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## **Integrated Workforce Capacity and Inventory Management Under Labour Supply Uncertainty**

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# Integrated Workforce Capacity and Inventory Management Under Labour Supply Uncertainty

## Abstract

In a manufacturing environment with volatile demand, inventory management can be coupled with dynamic capacity adjustments for handling the fluctuations more effectively. In this study, we consider the problem of integrated capacity and inventory management under non-stationary stochastic demand and capacity uncertainty. The capacity planning problem is investigated from the workforce planning perspective where the capacity can be temporarily increased by utilising contingent workers from an external labour supply agency. The contingent capacity received from the agency is subject to an uncertainty, but the supply of a certain number of workers can be guaranteed through contracts. There may also be uncertainty in the availability of the permanent and contracted workers due to factors such as absenteeism and fatigue. We formulate a dynamic programming model to make the optimal capacity decisions at a tactical level (permanent workforce size and contracted number of workers) as well as the operational level (number of workers to be requested from the external labour supply agency in each period), integrated with the optimal operational decision of how much to produce in each period. We analyse the characteristics of the optimal policies and we conduct an extensive numerical analysis that helps us provide several managerial insights.

## Keywords

inventory, capacity management, flexible capacity, workforce availability, supply uncertainty

## Disciplines

Business Administration, Management, and Operations | Operations and Supply Chain Management | Other Computer Sciences

# Integrated Workforce Capacity and Inventory Management Under Temporary Labor Supply Uncertainty

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## Abstract:

In a manufacturing environment with volatile demand, inventory management can be coupled with dynamic capacity adjustments for handling the fluctuations more effectively. In this study, we consider the problem of integrated capacity and inventory management under non-stationary stochastic demand and flexible capacity uncertainty. The capacity planning problem is investigated from the workforce planning perspective where the capacity can be temporarily increased by utilizing contingent workers from an external labor supply agency. The contingent capacity received from the agency is subject to an uncertainty, but the supply of a certain number of workers can be guaranteed through contracts. We formulate a dynamic programming model to make the optimal capacity decisions at a tactical level (permanent workforce size and contracted number of workers) as well as the operational level (number of workers to be requested from the external labor supply agency in each period), integrated with the optimal operational decision of how much to produce in each period. We analyze the characteristics of the optimal policies and we conduct an extensive numerical analysis that helps us provide several managerial insights.

*Keywords:* Inventory, Capacity Management, Flexible Capacity, Workforce Availability, Supply Uncertainty

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## 1 Introduction

Production and capacity decisions of manufacturing firms are significantly affected by demand volatility. In some industries, dynamic capacity adjustments arise as an effective tool

for handling this volatility. The production capacity can be temporarily increased by acquiring external capacity resources such as outsourcing, renting machinery, hiring contingent workers, etc. Effective utilization of such resources results in increased demand responsiveness and reduced operational costs. However, external capacity may not always be available at the desired quantity and/or quality in the environment. Therefore the uncertainty of external capacity supply should be considered in production planning. In this study, we consider the integrated planning of production/inventory and capacity under demand and external capacity supply uncertainties.

Capacity can be defined as the maximum amount of production that can be achieved by utilizing internal and external resources, whereas capacity flexibility stands for the ability to change the capacity temporarily. Especially when the inventory holding and/or back-ordering costs are high, capacity flexibility may prove to be an efficient tool for meeting the volatile demand. We consider labor intensive manufacturing environments and hence we consider capacity in terms of the workforce. Throughout the text, we use the terms “workers” and “capacity” interchangeably. We classify the production capacity under two main categories: Permanent capacity and contingent capacity. Permanent capacity is formed by the company’s own workforce under a steady payroll, whereas the contingent capacity is formed by the workers that can be acquired temporarily from an external labor supply agency (ELSA). Manufacturer’s request for contingent workers may be totally or partially unmet by the ELSA due to the lack of availability and/or skill requirements. In case there is a high demand for contingent workers in the market at the time of the request, or if the manufacturer requires the workers in short notice, the risk of the request not being met in full terms is higher. Moreover, ELSAs may not be willing to fulfill a specific request at a specific time, considering potentially better options. Therefore, the availability of contingent workers may be a major concern when the manufacturer relies on external capacity for production.

A labor supply contract between the manufacturer and the ELSA is a possible way of alleviating the impacts of labor supply uncertainty on the manufacturer where the manufacturer pays a certain fee per contracted worker per period (reservation cost), and the ELSA is committed to provide the required number of workers up to the contracted quantity to the manufacturer with certainty with an additional fee per worker requested (utilization cost). Note that this type of contracting is known as “option contracting”. The manufacturer may still request temporary workers in addition to the contracted workers, but the supply of those

workers are subject to uncertainty. Under this setting, we classify contingent workers under two categories: contracted workers and temporary workers.

Dynamic adjustments of the permanent capacity, such as hiring or firing, are generally too costly. Moreover, such adjustments tend to have negative effects on the efficiency of workers due to the social and motivational effects. Therefore we consider the determination of the permanent capacity level as a tactical decision that is made at the beginning of the planning horizon and not changed until the end of it. Utilizing flexible capacity is a means of overcoming these issues, and we consider this as one of the two main operational tools of coping with fluctuating demand, along with holding inventory. Consequently, the decisions that we consider are the determination of the permanent workforce size and the number of contracted workers from the ELSA at the beginning of the planning horizon, as well as the number of temporary workers to request from the ELSA and the production quantity in each period.

There exists a significant usage of flexible workforce in many countries. For example, 6.6% of the active labor force of the Netherlands was composed of flexible workers (temporary, standby, replacement, and such other workers) in 2003 (Beckers, 2005). US Bureau of Labor Statistics (2006) indicates that in February 2005 there were 14.8 million flexible workers (independent contractors, on-call workers, temporary help agency workers, and workers provided by contract firms) constituting 10.7% of total employment. Aside from the workers with alternative work arrangements as indicated above, contingent workers accounted for 4.1% of the total US employment. In March 2006, 7.9% of the active labor force in Turkey was composed of contingent workers (Turkish Statistical Institute, 2006). Contingent workers can be hired anytime and are generally paid for labor hours. The wage rate for contingent workers tend to be lower than that of their permanent counterparts, however their costs to the hiring firms are generally higher. Productivity of contingent workers may vary for industries requiring different levels of skills, with the productivity loss increasing in skill requirements.

## 2 Literature Review

Capacity planning has been analyzed extensively in all levels of decision making. An in depth review, presenting the formulation and solution of strategic capacity problems, is provided by Van Mieghem (2003). Holt et al. (1960) pioneered the research in the field of

workforce planning and flexibility, with their seminal work analyzing the trade-off between keeping large permanent workforce levels and frequent capacity adjustments. Our model is considering the same problem in essence, extending it to the case of demand and capacity supply uncertainty. Wild and Schneeweiss (1993) analyze manpower capacity planning with a hierarchical approach using stochastic dynamic programming.

In a particularly relevant work, Milner and Pinker (2001) consider the problem of designing labor supply contracts between firms and ELSAs under demand and temporary labor supply uncertainty. The authors consider a single period setting where the supply uncertainty is either in terms of productivity loss or unavailability. In the former case, if the labor request that is placed after demand materialization exceeds the contracted quantity, it is met with certainty by the ELSA at a higher cost. In the latter case the unavailability is a function of the number of temporary workers available in the market and the fee the firm pays to the ELSA per temporary worker. In our work, we consider a multi-period setting and we focus on several unavailability structures of labor supply. Moreover, in our model, capacity decisions are made before the demand is materialized, which implicitly takes the supply lead time into account since it can be considered that contingent workers are requested at the end of the previous period.

Among the papers that consider integrated production and capacity planning, the following papers are relevant to our work. Pinker and Larson (2003) consider the problem of managing permanent and contingent workforce levels under uncertain demand where inventory holding is not allowed. The sizes of regular and temporary labor are decision variables that are fixed throughout the planning horizon, but the capacity level may be adjusted by setting the number of shifts for each class of workers. Dellaert and de Kok (2004) investigate the integrated flexible capacity and production planning problem considering a production capacity composed of long-term contract workers and temporary workers. The approach of planning capacity and production in an integrated manner outperforms the decoupled approach. Hu et al. (2004) also investigate an integrated flexible capacity and production planning problem on a continuous-time framework under Markov-modulated demand. In a similar problem, Tan and Gershwin (2004) study production and subcontracting strategies with limited production capacity and fluctuating demand, considering lead time sensitive customers. Atamturk and Hochbaum (2001) focus on the integrated capacity and production planning problem under a non-stationary deterministic demand setting exploiting the trade-offs between capacity expansions, subcontracting and carrying inventory. Angelus

and Porteus (2002) present a simultaneous capacity and production planning problem for short life-cycle products, considering capacity expansions as well as contractions. Yang et al. (2005) consider a production/inventory system under Markovian internal capacity levels and outsourcing option, where the outsourcing decision is made after observing the realized capacity and the demand.

