The Inevitability of Market-Wide Underpriced Risk

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The Inevitability of Market-Wide Underpriced Risk

Abstract
Lenders are frequently accused of mispricing the put option embedded in nonrecourse lending. Prior research shows one lender's incentives to underprice. Here, we identify the conditions for a marketwide underpricing equilibrium. We demonstrate that, in a market with many players, given sufficient time, a race to the bottom and marketwide mispricing are inevitable. Underpricing occurs because bank managers and shareholders exploit mispriced deposit insurance. We show that the probability of the underpricing equilibrium increases with time since the previous market crash and that the more volatile the underlying asset market, the more likely it is subject to underpricing.

Disciplines
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The Inevitability of Market-Wide Underpricing of Mortgage Default Risk

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Real Estate Economics

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JEL Classification: G12, G13, G21
The Inevitability of Market-Wide Underpricing of Mortgage Default Risk

Lenders are frequently accused of mispricing the put option imbedded in non-recourse lending (Herring and Wachter, 1999 and 2003). Prior research (Pavlov and Wachter, 2004) shows one lender’s incentives to underprice. Here we identify the conditions for a market-wide underpricing equilibrium. We demonstrate that in a market with many players, given sufficient time, a race to the bottom and market-wide mispricing are inevitable. Underpricing occurs because bank managers and shareholders exploit mispriced deposit insurance. We show that the probability of the underpricing equilibrium increases with time since the previous market crash and that the more volatile the underlying asset market, the more likely it is subject to underpricing.

JEL Classification: G12, G13, G21
1. Introduction

The impact of bank lending on real estate crashes and the market-wide mispricing of real estate risk has been observed in the literature. In this paper we identify the conditions under which many lenders and the conditions under which all lenders rationally choose to underprice the put option imbedded in non-recourse lending. The contribution of this paper over the previous literature is the analysis of the underpricing behavior of lenders in the context of many banks and the demonstration of the competitive equilibrium in which all lenders underprice. Underpricing occurs because bank managers and shareholders exploit underpriced deposit insurance.

We construct a simple model of two competitive equilibria – “good” and “underpricing.” In the good equilibrium, lenders accurately price the imbedded put option and asset prices reflect their fundamental value. In the underpricing equilibrium, lenders rationally misprice the put option. We further analyze the mechanism that leads to the economy switching between the two equilibria and study the economic conditions that make the underpricing equilibrium more likely to occur. We find that if the correctly pricing lenders cannot break-even in the good state, all lenders switch to the underpricing equilibrium. The probability of entering the underpricing equilibrium increases with the value of the put option. For instance, the value of the put option is higher in more volatile asset markets, such as markets with low elasticity of supply. The probability of entering the underpricing equilibrium increases with the time since the last market crash. Markets with longer cycles are more likely to experience a higher degree of underpricing.
In Section 2, we derive a model that shows that one, two, or more rational lenders may choose to underprice the put option, and the impact for correctly pricing lenders. Section 3 presents the major result of the model, showing that under certain conditions all lenders underprice. Section 4 analyzes the economic environment that makes the underpricing equilibrium more likely to occur. Implications for market outcomes are presented in Section 5 and Section 6 concludes.

2. Markets with limited underpricing

We assume banks are financial intermediaries that accept deposits and make loans to investors (borrowers) who purchase risky asset (properties). In this framework, Allen and Gale (1998 and 1999) and Pavlov and Wachter (2004) show that correctly priced loans have no impact on asset markets. If the put option imbedded in the non-recourse loans is mispriced, however, efficient asset markets incorporate this mistake into the asset price. On its own, or in combination with other factors, this phenomenon can lead to asset price bubbles. They do not focus on the lending behavior that could lead to this outcome. We extend this work by focusing on the lending behavior that leads to underpricing of the imbedded put option. We examine the circumstances under which industry-wide underpricing of the default spread is likely to occur.

We further assume that investors purchase assets each period, and following the realization of the payoff either sell the asset and return the loan or (since these are non-
recourse loans) default. In the high payoff state, all loans are repaid. In the low payoff state, investors default and the lenders absorb the losses. Thus, assets are held for one period and loans are extended for one period. Nonetheless, this is repeated over infinite number of periods.

