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Abstract

International consumption risk sharing studies often generate counterfactual implications for asset return behavior with potentially misleading results. We address this contradiction using data moments of consumption and asset returns to fit a canonical international consumption risk sharing framework. Introducing persistent consumption risk, we find that its correlation across countries is more important for risk sharing than that of transitory risk. To identify these risk components, we jointly exploit the comovement of equity returns and consumption. This identification implies high correlations in persistent consumption risk, suggesting a strong degree of existing risk sharing despite low consumption correlations in the data.

Keywords

financial integration, international risk sharing, global asset pricing

Disciplines

Economics | Finance | Finance and Financial Management | International Economics

Evaluating International Consumption Risk Sharing Gains: An Asset Return View*

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October, 2012

Abstract

Multi-country consumption risk sharing studies that match the equity premium typically find very large gains from risk-sharing. However, these studies usually generate counterfactual implications for the risk-free rate and asset return variability. In this paper, we modify a canonical risk-sharing model to generate asset return behavior closer to the data and then consider the effects on welfare gains. To better fit asset return behavior, we introduce persistent consumption risk, finding that the welfare gains depend critically on the international correlation in this persistent risk. We then provide a new identification for this risk by jointly exploiting the data correlation for equity returns and for consumption. This identification implies high correlation in persistent consumption risk, suggesting a strong degree of diversification despite low correlations in transitory risk. As such, our findings show that matching equity returns can imply lower international risk sharing gains than previously thought.

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1 Introduction

How much welfare improvement can be generated by optimal international consumption risk-sharing? The obvious importance of this question has motivated a significant body of research.[1] As this literature shows, international risk-sharing gains depend directly upon the value of consumption risk and the ability to diversify across countries. Clearly, asset prices in international financial markets provide a direct measure of this consumption risk. Nevertheless, consumption risk-sharing studies often ignore asset return implications. Indeed, assumptions about risk and intertemporal substitution in consumption often generate counterfactual implications for the magnitude of asset returns.[2]

This gap between models of international risk-sharing gains and asset return behavior appears significant given advances in consumption-based asset pricing. Specifically, several lines of research have demonstrated that introducing persistent variation into the intertemporal marginal rate of substitution in consumption helps models fit asset return behavior.[3] This variation contrasts with the typical view in many international risk-sharing gains studies that all uncertainty is transitory. [4]

In this paper, we begin to bridge this gap using a canonical international consumption risk-sharing framework in the tradition of Obstfeld (1994b). Observed consumption and asset return moments benchmark the current degree of implied risk sharing. The fully diversified international risk sharing equilibrium is then derived using the parameters determined from these data moments. Comparing the lifetime utility from the current economy to that of the optimal risk sharing economy provides the welfare gains measure. Most of these studies assume that consumption only varies due to transitory shocks around a trend. As is well-

[1] Surveys that discuss the literature on international risk-sharing welfare gains include Tesar (1995), Lewis (2011), and Coeurdacier and Rey (forthcoming).

[2] See the discussion in Obstfeld (1994b) and Lewis (2000).

[3] Campbell and Cochrane (1999) employ habit persistence with time-varying risk aversion in preferences. Bansal and Yaron (2004) assume that consumption growth has a persistent component. Barro (2006) follows Reitz (1988) in assuming that investors price disaster risk.

[4] Colacito and Croce (2010) and Stathopoulos (2012) are important exceptions. Below we describe how our analyses differ.

known, however, consumption-based asset pricing models with only transitory risk cannot generate the size of the equity premium and the risk-free rate, not to mention the variability in asset returns. Therefore, to better fit these moments, we introduce low frequency variation in consumption risk in the form of a small autoregressive component in consumption following Bansal and Yaron (2004). We choose this approach because it incorporates the same recursive preferences as our canonical framework. As such, our analysis of persistent risk naturally nests the more typical transitory-only risk case as in Obstfeld (1994b).[5]

While persistent consumption risk helps to explain asset returns better, it also carries important implications for diversification gains. We show that risk-sharing gains depend strongly on how much persistent risk can be diversified. If persistent consumption risk correlations are low and, hence, can be diversified under optimal international risk-sharing, the welfare gains are very large. Therefore, understanding the welfare gains from full risk-sharing requires identifying the current degree of diversification in each type of risk. While the data correlation of consumption across countries provides an obvious metric of overall diversification, it depends upon the correlation of both the transitory and persistent components.

We therefore develop an identification strategy that uses equity return correlations together with consumption correlations to decompose each type of risk. These correlations imply that the persistent risk correlations are very high and near one across our sample of advanced economies.[6] The intuition behind this result is straightforward. In the data, international correlations of equity returns are higher than those of consumption. In the model, equity return correlations depend more strongly on persistent risk than do consumption correlations. Therefore, viewing the data through the lens of the model yields high correlations in this persistent risk and correspondingly low correlations in transitory risk.

[5]In order to measure welfare gains when economies grow, Obstfeld (1994a) demonstrates the importance of decoupling risk aversion and intertemporal elasticity as in Epstein and Zin (1989) and Weil (1990) preferences. Bansal and Yaron (2004) also use Epstein-Zin preferences. On the other hand, Campbell and Cochrane (1999) consider habit-persistent preferences. Barro (2009) does allow for recursive preferences but does not target return volatility.

[6]The analysis below only covers advanced economies. Our finding that gains based on asset returns are modest is likely to be mitigated for emerging countries if their returns are less correlated with the world.

This result highlights a key finding of our paper. The high correlations between equity returns across countries in combination with low correlations in consumption growth imply that persistent risk is already highly diversified. At the same time, transitory consumption risk remains relatively undiversified, even more so than consumption correlations suggest. Nevertheless, the high degree of diversification in persistent risk suppresses the overall gains from international risk-sharing. As such, the risk sharing gains arise primarily from the transitory consumption risk and, as such, are more consistent with studies that ignore asset pricing considerations. Importantly, this result stands in contrast to a conventional view that disciplining consumption-based models to match the equity premium generates high welfare gains, even exceeding 100% of permanent consumption[7].

On the other hand, our finding that important consumption risk is highly diversified is reminiscent of results in exchange rate-based studies identified through a different channel. Brandt, Cochrane, and Santa Clara (2006) show that the lower volatility of exchange rates compared to equity returns implies a high degree of risk sharing. At the same time, the low international consumption correlations in the data point to low risk-sharing. Therefore, they pose this contradiction as a puzzle. By contrast, we jointly target consumption moments along with key asset pricing moments to identify the degree of risk-sharing implicit in cross-country correlations in consumption growth and equity returns. We show that the high data correlations in equity returns relative to consumption imply a high degree of risk sharing in persistent risk but not transitory risk.

Similarly, several papers have considered the effects of persistent consumption risk on exchange rate behavior or the foreign exchange risk premium. Among these studies, Colacito and Croce (CC) (2011) assume long run risk to generate persistent variation in the intertemporal marginal rate of substitution in consumption as we do, and is thus the most related.[8]

[7] For example, see the discussions in Obstfeld (1994b), Lewis (2000), and, more recently, Courdacier and Rey (forthcoming).

[8] Another set of papers considers the effect of habit persistent preferences. Verdelhan (2010) uses these preferences to examine the foreign exchange risk premium anomaly. Stathopolous (2012) builds a model to match exchange rate variability and other key moments.

Moreover, they find that the long run risk components across countries are highly correlated, as we do. Nevertheless, our approaches differ in a number of significant ways. First, CC use the data to estimate the parameters of a complete markets model. By contrast, we use the data to fit a benchmark Euler equation without assuming complete markets. We then measure the gains of moving to a complete markets optimal risk sharing equilibrium. Second, CC assume differences in goods preferences between countries to determine exchange rate behavior in their model. We do not take a stand on the reasons for exchange rate variability, but measure its effects on consumption risk through the data.[9] Third, CC impose symmetry in their two country model on the stochastic processes and home bias preferences. This paper, on the other hand, allows countries to differ in the stochastic nature of consumption, but instead treats preferences as identical across all countries. Given these and other distinctions between our approaches, we view our results as complementary to theirs.

