



1-1-2005

## Understanding and Allocating Investment Risks in a Hybrid Pension Plan

Peter Albrecht

*University of Mannheim*, [p.albrecht@bwl.uni-mannheim.de](mailto:p.albrecht@bwl.uni-mannheim.de)

Joachim Coche

*European Central Bank*, [Joachim.coche@ecb.int](mailto:Joachim.coche@ecb.int)

Raimond Maurer

*Goethe University*, [RMAURER@WIWI.UNI-FRANKFURT.DE](mailto:RMAURER@WIWI.UNI-FRANKFURT.DE)

Ralph Rogalla

*Goethe University*, [Rogalla@wiwi.uni-frankfurt.de](mailto:Rogalla@wiwi.uni-frankfurt.de)

Follow this and additional works at: [https://repository.upenn.edu/prc\\_papers](https://repository.upenn.edu/prc_papers)

 Part of the [Economics Commons](#)

Albrecht, Peter; Coche, Joachim; Maurer, Raimond; and Rogalla, Ralph, "Understanding and Allocating Investment Risks in a Hybrid Pension Plan" (2005). *Wharton Pension Research Council Working Papers*. 384.

[https://repository.upenn.edu/prc\\_papers/384](https://repository.upenn.edu/prc_papers/384)

The published version of this Working Paper may be found in the 2006 publication: *Restructuring Retirement Risks*.

This paper is posted at ScholarlyCommons. [https://repository.upenn.edu/prc\\_papers/384](https://repository.upenn.edu/prc_papers/384)  
For more information, please contact [repository@pobox.upenn.edu](mailto:repository@pobox.upenn.edu).

---

## Understanding and Allocating Investment Risks in a Hybrid Pension Plan

### Abstract

This paper analyzes pension plan costs and investment strategies in the context of alternative hybrid pension plans which are optimal either from the perspective of the plan sponsor or the beneficiaries. The focus is in particular on how the introduction of minimum and maximum limits for pension benefits as well as minimum guarantees and caps on the return of the members' individual investment accounts affect investment decisions and plan costs. Within a comparative static analysis framework, it is shown that for low to medium risk portfolios, minimum benefit guarantees tend to be more expensive than minimum return guarantees while for the latter costs increase exponentially with investment risk. The study also finds that portfolio choices of the sponsor and the beneficiaries show substantial differences depending on the exact plan design and the beneficiaries' risk aversion. Combining minimum return guarantees and caps on investment returns emerged as a possible means to reduce such differences, to share investment risks and returns more equally between sponsor and beneficiaries, and to keep pension plan costs under control.

### Disciplines

Economics

### Comments

The published version of this Working Paper may be found in the 2006 publication: [Restructuring Retirement Risks](#).

# **Restructuring Retirement Risks**

Edited by

David Blitzstein, Olivia S. Mitchell,  
and Stephen P. Utkus

**OXFORD**  
UNIVERSITY PRESS

**OXFORD**  
UNIVERSITY PRESS

Great Clarendon Street, Oxford OX2 6DP

Oxford University Press is a department of the University of Oxford.  
It furthers the University's objective of excellence in research, scholarship,  
and education by publishing worldwide in

Oxford New York

Auckland Cape Town Dar es Salaam Hong Kong Karachi

Kuala Lumpur Madrid Melbourne Mexico City Nairobi

New Delhi Shanghai Taipei Toronto

With offices in

Argentina Austria Brazil Chile Czech Republic France Greece

Guatemala Hungary Italy Japan Poland Portugal Singapore

South Korea Switzerland Thailand Turkey Ukraine Vietnam

Oxford is a registered trade mark of Oxford University Press  
in the UK and in certain other countries

Published in the United States  
by Oxford University Press Inc., New York

© Pension Research Council, The Wharton School,  
University of Pennsylvania, 2006

The moral rights of the authors have been asserted

Database right Oxford University Press (maker)

First published 2006

All rights reserved. No part of this publication may be reproduced,  
stored in a retrieval system, or transmitted, in any form or by any means,  
without the prior permission in writing of Oxford University Press,  
or as expressly permitted by law, or under terms agreed with the appropriate  
reprographics rights organization. Enquiries concerning reproduction  
outside the scope of the above should be sent to the Rights Department,  
Oxford University Press, at the address above

You must not circulate this book in any other binding or cover  
and you must impose the same condition on any acquirer

British Library Cataloguing in Publication Data  
Data available

Library of Congress Cataloging in Publication Data  
Data available

Typeset by SPI Publisher Services, Pondicherry, India

Printed in Great Britain

on acid-free paper by

Biddles Ltd., King's Lynn, Norfolk

ISBN 0-19-920465-9 978-0-19-920465-6

1 3 5 7 9 10 8 6 4 2

## Chapter 11

# **Understanding and Allocating Investment Risks in a Hybrid Pension Plan**

---

*Peter Albrecht, Joachim Coche, Raimond Maurer, and Ralph Rogalla*

First introduced in the USA by the Bank of America in 1985, hybrid types of pension plans altered the traditional form of pension plan design in the developed world.<sup>1</sup> The term ‘hybrid’ pension plan subsumes plans with elements of both defined contribution (DC) and defined benefit (DB) plans. The motivation for hybrid plans is to combine the best characteristics of DB and DC plans while circumventing their major disadvantages. Most include a DC-type individual account, but also provide minimum and/or maximum annuity benefits at retirement using a DB-type formula. Additionally, investment returns credited to the individual accounts may be subject to return guarantees and/or return caps.<sup>2</sup>

When setting up and running a pension plan, the costs implied by the specific plan design, as well as the asset allocation decision for the accumulated funds, are of major importance. In a pure DC plan, plan members have extensive control over their accounts’ investment strategy (subject to the investment menu they are offered). This enables participants to shape their portfolio’s risk/return profile to their individual risk preferences. The sponsor only promises to make a certain contribution to the account, so the investment risk is therefore completely borne by the members; consequently, the plan sponsor tends to be rather indifferent toward the individual’s investment policy, as it poses no cost implications. By contrast, in a pure DB plan, the sponsor is obliged to provide adequate funds to cover the plan liabilities, so he is fully exposed to capital market risk. The asset allocation decision has direct cost implications for his funding situation. In a hybrid pension plan, both parties have an interest in influencing the plan’s investment policy. This can result in a conflict of interest, which is the object of investigation of this chapter. At the same time, we scrutinize the costs inherent in different DB-type elements.

To do so, we construct several hypothetical hybrid pension plans, make assumptions about key parameters, and the optimal investment strategy for particular objective functions. These include cost minimization from the perspective of plan sponsor versus maximizing risk-adjusted pension benefits from the perspective of plan members. Although the design of the plan

and the assumed parameters do not exactly match actually particular pension plans, the models draw on real-world elements. In particular, the formulation presented here draws on prior analysis of the European Central Bank retirement plan,<sup>3</sup> though the model plan developed here is less complex.<sup>4</sup>

This chapter is organized as follows. First, we discuss the main elements of the hybrid plan evaluated including the minimum pensions guarantee, maximum pension limits, and the return guarantee/caps. Next, we focus on technical aspects of the model and the decision-making process assumed. Finally, we analyze the optimal investment strategy, both from the perspective of the plan sponsor as well as plan members.

