value of installed capital, or *marginal* q, and satisfies the following Euler equation:

$$q_t = \mathrm{E}_t \left[M_{t,t+1} \left(\Pi_K \left(K_{t+1}, A_{t+1}, W_{t+1} \right) + \left(1 - \delta \right) q_{t+1} - \Phi_K \left(I_{t+1}, K_{t+1} \right) \right) \right]. \tag{6}$$

The computation of optimal investment policies requires combining the expressions in (5) and (6). However, under general conditions, there exist no explicit closed form solution. Nevertheless, these policies can be further characterized by inverting the marginal cost of investment in (5) as:

$$\frac{I_t}{K_t} = \widetilde{G} \left(K_t, q_t \right).$$

Most of the literature follows Hayashi (1982) and assumes linear homogeneity (in I and K) for the functions $\Pi(\cdot)$ and $\Phi(\cdot)$ to obtain a linear investment policy from (5) under quadratic adjustment costs:

$$\frac{I_t}{K_t} = \alpha_0 + \alpha_1 q_t. \tag{7}$$

Under these assumptions marginal q equals average Q - i.e. ratio of market value to replacement cost of capital - and the investment equation in (7) can be estimated directly from the data.

With less restrictive conditions, however, marginal q is no longer directly observable. Nevertheless, as long as the process for the stochastic variables is Markov, the law of motion (6) implies that the marginal value of installed capital can be written as $q_t = q\left(K_t, Z_t\right)$, where the vector Z denotes all state variables other than capital and captures possible shocks to firm productivity, firm output demand, firm wages, and aggregate state variables, i.e. $Z_t = \{A_t, W_t, \Omega_t\}$.

In general, the optimal rate of investment can always be characterized by the following state variable representation:

$$\frac{I_t}{K_t} = G \left(K_t, Z_t \right) \tag{8}$$

where the explicit form for the function $G(\cdot)$ depends on the specific functional forms of $\Pi(\cdot)$ and $\Phi(\cdot)$, and may not be readily available in most circumstances. However, given the measurability of investment, the unknown investment policy $G(\cdot)$ can be directly estimated as function of its underlying state variables K and Z as long as they are also measurable.[7]

The appeal of Tobin's Q lies on the belief that it serves as a forward-looking measure of investment opportunities that summarizes all information about the expected future profitability and discount rates. However, this information is also incorporated in the characterization of the optimal investment policy as function of the underlying state variables. A key difference is that the policy function characterization is **always** correct, while the use of average Q relies on an unlikely combination of assumptions. The following simple numerical example illustrates this point.

2.2.2 A Simple Example

As a special case of the dynamic investment model consider the case where:

$$Y = AK^{0.85} \tag{9}$$

and

$$\Phi(I, K) = I + \frac{b}{2}\left(\frac{I}{K}\right)^2 K, \qquad b > 0 \tag{10}$$

Note that this is a relatively small deviation from the standard homegeneous case with quadratic adjustment costs.

Figure 1 plots the optimal policy rules for the investment rate, I/K, against average Q, firm size, and firm sales-to-capital ratio, respectively.

Even in this simple case with a relatively small deviation from the extreme homogenous model with quadratic adjustment costs the relationship between investment and Tobin's Q is significantly weaker. Moreover this relationship seems much less precise

[7]When item (i) of Assumption 2 holds for any level of investment excluding $I^*(K)$, the optimal investment policy may be a discontinuous function. Nonetheless, it still admits the representation in (8), and it can be directly estimated as function of its underlying state variables.

than that with size and sales. Furthermore, these variables also more clearly identify the position of each firm over the state-space, $K \times A$, and as such are very informative about the optimal investment policy than Tobin's Q

Figure 2 makes this even clearer by depicting the difference in the model's actual and fitted values of I/K from standard Q regressions and comparing them with the outcome of fitting a second order polynomial in $\ln(K)$ and $\ln(Y/K)$, respectively. Again, even in this simple example with quadratic adjustment costs, a relatively small departure from constant returns to scale renders average Q much less informative about investment.

Although very simple, the example illustrates how approximated policy functions then offer a much more robust description of investment behavior and are thus likely to be more informative to infer underlying structural parameters.