Our work is closely related to the problems considered by Tan and Alp (2005), Alp and Tan (2007), and Mincsovcics et al. (2006). These three papers consider settings similar to ours, ignoring the labor supply uncertainty. Tan and Alp (2005) focus on the operational decisions under the existence of fixed costs for initiating production and for using contingent capacity. Alp and Tan (2007) extend this analysis by including the tactical level decision of determining the permanent capacity levels. Finally, Mincsovcics et al. (2006) model and analyze the problem under a lead time associated with the acquisition of contingent capacity.

Considering the field of production/inventory planning under random capacity/yield, Yano and Lee (1995) provide an extensive review of the literature on lot sizing under random production or procurement yields. Ciarallo et al. (1994) analyze the optimality of extended myopic policies under uncertain capacity and uncertain demand in a periodic review setting. Kouvelis and Milner (2002) analyze the joint effects of demand and supply uncertainty on capacity and outsourcing decisions in multi-stage supply chains. Authors indicate that as the supply uncertainty increases capacity investments increase. In a problem relevant to ours, Schmitt and Snyder (2006) consider a system with supply disruptions. The concept of reservation from a reliable supplier is similar to the contracting concept in our study. Different than this stream of research, we also consider a fixed permanent capacity. Moreover, the capacity decision which is subject to uncertainty and the production decisions are separate variables in our model.

### 3 Model Formulation

In this section, we provide a dynamic programming model that can be used to solve the integrated capacity and inventory management problem under consideration. We first present our basic definitions, assumptions and settings.

We define capacity position,  $w$ , as the total amount of capacity requested by the manufacturer. Capacity level is defined as the production capacity observed after the labor supply uncertainty is resolved. If the capacity position is less than or equal to the permanent pro-

duction capacity plus the contracted capacity ( $w \leq U + V$ ), the capacity level is equal to the capacity position. On the other hand, if  $w > U + V$  then the capacity level will be between  $U + V$  and  $w$ . The permanent and contracted capacity levels are determined at the beginning of the planning horizon and they are fixed and fully available throughout the planning horizon. The unmet demand is fully backlogged. The costs under consideration are inventory holding and backordering costs, and unit costs of permanent, contracted and temporary capacity, which are all non-negative. We assume that there are no shortages of raw material and the lead time of production and acquiring external capacity can be neglected. There are no fixed costs for initiating production and no material related costs are considered in the model. The notation is summarized in Table 1. Further explanation of notation will be provided as need arises.

Table 1: Summary of Notation

$T$	: Number of periods in the planning horizon
$U$	: Size of available permanent capacity
$V$	: Size of available contracted capacity
$c_p$	: Unit cost of permanent capacity per period
$c_r$	: Unit reservation cost of contracted capacity per period
$c_u$	: Unit utilization cost of contracted capacity per period
$c_{cw}$	: Total unit cost of contracted capacity ( $c_{cw} = c_r + c_u$ )
$c_{tw}$	: Unit cost of temporary capacity per period
$h$	: Inventory holding cost per unit per period
$b$	: Penalty cost per unit of backorder per period
$\alpha$	: Discounting factor ( $0 < \alpha \leq 1$ )
$w_t$	: Capacity position in period $t$
$\eta_t$	: Capacity level in period $t$
$N_t$	: Temporary capacity requested in period $t$
$m_t$	: Temporary capacity realized in period $t$
$Q_t$	: Number of items produced in period $t$
$Z_t$	: Random variable denoting the demand in period $t$
$G_t(z)$	: Distribution function of $Z_t$
$P_t(m_t, N_t)$	: Probability function of receiving $m_t$ workers when $N_t$ workers are requested
$x_t$	: Inventory position at the beginning of period $t$ before production
$y_t$	: Inventory position in period $t$ after production
$f_t(x_t)$	: Minimum total expected cost of operating the system in periods $t, t + 1, \dots, T$ , given the system state $x_t$

The cost of permanent capacity is independent of the production quantity and paid each period even if there is no production. The unit cost of permanent capacity is  $c_p$  per period. Therefore the total permanent capacity cost for a workforce of size  $U$  is  $c_p U$  per period.



In the particular contract type that we consider, each contracted worker costs  $c'_r$  per period, independent of the utilization. There is also an additional cost component  $c'_u$  for each contracted worker utilized in production per period. Consequently, the cost of a utilized contracted worker per period,  $c'_{cw}$ , is  $c'_{cw} = c'_r + c'_u$ . Also let  $c'_{tw}$  be the cost of a hired temporary worker per period. In order to synchronize the production quantity with the number of workers, we redefine the “unit production” as the number of actual units that an average permanent worker can produce per period. We also define the cost of production by contingent workers in the same unit basis, where the cost for contingent workers is related to their productivity. Consequently, the term “ $N$  workers are requested” stands for requesting workers that are sufficient to produce  $N$  units. Considering that productivity rates of permanent, contracted and temporary workers may differ, let  $\lambda_{cw}$  and  $\lambda_{tw}$  be the average productivity rates of contracted workers and temporary workers, respectively, relative to the productivity of permanent workers. The model is valid for all values of  $\lambda_{cw} > 0$  and  $\lambda_{tw} > 0$ , however it is likely that  $0 < \lambda_{tw} \leq \lambda_{cw} \leq 1$ . Assuming that the productivity rates remain approximately unchanged in time, the unit production cost by contracted workers,  $c_{cw}$ , can be written as  $c_{cw} = c'_{cw}/\lambda_{cw}$ , where the production-equivalent unit reservation and utilization costs by contracted workers,  $c_r$  and  $c_u$ , being  $c_r = c'_r/\lambda_{cw}$  and  $c_u = c'_u/\lambda_{cw}$ , respectively. Hence, the total reservation cost of contracted workers is  $c_r V$  for a total contracted capacity of  $V$  production units. Similarly, the unit production cost by temporary workers,  $c_{tw}$ , can be written as  $c_{tw} = c'_{tw}/\lambda_{tw}$ .

The amount of temporary workers received in period  $t$ ,  $m_t$ , depends on the requested quantity,  $N_t$ , with a probability function of  $P_t(m_t, N_t)$ . While  $P_t(m_t, N_t)$  is a mass function, we denote it as a density function in our model for notational simplicity. The total cost of temporary workers is  $c_{tw} m_t$  if the firm observes  $m_t$  temporary workers, regardless of whether they are utilized or not. Demand in period  $t$ ,  $Z_t$ , has distribution  $G_t(z)$ . We consider a planning horizon of  $T$  periods.

The order of events is as follows. At the beginning of the planning horizon, permanent and contracted capacity levels,  $U$  and  $V$ , are determined. At the beginning of each period  $t = 1, \dots, T$ , the inventory level  $x_t$  is observed, and the capacity position decision is made as  $w_t$ . If the capacity position  $w_t > U + V$ , then a temporary capacity of  $N_t = w_t - U - V$  is requested from the ELSA, which delivers a realization,  $m_t$ , bringing the capacity level,  $\eta_t$ , to  $\eta_t = U + V + m_t$ . If  $w_t \leq U + V$ , then  $N_t = m_t = 0$ . A production decision  $Q_t \leq U + V + m_t$  is made to raise the inventory level to  $y_t = x_t + Q_t$ . At the end of the period, the demand  $z_t$

is realized and met. Remaining inventory is carried to the next period at a cost of  $h$  per unit and any unmet demand is backordered at a unit cost of  $b$ . The minimum cost of operating the system from period  $t$  until the end of the planning horizon is denoted by  $f_t(x_t, U, V)$ , where we drop  $U$  and  $V$  from the notation for brevity whenever appropriate. We assume an ending condition of  $f_{T+1} = 0$ . We model our integrated capacity and inventory management problem as follows:

$$f_t(x_t, U, V) = U c_p + V c_r + \min_{w_t \geq 0} \{H_t(w_t | x_t, U, V)\} \text{ for } t = 1, 2, \dots, T$$

$$\text{and } f_0(x_1, U, V) = \min_{U \geq 0, V \geq 0} \{f_1(x_1, U, V)\}$$

where

$$H_t(w_t | x_t, U, V) = \begin{cases} \varphi_t(w_t | x_t) & \text{if } 0 \leq w_t \leq U \\ (w_t - U)c_u + \varphi_t(w_t | x_t) & \text{if } U < w_t \leq U + V \\ \gamma_t(w_t - U - V | x_t) & \text{if } U + V < w_t \end{cases} . \quad (1)$$

In equation (1),  $\varphi_t(w_t | x_t) = \min_{y_t: x_t \leq y_t \leq x_t + w_t} \{L_t(y_t) + \alpha E[f_{t+1}(y_t - z_t)]\}$  is the production decision function that attains the minimum total expected cost of operations excluding the immediate labor costs, where  $L_t(y_t) = h \int_0^{y_t} (y_t - z_t) dG_t(z) + b \int_{y_t}^{\infty} (z_t - y_t) dG_t(z)$  is the regular convex loss function, and

$$\gamma_t(N_t | x_t) = V c_u + \int_0^{N_t} (c_{tw} m_t + \varphi(U + V + m_t)) P_t(m_t, N_t) dm_t \quad (2)$$

is the expected minimum cost of operations when  $N_t$  temporary workers are requested. Hence we refer  $H_t(w_t | x_t, U, V)$  as the “decision function” where we drop  $U$  and  $V$  from the notation for brevity.

