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Keywords
forest planning, scenario tree, stochastic integer programming, branch-and-fix coordination

Disciplines
Business | Business Administration, Management, and Operations | Business Analytics | Business and Corporate Communications | Management Information Systems | Operations and Supply Chain Management

Forestry Management under uncertainty*

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April 5, 2009

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1 Introduction

The paper considers a problem of timber harvesting and road building under uncertainty in Chile. The forest industry is Chile's second largest source of exports, surpassed only by copper mining. According to data from INFOR (Instituto de Investigación Forestal de Chile - the Chilean Institute of Forest Research), forestry exports totaled US 3.1 billion in 2004, a growth of 28% over the previous year. Such a figure confirms the magnitude of the industry and underlines the importance of providing its planners with efficient decision-making tools. Forest companies must plan the sustainable harvest of their resources over a given time horizon. Cut timber is then sold in a specific market to meet demand. The main objective of the companies is to maximize profit while complying with environmental regulations and preserving the quality of life of their workers. One of the main difficulties encountered in planning harvesting operations is the stochasticity of future timber sale prices. The problem is therefore a Stochastic Programming (SP) problem (see [4] for a good book on this subject). Often the problems require using binary variables, thus Stochastic Integer Programming (SIP) approaches must be used. See [11] for a survey on the subject, see also [12, 13] for books describing algorithms and presenting applications.

The paper presents a multistage SIP-based mixed 0-1 model for timber harvest and road construction under uncertainty. The approach is validated using a real-life instance. A Branch-and-Fix Coordination (BFC) approach is used for solving the problem. Binary and continuous variables are allowed at any stage of the time horizon. Extensive computational comparisons are performed between the average scenario approach and the proposed BFC approach. The latter clearly dominates the average scenario approach as well as the plain use of a state-of-the-art optimization engine.

The paper is organized as follows. Section 2 introduces the stochastic setting of the SIP problem. Section 3 presents the problem. Section 4 introduces the related mixed 0-1 Deterministic Equivalent Model (DEM). Section 5 introduces the BFC specialization. Section 6 reports computational results and Section 7 presents conclusions.

2 Stochastic programming approach

Uncertainty is present in almost all dynamic systems, but it traditionally has not been explicitly included in the models due to the resulting complexity inherent in the problems to be solved. Uncertainty may be due to the lack of reliable data or the presence of measurement errors, or may take the form of parameters representing information about the future. As an example, in electricity supply system planning, uncertainty arises mainly in terms of future demand and prices, contributions to supply or the availability of generation and network components. Other areas in which uncertainty plays a significant role include investment planning, supply chain planning, production planning, site and road location, transportation, public policy, finance, vehicle routing, etc.

In traditional deterministic optimization, the parameters of the problem are assumed to be known with certainty. In stochastic optimization, however, this assumption is relaxed. Only the probability distributions of the parameters are known, and are generally supposed to be discrete with a finite number of possible states. With this assumption, uncertainty in an optimization problem that evolves over time can be modeled by means of a scenario tree representing all significant realizations of the stochastic parameters.

By using a scenario tree-based methodology as described below we can include the risk of making a bad decision in the actual model. In contrast to stochastic programming (SP), deterministic programming involves the unsatisfactory technique of replacing stochastic parameters with average values and optimizing the resulting model. This approach merely provides the optimal solution of the average scenario, which may not even exist as such. Its substitution into the various scenarios risks generating a poor result for the objective function and, in some cases, the solution will turn out to be infeasible. SP, on the other hand, takes every scenario into account without being subordinated to any one of them, adopting policies that are more consistent and, probably, feasible for all scenarios.

Consider the following deterministic model:

$$\begin{aligned} &\max cx + ay \\ &\text{s.t. } Ax + By = b \\ &\quad x \in \{0,1\}^n \, , y \geq 0 \, , \end{aligned} \tag{1}$$

where c and a are the coefficient vectors of the objective function, b is the right hand side vector, A and B are the constraint matrices, x and y are the 0–1 and continuous variables, respectively, and n is the number of 0–1 variables. The model will be extended to incorporate the uncertainty of some of its parameters. This will require the formalization of some definitions and notation.

Definition 1: A **stage** in a given time horizon is a period (or set of consecutive periods) in which the stochastic parameters take on a given value, that is, in which uncertainty realizes along the time horizon. Note: In this paper we consider that each stage consists of exactly one period.

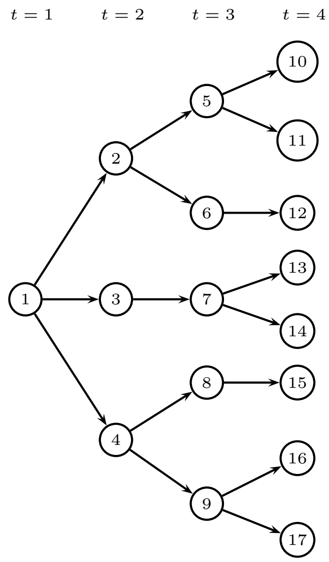
Definition 2: A **scenario** is a particular realization of uncertainty through the whole time horizon.

Note that in addition to the parameters that are specific to each scenario, there are numerous deterministic parameters common to all scenarios. Thus, when referring to a scenario we are referring not only to the parameters specific to that scenario but also to the deterministic ones.

A scenario tree is depicted in Figure 1. The particular tree is composed of 8 scenarios (represented by paths from the root node to the leaves) that are numbered $10, \ldots, 17$. For example, the path $\{1, 4, 9, 16\}$ represents one scenario and it is customary to call it scenario 16. Associated with each node in the tree a decision must be made for the scenarios that are identical in all their realizations up to the related node.

Definition 3: A **scenario group** for a given stage is the set of scenarios in which uncertainty has been realized identically up to that stage.

In Figure 1, for instance, scenarios 10, 11 and 12 form a scenario group for stage 2. They branch out of node 2 at stage 2, but have identical realizations of uncertainty up to stage 2.



$$\mathcal{T} = \{1,2,3,4\}\,;\ \mathcal{T}^- = \{1,2,3\}$$
$$\Omega = \Omega^1 = \{10,11,\ldots,17\}$$
$$\Omega^2 = \{10,11,12\};\ \Omega^4 = \{15,16,17\}$$
$$\mathcal{G}^2 = \{2,3,4\};\ \mathcal{G}^3 = \{5,6,7,8,9\}$$

Figure 1: Scenario Tree

A key objective of our approach is to comply with the non-anticipativity principle, see [4, 10], according to which if two different scenarios are identical up to a given stage in the time horizon, the values of the decision variables must be identical up to that stage. This principle guarantees that the solution obtained from the model up to a given stage does not depend on information that was not yet available. For clarification purposes let us apply the principle to the tree shown in Figure 1. Assume that variables x_1, x_2, x_3, x_4 correspond to decisions made in time periods 1,2,3,4, respectively. Since these variables belong to all scenarios, we add superscripts to differentiate them. Thus: x_t^{12} identifies the variable x in scenario 12 for stage t, and so on. Using this notation, the non-anticipativity constraints are as follows: First stage: $x_1^{10} = \cdots = x_1^{17}$, second stage: $x_2^{10} = x_2^{11} = x_2^{12}$, $x_2^{13} = x_2^{14}$, $x_2^{15} = x_2^{16} = x_2^{17}$, and third stage: $x_3^{10} = x_3^{11}$, $x_3^{13} = x_3^{14}$, $x_3^{16} = x_3^{17}$.

The following notation will be used:

- $\mathcal{T}$, set of stages in the time horizon.
- $\mathcal{T}^-$, set of stages excluding the last one.