2.3 Discussion

Direct estimation of the policy functions has other important benefits. First, unlike Q-type regressions which are based on an optimality condition where Q and investment are determined simultaneously, state variables are, by construction, pre-determined at the time current investment is chosen. Although endogeneity issues are not eliminated, our method represents a distinct improvement over standard Q-regressions. Second, policy function estimation also minimizes the measurement error concerns induced by potential stock market misvaluations (Blanchard, Rhee and Summers, 1993; Erickson and Whited, 2000).

Our methodology builds on the idea that a model is described not only by its restrictions on functional forms, but also, and most importantly, by its state variables. Different classes of investment models often lead to different sets of state variables. The importance of various classes of investment models can then be assessed through a statistical variance decomposition of their corresponding state variable representation of investment.[8]

[8]For example, a model with non trivial capital structure decisions (e.g. Hennessy and Whited (2005),

3 Empirical Estimation

This section discusses how to use the method above to construct empirical counterparts to the policy functions, $G(K, Z)$. First, we note that under general conditions we can approximate the optimal investment policy arbitrarily closely with the following tensor product representation:

$$\frac{I}{K} = \sum_{i_k=0}^{n_k} \sum_{i_z=0}^{n_z} c_{i_k,i_z} k^{i_k} z^{i_z} + \epsilon_{it} \tag{11}$$

where $z = \log(Z)$ and $k = \log(K)$ and ϵ_{it} is the approximation error.[9] Once estimated, the approximation coefficients c_{i_k,i_z} can be used to infer the underlying structural parameters of the model, or at the very least, place restrictions on the nature of technology and adjustment costs. We investigate several parameterizations of the model in the next section.

The choice of the polynomial order can be made according to standard model selection techniques based on a measure of model fit such as adjusted R^2 or Akaike information criterion (AIC). We show below that a second order polynomial is often sufficient, and higher order terms are generally not important to improve the quality of the approximation. The low order of approximation mitigates the need to use orthogonal polynomials, simplifying the interpretation of the estimated coefficients and their relationships with the underlying structural parameters of the model.

3.1 Measurement

Empirical implementation of (11) requires measurement of the state variables, most importantly of the possible components of the exogenous state Z. This can be achieved by imposing the theoretical restrictions implied by the model. For example, under the common assumption that the sources of uncertainty are in firm technology and demand

Gomes and Schmid (2010)) implies that financial leverage is also a state variable in (8).

[9]Non-smooth investment policies may require several high order polynomial terms to better capture nonlinearities in investment. More generally, although not pursued in this paper, optimal policies can also be estimated using a full nonparametric approach.

(i.e. $Z = \{A\}$) we can measure these shocks directly from observed sales by inverting the revenue function $Y = F(Z, K, N)$.[10]

In this case we can work instead with the polynomial approximation:

$$\frac{I}{K} = \sum_{i_k=0}^{n_k} \sum_{i_y=0}^{n_y} \sum_{i_n=0}^{n_n} g_{i_k,i_y,i_n} k^{i_k} y^{i_y} n^{i_n} + \epsilon_{it}. \tag{12}$$

The investment policy is now represented as a direct function of three observable variables, including capital, sales and labor, and can be readily estimated from the data.[11]

Finally, since the right hand side variables are all in logs, we can - without any loss of generality - scale employment and sales by the capital stock and estimate a version of (12) using $\ln(Y/K)$ and $\ln(N/K)$. This transformation allows us to make our results more directly comparable with the existing literature.

3.2 Time and Firm Fixed Effects

Any *aggregate* state variables, Ω, can also be part of the exogenous state Z. These variables may include, among others, shocks to the stochastic discount factor, aggregate wages, and input prices. Given a large panel of firms, complete knowledge of the aggregate state variables in Ω is not necessary for the purpose of estimating investment policies. To the extent that variation in these variables affects all firms equally, it can be easily captured by allowing the constant term $g_{0,0,0}$ in (12) to be time-specific.[12,13]

In addition, it is also natural to expect differences in firms' natural rate of investment, $I^*(K)/K$, mainly due to variations in the depreciation rates on their assets. We can

[10] Alternatively, we could also estimate Z directly (e.g. Olley and Pakes (1996)) and use a two stage approach. However this requires specification of the precise revenue function and adds a number of econometric problems, most significantly, endogeneity. However, since we are interested in characterizing investment, exact knowledge of Z is not required.