### **Designing a Hybrid Pension Plan**

The pension arrangement analyzed in this study is taken to be a mandatory plan whose members do not contribute to any other public or private pension scheme. It is a noncontributory funded pension plan, consisting of two types of accounts. First, every plan member owns an individual account endowed by the plan sponsor with an assumed payment of 17 percent of the members' annual salary,<sup>5</sup> representing the employer's regular plan contributions. In addition to this, the plan sponsor owns a separate account, called the *contingency reserve*, which plays the role of a settlement account for transfers to or from the individual accounts. The funds in both the individual accounts as well as the contingency reserve represent total plan assets. All plan funds must be invested in the same asset allocation, and the return on this portfolio is credited *pro rata* to the individual accounts and the contingency reserve.

In addition to the plan sponsor's pledge to finance the individual accounts with regular contributions, the plan design includes a combination of additional guarantees and/or limits. These are related to the level of benefits at retirement and/or to the asset return credited to the individual accounts. Incorporating this element influences plan obligations, and it may require additional payments from the sponsor (in addition to regular contributions). These supplementary contributions may be triggered in two cases. First, when guaranteeing a minimum return on the plan assets, the plan sponsor must cover shortfalls below the target return by replenishing the individual accounts through supplementary contributions. Second, supplementary contributions may be needed if there are guaranteed minimum pension benefits. Specifically, we posit that if the market value of the total plan assets falls below 90 percent of the actuarial present value of the plan liabilities (i.e. the solvency ratio falls below 0.9), the plan sponsor must immediately endow the contingency reserve with enough funds to reestablish a solvency ratio of one.

In this plan, participants cannot withdraw funds during the accumulation phase. Members leaving the plan before retirement (e.g. due to

workforce turnover) may either leave their funds in the plan, or receive the balance of their individual account as a lump sum (limited to the actuarial value of the maximum pension where applicable). At the retirement age of 65, the available funds are converted into a life annuity. This conversion may be subject to the guaranteed minimum pension benefits and to maximum pension limits, depending on the exact design of the benefit structure, to be discussed subsequently.

**Benefit Structure.** In what follows, six distinct hybrid pension plan benefit designs are scrutinized. Every plan is characterized by a unique combination of the elements mentioned earlier. The reason to compare these designs is to investigate their differential effects on plan costs and pension benefit levels, and implications for optimal plan asset allocation.

In Case I, which we term the ‘benchmark’ design, the pension plan consists of an individual account for every plan member endowed by the plan sponsor with regular contributions of 17 percent of the current salary. These funds are invested in the capital markets. Beneficiaries are protected from return shortfalls by an annual capital guarantee, i.e. a guaranteed yearly minimum return of 0 percent.<sup>6</sup> In case the funds earn less than 0 percent in any given year, the sponsor must make additional contributions. If the funds accumulated over a plan member’s career are insufficient to pay for an adequate pension, this plan also will guarantee a minimum level of pension benefits, corresponding to 2 percent of the career-average salary per year of service. In addition, this plan limits the maximum level of benefits to 2 percent of the beneficiaries’ final salary (times years of service).<sup>7</sup> In the event of a member either leaving the plan or retiring, any funds in the individual account that exceed the actuarial value of the maximum benefits are transferred to the contingency reserve.

The subsequent Cases II–V are constructed by eliminating certain plan elements, compared to the benchmark case. Case II excludes the capital guarantee, and in Case III, the maximum benefits are also removed. Case IV eliminates the minimum benefit from the benchmark case, while Case V only includes the annual capital guarantee. Case VI includes the annual capital guarantee and additionally a return cap of 10 percent per year, but provides no further benefit elements relating to salary and years of service. If the asset return on the funds in the individual accounts in any year exceeds the 10 percent level, the excess return will be credited to the contingency reserve. Case VII does not include any guarantees or caps and, therefore, can be interpreted as a pure defined contribution plan. Table 11-1 summarizes the various plan designs.

The minimum rate of return guarantee increases the complexity of the pension plan substantially. More specifically, the minimum rate of return guarantee *may* introduce an asymmetric link between assets and liabilities. Suppose the value of a given investment account corresponds to a pension

TABLE 11.1 Summary of Hybrid Pension Plan Designs

	<i>Case I</i>	<i>Case II</i>	<i>Case III</i>	<i>Case IV</i>	<i>Case V</i>	<i>Case VI</i>	<i>Case VII</i>
Individual account	✓	✓	✓	✓	✓	✓	✓
Minimum benefits	✓	✓	✓	—	—	—	—
Maximum benefits	✓	✓	—	✓	—	—	—
Capital guarantee	✓	—	—	✓	✓	✓	—
Return cap	—	—	—	—	—	✓	—

*Source:* Authors' compilations.

*Notes:* Minimum benefits are defined as 2 percent of career-average salary per year of service; maximum benefits are defined as 2 percent of final salary per year of service; capital guarantee refers to a guaranteed minimal return of 0 percent per year for the individual accounts; the return cap limits the annual return credited to the individual accounts to 10 percent.

payment in-between the minimum and the maximum pension limit. In this situation, a high asset return in any given year permanently increases the sponsors' liabilities for the current and future years. Negative returns in subsequent years do not decrease the liability as the minimum rate of return guarantee requires the sponsor to replenish the investment account. Thus the high asset return in the first year had a permanent effect on the liabilities.<sup>8</sup> However, in a situation where the investment account corresponds to a pension payment either below the minimum pension guarantee or above the maximum pension limit, asset returns do not have an immediate effect on the sponsor's liabilities.

**Asset Liability Modeling and the Pension Decision-Making Process.** Next we evaluate the asset–liability model and decision-making process needed to determine the fund's asset allocation behavior. To do so, we describe the key assumptions about how assets and liabilities are projected forward, and then specify decision rules used either by the plan sponsor or by the beneficiaries to identify the optimal asset allocation. Regardless of whether the asset allocation decision is made by the sponsor or by the beneficiaries, a two-step heuristic method is applied which is often found in practical decision-making formats. In the first step, the set of mean-variance efficient asset allocations is determined using a standard Markowitz-type portfolio optimization. In the second step, all portfolios from the efficient frontier are assessed against a projection of asset and liabilities over a horizon of thirty years.

To project the return and risk effects of a certain asset allocation over time, it is necessary to specify the stochastic processes governing asset class returns, interest rates for maturities of three months (representing money market investments) and ten years, as well as inflation rates. The

difference between the nominal ten-year interest rate and the inflation rate (i.e. the real ten-year interest rate) is used to discount future pension liabilities. The stochastic dynamics of the (uncertain) market values of the assets are modeled as geometric Brownian motion, which implies that the log return of every asset is independent and identically normally distributed. Long- and short-term interest rates as well as the inflation rate are modeled using the multidimensional Ornstein/Uhlenbeck process, to cover the empirically observable mean reversion characteristics in these time series.<sup>9</sup>

The investment universe comprises the broad asset classes, money market instruments, euro area bonds, worldwide diversified equities, and emerging market equities. A regime-switching model is used to derive expected returns for the fixed-income asset classes (i.e. money market instruments and Eurobonds). This technique allows consistent generation of yield curve projections contingent on expectations about economic activity (Bernadell et al. 2005). In the long-term projection of the macro-economic environment, we rely on the economist intelligence unit (EIU) as an external provider of forecasts for the Euro area, the US, and Japan.<sup>10</sup> Expected returns on equity investments are approximated by add-ons to the long-term yields on government bonds. In the analysis, the equity risk premium is fixed at 2.5 percent annually for worldwide diversified equity. Reflecting higher risk of emerging market investments we assume an equity risk premium of 4 percent for this asset class. All asset classes are subject to short selling constraints and, in addition, the investment in emerging market equity is restricted to a maximum of 5 percent of overall investments.