- ω, scenario.
- Ω, set of timber sale price scenarios.
- $\mathcal{G}$, set of scenario groups.
- $\mathcal{G}^t$, set of scenario groups in period t, for $t \in \mathcal{T}$.
- Ω^g, set of scenarios that belong to group g, for $g \in \mathcal{G}$.
- w^ω, weight assigned to scenario $\omega \in \Omega$, such that $\sum_{\omega \in \Omega} w^\omega = 1$.

For illustrative purposes, see Figure 1.

In what follows, we do not distinguish between a scenario (or a group) and the corresponding node in the tree with the same number. So for instance, "node 4" or "scenario group 4 for stage 2" will both refer to the set of scenarios $\{15, 16, 17\}$, and node 2 represents $\Omega^2 = \{10, 11, 12\}$.

We can now formulate the structured mixed 0–1 Deterministic Equivalent Model (DEM) of the stochastic version of problem (1) to maximize the expected value of the objective function. It is as follows:

$$\max Q_E = \sum_{\omega \in \Omega} w^\omega \left(c^\omega x^\omega + a^\omega y^\omega \right) \tag{2}$$

$$\text{s.t. } Ax^\omega + By^\omega = b^\omega \qquad \forall \omega \in \Omega \tag{3}$$

$$x_t^\omega = x_t^{\omega'} \qquad \forall t \in \mathcal{T}^-, \forall g \in \mathcal{G}^t, \forall \omega, \omega' \in \Omega^g \tag{4}$$

$$y_t^\omega = y_t^{\omega'} \qquad \forall t \in \mathcal{T}^-, \forall g \in \mathcal{G}^t, \forall \omega, \omega' \in \Omega^g \tag{5}$$

$$x^\omega \in \{0,1\}^n, y^\omega \geq 0 \qquad \forall \omega \in \Omega. \tag{6}$$

We can observe that in the above model, the only constraints linking the various scenarios are the non-anticipativity constraints (4) and (5). These equations complicate the problem considerably. Notice in particular the presence of the 0–1 variables in the constraints associated with any stage. It is thus unlikely (and we verified this in practice) that a commercial optimization software package will be able to solve a large-scale real-life problem in a satisfactory manner. As is typical in mixed integer programming, see [15], we may want to separate the original problem into smaller-sized subproblems that are less complicated to solve. Stated explicitly, if we relax the non-anticipativity constraints, the stochastic problem can be separated in such a way that we can then solve a single deterministic subproblem for each scenario $\omega \in \Omega$, expressed as follows:

$$\begin{aligned} &\max c^\omega x^\omega + a^\omega y^\omega \\ &\text{s.t. } Ax^\omega + By^\omega = b^\omega \\ &\qquad x^\omega \in \{0,1\}^n, y^\omega \geq 0. \end{aligned} \tag{7}$$

However, the individual solutions, x^ω and y^ω, obtained for each scenario are highly unlikely to satisfy the non-anticipativity constraints, particularly if the variance between the parameters associated with the different scenarios is high.

An efficient method for solving the $|\Omega|$ subproblems that arise when the constraints (4) and (5) are relaxed from the model but taken into account by the algorithm, can be a Branch-and-Fix Coordination (BFC) approach, see [1, 2]. See also a specialization of BFC to our problem in section 5.

As mentioned above, the performance of the stochastic approach will be compared to the solution obtained from the average scenario, that is, the solution to the problem:

$$\begin{aligned} \max\ & \overline{c}x + \overline{a}y \\ \text{s.t.}\ & Ax + By = \overline{b} \\ & x \in \{0,1\}^n, y \geq 0\,, \end{aligned} \tag{8}$$

where $\overline{a} = \sum_{\omega\in\Omega} w^\omega a^\omega$, $\overline{b} = \sum_{\omega\in\Omega} w^\omega b^\omega$ and $\overline{c} = \sum_{\omega\in\Omega} w^\omega c^\omega$. Thus, the vectors $\overline{a}$, $\overline{b}$ and $\overline{c}$ are the weighted sum of the vectors for each individual scenario.

3 Forest problem description

Only in the last 30 years have the twin problems of planning forest harvest and access road construction been addressed jointly using mathematical optimization models and computational tools. Previously, the planning of these activities was conducted manually and it relied on little more than the experience of the personnel involved. The advantages of integrating the two processes in a single mixed 0-1 model were demonstrated in [14] by obtaining solutions from 15% to 45% better than with models optimizing the processes separately, see also [9].

The logistics needed to develop efficient forestry planning are highly complex and must be firmly based on efficient mathematical models that can support the decision making process. Various relevant studies exist on the different phases of forestry planning, see for example [5] and [6] and, particularly, the problem of access road construction and timber harvest policies, see [8]. In essence, the problem can be formulated on the basis of a division of the forest into harvest units that we shall call cells, see Figure 2. Thus, for a chosen time horizon one must determine for each time period which cells will be cut, which roads need to be constructed for accessing those cells, and the quantity of wood to be transported from one point to another. These decisions are made via an optimization criterion that typically consists of maximizing the expected net return. It is important to emphasize that there are a number of sources of stochasticity in this problem, including future wood prices, the risk of forest fires and other relevant hazards. The approach developed in this paper analyzes the decision-making under uncertainty in wood selling prices. We assume that these prices can be modeled over time by means

Table 1: Annual Price of Pulpwood, 1994-2003

Year	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
Price [US$/m3]	40	40	42	47	49	55	50	34	41	43

of a set of scenarios with different associated weights, i.e., probabilities. In mathematical terms, the Deterministic Equivalent Model (DEM) of the stochastic version of the problem can be formulated as a large-scale structured mixed 0–1 linear program. As such it is difficult to solve due to its size and the presence of several thousands of 0–1 variables. Approaches to solving the deterministic (one simple scenario) problem can be found in [3], where model strengthening schemes and decomposition techniques such as Lagrangean relaxation are employed to obtain very good solutions in reasonable computation times with low residual gaps. See [7] for solving deterministic machinery location and road design problems in forestry management.

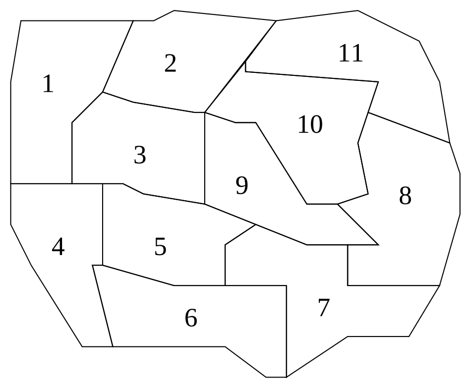


Figure 2: Division of Forest into Harvest Cells

The selling prices of forest products are a central element in forestry planning. Price fluctuations impact directly on profits from sales and figure prominently in planners' decision-making. The role played by randomness in a problem such as the one we propose to solve in this work is closely linked to the length of the chosen time horizon. Planners who must make strategic decisions are therefore concerned to study price variations given that their horizons will be in the neighborhood of 5 years. For operational planners, on the other hand, whose decisions relate to horizons of days or weeks, uncertainty is not a central factor. Historically, wood prices have experienced significant fluctuations, as exemplified in the selling price for pulpwood between 1994 and 2003, shown here in Table 1.

4 Mathematical formulation

The tactical planning horizon for our forestry problem is four years. The objective is to determine the optimal harvest and access road construction policy that will maximize expected net profit and satisfy the constraints for all scenarios. The company is assumed to own its own timber land, which is subdivided into cells for harvesting using geographic information systems. Given the planning horizon it is assumed that the entire forest is suitable for harvesting, implying that the age of the trees in the area under study is over 22 years approximately. The model we present is a simplified version of the one actually used for planning, shown in [3]. The simplification contains the main elements of the problem and makes it more convenient for highlighting the stochastic nature introduced in the model. We present below the simplified model. For convenience, we define the following terms used in describing the harvesting process and our mathematical model:

Origins: Specific points called nodes located within the forest where wood from the surrounding harvest cells is processed. Each node is therefore assigned to a set of cells, and each cell is associated with a single origin.