[11] The coefficients g_{i_k,i_z,i_n} are now convolutions of the structural parameters of the revenue function and the approximation coefficients c's.

[12] For comparison with the existing literature, we focus our analysis on unobserved aggregate variation that enters only linearly the investment policy. It is straightforward to allow for unobserved aggregate variation to enter non-additively the investment policy, with time-specific slope coefficients.

[13] Additional industry-level fixed effects can also be used to capture industry-level state variables.

capture firm heterogeneity in depreciation rates, i.e. $\delta = \delta_j$, by allowing the constant term $g_{0,0,0}$ in (12) to have a firm-specific component.

3.3 Estimated Policy Functions

We can now implement our methodology to estimate the empirical policy function and compare its performance with standard Q regressions. All details concerning the data and the construction of the variables are provided in the Appendix. Table 1 reports the key summary statistics including mean, standard deviation and main percentiles for the primary variables of interest. Our goal is to identify a parsimonious polynomial representation both in terms of variables and an order of approximation that provides the best overall fit for investment empirically and can be used to evaluate our structural model.

Table 2 shows the empirical estimates for various polynomial approximations to the investment policy (12). All estimates use time and firm fixed effects to account for potential aggregate shocks and firm differences in average investment rates. Generally, we find that first and second order terms are all strongly statistically significant, except for the second order term in the employment-to-capital ratio, which is significant only at the ten percent level. Moreover, it is generally the case that adding the employment-to-capital ratio leaves the overall fit of the regression virtually unaffected.[14]

We conclude that a second order polynomial approximation that uses firm size and the sales-to-capital ratio (column 2) offers the best parsimonious empirical representation of investment. For comparison, the last column of Table 2 also shows that this approximate policy function fits the investment data much better than a standard Q regression. Even a state variable approximation that use only first order terms has a much higher adjusted R^2.

[14]Interaction and higher order terms are generally not statistically significant and do not improve the overall quality of the approximation. We omit these results for conciseness but they are available upon request.

4 Structural Estimation and Inference

We now use the information from the estimated policy functions to structurally estimate the key adjustment cost parameters using indirect inference. The first step is the specification of functional forms for general sales and adjustment cost functions that satisfy Assumptions 1 and 2. We then report the elasticity of each moment with respect to each structural parameter and show how policy function estimates can be more informative than the coefficient from Q-type regressions.

4.1 Model Parameterization

We assume that the technology exhibits decreasing returns or, alternatively, that markets are not perfectly competitive. Specifically, sales are given by the decreasing returns to scale function:

$$Y = A\left(K^{\alpha} N^{1-\alpha}\right)^{\gamma}$$

where $\alpha \in (0,1)$ and $\gamma < 1$ captures the degree of returns to scale. The stochastic process for A is of the AR(1) form:

$$\ln A_t = (1-\rho)\ln \bar{A} + \rho \ln A_{t-1} + \sigma \zeta_t$$

where $|\rho| < 1$, $\sigma > 0$ and ζ follows a truncated i.i.d. normal with zero mean and unit variance. We assume that the unit labor cost W is constant and normalized to one.

A general adjustment cost function that satisfies Assumption 2 is:

$$\Phi\left(I, K\right) = I + \begin{cases} aK + \frac{b}{v}\left(\frac{I - I^{*}(K)}{K}\right)^{v} K & \text{if } I \neq I^{*}(K) \\ 0 & \text{if } I = I^{*}(K) \end{cases} \tag{13}$$

where a, b are all non-negative, and $v \in \{2, 4, 6, ...\}$. We normalize the relative price of investment to one and assume that adjustment costs apply to net capital formation, $I^{*}\left(K\right) = \delta K$. We have non-convex fixed cost of investment when a is positive. Note that standard smooth quadratic adjustment costs are obtained as special case of (13) when $v = 2$ and $a = 0$.