Now we consider the last period problem in particular. Let  $\hat{y}_T$  be the minimizer of  $L_T(y_T)$  and  $y_T^*$  be the optimal inventory level after production under a realized capacity of  $\eta_T = U + V + m_T$  in the last period. Then we have

$$y_T^* = \begin{cases} x_T + \eta_T & \text{if } x_T + \eta_T \leq \hat{y}_T \\ \hat{y}_T & \text{if } x_T \leq \hat{y}_T \leq x_T + \eta_T \\ x_T & \text{if } \hat{y}_T < x_T \end{cases}$$

and

$$\varphi_T(w_T | x_T) = \begin{cases} J_T(x_T + w_T) & \text{if } x_T + w_T \leq \hat{y} \\ J_T(\hat{y}) & \text{if } x_T \leq \hat{y} \leq x_T + w_T \\ J_T(x) & \text{if } \hat{y} < x_T \end{cases} . \quad (3)$$

Then substituting (3) in (2), we obtain the cost function for utilizing temporary capacity as follows:

$$\begin{aligned} \gamma_T(N_T|x_T) = & Vc_u + \int_0^{N_T} m_T c_{tw} P_T(m_T, N_T) dm_T + \int_0^{\hat{y}_T - U - V} L(x_T + \eta_T) P_T(m_T, N_T) dm_T \\ & + \int_{\hat{y}_T - U - V}^{N_T} L(\hat{y}_T) P_T(m_T, N_T) dm_T. \end{aligned}$$

This implies that if  $\hat{y}_T$  is less than or equal to the capacity level,  $x_T + U + V + m_T$ , then it is optimal to produce up to  $\hat{y}_T$  leaving a portion of the available capacity unutilized. We note that this property would hold for any period  $t$ , if  $L_t(y_t) + \alpha E[f_{t+1}(y_t - z_t)]$  was convex, which does not hold in general.

### 3.1 Supply Uncertainty Structures

In this section we model different supply uncertainty structures to reflect possible responses of an ELSA to workforce requests, which may differ according to the factors such as the size of available temporary worker pool, capability of finding skilled workers, competition in the environment, demand structure of different customers, and opportunities in alternative options. We use the following structures for modeling the supply uncertainty, given that  $N$  temporary workers are requested from the ELSA by the manufacturing firm.

**All-or-nothing availability:** The firm receives  $N$  contingent workers with probability  $p$  and does not receive any worker with probability  $(1 - p)$ . This may happen when the ELSA has better offers from other firms and therefore rejects the offer of the firm. Here  $1 - p$  can be considered as the probability of ELSA having better alternatives. It may also be the case that while the ELSA is able to supply the firm's request partially, such a partial supply is not acceptable by the firm, which might be the case, e.g., in assembly lines.

**Partial availability:**

- **Uniform availability:** Under this model the firm has equally likely chance of acquiring 0 to  $N$  workers, where the ELSA attempts to be "fair" to all requests based on the available temporary labor pool size. Note that the expected number of workers acquired increases as the number requested increases.
- **Normal availability:** In this case the number of workers to be received is distributed approximately with a (discrete) Normal distribution, the realization never exceeding  $N$ .
- **Decreasing availability:** In this case the ELSA has a limited temporary worker pool size,  $K$ , and a relatively stable market so that as  $N$  increases, the probability

of acquiring each worker decreases. In particular, we model this situation using a Binomial distribution with a decreasing success probability that equals to  $\frac{Max\{K-N,0\}}{K}$ .

- **Moderate availability:** Under this uncertainty structure we model an ELSA with a limited pool size,  $K$ , favoring moderate-sized demand. Lower demands are not preferred by the ELSA in order to prevent the temporary worker pool size from shrinkage, while higher demands have a lower chance of being met due to the scarcity of supply. The number of workers acquired has a Binomial distribution with a success probability of  $\frac{\cos(2\Pi N/K-\Pi)+1}{2}$ , for  $N \leq K$ , and 0 otherwise.
- **Increasing availability:** Under this setting we model an ELSA that favors larger-sized requests. The ELSA attempts to avoid the division of its workforce for this purpose and tries to satisfy larger-sized requests to a great extent, meeting requests that exceed a certain upper bound,  $K$ , with certainty. In particular, we model this situation using a Binomial distribution with an increasing success probability that equals to  $\frac{Min\{N,K\}}{K}$ .
- **High-Low availability:** This structure of uncertainty models an ELSA favoring requests that are either low or high. The underlying reason for such a preference may be a competitive environment where the ELSA wants to meet larger-sized requests to a great extent, meeting requests that exceed a certain upper bound,  $K$ , with certainty, but also does not want to turn down smaller-sized requests that can relatively easily be met. The ELSA may then deter from committing a moderate size of its workers to a firm, considering the chance of a larger-sized demands from other customers. The number of workers acquired by the firm has a binomial distribution with a success probability of  $\frac{\sin(2\Pi N/K+\Pi/2)+1}{2}$  for  $N \leq K$ , and 1 otherwise.

## 4 All-or-Nothing Type Contingent Capacity Availability

In this section we characterize the structure of the optimal policy for the all-or-nothing case for given  $U$  and  $V$ . The following theorem characterizes the optimal inventory and capacity management policy when  $p$  is reasonably large ( $p \geq c_u/c_{tw}$ ). Relatively low values of  $p$  would not be sustainable for the operations anyway, since a certain reliability of ELSA is necessary.

**Theorem 1** *If  $p \geq c_u/c_{tw}$  then (i) the multi-period decision function  $H_t(w_t|x_t)$  is convex in  $w_t$ , (ii) the optimal production policy is of state-dependent order-up-to type and the optimal order up-to levels can be stated as:*

$$y_t^*(x_t) = \begin{cases} y_{tc}^* & \text{if } x_t < y_{tc}^* - U - V \\ x_t + U + V & \text{if } y_{tc}^* - U - V < x_t \leq y_{tv}^* - U - V \\ y_{tv}^* & \text{if } y_{tv}^* - U - V < x_t \leq y_{tv}^* - U \\ x_t + U & \text{if } y_{tv}^* - U < x_t \leq y_{tp}^* - U \\ y_{tp}^* & \text{if } y_{tp}^* - U < x_t \leq y_{tp}^* \\ x_t & \text{if } y_{tp}^* < x_t \end{cases}$$

where  $y_{tp}^*$ ,  $y_{tv}^*$  and  $y_{tc}^*$  are three critical numbers that are independent of the starting inventory levels for each period  $t$ , and they refer to production with permanent capacity only, production with permanent and contracted capacity only, and production with permanent, contracted and temporary capacity, respectively, and (iii) the optimal capacity ordering decision is given by  $w_t^*(x_t) = y_t^*(x_t) - x_t$ .

**Proof :** See Appendix.  $\square$

**Corollary 1** *In the special case of  $V = 0$ ,  $H_t(w_t|x_t)$  is convex in  $w_t$  for all  $x_t$  and  $t$ .*

Theorem 1 states that the optimal production decision determine the capacity ordering decision. When the starting inventory level is low and use of temporary workers is required for production, the optimal number of temporary workers to be ordered is as much as necessary for materializing the optimal production quantity. The realized capacity level is fully used for production irrespective of whether all of the temporary workers ordered are received or not.

## 5 Partial Contingent Capacity Availability

In this section we analyze the partial availability cases mainly based on numerical analysis, as they are analytically intractable. In the case of uniform supply uncertainty, we show that the last period's cost function is convex in the capacity position  $w$  for a certain condition on cost coefficients, while the multi-period cost function is observed to be non-convex. Under other uncertainty types, we observe that the problem is non-convex both in single- and multi-period cases. While we presented our model as a finite horizon model, our numerical results are conducted for the case of  $T \rightarrow \infty$  yielding an infinite horizon model, in order to keep the

results unaffected from the end-of-horizon condition. In the problem settings we consider, we observe that the solution of the finite horizon problem converges to that of the infinite horizon problem rapidly. We drop subscript  $t$  when we refer to an infinite horizon solution. Similarly, we consider a stationary labor supply uncertainty distribution function,  $P(m, N)$ . In the results that we present, we use the term “increasing” (“decreasing”) in the weak sense to mean “non-decreasing” (“non-increasing”). We provide intuitive explanations to all of our results below and our findings are verified through several numerical studies. However, like any experimental result, one should be careful about generalizing them, especially for extreme values of problem parameters.

