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Abstract

I have examined the $B(E2)$'s in ^{12}Be connecting the first 2^+ state to the first two 0^+ states. I find that they can be understood in the simple model that has been successful for a variety of other observables in this nucleus, but only if the 2^+ state has an excited-core component.

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$B(E2)$ values in ^{12}Be and core excitation

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I have examined the $B(E2)$'s in ^{12}Be connecting the first 2^+ state to the first two 0^+ states. I find that they can be understood in the simple model that has been successful for a variety of other observables in this nucleus, but only if the 2^+ state has an excited-core component.

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I. INTRODUCTION

It is now fairly well established that the ground state (g.s.) and 2^+ first-excited state of ^{12}Be are dominated by two-neutron excitations into the sd shell. This fact was first demonstrated [1] with the $^{10}\text{Be}(t,p)$ reaction, in which the g.s. is about five times as strong as the pure p -shell state would be, and the 2^+ state is about 20 times the p -shell expectation. The analysis of the (t,p) data and a calculation [2] of the ^{12}Be - ^{12}O energy difference determined the g.s. p -shell component to be about 32(3)%. A similar percentage was subsequently confirmed [3,4] in other independent processes.

For the 2^+ state, the (t,p) reaction demonstrated sd -shell dominance, but did not determine the p -shell admixture, but certainly it is less than about 0.3. Reanalysis [5] of data for the $^{14}\text{C}(p,t)$ reaction [6] suggested that the second $T = 2$ peak contained both the 2^+ and the excited 0^+ states, with the physical 2^+ state containing about 19(9)% of the p -shell 2^+ state. If all the cross section of the second peak had been attributable to the 2^+ alone, this p -shell percentage would have been 24(8)%.

So, it seems reliably established that the g.s. has about 65%–70% $(sd)^2$ and the 2^+ has 70%–90% $(sd)^2$. A simple shell-model calculation gave the d^2/s^2 ratio = 0.22/0.78 for the g.s. [5] and the d^2/ds ratio = 0.13/0.87 for 2^+ [7]. In our model, the excited 0^+ at 2.24 MeV is just the orthogonal admixture of the two basis states: $(sd)^2$ and p -shell. These wave functions are listed in Table I.

Now, consider the $E2$ transitions connecting the 2^+ to the two 0^+ states. These two $B(E2)$'s (Table II) have recently been measured. For $0^+ \rightarrow 2^+$ they are $7.0(6) e^2 \text{fm}^4$ for the excited 0^+ state [8] and $40 \pm 11 \pm 4 e^2 \text{fm}^4$ for the g.s. [9]. The shell model of Ref. [8] predicts $B(E2; 0_2 \rightarrow 2) = 25 e^2 \text{fm}^4$ with “standard” effective charges $e_p = 1.3e$, $e_n = 0.5e$. Reducing them to $e_p = 1.2e$ and $e_n = 0.2e$ still gives a large $B(E2) = 9.13 e^2 \text{fm}^4$ [8], but now comparable to the experimental value. With antisymmetrized molecular dynamics (AMD), the calculated $B(E2)$ underestimates the observed value by about a factor of three [8]. Using single particles in a deformed potential, with approximately equal mixing between $1/2^+$ and $1/2^-$ Nilsson orbitals, Hamamoto and Shimoura [10] obtain $B(E2; 0_2 \rightarrow 2) = 141 e_n^2 \text{fm}^4$. Thus, to fit the observed value of $7.0(6) e^2 \text{fm}^4$ would require a neutron effective charge of $e_n = 0.22(1)e$, quite a small value.

Umaya *et al.* [11] performed mixed-shell shell-model calculations for some neutron-rich Be and C nuclei, using two different psd interactions SFO [12] and PSDMK [13].