4.2 Estimation Results

Most of the other structural parameters can be quite accurately estimated directly from unconditional moments of variables such as sales and/or profits without resorting to indirect inference methods. We thus fix a number of these accessory technology parameters to what are more or less consensual values in the literature. Specifically we set the degree of decreasing returns, $\gamma = 0.85$, and $\alpha = 0.35$ implying a capital share $(\alpha\gamma)$ of 0.30 in line with the numerical values used in previous studies (Gomes, 2001). Moreover, values like the the average depreciation rate, δ, and discount factor, M, are largely immaterial for our results. We set their values at 0.10 and 0.95, respectively. Throughout our analysis, we also set the persistence and the standard deviation of the technology shocks, ρ and σ, respectively, to 0.80 and 0.10. Although it is straightforward to include these parameters in the structural estimation exercise, they are usually best identified from the variance and persistence of profits or revenues and do are not generally crucial to the identification of adjustment costs parameters.

The algorithm for indirect inference is now well understood. First, given a specific set of parameter values, we solve numerically the problem of the firm in (3) using standard value function iteration techniques. We then generate multiple panels of simulated data using the optimal policy and value functions of the model. Next, we estimate the regression coefficients from both standard Q regressions and polynomial approximations to the optimal investment policy in each panel and compare the average estimate to those obtained in the Compustat dataset. The method then picks the model parameters that make the actual and simulated moments as close to each other as possible.[15]

For each parameterization of the adjustment cost function we simulate 100 artificial panels of 500 firms each with 390 years of data. We estimate the investment polynomial

[15] For a detailed description in a very general setting see Warusawitharana and Whited (2015).

regressions using the last 39 years of simulated data, which corresponds to the time span of the actual data sample. We then report the average coefficient estimates and standard errors across artificial panels.

Table 3 shows the estimated parameter values and compares the implied moments in the artificial data with our own empirical estimates. The table shows that a model with quadratic adjustment costs but also a small amount of fixed costs does well in matching all regression coefficients found in the data. This model is able to both generate a weak sensitivity of investment to Q and produce the coefficients from empirical policy function estimates.

4.3 Moment Elasticities

Our results offer compelling evidence that Q regressions are much less informative than estimates of the investment policy function. For example, the coefficient estimates on Q regressions are quite similar across alternative adjustment cost parameterizations ranging only from a minimum of 0.05 in the specification without adjustment costs to a maximum of about 0.08 across all parameterizations. On the other hand, the coefficients on the polynomial approximation exhibit substantial variation. For instance, the coefficients on the linear terms in firm size and sales range from -0.24 to -0.06, and 0.13 to 7.51, respectively.

We follow Hennessy and Whited (2007) and use the simulated model to measure the elasticity of key theoretical moments with respect to the various parameters.[16] Formally, the elasticity of moment x with respect to parameter κ is computed as:

$$\xi_{x,\kappa} = \frac{x\left(\widehat{\kappa}\left(1+\varepsilon\right);\theta\right) - x\left(\widehat{\kappa}\left(1-\varepsilon\right);\theta\right)}{2\varepsilon x\left(\widehat{\kappa}\right)}$$

where $\widehat{\kappa}$ is the baseline value of κ, ε is the percent deviation from the baseline, and θ is

[16] Intuitively, if the elasticity of a particular theoretical moment to a particular parameter is low, then that moment is an unreliable guide to inferring the true value of the underlying structural parameter.

a vector of the other structural parameters.[17]

We report the elasticity of the following conditional and unconditional moments: (1) the coefficient estimate from a standard Q-type investment regression; (2) the coefficient estimates from the investment policy function approximation; and (3) standard unconditional moments of the investment distribution such as the standard deviation and autocorrelation. We use our parameter estimates as our baseline.

Table 4 reports our findings. For completeness we include also the elasticities with respect to the technology parameters γ and α. The Table shows that most coefficients are quite sensitive to the degree of returns to scale, γ. Perhaps unsurprisingly, the capital elasticity α has a larger effect on unconditional moments of the investment distribution and the polynomial terms in $\ln K$.

The main conclusion however is that investment adjustment cost parameters are generally much better identified from estimated policy function coefficients, which exhibit higher elasticities than the coefficient from a standard Q-regression. To us, this suggests that full estimation of a structural model, should primarily target unconditional moments of the investment distribution together with the approximate investment policy function implied by the model. By contrast, the slope of a Q regression is generally less informative about model parameters.

