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Abstract

Motivated by the growing practice of using social network data in credit scoring, this study analyzes the impact of using network based measures on customer score accuracy and on tie formation among customers. We develop a series of models to compare the accuracy of customer scores obtained with and without network data. We also investigate how the accuracy of social network based scores changes when individuals can strategically construct their social networks to attain higher credit scores. We find that, if individuals are motivated to improve their scores, they may form fewer ties and focus them on more similar partners. The impact of such endogenous tie formation on the accuracy of consumer credit scores is ambiguous. Scores can become more accurate as a result of modifications in social networks, but this accuracy improvement may come with greater network fragmentation. The threat of social exclusion in such endogenously formed networks provides incentives to low type members to exert effort that improves everyone's creditworthiness. We discuss implications for both managers and public policy.

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Noscitur ex sociis.

Latin proverb

You are known by the company you keep.

English proverb

1 Introduction

When a consumer applies for credit, attempts to re-finance a loan, or wants to rent a house, potential lenders often seek information about the applicant's financial background in the form of a credit score provided by a credit bureau or other analysts. A consumer's score can influence both the lender's decision to extend credit and the terms of the credit. In general, consumers with high scores are more likely to obtain credit, and to obtain it with better terms, including the annual percentage rate (APR), the grace period, and other contractual obligations of a loan (Rusli, 2013). Given that people use credit for a range of undertakings that affect social and financial mobility, such as purchasing a house, starting a business, or obtaining higher education, credit scores have a considerable impact on the access to opportunities and hence on social inequality among citizens.

Until recently, assessing consumers' creditworthiness relied solely on their financial history. The financial credit score popularized by Fair, Isaac and Corporation (FICO), for example, relies on three key data to determine access to credit: consumers' debt level, length of credit history, and regular and on-time payments. Together, these elements account for about 80% of the FICO score. Within the past few years, however, the credit scoring industry has witnessed a dramatic change in data sources (Chui, 2013; Jenkins, 2014; Lohr, 2015). An increasing number of firms rely on network based data to assess consumer creditworthiness. One such company, Lenddo, is reported to assign credit scores based on information in users' social networking profiles, such as education and employment history, how many followers they have, who they are friends with, and information about those friends (Rusli, 2013). Similar to Lenddo, a growing number of start-ups specialize in using data from social networks. Such firms claim that their social network based credit scoring and financing practices broaden opportunities for a larger portion of the population and may benefit low-income individuals who would otherwise find it hard to obtain credit.

Our study is motivated by the growing use of such practices and investigates whether a move to network based credit scoring affects financing inequality. In particular, we address the following questions. First, from the perspective of lenders, is there an advantage to using network based measures rather than measures based only on an individual's data? Second, as the use of social network data becomes common practice, how may consumers' endogenous network formation influence the accuracy of credit scores? Third, how does peer pressure operate in network based credit scoring? Finally, and most importantly for public policy, how do these scores influence inequality in access to financing?

1.1 Main insights

Access to financing is correlated with one's credit score. Following Demirgüç-Kunt and Levine (2009), we assume that credit scores can influence access to financing at both the extensive and intensive margins, i.e., by increasing the number of individuals who are considered eligible for financing as well as by providing access to credit at better terms. Although network based scoring can affect access to financing at the extensive and intensive margin, the impact on each might be uneven for different segments of society.

We first develop a model with continuous risk types incorporating network based data (Section 2). Under the assumption of homophily, the notion that people are more likely to form social ties with others who are similar to them, we show that network data provide additional information about individuals and reduce the uncertainty about their creditworthiness. We find that the accuracy of network based scores is dependent mostly on information from the direct ties, i.e., the assessed consumers' ego-network. This implies that credit-scoring firms can assess an individual's creditworthiness efficiently using data from a subset of the overall network.

In Section 3, we extend our model to allow consumers in a network to form ties strategically in order to improve their credit scores. We find that they may then choose not to connect to people with lower scores. This can result in social fragmentation within a network: individuals with better access to financing opportunities choose to segregate themselves from individuals with worse financing opportunities. As a result, individuals self-select into highly homogeneous yet smaller sub-networks. The impact of such social fragmentation on credit scoring accuracy is ambiguous. On the one hand, scores may more accurately reflect borrowers' risk as each agent will be located in a more homogeneous ego-network. On the other hand, scores may become less accurate because smaller ego-networks provide fewer data points and hence less information on each person. How important financial scores are relative to social relationships determines whether strategic tie formation improves or harms credit score accuracy. When accuracy declines, network based scoring could put deserving individuals with low financing opportunities in further hardship. This result supports concerns about social credit scoring from consumer advocates and regulators like the Consumer Financial Protection Bureau and the Federal Trade Commission (Armour, 2014).

In Sections 2 and 3, we study environments where all individuals in the society, independent of their type, have similar needs for financing. We relax this assumption in Section 4 where we introduce a formulation with discrete risk types that may vary in their needs for financing. When studying this environment, we pay particular attention to the strategic formation of social ties. An important result is the emergence of social exclusion or discrimination *among* low type individuals. They avoid associating with one another, because such associations signal even more strongly to lending institutions that their type is low. Such within group discrimination is different from between-group discrimination studied commonly in the literature (e.g. Arrow, 1998; Becker, 1971; Phelps, 1972).

In Section 5, again within a discrete setting, we allow individuals to exert effort to improve their true creditworthiness or 'type'. When social ties motivate effort, social credit scoring may benefit individuals with poor financial health in two ways: not only by letting them benefit from a positive signal from social ties with others having a stronger financial footing, but also by motivating them to invest more in their own financial health. We consider environments with explicit discrimination and with homophily. We find that when there are complementarities between the effort exerted by individuals, the between-group connections can motivate effort and thus lead to increased social mobility in both environments. The within-group connections also improve effort in a discriminatory environment. In contrast, when homophily is the only factor determining tie formation, a high number of low type friends who exert low effort will reduce an individual's desire to exert effort. In Section 6, we analyze another way individuals can exert effort to improve their financial outcomes: actively networking to endogenously alter the probability of meeting people with high creditworthiness. Our analysis demonstrates that low types exert effort to meet others more aggressively than the high types only when they are in dire need of improving credit access. Otherwise, high types exert higher effort.

1.2 Related literature

Though motivated by and couched in terms of social credit scoring, the insights we develop go beyond that realm. Our models involve a relatively abstract notion of customer attractiveness or 'type' that has two properties: (1) social relationships are homophilic with respect to types and (2) a third party like a firm or society at large values higher types more and bestows some rewards (external to social relationships) that are monotonically increasing with one's type. The notion of homophily in customer value, i.e., the notion that attractive prospects or customers are more likely to be connected to one another than to unattractive ones, and vice versa, underlies social customer scoring in predictive analytics (e.g., Benoit and Van den Poel, 2012; Goel and Goldstein, 2013; Haenlein, 2011). It also is the basis for targeting friends and other network connections of valuable customers in new product launch (e.g., Haenlein and Libai, 2013; Hill et al., 2006), in targeted online advertising (Bagherjeiran et al., 2010; Bakshy et al., 2012; Liu and Tang, 2011), and in customer referral programs (e.g., Kornish and Li, 2010; Schmitt et al., 2011). The basic insights also apply to employment settings, where firms have long used employee referral programs to attract better applicants (e.g., Castilla, 2005) and many have started to use social network data to gain more information about applicants' character and work ethic (e.g., Roth et al., 2013).

The model construct that we label 'social credit score' captures a customer's attractiveness or type as perceived by a firm based on social network information, in which the firm bestows some benefits that are monotonically increasing with type. Hence, our insights about social credit scoring can also be interpreted as pertaining to consumers' social status more broadly, i.e., their "position in a social structure based on esteem that is bestowed by others" (Hu and Van den Bulte, 2014, p. 510). As such, our analysis involving endogenous tie formation adds not only to research traditions in economics and sociology (e.g., Ball et al., 2001; Podolny, 2008) but also to the recent work in marketing on how status considerations affect consumers' networking behavior (Lu et al., 2013; Toubia and Stephen, 2013), their acceptance of new products (Iyengar et al., 2015) and their appeal as customers (Hu and Van den Bulte, 2014).

Even when limited to the realm of financial credit scoring, our analysis relates to several streams of recent work. First is the large and growing amount of work on micro-finance and, more specifically, how group lending helps improve access to capital by reducing the negative consequences of information asymmetries between creditor and debtor (e.g., Ambrus et al., 2014; Bramoullé and Kranton, 2007a,b; Stiglitz, 1990; Townsend, 1994). Our analysis focuses on individual loans rather than group loans, and on a priori customer scoring rather than a posteriori compliance through group monitoring and social pressure. Hence, our result that social credit scoring can lead people to form their network ties differently and to exert more effort in improving their financial health is different from yet dovetails with the evidence by Feigenberg et al. (2010) that group lending tends to trigger changes in network structure that in turn reduce loan defaults. The two different kinds of "social financing" practices acting at two different stages of the loan (customer selection and terms definition vs. compliance) can both lead to improved outcomes mediated through endogenous changes in network structure.

Second, we provide new insights on the risk of discrimination and exclusion triggered by social financing (Ambrus et al., 2014; Armour, 2014). Our model allows for the possibility of discrimination against less creditworthy individuals. There are two ways through which such discrimination can come about. The first is that individuals may be subject to discrimination based on type. In an endogenous network, borrowers will be more selective in forming relationships, and may prefer to form relationships with higher-type individuals to protect their credit score. Formation of networks in order to attain a high credit score can be an indirect way of discrimination because some individuals are systematically excluded from others' networks. The second is that individuals may observe each other's effort to improve their score and may discriminate based on personal effort. Any low-type individual who does not exert effort may face disengagement by fellow low-type contacts who do exert effort and who want to disassociate their own credit score from his.

Third, our work is also relevant to ongoing debates on the impact of new social technologies on social integration versus balkanization. Rosenblat and Mobius (2004) find that a reduction in communication costs decreases the separation between individuals but increases the separation between groups. Along similar lines, van Alstyne and Brynjolfsson (2005) find that the internet can lead to segregation among different types of individuals. In this study, we identify conditions under which network based credit scoring (and customer scoring in general) may foster or harm integration within vs. between groups.

Finally, our work will be of topical interest to the growing number of scholars seeking to better understand consumers' financial behaviors, especially the role of homophily (Galak et al., 2011) and trust signaling (e.g., Herzenstein et al., 2011; Lin et al., 2013) in gaining access to credit. It will also be of interest to researchers focusing on the practices in emerging economies where consumer finance and access to credit are particularly important yet the traditional credit scoring apparatus is found lacking. Creditors in these markets often seek to enrich scores based on individual's history with additional information (e.g., Guseva and Rona-Tas, 2001; Rona-Tas and Guseva, 2014; Sudhir et al., 2014).

The rest of the article develops as follows. In Section 2, we present a benchmark model of data collection from networks to assess one's creditworthiness, and then provide justification for the emergence of this industry. In Section 3, we investigate the possibility of networks forming endogenously to the social credit scoring practice. We extend our model to allow individuals to vary in their financing needs in Section 4. We consider the possibility of social mobility through effort in Section 5. We extend the model in several directions in Section 6 and conclude with implications for public policy and marketing practice in Section 7.

2 Model with Exogenous Network

Consider a society with a large population S of individuals. Each individual i is represented with a type x_i, and x_i follows $N(0, q^{-1})$ across individuals, with precision $q > 0$. We assume that each agent knows his own type and discovers that of fellow consumers upon meeting them.

The process of forming friendships is specified as follows. Each pair of individuals meet with a very small independent probability of $\nu > 0$. Between i and j there is an independent match value $m_{ij} \sim \chi_2$. A friendship between i and j creates utility $m_{ij} - |x_i - x_j|$ for either individual. So, our model features homophily based on preference rather than opportunity (Zeng and Xie, 2008): Individuals enjoy the company of others like them more than that of others unlike them. Person i accepts the formation of a friendship tie with j iff they have met and:

$$m_{ij} > |x_i - x_j|. \tag{1}$$

Upon mutual consent of both parties, a friendship tie is created. The assumption of a χ_2 distribution implies that the probability i and j become friends upon meeting is:

$$Pr(m_{ij} > |x_i - x_j|) = e^{-|x_i - x_j|^2/2}. \tag{2}$$

Let G denote the set of friendships (ties) in society and n_i denote the number of friends of i, or, the *degree* of i under G. The expected number of friends for i is $\mathbb{E}(n_i|x_i) = S\nu\sqrt{\frac{q}{q+1}}e^{-\frac{q}{1+q}{x_i}^2/2}$.[1] In order to represent an environment with sufficient uncertainty about creditworthiness of individuals, we make three assumptions: (i) the society is large ($S \to +\infty$), (ii) the probability that any pair of individuals meet is very small ($\nu \to 0$), and (iii) types are diffuse ($q \to 0$.) These three properties characterize a society with sufficient uncertainty about individuals. They also allows us to assume that the product term $S\nu\sqrt{\frac{q}{q+1}}$ holds a constant, which we denote by N. [2]

Suppose that friendships in the society have been formed. The lender is interested in updating its information about the types of individuals using signals collected from the network. For any individual i, the lender may observe a noisy signal y_i about his type:

$$y_i = x_i + \varepsilon_i \tag{3}$$

where $\varepsilon_i \sim N(0, c^{-1})$ and is independent across individuals. The firm observes the signals of a finite set of individuals $\mathbf{y}$, which we refer to as the vector of signals as well. For these individuals, the firm may observe the presence or absence of a tie. We use $g \equiv (g^1, g^0)$ to denote such information. Specifically, g^1 is the set of the dyads which the lender knows are friends, and g^0 is the set of the dyads which the lender knows are not friends. Furthermore, for each person in $\mathbf{y}$, we allow g^0 to

[1] $\mathbb{E}(n_i|x_i) = S\int_{-\infty}^{+\infty} \nu e^{-(t-x_i)^2/2}\sqrt{\frac{q}{2\pi}}e^{-qt^2/2}dt = S\nu\sqrt{\frac{q}{q+1}}e^{-\frac{q}{1+q}{x_i}^2/2}$.

[2] In a small society where everyone is likely to be friends with others, or in a society where each type is organized in perfectly homogeneous and mutually disconnected sub-graphs ("components"), there is little to no uncertainty about an individual's type, implying that network based scores are less useful.

include all the dyads that involve him and someone outside $\mathbf{y}$. [3]

We first present some properties about the firm's posterior on the types of individuals in a network. Together with the nodes in $\mathbf{y}$, the ties in g^1 define a sub-network involving only nodes on which a signal is observed. In this sub-network, let d_i be the degree of i, and $r(i,j)$ be the length of the shortest path (i.e., geodesic distance) between i and j.

Proposition 1. *Let vector* $\mathbf{x}$ *indicate the types of individuals contained in vector* $\mathbf{y}$*.* $\Pr(\mathbf{x}|g,\mathbf{y})$ *is a multivariate normal density with precision matrix* $\mathbf{\Sigma}^{-1}$*:*

$$\begin{aligned}(\mathbf{\Sigma}^{-1})_{ii} &= c+d_i \\ (\mathbf{\Sigma}^{-1})_{ij} &= -\mathbf{1}_{\{ij\in g^1\}}\end{aligned}$$

and mean vector μ*:*

$$\mu = c\mathbf{\Sigma}\mathbf{y}. \tag{4}$$

Proposition 1 states that the lender's beliefs about the types of individuals in the network follow a multivariate normal distribution the parameters of which depend on the network structure. So, two individuals with identical individual signals (such as personal financial history) may obtain different network based scores because of social connections. These individuals would obtain similar financing opportunities if credit scores relied solely on individual history. In the new regime, despite identical individual financial histories, it is possible that they will have unequal access to financing because of the score gains and losses from the social network.

Equation (4) shows that the weight that a contact j's signal receives depends on his location within the network. Proposition 2 states an upper bound on the weight of connection j's signal on i's posterior mean. When all else is equal, the upper bound on the weight of j decreases in the distance $r(i,j)$. If i and j are not connected in the sub-network, the weight is zero.

Proposition 2. *For all* $i \neq j$ *and* $r(i,j) < +\infty$*, the weight matrix of Proposition 1 satisfies*

$$c\mathbf{\Sigma}_{ij} < \frac{c}{c+d_i}\frac{\delta^{r(i,j)}}{1-\delta}\ ,$$

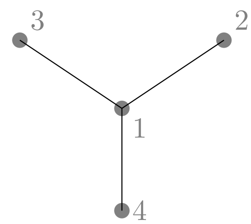
where

$$\delta \equiv \frac{\max_{k\in\mathbf{y}}\{d_k\}}{c+\max_{k\in\mathbf{y}}\{d_k\}}.$$

[3] This type of information arises when the lender observes all of i's friends and their signals, which implies that i is not friends with the rest of the society. Corollary 1 demonstrates an example of such a situation.

To generate further insights about how the weight of a connection's signal changes with distance, we follow with two examples.

