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CAPITAL ACCUMULATION AND UNCERTAIN LIFETIMES WITH ADVERSE SELECTION

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ABSTRACT

This paper examines the implications of adverse selection in the private annuity market for the pricing of private annuities and the consequent effects on consumption and bequest behavior. With privately known heterogeneous mortality probabilities, adverse selection causes the rate of return on private annuities to be less than the actuarially fair rate based on population average mortality. However, a fully funded social security system with compulsory participation can offer an implied rate of return equal to the actuarially fair rate based on population average mortality. Thus, since social security offers a higher rate of return than private annuities, consumers cannot completely offset the effects of social security by transacting in the private annuity market. Using an overlapping generations model with uncertain lifetimes, we demonstrate that the introduction of actuarially fair social security reduces the steady state rate of return on annuities and raises the steady state levels of average bequests and average consumption of the young. The steady state national capital stock rises or falls according to the strength of the bequest motive.

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Uncertainty about an individual's date of death affects the individual's consumption and portfolio behavior as well as the bequest ultimately left to the consumer's heirs. The early literature on lifetime uncertainty[1] focused on the effects of stochastic lifetimes on individual consumption and portfolio behavior, ignoring the effects on the bequests received by subsequent generations.[2] Much recent attention has been devoted to the effects of stochastic lifetimes on bequests and the implications for the distribution of wealth and the evolution of the capital stock. Sheshinski and Weiss (1981) extended the Modigliani-Brumberg (1954) - Samuelson (1958) - Diamond (1965) overlapping generations model to include uncertain lifetimes. They assumed that all consumers are identical and, furthermore, that all consumers in a given cohort die at the same date, thereby leaving identical bequests. However, if consumers die at different dates, then they will, in general, leave bequests of different sizes. Abel (1985) and Eckstein, Eichenbaum and Peled (1985) exploited the intra-cohort variation in ex post mortality experiences to analyze the steady state distributions of bequests, consumption and wealth in models without private annuity markets and with consumers without bequest motives.[3] Abel (1985) also shows that the introduction of fully funded social security crowds out steady state private wealth by more than one-for-one and that it reduces all central moments of the steady state distribution of wealth.

The effects of social security in the presence of uncertain lifetimes have been studied by Sheshinski and Weiss (1981) and Abel (1985) in models in which there is no private annuity market. However, if a competitive annuity market were introduced into these models, social security would then have no effect because the rate of return on private annuities would be the same as the rate of return implicit in actuarially fair fully funded social security; thus consumers would exactly offset the effects of social security by adjusting their purchases of private annuities. In this paper we introduce a private market for annuities and demonstrate that with privately-known heterogenous mortality probabilities, social security does have real effects on the allocation of consumption. The reason is that adverse selection drives the rate of return on competitively supplied annuities below the actuarially fair rate of return based on the population average ex ante mortality probability; however, because the social security system is compulsory, it is immune to adverse selection and a fully funded system can offer a rate of return equal to the actuarially fair rate based on population average mortality.

Eckstein, Eichenbaum and Peled (1985b) examine the welfare-enhancing role of mandatory social security when the private annuity market is subject to adverse selection.[4] However, there are two features of their model which make it unsuitable for our purposes. First, because they assume that consumers have no bequest motive, the availability of annuities implies, as noted by Yaari (1965), that consumers will hold all of their savings as annuities, and hence there will be no private intergenerational transfers in the form of bequests. Second, because the consumption good is a non-producible, non-storable endowment, aggregate savings is zero in every period; the saving of the young is exactly offset by the dissaving of the old. In contrast, in the model presented below, the specification of the utility function with a bequest motive introduces a non-trivial portfolio allocation problem and leads to intergenerational transfers in the form of bequests. Secondly, in the model below, the consumption good can be invested at a rate of return R so that aggregate saving need not be zero. Thus, this model can be used to analyze the effects of social security on capital accumulation.

In section I, we examine the optimal consumption and portfolio behavior of an individual consumer, taking as given the rate of return on private annuities and the consumer's inheritance received from his parent. Using the derived demands for private annuities by consumers with different mortality probabilities, we study, in section II, the determination of the rate of return on private annuities. In section III, we analyze the steady state effects of introducing actuarially fair fully funded social security. We show that the introduction of fully funded social security leads to an increase in the steady state consumption of young consumers, an increase in the steady state level of bequests, and to a reduction in the rate of return on private annuities. Finally, we show that depending on strength of the bequest motive, an actuarially fair increase in social security taxes will crowd out private capital by greater than or less than one-for-one.

1. Consumption and Portfolio Behavior of an Individual

Consider a consumer who may live either one period (with probability $p>0$) or two periods (with probability $1-p>0$). Let I be the initial wealth held by the consumer at the beginning of his life. (The determination of I will be discussed below.) During the first period of life the consumer earns a fixed labor income Y, pays a social security tax T, and consumes an amount c_1. At the end of the first period, the consumer chooses a portfolio of annuities and riskless bonds. Let Q be the amount of annuities held in the portfolio; the remainder of the portfolio, $I + Y - T - c_1 - Q$, is held in the form of riskless bonds. A one dollar annuity pays A dollars to the consumer in the following period if he survives; if the consumer dies young, the annuity pays nothing to his heir. A one dollar riskless bond yields R dollars in the following period to the consumer, if he survives, or to his heir, if the consumer dies young. As shown in section II, A>R in a competitive annuity market.

At the beginning of the second period, the consumer gives birth to an heir and then the uncertainty about the length of the consumer's life is resolved. If the consumer dies at the beginning of the second period, his heir receives a bequest, B^D, consisting of the consumer's riskless bonds with accrued interest,

$$B^D = (I + Y - T - c_1 - Q)R \tag{1}$$

If the consumer survives to the end of the second period, he receives a social security payment S, consumes an amount c_2, and gives the remainder of his wealth, B^s, to his heir. Since all uncertainty is resolved at the beginning of the second period, the consumer who lives for two periods knows at the beginning of the second period that he will leave a bequest of B^s where

$$B^s = (I + Y - T - c_1 - Q)R + QA + S - c_2 \tag{2}$$

We assume that the consumer who survives gives his heir the bequest B^s at the beginning of the second period. Thus, regardless of whether the consumer lives one period or two periods, the intergenerational transfer from the consumer to his heir takes place at the beginning of the second period i.e., at the beginning of the first period of the heir's life.