5 Cases with More State Variables and Model Misspecification

We have illustrated our approach for the (popular) class of investment models where optimal policies depend only on two state variables, size (K) and exogenous demand/productivity shocks (A). However, our core insight of directly estimating optimal policy functions is easily expanded to much larger classes of models. In this section we describe how to

[17] We generally use $\varepsilon = 0.1$, except for the curvature of the adjustment costfunction where we use $\varepsilon = 1$ and consider a one sided deviation only.

adapt our procedure to two specific examples: (i) a model with financial leverage, and (ii) a model with firm-specific shocks to the price of variable inputs such as wages.

5.1 Capital Market Imperfections and Leverage

Our basic approach can be easily extended to models with financial frictions. Most modifications of the firm problem (3), that allow for frictions such as tax benefits of debt, collateral requirements and costly external financing, also imply that firm debt, B, becomes an additional state variable for the optimal investment policy. Formally, these models imply that:[18]

$$\frac{I}{K} = G\left(K, B, Z\right). \tag{14}$$

It follows that in this case we can generalize our procedure by augmenting the approximate policy function (12) with additional terms including corporate debt:

$$\frac{I}{K} = \sum_{i_k=0}^{n_k} \sum_{i_y=0}^{n_y} \sum_{i_n=0}^{n_n} \sum_{i_b=0}^{n_b} g_{i_k,i_y,i_n,i_b} k^{i_k} y^{i_y} n^{i_n} b^{i_b} + \epsilon_{it}. \tag{15}$$

5.2 Labor Market Shocks and Cash Flow

Aggregate variation in the price of variable inputs, such as labor, will be captured by adding simple time effects to (11). However, if some of these shocks are firm-specific, the set of state variables, Z, would now need to be expanded to also include the firm level wage rate, W (i.e. $Z = \{A, W\}$). Since direct evidence on firm level labor costs is often sparse it is often best to again use theory to infer these shocks directly from observed cash flow data.

For example, if the production function, $F\left(A, K, N\right)$, is Cobb-Douglas, operating profits become $\Pi = ZK^{\theta}$, where Z captures joint information about A and W, and can

[18] Examples of models where net financial liabilities represents an additional state variable for the optimal investment policy include Whited (1992), Bond and Meghir (1994), Gilchrist and Himmelberg (1999), Moyen (2004), Hennesy and Whited (2007), Hennessy, Levy, and Whited (2007), Gomes and Schmid (2010), Bustamante (2011), and Bolton, Chen, and Wang (2012), among others.

be directly constructed from:

$$z \equiv \ln Z = \log \Pi - \theta \log(K).$$

The investment policy is now be approximated as:

$$\frac{I}{K} = \sum_{i_k=0}^{n_k} \sum_{i_\pi=0}^{n_\pi} d_{i_k,i_\pi} k^{i_k} \pi^{i_\pi} + \epsilon_{it},$$

using only data on log operating profits, $\pi = \log \Pi$ and the stock of capital.

5.3 Model Misspecification and Tobin's Q

The thrust of our argument is that marginal Q should matter a lot more than average Q for investment policies. Theoretically, any information contained in *marginal* Q will be spanned by the state variables characterizing the optimal investment policy. Moreover, because these are much easier to measure accurately, empirical policy functions offer a much better description of the investment data.[19]

How useful is then Tobin's average Q? It remains true that Tobin's average Q remains an endogenous variable in the model which retains some (but generally far from perfect) correlation with investment behavior. As such is it possible that this variable may isolate additional investment variation due to some omitted state variables.

Table 5 investigates this possibility empirically by reporting the results of a covariance analysis (ANCOVA) as in Lemmon and Roberts (2008). Specifically, we compute a normalized Type III partial sum of squares for several variables or groups of variables - including Q - in the investment specification.[20]

Unsurprisingly, firm fixed effects account for a large fraction of the variation in investment **levels** - in the long run all cross-sectional variation in levels is accounted by

[19] As a corollary, estimating the underlying process for investment as a function of state variables offers an estimate of marginal Q (see Gala (2015).

[20] Type III sum of squares is not sensitive to the ordering of the covariates, and our panel is unbalanced.

variation in the depreciation rate, δ_j. A more meaningful decomposition of the variation in investment **changes** however, shows that our baseline polynomial in firm sales and size accounts for 97 percent of the investment variation.

