8-2017

Omnichannel Retail Operations with Buy-Online-and-Pick-up-in-Store

Fei Gao
University of Pennsylvania

Xuanming Su
University of Pennsylvania

Recommended Citation

Gao, F., & Su, X. (2017). Omnichannel Retail Operations with Buy-Online-and-Pick-up-in-Store. *Management Science, 63* (8), 2478-2492. http://dx.doi.org/10.1287/mnsc.2016.2473

Omnichannel Retail Operations with Buy-Online-and-Pick-up-in-Store

Abstract

Many retailers have recently started to offer customers the option to buy online and pick up in store (BOPS). We study the impact of the BOPS initiative on store operations. We build a stylized model where a retailer operates both online and offline channels. Customers strategically make channel choices. The BOPS option affects customer choice in two ways: by providing real-time information about inventory availability and by reducing the hassle cost of shopping. We obtain three findings. First, not all products are well suited for in-store pickup; specifically, it may not be profitable to implement BOPS on products that sell well in stores. Second, BOPS enables retailers to reach new customers, but for existing customers, the shift from online fulfillment to store fulfillment may decrease profit margins when the latter is less cost effective. Finally, in a decentralized retail system where store and online channels are managed separately, BOPS revenue can be shared across channels to alleviate incentive conflicts; it is rarely efficient to allocate all the revenue to a single channel.

Keywords

retail operations, omnichannel, strategic customer behavior, decentralization

Disciplines

Finance and Financial Management | Social and Behavioral Sciences

Omnichannel Retail Operations with Buy-Online-and-Pickup-in-Store

Fei Gao

The Wharton School, University of Pennsylvania, Philadelphia, PA, 19104

feigao@wharton.upenn.edu

Xuanming Su

The Wharton School, University of Pennsylvania, Philadelphia, PA, 19104

xuanming@wharton.upenn.edu

Abstract

Many retailers have recently started to offer customers the option to buy online and pick up in store (BOPS). We study the impact of the BOPS initiative on store operations. We build a stylized model where a retailer operates both online and offline channels. Consumers strategically make channel choices. The BOPS option affects consumer choice in two ways: by providing real-time information about inventory availability and by reducing the hassle cost of shopping. We obtain three findings. First, not all products are well-suited for in-store pickup; specifically, it may not be profitable to implement BOPS on products that sell well in stores. Second, BOPS enables retailers to reach new customers, but for existing customers, the shift from online fulfillment to store fulfillment may decrease profit margins when the latter is less cost effective. Finally, in a decentralized retail system where store and online channels are managed separately, BOPS revenue can be shared across channels to alleviate incentive conflicts; it is rarely efficient to allocate all the revenue to a single channel.

Keywords: retail operations, omnichannel, strategic consumer behavior, decentralization

1 Introduction

As consumers become accustomed to online shopping, brick-and-mortar retailers have increasingly supplemented their shops with online businesses (Financial Times, 2013). The online channel has traditionally been viewed as a separate way to sell products. Today, however, many retailers have realized the need to integrate their existing channels to enrich customer value proposition and improve operational efficiency. As a result, there is an emerging focus on "omnichannel retailing" with the goal of providing consumers with a seamless shopping experience through all available shopping channels (Bell et al., 2014; Brynjolfsson et al., 2013; Rigby, 2011). When asked about omnichannel priorities, the retailers surveyed by Forrester Research reported that fulfillment initiatives ranked higher than any other channel integration program; moreover, among all omnichannel fulfillment initiatives, allowing customers to buy online and pick up in store (BOPS) is regarded as the most important one (Forrester, 2014). According to Retail Systems Research (RSR), as of June 2013, 64% of retailers have implemented BOPS (RSR, 2013).

Retailers benefit from allowing customers to pick up their online orders in store. Specifically, BOPS generates store traffic and potentially increases sales (New York Times, 2011). According to a recent UPS study, among those who have used an in-store pickup option, 45% of them have made a new purchase when picking up the purchase in store (UPS, 2015). Typically, a substantial amount of store sales is generated through such cross-selling: it is estimated that, on average, when a customer comes to the store intending to buy \$100 worth of merchandise, they leave with \$120 to \$125 worth of merchandise (Washington Post, 2015). Thus, unsurprisingly, more and more retailers are starting to offer the BOPS functionality on their websites (RSR, 2013).

A key challenge facing retailers is to choose the right set of products for BOPS. Most retailers generally agree that BOPS should not be a blanket functionality that is blindly applied to all products across all categories. According to senior vice president and general manager of Walmart.com, Steve Nave, one of the reasons for being selective is to focus on those products that would "drive more customers into the stores" (Time, 2011). On the websites of major retailers such as Toys R Us, customers will find that some products are not eligible for BOPS. For example, new releases such as the LEGO Star Wars Sith Infiltrator are not available for store pickup. These items are sold in stores, but shoppers need to check their local stores for availability if they do not wish to wait for online delivery. In contrast, for most of the extensive line of LEGO products sold on toysrus.com, the BOPS option is available. Since retailers typically carry large numbers of SKUs online, a key challenge is to understand the main criteria for selecting which product to allow for in-store pickup.

Many retailers regard BOPS as a way to reach new customers, as this new fulfillment option has become increasingly popular among shoppers (New York Times, 2011). With BOPS, consumers experience instant gratification, avoid shipping and delivery changes, and enjoy the convenience of hassle-free shopping (their items have already been picked and packed by store staff by the time they arrive). With this unique combination that has never been offered before, it is not unrealistic for retailers to expect market expansion. However, another more pessimistic view is that BOPS simply shifts customers from online fulfillment to store fulfillment; customers who use BOPS would have purchased online anyway. To understand the impact of BOPS on retailers' bottom-lines, it is useful to distinguish between the demand cannibalization and demand creation scenarios described above.

The advent of BOPS blurs the distinction between store and online operations. Although BOPS orders originate online, they are fulfilled using inventory in retail stores. Consequently, a successful BOPS implementation requires good coordination between the online and offline channels. Very few retailers have completely dismantled their online and offline channel silos, maintaining a single accounting ledger with an associated organizational structure for all sales regardless of channel (Forrester, 2014). When online and offline channels are operated by separate teams, the company needs to decide how to allocate BOPS revenue. On this subject, there is no consensus: 46% of retailers allow the online channel to receive full credit for the transaction, 31% award full credit to the store channel, and 23% divide them between channels (Forrester, 2014). This lack of consensus is not surprising, since both channels have legitimate reasons to claim credit. The store incurs operating costs for fulfilling demand, while the online channel is the source of demand in the first place.

In this paper, we focus on the following research questions:

1. For what types of product will the BOPS option be profitable?

2. How does BOPS impact the retailer's customer base?

3. How should BOPS revenue be allocated between store and online channels?

To address these questions, we develop a stylized model that captures essential elements of omni-channel retail environments. There is a retailer who operates online and store channels, with the goal of maximizing total expected profit over both channels. Consumers strategically choose among buying online, buying in store, and buying online for store pickup, to maximize individual utility. We first analyze the centralized system; specifically, for a particular product with given financial parameters, we study optimal inventory decisions under BOPS and examine the impact of BOPS on total profits. Using these results, we determine whether a product should be carried in store and whether BOPS should be offered. Finally, we consider the decentralized system and examine how to allocate BOPS revenue between channels.

Our first main finding is that BOPS may not be suitable for all products. Specifically, for products that are bestsellers in retail stores, the benefit of BOPS may be outweighed by the drawback, which is as follows. Since products that are available for store-pickup must be in stock, BOPS indirectly discloses real-time store inventory status. Customers initiating a BOPS order online and finding that the desired item is out of stock will not visit the store. In this way, stockouts of blockbuster products may drive customers away, thus reducing store traffic and cross-selling opportunities. In other words, BOPS may compromise the function of bestselling products in attracting customers to the store.

Our second result is that BOPS helps retailers expand their market coverage. As BOPS mitigates the stockout risk and hassle costs during the shopping journey, more people will be willing to consider buying from the retailer. However, apart from attracting new customers, BOPS can also sway the channel choices of existing customers. Among these existing customers, some who had waited for their orders to be shipped to them (i.e., online fulfillment) may now choose to pick up their orders (i.e., store fulfillment) instead. This shift is unprofitable if the profit margin is lower in stores compared to the online channel.

Finally, in decentralized systems where store and online channels are operated by separate entities, we identify a misalignment of incentives. Specifically, the store neglects the fact that potential BOPS customers may purchase online instead when there is no stock in the store for them to pick up. Online sales generate value for the company but the store is not explicitly compensated when they occur. Consequently, the store stocks too much inventory if they retain 100% of BOPS revenue. To correct for this potential incentive problem, it is optimal to give the store channel partial credit for fulfilling BOPS demand.

2 Literature Review

This paper studies the management of online and offline channels. With the advent of e-commerce around the turn of the century, many manufacturers or suppliers introduced a direct online selling channel, which competes with their own retail partners. Much of the literature on channel management studies this type of business setting. Chiang et al. (2003) study a price setting game between a manufacturer and its independent retailer. They find the manufacturer is more profitable even if no sales occur in the direct channel, because the manufacturer can use the direct channel to improve the functioning of the retail channel by preventing the prices from being too high and thus leading to more sales or orders from the retailer. Some other papers also study the pricing game, but are more concerned with specific pricing mechanisms, e.g., price matching between channels (Cattani et al., 2006) and personalized pricing (Liu and Zhang, 2006). Apart from pricing, Tsay and Agrawal (2004) consider firms' sales effort and find that both parties can benefit from the addition of a direct channel. Chen et al. (2008) study service competition between the two channels and characterize the optimal channel strategy for the supplier. Netessine and Rudi (2006) study the practice of drop-shipping, where the supplier stocks and owns the inventory and ships products directly to customers at retailers' request. In contrast to this stream of work, our paper focuses on a single retailer who manages both online and store channels, as commonly observed in retail environments today.

Omnichannel management has received a lot of attention in industry; the topic is broadly surveyed in Bell et al. (2014), Brynjolfsson et al. (2013) and Rigby (2011). In addition, there are a few other papers. Ansari et al. (2008) empirically study how customers migrate between channels in a multichannel environment and the role of marketers in shaping migration through their communications strategy. Chintagunta et al. (2012) study consumer channel choice in grocery stores and empirically quantify the relative transaction costs when households choose between the online and offline channels. Ofek et al. (2011) focus on the impact of product returns on a multichannel retailer, and use a theoretical model to examine how pricing strategies and physical store assistance levels change as a result of the additional online outlet. Gallino and Moreno (2014) empirically investigate the impact of BOPS on a retailer's sales in both online and offline channels. Interestingly, they find that instead of increasing online sales, the implementation of BOPS is associated with a reduction in online sales and an increase in store sales and traffic. While most papers in this area are empirical, our paper develops a tractable theoretical framework. Using our model, we study omnichannel inventory management and channel coordination within the firm.

There are models in operations management that study the role of inventory availability on demand, given that consumers have to incur hassle costs and bear the stockout risk when visiting stores. Dana and Petruzzi (2001) is the first paper that extends the classic newsvendor model by assuming demand is a function of both price and inventory level. Since then, many researchers have investigated how to attract people to pay the hassle cost to come to the store: Su and Zhang (2009) show that it is always beneficial to the retailer if he can credibly make an ex ante quantity commitment. Yin et al. (2009) compare the efficacy of two different in-store inventory display formats to manipulate consumer expectations on the availability. Allon and Bassamboo (2011) explore the issue of cheap talk when the information shared by the retailer is not verifiable. Alexandrov and Lariviere (2012) examines the role of reservations in the context of revenue management. Cachon and Feldman (2015) focus on retailer's pricing issue and find that a strategy that embraces frequent discounts is optimal. In this paper, we study a new way to attract customers to store, BOPS, by which the retailer shares with customers the real-time information about store inventory status.

A critical feature of our model is that customers will make additional purchases once they enter the store. There are many papers in marketing (e.g., Li et al. (2005); Akçura and Srinivasan (2005); Li et al. (2011)) and operations management (e.g., Netessine et al. (2006); Gurvich et al. (2009); Armony and Gurvich (2010)) examining how to make use of the cross-selling opportunities during the interaction with consumers. In this paper, instead of studying the design of a cross-selling strategy, we will focus on the impact of cross-selling benefits on the implementation of the new omnichannel fulfillment strategy, i.e., BOPS.

This paper provides a different angle to the stream of work on strategic customer behavior in retail management. Su (2007) study a dynamic pricing problem with a heterogeneous population of strategic as well as myopic customers and show that optimal price paths could involve either markups or markdowns. Aviv and Pazgal (2008) study two types of markdown pricing policies (i.e., contingent and announced fixed-discount) in the presence of strategic consumers. There is a rich body of work on operational strategies that consider strategic customers: e.g., capacity rationing (Liu and van Ryzin, 2008), supply chain contracting (Su and Zhang, 2008), quick response (Cachon and Swinney, 2009), opaque selling (Jerath et al., 2010), posterior price matching (Lai et al., 2010) and product rollovers (Liang et al., 2014). Most of the existing literature consider models in which consumers decide whether to make an immediate purchase or to wait for future discounts. In contrast, our paper concentrates on consumers' channel choice. In other words, instead of studying the decision of *when* to buy, we pay attention to the decision of *where* to buy.

The topic of decentralization has been studied by many researchers in the operations management area. There are two streams of literature, focusing on *interfirm* and *intrafirm* coordination. For the former, there is a large body of literature on the design of optimal contracts to optimize supply chain performance; see, for example, Lee and Whang (1999); Taylor (2002); Cachon and Lariviere (2005). Readers can refer to Cachon (2003) for a comprehensive review. The other research stream addresses conflicts of interest within a firm. Harris et al. (1982) study an intrafirm resource allocation problem where different divisional managers of the firm possess private information that is not available to the headquarters. Porteus and Whang (1991) examine different incentive plans to coordinate the marketing and manufacturing managers of the firm. A similar cross-functional coordination problem is also studied by Kouvelis and Lariviere (2000). In retail assortment planning, Cachon and Kök (2007) study category management, where each category is managed separately by different managers. In line with this research stream, we look at incentive conflicts between the store channel and the headquarters of an omnichannel retailer.

3 Model

There is a retailer who sells a product through two channels, store and online, at price p. In the store channel, the retailer faces a newsvendor problem: there is a single inventory decision q to be made before random demand is realized, so there may eventually be unmet demand or leftovers in the store. The unit cost of inventory is c, and the salvage value of leftover units is normalized to zero. The online channel is modeled exogenously: the retailer simply obtains a net profit margin w from each unit of online demand. This model focuses on store operations and can be separately applied to any particular product the retailer carries.

The market demand D is random and follows a continuous distribution F and density f. Consumers choose between store and online channels to maximize their utility. Each individual consumer has valuation v for the product. When shopping in store, each consumer incurs hassle cost h_s (e.g., traveling to the store or searching for the product in aisles); similarly, when shopping online, each consumer incurs hassle cost h_o (e.g., paying shipping fees or waiting for the product to arrive). There is a key difference between store and online hassle costs: h_s is incurred *before* customers find and purchase the product in the store, whereas h_o is incurred *after* customers make the purchase online. To ensure that consumers are willing to consider both channels, we assume that both hassle costs are smaller than the surplus $v - p$.

We first consider the scenario before BOPS is introduced; here, each consumer makes a choice between shopping online directly or going to the store. If she chooses to buy online directly, her payoff is simply given by

$$u_o = v - p - h_o.$$

On the other hand, if she chooses to go to the store, her payoff is

$$u_s = -h_s + \hat{\xi}(v - p) + (1 - \hat{\xi})(v - p - h_o).$$

To understand this expression, note that the consumer first incurs the hassle cost h_s upfront. Then, once she is in the store, she may encounter two possible outcomes: (1) if the store has inventory, then she can make a purchase on the spot and receive payoff $v - p$; (2) if the store is out of stock, she can go back to buying the product online and receive payoff $v - p - h_o$. The consumer expects the former to occur with probability $\hat{\xi}$. Based on this belief, the consumer compares the expected utility from each channel and chooses accordingly.

Next, we consider retailer's decision problem. First, the retailer anticipates that a fraction $\hat{\phi} \in [0, 1]$ of customers will visit the store; i.e., if total demand is D, the retailer expects that the number of customers coming to the store will be $\hat{\phi}D$. Given this belief, the retailer's profit function is

$$\pi(q) = pE \min\left(\hat{\phi}D, q\right) - cq + rE\left(\hat{\phi}D\right) + wE\left(\left(1 - \hat{\phi}\right)D\right) + wE\left(\hat{\phi}D - q\right)^+. \tag{1}$$

Given the store inventory level q, the newsvendor expected profit from selling the product in the store channel is shown in the first two terms above. In addition, since customers tend to make additional purchase when they come to the store (UPS, 2015; Washington Post, 2015), there is an additional profit r from every customer coming to the store; this is the third term in the profit function above. The last two terms above show the retailer's online profit; the fourth represents profit from customers who shop online directly and the last represents profit from customers who switch to online after encountering stockouts in store. With the profit function above, the retailer chooses q to maximize expected profit.

To study the strategic interaction between the retailer and the consumers, we shall use the notion of rational expectations (RE) equilibrium (see Su and Zhang (2008, 2009); Cachon and Swinney (2009)). One important feature of a RE equilibrium is that beliefs must be consistent with actual outcomes. In other words, the retailer's belief $\hat{\phi}$ must coincide with the true proportion ϕ of consumers choosing the store channel, and consumers' beliefs over in-store inventory availability probability $\hat{\xi}$ must agree with the actual in-stock probability corresponding to the retailer's chosen quantity q. According to Deneckere and Peck (1995) and Dana (2001), this probability is given by $A(q) = E\min(\phi D, q)/E(\phi D)$ where $\phi > 0$. The reason is as follows. Conditional on her own presence in the market, an individual consumer's posterior demand density is $g(x) = xf(x)/ED$. Therefore, given this posterior demand density, the availability probability is $\int (\min(\phi x, q)/\phi x)\, g(x)\, dx = A(q)$, since the product is available with probability $\min(\phi x, q)/\phi x$ when there are x consumers in the market. Then, we have the following definition for a RE equilibrium. Henceforth, we refer to the RE equilibrium as "equilibrium" for brevity.

Definition 1. *A RE equilibrium $(\phi, q, \hat{\xi}, \hat{\phi})$ satisfies the following:*

i Given $\hat{\xi}$, if $u_s \geq u_o$, then $\phi = 1$; otherwise $\phi = 0$;

ii. Given $\hat{\phi}$, $q = \arg\max_q \pi(q)$, where $\pi(q)$ is given in (1);

iii. $\hat{\xi} = A(q)$;

iv. $\hat{\phi} = \phi$.

Conditions (i) and (ii) state that under beliefs $\hat{\xi}$ and $\hat{\phi}$, consumers and the retailer are choosing the optimal decisions. Conditions (iii) and (iv) are the consistency conditions.

First, it is easy to see that there always exists a *nonparticipatory* equilibrium $(0, 0, 0, 0)$. If the retailer expects no one comes to the store to buy the product, he will stock nothing there, i.e., $q = 0$; If consumers believe there is no inventory in the store, they will not come, i.e., $\phi = 0$. In the end, we have a self-fulfilling prophecy and beliefs are trivially consistent with the actual outcome.

Is there any *participatory* equilibrium where consumers are willing to visit store and the retailer has stock in the store as well (i.e., $\phi = 1$ and $q > 0$)? When such an equilibrium exists, it Pareto-dominates the nonparticipatory equilibrium, because it generates positive payoffs for both the retailer and consumers. We shall adopt the Pareto dominance equilibrium selection rule.

The following proposition gives the equilibrium result; we use the superscript $(\cdot^\circ)$ to denote the equilibrium outcome for this basic scenario. All proofs are presented in Supplementary Appendix A.