Existing roads: Roads in available condition from the start of the time horizon and suitable for logging truck traffic.

Potential roads: Roads that do not exist at the start of the time horizon but may be built at any time period. Once such a road is built, it is available until the end of the time horizon.

Intersections: Points between roads.

Exits: Points in the forest by which the wood extracted in each time period is transported out of the zone and distributed to the various demand points.

Let the following notation for the DEM below:

Sets

$\mathcal{T}$: time horizon $\{t\}$.

$\mathcal{O}$: harvest origins $\{o\}$.

$\mathcal{J}$: intersection nodes $\{j\}$.

$\mathcal{S}$: wood exit nodes $\{s\}$.

$\mathcal{H}$: harvest cells $\{h\}$.

$\mathcal{H}_o$: harvest cells associated with origin o.

$\mathcal{R}^E$: existing roads at start of the time horizon $\{k, l\}$.

$\mathcal{R}^P$: potential roads $\{k,l\}$.

$\mathcal{K} = \mathcal{R}^E \bigcup \mathcal{R}^P$.

Deterministic parameters

a_h^t: productivity of cell h, if it is harvested in period t [m3/Ha].

A_h: area of the cell h to be harvested [Ha].

$U_{k,l}^t$: flow capacity of an existing or a potential road constructed in arc (k,l) at time period t [m3].

P_h^t: harvesting cost of one hectare of cell h in time period t [dollars/Ha].

Q_o^t: unit production cost at origin o in time period t [dollars/m3].

$C_{k,l}^t$: construction cost of one road in arc (k,l) at time period t [dollars].

$D_{k,l}^t$: unit transport cost through arc (k,l) in time period t [dollars/m3].

Stochastic parameters

$R_s^{t,\omega}$: sale price at exit s in time period t under scenario ω [dollars/m3].

$\underline{Z}^{t,\omega}$, $\overline{Z}^{t,\omega}$: lower and upper demand bounds in period t under scenario ω [m3], respectively.

Variables

$\delta_h^{t,\omega}$: 0–1 variable such that its value is 1, if cell h is harvested in period t under scenario ω; 0, otherwise.

$\gamma_{k,l}^{t,\omega}$: 0–1 variable such that its value is 1, if road in arc (k,l) is built in period t under scenario ω; 0 otherwise.

$f_{k,l}^{t,\omega}$: flow of wood transported through arc (k,l) in period t under scenario ω [m3].

$z_s^{t,\omega}$: total demand timber at exit s in period t under scenario ω [m3].

Constraints

- Flow balance equations at the nodes:

  1. At origin nodes:

$$\sum_{h\in\mathcal{H}_o} a_h^t A_h \delta_h^{t,\omega} + \sum_{(k,o)\in\mathcal{K}} f_{k,o}^{t,\omega} - \sum_{(o,k)\in\mathcal{K}} f_{o,k}^{t,\omega} = 0\,, \quad \forall o \in \mathcal{O}, t \in \mathcal{T}, \omega \in \Omega$$

  2. At intersection nodes:

$$\sum_{(k,j)\in\mathcal{K}} f_{k,j}^{t,\omega} - \sum_{(j,k)\in\mathcal{K}} f_{j,k}^{t,\omega} = 0\,, \quad \forall j \in \mathcal{J}, t \in \mathcal{T}, \omega \in \Omega$$

3. At destination nodes:

$$z_s^{t,\omega} = \sum_{(k,s)\in\mathcal{K}} f_{k,s}^{t,\omega}\,, \quad \forall s \in \mathcal{S}, t \in \mathcal{T}, \omega \in \Omega$$

- Wood demand bounds:

$$\underline{Z}^{t,\omega} \leq \sum_{s\in\mathcal{S}} z_s^{t,\omega} \leq \overline{Z}^{t,\omega}\,, \quad \forall t \in \mathcal{T}, \omega \in \Omega$$

- Construction of potential roads:

1. Road flow capacity:

$$f_{k,l}^{t,\omega} \leq U_{k,l}^t \sum_{1\leq\tau\leq t} \gamma_{k,l}^{\tau,\omega}\,, \quad \forall (k,l) \in \mathcal{R}^P, t \in \mathcal{T}, \omega \in \Omega$$

2. A potential road can be built at most once in the time horizon:

$$\sum_{t\in\mathcal{T}} \gamma_{k,l}^{t,\omega} \leq 1\,, \quad \forall (k,l) \in \mathcal{R}^P, \omega \in \Omega$$

- Existing roads flow capacity:

$$f_{k,l}^{t,\omega} \leq U_{k,l}^t\,, \quad \forall (k,l) \in \mathcal{R}^E, t \in \mathcal{T}, \omega \in \Omega$$

- A cell can be harvested at most once in the time horizon:

$$\sum_{t\in\mathcal{T}} \delta_h^{t,\omega} \leq 1\,, \quad \forall h \in \mathcal{H}, \omega \in \Omega$$

- Non-anticipativity constraints for the 0–1 variables:

$$\delta_h^{t,\omega} = \delta_h^{t,\omega'}$$
$$\gamma_{k,l}^{t,\omega} = \gamma_{k,l}^{t,\omega'}$$

$\forall t \in \mathcal{T}^-, \forall g \in \mathcal{G}^t, \forall \omega, \omega' \in \Omega^g, \omega \neq \omega', \forall h \in \mathcal{H}, \forall (k,l) \in \mathcal{R}^P.$

- Non-anticipativity constraints for the continuous variables:

$$f_{k,l}^{t,\omega} = f_{k,l}^{t,\omega'}$$
$$z_s^{t,\omega} = z_s^{t,\omega'}$$

$\forall t \in \mathcal{T}^-, \forall g \in \mathcal{G}^t, \forall \omega, \omega' \in \Omega^g, \omega \neq \omega', \forall s \in \mathcal{S}, \forall (k,l) \in \mathcal{K}.$

- Nonnegativity and integrality of the variables:

$$f_{k,l}^{t,\omega}\,, z_s^{t,\omega} \geq 0\,, \quad \forall\, s \in \mathcal{S}, (k,l) \in \mathcal{K}, t \in \mathcal{T}, \omega \in \Omega$$

$$\delta_h^{t,\omega}\,, \gamma_{k,l}^{t,\omega} \in \{0,1\}\,, \quad \forall\, h \in \mathcal{H}, (k,l) \in \mathcal{R}^P, t \in \mathcal{T}, \omega \in \Omega$$

Objective Function

$$\max Z = T_1 - T_2 - T_3 - T_4 - T_5,$$

where the terms on the right hand side of the function are as follows:

T_1: Wood sale revenue.

$$T_1 = \sum_{\omega \in \Omega} \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}} w^\omega R_s^{\omega,t} z_s^{t,\omega}$$

T_2: Wood harvest cost.

$$T_2 = \sum_{\omega \in \Omega} \sum_{h \in \mathcal{H}} \sum_{t \in \mathcal{T}} w^\omega P_h^t A_h \delta_h^{t,\omega}$$

T_3: Production costs at origin nodes.

$$T_3 = \sum_{\omega \in \Omega} \sum_{o \in \mathcal{O}} \sum_{t \in \mathcal{T}} w^\omega Q_o^t \left(\sum_{h \in \mathcal{H}_o} a_h^t A_h \delta_h^{\omega,t} \right)$$

T_4: Potential road construction cost.

$$T_4 = \sum_{\omega \in \Omega} \sum_{(k,l) \in \mathcal{R}^P} \sum_{t \in \mathcal{T}} w^\omega C_{k,l}^t \gamma_{k,l}^{t,\omega}$$

T_5: Wood transport cost.

$$T_5 = \sum_{\omega \in \Omega} \sum_{(k,l) \in \mathcal{K}} \sum_{t \in \mathcal{T}} w^\omega D_{k,l}^t f_{k,l}^{t,\omega}$$

5 Branch-and-Fix Coordination scheme

Solving the stochastic model (2)–(6) using a traditional branch-and-bound approach is extremely difficult for solvers and commercial optimization packages. This is due mainly to the presence of the non-anticipativity constraints (4) and (5). The truly complicated group of constraints is (4), which requires that the 0-1 variables remain the same throughout given scenarios.

