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Abstract

In ^{30}Mg, existing data for *B(E2)* strengths connecting the ground and excited 0+ states to the first 2+ state have been used, together with earlier shell-model predictions of normal and intruder *E*2 strengths, to estimate the intruder-normal state mixing in the 0+ and 2+ states. Resulting mixing is small, as expected, and for the ground state my value of 0.11(7) has a larger uncertainty, but is in quantitative agreement with the estimate of 0.0319(76) obtained earlier from the measured *E*0 strength connecting the 0+ states.

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Intruder–normal-state mixing in ^{30}Mg

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In ^{30}Mg, existing data for $B(E2)$ strengths connecting the ground and excited 0^+ states to the first 2^+ state have been used, together with earlier shell-model predictions of normal and intruder $E2$ strengths, to estimate the intruder-normal state mixing in the 0^+ and 2^+ states. Resulting mixing is small, as expected, and for the ground state my value of 0.11(7) has a larger uncertainty, but is in quantitative agreement with the estimate of 0.0319(76) obtained earlier from the measured $E0$ strength connecting the 0^+ states.

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I. INTRODUCTION

In neutron-rich nuclei near neutron number $N = 20$, the structure of the low-lying states is changing rapidly with changing neutron number. This region of nuclei has been called the "island of inversion." For many of these nuclei, intruder neutron excitations into the fp shell are important in order to reproduce the properties of the low-lying states. But, the extent to which such intruder configurations mix into the ground states (g.s.) of the even-A nuclei is still a matter of some debate. The lowering of the (sd)-(fp) shell gap has been attributed to deformation and/or pairing. Many calculations agree that ^{34}Mg, which must have at least two neutrons in the fp shell, has a deformed g.s., while ^{30}Mg is probably spherical but β soft. In between these two is ^{32}Mg, about which there is a large difference of opinion. Many groups have claimed that the low energy of the first 2^+ state and the large $B(E2)$ connecting it to the g.s. require the g.s. and first 2^+ state to be dominated by the $(fp)^2$ intruder configuration. However, Ref. [1] found that in both ^{30,32}Mg, the 2^+ energy and the $B(E2)$ could be understood with spherical states. Also, information from the recent ^{30}Mg(t,p) experiment [2] contradicts the conventional explanation. This reaction, in reverse kinematics, was used to locate the excited 0^+ state at $E_x = 1.058$ MeV [2]. Straightforward analysis [3] of the cross-section ratio for the two 0^+ states, in a two-state model, demonstrated that the g.s. of ^{32}Mg is predominantly sd shell and the excited 0^+ state has most of the $(fp)^2$ intruder configuration. The observed exc/g.s. ratio was much too large for the g.s. to be mostly the intruder. Analysis of those data obtained a value of 19(2)% for the intruder admixture in the g.s. [3]. The same model was reasonably successful in accounting for the g.s. to 2^+ $B(E2)$ in this nucleus. Analysis with this g.s. wave function demonstrated [4] that the $B(E2)$ could be understood with a 2^+ state that was also largely sd shell. It remains to be seen whether mixed-shell shell-model calculations can reproduce this behavior.

I turn now to ^{30}Mg, where the mixing is expected to be small. Both shell-model (sm) [5–9] and Hartree-Fock-Bogoliubov (HFB) [10–12] calculations suggest that its g.s. is almost pure sd shell. This view is supported by a measurement [13] of the $E0$ strength connecting the g.s. and excited 0^+ state at 1.789 MeV. A simple model of this $E0$ strength resulted in an estimate of the mixing intensity, $b^2 = 0.0319(76)$ [13]. In ^{30}Mg, the $B(E2)$'s are known for both 0^+ states to the first 2^+ state at 1.481 MeV [14,15]. These are listed in Table I, along with shell-model predictions [7] for the normal sd-shell transition and for the intruder $(fp)^2$ one. Caurier *et al.* [7], performed sm calculations for ^{30}Mg totally within the sd shell and for two nucleons in the fp shell. They predicted energies and $B(E2)$ values for the unmixed states. The yrast experimental $B(E2)$ is only slightly larger than the sd sm value (also listed in Table I), and significantly smaller than in ^{32}Mg. Here, I investigate whether these two experimental $B(E2)$'s [14,15] can be understood in a simple two-state mixing model, and used to obtain another estimate of the intruder-normal state mixing.