Let the consumer's utility function be

$$U(c_1) + p\delta V(B^D) + (1-p)\delta U(c_2) + (1-p)\delta V(B^S) \tag{3}$$

where δ ($0<\delta\leq 1$) is the one-period discount factor, U( ) is a strictly concave utility index of the consumer's own consumption and V( ) is a strictly concave index of utility derived from leaving a bequest. The utility function in (1) is simply the expected value of utility, where the only stochastic element is the consumer's date of death.[5]

The consumer maximizes the utility function in (3) subject to his lifetime budget constraint. The lifetime budget constraint is obtained by first substituting (1) into (2) to obtain

$$B^s = B^D + QA + S - c_2 \tag{4}$$

Then combining (1) and (4) to eliminate Q yields

$$B^s = A(I + Y - T) + S - Ac_1 - c_2 - \left(\frac{A-R}{R}\right) B^D \tag{5}$$

Substituting the lifetime budget constraint (5) into the utility function (3) and differentiating with respect to c_1, c_2, and B^D, respectively, yields

$$U'(c_1) = (1-p)\delta A\, V'(B^s) \tag{6a}$$

$$U'(c_2) = V'(B^s) \tag{6b}$$

$$pV'(B^D) = (1-p)\, \frac{A-R}{R}\, V'(B^s) \tag{6c}$$

Annuities are said to be actuarially fair if the expected rate of return on an annuity, $(1-p)A$, is equal to the rate of return on riskless bonds, R. Note that actuarial fairness implies that $\frac{A-R}{R} = \frac{p}{1-p}$ so that from (6c) we obtain $V'(B^D) = V'(B^s)$ and hence $B^D = B^s$. Furthermore, since $B^D = B^s$, it follows from (4) that $c_2 = QA + S$.[6] Thus, if the rate of return on private annuities is actuarially fair, the consumer's portfolio consists of: (1) riskless bonds which will be given to his heir as a bequest; and (2) annuities which, along with the social security payment S, will be used to provide for second-period consumption.

If the expected rate of return on annuities is smaller than the riskless rate of return R, i.e., $A < \frac{R}{1-p}$ then it follows from (6c) that $V'(B^D) < V'(B^s)$ so that $B^D > B^s$ and (from (4)) $c_2 > QA + S$. In this case the consumer does not use annuities to provide for all of second-period consumption; some of second-period consumption is provided for by riskless bonds which have a higher expected rate of return than annuities.[7]

In order to obtain explicit solutions for the optimal levels of consumption and bequests, we assume that $U(c)=\frac{c^{1-\sigma}-1}{1-\sigma}$ and $V(B)=\lambda\frac{B^{1-\sigma}-1}{1-\sigma}$, as in Hakansson (1969), Fischer (1973) and Richard (1975), where $\lambda \geq 0$ indicates the strength of the bequest motive and $\sigma > 0$. Therefore the utility function in (3) is homothetic, and the income expansion path for c_1, c_2, B^D and B^s is a ray through the origin. The optimal value of each of these variables, as well as the demand for contingent second-period income QA+S, is proportional to the expected present value of lifetime resources $I + Y - T + A^{-1}S$. It is shown in Appendix A that

$$c_1\ (p,A) = \phi(p,A)\ (I + Y - T + A^{-1}S) \tag{7}$$

where

$$0 < \phi(p,A) < 1$$

and

$$Q(p,A) + A^{-1}S = q_1(p,A)(I + Y - T + A^{-1}S) \tag{8}$$

where

$$q_1(p,A) < 1.$$

Explicit expressions for $\phi(p,A)$ and $q_1(p,A)$ are presented in Appendix A. It can be shown that $\partial q_1/\partial p < 0$. Also, $q_1(p,A)$ will be positive if and only if

$$A > \frac{R}{1-p}\left[(1-p) + p\left(\frac{\lambda^{\frac{1}{\sigma}}}{1+\lambda^{\frac{1}{\sigma}}}\right)^{\sigma}\right] \geq R \tag{9}$$

If $S=0$, then (9) is necessary and sufficient for a positive demand for annuities. With actuarially fair annuities ($A=\frac{R}{1-p}$), (9) is satisfied.

It is convenient to rewrite (8), the demand for private annuities as

$$Q(p,A) = q_1(p,A)(I + Y - T) - q_2(p,A)S \tag{10a}$$

where

$$q_2(p,A) = [1-q_1(p,A)]A^{-1} \tag{10b}$$

Since $q_1(p,A) < 1$ and $\partial q_1/\partial p < 0$, it follows from (10b) that $q_2(p,A) > 0$ and that $\frac{\partial q_2}{\partial p} > 0$. Therefore, if $I+Y-T > 0$ and $S \geq 0$, then

$$\frac{\partial Q}{\partial p} < 0. \tag{11}$$

II. Equilibrium in the Private Annuity Market

Suppose that consumers are characterized by different probabilities, p, of dying young. We will refer to a consumer with a probability p of dying young as a type p consumer. Except for the difference in p, all consumers have identical utility functions. Let $H(p)$ be the fraction of young consumers with probability of dying young less than or equal to p. The support of the distribution $H(p)$ is $[p^L, p^H]$, where $0 < p^L < p^H < 1$. We restrict the range of values of p in the population by assuming that that

$$\frac{\lambda}{(1-p^H)(1+\lambda^{\frac{1}{\sigma}})^{\sigma}+p^H\lambda}\, p^H < p^L < p^H \tag{12}$$

The effect of this assumption is to guarantee that condition (9) is satisfied so that if $S\geq 0$ is sufficiently small, then all consumers will have a positive demand for annuities.[8]

