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# Properties of the Lowest $1/2^+$, $T = 3/2$ States in $A = 11$ Nuclei

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## Abstract

Analysis of energies and widths of the lowest $1/2^+$ $T = 3/2$ states in $A = 11$ nuclei suggests that the excitation energy in $^{11}C$ should be about 200 keV below the energy in the literature, and the width should be 4 to 5 times the literature value. Properties of the state in $^{11}B$ and $^{11}N$ are in agreement with the present model.


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## Comments


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# Properties of the lowest $1/2^+$, $T = 3/2$ states in $A = 11$ nuclei

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Abstract. Analysis of energies and widths of the lowest $1/2^+$ $T = 3/2$ states in $A = 11$ nuclei suggests that the excitation energy in $^{11}$C should be about 200 keV below the energy in the literature, and the width should be 4 to 5 times the literature value. Properties of the state in $^{11}$B and $^{11}$N are in agreement with the present model.

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## I. INTRODUCTION

The lowest $T = 3/2$ states in $^{11}$B and $^{11}$C [1] have been a puzzle for a long time. In a study of the low-lying levels of the $A = 11$ isospin quartet [2], we found that the $1/2^-$ and $5/2^+$ states behaved appropriately in the four nuclei, but there was a problem for the $1/2^+$ states. For the ground state (g.s.) of $^{11}$N, the various experimental determinations of its energy and width did not agree within the assigned uncertainties. And, for the known ($^{11}$B) and supposed ($^{11}$C) $1/2^+$ states the experimental widths were only 1/3 (or less) of the values expected. Barker disagreed [3], arguing that these states could have lost width by mixing with $T = 1/2$ states—ignoring the fact that if these states lost width by mixing, then some nearby states would have acquired this missing width. No such states are known.

We have previously used a simple potential model [4] to compute energies of $0^+$, $T = 2$ states using a nuclear plus Coulomb potential to couple states in nuclei $A - 1$ to single nucleons to produce the $T = 2$ nuclei $A$. The model has worked reasonably well. The present situation for $A = 11$, 12 is similar to that for $A = 15$, 16 [5]. In the latter, the $0^+$, $T = 2$ state was not known in $^{16}$F [6], and the $1/2^+$, $T = 3/2$ state was unknown in $^{15}$O [7]. Also, the energy of the $1/2^+$ g.s. of $^{15}$F was poorly defined because of its large width [8,9]. For $A =$ 11, 12 the $0^+$, $T = 2$ state in $^{12}$N has not been identified [1], and the $1/2^+$, $T = 3/2$ state in $^{11}$C is questionable. Similar to $^{15}$F, the g.s. of $^{11}$N [10,11] is too wide to provide a precise energy for it.

For $A = 15$, 16, we were able to use the known masses and relationships among the masses in our model to put constraints on the unknown energies and on the percentage of $s^2$ component in the $0^+$, $T = 2$ state (assumed equal for all five $T = 2$, $A = 16$ nuclei) [12]. That procedure also provided "best" values for the energy of the g.s. of $^{15}$F. Here, we have attempted to apply that technique to $A = 11$, 12.

## II. $^{11}$B

The best evidence for the lowest $T = 3/2$ state in $^{11}$B comes from the $^{10}$Be($p$,$\gamma$) reaction [13]. Those authors found a $T = 3/2$ state with $J^\pi = 1/2^+$ or $(3/2^+)$ at an excitation energy of 12.55(3) MeV and with a width of 230(65) keV. This width persisted in the compilations for more than 30 years [1], until those data were refit [14] and it was found that the data required a broad peak in order to explain the cross section. And this width could come only from the $1/2^+$, $T = 3/2$ state. The resulting excitation energy and width were 12.61(5) MeV and 640(33) keV, respectively, rather than the 210(20) keV width listed in the compilations [1]. Alternative fits with various assumptions (e.g., energy-dependent width vs constant width, four states vs three) gave widths of about 730 and 700(100) keV. Barker later refit the ($p$,$\gamma$) data and provided a width "of order 600 keV" [15]. So, the $^{11}$B puzzle was solved, but the $^{11}$C problem remained.