In our goal to provide the best fit between the model and data, we use Simulated Method of Moments (SMM) for seven industrialized countries to anchor our calibration approach. In particular, we target the means and standard deviations of equity returns and the risk-free rate, along with moments from consumption or dividends. We analyze two different versions of the model that successively improve on the fit for asset return implications. As such, our results contribute to a growing literature that examines persistent consumption risk in a panel of countries. However, studies in this literature focus on the individual asset pricing relationships for each country without considering the international implications. For example, Nakamura et al (2012) estimate a long run risk model in a panel of countries and generate the asset returns for each country.[10] By contrast, we develop a framework that can be used to evaluate international asset pricing and the associated welfare gains.

The structure of the paper is as follows. Section 2 describes the basic risk-sharing

[9] Below we describe our treatment of real exchange rate variations in more detail.

[10] Other papers consider the effects of disaster risk. For example, Nakamura, Steinsson, Barro, and Ursua (2010) and Farhi, Fraiberger, Gabaix, Ranciere, and Verdelhan (2009) examine the impact on the equity premium and currency markets, respectively.

framework and its relationship with returns. In Section 3, we evaluate that framework under the assumption that all equity returns pay out consumption. Section 4 considers the Bansal and Yaron (2004) model based upon dividend data. Section 5 extends the analysis in several ways including differing means, population sizes and a wider set of countries. Section 6 gives concluding remarks.

2 Risk-Sharing and Returns: The Framework

Given the degree of integration across countries, what are the benefits to complete international risk-sharing? The obvious importance of this question has motivated a large literature that studies the gains from consumption risk-sharing as noted above[11]. These studies typically evaluate the benefits of risk sharing by comparing implications for welfare from observed consumption to that of an alternative fully integrated world economy. While the details of the studies differ, the welfare gains calculations follow a common approach. The approach compares the utility from a consumption path in a benchmark economy to that of a fully diversified economy. To summarize this approach, first define C_t^B and W_t^B as the benchmark economy consumption and wealth at time t and C_t^* and W_t^* as their counterparts in the fully diversified economy. The approach then compares the life-time utility, or value function, in a benchmark economy, $V(C_t^B, W_t^B)$, to the value function in a fully diversified economy, $V(C_t^*, W_t^*)$. Specifically, the welfare gains at some initial time period 0 are given by Δ in the following equation:

$$V((1+\Delta)C_0^B, (1+\Delta)W_0^B) = V(C_0^*, W_0^*) \tag{1}$$

[11] These gains are also related to the literature on consumption risk sharing. Backus, Kehoe and Kydland (1992) observed that consumption correlations are lower than output correlations, thus violating the implications of perfect risk-sharing. Explanations range from incomplete markets (e.g., Baxter and Crucini (1995)), hedging labor risk (e.g., Baxter and Jermann (1997), Heathcote and Perri (2008)), hedging non-tradeable (e.g., Stockman and Tesar (1995)), and transactions costs (e.g, Tesar and Werner (1995)).

As such, welfare gains are the percentage increases in current permanent consumption and wealth required to increase welfare to that of the full risk sharing economy.

The asset pricing implications in these papers are generally counterfactual, however. In particular, the equity premium is too low (Mehra-Prescott (1985)), the risk free rate is too high (Weil (1989)), and the volatility of asset returns are too low (Campbell and Shiller (1988)). In this paper, we examine how risk sharing gains are affected when the consumption process better matches asset return behavior than under the standard model. While asset return behavior is clearly only one way to discipline the model, it is arguably the most important for the question at hand. Trade in international capital markets is often viewed as the primary mechanism for sharing risks globally. As such, the prices of assets in these markets reflect equilibrium views toward risk.

Asset returns generally depend upon the trend growth rate in consumption, raising additional considerations. As shown by Obstfeld (1994a,b), time-additive constant relative risk aversion (CRRA) preferences cannot be used to accurately evaluate welfare gains in the presence of consumption growth. Gains to future certainty equivalent consumption become more important as the intertemporal elasticity of consumption rises. On the other hand, higher IES implies lower risk aversion under constant relative risk aversion utility, dampening the value of reduced volatility. Counter-intuitively, risk sharing gains may appear to decline as risk aversion increases. Therefore, we assume consumers in each country have recursive preferences that decouples risk-aversion and IES. Following Epstein and Zin (1989) and Weil (1990), preferences are given by:

$$U^j(C_t, U_{t+1}) = \left\{ C_t^{\left(\frac{1-\gamma}{\theta}\right)} + \beta E_t \left[\left(U^j_{t+1} \right)^{1-\gamma} \right]^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \tag{2}$$

where C_t is the consumption at time t, U^j_{t+1} is the utility function at $t+1$; $0 < \beta < 1$ is the time discount rate; $\gamma \geq 0$ is the risk-aversion parameter; $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$ for $\psi \geq 0$, the intertemporal elasticity of substitution; and where $\mathrm{E}_t(\cdot)$ is the expectation operator conditional on the information set at time t[12].

Determining welfare gains as in equation (1) then requires a solution for the value function in terms of the current economy and the risk-sharing economy. For the Epstein-Zin utility, it is well-known that the value function is homogeneous of degree one in consumption and wealth, W_t, and can be written as: $V(C_t, W_t) = (W_t/C_t)^{\left(\frac{1}{1-\left(\frac{1}{\psi}\right)}\right)} C_t$.[13] Also, according to the budget constraint, wealth is given by $W_t = P_t + C_t$ where P_t is the price of an asset paying out consumption in all future periods. We arbitrarily denote the period when the economy moves to the full risk sharing equilibrium as $t = 0$. Then, substituting the form of the value function into the basic welfare gain relationship in equation (1) implies that the welfare gain, Δ, can be expressed as:

$$(1+\Delta) = \left\{\frac{W_0^*/C_0^*}{W_0^B/C_0^B}\right\}^{\left(\frac{1}{1-\left(\frac{1}{\psi}\right)}\right)} \left(\frac{C_0^*}{C_0^B}\right) = \left\{\frac{Z_0^*+1}{Z_0^B+1}\right\}^{\left(\frac{1}{1-\left(\frac{1}{\psi}\right)}\right)} \left(\frac{C_0^*}{C_0^B}\right) \tag{3}$$

where $Z^* = (P^*/C^*)$ and $Z^B = \left(P^B/C^B\right)$ are the price-dividend ratios for the consumption asset prices under the full risk sharing and the benchmark economies, respectively. Therefore, as equation (3) shows, welfare gains can be computed directly from consumption levels and the price of the consumption asset in the benchmark and risk sharing economies.

2.1 Solving Asset Prices and Consumption in the Economies

Calculating international risk-sharing gains requires the price of an asset that pays consumption in all future periods in both the benchmark and perfect risk-sharing economies. For this reason, asset price determination is an important calculation in our analysis. We discipline our prices by calibrating the parameters to consumption and asset return moments. Following Epstein and Zin (1989), any asset ℓ must satisfy the first-order Euler condition in

[12] As described by Epstein and Zin (1989), this utility function reduces to standard time-additive CRRA preferences when $\gamma = \frac{1}{\psi}$.

[13] For example, see Campbell (1993).

the benchmark economy:

$$E_t\left\{\beta^{\theta}(C_{t+1}/C_t)^{\left(-\frac{\theta}{\psi}\right)}(R^P_{t+1})^{(\theta-1)}R^{\ell}_{t+1}\right\}=1 \quad (4)$$

where R^P_{t+1} is the gross return on the market portfolio paying out consumption and R^{ℓ}_{t+1} is the gross return on asset ℓ. We use the Euler equation to derive analytical solutions for asset returns and calibrate benchmark model parameters to match the observed data. Using these parameters, we calculate the benchmark economy price-to-consumption ratio, Z^B_0, implied by the current data.

The welfare gain in equation (3) shows that we also require price-to-consumption and consumption levels in the full risk sharing economy, Z^*_0 and C^*_0, respectively.[14] In the full risk sharing equilibrium, however, countries pool their consumption streams into aggregate world consumption and then share the same consumption growth. Therefore, the price of the world consumption good Z^* is a mutual fund of all countries' consumption processes. Defining the world mutual fund payout as $C^w_t \equiv \Sigma^J_{j=1}C^{Bj}_t$ and its price in the risk sharing economy as P^{w*}_t, the price-dividend ratio for the total economy is $Z^*_t \equiv P^{w*}_t/C^w_t$.