As an alternative to the direct optimization of the stochastic model by state-of-the-art-optimization engines, we adopt a specialization of the Branch-and-Fix Coordination (BFC) approach introduced in [1, 2]. BFC is especially designed to coordinate the choice of the branching variable and the branching node in the Branch-and-Fix (BF) tree associated with each scenario, so that constraints (4) are satisfied when the 0–1 variables are branched on 0 or 1. Once all 0–1 variables are fixed, BFC solves the resulting Linear Programming (LP) problem to satisfy constraints (5).

5.1 Introduction and additional definitions

Let us set $x_t^\omega = (\delta_h^{t,\omega}, \gamma_{k,l}^{t,\omega})$ and $y_t^\omega = (f_{k,l}^{t,\omega}, z_s^{t,\omega})$ in the generic model (2)–(6), to represent the above forestry problem.

Let $\mathcal{R}^\omega$ be the BF tree associated with scenario ω and $\mathcal{A}^\omega$ the set of active nodes, i.e., those nodes in which there still exist x–variables not fixed to 0-1 and whose solution value is better than the value of the incumbent solution. Also, we denote by $\mathcal{I}$ the set of indices of the 0–1 variables x in any scenario group, and by $(x_t^\omega)_i$ the ith variable of the vector x_t^ω, for $t \in \mathcal{T}$, $i \in \mathcal{I}$, $\omega \in \Omega$.

We use the tree in Figure 3 to illustrate new concepts. It has three scenarios with three stages and four decision variables: x_1 in the first stage, x_2 and z_2 in the second stage, and x_3 in the third stage. In the bottom section of the figure is depicted the development of the branch-and-fix trees for each scenario, with nodes 1, 2 and 3 corresponding to the LP relaxation of problem (7) for scenarios 1, 2 and 3, respectively. As an illustration, let the variable branching order be x_1, x_2, z_2, x_3.

Definition 4. Given index $i \in \mathcal{I}$, stage $t \in \mathcal{T}^-$, a scenario group $g \in \mathcal{G}_t$, and two different scenarios ω and ω' in the same group Ω_g , variables $(x_t^\omega)_i$ and $(x_t^{\omega'})_i$ are said to be *common* variables for scenarios ω and ω'.

Notice that two *common* variables both have nonzero coefficients in the *non-anticipativity* constraint related to a given scenario group.

For convenience, let us delete the subindex i in the variables shown in Figure 3. As an example, variables x_2^1 and x_2^2 in the figure are common variables for scenarios 1 and 2 at time period $t = 2$, since x_2 must be fixed to the same 0-1 value in those two scenarios (notice that they evolve identically up to stage 2). Variables x_2 and z_2 corresponding to time (or stage) $t = 2$ are not common to scenarios 1 and 3, since these scenarios belong to different scenario groups, namely groups b and c, at time 2. Variables x_3^1 and x_3^3 corresponding to time 4 are not common variables for scenarios 1 and 3 at time 4 because time 4 is not in T^-.

Definition 5. Given index $i \in \mathcal{I}$, stage $t \in \mathcal{T}^-$, scenario group $g \in \mathcal{G}_t$, and two scenarios ω,$\omega' \in \Omega_g$, nodes $a \in \mathcal{A}^\omega$ and $a' \in \mathcal{A}^{\omega'}$ are said to be *twin* nodes with respect to scenario group g if on the paths from the root nodes to these nodes in each of the two *BF* trees $\mathcal{R}^\omega$ and $\mathcal{R}^{\omega'}$, the *common* variables, if any, $(x_t^\omega)_i$ and $(x_t^{\omega'})_i$, have been branched on at the same value.

For example, nodes 10 and 12 in Figure 3 are twin nodes with respect to scenario group b because both have fixed the values of their common variables x_1 and x_2 to 0. Nodes 16 and 19, however, are not twin, because their common variable z_2 has been branched on at opposite values. Nodes 11 and 15 are not twin either, since although variable x_2 has been branched on at the same value, scenarios 1 and 3 do not belong to the same scenario group at stage 2 (scenario 1 belongs to scenario group b and scenario 3 belongs to scenario group c).

Definition 6. A *Twin Node Family (TNF)*, say, $\mathcal{J}_f$, is a set of nodes such that any node is a *twin* node to all other nodes in the family. $\mathcal{F}$ will denote the set of all such families, with $f \in \mathcal{F}$.

As an example, nodes 4, 6 and 8 in Figure 3 constitute a twin node family.

Definition 7: A *candidate* TNF is a TNF whose members have not yet fixed all of their common variables.

As an example nodes 4 and 6 in Figure 3 constitute a candidate TNF, since their scenario trees have in common variables x_2^1 and x_2^2 that have not yet been branched on. On the other hand, nodes 23 and 27 do not, since the common variables of their scenario trees have all been fixed already. We want to point out that the TNF consisting of nodes 4, 6 and 8 is not a candidate TNF, since the nodes do not have any variable in common that has not yet been branched on: the only common variable is x_1 and it has already been branched on.

Definition 8. Given $g \in \mathcal{G}_t, t \in \mathcal{T}^-, i \in \mathcal{I}$, a *TNF integer set* is a set of *TNFs* where all x variables take integer values, there is one node in each *BF* tree and the *non-anticipativity* constraints (4) $(x_t^\omega)_i - (x_t^{\omega'})_i = 0$ are satisfied, $\forall \omega, \omega' \in \Omega_g$.

As an example the nodes 22, 26 and 33 in Figure 3 constitute a TNF integer set, since all their (integer) variables have taken integer values, the common variables have taken the same values ($x_1^1 = x_1^2 = x_1^3 = 0$, $x_2^1 = x_2^2 = 1$ and $z_2^1 = z_2^2 = 0$) and there is one node per BF tree. Nodes 22, 28 and 33 are *not* a TNF integer set, since the common variables z_2^1 and z_2^2 have taken different values (0 and 1, respectively).

Some of the TNFs are: $\mathcal{J}_1 = \{1, 2, 3\}$, $\mathcal{J}_2 = \{4, 6, 8\}$ (non-candidate), $\mathcal{J}_3 = \{4, 6\}$, $\mathcal{J}_4 = \{8\}$, $\mathcal{J}_5 = \{5, 7, 9\}$ (non-candidate), $\mathcal{J}_6 = \{5, 7\}$, $\mathcal{J}_7 = \{9\}$, $\mathcal{J}_8 = \{10, 12\}$, etc.