## III. $^{11}$C

Here, a state at $E_x = 12.16(4)$ MeV has been assigned $T = 3/2$ in several reactions [1], but $J^\pi$ has never been assigned. But, of the known states, it is the only candidate to be the required $1/2^+$ state. This state was observed in the reactions $^{11}$B($^3$He,$t$), $^9$Be($^3$He,$n$), and in $^{10}$B($p$,$p'$) resonance inelastic scattering [16]. In the latter the width is all for decay to the $0^+$, $T = 1$ state of $^{10}$B. The ($^3$He,$t$) data are especially compelling because they were compared to results of the inelastic reaction $^{11}$B($^3$He, $^3$He$'$) leading to the $^{11}$B state discussed above. All these reactions found a small width (as did the inelastic reaction for the $^{11}$B state).

Earlier [4] we found that the value of $\alpha^2$ (the $s^2$ component) in $^{12}$O(g.s.) needed to explain its Coulomb energy was 53(3)%. And, as noted above, our model assumes this component is the same in the five $A = 12$ nuclei. Here, we present our results for various values of this parameter. We use the symbol of a nucleus to represent the mass excess of that nucleus, and an asterisk to denote the lowest $T = 3/2$ state in a $T_z = \pm 1/2$ nucleus. We define $\Delta_B$ and $\Delta_C$ so that $^{11}B^* = {}^{11}B^*$ (Ref. [1]) + $\Delta_B$, $^{11}C^* = {}^{11}C^*$ (Ref. [1]) + $\Delta_C$. As noted above, refitting the $^{10}$Be($p$,$\gamma$) data provided $\Delta_B = 50(50)$ keV [14]—a small correction.

The $^{11}B^*$ and $^{11}C^*$ masses are needed as input to compute the energy of the lowest $0^+$, $T = 2$ state of $^{12}$C. If we require that the model fits this energy exactly within the uncertainties, we arrive at a constraint connecting $\Delta_B$, $\Delta_C$, and $\alpha^2$ represented in Fig. 1. We have temporarily suppressed the uncertainties in the figure, but we return to them shortly. First, we note that the small correction $\Delta_B$ from the ($p$,$\gamma$) refit [14] (horizontal dashed lines) is consistent with a wide range of values of $\alpha^2$. Secondly, the required value of $\Delta_C$ is negative; that is, the "best" excitation energy in $^{11}$C is below the one in the compilation [1]. For $\alpha^2$ in the previously mentioned

 

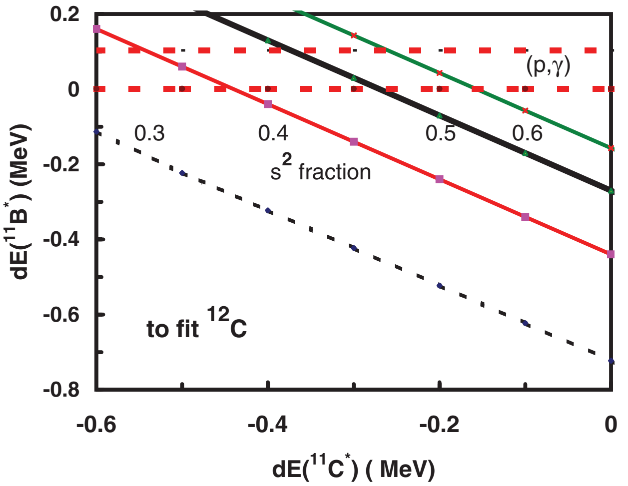


FIG. 1. (Color online) Plot of energy correction in $^{11}$B vs the correction in $^{11}$C needed to fit the $0^+$, $T = 2$ energy in $^{12}$C for various values of $\alpha^2$, the $s^2$ fraction in the $0^+$ state. Horizontal dashed lines represent the 50(50) keV $^{11}$B correction from Ref. [14].

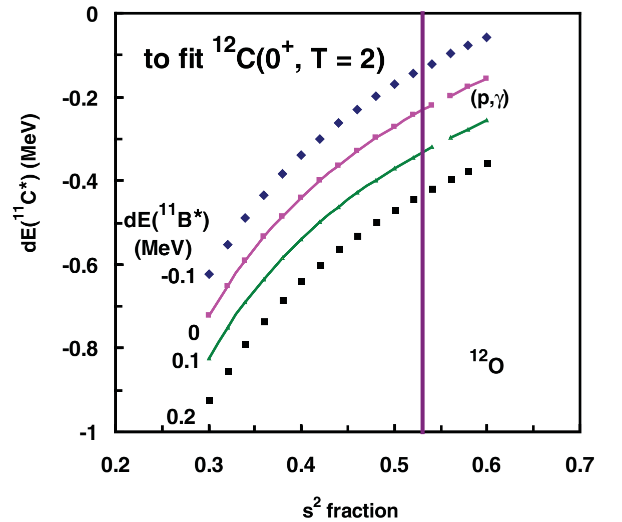


FIG. 2. (Color online) Same information as in Fig. 1, but plot of the $^{11}$C correction vs $\alpha^2$, for various $^{11}$B corrections. Vertical line at $\alpha^2 = 0.53$ is from Ref. [4].