We first solve for the consumption level for each country, C^{j*}_t.[15] This level depends upon the value of each country's benchmark consumption in the risk sharing economy. That is, to buy into the world mutual fund, investors in each country sell off claims to their own consumption stream valued at P^{j*}_t.[16] They then seek to buy the highest claims on the aggregate world consumption stream valued at P^{w*}_t. That is, defining ϖ^{jw} as the claim of country j on world consumption in period $t=0$ and normalizing the number of shares in each country to 1, the investor in country j faces the constraint:

$$\left(C^w_0+P^{w*}_0\right)\varpi^{jw}\leq\left(C^{jB}_0+P^{j*}_0\right) \quad (5)$$

[14]We describe these relationships as in a decentralized asset market here. In Appendix A, we show that this equilibrium also solves the social planner problem.

[15]For now, we assume that each country has a single representative agent, thereby implicitly assuming equal population weights. Below, we relax this assumption.

[16]We could alternatively have assumed countries can sell off claims to their output or factor resources. In the text, we evaluate the gains from sharing consumption because we can then condition on the current level of integration based on the Euler equation.

where P_t^{w*} is the time t price of the mutual fund in world markets and where P_t^{j*} is the price of country j's benchmark consumption in the full risk sharing economy. Clearly, for a utility-maximizing investor, the portfolio constraint holds with equality so that: $\varpi^{jw} = (C_0^{jB} + P_0^{j*})/(C_0^w + P_0^{w*})$. Therefore, all countries receive payouts of aggregate world consumption albeit with differing shares depending on the value of their benchmark consumption stream in the full risk sharing economy. As a result, the price-consumption ratio in the full risk sharing equilibrium is common across all countries: $Z_t^{j*} \equiv P_t^{j*}/C_t^{j*} = (\varpi^{jw} P_t^{w*}) / (\varpi^{jw} C_t^w) = Z_t^*$, $\forall j$.

To determine the consumption asset prices in the risk sharing economy, P_t^{w*} and P_t^{j*}, we again use the Euler equation (4). In the full risk-sharing economy, the market portfolio becomes the world consumption asset. Defining the world mutual fund payout as C_t^w and recalling that its price is P_t^w, the return on the consumption asset is $R_{t+1}^P = R_{t+1}^{w*} \equiv \left(P_{t+1}^{w*} + C_{t+1}^w\right)/P_t^{w*}$. Since this equation must be satisfied for all returns, we can solve for the price of the world consumption P_t^{w*} by setting: $R_{t+1}^{\ell} = R_{t+1}^{w*}$ in the Euler equation. Similarly, the prices of the benchmark consumption stream for each country, P_t^{j*}, can be determined by setting $R_{t+1}^{\ell} = R_{t+1}^{j*}$, where $R_{t+1}^{j*} = \left(P_{t+1}^{j*} + C_{t+1}^{jB}\right)/P_t^{j*}$.

$$E_t \left\{ \beta^{\theta} (C_{t+1}^w / C_t^w)^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^{w*})^{(\theta-1)} R_{t+1}^{j*} \right\} = 1 \tag{6}$$

These two prices along with the price-dividend ratio determine the welfare of each country under the full risk sharing economy. Next, we describe how we use the Euler equation to discipline the welfare gains calculations.

2.2 Matching Asset Returns with Consumption

We consider a canonical consumption risk-sharing welfare gain model based upon common preferences calibrated to benchmark consumption processes from the data. We focus upon the observed consumption since it is an equilibrium variable and may be generated from any general production process.

A standard approach in the literature is to evaluate the gains from sharing risk of temporary consumption variations around a trend.[17] For example, Obstfeld (1994a) specifies the process as a trend plus transitory disturbance as in:

$$g_{c,t+1} = \mu + \eta_{t+1} \tag{7}$$

where $g_{c,t+1}$ is the change in the logarithm of consumption, μ is the mean growth rate, and η_{t+1} is an i.i.d. stationary process with mean zero. However, an extensive literature has shown that a consumption process with purely transitory disturbances generates counterfactual implications for asset returns.

In order to address these inconsistencies, several approaches have been suggested that incorporate some persistent consumption risk. As noted earlier, these approaches include habit persistence (Campbell and Cochrane (1999)), long-run risk (Bansal and Yaron (2004), and disaster risk (Reitz (1988), Barro (2006,2009)). Among these, the "long run risk" approach of Bansal and Yaron (2004) is the only one that both use recursive preferences and targets asset return variability. Therefore, following the long-run risk approach, we specify a persistent stochastic component x_t^j in consumption growth.[18]

$$\begin{aligned} g^j_{c,t+1} &= \mu^j + x^j_t + \eta^j_{t+1} \\ x^j_{t+1} &= \rho^j x^j_t + e^j_{t+1} \end{aligned} \tag{8}$$

where $\eta^j_{t+1} \backsim N(0, \sigma^j)$ and $e^j_{t+1} \backsim N(0, \sigma^j_e)$.

Since deviations from annual consumption growth look close to transitory, the persistent component in consumption must be small. Because persistence is difficulty to detect at the annual level, we follow Bansal and Yaron (2004) in assuming that consumption decisions are made at the monthly frequency. We then choose the consumption parameter values that come closest to generating the consumption and asset return moments we observe in the

[17] See for example the survey in Tesar (1995) or van Wincoop (1994).

[18] Some studies consider an autoregressive consumption growth process but with no transitory component. For example, see van Wincoop (1999).

data. We find the implied persistent risk variance to be quite small consistent with the low autocorrelation in consumption data. Nevertheless, we come closer to fitting the asset return moments across countries than with standard transitory only risk[19].

2.3 Identifying the Benchmark Model Parameters

We calibrate preference parameters to values from the literature and then fit the consumption process parameters in equation (8) to obtain the closest match between the model implied asset returns and data based on the Euler equation (4). We base our analysis on a general Euler equation since this relationship holds for any level of current integration in the benchmark economy. For example, if domestic investors hold foreign assets, these assets are also priced according to this Euler equation. Moreover, the consumption process measured by the data is an equilibrium result based upon the current level of integration of goods and asset trade in the world.

We then use the parameters to determine the utility in the benchmark economy. In particular, we use the Euler equation (4) to solve the price-consumption ratio as a function of the preference parameters, ψ, β, γ and consumption process parameters, μ^j, σ^j, and σ_e^j. As noted in equation (3), these price-consumption ratios, Z^B together with consumption, C^B, determine the benchmark welfare. For much of our analysis, we normalize the initial period benchmark consumption levels for country, C_B, to equal one.[20]

Finally, we must calculate the utility in the full risk-sharing economy. As equation (3) highlights, the welfare in this economy requires calculating the price-consumption ratio for the consumption asset in the full risk sharing economy, Z^*, as well as the consumption level in this equilibrium, C^*. As a pooled basket of individual consumption processes, the variance of the world mutual fund depends directly on the consumption correlation across countries. When the consumption correlation embodies transitory risk only, the empirical correlation

[19]Lewis and Liu (2012) show the asset return implications under a standard model with only transitory risk

[20]Implicitly, this normalization assumes that all countries are equal in size. In Section 4.2, we consider the effect of relaxing this assumption.

in consumption provides a unique historical measure. However, when consumption includes a persistent component, this measure depends upon two sources of risk, the transitory shock η^j_{t+1} and the persistent shock, e^j_{t+1}. Therefore, the price-dividend ratio in the risk-sharing economy, Z^*_t, depends not only upon the consumption process parameters in the benchmark economy for all countries, but also upon the cross country correlation matrix for the transitory shock, η^j_{t+1}, and that of the persistent shock, e^j_{t+1}.[21]

Intuitively, the price of the world mutual fund depends upon the sum of growth rates, μ^ℓ, and the volatility of consumption characterized by the world variance-covariance matrix with components equal to the standard deviations, σ^ℓ and σ^ℓ_e for all countries and the correlations across those countries. Therefore, to determine welfare for the risk-sharing economy, we must identify the cross-country correlations of transitory shocks, η, and the persistent shocks, e. The following example demonstrates the importance of these correlations for welfare gains.

2.4 Preliminary Example

To illustrate the impact of the correlation of persistent shocks, we begin with a three country example using consumption data for the United States, the United Kingdom, and Canada. Focusing on three countries allows us to demonstrate the effects of asymmetry and multiple countries parsimoniously. Below, we extend this analysis to seven OECD countries.

Table 1, Panel A shows the means and standard deviations for consumption in this three country set, along with the first order autocorrelation, and cross-country correlation. The mean growth rates range from 1.96% for Canada to 2.08% for the U.S. However, the standard deviations in all three countries are large and are close to the mean growth rates. For this reason, we assume in the preliminary analysis that the mean growth rates are equal across countries. The table shows that the first order autocorrelations are lowest for the U.S. at 0.27 and highest for the U.K. at 0.40. The table also reports the correlation matrix for consumption ranging from 0.32 for Canada-UK to 0.63 for US-Canada.