Following the premise that depositors are insured and, therefore, can rationally choose to remain uninformed, we assume that the cost function for each lender is:

\[ c(y) = y^2 + 1 + vy = d(y)v + vy, \]  \hspace{1cm} (1)

where \( y \) is the number of loans made by each bank, \( v \) is the value of the put option imbedded in each loan, and \( d(y) \) is the deposit rate. This cost function assumes the depositors are uninformed because the deposit rate, \( d(y) = y + 1/y \), is independent of the pricing of the credit risk by the bank. Instead, the deposit rate takes a \( U \)-shape with respect to the size of the lender. Very small banks may lack brand recognition and may not offer sufficient network infrastructure, while very large banks may have to increase their deposit rates to attract customers who would otherwise prefer a different lender.

The deposit rate we assume, and specifically its independence of the lending risk, is crucial for our results. We motivate this deposit rate either through assuming uninformed depositors who cannot directly lend to the users of capital, or the presence of deposit insurance. Note that the deposit insurance need not be mispriced on average. Our results depend only on the assumption that the deposit insurance is independent of the lending
activity of the bank. The cost of the insurance, correctly priced on average or not, becomes part of the fixed costs of lending, which are normalized to 1 in Equation (1).\textsuperscript{5}

The second component of the cost function (1) is the value of the put option imbedded in each loan times the number of loans. Since the lender provides this put option with every loan, it is part of the cost of lending. In our setting, the value of the put option is exogenous, as we do not specify the exact payoffs to the underlying asset in each state. If these payoffs are available, the computation of the put option is immediate through a non-arbitrage condition. (See Allen and Gale (1999), Allen (2001) and Pavlov and Wachter (2004) for examples.)

The interest rate charged on the loan, \(i\), incorporates the value of the put option and the deposit rate:

\[
i = \frac{v + d}{\delta}
\]  

where \(\delta\) is the probability of the loan, including interest, is repaid. The division by \(\delta\) reflects the assumption that interest is collected only in the good state. This specification also assumes the losses in case of default exceed the interest payment on the loan. This specification arises from the assumption of risk-neutral lenders whose only marginal cost is the deposit rate. Thus, the value of the put option becomes part of the cost of capital for the borrowers.\textsuperscript{6}
The expected profits to the bank, \( \pi_0 \), are:

\[
\pi_0 = \delta i y - (y^2 + 1 + vy) = (v + d(y))y - d(y)y - vy = 0
\]  

(3)

By equating the expected marginal revenue to the expected marginal cost we obtain:

\[
\delta i = 2y + v
\]  

(4)

Substituting \( \delta i \) into the expected profit condition (3), we determine profits of the bank in the good state, \( \pi \), as:

\[
\pi = (2y + v)y - (y^2 + 1) = y^2 + vy - 1
\]  

(5)

Zero expected profit means that in the good state profits are \( v \), which implies that optimal output is one. The optimal number of loans (=1) is a result of normalization and can be set to any number by changing the fixed cost in the cost function of the lender.

This leads to the proposition:

**Proposition 1:** Profits in the good state are an increasing function of the optimal output, \( y \).

To determine the optimal output, we aggregate supply and demand:
\[ a - b(2y + v) = my \]
\[ y = \frac{a - bv}{m + 2b} \]

where \( a \) and \( b \) determine the industry demand for loans, \( a - b(E(i)) \), as a function of the expected interest rate, and the total number of banks, \( m \). The equilibrium number of banks, \( m \), is determined by finding the maximum \( m \) that will allow the participating banks to break-even. In this case,

\[ m = a - b(2 + v) \]

and the optimal output, \( y \), equals one. This leads to zero expected net profit with the profit in the good state determined as the value of the option, \( v \).

