# Reexamining $^{18}$Na and $^{19}$Mg

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# Reexamining $^{18}$Na and $^{19}$Mg

## Abstract

New results for energies of resonances in $^{18}$Na have led us to reexamine the problems of $^{18}$Na and $^{19}$Mg. We have calculated the effect of the new data on energy and decay width of $^{19}$Mg (ground state).

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## Comments

# Reexamining $^{18}$Na and $^{19}$Mg

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New results for energies of resonances in $^{18}$Na have led us to reexamine the problems of $^{18}$Na and $^{19}$Mg. We have calculated the effect of the new data on energy and decay width of $^{19}$Mg (ground state).

Earlier [1], we calculated the mass excess of the ground state (g.s.) of $^{19}$Mg, assumed to be the mirror of $^{19}$N(g.s.). Those calculations needed six core states in $^{18}$N/$^{18}$Na. These are the six states whose dominant configuration is a $p_{1/2}$ hole coupled to the three lowest $(sd)^3$ states in $^{19}$O/$^{19}$Na, having $T = 3/2$ and $J^\pi = 5/2^+$, $3/2^+$, and $1/2^+$. These states were not known in $^{18}$Na, and thus it was necessary to compute their energies. Candidates for four of the six were known [2] in $^{18}$N, and could be used as input to a potential model to compute their mirrors in $^{18}$Na. But two had no candidates in $^{18}$N. These were the $0^-$ and $1_2^-$ states whose parentage involves the $1/2^+$ $T = 3/2$ state of $A = 19$. Despite the small amount of $s^2$ in $^{19}$Mg(g.s.), the results were sensitive to the inclusion of these two states. Because they were unknown in $A = 18$, we did the $^{19}$Mg calculations assuming two different sets of theoretical energies for them in $^{18}$N. The mass excess of $^{19}$Mg(g.s.) differed by only 6 keV in the two calculations, but leaving out these two core states caused a shift of about 100 keV. We concluded [1] that inclusion of these two core states was important, but their precise energies were not. The known and computed energies in $^{18}$N/$^{18}$Na are listed in Table I.

Our prediction for the $^{19}$Mg(g.s.) energy was $E_{2p} = 0.87(7)$ MeV—a significantly narrower range than previous estimates [3,4]. A subsequent experiment [5] gave $E_{2p} = 0.75(5)$ MeV, just at the $1\sigma$ limit of our calculation. We also calculated the expected decay width. At our estimated energy, our width for simultaneous $2p$ decay was 3.3 meV, with a large uncertainty arising from the 70-keV estimated uncertainty in the calculated energy. At the experimental energy, our $\Gamma_{sim}$ was 0.08 to 0.80 meV [6]. The measured width [5] was $0.114^{+0.068}_{-0.031}$ meV. We also estimated the width for sequential decay through the tails of the expected broad $s$-wave resonances. This estimate is extremely sensitive to the very-low-energy [below the $^{18}$Na(g.s.)] behavior of the resonance profile. For the $0^-$ state alone we found that reasonable variations in this profile could produce variation of almost a factor of $10^3$ in the computed sequential width. With our assumed $0^-$ energy, our estimated range [1] was 1.5 $\mu$eV to 1.2 meV. We concluded that these sequential decays through tails of higher-lying resonances were large enough that they should be included, but that they would probably not dominate. At the experimental energy of 0.75(5) MeV, our range of sequential width becomes about 0.039 $\mu$eV to 0.096 meV.

Some of the uncertainty in our energy calculation was due to the fact that the energies of the relevant core states in $^{18}$Na were not known, but had been calculated. Since that time, additional information has been obtained on levels of $^{18}$Na, using the $^{17}$Ne $+$ $p$ resonance reaction [7]. The $2^-$ and $3^-$ states arising from the $^{19}$Na($5/2^+$) $\times$ $(1p)^{-1}$ coupling were clearly observed, as were the $0^-$ and $1^-$ states mentioned earlier. The $1^-$ g.s. and the $2^-$ state of the configuration $^{19}$Na($3/2^+$) $\times$ $(1p)^{-1}$ were not observed, and they were not expected to be because of their extremely small spectroscopic factors to $^{17}$Ne(g.s.)