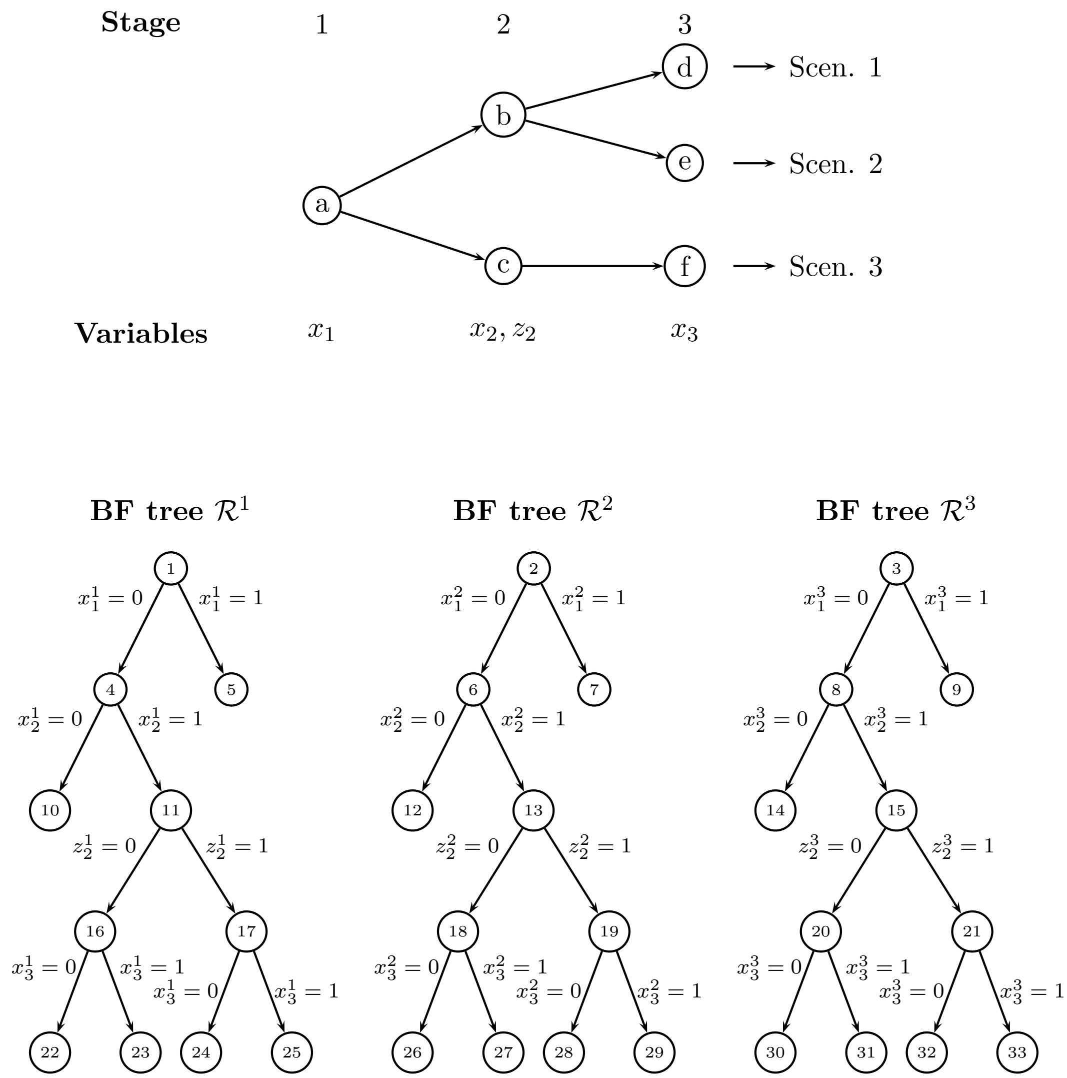
5.2 BFC algorithm major steps

The BFC algorithm may be broken down into 9 major steps.

Step 1: Solve the LP relaxation of problem (7) for each scenario. If all non-anticipativity and all integrality constraints are satisfied then stop; the optimal solution to the original problem has been obtained. Otherwise, a lower bound for the optimal solution has been found.

Step 2: Selection of the branching common x variable for a given scenario group, according the *greatest smaller deterioration* criterion, see below.

Step 3: *Depth first* strategy to use. Selection of the candidate TNF by branching on the chosen 0–1 variable in a given stage, according to the criterion: *greatest solution value* first (see below) and, later, the other branch.



Variable Branching Order: x_1, x_2, z_2, x_3

Figure 3: Branch and Fix Coordination

Step 4: Bound the just created TNF, by optimizing the LP relaxation of the problems associated with its nodes. If the LP model is infeasible or its solution value is not better than the incumbent, then TNF is pruned. Go to Step 9.

Step 5: If, by considering the solution of the relaxed problem in Step 4, the TNF is not an integer TNF, then go to Step 2 (selection of a new common x branching variable) for continuing the branching phase).

If the non-nanticipativity constraints (5) are satisfied by the common y variables, then go to Step 7. Otherwise, go to Step 6.

Step 6: Optimize the LP relaxation of the original model that results from fixing the common x variables to their 0–1 values obtained in step 4. If the current solution is better that the incumbent solution, then update the incumbent solution and the active node sets.

If all 0–1 variables have been branched on, then discard the TNF integer set (otherwise a better solution can be found by branching on the not-yet branched on common x variables) and go to Step 8; otherwise, go to Step 2.

(Notice that constraints (5) are relaxed in the LP relaxation problems of Steps 1 and 4, but they are forced to be satisfied in the LP problem of Step 6).

Step 7: The current TNF is pruned. If the current solution is better than the incumbent solution, then update the incumbent solution and the active node sets.

Step 8: If the sets of active nodes are empty (i.e., there is no any active node with a fractional x variable), then STOP, since the incumbent solution is optimal.

Step 9: If the current branching variable x has been branched on in both directions, then backtrack and repeat Step 9. Otherwise, "Branch on x at one" if it has been "branched on at zero", and vice versa and, in any case, go to Step 4.

5.3 Branching variable selection

Various criteria exist for choosing the branching variable, see [15], such as the variable whose smaller distance to 0 or 1 is the largest, the variable with the largest coefficient in the objective function, meaning that its contribution to the maximization is the largest, a weighted combination of the above two criteria, etc. For the problem under study in this work we have selected the first criterion. Thus, we compute the expected distance separating it from 0 and $|\mathcal{J}_f|$ taking into account the contribution of the nodes in the given TNF $\mathcal{J}_f$, that is, we compute the following indicator for variable x_k:

$$\Delta_k = \min \left\{ \sum_{n \in \mathcal{J}_f} x_k^n,\ |\mathcal{J}_f| - \sum_{n \in \mathcal{J}_f} x_k^n \right\}, \tag{9}$$

and the branching variable x_{k^*} to select is such that

$$k^* = \arg\max_k \{\Delta_k\}\,. \tag{10}$$

5.4 Branching Twin Node Family selection

Let $Z_{LP}^{n,k}$ be the value of the objective function of the LP problem solved at node n by fixing the chosen branching variable to $k \in \{0,1\}$ in the current TNF. Thus the LP solution value of the TNF $\mathcal{J}_f$ to create in case of branching on the k value will be

$$Z_{LP,f}^{k} = \sum_{n \in \mathcal{J}_f} w^n Z_{LP}^{n,k}.$$

Thus, since the original problem is one of maximization, the TNF $\mathcal{J}_f^{k^*}$ to create first will be

$$\mathcal{J}_f^{k^*} = \arg \max_{k \in \{0,1\}.} \left\{ Z_{LP,f}^{k} \right\}.$$

6 A proof of concept case: Los Copihues Forest

The forest property called Los Copihues, owned by the Chilean company Sociedad Forestal Millalemu S.T., is one of the country's largest wood stands destined for harvest. Covering some 300 hectares, the property has been divided into $|\mathcal{H}| = 25$ harvest cells based on company maps. The other dimensions of the problem are as follows: $|\mathcal{O}| = 9$ origins, $|\mathcal{J}| = 3$ junction nodes, $|\mathcal{S}| = 1$ exit node, $|\mathcal{R}^E| = 6$ existing roads and $|\mathcal{R}^P| = 14$ potential roads.