Proposition 1 has immediate implications for the behavior of bank managers. To induce effort, shareholders may offer managers profitability or market share – based compensation. While such compensation may solve the agency frictions when effort is costly and unobservable, in our setting it induces managers to underprice the put option. In other words, some managers may choose to underprice the put option to increase the bank’s volume of lending and profitability in the good state even though such behavior can be devastating for the bank in the bad state. To prevent underpricing, shareholders may impose penalties on underpricing managers (such as the managers loosing their job). While this may work in the long run, underpricing is not likely to be discovered unless
the bad state occurs. Therefore, a manager with a short time horizon may optimally prefer to maximize his or her compensation in the short run and accept the risk of losing their job in case the bad state occurs and their underpricing behavior is discovered.8

The compensation scheme we assume is as follows:

- if managers do not underprice, they receive their salary both in the good and in the bad states,
- if managers underprice, they receive a salary plus bonus in the good state, and lose their job in the bad state as their underpricing is discovered.

Since our model is static, we use the terms long and short horizon only to distinguish between managers who lightly weigh the penalty, given their utility functions, that they receive if they underprice and are discovered, from ones who heavily weigh the penalty.9

The above compensation scheme results in the following condition for underpricing:

\[
\delta(B(\pi_u) + S) + \delta V(T) > S + \delta B(\pi) + V(T),
\]

where \(T\) denotes the time horizon of the manager, and \(V(T)\) denotes the value to the manager of keeping their job, or alternatively the value of the loss for the manager if they underprice and are discovered. Notice that \(V(T)\) includes the entire value to the manager of avoiding the penalty of discovered underpricing under expected optimal behavior in the future. Solving for the continuation value we obtain:
Given that $V$ is an increasing function of $T$, and $B$ is an increasing function of $\pi$, we summarize the above reasoning in the following proposition:

\textit{Proposition 2: If underpricing increases profits in the good state, }$\pi_u > \pi$, \textit{there exists a time horizon }$T^*$\textit{ such that managers with time horizons shorter than }$T^*$\textit{ will underprice the put option. Furthermore, }$T^*(\pi_u - \pi)$\textit{ is an increasing function of the additional profits obtained in the good state if the put option is underpriced.}

The intuition behind Proposition 2 is based on the tradeoff between increased profits in the good state and discovery of underpricing in the bad state. Long-term managers have a great deal to lose if they underprice and are discovered. Thus, long-term managers would not underprice. Short-term managers, however, have relatively little to lose if their underpricing is discovered. For them the benefit of increased profits in the good state is sufficient to underprice and risk discovery.

Furthermore, the above compensation scheme is consistent with maximizing shareholder value. Thus, shareholders with limited liability may provide incentives for the managers to underprice. This possibility depends on the equity stake compared to the payoff from underpricing in the good state. Therefore, Proposition 2 holds even if there is no agency
friction or informational asymmetry between managers and shareholders. Furthermore, we can phrase Proposition 2 in terms of shareholders’ equity:

**Proposition 2a:** If underpricing increases profits in the good state, \( \pi_u \geq \pi \), there exists a bank equity level, \( E^* \), such that shareholders of banks with lower equity prefer underpricing. Furthermore, \( E^*(\pi_u - \pi) \) is an increasing function of the additional profits obtained in the good state if the put option is underpriced.

In what follows we focus on bank management and use the first form of Proposition 2 in the remainder of this paper.

We next show that when one lender underprices, this lowers the profits of the remaining industry participants in the good state, \( \pi^g \). In the presence of one lender that underprices the put, the marginal revenue for the bank that underprices is \( 2y_u \). Thus, the interest rate charged on the loans is:

\[
\delta i = 2y_u^g + v = 2y_u
\]  

(10)

where \( y_u \) denotes the number of loans made by the bank that underprices the put option and \( y_u^g \) denotes the number of loans made by the banks that do not underprice the option (good banks), when there is one bank that underprices. Equating aggregate demand and supply, we obtain the following equation for the output of the lenders:
\[ a - b(2y_u) = (m - 1)y_u^e + y_u \]  

(11)

Substituting the expression for \( y_u^e \) from equation (10) into equation (11), leads to the following expression for the optimal output of the bank that underprices:

\[ y_u = \frac{a(2 + v) - v(1 + b(2 + v))}{2(a - bv)} \]  

(12)

Equation (7) implies that \( a - bv = m + 2b > 1 \). It is then easily verified that \( y_u > l = y \).