Example 1. *For a simple example, consider a star network g^1 that is centered at 1.*

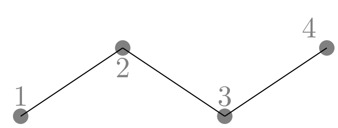


With $c = 1$, $c\mathbf{\Sigma}$ equals:

$$\begin{pmatrix} 0.4 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.6 & 0.1 & 0.1 \\ 0.2 & 0.1 & 0.6 & 0.1 \\ 0.2 & 0.1 & 0.1 & 0.6 \end{pmatrix}$$

By Proposition 1, this is a "weight" matrix, suggesting that to calculate the posterior mean of x_1, for example, the firm should weigh the signals (y_1, y_2, y_3, y_4) by $(0.4, 0.2, 0.2, 0.2)$. Note, further, that direct neighbors (friends) for nodes 2, 3, and 4 receive more weight than indirect neighbors (friends of friends).

Example 2. *Consider the following g^1.*



With $c = 1$, the weight matrix is:

$$\begin{pmatrix} 0.62 & 0.24 & 0.10 & 0.05 \\ 0.24 & 0.48 & 0.19 & 0.10 \\ 0.10 & 0.19 & 0.48 & 0.24 \\ 0.05 & 0.10 & 0.24 & 0.62 \end{pmatrix}$$

Note that direct neighbors are weighed more heavily than indirect neighbors, and that direct neighbors need not receive equal weight. For instance, the updating of x_2 weighs the signal from node 1 more heavily than that from node 3.

The above examples convey the intuition that distant signals on average receive lower weight in firm's updating of the beliefs about a consumer's type. In examples 1 and 2, the weight of the signal of an individual who is two links away is always lower than the weight of the individual who is only one link away. In the second example, although individual 2 is at equal distance to persons 1 and 3, their signals receive different weights: Individual 3's signal is diluted as he is linked to individual 4.

Propositions 1 and 2 together imply that agents who have lower distances to high type individuals can receive a more favorable posterior in credit score assessment. Conversely, proximity to individuals with low signals may hurt an individual's assessment. Individuals cannot choose their distance as we have not yet considered active selection of friendship ties to attain such benefits (see Section 3).

In the remainder of the paper, we assume that when evaluating a particular i, the firm observes the complete ego-network of i, i.e., all the ties $ij \in G$, and receives a signal on each of i's friends. We collect the signals in the vector $\mathbf{y}_i$, which we will refer to as the set of i's friends as well. Note that this imposes an additional assumption on the previous analysis: We now require that g^1 equals the complete set of i's direct ties. The posterior belief of the firm about an individual's type can then be stated as a special case of Proposition 1.

Corollary 1. *For the evaluation of i,* $\Pr(x_i|\mathbf{y}_i)$ *is normal with precision*

$$\rho_i = \left(c + \frac{c}{c+1} n_i\right), \tag{5}$$

and mean

$$\mu_i = \frac{1}{\rho_i}\left[c y_i + \frac{c}{c+1}\sum_{ij\in G} y_j\right].$$

Corollary 1 states that when an individual has a higher number of connections, the posterior about his type will have higher precision. The assessment of an individual with a higher degree is likely to be closer to this true type, x_i.[4] More importantly, (5) implies that the precision of lender's beliefs is higher than the precision of the individual signal of i, even with data only from the direct relationships of i. The corollary thus states useful information about the efficiency of risk assessment based on network data. If gathering data on the whole network is impossible or costly, efficiency gains can still be attained by using data from the focal consumer's immediate

[4]Notice that $\rho_i = 1/E((\mu_i - x_i)^2|\mathbf{y}_i)$, which is the inverse of the conditional mean squared error. Since in (5) ρ_i is increasing in n_i, the conditional mean squared error is decreasing with n_i.

neighbors. Remember from Proposition 2 that first degree contacts of i receive a greater weight, and that data from longer paths in the network are expected to receive gradually lower weights in the beliefs about one's credit-worthiness.

3 Endogenous Tie Formation

We next study individuals' incentives to form network ties in order to improve their scores. This suggests that the probability that two agents will become friends depends on their type, x_i, and the expected utility from improving their credit score.

Facing network based scoring, an individual has an incentive not to form ties with low types in order to achieve a more favorable score. Such endogenous tie formation involves a trade-off between utility from friendship ties with people one likes and utility from a high score. To formally express this, we assume that the posterior mean μ_i enters the utility additively. The utility of individual i is:

$$U_i = \sum_{ij \in G} (m_{ij} - |x_i - x_j|) + \alpha \mu_i, \tag{6}$$

where the first part of the utility, $(m_{ij} - |x_i - x_j|)$, indicates a social utility taking into consideration homophily; and the second part, $\alpha\mu_i$, indicates how much i enjoys having a high posterior mean. Here, α calibrates the relative importance an individual places on receiving a high credit score vs. the utility from friendship ties with people he likes. All individuals gain utility from their posterior credit score at rate α.[5] If $\alpha = 0$, the individual cares only about forming friendships for social utility. If $\alpha \to +\infty$, then the agent cares little about social utility but highly about improving his score.

Parameter α can also be interpreted as a measure of the desire for status. How much people care about how highly others evaluate them (i.e., generate a posterior about their type based on characteristics of their network) captures the importance people place on their position in a social structure based on esteem that is bestowed by others, i.e., their status. Let each individual i adopt a tie formation rule a priori (i.e., before meeting j) which states that he will accept friendship with

[5]To allow for the possibility that some agents may have no interest in improving their scores when they meet others with similar types, Section 4 presents a discrete formulation of our matching model and we provide a special case where the high types have zero utility from credit scores.

j iff

$$\begin{cases} m_{ij} > \eta_i |x_i - x_j| & \text{for } x_j \geq x_i, \\ m_{ij} > \lambda_i |x_i - x_j| & \text{for } x_j < x_i. \end{cases}$$

The parameters λ_i and η_i represent the degree to which i is willing to accept a lower and a higher type individual as a friend. These parameters are not exogenous but will be chosen simultaneously[6] and optimally by individuals. Although individual i would prefer to be friends with others similar to him, which was expressed in (1), he may have additional utility from adding high type or removing low type friends due to the improvement in his credit assessment. This suggests that individuals will form relationships with others who have lower types only if the match value m_{ij} yields sufficiently high utility.

Comparing (6) with (1), a greater (lesser) desire to link to individuals with higher (lower) types would indicate that an agent should pick $\eta_i \leq 1$ and $\lambda_i \geq 1$. [7] Remember that forming a friendship tie requires mutual consent: for i and j to become friends, i should want to connect with j and j should want to connect with i. [8] Thus η_i becomes irrelevant and λ_i becomes the parameter that sets the level of mixing with 'others'. In the rest of the paper we omit any further references to η_i.

Consider the symmetric case where $\lambda_i = \lambda$ for all i. If everyone in the society applies the same rule with common λ, a friendship is established after meeting iff $m_{ij} > \lambda |x_i - x_j|$. With the common rule in place, the probability of becoming friends after meeting becomes:

$$Pr(|x_i - x_j|, \lambda) = e^{-\lambda |x_i - x_j|^2/2}.$$

Compared to the tie formation probability in an exogenous setting (given by Equation 2), individuals will be more selective in linking to others. Fewer ties will be formed in the endogenous case.

[6]Note that in this model individuals form ties simultaneously. A model with sequential friendship formation would also need to consider, in addition to tie formation rules, rules about the order in which individuals form ties, and would need to assume that individual beliefs about firms' financial assessment are consistent with equilibrium outcomes.

[7]The benefits from network based scoring is measured by the difference between one's expected posterior mean and one's individual signal. This difference increases in λ_i (i.e., the rate at which the individual rejects ties with low-type friends) and decreases in η_i (i.e., the rate at which the individual adds high-type friends). Choosing $\eta_i > 1$ is worse than $\eta_i = 1$, because it decreases both the expected score benefit and the social utility of a tie. Similarly, choosing $\lambda_i < 1$ rather than $\lambda_i = 1$ would decrease the utility from a higher credit score and the social utility of a well-matching tie. Together, these two arguments imply that: (i) any symmetric equilibrium derived with restrictions is still an equilibrium even if we allow $\eta_i > 1$ or $\lambda_i < 1$; and more importantly, (ii) there is no symmetric equilibria where $\eta > 1$ or $\lambda < 1$.

[8]If we allowed individuals to form friendships without the consent of the other, then we would be in a trivial world where everyone can link to anyone to improve his own score. In such a world, the benefits of network based scoring are limited since a connection to a high-type is not informative of one's type.

3.1 Credit Scoring with Endogenous Tie Formation

In this section we complete the analysis of endogenous relationship formation using an equilibrium concept. We use (λ, λ_i) to denote the common rule with the possible deviation of i. The expected utility of i becomes:

$$\mathbb{E}\left(U_i|x_i, \lambda, \lambda_i\right) = \mathbb{E}\left(\sum_{ij \in G} m_{ij} - |x_i - x_j| \,\middle|\, x_i, \lambda, \lambda_i\right) + \alpha \mathbb{E}\left[\mu_i(\lambda)|x_i, \lambda, \lambda_i\right]. \tag{7}$$

where $\mu_i(\lambda) = \mathbb{E}(x_i|\mathbf{y}_i, \lambda)$ is the lender's posterior. Each individual calculates his expected utility from being in a friendship network before the network is formed, implying that expected utility will depend on the friendship rule (λ, λ_i) adopted. The expectation $\mathbb{E}(\cdot)$ is taken before meeting others. We first display a version of Corollary 1 under a symmetric rule. In the following, when λ_i conforms with the common rule, we omit λ_i in the expectation conditionals.

Lemma 1. *Under a common relationship formation rule λ, the posterior $\Pr(x_i|\mathbf{y}_i, \lambda)$ is normal with precision*

$$\rho_i(\lambda) = \left(c + \frac{c\lambda}{c+\lambda} n_i\right), \tag{8}$$

and mean

$$\mu_i(\lambda) = \frac{1}{\rho_i(\lambda)}\left[c y_i + \frac{\lambda c}{c+\lambda} \sum_{ij \in G} y_j\right].$$

Compared to Corollary 1, in Lemma 1, ρ_i and μ_i are scaled by the selection rule λ. When borrowers are more selective in forming friendships with lower types (when λ is higher), a financial institution will put more weight on friends' signals to update beliefs about the type of an individual (i.e., to calculate the posterior). In broad terms, this selectivity addresses our second main research question: When individuals begin reacting to an environment with network based scoring, are scores going to be less precise or even more precise? In other words, can assessments based on network data yield better assessment of individual data? Our answer to this question is a qualified yes. We explain the mechanism through which this improvement can be achieved via a lemma and then a proposition.

Lemma 2. *The expected degree under a symmetric rule λ satisfies*

$$\mathbb{E}(n_i|\lambda) = \frac{N}{\sqrt{\lambda}}. \tag{9}$$

A lower rate of mixing between types (a higher λ) results in a smaller number of ties per person. Ties are formed only between individuals who are highly similar to each other in type. Such self-selection reduces the expected number of connections among consumers but increases the information value of any single link and the signal it conveys. The net effect on the formation of ties is not clear yet. We address it next.

Proposition 3 shows that, under the limits of S, ν and q, there is a symmetric equilibrium λ^* where $\lambda_i = \lambda^*$ which maximizes (7) for any individual i, given that $\lambda = \lambda^*$ is the common rule adopted by everyone else. In other words, there exists a common tie formation rule no individual wants to deviate from, with which the lender's posterior is consistent.

Proposition 3. *For $0 < \alpha < N$, there exists at least one symmetric equilibrium, and any symmetric equilibrium λ^* must satisfy*

$$1 < \lambda^* < \left(1 - \frac{\alpha}{N}\right)^{-1}. \tag{10}$$

Corollary 2. *If $c \geq \sqrt{\frac{N}{N-\alpha}}$, then $\mathbb{E}\left[\rho_i(\lambda^*)|\lambda^*\right] > \mathbb{E}\left[\rho_i(1)|\lambda = 1\right]$, where $\rho_i \equiv Prec(x_i|\mathbf{y}_i, \lambda)$. On average, the network based score becomes more accurate when consumers are averse to connecting with lower type peers. Otherwise, if $c \leq 1$, then $\mathbb{E}\left[\rho_i(\lambda^*)|\lambda^*\right] < \mathbb{E}\left[\rho_i(1)|\lambda = 1\right]$. On average, the network based scores are less accurate.*

Social credit scoring changes the incentives of individuals to form relationships. There are two directions of change. Compared to the exogenous setting ($\lambda = 1$), in the endogenous setting with $\lambda = \lambda^* > 1$, individual relationships are formed more selectively. This has several consequences. First, relationships are more strongly homophilous, that is, individuals form relationships with others who are closer to their own type. This first effect has a positive impact on network scores for lenders: The accuracy of their assessment will improve as a result of obtaining signals from closer types. Network based scores will prove to be even more precise due to data from others who are expected to be more similar in type.

Second, individuals will reject friendship ties with others who have lower types, implying that ego-networks will shrink in size (Lemma 2). This second effect has a negative impact on the accuracy of network scoring. The two forces, the homogenization and the shrinkage of ego-networks, work against each other. The net effect is ambiguous.

Corollary 2 identifies a further condition, which we interpret using the parameter α, to characterize situations in which the net effect is positive and network score accuracy improves with endogenous tie formation. For some sufficiently small α, lenders may benefit from using network based credit scoring as it becomes even more precise with self-selection of individuals to form networks to improve their credit scores. The improvement in precision is conditional on consumers placing sufficiently low weight on financial outcomes relative to the utility derived from social connections. Paradoxically, when individuals care greatly about their score or status, they may reduce the size of their social networks so much that network based scoring becomes less reliable in equilibrium.

Can societal tissue make network based scoring more effective in some societies than others? Corollary 2 states that the parameter range under which network based scores are more precise is larger when the average number of friends is higher. If everything else remains the same, the benefits of network based scoring may be greater in societies where people maintain a large number of connections, which are likely to be societies with collectivist cultures (Hofstede, 2001). Interestingly, several start-ups turning to social scoring have been growing in countries known to have collectivist cultures where the density of relationships is generally higher. Lenddo, for instance, operates in Mexico, Colombia, and the Philippines, and reports that Mexico is its fastest growing market.[9]

3.2 Lending Rates with Endogenous Network Formation

We now relate our scoring formulation to lending rates, i.e., access to finance at the intensive margin. The discussion in this section implies that network based scoring affects the rates at which individuals can borrow, even if these individuals would qualify to receive credit using the individual score system. For simplicity and concreteness of discussion, we specify the perceived probability of repayment of credit by individual i, P_i as

$$P_i = \frac{1}{1 + e^{-\mu_i}}$$

which increases from 0 to 1 as the lender's assessment of the borrower's posterior mean, μ_i, increases from $-\infty$ to $+\infty$. Consider a risk-neutral lender who earns a rate of r_o from a non- risky investment. Let r_i be the lending rate to be charged to individual i with type x_i. The firm determines the rate by solving:

$$P_i \cdot (1 + r_i) + (1 - P_i) \cdot 0 = 1 + r_o.$$

This formulation takes into account not only the expected creditworthiness of an individual, μ_i, but also the outside options of the lender, r_o. For $r_o = 0$, the borrowing rate for i equals the log

[9] `http://techonomy.com/2014/02/lenddos-borrowers-mexico-philippines-get-credit-via-facebook/`

odds of default vs. repayment:

$$r_i = \frac{1 - P_i}{P_i} = e^{-\mu_i}. \tag{11}$$

As the consumer's likelihood of a default increases, he faces a higher borrowing rate. Notice that the financial utility of consumers given in Equation (6) can be derived by assuming that the lending rate enters the utility through $-\alpha \log(r_i)$. If lending rates can be interpreted within the context of economic opportunities available to consumers, then an individual with a better network score will be likely to receive a loan on better terms. This links network based credit scores to financing access at the intensive margin.

4 Role of Signals from Social Contacts

In the preceding sections, we developed a model with continuous types and assumed that every individual had identical incentives to improve his credit score. In reality, there may be differences among individuals about how much utility they can gain from improving their credit score conditional on their type. In this section, we introduce a discrete version of the model to allow for this possibility. The discrete version allows us to analyze in greater detail how the firm utilizes signals of low vs. high type friends when assessing an individual's creditworthiness. This enables us to disentangle and contrast the role of high and low type contact signals in the network.