## 5.1 Optimal production and capacity ordering policies

In this section, we provide an analysis of the cost functions,  $f_t(x_t)$  and  $H_t(w_t|x_t)$ , and the characteristics of the optimal production and capacity ordering policies for different forms of supply availabilities. The demand has a Poisson distribution with a mean of 10 in every period. Mean supply is taken as  $N/2$  in the Normal availability case. We denote the (discrete) Normal availability case with a Coefficient of Variation of CoV as Normal[CoV]. We take  $K = 20$  in the availability structures with Binomial distribution.

Our numerical analysis shows that  $f_t(x_t)$  is non-convex. However, in all problem instances that we solved, this function is quasi-convex. On the other hand, the decision function,  $H_t(w_t)$ , is not necessarily (quasi-)convex (see Figure 1 for an infinite horizon problem instance). Nevertheless, we show that the last period’s decision function is convex under Uniform availability when  $c_{tw}$  is at least  $2c_u$ .

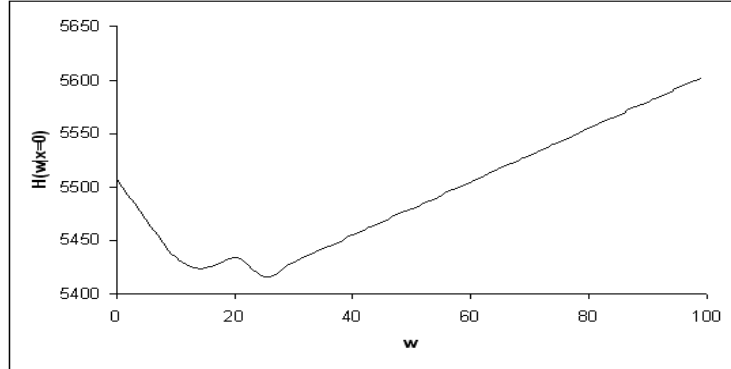
**Theorem 2** *Under Uniform availability, the last period’s decision function  $H_T(w_T|x_T)$  is convex for all  $x_T$  when  $c_u \leq c_{tw}/2$ .*

**Proof:** See Appendix.

**Corollary 2** *In the special case of  $V = 0$ ,  $H(w|x)$  is convex for all  $x$  under Uniform availability.*

Recall from Section 3 that if the capacity position is set to values greater than the “ensured” capacity (permanent plus contracted) then the optimal production decision depends on a particular realization of the capacity level (which is a random variable). The uncertainty

Figure 1:  $H(w|x = 0)$  vs. Capacity Position - High-Low Availability,  $c_p = 2.5$ ,  $c_{tw} = 3.5$ ,  $h = 1$ ,  $b = 5$ ,  $U = 10$



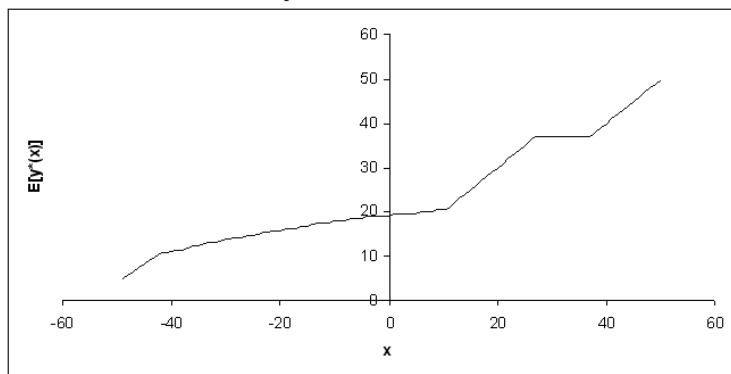
in the capacity level vanishes if the capacity position is lower than the ensured capacity. Accordingly, we define the expectation of the optimal order-up-to levels,  $E[y^*(x)]$ . For different availability structures,  $E[y^*(x)]$  depicts different characteristics considered when the optimal capacity position requires usage of temporary workers. In all availability structures considered, there exists a threshold starting inventory level value before which, also temporary workers are utilized and after which, only ensured capacity is utilized. The latter region can further be divided into five smaller regions in the optimal policy as follows: (i) all ensured capacity is utilized for production, (ii) inventory is raised to a fixed critical order-up-to level where all of the permanent workers and a portion of the contracted workers are used for production, (iii) only all of the permanent workers are used for production, (iv) inventory is raised to another fixed critical order-up-to level where a portion of the permanent workers are used for production, and (v) no production takes place. For the special case of no temporary workers, Tan and Alp (2005) prove that this policy is indeed optimal.

Next, we analyze  $E[y^*(x)]$  with respect to lower values of  $x$ , where the optimal policy requires the use of temporary workers. For the special case of  $V = 0$  and deterministic labor supply, Tan and Alp (2005) show that it is optimal to produce up to a certain value when temporary workers are utilized. Figures 2a and 2b depict  $x$  versus  $E[y^*(x)]$  graphs for two problem instances, one with Uniform and the other with Increasing availability, respectively. In Figure 2a, we observe that the expected order-up-to level increases as the starting inventory level increases for low values of  $x$ , contrary to the results of Tan and Alp (2005). This structure is observed for all problem instances considered with Uniform

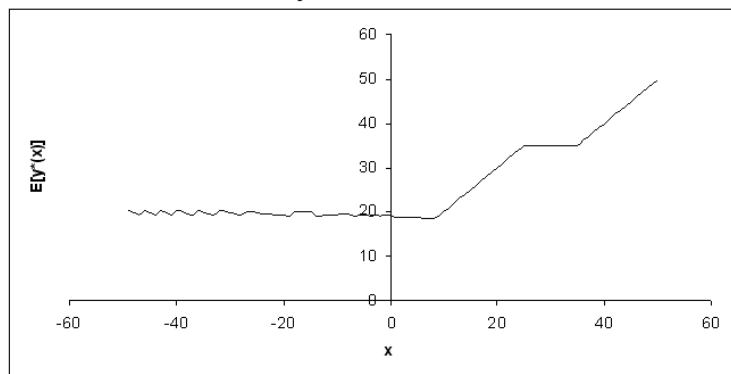
availability. This is because Uniform distribution is platykurtic (has negative kurtosis). The variability increases so high for increased values of  $N$  that the system tries to avoid ordering too high. In Normal, Increasing, and High-Low availability structures,  $E[y^*(x)]$  values fluctuate around a certain level for low values of starting inventory levels, maintaining a general order-up-to level, in line with the results of Tan and Alp (2005). In Decreasing availability structure,  $E[y^*(x)]$  is increasing in the starting inventory level for very low values of inventory levels since acquiring large number of workers (larger than  $K$ ) is not possible in this structure. As the need for temporary workers decreases, an order-up-to level behavior is observed similar to the previous cases. In Moderate availability structure,  $E[y^*(x)]$  is also increasing in  $x$  for very low inventory levels, as the manufacturer constantly requests the level yielding the highest expected capacity in order to raise the inventory level. After a critical level, a similar order-up-to level behavior is observed.