Invoking Proposition 1, we conclude that the profits of the underpricing bank are higher relative to the base case, i.e., \( \pi_u > \pi \).

Using equation (10) we obtain an expression for the output of the banks that do not underprice in the presence of one underpricer:

\[ y_u^e = 1 - \frac{v}{2(a - bv)} \]  

(13)

Thus, the optimal output of the correctly pricing banks is below the output in the base case, \( y_u^e < l = y \). Invoking Proposition 1, we conclude that the profits of the correctly pricing banks are lowered by the presence of an underpricer, i.e., \( \pi > \pi_u^e \).

We summarize the above arguments into the following proposition:
Proposition 3: In the good state, the bank that underprices the put receives higher profits and the other banks receive lower profits relative to the base case, i.e.,

$$\pi_u > \pi > \pi_u^g$$

(14)

Combining Propositions 2 and 3 directly yields the following result:

Result 1: There exists a time horizon $T_1^*$ such that a manager with shorter time horizon, $T < T_1^*$, chooses to underprice the put option. Such a choice leads to lower profits in the good state for the banks that price the put option correctly (they still have zero expected profits over all states) lower compensation for the managers of the banks that price correctly, and lower lending rates.

An important implication of Result 1 is that the presence of an underpricing manager leads to lower lending rates. Interest rates no longer fully reflect the value of the put option imbedded in the loans. We will come back to this point below.

Conditional on one lender underpricing the put option, we now examine the incentives for a second lender to underprice. We introduce the following additional notation:

$$y_{uu} = \text{number of loans made by each of the two banks that underprice the put option.}$$
\[ y^g_{uu} = \text{number of loans made by the banks that do not underprice the option (good banks), when there are two banks that underprice.} \]

\[ \pi_{uu} = \text{profit of the banks that underprice the put} \]

\[ \pi^g_{uu} = \text{profit of the banks that price correctly in the presence of two banks that underprice} \]

We formulate the following proposition:

**Proposition 4:** In the presence of one underpricer, a bank can increase the good state profits by underpricing. Furthermore, the other banks who price correctly in the presence of two underpricers receive lower profits relative to the case of one underpricer, i.e.,

\[ \pi_u > \pi_{uu} > \pi^g_u > \pi^g_{uu} \]  \hspace{1cm} (15)

**Proof:**

Analogously to the proof of Proposition 3, we equate the demand and supply:

\[ a - b(2y_{uu}) = (m - 2)y^g_{uu} + 2y_{uu} \]  \hspace{1cm} (16)

This leads to the following expression for the optimal output of a bank that underprices:
Comparing expression (17) to (12) shows that \( y_{uu} < y_u \) which verifies the first part of Proposition 4.

Comparing expression (17) with that for \( y_u^g \) given by (13), verifies that \( y_{uu} > y_u^g \), which proves the second inequality of Proposition 4. Equating the marginal costs of the underpricing and correctly pricing banks, we obtain that

\[
y_{uu}^g = y_{uu} - \frac{\nu}{2}
\]  

(18)

It is easily verified that \( y_{uu}^g < y_u^g \) which proves the third inequality of Proposition 4.

Q.E.D.

Combining Propositions 2 and 4 we obtain the following result:

\textbf{Result 2:} In the presence of one underpricer, there exists a time horizon \( T_2^* \) such that a manager with shorter time horizon, \( T < T_2^* \), chooses to switch from pricing correctly to underpricing. Such a choice leads to negative expected profit for both underpricing banks, lower profits for the underpricing banks in the good state, lower profits in the good state for the banks that price the put option correctly (they still have zero expected
profits over all states), lower compensation for the managers of the banks that price correctly, and lower interest rates.