range, the value of $\Delta_C$ is about $-0.23$ MeV. The uncertainty in this value can perhaps be seen better in Fig. 2, where we replot this constraint differently. Here we plot $\Delta_C$ vs $\alpha^2$ for various values of $\Delta_B$. Recall that the earlier estimate for $\Delta_B$ is 0 to 0.1 MeV [14]. The vertical line at $\alpha^2 = 0.53$ is the value required to fit the $^{12}$O Coulomb energy. The best-fit value for $\Delta_C$ from this analysis is $\Delta_C = -0.27(10)$ MeV, where the uncertainty contains contributions from uncertainties in the various energies and in the value of $\alpha^2$. The result for $\Delta_C$ is negative, but with a disappointingly large uncertainty. Smaller uncertainties in the relevant excitation energies would be a great help.

## IV. $^{11}$N

For $^{11}$N(g.s.), the experimental $p + {}^{10}$C resonance energies [10,11,17–20] cover the range from 1.27 to 1.63 MeV, with widths ranging from 0.24(24) to 1.44(2) MeV. Theoretical resonance energies [4,21–23] span a similar range, but calculated widths are all about 0.8 MeV or larger. These are summarized in Table I and Figs. 3 and 4. In Ref. [21], the large uncertainty in the predicted width comes largely from the uncertainty in predicted energy. We also list three "averages" of the experimental results. The most recent $A = 11$ compilation [20] averaged results of the three experiments with the best resolution to get $E_p = 1.49(6)$ MeV, $\Gamma = 0.83(3)$ MeV. The mass evaluation [24] has an average of $E_p = 1.315(46)$ MeV. If we average all five experimental values, the results are $E_p = 1.41(10)$ MeV, $\Gamma = 0.78(11)$ MeV. Our predictions [4] were 1.35(7) and 0.87(10) MeV, respectively.

So far here, we have not made use of the isobaric multiplet mass equation (IMME). If we use the uncorrected energies for $A = 11$, we can compute the value of $d$—the coefficient of a possible cubic term in the IMME. For $A = 12$, $T = 2$, the result was $d = -8.4(17)$ keV. Thus, those masses do not require a nonzero value for $d$. With $d = 0$ in $A = 11$, $T = 3/2$, the masses obey a simple relation: $^{11}\mathrm{N} = {}^{11}\mathrm{Be} - 3\ {}^{11}\mathrm{B}^* + 3\ {}^{11}\mathrm{C}^*$. The mass tables [24] list a $^{11}$N mass

TABLE I. Resonance energies and widths (both in MeV) of $^{11}$N(g.s.).

| | Label | Method | $E_r$ | $\Gamma$ | Ref. |
|---|---|---|---|---|---|
| Expt. | 1 | $p + {}^{10}$C elastic | 1.30(4) | $0.99^{+0.10}_{-0.20}$ | [17] |
| | 2 | $p + {}^{10}$C elastic | $1.27^{+0.18}_{-0.05}$ | 1.44(2) | [10] |
| | 3 | $^{10}$B($^{14}$N,$^{13}$B) | 1.63(5) | 0.4(1) | [18] |
| | 4 | $^{14}$N($^3$He,$^6$He) | 1.31(5) | 0.24(24) | [19] |
| | 5 | $p + {}^{10}$C elastic | 1.54(2) | 0.83(3) | [11] |
| Ave. | 7 | Compilation | 1.49(6) | 0.83(3) | [20] |
| | 8 | Mass evaluation | 1.315(46) | | [24] |
| | 9 | Present | 1.41(10) | 0.78(11) | Present |
| Calc. | 11 | Mirror of $^{11}$Be | 1.35(7) | 0.87(10) | 4 |
| | 12 | Mirror of $^{11}$Be | 1.60(22) | $1.58^{+0.75}_{-0.52}$ | 21 |
| | 13 | Mirror of $^{11}$Be | 1.2 | 1.1 | 22 |
| | 14 | Mirror of $^{11}$Be | 1.34 | 1.47 | 23 |