A consumer's probability of dying young, p, is independent of the p of his parent. Moreover, we assume that each individual knows his own value of p but that annuity companies and the government are unable to determine an individual consumer's p. We assume that there is no aggregate uncertainty: a fraction p of each cohort of type p consumers will die young. Finally, we assume that annuity companies cannot determine whether an individual consumer holds annuities from other insurance companies. The effect of this assumption is that the equilibrium in the annuity market will be a pooling equilibrium rather than a separating equilibrium.[9]

Assuming that annuity companies are risk-neutral and perfectly competitive, the expected profits of annuity companies must be equal to zero. Let M(p,A) be the expected profit per dollar of annuity with rate of return A issued to a type p consumer. Therefore

$$M(p,A) = R - (1-p)A \tag{13}$$

so that $\frac{\partial M}{\partial p} = A > 0$. It is obvious that the equilibrium rate of return on annuities, A, must lie between $\frac{R}{1-p^L}$ and $\frac{R}{1-p^H}$: if A were less than $\frac{R}{1-p^L}$, then an annuity company could offer a rate higher than A and profitably attract all buyers of annuities; if A were greater than $\frac{R}{1-p^H}$, then annuity companies would suffer expected losses on all annuities sold.

We will now show that the competitive rate of return on annuities, A, must be less than $\bar{A}$, the actuarially fair rate based on population average mortality, where $\bar{A} \equiv \frac{R}{1-\bar{p}}$ and $\bar{p} \equiv \int_{p^L}^{p^H} p\,dH(p)$ is the population average probability of dying young. First, we state the following well-known lemma.

LEMMA. Suppose that $f(p) \lessgtr 0$ as $p \lessgtr p^*$ and that $\int_{p^L}^{p^H} f(p)dH(p) = 0$. If $g(p)$ is strictly increasing, then $\int_{p^L}^{p^H} f(p)g(p)dH(p) \geq 0$, with strict inequality if $dH(p)$ is not degenerate.[10]

Let $\pi(A;I^*+Y-T,S)$ be the expected profit of the annuity industry if the private annuity rate of return is A. Observe that

$$\pi(A;I^*+Y-T,S) = \int_{p^L}^{p^H} M(p,A)Q^*(p,A)dH(p) \tag{14}$$

where $Q^*(p,A) = q_1(p,A)(I^* + Y - T) - q_2(p,A)S$ is average annuity demand of type p consumers and I^* is the average inheritance received at birth. Using the relation $M(p,A) = M(p,\bar{A}) + (1-p)(\bar{A}-A)$ which follows from (13), we can rewrite (14) as

$$\pi(A;I^*+Y-T,S) = \int_{p^L}^{p^H} M(p,\bar{A})Q^*(p,A)dH(p) + (\bar{A}-A)\int_{p^L}^{p^H} (1-p)Q^*(p,A)dH(p) \tag{15}$$

Since $\int_{p^L}^{p^H} M(p,\bar{A})dH(p) = 0$ (from (13) and the definition of $\bar{A}$) and since $\frac{\partial Q^*}{\partial p} < 0$ (from (11)), the lemma implies that the first integral in (15) is negative. Since (for S sufficiently small) the second integral in (15) is positive, it follows that if $A \geq \bar{A}$, then $\pi(A) < 0$. The result that $\pi(\bar{A}) < 0$ is, of course, a consequence of adverse selection. Therefore, the equilibrium rate of return A must lie in the open interval $(\frac{R}{1-p^L}, \frac{R}{1-\bar{p}})$.

The equilibrium rate of return on private annuities, $\hat{A}$, must be a root of the equation $\pi(A) = 0$. Since $\frac{R}{1-p^L} < \hat{A} < \frac{R}{1-\bar{p}}$, $\pi(\frac{R}{1-p^L}) > 0 > \pi(\frac{R}{1-\bar{p}})$ and $\pi(A)$

is a continuous function of A, there is at least one root of $\pi(A) = 0$ between $\frac{R}{1-p^L}$ and $\frac{R}{1-\bar{p}}$ for which $\pi'(\hat{A}) \leq 0$. We demonstrate in Appendix B that in the case of logarithmic utility ($\sigma = 1$), $\pi(A)$ is strictly concave for $A > R$ and thus there is a unique root $\hat{A}$ of $\pi(A) = 0$ in $(\frac{R}{1-p^L}, \frac{R}{1-\bar{p}})$ and $\pi'(\hat{A}) < 0$.[11]

The equilibrium annuity rate of return, $\hat{A}$, can be expressed as a function of I^*+Y-T and S. Observe from (10a) that $Q^*(p,A)$ is a linearly homogeneous function of I^*+Y-T and S. Therefore, from (14), $\pi(A;I^*+Y-T,S)$ also linearly homogeneous in I^*+Y-T and S, so that if $\hat{A}$ satisfies $\pi(A;I^*+Y-T,S) = 0$, it also satisfies $\pi(A;\beta(I^*+Y-T),\beta S) = 0$ for any $\beta > 0$. Hence $\hat{A}$ can be written as

$$\hat{A} = \hat{A}(I^*+Y-T,S) \tag{16}$$

where $\hat{A}(.,.)$ is homogeneous of degree zero.

To demonstrate that $\partial\hat{A}/\partial S < 0$, recall from (10a) that an increase in social security benefits leads type p consumers to reduce their demand for private annuities by $q_2(p,A)$. Since $\frac{\partial q_2}{\partial p} > 0$, consumers with high p reduce their annuity demands by more than low p consumers. Furthermore, since high p consumers begin with a lower demand for annuities than low p consumers, the percentage reduction in annuity demand is greatest for high p consumers. Now, since it is the annuities sold to the high p consumers on which annuity companies expect positive profits, this shift in the composition of annuity holders away from the profitable (high p) consumers leads to a reduction in expected profits. In order to restore zero expected profits, the equilibrium rate of return $\hat{A}$ must fall (since $\pi'(\hat{A}) < 0$). Thus, the partial derivatives of $\hat{A}(I^*+Y-T,S)$ are

$$\frac{\partial\hat{A}}{\partial S} < 0 \tag{17a}$$

$$\frac{\partial \hat{A}}{\partial(I^{*}+Y-T)} = \frac{-S}{I^{*}+Y-T}\frac{\partial \hat{A}}{\partial S} \gtrless 0 \quad \text{as} \quad S \gtrless 0 \tag{17b}$$

where (17a) follows from applying Euler's Theorem to $\hat{A}(.,.)$ which is homogeneous of degree zero.