We now generalize to the case when \( K \) lenders underprice the put option. We adopt the following notation:

\[
y_k = \text{number of loans made by each of the banks that underprice the put option.}
\]

\[
y^g_k = \text{number of loans made by the banks that DO NOT underprice the option (good banks), when there are } K \text{ banks that underprice.}
\]

\[
\pi_k = \text{profit of the banks that underprice the put}
\]

\[
\pi^g_k = \text{profits of the banks that price correctly in the presence of } K \text{ banks that underprice}
\]

Equating demand and supply with \( K \) underpricers, we obtain the following equilibrium condition:

\[
a - b(2y_k) = (m - K)y^g_k + Ky_k
\]  

(19)

Analogously to Propositions 3 and 4, this leads to the following expressions for \( y_k \) and \( y^g_k \):

\[
y_k = \frac{a(2 + \psi) - \psi(K + b(2 + \psi))}{2(a - b\psi)}
\]  

(20)
and

\[ y^g_k = 1 - \frac{Kv}{2(a - bv)} \]  

(21)

To determine the underpricing incentive we compare the output of \( K \) underpricers with the output of a good bank in the presence of \( K-1 \) underpricers:

\[ y_k - y^g_k = \frac{v}{2} \left( 1 - \frac{1}{(a - bv)} \right) > 0 \]  

(22)

Combined with Propositions 1 and 2, this leads to the following result:

**Result 3:** In the presence of \( K \) underpricers, there exists a time horizon \( T_{k^*} \) such that a manager with a shorter time horizon, \( T<T_{k^*} \), chooses to switch from pricing correctly to underpricing. Such a switch leads to negative expected profits for all underpricing banks, lower profits for all underpricing banks in the good state, lower profits in the good state for the banks that price the put option correctly (they still have zero expected profits over all states), lower compensation for the managers of the banks that price correctly, and lower interest rates.

3. Underpricing Equilibrium
The management compensation scheme described above assumes that the bank is in business. A necessary condition for staying in business is that the bank makes positive profits at least in the good state:

\[
\pi^g_k = (y^g_k)^2 + vy^g_k - 1 > 0
\]  

(23)

In other words,

\[
y^g_k = \frac{-v + \sqrt{4 + v^2}}{2} > 0.
\]  

(24)

Substituting (21) into (24) results in the following condition for \( K \):

\[
K < K_{\text{max}} = \frac{(a - bv)(2 + v - \sqrt{4 + v^2})}{v}
\]  

(25)

where \( K_{\text{max}} \) denotes the maximum number of underpricers that will allow for the good banks to make positive profits in the good state. This maximum will be binding only if \( K_{\text{max}} \) is less than the total number of banks, \( m \). Using expression (7) for \( m \), we obtain the following condition for \( K_{\text{max}} < m \):

\[
a > \frac{bv\sqrt{4 + v^2}}{\sqrt{4 + v^2} - 2}
\]  

(26)
which simply states that the total debt market has to be large relative to the value of the put option.

If a correctly-pricing bank cannot earn positive profits even in the good state, i.e. \( K > K_{\text{max}} \), the manager faces certain dismissal. If, however, the manager underprices the put, the manager receives salary plus bonus in the good state and only loses his or her job if the bad state occurs. It is thus clear that all remaining managers will underprice the put regardless of their time horizon.

This allows us to formulate the following result:

**Result 4:** For a sufficiently large market (condition (26)), if the number of underpricing banks exceeds \( K_{\text{max}} \) all remaining managers will switch to underpricing regardless of their time horizon. The new equilibrium has the following characteristics:

1. The interest rate on the loans reflects only the deposit rate and does not include the put option at all.
2. The new interest rate is lower relative to the base case.
3. All banks make the same number of loans.
4. All banks make positive profits in the good state.
5. All banks have negative expected profits.
6. The deposit rate is higher relative to the base case.
7. More loans are made by each bank relative to the base case.
Proof:

Let \( y^* \) denote the optimal number of loans when all banks underprice. The marginal cost to the bank only reflects the cost of obtaining new deposits, \( 2y^* \), and does not include the put option. Equating aggregate supply and demand provides the following equation for the optimal output:

\[
a - 2by = my
\]  

(27)

Substituting (7) into (27) provides the following expression for the optimal output:

\[
y^* = \frac{a}{a - bv}
\]  

(28)

This is greater than the optimal output in the base case (=1). For a large debt market, i.e., \( a \gg bv \), \( y^* \) approaches the output in the base case.