Proposition 1. *If* $h_s \leq \xi^\circ \cdot h_o$ *and* $p - c > w$*, then customers visit store and* $q^\circ = \bar{F}^{-1}(\frac{c}{p-w})$*. Otherwise, no one comes to store and* $q^\circ = 0$*. Here,* $\xi^\circ = \frac{E\min\left(D, \bar{F}^{-1}\left(\frac{c}{p-w}\right)\right)}{ED}$ *is the equilibrium in-stock probability at the store.*

According to Proposition 1, in order to have positive sales in the store, we need to ensure that the store channel is attractive to both the consumers and the retailer. As for the retailer, it is profitable for him to sell through the store channel only if he could get a higher margin by selling offline than online (i.e., $p - c > w$). Nonetheless, even if store fulfillment is attractive to the retailer, store sales can occur only if consumers are willing to pay a visit to the store. The first condition of Proposition 1 ensures that: (i) the store in-stock probability is large enough, and (ii) the online hassle cost dominates the store hassle cost, so that when put together, consumers are willing to risk encountering stockouts and come to the store. Under the conditions of Proposition 1, there is a participatory equilibrium.

Now, we turn to the scenario where the retailer implements BOPS on the product. With this added functionality, consumers assess information online and face one of two possible situations. The first possibility is that the product is out of stock at the store and BOPS is not an option; in this case, the consumer simply buys from the online channel. The other possibility is that the product is in stock and BOPS is feasible; in this case, the consumer chooses where to shop and we discuss this decision problem below. We stress that with the introduction of BOPS, consumers no longer have to form beliefs about inventory availability because this information is immediately accessible online. In other words, a useful by-product of BOPS is inventory availability information, which is provided on a real-time basis.

When BOPS is a viable option, the consumer faces a choice between three alternatives: buy online, buy in store, or use BOPS. To distinguish between the last two options, we introduce a new model parameter h_b, which is the hassle cost associated with using BOPS. Although BOPS consumers still need to go to the store after making their purchases online, the process is different from buying in store; for example, BOPS consumers do not search for products in store because their orders would already have been picked and packed by store staff. Therefore, the BOPS hassle cost h_b differs from the store and online hassle costs h_s, h_o. With this setup, all three alternatives yield the utility $v-p-h_i$, where the hassle cost h_i corresponds to the shopping mode chosen by the consumer. In other words, utility maximization boils down to choosing the shopping mode with the lowest cost. When the online channel offers the lowest hassle cost, consumers never go to the store. When the store hassle cost is lowest, consumers buy in store but only after verifying online that the product is in stock. When the BOPS hassle cost is lowest, consumers place orders online for store pickup.

We are now ready to write down the retailer's profit function with BOPS. When consumers choose to go to the store (i.e., when the online hassle cost h_o exceeds either the store hassle cost h_s or the BOPS hassle cost h_b), the profit function is

$$\pi(q) = pE\min(D,q) - cq + rE\min(D,q) + wE(D-q)^+ .$$

This is because when the store inventory level is q, there are on average $E\min(D,q)$ customers who come to the store, since they come to the store only when a corresponding unit is available. The first two terms above correspond to the newsvendor profit from selling the product and the third term corresponds to the additional cross-selling profit. Finally, when demand exceeds store inventory, customers who find that the store is out of stock can still choose to buy online; this yields the last term. In the other case where all consumers prefer shopping in the online channel (i.e., when h_o is the smallest hassle cost), the retailer will stock nothing in the store (i.e., $q=0$) and earn an expected profit $\pi = wED$.

We use superscript $(\cdot^*)$ to denote the market outcome with BOPS, which is given in the following proposition:

Proposition 2. *If $\min(h_s,h_b) \leq h_o$ and $p-c > w-r$, then customers visit the store and $q^* = \bar{F}^{-1}\left(\frac{c}{p+r-w}\right)$. Otherwise, no one comes to store and $q^*=0$.*

Proposition 2 (after BOPS) differs from Proposition 1 (before BOPS) in three significant ways. First, the condition $h_s \leq \xi^\circ \cdot h_o$ in Proposition 1 is weakened to $\min(h_s,h_b) \leq h_o$ in Proposition 2; in particular, the term corresponding to the in-stock probability ξ° vanishes. This discrepancy suggests that the risk of stockouts is no longer of concern after the introduction of BOPS. Indeed, BOPS provides real-time inventory information that essentially guarantees availability once an order is placed. In this way, BOPS attracts consumers to the store. The second difference is that the condition $p-c>w$ in Proposition 1 is weakened to $p-c>w-r$ in Proposition 2. The former condition requires the margin to be higher in store than online for the retailer to carry the product in store, but the latter condition combines the store margin with the cross-selling benefit. In other words, BOPS makes it more attractive for retailers to carry products in store; due to the cross-selling benefit r, the retailer may wish to carry a product in store even when the store margin is lower than the online margin. However, inventory now becomes more important: with BOPS, the retailer loses both the product margin $p-c$ as well as the cross-selling benefit r in the event of a stockout. This brings us to the third difference between Propositions 1 and 2: the critical fractile and hence the in-stock probability is higher with BOPS. This occurs because the underage cost increases from $p-c-w$ to $p-c-w+r$ after the introduction of BOPS. Consequently, the retailer has the incentive to increase inventory to lure more customers to the store.

In summary, we find that BOPS impacts store operations in two main ways. First, BOPS expands the set of products that may be offered at the store. For such products, omnichannel consumers who were previously unwilling to buy in store can be swayed by BOPS to visit the store. These products may originally be online exclusives but are now profitable to bring to retail stores. Second, for products that were originally carried at the store, BOPS leads to an increase in the in-stock probability. In other words, the store channel stocks more inventory, and would consequently end up with more leftover inventory. Excess inventory appears to be an inevitable downside of BOPS implementations (Reuters, 2014), but despite this downside, the next section shows that BOPS may be accompanied by increased profits.

4 Information Effect and Convenience Effect

In this section, we compare the market outcomes before and after the introduction of BOPS (i.e., Propositions 1 and 2). With the following comparison results, we are able to identify two main effects of BOPS, i.e., the information effect and the convenience effect.

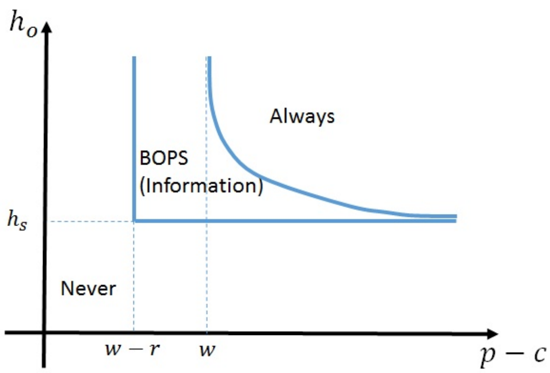
Let us first describe the framework for our analysis. Based on Propositions 1 and 2, we note there are three parameter regions. In some cases, consumers who were initially unwilling to visit the store will find the trip more appealing after the BOPS option is made available. In some other cases, BOPS will have no impact on channel choice: consumers always prefer a particular channel regardless of the BOPS option. These possibilities are summarized in Figure 1 below. Specifically, our three parameter regions are as follows:

i. In the "Always" regions (i.e., $h_s \leq \frac{E\min\left(D,\bar{F}^{-1}\left(\frac{c}{p-w}\right)\right)}{ED}h_o$ and $p-c>w$), consumers always buy the product in store, regardless of the implementation of BOPS;

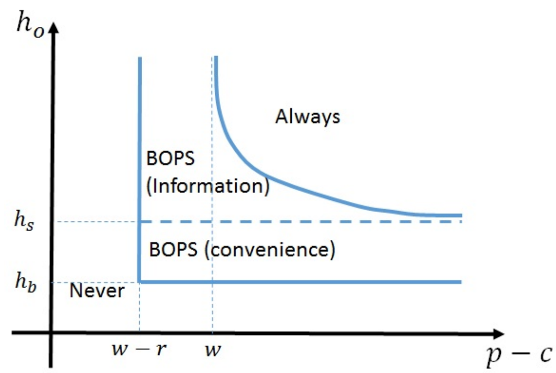
ii. In the "Never" regions (i.e., $\min(h_s,h_b)>h_o$ or $p-c\leq w-r$), consumers never come to the store, preferring to buy the product online;

iii. In the "BOPS" regions (i.e., $h_s > \frac{E\min\left[D,\bar{F}^{-1}\left(\frac{c}{p-w}\wedge 1\right)\right]}{ED}h_o$, $\min(h_s,h_b)\leq h_o$ and $p-c>w-r$), consumers come to the store only if BOPS is available. The "BOPS" regions are further labeled "Information" or "Convenience" as discussed below.

Figure 1: Do consumers buy the product in store?



(a) $h_b > h_s$



(b) $h_b \leq h_s$

In the following analysis, we will examine the impact of BOPS on the retailer's profit by separately considering different parameter regions in Figure 1. We begin with the following proposition.

Proposition 3. *If* $h_s \in \left(\frac{E\min\left[D,\bar{F}^{-1}\left(\frac{c}{p-w}\wedge 1\right)\right]}{ED}h_o, h_o\right]$ *and* $p-c>w-r$*, then customers visit the store only if BOPS is available. Further, BOPS increases total profit (i.e.,* $\pi^* > \pi^\circ$*).*

The conditions in Proposition 3 correspond to the "BOPS" regions labeled "Information" in Figures 1a and 1b. In these parameter regions, BOPS influences consumer shopping behavior through the information sharing mechanism discussed in the previous section. By revealing real-time information about store inventory status, BOPS draws additional customers to the store; these customers were previously unwilling to visit the store because they were discouraged by the possibility of stockouts. In such cases, Proposition 3 confirms that BOPS leads to increased profit for the retailer. This increase in profit arises because the store profit margin $p-c$, combined with the cross-selling benefit r, exceeds the online margin w. In other words, through information provision, BOPS brings about a demand shift to the more profitable store channel.

There is a subtle difference between the two "BOPS (Information)" regions of Figures 1a and 1b. Although demand shifts to the store in both cases, they occur in different ways. In the "BOPS (Infomation)" region of Figure 1a, since the pickup hassle cost h_b exceeds the store hassle cost h_s, offering BOPS induces consumers to buy in store after verifying availability online, without actually using the BOPS functionality. On the other hand, in the corresponding region of Figure 1b, consumers indeed buy online and pickup in store when the option is available. We separately discuss these two behaviors in the next two paragraphs.

When consumers verify availability online without actually using the BOPS functionality, BOPS simply serves as a source of information. The same market outcome arises if the retailer simply provides real-time availability information on the website (i.e., directly showing whether or not store is in stock). This strategy has been adopted by retailers such as Gap and Levi's. Our model can be applied to study this pure information sharing mechanism, which can be regarded as a special case with $h_b > h_s$. In this special case, BOPS generates an interesting dynamic: after the implementation of BOPS as an added online functionality, online sales may *decrease*, while store sales may increase. This phenomenon was first identified by Gallino and Moreno (2014), who undertake a comprehensive empirical study of a US retailer with a recent BOPS implementation.

On the other hand, when the pickup process is relatively hassle-free, consumers will indeed buy online and pickup in store. In this case, apart from eliminating the risk of stockouts as described above, BOPS also provides consumers with a more convenient means of shopping. In this sense, comparing the two "BOPS (Information)" regions in Figures 1a and 1b, consumer surplus is higher in the latter than in the former.

The next proposition examines the "BOPS" region labeled "Convenience" in Figure 1b.

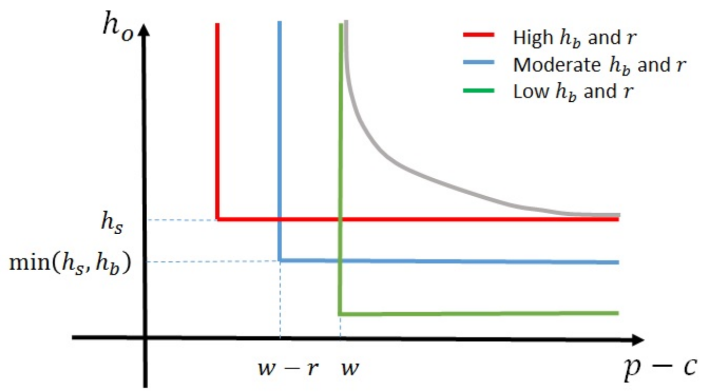
Proposition 4. *If $h_b \leq h_o < h_s$ and $p - c > w - r$, customers visit the store only if BOPS is available. Further, BOPS increases total profit (i.e., $\pi^* > \pi^\circ$).*

The above result highlights the importance of shopping convenience for BOPS to attract consumers to the store. By additionally providing convenience, BOPS becomes more powerful than a pure information sharing mechanism. In the "BOPS (Convenience)" region of Figure 1b, a pure information sharing mechanism can never attract customers to the store; even if customers are guaranteed availability in store, they still prefer to buy online because the online hassle cost is lower than the store hassle cost (i.e., $h_o < h_s$). However, once BOPS is available and provides convenience that trumps an online order (i.e., $h_b < h_o$), customers may now prefer to buy online and pick up in store. This shopping mode benefits the retailer because customers may buy additional products (yielding profit r) when they pick up their products. As long as the store margin $p-c$ and cross-selling benefit r exceeds the online margin w, the convenience dimension of BOPS will lead to increased profit for the retailer.

Proposition 4 provides a word of caution for retailers. Although making the pickup process more convenient is potentially a good way to improve the profitability of BOPS, retailers should exercise care in preserving the cross-selling benefit. In particular, some retailers have introduced drive-through service that allows customers to receive their orders without leaving their cars (New York Times, 2012; Bloomberg, 2012). Although this will help to reduce hassle in the pickup process, it will also prevent people from entering the store and thus lead to a loss of the cross-selling benefit r. According to Proposition 4, if the margin from selling this particular product in the store is very high (i.e., $p - c > w$), then it is still profitable for the retailer to implement BOPS even if $r = 0$. However, if profit margins are lower in store than online, then the cross-selling profit r plays an important role; in this case, drive-through service may hurt the retailer's overall profit by neutralizing the advantages of cross-selling.

There is a delicate balance between pickup convenience and cross-selling potential. While consumers appreciate a more convenient pickup process, retail managers wishing to make the most out of the cross-selling opportunity may choose to locate the pick-up counter at far corners of the store so that shoppers have to walk through the entire store before picking up their online orders (Retail Dive, 2015). This tradeoff is illustrated in Figure 2. At one extreme, setting the pickup counter at the back of store maximizes both pickup cost h_b and cross-selling benefit r (i.e., the red L-line). At the other extreme, providing drive through pickup service minimizes both h_b and r (i.e., the green L-line). As both h_b and r increase, the L-line in Figure 1 moves up and left, and the "BOPS" region changes. The optimal location of the L curve depends on retailer's portfolio of products. According to Figure 2, if most of the retailer's products have high store profit margins (i.e., $p - c$ is large) and can be easily purchased online (i.e., h_o is small), then the retailer should seek to make the pickup process more convenient; in contrast, if most of the retailer's products have low store profit margins (i.e., $p - c$ is small) and are difficult to purchase online (i.e., h_o is large), then setting the pickup counter far from the store entrance is a better strategy.

Figure 2: Impacts of BOPS hassle cost h_b and cross-selling benefit r



The next proposition tells a different side of the story. Although BOPS brings about many benefits, it may lead to reduced profits in some cases.

Proposition 5. *If* $h_s \leq \frac{E\min\left(D, \bar{F}^{-1}\left(\frac{c}{p-w}\right)\right)}{ED} h_o$ *and* $p - c > w$*, customers visit the store regardless of the implementation of BOPS. Further, if* $r > 0$*, then BOPS decreases total profit (i.e.,* $\pi^* < \pi^\circ$*).*

The conditions of Proposition 5 correspond to the "Always" regions in Figures 1a and 1b. In these regions, consumers choose to visit the store regardless of BOPS. Here, BOPS has an important but easily overlooked effect. Prior to the introduction of BOPS, all customers were already willing to visit the store, but after BOPS is made available, fewer consumers will come to the store. This is because customers who attempt to place an order online but find that the item is not in stock for pickup will no longer go to the store. As store traffic decreases, the retailer loses the potential profit from cross-selling. The loss of cross-selling benefits (i.e., whenever $r > 0$) leads to a reduction in total profits, as shown in Proposition 5. Since BOPS can be selectively implemented, our result suggests that the BOPS option should not be offered on products that have been attracting considerable demand to the store.

Our results differ from existing findings in the literature because we study a different information sharing mechanism. In our model, BOPS provides real-time information about the store inventory status, i.e., customers are informed that the product is available until inventory runs out. However, in Su and Zhang (2009), the focus is on quantity commitment, i.e., the retailer commits to an initial inventory level in the store. With quantity commitment, the retailer may be able to use a small amount of store inventory to attract a large number of customers to visit the store.

In particular, when the cross-selling benefit is very large, the retailer may still choose to stock the product in the store even if it is more profitable to sell online, with the hope of attracting customers to make additional purchases in store. Such a "loss leader strategy" is no longer feasible when the retailer implements BOPS, because customers have access to real-time store inventory information and will not visit the store after the product is out of stock. Therefore, we find that BOPS, by providing real-time inventory information, may decrease profits, while Su and Zhang (2009) find that quantity commitment is generally valuable.

In summary, BOPS has two effects: it provides customers with real-time information about in-store inventory availability and it introduces a new shopping mode that may add convenience to customers. The former effect (information effect) helps attract customers to the store by letting them know about inventory availability, but it is a double-edged sword in that when inventory is not available, it turns away customers who might be willing to visit the store. The latter effect (convenience effect) applies when customers use the store pickup functionality, as opposed to simply using BOPS as a source of availability information; it draws customers to the store and may even open up new sources of demand.

When put together, the information and convenience effects of BOPS yield different profit implications. Figures 1a and 1b present a clear distinction: in the "BOPS" regions, BOPS leads to higher profits, but in the "Always" regions, BOPS leads to lower profits. The difference between these two regions is that, prior to the introduction of BOPS, consumers were already willing to visit the store in the latter but not in the former. These results suggest that BOPS should be offered for products with weak store sales but not those with strong records to begin with. In other words, it is likely profitable to implement BOPS on in-store "underdogs" but may not be so for in-store "favorites."

5 Heterogeneous Customers

In this section, we incorporate customer heterogeneity. For example, some customers may reside further away from the store than others; some may be more impatient than others and are thus more averse to waiting for online delivery. In our model, the store and online hassle costs h_s, h_o may now differ across customers. Specifically, customers are uniformly distributed across the following "square" $\{(h_s, h_o) | h_s \in [0, H], h_o \in [0, H]\}$, where $H > v - p$ (i.e., some customers have a prohibitively high hassle cost in one channel). The goal is to study the impact of BOPS on a retailer's customer base in such a heterogeneous market.

We begin by considering the scenario in the absence of BOPS. In this case, each customer has three options: go to the store, buy online, or leave the market. The corresponding utilities are:

$$\begin{aligned} u_s &= -h_s + \hat{\xi}(v-p) + (1-\hat{\xi})u_o^+, \\ u_o &= v - p - h_o, \\ u_l &= 0, \end{aligned}$$

where $\hat{\xi}$ denotes the belief about store inventory availability as before. Note that customers who find the store out of stock will buy online only if doing so is preferred over leaving the market. As customers make utility-maximizing choices (which depend on their hassle costs h_s, h_o), the market is divided into four segments, as depicted in Figure 3(a). Specifically, there are "pure online" customers (who buy online directly), "store→online" customers (who visit the store but switch online when the store is out of stock), "pure store" customers (who visit the store exclusively), as well as customers who simply leave the market. Denote the fractions of these four types of customer as α_o, α_{so}, α_s, and α_l, respectively. Given that consumers are uniformly distributed in $\{(h_s, h_o)|h_s \in [0, H], h_o \in [0, H]\}$, we can find that $\alpha_o = \frac{v-p}{H} - \frac{\hat{\xi}(v-p)^2}{2H^2}$, $\alpha_{so} = \frac{\hat{\xi}(v-p)^2}{2H^2}$, $\alpha_s = \frac{\hat{\xi}(v-p)(H-(v-p))}{H^2}$, and $\alpha_l = \frac{[H-(v-p)][H-\hat{\xi}(v-p)]}{H^2}$.