III. The Steady State Effects of Changes in Social Security

Let $B_t^{*}(p)$ denote the actual ex post bequests (per capita) left by the group of type p consumers born at the beginning of period t. Letting $B_t^D(p)$ denote the bequests (per capita) of the consumers who died young and $B_t^S(p)$ denote the bequests (per capita) of the consumers who survived two periods, and recalling that a fraction p of type p consumers dies young, we obtain

$$B_t^{*}(p) = pB_t^D(p) + (1-p)B_t^S(p) \tag{18}$$

The homotheticity of preferences implies that $B_t^D(p)$ and $B_t^S(p)$ are each proportional to the expected present value of lifetime income so that (18) may be rewritten as

$$B_t^{*}(p) = \theta(p,A_t)[I_t^{*}+Y-T+A_t^{-1}S] \tag{19}$$

where I_t^{*} is the initial bequest (per capita) received at birth; A_t is the rate of return on annuities purchased at the end of period t (and which pay off in period t+1). An expression for $\theta(p,A_t)$ is given in Appendix A. We will assume that $0<\theta(p,A_t)<1$ for $p^L \leq p \leq p^H$.[12]

Define $B_t^{*} \equiv \int_{p^L}^{p^H} B_t^{*}(p)dH(p)$ to be the average bequest left by members of the generation born at time t. It follows from (19) that

$$B_t^{*} = \bar{\theta}(A_t)[I_t^{*} + Y - T + A_t^{-1}S] \tag{20a}$$

where

$$\bar{\theta}(A_t) = \int_{p^L}^{p^H} \theta(p, A_t)\,dH(p) \tag{20b}$$

In the steady state $B_t^* = I_t^*$ so that (dropping the time subscript) (20a) may be rewritten as

$$B^* = \frac{\bar{\theta}(A)}{1-\bar{\theta}(A)}(Y-T+A^{-1}S) \tag{21}$$

We assume that $0<\bar{\theta}(A)<1$.[13]

A fully funded social security system operates by collecting T from each young consumer and investing the proceeds in riskless capital earning a gross rate of return R. In the following period the social security tax cum interest, RT, is divided equally among the surviving consumers. Since a fraction $1-\bar{p}$ of the consumers survives to the second period, the payment S received by each surviving consumer is

$$S = \bar{A}T \tag{22}$$

where we recall that $\bar{A}$ is defined as $R/(1-\bar{p})$. Equation (22) shows that the marginal rate of return implicit in the social security system, $\frac{dS}{dT}$, is $\bar{A}$ which, as we have shown in Section II, is greater than A, the equilibrium rate of return on private annuities. Therefore, an actuarially fair increase in social security taxes and benefits increases the expected present value lifetime income $I + Y - T + A^{-1}S$, for a given level of inherited wealth I.

We will confine our attention to a small increase in S and T starting from an initial steady state in which S=0. It follows immediately from (17b) that $\left.\frac{\partial \hat{A}}{\partial(I^*+Y-T)}\right|_{S=0} = 0$ so that

$$\left.\frac{dA}{dS}\right|_{S=0} = \frac{\partial A}{\partial S} < 0 \tag{23}$$

Thus, an increase in fully funded social security reduces the steady state rate of return on annuities.

Henceforth, we assume that $\sigma=1$ (logarithmic utility) so that, as shown in Appendix D, $\bar{\theta}'(A) \leq 0$. The state state level of bequests is found by substituting (22) into (21) to obtain

$$B^* = \frac{\bar{\theta}(A)}{1-\bar{\theta}(A)} (Y+(A^{-1}-\bar{A}^{\ -1})S) \tag{24}$$

Differentiating (24) with respect to S yields

$$\frac{dB^*}{dS}\Big|_{S=0} = \frac{\bar{\theta}(A)}{1-\bar{\theta}(A)}[A^{-1} - \bar{A}^{\ -1}] + \frac{\bar{\theta}'(A)}{(1-\bar{\theta}(A))^2} \ Y \ \frac{dA}{dS}\Big|_{S=0} > 0 \tag{25}$$

where the inequality follows from $A<\bar{A}$, $\bar{\theta}'(A)\leq 0$ and (23). The increase in B^* occurs for two reasons. First, since social security pays a higher rate of return than private annuities, the introduction of social security raises (by $(A^{-1} - \bar{A}^{\ -1})dS$) the expected present value of lifetime resources for a given initial wealth. Second, the fall in A causes the share of lifetime resources passed on as bequests, $\bar{\theta}(A)$, to rise. Therefore, the factor $\frac{\bar{\theta}(A)}{1-\bar{\theta}(A)}$ in (21) rises.