The projection of liabilities is based on a discontinuance valuation method usually applied by plan actuaries; this relies on the assumption that service of each participant ceases on the respective valuation date. It assumes that at a given valuation date the individual investment accounts are translated into a (usually deferred) life annuity with inflation-adjusted payments, whereby the minimum and maximum pension limits laid out earlier are applied. The real discount rates used for this exercise are the real ten-year interest rates determined by the asset model. Discontinuance valuation is performed for each year over the thirty-year analysis horizon (Bacinello 2000). The valuation of liabilities requires projecting population dynamics comprising the evolution of the number and composition of staff, salaries, number of retirees, and dependents. For this purpose a hypothetical population comprising initially of 1,000 staff members is constructed. The population is evolved forward using an inhomogeneous, discrete-time Markov chain. Transition probabilities are derived using assumptions for the company's recruitment, promotion and turnover patterns, evolution of salaries as a function of consumer price inflation, and mortality rates.

TABLE 11-2 Parameter Assumptions for Asset Returns, Interest Rates, and Inflation Dynamics

	Mean	Volatility	Correlations		
			Eurobonds	Global equities	EM equities
Eurobonds	5.1	3.7	1	—	—
Global equities	7.6	17.9	0.21	1	—
EM equities	9.1	27.5	0.1	0.73	1

	$\theta$	$\kappa$	$\sigma$	Correlations of innovations		
				3-m interest rate	10-y interest rate	Inflation rate
3-m interest rate	0.043	0.114	0.012	1	—	—
10-y interest rate	0.05	0.075	0.01	0.8461	1	—
Inflation rate	0.02	0.286	0.011	0.7757	0.8103	1
Eurobonds	—	—	—	0.0683	0.1396	0.0740
Global equities	—	—	—	-0.0100	0.0000	0.0100
EM equities	—	—	—	-0.0100	0.0000	0.0100

Source: Authors' calculations.

Notes: Return expectations are derived using yield curve projections as laid down in Bernadell et al. (2005) as well as the assumption of equity risk premia of 2.5 and 4% for global equities and emerging markets (EM) equities. Furthermore, the return dynamics are assumed to follow a geometric Brownian motion. The three-month and ten-year interest rates as well as the inflation rate are modeled using the process specified by  $dX_t = \kappa(\theta - X_t)dt + \sigma dW_t$  where  $X_t$  is the value of the Ornstein/Uhlenbeck process in  $t$ , kappa ( $\kappa$ ) is the speed of mean-reversion, theta ( $\theta$ ) is the long-run mean, and sigma ( $\sigma$ ) is the volatility of changes of the process.  $dW_t$  is the increment of a standard Wiener process. The above estimates are made on the basis of monthly data from January 1986 throughout December 2002 for the JP Morgan European Bond index (Eurobonds), MSCI World ex EMU index (Global Equities), and MSCI Emerging Markets Free index (EM Equities), as well as German inflation rates, three-month Euribor and REX ten-year yields.

Comparing the value of liabilities with the projected value of assets at the respective valuation date allows for the evolution of the plan's solvency ratio to be determined and supplementary contributions to be made by the sponsor and average benefits. Given the complexity of the plan design, solutions are determined using Monte Carlo simulation over 1,000 simulation runs. In the process, we make a number of specific assumptions about selection criteria used to determine the plan's optimal asset allocation. To this end, two different regimes are introduced. Under the first regime, arguably the standard for hybrid pension plans, the plan sponsor is solely responsible for the investment strategy. Correspondingly, the second regime assumes that decisions are made by the beneficiaries. In both

cases, investment decisions apply simultaneously to all individual investment accounts and the contingency reserve.

For the sponsor, we assume the objective is to minimize the costs of running the plan. More specifically the sponsor is modeled as minimizing the worst-case value of discounted supplementary contributions, where the worst-case value is defined as the 5 percent quantile of the distribution of the sum of discounted supplementary contributions over the 30-year investment horizon. Thus, decision criteria other than costs (such as plan solvency) are not considered explicitly. Plan funding is accounted for by the solvency rule, as specified later, according to which the funding ratio cannot fall short of 90 percent in any single year. More formally, let  $SC_t$  be the total amount of supplementary contributions to be made by the plan sponsor in period  $t$  and  $r$  the appropriate discount rate, then the objective function is given by:

$$\min VaR_{5\%} \left[ \sum_{t=1}^{30} \frac{SC_t}{(1+r)^t} \right] \quad (1)$$

Investment decisions for the plan are made collaboratively for all investment accounts. These decisions may be made in the context of an investment committee composed of staff representatives. Such a body is assumed to maximize the expected value of the constant-relative-risk-aversion (CRRA) utility function  $u(PBF)$  with risk-aversion parameter  $\gamma > 0$ .

$$\max E[u(PBF)] = \max E \left[ \left( \frac{PBF^{1-\gamma}}{1-\gamma} \right) \right] \quad (2)$$

Utility is defined over the pension benefit factor  $PBF$  which refers to pension payments per year service expressed as the percentage of final salary at time of retirement. Factor  $PBF$  comprises all simulation runs and all plan members retiring over the thirty-year investment horizon.

### **The Plan Sponsor’s Investment Decision**

We next take the perspective of the plan sponsor, to evaluate the interrelation between asset allocation in the individual pension accounts and the resulting plan costs measured in terms of supplementary contributions by the plan sponsor. Figure 11-1 depicts the worst-case supplementary contributions for Cases I–IV for different portfolio allocations, and Figure 11-2 for Cases V and VI. Worst-case costs are measured as the 5 percent value at risk of the supplementary contributions, i.e. the present value of contributions by the plan sponsor exceeding the regular payments of 17 percent of the salaries. Portfolio allocations are represented by the mean-variance efficient portfolio returns. Details of the cost-optimal asset allocations, including the asset weights for cash, Eurobonds, global, and emerging

market equities appear in Panel 1 of Table 11-3. Panel 2 contains the distributional characteristics of the discounted supplementary contributions for the cost-optimal asset allocations. Finally, Panel 3 reports the pension benefits for these allocations in terms of certainty equivalents.<sup>11</sup> These certainty equivalents are calculated according to the utility function stated earlier and for four different parameters of relative risk aversion.

TABLE 11-3 Optimal Investment Decisions: The Plan Sponsor’s Perspective

	<i>Case I</i>	<i>Case II</i>	<i>Case III</i>	<i>Case IV</i>	<i>Case V</i>	<i>Case VI</i>
<i>Panel 1: Cost-optimal asset allocation (%)</i>						
Mean return	5.57	5.70	5.76	5.63	4.55	5.95
Volatility	4.68	5.23	5.53	4.94	2.17	6.53
Cash	0.00	0.00	0.00	0.00	71.28	0.00
Eurobonds	84.27	79.16	76.61	81.72	28.04	68.95
Global equities	10.73	15.84	18.39	13.28	0.68	26.05
Emerging markets equities	5.00	5.00	5.00	5.00	0.00	5.00
<i>Panel 2: Distributional characteristics of DSC with optimal asset allocation (%)</i>						
Mean DSC	19.64	18.44	22.14	9.24	13.02	7.00
Std. DSC	13.03	12.89	12.60	9.62	8.01	9.17
5%-VaR DSC	42.49	41.50	43.95	27.01	28.04	25.87
25%-Q DSC	10.27	9.04	12.71	0.00	6.95	0.00
50%-Q DSC	17.56	16.12	19.87	6.70	12.15	2.77
75%-Q DSC	26.92	25.65	29.41	15.17	17.96	11.39
<i>Panel 3: Distributional characteristics of PB with optimal asset allocation (%)</i>						
Mean PB	1.875	1.872	2.257	1.793	1.531	1.939
Std. PB	0.045	0.050	0.415	0.102	0.205	0.307
Certainty equivalent ( $\gamma = 1$ )	1.874	1.872	2.223	1.790	1.517	1.915
Certainty equivalent ( $\gamma = 5$ )	1.872	1.869	2.121	1.776	1.470	1.827
Certainty equivalent ( $\gamma = 10$ )	1.869	1.866	2.040	1.750	1.421	1.726

Source: Authors’ calculations.