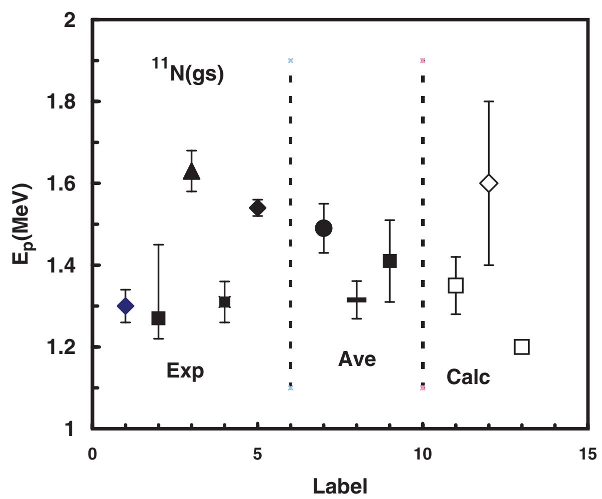


FIG. 3. Resonance energies for $^{11}$N(g.s.) from various sources. Labels are as in Table I.

excess that translates to $E_p = 1.312(50)$ MeV. With a $^{10}$C mass excess [24] of 15.699 MeV, and our definition of $\Delta_C$, the IMME provides $E_p[^{11}\text{N(g.s.)}] = 1.79(19)$ MeV + $3\Delta_C$. (Without the $^{11}$B correction, this value was 1.94(13) MeV.) Without $\Delta_C$ this value is far higher than any previous values for $^{11}$N(g.s.), although with a large uncertainty. Still, this is some confirmation of the need for a nonzero, negative, value of $\Delta_C$.

Recall from above that the IMME, with $d = 0$, requires $E_p$ [$^{11}$N(g.s.)] = 1.79(19) MeV + $3\Delta_c$. The three averages in Table I for $^{11}$N would then yield $\Delta_c = -0.10(7)$ to $-0.16(7)$ MeV—smaller (in absolute value) than, but approximately consistent with, the other analysis presented in Sec. III above. These two proposed energy corrections for $^{11}$C are summarized in Table II.

We make no further use of the IMME, but we do note that our model automatically satisfies the IMME with $d = 0$. The recent correction to the $^{11}$Be(g.s.) mass [25] is too small to have a noticeable effect on the energies discussed here.

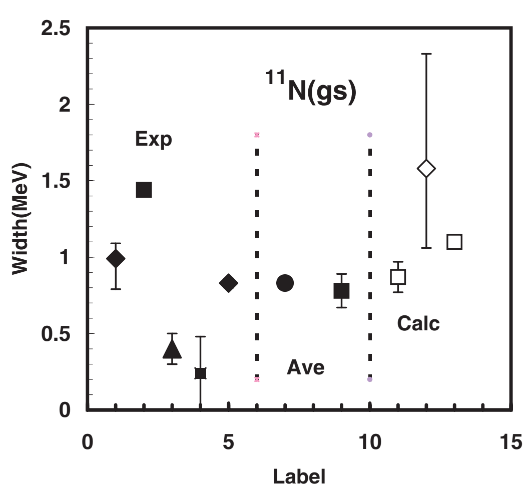


FIG. 4. As Fig. 3, but for the widths.

TABLE II. Proposed energy correction in $^{11}$C.

| Fit using | $\Delta_C$ (MeV) |
|---|---|
| $^{11}$Be, $^{11}$B*; $^{12}$Be, $^{12}$C**, $^{12}$O | −0.27(10) |
| IMME: $^{11}$Be, $^{11}$B*, $^{11}$N | −0.13(7) |
| Weighted average | −0.18(6) |

## V. DISCUSSION

The spectroscopic factors for the four $1/2^+$, $T = 3/2$ states are listed in Table III. For all but $^{11}$Be, these are obtained from the expression $C^2S = \Gamma_{expt}/\Gamma_{sp}$, where $C^2 = 1/3$, $2/3$, and 1 for $^{11}$B, $^{11}$C, and $^{11}$N, respectively. Estimates of $\Gamma_{sp}$ are listed in the table. They were calculated using a Woods-Saxon potential (plus Coulomb), with $r_0 = 1.25$ fm and $a = 0.65$ fm. The depths were adjusted to reproduce the observed energies.