For the wave functions of the first 0^+ and 2^+ states in ^{12}Be , results for the two calculations were quite different, and neither was close to the wave functions now in common use. The SFO results were closer, but had 58.5% p shell in the g.s. and only 4.4% in the 2^+ state. However, predictions for the g.s. $B(E2)$ were much better: $34.4 e^2 \text{fm}^4$ with SFO and 31.0 with PSDMK, close to the later experimental value of $40 \pm 11 \pm 4$ [9]. They did not give a calculated value for $0_2^+ \rightarrow 2^+$.

The aim here is to investigate whether these $B(E2)$'s can be understood within the framework of the same model that has been successful for several other properties of ^{12}Be .

II. THE MODEL

In the simplest model the wave functions are

$$\begin{aligned} \text{g.s.} &= a {}^{10}\text{Be}(p\text{-shell g.s.}) \times v(sd)_0^2 \\ &\quad + b {}^{12}\text{Be}(p\text{-shell g.s.}), \\ 2^+ &= A {}^{10}\text{Be}(p\text{-shell g.s.}) \times v(sd)_2^2 \\ &\quad + B {}^{12}\text{Be}(p\text{-shell } 2_1^+), \\ 0_{\text{exc}}^+ &= -b {}^{10}\text{Be}(p\text{-shell g.s.}) \times v(sd)_0^2 \\ &\quad + a {}^{12}\text{Be}(p\text{-shell g.s.}). \end{aligned}$$

Later I add another term to the 2^+ wave function: $C {}^{10}\text{Be}(p\text{-shell } 2_1^+) \times v(sd)_0^2$.

Define $M^2(E2) = (2J_i + 1)B(E2; i \rightarrow f)$ so that the value of M is independent of the direction of the transition. Then, in an obvious notation, for the two-component 2^+ state, the physical $E2$ amplitudes are

$$\begin{aligned} M(\text{exc}) &= -bA M(sd) + aB M(1p); \\ M(\text{g.s.}) &= aA M(sd) + bB M(1p), \end{aligned}$$

where $M(sd)$ is the $E2$ amplitude connecting the 0_1^+ and 2_1^+ $(sd)^2$ states, and $M(1p)$ is that for the p -shell 0^+ and 2^+ states. In terms of shell-model amplitudes, we have

$M(E2) = e_p A_p + e_n A_n$, where A_p and A_n are bare shell-model $E2$ transition matrix elements. In the present space, $M(sd) = e_n A_n(sd)$, because we have no sd -shell protons; and $M(1p) = e_p A_p(1p)$, because the neutron p shell is filled. The value of $M(1p)$ is about $5.88 e \text{fm}^2$ from a shell-model calculation [14] totally within the p shell, using a proton effective charge of $e_p = 1.2e$ (the neutron shell is full; hence, no n participation.) This value of $M(1p)$ corresponds

TABLE I. Energies and wave-function intensities in ^{12}Be .

J^π	Space	State	E_x (MeV)	s^2	d^2	p shell	Reference	
0^+	$(sd)^2$	0_1^+	0.20	0.78	0.22	—		
	$(sd)^2$	0_2^+	4.35	0.22	0.78	—		
	$(sd)^2 + p$ shell	g.s.			0.53	0.15	0.32	[2,5]
		exc 0^+			-0.25	-0.07	0.68	
2^+	$(sd)^2$	2_1^+	3.63	ds	d^2	—		
	$(sd)^2$	2_2^+	5.42	0.87	0.13	—		
	$(sd)^2 + p$ shell				0.13	0.87	—	
			2.11 MeV		0.71	0.10	0.19	[7]

to $B(E2; 2^+ \rightarrow 0^+) = 6.92 e^2 \text{fm}^4$ for the p -shell ^{12}Be . With typical 0_1^+ and 2_1^+ states within the $(sd)^2$ space, the sd -shell $E2$ matrix element is $M(sd) \sim 4.5 e \text{fm}^2$ for an effective charge of $e_n = 0.5e$. (With an effective charge of $e_n = 0.5e$ and the pure sd -shell wave functions of Table I, $M(sd)$ is $4.33 e \text{fm}^2$.) If the effective charge is $e_n = (Z/A)e$, we have $M(sd) \sim 3.3 e \text{fm}^2$. This is a pure neutron excitation. We would hope to be able to find a solution with $M(sd)$ between these two limits.