On the supply side, the retailer faces the following profit function:

$$\pi(q) = pE\min\left(\left(\hat{\alpha}_s + \hat{\alpha}_{so}\right)D, q\right) - cq + rE\left(\hat{\alpha}_s + \hat{\alpha}_{so}\right)D + wE\hat{\alpha}_o D + wE\frac{\hat{\alpha}_{so}}{\hat{\alpha}_s + \hat{\alpha}_{so}}\left(\left(\hat{\alpha}_s + \hat{\alpha}_{so}\right)D - q\right)^+ \tag{2}$$

where $\hat{\alpha}_o$, $\hat{\alpha}_{so}$, $\hat{\alpha}_s$, and $\hat{\alpha}_l$ denote the retailer's beliefs over the α's above. Given the store inventory level q, the newsvendor expected profit from selling the product in the store channel is shown in the first two terms above. The third term captures the additional cross-selling profit r from every customer coming to the store. The last two terms above show the retailer's online profit. The fourth represents profit from customers who shop online directly; the last represents profit from customers who switch to online after encountering stockouts in store, in which case we assume store customers have equal chance of being rationed. With the profit function above, the retailer chooses q to maximize expected profit.

Definition 2. *A RE equilibrium* $(\alpha_o, \alpha_{so}, , \alpha_s, \alpha_l, q, \hat{\xi}, \hat{\alpha}_o, \hat{\alpha}_{so}, \hat{\alpha}_s, \hat{\alpha}_l)$ *satisfies the following:*

i Given $\hat{\xi}$*, then* $\alpha_o = \frac{v-p}{H} - \frac{\hat{\xi}(v-p)^2}{2H^2}$, $\alpha_{so} = \frac{\hat{\xi}(v-p)^2}{2H^2}$, $\alpha_s = \frac{\hat{\xi}(v-p)(H-(v-p))}{H^2}$, *and* $\alpha_l = \frac{[H-(v-p)][H-\hat{\xi}(v-p)]}{H^2}$;

ii. Given $\hat{\alpha}_o, \hat{\alpha}_{so}, \hat{\alpha}_s$ *and* $\hat{\alpha}_l$, $q = \arg\max_q \pi(q)$, *where* $\pi(q)$ *is given in (2);*

iii. $\hat{\xi} = A(q)$, *where* $A(q) = \frac{E\min((\alpha_s+\alpha_{so})D, q)}{E(\alpha_s+\alpha_{so})D}$;

iv. $\hat{\alpha}_s = \alpha_s$, $\hat{\alpha}_o = \alpha_o$, $\hat{\alpha}_{so} = \alpha_{so}$ *and* $\hat{\alpha}_l = \alpha_l$.

The following proposition gives the RE equilibrium. As before, we use the superscripts $\cdot^\circ$ and $\cdot^*$ to denote the no-BOPS and BOPS scenario, respectively.

Proposition 6. *If* $p-c > w\frac{v-p}{2H-(v-p)}$*, then there are customers visiting store (*$\alpha_s^\circ = \frac{\xi^\circ(v-p)(H-(v-p))}{H^2} > 0$, $\alpha_{so}^\circ = \frac{\xi^\circ(v-p)^2}{2H^2} > 0$*) and* $q^\circ = (\alpha_s^\circ + \alpha_{so}^\circ)\,\bar{F}^{-1}\left(\frac{c}{p-w\frac{v-p}{2H-(v-p)}}\right)$*, where the equilibrium store in-stock probability is* $\xi^\circ = \frac{\min\left(D, \bar{F}^{-1}\left(\frac{c}{p-w\frac{v-p}{2H-(v-p)}}\right)\right)}{ED}$*. Otherwise, no one comes to store and* $q^\circ = 0$, $\xi^\circ = 0$.

Next, we turn to the scenario where the retailer implements BOPS on the product. When a customer uses BOPS, she experiences hassle in both online and offline worlds. For example, she needs to go through the online payment process and she also has to go to the store to pick up the product. As a result, we assume the hassle cost of using BOPS is given by $h_b = \beta_s h_s + \beta_o h_o$, where $\beta_s, \beta_o \in (0, 1)$. Further, as explained before, all customers have access to information about store inventory status before they visit the store.

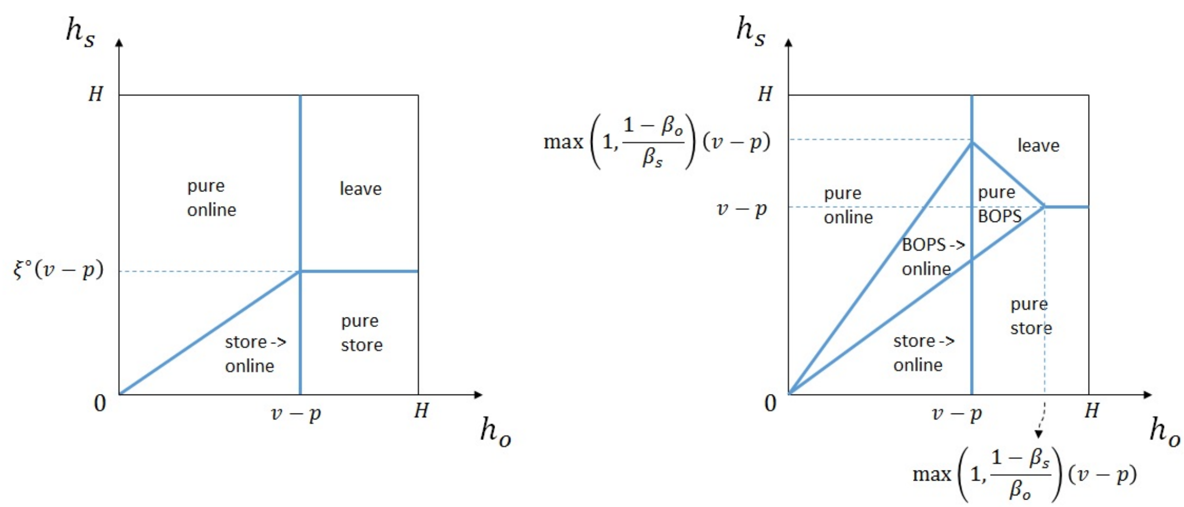
Now, we consider customer choice in the presence of BOPS. There are two cases to consider. First, when the store is in stock, the consumer faces a choice between four alternatives: buy online (with payoff $v - p - h_o$), buy in store (with payoff $v - p - h_s$), use BOPS (with payoff $v - p - h_b$), or leave (with payoff 0). Based on their individual hassle costs, consumers choose their shopping mode with the highest payoff. Second, when the store is out of stock, only customers with $h_o < v - p$ will choose to buy online, while the rest will leave. With the above decisions, the market is divided into six segments, as depicted in Figure 3(b). In addition to the four segments described before, we now see "BOPS→online" customers (who use BOPS if the store is in stock but switch online otherwise) and "pure BOPS" customers. Denote the fraction of these two new types of customer as α_{bo}^* and α_b^*. We can calculate the sizes of these six customer segments as follows:

$$\alpha_i^* = \iint_{A_i^*} \frac{1}{H^2} dh_s dh_o, \; i = o, so, s, l, bo, b$$

where

$$\begin{aligned}
A_o^* &= \{(h_s, h_o) \,| v - p - h_o > \max(v - p - h_s, v - p - h_b, 0)\} \\
A_{so}^* &= \{(h_s, h_o) \,| v - p - h_s > \max(v - p - h_o, v - p - h_b, 0), v - p - h_o > 0\} \\
A_s^* &= \{(h_s, h_o) \,| v - p - h_s > \max(v - p - h_o, v - p - h_b, 0), 0 > v - p - h_o\} \\
A_l^* &= \{(h_s, h_o) \,| 0 > \max(v - p - h_o, v - p - h_s, v - p - h_b)\} \\
A_{bo}^* &= \{(h_s, h_o) \,| v - p - h_b > \max(v - p - h_o, v - p - h_s, 0), v - p - h_o > 0\} \\
A_b^* &= \{(h_s, h_o) \,| v - p - h_b > \max(v - p - h_o, v - p - h_s, 0), 0 > v - p - h_o\}
\end{aligned}$$

Figure 3: Market Segmentation with and without BOPS



(a) Without BOPS

(b) With BOPS

Next, the retailer's profit function can be expressed as follows:

$$\begin{aligned}
\pi(q) =& pE\min\left(\left(\alpha_s^* + \alpha_b^* + \alpha_{so}^* + \alpha_{bo}^*\right)D, q\right) - cq + rE\min\left(\left(\alpha_s^* + \alpha_b^* + \alpha_{so}^* + \alpha_{bo}^*\right)D, q\right) \\
& wE\alpha_o^* D + wE\frac{\alpha_{so}^* + \alpha_{bo}^*}{\alpha_s^* + \alpha_b^* + \alpha_{so}^* + \alpha_{bo}^*}\left(\left(\alpha_s^* + \alpha_b^* + \alpha_{so}^* + \alpha_{bo}^*\right)D - q\right)^+
\end{aligned}$$

To understand this expression, note when the store inventory level is q, there are on average $E\min((\alpha_s^* + \alpha_b^* + \alpha_{so}^* + \alpha_{bo}^*)\,D, q)$ customers who come to the store, since they come to the store only when a corresponding unit is available. The first two terms above correspond to the newsvendor profit from selling the product and the third term corresponds to the additional cross-selling profit. The last two terms represent the profit from the online channel: the fourth term is the profit from those who buy online directly, while the fifth term is the profit from those who prefer to go to store but buy online instead because of store stockouts.

Proposition 7. *When there is BOPS, the market outcome is given as follows:*

- *if* $p - c > w\frac{\alpha^*_{so}+\alpha^*_{bo}}{\alpha^*_s+\alpha^*_b+\alpha^*_{so}+\alpha^*_{bo}} - r$, *then there are customers visiting store and* $q^* = (\alpha^*_s + \alpha^*_b + \alpha^*_{so} + \alpha^*_{bo})\,\bar{F}^{-1}\left(\frac{c}{p+r-w\frac{\alpha^*_{so}+\alpha^*_{bo}}{\alpha^*_s+\alpha^*_b+\alpha^*_{so}+\alpha^*_{bo}}}\right)$;
- *if* $p - c \leq w\frac{\alpha^*_{so}+\alpha^*_{bo}}{\alpha^*_s+\alpha^*_b+\alpha^*_{so}+\alpha^*_{bo}} - r$, *then no one ever comes to store and* $q^* = 0$.

With a homogeneous population of customers, we identified three possible scenarios (as shown in Figure 1 in Section 4). Consumers may: (i) always visit the store, (ii) never visit the store, or (iii) visit the store only after BOPS is implemented. With a heterogeneous consumer population, these types of behavior may coexist, as shown in Figure 4 (which is obtained from comparing Figures 3(a) and 3(b)). In other words, there are three types of customers, each exhibiting a specific response to BOPS. Moreover, the retailer's profit from each type of customers follows the same pattern as before. If a consumer always visits the store (as those in the "Always" region), then the retailer's profit from this customer decreases after BOPS is implemented; she stops coming to the store once she knows that the store is out of stock and thus the retailer loses the potential cross-selling benefit. Next, if a consumer visits the store only after BOPS is implemented (as those in the "BOPS" region), then BOPS increases retailer's profit. Finally, if a consumer never shops in the store (as those in the "Never" region), then offering BOPS does not affect the retailer's profit.

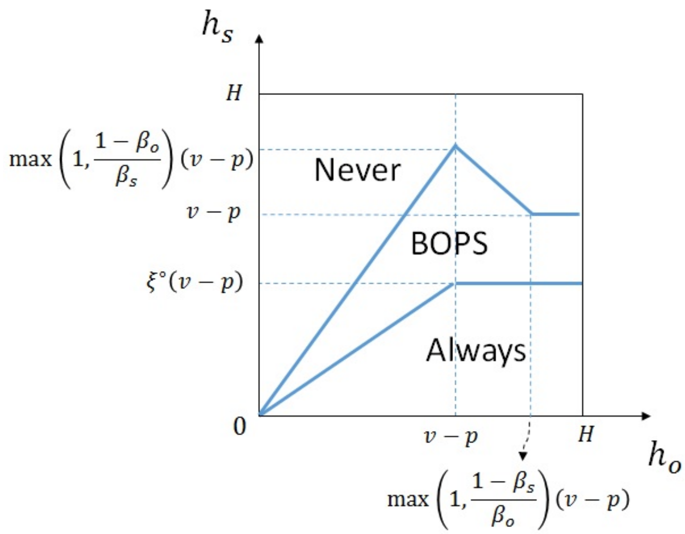
The next proposition shows the impact of BOPS on the retailer's overall customer base.

Proposition 8.

i. BOPS helps to expand market coverage, i.e., $\alpha^*_s + \alpha^*_o + \alpha^*_b + \alpha^*_{so} + \alpha^*_{bo} > \alpha^\circ_s + \alpha^\circ_o + \alpha^\circ_{so}$;

ii. Suppose $r = 0$. *If there are customers visiting store when there is no BOPS, then there exists* $\bar{w}$ *such that the implementation of BOPS decreases total profit (i.e.,* $\pi^* < \pi^\circ$*) if* $\beta_s + \beta_o < 1$ *and* $w > \bar{w}$.

Figure 4: When do consumers go to store?



Before BOPS is implemented, customers who face high hassle costs in both store and online channels do not consider purchasing from the retailer. Part (i) of Proposition 8 shows that BOPS could provide a way for the retailer to reach these customers. By alleviating the risk of stockouts and reducing the hassle of shopping, BOPS could attract some new customers to join the market. This market expansion effect could also be seen from the reduction of the "leave" region in Figure 3(b) compared to Figure 3(a).

While reaching out to new customers, BOPS may change the behavior of existing customers. Specifically, the more convenient BOPS is, the more existing online customers will choose to pick up their orders in store; this shift will hurt profits if the store profit margin is lower than the online profit margin. This potential drawback of BOPS may exist even when $r = 0$, as shown in Proposition 8(ii). In other words, apart from possibly eliminating cross-selling opportunities (when $r > 0$) as discovered earlier, BOPS has another potential drawback of shifting demand to a less profitable store channel.

6 Decentralized System

In this section, we study the scenario where the store and online channels are operated by two separate teams, with the goal of understanding how BOPS revenue should be allocated. We begin by examining the case with homogeneous customers and then later extrapolate our findings to the case with heterogeneous customers. (A detailed analysis of the decentralized system for the heterogeneous market is given in Supplementary Appendix B.) We assume that the conditions in Proposition 2 hold (i.e., $\min(h_s, h_b) \leq h_o$ and $p - c > w - r$) and BOPS hassle cost is lowest (i.e., $h_b < \min(h_s, h_o)$); otherwise, there would be nobody using BOPS and the issue of revenue allocation becomes irrelevant as the system collapses into two independent channels.

We use $\theta \in [0, 1]$ to denote the share of BOPS revenue that the store obtains. In other words, for every customer who purchases online and picks up in the store, the store earns θp from selling the product. Note that once the customer comes to the store, the store could also get an additional profit r through cross-selling. Therefore, the total revenue that the store could receive from fulfilling each unit of BOPS demand is $\theta p + r$. Then, the store's expected profit as a function of the stocking decision $\tilde{q}$ is given below. Here, we use the "tilde" symbol $(\tilde{\cdot})$ to denote the decentralized case.

$$\tilde{\pi}_s = (\theta p + r)\, E \min(D, \tilde{q}) - c\tilde{q}$$

Proposition 9. *In the decentralized system, the store will stock $\tilde{q}^*$ which is given as follows:*

- *If $\theta p + r - c > 0$, then $\tilde{q}^* = \bar{F}^{-1}\left(\frac{c}{\theta p + r}\right)$;*
- *If $\theta p + r - c \leq 0$, then $\tilde{q}^* = 0$.*

In practice, according to the survey conducted by Forrester Research (Forrester, 2014), the two most common revenue sharing schemes are either giving the store full credit (i.e., $\theta = 1$) or letting the online channel keep all the revenue (i.e., $\theta = 0$). According to Proposition 9, when $\theta = 1$, the store will definitely stock the product to serve BOPS users, because he can not only receive a positive profit margin from selling this particular product, but he may also be able to cross sell other products to those who come to store for pickup. However, if $\theta = 0$, BOPS customers represent a pure cost to the store; then, the store may choose not to stock the product in store, unless the cross-selling benefit is large enough to offset the loss from serving BOPS customers (i.e., $r > c$).

What is the optimal share of BOPS revenue that should be allocated to the store? We use $\tilde{\pi}^*(\theta)$ to denote total profit in the decentralized system with revenue allocation θ. For any given θ, the next proposition compares decentralized and centralized inventory decisions and shows that the store is usually either overstocked or understocked, relative to the centralized benchmark. However, there is an optimal revenue share, under which the decentralized system achieves the centralized optimal profit.

Proposition 10. *Total profit $\tilde{\pi}^*(\theta)$ is quasiconcave in θ. Moreover,*

- *if $\theta < \frac{p-w}{p}$, then $\tilde{q}^* < q^*$ and $\tilde{\pi}^*(\theta) < \pi^*$;*
- *if $\theta = \frac{p-w}{p}$, then $\tilde{q}^* = q^*$ and $\tilde{\pi}^*(\theta) = \pi^*$;*
- *if $\theta > \frac{p-w}{p}$, then $\tilde{q}^* > q^*$ and $\tilde{\pi}^*(\theta) < \pi^*$.*

Proposition 10 points out an incentive conflict between the store channel and the retail organization. If the store channel obtains a large share of BOPS revenue, they tend to stock too much. This is because the store channel considers only store profits but neglects the fact that customers may still be willing to shop online after the store runs out of inventory for customers to pick up. In contrast, if the store is allocated only a small share of BOPS revenue, they tend to stock too little. Since the store channel incurs the inventory cost for fulfilling BOPS demand, it is natural to suppress inventory to decrease exposure to potential losses. In general, it is optimal to give the store partial credit for the revenue earned from BOPS customers. In fact, our result also shows that such a simple revenue sharing mechanism is sufficient for the retailer to fully coordinate the store and online channels.

Proposition 10 shows that the optimal revenue share $\theta^* = \frac{p-w}{p}$ coordinates the decentralized system and achieves the centralized optimal profit π^*. With this optimal revenue share, the incentives of the store channel are aligned with the entire organization. According to Proposition 9, the decentralized store channel holds stock if and only if $\theta p + r - c > 0$. However, from the perspective of the entire organization, as we have shown in the previous section, it is optimal to stock the product in the store as long as $p - c > w - r$, i.e., whenever the store margin $p - c$, combined with the cross-selling benefit r, exceeds the online margin w. The revenue share θ that aligns both sets of incentives is precisely $\theta^* = \frac{p-w}{p}$.

Note that the optimal share for the store channel $\theta^* = \frac{p-w}{p}$ is decreasing in w. The reason is as follows. As the online margin w increases, it becomes more profitable to sell through the online channel. However, in the absence of BOPS, the store will continue to stock the same amount of inventory since they tend to neglect the profit from online orders. Through sharing the BOPS revenue, the retail organization has a natural way to correct for the misaligned incentive. By allocating less revenue to the store channel for fulfilling BOPS demand, the retail organization can induce the store channel to lower their inventory level. This compensates for the incentive conflict and allows more demand to flow to the more profitable online channel.

Although Proposition 10 shows that the optimal revenue sharing mechanism will achieve full centralized profits in a homogeneous market, Appendix B presents a different result for a heterogeneous market. Specifically, when the proportion of customers who use the BOPS functionality is too low, the amount of BOPS revenue to be shared between the channels may be insignificant and as a result, a simple revenue sharing mechanism cannot fully coordinate the omnichannel retail system. The analysis in Appendix B highlights the benefit as more customers adopt BOPS: the increased stream of BOPS revenue can provide headquarters with more leverage to alleviate incentive conflicts between the store and online channels.

Omnichannel retailers who recognize the incentive conflicts brought about by BOPS have begun to experiment with simple revenue sharing schemes. A common and simple approach is to assign full credit for the sale of a BOPS item to both store and online channels. In other words, there is some double counting on internal books that are subsequently adjusted for. Depending on accounting protocols, this method is usually akin to allocating equal revenue shares to each channel. Since the optimal revenue share may not be 50%, the simple heuristic described above has room for improvement, and our analysis provides a possible way to think about how to do so.

In practice, a retailer may carry a large number of SKUs with different prices and margins. Admittedly, it would be impractical to set a different revenue sharing parameter θ for each SKU. Instead, a retailer may want to have a common θ for a group of products. In such case, since the optimal profit function $\tilde{\pi}^*(\theta)$ is quasiconcave in θ, the optimal θ^*'s given in Proposition 10 for the group of products could serve as benchmarks for the retailer to find a common compromise θ that is close enough to each of the different θ^*'s.