Next we examine the effect of social security on the steady state level of average consumption of the young, c_1^* , where

$$c_1^* = \bar{\phi}(A)(B^*+Y-T+A^{-1}S) \tag{26a}$$

where

$$\bar{\phi}(A) \equiv \int_{p^L}^{p^H} \phi(p,A)dH(p) < 1 \tag{26b}$$

With logarithmic utility, $\bar{\phi}(A) = \bar{\phi}$ is invariant to A. Therefore, the effect of social security on c_1^* is proportional to the effect on $B^*+Y-T+A^{-1}S$ which increases as a result of three effects: (1) B^* rises as shown in (25); (2) since the gross return on social security, S/T, exceeds A, it follows that $-T+A^{-1}S$ rises for a given A; and (3) A falls as shown in (23) so that, $A^{-1}S$, the present value of the social security payment, rises. Therefore, the expected present value of lifetime resources rises and a fortiori the average consumption of young consumers also rises.[14]

The steady state private capital stock at the end of a period is equal to the saving of young consumers $B^*+Y-T-c_1^*$. In a fully funded social security system, the end-of-period capital stock held by the government is T. The steady state national capital stock K^* is the sum of private capital and government capital

$$K^*=B^*+Y-c_1^*. \tag{27}$$

Substituting (24) and (26a) into (27) yields

$$K^*=Y + \frac{\bar{\theta}(A)-\bar{\phi}}{1-\bar{\theta}(A)}[Y+(A^{-1}-\bar{A}^{-1})S] \tag{28}$$

Differentiating (28) with respect to S, we obtain

$$\frac{dK^*}{dS}\Big|_{S=0} = Y\frac{1-\bar{\phi}}{(1-\bar{\theta}(A))^2}\bar{\theta}'(A)\frac{dA}{dS}\Big|_{S=0} + \frac{\bar{\theta}(A)-\bar{\phi}}{1-\bar{\theta}(A)}(A^{-1} - \bar{A}^{-1}) \tag{29}$$

Since $\bar{\theta}'(A)\leq 0$ it follows from (23) that the first term on the right hand side of (29) is positive. Since $A^{-1}>\bar{A}^{-1}$ the second term on the right hand side of (29) will be positive if $\bar{\phi}<\bar{\theta}(A)<1$. In this case, the right hand side of (29) is unambiguously positive so that the introduction of fully funded social security will increase the steady state national capital stock. Appendix C provides conditions under which $\bar{\phi}<\bar{\theta}(A)<1$. Intuitively, the bequest motive as measured by λ must be sufficiently strong so that a larger share of lifetime resources is devoted to bequests than to first-period consumption.

In the case in which $\bar{\theta}<\bar{\phi}$, the first term on the right hand side of (29) remains positive, but the second term is negative. Observe that if $\lambda=0$, then $\bar{\theta}(A)\equiv\bar{\theta}'(A)\equiv 0$ and the first term on the right hand side of (29) becomes zero. Thus, if the bequest motive is sufficiently weak, then an increase in fully funded social security will reduce the total national capital stock in the steady state. Thus we have shown

$$\left.\frac{dK^*}{dS}\right|_{S=0} \begin{cases} < 0 & \text{if } \lambda \text{ is small} \\ > 0 & \text{if } \bar{\phi}(A) < \bar{\theta}(A) < 1 \end{cases} \tag{30}$$

IV. Conclusion

In this paper we have developed an overlapping generations model based on individual utility maximization subject to uncertainty about the date of death. We used this model to examine the dynamic behavior of consumption and bequests in an economy with consumers who have different probabilities of dying. Even though there are markets in annuities and in riskless bonds, consumers are unable to offset the introduction of actuarially fair social security. The reason is that adverse selection in the private annuity market leads to a rate of return on private annuities which is lower than the rate of return implicit in compulsory social security.

The introduction of actuarially fair social security raises the steady state average levels of bequests and first-period consumption; it reduces the steady state rate of return on private annuities. If the bequest motive is sufficiently weak, then an increase in fully funded social security benefits reduces private wealth by more than one-for-one. With a sufficiently strong bequest motive, an increase in social security taxes crowds out private wealth by less than one-for-one.

Footnotes

1. The seminal work in this area is Yaari (1965), which provided the framework for later work by Hakansson (1969), Fischer (1973), Richard (1975), Levhari and Mirman (1977), Barro and Friedman (1977) and Kotlikoff and Spivak (1981).

2. Kotlikoff and Spivak (1981) examine the role of the family in providing an (incomplete) annuities market but stop short of a full-scale overlapping generations model in which the intra-cohort distribution of bequests is determined endogenously.

3. See Kotlikoff, Shoven and Spivak (1983) and Karni and Zilcha (1984) for interesting extensions of the overlapping generations model in which consumers within a cohort have different ex post mortality experiences.

4. Their analysis is more general than an analysis of annuity markets which are based on lifetime uncertainty; it applies more generally to mandatory insurance as a partial remedy for adverse selection in insurance markets. In particular, Eckstein, Eichenbaum and Peled pay careful attention to various concepts of equilibrium.

5. We follow Yaari (1965), Hakansson (1969), Fischer (1973) and Richard (1975) in specifying utility as a function of the size of the bequest left to one's heir. An alternative formulation which also gives rise to a bequest motive is to specify utility as a function of one's heir's utility as in Barro (1974) and Drazen (1978).

   The specification of utility as a function of the size of the bequest left to one's heir was chosen for tractability. The substantive results of this paper do not depend on choosing this specification rather than the specification suggested by Barro (1974). In particular, the fact that social security affects consumption and capital accumulation depends, not on the particular specification of the bequest motive, but rather on the fact that adverse selection drives a wedge between the rates of return on social security and on private annuities. In the absence of adverse selection, fully funded social security would not affect consumption regardless of whether the bequest motive is specified as in this paper or as in Barro (1974).

6. Sheshinski and Weiss (1981) have derived a similar result in a model which is similar in spirit, but different in detail from the model in this paper.

7. If $A > \frac{R}{1-p}$, then all of the results in this paragraph are reversed.

8. To derive this implication, we observe that (as will be argued below) competition in the annuity market will prevent the rate of return A from being less than $\frac{R}{1-p^L}$. Let $N(p,A)$ be the numerator of $q_1(p,A)$ in (A-8b), i.e., $N(p,A) = 1 + \lambda^{\frac{1}{\sigma}}[1 - (\frac{1-p}{p})^{\frac{-1}{\sigma}} (\frac{A-R}{R})^{\frac{-1}{\sigma}}]$. Observe that $\partial N/\partial p < 0$ and

$\partial N/\partial A > 0$. Next observe that $N(p^H, \frac{R}{1-p^L})$ will be positive if and only if

$\frac{p^L}{1-p^L} > \left(\frac{\lambda^{\frac{1}{\sigma}}}{1+\lambda^{\frac{1}{\sigma}}}\right)^{\sigma}\left(\frac{p^H}{1-p^H}\right)$ which will be true if and only if equation (12)

holds. Note that the term on the left of the first inequality in (12) is less than p^H since this term can be written as

$\{1 + (1-p^H)[(1 + \lambda^{\frac{-1}{\sigma}})^{\sigma}-1]\}^{-1}p^H$. Therefore, given p^H, λ, and σ, the set of possible values for p^L is not empty.