III. CALCULATIONS

Oddly enough, the $B(E2)$ from the excited 0^+ state is very close to the value it would have (Table III) if the 2^+ state were pure $^{10}\text{Be}(p\text{-shell g.s.}) \times \nu(sd)_2^2$, that is, if $B = 0$ in the 2^+ wave function. However, if we ignore the presence of any p -shell amplitude in the 2^+ , it is obvious that the ratio of $B(E2)$'s connecting the 0^+ states to the 2^+ is independent of the d^2/s^2 ratio in the 0^+ state and independent of the ds/d^2 ratio in the 2^+ . Also, the ratio is independent of the value of the $2^+ \rightarrow 0^+$ $(sd)^2$ $E2$ matrix element [and, of course, independent of $M(1p)$]. Furthermore, the ratio of $B(E2)$'s is just equal to the $(sd)^2$ intensity ratio in the two 0^+ states. The measured ratio of 0.175(52) is significantly smaller than the ratio of 0.32/0.68 expected. Also, the sum of the two experimental $B(E2)$'s is too large for the assumption of no p -shell component in the 2^+ state.

I use the procedure outlined below. Throughout I adopt the 0^+ values of $a^2 = 0.68$, $b^2 = 0.32$ from Refs. [2] and [5]. The equations for $M(\text{exc})$ and $M(\text{g.s.})$ given above can be rewritten as

$$AM(sd) = aM(\text{g.s.}) - bM(\text{exc}) \text{ and}$$

$$BM(1p) = aM(\text{exc}) + bM(\text{g.s.}).$$

From the experimental $B(E2) = 7.0(6) e^2 \text{fm}^4$, we have $M(\text{exc}) = \pm 2.65(11) e \text{fm}^2$ (Table II). Clearly, the negative square root must be chosen for $M(\text{exc})$. With a , b known,

TABLE II. Experimental $B(E2; 0^+ \rightarrow 2^+)$ in ^{12}Be .

Transition	$B(E2)$ ($e^2 \text{fm}^4$)	Reference	$M(E2)$ ($e \text{fm}^2$)
$0_2 \rightarrow 2_1$	7.0(6)	[8]	$\pm 2.65(11)$
$\text{g.s.} \rightarrow 2_1$	40(12)	[9]	6.32(92)

results are $BM(1p) = 1.39(53) e \text{fm}^2$ and $AM(sd) = 6.71(98) e \text{fm}^2$, listed in Table IV. Recall that we are using $M(1p) = 5.88 e \text{fm}^2$. Thus, $B = 0.236(90)$, $B^2 = 0.056(42)$. Then, with A , B related via $A^2 + B^2 = 1$, we can compute $M(sd) = 6.9(10) e \text{fm}^2$. Note that, even for such small values of B^2 , the value of $M(sd)$ needed becomes unreasonably large. This is not surprising, because this experimental $B(E2)$ is very close to the result for $B = 0$. Thus, even though we obtain an approximate fit to both $B(E2)$'s with only two terms in the 2^+ wave function, the results are unsatisfactory: The values of B^2 are very small, and the required value of $M(sd)$ is larger than seems reasonable. In the two-component 2^+ , the sum of the two $B(E2)$'s is $\Sigma B(E2) = A^2 M^2(sd) + B^2 M^2(1p)$. Here, too, the experimental value is larger than the calculated one for any reasonable range of $M(sd)$.