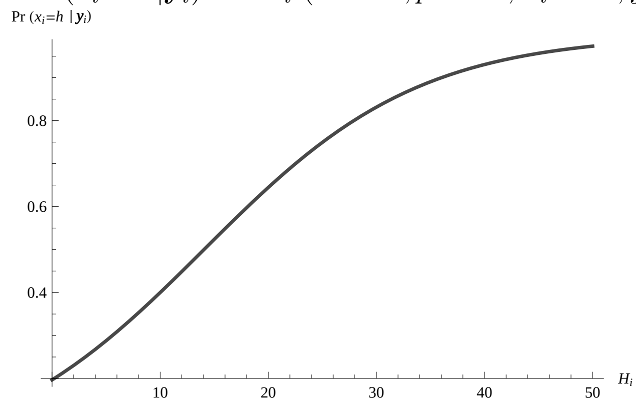
4.1 Credit Scoring and Tie Formation with High and Low Types

Consider a society with two types of borrowers: high types (h) and low types (ℓ) where the prior is uniform, with $Pr(x_i = \ell) = Pr(x_i = h) = \frac{1}{2}$. Whereas high types have a low risk of credit default, low types have a higher risk. With probability ν, any two individuals will meet. Upon meeting, they learn each other's type and their match value $m_{ij} > 0$, which is i.i.d. across pairs, with positive distribution density f. For i, the utility of becoming friends with j is

$$m_{ij} - \mathbf{1}_{\{x_j \neq x_i\}}, \tag{12}$$

where the disutility of becoming friends with a different type is normalized to 1. The utility of not becoming friends is 0. Given the specification, the probability that two same-type consumers will become friends conditional on meeting is 1, while the probability of two different types becoming friends is $p \equiv \Pr(m_{ij} > 1) < 1$; hence the network features preference-based homophily. We retain

Figure 1: $\Pr(x_i = h|\mathbf{y}_i)$ vs. H_i ($\varepsilon = 0.4, p = 0.6, L_i = 10, y_i = -1$)



the assumptions $S \to +\infty$ and $\nu \to 0$ and set $S\nu = N$ for some positive number N. With the discrete formulation, the expected number of friends for any type is $\frac{1}{2}S\nu(1+p)$: increasing the degree of homophily (a lower p) reduces the expected number of friends.

Network based score. We assume that the lender may observe a signal y_i which is -1 or 1, indicating a low or high type. The signal is credible but incorrect with probability $\varepsilon < \frac{1}{2}$. This implies, for example, that if the lender receives a signal from an ℓ-type consumer, with probability $1-\varepsilon$ it observes $y_i = -1$ and with the remaining probability it observes $y_i = 1$. Let $\mathbf{y}_i$ be the collection of signals from i and the friends of i. We first explore how the firm perceives the probability of an agent being of h-type conditional on the structure of his social network.

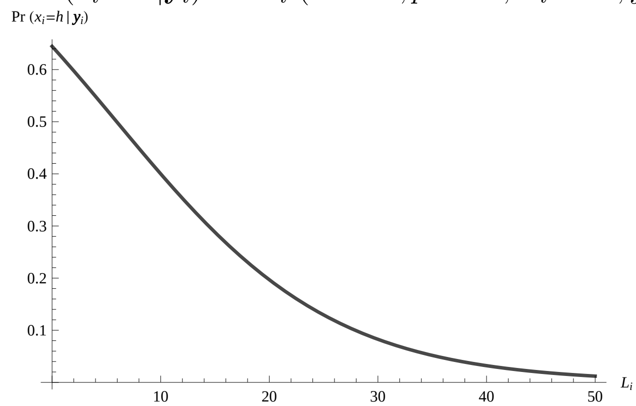
Lemma 3. *In evaluating i, the posterior for him to be high type is*

$$\Pr(x_i = h|\mathbf{y}_i) = \left[1 + \left(\frac{\varepsilon}{1-\varepsilon}\right)^{y_i} \left(\frac{\varepsilon p + (1-\varepsilon)}{\varepsilon + (1-\varepsilon)p}\right)^{L_i} \left(\frac{\varepsilon + (1-\varepsilon)p}{\varepsilon p + (1-\varepsilon)}\right)^{H_i}\right]^{-1} \tag{13}$$

where y_i is the signal observed for agent i, H_i is the number of friends with high signal, L_i is the number of friends with low signal.

Lemma 3 suggests that low and high type signals observed for an individual's social connections affect the lender's assessment of that individual's creditworthiness in different directions. Notice that $\left(\frac{\varepsilon+(1-\varepsilon)p}{\varepsilon p+(1-\varepsilon)}\right) < 1$ and $\left(\frac{\varepsilon p+(1-\varepsilon)}{\varepsilon+(1-\varepsilon)p}\right) > 1$. Thus, high type signals increase the likelihood that an agent will be categorized as being of high type, whereas low type signals reduce this likelihood. Figures 1 and 2 illustrate how $\Pr(x_i = h|\mathbf{y}_i)$ changes with H_i and L_i. The firm would prefer to extend credit to ℓ-types with a higher number of h-type connections, if everything else remained the

Figure 2: $\Pr(x_i = h|\mathbf{y}_i)$ vs. L_i ($\varepsilon = 0.4, p = 0.6, H_i = 10, y_i = -1$)



same. This suggests that in a given network where ℓ-types are fairly segregated from the h-types due to homophily, ℓ-types who are bridges between ℓ-types and h-types may be favored by the lender (compared to ℓ-types surrounded by the same-types). Put differently, in-group centrality of ℓ-types will hurt their financing opportunities whereas between-group centrality will improve them.

Endogenous Network Formation. Eq. (13) applies only when tie formation is based only on social utility and excludes the credit score ($\alpha = 0$). We now consider the case where consumer utility includes credit score. We construct the utility of a borrower similar to Section 3.2. P_i is the firm's assessment of borrower i's probability of repayment, which we may take as the posterior probability that i is a high type. The lending rate for borrower i is again given by $r_i = \frac{1-P_i}{P_i}$. Since the lending rate enters the utility additively through $-\alpha_{x_i}\log(r_i)$, we have

$$U_i = \sum_{ij\in G}\left(m_{ij} - \mathbf{1}_{\{x_i\neq x_j\}}\right) + \alpha_{x_i}R_i \tag{14}$$

where $R_i \equiv \log\left(\frac{P_i}{1-P_i}\right)$. A higher R_i implies a lower risk of extending credit to an individual. Further, the parameter α_{x_i} calibrates the importance of improving access to financing. Notice that this formulation allows low and high types to have two different levels of financial need. When $\alpha_h < \alpha_\ell$, high types' utility is less dependent on improving financing compared to the low types. When $\alpha_h = \alpha_\ell$, both types have identical financial needs. The exposition here mirrors our continuous-type model, except that different types may weigh financial concerns (represented by R_i) differently when forming ties.

Let individuals choose tie formation rules before the meeting process. Intuitively, given the network based score, individuals will be more selective towards low types and less selective towards high types. Due to the simplicity of the discrete-type model, friendship rules we allow are general and flexible. More specifically, two high types will continue to form a tie with probability 1 after they meet. As to the friendship between low types, a low type i will set a threshold θ_i and accept another low type j iff:

$$m_{ij} - \theta_i > 0.$$

Since friendships are formed based on mutual consent, a friendship between a high and low type can only be formed when the high type accepts friendship. A high type i will accept a low type j iff:

$$m_{ij} - \beta_i > 0.$$

As in the continuous case, social credit scoring makes individuals wary of forming ties with low types. In the discrete case low and high types are allowed to differ in their need for financing, and low types face discrimination or social rejection from both low and high types. This result is interesting since discrimination is often thought to take place between groups, or is believed to be exercised by one group on another. Interestingly, 'within-group' discrimination arises endogenously with the use of the network based scoring for the low types, in addition to the more common between-group discrimination. Within-group discrimination may make the surviving within-group ties more valuable, as we will see next in Lemma 4.

Let's define a symmetric profile characterized by two thresholds (θ, β), where $\theta_i = \theta$ for all low type i and $\beta_i = \beta$ for all high type i. Let $(\theta, \beta, \theta_i)$ denote a symmetric profile except for possible deviation of a low type i. Let $\mathbb{E}(U_i|\ell, \theta, \beta, \theta_i)$ represent the expected utility prior to the meeting process for a low-type individual i:

$$\mathbb{E}(U_i|\ell, \theta, \beta, \theta_i) = \mathbb{E}\left(\sum_{ij \in G} m_{ij} - \mathbf{1}_{\{x_i \neq x_j\}}|\ell, \theta, \beta, \theta_i\right) + \alpha_{x_i=\ell}\mathbb{E}\left[R_i(\theta, \beta)|\ell, \theta, \beta, \theta_i\right]$$

where the lender's posterior assessment is $P_i(\theta, \beta) = \Pr(x_i = h|\mathbf{y}_i, \theta, \beta)$, consistent with the profile. Similarly, $\mathbb{E}(U_i|h, \theta, \beta, \beta_i)$ is the corresponding expected utility of a high type. Using this utility formulation, we first lay out the lender's prior about individuals' types in Lemma 4.

Lemma 4. *Let (θ, β) be the symmetric criterion, $p_\theta \equiv \Pr(m_{ij} > \theta)$ the probability of two ℓ-types forming tie, and $p_\beta \equiv \Pr(m_{ij} > \beta)$ be the probability of a tie formation between h and ℓ types. Then the posterior probability of i being high type is*

$$\Pr(x_i = h|\mathbf{y}_i, \theta, \beta) = \left[1 + \left(\frac{\varepsilon}{1-\varepsilon}\right)^{y_i}\left(\frac{\varepsilon p_\beta + (1-\varepsilon)p_\theta}{\varepsilon + (1-\varepsilon)p_\beta}\right)^{L_i}\left(\frac{\varepsilon p_\theta + (1-\varepsilon)p_\beta}{\varepsilon p_\beta + (1-\varepsilon)}\right)^{H_i} e^{\frac{1}{2}N(1-p_\theta)}\right]^{-1} \tag{15}$$

where H_i is the number of friends with high signal, and L_i is the number of friends with low signal.

Lemma 4 presents a slightly different result compared to Lemma 3 in decomposing the contributions of high and low signals. When individuals form ties endogenously, the probability of a favorable risk assessment, $P_i(\theta, \beta)$ (or the corresponding $R_i(\theta, \beta)$), is increasing in the number of high signals (i.e., H_i) for any level of p_θ. In contrast, $R_i(\theta, \beta)$ increases in the number of friends with low signals (i.e., L_i) only if p_θ is sufficiently small[10], and decreases in L_i otherwise. In other words, when ℓ-types are very selective in forming ties amongst themselves (p_θ low), then in-group ties help to achieve a more favorable assessment from the firm, as low types have fewer ties than the high types and a large friendship circle becomes a conspicuous signal, suggesting that one is more likely to be a h-type. That is the reason low-type signals can increase the high type perception, $P_i(\theta, \beta)$. But when low types are less selective towards other own types, the negative signal begins to dominate the positive impact from size of social circle and P_i decreases in L_i.

We now turn to the impact of how selective low types are in forming ties amongst themselves, characterized by the selection rule θ. $R_i(\theta, \beta)$ is not always decreasing in L_i. In particular, we can define a value $\overline{\theta}(\beta)$ such that the *expected* effect of an additional low type *friend* on $R_i(\theta, \beta)$ is positive iff $\theta > \overline{\theta}(\beta)$. Formally, $\overline{\theta}$ can be defined as

$$\left(\frac{\varepsilon p_\beta + (1-\varepsilon)p_{\overline{\theta}}}{\varepsilon + (1-\varepsilon)p_\beta}\right)^{1-\varepsilon} \left(\frac{\varepsilon p_{\overline{\theta}} + (1-\varepsilon)p_\beta}{\varepsilon p_\beta + (1-\varepsilon)}\right)^{\varepsilon} = 1$$

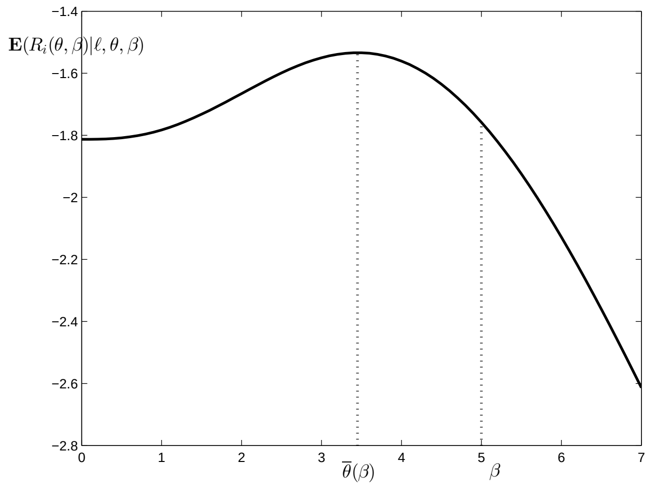
It can be easily shown that $0 < \overline{\theta}(\beta) < \beta$. We detail how an individual's odds of a favorable risk assessment vary with respect to the selectivity of ℓ-types in Lemma 5.

Lemma 5. *The expected log odds for a low type under a common tie formation criterion (θ, β), $\mathbb{E}\left[R_i(\theta, \beta)|\ell, \theta, \beta\right]$, is strictly quasi-concave in θ and achieves its maximum at $\overline{\theta}(\beta)$. Further, $0 < \overline{\theta}(\beta) < \beta$.*

Figure 3 plots a numerical example for the expected log odds of repayment as a function of θ. Notice that very high or very low levels of within-group selectivity results in lower expected odds; whereas medium levels of selectivity among low types yield the most favorable risk assessment for them. The inverse U-curve relationship stems from two competing forces that shape low-type borrowers' chances of receiving a loan. As the level of selectivity begins to increase from zero, the expected assessment improves at first. Consumers benefit from disassociating themselves from ℓ-types, improving the appearance of being an h-type. As selectivity increases further, however, a

[10]Precisely, when $p_\theta < p_\beta + \frac{\varepsilon}{1-\varepsilon}(1-p_\beta)$.

Figure 3: Expected Log Odds of Repayment vs. Selectivity θ.($\varepsilon = 0.2$, $f \sim \Gamma(3,2)$, $\beta = 5$).



second and competing effect starts to dominate: Individuals' ego-networks start to shrink extensively. Recall that the size of a borrower's network becomes a conspicuous signal of his type when individuals can form ties endogenously. Extreme selectivity leads to a smaller number of ties and so reveals the true low type of a borrower, reducing his chances of a favorable credit assessment.

Lemma 6. *The expected log odds for a low type is strictly decreasing in β for $\theta < \beta$.*

The lemma states that unlike the within-group exclusion which helps low types to some degree, between-type exclusion strictly reduces their chances of improving their financial outcomes. As high types exclude lower types from their networks, the latter's chances of a favorable assessment from the firm goes down, resulting in further hardship for this segment.

We will seek for a symmetric equilibrium where no individuals have ex-ante incentive to deviate, and company's posterior is consistent with their equilibrium behaviors. More precisely, (θ^*, β^*) is a symmetric equilibrium if for all i, $\mathbb{E}(U_i|\ell, \theta^*, \beta^*, \theta_i)$ (or $\mathbb{E}(U_i|h, \theta^*, \beta^*, \beta_i)$, depending on i's type) is maximized by $\theta_i = \theta^*$ (or $\beta_i = \beta^*$). While ensuring that there will be no unilateral deviation, a Nash equilibrium in social networks does not necessarily allow for mutual improvement in the utility of individuals. For example, a very high acceptance criteria such as $\theta^* = \infty$ can always be part of an equilibrium, because if no ℓ-type accepts another ℓ-type, an ℓ-type would have no incentive from deviating from this threshold unilaterally. We remove "unintuitive" equilibria similar to the one described from consideration. Formally, we will not consider tie formation criteria (θ^*, β^*) an equilibrium if there is another profile (θ^{**}, β^*) such that (i) low types are better off, (ii) given that high types choose β^* and every other low type chooses θ^{**}, a low type is willing to set his criterion

θ^{**} as well. Similarly, we do not consider (θ^*, β^*) an equilibrium if there is a profile (θ^*, β^{**}) with unintuitive properties alike.

Notice that from Lemma 5, for any equilibrium, $\theta^* < \beta^*$ should hold. In words, when both low and high types need financing, regardless of how dire the needs of the low types are (i.e., independent of the value of α_ℓ), low types will face *within* and *between* group exclusion. More importantly, since high types are more successful in tie formation, they afford to be selective in forming friendships. The low types, on the contrary, cannot be picky choosers: If they set the friendship threshold too high, they find themselves on the downhill side of the expected log odds curve (Figure 3). They would achieve a higher score and higher social utility by being less selective. As a result, the within-group discrimination against low types is always lower than the between-group discrimination against them. This result is formally stated in Proposition 4.

Proposition 4. *Suppose* $\alpha_h, \alpha_\ell > 0$*. In any symmetric equilibrium* (θ^*, β^*)*, we have* $0 < \theta^* < \overline{\theta}(\beta^*)$ *and* $\beta^* > 1$*.*

In summary, two forces influence the network-based score in equilibrium to be more or less diagnostic for detecting a low type. Compared to the scenario before people react, higher exclusion amongst low types make social network based scoring less powerful, by Lemma 5. In a similar vein, higher levels of exclusion on low types by high types increase the accuracy of the scores by Lemma 6.

4.2 Special Case: Lower Financing Needs for High Types

Up to now, we focused on an environment where the high types need financing. In reality, it is often the case that the need for financing (i.e., obtaining a credit or a loan) is markedly more severe for low types. To address this possibility, we provide the outcomes from the special case when $\alpha_h = 0$. Notice, by continuity, this implies that similar results would hold if α_h is a very small positive number. Note that when $\alpha_h = 0$, β is no longer material, and high types form a tie with low type only when $m_{ij} > 1$ (i.e., $\beta = 1$).