Notes: Asset weights in percent; DSC (i.e. discounted supplementary contributions): contributions required on top of fixed regular contributions to fully fund the pension plan (in percent of expected discounted regular contributions); PB (i.e. pension benefits): attainable income replacement factor (in percent of final salary per year of service); Gamma ( $\gamma$ ): parameter of risk aversion in a constant relative risk aversion (CRRA-) utility function of the type:  $u(W) = W^{1-\gamma}/(1-\gamma)$ ; Objective function: Minimize the 5 percent VaR of DSC; Q: quantile; Case I: DC + minimum benefits + maximum benefits + capital guarantee; Case II: DC + minimum benefits + maximum benefits; Case III: DC + minimum benefits; Case IV: DC + maximum benefits + capital guarantee; Case V: DC + capital guarantee; Case VI: DC + capital guarantee + 10% cap on asset return.

Focusing first on the benchmark, Figure 11-1 shows that the worst-case plan costs for Case I follow a U-shaped curve. With increasing expected portfolio returns, the costs first decrease and then rise, resulting in minimum supplementary contributions for an asset allocation with an expected return of 5.57 percent. This portfolio consists of about 84 percent bonds and 16 percent equities. The minimum supplementary contributions amount to 43 percent of the expected regular contributions. Hence, for every discounted Euro the sponsor regularly paid into the plan, additional payments of 43 discounted cents are required to cover the costs of the plan. The U-shape of the cost curve can be directly related to the guarantees included in Case I. Investing in portfolios mainly consisting of cash or bonds will result in assets not being able to generate enough return to cover the costs of the guaranteed minimum benefits. These costs have to be borne by the plan sponsor. As the expected return on the portfolio increases, it becomes more and more likely that the funds will suffice to at least pay the minimum pension without further contributions by the plan sponsor. The rise in expected portfolio return is in turn accompanied by an increase in return volatility, which induces costs resulting from falling short of the guaranteed minimum annual asset return of 0 percent. From a certain level of volatility onwards, these newly induced costs overcompensate the cost savings related to the minimum pension benefits and the overall costs increase again.

Changing the structural design of the plan has some interesting effects. Eliminating the annual return guarantee for the individual accounts in Case II cuts the amount of supplementary contributions, especially in the case of a more risky asset allocation. However, the asset allocation which minimizes costs is only slightly different compared to Case I (i.e. about 5 percent less bonds and more equities). The worst-case costs only fall from 43 to 42 percent of regular contributions. This results from the fact that for a low-risk asset allocation the costs from the annual return guarantee are relatively low.<sup>12</sup>

In Case III, not only the capital guarantee but also the maximum pension regulation is eliminated; now the plan is basically DC but the beneficiaries are protected by a DB minimum pension benefit in the event of adverse capital market developments. Therefore, the amount of supplementary contributions increases compared to Case II, as there are no longer funds in excess of those needed to provide maximum pension benefits which could be credited to the plan sponsor. Looking at the cost minimizing asset allocation, the equity exposure is slightly further increased to about 13 percent with overall worst-case supplementary contributions of 44 percent.

Case IV shows a quite different curve. The general U-shape is maintained showing a minimum of supplementary contributions for a portfolio return of 5.6 percent, which corresponds to an asset allocation of 82 percent

bonds and 18 percent equities. While the asset allocation is comparable to Cases I–III, the level of supplementary contributions with 27 percent of regular contributions is substantially lower than in the previous cases. Additionally, the left branch of the cost curve (low-risk portfolios) is nearly flat while the right branch (more risky portfolios) shows a strong increase. Economically, this can be explained as follows. As discussed for Case I, the predominant source of costs, especially when investing in low-risk allocations, is the minimum benefit guarantee. This guarantee is not included in Case IV, leading to substantially lower costs compared to Case I. Increasing the expected return and the volatility of the portfolio now has two opposing effects. The higher the (expected) portfolio return, the more often the plan sponsor will profit from cashing in funds from the individual retirement accounts that exceed the amount necessary to cover the maximum pension benefits. Contrarily, the higher the return volatility, the more often supplementary contributions will be triggered due to the annual capital guarantee. Since the former effect dominates the latter for less risky portfolios, the overall costs first decrease with increasing

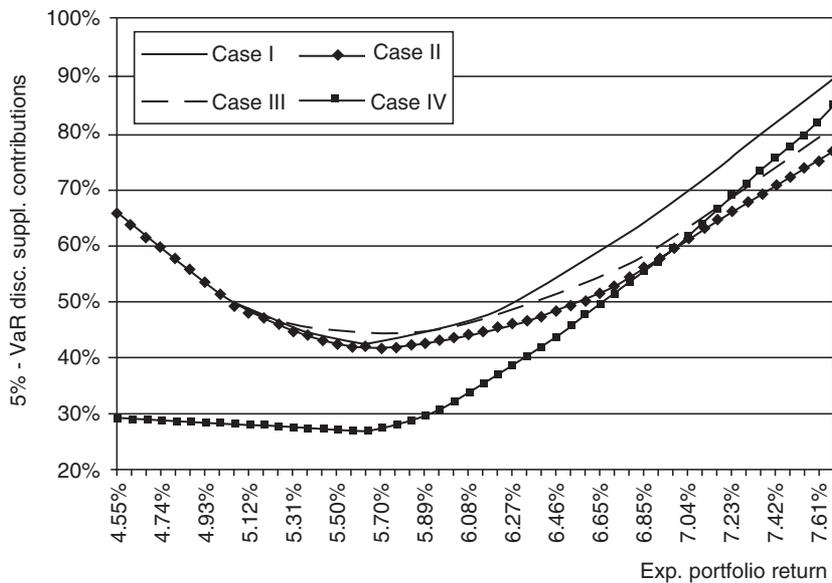


Figure 11-1. Worst-case plan costs vs. asset allocation.

Source: Authors' calculations.

Notes: Discounted supplementary contributions (DSC) in percent of expected discounted regular contributions; Case I: DC + minimum benefits + maximum benefits + capital guarantee; Case II: DC + minimum benefits + maximum benefits; Case III: DC + minimum benefits; Case IV: DC + maximum benefits + capital guarantee.

expected portfolio return. For more risky allocations the latter effect dominates the former, which leads to rapidly growing contributions. As the cost impact of the minimum benefit guarantee is diminishing for increasing portfolio returns, Cases I and IV hardly differ for highly risky portfolios.

We now turn to the cases with no explicit defined minimum or maximum benefit, depicted in Figure 11-2. Case V shows a plan with unlimited upside potential but with a shortfall protection resulting from the annual return guarantee. It is clear that such a plan design results in increasing supplementary contributions, the higher the equity exposure. Hence, minimizing the costs in terms of supplementary contributions leads to the minimum volatility portfolio, consisting of 71 percent cash, 28 percent bonds, and only 1 percent equities. The resulting costs amount to 28 percent of regular contributions.