So, what is wrong with this simple model, and why is it unable to account for these two $B(E2)$'s? I investigate next the possible presence of a component of the ^{12}Be 2^+ state that has a ^{10}Be excited core, specifically $^{10}\text{Be}(2^+) \times \nu(sd)_0^2$. We know this term is present at some level, but it has been ignored before because it has no direct one-step route in the (t, p) reaction. With this new term for the 2^+ , we have

$$2^+ = A {}^{10}\text{Be}(p\text{-shell g.s.}) \times \nu(sd)_2^2 + B {}^{12}\text{Be}(p\text{-shell } 2_1^+)$$

$$+ C {}^{10}\text{Be}(p\text{-shell } 2_1^+) \times \nu(sd)_0^2.$$

Then the $E2$ amplitudes are

$$M(\text{g.s.}) = aAM(sd) + b[BM(1p) + CM(^{10}\text{Be})];$$

$$M(\text{exc}) = -bAM(sd) + a[BM(1p) + CM(^{10}\text{Be})],$$

where $M(^{10}\text{Be}) = 5B(E2; 2^+ \rightarrow 0^+)$ in ^{10}Be . The new term could contribute significantly here, because the ^{10}Be $B(E2)$ is large, about $9.2 e^2 \text{fm}^4$ [15]. I expect C^2 to be small, certainly < 0.3 from results of the (t, p) reaction. With this third component in the 2^+ state, a fit to the excited 0^+ $B(E2)$ is underdetermined. An extra complication here is that the C term also contributes (but only slightly) to the $^{14}\text{C}(p, t)$ reaction

TABLE III. Calculated $B(E2; 0_2 \rightarrow 2_1)$ with pure $(sd)^2$ 2^+ .

e_n	$M(sd)$ ($e \text{fm}^2$)	$B(E2)$ ($e^2 \text{fm}^4$)
$0.5e$	4.5	6.48
$(Z/A)e$	3.3	3.43

TABLE IV. Results for two-component 2^+ state. Units of M are $e \text{ fm}^2$.

$M(\text{exc})$	$M(\text{g.s.})$	$AM(sd)$	$BM(1p)$	B^a	$M(sd)$
$\pm 2.65(11)$	6.32(92)	6.71(98)	1.39(53)	0.236(90)	6.9(10)

^aComputed from value of $BM(1p)$ in fourth column, using $M(1p) = 5.88 e \text{ fm}^2$ (see text).

through core excitation [16] in $^{14}\text{C}(\text{g.s.})$. So I withhold the (p,t) constraint until later.

A set of approximate solutions have $B \sim C$, close to the parameter contour that would maximize the sum of the two $B(E2)$'s. I mentioned above that, in the simplest model, this sum is larger experimentally than expected for reasonable values of $M(sd)$. Results of one such search are displayed in Fig. 1, where I plot vs A^2 the value of $M(sd)$ that fits the excited 0^+ $B(E2)$ together with the calculated and experimental g.s. $M(E2)$. Note that the calculated and experimental values agree to better than 1σ for a range of A^2 values just below $A^2 = 0.67$. The horizontal lines crossing the $M(sd)$ curve are at $M(sd) = 4.5$ and $3.3 e \text{ fm}^2$, the values corresponding to $e_n = 0.5e$ and $(Z/A)e$, respectively. Agreement with $M(\text{g.s.})$ can be had for most of the $M(sd)$ range between these two limits. If we now impose a constraint from (t,p) that $A^2 = 0.70(7)$, our solution can be limited to the upper end of the allowed range in Fig. 1. For $A^2 = 0.66$, $B^2 = 0.15$, $C^2 = 0.19$, we have $M(sd) = 3.61(25) e \text{ fm}^2$. Furthermore, if I use these values of B, C to estimate the 2^+ cross section in $^{14}\text{C}(p,t)$, I find that the 2^+ wave function contains 18% of the p -shell 2^+ cross section. These results are summarized in Table V. These results are reasonably close to the estimates of Ref. 7 (in Table I), but the 2^+ p -shell component is now about 15%, rather than 19(9)%, and the sd -shell intensity of 85% (instead of 81%) has an excited-core component.