To represent the variability of wood prices over time, 18 possible price and demand scenarios were defined as shown in Table 2. See the scenario tree on Figure 4.

Notice that scenarios which consider higher demand markets imply higher timber prices as well as higher demand, which is indicated by higher lower and upper bounds on possible sales.

- Each scenario is comprised of $\mathcal{T} = 4$ annual periods, where $P1, P2, P3$ and $P4$ give the timber sales prices corresponding to the years 2004, 2005, 2006 and 2007, respectively. Common to all scenarios is that the price for 2004 is 45 US$/ m3.
- The columns LB and UB to the right of each annual price show the lower and upper demand bounds for each period in the time horizon, respectively.

The scenarios were defined to embrace a range of prices that oscillate between 20 US$/m3 and 68 US$/m3. The demand bounds are directly proportional to the sale price in each period so that demand is high when prices are high in order to increase sales revenues. Figure 4 shows graphically the definition of scenarios shown in Table 2. We choose to compare scenarios only by timber price, and do not consider demand bounds, to simplify the figure.

In addition, Table 3 displays the prices and demand bounds summed over the 4 periods. There is an observable decrease in the aggregate values from scenario 1, which has the best prices and demand figures, to scenario 18, where prices and demand are the lowest.

Table 2: Wood prices and corresponding lower (LB) and upper (UB) demand bounds, by period

Sce. num.	**P1** US$	**LB** m3	**UB** m3	**P2** US$	**LB** m3	**UB** m3	**P3** US$	**LB** m3	**UB** m3	**P4** US$	**LB** m3	**UB** m3
1	45	30000	40000	60	27000	50000	65	28000	52000	68	25000	50000
2	45	30000	40000	60	27000	50000	65	28000	52000	57	20000	51000
3	45	30000	40000	60	27000	50000	55	26000	50000	62	25000	48000
4	45	30000	40000	60	27000	50000	55	26000	50000	50	26000	49000
5	45	30000	40000	60	27000	50000	45	24000	44000	58	26000	48000
6	45	30000	40000	60	27000	50000	45	24000	44000	48	22000	40000
7	45	30000	40000	45	15000	33000	52	27000	45000	55	21000	50000
8	45	30000	40000	45	15000	33000	52	27000	45000	44	19000	42000
9	45	30000	40000	45	15000	33000	45	25000	41000	50	20000	36000
10	45	30000	40000	45	15000	33000	45	25000	41000	42	15000	30000
11	45	30000	40000	45	15000	33000	30	20000	32000	48	15000	28000
12	45	30000	40000	45	15000	33000	30	20000	32000	42	13000	24000
13	45	30000	40000	30	10000	18000	45	22000	40000	40	12000	18000
14	45	30000	40000	30	10000	18000	45	22000	40000	30	12000	18000
15	45	30000	40000	30	10000	18000	30	15000	25000	40	13000	18000
16	45	30000	40000	30	10000	18000	30	15000	25000	30	12000	17000
17	45	30000	40000	30	10000	18000	22	12000	22000	35	11000	16000
18	45	30000	40000	30	10000	18000	22	12000	22000	20	10000	15000

The numerical experiments that we designed for solving the stochastic version of the Los Copihues forest planning problem involve making comparisons between the performance of a solution adopted by a stochastic planner and a solution applied by a planner guided only by the average values of the uncertain parameters.

The experiments for the average scenario approach assume that the planner must make all decisions at the start of the time horizon, that is, without any knowledge of the realizations of uncertainty but knowing the range of scenarios by which it may evolve.

Three probabilistic configurations of the scenario tree were studied:

- Equiprobable scenarios
- Probabilities weighted towards high price scenarios
- Probabilities weighted towards low price scenarios

Table 3: Prices and lower (LSB) and upper demand bounds (USB) aggregated over time

Sce.	**Prices(US$/m3)**	**LSB (m3)**	**USB (m3)**
1	238	110000	192000
2	227	105000	193000
3	222	108000	188000
4	210	109000	189000
5	208	107000	182000
6	198	103000	174000
7	197	93000	168000
8	186	91000	160000
9	185	90000	150000
10	177	85000	144000
11	168	80000	133000
12	162	78000	129000
13	160	74000	116000
14	150	74000	116000
15	145	68000	101000
16	135	67000	100000
17	132	63000	96000
18	117	62000	95000

The dimensions of the stochastic problem, DEM compact representation are as follows: 22699 constraints; 2808 0–1 variables; 1512 continuous variables; and 56737 nonzero elements in the constraint matrix.

As a first approximation to a solution, the stochastic problem configuration with equiprobable scenarios was run directly using CPLEX 8.1. After 500 minutes (8 hours and 20 minutes) on an HP Celeron with an Intel 2.40 GHz processor and 512 mb of RAM, the residual gap had not fallen below 12% . (It was this experiment that motivated the implementation of the BFC approach in the search for the optimal solution.)

6.1 Equiprobable configuration

In this experiment it is assumed that the uncertain parameters evolve equiprobably. For example, in the scenario group G-19 (year 2004, first period) with the price at 45 US$/m3, there is a probability of 1/3 that the price in the next period will be 60 US$/m3 (scenario group G-20), a probability of 1/3

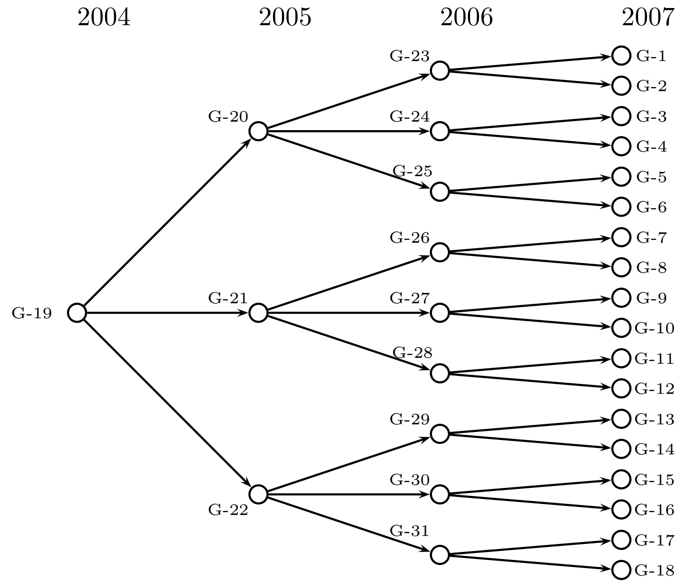


Figure 4: Scenario tree with 18 scenarios for Los Copihues case

that it will be 45 US$/m3 (scenario group G-21) and a probability of 1/3 that it will be 30 US$/m3 (scenario group G-22) (see Figure 5). We use the same probabilities for increasing and decreasing prices in the following period.