The profit in the good state is given by:

\[
\pi = y^* - 1 = \left( \frac{a}{a - bv} \right)^2 - 1
\]  

(29)

Expression (29) is greater than zero, but approaches zero for a large debt market. In other words, the profits in the good state are small and approach zero, while the losses in the bad state remain unchanged. This leads to negative expected profits for the lenders.
Using Equation (1), we obtain the following expression for the deposit rate in the new equilibrium:

\[ d(y^*) = \frac{a}{a-bv} + \frac{a-bv}{a} \] (30)

Since \( a - bv = m + 2b > 1 \), \( a \) is greater than 1, and it is easily verified that \( d(y^*) \) is greater than 2, which is the deposit rate in the base case. Expression (30) also suggests that for a large market, i.e. \( a >> bv \), \( d(y^*) \) approaches the deposit rate in the base case.

Finally, we compare the interest rates charged on loans in the base case and the underpricing equilibrium. The base case interest rate is given by (4), while the new interest rate, \( i^* \), is

\[ i^* = \frac{2a}{a-bv}. \] (31)

It is easily verified that the new interest rate is lower than the interest rate in the base case, \( i^* < i \).

Q.E.D.
4. Determinants of $K_{\text{max}}$

In what follows we examine the influence of the economic environment on the critical number of underpricing banks, $K_{\text{max}}$. We summarize the results in the following statement:

Result 5: The following relationships hold:

$$\frac{\partial K_{\text{max}}}{\partial v} < 0, \quad \frac{\partial K_{\text{max}}}{\partial a} > 0, \quad \text{and} \quad \frac{\partial K_{\text{max}}}{\partial b} < 0$$

Proof:

Based on equation (25), we compute the following derivatives:

$$\frac{\partial K_{\text{max}}}{\partial v} = -\frac{a \left( \frac{4}{\sqrt{4+v^2}} - 1 \right) + bv \left( \frac{v}{\sqrt{4+v^2}} - 1 \right)}{v^2} < 0$$

$$\frac{\partial K_{\text{max}}}{\partial a} = \frac{2 + v - \sqrt{4+v^2}}{v} > 0$$

$$\frac{\partial K_{\text{max}}}{\partial b} = -(2 + v - \sqrt{4+v^2}) < 0$$

Q.E.D.

The most important claim of Result 5 is that the larger the option value, the smaller the critical number of underpricers. In other words, highly volatile markets are substantially more likely to enter the equilibrium in which all lenders completely ignore the put option...
in the loans they are providing. For instance, markets with low elasticity of supply are naturally more volatile.

Following a low realization all underpricing managers are discovered and replaced. Let $G_0(T)$ denote the cumulative distribution function of the time horizons of the managers following a low realization. Notice that $G_0(T_i) = 0$, i.e., there are no managers with time horizons shorter than $T_i$, defined in Result 1 as the cutoff point for a single manager to underprice. Each time period, the time horizon for all managers goes down by one. Thus, the cumulative distribution function $t$ intervals after the last low realization, $G_t$, can be expressed as follows:

$$G_t(T) = G_{t-1}(T + 1) = G_0(T + t).$$  \hspace{1cm} (33)

The number of underpricing managers, $K_t$, at time $t$ is the solution to the following equation:

$$K_t = mG_t(T(K_t)) = mG_0(T(K_t) + t).$$  \hspace{1cm} (34)

Proposition 5: The number of underpricing managers, $K_t$, is an increasing function of time since the last low realization.

Proof:
Notice that the critical time horizon that induces underpricing, $T(K_t)$, is a function of the number of underpricing managers, $K_t$. Let $\Delta \pi_k = \pi_k - \pi_k^g$ denote the change in the good state profits and $\Delta y_k = y_k - y_k^g$ denote the change in optimal output for a bank that begins to underprice in the presence of $K-1$ underpricers. Equation (22) shows that $\Delta y_k$ is independent of $K$. Since the profit is a quadratic function of the optimal output (Equation (5)), the change in profits is an increasing function of the optimal output:

$$\Delta \pi_k = \Delta y_k^2 + (2y_{k-1}^g + v)\Delta y_k$$  \hspace{1cm} (35)$$