Proposition 5. *When* $\alpha_h = 0$*, there exists a unique equilibrium among low types such that* $0 < \theta^* < \overline{\theta}(1)$*.*

Proposition 5 suggests that when high types put no or very little weight on access to financing, high types may reject many social ties with low types due to homophily. In addition, due to financial concerns, ℓ-type individuals are systematically excluded even from the networks of others similar to them. Put differently, existing financial inequality breeds within-group discrimination and social isolation among those of lower type and greater need.

4.3 Explicit Discrimination against Low Types

We have shown how strategic discrimination against low types may emerge endogenously even in the presence of non-strategic homophily among low types. To extend the discussion on discrimination, we analyze an environment with exogenous discrimination against ℓ-types. To formally express such discrimination, we construct the utility for i of becoming friends with j in a manner similar to but different from the specification in Equation (12):

$$m_{ij} - \mathbf{1}_{\{x_j=\ell\}}.$$

Keeping the discrete matching formulation with this slight modification, the probability that two h-type individuals will become friends conditional on meeting is 1 and the probability that any other type of pairs will become friends is $p_1 \equiv \Pr(m_{ij} > 1) < 1$. The social utility is penalized whenever one becomes friends with an individual who is an ℓ-type.

Parallel to Lemma 3, the following lemma gives the posterior before individuals strategically form their social ties to obtain better network based score. Notice that mathematically the Lemma is a special case of Lemma 4 where $p_\theta = p_\beta = p_1$.

Lemma 7. *Let $p_1 \equiv \Pr(m_{ij} > 1)$ be the probability of formation of a tie with at least one low type. Then,*

$$\Pr(x_i = h|\mathbf{y}_i) = \left[1 + \left(\frac{\varepsilon}{1-\varepsilon}\right)^{y_i} \left(\frac{p_1}{\varepsilon + (1-\varepsilon)p_1}\right)^{L_i} \left(\frac{p_1}{\varepsilon p_1 + (1-\varepsilon)}\right)^{H_i} e^{\frac{1}{2}N(1-p_1)}\right]^{-1} \tag{16}$$

where H_i is the number of friends with high signal, L_i is the number of friends with low signal.

The lemma says that having a friend with a low signal actually improves one's score. When explicit discrimination is present, the expected number of friends varies for each type: For a high type, the expected degree is $\frac{1}{2}S\nu(1+p)$, whereas for a low type it is $S\nu p$. Similar to the endogenous rise of discrimination, a larger social network is a conspicuous signal. An individual with a larger network emits a stronger signal that he is a high type. Since in expectation low types have a smaller social circle, any tie becomes a signal of being high type.

Endogenous Network Formation. What happens when both exogenous discrimination and endogenous tie formation are at work? Lemma 7 implies that individuals will be less selective towards low types in an attempt to obtain better scores. Similar to the thresholds we defined for the homophily case, we let low types choose a criterion $\theta_i \leq 1$ towards their same type fellows, and let high types choose $\beta_i \leq 1$ towards low types. High types continue to form ties with probability one upon meeting, and a tie between two different types forms only when the high type accepts the low type. It is not difficult to see that Lemma 5 and 6 can be stated here without change. Further, a result can be derived that corresponds to Proposition 4.

Proposition 6. *When low-types are exogenously discriminated against and $\alpha_\ell, \alpha_h > 0$, in a symmetric equilibrium, $\overline{\theta}(\beta^*) < \theta^* < 1$ and $\beta^* < 1$.*

5 Effort to Become a High Type

Our results thus far relied on the assumption that individuals in society are endowed with 'types' that cannot be changed. In other words, we assumed that there is no social mobility. Although some indicators of type (e.g., family, race, birth place, country of origin) cannot be altered, other potential indicators, such as occupation or financial discipline, can be improved if low types exert effort (e.g., by investing in education). In this section, we extend our discussion to allow for this possibility. An array of factors may force ℓ-type individuals to exert effort, but we will focus on factors endogenous to tie formation such as the reduction of borrowing costs and the threat of social exclusion.

We model the mechanism in the following fashion. Consider a friends network G among ℓ and h type individuals. Let G_ℓ denote the sub-network among the low types. Further, let H_i denote the number of h-type contacts of a low-type i, which collectively are represented with the vector $\mathbf{H}$ for all the low types. Similarly, let L_i denote the number of ℓ-type contacts of a low-type i. Each low-type individual may then exert effort $e_i \geq 0$ such that with probability e_i he will become a high type. Notice, the effort therefore projects types of contacts one may have in the future. We assume that given the network and the parameters of our model, $e_i \leq 1$ for all low type i. High type consumers exert zero effort and remain high types.

The utility that a low-type individual i derives from exerting effort e_i consists of two parts:

$$U_i(\mathbf{e}, G) = \sum_{ij \in G} \Big\{ m_{ij} - \mathbf{1}_{\{x_j = \ell\}}(1 - e_j) \Big\} + u_i \tag{17}$$

where

$$u_i = ae_i - b\left[\frac{e_i}{2} - \phi\left(H_i + \sum_{ij\in G, x_j=\ell} e_j\right)\right] e_i \tag{18}$$

The term in curly brackets in equation (17) captures individual i's expected social utility under the assumption of explicit discrimination (Section 4.3) and exertion of own and friends' effort. Given the effort of a friend e_j, there is $1 - e_j$ probability that j will remain a low type, in which case i's utility from forming ties with j will be discounted by a unit normalized to 1.

The term u_i expresses the non-social benefits and costs of exerting effort. First, term ae_i captures the expected intrinsic benefits of becoming a high type. Second, the cost of effort is captured with the marginal cost $be_i/2$ that is increasing in effort. Third, under social network-based scoring, a (potential) high-type friend j has a positive effect on i's credit score and thus reduces i's financing burden. We formally express this "network effect" by allowing the marginal cost of effort for i to decrease in the number of the high-type friends he has and in the efforts of his low-type friends to become high types, at rate $\phi > 0$. Alternatively, $b\phi\left(H_i + \sum_{ij\in G, x_j=\ell} e_j\right) e_i$ can be thought of as an interaction term, representing how the return to one's own effort (e_i) is expected to be amplified by the number of friends one expects will be considered high-type. Some investors, for instance, may prefer if they have friends who are also invited to participate in exclusive investment opportunities (Bursztyn et al., 2014). In a very different setting, one is likely to gain admission to an exclusive bar or dance club if both oneself and the rest of one's party is dressed attractively.

It is important to make two notes here. First, the derivation of the functional form of u_i is a"reduced-form" approach to motivate the complementarity between one's effort and his friends' efforts. It is possible to derive this form of complementarity based on the results provided in the earlier sections. (In the Web Appendix, we offer a more detailed description of how equation (18) can be derived from this channel.) As demonstrated in Section 4, under network-based credit scoring with non-zero financing needs for both types, low types will face both within-group and between-group discrimination. Under such pressure, ℓ-type individuals would exert effort to increase their social and credit scoring utility from friendships. The benefits to exerting effort depend on the expected number of low and high type friends.

Second, notice that it is also possible to consider alternate specifications of social utility. For instance, we could also investigate an environment with pure homophily instead of discrimination, in which case Equation (17) would be replaced with:

$$U_i(\mathbf{e}, G) = \sum_{ij\in G}\left\{m_{ij} - e_i \mathbf{1}_{\{x_j=\ell\}}(1-e_j) - (1-e_i)\left[\mathbf{1}_{\{x_j=h\}} + \mathbf{1}_{\{x_j=\ell\}} e_j\right]\right\} + u_i. \tag{19}$$

In an environment with homophily, individual i will become a high type with probability e_i, in which case there will be a disutility for a tie with individual j who, after exerting effort e_j, remains a low type (which happens with probability $1-e_j$). With probability $1-e_i$, individual i will remain a low type, in which case he will face a disutility from ties with high types (including low types who become high types after exerting effort e_j).

Next, given the utility form in (18), we will first derive the optimal effort level in a given network.

Effort in an Exogenous Network. We are interested in the Nash equilibrium when people simultaneously choose their efforts when the network is exogenously given. Proposition 7 summarizes the optimal level of effort for an individual conditional on his social network, following Ballester et al. (2006).

Proposition 7. *Let $\mathbf{A}_\ell$ be sociomatrix (i.e., the adjacency matrix) of G_ℓ.*
(i) Under a discriminating social utility, if the largest-magnitude eigenvalue of $\mathbf{A}_\ell$ is smaller than $|\phi|^{-1}$, then the equilibrium effort is

$$\begin{aligned}\mathbf{e}^* =& (\mathbf{I}-\phi\mathbf{A}_\ell)^{-1}(ab^{-1}+\phi\mathbf{H})\\ =& (\mathbf{I}+\phi\mathbf{A}_\ell+\phi^2\mathbf{A}_\ell^2+...)(ab^{-1}+\phi\mathbf{H}).\end{aligned} \tag{20}$$

(ii) Under a homophilic social utility, if the largest-magnitude eigenvalue of $\mathbf{A}_\ell$ is smaller than $|2b^{-1}+\phi|^{-1}$, the equilibrium effort is:

$$\begin{aligned}\mathbf{e}^* &= \left[\mathbf{I}-(2b^{-1}+\phi)\mathbf{A}_\ell\right]^{-1}\left[(a+\mathbf{H}-\mathbf{L})b^{-1}+\phi\mathbf{H}\right]\\ &= \left[\mathbf{I}+(2b^{-1}+\phi)\mathbf{A}_\ell+(2b^{-1}+\phi)^2\mathbf{A}_\ell^2+...\right]\left[(a+\mathbf{H}-\mathbf{L})b^{-1}+\phi\mathbf{H}\right].\end{aligned} \tag{21}$$

Proposition 7 states that the effort exerted by individuals to improve their score relies on several factors. A discriminatory environment and an environment with homophily differ in the role of the low types in inducing effort. In both environments, an individual with higher number of high-type friends is likely to exert more effort, as his overall cost of borrowing is lower. In an environment with discrimination, if two ℓ-type individuals are connected to the same number of h-type friends, the one with higher number of ℓ-type friends is incentivized to exert more effort. This is perhaps surprising, as sufficiently high within-group connectivity can be a stronger motivator of effort. In contrast, in homophily, increasing proportions of low type friends can reduce effort due to enhanced social utility when an individual with low-type friends remains low type with low effort.

Observation *The expression for the equilibrium level of effort given in Equations (20)* *and (21) are forms of Bonacich centrality. The effort exerted by an agent to improve his credit score is proportional to his centrality measure.*

Further, the Nash effort in both environments is proportional to the Bonacich centrality measure, which is the 'summed connections to others, weighted by their centralities of connections to others' (Bonacich, 1987, p. 1172). With a discriminating social utility, an individual who is located at the center of a social network is likely to be exposed to higher positive network effects, therefore may exert greater effort. As a result, individuals who are more central in the network are more prone to social mobility when there are complementarities. In an environment with pure homophily, there will be two conflicting forces determining centrality and social mobility relationship. First, being central in a network of high types and low types who exert effort can increase an individual's chances of social mobility. Second, if a low type individual is central among other low types who exert little effort, he will reduce his effort to 'fit' and be similar to his network to enhance his social utility. Therefore, in tie formation based on homophily, it is possible for central low types to exert low effort leading to 'permanent' low class membership and stagnant financial hardship.

Effort with Endogenous Network Formation among Low Types Under Discriminating Utility. As we have specified in (17)–(19), the friendship utility of a friend of i depends on the effort that i will exert. Hence the effort of i plays an important role in his friends' network formation. Moreover, in the last section we saw that i's effort depends on his position in the network. This mutual dependence between the network position and effort suggests the possibility of multiple stable situations. With discriminating social utility, for example, in one society, people may exert low effort, and as a result, may become sparsely connected. This in turn gives little incentive for them to exert effort. Conversely, in another society, people may exert high effort and thus may become more densely connected, reinforcing their high-effort behavior.

To further explore how effort mitigates the likelihood of exclusion, we consider a two-stage game under the discrimination environment. In the first stage, individuals choose friends and friendships are formed bilaterally. In the second stage, individuals exert efforts. Let $\mathbf{e}^*(G)$ be the Nash effort for a given network G, which is characterized in Proposition 7. The first-stage reduced form utility for i depends on G only:

$$U_i\left(\mathbf{e}^*(G), G\right).$$

We look for pairwise-stable networks G under U. G is pairwise stable if (i) for any $ij \in G$, we have both $U_i(G) > U_i(G - ij)$ and $U_j(G) > U_j(G - ij)$; (ii) for any $ij \notin G$, either $U_i(G) \geq U_i(G + ij)$ or $U_j(G) \geq U_j(G + ij)$. Example 3 provides an application of different stability outcomes in equilibrium.

Example 3. *Consider a society with four low-type individuals featured with explicit discrimination, and assume* $a = 1$*,* $b = 5$*, and* $m_{ij} = \frac{1}{2}$ *for all* i, j*. Let* $\phi b = \frac{1}{5}$*. It can be easily verified that both the empty network and the complete network are pair-wise stable. For the empty network, each individual exerts effort* $\frac{1}{5}$ *and obtains utility of* $\frac{1}{10}$*. For the complete network, each individual exerts effort* $\frac{1}{2}$ *and has utility* $\frac{5}{8}$*.*

The example demonstrates that the empty network is pair-wise stable because everyone exerts very low effort, and a single link between a pair won't generate a change sufficiently large. The disutility of friendship with a low type (which is normalized to 1) prevents any pair from becoming friends. Moreover, a complete network is pair-wise stable because everyone exerts reasonable effort. The effort reduces the disutility of friendship between low types, and the friendship utility between any pair is exactly zero. Breaking any one link increases the costs of effort for the pair, and they will decrease their efforts. This leads to higher costs for their friends and eventually the effort of everyone will decrease. As a result, everyone receives less utility from both the friendship and effort.

Overall, the example suggests that the network structure in different societies may facilitate social pressure to exert effort at different rates. In particular, in societies where network structure is sparse, it is expected to be less effective and social mobility may remain limited. In contrast, in denser societies, social pressure can be more effective, motivating higher levels of social mobility. The difference suggests that network based scoring practices are expected to reach different levels of success in different societies; and the performance is conditional on the network structure of society.

6 Extensions

6.1 Uncertainty about Friends' Types

In our main model, the underlying assumption was that upon meeting, individuals learn about each others' types with certainty. In reality, types may be observed with some noise. Let's consider the case when individuals meet others but observe their types imperfectly. Let individual i observe a signal of x_j upon meeting with j, which is correct with probability $1 - \tau$ with $0 < \tau < \frac{1}{2}$. This implies that the added utility from homophily relies on how the uncertainty about the other's type is resolved: expected social utility is $m_{ij} - \tau$ if the signal is the same as one's own type, and

$m_{ij} - 1 + \tau$ otherwise. Respectively, probabilities $p_\tau \equiv P(m_{ij} > \tau)$ and $p_{1-\tau} \equiv P(m_{ij} > 1 - \tau)$ define how likely two individuals are to become friends upon meeting.

Compared to the benchmark model, the added uncertainty implies that ties will be less informative for the firm to predict an individual's type. To see this, first notice that under this formulation, the probability that two individuals of the same type will form ties upon meeting is

$$q_s \equiv (1-\tau)^2 p_\tau + (1 - (1-\tau)^2) p_{1-\tau}, \tag{22}$$

and two individuals of opposite types will form a tie is

$$q_d \equiv \tau^2 p_\tau + (1 - \tau^2) p_{1-\tau}. \tag{23}$$

Utilizing these probabilities, we can formulate how the firm will assess a borrower's type to be high as given in Lemma 8.[11]

Lemma 8. *When individuals learn about each others' types with uncertainty,*

$$\Pr(x_i = h | \mathbf{y}_i) = \left[1 + \left(\frac{\varepsilon}{1-\varepsilon} \right)^{y_i} \left(\frac{\varepsilon q_d + (1-\varepsilon) q_s}{\varepsilon q_s + (1-\varepsilon) q_d} \right)^{L_i} \left(\frac{\varepsilon q_s + (1-\varepsilon) q_d}{\varepsilon q_d + (1-\varepsilon) q_s} \right)^{H_i} \right]^{-1}$$

where H_i is the number of friends with high signal, L_i is the number of friends with low signal.

We are interested in how presence of noise in detecting each other's true types in social relationships may influence the firm's ability to rely on social credit scores. We compare Lemma 8 with Lemma 3. Since $p_1 < q_d < q_s < 1$, $1 < \frac{\varepsilon q_d + (1-\varepsilon) q_s}{\varepsilon q_s + (1-\varepsilon) q_d} < \frac{\varepsilon p_1 + (1-\varepsilon)}{\varepsilon + (1-\varepsilon) p_1}$ and $\frac{\varepsilon + (1-\varepsilon) p_1}{\varepsilon p_1 + (1-\varepsilon)} < \frac{\varepsilon q_s + (1-\varepsilon) q_d}{\varepsilon q_d + (1-\varepsilon) q_s} < 1$. In words, signals from contacts carry less weight to form beliefs about an individual's type when types cannot be perfectly observed in friendship.

