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Critical Properties of Spin-Glasses

A. Brooks Harris

University of Pennsylvania, harris@sas.upenn.edu

Thomas C. Lubensky

University of Pennsylvania, tom@physics.upenn.edu

Jing-Huei Chen

University of Pennsylvania

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Abstract

The critical properties of the model of a spin-glass proposed by Edwards and Anderson are studied using the renormalization group. The critical exponents are calculated in $6-\epsilon$ spatial dimensions. It is argued that a tricritical point can exist where the nonordering field is the skewness of the distribution of J .

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Critical Properties of Spin-Glasses*

A. B. Harris, T. C. Lubensky,† Jing-Huei Chen

Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19174

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The critical properties of the model of a spin-glass proposed by Edwards and Anderson are studied using the renormalization group. The critical exponents are calculated in $6 - \epsilon$ spatial dimensions. It is argued that a tricritical point can exist where the nonordering field is the skewness of the distribution of J .

Although spin-glasses, such as dilute solutions of Mn in Cu,^{1,2} have been studied experimentally for many years, only recently have formulations been given in terms of a microscopic Hamiltonian.³⁻⁶ Even so, the spin-glass transition has not been successfully related to the usual picture of phase transitions as we shall do here. As in Refs. 3-6 we consider the spin Hamiltonian, $\mathcal{H}$, given by

$$\mathcal{H}/kT = - \sum_{\mathbf{r}, \mathbf{r}'} K(\mathbf{r}, \mathbf{r}') \tilde{S}(\mathbf{r}) \cdot \tilde{S}(\mathbf{r}'), \quad (1)$$

where $\tilde{S}(\mathbf{r}) = S_1(\mathbf{r}), S_2(\mathbf{r}), \dots, S_m(\mathbf{r})$ is a classical m -component spin of unit magnitude at the lattice point $\mathbf{r}$, and $K(\mathbf{r}, \mathbf{r}') = J(\mathbf{r}, \mathbf{r}')/kT$, where $J(\mathbf{r}, \mathbf{r}')$ is a random variable with a probability distribution $P(\mathbf{r}, \mathbf{r}'; J)$, and $J(\mathbf{r}, \mathbf{r}')$ is assumed to be a finite-ranged interaction. We treat a quenched random system where the average free energy is calculated as the average, denoted $[\]_{av}$, over all configurations of $J(\mathbf{r}, \mathbf{r}')$:

$$F = [F(\{J\})]_{av}. \quad (2)$$

According to mean-field theory one expects a ferromagnetic or antiferromagnetic state if $[J(\mathbf{r}, \mathbf{r}')]_{av}$ is sufficiently large in magnitude. If $[J(\mathbf{r}, \mathbf{r}')]_{av}$ is zero, Edwards and Anderson (EA)³ argue that there will still be a transition at a freezing temperature T_f to an ordered state characterized by a new order parameter,

$$q(\mathbf{r}) = [\langle \tilde{S}(\mathbf{r}) \rangle_{\{J\}} \cdot \langle \tilde{S}(\mathbf{r}) \rangle_{\{J\}}]_{av}, \quad (3)$$

where $\langle \tilde{S}(\mathbf{r}) \rangle_{\{J\}}$ is the thermal average of $\tilde{S}(\mathbf{r})$ for a given configuration $\{J\}$. Note that q is by definition a positive quantity. This will be important in what follows. EA calculate the properties of this spin-glass phase transition using mean-field theory and a Gaussian random distribution of J 's centered about $[J(\mathbf{r}, \mathbf{r}')]_{av} = 0$. They find a continuous transition with an order-parameter exponent of $\beta = 1$ and a finite discontinuity in the slope of the specific heat, $dC(T)/dT$, at $T = T_f$, so that $\alpha = -1$. Similar results were found by other more detailed calculations.⁴⁻⁶ A straightforward generalization of the EA treatment to in-

clude an external field conjugate to the order parameter yields susceptibility and correlation-length exponents $\gamma=1$ and $\nu=\frac{1}{2}$. These exponents should be valid for spatial dimensionality, d , greater than a critical value, d_c . The value of d_c may be determined as the value of d for which the scaling relation $2\beta+\gamma=d_c\nu$ is satisfied by the mean-field values of the exponents given above. Thus $d_c=6$ for the spin-glass problem. The same argument was used by Toulouse⁷ to correctly predict $d_c=6$ for the percolation problem⁸ and by other authors in connection with similar random systems.^{9,10} Deviations from mean-field theory occur for $d < d_c$ and are of order $d_c - d = \epsilon$ for $d \rightarrow d_c^-$.