Figure 2: Expected Order-Up-To Level vs. Starting Inventory Level

a. Uniform Availability,  $c_p = 2.5$ ,  $c_{tw} = 3.5$ ,  $h = 1$ ,  $b = 5$   $U = 10$



b. Increasing Availability,  $c_p = 2.5$ ,  $c_{tw} = 3.5$ ,  $h = 1$ ,  $b = 5$   $U = 10$



As pointed out in Section 3, not every worker that is paid for, even a temporary one, is

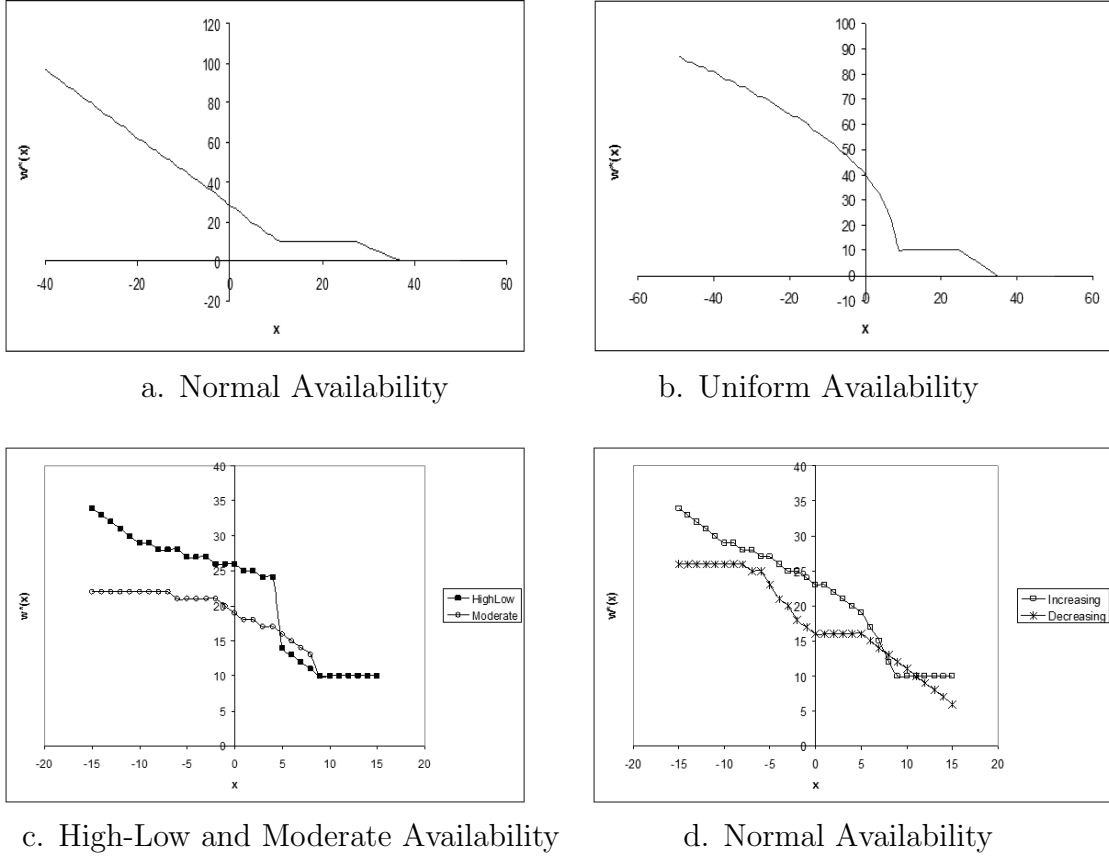


utilized in the optimal solution. The decision maker sets the capacity position considering the expected outcome, consequently the production decision is made after observing the capacity level. We call the difference between the capacity level and the optimal production level as the surplus of temporary capacity (STC). In what follows, we examine the effect of availability structure on the STC. Uniform availability yields the highest STC among all availability structures, since it is platykurtic. In a particular problem instance, the total expected STC under Uniform availability is 3.6% of the realized temporary capacity, whereas this value is 0.4% under Normal availability. The STC values in Increasing, Decreasing, High-Low, and Moderate availability structures are very close to zero since the level of the capacity position is set to a value which produces high success probability of acquiring the desired capacity level value.

We investigate the optimal capacity position decision as a function of the starting inventory under different availability structures in order to develop managerial insights as to the optimal capacity ordering policy. Under Normal availability, the optimal capacity ordering decision is following a monotone decreasing pattern in  $x$  for small  $x$  values (i.e. when temporary capacity is called for) as illustrated in Figure 3a, so that a certain expected order-up-to point is reached by making use of (most of) this capacity. Under the availability structures that assume Binomial distribution, the capacity ordering decision is making the best use of higher success probabilities to assure sufficient capacity in order to be able to produce the optimal amount, surplus of temporary capacity being mostly zero. For example, High-Low availability case avoids moderate sized orders due to low availability rates, therefore the optimal capacity position faces a steep fall at a certain point where a large order would otherwise result in surplus of temporary capacity. See Figures 3c and 3d for an illustration. Nevertheless, such a capacity ordering policy does not hold in the Uniform availability structure, due to the reason discussed before. See Figure 3b.

In what follows we investigate the impact of labor supply uncertainty, demand variability, and cost parameters on operational and tactical decisions. In our experimental setting, we consider an infinite horizon problem with a seasonal demand pattern following a cycle of 4 periods, the expected demands being 10, 15, 10 and 5, respectively. Unless otherwise noted, we assume that the demand has a Poisson distribution,  $h = 1$ ,  $c_p = 2.5$ ,  $c_r + c_u = 3$ ,  $c_{tw} = 3.5$ ,  $\alpha = 0.99$ . In addition to the partial availability structures presented in Section 5.1, we also consider three more Normal availability structures, Normal[0.1], Normal[0.15], and Normal[0.2] with mean values of  $N/2$ , and a deterministic labor supply structure. In

Figure 3: Capacity Position vs. Starting Inventory Level



some of our experiments, we assume Normal demand with CoV values of 0.1, 0.2, and 0.3 and Gamma demand with CoV values of 0.5, 1.0, and 1.5, to investigate the effect of demand variability on flexible capacity management.

## 5.2 Effect of Labor Supply Uncertainty

In this section, we investigate the effects of labor supply uncertainty on flexible capacity and production management. Table 2 illustrates the change in average inventory level and the contribution of temporary workers in production under deterministic labor supply, and Normal and Uniform availability structures. The average inventory level carried increases as we switch from deterministic labor supply to uncertain supply. Under Uniform availability, the average inventory levels carried increase drastically when  $U = 6$ . This is because the probability of observing low capacity levels is much higher when compared to other availability structures and the system tries to avoid backorders originating from this by holding higher

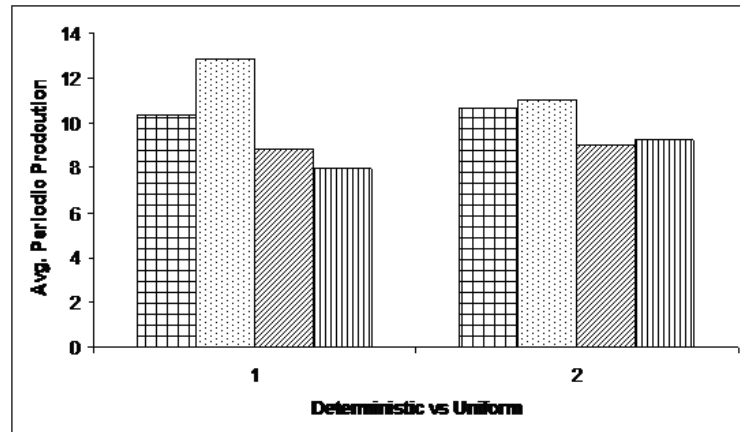
Table 2: Comparison of Supply Structures.  $U = 6, V = 0$

Parameters	Criteria	$U = 6, V = 0$			$U = 10, V = 2$		
		Deterministic	Normal[0.15]	Uniform	Deterministic	Normal[0.15]	Uniform
$c_{tw} = 1.5$ $b = 50$	Ave. Inv. Lev.	7.38	7.99	11.34	7.44	7.59	8.35
	% Temporary	40.88	41.22	40.3	7.52	8.49	5.07
$c_{tw} = 4.5$ $b = 50$	Ave. Inv. Lev.	7.58	8.22	12.01	8.73	8.80	9.20
	% Temporary	40.04	40.05	40.05	2.59	2.9	1.88

inventory levels. Nevertheless, this is not the case when  $U = 10$  and  $V = 2$ , since the system depends less on the temporary workers in this case. Finally, the average production made with temporary workers is not affected much by different problem parameters considered when  $U = 6$ , since the low permanent capacity level is almost always fully utilized anyway.

When the level of ensured capacity is sufficient to produce the average demand and the temporary labor supply has high variability, we observe that the manufacturer spreads the total production among periods, rather than utilizing flexible capacity against the demand seasonality (see Figure 4).