Equation (21) shows that $y_k^g$ is a decreasing function of $K$. Since $\Delta y_k$ is independent of $K$, $\Delta \pi_k$ is a decreasing function of $K$. Invoking Proposition 2, we find that

$$\frac{\partial T}{\partial K} < 0.$$  \hspace{1cm} (36)$$

Differentiating equation (34) with respect to time gives:

$$\frac{\partial K}{\partial t} = mg(T(K_t + t)) \left( \frac{\partial T}{\partial K} \frac{\partial K}{\partial t} + 1 \right),$$  \hspace{1cm} (37)$$

where $g$ denotes the probability density function of $T$. Rearranging (37) we obtain:
\[
\left(1 - mg(T(K_0 + t))\frac{\partial T}{\partial K}\right) \frac{\partial K}{\partial t} = mg(T(K_0 + t))
\]  
(38)

The right hand side of Equation (38) is positive. Equation (36) suggests that

\[
\left(1 - mg(T(K_0 + t))\frac{\partial T}{\partial K}\right) > 0
\]  
(39)

Therefore,

\[
\frac{\partial K}{\partial t} > 0
\]  
(40)

Q.E.D.

Since the number of underpricing managers is an increasing function of time since the last low realization, it is only a matter of time until \(K_t\) reaches the critical point \(K_{\text{max}}\) at which all managers switch to underpricing regardless of their time horizon. We summarize the above reasoning in the following result:

**Result 6:** The number of underpricing managers increases with time since the last market crash (i.e. low realization). Given enough time without a crash, the critical level of underpricing, \(K_{\text{max}}\), is achieved and all managers switch to underpricing.
Result 6 immediately leads to the following important conclusion:

Result 7: Asset markets with longer cycles suffer, on average, a higher degree of underpricing.

5. Implications for Market Outcomes

The immediate empirical implication of Result 4 for market outcomes is that underpricing of the put option results in inflated asset prices. Because outsiders do not observe the value of the put option or the fundamental price of the asset, the spread of the loan rate over the deposit rate could be used as a proxy for the underpricing and, therefore, the inflated asset price.

This spread compensates the lenders for providing the put option imbedded in their loans. During periods of market-wide underpricing, lenders require little or no compensation for the put option, and the spread of lending over deposit rates is reduced, which is reflected in higher market price of the underlying asset.

Furthermore, periods of widespread underpricing are associated with increased lending activity. In order for lenders to raise enough capital to support this increased lending activity they increase deposit rates. This leads to a second testable implication that
deposit rates are positively correlated with asset prices. We summarize the above reasoning in the following empirical prediction:

**Result 8:** The spread between lending rates and deposit rates is negatively correlated with asset prices. Deposit rates are positively correlated with asset prices.

Note that all market implications of our model relate one or more symptoms of underpricing to observed asset prices. Since the value of the put option is unobservable by outsiders, the above conclusions cannot be tested directly. The model yields indirect empirical implications which can identify underpricing and asset bubbles, in particular, rising asset prices which are positively correlated to demand deposit rates and negatively correlated to lending rate to deposit rate spreads. Such a narrowing of the spread will, in turn, drive out non-bank lenders who cannot underprice the default risk. Consequently, investors will seek funding from the underpricing bank sector or exit the market.

Furthermore, although not part of our model, regulators likely exercise excessive scrutiny following a negative demand shock. This, in turn, raises the lending costs of banks and reduces their lending activity. If, despite the excessive scrutiny, regulators are still unable to discriminate between underpricing and correctly pricing managers, banks are forced to make only the very best loans following a negative demand shock. This may cause a shift in funding in the opposite direction, from the banking sector to other sources.
6. Conclusions and Policy Implications

In this paper we identify the conditions which give rise to market wide mispricing of real estate risk. In the presence of demand deposit insurance, lending officials of banks may be induced to underprice risk to gain short term profits. Such underpricing behavior forces a race to the bottom across the lending institutions, with market wide consequences, when the number of underpricers reaches a critical level. The longer the underlying real estate cycle, the greater the value of the put option and the elasticity of demand for bank loans, the greater the probability that the market will enter into an equilibrium in which all banks underprice risk.