In this paper we will use the renormalization group^{11,12} (RG) to analyze the spin-glass transition. As predicted above, this analysis will lead to an ϵ expansion in $6-\epsilon$ dimensions. We begin with a Hamiltonian of Eq. (1) with only nearest-neighbor interactions $K(\vec{r}, \vec{r}+\vec{\delta})$. As is convenient in treating random systems,³ we evaluate the partition function of the system replicated n times:

$$Z^{(n)} = \int d\{K\} P(\{K\}) \text{Tr}_s \exp(-\mathcal{H}^{(n)}/kT), \quad (4)$$

where

$$\frac{\mathcal{H}^{(n)}}{kT} = - \sum_{\alpha=1}^n \sum_{\vec{r}, \vec{\delta}} K(\vec{r}, \vec{r}+\vec{\delta}) \vec{S}^\alpha(\vec{r}) \cdot \vec{S}^\alpha(\vec{r}+\vec{\delta}) \quad (5)$$

and where Tr_s is a trace over all spin variables. The free energy of Eq. (2) is then given by

$$F = \lim_{n \rightarrow 0} (-kT/n) \ln Z^{(n)}.$$

The integral over $\{K\}$ in Eq. (4) can be carried out formally for independent interactions and the result is

$$Z^{(n)} = \text{Tr}_s e^{-\tilde{\mathcal{H}}/kT}, \quad (6a)$$

$$\frac{\tilde{\mathcal{H}}}{kT} = - \sum_{k=1}^{\infty} \sum_{\vec{r}, \vec{\delta}} C_k \left[\sum_{\alpha=1}^n \vec{S}^\alpha(\vec{r}) \cdot \vec{S}^\alpha(\vec{r}+\vec{\delta}) \right]^k, \quad (6b)$$

where C_k is the k th cumulant of the distribution $P(K)$: $C_1 = [K]_{\text{av}}$, $C_2 = [K^2]_{\text{av}} - [K]_{\text{av}}^2$.

For $C_1 \neq 0$, the model described above has been used to study the critical properties of dilute quenched random systems.¹³ We now consider the spin-glass in which $C_1 = 0$. For simplicity, we consider first the case of a Gaussian random distribution for which $C_k = 0$ for all $k > 2$. In this case Eq. (6b) simplifies and $\tilde{\mathcal{H}}/kT$ becomes

$$\sum_{\vec{r}, \vec{\delta}} C_2 \sum_{\alpha \neq \beta} \sum_{i,j} Q_{ij}^{\alpha\beta}(\vec{r}) Q_{ij}^{\alpha\beta}(\vec{r}+\vec{\delta}) + \sum_{\vec{r}, \vec{\delta}} C_2 \sum_{\alpha} [\vec{S}^\alpha(\vec{r}) \cdot \vec{S}^\alpha(\vec{r}+\vec{\delta})]^2, \quad (7)$$

where $\tilde{Q} \equiv Q_{ij}^{\alpha\beta}(\vec{r}) = S_i^\alpha(\vec{r}) S_j^\beta(\vec{r})(1 - \delta_{\alpha\beta})$. In the limit $n \rightarrow 0$ the order parameter $\langle Q_{ij}^{\alpha\beta} \rangle$ is directly related to the order parameter q of EA:

$$q = \sum_{i=1}^m \langle Q_{ii}^{\alpha\beta} \rangle, \quad \alpha \neq \beta. \quad (8)$$

To construct a field-theoretic formulation we need to express $Z^{(n)}$ in terms of the tensor $\tilde{Q}$ which orders at the spin-glass transition. Accordingly, we introduce a probability distribution for $\tilde{Q}$ via

$$P(\{\tilde{Q}\}) = \text{Tr}_s \prod_{\vec{r}, \alpha, \beta} \delta(\tilde{Q}^{\alpha\beta}(\vec{r}) - \vec{S}^\alpha(\vec{r}) \cdot \vec{S}^\beta(\vec{r})) \exp\{C_2 \sum_{\vec{r}, \vec{\delta}} \sum_{\alpha} [\vec{S}^\alpha(\vec{r}) \cdot \vec{S}^\alpha(\vec{r}+\vec{\delta})]^2\}. \quad (9)$$