II. THE MODEL

I define wave functions

$$\Psi(\text{g.s.}) = a\Phi_{0N} + b\Phi_{0I}, \quad \Psi(\text{exc}) = -b\Phi_{0N} + a\Phi_{0I};$$
$$\Psi(2^+) = A\Phi_{2N} + B\Phi_{2I}.$$

The two 0^+ states are obviously orthogonal. Normalization requires $a^2 + b^2 = 1$, and $A^2 + B^2 = 1$. Here, Φ_{0N} and Φ_{2N} are, respectively, the wave functions of the g.s. and first 2^+ state of ^{30}Mg from a shell-model calculation totally within the sd shell. The intruder states, labeled I, are more complicated. They consist of two fp-shell nucleons coupled to a complete set of sd-shell $A = 28$ states, subject only to the total wave function having good J^π and isospin [16]. It appears that the intruder g.s. contains components with $J = 2$ for both the core and the fp-shell pair—and presumably also $J = 4$ and 6. The 2^+ intruder state could then presumably contain terms all the way from 0×2 to 8×6, where the first factor refers to J of the core and the second one to J of the fp-shell pair. I have seen no indications of the likely magnitudes of these various terms. And the fp-shell nucleons were not restricted to be neutrons, although it turned out that they were mostly neutrons. The number quoted for the 0^+ intruder in ^{32}Mg is 1.95 fp-shell neutrons and 0.05 protons [17].

Luckily, we do not need the detailed wave functions because the $B(E2)$'s connecting normal states and connecting intruder states are given [7], together with the statement that the $B(E2)$ transitions between N and I vanish. We define $B(E2;\, i \rightarrow f) = M^2/(2J_i + 1)$, so that if $J_i = 0$, $B(E2) = M^2$.

TABLE I. Relevant $B(E2)$ values (e^2fm^4) in ^{30}Mg.

Source	Transition	$B(E2)$	Ref.	$B(E2)$ Sum	Amp. ratio
Exp.	g.s. $\rightarrow 2_1^+$	295(26)	15	348(27)	2.36(17)
	$0_{exc}^+ \rightarrow 2_1^+$	53(6)	14		
Calc.	$0 \rightarrow 2, N$	265	7	825	1.45
	$0 \rightarrow 2, I$	560	7		

Then we have

$$M(\text{g.s.}) = aAM_N + bBM_I, \quad M(\text{exc}) = -bAM_N + aBM_I$$

and the two terms will be constructive for the g.s.

Here I take the unmixed $E2$ amplitudes from the shell-model calculations. With the normal and intruder $B(E2)$'s from Ref. [7] (listed in Table I), the M's are $M_N = 16.3$ and $M_I = 23.7$, both in $e$fm^2.

III. ANALYSIS AND RESULTS

I first attempt a fit with the model given above for the 2^+ state and the two 0^+ states, with b and B allowed to vary. With only two unknowns (the 0^+ and 2^+ mixing) and two known experimental $B(E2)$ values, it is a simple matter to obtain a unique solution for the mixing from the $E2$ data. Actually, it is easier to fit the $B(E2)$ sum and the ratio of the experimental M's, because we have

$$\sum \equiv B(E2;\ \text{g.s.} \rightarrow 2_1^+) + B(E2;\ 0_{exc}^+ \rightarrow 2_1^+)$$
$$= A^2M_N^2 + B^2M_I^2 = 348(27)\ e^2\text{fm}^4.$$

This relation gives B^2 directly (using $A^2 + B^2 = 1$): $B^2 = 0.281(92)$.