This variance decomposition shows that, in this sample of publicly traded firms, only about 4 percent of the explained variation in investment levels can be accounted by the covariation with firm financial leverage. Similarly, financial leverage accounts only for about 4 percent of the overall explained variation in investment changes, while 94 percent is attributable to our core state variables alone.[21]

Column (4) shows that only 7 percent of the overall variation in investment can be attributed to Tobin's Q, while 32 percent is attributable to the state variable polynomial. A similar decomposition of investment rates changes is more stark. Tobin's Q accounts only for 2 percent of the overall explained variation in investment changes, while 96 percent is attributable to our core state variables alone. Overall, it seems that Tobin's Q incorporates fairly little "independent" information for investment.

6 Conclusion

How can we understand the investment of small private firms where information about Tobin's Q is not available? This paper proposes an asset price-free alternative relying on the insight that the optimal investment policy is a function of much more easily measurable state variables. Under very general assumptions about the nature of technology and markets, our approach ties investment rates directly to firm size, sales or cash flows, and, in the presence of financial market frictions, measures of net liabilites. Empirically, our methodology also appears superior to Q theory even for samples of large publicly traded firms.

[21] Because leverage is itself an endogenous variable, it can still affect investment indirectly through feedbacks on firm size and sales. Gala and Gomes (2012) investigate in more details the implications of capital market imperfections on the direct estimation of alternative investment policies.

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A Appendix

Our data comes from the combined annual research, full coverage, and industrial COMPUSTAT files. To facilitate comparison with much of the literature our initial sample is made of an unbalanced panel of firms for the years 1972 to 2010, that includes only manufacturing firms (SIC 2000-3999) with at least five years of available accounting data.

We keep only firm-years that have non-missing information required to construct the primary variables of interest, namely: investment, I, firm size, K, employment, N, sales revenues, Y, and Tobin's Q. Firm size, or the capital stock, is defined as net property, plant and equipment. Investment is defined as capital expenditures in property, plant and equipment. Employment is the reported number of employees. Sales are measured by net sales revenues. In our implementation these variables are scaled by the beginning-of-year capital stock. Finally, Tobin's Q is measured by the market value of assets (defined as the book value of assets plus the market value of common stock minus the book value of common stock) scaled by the book value of assets.[22] We use also standard measures of cash flow, CF, defined as earnings before extraordinary items plus depreciation; and net corporate debt, B, computed as the sum of short-term plus long-term debt minus cash and short-term investments.

Our sample is filtered to exclude observations where total capital, book value of assets, and sales are either zero or negative. To ensure that our measure of investment captures the purchase of property, plant and equipment, we eliminate any firm-year observation in which a firm made an acquisition. Finally, all primary variables are trimmed at the 1st and 99th percentiles of their distributions to reduce the influence of any outliers, which are common in accounting ratios. This procedure yields a base sample of 32,890 firm-years observations.

[22]Erickson and Whited (2006) show that using a perpetual inventory algorithm to estimate the replacement cost of capital and/or a recursive algorithm to estimate the market value of debt barely improves the measurement quality of the various proxies for Q.

Table 1: **Summary Statistics**

This table reports summary statistics for the primary variables of interest from Compustat over the period 1972-2010. The investment rate, I/K, is defined as capital expenditures in property, plant and equipment scaled by the beginning-of-year capital stock. The capital stock, K, is defined as net property, plant and equipment. Firm size, $\ln(K)$, is the natural logarithm of the beginning-of-year capital stock. The sales-to-capital ratio, $\ln(Y/K)$, is computed as the natural logarithm of end-of-year sales scaled by the beginning-of-year capital stock. The employment-to-capital ratio, $\ln(N/K)$, is defined as the natural logarithm of the number of employees scaled by the capital stock. The cash flow rate, CF/K, is calculated as the sum of end-of-year earnings and depreciation scaled by the beginning-of-year capital stock. Tobin's Q is defined as the market value of assets scaled by the book value of assets.