There are two observations related to this finding. First, the level of information sharing between individuals can change the appropriateness of a social network for credit scoring. For example, if an online network allows individuals to frequently communicate and exchange in depth information, this may positively influence the efficiency of credit assessment by reducing the uncertainty about friends' types. Second, the ability of peers to observe each other's types may correlate with the characteristics of the network, including tie strength. For example, the parameter τ could reflect the strength of ties correlating with the ability to convey complex or subtle information (Van den

[11] The derivation of this Lemma follows the derivation of Lemma 4.

Bulte and Wuyts, 2007, pp. 71-72) and hence with one's ability to observe a friend's type. Next, in Section 6.2, we detail this discussion.

6.2 Friendship Formation and Strength of Ties

In Section 6.1, we maintained the assumption that all relationships carry equal information and pointed out that the informativeness of a link may relate to tie strength. We will adjust the earlier model slightly to extend the earlier discussion.

Specifically, let's assume that individuals can form weak and strong ties, and that they learn about others' type with certainty only if they have strong ties with them. After meeting, a match value $m_{ij} > 0$ and the tie type are randomly determined. If the tie is strong, individuals obtain the utility $m_{ij} - \mathbf{1}_{\{x_i \neq x_j\}}$ by forming a friendship. If the tie is weak, types remain unknown, and the social utility of forming a tie is $m_{ij} - \kappa$. The parameter κ captures the disutility from forming a weak tie.

Since weak ties do not carry information about the type or the type difference between the ego and the friend, a firm cannot use them to update its posterior belief about an individual's type. Only the strong ties will reveal information about a contact's type and become eligible for the firm to use to determine the social score.

The general implication is straightforward. Since strong ties are more homophilous than weak ties and since they provide a greater ability to learn about one's contacts, the accuracy of social scoring increases with the relative prevalence of strong vs. weak ties.

6.3 Effort to Enhance Probability of Meeting High Types

In Section 5, the model was built such that the low type individuals exerted effort to climb social ladders by improving their type. It is possible, under some circumstances, that individuals cannot change their type but can exert effort to increase the probability of meeting high types. Networking is an example of such directed effort. In this section we explore this possibility which also allows us to endogenize the probability of meeting between two individuals.

We use the settings of the discrete-type model in Section 4 and allow individual i to choose an effort level e_i. Conditional on the effort exerted, the individual is likely to meet another person randomly with probability $\frac{M}{S}e_i$, where M is a constant that calibrates the chance of meeting another person proportional to the effort exerted in a society of size S. A meeting between i and j happens when either of the two individuals "runs into" the other. Suppose a common effort e is exerted by everyone but i. Then the expected number of meetings for i becomes:

$$S\left[1-(1-e_i\frac{M}{S})(1-e\frac{M}{S})\right]=\left(e_i+e-\frac{e_ieM}{S}\right)M.$$

When $S\to+\infty$, the expected number of meetings go to $(e_i+e)M$.

First consider the scenario of exogenous tie formation where an individual's utility only depends on the social utility from friendships. Recall that, upon meeting with j, i always forms a tie if j is of the same type, and forms a tie iff $m_{ij}>1$ if j is of the different type. Let f be the density of the matching value distribution. The expected social utility for i, given that a symmetric effort e is used except for possible deviation of i to e_i, can be derived from

$$\mathbb{E}\left(\sum_{ij\in G} m_{ij}-\mathbf{1}_{\{x_i\neq x_j\}}\middle| x_i,e,e_i\right)=(e_i+e)\frac{M}{2}\left(\int_0^\infty tf(t)dt+\int_1^\infty (t-1)f(t)dt\right)$$

Let Λ denote the term in the last parenthesis. Let $\frac{1}{2}e_i^2$ be the cost of effort. The equilibrium effort is then given by

$$e^*=\frac{M\Lambda}{2} \tag{24}$$

Under this common effort level, the firm's posterior on type is again given by (13) in Lemma 3.

Next, we will set this equilibrium effort level as the baseline, and compare it to that when social relationships affect financial benefits. Since the credit score introduces asymmetric desirability of low-type and high-type friends, the effort levels exerted by low types and high types will be different in general. In principle, the firm's posterior needs to incorporate the difference in efforts. Here for simplicity, we focus on how effort level will differentiate between types but leave out how it would affect firm's posterior assessment.

Formally, we let the credit score enter utility additively through $\alpha_{x_i}R_i$ with P_i simply given by (13). We characterize a symmetric equilibrium, by which we mean the effort pair (e_ℓ^*,e_h^*) where every low type chooses e_ℓ^* and every high type chooses e_h^* such that no individual has incentive to deviate.

The following proposition summarizes how the motivation of individuals to meet others change compared to the effort they would exert simply to maximize their utility from friendships.

Proposition 8. *(i) for $\alpha_\ell=\alpha_h>0$, $e_h^*>e^*>e_\ell^*$, and*
(ii) for α_ℓ sufficiently larger than α_h, $e_\ell^*>e_h^*\geq e^*$.

The proposition suggests that when both ℓ and h type individuals have identical needs for financing, high types exert higher levels of effort to increase their probability of meeting others compared to low types and compared to the effort exerted when individuals only want to maximize social utility. This is because high types have higher marginal return on effort than the low types (i.e., are more likely to form new ties as a result of effort). As a result, independent of their financial needs, high types always exert more effort than they would when they earn utility from improving their access to credit in addition to the gains in social utility. Low types, in contrast, have lower returns, but when high types make an effort to meet others, they also benefit from it. With some probability, a meeting will take place between a low and a high type and a friendship will be formed if m_{ij} is sufficiently high.

If, on the other hand, the low types' utility from improving their credit scores is very high (α_ℓ very high), this pattern result could reverse. Low types would feel an immense pressure to increase the probability of becoming friends with high types, resulting in a higher level of effort exerted by low types compared to that of high types.

7 Conclusion

7.1 Main Insights

Increasing access to financing is important in many countries where institutions and contract enforcement are weak (e.g., Feigenberg et al., 2010; Rona-Tas and Guseva, 2014). In low-income countries, in particular, part of the credit access problem stems from the fact that reliable data on financial history do not exist, are limited, costly to collect, or hard to verify. In these countries, lenders tend to be very conservative in accepting borrowers' credit applications. This, of course, makes it even harder for individuals who are in financial hardship to obtain credit and generate a financial track record. Group lending has proven to be a popular way to address this problem. An alternative and possible complement is to use additional available data to assess individuals' creditworthiness. Using social data is one such option.

Motivated by the importance of consumer access to credit and by the increasing use of network based credit scoring, we analyzed the potential implications of such practices for consumers. Our study shows that there are indeed benefits to collecting information from a consumer's network rather than only individualized data. Simply put, when people have an above-average chance of interacting with others of similar creditworthiness, then network ties provide additional reliable signals about a consumer's true creditworthiness. Hence, social scoring can reduce lenders' misgivings about engaging people with limited personal financial history, which include many who are economically disadvantaged and "underbanked".

As these new scoring methods gain popularity, consumers may adapt their personal networks, which in turn may affect the usefulness of these scores. If one's network can influence one's financing chances, some individuals, particularly those in more dire need of improving their credit score, may be inclined to form social ties more selectively. If all consumers behave in this manner and forming social ties requires mutual agreement, the end result of such behavior will be social fragmentation into sub-networks where people connect only to others who are very similar to them. Though we expect that such fragmentation and balkanization will be deemed socially undesirable by many, its implications for network scoring accuracy is not straightforward. People will have fewer ties conveying information about one's contacts useful in updating lenders' prior beliefs, but each of the ties will be more informative. We find, however, that there are situations in which social scoring is beneficial even when consumers adjust their networks. Specifically, when consumers place sufficiently low importance on the posterior mean of the firm, higher accuracy in risk assessment with network based scoring is possible even when individuals form their ties endogenously.

To focus on the role of connections to consumers with different levels of financial strength in the emergence of balkanized societal structures, we introduce discrete types and discrete type matching. Unsurprisingly, connections to individuals with high type signals have an overall positive impact. More interesting is that the impact of connections to low type signal individuals can be positive or negative, depending on the tie formation rules used in society. We find that consumers with poor financial health and in great need for credit would prefer others not to be too selective but also not to be too liberal in their willingness to associate with people having poor financial health and a great need for credit. As a result, disadvantaged consumers would prefer some intermediate level of ostracism and social isolation.

In our extensions, we discussed two scenarios which may reduce the reliability of social scores. First, if individuals cannot observe their social contacts' types perfectly upon meeting, the added noise will imply that in formation of social networks homophily will play a lesser role. As a result, firms' ability to detect a borrower's type by looking at his friends will be limited. In a similar vein, if the network consists mainly of weak rather than strong ties, then this will also reduce social scores' diagnosticity, since strength correlates with how well individuals know each other. In both of these scenarios, contacts' signals carry lower value to the firm in assessing the risk of a borrower.

We also considered the possibility of exerting effort in two different ways. First, we move away from the static type model and allow individuals to improve their type. We find that when there is discrimination against low types, both low and high type contacts play a role in motivating effort, but high types, in general, have a stronger effect. In an environment with only homophily, these results hold as well, unless an individual is highly embedded in a network with many low-type friends who exert low effort. Such individuals are not motivated to exert effort towards improving themselves and are more likely to remain a low type. Second, when types are sticky and cannot be altered, we allow individuals to exert effort to improve their chances of meeting other people. This second model shows that individuals' networking effort will depend on their need for financing. When high and low types have comparable needs for financing, high types have higher returns on their effort of creating new ties and thus exert more effort to meet others. Since the types are revealed only after meeting, low types' likelihood of running into a higher type increases when high types exert effort too. Therefore they choose to free ride on others' efforts. This outcome reverses when low types are in dire need of financing, and they become the primary driver of meetings in society.

One possible outcome of social scoring which is not addressed in this research is the possibility that individuals strategically manipulate the perception of their type by trading friendships for financial access. In particular, realizing their 'higher' financial status, high types may want to offer their friendships in exchange for monetary rewards. To model an environment where friendships are traded, we could need to consider several additional layers of complexity. First, rationally, traded friendships would need to be formed such that the credit scoring firm should not be able to distinguish a fake relationship from a true friendship. Otherwise, low types would have no incentive to pay for a high type's friendship. Second, high types must be financially motivated and the benefit from forming a friendship with a low type must exceed the losses from less favorable risk assessment. Third, trading friendships must be rare enough that a credit scoring firm still benefits and desires to use data from the social networks. Altogether, modeling an environment of this sort would require a fairly complicated model which goes beyond the purposes of the current study. Despite the complication, our expectation for the findings, would be fairly simple: In line with the extensions we discussed in Sections 6.1 and 6.2, if social ties have lower informative value and homophily is diluted, social credit scores will be less diagnostic in detecting one's true credit worthiness.

7.2 Implications for Public Policy

The link between credit scores and income is hard to ignore.[12] It is reported that most U.S. consumers with an income lower than $60K have a poor credit score.[13] Moreover, a significant portion of the individualized credit score calculation relies on an individual's existing debt level.

[12] This is so even though FICO and other leading institutions state that income is not a part of one's individual credit score, as it is a self-reported item of assessment.

[13] http://www.creditsesame.com/about/press/consumers-who-earn-60000-or-less-have-dangerously-high-credit-usage-levels-according-to-credit-sesame/

Someone owing higher amounts, all else equal, is expected to have a lower credit score. With network based assessment, it is possible for immigrants, "underbanked" consumers, recent college graduates, and other individuals who do not have a credit history but are creditworthy to signal this to lenders with higher accuracy. The benefits introduced through network based systems may help overcome a portion of the financing problems, particularly if networks are created based on attributes correlated to financial health.

However, our analysis also raises an important concern about discrimination against already financially disadvantaged and underbanked groups. For instance, the U.S. Equal Credit Opportunity Act (ECOA) prohibits lenders to discriminate based on sex, race, color, religion, national origin, or age. To the extent that some of these characteristics correlate with creditworthiness and that homophily along those dimensions correlate with homophily along levels of creditworthiness, there is a concern that a side-effect of social credit scoring may be discrimination in access to credit along characteristics prohibited by the ECOA (National Consumer Law Center, 2014, pp. 27-29). Aside from strict legality, there is a concern that social scoring opens an additional back door to discrimination along dimensions that many may find objectionable (Dixon and Gelman, 2014; Pasquale, 2015).

Matters are even more complex since our results also show that social scoring may lead people with low creditworthiness to prefer being discriminated against- in tie formation at least- to some moderate extent. So, moderate levels of discrimination and social ostracism by fellow consumers may actually help rather than harm disadvantaged consumers. Also, one hitherto ignored societal benefit of social scoring is that it can motivate rather than demotivate financially disadvantaged citizens to greater exert effort to improve their creditworthiness. The financial discrimination and social exclusion implications of social credit scoring, and how they balance against its benefits, warrant attention from policy makers and researchers alike.

Finally, our study's findings are of interest to policy makers keen on understanding the mutual interaction between social status and network structure. As we noted at the outset, our mathematical analysis of credit scoring applies to social status broadly. Some people command less respect than others. Differences in status are rarely based solely on differences in true but hard-to-observe ability or character. Often, people use the company others keep as a signal when assessing the respect they deserve. Our analysis of the benefits and challenges of social credit scoring – including improved diagnosticity paired with the risk of unwitting discrimination and the seeming paradox of optimal ostracism – extends to situations where citizens, employees, or customers are valued and accorded status based on the company they keep.

7.3 Implications for Management

To managers in the financial industry, our analysis suggests that lenders can expect to reduce their risk by incorporating network based measures in the short run. This dovetails with new governmental policies on risk. For example, as part of the regulations posed by the Basel Committee on Banking and Supervision, banks in Europe have been encouraged to reduce the level of risk they undertake (Sousa et al., 2013). Regulations in the banking industry encourage financial institutions to better manage risk in the U.S. as well. These regulations have come at a time when big data analytics are enabling financial institutions to access larger and richer datasets. Indeed, it has been reported that social media and social network data are being used not only by start-ups, but also by established and more institutionalized credit scoring firms, such as Experian (Armour, 2014). The trend to use social data may prove to be useful in the post crisis environment.

Our study also offers some insight to managers outside the financial industry who use social scoring for targeting customers when launching new products, targeting ads, or designing referral programs. (i) The effectiveness of social scoring need not decrease when customers purposely adapt their networks in order to improve their score and their access to the benefits it entails. (ii) Marketers do not need information on the complete network. Data on the focal consumer's immediate contacts already provide an improvement in scoring accuracy. (iii) Social scoring is likely to be most diagnostic in societies and communities (online or not) where consumers maintain many strong rather than weak ties. (iv) Smart marketers will go beyond generic ties and seek to leverage specific ties that correlate highly with the traits they seek in their target customers. A car manufacturer like Audi, for instance, will benefit from focusing on Twitter connections pertaining to cars (personal communication). (v) The benefits of social scoring to the marketer are greater when the benefits of having a high score matters little to customers, or at least has little impact on who they choose to form ties with. More generally, the benefit of social scoring are greater when it involves networks of ties that not only exhibit great homophily but also are built and maintained for intrinsic rather than extrinsic reasons. Examples of the former used in social scoring include telephone call data and kinship data (Benoit and Van den Poel, 2012; Hill et al., 2006). Examples of the latter are many ties in general-purpose online social networking platforms, where linking is very easy and often between casual contacts. (vi) Customers with a high number of connections (degree centrality) in an undirected network like Facebook or LinkedIn are not necessarily the most attractive. The reason is not only that centrality in such networks cannot distinguish between opinion seekers and opinion leaders (indegree vs. outdegree centrality), but also that –as our analysis shows– the most active networkers may be either high-type or low-type customers, depending on whether low-types value the benefits of a high consumer score more than high-types do. (vii) Marketers should be concerned that social customer scoring may create the impression of unfair discrimination. This is not only a legal and an ethical issue, but also a commercial one. For instance, in January 2015, users of WeChat, the Chinese chat app, protested against discrimination after they were not targeted to seer an ad for BMW, the luxury car maker – with some believing that the targeting algorithm involved social scoring based on who potential targets were connected to (Clover, 2015). Since social scoring uses inputs beyond the individual's traits and history, it requires marketers to balance improved diagnosticity against actual and perceived fairness.