Then, defining $r = M(\text{g.s.})/M(\text{exc})$, we have $r = (1 + xyR)/(yR - x)$, where $x = b/a$, $y = B/A$, and $R = M_I/M_N$. The experimental value of $r = 2.36(17)$, combined with the value of y computed from the value of B^2 above, leads to $b^2 = 0.109(16)$. This value of b^2 is significantly larger than the estimate [13] of 0.0319(76) from the $E0$ analysis. For any given value of B^2, the allowed range of b^2 is quite small. But, for values computed for B^2 within its 1σ range, b^2 can be significantly different. This behavior

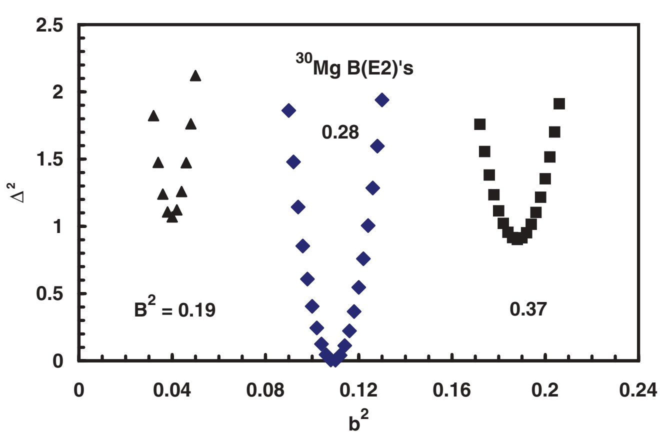

FIG. 1. (Color online) Plot of Δ^2 (defined in the text) vs b^2 for B^2 at its central value and $\pm 1\sigma$.

TABLE II. Mixing intensities in 0^+ and 2^+ states in ^{30}Mg.

Model	g.s.	2^+
Fit sum and ratio	0.109(16)[a]	0.281(92)
Use b^2 from $E0$	0.0319(76)[b]	0.180(14)

[a]For the full range of B^2, b^2 is 0.11(7).
[b]Held fixed at the value from Ref. [13].

is demonstrated in Fig. 1, where I have plotted Δ^2 vs b^2 for three values of B^2—its central value and $\pm 1\sigma$. The definition of Δ^2 is

$$\Delta^2 = \{[M(\text{g.s.})_{exp} - M(\text{g.s.})_{calc}]/\Delta M(\text{g.s.})\}^2$$
$$+ \{[M(\text{exc})_{exp} - M(\text{exc})_{calc}]/\Delta M(\text{exc})\}^2.$$

The allowed range of b^2 is thus from 0.04 to 0.19, with "best" fit at $b^2 = 0.11$. This range certainly overlaps the range of b^2 from the $E0$ analysis, but the uncertainty is disappointingly large (even though the upper limit of b^2 is still reasonably small). Results are listed in Table II.

We could ask what value of B is required if we use the value of $b^2 = 0.0319(76)$ from the $E0$. Results are plotted in Fig. 2. For the entire range of B^2 from 0 to about 0.3, the computed $B(E2)$ for the g.s. is just slightly more than 1σ below the experimental value. However, the calculated value for the excited state varies rapidly with B^2, so that only a narrow range of B^2 is allowed. Thus, using b^2 from the $E0$ analysis, the resulting value of B^2 is 0.180(14), somewhat smaller than the value required by fitting the sum, but consistent with it.

There is another way to estimate the g.s. mixing. Given 0^+ mixing coefficients a and b, and an energy separation of E, the matrix element responsible for the mixing is $V = abE$. If I take $V = 0.415$ MeV [4] from the 0^+ mixing in ^{32}Mg (probably not correct, but perhaps a reasonable approximation), the equation can be used to determine b^2. The result is 0.057 (with an uncertainty that is difficult to estimate), reasonably close to the other two estimates. All three values of b^2 are listed in Table III.