Figure 4: Periodic Production-Deterministic Supply vs. Uniform Availability.  $U = 10, V = 0, c_{tw} = 3.5, b = 50$



### 5.3 Optimal Contracted Capacity Level

In this section we analyze the effects of the problem parameters  $c_r, c_u, c_{tw}, b$ , labor supply uncertainty, and demand uncertainty on the optimal size of contracted capacity for a given permanent capacity. This analysis provides insights on the number of contingent workers

Table 3: Optimal Contracted Capacity Level ( $V^*$ ).  $U = 6$ ,  $c_{tw} = 3.5$

$b$	$c_r$	Normal[0.1]	Normal[0.15]	Normal[0.2]	Normal[0.25]	Uniform
2.5	0.6	3	4	4	4	5
2.5	1.2	2	2	2	3	4
2.5	1.8	2	2	2	2	3
2.5	2.4	2	2	2	2	3
2.5	3	2	2	2	2	3
5.5	0.6	3	4	4	4	5
5.5	1.2	2	2	2	2	4
5.5	1.8	2	2	2	2	3
5.5	2.4	1	2	2	2	3
5.5	3	1	2	2	2	3
50	0.6	3	3	4	4	7
50	1.2	1	2	2	2	5
50	1.8	1	1	2	2	4
50	2.4	1	1	1	2	3
50	3	1	1	1	2	3

to contract when the manufacturer operates with a suboptimal permanent capacity level. Table 3 depicts the optimal contracted capacity size under different labor supply uncertainty structures. First of all, as the reservation cost,  $c_r$  gets larger, naturally,  $V^*$  decreases. Nevertheless, even when  $c_r = c_{cw}$  (which makes the contracted capacity practically equivalent to permanent capacity), we observe that keeping contracted workers may still be beneficial depending on other cost parameters. As the labor supply uncertainty increases, the system prefers contracting higher capacities as expected. In the Normal availability cases, the system carries higher safety stock to avoid backorders as the backordering cost increases. This leads to system's preference in higher capacity flexibility in order to avoid idle capacity costs. However, since all system parameters interact in the optimal decisions, this result cannot be generalized. For example, the uncertainty of the labor supply in the Uniform availability dominates this affect and the system prefers higher contracted capacity levels in order to decrease temporary workers usage, as discussed before.

The effect of demand variability on the optimal contracted capacity heavily interact with cost parameters. In Normal availability structure,  $V^*$  decreases as the demand variability increases when  $c_{tw}$  is not much larger than  $c_{cw}$ , as illustrated in Table 4 for  $c_{tw} = 3.5$  in order to avoid unutilized contracted capacity. On the other hand, when  $c_{tw}$  is significantly larger than  $c_{cw}$ , the opposite behavior is observed since the system tries to avoid using expensive temporary labor, as illustrated in Table 4 for  $c_{tw} = 7.5$ . However, labor supply uncertainty structure also plays an important role in this interaction. For example, in Increasing availability structure, the system reserves higher contracted capacity as the demand variability increases since acquiring a small number workers from the ELSA is not probable.

Table 4: Effect of Demand Uncertainty on Optimal Contracted Capacity Level.  $U = 4$ ,  $b = 50$ .

			Demand Distribution					
Labor Supply	$c_{tw}$	$c_r$	Normal[0.1]	Normal[0.2]	Normal[0.3]	Gamma[0.5]	Gamma[1.0]	Gamma[1.5]
Normal[0.25]	3.5	0.6	8	8	7	6	4	2
		1.2	6	5	5	4	2	0
		1.8	6	5	4	3	2	0
Normal[0.25]	7.5	0.6	10	10	11	11	12	13
		1.2	9	10	10	10	11	12
		1.8	7	8	8	9	9	10
Increasing	3.5	0.6	8	9	9	8	11	11
		1.2	6	7	8	7	10	10
		1.8	6	6	7	6	8	8

Table 5: Effect of Temporary Labor Cost and Uncertainty on Ensured Capacity ( $U^*$ ,  $V^*$ ).  $c_p = 2.5$ ,  $c_r = 0.6$ ,  $b = 50$

$c_{tw}$	Norm[0.1]	Norm[0.15]	Norm[0.2]	Norm[0.25]	Uniform
2.5	(2,0)	(4,0)	(6,0)	(6,0)	(8,3)
3.5	(8,1)	(8,2)	(8,2)	(10,0)	(8,4)

## 5.4 Optimal Permanent and Contracted Capacity Decisions

In this section, we investigate the optimal levels of permanent and contracted capacity under various settings. Table 5 illustrates the effect of temporary labor cost and labor supply uncertainty on the optimal capacity levels (permanent and contracted). We observe that as the labor supply uncertainty increases, the level of ensured capacity also increases in line with our observation in Section 5.3. When there is no labor supply uncertainty, Alp and Tan (2007) show that, for  $c_p = c_{tw}$  the optimal permanent capacity level is zero. Nevertheless, this does not turn out to be the case under labor supply uncertainty in order to hedge against this uncertainty. Moreover, it may be optimal to reserve contracted capacity even when the cost of a contracted worker is higher than that of a temporary worker, when the supply uncertainty is high, as is the case under Uniform availability with  $V^* = 3$ . Finally we note that the ensured capacity level increases as  $c_{tw}$  increases, as expected.

We observe that the effects of demand variability and backordering cost on the optimal permanent and contracted capacity levels (see Table 6) are in line with those on the optimal contracted capacity of Section 5.3.

Table 6: Effect of Backordering Cost on Ensured Capacity.  $c_p = 2.5$ ,  $c_r = 0.6$ .

Supply Uncertainty		Normal[0.1]		Uniform	
Demand Uncertainty		Normal[0.2]	Normal[0.3]	Normal[0.2]	Normal[0.3]
$c_{tw}$	$b$	$(U^*, V^*)$	$(U^*, V^*)$	$(U^*, V^*)$	$(U^*, V^*)$
2.5	2.5	(4,0)	(2,0)	(8,0)	(8,0)
2.5	5.5	(4,0)	(0,0)	(8,1)	(8,1)
2.5	50	(2,0)	(0,0)	(8,3)	(8,3)
3.5	2.5	(10,0)	(10,0)	(10,0)	(8,3)
3.5	5.5	(10,0)	(8,1)	(10,1)	(8,4)
3.5	50	(8,1)	(8,0)	(8,4)	(8,5)

## 6 Conclusions

In this study, we consider the problem of integrated capacity and inventory management under non-stationary stochastic demand and temporary capacity uncertainty. We investigate the problem under the workforce planning framework. The focus of the paper is modeling and analyzing the effects of temporary labor uncertainty. We model a number of possible availability structures for this purpose: All-or-nothing, Uniform, Normal, Decreasing, Increasing, Moderate, and High-Low. Our model and analysis provide insights on the optimal usage of all capacity means coupled with inventory management in this environment. In the tactical level, these means are contracting a number of contingent workers whose availability is ensured by a reservation cost and determining the optimal level of permanent capacity. In the operational level, the decisions to make are determining the number of workers to be requested from the external labor supply agency and the quantity of production in each period.

We show for the all-or-nothing type availability that the resulting cost function is convex under a reasonable condition and the optimal production policy is of state-dependent order-up-to type, which dictates the capacity ordering decision. In the case of uniform supply uncertainty, we show that the last period's cost function is convex in the capacity position for a certain condition on cost coefficients, while the multi-period cost function is observed to be non-convex. Under other uncertainty types, we observe that the cost function is non-convex both in single- and multi-period cases.

In all availability structures considered, there exists a threshold starting inventory level value before which, also temporary workers are utilized and after which, only ensured ca-

capacity is utilized. The latter region can further be divided into five smaller regions in the optimal policy. For the former region, we observe that for some uncertainty structures the expected order-up-to level increases as the starting inventory level increases for low values of  $x$ , contrary to the results of Tan and Alp (2005), who show that for the case of deterministic labor supply it is optimal to produce up to a certain value when temporary workers are utilized.

We also show that not every temporary worker that is paid for is utilized in the optimal solution. Such a surplus of temporary capacity is the highest for the Uniform availability structure, followed by the Normal availability structure. The surplus in Increasing, Decreasing, High-Low, and Moderate availability structures are very close to zero. We observe that Uniform availability performs worst among all availability structures that we considered in all of our experiments. This is because Uniform distribution is platykurtic (has negative kurtosis). The absence of any “peak” in Uniform distribution makes it difficult to manage this availability structure, especially when higher number of workers are required. Increasing, Decreasing, High-Low and Moderate availability structures are easier to manage, since the level of the capacity position can be set to a value which produces high success probability of acquiring the desired capacity level in those cases. Nevertheless this holds only as long as the parameters of the problem are appropriate in the sense that such high success probabilities are attainable. This explains, for example, why Moderate and Decreasing availability cases perform worse than Increasing and High-Low cases for low values of  $x$ : it is not possible (or it is very unlikely) to acquire high number of temporary workers. In the Normal availability case, the performance deteriorates as the variability increases.

Our analysis provides insights on the number of contingent workers to contract for any given permanent capacity level. This situation might be useful to represent the manufacturers that operate under a suboptimal permanent capacity level. Since all problem parameters interact in making the optimal contracting decision, it is not possible to derive results that are valid everywhere, except for the following: The optimal number of contingent workers to contract increases as **(i)** reservation cost decreases, **(ii)** temporary labor cost increases, and **(iii)** the labor supply uncertainty increases. Moreover, even when the reservation cost constitutes 100% of the contracted worker cost, we observe that keeping contracted workers may still be beneficial.