The trace over $\vec{S}$ in Eq. (9) could be performed explicitly. However, since we wish to develop a continuum theory, we observe that the following form for $P(\tilde{Q}(\vec{r}))$ will reproduce the constraint that $Q_{ij}^{\alpha\beta}(\vec{r})$ be obtained from spins of unit length according to the definition below Eq. (7):

$$P(\tilde{Q}(\vec{r})) \sim \exp[a \text{Tr} \tilde{Q}^2 + w \text{Tr} \tilde{Q}^3 - b (\text{Tr} \tilde{Q}^2)^2], \quad (10)$$

where

$$\text{Tr} \tilde{Q}^2 = \sum_{\alpha, \beta, i, j} Q_{ij}^{\alpha\beta} Q_{ji}^{\beta\alpha},$$

etc. This equation does not include nonlocal effects present in Eq. (9). These will contribute to the $\text{Tr}(\nabla \tilde{Q})^2$ term in Eq. (11b). Following the approach used by Wilson and Kogut¹⁴ for the Ising model, we note that if a and b tend to infinity in the appropriate ratio, then the normalization of $\text{Tr} \tilde{Q}^2$ is fixed. Furthermore, for fixed $\text{Tr} \tilde{Q}^2 = n(n-1)$, $\text{Tr} \tilde{Q}^3$ is a maximum when $Q_{ij}^{\alpha\beta}$ is of the form $S_i^\alpha S_j^\beta$ with $\vec{S}^\alpha \cdot \vec{S}^\alpha = 1$. Thus in the limit when w , a , and b become infinite, in the appropriate way Eq. (10) is valid. A similar scheme has been used by Priest and Lubensky¹⁵ for generalizations of the Potts model. Us-

ing Eq. (10) and taking the continuum limit we obtain

$$Z^{(n)} = \int \mathcal{D}\mathbf{Q} e^{-\mathcal{H}} \quad (11a)$$

$$\mathcal{H} = \int d^d \mathbf{r} \left[\frac{1}{4} r \text{Tr} \tilde{\mathbf{Q}}^2 + \frac{1}{4} \text{Tr} (\nabla \tilde{\mathbf{Q}})^2 - w \text{Tr} \tilde{\mathbf{Q}}^3 + u (\text{Tr} \tilde{\mathbf{Q}}^2)^2 + \sum_i v_i F_i(\tilde{\mathbf{Q}}) \right], \quad (11b)$$

where $r \sim T - T^*$, and $T^* \sim [(\delta J)^2]_{\text{av}}^{1/2}$ is the mean-field transition temperature. The $F_i(\tilde{\mathbf{Q}})$ are the fourth-order invariants other than $(\text{Tr} \tilde{\mathbf{Q}}^2)^2$ which are generated by repeated iterations of the RG. For the $m=1$, Ising case $F_1(\tilde{\mathbf{Q}}) = \text{Tr} \tilde{\mathbf{Q}}^4$, $F_2(\tilde{\mathbf{Q}}) = \sum_{\alpha\beta\delta} (Q_{11}^{\alpha\beta})^2 (Q_{11}^{\alpha\delta})^2$, and $F_3(\tilde{\mathbf{Q}}) = \sum_{\alpha\beta} (Q_{11}^{\alpha\beta})^4$. For $m \geq 2$ there are many F_i 's. The Hamiltonian of Eq. (11b) may also be used when $P(J)$ is non-Gaussian provided $[J]_{\text{av}} = 0$. In this case the initial values of the potentials will depend on the details of $P(J)$.

The mean-field minima of Eq. (11b) with $v_i = 0$ can be located by setting $Q_{ij}^{\alpha\beta} = m^{-1} q \delta_{ij} I_{\alpha\beta}$, where $I_{\alpha\beta}$ is a symmetric off-diagonal tensor with unit entries. Then we have

$$\frac{m^2 \mathcal{H}}{n(n-1)\Omega kT} = \frac{1}{4} m r q^2 - w(n-2)q^3 + n(n-1)uq^4, \quad (12)$$

where Ω is the volume. The extrapolation of this mean-field Hamiltonian into the regime $n < 1$ is ambiguous, as may also be the case in Ref. 6. There is, however, no ambiguity for $n > 1$. We, therefore, calculate physical quantities such as susceptibilities or specific heats, etc., for $1 < n < 2$ and analytically continue the results to $n=0$. In so doing, no anomalies as a function of n are encountered. For instance, consider the calculation of the order parameter. Remembering that q must be positive, we see^{8,15} that Eq. (12) predicts a first-order transition whenever $(n-2)w > 0$ and a second-order transition when $(n-2)w < 0$. For $w > 0$ and $n < 2$ one has a second-order transition with $q = mr/[6(n-2)w]$ and we believe this result can be extended to $n=0$.

We now discuss the ϵ expansion in $6-\epsilon$ dimensions. The recursion relations are obtained in the standard way and in the notation of Ref. 12 are

$$r' = b^{2-\eta} \{ r - 36(n-2)mw^2[A(0) - 2K_6 r \ln b] \}, \quad (13a)$$

$$w' = b^{\epsilon/2-3\eta/2} \{ w + 36[(n-3)m+1]w^3 K_6 \ln b \}, \quad (13b)$$

$$\eta = 12(n-2)mw^2 K_6. \quad (13c)$$

These equations have a stable fixed point with $(w^*)^2 = -\epsilon \{ 36K_6[(n-4)m+2] \}^{-1}$ whenever $n < 4$

$-(2/m)$. The exponents for $n \rightarrow 0$ are listed in Table I.