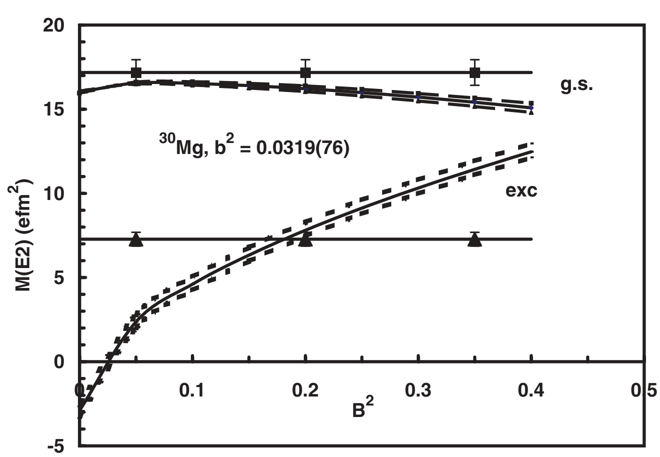

FIG. 2. Experimental and calculated $M(E2)$'s in ^{30}Mg connecting the g.s. (top) and excited 0^+ (bottom) to the first 2^+ state. Calculations use $b^2 = 0.0319(76)$ from the $E0$ analysis of Ref. [13]. Dashed lines represent the uncertainty in b^2.

TABLE III. Intruder intensity in ^{30}Mg (g.s.) from various sources.

Source	b^2	Ref.
$E0$	0.0319(76)	13
$E2$'s	0.11(7)	Present
g.s.-0^+_{exc} energy	0.057	Present

Thus, we have three independent values of the intruder component mixing into the g.s. of ^{30}Mg, and they all agree.

IV. THE SECOND 2+ STATE

The two-component picture of the first 2^+ state can be used to make some estimates of the properties of the second 2^+ state, which is currently unknown, but could be the state at 2.465 MeV [14,18]. Assuming the second 2^+ state to be the orthogonal linear combination to the first one, viz.,

$$\Psi(2_2^+) = -B\Phi_{2N} + B\Phi_{2I},$$

and using the g.s. and 0^+_{exc} wave functions from above, we can estimate the $E2$ strength to the two 0^+ states. Results are $B(E2;\,\mathrm{g.s.} \rightarrow 2_2^+) = 2$ to 8 $e^2\mathrm{fm}^4$, with a large uncertainty, and $B(E2;\,0^+_{\mathrm{exc}} \rightarrow 2_2^+) = 475$ or 500 $e^2\mathrm{fm}^4$. Thus, within this model, the second 2^+ state would have a very small $B(E2)$ to the g.s., but a very large one to the excited 0^+ state. Note that, in this simple model, the first two 2^+ states preserve the summed $E2$ strength of the normal and intruder $0 \rightarrow 2$ transitions. It might be worthwhile to look for a g.s. branch from the 2.465-MeV state, which decays primarily to the first 2^+ at 1.481 MeV—a transition that is probably mostly $M1$ [14]. Even with the very small g.s. $B(E2)$, a g.s. decay branch from the 2.465-MeV state would still be favored over a branch to the excited 0^+ state by a factor of 2 to 10, because of the E^5 factor.

V. SUMMARY

In ^{30}Mg, the $B(E2)$'s connecting the first 2^+ state to the g.s. and excited 0^+ state have been previously measured. I have used a simple model, employing two-state mixing for the 0^+ states and the first 2^+ state, and $B(E2)$'s from a shell-model calculation, to estimate the mixing. The g.s. mixing is small, as expected, and is consistent with an earlier estimate from analysis of the $E0$ strength in a similar two-state model. I suggest that the second 2^+ state should have a very weak g.s. branch.

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