Since we have shown that (12) implies that $N(p^H, \frac{R}{1-p^L}) > 0$, it follows from $\partial N/\partial p < 0$ and $\partial N/\partial A > 0$ that $N(p,A) > 0$ for $p \leq p^H$ and $A \geq \frac{R}{1-p^L}$ if (12) holds. Therefore, equation (12) implies that $q_1(p,A) > 0$, since the denominator of the right hand side of (A-8b) is positive.

9. The Rothschild and Stiglitz (1976) demonstration that there cannot be a pooling equilibrium depends on their assumption "that customers can buy only one insurance contract". As they point out themselves, "this is an objectionable assumption" (p. 632). The appropriate equilibrium concept in the presence of monitoring of purchases from other companies still requires further research. The equilibrium described in this paper has some desirable characteristics and is suitable for our purposes.

10. The proof of this lemma is

$$\int_{p^L}^{p^H} f(p)g(p)dH(p) = \int_{p^L}^{p^*} f(p)g(p)dH(p) + \int_{p^*}^{p^H} f(p)g(p)dH(p) \geq$$

$$g(p^*)\left[\int_{p^L}^{p^*} f(p)dH(p) + \int_{p^*}^{p^H} f(p)dH(p)\right] = 0. \qquad \text{q.e.d.}$$

11. More generally, when σ is not equal to one, we have not ruled out multiple roots of $\pi(A) = 0$ in the interval $(\frac{R}{1-p^L}, \frac{R}{1-\bar{p}})$. Nonetheless, we can rule out as possible equilibria those roots for which $\pi'(A) > 0$ by observing that if such an $\hat{A}$ were the prevailing rate of return on private annuities, a firm could offer a slightly higher rate of return and profitably attract all annuity purchases. Thus the equilibrium rate $\hat{A}$ is characterized by $\pi'(\hat{A}) \leq 0$. Henceforth, we assume that this inequality

holds strictly.

12. See Appendix C for conditions under which $\Theta(p,A)<1$.

13. In Appendix C, we present conditions under which $0<\bar{\Theta}(A)<1$. These conditions guarantee that $B^*>0$ if $Y-T+A^{-1}S>0$ and will guarantee that B_t^* approaches the steady state B^* monotonically.

14. It can also be shown that wth with logarithmic utility the introduction of actuarially fair social security leads to an increase in the amount of riskless bonds held in the portfolios of young consumers. This result follows from the fact that riskless bond holdings are proportional to $\frac{A}{A-R}(I+Y-T+A^{-1}S)$. (Substituting (A-3b) into (A-5b) in Appendix A and then setting σ equal to 1 yields $\theta^D = [1+\delta(1-p+\lambda)]^{-1}p\delta\lambda R\frac{A}{A-R}$.) Since the introduction of social security leads to a reduction in A, the factor $\frac{A}{A-R}$ rises. We have already shown that the steady state expected present value of lifetime income rises with the introduction of social security.

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Appendix A

In this Appendix we calculate the optimal values of c_1, c_2, B^D and B^S for the case in which U() and V() each have constant relative risk aversion equal to σ. Because $U'(c) = c^{-\sigma}$ and $V'(B) = \lambda B^{-\sigma}$, the first-order conditions (6a-c) may be rewritten as

$$c_1 = [(1-p)\delta\lambda A]^{-\frac{1}{\sigma}} B^S \tag{A-1a}$$

$$c_2 = \lambda^{-\frac{1}{\sigma}} B^S \tag{A-1b}$$

$$B^D = [\frac{1-p}{p} (\frac{A-R}{R})]^{-\frac{1}{\sigma}} B^S \tag{A-1c}$$

Substituting (A-1a-c) into the lifetime budget constraint (5) yields

$$B^S = A(I+Y-T)+S-\{A[(1-p)\delta\lambda A]^{-\frac{1}{\sigma}}+\lambda^{\frac{-1}{\sigma}}+(\frac{1-p}{p})^{-\frac{1}{\sigma}}(\frac{A-R}{R})^{1-\frac{1}{\sigma}}\} B^S \tag{A-2}$$

Re-writing (A-2), we obtain

$$B^S = \theta^s(p,A)(I+Y-T + A^{-1}S) \tag{A-3a}$$

where

$$\theta^s(p,A) = A\{1+\lambda^{-\frac{1}{\sigma}}+[(1-p)\delta\lambda]^{-\frac{1}{\sigma}} A^{1-\frac{1}{\sigma}} + (\frac{1-p}{p})^{-\frac{1}{\sigma}}(\frac{A-R}{R})^{1-\frac{1}{\sigma}}\}^{-1} \tag{A-3b}$$

Substituting (A-3) into (A-1a) and simplifying yields

$$c_1 = \phi(p,A)(I+Y-T+A^{-1}S) \tag{A-4a}$$

where

$$\phi(p,A) = \{1+\delta^{\frac{1}{\sigma}} A^{\frac{1}{\sigma}-1} [(1-p)^{\frac{1}{\sigma}}(1+\lambda^{\frac{1}{\sigma}})+p^{\frac{1}{\sigma}}\lambda^{\frac{1}{\sigma}}(\frac{A-R}{R})^{1-\frac{1}{\sigma}}]\}^{-1} \tag{A-4b}$$

To obtain an expression for B^D, substitute (A3-a) into (A-1c) which yields

$$B^D = \theta^D(p,A)(I+Y-T+A^{-1}S) \qquad \text{(A-5a)}$$

where

$$\theta^D(p,A) = \left(\frac{1-p}{p}\right)^{-\frac{1}{\sigma}}\left(\frac{A-R}{R}\right)^{-\frac{1}{\sigma}}\theta^S(p,A) \qquad \text{(A-5b)}$$