As in Case V, Case VI offers an annual capital guarantee and therefore protection against return shortfalls. However, the upside potential is limited due to the 10 percent return cap. This structural change has a significant impact on the shape of the cost curve. While the amount of supplementary contributions in Cases V and VI is approximately equal for low-risk asset allocations, the costs in Case VI begins to decrease again

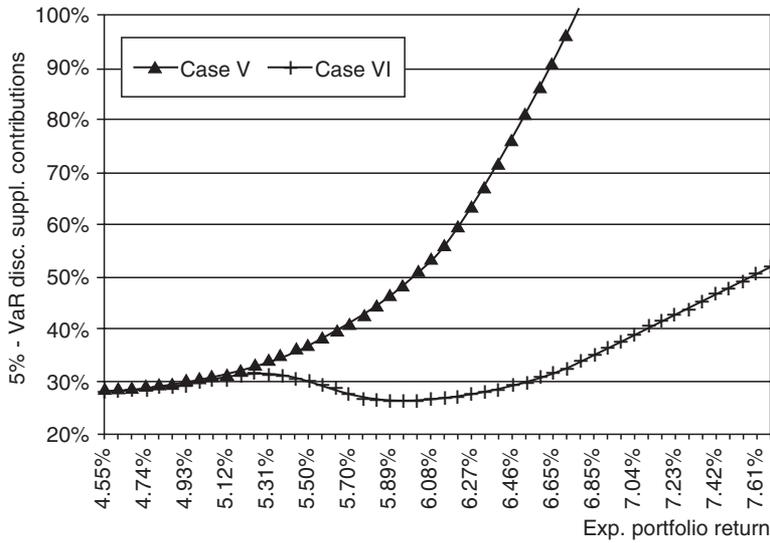


Figure 11-2. Worst-case plan costs vs. asset allocation.

Source: Authors' calculations.

Notes: Discounted supplementary contributions (DSC) in percent of expected discounted regular contributions; Case V: DC + capital guarantee; Case VI: DC + capital guarantee + 10% cap on asset return.

for increasing portfolio return and volatility. For these allocations, the increasing costs resulting from the capital guarantee are overcompensated by the profits the plan sponsor can generate through cashing in any returns that exceed the 10 percent cap. With even more increasing volatility, the costs of the capital guarantee increase disproportionately, leading to overall growing supplementary contributions. With minimum worst-case costs of 26 percent resulting from investing in about 69 percent bonds and 21 percent equities, this case proves to be the cheapest of all plan designs discussed. This holds for the minimum cost asset allocation and especially for all portfolios with high expected returns and volatilities.

**Implications for Plan Beneficiaries.** The implications of investing in the cost-minimizing portfolios for expected pension benefits can now be derived (see Figure 11-3). Panel 3 of Table 11-3 summarizes expected pension benefit factors and their standard deviation, expressing the pension benefits as a percent of final salary per year of service. In order to relate the whole probability distribution of the pension benefit factors to the risk aversion of a representative beneficiary, the pension factor certainty equivalents are calculated for a range of risk-aversion parameters using the standard CRRA utility function. This allows a direct evaluation of the cost-optimal asset allocation for the various plan designs, Cases I–VI, from the perspective of plan members with different levels of risk aversion. Figure 11-3 depicts the certainty equivalents for all parameters of risk aversion from one to ten in half steps. Additionally, numerical results for selected levels of risk aversion ( $\gamma = 1, 5, \text{ and } 10$ ) are presented in Panel 3 of Table 11-3.

The figure shows that Case III results in the highest pension benefit factors for all levels of risk aversion under scrutiny, with the mean benefit factor being 2.257 percent (see also Table 11-3, Panel 3). At the same time, Case V always produces the lowest factors, on average 1.531 percent. This is an interesting result, as both cases show structural similarities. Cases III and V both offer downside protection to the beneficiaries, Case III by means of guaranteed minimum pension benefits, Case V with the annual capital guarantee for the individual accounts. Neither case limits the upside potential.

An explanation for this can be found when looking at the different cost-minimal asset allocations. Optimizing the amount of supplementary contributions in Case V, the plan sponsor will only invest in cash and bonds, resulting in the lowest risk exposure with respect to the capital guarantee. With highly conservative risk and return profile of the assets simultaneously low pension benefits are expected. Such an asset allocation, however, is not appropriate in Case III, since its return expectations are insufficient to cover the costs resulting from the guaranteed minimum benefits, i.e. 2 percent of the career-average salary per year of service.

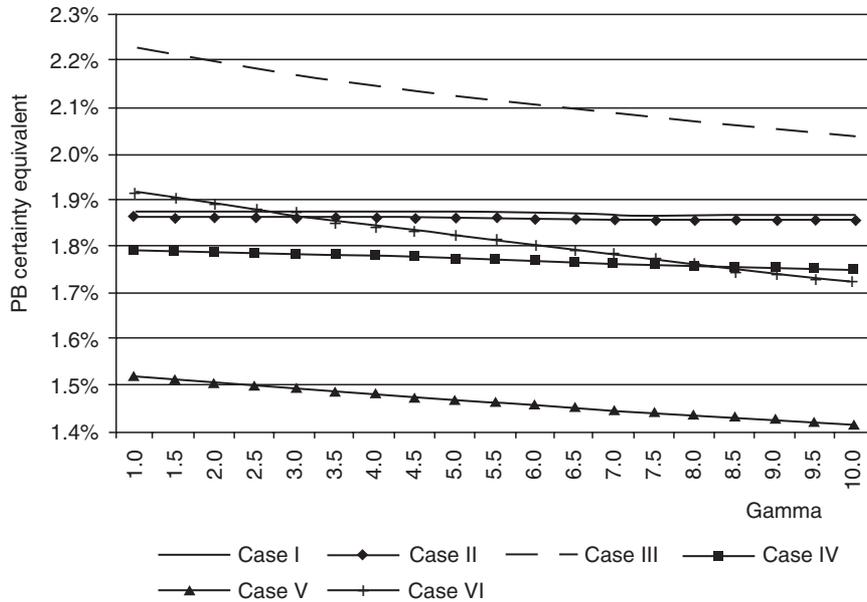


Figure 11-3. Certainty equivalents of pension benefit factors for different plan designs with sponsor’s optimal asset allocation.

Source: Authors’ calculations.

Notes: PB certainty equivalent: certain pension benefit (PB) factor, i.e. retirement income in percent of final salary per year of service, that has the same utility to the beneficiary as the random pension benefit factor provided by the pension plan. Gamma: parameter of risk aversion in a constant relative risk aversion (CRRA-) utility function of the type:  $u(W) = W^{1-\gamma}/(1 - \gamma)$ ; Case I: DC + minimum benefits + maximum benefits + capital guarantee; Case II: DC + minimum benefits + maximum benefits; Case III: DC + minimum benefits; Case IV: DC + maximum benefits + capital guarantee; Case V: DC + capital guarantee; Case VI: DC + capital guarantee + 10% cap on asset return.

Rather, it is necessary to implement a portfolio strategy that offers higher mean returns, coming at the cost of higher volatility. This, in turn, leads to substantially higher supplementary contributions, since the plan sponsor fully bears the downside volatility while only the beneficiaries profit from the upside volatility.

Implementing a maximum benefit cap (2 percent of final salary per year of service) results in considerably reduced volatilities of the pension benefit factors, i.e. 0.045 percent for Case I, 0.05 percent for Case II, and 0.102 percent for Case IV (see Table 11-3, Panel 3). Consequently, the certainty equivalents of the pension benefit factors are nearly constant for the various risk-aversion coefficients reported in Figure 11-3. Among these cases, Case I

offers the highest pension benefits but is also the most costly design. Case II only offers slightly lower benefits combined with slightly lower costs.