Observed energies in $^{18}$Na are compared with our earlier predictions in Table II. The $3^-$ resonance was by far the most cleanly observed, at a resonance energy of $E_p = 2.084(5)$ MeV, compared to our prediction of 2.133 MeV—quite good agreement. The observed width of the $3^-$ resonance was $\Gamma_{exp} = 42(10)$ keV. We have used our potential model to compute the single-particle (sp) width for $\ell = 2$ at this energy. The result of 56 keV can be used to compute a spectroscopic factor, $S = \Gamma_{exp}/\Gamma_{sp} = 0.75(18)$. This value is remarkably close to the spectroscopic factor from the shell-model (sm) calculations of 0.70. The $2^-$ state of the same configuration was observed as a resonance at $E_p = 1.552(5)$ MeV, compared to our prediction of 1.521 MeV—again quite good agreement. However, the width observed was only 5(3) keV. With a computed sp width of 15 keV, the resulting spectroscopic factor

TABLE I. Energies (MeV) of the relevant states in $^{18}$Na/$^{18}$Na.

| $J_n^{\pi}$ [a] | $E_x(^{18}$N)[b] | $E_p(^{18}$Na)[c] | $S$ to $^{19}$Mg(g.s.)[d] |
|---|---|---|---|
| $1^-$ | 0.00 | 1.382 | 0.0048, 0.029 |
| $(2_1^-)$ | 0.115 | 1.521 | 1.298 |
| $(2_2^-)$ | 0.588 | 1.919 | 0.050 |
| $(3^-)$ | 0.747 | 2.133 | 1.794 |
| $0_1^-$ | 0.662 sm | 1.574 | 0.113 |
|  | 1.380 wc | 2.102 |  |
| $1_2^-$ | 1.167 sm | 2.081 | 0.348, 0.003 |
|  | 1.870 wc | 2.638 |  |

[a]Reference [2].
[b]Reference [2], unless otherwise noted; sm and wc stand for shell model and weak coupling, respectively, from Ref. [1].
[c]Calculated in Ref. [1].
[d]Reference [1]. When two $S$'s are listed, they are for $\ell = 0$, then 2.

TABLE II. Results for $^{18}$Na $= {}^{17}$Ne(g.s.) $+ p$ (Energies in MeV, widths in keV).

| State | $E_p$ | | $\ell$ | $S_{calc}$[c] | $\Gamma_{sp}$ | $\Gamma_{exp}$[b] | $S_{exp} = \Gamma_{exp}/\Gamma_{sp}$ |
|---|---|---|---|---|---|---|---|
| | Calc.[a] | Exp.[b] | | | | | |
| $1_1^-$ | 1.382 | – | 2 | 0.012 | 9.6 | $< 1$ | – |
| | | | 0 | 0.005 | 420 | $< 1$ | $< 0.0024$ |
| $2_1^-$ | 1.521 | 1.552(5) | 2 | 0.70 | 15 | 5(3) | 0.33(20) |
| $0_1^-$ | 1.57–2.10 | 1.842(40) | 0 | 0.83 | 1170 | 300(100) | 0.26(9) |
| $2_2^-$ | 1.919 | – | 2 | 0.012 | 39 | $< 1$ | $< 0.026$ |
| $1_2^-$ | 2.08–2.64 | 2.030(20) | 2 | $\sim 0$ | 48 | – | – |
| | | | 0 | 0.83 | 1600 | 900(100) | 0.56(6) |
| $3_1^-$ | 2.133 | 2.084(5) | 2 | 0.70 | 56 | 42(10) | 0.75(18) |

[a]Reference [1].
[b]Reference [7].
[c]Reference [1].

is only $S(2^-) = 0.33(20)$, rather than the 0.70 expected. Of course, the discrepancy is less than a $2\sigma$ effect.

The $0^-$ and $1^-$ $s$-wave resonances were observed at energies of $E_p = 1.842(40)$ and 2.030(20) MeV, with widths of 300(100) and 900(100) keV, respectively. The definition of the resonance energy of such a broad resonance is not straightforward. Reference [7] does not state if the resonance energies correspond to a phase shift of $\pi/2$, or a maximum in the energy derivative of the phase shift, or a maximum in the cross section, or something else. These different definitions can produce resonance energies that differ by 100–300 keV for broad resonances. This $0^-$, $1^-$ pair are closer to one another in the experiment than in the sm calculation, but the $(2J + 1)$-weighted energy centroid is in reasonable agreement. The sm has it at 1.954 MeV: experimentally it is 1.983(18). Recall that we were unable to make predictions for the energies of these two states in $^{18}$Na because they are not known in $^{18}$N. But, we can still compute their spectroscopic factors from the measured widths and our computed sp widths. These are also given in Table II. For the $1^-$, the sp width is only an estimate because it is so wide. These two spectroscopic factors should be about 0.83 from the sm calculations, but the values from $\Gamma_{exp}/\Gamma_{sp}$ are only 0.26(9) for the $0^-$ and 0.56 (with larger uncertainty) for $1^-$. Because of the relatively small uncertainty for the $0^-$ this is a serious discrepancy. Thus, three of the four resonances observed by Ref. [7] are significantly narrower than expected.