7 Conclusion

In this paper, we study a specific omnichannel fulfillment strategy: buy-online-and-pickup-in-store (BOPS). We develop a stylized model that captures essential elements of omnichannel retail environments; in particular, consumers strategically choose channels for purchase and fulfillment. We find that BOPS attracts consumer demand through an *information* effect and a *convenience* effect. The former effect arises because BOPS reveals real-time information about store inventory availability. Products that are available for store-pickup must be in stock. With this assurance, customers are more willing to visit the store. The latter effect arises because BOPS offers a new and possibly more convenient mode of shopping. By helping consumers pick out items and move them to checkout counters, BOPS reduces the hassle of shopping.

Even though BOPS is a popular fulfillment option among consumers, we find that retailers need to be cautious when implementing it. Retailers can benefit from this new fulfillment strategy by being selective when choosing the set of products eligible for in-store pickup. BOPS can help attract more customers to the store and thus boost the sales of products that were previously not selling well; however, for store bestsellers, offering the BOPS option may have the unintended consequence of reducing store traffic. Moreover, although BOPS can be a good strategy for a retailer to build up its customer base, BOPS may at the same time drive existing online customers to the store channel where the profit margin might be lower compared to the online channel. Finally, retailers with decentralized operations can maximize profits by allocating BOPS revenue between the online and store channels appropriately, and giving full credit to either channel is seldom optimal.

To simplify the analysis, we have imposed two assumptions in our model: (1) The online channel is exogenous and always in stock, and (2) all customers check the information online when BOPS is offered. As robustness checks, we have built two model extensions to relax the assumptions above. Specifically, in Supplementary Appendix C, we consider the case where the retailer has limited inventory in both the store and online channels; in Supplementary Appendix D, we consider customers who simply head to the store by default (e.g., they may forget or simply not care to check websites beforehand). Our key results remain valid in these model extensions.

Beyond the scope of our current analysis is the potential impact of BOPS on operational costs. On the one hand, BOPS helps to reduce online shipping costs since it transfers the burden of last-mile delivery to customers. On the other hand, BOPS is accompanied by new fulfillment responsibilities wherein a retailer's comparative advantage may not lie. For example, stores need to train their workforce to perform pick-and-pack tasks in a timely fashion (Forrester, 2014), and to handle increased demand, stores need to hire more employees to deal with online orders (Business Insider, 2012). A more careful cost-based analysis of BOPS is left as an interesting topic for future research.

Some products, such as clothes, have non-digital attributes that can be communicated only in the offline channel (Lal and Sarvary, 1999). Customers who buy online may not know how much they like the product, and some online purchases may eventually be returned. Gao and Su (2015) develop a similar model where a retailer could deliver product information in an omnichannel environment. One of the information mechanisms they study is virtual showrooms, by which consumers can virtually try on a product online; for example, on the website of UK luxury shirt brand Thomas Pink, consumers can check the fit of a shirt through a digital avatar. Such informational mechanisms can be a double-edged sword: while they help to reduce online returns, omnichannel channel consumers may not patronize the store after a negative virtual try-on experience. Additional research can further clarify the effectiveness of omnichannel informational strategies.

The omnichannel strategy discussed in this paper, BOPS, addresses purchases that originate online but are completed in the store. In the spirit of Bell et al. (2014), who provide a framework for omnichannel retail, BOPS customers receive information online but their demand is fulfilled in the store. The reverse type of shopping behavior is where customers research the product in stores and then shop online, usually at lower prices. This is known as showrooming behavior and has been critiqued widely: showroomers and e-tailers are accused of free-riding on inventory displays at brick-and-mortar stores (Wall Street Journal, 2012a,b). Recent research by Balakrishnan et al. (2014) shows that showrooming behavior intensifies retail competition, and Mehra et al. (2013) studies how a brick-and-mortar retailer can counteract showrooming behavior through strategies such as price matching and retail club memberships. Taking the e-tailer's perspective, Bell et al. (2013) empirically studies the value of providing offline showrooms to mitigate customer uncertainty. We hope that our model in this paper can contribute to this exciting line of research.

References

Akçura, M. T. and K. Srinivasan (2005). Research note: customer intimacy and cross-selling strategy. *Management Science 51*(6), 1007–1012.

Alexandrov, A. and M. A. Lariviere (2012). Are reservations recommended? *Manufacturing & Service Operations Management 14*(2), 218–230.

Allon, G. and A. Bassamboo (2011). Buying from the babbling retailer? the impact of availability information on customer behavior. *Management Science 57*(4), 713–726.

Ansari, A., C. F. Mela, and S. A. Neslin (2008). Customer channel migration. *Journal of Marketing Research 45*(1), 60–76.

Armony, M. and I. Gurvich (2010). When promotions meet operations: Cross-selling and its effect on call center performance. *Manufacturing & Service Operations Management 12*(3), 470–488.

Aviv, Y. and A. Pazgal (2008). Optimal pricing of seasonal products in the presence of forward-looking consumers. *Manufacturing & Service Operations Management 10*(3), 339–359.

Balakrishnan, A., S. Sundaresan, and B. Zhang (2014). Browse-and-switch: Retail-online competition under value uncertainty. *Production and Operations Management 23*(7), 1129–1145.

Bell, D., S. Gallino, and A. Moreno (2013). Inventory showrooms and customer migration in omni-channel retail: The effect of product information. Working paper, Unversity of Pennsylvania.

Bell, D., S. Gallino, and A. Moreno (2014). How to win in an omnichannel world. *MIT Sloan Management Review 56*(1).

Bloomberg (2012, Sep 4). Now at Tesco: Drive-through grocery pickup.

Brynjolfsson, E., Y. J. Hu, and M. S. Rahman (2013). Competing in the age of omnichannel retailing. *MIT Sloan Management Review 54*(4), 23–29.

Business Insider (2012, Sep 25). Toys "R" Us is hiring 45,000 employees for the holiday season.

Cachon, G. and P. Feldman (2015). Price commitments with strategic consumers: Why it can be optimal to discount more frequently ... than optimal. *Manufacturing & Service Operations Management 17*(3), 399–410.

Cachon, G. P. (2003). Supply chain coordination with contracts. *Handbooks in Operations Research and Management Science 11*, 227–339.

Cachon, G. P. and A. G. Kök (2007). Category management and coordination in retail assortment planning in the presence of basket shopping consumers. *Management Science 53*(6), 934–951.

Cachon, G. P. and M. A. Lariviere (2005). Supply chain coordination with revenue-sharing contracts: strengths and limitations. *Management Science 51*(1), 30–44.

Cachon, G. P. and R. Swinney (2009). Purchasing, pricing, and quick response in the presence of strategic consumers. *Management Science 55*(3), 497–511.

Cattani, K., W. Gilland, H. S. Heese, and J. Swaminathan (2006). Boiling frogs: Pricing strategies for a manufacturer adding a direct channel that competes with the traditional channel. *Production and Operations Management 15*(1), 40–56.

Chen, K.-Y., M. Kaya, and Ö. Özer (2008). Dual sales channel management with service competition. *Manufacturing & Service Operations Management 10*(4), 654–675.

Chiang, W.-y. K., D. Chhajed, and J. D. Hess (2003). Direct marketing, indirect profits: A strategic analysis of dual-channel supply-chain design. *Management Science 49*(1), 1–20.

Chintagunta, P. K., J. Chu, and J. Cebollada (2012). Quantifying transaction costs in online/off-line grocery channel choice. *Marketing Science 31*(1), 96–114.

Dana, J. D. (2001). Competition in price and availability when availability is unobservable. *Rand Journal of Economics*, 497–513.

Dana, J. D. and N. C. Petruzzi (2001). Note: The newsvendor model with endogenous demand. *Management Science 47*(11), 1488–1497.

Deneckere, R. and J. Peck (1995). Competition over price and service rate when demand is stochastic: A strategic analysis. *The RAND Journal of Economics*, 148–162.

Financial Times (2013, Dec. 27). US bricks-and-mortar retailers seek online success.

Forrester (2014). Customer desires vs. retailer capabilities: Minding the omni-channel commerce gap. Technical report, Forrester Consulting.

Gallino, S. and A. Moreno (2014). Integration of online and offline channels in retail: The impact of sharing reliable inventory availability information. *Management Science 60*(6), 1434–1451.

Gao, F. and X. Su (2015). Online and offline information for omnichannel retailing. Working paper, Unversity of Pennsylvania.

Gurvich, I., M. Armony, and C. Maglaras (2009). Cross-selling in a call center with a heterogeneous customer population. *Operations research 57*(2), 299–313.

Harris, M., C. H. Kriebel, and A. Raviv (1982). Asymmetric information, incentives and intrafirm resource allocation. *Management Science 28*(6), 604–620.

Jerath, K., S. Netessine, and S. K. Veeraraghavan (2010). Revenue management with strategic customers: Last-minute selling and opaque selling. *Management Science 56*(3), 430–448.

Kouvelis, P. and M. A. Lariviere (2000). Decentralizing cross-functional decisions: Coordination through internal markets. *Management Science 46*(8), 1049–1058.

Lai, G., L. G. Debo, and K. Sycara (2010). Buy now and match later: Impact of posterior price matching on profit with strategic consumers. *Manufacturing & Service Operations Management 12*(1), 33–55.

Lal, R. and M. Sarvary (1999). When and how is the internet likely to decrease price competition? *Marketing Science 18*(4), 485–503.

Lee, H. and S. Whang (1999). Decentralized multi-echelon supply chains: Incentives and information. *Management Science 45*(5), 633–640.

Li, S., B. Sun, and A. L. Montgomery (2011). Cross-selling the right product to the right customer at the right time. *Journal of Marketing Research 48*(4), 683–700.

Li, S., B. Sun, and R. T. Wilcox (2005). Cross-selling sequentially ordered products: An application to consumer banking services. *Journal of Marketing Research 42*(2), 233–239.

Liang, C., M. Cakanyildirim, and S. P. Sethi (2014). Analysis of product rollover strategies in the presence of strategic customers. *Management Science 60*(4), 1033–1056.

Liu, Q. and G. J. van Ryzin (2008). Strategic capacity rationing to induce early purchases. *Management Science 54*(6), 1115–1131.

Liu, Y. and Z. J. Zhang (2006). Research note-the benefits of personalized pricing in a channel. *Marketing Science 25*(1), 97–105.

Mehra, A., S. Kumar, and J. S. Raju (2013). "Showrooming" and the competition between store and online retailers. Working paper, Unversity of Pennsylvania.

Netessine, S. and N. Rudi (2006). Supply chain choice on the internet. *Management Science 52*(6), 844–864.

Netessine, S., S. Savin, and W. Xiao (2006). Revenue management through dynamic cross selling in e-commerce retailing. *Operations Research 54*(5), 893–913.

New York Times (2011, March 10). Wal-mart has a web plan to bolster in-store sales.

New York Times (2012, July 4). Luring online shoppers offline.

Ofek, E., Z. Katona, and M. Sarvary (2011). "Bricks and clicks": The impact of product returns on the strategies of multichannel retailers. *Marketing Science 30*(1), 42–60.

Porteus, E. L. and S. Whang (1991). On manufacturing/marketing incentives. *Management Science 37*(9), 1166–1181.

Retail Dive (2015, June 25). Delivering convenience: The keys to buy online, pickup in store.

Reuters (2014, Nov 28). Will in-store pick-up of online sales pay off for retailers?

Rigby, D. (2011). The future of shopping. *Harvard Business Review 89*(12), 65–76.

RSR (2013). Omni-channel 2013: The long road to adoption.

Su, X. (2007). Intertemporal pricing with strategic customer behavior. *Management Science 53*(5), 726–741.

Su, X. and F. Zhang (2008). Strategic customer behavior, commitment, and supply chain performance. *Management Science 54*(10), 1759–1773.

Su, X. and F. Zhang (2009). On the value of commitment and availability guarantees when selling to strategic consumers. *Management Science 55*(5), 713–726.

Taylor, T. A. (2002). Supply chain coordination under channel rebates with sales effort effects. *Management Science 48*(8), 992–1007.

Time (2011, March 11). Why retailers prefer "ship-to-store" over plain old shipping to the customer's home.

Tsay, A. A. and N. Agrawal (2004). Channel conflict and coordination in the e-commerce age. *Production and Operations Management 13*(1), 93–110.

UPS (2015, June). UPS online shopping study: Empowered consumers changing the future of retail.

Wall Street Journal (2012a, April 11). Can retailers halt ‘showrooming’?

Wall Street Journal (2012b, Jan 23). Showdown over ‘showrooming’.

Washington Post (2015, Jan 13). From new mobile offerings to a potential off-price business, a look at whats in store for macys.

Yin, R., Y. Aviv, A. Pazgal, and C. S. Tang (2009). Optimal markdown pricing: Implications of inventory display formats in the presence of strategic customers. *Management Science 55*(8), 1391–1408.

Supplementary Appendix

A Proofs

Proof of Proposition 1: Let's look for participatory RE equilibrium, where $\phi = 1$ and $q > 0$. All we need to do is to check the four conditions specified in Definition 1.

First, we look at retailer's problem: Given belief $\hat{\phi}$, the retailer maximizes total profit $\pi = pE\min(\hat{\phi}D, q) - cq + rE(\hat{\phi}D) + wE((1-\hat{\phi})D) + wE(\hat{\phi}D - q)^+ = (p-w)E\min(\hat{\phi}D, q) - cq + (\hat{\phi}r + w)ED$, which is a typical newsvendor problem (plus a constant $(\hat{\phi}r + w)ED$), and therefore the optimal order quantity q° is given by $\bar{F}\left(\frac{q^\circ}{\hat{\phi}}\right) = \frac{c}{p-w} \wedge 1$, where $x \wedge y$ means $\min(x, y)$.

Since in equilibrium, the retailer's belief is consistent with the outcome, we have $\hat{\phi} = \phi = 1$. Thus, $q^\circ = \bar{F}^{-1}\left(\frac{c}{p-w} \wedge 1\right)$. Since $q > 0$ in the participatory equilibrium, we must have $p - c > w$ and thus $q^\circ = \bar{F}^{-1}\left(\frac{c}{p-w}\right)$.

Note in equilibrium, we also need consumer's belief to be consistent with the outcome, i.e., $\hat{\xi}^\circ = A(q^\circ) = E\min(\phi D, q^\circ)/E(\phi D) = E\min(D, \bar{F}^{-1}(\frac{c}{p-w}))/ED$.

Finally, we go back to consumer's decision. To ensure $\phi = 1$, we need $u_s \geq u_o$, i.e., $h_s \leq \hat{\xi}^\circ h_o$. Therefore, we need $h_s \leq \frac{E\min(D, \bar{F}^{-1}(\frac{c}{p-w}))}{ED} h_o$.

Based on the analysis above, we find the conditions for a participatory equilibrium are $h_s \leq \frac{E\min(D, \bar{F}^{-1}(\frac{c}{p-w}))}{ED} h_o$ and $p - c > w$. And the equilibrium outcome is $\phi = 1$ and $q^\circ = \bar{F}^{-1}\left(\frac{c}{p-w}\right)$. □

Proof of Proposition 2: If $\min(h_s, h_b) \leq h_o$, then the profit function $\pi = pE\min(D, q) - cq + rE\min(D, q) + wE(D - q)^+ = (p + r - w)E\min(D, q) - cq + wED$, which is a typical newsvendor problem (plus a constant wED). Thus, the optimal order quantity q^* is given by $\bar{F}(q^*) = \frac{c}{p+r-w} \wedge 1$. Then, if $p - c > w - r$, we have $q^* = \bar{F}^{-1}\left(\frac{c}{p+r-w}\right)$; otherwise, $q^* = 0$.

If $\min(h_s, h_b) > h_o$, then no customer comes to store and thus the retailer will stock nothing in the store, i.e., $q^* = 0$. □

Proof of Proposition 3: If $h_s \in \left(\frac{E\min\left[D,\bar{F}^{-1}\left(\frac{c}{p-w}\wedge 1\right)\right]}{ED}h_o, h_o\right]$ and $p-c > w-r$, then

$$\begin{aligned}\pi^* &= pE\min(D,q^*) - cq^* + rE\min(D,q^*) + wE(D-q^*)^+ \\ &= (p+r-w)E\min(D,q^*) - cq^* + wED \\ &> wED \\ &= \pi^\circ\end{aligned}$$

where the inequality is due to the fact that $q^* > 0$. This completes the proof. □

Proof of Proposition 4: If $h_b \le h_o < h_s$ and $p-c > w-r$, then $q^* = \bar{F}^{-1}\left(\frac{c}{p+r-w}\right) > 0$ because of Proposition 2. Then,

$$\begin{aligned}\pi^* &= pE\min(D,q^*) - cq^* + rE\min(D,q^*) + wE(D-q^*)^+ \\ &= (p+r-w)E\min(D,q^*) - cq^* + wED \\ &> wED \\ &= \pi^\circ\end{aligned}$$

where the inequality is due to the fact that $q^* > 0$. This completes the proof. □

Proof of Proposition 5: If $h_s \le \frac{E\min\left(D,\bar{F}^{-1}\left(\frac{c}{p-w}\right)\right)}{ED}h_o$ and $p-c > w$, we have $q^\circ \ne q^*$ when $r > 0$. Then

$$\begin{aligned}\pi^\circ &= pE\min(D,q^\circ) - cq^\circ + rED + wE(D-q^\circ)^+ \\ &= (p^\circ - w)E\min(D,q^\circ) - cq^\circ + rED + wED \\ &> (p^* - w)E\min(D,q^*) - cq^* + rED + wED \\ &> (p^* - w)E\min(D,q^*) - cq^* + rE\min(D,q^*) + wED \\ &= p^*E\min(D,q^*) - cq^* + rE\min(D,q^*) + wE(D-q^*)^+ \\ &= \pi^*\end{aligned}$$

where the first inequality is due to the strict concavity of the newsvendor profit function and q° is the unique maximizer of π°, and the second inequality is because $ED > E\min(D,q)$ for any q. This completes the proof. □

Proof of Proposition 6: Note nonparticipatory equilibrium, $(\frac{v-p}{H}, 0, 0, 0, 0, 0, \frac{v-p}{H}, 0, 0, 0)$, always exists. Same as what we did in the base model, we are going to look for participatory equilibrium, where $\alpha_s^\circ > 0, \alpha_{so}^\circ > 0$ and $q^\circ > 0$. All we need to do is to check the four conditions specified in Definition 2.

First, we look at retailer's problem: Given belief $\hat{\alpha}_o, \hat{\alpha}_s, \hat{\alpha}_{so}$, the retailer maximizes total profit $\pi = pE\min\left(\left(\hat{\alpha}_s + \hat{\alpha}_{so}\right)D, q\right) - cq + rE\left(\hat{\alpha}_s + \hat{\alpha}_{so}\right)D + wE\hat{\alpha}_o D + wE\frac{\hat{\alpha}_{so}}{\hat{\alpha}_s+\hat{\alpha}_{so}}\left(\left(\hat{\alpha}_s + \hat{\alpha}_{so}\right)D - q\right)^+ = \left(p - w\frac{\hat{\alpha}_{so}}{\hat{\alpha}_s+\hat{\alpha}_{so}}\right)E\min((\hat{\alpha}_s + \hat{\alpha}_{so})D, q) - cq + ((\hat{\alpha}_s + \hat{\alpha}_{so})r + (\hat{\alpha}_o + \hat{\alpha}_{so})w)ED$, which is a typical newsvendor problem (plus a constant $((\hat{\alpha}_s + \hat{\alpha}_{so})r + (\hat{\alpha}_o + \hat{\alpha}_{so})w)ED$), and therefore the optimal order quantity q° is given by $\bar{F}\left(\frac{q^\circ}{\hat{\alpha}_s+\hat{\alpha}_{so}}\right) = \frac{c}{p-w\frac{\hat{\alpha}_{so}}{\hat{\alpha}_s+\hat{\alpha}_{so}}} \wedge 1$.