When the optimal permanent capacity level can be optimized as well as the contracted capacity, we observe that the level of ensured capacity (permanent + contracted) increases as

(i) temporary labor cost increases, and (ii) the labor supply uncertainty increases. When the cost of temporary labor cost is equal to the cost of permanent labor, the optimal permanent capacity level may be positive, in order to hedge against supply uncertainty. Moreover, it may be optimal to reserve contracted capacity even when the cost of a contracted worker is higher than that of a temporary worker, when the supply uncertainty is high.

This research can be extended by considering the perspective of the external labor supply agency. In such a context, the optimal capacity planning of the ELSA and the contract design problem between the ELSA and the manufacturer might be of interest.

## Appendix

### Proof Theorem 1

We begin by proving the convexity of the single period cost function,  $H(w|x)$ . In all-or-nothing availability,  $H(w|x)$  can be rewritten as follows:

$$H(w|x) = \begin{cases} \min_{y:x \leq y \leq x+w} \{L(y)\} & \text{if } 0 \leq w \leq U \\ (w - U)c_u + \min_{y:x \leq y \leq x+w} \{L(y)\} & \text{if } U < w \leq U + V \\ Vc_u + p(c_{tw}(w - U - V) + \min_{y:x \leq y \leq x+w} \{L(y)\}) \\ \quad + (1 - p) \min_{y:x \leq y \leq x+U+V} \{L(y)\} & \text{if } U + V < w \end{cases} \quad (4)$$

Let  $\hat{y}$  be the minimizer of the convex function  $L(y)$ , which is known to be  $G^{-1}(\frac{b}{h+b})$  from the classical newsvendor solution. Note that when  $\hat{y} \leq x$  we have  $\min_{y:x \leq y \leq x+w} \{L(y)\} = L(x)$  which implies that optimal production quantity is zero. When  $\hat{y} \geq x$ , we can write  $H(w|x)$  by using equations (3) and (4) as follows:

**Case I.** ( $0 \leq w \leq U$ )

$$H(w|x) = \begin{cases} L(x+w) & \text{if } 0 \leq w \leq \hat{y} - x \leq U \text{ or } 0 \leq w \leq \hat{y} - x \leq U \\ L(\hat{y}) & \text{if } \hat{y} - x < w \leq U \end{cases}$$

**Case II.** ( $U < w \leq U + V$ )

$$H(w|x) = \begin{cases} (w - U)c_u + L(x+w) & \text{if } U < w \leq \hat{y} - x \leq U + V \\ & \text{or } U < w \leq U + V \leq \hat{y} - x \\ (w - U)c_u + L(\hat{y}) & \text{if } U \leq \hat{y} - x < w \leq U + V \\ & \text{or } \hat{y} - x \leq U < w \leq U + V \end{cases}$$

**Case III.** ( $U + V < w$ )

$$H(w|x) = \begin{cases} Vc_u + p((w - U - V)c_{tw} + L(x+w)) \\ \quad + (1 - p)L(x + U + V) & \text{if } U + V < w \leq \hat{y} - x \\ Vc_u + p((w - U - V)c_{tw} + L(\hat{y})) \\ \quad + (1 - p)L(\psi(x)) & \text{if } U + V \leq \hat{y} - x < w \\ & \text{or } \hat{y} - x \leq U + V < w \end{cases}$$



where

$$\psi(x) = \begin{cases} x + U + V & \text{if } x \leq \hat{y} - U - V \\ \hat{y} & \text{otherwise} \end{cases}.$$

Note that  $\psi(x)$  is constant in  $w$ . Therefore  $H(w|x)$  is convex in  $w$  in all of the above regions, for all values of  $x$ , which follows from the convexity of  $L(\cdot)$ . To conclude the convexity of  $H(w|x)$  we need to show that convexity is preserved in transition points  $w = U$  and  $w = U + V$ . We denote the respective regions by the following subscripts:  $I$  ( $0 \leq w \leq U$ ),  $II$  ( $U < w \leq U + V$ ) and  $III$  ( $U + V < w$ ). The following first order condition is sufficient:

$$\frac{dH_I(w|x)}{dw} \leq \frac{dH_{II}(w|x)}{dw} \leq \frac{dH_{III}(w|x)}{dw}$$

For the first transition point we need to check the above inequalities for values of  $x$  below and above  $\hat{y} - U$ . If  $x + U \leq \hat{y}$  then we have  $\lim_{w \rightarrow U^-} \frac{dH_I(w|x)}{dw} = L'(x + U)$  from the first region of Case I, and  $\lim_{w \rightarrow U^+} \frac{dH_{II}(w|x)}{dw} = c_u + L'(x + U)$  from the first region of Case II. If  $x + U \geq \hat{y}$  then we have  $\lim_{w \rightarrow U^-} \frac{dH_I(w|x)}{dw} = 0$  from the second region of Case I, and  $\lim_{w \rightarrow U^+} \frac{dH_{II}(w|x)}{dw} = c_u$  from the second region of Case II. Since  $c_u > 0$ , the convexity is preserved at the junction point  $U$ . If  $x + U + V \leq \hat{y}$  then we have  $\lim_{w \rightarrow (U+V)^-} \frac{dH_{II}(w|x)}{dw} = c_u + L'(x + U + V)$  from the first region of Case II, and  $\lim_{w \rightarrow (U+V)^+} \frac{dH_{III}(w|x)}{dw} = pc_{tw} + pL'(x + U + V)$  from the first region of Case III. If  $x + U + V \geq \hat{y}$  then we have  $\lim_{w \rightarrow (U+V)^-} \frac{dH_{II}(w|x)}{dw} = c_u$  from the second region of Case II, and  $\lim_{w \rightarrow (U+V)^+} \frac{dH_{III}(w|x)}{dw} = pc_{tw}$  from the second region of Case III. If  $pc_{tw} > c_u$ , the convexity is preserved at the junction point  $U + V$  since  $L(x + U + V) < 0$  when  $x + U + V \leq \hat{y}$ .

After proving the convexity of the decision function, we now characterize the optimal policy of the single period problem. Recall that,

$$H(w|x) = \begin{cases} \varphi(w|x) & \text{if } 0 \leq w \leq U \\ (w - U)c_f + \varphi(w|x) & \text{if } U \leq w \leq U + V \\ Vc_f + pc_c((w - U - V) + \varphi(w|x)) \\ \quad + (1 - p)\varphi(U + V|x) & \text{if } U + V < w \end{cases}.$$

If  $\hat{y} - U \leq x < \hat{y}$  then the value of  $w$  minimizing  $H(w|x)$  is in region ( $I$ ) and it is the minimizer of  $L(x + w)$ . From the classical newsboy solution we derive the optimal capacity position as:

$$w^*(x) = \hat{y} - x = G^{-1}\left(\frac{b}{h + b}\right) - x = y_p^* - x.$$

We let  $y_p^* = \hat{y}$ . If  $y_p^* - U - V < x < y_p^* - U$ , then the minimizer of the function  $H(w|x)$  is in region ( $II$ ). From the first order condition, we have

$$0 = c_u + L'(x + w).$$

Note that the optimality equation may not be satisfied even if  $x$  is in the above region, particularly if  $c_u + L'(x + w) > 0$ . In that case the optimal policy is to produce at full permanent capacity,  $w^* = U$ , the resulting order up-to level is  $x + U$ . Otherwise using the solution of the optimality equation the optimal capacity position is found as  $w^*(x) = G^{-1}(\frac{b-c_f}{b+h}) - x$  and the corresponding order up-to level is  $y_v^* = G^{-1}(\frac{b-c_f}{b+h})$ . Note that for non-negative  $c_f$ ,  $y_v^* \geq y_p^*$ . The optimal capacity policy for this particular region can be found as:

$$w^*(x) = \begin{cases} y_v^* - x & \text{if } x \leq y_v^* - U \\ U & \text{if } y_v^* - U \leq x < y_p^* - U \end{cases} .$$

If  $x < y_v^* - U - V$  then the minimizer of  $H(w|x)$  is in region *III*. Similarly, we obtain

$$w^*(x) = \begin{cases} y_c^* - x & \text{if } x \leq y_c^* - U - V \\ U + V & \text{if } y_c^* - U - V \leq x < y_v^* - U - V \end{cases}$$

where  $y_c^* = G^{-1}(\frac{b-c_c}{b+h})$ . For  $0 < c_f < c_c$ , the optimal values for the above functions have the following relation:  $y_p^* > y_v^* > y_c^*$ . Using this above property, the single period state dependent order up-to can be written as

$$y^*(x) = \begin{cases} y_c^* & \text{if } x < y_c^* - U - V \\ x + U + V & \text{if } y_c^* - U - V < x \leq y_v^* - U - V \\ y_v^* & \text{if } y_v^* - U - V < x \leq y_v^* - U \\ x + U & \text{if } y_v^* - U < x \leq y_p^* - U \\ y_p^* & \text{if } y_p^* - U < x \leq y_p^* \\ x & \text{if } y_p^* < x \end{cases} .$$