[21] We detail this decomposition in Appendix B.

We then use the approach described in the next section to get the best fit of the country-specific consumption parameters, $\mu^\ell, \sigma^\ell, \sigma_e^\ell$, and then determine the gains from risk-sharing. Typically, the diversification gains would be determined from the consumption correlations. However, the consumption correlations in Panel A do not identify the correlation between transitory shocks, $Corr(\eta^\ell, \eta^j)$, separately from the persistent shocks, $Corr(e^\ell, e^j)$. To illustrate the impact of the correlation in persistent shocks, therefore, we assume the correlation between transitory shocks are given by the data correlations as in standard literature. We then consider a wide range of persistent risk correlations to understand the effects of this risk.

Table 1, Panel B illustrates the effects of persistent consumption risk correlation on the welfare gains. The top numbers for each country report the gains as a percent of permanent consumption while the numbers in parentheses below give the percent of the country's share in world output, ϖ^j. For reference, the first column gives the results using the same parameter estimates when there is no persistent risk so that $\sigma_e = 0$. The following five columns report welfare gains assuming correlations between persistent consumption ranging from 0 to 1, implying a decreasing ability to diversify this risk. When the correlation is zero, the gains increase dramatically for all countries relative to the case with no persistent risk. For example, the gains for the U.S are 10.2% when $\sigma_e = 0$ but increase to 70% if persistent shocks are uncorrelated. As the estimates show for increasing correlations of $Corr(e_t^i, e_t^w)$, the U.S. gains decline steadily to about 8%. Similar patterns hold for the other countries.

2.5 Identifying Persistent Risk Correlation

The example in Table 1 shows that international risk sharing gains depend crucially upon the correlation in persistent consumption risk. We now show that the basic model framework together with asset return and consumption data provide an identification for this correlation.

The identification follows naturally from covariances in consumption growth and equity returns in the benchmark economy as we summarize next. Appendix D details the derivation. First, note that the covariance in consumption growth across countries using equation (8) can be written:

$$Cov(g_c^i, g_c^j) = \sigma^i \sigma^j Corr(\eta^i, \eta^j) + \frac{\sigma_e^i \sigma_e^j}{1-\rho^2} Corr(e^i, e^j) \tag{9}$$

where $Corr(\ , \ )$ is the correlation operator. Thus, the observed covariance is comprised of two sources of correlation: the component due to the temporary shock, η, and to the persistent shock, e, where $1-\rho^2$ adjusts for the autocorrelation.

We now turn to the correlation of equity returns generated by the model. The Campbell-Shiller (1989) approximation implies that equity returns for country i can be written in the form:

$$R_{t+1}^i = a_0^i + a_1^i x_t^i + a_2^i e_{t+1}^i + \eta_{t+1}^i \tag{10}$$

where a_0^i, a_1^i, a_2^i are constants. Calculating the covariance of equity returns across countries provides a second observable variable that depends upon both temporary and persistent shock correlations:

$$Cov(R^i, R^j) = \sigma^i \sigma^j Corr(\eta^i, \eta^j) + \left[\frac{a_1^i a_1^j}{1-\rho^2} + a_2^i a_2^j \right] \sigma_e^i \sigma_e^j Corr(e^i, e^j) \tag{11}$$

Note that equity covariances and consumption covariances depend upon the transitory correlation, $Corr(\eta^i, \eta^j)$, in the same way. However, the variability in returns also depends upon the current level of persistence risk through the two terms in square brackets in equation (11). First, it depends upon the current level of persistent risk, x_t, measured by the autoregressive effect $a_1^i a_1^j/(1-\rho^2)$. Second, it depends upon the current innovation in persistent risk through $a_2^i a_2^j$.

Given the two observable covariances in consumption growth in equation (9) and equity returns in equation (11), we can identify the two sets of correlations, $Corr(\eta^i, \eta^j)$ and $Corr(e^i, e^j)$, for each pair of covariances across countries. Combining the consumption covariances in equation (9) with the equity covariance in equation (11), we solve for the correlation in the persistent shock as:

$$Corr(e^i, e^j) = D_o \frac{\sigma_R^i \sigma_R^j}{\sigma_e^i \sigma_e^j} \left[Corr(R^i, R^j) - \frac{\sigma_c^i \sigma_c^j}{\sigma_R^i \sigma_R^j} Corr(g_c^i, g_c^j) \right] \tag{12}$$

where $D_o \equiv \left[\frac{a_1^i a_1^j - 1}{1-\rho^2} + a_2^i a_2^j\right]^{-1}$. In Appendix D.1, we show that $D_o > 0$.

Equation (12) highlights the implications of consumption and equity covariances for the correlation on persistent risk. As the correlation in equity returns, $Corr(R^i, R^j)$, increases relative to the correlation in consumption, $Corr(g_c^i, g_c^j)$, the implied correlation of persistent shocks rises. Furthermore, this effect is exacerbated since the variability in equity returns, σ_R^i, significantly exceeds the variability in consumption, σ_c^i, in the data.

We next use this identification approach to pin down the empirically appropriate persistent consumption risk correlation.

2.6 Fitting Parameters: Treating equity as consumption asset

We now describe our approach to provide the best parameter fit to match asset return moments to the model. Given these parameters, we then identify the correlation in persistent consumption risk.

Since we assume countries have common preferences, we require a measure of consumption that incorporates potential risk in purchasing power variations across countries. For this purpose, we analyze annualized consumption growth adjusted for purchasing power parity deviations in the Penn World Tables following Obstfeld (1994b). For dividend and equity return data, we use quarterly data through 2009 from the Total Market Indices in Datastream-Thomson Financial while our risk-free rates are from the IMF's International Financial Statistics. We follow Colacito and Croce (2010) in restricting the asset return sample to begin in 1970. We deflate all asset returns using the common good deflator that incorporates real exchange rate risk through PPP deviations. We return to the implications for exchange rate variation in these data below. Other details of the data construction are in Appendix C.

2.6.1 Simulated Method of Moments

The persistent component in consumption must be small since deviations from annual consumption growth look close to transitory. As pointed out by Colacito and Croce (2011), estimating long run risk in international data is difficult since most countries except the UK and the US do not have sufficiently long time periods. Since we consider a multiple country approach, we calibrate, rather than estimate, our parameters. At the same time, we want to discipline our framework as tightly as possible. Therefore, we proceed in two steps. First, we restrict our preference parameters to those found by others in the long run risk literature using a longer time series. Second, we use a Simulated Method of Moments (SMM) approach to generate consumption parameter values that come closest to fitting the model-implied consumption and asset return moments to those we observe in the data. Here we briefly summarize this identification, relegating the details to Appendix C.

We analyze consumption decisions at the monthly frequency, following Bansal and Yaron (2004). According, we first calibrate the monthly growth rates, μ, to the annual means of consumption growth. We then implement Simulated Method of Moments (SMM) to provide the best fit to the parameters for each country. That is, for every set of parameter values, we first solve the model using the analytical solutions for returns in the benchmark economy. We then compute the difference between a targeted set of model generated moments and the data return and consumption moments. We weight these moments equally to give the same importance to consumption and returns. The set of parameter values that minimizes this difference is the SMM fit.

We target six data moments for each country: the standard deviation and auto-correlation of annual consumption growth, the mean equity premium, the mean risk free rate, the standard deviations of the market return and the risk free rate. Using these six moments per country, we use SMM to obtain three parameters for each country: (a) the standard deviation of the transitory component of consumption, σ^j; (b) the standard deviation of the persistent component, σ_e^j, and (c) the autocorrelation of the persistent risk component, ρ^j.

In all our estimates, we find that the autocorrelation parameters ρ^j are quite similar to each other. Therefore, in the reported results we set $\rho^j = \rho$ for all j for parsimony.

Our SMM analysis requires a set of preference parameters. We consider a range for the risk aversion parameter as $\gamma \in \{4, 10\}$ and for the intertemporal elasticity of substitution as $\psi \in \{0.5, 1.5\}$. Higher IES and risk aversion parameters help deliver the higher equity premia and lower risk-free rates observed in the data.[22] For this reason, we restrict our attention to the higher end of our parameter range with $\psi = 1.5$ and $\gamma = 10$ and assume $\beta = .985$ annually, numbers that are also consistent with Bansal and Yaron (2004).