Appendix: Proofs

Proof of Proposition 1: Because once conditional on the types $\mathbf{x}$, the signals $\mathbf{y}$ are independent of the network, we have $\Pr(\mathbf{y}|\mathbf{x}) = \Pr(\mathbf{y}|g, \mathbf{x})$. Using Bayes' rule we have

$$\begin{aligned} \Pr(\mathbf{x}|g, \mathbf{y}) &\propto \Pr(\mathbf{x}) \Pr(g, \mathbf{y}|\mathbf{x}) \\ &= \Pr(\mathbf{x}) \Pr(\mathbf{y}|\mathbf{x}) \Pr(g|\mathbf{x}) \end{aligned}$$

Thus

$$\begin{aligned} \Pr(\mathbf{x}|g, \mathbf{y}) \propto & \prod_{i \in \mathbf{y}} e^{-q x_i{}^2/2} \times \prod_{i \in \mathbf{y}} e^{-c(y_i - x_i)^2/2} \times \prod_{ij \in g^1} \nu e^{-(x_i - x_j)^2/2} \times \\ & \prod_{ij \in g^0 : i, j \in \mathbf{y}} [1 - \nu e^{-(x_i - x_j)^2/2}] \times \prod_{ij \in g^0 : j \notin \mathbf{y}} \left(1 - \frac{\mathbb{E}(n_i|x_i)}{S}\right). \end{aligned} \tag{A.1}$$

In the expression above, $(1 - \mathbb{E}(n_i|x_i)/S)$ is the probability that "i is not friends with j" for some i whose type is x_i and some j whose type is unknown. Fix some $i \in \mathbf{y}$ and consider the term $\prod_{ij \in g^0 : j \notin \mathbf{y}} (1 - \mathbb{E}(n_i|x_i)/S)$. If $\{ij \in g^0 : j \notin \mathbf{y}\}$ is not empty, then by our assumption on the information structure, it multiples across everyone in the rest of the society. So its value under the limits of S, ν and q is

$$\lim_{S \to \infty, \mathbb{E}(n_i|x_i) \to N} \left(1 - \frac{\mathbb{E}(n_i|x_i)}{S}\right)^{S - |\mathbf{y}|} = e^{-N}$$

which isn't a function of $\mathbf{x}$ thus does not contribute to the conditional density. Notice that the rest of the terms in the right hand side of (A.1) multiple across finite items. It is easy to see that as $\nu \to 0$ and $q \to 0$,

$$\Pr(x|g, y) \propto \prod_{i \in \mathbf{y}} e^{-c(y_i - x_i)^2/2} \times \prod_{ij \in g^1} e^{-(x_i - x_j)^2/2}. \tag{A.2}$$

This implies that $\Pr(\mathbf{x}|g, \mathbf{y})$ is a multivariate normal density $N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. To find the parameters $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$, all we need to do is matching the coefficients. The coefficients of $x_i^2, x_i x_j$ and x_i in the quadratic form $-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})$ are $-\frac{1}{2}(\boldsymbol{\Sigma}^{-1})_{ii}$, $-(\boldsymbol{\Sigma}^{-1})_{ij}$ and $(\boldsymbol{\Sigma}^{-1})_{i1}\mu_1 + (\boldsymbol{\Sigma}^{-1})_{i2}\mu_2 + ...$, while the corresponding coefficients in the right hand side of (A.2) are $-\frac{1}{2}(c+d_i)$, $1_{\{ij \in g^1\}}$ and cy_i. Matching them gives us the results in the Proposition. $\square$

Proof of Corollary 1: This is just a special case of Proposition 1, where i is fixed and $\mathbf{y} = \{j|\, ij \in G\}$, $g^1 = \{ij|\, ij \in G\}$ and $g^0 = \{ij|\, ij \notin G, j \neq i\}$. $\square$

Proof of Proposition 2: Let $\mathbf{D}$ be the diagonal matrix where $D_{ii} = c + d_i$, and $\mathbf{B} = \mathbf{D}^{-1}\mathbf{A}$ where $\mathbf{A}$ is the adjacency matrix of g^1. We can express the precision matrix by

$$\boldsymbol{\Sigma} = (\mathbf{I} - \mathbf{B})^{-1}\mathbf{D}^{-1}$$

Let $\mathbf{B}_0$ denote the matrix $\mathbf{B}$ when $c = 0$. Since $\mathbf{B}_0$ is a stochastic matrix (i.e., each row summing up to 1), its largest-magnitude eigenvalue is 1. When $c > 0$, $\mathbf{B}$ is non-negative and it is easy to see that

$$\mathbf{B} < \delta\mathbf{B}_0$$

By the Perron-Frobenius Theorem, we know that the largest-magnitude eigenvalue of $\mathbf{B}$ is smaller than that of $\delta\mathbf{B}_0$, which is δ. Given that $\delta < 1$, we may write

$$\boldsymbol{\Sigma} = (\mathbf{I} + \mathbf{B} + \mathbf{B}^2 + ...)\mathbf{D}^{-1}$$

Because for any $k \geq 1$, $\mathbf{B}^k$ is non-negative and $\left\|\mathbf{B}^k\right\| < \delta^k$, we have,

$$(\mathbf{B}^k)_{ij} < \delta^k$$

Now consider a node j whose distance from i in the sub-network defined by g^1 is $r(i,j) \geq 1$. Because $\mathbf{A}$ is the adjacency matrix of g^1, and there is no path between i and j whose length is less than $r(i,j)$, we know $(\mathbf{B}^k)_{ij} = 0$ for all $k < r(i,j)$. Hence an upper bound of $(\mathbf{I} + \mathbf{B} + \mathbf{B}^2 + ...)_{ij}$ is

$$\sum_{k=r(i,j)}^{+\infty} \delta^k = \delta^{r(i,j)}/(1-\delta).$$

$\square$

Proof of Lemma 1: Derivation of the lemma follows similarly to the proof of Corollary 1. $\square$

Proof of Lemma 2: Under a symmetric rule λ, i and j become friends iff they have met and $m_{ij} > \lambda(x_i - x_j)^2$. Thus

$$\mathbb{E}(n_i|x_i,\lambda) = S\int_{-\infty}^{+\infty} \nu e^{-\lambda(t-x_i)^2/2}\sqrt{\frac{q}{2\pi}}e^{-qt^2/2}dt = S\nu\sqrt{\frac{q}{q+\lambda}}e^{-\frac{\lambda q}{\lambda+q}x_i{}^2/2}$$

Recall that $S\nu\sqrt{\frac{q}{q+1}} = N$. Taking $q \to 0$ gives the result. □

Proof of Proposition 3: For notational simplicity, the expectation sign $\mathbb{E}(\cdot)$ throughout this proof refers to the conditional expectation $\mathbb{E}(\cdot|x_i,\lambda,\lambda_i)$, which is computed conditional on the type x_i and a symmetric rule λ except for possible deviation of i to λ_i. Similarly, the notation $\Pr(\cdot)$ also refers to the probability with the same conditionals.

First let us calculate the expected social utility, $\mathbb{E}\sum_{ij\in G}(m_{ij} - |x_j - x_i|)$, which we will denote more compactly as $\mathbb{E}u_i$. For any j we have:

$$\begin{aligned}\Pr(x_j, ij\in G) &= \Pr(x_j)\Pr(ij\in G|x_j) \\ &= \sqrt{\frac{q}{2\pi}}e^{-qx_j^2/2}\times\begin{cases}\nu e^{-\lambda_i(x_i-x_j)^2/2} & \text{if } x_j\le x_i\\ \nu e^{-\lambda(x_i-x_j)^2/2} & \text{if } x_j > x_i.\end{cases}\end{aligned} \tag{A.3}$$

(A.3) enables us to calculate the probability of being friends with j:

$$\begin{aligned}\Pr(ij\in G) &= \int_{-\infty}^{+\infty}\Pr(x_j, ij\in G)dx_j \\ &= \frac{1}{2}\left(\frac{1}{\sqrt{\lambda_i}}+\frac{1}{\sqrt{\lambda}}\right)\nu\sqrt{\frac{q}{q+1}}e^{-\frac{q}{q+1}x_i^2/2}\end{aligned}$$

and in particular, its limiting value:

$$S\Pr(ij\in G)\to\frac{1}{2}N\left(\frac{1}{\sqrt{\lambda_i}}+\frac{1}{\sqrt{\lambda}}\right) \tag{A.4}$$

In a similar way, (A.3) also enables us to calculate the conditional type difference and its limiting value:

$$\begin{aligned}\mathbb{E}\left(-|x_j - x_i|\middle| ij\in G\right) &= \int_{-\infty}^{+\infty} -|x_j - x_i|\Pr(x_j|ij\in G)dx_j \\ &\to \sqrt{\frac{2}{\pi}}\left(\frac{1}{\lambda_i}+\frac{1}{\lambda}\right)\Big/\left(\frac{1}{\sqrt{\lambda_i}}+\frac{1}{\sqrt{\lambda}}\right)\end{aligned} \tag{A.5}$$

Next we turn to the matching value. We have

$$\begin{aligned}
\Pr(m_{ij}, ij \in G) &= \Pr(m_{ij}) \Pr(ij \in g | m_{ij}) \\
&= m_{ij} e^{-m_{ij}^2/2} \left(\nu \sqrt{\frac{q}{2\pi}} \int_{x_i - m_{ij}/\sqrt{\lambda_i}}^{x_i + m_{ij}/\sqrt{\lambda}} e^{-q x_j^2/2} dx_j \right)
\end{aligned}$$

So,

$$S \Pr(m_{ij}, ij \in G) \to \frac{N}{\sqrt{2\pi}} m_{ij}^2 e^{-m_{ij}^2/2} \left(\frac{1}{\sqrt{\lambda}} + \frac{1}{\sqrt{\lambda_i}} \right)$$

which, with (A.4), implies that

$$\Pr(m_{ij} | ij \in G) \to \sqrt{\frac{2}{\pi}} m_{ij}^2 e^{-m_{ij}^2/2}$$

This is the density of a χ_3 distribution. So we have

$$\mathbb{E}\left(m_{ij} \middle| ij \in G \right) \to 2\sqrt{\frac{2}{\pi}} \tag{A.6}$$

The expected social utility can be computed by summing over i's expected social utility from each j in the society:

$$\begin{aligned}
\mathbb{E}u_i &= \sum_{j \neq i} \Pr(ij \in G) \left[\mathbb{E}\left(-|x_j - x_i| \middle| ij \in G \right) + \mathbb{E}\left(m_{ij} \middle| ij \in G \right) \right] \\
&= S \Pr(ij \in G) \left[\mathbb{E}\left(-|x_j - x_i| \middle| ij \in G \right) + \mathbb{E}\left(m_{ij} \middle| ij \in G \right) \right]
\end{aligned}$$

Equipped with (A.4), (A.5) and (A.6), we are able to find its limiting value,

$$\mathbb{E}u_i \to \frac{N}{\sqrt{2\pi}} \left[2\left(\frac{1}{\sqrt{\lambda_i}} + \frac{1}{\sqrt{\lambda}} \right) - \left(\frac{1}{\lambda_i} + \frac{1}{\lambda} \right) \right] \tag{A.7}$$

A nice intuitive result from this is that the social utility is maximized at $\lambda_i = 1$. Any deviation from that "distorts" the friendship formation and is suboptimal in terms of social utility.

Next we look at the expected utility from the network based score. The bias from using network based scoring is:

$$\begin{aligned}
\mathbb{E}\mu_i(\lambda) - x_i &= \mathbb{E}\left[\frac{\lambda \sum_{ij \in G}(x_j - x_i)}{c + \lambda + \lambda n_i} \right] \\
&= \mathbb{E}\left[\frac{\lambda}{c + \lambda + \lambda n_i} \mathbb{E}\left(\sum_{ij \in G} (x_j - x_i) \middle| n_i \right) \right] \\
&= \mathbb{E}\left[\frac{\lambda n_i}{c + \lambda + \lambda n_i} \mathbb{E}\left(x_j - x_i \middle| ij \in G \right) \right] \\
&= \mathbb{E}\left(\frac{\lambda n_i}{c + \lambda + \lambda n_i} \right) \mathbb{E}\left(x_j - x_i \middle| ij \in G \right)
\end{aligned}$$

The first equality comes from the fact that y_i (and y_j) are unbiased signals of x_i (and x_j). The second equality makes use of the iterated law of expectation. The last equality makes use of the fact that $\mathbb{E}(x_j - x_i|ij \in G)$ is not a function of n_i.

Using (A.3), we may calculate, in a way similar to (A.5),

$$\mathbb{E}\left(x_j - x_i\Big|ij \in G\right) \to \sqrt{\frac{2}{\pi}}\left(\frac{1}{\lambda} - \frac{1}{\lambda_i}\right) \Big/ \left(\frac{1}{\sqrt{\lambda}} + \frac{1}{\sqrt{\lambda_i}}\right)$$

So under the limits, we have the equality:

$$\mathbb{E}\mu_i(\lambda) - x_i = \underbrace{\mathbb{E}\left(\frac{\lambda n_i}{c + \lambda + \lambda n_i}\right)}_{\varphi} \times \underbrace{\sqrt{\frac{2}{\pi}}\left(\frac{1}{\lambda} - \frac{1}{\lambda_i}\right) \Big/ \left(\frac{1}{\sqrt{\lambda}} + \frac{1}{\sqrt{\lambda_i}}\right)}_{\xi} \tag{A.8}$$

It is a bit difficult to find an explicit expression for φ even under limits, so we will deal with it implicitly. From this point on, notations $\mathbb{E}U_i$, $\mathbb{E}u_i$ and $\mathbb{E}\mu_i$ all refer to their limiting values.

As a first step to find the equilibrium, we look at the "best response" correspondence for i, that is, the value of λ_i that maximizes $\mathbb{E}U_i$ for any λ. We will make use of the derivative of $\mathbb{E}U_i$:

$$F(\lambda_i, \lambda) := \frac{\partial \mathbb{E}u_i}{\partial \lambda_i} + \alpha \frac{\partial(\mathbb{E}\mu_i(\lambda) - x_i)}{\partial \lambda_i}$$

By (A.8),

$$\frac{\partial(\mathbb{E}\mu_i(\lambda) - x_i)}{\partial \lambda_i} = \frac{\partial \varphi}{\partial \lambda_i}\xi + \frac{\partial \xi}{\partial \lambda_i}\varphi$$

Notice that (i) ξ has the same sign as $\lambda_i - \lambda$, (ii) $\partial\xi/\partial\lambda_i > 0$, and (iii) $0 < \varphi < 1$, (iv) $\partial\varphi/\partial\lambda_i < 0$. The first three points are easy to be seen. The last point can be seen by noticing that n_i is binomially distributed, and under the limits, Poisson distributed with the mean given in (A.4).

Using (i)-(iv), we see two useful properties for the second component of F:

$$\frac{\partial(\mathbb{E}\mu_i(\lambda) - x_i)}{\partial \lambda_i} < \frac{\partial \xi}{\partial \lambda_i}, \quad \text{for } \lambda_i \geq \lambda \tag{A.9}$$

and

$$\frac{\partial(\mathbb{E}\mu_i(\lambda) - x_i)}{\partial \lambda_i} > 0, \text{ at } \lambda_i = 1 \tag{A.10}$$

Next we will translate these two properties into two properties of F. First, (A.10) tells us that

$$F(1, \lambda) > 0 \tag{A.11}$$

because $\frac{\partial \mathbb{E}u_i}{\partial \lambda_i} = 0$ at $\lambda_i = 1$, by (A.7).

To obtain the second property, we need to look at a simpler case of F where φ is ignored in the derivative:

$$\widetilde{F}(\lambda_i, \lambda) := \frac{\partial \mathbb{E}u_i}{\partial \lambda_i} + \alpha \frac{\partial \xi}{\partial \lambda_i}$$

It can be easily verified that as long as $\alpha < N$, there is an invariant solution to $\widetilde{F}(\cdot, \lambda) = 0$:

$$\lambda^o \equiv \left(1 - \frac{\alpha}{N}\right)^{-1}$$

and $\widetilde{F}(\lambda_i, \lambda) \leq 0$ for $\lambda_i \geq \lambda^o$. Together with (A.9), this tells us that

$$F(\lambda_i, \lambda) < \widetilde{F}(\lambda_i, \lambda) \leq 0, \text{ for any } \lambda_i \geq \max(\lambda, \lambda^o) \tag{A.12}$$

These two properties about F are sufficient to derive the proposition. Define $\Xi_i(\lambda) := \text{argmax}_{\lambda_i \geq 1} \mathbb{E} U_i$ as the "best response" correspondence. Using Berge's Theorem one can show that it is upper semi-continuous. Furthermore, (A.11) and (A.12) imply

$$1 < \Xi_i(\lambda) < \max(\lambda, \lambda^o)$$

This tells us that any fixed point of $\Xi_i(\cdot)$ must be between 1 and λ^o. Using Kakutani Fixed-Point Theorem one can show that a fixed point exists. □

Proof of Corollary 2: For the precision,

$$\begin{aligned} \mathbb{E}[\rho_i(\lambda)|\lambda] &= c + \frac{c\lambda}{c+\lambda} \mathbb{E}_\lambda n_i \\ &= c + \frac{c\sqrt{\lambda}}{c+\lambda} N \end{aligned}$$

The first equality uses (8) for the expression of $\rho_i(\lambda)$. The second equality comes from (9).