Since in equilibrium, the retailer's belief is consistent with the outcome, we have $\hat{\alpha}_i = \alpha_i^\circ$, $i = o, s, so$. Note, in any participatory equilibrium, $\xi > 0$. Then, $\frac{\alpha_{so}^\circ}{\alpha_s^\circ+\alpha_{so}^\circ} = \frac{v-p}{2H-(v-p)}$. Thus, $q^\circ = (\alpha_s^\circ + \alpha_{so}^\circ)\bar{F}^{-1}\left(\frac{c}{p-w\frac{v-p}{2H-(v-p)}} \wedge 1\right)$. Since $q^\circ > 0$ in the participatory equilibrium, we must have $p - c > w\frac{v-p}{2H-(v-p)}$ and thus $q^\circ = (\alpha_s^\circ + \alpha_{so}^\circ)\bar{F}^{-1}\left(\frac{c}{p-w\frac{v-p}{2H-(v-p)}}\right)$.

Note in equilibrium, we also need consumer's belief to be consistent with the outcome, i.e., $\hat{\xi} = A(q^\circ) = E\min((\alpha_s^\circ + \alpha_{so}^\circ)D, q^\circ)/E((\alpha_s^\circ + \alpha_{so}^\circ)D) = E\min(D, \bar{F}^{-1}(\frac{c}{p-w\frac{v-p}{2H-(v-p)}}))/ED$, which is indeed greater than 0. Thus, there are customers who are willing to come to store.

Based on the analysis above, we find the condition for a participatory equilibrium is $p - c > w\frac{v-p}{2H-(v-p)}$. And the equilibrium store inventory level $q^\circ = \bar{F}^{-1}\left(\frac{c}{p-w\frac{v-p}{2H-(v-p)}}\right)$. $\square$

Proof of Proposition 7: Note the retailer's profit can be expressed as

$$\pi = \left(p + r - w\frac{\alpha_{so}^* + \alpha_{bo}^*}{\alpha_s^* + \alpha_b^* + \alpha_{so}^* + \alpha_{bo}^*}\right)E\min\left(\left(\alpha_s^* + \alpha_b^* + \alpha_{so}^* + \alpha_{bo}^*\right)D, q\right) - cq + w(\alpha_o^* + \alpha_{so}^* + \alpha_{bo}^*)ED$$

which is a typical newsvendor problem (plus a constant $w(\alpha_o^* + \alpha_{so}^* + \alpha_{bo}^*)ED$). Thus, the optimal store inventory level is given by $q^* = \left(\alpha_s^* + \alpha_b^* + \alpha_{so}^* + \alpha_{bo}^*\right)\bar{F}^{-1}\left(\frac{c}{p+r-w\frac{\alpha_{so}^*+\alpha_{bo}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}} \wedge 1\right)$ $\square$

Proof of Proposition 8: Denote $A_l^\circ = \{(h_s, h_o)|\max(v - p - h_o, -h_s + \xi^\circ(v - p)) < 0\}$ and $A_l^* = \{(h_s, h_o)|\max(v - p - h_o, v - p - h_s, v - p - \beta_s h_s - \beta_o h_o) < 0\}$. To prove part (i), we simply need to show that $A_l^* \subset A_l^\circ$. Since $\xi^\circ < 1$, for any $(h_s, h_o) \in A_l^*$, we will have $(h_s, h_o) \in A_l^\circ$. Thus, $A_l^* \subset A_l^\circ$.

Next, let's look at part (ii). To simplify notation, we denote $\Delta^\circ = \frac{v-p}{2H-(v-p)}$ and $\Delta^* = \frac{\alpha_{so}^*+\alpha_{bo}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}$. Note if $\beta_s + \beta_o > 1$, we have $\Delta^\circ < \Delta^*$. Thus, $\frac{p-c}{\Delta^\circ} > \frac{p-c}{\Delta^*}$.

If there are customers visiting store when there is no BOPS, we need to have $w < \frac{p-c}{\Delta^\circ}$. The following analysis assumes this condition holds.

Suppose $r = 0$. If $w \geq \frac{p-c}{\Delta^*}$ (this is possible since $\frac{p-c}{\Delta^\circ} > \frac{p-c}{\Delta^*}$), then no one comes to store when there is BOPS . In this case,

$$
\begin{aligned}
\pi^* &= wE\left(\alpha_{so}^* + \alpha_{bo}^* + \alpha_o^*\right) D \\
&= wE\left(\alpha_{so}^\circ + \alpha_o^\circ\right) D \\
&< pE\min\left(\left(\alpha_s^\circ + \alpha_{so}^\circ\right) D, q\right) - cq + rE\left(\alpha_s^\circ + \alpha_{so}^\circ\right) D + wE\alpha_o^\circ D + wE\frac{\alpha_{so}^\circ}{\alpha_s^\circ + \alpha_{so}^\circ}\left(\left(\alpha_s^\circ + \alpha_{so}^\circ\right) D - q\right)^+ \\
&= \pi^\circ
\end{aligned}
$$

where the inequality is because $q = 0$ is also a feasible but not the optimal solution to the case where there is no BOPS. If $w < \frac{p-c}{\Delta^*}$, then there are consumers visiting store when there is BOPS. Note (1) $\pi^\circ - \pi^*$ is continuous in w, and (2) $\pi^\circ > \pi^*$ when $w = \frac{p-c}{\Delta^*}$. Thus, the analysis above implies that there exists $\bar{w} < \frac{p-c}{\Delta^*}$ such that $\pi^\circ > \pi^*$ if $w > \bar{w}$. □

Proof of Proposition 9: Note $\tilde{\pi}_s$ is a typical newsvendor profit function. Then, the optimal order quantity $\tilde{q}^*$ is given by $\bar{F}(\tilde{q}^*) = \frac{c}{\theta p + r} \wedge 1$. Thus, if $\theta p + r - c > 0$, we have $\tilde{q}^* = \bar{F}^{-1}\left(\frac{c}{\theta p + r}\right)$, otherwise, $\tilde{q}^* = 0$. □

Proof of Proposition 10: In the decentralized system, for any θ, denote optimal stock levels in the store as $\tilde{q}^*(\theta)$. Suppose we have the same stock level in the centralized system. Then clearly we will achieve exactly the same total profit as $\tilde{\pi}^*(\theta)$, since the BOPS revenue is just shared between channels in the decentralized case. Thus, we must have $\tilde{\pi}^*(\theta) \leq \pi^*$. And due to the fact that π is strictly concave in q and $q^* > 0$ (because of the assumptions that $h_b < \min(h_s, h_o)$ and $p - c > w - r$), we have $\tilde{\pi}^*(\theta) = \pi^*$ if only if $\tilde{q}^* = q^*$.

Next, let's compare the optimal store inventory levels in both the centralized and decentralized systems:

- If $\theta < \frac{p-w}{p}$, then $\frac{c}{\theta p + r} > \frac{c}{p+r-w}$. Thus, $\tilde{q}^* = \bar{F}^{-1}\left(\frac{c}{\theta p + r}\right) < \bar{F}^{-1}\left(\frac{c}{p+r-w}\right) = q^*$. Since $\tilde{q}^* \neq q^*$, $\tilde{\pi}^* < \pi^*$.

- If $\theta = \frac{p-w}{p}$, then $\frac{c}{\theta p + r} = \frac{c}{p+r-w}$. Thus, $\tilde{q}^* = \bar{F}^{-1}\left(\frac{c}{\theta p + r}\right) = \bar{F}^{-1}\left(\frac{c}{p+r-w}\right) = q^*$. Since $\tilde{q}^* = q^*$, $\tilde{\pi}^* = \pi^*$.

- If $\theta > \frac{p-w}{p}$, then $\frac{c}{\theta p+r} < \frac{c}{p+r-w}$. Thus, $\tilde{q}^* = \bar{F}^{-1}\left(\frac{c}{\theta p+r}\right) > \bar{F}^{-1}\left(\frac{c}{p+r-w}\right) = q^*$. Since $\tilde{q}^* \neq q^*$, $\tilde{\pi}^* < \pi^*$.

Finally, let's prove $\tilde{\pi}^*$ is quasiconcave in θ.

Let's first show that $\tilde{\pi}^*$ is nondecreasing in $\theta < \frac{p-w}{p}$. If $\theta \leq \frac{c-r}{p}$, then $\tilde{q}^* = 0$ and thus $\tilde{\pi}^* = wED$, which is independent of θ. If $\theta \in (\frac{c-r}{p}, \frac{p-w}{p})$, then $\frac{\partial \tilde{\pi}^*}{\partial \theta} = \frac{\partial \tilde{\pi}^*}{\partial q}\frac{\partial \tilde{q}^*}{\partial \theta} > 0$, because $\frac{\partial \tilde{\pi}^*}{\partial q} > 0$ if $q < \tilde{q}^*$ and $\frac{\partial \tilde{q}^*}{\partial \theta} > 0$.

Then, let's show that $\tilde{\pi}^*$ is nonincreasing in $\theta > \frac{p-w}{p}$. Note $\frac{\partial \tilde{\pi}^*}{\partial \theta} = \frac{\partial \tilde{\pi}^*}{\partial q}\frac{\partial \tilde{q}^*}{\partial \theta} < 0$, because $\frac{\partial \tilde{\pi}^*}{\partial q} < 0$ if $q > \tilde{q}^*$ and $\frac{\partial \tilde{q}^*}{\partial \theta} > 0$.

Thus, we can conclude that $\tilde{\pi}^*$ is quasiconcave in θ. □

B Decentralized System in Heterogeneous Market

Similar to what we did in Section 6, we only look at the case where there are some customers using BOPS, i.e., $p - c > w\frac{\alpha_{so}^*+\alpha_{bo}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*} - r$ and $\alpha_b^* + \alpha_{bo}^* > 0$ (see Proposition 7); otherwise, there would be nobody using BOPS and the issue of revenue allocation becomes irrelevant as the system collapses into two independent channels.

With revenue sharing parameter θ, the store channel's profit is given as follows

$$\tilde{\pi}_s = \left(\frac{\alpha_s^* + \alpha_{so}^*}{\alpha_s^* + \alpha_b^* + \alpha_{so}^* + \alpha_{bo}^*} + \frac{\alpha_b^* + \alpha_{bo}^*}{\alpha_s^* + \alpha_b^* + \alpha_{so}^* + \alpha_{bo}^*}\theta\right) pE\min\left(\left(\alpha_s^* + \alpha_b^* + \alpha_{so}^* + \alpha_{bo}^*\right)D, q\right) - cq$$
$$+ rE\min\left(\left(\alpha_s^* + \alpha_b^* + \alpha_{so}^* + \alpha_{bo}^*\right)D, q\right)$$

Proposition B.1. *In the decentralized system, the store will stock $\tilde{q}^*$ which is given as follows:*

- *If* $\left(\frac{\alpha_s^*+\alpha_{so}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*} + \frac{\alpha_b^*+\alpha_{bo}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}\theta\right)p + r - c > 0$, *then*
$$\tilde{q}^* = \left(\alpha_s^* + \alpha_b^* + \alpha_{so}^* + \alpha_{bo}^*\right)\bar{F}^{-1}\left(\frac{c}{\left(\frac{\alpha_s^*+\alpha_{so}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*} + \frac{\alpha_b^*+\alpha_{bo}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}\theta\right)p+r}\right);$$
- *If* $\left(\frac{\alpha_s^*+\alpha_{so}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*} + \frac{\alpha_b^*+\alpha_{bo}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}\theta\right)p + r - c \leq 0$, *then* $\tilde{q}^* = 0$.

Proposition B.2. *Total profit $\tilde{\pi}^*(\theta)$ is quasiconcave in θ. Moreover,*

- *If* $\frac{\alpha_b^*+\alpha_{bo}^*}{\alpha_{so}^*+\alpha_{bo}^*} < \frac{w}{p}$, *then* $\forall\theta \in [0,1]$, $\tilde{q}^* > q^*$ *and* $\tilde{\pi}^*(\theta) < \pi^*$.

- *If* $\frac{\alpha_b^*+\alpha_{bo}^*}{\alpha_{so}^*+\alpha_{bo}^*} \geq \frac{w}{p}$, *then*
  - *if* $\theta < \frac{(\alpha_b^*+\alpha_{bo}^*)p-(\alpha_{so}^*+\alpha_{bo}^*)w}{(\alpha_b^*+\alpha_{bo}^*)p}$, *then* $\tilde{q}^* < q^*$ *and* $\tilde{\pi}^*(\theta) < \pi^*$*;*
  - *if* $\theta = \frac{(\alpha_b^*+\alpha_{bo}^*)p-(\alpha_{so}^*+\alpha_{bo}^*)w}{(\alpha_b^*+\alpha_{bo}^*)p}$, *then* $\tilde{q}^* = q^*$ *and* $\tilde{\pi}^*(\theta) = \pi^*$*;*
  - *if* $\theta > \frac{(\alpha_b^*+\alpha_{bo}^*)p-(\alpha_{so}^*+\alpha_{bo}^*)w}{(\alpha_b^*+\alpha_{bo}^*)p}$, *then* $\tilde{q}^* > q^*$ *and* $\tilde{\pi}^*(\theta) < \pi^*$*.*

Proposition B.2 shows that if there are not many people using BOPS (i.e., $\frac{\alpha_b^*+\alpha_{bo}^*}{\alpha_{so}^*+\alpha_{bo}^*} < \frac{w}{p}$ or $\alpha_b^* p + \alpha_{bo}^*(p-w) < \alpha_{so}^* w$), then there is not enough BOPS revenue for the two channels to share and thus for the headquarters to fully correct store channels' incentive: Even if the store channel is given no credit for fulfilling BOPS demand, i.e., $\theta = 0$, they still stock more than the system optimal level.

B.1 Proofs

Proof of Proposition B.1: Note $\tilde{\pi}_s$ is a typical newsvendor profit function. Then, the optimal order quantity $\tilde{q}^*$ is given by $\bar{F}(\frac{\tilde{q}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}) = \frac{c}{\left(\frac{\alpha_s^*+\alpha_{so}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}+\frac{\alpha_b^*+\alpha_{bo}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}\theta\right)p+r} \wedge 1$. Thus, if $\left(\frac{\alpha_s^*+\alpha_{so}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}+\frac{\alpha_b^*+\alpha_{bo}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}\theta\right)p+r-c > 0$, then $\tilde{q}^* = (\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*)\,\bar{F}^{-1}\left(\frac{c}{\left(\frac{\alpha_s^*+\alpha_{so}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}+\frac{\alpha_b^*+\alpha_{bo}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}\theta\right)p+r}\right)$, otherwise, $\tilde{q}^* = 0$. □

Proof of Proposition B.2: In the decentralized system, for any θ, denote optimal stock levels in the store as $\tilde{q}^*(\theta)$. Suppose we have the same stock level in the centralized system. Then clearly we will achieve exactly the same total profit as $\tilde{\pi}^*(\theta)$, since the BOPS revenue is just shared between channels in the decentralized case. Thus, we must have $\tilde{\pi}^*(\theta) \leq \pi^*$. And due to the fact that π is strictly concave in q and $q^* > 0$ (because of the assumptions that $p - c > w\frac{\alpha_{so}^*+\alpha_{bo}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*} - r$), we have $\tilde{\pi}^*(\theta) = \pi^*$ if only if $\tilde{q}^* = q^*$.

Next, let's compare the optimal store inventory levels in both the centralized and decentralized systems:

- If $\frac{\alpha_b^*+\alpha_{bo}^*}{\alpha_{so}^*+\alpha_{bo}^*} < \frac{w}{p}$, then $\forall\theta \in [0,1]$, we have $\left(\frac{\alpha_s^*+\alpha_{so}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}+\frac{\alpha_b^*+\alpha_{bo}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}\theta\right)p+r > p+r-w\frac{\alpha_{so}^*+\alpha_{bo}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}$. Therefore, $\tilde{q}^* > q^*$. Since $\tilde{q}^* \neq q^*$, $\tilde{\pi}^* < \pi^*$.
- If $\frac{\alpha_b^*+\alpha_{bo}^*}{\alpha_{so}^*+\alpha_{bo}^*} \geq \frac{w}{p}$, then

– if $\theta < \frac{(\alpha_b^*+\alpha_{bo}^*)p-(\alpha_{so}^*+\alpha_{bo}^*)w}{(\alpha_b^*+\alpha_{bo}^*)p}$, then $\tilde{q}^* = \bar{F}^{-1}\left(\frac{c}{\left(\frac{\alpha_s^*+\alpha_{so}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}+\frac{\alpha_b^*+\alpha_{bo}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}\theta\right)p+r}\right) < \bar{F}^{-1}\left(\frac{c}{p+r-w\frac{\alpha_{so}^*+\alpha_{bo}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}}\right) = q^*$. Since $\tilde{q}^* \neq q^*$, $\tilde{\pi}^* < \pi^*$.

– if $\theta = \frac{(\alpha_b^*+\alpha_{bo}^*)p-(\alpha_{so}^*+\alpha_{bo}^*)w}{(\alpha_b^*+\alpha_{bo}^*)p}$, then $\tilde{q}^* = \bar{F}^{-1}\left(\frac{c}{\left(\frac{\alpha_s^*+\alpha_{so}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}+\frac{\alpha_b^*+\alpha_{bo}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}\theta\right)p+r}\right) = \bar{F}^{-1}\left(\frac{c}{p+r-w\frac{\alpha_{so}^*+\alpha_{bo}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}}\right) = q^*$. Since $\tilde{q}^* = q^*$, $\tilde{\pi}^* = \pi^*$.

– if $\theta > \frac{(\alpha_b^*+\alpha_{bo}^*)p-(\alpha_{so}^*+\alpha_{bo}^*)w}{(\alpha_b^*+\alpha_{bo}^*)p}$, then $\tilde{q}^* = \bar{F}^{-1}\left(\frac{c}{\left(\frac{\alpha_s^*+\alpha_{so}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}+\frac{\alpha_b^*+\alpha_{bo}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}\theta\right)p+r}\right) > \bar{F}^{-1}\left(\frac{c}{p+r-w\frac{\alpha_{so}^*+\alpha_{bo}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}}\right) = q^*$. Since $\tilde{q}^* \neq q^*$, $\tilde{\pi}^* < \pi^*$.

Finally, let's prove $\tilde{\pi}^*$ is quasiconcave in θ.

Let's first show that $\tilde{\pi}^*$ is nondecreasing in $\theta < \frac{p-w}{p}$. If $\theta \leq \frac{(\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{so}^*)(c-r)-(\alpha_s^*+\alpha_{so}^*)p}{(\alpha_b^*+\alpha_{bo}^*)p}$, then $\tilde{q}^* = 0$ and thus $\tilde{\pi}^* = wE(\alpha_o^*+\alpha_{so}^*+\alpha_{bo}^*)D$, which is independent of θ. If $\frac{(\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{so}^*)(c-r)-(\alpha_s^*+\alpha_{so}^*)p}{(\alpha_b^*+\alpha_{bo}^*)p} < \theta < \frac{(\alpha_b^*+\alpha_{bo}^*)p-(\alpha_{so}^*+\alpha_{bo}^*)w}{(\alpha_b^*+\alpha_{bo}^*)p}$, then $\frac{\partial\tilde{\pi}^*}{\partial\theta} = \frac{\partial\tilde{\pi}^*}{\partial q}\frac{\partial\tilde{q}^*}{\partial\theta} > 0$, because $\frac{\partial\tilde{\pi}^*}{\partial q} > 0$ if $q < \tilde{q}^*$ and $\frac{\partial\tilde{q}^*}{\partial\theta} > 0$.

Then, let's show that $\tilde{\pi}^*$ is nonincreasing in $\theta > \frac{(\alpha_b^*+\alpha_{bo}^*)p-(\alpha_{so}^*+\alpha_{bo}^*)w}{(\alpha_b^*+\alpha_{bo}^*)p}$. Note $\frac{\partial\tilde{\pi}^*}{\partial\theta} = \frac{\partial\tilde{\pi}^*}{\partial q}\frac{\partial\tilde{q}^*}{\partial\theta} < 0$, because $\frac{\partial\tilde{\pi}^*}{\partial q} < 0$ if $q > \tilde{q}^*$ and $\frac{\partial\tilde{q}^*}{\partial\theta} > 0$.

Thus, we can conclude that $\tilde{\pi}^*$ is quasiconcave in θ. $\square$

C Model Extension: Endogenous Online Channel

In this section, we relax the assumption in the original model that online is exogenous. Suppose both online and offline follow newsvendor setup. Retail price is p, which is the same across both channels. Cross-selling benefit in store is r. Unit inventory costs are c_s and c_o in the store and online channel, both of which are smaller than p. Retailer needs to decide inventory levels q_s and q_o in the store and online channel.