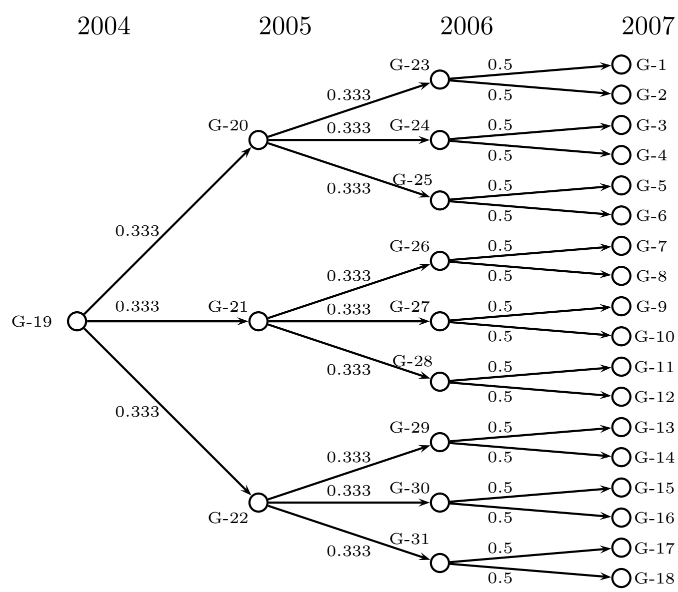


Figure 5: Probabilities for Equiprobable Configuration

Table 4: Equiprobable Scenarios

	Z_{AVSC}	Z_{BFC}	ABS GAP	REL GAP %
SC 1	7860376.2	8141684.5	281308.3	3.6
SC 2	74986706.4	7832291.9	335585.6	4.5
SC 3	7272765.9	7681257.3	408491.9	5.6
SC 4	6876035.1	7248863.7	372828.7	5.4
SC 5	6751277.3	7288986.5	537709.3	8.0
SC 6	6420668.3	6913420.5	492752.3	7.7
SC 7	6440966.1	6739744.0	298777.9	4.6
SC 8	6077296.2	6359584.0	282287.8	4.6
SC 9	6003190.1	6111671.1	108481.1	1.8
SC 10	Infeasible	5604078.0	-	-
SC 11	Infeasible	4945591.0	-	-
SC 12	Infeasible	4541990.9	-	-
SC 13	Infeasible	4324647.4	-	-
SC 14	Infeasible	4149814.2	-	-
SC 15	Infeasible	3335188.5	-	-
SC 16	Infeasible	3067968.2	-	-
SC 17	Infeasible	2866035.9	-	-
SC 18	Infeasible	2593300.9	-	-
tt	-	11942	-	-
Z_{IP}	-	5541451.0	-	-

The results of the comparison between the average scenario solution and the stochastic solution obtained by the BFC approach are summarized in Table 4 for all scenarios. The average scenario based solutions were obtained by simulating what happens in a given scenario, say, $\omega \in \Omega$, when applying the average scenario solution (AVSC). The headings are as follows: Z_{AVSC}, solution value of the average scenario approach for each scenario; Z_{BFC}, solution value of the BFC approach for each scenario; ABS GAP $=Z_{BFC}$ - Z_{AVSC}; REL GAP = ABS GAP / Z_{AVSC} (%); tt, total elapsed time (secs.) for the solution value in all scenarios; Z_{IP}, expected solution value for the whole set of scenarios.

We considered a deterministic decision maker, which uses average values as inputs, and a stochastic decision maker, who uses the proposed Branch-and-Fix Coordination (BFC) approach. In order to compare both approaches we took both solutions, the deterministic solution being one vector of decisions, while the stochastic solutions leads to one vector per scenario. We then evaluated the performance of the solutions under each scenario. For illustration purposes, let us assume the occurrence of scenario 5. In this case, the solution vector of the average scenario when substituted in the scenario yields the solution value $Z_{AVSC} = 6751277.3$. The BFC solution for scenario 5 gives the value $Z_{BFC} = 7288986.5$. The relative difference is given by REL GAP = 8.0%.

Notice that for the scenarios 10 through 18, the Z_{AVSC} column shows 'infeasible'. This means that when the AVSC solution vector is substituted into these scenarios a demand constraint is violated, thus rendering the solution mathematically infeasible. This typically occurs because given the structure of the solution of the AVSC approach, in latter periods a demand bound is violated, either the lower or the upper bound. This is due to the fact that the deterministic AVSC does not consider the stochastic aspects of demand and prices, and may get locked in its initial periods into productions which may later lead to not being able to satisfy the demand bounds of the model.

6.2 Probabilities weighted towards high price scenarios

In a way similar to the equiprobable configuration, we now compare the performance of the average scenario and BFC approaches for each scenario with the probabilities slanted towards the better-price scenarios. Since the demand bounds will also be higher than those for equiprobable scenarios, we find that the volume of wood harvested in the average scenario approach is greater as well.

Figure 6 sets out the probabilities employed. This is similar to the equiprobable configuration. Table 5 compares the performances of AVSC and BFC for each scenario.

6.3 Probabilities weighted towards low price scenarios

As for the preceding simulations, we compare the performance of AVSC and BFC for each scenario when the probabilities are slanted towards the lower price scenarios. Since the demand bounds will also be lower than those for equiprobable scenarios, we would expect that the harvested volume is lower as well.

Figure 7 shows the probabilities employed while Table 6 summarizes the comparisons between both approaches AVSC and BFC.

6.4 Discussion of the results

Equiprobable Configuration

In aggregate terms, we observe that BFC always returns feasible solutions up to 8% better than AVSC. The ABS GAP indicator measures the difference between BFC and AVSC, with REL GAP giving the percentage difference. Notice that in scenario 9, BFC is only 1.8% better than AVSC, a phenomenon