Table 2, Panel A shows the resulting SMM-generated parameters of (σ^j, σ_e^j) along with the monthly calibrated means of consumption. The monthly growth rates, μ^j, are near 0.17% for all three countries. The transitory risk standard deviation ranges from 0.6% for the U.K. to 0.9% for the U.S. As expected, persistent consumption measured by σ_e is only a small fraction of transitory volatility. This persistent consumption risk is lowest for Canada at .026%. The U.S. has only marginally higher persistent risk variability but has the highest overall variability at 0.929% monthly. As a result, Canada will have the most valuable benchmark consumption stream in the full risk sharing economy, as reported below.

Table 2, Panel B gives the targeted moments for asset returns and consumption used to fit these parameters while Panel C reports the implied moments from our simulation. Although the standard asset pricing puzzles are present in our results, the moments improve relative to the purely transitory consumption risk in the literature. For example, the equity premium ranges between 1.1% to 1.6% in the model, substantially higher then the 35 basis points found by Mehra and Prescott (1985), but still lower than the data.[23] Similarly, the risk-free rate in the model is lowered so that the means for the U.S. and Canada are close to their data counterparts, although the rate is now too low for the U.K.[24] Similarly, the

[22] Lewis and Liu (2012) show how these moments change with varying preference parameters under i.i.d. disturbances.

[23] When we assume no persistent risk, our model generates equity premium numbers ranging from 20 to 30 basis points, consistent with Mehra and Prescott (1985).

[24] See Weil (1989) for a discussion of the risk-free rate puzzle. Indeed, our framework without persistent risk generates means for the risk-free rates in the range of 3% to 6%.

standard deviation of equity returns increases but remains too low compared to the data. Finally, although the model without persistent risk implies a constant risk-free rate, the table shows that persistent risk generates some risk-free rate volatility. Overall, while the model falls short of fitting the data moments, the addition of persistent consumption risk moves the model in the direction of higher equity premium, lower risk-free rate, and more volatile asset returns.

Panel C also shows the fit for consumption moments. The implied consumption volatility is higher than the data for all three countries. In the data, the standard deviation is about 1.7, but the model generates higher volatility ranging from 2.9 for the U.S. to 2.2 for Canada. Below we demonstrate the effects of this over-statement of consumption volatility on the risk-sharing gains. On the other hand, the implied consumption autocorrelations fit the data quite well for all three countries.

2.6.2 Identifying consumption correlations and welfare gains

We can measure the welfare gains given these consumption and preference parameters once we identify the correlations in persistent consumption risk. For this purpose, Table 3, Panel A reports the equity return correlations in the data. The correlations between equity returns are generally higher than the correlations between consumption growth rates in Table 1.[25] In particular, the equity return correlations are higher than 0.5. By contrast, the correlations between consumption growth rates are generally lower. This pattern between equity return correlations and consumption correlations is even more pronounced when we expand the set of countries below. As a result, the relationship between equity and consumption correlations in equation (12) generates high correlations for the persistent consumption risk, $Corr(e^i, e^j)$. Indeed, across all seven of our countries studied and all versions of our model, the implied correlations for persistent risk are never below 0.8.

[25]Dumas et al (2003) also find that equity correlations across countries are higher than output correlations, and use this observation to analyze the degree of integration. Bansal and Lundblad (2002) use the high international correlation in equity returns to argue that cash flow growth rates contain a small predictable component.

Table 3, Panel B then shows the implied correlations for persistent and transitory risk. As expected, the combinations of consumption and equity covariances imply a very high degree of correlation in persistent risk. In the interest of parsimony, we report only the correlation of each country against the world, values that are all near one. Panel B also shows the implied correlation between the transitory risk components. Comparing these correlations to the consumption correlations in Table 1 shows that the high correlations on persistent risk generate slightly lower correlations on the transitory risk. For example, the total correlation between Canadian and U.K. consumption is 0.32 in Table 1A, but the transitory correlation in Table 3B is only 0.29.

Table 3, Panel C reports the implied standard deviations for the world mutual fund. This measure for transitory risk, σ^*, is less than 0.6%, clearly lower than the 0.63% to 0.92% given in Table 2A for each individual country. By contrast, the implied world standard deviation of the persistent risk, σ_e^*, is 0.028%, a number within the range shown for the countries. This comparison highlights the high degree of risk-sharing in the long run risk component.

Panel D of Table 3 gives the welfare gains based upon the implied consumption correlations. Since the identified correlations on the persistent component are essentially equal to one, persistent risk is already fully diversified thereby attenuating the welfare gains. The gains range from 7.8% for Canada to 9.4% for the U.K., far lower than the levels in Table 1 reported for higher diversification potential in persistent risk.

The gains in equation (3) arise from two components. The first component is the gain from the change in the wealth-to-consumption ratio: $\left\{\frac{W_0^{j*}/C_0^{j*}}{W_0^{jB}/C_0^{jB}}\right\}^{\frac{1}{1-\frac{1}{\psi}}}$. We report these percentage gains in Table 3D in the rows labeled "Gain from W^j/C^j" for each country. Table 2 shows that the Canadian process has lower persistent consumption risk so that the wealth-to-consumption ratio for Canada declines in the risk-sharing economy and the "gain" is actually a loss of 4%.

The second component, C^{j*}/C^{jB}, captures the compensation to countries such as Canada with better diversification potential. The change in the initial consumption allocation reflects the value of each country's endowment at world prices, $\varpi^j = \left(C_0^{Bj} + P_0^{j*}\right) / \left(C_0^w + P_0^{w*}\right)$. Thus, this component is greater for countries with higher endowments and prices. In this case, the consumption in Canada's benchmark economy is the most valuable and therefore the percent gain is positive at 12%.

Since the correlation of persistent risk is close to one, the declining value of the wealth-to-consumption gains to Canada is offset by the price effect and the gains are net 7.8%. By contrast, both the U.S. and the U.K. gain from the world wealth-to-consumption ratio, but lose from initial risk-sharing consumption relative to closed economy consumption at −7% and −5%, respectively.

2.7 Relationship to Risk-Sharing Models Based Upon Exchange Rate Variation

The results in Table 3 indicate a high correlation in persistent risk that is nevertheless consistent with a low correlation in consumption data across countries. High correlation of long run risk has also been found in other studies such as Colacito and Croce (2010,2011). In these studies, complete markets is assumes so that exchange rate behavior identifies the co-movements in marginal utility growth across countries[26]. By contrast, we identify the co-movement of the marginal utility in consumption using the international correlation of asset returns and consumption. We choose this identification because our goal is to evaluate the gains from moving to full-risk sharing. As such, we do not want to assume complete markets.

Although our framework does not restrict the exchange rate behavior, relative price movements do affect the variability of the consumption growth in our data and, hence, are captured in our welfare gains measures. Specifically, our consumption growth data adjust for purchasing power parity deviations based on a panel set of multi-country World Bank pricing

[26] Risk in this model derives from relative price variations across countries, a risk channel first articulated by Cole and Obstfeld (1991).

surveys across a wide cross-section of goods. These adjustments deflate the consumption in each country by a single numeraire goods basket. Thus, we could rewrite consumption growth in country i as: $g^i_{c,t+1} = \ln(C^{im}_{t+1}/C^{im}_t) - \pi^{in}_{t+1}$ where C^{im}_t is the consumption of country i measured in local nominal "monetary" currency units and π^{in} is the inflation rate of that currency in units of the numeraire consumption basket. If the consumption growth rate were measured in local goods market units ignoring effects from international trade, the consumption growth rate would be: $g^{iL}_{c,t+1} = \ln(C^{im}_{t+1}/C^{im}_t) - \pi^{iL}_{t+1}$ where π^{iL}_{t+1} measures inflation using the local index only. The difference between the goods index of the local market and the foreign market is then the real exchange rate of country L to the numeraire consumption basket. Defining the real exchange rate relative to the world numeriare as q^i_t, then $\ln(q^i_{t+1}/q^i_t) = \pi^{iL}_t - \pi^{in}_t$.[27] As such, the typical measured consumption growth will differ from the consumption growth including purchasing power variations according to:

$$g^i_{c,t+1} = g^{iL}_{c,t+1} - ln(q^i_{t+1}/q^i_t)$$

Thus, real exchange rate movements that lower the real value of consumption are captured as consumption risk in our framework. Since these variations also affect the real value of assets to the consumer, we deflate asset returns in the same way. Table 3, Panel E reports the standard deviations for annual changes in the real exchange rate implicit in our consumption data using the U.S. as a numeraire. The standard deviation ranges from 1.24% for the Canada-U.S. rate to 3% for the U.K.-U.S. rate. Clearly, exchange rate movements contribute significant variation to the consumption growth measures.