If $c \leq 1$, then the precision is decreasing in λ after 1. So it is smaller at λ^* than at 1. If $c \geq \sqrt{\frac{N}{N-\alpha}} > 1$, then the precision is no larger at 1 than at $\left(1 - \frac{\alpha}{N}\right)^{-1}$, which is the upper bound of λ^*. Noticing that the precision is also quasi-concave in λ, we see it is smaller at 1 than at λ^*. □

Proof of Lemma 3: Using the definition of conditional probability, we have

$$\Pr(x_i = h, \mathbf{y}_i) = \Pr(\mathbf{y}_i | x_i = h) \Pr(x_i = h)$$

The prior $\Pr(x_i = h) = 1/2$ by our assumption. The likelihood $\Pr(\mathbf{y}_i | x_i = h)$ has three parts: (i) the probability that i is friends with those whose signals are collected in $\mathbf{y}_i$ and these friends have the signals as collected in $\mathbf{y}_i$, (ii) the probability that i is not friends with anyone outside $\mathbf{y}_i$, (iii) the probability that i's own signal is as that collected in $\mathbf{y}_i$. Formally,

$$\begin{aligned}\Pr(\mathbf{y}_i|x_i=h) &= \sum_{\mathbf{x}_i}\left[\prod_{ij\in G}\nu\left(\mathbf{1}_{\{x_j=h\}}+p_1\mathbf{1}_{\{x_j=\ell\}}\right)\prod_{ij\in G}\Pr(y_j|x_j)\right]\times \\ &\quad \prod_{ij\notin G,j\neq i}\left(1-\frac{1}{2}\nu(1+p_1)\right)\times\Pr(y_i|x_i=h)\times\frac{1}{2}\end{aligned}$$

where $\sum_{\mathbf{x}_i}$ is the summation across all possible vectors of friends' types, which contains 2^{n_i} items.

Another way of expressing the probability $\Pr(x_i=h,\mathbf{y}_i)$ is

$$\begin{aligned}\Pr(x_i=h,\mathbf{y}_i) &= \prod_{ij\in G}\nu\left[\Pr(y_j|x_j=h)+p_1\Pr(y_j|x_j=\ell)\right]\times \\ &\quad \prod_{ij\notin G,j\neq i}\left(1-\frac{1}{2}\nu(1+p_1)\right)\times\Pr(y_i|x_i=h)\times\frac{1}{2} \qquad \text{(A.13)}\end{aligned}$$

Similarly we can find the corresponding expression for $\Pr(x_i=\ell,\mathbf{y}_i)$.

$$\begin{aligned}\Pr(x_i=\ell,\mathbf{y}_i) &= \prod_{ij\in G}\nu\left[p_1\Pr(y_j|x_j=h)+\Pr(y_j|x_j=\ell)\right]\times \\ &\quad \prod_{ij\notin G,j\neq i}\left(1-\frac{1}{2}\nu(1+p_1)\right)\times\Pr(y_i|x_i=\ell)\times\frac{1}{2}\end{aligned}$$

Hence we may compute the following ratio:

$$\frac{\Pr(x_i=\ell|\mathbf{y}_i)}{\Pr(x_i=h|\mathbf{y}_i)}=\left(\frac{\varepsilon}{1-\varepsilon}\right)^{y_i}\left(\frac{\varepsilon p_1+1-\varepsilon}{\varepsilon+p_1-\varepsilon p_1}\right)^{L_i}\left(\frac{p_1-\varepsilon p_1+\varepsilon}{1-\varepsilon+\varepsilon p_1}\right)^{H_i}$$

This ratio, together with $\Pr(x_i=\ell|\mathbf{y}_i)+\Pr(x_i=h|\mathbf{y}_i)=1$, proves the proposition. □

Proof of Lemma 4: Similar to the proof of Lemma 3, we can find the expression $\Pr(x_i=h,\mathbf{y}_i)$ by replacing p_1 in (A.13) with p_β. The expression for $\Pr(x_i=\ell,\mathbf{y}_i)$ is

$$\begin{aligned}\Pr(x_i=\ell,\mathbf{y}_i) &= \prod_{ij\in G}\nu\left[p_\beta\Pr(y_j|x_j=h)+p_\theta\Pr(y_j|x_j=\ell)\right]\times \\ &\quad \prod_{ij\notin G,j\neq i}\left(1-\frac{1}{2}\nu(p_\theta+p_\beta)\right)\times\Pr(y_i|x_i=\ell)\times\frac{1}{2}\end{aligned}$$

So the ratio becomes:

$$\frac{\Pr(x_i = \ell|\mathbf{y}_i)}{\Pr(x_i = h|\mathbf{y}_i)} = \left(\frac{1-\nu(p_\theta + p_\beta)/2}{1-\nu(1+p_\beta)/2}\right)^{S-n_i} \times \left(\frac{\varepsilon}{1-\varepsilon}\right)^{y_i} \left(\frac{\varepsilon p_\beta + p_\theta - \varepsilon p_\theta}{\varepsilon + p_\beta - \varepsilon p_\beta}\right)^{L_i} \left(\frac{p_\beta - \varepsilon p_\beta + \varepsilon p_\theta}{1-\varepsilon+\varepsilon p_\beta}\right)^{H_i}$$

Taking limits of S and ν completes the proof. □

Proof of Lemma 5: We want to study the expected log odds as a function of θ. By Lemma 4, we have,

$$\mathbb{E}\left[R_i(\theta,\beta)|\ell,\theta,\beta\right] = (1-2\varepsilon)\log\left(\frac{\varepsilon}{1-\varepsilon}\right) - \frac{1}{2}N\left[\varepsilon p_\beta + (1-\varepsilon)p_\theta\right]\log\left(\frac{\varepsilon p_\beta + (1-\varepsilon)p_\theta}{\varepsilon + (1-\varepsilon)p_\beta}\right) - \frac{1}{2}N\left[\varepsilon p_\theta + (1-\varepsilon)p_\beta\right]\log\left(\frac{\varepsilon p_\theta + (1-\varepsilon)p_\beta}{\varepsilon p_\beta + (1-\varepsilon)}\right) - \frac{1}{2}N(1-p_\theta) \tag{A.14}$$

Since $p_\theta = \int_\theta^{+\infty} f(t)dt$ where f is the density of the matching value, the derivative of the above expected log odds w.r.t. θ is

$$\begin{aligned}\frac{\partial\mathbb{E}\left[R_i(\theta,\beta)|\ell,\theta,\beta\right]}{\partial\theta} &= \frac{\partial\mathbb{E}\left[R_i(\theta,\beta)|x_i=\ell,\theta,\beta\right]}{\partial p_\theta}\cdot\frac{\partial p_\theta}{\partial\theta} \\ &= \frac{1}{2}N\left[(1-\varepsilon)\log\left(\frac{\varepsilon p_\beta + (1-\varepsilon)p_\theta}{\varepsilon + (1-\varepsilon)p_\beta}\right) + \varepsilon\log\left(\frac{\varepsilon p_\theta + (1-\varepsilon)p_\beta}{\varepsilon p_\beta + (1-\varepsilon)}\right)\right] f(\theta) \qquad \text{(A.15)}\end{aligned}$$

Note that the derivative is strictly increasing in p_θ, thus strictly decreasing in θ. By the definition we gave to $\overline{\theta}$, the derivative is zero at $\overline{\theta}(\beta)$. So we can conclude that the expected log odds as a function of θ is quasi-concave with the maximum attained at $\overline{\theta}(\beta)$. □

Proof of Lemma 6: We want to study the expected log odds as a function of β. First, the expected log odds is expressed as in (A.14). We take its derivative w.r.t. β:

$$\frac{\partial\mathbb{E}\left[R_i(\theta,\beta)|x_i=\ell,\theta,\beta\right]}{\partial\beta} = \frac{1}{2}N\left[\varepsilon\log\left(\frac{\varepsilon p_\beta + (1-\varepsilon)p_\theta}{\varepsilon + (1-\varepsilon)p_\beta}\right) + (1-\varepsilon)\log\left(\frac{\varepsilon p_\theta + (1-\varepsilon)p_\beta}{\varepsilon p_\beta + (1-\varepsilon)}\right) + \frac{\varepsilon^2 - (1-\varepsilon)^2 p_\theta}{\varepsilon + (1-\varepsilon)p_\beta} + \frac{(1-\varepsilon)^2 - \varepsilon^2 p_\theta}{\varepsilon p_\beta + (1-\varepsilon)}\right] f(\beta)$$

Since f is positive, we focus on the term within the brackets. Using the inequality $\log(t) < t-1$ except for $t = 1$, we have

$$[...] < \frac{\varepsilon^2 p_\beta + \varepsilon(1-\varepsilon)p_\theta}{\varepsilon + (1-\varepsilon)p_\beta} + \frac{\varepsilon(1-\varepsilon)p_\theta + (1-\varepsilon)^2 p_\beta}{\varepsilon p_\beta + (1-\varepsilon)} - \frac{\varepsilon(1-\varepsilon)p_\beta + (1-\varepsilon)^2 p_\theta}{\varepsilon + (1-\varepsilon)p_\beta} - \frac{\varepsilon(1-\varepsilon)p_\beta - \varepsilon^2 p_\theta}{\varepsilon p_\beta + (1-\varepsilon)}$$

Notice that the right side is (linearly) decreasing in p_θ. Recall the condition of the Lemma is $\theta < \beta$ which implies $p_\theta > p_\beta$. Hence we may replace p_θ by p_β on the right hand side of the above inequality,

$$[...] < \left[\frac{\varepsilon^2 - (1-\varepsilon)^2}{\varepsilon + (1-\varepsilon)p_\beta} + \frac{(1-\varepsilon)^2 - \varepsilon^2}{\varepsilon p_\beta + (1-\varepsilon)}\right] p_\beta$$

which is smaller than zero because the denominator in the first term is smaller than that of the second. □

Proof of Proposition 4: For notational simplicity, we omit the (θ, β) in the conditional of any expectation operator. We also use R_i short for $R_i(\theta, \beta)$.

We start with a low type person. With $P_i(\theta, \beta)$ given by (15), we can easily write down i's expected credit score for any $\theta_i \geq \theta$:

$$\begin{aligned}\mathbb{E}(R_i|\ell, \theta_i) \quad = \quad & (1-2\varepsilon)\log\left(\frac{\varepsilon}{1-\varepsilon}\right) - \frac{1}{2}N\left[\varepsilon p_\beta + (1-\varepsilon)\int_{\theta_i}^{+\infty} f(t)dt\right]\log\left(\frac{\varepsilon p_\beta + (1-\varepsilon)p_\theta}{\varepsilon + (1-\varepsilon)p_\beta}\right) - \\ & \frac{1}{2}N\left[\varepsilon\int_{\theta_i}^{+\infty} f(t)dt + (1-\varepsilon)p_\beta\right]\log\left(\frac{\varepsilon p_\theta + (1-\varepsilon)p_\beta}{\varepsilon p_\beta + (1-\varepsilon)}\right) - \frac{1}{2}N(1-p_\theta)\end{aligned}$$

Thus for any $\theta_i \geq \theta$,

$$\frac{\partial \mathbb{E}(R_i|\ell, \theta_i)}{\partial \theta_i} = \frac{1}{2}N\left[(1-\varepsilon)\log\left(\frac{\varepsilon p_\beta + (1-\varepsilon)p_\theta}{\varepsilon + (1-\varepsilon)p_\beta}\right) + \varepsilon\log\left(\frac{\varepsilon p_\theta + (1-\varepsilon)p_\beta}{\varepsilon p_\beta + (1-\varepsilon)}\right)\right] f(\theta_i) \qquad \text{(A.16)}$$

Next for the social utility $\mathbb{E}\left(\sum_{ij\in G} m_{ij} - 1_{\{x_j=h\}}|\ell, \theta_i\right)$, which we will use $\mathbb{E}(u_i|\ell, \theta_i)$ as a shorthand for, we have for any $\theta_i \geq \theta$,

$$\mathbb{E}(u_i|\ell, \theta_i) = \frac{1}{2}N\left(\int_\beta^{+\infty}(t-1)f(t)dt + \int_{\theta_i}^{+\infty} tf(t)dt\right)$$

Thus for any $\theta_i \geq \theta$,

$$\frac{\partial \mathbb{E}(u_i|\ell, \theta_i)}{\partial \theta_i} = -\frac{1}{2}N\theta_i f(\theta_i) \qquad \text{(A.17)}$$

First let us show that $\theta^* > 0$. Consider the case where every low type chooses $\theta = 0$. It is easy to see that at $\frac{\partial \mathbb{E}(u_i|\ell,\theta_i)}{\partial \theta_i} = 0$ but $\frac{\partial \mathbb{E}(R_i|\ell,\theta_i)}{\partial \theta_i} > 0$ at $\theta_i = 0$ for any $\beta \geq 1$. Hence $\frac{\partial \mathbb{E}(U_i|\ell,\theta_i)}{\partial \theta_i} > 0$ and the low type wants to increase θ_i above 0 and be more exclusive towards his fellows. This incentive to deviate means $\theta = 0$ cannot be part of an equilibrium.

Second, to show that $\theta^* < \overline{\theta}(\beta^*)$, we use our refinement. Suppose (θ^*, β^*) is an equilibrium where $\theta^* \geq \overline{\theta}(\beta^*)$. Now consider a behavior θ^{**} that is smaller than but sufficiently close to $\overline{\theta}(\beta^*)$ for the low type. Every low type will be better off in (θ^{**}, β^*) than in (θ^*, β^*), because (i) by Lemma 5, we know that $\mathbb{E}(R_i|\ell)$ is quasi-concave in θ and differentiably maximized at $\overline{\theta}(\beta)$, (ii) the social utility

$$\mathbb{E}(u_i|\ell) = \frac{1}{2}N\left(\int_\beta^{+\infty}(t-1)f(t)dt + \int_\theta^{+\infty} tf(t)dt\right)$$

is strictly decreasing in θ:

$$\frac{\partial \mathbb{E}(u_i|\ell)}{\partial \theta} = -\frac{1}{2}N\theta f(\theta) \tag{A.18}$$

Further, given that every other low type chooses $\theta = \theta^{**}$ and every high type chooses $\beta = \beta^*$, a low type i has no incentive to increase his criterion θ_i beyond θ^{**} because (i) by (A.17) $\frac{\partial \mathbb{E}(R_i|\ell,\theta_i)}{\partial \theta_i} < 0$ and, (ii) by (A.16) and our choice of θ^{**}, $\frac{\partial \mathbb{E}(u_i|\ell,\theta_i)}{\partial \theta_i} < 0$ so. Nor does he have incentive to lower the criterion, because doing so changes nothing (friendship must be mutual). We conclude that (θ^*, β^*) fails the refinement.

Lastly, we turn our attention to the high types. We want to show that $\beta = 1$ cannot be part of an equilibrium. The argument is similar to that for the low types. Briefly, consider a symmetric profile $(\theta, \beta = 1)$. To be an equilibrium, it must be that $\theta < \overline{\theta}(1)$. This will imply that $\frac{\partial \mathbb{E}(R_i|h,\beta_i)}{\partial \beta_i} > 0$ at $\beta_i = 1$. But $\frac{\partial \mathbb{E}(u_i|h,\beta_i)}{\partial \beta_i} = 0$ at $\beta_i = 1$. Hence a high type wants to raise her β_i above 1. This incentive to deviate means that $\beta = 1$ cannot be part of an equilibrium. □

Proof of Proposition 5: Let Θ be the set of equilibria without refinement. Given that $\alpha_h = 0$ and $\beta = 1$, we are effectively looking for the point(s) in Θ that maximizes the expected total utility of low types.

Using (A.15) with $\beta = 1$, we see that $\frac{\mathbb{E}(R_i|\ell)}{\partial \theta}/f(\theta)$ is strictly decreasing in θ and equals 0 at $\theta = \overline{\theta}$. Using (A.18), we see $\frac{\mathbb{E}(u_i|\ell)}{\partial \theta}/f(\theta)$ is strictly decreasing in θ and equals 0 at $\theta = 0$. These imply that

$$\frac{\partial \mathbb{E}(u_i|\ell)}{\partial \theta} + \alpha_\ell \frac{\partial \mathbb{E}(R_i|\ell)}{\partial \theta}$$

is strictly decreasing and has a single point within $(0, \overline{\theta})$ where it is zero. It is also where $\mathbb{E}(U_i|\ell)$ is maximized. Denote this point by θ^*. We would be done if θ^* is shown to be an equilibrium without refinement. By comparing (A.15) with (A.16) and (A.18) with (A.17), it is not difficult to see that at $\theta_i = \theta^*$ and $\theta_i \geq \theta^*$,

$$\frac{\partial \mathbb{E}(u_i|\ell,\theta_i)}{\partial \theta_i} + \alpha_\ell \frac{\partial \mathbb{E}(R_i|\ell,\theta_i)}{\partial \theta_i} \leq 0$$

and the inequality is strict for $\theta_i > \theta^*$. This implies that a low type i has no incentive to deviate if every other low type chooses θ^*. □

Proof of Lemma 7: Mathematically it is really a special case of Lemma 4, with $p_\beta = p_\theta = p_1$. □

Proof of Proposition 6: The arguments closely resemble those of the proof of Proposition 4. Here we discuss them briefly.