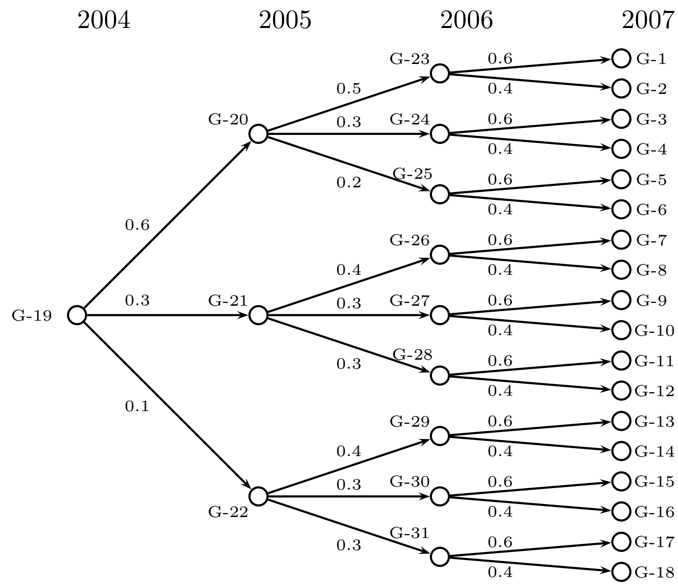


Figure 6: High Probabilities for High Price Scenarios

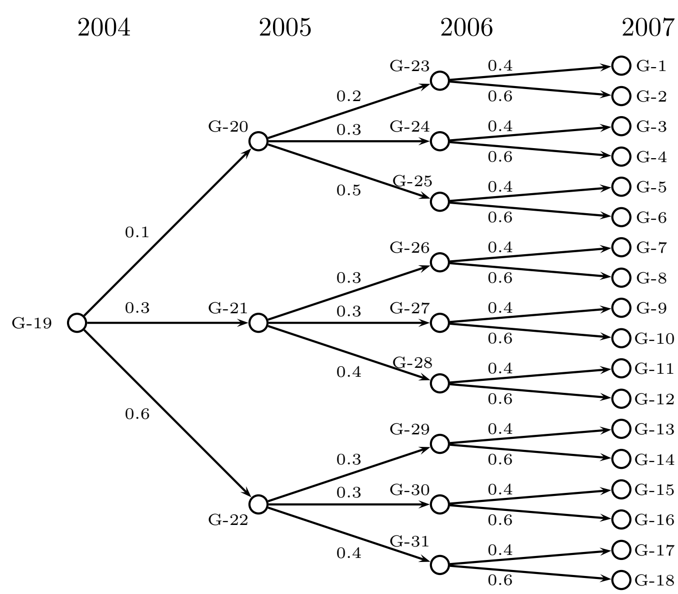


Figure 7: High Probabilities for Low Price Scenarios

Table 5: Probabilities Weighted Towards High Prices

	Z_{AVSC}	Z_{BFC}	ABS GAP	REL GAP%
SC 1	8401129.1	8162086.3	-239042.9	-2.8
SC 2	7946073.4	7865469.1	-80604.3	-1.0
SC 3	7779746.9	7674062.6	-105684.3	-1.4
SC 4	7283322.5	7273159.4	-10163.1	-0.1
SC 5	7241102.1	7318535.0	77432.9	1.1
SC 6	Infeasible	6919073.0	-	-
SC 7	Infeasible	6744774.0	-	-
SC 8	Infeasible	6363873.7	-	-
SC 9	Infeasible	6117076.8	-	-
SC 10	Infeasible	5662547.3	-	-
SC 11	Infeasible	4944700.8	-	-
SC 12	Infeasible	4607232.8	-	-
SC 13	Infeasible	4044687.9	-	-
SC 14	Infeasible	3801413.5	-	-
SC 15	Infeasible	3193279.8	-	-
SC 16	Infeasible	3032666.9	-	-
SC 17	Infeasible	2798942.6	-	-
SC 18	Infeasible	2531468.4	-	-
tt	-	12123	-	-
Z_{IP}	-	6715210.6	-	-

that may be explained by the fact that the average scenario is quite similar to Scenario 9 in the equiprobable configuration. The virtues of the stochastic solution are also evident when considering that the solution vector of AVSC for scenarios 10 through 18 proved upon evaluation to be infeasible.

In short, for the equiprobable configuration, BFC is always a better alternative than the average scenario. This is so in half of the instances because the objective function value is better, and in the other half because the average scenario is simply infeasible.

Probabilities Weighted towards High Prices

In this configuration the probabilities favor high price scenarios, which results in the average scenario also displaying better prices and higher demand bounds than for the equiprobable configuration, making full use of the available forest resources. Thus, in scenarios 1 through 4 AVSC performs better

Table 6: Probabilities Weighted Towards Low Prices

	Z_{AVSC}	Z_{BFC}	ABS GAP	REL GAP%
SC 1	Infeasible	8140719.9	-	-
SC 2	Infeasible	7852042.5	-	-
SC 3	Infeasible	7623354.5	-	-
SC 4	Infeasible	7287829.7	-	-
SC 5	Infeasible	7280768.5	-	-
SC 6	Infeasible	6905772.5	-	-
SC 7	5447432.9	6731022.2	1283589.3	23.6
SC 8	5176245.5	6354119.3	1177870.8	22.8
SC 9	5101141.7	6107012.7	1005871.0	19.7
SC 10	4903914.5	5666319.3	762404.8	15.5
SC 11	4573925.9	4866508.9	292583.0	6.4
SC 12	Infeasible	4571835.1	-	-
SC 13	Infeasible	4257515.9	-	-
SC 14	Infeasible	4125105.1	-	-
SC 15	Infeasible	3334782.0	-	-
SC 16	Infeasible	3182105.0	-	-
SC 17	Infeasible	2930051.7	-	-
SC 18	Infeasible	2657131.6	-	-
tt	-	13357	-	–
Z_{IP}	-	4408573.6	-	-

than BFC by up to 2.8%. However, in 13 scenarios the AVSC solution turns out to be infeasible whereas the stochastic solution is always feasible. The AVSC infeasibility is attributable to the fact that in periods 2, 3 and 4 this solution proposes levels of demand above the upper bounds permitted for those periods.

When probabilities are weighted towards high prices, BFC is better in 13 out of the 18 scenarios for reasons of infeasibility, while in 4 instances AVSC is a better indicator though only marginally so.

Probabilities Weighted towards Low Prices

In this configuration the probabilities favor low price scenarios, which implies that the average scenario also displays inferior prices and lower demand bounds than the equiprobable configuration. The AVSC solution is infeasible in scenarios 1 through 6 given that it proposes harvesting less than the defined lower demand bound. The infeasibility for scenarios 12 through 18 is due to the fact that AVSC proposes cutting more than what is permitted. Finally, the BFC approach is clearly superior in terms of planning to the average scenario, in some instance as much as 23.6% better.

To sum up, with high probabilities for low prices, BFC always performs better than AVSC.

It is of interest to analyze the possible significance to a forest manager of the previously discussed analysis. As was seen, in all three cases the BFC approach led to better solutions than the deterministic approach under most scenarios, often by significant margins. (In a few scenarios for the case of probabilities weighted toward high prize scenarios the deterministic approach led to slightly better solutions for several scenarios). But what was more significant, the deterministic approach could not find feasible solutions in multiple scenarios for all cases. The reason for these results is that in the deterministic case, where average expectations of market conditions are taken, the possible fluctuations of future market conditions are not considered. This leads in multiple cases to decisions in the first periods that lead to conditions in future periods in relation to timber availability that make it difficult to react well under specific scenarios. In contrast, the BFC approach looks ahead at possible scenarios and protects itself against possible fluctuations. A forest manager might well want to consider these future possible fluctuations and protect his firm against them. The BFC approach allows for this possibility. For instance, in Table 2 we see that scenarios 13 and 14 are identical for periods 1, 2 and 3 but they differ in the lower bounds for period 4 and on top of that the prices change from 45 dollars to 40 and 30. This gap in the prices explain why the average scenario is infeasible for period 4, specifically the average solution takes advantage of the higher price in period 3 and produce too much wood leaving an insufficient amount for covering the lower bound in period 4.

7 Conclusions

In this study we have satisfactorily formulated and solved a stochastic version of the forest harvest and road construction planning problem. Specifically, by using the BFC methodology for multistage stochastic integer programming, the problem was solved assuming randomness of wood sale prices. In this sense, we may conclude that the objective of incorporating uncertainty into the forest planning problem, as proposed at the beginning of this paper, was successfully accomplished.

The principal contribution of this work consists in formally incorporating uncertainty into a problem of the type described above through the analysis of discrete scenarios, as opposed to the traditional approach of using average values for uncertain parameters. This more sophisticated design greatly complicates the solution process, given that it increases considerably both the dimensions of the original mixed-integer programming model and the range of constraints to be satisfied. This is particularly true because of the so-called non-anticipativity constraints, which conceptually require that the decisions made by the planner at every moment do not depend on information not yet available.

To get a feeling for the difficulties involved we initially attempted to solve the problem directly by using a state-of-the-art optimization engine, but our lack of success was evident in the fact that after eight hours of running it for the equiprobable scenarios configuration the residual gap had still not fallen below 12%. As an alternative, various decomposition methods were examined that would facilitate the handling of the 0-1 variables, which constitute the heart of the problem (the linear relaxation optimization being fairly rapid). Thus, we proposed a Branch-and-Fix Coordination algorithmic approach especially designed to coordinate the searching, branching and pruning of the branch-and-fix trees for each scenario. The algorithm found the optimal solution of the problem in less than four hours, a very satisfactory result compared to the direct approach.

To underline the true value of the stochastic solution as compared to the traditional planning approach of using the average for uncertain parameters, we conducted three simulations under the assumption that the decisions for the entire planning horizon are adopted at the first time period in the average scenario approach. The BFC approach performed very well compared to the average scenario approach in the three simulations. Among its main advantages, BFC always returns sufficiently good solutions for all scenarios at once, and the solutions are always feasible. This is not the case with the average scenario, which for many scenarios yielded infeasible solutions.

Regarding future research, it would be interesting to study the application of the algorithmic methodology developed here to the problem of forestry planning in which the uncertainty of forest fires is incorporated into scenarios jointly with the wood sale price evolution over time.

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