We have intentionally imposed the least amount of structure to allow the real exchange rate to reflect possible inefficiencies in the goods market and the asset market. For this reason, we take the consumption and asset return as given and determine the current level of integration using only the Euler equation (4). Nevertheless, our measured real exchange rate

[27] Note that in a one good world, the real exchange rate has the natural interpretation as a deviation from purchasing power parity. In this case, $q_t = (S_t P_t / P^n_t)$ where S_t is the nominal exchange rate, P_t is the measured price index in country i, and P^n_t is the price index of the numeraire. Clearly then $\ln(q^i_{t+1}/q^i_t) = \ln(S^i_{t+1}/S^i_t) + \pi^{iL}_t - \pi^n_t$ where π^n_t is the inflation rate in the numeraire currency.

variations are consistent with standard explanations for purchasing power parity deviations such as transactions costs and non-tradeable goods.[28]

Overall, we treat real exchange rate variations from the data as additional sources of consumption risk. We then consider the welfare gains from moving to an optimal risk sharing economy that reduces the deleterious effects of real exchange rate movements on consumption.

2.8 Summary: Persistent Risk with Consumption-Paying Equity

In this section, we examined the gains from risk sharing when asset returns are used to discipline consumption parameters. We assumed that equity pays out consumption as measured by the data. To determine the correlations of persistent versus transitory consumption risk across countries, we used data on correlations in equity and consumption. Since cross-country equity correlations are higher than consumption correlations and since the volatility of equity is higher than consumption, the model implied high correlations in persistent consumption risk. As a result, this risk is almost completely diversified, even without full international risk-sharing.

Although this model generated better asset pricing implications than the transitory-only case, the fitted asset return and consumption moments remain far from the data. In the next section, we address a revised version of this model to improve the fit.

3 Risk Sharing and Dividend-Paying Asset Returns

So far, we have assumed that equity returns pay out consumption, following in the tradition of Mehra and Prescott (1985) and Obstfeld (1994b) among many others. However, as the analysis above shows, even persistent consumption risk does not generate a sufficiently high equity premium or volatility in returns. Moreover, the better fit for asset returns comes at

[28]We allow for transactions costs because Fitzgerald (forthcoming) shows that they are an important source for reducing risk-sharing. Differing prices of non-tradable such as housing also affect deviations in the measured price index across countries.

the cost of higher variability in consumption than observed in the data. Bansal and Yaron (2004) (hereafter BY) have argued that equity returns are better explained when persistence consumption risk depends upon dividend payments. In this section, we employ this framework to identify consumption risk and then re-examine the implications for international consumption risk-sharing.

3.1 Persistent Consumption Risk and Dividends

We now reconsider the consumption growth process with persistent consumption risk specified in equation (8), augmented with an additional dividend process. In order to match asset return behavior, BY fit the behavior of dividends and consumption growth rates to the implied estimates of asset return moments. For this purpose, they assume that the growth rate of dividends, $g_{d,t}$, depends upon the persistent component of consumption. Using a superscript to identify the country j, we rewrite their assumed dividend process as:

$$g^j_{d,t+1} = \mu^j_d + \phi^j x^j_t + u^j_{t+1} \tag{13}$$

where $u^j_{t+1} \sim N(0, \sigma^j_u)$, $u^j_{t+1} \perp \eta^j_{t+1} \perp e^j_{t+1}$ and μ^j_d is the growth rate of dividends. Note that in equation (13), dividends depend upon persistent consumption risk according to the coefficient ϕ^j. As BY have observed, the variability in dividends along with this "leverage" coefficient helps generate greater variability in persistent consumption growth, thereby generating better fit to asset returns. In our analysis, we follow BY in setting ϕ^j to 3.

3.2 Identifying Country-Specific Consumption Risk with Dividends

We now amend our asset return framework to assume that equity pays the dividend process specified in equation (13). We use the model to provide fitted values for the dividend parameters along with new estimates of the original consumption parameters. For this purpose, we add the standard deviation and autocorrelation of dividends to the set of target moments. As with consumption growth, we first calibrate the monthly growth rate of dividends. We then use SMM to fit the four parameters $\left[\sigma^j, \sigma_e^j, \rho^j, \sigma_d^j,\right]$ to eight moments: the set of six consumption and asset return moments studied before, but now augmented by the two new dividend moments. As above, the fitted values for ρ^j are quite similar to each other across countries so we restrict them to be equal in our reported results.

Table 4, Panel A reports the parameter estimates. Compared to the consumption asset model in Table 2, the variability due to persistent risk is higher for all three countries at around 0.04%. As equation (13) shows, this higher volatility is in part generated by the greater volatility of dividends as well as the leverage ratio, ϕ^j. As a result of this higher variability in persistent risk, the model does not push the overall consumption variance to be as high as in the consumption asset case.

Table 4, Panel B shows the new target moments for dividends used in this version of the model. These moments in addition to the set of asset and consumption moments reported in Table 2, Panel B give the eight targets for SMM.

Panel C provides the best fit parameters from SMM. The fitted equity premium is now close to the data at 5% for the U.S. and 6.5% for Canada, though the number for the U.K. is somewhat larger than the data. Similarly, the implied risk-free rates are now closer to the data. Importantly, the standard deviation of equity is close to the data with implied estimates between 15% to 18.5%. The standard deviation of the risk free rate is also higher, though still considerably lower than the data suggest.[29] The model also tends to predict a more volatile dividend process for the U.S. and Canada as well as somewhat greater persistence. As in the consumption asset case, the implied consumption volatility and autocorrelation is higher than in the data. Nevertheless, compared to the prior model, the dividend-based model gets closer to matching the target asset moments.

[29] BY address this issue by assuming stochastic volatility. For parsimony, we do not include this risk in the present paper. Nevertheless, the high degree of correlation across countries in volatility measures suggests that this risk is also highly diversified.

3.3 Identifying Persistent Risk Correlation when Equity Pays Dividends

We now use the newly fitted parameters to re-evaluate international risk-sharing gains. As before, we require additional restrictions from equity returns to identify the correlation in persistent consumption risk. When equity pays out dividends, we show in Appendix D.2 that the Campbell-Shiller approximation implies that equity returns for country i can be written:

$$R^i_{t+1} = b^i_0 + b^i_1 x^i_t + b^i_2 e^i_{t+1} + u^i_{t+1} \tag{14}$$

where b^i_0, b^i_1, b^i_2 are constants. Note that by contrast to the case when equity pays out consumption, returns now depend upon the innovation to dividend growth, u, instead of the innovation to transitory consumption, η. Using this relationship, we calculate the implied covariance as in equation (12) and find a simlar relationship as before. In this case, however, a higher correlation in equity returns than dividends generates the high persistent consumption risk correlation.

Table 4, Panel D reports the dividend correlation in the data. Consistent with the pattern observed between equity returns and consumption, the correlations between equity returns are higher than dividends. Furthermore, as previously reported, the standard deviations of dividends are smaller than the standard deviations of equity returns. As a result, Panel E shows that the implied correlations on persistent consumption risk are all close to one. The high correlations on persistent risk also identify a lower correlation on transitory risk, as before.

Panel F of Table 4 gives the gains implied by this decomposition, ranging from 2.7% for the U.S. and Canada to 4.2% for the U.K. Notably, these levels are consistent with those found in the risk sharing literature ignoring asset returns (e.g., Tesar (1995), van Wincoop (1994)). When equity is assumed to pay dividends instead of consumption, the implied standard deviation on persistent consumption, σ_e, is lowest for the U.K. Thus, many of the features previously observed for the lowest persistent risk country, Canada, hold here for the U.K. In particular, the U.K. has the highest share of world output at 38.3%. Furthermore, the percentage certainty equivalent change from the wealth-to-consumption ratio worsens for the U.K. at −9% while both the U.S. and Canada gain at 13% and 9%, respectively. At the same time, the U.K. gains from an improvement of initial consumption of 15% relative to the closed economy, while this ratio is lower for both the U.S. and Canada.