	Obs	Mean	Std. Dev.	25th	50th	75th
I/K	32,890	0.26	0.24	0.12	0.20	0.32
$\ln K$	32,890	3.56	2.27	1.88	3.44	5.20
$\ln \frac{Y}{K}$	32,890	1.72	0.77	1.23	1.69	2.18
$\ln \frac{N}{K}$	32,890	-3.16	1.00	-3.81	-3.15	-2.46
Q	32,890	1.56	1.00	0.95	1.24	1.80

Table 2: **Empirical Investment Policies**

This table reports empirical estimates from the investment regression specification:

$$\frac{I_{jt+1}}{K_{jt}} = \beta X_{jt} + \delta_j + \eta_t + \varepsilon_{jt+1}$$

where the left-hand-side is end-of year capital expenditures scaled by beginning-of-year property, plant and equipment, δ_j is a firm fixed effect, η_t is a year fixed effect, and X denotes a set of explanatory variables including average Q, firm size, $\ln K$, sales-to-capital ratio, $\ln(Y/K)$, and employment-to-capital ratio, $\ln(N/K)$. Standard errors are clustered by firm and reported in parenthesis. $\overline{R}^2$ denotes the adjusted R^2 and AIC is the adjusted Akaike Information Criterion. The sample period is 1972 to 2010.

	(1)	(2)	(3)	(4)	(5)
Q					0.078
					(0.004)
$\ln \frac{Y}{K}$	0.204	0.100	0.222	0.113	
	(0.008)	(0.010)	(0.007)	(0.010)	
$\ln K$	-0.014	-0.021	-0.018	-0.027	
	(0.003)	(0.005)	(0.003)	(0.005)	
$\ln \frac{N}{K}$			-0.034	-0.060	
			(0.006)	(0.018)	
$(\ln \frac{Y}{K})^2$		0.029		0.030	
		(0.003)		(0.003)	
$(\ln K)^2$		0.002		0.002	
		(0.001)		(0.001)	
$(\ln \frac{N}{K})^2$				-0.004	
				(0.002)	
$\overline{R}^2$	0.38	0.39	0.38	0.39	0.29
AIC	-12,887	-13,229	-12,955	-13,292	-8,430
Obs	32,890	32,890	32,890	32,890	32,890

Table 3: **Estimated Moments and Parameters**

This table reports results from estimating the baseline model using investment regressions from simulations using 100 artificial panels of 500 firms each with 39, which corresponds to the time span of the actual data sample from Compustat. The top panel reports the average regression coefficient estimates and standard errors for the data and across artificial panels. The bottom panel reports the estimated parameter values as well as the implied χ^2 statistic.

PANEL A

	Data Moments		Simulated Moments	
Q	0.078		0.057	
	(0.004)		(0.001)	
$\ln \frac{Y}{K}$		0.100		0.132
		(0.010)		(0.005)
$\ln K$		-0.021		-0.058
		(0.005)		(0.002)
$(\ln \frac{Y}{K})^2$		0.029		0.025
		(0.003)		(0.003)
$(\ln K)^2$		0.002		0.002
		(0.001)		(0.002)

PANEL B

Estimated Parameters			
a	b	ν	χ^2
0.010	0.75	2	0.0028

Table 4: **Sensitivity of Model Moments to Parameters**

This table presents elasticities of model moments with respect to key model parameters. The parameters values are those estimated in Section 4. The set of moments include: (1) the coefficient estimate from a standard Q-type investment regression; (2) the coefficient estimates from the investment policy function approximation; (3) moments of the investment distribution such standard deviation (Std) and autocorrelation (AR).

	Moments	γ	α	a	b	ν
1	Q	7.92	0.23	-0.46	-0.33	0.49
2	$\ln \frac{Y}{K}$	4.76	0.01	-0.55	-0.75	3.44
	$\ln K$	-0.20	-0.65	-0.31	-0.16	1.36
	$(\ln \frac{Y}{K})^2$	3.22	-0.01	-0.27	-1.30	1.49
	$(\ln K)^2$	10.04	3.08	2.26	9.39	4.13
3	Std I/K	3.41	-0.62	-0.22	-0.35	2.24
	AR I/K	6.84	-1.09	-1.07	-0.30	0.50

Table 5: **Empirical Variance Decompositions**

This table presents a variance decomposition of several polynomial specifications for both the levels (Panel A) and changes (Panel B) in investment. We compute the Type III partial sum of squares for each effect in the model and then normalize each estimate by the sum across the effects, forcing each column to sum to one. For example, in specification (4) of Panel A, 7% of the explained sum of squares captured by the included covariates can be attributed to Tobin's Q. Similarly, in specification (4) of Panel B, 2% of the explained investment changes can be attributed to changes in Tobin's Q. Firm FE are firm fixed effects. Year FE are calendar year fixed effects. Q denotes Tobin's Q. "Sales and Size" denotes the second order polynomial in firm size, $\ln(K)$, and sales-to-capital ratio, $\ln(Y/K)$. "Cash Flow" denotes a second order polynomial in firm cash flow-to-capital ratio, CF/K. "Leverage" denotes a second order polynomial in firm net leverage, B/K. $\overline{R}^2$ denotes adjusted R^2. The sample period is 1972 to 2010.