For a given value of A , the predictions are a very slow function of B and C , as long as we have $A^2 + B^2 + C^2 = 1$. So, we have not really independently determined B and C ,

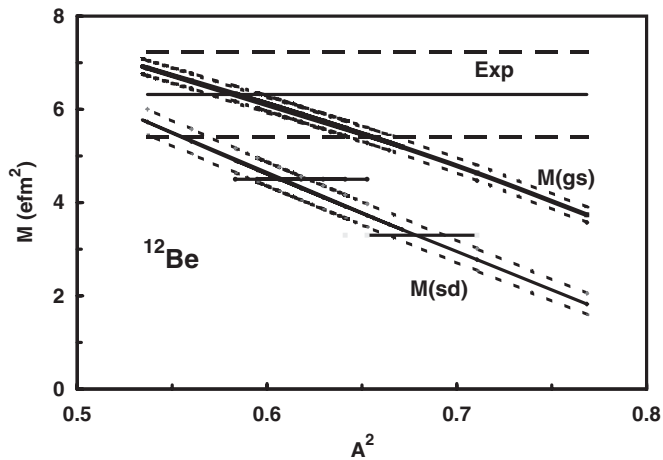


FIG. 1. The experimental value of $M(\text{g.s.}) = 6.32(92) e \text{ fm}^2$; plotted vs A^2 is the value of $M(sd)$ that fits the $0^+_{\text{exc}} \rightarrow 2^+ B(E2)$, and the value of $M(\text{g.s.})$ computed with this $M(sd)$, but now for a three-component 2^+ state. Horizontal lines crossing $M(sd)$ are at 3.3 and $4.5 e \text{ fm}^2$ (see text).

but some combination of them. To get the $B(E2)$ sum large enough we need C slightly greater than B .

IV. POSSIBLE REFINEMENTS

Some minor components that are known to exist in the wave functions have been omitted in the current treatment. These include

- (i) $(sd)^2$ configurations involving the $d_{3/2}$ orbital, abbreviated as d' here,
- (ii) the configuration $^{10}\text{Be}(2^+) \times (sd)_2^2$, which we call 2×2 , and
- (iii) configurations with four sd -shell neutrons coupled to states in ^8Be , which we call $4 \hbar\omega$.

Inclusion of the d' components would increase the $(sd)^2$ (t,p) cross section by a few percent, because of constructive coherence. Neither of the other two components has a direct one-step route in the (t,p) reaction. Their inclusion would thus slightly decrease the (t,p) predictions, through renormalization. Inclusion of all three components should leave the (t,p) cross sections basically unchanged.

Each of the three components, if included, would lead to a renormalization of the quantity called $M(sd)$ above, thereby widening its allowed range. We do not use a calculated value for this quantity, but treat it as a free parameter, within certain limits. Because the resulting fitted value was within the previous allowed range, it would naturally also be within a wider allowed range.

We conclude that omission of these three components in our simple model has no appreciable effect on the outcome. Of course, even the simplest $(0+2) \hbar\omega$ shell-model calculation would include the first two. It is likely that such calculations should be expanded to include $4 \hbar\omega$. We remarked in the Introduction that no shell-model calculations have been successful in explaining both the $B(E2)$'s that are the subject of the present analysis.

V. SUMMARY

I have calculated the $B(E2)$'s connecting the first 2^+ state of ^{12}Be with the first two 0^+ states, using previous wave functions for the latter and both two- and three- component models for the former. With two components in the 2^+ state,

TABLE V. Wave-function intensities for three-component 2^+ state.

Component	Intensity
$^{10}\text{Be}(p\text{-shell g.s.}) \times v(sd)_2^2$	0.66
$^{12}\text{Be}(p\text{-shell } 2^+_1)$	0.15
$^{10}\text{Be}(p\text{-shell } 2^+_1) \times v(sd)_0^2$	0.19

the model requires the sd -shell $E2$ amplitude to be larger than is reasonable. With three components, the calculations are underdetermined, but agreement can be achieved for a range of the two small components, with about 66% of the dominant one. These are listed in Table V.

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