The difficulty with $^{11}$C is apparent. The spectroscopic factor derived from its width is only about 20% of $S$ for the other three nuclei—and the $S$'s should all be equal. As pointed out above, if the $^{11}$C state loses width (spectroscopic strength) by mixing with $T = 1/2$ states, then one or more of them should exhibit this strength, and none do. We recall that the $1/2^+$, $T = 3/2$ state in $^{15}$O has also never been identified. An early candidate turned out to have $T = 1/2$, as demonstrated by its large width for a decay that would be forbidden for a $T = 3/2$ state.

The problem in $^{11}$C is not with the $sp$ widths. Barker [3] used a potential model to compute $\Gamma_{sp}$ for states at the experimental energies. His values (last column of Table III) are similar to ours. For $^{11}$C, his $sp$ width is actually 12% larger than ours. We thus expect a $1/2^+$, $T = 3/2$ state near 12 MeV in $^{11}$C, with a width of about 1.2 MeV and $C^2S_p \sim 0.50$.

We have given considerable thought to finding a reaction to make these states in $^{11}$C (and $^{15}$O). The $(p,t)$ reaction does not work, because the targets do not contain the $2s_{1/2}$ nucleon that is the main feature of these states. The ($^3$He,$t$) reaction populates both $T = 1/2$ and $3/2$ states, as does ($^3$He,$n$). Finding a state at roughly the expected energy that preferentially decays to the $0^+$, $T = 1$ state of $^{10}$B in the $^{10}$B$(p,p')$ reaction was encouraging, but the width reported there is also too small by about a factor of five. In the ($^3$He,$n$) reaction the background (both real and from $T = 1/2$ states) is a serious problem. This reaction does have the advantage that cross-section ratios for different $T = 3/2$ final states should be approximately the same in ($^3$He,$n$) and $(t,p)$ on the same target and under similar kinematic conditions. Thus, for a $^9$Be target, we expect the ratio $\sigma(1/2^+)/\sigma\ (1/2^-)$ in ($^3$He,$n$) to be roughly equal to the same ratio in $(t,p)$. In the latter, the ratio at the peak angle and the ratio of angle-integrated cross sections were both about 0.22. The best candidate might be $^{10}$C$(d,p)$ in reverse kinematics. In that reaction, $C^2S_n$ would be about 0.25.

There is one last possibility to consider—could interference between overlapping $T = 1/2$ and $T = 3/2$ states cause a broad negative dip? If so, the void between 11.44 and 12.16 MeV in $^{11}$C could actually be the negative profile of the $1/2^+$,

TABLE III. Widths (in MeV) and spectroscopic factors for $1/2^+$, $T = 3/2$ states in $A = 11$ nuclei.

| Nuclei | $E_x$ (MeV) | $\Gamma$ | $\Gamma_{sp}$ | $S$ | $\Gamma_{sp}$ (Ref. [3]) |
|---|---|---|---|---|---|
| $^{11}$Be | 0 | | | 0.80[e] | |
| $^{11}$B | 12.61(5)[a] | 0.640(33)[a] | ~2.4 | 0.80 | 2.31 |
| $^{11}$C | 12.16(4)[b] | 0.27(5)[b] | ~2.4 | 0.16 | 2.69 |
| $^{11}$N | 0[c] | 0.83(3)[d] | ~1.3 | ~0.64 | 1.42 |

[a]Includes the correction from Ref. [14].
[b]Ref. [1].
[c]$E_p = 1.32$ to 1.49 MeV (averages in Table I).
[d]Average in Table I.
[e]As averaged in Ref. [4].

$T = 3/2$ state. But, would this interference be about the same in, say, ($^3$He,$t$) and ($p$,$p$')?

## VI. CONCLUSION

We have noted here that the global averages of the energy and width of $^{11}$N(g.s.) are consistent with the calculations. We conclude that the small correction found earlier [14] for the energy of the $1/2^+$, $T = 3/2$ state in $^{11}$B is consistent with the current analysis, and that the previous problem [2] with the width in $^{11}$B has been solved [14]. A larger, negative energy correction [180(60) keV] is needed for $^{11}$C. That finding presents a problem, because in $^{11}$C there is nothing between 11.44 and 12.16 MeV. The width in $^{11}$C should be 4 to 5 times the currently accepted value. From inspection of the relevant spectra, it is difficult to see how the $1/2^+$ width could be several times the estimate in the compilation [1], especially if the energy is shifted lower. Of course, inspection of the spectra also provided a small width for $^{11}$B*, which we now know was a factor of three too small. It would be very useful to find a way to settle this width question in $^{11}$C.

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