3.4 Persistent Risk and Risk-Sharing Gains: Uncovering the Channels

Comparing the gains from risk-sharing when equity pays consumption and when equity pays dividends highlights a surprising pattern. Although the dividend case implies greater persistent risk, the risk-sharing gains are uniformly lower. That is, Table 3C shows that the gains from risk-sharing range from around 8% to 9.5% when equity pays consumption. But Table 4F reports the counterpart gains when equity pays dividends at around 3% to 4%. This result might seem counterintuitive since greater persistent risk should make international diversification more valuable.

A problem with this comparison is that the two scenarios differ in other respects as well. Importantly, the model implied consumption variability is lower in the dividend asset case than the consumption asset case. The standard deviation of monthly consumption ranges from 0.9% to 0.7% when equity pays consumption (Table 2A) but is lower at 0.5% to 0.6% when equity pays dividends (Table 4A). Thus, the lower gains may simply reflect lower overall consumption risk.

To disentangle these two effects, we conduct a thought experiment. We constrain the data consumption volatility, σ_{gc}, to the U.S. estimates in the two asset cases. We then increase the volatility of persistent risk and recalculate the gains. Figure 1a illustrates the results. Strikingly, the gains for both cases decline as the persistent risk increases. Moreover, due to the lower volatility, the dividend case remains everywhere below the consumption asset.

The triangles mark the fitted numbers from the table. Clearly, whether the data volatility were higher, as in the consumption case or lower as as in the dividend case, higher persistent risk would reduce welfare gains.

Therefore, the lower gains in the dividend case arise from greater persistent risk and not lower consumption volatility. The intuition is clear. When persistent risk is almost perfectly diversified, an increase in the volatility of persistent risk dampens the gains to diversification. To fit the overall consumption variance given by the data, greater persistent volatility implies lower transitory volatility. Therefore, the transitory diversifiable risk is reduced.

To verify this conjecture, we consider a counterfactual experiment. We conduct the same experiment as depicted in Figure 1a, but assume instead that the correlation on persistent risk is 0.8 instead of 1. Thus, some persistent risk can be diversified. The pattern is shown in Figure 1b. As the volatility of the persistent shock, σ_e, increases, the welfare gains now increase since this risk is diversifiable. When persistent risk can be diversified, the dividend asset case has the greatest gains, highlighting once again the role of persistent risk.

3.5 Summary: Persistent Risk with Dividend-Paying Equity

In this section, we re-evaluated the model assuming that equity pays out dividends as measured by the data. This version of the model provided better fits for asset return and consumption moments. It also required a new identification of the correlations of persistent consumption across countries based on dividends. As with the consumption case, we found that persistent consumption risk is almost completely diversified, even without fully open markets.

Comparing the two versions of the model generated surprising results. Despite greater persistent risk when equity pays dividends, the gains were lower than the consumption asset case. Further analysis yielded a straightforward explanation, however. Higher volatility in persistent risk implies lower volatility in transitory risk measured by the data. Since the model implies persistent risk is essentially diversified, the lower risk on the diversifiable component means lower gains.

4 Risk-Sharing Gains and Other Considerations

In order to highlight the key features of risk-sharing with persistent consumption risk across countries, we have focused upon a number of simplifying assumptions. First, we have assumed that all countries have the same mean growth rates. Second, we have treated all countries as though they are the same size. Third, we have used observed equity correlation jointly with consumption correlation to identify the correlation in persistent risk, while an alternative would be to use the risk free rate. Fourth, we have considered a small set of three countries. However, our model framework can easily accommodate all these assumptions. In this section, we analyze the results of relaxing these four assumptions.

4.1 Differing Means

In the quantitative analysis, we have so far assumed common growth rates across countries. Here we consider the effects of relaxing this assumption. The effects on welfare gains are straightforward. The price of a country's output in the risk sharing equilibrium is increasing in the mean growth rate. At the same time, a higher growth rate economy will not benefit as much from the common growth rate in the open economy since it must share the lower growth rates of the others.

Table 5, Panel A shows that this intuition holds in our quantitative analysis as well. The top row repeats the mean annualized growth rates in Table 1 showing that the U.S. has the higher growth rate in the sample at about 2.1%. Under the sections labeled "2. Equity paying Consumption" and "3. Equity paying Dividends", the table reports the gains analysis with differing μ^i for Table 3 when equity pays consumption and for Table 4 when equity pays dividends, respectively.

Compared to the common means analysis, the U.S. receives a greater share of world output but also has a lower welfare gain than the other countries. For the dividend asset case, for example, when mean growth rates are common as in Table 4F, the share of world output is 30.2% but this share increases to 31.3% with the higher U.S. mean in Table 5A. At the same time, the gains to the U.S. decline from 2.7% with common means to 2.3% with the differing means. Overall, allowing for differing means imply a higher world share for high growth countries, but also lower welfare gains as they share in a lower growth world economy.

4.2 Differing Sizes

Above we treated the three countries as though they were all the same size, though this assumption is clearly counterfactual. Since our consumption data are measured in per capita units, we can easily recover aggregate consumption by multiplying population size. Accounting for differing sizes requires a modification of our framework. Here we describe the modified equilibrium as a decentralized economy. Appendix A shows that this equilibrium is also the solution to a social planner's problem that puts equal weight on each person in the population.

Table 5, Panel B reports the results assuming differing country sizes. The first row shows that the U.S. has the largest share of population at 70%, followed by the U.K. and then Canada. When we calculate the equilibrium using these parameters, the allocations implied by the decentralized economy do not provide a steady-state equilibrium because one or more countries have unbounded utility[30]. Therefore, we instead characterize the range of Pareto efficient allocations. That is, we calculate the gains assuming each country individually receives all the surplus while leaving every other country indifferent. Thus, we can determine the upper bound in gains for country j by calculating the gains from receiving

[30]For an equilibrium to exist we require that lifetime utility be bounded and rational along the equilibrium path for each country or that, $U^j(C_t^j, E_t[U_{t+1}^j]) \in \mathbb{R}, < \infty$. This condition may be violated in the risk-sharing economy if the price of the consumption tree from country j in world markets, P^{*j}, goes to infinity. Infinite prices can result from if a country's process is very valuable relative to the rest of the world.

all of the initial surplus consumption allocation while making all other countries indifferent; that is, by setting $\Delta^i = 0, \forall i \neq j$. For this calculation, we use equation (3) to first solve for $\widehat{C}_0^{i*}$, the initial consumption allocation for country i residents that implies no gains for residents of all countries except j,

$$1 = \left(\frac{\widehat{C}_0^{i*}}{C_0^{iB}} \right) \left\{ \frac{W_0^*/C_0^*}{W_0^{iB}/C_0^{iB}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \tag{15}$$

where W_0^{iB}/C_0^{iB} are calculated from the benchmark economy as above. Similarly, the risk sharing equilibrium wealth-to-consumption ratios W_0^*/C_0^* must be the same among the set of efficient allocations since state prices are equalized in the competitive equilibrium. The upper bounds on country j welfare among this set of allocations is then given by:

$$(1 + \Delta_{Max}^j) = \left(\frac{\widehat{C}_{0,Max}^{j*}}{C_0^{iB}} \right) \left\{ \frac{W_0^{j*}/C_0^{j*}}{W_0^{jB}/C_0^{jB}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} = \left(\frac{C_0^w - \sum_{i \neq j} n^j \widehat{C}_0^{i*}}{n^j C_0^{iB}} \right) \left\{ \frac{W_0^{j*}/C_0^{j*}}{W_0^{jB}/C_0^{jB}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \tag{16}$$

The set of efficient allocations for residents in each country i are then bracketed by the minimum consumption, $\widehat{C}_0^{i*}$, that yields zero welfare gains and the maximum consumption, $\widehat{C}_{0,Max}^{i*}$, that gives all the world welfare surplus to country i. Note that whether the country has the minimum consumption in equation (15) or the maximum consumption in equation (16), the change in the wealth-to-consumption ratio is the same.

Table 5 Panel B reports the results of these calculations under the sections labeled 2 and 3 assuming equity pays consumption and equity pays dividends, respectively. The first row shows the gains from the improvement in the wealth-to-consumption ratio. As before, these changes are positive for the U.S. and U.K., but negative for Canada in the consumption asset case under 2, while this pattern is reversed for the U.K. in the dividend asset case under 3. The remaining rows show the range in gains depending upon which country receives all the surplus. Under the consumption asset case, the U.S. receives as much as 71% of world output when Americans receive all the surplus, but that share declines to 65% when the U.S. receives no gains. By contrast, Canada loses on the wealth-to-consumption ratio but if compensated to the maximum share of 13% of world output, receives a large 78.6%

gain. Similar patterns hold for the dividend asset case under section 3 but since the U.K. has better hedge properties, its role switches with Canada.