The question naturally arises: How much does our $^{19}$Mg(g.s.) energy prediction change when these experimental energies are used in the calculation? The answer is not much. The $2^-$ is 31(5) keV higher than predicted, and the $3^-$ is 49(5) keV lower, so the shifts in $^{19}$Mg(g.s.) from their new energies go in opposite directions. The prediction was not very sensitive to the exact locations of the $s$-wave resonances, only to their inclusion. Still using the predicted energies for the other two states, but the experimental energies for the four

resonances, causes a shift of 26 keV lower in the predicted $^{19}$Mg(g.s.) mass, leading to $E_{2p} = 0.84(7)$, compared to the earlier result of 0.87(7) MeV. The shift is well within our estimated uncertainty of 70 keV in the calculation.

Recall from above that, with the known $E_{2p} = 0.75(5)$ MeV for $^{19}$Mg(g.s.), our estimate of the sequential decay width through the extreme low-energy tail of the $0^-$ $s$-wave resonance covered the range 0.039 $\mu$eV to 0.096 meV. These were for our earlier assumed energy for the $0^-$ state. If we use the new experimental $0^-$ energy, our limits become 0.056 $\mu$eV to 0.13 meV. A similar calculation for the decay through the tail of the $1^-$ resonance at its experimental energy gives a sequential width of 0.13 $\mu$eV to 0.32 meV. The authors of Ref. [7] also computed these widths. They get 0.24 meV for decay through the $0^-$ tail, and 0.41 meV for decay through the $1^-$. These are significantly larger than the experimental value of 0.11 meV. They are also slightly larger than our upper limits, but close to them. Reference [7] suggests that $^{19}$Mg must have less $s$-wave strength than is present in the calculations. But this quantity is already quite small—only about 10% of the total $sd$ occupancy is in the $s$ orbital in typical sm calculations.

Perhaps the reason why our simple procedure works so well for $^{19}$Mg can be understood by comparison with our recent work [8] concerning the $^{20}$O/$^{20}$Mg energy difference. In that case, a full $(sd)^4$ calculation and a severely truncated calculation that included only the three lowest states of $^{19}$O/$^{19}$Na produced virtually identical results. By far the dominant component of the g.s. of $^{19}$N ($^{19}$Mg) is a proton (neutron) hole in the g.s. of $^{20}$O ($^{20}$Mg). Because three $T = 3/2$ states of $A = 19$ were sufficient for the $0^+$, $T = 2$ g.s. of $A = 20$, it is not surprising that the $^{19}$N/$^{19}$Mg calculation needs only the six states that arise from coupling a $p_{1/2}$ hole to these three states. These are the six core states in $^{18}$N/$^{18}$Na that we have included. We noted in Ref. [1] that it was important to include all six.

[1] H. T. Fortune and R. Sherr, Phys. Rev. C **76**, 014313 (2007).

[2] D. R. Tilley, H. R. Weller, C. M. Cheves, and R. M. Chasteler, Nucl. Phys. A **595**, 1 (1995).

[3] L. V. Grigorenko, I. G. Mukha, and M. V. Zhukov, Nucl. Phys. A **713**, 372 (2003).

[4] L. V. Grigorenko, R. C. Johnson, I. G. Mukha, I. J. Thompson, and M. V. Zhukov, Phys. Rev. Lett. **85**, 22 (2000).

[5] I. Mukha *et al*., Phys. Rev. Lett. **99**, 182501 (2007).

[6] H. T. Fortune and R. Sherr, Phys. Rev. C **83**, 057301 (2011).

[7] M. Assie *et al*., Phys. Lett. (in press).

[8] H. T. Fortune, R. Sherr, and B. A. Brown, Phys. Rev. C **85**, 054304 (2012).