In general, it can be concluded that hybrid plans that offer the highest expected pension benefits tend to cause the highest amount of supplementary contributions. Yet there are two exceptions: Case V offers by far the lowest pension benefits, but even given optimal asset allocation patterns, additional costs are not small. By contrast, Case VI has the lowest supplementary contribution, and will lead to expected pensions benefits that exceed all but one other case. The rather high volatility of the pension benefit factor, however, causes the certainty equivalents to drop below those of most other cases for higher levels of risk aversion.

### **Beneficiaries' Investment Decisions**

In this section we assume that the asset allocation decisions are made by the plan participants, rather than the plan sponsor; here, the plan members' objective function is to maximize the expected utility of pension benefits by choosing an appropriate asset allocation. This analysis is undertaken for Cases I–VI and also for Case VII, a pure defined contribution plan. Our interest here is to look at the resulting pension benefits for plan members with different levels of risk aversion, as well as the composition of the optimal asset allocation. For simplicity, we assume that the asset allocation decision made by the beneficiaries and their cost impact have no repercussive effects on plan member salaries; neither will rising supplementary contributions lead to lower salaries/salary increases nor will reductions of plan costs be passed on to the workers.

As earlier, we represent the plan's portfolio allocations by the mean-variance efficient portfolio returns; details of the benefit-optimal investment weights (i.e. the mean and volatility of asset returns, mean and certainty equivalents of pension benefit factors for plan members as well as the resulting costs in terms of supplementary contributions for the plan sponsor) appear in Table 11-4. The first Panel contains the results for a representative plan member with a low coefficient of risk aversion ( $\gamma = 1$ ), while the other panels show findings for a medium ( $\gamma = 5$ ) and a high ( $\gamma = 10$ ) coefficient. Table 11-5 provides details regarding the investment weights.

The results show that, independent of risk aversion, plan beneficiaries would opt to invest in the asset allocation that offers the highest or almost the highest expected return and the highest or almost the highest volatility in Cases I–V. Table 11-5 indicates that the optimal asset allocation consists of 100 percent stocks. This is because beneficiaries are protected against downside volatility of the international equity markets by guaranteed minimum pension benefits and the annual capital guarantee. The value of this downside protection *ceteris paribus* increases with the volatility. By analogy to option pricing theory, the minimum pension benefit and the capital

TABLE 11-4 Optimal Investment Decision: The Plan Participants' Perspective

	<i>Case I</i>	<i>Case II</i>	<i>Case III</i>	<i>Case IV</i>	<i>Case V</i>	<i>Case VI</i>	<i>Case VII</i>
<i>Panel 1: Low level of risk aversion (<math>\gamma = 1</math>) (%)</i>							
Mean return	7.68	7.68	7.68	7.68	7.68	7.68	7.68
Volatility	17.27	17.27	17.27	17.27	17.27	17.27	17.27
Mean DSC	30.91	24.60	29.80	26.58	126.85	14.29	0.00
5%-VaR DSC	87.93	75.71	79.28	84.52	246.62	51.33	0.00
Mean PB	1.974	1.915	3.937	1.949	6.890	1.948	3.793
Certainty equivalent	1.974	1.914	3.355	1.948	6.014	1.918	3.093
<i>Panel 2: Medium level of risk aversion (<math>\gamma = 5</math>) (%)</i>							
Mean return	7.68	7.61	7.68	7.68	7.68	5.89	6.33
Volatility	17.27	16.83	17.27	17.27	17.27	6.18	8.74
Mean DSC	30.91	24.14	29.80	26.58	126.85	7.35	0.00
5%-VaR DSC	87.93	74.14	79.28	84.52	246.62	26.06	0.00
Mean PB	1.974	1.915	3.937	1.949	6.890	1.937	2.543
Certainty equivalent	1.973	1.909	2.449	1.947	4.011	1.827	2.007
<i>Panel 3: High level of risk aversion (<math>\gamma = 10</math>) (%)</i>							
Mean return	7.68	7.61	7.61	7.68	7.68	5.76	5.76
Volatility	17.27	16.83	16.83	17.27	17.27	5.53	5.53
Mean DSC	30.91	24.14	29.22	26.58	126.85	7.98	0.00
5%-VaR DSC	87.93	74.14	77.68	84.52	246.62	26.31	0.00
Mean PB	1.974	1.915	3.847	1.949	6.890	1.930	2.155
Certainty equivalent	1.973	1.902	2.160	1.944	2.990	1.729	1.760

*Source:* Authors' calculations.

*Notes:* DSC (i.e. discounted supplementary contributions): contributions required on top of fixed regular contributions to fully fund the pension plan (in percent of expected discounted regular contributions); PB (i.e. pension benefits): attainable income replacement factor (in percent of final salary per year of service); Gamma ( $\gamma$ ): parameter of risk aversion in a constant relative risk aversion (CRRA-) utility function of the type:  $u(W) = W^{1-\gamma}/(1-\gamma)$ ; Objective function: maximize the expected utility of pension benefits using a CRRA-utility function defined over the pension benefits in percent of final salary per year of service; Case I: DC + minimum benefits + maximum benefits + capital guarantee; Case II: DC + minimum benefits + maximum benefits; Case III: DC + minimum benefits; Case IV: DC + maximum benefits + capital guarantee; Case V: DC + capital guarantee; Case VI: DC + capital guarantee + 10% cap on asset return; Case VII: DC.

guarantee can be interpreted as a call option, for which the value is also positively related to the volatility of the underlying.

Looking at the level of supplementary contributions associated with these asset allocations, it would appear that costs for the plan sponsor would be prohibitively high. This is particularly true for Case V, in which the members' individual accounts are protected against negative fluctuations in the capital markets while at the same time offering full participation in positive returns. Here, the certainty equivalents of the pension

TABLE 11-5 Optimal Asset Allocations: Participant Perspectives

	<i>Case I</i>	<i>Case II</i>	<i>Case III</i>	<i>Case IV</i>	<i>Case V</i>	<i>Case VI</i>	<i>Case VII</i>
<i>Panel 1: Low level of risk aversion (<math>\gamma = 1</math>) (%)</i>							
Cash	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Eurobonds	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Global equities	95.00	95.00	95.00	95.00	95.00	95.00	95.00
EM equities	5.00	5.00	5.00	5.00	5.00	5.00	5.00
<i>Panel 2: Medium level of risk aversion (<math>\gamma = 5</math>) (%)</i>							
Cash	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Eurobonds	0.00	2.55	0.00	0.00	0.00	71.50	53.63
Global equities	95.00	92.45	95.00	95.00	95.00	23.50	41.37
EM equities	5.00	5.00	5.00	5.00	5.00	5.00	5.00
<i>Panel 3: High level of risk aversion (<math>\gamma = 10</math>) (%)</i>							
Cash	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Eurobonds	0.00	2.55	2.55	0.00	0.00	76.61	76.61
Global equities	95.00	92.45	92.45	95.00	95.00	18.39	18.39
EM equities	5.00	5.00	5.00	5.00	5.00	5.00	5.00

Source: Authors' calculations.

Notes: Objective function: maximize the expected utility of pension benefits using a CRRA-utility function defined over the pension benefits in percent of final salary per year of service; Case I: DC + minimum benefits + maximum benefits + capital guarantee; Case II: DC + minimum benefits + maximum benefits; Case III: DC + minimum benefits; Case IV: DC + maximum benefits + capital guarantee; Case V: DC + capital guarantee; Case VI: DC + capital guarantee + 10% cap on asset return; Case VII: DC.

benefit factors vary between 6.014 percent for a low-risk aversion ( $\gamma = 1$ ) and 2.99 percent for a higher risk aversion ( $\gamma = 10$ ). These high pension benefits are associated with worst-case (mean) supplementary contributions of 247 percent (127 percent).