An interesting possibility exists if $P(J)$ is not symmetric in J . If $[J^3]_{\text{av}}$ is nonvanishing, there will be an extra term in the Hamiltonian proportional to $-[J^3]_{\text{av}} \text{Tr} \tilde{\mathbf{Q}}^3$. Thus, if the distribution is sufficiently skewed to the antiferromagnetic side, the sign of w in Eq. (11) can change. In this case, there would be a first-order transition within mean-field theory for $n < 2$. Thus the point $w=0$ would correspond to a tricritical point with the skewness of the distribution acting as the non-ordering field. The tricritical exponents can be determined from the recursion relations in $4-\epsilon$ dimensions with $w=0$. To first order in ϵ they are of the form

$$u' = b^\epsilon [u - 128u^2 K_d \ln b - \sum_i A_i u v_i - \sum_{ij} B_{ij} v_i v_j], \quad (14a)$$

$$v_i' = b^\epsilon [v_i - 192u v_i K_d \ln b - \sum_{ij} C_{ij} v_i v_j], \quad (14b)$$

$$r' = b^2 [r - 32ru K_d \ln b + Du], \quad (14c)$$

where A_i , B_{ij} , C_{ij} , and D are constants of order $\ln b$. Thus, at the "Heisenberg" fixed point (HFP) with $u^* = \epsilon/(128K_d)$, $v_i^* = 0$ for all i , one has $\lambda_u = -\epsilon$, $\lambda(v_i) = -\frac{1}{2}\epsilon$, for all i , so that the HFP is stable. The other fixed points are assumed to be unstable or unphysical. Also, to lowest order in ϵ near the HFP, one has

$$w' = b^{1+\epsilon/2} w [1 - 96K_d u \ln b], \quad (15)$$

from which one can determine the crossover exponent φ_w defined so that the free energy depends on the variable w as w/r^{φ_w} . The tricritical ex-

TABLE I. Values of exponents correct to first order in ϵ . Other exponents are obtained by $\alpha = 2 - d\nu$, $\beta = \frac{1}{2}(d + \eta - 2)\nu$, $\gamma = (2 - \eta)\nu$.

Exponent	Critical point at $d = 6 - \epsilon$	Tricritical point at $d = 4 - \epsilon$
ν	$\frac{1}{2} + 5m\epsilon/12(2m-1)$	$\frac{1}{2} - \epsilon/16$
η	$-m\epsilon/3(2m-1)$	0
φ_w		$\frac{1}{2} - \frac{3}{16}\epsilon$

ponents deduced from Eqs. (14) and (15) are listed in Table I.

There are other cooperative phenomena which must be re-examined in light of the possible existence of the spin-glass state. In particular we are now considering the competition between ferromagnetic or antiferromagnetic ordering and spin-glass ordering that will occur at some critical value of $[J]_{av}$. We are also considering the possibility of other types of spin-glass ordering that may occur in other types of random systems, e.g., systems with random uniaxial anisotropy.¹⁶

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Temperature Dependence of Electric Field Gradients in Noncubic Metals*

P. Jena

Department of Physics, Northwestern University, Evanston, Illinois 60201

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Temperature dependence of the nuclear quadrupole frequency, ν_Q , of noncubic metals has been studied theoretically. It is shown that the electronic contribution to the field gradient is largely responsible for the observed $T^{3/2}$ behavior. The conclusions are general and apply to all noncubic metals.

The study of electric field gradients, eq , in metals is of great importance since it not only provides a detailed knowledge of the electronic wave functions in the occupied Fermi volume, but can also yield valuable information regarding the nuclear quadrupole moment, Q . Experiments using a variety of probes, such as nuclear magnetic resonance, time-differential perturbed angular correlation, and Mössbauer effect, have been performed on both pure noncubic metals and alloys to study the distribution of eq . In several systems the sign of the nuclear quadrupole coupling, $\nu_Q = e^2qQ/h$, has also been determined.

Recently the temperature dependence of ν_Q of several metals, such as Cd,^{1,2} Zn,³ In,⁴ Sb,⁵ and Ga,⁶ has been studied experimentally. An analysis of these results reveals the interesting feature that ν_Q generally decreases⁷ as $T^{3/2}$ for all these metals, namely,

$$\nu_Q \simeq \nu_Q^0 (1 - \alpha T^{3/2}), \quad (1)$$

where ν_Q^0 is the value of the nuclear quadrupole frequency at $T = 0^\circ\text{K}$ and α is a constant. Since the electronic structures of all these metals are very different from each other, this "universal" form of the temperature dependence suggests that