First, consider a candidate profile (θ, β) with $\theta \leq \overline{\theta}(\beta)$. One can show that a low type has incentive to increase his criterion because doing so increases both his social utility and credit score. So it cannot be an equilibrium.

Second, we filter $\theta^* = 1$ with refinement. Suppose that (θ^*, β^*) is an equilibrium. Compare it with (θ^{**}, β^*) where θ^{**} is smaller but sufficiently close to 1. One can show that low types are better off under θ^{**} and no single low type has incentive to increase criterion under θ^{**}.

Lastly, consider a candidate profile (θ, β) with $\beta = 1$. From the above we know for it to be an equilibrium it must be that $\theta > \overline{\theta}(\beta)$, which says that removal of one low type friend strictly increases one's expected credit score. Hence a high type has incentive to raise her criterion above 1. So it cannot be an equilibrium. □

Proof of Proposition 7: First we look at the case of discrimination. Taking first-order condition w.r.t. e_i, we have for each i:

$$e_i^* = ab^{-1} + \phi(H_i + \sum_{ij \in G, x_i = \ell} e_j^*)$$

which we may write in the matrix form:

$$\mathbf{e}^* = ab^{-1} + \phi \mathbf{H} + \phi \mathbf{A}_\ell \mathbf{e}^*$$

This implies

$$(\mathbf{I} - \phi \mathbf{A}_\ell)\, \mathbf{e}^* = ab^{-1} + \phi \mathbf{H}$$

By Perron-Frobenius Theorem, the largest-magnitude eigenvalue of $\mathbf{A}_\ell$ is real and positive. Furthermore, if this eigenvalue is smaller than $|\phi|^{-1}$, then $\|\phi \mathbf{A}\| < 1$, which implies that the series $\sum_{k=0}^{\infty} \phi^k \mathbf{A}_\ell^k$ exists. One can readily check that the series is the inverse of $(\mathbf{I} - \delta \mathbf{A}_\ell)$.

The case of homophily can be proved similarly. In particular, the first-order condition is

$$e_i^* = (H_i - L_i)b^{-1} + 2b^{-1} \sum_{ij \in G, x_i = \ell} e_j^* + ab^{-1} + \phi(H_i + \sum_{ij \in G, x_i = \ell} e_j^*)$$

One can again write it into matrix form and solve for $\mathbf{e}^*$.

□

Proof of Lemma 8: Using

$$\begin{aligned} \Pr(x_i = h | \mathbf{y}_i) &= \prod_{ij \in G} \nu \left[q_s \Pr(y_j | x_j = h) + q_d \Pr(y_j | x_j = \ell) \right] \times \\ & \quad \prod_{ij \notin G, j \neq i} \left(1 - \frac{1}{2} \nu (q_s + q_d) \right) \times \Pr(y_i | x_i = h) \times \frac{1}{2} \end{aligned}$$

$$\begin{aligned}\Pr(x_i = \ell|\mathbf{y}_i) &= \prod_{ij\in G} \nu\left[q_d \Pr(y_j|x_j = h) + q_s \Pr(y_j|x_j = \ell)\right] \times \\ &\quad \prod_{ij\notin G, j\neq i} \left(1 - \frac{1}{2}\nu(q_s + q_d)\right) \times \Pr(y_i|x_i = \ell) \times \frac{1}{2}\end{aligned}$$

So the ratio is:

$$\frac{\Pr(x_i = \ell|\mathbf{y}_i)}{\Pr(x_i = h|\mathbf{y}_i)} = \left(\frac{\varepsilon}{1-\varepsilon}\right)^{y_i} \left(\frac{\varepsilon q_d + q_s - \varepsilon q_s}{\varepsilon q_s + q_d - \varepsilon q_d}\right)^{L_i} \left(\frac{q_d - \varepsilon q_d + \varepsilon q_s}{q_s - \varepsilon q_s + \varepsilon q_d}\right)^{H_i}$$

which, together with $\Pr(x_i = \ell|\mathbf{y}_i) + \Pr(x_i = \ell|\mathbf{y}_i) = 1$, gives us the result. □

Proof of Proposition 8: Again for notational simplicity, all expectation operators in this proof are conditional on the symmetric profile (e_ℓ, e_h). So for example, $\mathbb{E}(U_i|\ell, e_i)$ actually refers to $\mathbb{E}(U_i|\ell, e_\ell, e_h, e_i)$ which is the expected utility of a low type when he chooses e_i while everyone else follows (e_ℓ, e_h).

Given (13), we see that for any individual i, an additional high type friend increases (and an additional low type friend decreases) the expected utility from credit score by

$$D_{x_i} = \alpha_{x_i}\left[\varepsilon \log\left(\frac{\varepsilon + (1-\varepsilon)p_1}{\varepsilon p_1 + (1-\varepsilon)}\right) + (1-\varepsilon)\log\left(\frac{\varepsilon p_1 + (1-\varepsilon)}{\varepsilon + (1-\varepsilon)p_1}\right)\right]$$

It is not difficult to see that the friendship formation criteria will be: a low type accepts another low type iff $m_{ij} > D_\ell$, a high type accepts a low type iff $m_{ij} > 1 + D_h$, a low type accepts a high type iff $m_{ij} > 1 - D_\ell$, and a high type accepts another high type iff $m_{ij} > -D_h$ (which always holds). Using these criteria and the requirement that a tie is formed upon mutual acceptance, the expected utility of a low type when he chooses e_i is

$$\mathbb{E}(U_i|\ell, e_i) = (e_i + e_\ell)M\int_{D_\ell}^{\infty}(t - D_\ell)f(t)dt + (e_i + e_h)M\int_{1+D_h}^{\infty}(t - 1 + D_\ell)f(t)dt,$$

and the expected utility of a high type when she chooses e_i is

$$\mathbb{E}(U_i|h, e_i) = (e_i + e_\ell)M\int_{1+D_h}^{\infty}(t - 1 - D_h)f(t)dt + (e_i + e_h)M\int_{0}^{\infty}(t + D_h)f(t)dt.$$

Using first-order conditions, we have, in an equilibrium,

$$e_\ell^* = M\int_{D_\ell}^{\infty}(t - D_\ell)f(t)dt + M\int_{1+D_h}^{\infty}(t - 1 + D_\ell)f(t)dt. \tag{A.19}$$

and

$$e_h^* = M\int_{1+D_h}^{\infty}(t - 1 - D_h)f(t)dt + M\int_{0}^{\infty}(t + D_h)f(t)dt. \tag{A.20}$$

Comparing (A.19), (A.20) and (24), one can verify that: (i) $e^*_\ell < e^* < e^*_h$ for $\alpha_\ell = \alpha_h > 0$, (ii) $e^*_\ell > e^*_h$ iff

$$\int_{1+D_h}^{D_\ell} (D_\ell - t) f(t) dt > \int_0^{1+D_h} D_h f(t) dt + \int_0^{1+D_h} t f(t) dt$$

which holds for D_ℓ sufficiently large. $\square$

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Web Appendix

Justification of the functional form of credit score utility (Section 5)

This appendix sketches a micro-model that justifies how equation (18) can be derived from the models and results of Section 4.

We assume the following two-stage game structure: Before the game starts, a network has been exogenously formed. After the network is formed, the use of social network data in credit scoring is suddenly announced. In stage 1 of the game, low type individuals are given the opportunity to exert effort to improve their types. If a low type exerts effort e, then with probability e she will become a high type. After individuals make effort decisions, in stage 2 new types are revealed, individuals are given the opportunity to drop friendships, and credit score utility is realized on the basis of the final network. Based on the general results of Section 4, we assume that high types who care about their credit score, will drop low type friends with some probability p_{HL} and low types will, similarly, drop other low types with probability p_{LL}. The specific values and relationships between p_{HL} and p_{LL} are not crucial for the analysis that follows, so these assumptions are very general.

Recall that the general form of credit utility $R_i = -\alpha log(\frac{1-P_i}{P_i})$, $P_i = (1 + k_0 k_1^{y_i} k_2^{h_i} k_3^{l_i})^{-1}$ that is implied by the general functional form of results of Section 4 is:

$$R_i = K_0 + y_i K_1 + h_i K_2 + l_i K_3$$

where $y_i \in \{-1, 1\}$, K_i are (positive or negative) constants, and h_i, l_i denote the number of friends with high and low signals. If we assume that the actual numbers of individual i's high and low friends in stage 1 are H_i, L_i respectively, and that ℓ-type friend j exerts effort e_j, the expected numbers of individual i's high and low type friends in stage 2 (before any friendship ties are dropped) are $H_i + \sum_{ij \in G_\ell} e_j$ and $L_i - \sum_{ij \in G_\ell} e_j$ respectively. Under these assumptions, individual i's expected stage 2 credit score utility is approximately[14] given by:

$$u_{credit} = \begin{cases} \\ K_0 + K_1 + (H_i + \sum_{ij \in G_\ell} e_j) K_2 + (L_i - \sum_{ij \in G_\ell} e_j) p_{HL} K_3 & \text{if } i \text{ becomes high type in stage 2} \\ K_0 - K_1 + (H_i + \sum_{ij \in G_\ell} e_j) p_{HL} K_2 + (L_i - \sum_{ij \in G_\ell} e_j) p_{LL} K_3 & \text{if } i \text{ remains low type in stage 2} \end{cases}$$

Therefore, low type individual i who is pondering what level of effort to exert, is faced with expected gains

[14] Signals are noisy and, thus, the number of friends with high and low signals is not always the same as the actual number of friends with high and low types. For the sake of simplicity, we assume here that the expected number of friends with high and low signals in stage 2 is approximately equal to the expected number of friends with high and low types in stage 2 respectively.

of:

$$u_i = e_i \left[K_0 + K_1 + (H_i + \textstyle\sum_{ij\in G_\ell} e_j)K_2 + (L_i - \textstyle\sum_{ij\in G_\ell} e_j)p_{HL}K_3\right]$$
$$+(1-e_i)\left[K_0 - K_1 + (H_i + \textstyle\sum_{ij\in G_\ell} e_j)p_{HL}K_2 + (L_i - \textstyle\sum_{ij\in G_\ell} e_j)p_{LL}K_3\right]$$

which can be rewritten as:

$$u_i = \left[K_0 - K_1 + (H_i + \textstyle\sum_{ij\in G_\ell} e_j)K_2 + (L_i - \textstyle\sum_{ij\in G_\ell} e_j)p_{HL}K_3\right]$$
$$+e_i\left[2K_1 + H_i(1-p_{HL})K_2 + L_i(p_{HL}-p_{LL})K_3\right.$$
$$\left.+\left(\textstyle\sum_{ij\in G_\ell} e_j\right)[(1-p_{HL})K_2 - (p_{HL}-p_{LL})K_3]\right]$$

The returns to own effort, thus, have:

1. a term $\left[K_0 - K_1 + (H_i + \sum_{ij\in G_\ell} e_j)K_2 + (L_i - \sum_{ij\in G_\ell} e_j)p_{HL}K_3\right]$ that does not depend on one's own effort, which we may omit.
2. a second term $e_i\left[2K_1 + H_i(1-p_{HL})K_2 + L_i(p_{HL}-p_{LL})K_3\right]$ that multiplies one's own effort by an expression that has the general form $a + c_1 H_i + c_2 L_i$.
3. a third term $e_i\left(\sum_{ij\in G_\ell} e_j\right)[(1-p_{HL})K_2 - (p_{HL}-p_{LL})K_3]$ that interacts own effort e_i with the expected sum $\left(\sum_{ij\in G_\ell} e_j\right)$ of every ℓ-type friend's effort, times a constant.

If we assume a quadratic cost of effort $\frac{be_i^2}{2}$, the above analysis sketch justifies the form of equation (18) used in the text:

$$u_i = (a + c_1 H_i + c_2 L_i)\, e_i + b\phi\left(\sum_{x_j=\ell} e_j\right) e_i - \frac{be_i^2}{2}$$

Technical Note about Outside Options

This technical note addresses the possibility of individuals choosing to remain outside of the network. Let's assume that a friendship network is formed exogenously based on social utility as in our baseline discrete-type model. The probability of same-type tie after meeting is 1 while the probability of different-type tie after meeting is $p_1 < 1$.

Given the network, each individual may opt in social-network-based scoring system or opt out. If she opts out, the lender only observes her individual signal (y_i), otherwise the lender sees the signals on her friends as well.

To simplify the analysis, assume that the individual observes her own signal and her friends' signals before making the in-or-out decision. So individual's in-or-out decision is a function of her own signal and signals of her friends: $o_i = \widehat{o}(y_i, H_i, L_i) \in \{1, 0\}$. Strictly speaking, o_i could be a function of her own type as well. But since the lender's posterior only depends on the signals, there is no reason for a high type and low type to choose differently if they have the same signal and signals of friends. This point will become clearer as we go deeper into the analysis.

If the individual opts in, then lender's posterior is

$$P_i = \Pr(x_i = h|\mathbf{y}_i, o_i = 1)$$

Since o_i is a function of $\mathbf{y}_i$, $P_i = \Pr(x_i = h|\mathbf{y}_i)$, which we have derived in our baseline discrete-type model,

$$P_i = \left[1 + \left(\frac{\varepsilon}{1-\varepsilon}\right)^{y_i} \left(\frac{\varepsilon + (1-\varepsilon)p_1}{\varepsilon p_1 + (1-\varepsilon)}\right)^{H_i - L_i}\right]^{-1}. \tag{A.21}$$

If she opts out, the lender's posterior for her would be

$$P_i = \Pr(x_i = h|y_i, o_i = 0)$$

which depends on $y_i \in \{1, -1\}$.

An equilibrium is some individual behavior rule $\widehat{o}$ such that (i) lender's posterior is consistent with the behavior, (ii) the behavior is optimal for the individuals.

Proposition 9. *There is no equilibrium in which some people opt out.*

The intuition for the Proposition 9 is in "The Market for Lemons" (Akerlof 1979). The lender treats someone that opts out as an average in the pool of the people that opt out. The person that is above that average in the pool thus finds it undesirable to opt out and subsequently be treated so. This leads to a "market failure" where no one likes to opt out, and the lender puts the worst possible belief (i.e., $P_i = 0$) on the off-equilibrium action of opting out.

Proof. If an individual opts in, she will receive a score given by (A.21) which depends on both her own signal and the relative number of friends she has with high signals. If she opts out, she will receive a score which only depends on her own signal. So it is easy to see that $\widehat{o}$ should be a cut-off rule. In particular, for someone with $y_i = 1$,

$$\widehat{o}(y_i = 1, H_i, L_i) = 1 \text{ iff } H_i - L_i \geq \Delta$$

for some Δ. Suppose that Δ is finite. In other words, both $\{y_i = 1, o_i = 0\}$ and $\{y_i = 1, o_i = 1\}$ happen for nonzero proportions of the society. We will show this cannot be the case.

Let us consider someone with $y_i = 1$ and on the edge of opting out: $H_i - L_i = \Delta$, it must be that her opt-in score is no less than her opt-out score:

$$\begin{aligned}\left[1 + \left(\frac{\varepsilon}{1-\varepsilon}\right)^{y_i} \left(\frac{\varepsilon + (1-\varepsilon)p_1}{\varepsilon p_1 + (1-\varepsilon)}\right)^{\Delta}\right]^{-1} &\geq \Pr(x_i = h|y_i = 1, o_i = 0) \\ &= \Pr(x_i = h|y_i = 1, H_i - L_i < \Delta)\end{aligned} \tag{A.22}$$

Noticing that

$$\Pr(x_i = h | y_i = 1, H_i - L_i = \mathit{\Delta}) = \left[1 + \left(\frac{\varepsilon}{1-\varepsilon}\right)^{y_i} \left(\frac{\varepsilon + (1-\varepsilon)p_1}{\varepsilon p_1 + (1-\varepsilon)}\right)^{\mathit{\Delta}}\right]^{-1}$$

and because $\mathit{\Delta}$ is finite,

$$\Pr(x_i = h | y_i = 1, H_i - L_i = \mathit{\Delta}) > \Pr(x_i = h | y_i = 1, H_i - L_i < \mathit{\Delta})$$

So we must have the exact opposite of (A.22). By contradiction, it must be that $\mathit{\Delta}$ is infinite. If $\mathit{\Delta}$ is $+\infty$, that is, everyone opts out, then the opt-out score $\Pr(x_i = h | y_i = 1, o_i = 0) = \Pr(x_i = h | y_i = 1)$, which equals $1 - \varepsilon$. However, in this case an individual with enough many friends with high signals would like to opt in and get a score given by (A.21). The only possible equilibrium left is then $\mathit{\Delta} = -\infty$, everyone opts in. The equilibrium is easily supported by the lender putting the worst belief on opting out, which is off equilibrium path.

A similar argument applies to someone with $y_i = 0$. To conclude, there is no equilibrium in which someone opts out and everyone opts in. □

Reference:

Akerlof, George A. "The Market for Lemons: Quality uncertainty and the market mechanism." The Quarterly Journal of Economics 84.3 (1970): 488-500.