C.1 Homogeneous Market

Consumers setup is the same as before: They have valuation v for the product and hassle costs h_s, h_o, h_b. We assume when the online channel is out of stock, those who are willing to buy online will leave for other websites to buy the product at the same price. Therefore, given belief about store inventory availability $\hat{\xi}$, consumer's utility from visiting store is the same as before, i.e., $u_s = -h_s + \hat{\xi}(v-p) + (1-\hat{\xi})(v-p-h_o)$. Consumers compare the expected utility from each channel and choose accordingly.

Retailer has belief $\hat{\phi}$ about the fraction of customers who visit store. Given this belief, the retailer's profit function is

$$\pi(q_s, q_o) = pE\min\left(\hat{\phi}D, q_s\right) - c_s q_s + rE\hat{\phi}D + pE\min\left((1-\hat{\phi})D + \left(\hat{\phi}D - q_s\right)^+, q_o\right) - c_o q_o \quad \text{(C.1)}$$

Definition C.1. *A RE equilibrium $(q_s, q_o, \phi, \hat{\phi}, \hat{\xi})$ satisfies the following:*

i Given $\hat{\xi}$, if $u_s \geq u_o$, then $\phi = 1$; otherwise $\phi = 0$;

ii. Given $\hat{\phi}$, $(q_s, q_o) = \arg\max \pi(q_s, q_o)$, where $\pi(q_s, q_o)$ is given in (C.1);

iii. $\hat{\xi} = A(q_s)$;

iv. $\hat{\phi} = \phi$.

The following proposition gives the RE equilibrium.

Proposition C.1. *If $h_s \leq \frac{E\min\left(D, \bar{F}^{-1}\left(\frac{c_s}{p}\right)\right)}{ED} h_o$ and $c_s < c_o$, then customers visit store and $q_s^\circ = \bar{F}^{-1}(\frac{c_s}{p})$ and $q_o^\circ = 0$. Otherwise, no one comes to store and $q_s^\circ = 0$ and $q_o^\circ = \bar{F}^{-1}(\frac{c_o}{p})$.*

With BOPS, retailer's profit function is as follows:

- If $\min(h_s, h_b) > h_o$, then no one comes to store, and thus

$$\pi = pE\min(D, q_o) - c_o q_o$$

- If $\min(h_s, h_b) \leq h_o$, then consumers come to store if it is in stock, and thus

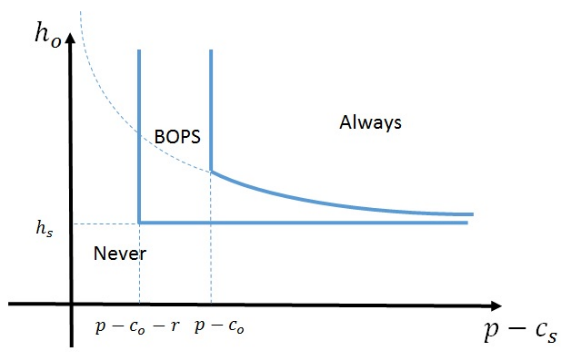
$$\pi = (p+r)\,E\min(D, q_s) - c_s q_s + pE\min\left((D-q_s)^+, q_o\right) - c_o q_o$$

Proposition C.2. *When there is BOPS,*

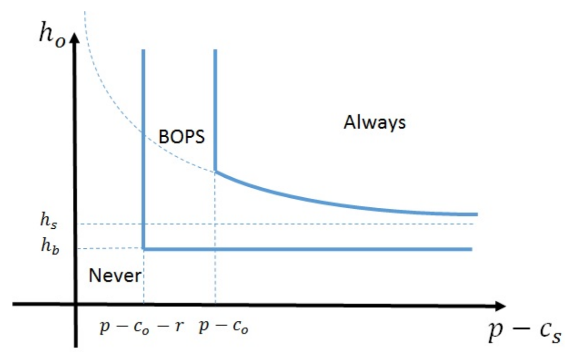
- *if* $\min(h_s, h_b) \le h_o$ *and* $c_s < c_o + r$*, then customers visit store and*
  - *if* $c_s \le \frac{p+r}{p} c_o$*, then* $q_s^* = \bar{F}^{-1}\left(\frac{c_s}{p+r}\right)$ *and* $q_o^* = 0$*;*
  - *if* $c_s > \frac{p+r}{p} c_o$*, then* $q_s^* = \bar{F}^{-1}\left(\frac{c_s - c_o}{r}\right)$ *and* $q_o^* = \bar{F}^{-1}\left(\frac{c_o}{p}\right) - \bar{F}^{-1}\left(\frac{c_s - c_o}{r}\right)$*;*
- *otherwise, no one comes to store and* $q_s^* = 0$ *and* $q_o^* = \bar{F}^{-1}\left(\frac{c_o}{p}\right)$*.*

Comparing Propositions C.1 and C.2, we can have the three regions as before, though the shape is different (see Figure C.1).

Figure C.1: Do consumers buy the product in store?



(a) $h_b > h_s$



(b) $h_b \le h_s$

In the "BOPS" and "Always" region, we can get similar comparison results as before:

Proposition C.3. *If* $h_s \in \left(\frac{E \min\left(D, \bar{F}^{-1}\left(\frac{c_s}{p}\right)\right)}{ED} h_o, h_o \right]$ *and* $c_s < c_o + r$*, then customers visit the store only if BOPS is available. Further, BOPS increases total profit (i.e.,* $\pi^* > \pi^\circ$*).*

Proposition C.4. *If* $h_b \le h_o < h_s$ *and* $c_s < c_o + r$*, customers visit the store only if BOPS is available. Further, BOPS increases total profit (i.e.,* $\pi^* > \pi^\circ$*).*

Proposition C.5. *If* $h_s \le \frac{E \min\left(D, \bar{F}^{-1}\left(\frac{c_s}{p}\right)\right)}{ED} h_o$ *and* $c_s < c_o$*, then customers visit store regardless of the implementation of BOPS. Further, if* $r > 0$*, then BOPS decreases total profit (i.e.,* $\pi^* < \pi^\circ$*).*

C.2 Heterogeneous Market

Similar as what we did for the homogeneous market, we assume when the online channel is out of stock, those who are willing to buy online will leave for other websites to buy the product at the same price. Thus, we could find that consumers behavior remains the same as before.

Given retailer's belief $\hat{\alpha}_o, \hat{\alpha}_{so}, \hat{\alpha}_s, \hat{\alpha}_l$, the retailer's profit function is

$$\begin{aligned}\pi\left(q_s, q_o\right) =& pE\min\left(\left(\hat{\alpha}_s+\hat{\alpha}_{so}\right)D, q_s\right) - c_s q_s + rE\left(\hat{\alpha}_s+\hat{\alpha}_{so}\right)D \\ &+ pE\min\left(\hat{\alpha}_o D + \frac{\hat{\alpha}_{so}}{\hat{\alpha}_s+\hat{\alpha}_{so}}\left(\left(\hat{\alpha}_s+\hat{\alpha}_{so}\right)D - q_s\right)^+, q_o\right) - c_o q_o\end{aligned} \tag{C.2}$$

Definition C.2. *A RE equilibrium $(\alpha_o, \alpha_{so},, \alpha_s, \alpha_l, q_s, q_o, \hat{\xi}, \hat{\alpha}_o, \hat{\alpha}_{so}, \hat{\alpha}_s, \hat{\alpha}_l)$ satisfies the following:*

i Given $\hat{\xi}$, then $\alpha_o = \frac{v-p}{H} - \frac{\hat{\xi}(v-p)^2}{2H^2}$, $\alpha_{so} = \frac{\hat{\xi}(v-p)^2}{2H^2}$, $\alpha_s = \frac{\hat{\xi}(v-p)(H-(v-p))}{H^2}$, and $\alpha_l = \frac{[H-(v-p)][H-\hat{\xi}(v-p)]}{H^2}$;

ii. Given $\hat{\alpha}_o, \hat{\alpha}_{so}, \hat{\alpha}_s$ and $\hat{\alpha}_l$, $(q_s, q_o) = \arg\max \pi(q_s, q_o)$, where $\pi(q_s, q_o)$ is given in (C.2);

iii. $\hat{\xi} = A(q_s)$, where $A(q_s) = \frac{E\min((\alpha_s+\alpha_{so})D, q_s)}{E(\alpha_s+\alpha_{so})D}$;

iv. $\hat{\alpha}_s = \alpha_s$, $\hat{\alpha}_o = \alpha_o$, $\hat{\alpha}_{so} = \alpha_{so}$ and $\hat{\alpha}_l = \alpha_l$.

Proposition C.6.

- *If $c_s < \frac{v-p}{2H-(v-p)}c_o + \frac{2H-2(v-p)}{2H-(v-p)}p$, then there are customers visiting store, specifically,*
  - *if $c_s < c_o$, then $\alpha_s^\circ = \frac{\xi_1^\circ(v-p)(H-(v-p))}{H^2}$, $\alpha_{so}^\circ = \frac{\xi_1^\circ(v-p)^2}{2H^2}$, $\alpha_o^\circ = \frac{v-p}{H} - \frac{\xi_1^\circ(v-p)^2}{2H^2}$, $q_s^\circ = (\alpha_s^\circ + \alpha_{so}^\circ)\bar{F}^{-1}(\frac{c_s}{p})$ and $q_o^\circ = \alpha_o^\circ \bar{F}^{-1}(\frac{c_o}{p})$, where $\xi_1^\circ = \frac{\min\left(D, \bar{F}^{-1}\left(\frac{c_s}{p}\right)\right)}{ED}$*
  - *if $c_s \geq c_o$, then $\alpha_s^\circ = \frac{\xi_2^\circ(v-p)(H-(v-p))}{H^2}$, $\alpha_{so}^\circ = \frac{\xi_2^\circ(v-p)^2}{2H^2}$, $\alpha_o^\circ = \frac{v-p}{H} - \frac{\xi_2^\circ(v-p)^2}{2H^2}$, $q_s^\circ = (\alpha_s^\circ + \alpha_{so}^\circ)\bar{F}^{-1}\left(\frac{c_s - \frac{v-p}{2H-(v-p)}c_o}{\frac{2H-2(v-p)}{2H-(v-p)}p}\right)$ and $q_o^\circ = (\alpha_o^\circ + \alpha_{so}^\circ)\bar{F}^{-1}(\frac{c_o}{p}) - \alpha_{so}^\circ F^{-1}\left(\frac{c_s - \frac{v-p}{2H-(v-p)}c_o}{\frac{2H-2(v-p)}{2H-(v-p)}p}\right)$, where $\xi_2^\circ = \frac{\min\left(D, F^{-1}\left(\frac{c_s - \frac{v-p}{2H-(v-p)}c_o}{\frac{2H-2(v-p)}{2H-(v-p)}p}\right)\right)}{ED}$*
- *If $c_s \geq \frac{v-p}{2H-(v-p)}c_o + \frac{2H-2(v-p)}{2H-(v-p)}p$, then no one ever comes to store and $q_s^\circ = 0$, $q_o^\circ = \frac{v-p}{H}\bar{F}^{-1}(\frac{c_o}{p})$, $\xi^\circ = 0$.*

With BOPS, retailer's profit function is as follows

$$\begin{aligned}\pi\left(q_s, q_o\right) =& pE\min\left(\left(\alpha_s^* + \alpha_b^* + \alpha_{so}^* + \alpha_{bo}^*\right)D, q_s\right) - c_s q_s + rE\min\left(\left(\alpha_s^* + \alpha_b^* + \alpha_{so}^* + \alpha_{bo}^*\right)D, q_s\right) \\ &+ pE\min\left(\alpha_o^* D + \frac{\alpha_{so}^* + \alpha_{bo}^*}{\alpha_s^* + \alpha_b^* + \alpha_{so}^* + \alpha_{bo}^*}\left(\left(\alpha_s^* + \alpha_b^* + \alpha_{so}^* + \alpha_{bo}^*\right)D - q_s\right)^+, q_o\right) - c_o q_o\end{aligned}$$

Proposition C.7. *Suppose $r = 0$. When there is BOPS, market outcome is given as follows:*

- *If $c_s < \frac{\alpha_{so}^* + \alpha_{bo}^*}{\alpha_s^* + \alpha_b^* + \alpha_{so}^* + \alpha_{bo}^*}c_o + \frac{\alpha_s^* + \alpha_b^*}{\alpha_s^* + \alpha_b^* + \alpha_{so}^* + \alpha_{bo}^*}p$, then there are customers visiting store, and*

- *if* $c_s < c_o$*, then* $q_s^* = (\alpha_s^* + \alpha_{so}^*)\bar{F}^{-1}(\frac{c_s}{p})$ *and* $q_o^* = \alpha_o^*\bar{F}^{-1}(\frac{c_o}{p})$
- *if* $c_s \geq c_o$*, then* $q_s^* = (\alpha_s^* + \alpha_{so}^*)\bar{F}^{-1}\left(\frac{c_s - \frac{\alpha_{so}^*+\alpha_{bo}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}c_o}{\frac{\alpha_s^*+\alpha_b^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}p}\right)$ *and* $q_o^* = (\alpha_o^* + \alpha_{so}^*)\bar{F}^{-1}(\frac{c_o}{p}) - \alpha_{so}^* F^{-1}\left(\frac{c_s - \frac{\alpha_{so}^*+\alpha_{bo}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}c_o}{\frac{\alpha_s^*+\alpha_b^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}p}\right)$,

- *If* $c_s \geq \frac{v-p}{2H-(v-p)}c_o + \frac{2H-2(v-p)}{2H-(v-p)}p$*, then no one ever comes to store and* $q_s^* = 0$*,* $q_o^* = \frac{v-p}{H}\bar{F}^{-1}(\frac{c_o}{p})$*.*

Proposition C.8.

i. BOPS helps to expand market coverage, i.e., $\alpha_s^* + \alpha_o^* + \alpha_b^* + \alpha_{so}^* + \alpha_{bo}^* > \alpha_s^\circ + \alpha_o^\circ + \alpha_{so}^\circ$*;*

ii. Suppose $r = 0$*. If there are customers visiting store when there is no BOPS, then there exists* $\bar{c}_s$ *such that the implementation of BOPS decreases total profit (i.e.,* $\pi^* < \pi^\circ$*) if* $\beta_s + \beta_o < 1$ *and* $c_s > \bar{c}_s$*.*

This shows that although BOPS could expand market coverage, it may still reduce total profit even if $r = 0$.

C.3 Decentralized System

Finally, let's look at decentralized system. Here, for simplicity of exposition, we only look at the homogeneous market. Same as before, we only consider the situation where consumers use BOPS when it is available. Then, with the revenue sharing parameter θ, store and online's profits are given as follows:

$$\tilde{\pi}_s = (\theta p + r)\, E \min(D, \tilde{q}_s) - c_s \tilde{q}_s$$
$$\tilde{\pi}_o = (1-\theta)\, pE \min(D, \tilde{q}_s) + pE \min\big((D - \tilde{q}_s)^+, \tilde{q}_o\big) - c_o \tilde{q}_o$$

Note the BOPS revenue is just free money to the online channel. So we only need to consider store's incentive. Comparing $\tilde{\pi}_s$ with the centralized profit funtion π, we can still find that the store channel ignores the fact that customers would buy online instead in case of stockouts (i.e., the second term in $\tilde{\pi}_o$), and thus there will be a profit loss in the decentralized system compared to the centralized case. Moreover, it is easy to find the following revenue sharing parameter will correct store's incentive and coordinate both channels:

- If $c_s \leq \frac{p+r}{p} c_o$, then $\theta^* = 1$
- If $c_s > \frac{p+r}{p} c_o$, then $\theta^* = \frac{c_o r}{(c_s - c_o)p} \in (0, 1)$.

C.4 Proofs

Proof of Proposition C.1: Note nonparticipatory equilibrium, $(0, \bar{F}^{-1}(\frac{c_o}{p}), 0, 0, 0)$, always exists. Same as what we did in the base model, we are going to look for participatory equilibrium, where $\phi^\circ = 1$ and $q_s^\circ > 0$. All we need to do is to check the four conditions specified in Definition C.1.

First, we look at retailer's problem: In the participatory equilibrium, retailer's belief is consistent, i.e., $\hat{\phi} = \phi^\circ = 1$. Then, the retailer's profit is $\pi = pE\min(D, q_s) - c_s q_s + rED + pE\min((D - q_s)^+, q_o) - c_o q_o = rED + pE\min(D, q_s + q_o) - c_s q_s - c_o q_o$, the optimal solution of which is easy to find as the follows

- if $c_s < c_o$, then $q_s^\circ = \bar{F}^{-1}(\frac{c_s}{p})$ and $q_o^\circ = 0$
- if $c_s \geq c_o$, then $q_s^\circ = 0$ and $q_o^\circ = \bar{F}^{-1}(\frac{c_o}{p})$.

Thus, to ensure participatory equilibrium, we need $c_s < c_o$.

Note in equilibrium, we also need consumer's belief to be consistent with the outcome, i.e., $\hat{\xi}^\circ = A(q_s^\circ) = E\min(\phi^\circ D, q_s^\circ)/E(\phi^\circ D) = E\min(D, \bar{F}^{-1}(\frac{c_s}{p}))/ED$.

Finally, we go back to consumer's decision. To ensure $\phi^\circ = 1$, we need $u_s \geq u_o$, i.e., $h_s \leq \hat{\xi}^\circ h_o$. Therefore, we need $h_s \leq \frac{E\min(D, \bar{F}^{-1}(\frac{c_s}{p}))}{ED} h_o$.

Based on the analysis above, we find the conditions for a participatory equilibrium are $h_s \leq \frac{E\min(D, \bar{F}^{-1}(\frac{c_s}{p}))}{ED} h_o$ and $c_s < c_o$. And the equilibrium outcome is $\phi^\circ = 1$, $q_s^\circ = \bar{F}^{-1}\left(\frac{c_s}{p}\right)$ and $q_o^\circ = 0$. □

Proof of Proposition C.2: First, let's look at the case where $\min(h_s, h_b) \leq h_o$. In this case, consumers come to store if it is in stock. Thus, the retailer's profit is

$$\pi = (p + r) E\min(D, q_s) - c_s q_s + pE\min\left((D - q_s)^+, q_o\right) - c_o q_o$$

First, by checking the Hessian matrix, we can easily find the profit function is jointly concave in (q_s, q_o). Moreover, note that $\frac{\partial \pi}{\partial q_s} = r\bar{F}(q_s) + p\bar{F}(q_s + q_o) - c_s$ and $\frac{\partial \pi}{\partial q_o} = p\bar{F}(q_s, q_o) - c_o$. Thus, by the first order condition and taking into account the constraints that $q_s \geq 0$ and $q_o \geq 0$, we can find the optimal solution that maximizes the profit function as follows:

- if $c_s \leq \frac{p+r}{p} c_o$, then $\bar{F}(q_s^*) = \frac{c_s}{p+r} \Rightarrow q_s^* = \bar{F}^{-1}(\frac{c_s}{p+r})$, and $q_o^* = 0$
- if $c_s \in (\frac{p+r}{p} c_o, c_o + r)$, then $\bar{F}(q_s^*) = \frac{c_s - c_o}{r} \Rightarrow q_s^* = \bar{F}^{-1}(\frac{c_s - c_o}{r})$, and $\bar{F}(q_s^* + q_o^*) = \frac{c_o}{p} \Rightarrow q_o^* = \bar{F}^{-1}(\frac{c_o}{p}) - \bar{F}^{-1}(\frac{c_s - c_o}{r})$
- if $c_s \geq c_o + r$, then $q_s^* = 0$ and $\bar{F}(q_o^*) = \frac{c_o}{p} \Rightarrow q_o^* = \bar{F}^{-1}(\frac{c_o}{p})$, in which case customers actually never come to store because store does not have any inventory.