Even though our model demonstrates the link between bank lending and real estate crashes, it does not capture all mechanisms that can lead to real estate price bubbles. The absence of short selling in real estate, and the ability of optimists to drive prices up can, for example, produce price bubbles even in the absence of underpricing. Nonetheless, the willingness and ability of the banking sector to provide underpriced funding exacerbates these inefficiencies. Even markets that do not have access to underpriced lending, such as the markets for raw land, can experience price run-ups in anticipation that future bidders for the developed land asset will have access to underpriced funds.

What can be done to mitigate the potential for market wide episodes of the underpricing of risk? In particular, are there public policy remedies to counter the private incentives for underpricing, given deposit insurance? Our model identifies three fundamental ways
to do so: limit the value of the put option, reduce shareholder and managerial incentives to underprice and monitor the lending activities of banks in order to detect and sanction underpricing.

First, any policy that reduces the value of the put option imbedded in non-recourse lending reduces the ability of the bank officers to expand through underpricing this option. This can be accomplished through less leverage and/or recourse lending. Alternatively, and less directly reducing the volatility of asset prices through, for example, more supply elasticity, can reduce the value of the put option.

Second, managerial and shareholder incentives for underpricing can reduced. To the extent contracts are not optimal and insurance is underpriced, or even just insensitive to the riskiness of the loans, episodes of underpricing are likely to occur. Bonuses calibrated to long term measures of profitability are preferable to short term measures that make no adjustment for reserving for shocks. In addition, the loan underwriting process can be modified so that officers whose compensation does not depend on loan originations oversee the lending activities. Moreover, if banks are profitable long term, there will be less incentive to underprice: banks with franchise value supported by barriers to entry may be protected from the temptation to underprice.

Third, regulation and monitoring of bank equity and lending behavior are necessary in the presence of deposit insurance which is not continuously repriced to reflect the bank’s level of risk. The regulators role in mispricing deposit insurance is of course the
fundamental source of incentives to underprice. This underlies the regulatory need to prevent individual banks from beginning or reinforcing the chain that leads to market wide underpricing. Regulations and sanctions that effectively require banks to operate with substantial amounts of shareholder funds at risk will help. The adoption of rules to require prompt regulatory response to replace supervisory discretion to respond will help stem spillover effects as institutions become decapitalized. The threat of sanctions will induce shareholders to recapitalize the bank in the face of short term negative shocks if banks are profitable long term and viable short term.
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References


1 Allen and Gale (1998 and 1999), Pavlov and Wachter (2004), among others.
2 For the purposes of our model, all loans are identical and all assets receive the high or the low payoff, which is the only uncertainty in this economy.
3 Note that in a one-period model, $v$ and $d$ are both dollar values and rates.
4 Potential depositors may prefer a smaller and (perceived to be) more responsive lender.
5 Furthermore, we only assume the presence of deposit insurance to justify the deposit rate implicit in the bank cost function. If depositors are willing to lend at this rate even in the absence of deposit insurance, none of our subsequent results change.
6 Typically, the cost of capital contains a risk premium which incorporates the risk of default and depends on the risk-aversion of the lenders. Our assumption of risk-neutral lenders simplifies the risk premium to just the value of the put option.
7 If the high payoff occurs and all loans are repaid, it is very difficult for regulators and/or shareholders to closely investigate the riskiness of the lending activities and penalize the bank managers if this riskiness is too high. In the bad state, however, the large number of defaults justifies scrutiny and underpricing will be detected. In an extension with heterogeneous loans this detection can be based on the proportion of loan defaults over time. In the good state, even with heterogeneous loans, it is statistically difficult to detect underpricing. Only with time and increased default experience would it be possible to detect underpricing. This process could lead to awareness of underpricing, which, in turn, could evoke regulatory response and precipitate an endogenous crash.
8 The correctly pricing manager who has a long-term horizon anticipates correctly that they will continue to hold their position after the bad state, in a second round of the repeated game implicit in the model.
9 In this framework, horizon is purely an illustrative concept, to motivate differential valuation of the underpricing penalty.
11 Note that lenders can underprice the risk through loan terms other than the lending rate spread. For instance, they can raise the loan-to-value ratios. In fact, since the asset prices are inflated, even just keeping the LTV ratio constant underprices the default risk.