The average bequest left by a type p consumer, $B^*(p)$, is equal to $pB^D + (1-p)B^S$. Therefore, from (A-3) and (A-5) it follows that

$$B^*(p) = \theta(p,A)(I+Y-T+A^{-1}S) \qquad \text{(A-6a)}$$

where

$$\theta(p,A) = \left[1-p + p\left(\frac{1-p}{p}\right)^{\frac{-1}{\sigma}}\left(\frac{A-R}{R}\right)^{\frac{-1}{\sigma}}\right]\theta^S(p,A) \qquad \text{(A-6b)}$$

Finally we calculate the demand for private annuities by substituting (A-1b) and (A-1c) into (4) to obtain

$$Q(p,A)+A^{-1}S = (1+\lambda^{-\frac{1}{\sigma}}-\left(\frac{1-p}{p}\right)^{\frac{-1}{\sigma}}\left(\frac{A-R}{R}\right)^{-\frac{1}{\sigma}})A^{-1}B^S \qquad \text{(A-7)}$$

Substituting (A-3a,b) into (A-7) yields

$$Q(p,A)+A^{-1}S = q_1(p,A)(I+Y-T+A^{-1}S) \qquad \text{(A-8a)}$$

where

$$q_1(p,A) = \frac{1+\lambda^{\frac{1}{\sigma}}\left[1 - \left(\frac{1-p}{p}\right)^{-\frac{1}{\sigma}}\left(\frac{A-R}{R}\right)^{-\frac{1}{\sigma}}\right]}{1+\delta^{-\frac{1}{\sigma}}(1-p)^{-\frac{1}{\sigma}}A^{1-\frac{1}{\sigma}}+\lambda^{\frac{1}{\sigma}}\left[1+\left(\frac{1-p}{p}\right)^{-\frac{1}{\sigma}}\left(\frac{A-R}{R}\right)^{1-\frac{1}{\sigma}}\right]} \qquad \text{(A-8b)}$$

Appendix B

In this Appendix we show that under logarithmic utility, ($\sigma = 1$), and with $I+Y-T>0$ and $S\geq 0$, $\pi(A)$ is strictly concave for $A>R$.

First we differentiate (14) twice with respect to A to obtain

$$\pi''(A) = \int_{p^L}^{p^H} [\frac{\partial^2 M}{\partial A^2}Q* + 2\frac{\partial M}{\partial A}\frac{\partial Q*}{\partial A} + \frac{\partial^2 Q*}{\partial A^2}M]\,dH(p) \tag{B1}$$

From (13), it follows that

$$\frac{\partial M}{\partial A} = -(1-p) \tag{B2}$$

$$\frac{\partial^2 M}{\partial A^2} = 0 \tag{B3}$$

Setting $\sigma=1$ in (A-8b) and differentiating with respect to A yields

$$\frac{\partial q_1}{\partial A} = \frac{p\delta\lambda R}{(A-R)^2}\phi(p) > 0 \tag{B4}$$

Differentiating (10b) with respect to A and using (B4) we obtain

$$\frac{\partial q_2}{\partial A} = -\phi(p)[A^{-2} + \frac{p\delta\lambda}{(A-R)^2}] \tag{B5}$$

It follows from (B4) and (B5) that

$$\frac{\partial Q}{\partial A} = \frac{p\delta\lambda R}{(A-R)^2}\phi(p)(I+Y-T)+\phi(p)[A^{-2}+\frac{p\delta\lambda}{(A-R)^2}]S \tag{B6}$$

Differentiating (B6) with respect to A yields

$$\frac{\partial^2 Q}{\partial A^2} = \frac{-2p\delta\lambda R}{(A-R)^3}\phi(p)(I+Y-T)-2\phi(p)[A^{-3}+\frac{p\delta\lambda}{(A-R)^3}]S \tag{B7}$$

Substituting (13), (B2), (B3), (B6), and (B7) into (B1) yields

$$\pi''(A) = -2\int_{p^L}^{p^H} \phi(p)\{\frac{p^2R^2\delta\lambda}{(A-R)^3}(I+Y-T)+[A^{-3}+\frac{p^2\delta\lambda}{(A-R)^3}]RS\}dH(p)<0, \text{ if } A>R \tag{B8}$$

Appendix C

In this Appendix we restrict our attention to the case of logarithmic utility ($\sigma = 1$) and derive conditions under which $\bar{\theta} < \bar{\phi}$ and conditions under which $\bar{\phi} \leq \bar{\theta} < 1$. We begin by defining the function $\gamma(p,q)$ as

$$\gamma(p,q) \equiv \lambda\left[\frac{p^2}{q} + \frac{(1-p)^2}{(1-q)}\right] \tag{C1}$$

Now define p^* ($p^L \leq p^* \leq p^H$) as the probability of dying young which is implicit in the rate of return on private annuities, i.e.,

$$R = (1-p^*)A \tag{C2}$$

From the definition of $\theta(p,A)$ in (A-6b) and using (A-3b) and (A-4b) it can be shown that with logarithmic utility ($\sigma = 1$)

$$\theta(p,A) = \phi(p)\delta\lambda A[(1-p)^2+p^2\frac{R}{A-R}] \tag{C3}$$

Then using (C1) and (C2), we may rewrite (C3) as

$$\theta(p,A) = \phi(p)\delta R\gamma(p,p^*) \tag{C4}$$

Differentiating $\gamma(p,q)$ twice with respect to p and q demonstrates that $\gamma(p,q)$ is strictly convex in p and in q so that

$$\sup_{p^L \leq p \leq p^H} \gamma(p,q) = \max[\gamma(p^L,q),\ \gamma(p^H,q)] \tag{C5}$$

$$\sup_{p^L \leq q \leq p^H} \gamma(p,q) = \max[\gamma(p,p^L),\ \gamma(p,p^H)] \tag{C6}$$