Then, let's look at the case where $\min(h_s, h_b) > h_o$. Then, consumers never come to store and the retailer's profit is $\pi = pE\min(D, q_o) - c_o q_o$, which is a newsvendor problem. Thus, $q_s^* = 0$ and $q_o^* = \bar{F}^{-1}(\frac{c_o}{p})$. □

Proof of Proposition C.3: If customers visit the store only if BOPS is available, then

$$\pi^* = (p+r)\, E\min\left(D, q_s^*\right) - c_s q_s^* > pE\min\left(D, q_o^\circ\right) - c_o q_o^\circ = \pi^\circ$$

where the inequality is because (q_s^*, q_o^*) rather than (q_s°, q_o°) maximizes π^*. □

Proof of Proposition C.4: Since consumer behavior is the same as Proposition C.3, the proof is also very similar to the one for Proposition C.3, and thus omitted. □

Proof of Proposition C.5: If $h_s \leq \frac{E\min\left(D, \bar{F}^{-1}\left(\frac{c_s}{p}\right)\right)}{ED} h_o$ and $c_s < c_o$, then $q_s^\circ = \bar{F}^{-1}(\frac{c_s}{p})$, $q_o^\circ = 0$, $q_s^* = \bar{F}^{-1}(\frac{c_s}{p+r})$ and $q_o^* = 0$. Then,

$$\begin{aligned}
\pi^\circ &= rED + pE\min\left(D, q_s^\circ\right) - c_s q_s^\circ \\
&> rED + pE\min\left(D, q_s^*\right) - c_s q_s^* \\
&> rE\min\left(D, q_s^*\right) + pE\min\left(D, q_s^*\right) - c_s q_s^* = \pi^*
\end{aligned}$$

where the first inequality is due to the fact that q_s° rather than q_s^* maximizes π°. □

Proof of Proposition C.6: Note nonparticipatory equilibrium always exists. Same as what we did in the base model, we are going to look for participatory equilibrium, where $\alpha_s > 0$, $\alpha_{so} > 0$ and $q_s^\circ > 0$.

First, let's look at retailer's problem. It is easy to verify that π is jointly concave in (q_s, q_o). Also, note that with $\hat{\alpha}_i = \alpha_i^\circ$, $i = s, o, so, l$,

- if $\frac{q_s}{\alpha_s^\circ+\alpha_{so}^\circ} > \frac{q_o}{\alpha_o^\circ}$, then

$$\frac{\partial \pi}{\partial q_s} = p\bar{F}\left(\frac{q_s}{\alpha_s^\circ+\alpha_{so}^\circ}\right) - c_s$$
$$\frac{\partial \pi}{\partial q_o} = p\bar{F}\left(\frac{q_o}{\alpha_o^\circ}\right) - c_o$$

- if $\frac{q_s}{\alpha_s^\circ+\alpha_{so}^\circ} \leq \frac{q_o}{\alpha_o^\circ}$, then

$$\frac{\partial \pi}{\partial q_s} = \frac{\alpha_s^\circ}{\alpha_s^\circ+\alpha_{so}^\circ}p\bar{F}\left(\frac{q_s}{\alpha_s+\alpha_{so}^\circ}\right) + \frac{\alpha_{so}^\circ}{\alpha_s^\circ+\alpha_{so}^\circ}p\bar{F}\left(\frac{q_o+\frac{\alpha_{so}^\circ}{\alpha_s^\circ+\alpha_{so}}q_s}{\alpha_o^\circ+\alpha_{so}^\circ}\right) - c_s$$
$$\frac{\partial \pi}{\partial q_o} = p\bar{F}\left(\frac{q_o+\frac{\alpha_{so}^\circ}{\alpha_s^\circ+\alpha_{so}^\circ}q_s}{\alpha_o^\circ+\alpha_{so}^\circ}\right) - c_o$$

Then, given the constraints $q_s \geq 0$ and $q_o \geq 0$, we can easily get the solution as follows:

- If $c_s < c_o$, then

$$q_s^\circ = \left(\alpha_s^\circ + \alpha_{so}^\circ\right)\bar{F}^{-1}\left(\frac{c_s}{p}\right)$$
$$q_o^\circ = \alpha_o^\circ \bar{F}^{-1}\left(\frac{c_o}{p}\right)$$

- if $c_s \geq c_o$ and $c_s < \frac{\alpha_{so}^\circ}{\alpha_s+\alpha_{so}^\circ}c_o + \frac{\alpha_s^\circ}{\alpha_s^\circ+\alpha_{so}^\circ}p$, then

$$q_s^\circ = \left(\alpha_s^\circ + \alpha_{so}^\circ\right)\bar{F}^{-1}\left(\frac{c_s - \frac{\alpha_{so}^\circ}{\alpha_s^\circ+\alpha_{so}^\circ}c_o}{\frac{\alpha_s^\circ}{\alpha_s^\circ+\alpha_{so}^\circ}p}\right)$$
$$q_o^\circ = \left(\alpha_o^\circ + \alpha_{so}^\circ\right)\bar{F}^{-1}\left(\frac{c_o}{p}\right) - \alpha_{so}^\circ\bar{F}^{-1}\left(\frac{c_s - \frac{\alpha_{so}^\circ}{\alpha_s^\circ+\alpha_{so}^\circ}c_o}{\frac{\alpha_s^\circ}{\alpha_s^\circ+\alpha_{so}^\circ}p}\right)$$

- if $c_s \geq \frac{\alpha_{so}^\circ}{\alpha_s^\circ+\alpha_{so}^\circ}c_o + \frac{\alpha_s^\circ}{\alpha_s^\circ+\alpha_{so}^\circ}p$, then

$$q_s^\circ = 0$$
$$q_o^\circ = \left(\alpha_o^\circ + \alpha_{so}^\circ\right)\bar{F}^{-1}\left(\frac{c_o}{p}\right)$$

Note in equilibrium, we also need consumer's belief to be consistent with the outcome, i.e., $\hat{\xi}^\circ = A(q_s^\circ) = E\min((\alpha_s^\circ + \alpha_{so}^\circ)D, q_s^\circ)/E((\alpha_s^\circ + \alpha_{so}^\circ)D)$. Thus, if $c_s < c_o$, then $\hat{\xi}^\circ = \xi_1^\circ$; if $c_s \geq c_o$ and $c_s < \frac{\alpha_{so}^\circ}{\alpha_s+\alpha_{so}^\circ}c_o + \frac{\alpha_s^\circ}{\alpha_s^\circ+\alpha_{so}^\circ}p$, then $\hat{\xi}^\circ = \xi_2^\circ$; otherwise, $\hat{\xi}^\circ = 0$. Here, we used the fact that for any $\hat{\xi} > 0$, we have $\frac{\alpha_{so}}{\alpha_s+\alpha_{so}} = \frac{v-p}{2H-(v-p)}$ and $\frac{\alpha_s}{\alpha_s+\alpha_{so}} = \frac{2H-2(v-p)}{2H-(v-p)}$.

Note $\hat{\xi}^{\circ}$ is independent of α's. Thus, to ensure equilibrium, we just need to replace $\hat{\xi}$ with $\hat{\xi}^{\circ}$ and set all the αs to satisfy (i) in Definition C.2. $\square$

Proof of Proposition C.7: First, when $r = 0$, we can find that π is jointly concave in (q_s, q_o). Also,

- if $\frac{q_s}{\alpha_s^*+\alpha_b^*+\alpha_{bo}^*+\alpha_{so}^*} > \frac{q_o}{\alpha_o^*}$, then

$$\begin{aligned}
\frac{\partial\pi}{\partial q_s} &= p\bar{F}\left(\frac{q_s}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}\right) - c_s \\
\frac{\partial\pi}{\partial q_o} &= p\bar{F}\left(\frac{q_s}{\alpha_o^*}\right) - c_o
\end{aligned}$$

- if $\frac{q_s}{\alpha_s^*+\alpha_b^*+\alpha_{bo}^*+\alpha_{so}^*} \leq \frac{q_o}{\alpha_o^*}$, then

$$\begin{aligned}
\frac{\partial\pi}{\partial q_s} &= \frac{\alpha_s^*+\alpha_b^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}p\bar{F}\left(\frac{q_s}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}\right) + \frac{\alpha_{so}^*+\alpha_{bo}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}p\bar{F}\left(\frac{q_o+\frac{\alpha_{so}^*+\alpha_{bo}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}q_s}{\alpha_o^*+\alpha_{so}^*+\alpha_{bo}^*}\right) - c_s \\
\frac{\partial\pi}{\partial q_o} &= p\bar{F}\left(\frac{q_o+\frac{\alpha_{so}^*+\alpha_{bo}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}q_s}{\alpha_o^*+\alpha_{so}^*+\alpha_{bo}^*}\right) - c_o
\end{aligned}$$

Then, given the constraints $q_s \geq 0$ and $q_o \geq 0$, we can easily get the solution as follows:

- If $c_s < c_o$, then

$$\begin{aligned}
q_s^* &= (\alpha_s^* + \alpha_b^* + \alpha_{so}^* + \alpha_{bo}^*)\,\bar{F}^{-1}\left(\frac{c_s}{p}\right) \\
q_o^* &= \alpha_o^*\bar{F}^{-1}\left(\frac{c_o}{p}\right)
\end{aligned}$$

- if $c_s \geq c_o$ and $c_s < \frac{\alpha_{so}^*+\alpha_{bo}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}c_o + \frac{\alpha_s^*+\alpha_b^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}p$, then

$$\begin{aligned}
q_s^* &= (\alpha_s^* + \alpha_b^* + \alpha_{so}^* + \alpha_{bo}^*)\,\bar{F}^{-1}\left(\frac{c_s-\frac{\alpha_{so}^*+\alpha_{bo}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}c_o}{\frac{\alpha_s^*+\alpha_b^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}p}\right) \\
q_o^* &= (\alpha_o^* + \alpha_{so}^* + \alpha_{bo}^*)\,\bar{F}^{-1}\left(\frac{c_o}{p}\right) - (\alpha_{so}^* + \alpha_{bo}^*)\,\bar{F}^{-1}\left(\frac{c_s-\frac{\alpha_{so}^*+\alpha_{bo}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}c_o}{\frac{\alpha_s^*+\alpha_b^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}p}\right)
\end{aligned}$$

- if $c_s \geq \frac{\alpha_{so}^*+\alpha_{bo}^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}c_o + \frac{\alpha_s^*+\alpha_b^*}{\alpha_s^*+\alpha_b^*+\alpha_{so}^*+\alpha_{bo}^*}p$, then

$$\begin{aligned}
q_s^* &= 0 \\
q_o^* &= (\alpha_o^* + \alpha_{so}^* + \alpha_{bo}^*)\,\bar{F}^{-1}\left(\frac{c_o}{p}\right)
\end{aligned}$$

$\square$

Proof of Proposition C.8: Since consumer behavior remains the same as before, part (i) remains valid as before.

Next, let's look at part (ii). To simplify notation, we denote $\Delta^\circ = \frac{v-p}{2H-(v-p)}c_o + \frac{2H-2(v-p)}{2H-(v-p)}p$ and $\Delta^* = \frac{\alpha^*_{so}+\alpha^*_{bo}}{\alpha^*_s+\alpha^*_b+\alpha^*_{so}+\alpha^*_{bo}}c_o + \frac{\alpha^*_s+\alpha^*_b}{\alpha^*_s+\alpha^*_b+\alpha^*_{so}+\alpha^*_{bo}}p$. Note if $\beta_s + \beta_o < 1$, we have $\Delta^\circ > \Delta^*$.

If there are customers visiting store when there is no BOPS, we need to have $c_s < \Delta^\circ$. The following analysis assumes this condition holds.

Suppose $r = 0$. If $c_s \geq \Delta^*$ (this is possible since $\Delta^\circ > \Delta^*$), then no one comes to store when there is BOPS. In this case,

$$
\begin{aligned}
\pi^* &= pE\min\left(\left(\alpha^*_o + \alpha^*_{so} + \alpha^*_{bo}\right)D, q^*_o\right) - c_o q^*_o \\
&= pE\min\left(\left(\alpha^\circ_o + \alpha^\circ_{so}\right)D, q^*_o\right) - c_o q^*_o \\
&< pE\min\left(\left(\alpha^\circ_s + \alpha^\circ_{so}\right)D, q^\circ_s\right) - c_s q^\circ_s + rE\left(\alpha^\circ_s + \alpha^\circ_{so}\right)D \\
&\qquad + pE\min\left(\alpha^\circ_o D + \frac{\alpha^\circ_{so}}{\alpha^\circ_s + \alpha^\circ_{so}}\left(\left(\alpha^\circ_s + \alpha^\circ_{so}\right)D - q^\circ_s\right)^+, q^\circ_o\right) - c_o q^\circ_o \\
&= \pi^\circ
\end{aligned}
$$

where the inequality is because $q_s = 0, q_o = q^*_o$ is also a feasible but not the optimal solution to the case where there is no BOPS. If $c_s < \Delta^*$, then there are consumers visiting store when there is BOPS. Note (1) $\pi^\circ - \pi^*$ is continuous in c_s, and (2) $\pi^\circ > \pi^*$ when $c_s = \Delta^*$. Thus, the analysis above implies that there exists $\bar{c}_s < \Delta^*$ such that $\pi^\circ > \pi^*$ if $c_s > \bar{c}_s$. □

D Model Extension: Default Channel Choice

Suppose a fraction λ of customers are nonstrategic, who head to store by default (because they may forget or they may not care about checking the website beforehand) and may consider buying from the online channel only if store is out of stock; the rest $1-\lambda$ are strategic as before. As a result, when there is BOPS, only a fraction $1-\lambda$ of customers will check online for store inventory information.

D.1 Homogeneous Market

Let's first consider the case when there is no BOPS. Assume all customers (including both nonstrategic and strategic customers) have the same online hassle cost h_o. Then, since $v - p - h_o \geq 0$, all nonstrategic customers will buy online if store is out of stock.

The retailer has a belief about the fraction of strategic customers who visit store, denoted as $\hat{\phi}$. Given this belief, his total profit is

$$\begin{aligned}\pi =& pE \min\left(\left(\lambda + \hat{\phi}(1-\lambda)\right) D, q\right) - cq + rE\left(\lambda + \hat{\phi}(1-\lambda)\right) D \\ &+ wE(1-\hat{\phi})(1-\lambda)D + wE\Big(\left(\lambda + \hat{\phi}(1-\lambda)\right) D - q\Big)^+\end{aligned}$$

The following proposition gives the RE equilibrium.

Proposition D.1. *If* $h_s \leq \frac{E\min\left(D, \bar{F}^{-1}\left(\frac{c}{p-w}\right)\right)}{ED} h_o$ *and* $p - c > w$*, then strategic customers visit store and* $q^\circ = \bar{F}^{-1}\left(\frac{c}{p-w}\right)$*. Otherwise, no strategic customer comes to store and* $q^\circ = \lambda \bar{F}^{-1}\left(\frac{c}{p-w} \wedge 1\right)$*.*

When there is BOPS, strategic customers choose their shopping channel given the current store inventory status.

- If $\min(h_s, h_b) \leq h_o$, strategic customers come to store if it is in stock and the retailer's profit is

$$\pi = pE\min(D, q) - cq + rE\lambda D + rE(1-\lambda)\min(D, q) + wE(D-q)^+$$

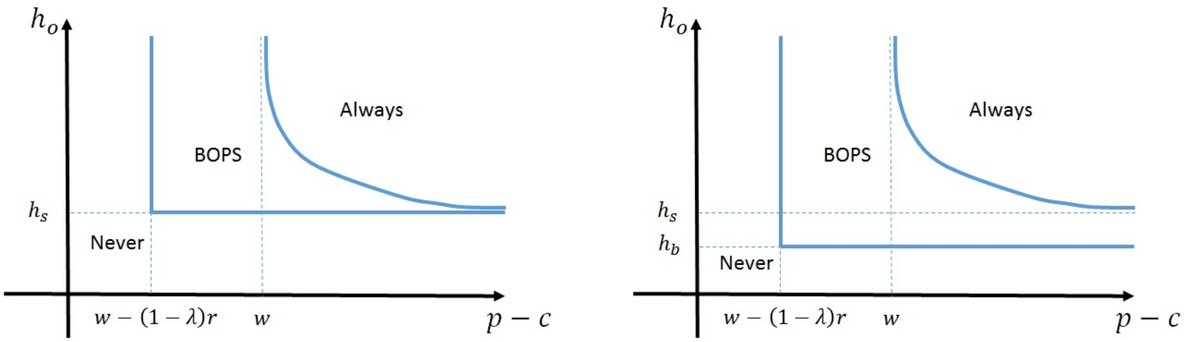
- If $\min(h_s, h_b) > h_o$, no strategic customers come to store and the retailer's profit is

$$\pi = pE\min(\lambda D, q) - cq + rE\lambda D + wE(1-\lambda)D + wE(\lambda D - q)^+$$

Proposition D.2. *When there is BOPS, if* $\min(h_s, h_b) \leq h_o$ *and* $p - c > w - (1-\lambda)r$*, then strategic customers visit store and* $q^* = \bar{F}^{-1}\left(\frac{c}{p+(1-\lambda)r-w}\right)$*; otherwise, no strategic customer comes to store and* $q^* = \lambda \bar{F}^{-1}\left(\frac{c}{p-w} \wedge 1\right)$*.*

Comparing Propositions D.1 and D.2, Figure D.1 shows the impact of BOPS on strategic customer's channel choice. Note it is similar to Figure 1 in Section 4, and it is easy to verify that the same insights still hold in this case, i.e., BOPS increases profit in the "BOPS" region as it helps to persuade strategic customers to the more profitable store channel, but BOPS decreases profit in the "Always" region because strategic customers do not come to store once it is out of stock and thus the retailer loses some cross-selling profits.

Figure D.1: Do strategic consumers buy the product in store?



(a) $h_b > h_s$ (b) $h_b \leq h_s$

D.2 Heterogeneous Market

Strategic customer's behavior is the same as before. As for nonstrategic customers, they always first go to store. When nonstrategic customers are in store, if store is in stock, they buy on the spot; if store is out of stock, they can either buy online instead (and obtain payoff $v-p-h_o$) or leave (and obtain payoff 0). Assume, nonstrategic consumers are heterogeneous in terms of h_o, and the distribution is the same as strategic customers, i.e., all nonstrategic customers are distributed uniformly on the line $[0,H]$. Then, when store is out of stock, a fraction $\frac{v-p}{H}$ of nonstrategic customers (i.e., those with $h_o \leq v-p$) will substitute to the online channel.

When there is no BOPS, we keep using notation α_o, α_s and α_{so} to denote the fraction of the pure online, pure store and store-to-online customers among strategic customers. The retailer has belief over these α's, denoted as $\hat{\alpha}_o$, $\hat{\alpha}_s$, and $\hat{\alpha}_{so}$. Given this belief, the retailer's profit is

$$\begin{aligned}\pi =& pE\min\left(\left(\lambda+\left(\hat{\alpha}_s+\hat{\alpha}_{so}\right)(1-\lambda)\right)D,q\right)-cq+rE\left(\lambda+\left(\hat{\alpha}_s+\hat{\alpha}_{so}\right)(1-\lambda)\right)D\\ &+wE\hat{\alpha}_o(1-\lambda)D+w\frac{\frac{v-p}{H}\lambda+\hat{\alpha}_{so}(1-\lambda)}{\lambda+\left(\hat{\alpha}_s+\hat{\alpha}_{so}\right)(1-\lambda)}E(\left(\lambda+\left(\hat{\alpha}_s+\hat{\alpha}_{so}\right)(1-\lambda)\right)D-q)^+\end{aligned}$$

Proposition D.3. *When there is no BOPS, the RE equilibrium is given as follows:*

- *If equation* $\xi = \frac{\min\left(D,\bar{F}^{-1}\left(\frac{c}{p-w\Delta^{\circ}(\xi)}\wedge 1\right)\right)}{ED}$ *has a solution* $\xi^{\circ} > 0$*, then there are strategic* *consumers visiting store and* ($\alpha_s^\circ = \frac{\xi^\circ(v-p)(H-(v-p))}{H^2} > 0$, $\alpha_{so}^\circ = \frac{\xi^\circ(v-p)^2}{2H^2} > 0$) *and* $q^\circ = (\lambda + (\alpha_s^\circ + \alpha_{so}^\circ)(1-\lambda))\,\bar{F}^{-1}\left(\frac{c}{p - w\Delta^\circ(\xi^\circ)}\right)$, *where* $\Delta^\circ(\xi) = \frac{v-p}{2H-(v-p)} + \frac{(v-p)(H-(v-p))\lambda}{H(2H-(v-p))\left[\lambda + \frac{2H-(v-p)}{2H^2}(v-p)\xi(1-\lambda)\right]}$;

- *otherwise, no strategic customer comes to store and* $q^\circ = 0$.