	(1)	(2)	(3)	(4)
A: Investment Levels (I/K)				
Firm FE	0.61	0.70	0.59	0.56
Year FE	0.06	0.07	0.07	0.05
Sales and Size	0.33	0.19	0.30	0.32
Cash Flow		0.03		
Leverage			0.04	
Q				0.07
$\overline{R}^2$	0.39	0.40	0.40	0.41
B: Investment Changes ($\Delta I/K$)				
Year FE	0.02	0.02	0.02	0.02
Sales and Size	0.98	0.97	0.94	0.96
Cash Flow		0.01		
Leverage			0.04	
Q				0.02
$\overline{R}^2$	0.30	0.30	0.31	0.31

Figure 1: Optimal Investment Policies with Decreasing Returns.
This figure shows scatter plots of the optimal investment rates, I/K, against average Q, firm size, $\ln K$, and firm sales-to-capital ratio, $\ln(Y/K)$, respectively, in a model with standard quadratic adjustment costs but decreasing returns to scale.

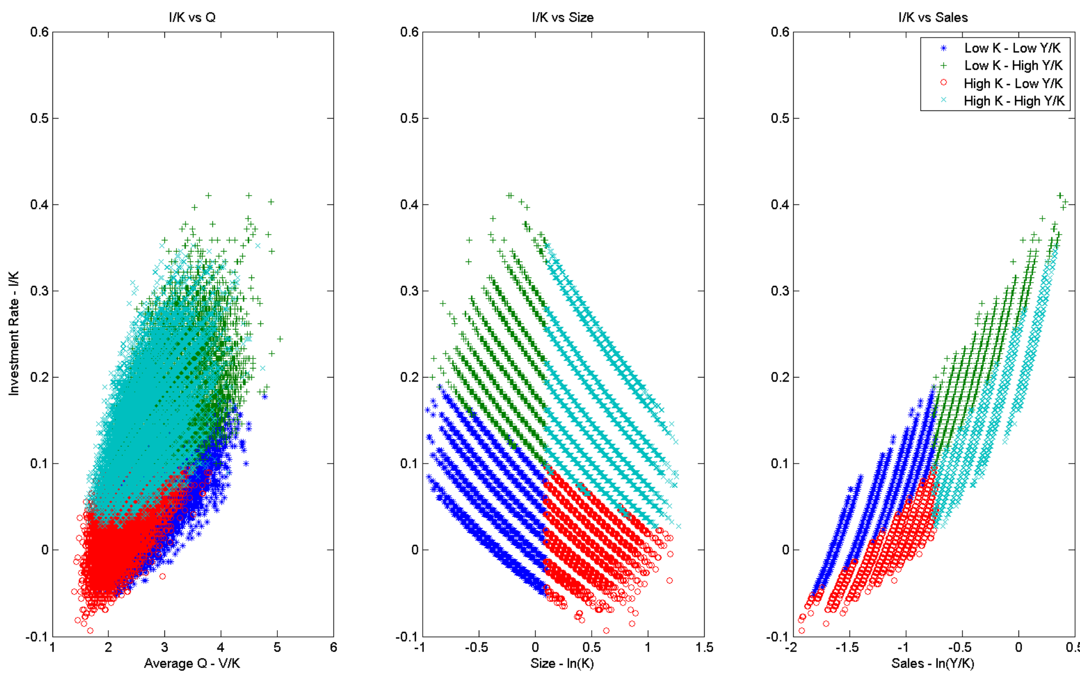

Figure 2: Actual versus Fitted Investment.
This figure compares the actual and fitted investment rates from simulated data when using regressions on average Q and a second order polynomial approximation of the optimal policy function. Simulated data from a simple dynamic investment model with quadratic adjustment costs but decreasing returns to scale.