Cases I, II, and IV limit the upside potential available to the beneficiaries by incorporating the maximum pension benefit restriction. This results in lower benefits and lower costs compared to Case V, yet plan members still have the incentive to choose portfolios with very high volatility. Even though the costs are substantially reduced, they are still intolerably high. For example in Case IV, i.e. Case V with incorporated maximum benefit limit, the worst-case (expected) supplementary contributions amount to 85 percent (27 percent), being about three times as high as in case the plan sponsor chooses the asset allocation.

Case VI produces a different picture: now, the annual capital guarantee and a 10 percent cap on the maximum annual asset return is credited to the beneficiaries' individual accounts. Beneficiaries with a low level of risk aversion ( $\gamma = 1$ ) will still invest in the maximum expected return/maximum volatility portfolio, but more risk-averse plan members will choose an

asset allocation with substantially reduced exposure to capital market risk. For a medium (high) level of risk aversion ( $\gamma = 5$  vs. 10), the allocation to bonds will increase from none (at  $\gamma = 1$ ), to 72 (79) percent (see Table 11-5). We compare this benefit optimal asset allocation from the members' perspective (with moderate-to-high risk aversion) to the cost optimal asset allocation from the sponsor's perspective in Panel 1 of Table 11-3. It is interesting that from both perspectives, the optimal investment strategy is nearly identical—to have high exposure to bonds and low exposure to equities. This results in quite similar cost implication in terms of supplementary contributions. If the plan sponsor were to set the asset allocation, the 5 percent Value-at-Risk of supplementary contributions would be 26 percent (Panel 2 of Table 11-3); if the representative member with a medium- or high-risk aversion selected the optimal asset allocation for him, this would result in supplementary contributions for the plan sponsor of 26 percent. Hence, if the benefit structure of the pension plan is designed according to Case VI, this will lead to 'harmony' of the members' and sponsor's interests, at least with respect to the asset allocation decision for a given plan design.

Case VI might seem to be most suitable for a hybrid pension plan, as it combines acceptable cost consequences for the sponsor and attractive pension benefit factors for the plan members. Nevertheless, a superior plan design exists. Case VII is a pure defined contribution plan with no capital guarantee and no return cap. By construction, this plan causes no additional costs in term of supplementary contributions for the sponsor. Additionally, as shown in Table 11-4, a pure DC plan provides higher mean pension benefits as well as certainty equivalents for all levels of risk aversion. This result is due to the specification of the floor/cap structure, i.e. a minimum rate of return of 0 percent per year and a return maximum of 10 percent per year. Setting the cap to an annual return of 12.5 percent and leaving the floor constant at 0 percent leads to the following results: For members with a low level of risk aversion ( $\gamma = 1$ ), the certainty equivalent for the benefit factor of the hybrid plan is still lower than in the case of a pure DC plan (2 percent compared to 3 percent). For members with medium to high levels of risk aversion ( $\gamma = 5$  or 10), the hybrid plan is more attractive than the pure defined contribution plan.<sup>13</sup> Yet when increasing the cap to 12.5 percent, the plan members will again choose the maximum volatility portfolio (i.e. 100 percent equities), independent of their level of risk aversion. Unfortunately this will cause unacceptable costs in terms of supplementary contributions for the plan sponsor.

## **Conclusions**

This chapter evaluates key properties of hypothetical hybrid pension plans in terms of their cost consequences, by adapting a pure defined

contribution scheme to include minimum and maximum limits for pension benefits, as well as minimum guarantees and return caps on individual investment accounts. We also explore optimal investment strategies from the perspectives of both plan sponsor and beneficiaries. We find that introducing DB elements substantially increases the overall costs of running the pension plan and has a major impact on the resulting optimal portfolios.

The investment strategy chosen and the additional plan costs show strong interrelationship. If only minimum rate of return guarantees are included in the plan design, additional costs increase exponentially as a function of higher expected asset return and volatility. Consequently, plan sponsors choose the minimum risk portfolio consisting of around 70 percent cash, around 30 percent bonds, and virtually no equities. Enhancing this plan with a cap on returns credited to the individual accounts leads to a U-shaped cost curve for a broad range of possible asset allocations. Here, the optimal portfolio consists of about 69 percent bonds and 31 percent equities, i.e. about 50 percent more equities than for any other plan design optimized from the sponsor's perspective. At the same time, with this design the additional costs are reduced to 26 percent from 28 percent in the case without the return cap.

For plan designs that guarantee minimum pension benefits, the implied additional costs (expected and worst-case values) are also U-shaped as a function of expected investment returns. Therefore, assuming the objective to minimize the worst-case value of additional costs, the sponsor will opt for asset allocations which deviate from the minimum risk allocation as well. These portfolios comprise between 77 and 84 percent bonds and between 16 and 23 percent equities. The additional costs for these plans lie in the range of 42 and 44 percent of regular contributions, i.e. about 50 percent above the costs of a plan only guaranteeing a minimum rate of return.

Taking beneficiaries' perspective, we also evaluate the utility implications of alternative DB elements and sponsor pension fund asset allocation decisions. To this end, certainty equivalents of pension benefits are calculated for a range of risk aversion parameters. Generally higher additional costs imply higher expected pension benefits, but the introduction of caps on credited asset returns allows cost reductions with only slightly lower certainty equivalents of random pension benefits. We also evaluate optimal investment choices from the beneficiaries' perspective, and we find that almost independent of risk aversion, plan members tend to select maximum return, maximum risk asset allocations where the plan either guarantees minimum pension benefits or minimum return guarantees. However, if the minimum rate of return guarantees are combined with a cap on the maximum return credited to individual accounts, risk-averse members opt for less risky asset allocations. In this case, the optimal asset allocation includes about 72 percent in bonds and 28 percent in equities.

Our results are directly relevant to the moral hazard problem faced by agencies' insurance pension plan defaults, including the PBGC in the USA, and the newly established Pension Protection Fund in the UK (see, e.g. Coronado and Liang 2006; McCarthy and Neuberger 2006; Warshawsky et al. 2006). Like the plan sponsor in this chapter, those organizations issue a put option on the value of the assets invested in the insured pension plans. They therefore should be interested in rather conservative pension fund asset allocations mainly concentrated in bonds. If, as for the beneficiaries in this chapter, the price of such an option (i.e. the insurance premium) is set independently of its value, and if the insured party can influence the value, there is a chance that the insured party will seek to boost the probability of exercising the option—by investing in high-risk assets or by underfunding the pension plan. A possible solution to this moral hazard problem is to implement funding requirements that take into account both current level of funding as well as investment risk, as for example is done for the German Individual Investment Accounts ('Riester' accounts; cf. Maurer and Schlag 2004).

The analysis presented in this study can be useful when discussing possible designs of hybrid pension plans. Some plan designs appear to be Pareto-inefficient (e.g. minimum and maximum pension benefits in combination with minimum rate of return guarantee) as they are dominated by others which imply lower additional costs and higher expected utility for plan members. Furthermore, if plan sponsors and beneficiaries are jointly responsible for investment decision, caps on investment returns may reduce conflicts of interest as asset allocations will diverge less between the parties.