Note a sufficient condition for the equation $\xi = \frac{\min\left(D, \bar{F}^{-1}\left(\frac{c}{p-w\Delta^\circ(\xi)} \wedge 1\right)\right)}{ED}$ having a positive solution is $p - w\Delta^\circ(0) > c$. The reason is as follows: If $\xi = 1$, the left hand side of the equation is strictly larger than the right hand side; if $\xi = 0$, when $p - w\Delta^\circ(0) > c$, the left hand side of the equation is strictly smaller than the right hand side. Since both sides of the equation are continuous and increasing in ξ, the equation must have a solution $\xi^\circ \in (0,1)$.

Next, let's look at the case when there is BOPS. Note nonstrategic customers will always first visit store, so they will not be influenced by the information effect. Let's keep using our original notations, $\alpha_s^*, \alpha_o^*, \alpha_b^*, \alpha_{so}^*, \alpha_{bo}^*$, to denote the fraction of pure store, pure online, pure BOPS, store-to-online and BOPS-to-online customers among strategic customers. Then, the retailer's profit function is

$$\begin{aligned}\pi =& pE\min\left(\left(\lambda + \left(\alpha_s^* + \alpha_{so}^* + \alpha_b^* + \alpha_{bo}^*\right)(1-\lambda)\right)D, q\right) - cq \\ &+ rE\lambda D + rE\frac{\left(\alpha_s^* + \alpha_{so}^* + \alpha_b^* + \alpha_{bo}^*\right)(1-\lambda)}{\lambda + \left(\alpha_s^* + \alpha_{so}^* + \alpha_b^* + \alpha_{bo}^*\right)(1-\lambda)}\min\left(\left(\lambda + \left(\alpha_s^* + \alpha_{so}^* + \alpha_b^* + \alpha_{bo}^*\right)(1-\lambda)\right)D, q\right) \\ &+ wE\alpha_o^*(1-\lambda)D + wE\frac{\frac{v-p}{H}\lambda + \left(\alpha_{so}^* + \alpha_{bo}^*\right)(1-\lambda)}{\lambda + \left(\alpha_s^* + \alpha_{so}^* + \alpha_b^* + \alpha_{bo}^*\right)(1-\lambda)}\left(\left(\lambda + \left(\alpha_s^* + \alpha_{so}^* + \alpha_b^* + \alpha_{bo}^*\right)(1-\lambda)\right)D - q\right)^+\end{aligned}$$

Proposition D.4. *With BOPS, the market outcome is given as follows:*

- *if* $p - c > w\Delta^* - r\frac{\left(\alpha_s^* + \alpha_{so}^* + \alpha_b^* + \alpha_{bo}^*\right)(1-\lambda)}{\lambda + \left(\alpha_s^* + \alpha_{so}^* + \alpha_b^* + \alpha_{bo}^*\right)(1-\lambda)}$, *then there are strategic customers visiting store and* $q^* = \left(\lambda + \left(\alpha_s^* + \alpha_{so}^* + \alpha_b^* + \alpha_{bo}^*\right)(1-\lambda)\right)\bar{F}^{-1}\left(\frac{c}{p - w\Delta^* + r\frac{\left(\alpha_s^* + \alpha_{so}^* + \alpha_b^* + \alpha_{bo}^*\right)(1-\lambda)}{\lambda + \left(\alpha_s^* + \alpha_{so}^* + \alpha_b^* + \alpha_{bo}^*\right)(1-\lambda)}}\right)$, *where* $\Delta^* = \frac{\frac{v-p}{H}\lambda + \left(\alpha_{so}^* + \alpha_{bo}^*\right)(1-\lambda)}{\lambda + \left(\alpha_s^* + \alpha_{so}^* + \alpha_b^* + \alpha_{bo}^*\right)(1-\lambda)}$;
- *otherwise, no strategic customer comes to store and* $q^* = 0$.

Proposition D.5.

i. BOPS helps to expand market coverage, i.e., $\lambda + (\alpha_s^* + \alpha_o^* + \alpha_b^* + \alpha_{so}^* + \alpha_{bo}^*)(1-\lambda) > \lambda + (\alpha_s^\circ + \alpha_o^\circ + \alpha_{so}^\circ)(1-\lambda)$;

ii. *Suppose $r = 0$. If $p - w\Delta^\circ(0) > c$, i.e., there are customers visiting store when there is no BOPS, then there exist $\bar{w}$ and $\bar{\beta}_s > 0$ such that the implementation of BOPS decreases total profit (i.e., $\pi^* < \pi^\circ$) if $\beta_s < \bar{\beta}_s$, $w > \bar{w}$ and $H > \frac{v-p}{\beta_o}$.*

This shows that although BOPS could expand market coverage, it may still reduce total profit even if $r = 0$.

D.3 Decentralized System

For simplicity of exposition, let's only look at the homogeneous market. In a decentralized system, assuming there are strategic customers using BOPS, the store's profit is

$$\tilde{\pi}_s = \underbrace{pE\lambda \min(D, \tilde{q}) + rE\lambda D}_{from\ nonstrategic\ customers} + \underbrace{(\theta p + r)\, E(1-\lambda) \min(D, \tilde{q})}_{from\ strategic\ customers} - c\tilde{q}$$

Comparing it with the centralized profit function π described above, we find the store's incentive is not fully aligned with the system's. With proper choice of the revenue sharing parameter θ, we should be able to correct store's incentive. However, we find that if λ is very large, we may not have enough BOPS revenue to correct store's incentive to overstock. Thus, a simple θ may not be enough to make sure we achieve the centralized profit level.

D.4 Proofs

Proof of Proposition D.1: Note there are two possible equilibrium outcomes

i. Participatory equilibrium: $\phi^\circ = 1$

ii. Nonparticipatory equilibrium: $\phi^\circ = 0$.

Let's first look for participatory equilibrium: First, we look at retailer's problem: Given belief $\hat{\phi}$, the retailer maximizes total profit $\pi = pE\min((\lambda + \hat{\phi}(1-\lambda))D, q) - cq + rE((\lambda + \hat{\phi}(1-\lambda))D) + wE((1-\hat{\phi})(1-\lambda)D) + wE((\lambda + \hat{\phi}(1-\lambda))D - q)^+ = (p-w)E\min((\lambda + \hat{\phi}(1-\lambda))D, q) - cq + ((\lambda + \hat{\phi}(1-\lambda))r + w)ED$, which is a typical newsvendor problem (plus a constant $((\lambda + \hat{\phi}(1-\lambda))r + w)ED$), and therefore the optimal order quantity q° is given by $\bar{F}\left(\frac{q^\circ}{\lambda + \hat{\phi}(1-\lambda)}\right) = \frac{c}{p-w} \wedge 1$. Since in equilibrium, the retailer's belief is consistent with the outcome, we have $\hat{\phi} = \phi^\circ = 1$. Thus,

$q^\circ = \bar{F}^{-1}\left(\frac{c}{p-w} \wedge 1\right)$. Since $q > 0$ in the participatory equilibrium, we must have $p - c > w$ and thus $q^\circ = \bar{F}^{-1}\left(\frac{c}{p-w}\right)$. Note in equilibrium, we also need consumer's belief to be consistent with the outcome, i.e., $\hat{\xi}^\circ = A(q^\circ) = E\min(D, q^\circ)/E(D) = E\min(D, \bar{F}^{-1}(\frac{c}{p-w}))/ED$. Finally, we go back to consumer's decision. To ensure $\phi^\circ = 1$, we need $u_s \geq u_o$, i.e., $h_s \leq \hat{\xi}^\circ h_o$. Therefore, we need $h_s \leq \frac{E\min(D,\bar{F}^{-1}(\frac{c}{p-w}))}{ED} h_o$. Based on the analysis above, we find the conditions for a participatory equilibrium are $h_s \leq \frac{E\min(D,\bar{F}^{-1}(\frac{c}{p-w}))}{ED} h_o$ and $p - c > w$. And the equilibrium outcome is $\phi^\circ = 1$ and $q^\circ = \bar{F}^{-1}\left(\frac{c}{p-w}\right)$.

Next, let's look for nonparticipatory equilibrium, i.e., $\phi^\circ = 0$. Note, given belief $\hat{\phi}$, the retailer's optimal store inventory level is given by $\bar{F}\left(\frac{q^\circ}{\lambda+\hat{\phi}(1-\lambda)}\right) = \frac{c}{p-w} \wedge 1$ as shown above. Since in equilibrium, the retailer's belief is consistent with the outcome, we have $\hat{\phi} = \phi^\circ = 0$. Thus, $q^\circ = \lambda\bar{F}^{-1}\left(\frac{c}{p-w} \wedge 1\right)$. Note in equilibrium, we also need consumer's belief to be consistent with the outcome, i.e., $\hat{\xi}^\circ = A(q^\circ) = E\min(\lambda D, q^\circ)/E(\lambda D) = E\min(D, \bar{F}^{-1}\left(\frac{c}{p-w} \wedge 1\right))/ED$. Finally, we go back to consumer's decision. To ensure $\phi^\circ = 0$, we need $u_s < u_o$, i.e., $h_s > \hat{\xi}^\circ h_o$. Therefore, we need $h_s \leq \frac{E\min(D,F^{-1}\left(\frac{c}{p-w}\wedge 1\right))}{ED} h_o$. Based on the analysis above, we find the conditions for a nonparticipatory equilibrium are $h_s \leq \frac{E\min(D,F^{-1}\left(\frac{c}{p-w}\wedge 1\right))}{ED} h_o$. And the equilibrium outcome is $\phi^\circ = 0$ and $q^\circ = \lambda\bar{F}^{-1}\left(\frac{c}{p-w} \wedge 1\right)$.

Finally, note the conditions for the participatory equilibrium and nonparticipatory equilibrium are mutually exclusive and collectively exhaustive. Thus, the equilibrium always exists and is unique. □

Proof of Proposition D.2: If $\min(h_s, h_b) \leq h_o$, then the profit function $\pi = pE\min(D, q) - cq + rE\lambda D + rE(1-\lambda)\min(D, q) + wE(D-q)^+ = (p + (1-\lambda)r - w)E\min(D, q) - cq + wED$, which is a typical newsvendor problem (plus a constant wED). Thus, the optimal order quantity q^* is given by $\bar{F}(q^*) = \frac{c}{p+(1-\lambda)r-w} \wedge 1$. Then, if $p - c > w - (1-\lambda)r$, we have $q^* = \bar{F}^{-1}\left(\frac{c}{p+r-w}\right)$; otherwise, $q^* = 0$.

If $\min(h_s, h_b) > h_o$, then no strategic customer comes to store and thus the retailer's profit is $\pi = pE\min(\lambda D, q) - cq + rE\lambda D + wE(1-\lambda)D + wE(\lambda D - q)^+ = (p-w)E\min(\lambda D, q) - cq + (r\lambda + w)ED$, which is a typical newsvendor problem (plus a constant $(r\lambda + w)ED$). Thus, the optimal order quantity $q^* = \lambda\bar{F}^{-1}(\frac{c}{p-w} \wedge 1)$. □

Proof of Proposition D.3: Let's first look for participatory equilibrium:

First, we look at retailer's problem: The retailer's profit can be expressed as

$$
\begin{aligned}
\pi &= pE\min\left(\left(\lambda+\left(\hat{\alpha}_s+\hat{\alpha}_{so}\right)(1-\lambda)\right)D,q\right)-cq+rE\left(\lambda+\left(\hat{\alpha}_s+\hat{\alpha}_{so}\right)(1-\lambda)\right)D\\
&\quad +wE\hat{\alpha}_o(1-\lambda)D+w\frac{\frac{v-p}{H}\lambda+\hat{\alpha}_{so}(1-\lambda)}{\lambda+(\hat{\alpha}_s+\hat{\alpha}_{so})(1-\lambda)}E((\lambda+(\hat{\alpha}_s+\hat{\alpha}_{so})(1-\lambda))D-q)^+\\
&= \left(p-w\frac{\frac{v-p}{H}\lambda+\hat{\alpha}_{so}(1-\lambda)}{\lambda+(\hat{\alpha}_s+\hat{\alpha}_{so})(1-\lambda)}\right)E\min\left(\left(\lambda+\left(\hat{\alpha}_s+\hat{\alpha}_{so}\right)(1-\lambda)\right)D,q\right)-cq\\
&\quad +\left(w\left(\tfrac{v-p}{H}\lambda+\left(\hat{\alpha}_o+\hat{\alpha}_{so}\right)(1-\lambda)\right)+r\left(\lambda+\left(\hat{\alpha}_s+\hat{\alpha}_{so}\right)(1-\lambda)\right)\right)ED
\end{aligned}
$$

which is a typical newsvendor problem (plus a constant $\left(w\left(\frac{v-p}{H}\lambda+\left(\hat{\alpha}_o+\hat{\alpha}_{so}\right)(1-\lambda)\right)+r\left(\lambda+\left(\hat{\alpha}_s+\hat{\alpha}_{so}\right)(1-\lambda)\right)\right)ED$), and therefore the optimal order quantity q° is given by $\bar{F}\left(\frac{q^\circ}{\lambda+(\hat{\alpha}_s+\hat{\alpha}_{so})(1-\lambda)}\right)=\frac{c}{p-w\frac{\frac{v-p}{H}\lambda+\hat{\alpha}_{so}(1-\lambda)}{\lambda+(\hat{\alpha}_s+\hat{\alpha}_{so})(1-\lambda)}}\wedge 1$.

Since in equilibrium, the retailer's belief is consistent with the outcome, we have $\hat{\alpha}_i=\alpha_i^\circ$, $i=o,s,so$. Thus, $\bar{F}\left(\frac{q^\circ}{\lambda+(\alpha_s^\circ+\alpha_{so}^\circ)(1-\lambda)}\right)=\frac{c}{p-w\Delta^\circ(\xi)}\wedge 1$

Note in equilibrium, we also need consumer's belief to be consistent with the outcome, i.e., $\hat{\xi}=\xi^\circ=A(q^\circ)=E\min((\lambda+\left(\alpha_s^\circ+\alpha_{so}^\circ\right)(1-\lambda))D,q^\circ)/E((\lambda+\left(\alpha_s^\circ+\alpha_{so}^\circ\right)(1-\lambda))D)=E\min(D,\bar{F}^{-1}(\frac{c}{p-w\Delta^\circ(\xi^\circ)}\wedge 1)/ED$. To ensure the existence of a participatory equilibrium, we need to make sure there exists $\xi^\circ>0$ such that $\xi^\circ=E\min(D,\bar{F}^{-1}(\frac{c}{p-w\Delta^\circ(\xi^\circ)}\wedge 1)/ED$.

What's left is to show that when equation $\xi=\frac{\min\left(D,\bar{F}^{-1}\left(\frac{c}{p-w\Delta^\circ(\xi)}\wedge 1\right)\right)}{ED}$ does not have a positive solution, nonparticipatory outcome (i.e., no strategic customers comes to store and $q=0$) is an equilibrium.

Note a necessary condition for the equation $\xi=\frac{\min\left(D,\bar{F}^{-1}\left(\frac{c}{p-w\Delta^\circ(\xi)}\wedge 1\right)\right)}{ED}$ not having a positive solution is $p-w\Delta^\circ(0)\le c$. Note $\Delta^\circ(0)=\frac{v-p}{H}$. Given $p-w\frac{v-p}{H}\le c$, it is easy to find that the retailer will set $q=0$ if no strategic customers visit store. Then, given $q=0$, strategic customers will indeed not come to store. Thus, this nonparticipatory outcome is a RE equilibrium. $\square$

Proof of Proposition D.4: The profit function can be expressed as

$$
\begin{aligned}
\pi &= \left[p-w\Delta^*+r\frac{\left(\alpha_s^*+\alpha_{so}^*+\alpha_b^*+\alpha_{bo}^*\right)(1-\lambda)}{\lambda+\left(\alpha_s^*+\alpha_{so}^*+\alpha_b^*+\alpha_{bo}^*\right)(1-\lambda)}\right]E\min\left(\left(\lambda+\left(\alpha_s^*+\alpha_{so}^*+\alpha_b^*+\alpha_{bo}^*\right)(1-\lambda)\right)D,q\right)-cq\\
&\quad +\left(w\left(\tfrac{v-p}{H}\lambda+\left(\alpha_o^*+\alpha_{so}^*+\alpha_{bo}^*\right)(1-\lambda)\right)+r\lambda\right)ED
\end{aligned}
$$

which is a typical newsvendor problem (plus a constant $\left(w\left(\frac{v-p}{H}\lambda+\left(\alpha_o^*+\alpha_{so}^*+\alpha_{bo}^*\right)(1-\lambda)\right)+r\lambda\right)ED$). Thus, the optimal order quantity

$$q^* = \left(\lambda + \left(\alpha_s^* + \alpha_{so}^* + \alpha_b^* + \alpha_{bo}^*\right)(1-\lambda)\right)\bar{F}^{-1}\left(\frac{c}{p - w\Delta^* + r\frac{\left(\alpha_s^*+\alpha_{so}^*+\alpha_b^*+\alpha_{bo}^*\right)(1-\lambda)}{\lambda+\left(\alpha_s^*+\alpha_{so}^*+\alpha_b^*+\alpha_{bo}^*\right)(1-\lambda)}} \wedge 1\right) \qquad \square$$

Proof of Proposition D.5: If $\frac{v-p}{\beta_o} < H$, then $\lim_{\beta_s \to 0} \frac{\alpha_{so}^*+\alpha_{bo}^*}{\alpha_s^*+\alpha_{so}^*+\alpha_b^*+\alpha_{bo}^*} > \frac{v-p}{H}$. Therefore, $\lim_{\beta_s \to 0} \Delta^* > \frac{v-p}{H}$. Since Δ^* is continuous in β_s, there exists $\bar{\beta}_s > 0$ such that $\Delta^* > \frac{v-p}{H}$ if $\beta_s < \bar{\beta}_s$. Note $\Delta^\circ(0) = \frac{v-p}{H}$.

Suppose $r = 0$. If $w \geq \frac{p-c}{\Delta^*}$ (this is possible since $\frac{p-c}{\Delta^\circ} > \frac{p-c}{\Delta^*}$), then no one comes to store when there is BOPS . In this case,

$$\begin{aligned}
\pi^* &= \left(w\left(\frac{v-p}{H}\lambda + \left(\alpha_o^* + \alpha_{so}^* + \alpha_{bo}^*\right)(1-\lambda)\right) + r\lambda\right) ED \\
&= \left(w\left(\frac{v-p}{H}\lambda + \left(\alpha_o^\circ + \alpha_{so}^\circ\right)(1-\lambda)\right) + r\lambda\right) ED \\
&< pE\min\left(\left(\lambda + \left(\alpha_s^\circ + \alpha_{so}^\circ\right)(1-\lambda)\right)D, q^\circ\right) - cq^\circ + rE\left(\lambda + \left(\alpha_s^\circ + \alpha_{so}^\circ\right)(1-\lambda)\right)D \\
&\qquad + wE\alpha_o^\circ(1-\lambda)D + w\frac{\frac{v-p}{H}\lambda + \alpha_{so}^\circ(1-\lambda)}{\lambda + \left(\alpha_s^\circ + \alpha_{so}^\circ\right)(1-\lambda)}E((\lambda + \left(\alpha_s^\circ + \alpha_{so}^\circ\right)(1-\lambda))D - q^\circ)^+ \\
&= \pi^\circ
\end{aligned}$$

where the inequality is because $q = 0$ is also a feasible but not the optimal solution to the case where there is no BOPS. If $w < \frac{p-c}{\Delta^*}$, then there are consumers visiting store when there is BOPS. Note (1) $\pi^\circ - \pi^*$ is continuous in w, and (2) $\pi^\circ > \pi^*$ when $w = \frac{p-c}{\Delta^*}$. Thus, the analysis above implies that there exists $\bar{w} < \frac{p-c}{\Delta^*}$ such that $\pi^\circ > \pi^*$ if $w > \bar{w}$. $\square$