It can also be shown that

$$\min_{p^L \leq p,\, q \leq p^H} \gamma(p,q) = \gamma(p,p) = \lambda \tag{C7}$$

Combining (C5), (C6) and (C7), we obtain

$$\lambda \leq \gamma(p,q) \leq \max[\gamma(p^L,p^H),\ \gamma(p^H,p^L)] \quad \text{for } p^L \leq p, q \leq p^H \tag{C8}$$

Note that for a given p^L (or p^H), $\gamma(p^L,p^H)$ and $\gamma(p^H,p^L)$ are maximized by maximizing p^H (or minimizing p^L). Recall, however, that we have restricted the values of p^L and p^H in (12) in order to assure positive demands for annuities by all consumers. Setting p^L equal to its lower bound $\frac{\lambda p^H}{1-p^H+\lambda}$ yields

$$\gamma(p^H,p^L) \leq \frac{1}{1+\lambda}(1-p^H+\lambda)(\lambda+p^H) \tag{C9}$$

Similarly, setting $p^H = \frac{1+\lambda}{p^L+\lambda}p^L$, we obtain

$$\gamma(p^L,p^H) \leq \frac{1}{1+\lambda}(1-p^L+\lambda)(\lambda+p^L) \tag{C10}$$

From (C9) and (C10), it follows that

$$\gamma(p,q) \leq \frac{1}{1+\lambda}[\lambda+\lambda^2+\max(p^H-p^{H^2},\ p^L-p^{L^2})] \tag{C11}$$

From (C11) it follows that

$$\gamma(p,q) < 1 \quad \text{if} \quad \lambda < p^L < p^H \tag{C12a}$$

$$\text{or if} \quad \lambda^2 < \frac{3}{4} \tag{C12b}$$

Thus from (C4) and (C12), and recalling the definitions of $\bar{\theta}$ and $\bar{\phi}$, we have

$$\bar{\theta} < \bar{\phi} \quad \text{if} \quad \delta R \leq 1 \text{ and if } \lambda < \max[p^L,(3/4)^{1/2}] \tag{C13}$$

Next we establish conditions under which $\bar{\phi} < \bar{\theta} < 1$. It follows immediately from (C4) that

$$\theta(p,A) \leq \delta R \sup_{p^L \leq p \leq p^H} \phi(p) \sup_{p^L \leq p, q \leq p^H} \gamma(p,q) \tag{C14}$$

Since $p^H-p^{H^2}-(p^L-p^{L^2}) = (p^H-p^L)(1-p^L-p^H)$ it follows from (C11) that if $p^L+p^H \leq 1$, then

$$\sup_{p^L \leq p,\, q \leq p^H} \gamma(p,q) \leq \frac{\lambda+\lambda^2+p^H-{p^H}^2}{1+\lambda} = (1+\lambda-p^H)\frac{\lambda+p^H}{\lambda+1} \tag{C15}$$

Using the definition of $\phi(p)$, (C15) may be rewritten as

$$\sup_{p^L \leq p,\, q \leq p^H} \gamma(p,q) \leq \delta^{-1}\left(\frac{1}{\phi(p^H)}-1\right)\frac{\lambda+p^H}{\lambda+1}\text{, if } p^L+p^H \leq 1. \tag{C16}$$

Using (C16) and the fact that $\sup_{p^L \leq p \leq p^H} \phi(p) = \phi(p^H)$, (C14) yields

$$\theta(p,A) \leq R\,(1-\phi(p^H))\frac{\lambda+p^H}{\lambda+1} < R,\text{ if } p^L + p^H \leq 1 \tag{C17}$$

Now suppose that $\delta = R = \lambda = 1$ and that $p^L+p^H \leq 1$. It follows from (C17) that $\theta(p,A) < 1$ and hence $\overline{\theta} < 1$. It follows from (C4) and (C8) that $\theta(p,A) \geq \phi(p)$ with strict inequality for $p \neq p^*$. Therefore $\overline{\theta} > \overline{\phi}$ so that we have established the existence of parameter values for which $\overline{\phi} < \overline{\theta} < 1$.

Appendix D

In this Appendix, we show that with logarithmic utility ($\sigma = 1$), $\bar{\theta}'(A) \leq 0$ if $S = 0$. It follows from the definition of $\bar{\theta}(A)$ in (20b) that

$$\bar{\theta}'(A) = \int_{p^L}^{p^H} \frac{\partial \theta(p,A)}{\partial A} dH(p) \tag{D1}$$

Differentiating (C3) with respect to A yields

$$\frac{\partial \theta(p,A)}{\partial A} = -\phi(p)\delta\lambda[pR+(1-p)(A-R)]\frac{M(p,A)}{(A-R)^2} \tag{D2}$$

Substituting (D1) into (D2) yields

$$\bar{\theta}'(A) = -\int_{p^L}^{p^H} f(p)g(p)dH(p) < 0 \text{ if } S=0 \tag{D3a}$$

where

$$f(p) = \frac{\lambda}{(A-R)^2}M(p,A)Q(p,A) \tag{D3b}$$

$$g(p) = \frac{[pR+(1-p)(A-R)]\delta}{Q(p,A)/\phi(p)} \tag{D3c}$$

The inequality in (D3a) follows immediately from the Lemma after observing that $\int_{p^L}^{p^H} f(p)dH(p) = \frac{\lambda}{(A-R)^2}\pi(A) = 0$ and showing that $g'(p) \geq 0$. Below, we show that if $S=0$, then $g'(p) > 0$.

Using (10a), (A4b) and (A8b) we find that with logarithmic utility and $S=0$

$$\frac{\partial (Q/\phi)}{\partial p} = -\delta(1+\lambda\frac{A}{A-R})(I+Y-T) \tag{D4}$$

Differentiating (D3c) with respect to p and using (D4) yields

$$g'(p) = \frac{1}{[Q(p,A)/\phi(p)]^2}\{\delta^2(1+2\lambda)R(I+Y-T)\} > 0 \tag{D5}$$