To conclude the convexity of the multi-period expected total cost function  $J_t(\cdot)$  it is sufficient to show that  $f(x)$ , single period minimum expected cost of operations for starting inventory level  $x$ , is convex in  $x$ . Using  $y^*(x)$  we can write  $f(x)$  as:

$$f(x) = \begin{cases} U c_p + V c_r + V c_u \\ \quad + p(c_{tw}(y_c^* - U - V - x) + L(y_c^*)) \\ \quad + (1-p)L(x + U + V) & \text{if } x < y_c^* - U - V \\ U c_p + V c_r + V c_u + L(x + U + V) & \text{if } y_c^* - U - V < x \leq y_v^* - U - V \\ U c_p + V c_r + (y_v^* - U - x)c_u + L(y_v^*) & \text{if } y_v^* - U - V < x \leq y_v^* - U \\ U c_p + V c_r + L(x + U) & \text{if } y_v^* - U < x \leq y_p^* - U \\ U c_p + V c_r + L(y_p^*) & \text{if } y_p^* - U < x \leq y_p^* \\ U c_p + V c_r + L(x) & \text{if } y_p^* < x \end{cases}$$

Similar to the convexity of the function  $H$ , it is straightforward to show that the function  $f(x)$  is convex. Then by regular inductive arguments, it can be shown that the results also

hold for any period  $t$ . For details, see Pac (2006).  $\square$

### Proof Theorem 2

The theorem is proved for a continuous Uniform distribution. Single period cost function for the uniform contingent labor uncertainty case can be written as:

$$H(w|x) = \begin{cases} \varphi(w|x) & \text{if } 0 \leq w \leq U \\ (w - U)c_u + \varphi(w|x) & \text{if } U < w \leq U + V \\ Vc_u + \int_0^N (mc_{tw} + \varphi(U + V + m|x)) \frac{1}{N} dm & \text{if } U + V < w \end{cases}$$

To prove that  $H(w|x)$  is convex it is sufficient to analyze the case with contingent capacity region and the corresponding transition point, since for  $w \leq U + V$  the function remains identical for all labor supply uncertainty types.

For  $w > U + V$

$$H(w|x) = \begin{cases} Vc_u + c_{tw} \frac{(w-U-V)}{2} + \int_0^{w-U-V} \frac{L(x+U+V+m)}{(w-U-V)} dm & \text{if } U + V < w \leq y_p^* - x \\ Vc_u + c_{tw} \frac{(w-U-V)}{2} + \int_0^{y_p^* - x - U - V} \frac{L(x+U+V+m)}{(w-U-V)} dm + \int_{y_p^* - x - U - V}^{w-U-V} \frac{L(y_p^*)}{(w-U-V)} dm & \text{if } y_p^* - x < w \end{cases}$$

We take the first derivative of the function to check the first order condition:

$$\frac{dH(w|x)}{dw} = \begin{cases} \frac{c_{tw}}{2} + \frac{L(x+w)}{(w-U-V)} - \int_0^{w-U-V} \frac{L(x+U+V+m)}{(w-U-V)^2} dm & \text{if } U + V < w \leq y_p^* - x \\ \frac{c_{tw}}{2} + \frac{L(y_p^*)(y_p^* - x - U - V)}{(w-U-V)^2} - \int_0^{y_p^* - x - U - V} \frac{L(x+U+V+m)}{(w-U-V)^2} dm & \text{if } y_p^* - x < w \end{cases}$$

At the transition point  $w = y_p^* - x$  the first derivatives are equal, therefore if the second derivative is non-negative at both sides of the transition point, the first order condition will

be satisfied.

$$\frac{d^2H(w|x)}{dw^2} = \begin{cases} \frac{L'(x+w)(w-U-V)^2 - 2L(x+w)(w-U-V)}{(w-U-V)^3} + \frac{2 \int_0^{w-U-V} L(x+U+V+m) dm}{(w-U-V)^3} & \text{if } U+V < w \leq y_p^* - x \\ \frac{2 \int_0^{y_p^* - x - U - V} L(x+U+V+m) dm}{(w-U-V)^3} - \frac{2L(y_p^*)(y_p^* - x - U - V)}{(w-U-V)^3} & \text{if } y_p^* - x < w \end{cases}$$

It is evident that  $\frac{d^2H(w|x)}{dw^2}$  is positive for  $w > y_p^* - x$ , because the first term in the numerator is greater than the second term, since it integrates  $L(x+U+V+m)$  over a region where the values are greater than the optimal  $L(y_p^*)$ , whereas the second term is equivalent to the integration of  $L(y_p^*)$  over the same region. For  $U+V < w \leq y_p^* - x$  we take the limit of the second derivative as  $w \rightarrow U+V$  and show that it is positive, and remains positive throughout the whole domain.

$$\lim_{w \rightarrow U+V} \frac{L'(x+w)(w-U-V)^2 - 2L(x+w)(w-U-V) + 2 \int_0^{w-U-V} L(x+U+V+m) dm}{(w-U-V)^3} = \frac{0}{0}$$

By using L'Hopital's Rule we get:

$$\frac{L''(x+w)(w-U-V)^2 + 2L'(x+w)(w-U-V) - 2L'(x+w)(w-U-V) - 2L(x+w) + 2L(x+w)}{3(w-U-V)^2} = \frac{L''(x+w)}{3} > 0$$

The second derivative is positive at  $U+V$ , we have to ensure that it remains positive for  $w > U+V$ . To do so we check the numerator of the second derivative, since the denominator is always positive for  $w > U+V$ . We take the derivative of the numerator and check if it is positive. Let us denote the numerator by  $\varpi(w)$ , then  $\frac{d\varpi(w)}{dw} = L''(x+w)(w-U-V)^2$ , which is positive for all  $w$ , hence the function is convex for  $w > U+V$ .

To conclude the convexity of  $H(w|x)$ , we need to show that the convexity is preserved at the transition point  $w = U+V$ . Since  $H(w|x)$  is dependent on the starting inventory level  $x$ , the first order condition should be satisfied for all  $x$ . It is sufficient to analyze the transition point for  $x < y_p^* - U - V$  and  $x \geq y_p^* - U - V$ . Note that we analyze the derivative of the function on both sides of the transition point. For the initial case  $H(w|x)$  takes the following form near the transition point.

$$H(w|x) = \begin{cases} c_u(w-U) + L(x+w) & \text{if } U < w \leq U+V, \\ Vc_u + c_{tw} \frac{w-U-V}{2} + \int_0^{w-U-V} \frac{L(x+U+V+m)}{w-U-V} dm & \text{if } U+V < w \leq y_p^* - x. \end{cases}$$

The first derivative for this region is:

$$\frac{dH(w|x)}{dw} = \begin{cases} c_u + L'(x+w) & \text{if } U < w \leq U+V, \\ \frac{c_{tw}}{2} + \frac{L(x+w)}{w-U-V} + \int_0^{w-U-V} \frac{L(x+U+V+m)}{(w-U-V)^2} dm & \text{if } U+V < w \leq y_p^* - x. \end{cases}$$

The first order condition for the above region is:

$$c_u + L'(x+w) \leq \frac{c_{tw}}{2} + \frac{L(x+w)(w-U-V) - \int_0^{w-U-V} L(x+U+V+m) dm}{(w-U-V)^2}.$$

Taking the limit as  $w \rightarrow U+V$  we get

$$c_u + L'(x+w) \leq \frac{c_{tw}}{2} + \frac{L'(x+w)}{2}$$

Note that the above inequality holds if  $c_u \leq \frac{c_{tw}}{2}$ .

For the second case ( $x \geq y_p^* - U - V$ ) the cost function takes the following form:

$$H(w|x) = \begin{cases} c_u(w-U) + L(y_p^*) & \text{if } y_p^* - x < w \leq U+V \\ Vc_u + c_{tw} \frac{w-U-V}{2} + L(y_p^*) & \text{if } U+V < w \end{cases}$$

The first derivative for this region is in the following form:

$$\frac{dH(w|x)}{dw} = \begin{cases} c_u & \text{if } y_p^* - x < w \leq U+V \\ \frac{c_{tw}}{2} & \text{if } U+V < w \end{cases}$$

For the first order condition to hold  $c_u$  must be less than or equal to  $\frac{c_{tw}}{2}$ . Therefore if  $c_u \leq \frac{c_{tw}}{2}$  then the single period cost function is convex in  $w$  for all starting inventory levels  $x$ .

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