## **Endnotes**

1. Pension promises in the USA have traditionally been either of the pure DB or pure DC type (Schieber 2003). In a DB scheme, the plan sponsor promises to the plan beneficiaries a final level of pension benefits. This level is usually defined according to a benefit formula, as a function of salary trajectory and years of service. Benefits are usually paid as a life annuity rather than as a lump sum. As Bodie et al. (1988) note, the foremost advantage of a DB plan is that it offers stable income replacement rates to retired beneficiaries. The major drawbacks of DB schemes include the lack of benefit portability when leaving the company and the complex valuation of plan liabilities. Moreover, the plan sponsor is exposed to substantial investment and longevity risk, which could result in significant contribution expenses. In a DC scheme, by contrast, the plan sponsor commits to paying funds into the beneficiaries' individual accounts according to a specified formula, e.g. a fixed percentage of annual salary. The most prominent feature of a DC scheme is its inherent flexibility: by construction, it is fully funded in individual accounts. The value of the pension benefits is simply determined as the market value of the backing assets. Therefore, the pension

benefits are easily portable in case of job change. Additionally, the beneficiaries have control over their funds' investment strategy and at retirement can usually take the money as a life annuity, a phased withdrawal plan, a lump sum payment, or some combination of these. While the employer is only obliged to make regular contributions, the employee bears the risk of uncertain replacement rates, especially caused by fluctuations in the capital markets (Bodie and Merton 1992).

2. An in-depth discussion of the implications of introducing hybrid pension plans in the USA can—among others—be found in Clark and Schieber (2004), Coronado and Copeland (2004), Johnson and Steuerle (2004), and Mitchell and Mulvey (2004).
3. The European Central Bank (ECB) operates a hybrid pension scheme; plan assets, which exist solely for the purpose of providing benefits for members of the plan and their dependents, are included in the other assets of the ECB. Benefits payable, resulting from the ECB's contributions, have minimum guarantees underpinning the DC benefits.
4. For example, we do not handle dependent benefits and we assume a simplified population model.
5. A contribution rate of 17 percent can be considered as reasonable assumption given the typical structure of European pension plans. For example, in Germany, contributions to the state-run pay-as-you-go pension system currently amount to 19.5 percent of salaries. As provisions for dependents' pensions are neglected in this study, reducing the contribution rate by 2.5 percent compared to the German state pension system seems a reasonable assumption.
6. Alternatively to a focus on absolute return, a minimum fixed rate of return guarantee could be applied to a relative rate of return. For example, Chile's private pension funds were long required to earn an annual real rate of return that depended on the average annual real rate of return earned by all of Chile's private pension funds (Pennacchi 1999). Or the guarantee may be applied to the account balance at the time of retirement, instead of the assumed annual basis.
7. As is typical for public employees, the wage path until retirement is nondecreasing and so the guaranteed minimum pension benefits will always be lower than the maximum pension benefit limit.
8. This link between assets and liabilities is in contrast to the analogy developed by Bodie and Davis (2000), who compare a pension plan to an equipment trust such as those set up by an airline to finance the purchase of airplanes. Here the equipment serves as specific collateral for the associated debt obligation. The borrowing firm's liability is not affected by the value of the collateral. So, for instance, if the market value of the equipment were to double, this would greatly increase the security of the promised payments, but it would not increase their size. As opposed to this scenario, in the scheme developed in this chapter, the value of the assets may well affect the liabilities as a high return in a given year may increase the value of the liabilities as outlined above.
9. A drawback of the Ornstein/Uhlenbeck process is the theoretically positive probability of negative nominal interest rates, but this is eliminated in the simulation procedure by cutting off the negative nominal interest rates.

10. The EIU forecasts are constructed with the aid of an econometric world model, maintained by the UK-based Oxford Economic Forecasting.
11. The certainty equivalent of a lottery is defined as the fixed payment that provides the same utility as the random lottery.
12. Analyzing the costs of Individual Account guarantees, Lachance and Mitchell (2004) argue that guarantee costs tend to be insensitive to the asset allocation in cases where the exercise of the guarantees is either extremely likely or extremely unlikely.
13. The certainty equivalents are 2.100 percent compared to 2.007 percent for  $\gamma = 5$ , and 1.913 percent compared to 1.760 percent for  $\gamma = 10$ .

## References

- Bacinello, Anna R. (2000). 'Valuation of Contingent-Claims Characterizing Particular Pension Schemes', *Insurance: Mathematics and Economics*, 27: 177–88.
- Bernadell, Carlos, Joachim Coche, and Ken Nyholm (2005). 'Yield Curve Prediction for the Strategic Investor', Working Paper, European Central Bank, forthcoming.
- Bodie, Zvi and E. Philip Davis (2000). *The Foundations of Pension Finance*. Cheltenham, UK: Edward Elgar, p. xvi.
- and Robert C. Merton (1992). 'Pension Benefit Guarantees in the United States: A Functional Analysis', in Raymond Schmitt (ed.), *The Future of Pensions in the United States*. Philadelphia, PA: University of Pennsylvania Press, pp. 194–234.
- Alan J. Marcus, and Robert C. Merton (1988). 'Defined Benefit versus Defined Contribution Pension Plans: What Are the Real Tradeoffs?' in Zvi Bodie, John B. Shoven, and David A. Wise (eds.), *Pensions in the U.S. Economy*. Chicago, IL: University of Chicago Press, pp. 139–62.
- Clark, Robert and Sylvester J. Schieber (2004). 'Adopting Cash Balance Pension Plans: Implications and Issues', *The Journal of Pension Economics and Finance*, 3(3): 271–95.
- Coronado, Julia L. and Nellie Liang (2006). 'The Influence of PBGC Insurance on Pension Fund Finances', this volume.
- and Philip C. Copeland (2004). 'Cash Balance Pension Plan Conversions and the New Economy', *The Journal of Pension Economics and Finance*, 3(3): 297–314.
- Johnson, Richard W. and Eugene Steuerle (2004). 'Promoting Work at Older Ages: The Role of Hybrid Pension Plans in an Aging Population', *The Journal of Pension Economics and Finance*, 3(3): 315–37.
- Lachance, Marie-Eve and Olivia S. Mitchell (2004). 'Understanding Individual Account Guarantees', in Olivia S. Mitchell and Kent Smetters (eds.), *The Pension Challenge. Risk Transfers and Retirement Income Security*. Oxford: Oxford University Press, pp. 159–86.
- Maurer, Raimond and Christian Schlag (2004). 'Money-Back Guarantees in Individual Account Pensions: Evidence from the German Pension Reform', in Olivia S. Mitchell and Kent Smetters (eds.), *The Pension Challenge. Risk Transfers and Retirement Income Security*. Oxford University Press, pp. 187–213.
- McCarthy, David and Anthony Neuberger (2006). 'The UK Approach to Insuring Defined Benefit Pension Plans', this volume.

- Mitchell, Olivia S. and Janemarie Mulvey (2004). 'Potential Implications of Mandating Choice in Corporate Defined Benefit Plans', *The Journal of Pension Economics and Finance*, 3(3): 339–54.
- Pennacchi, George G. (1999). 'The Value of Guarantees on Pension Fund Returns', *The Journal of Risk and Insurance*, 66(2): 219–37.
- Pesando, James E. (1996). 'The Government's Role in Insuring Pensions', in Zvi Bodie, Olivia S. Mitchell, and John A. Turner (eds.), *Securing Employer Based Pensions: An International Perspective*. Philadelphia, PA: University of Pennsylvania Press, pp. 286–305.
- Sylvester J. Schieber (2003). 'A Symposium on Cash Balance Pensions: Background and Introduction', Pension Research Council Working Paper, Wharton School, <http://rider.wharton.upenn.edu/~prc/wpcashbalance.html>
- Warshawsky, Mark J., Neal McCall, and John D. Worth (2006). 'Regulating Single Employer Defined Benefit Plans: A Modern Approach', this volume.