4.3 Identifying Persistent Risk Correlation with the Risk-Free Rate

Above, we calibrate our benchmark model to both the equity returns and the risk free rate. However, to identify persistent versus transitory consumption variations, we only exploit the international correlation of equity claims and of consumption growth. We choose to focus on equity return co-movements because our framework matches the equity return moments better than those of the risk-free rate. For example, even in the dividend model in Table 4, the standard deviation in model-implied equity returns are close to the data moments while the standard deviation of the risk free rate is only about 0.8% compared to 2% to 6% in the data. This discrepancy casts doubt on the reliability of the risk-free rate correlations to accurately identify the consumption risk components.

Nevertheless, we now show how risk-free rate co-movements could also identify the cross-country correlations in consumption risk components. Pricing the risk free asset is straightforward using the Euler equation in Equation 4, where $R^\ell = R_f$. Using the same methodology as equity returns and Campbell-Shiller approximation of the return on the consumption portfolio, r^p_{t+1}, Appendix D shows that the covariance of the risk free rate is affected by only the persistent risk. That is,

$$Cov(R^i_f, R^j_f) = \left[\frac{a^i_1 a^j_1}{1-\rho^2}\right] \sigma^i_e \sigma^j_e Corr(e^i, e^j) \qquad (17)$$

where $Corr(e^i, e^j)$ is the persistent risk correlation. Like the covariance of equity returns in Equation (11), the persistent correlation increases with the autoregressive parameters ρ and the size of the persistent risk σ_e. Unlike the covariance of equities, the covariance of the risk free rate does not contain a transitory risk correlation component. Therefore, we can identify the persistent risk correlation from the covariances of the risk free rate. Indeed, equation

(17) shows that the correlation of persistent risk equals the correlation of the risk-free rate.

Identifying the persistent risk correlation from the risk free rate implies this correlation is lower than that determined by equity returns. In particular, for the three country case of U.S., U.K., and Canada, we find that the correlation is highest between U.S. and U.K. at 0.63 and lowest between U.K. and Canada at 0.26.

The lower correlations of persistent consumption risk derived from the risk free rate would generate higher welfare gains than those derived from equity returns. Given the results in Table 1, we know that the welfare gains will not exceed the case when persistent risk was assumed to be 0.2 for all country pairs. This number then gives us an upper bound for the implied welfare gains. However, as noted above, the benchmark model does not provide a good fit for the risk-free rate volatility. Therefore, given the superior fit of the benchmark model in matching observed equity returns, we argue that identifying persistent risk jointly through equity returns and consumption growth provides a more robust calibration.

4.4 More Countries

In the analysis so far, we have focused upon a small group of three countries. This analysis demonstrates how the framework can expand the number of countries over the two-country models of Colacito and Croce (2010, 2012) and Stathopoulos (2012). In principle, however, the multi-country framework described in Section 2 applies to an arbitrary number of countries. To show the analysis with more countries, we now apply our framework over seven countries. For our analysis, we consider the three countries above and include Australia, France, Germany, and Japan.

As above, we consider the effects of persistent consumption risk under the two alternative assumptions that link equity returns to the data: (1) equity pays out consumption; and (2) equity pays out dividend. We first use the target moments to try to fit consumption parameters for the new countries. We then use these parameters together with the parameters for the U.S., U.K., and Canada above to re-evaluate the risk-sharing gains. In the interest of parsimony, we only report the results for the dividend asset case since it fits returns better.

We first implement our Simulated Method of Moments approach on consumption and asset return data for the four new countries. Table 6 Panel A reports the set of consumption and dividend parameters $[\mu, \sigma, \sigma_e, \sigma_{gc}, \mu_d, \sigma_d]$. Panels B and C give the set of target data moments and implied moments, respectively. The variability in persistent consumption risk, σ_e, is similar across countries. Although Japan has the lowest variability of persistent consumption, it also has the highest variability in transitory consumption risk. Note that to fit asset returns, implied consumption variability is higher than the data as found for the other three countries. Moreover, while the autocorrelation in consumption is close to the data for most countries, it is clearly too high to match the tiny data autocorrelation for Australia.

We next consider the implications for risk-sharing using the fitted parameters for the four new countries together with the corresponding parameters for the U.S., U.K., and Canada previously reported in Table 4. Panel D of Table 6 shows these results. The first column reports the data correlations between dividends for each country and the world, although the full matrix is used in estimation. The correlations demonstrate the low correlations in dividends relative to those between equity returns, noted earlier. All correlations are less than 0.55 and that of the UK is as low as 0.33. The next two columns report the implied correlations between the world and country persistent shock, e^i as well as the transitory shock, η^i. Once again, the correlations in persistent risk are very high and close to one.

The final columns show the welfare gains. As in the population-weighted case, the decentralized economy does not have a steady state equilibrium. We therefore report the range of Pareto efficient allocations. Under the column labeled "Gains", we report the maximum gain for the row country while setting the gains for all other countries equal to zero. The following three columns report the maximum share of output for that country when setting all other country gains to zero along with the gain due to increases in wealth-to-consumption "W/C" and the change in initial consumption "C^*/C^B". For example, the gain for the U.S.

is 127% when the gains are zero for all other countries so that residents receive 28% of world per capita income. The gains from wealth-to-consumption are only 16% while the gains from receiving initial consumption is 96%. On the other hand, the last column reports the lowest world consumption share so that the U.S. is not made worse off in the world economy. At 12%, this share is significantly lower than the maximum.

The welfare gains may appear high relative to earlier tables, but the reasons are clear. First, the reported gains are the maximum if all surplus were given to one country. For the U.S. gains, for example, dividing by seven would imply an average gain per country of only about 17%. Second, the gains are larger because there are more countries, increasing the potential gains from trade.

5 Conclusion

International asset returns incorporate market valuations of risk and these valuations are central to understanding potential gains from global consumption risk sharing. Nevertheless, many studies of the gains to international risk sharing ignore the implications of these markets. In this paper, we have begun to bridge this gap by noting how features that bring the model closer to data impact views about the benefits of risk sharing.

Low frequency variations in consumption risk are key to generating the size of equity premia and volatility of asset returns. In this paper, we consider these variations as a small but persistent component of consumption shocks. For this purpose, we use data on consumption, asset returns, and in the final version, dividends to determine the best fit for seven industrialized economies. Our analysis produces three main insights.

First, we find that the magnitude of risk-sharing gains depend inversely on the degree of correlation in persistent consumption risk across countries. In other words, the consumption risk-sharing gains increase with the ability to diversify persistent risk.

Second, we provide an identification for the persistent risk correlation using consumption and equity return correlations across countries. This identification implies high correlations on persistent risk and, hence, a low diversification potential. In the data, equity return correlations are higher than consumption correlations across countries. In the model, equity returns and, hence, their correlations depend more strongly on the persistent risk component than the transitory risk component. Taken together, the model implies a high correlation in persistent risk.

Third, we show that higher volatility in persistent risk reduces the implied gains from risk sharing. Once we disentangle the diversification benefits of transitory versus persistent risk, the intuition is clear. Greater volatility in persistent risk implies lower volatility in transitory risk. Since persistent risk is already highly diversified, only transitory risk can be shared. Higher persistent risk therefore implies lower diversifiable transitory risk, thereby reducing risk sharing gains. Thus, significant international consumption risk is already shared. As a result, consumption risk-sharing gains look more similar to those generated by models that do not target asset returns.

Overall, our results shed new light on conventional views about the gains from international consumption risk sharing when disciplined by asset returns. Calibrating models with common goods preferences to asset return moments such as the equity premium do not translate into significantly higher risk sharing gains, in contrast to a conventional view. Our finding that there is significant risk-sharing is also consistent with studies that identify long run risk through exchange rates. By contrast, we do not assume markets are complete in the data. As such, our approach provides a new identification for measuring the gains from international consumption risk sharing.

References

[1] Backus, David K; Kehoe, Patrick J; Kydland, Finn E., *International Real Business Cycles*, Journal of Political Economy, vol. 100, no. 4, August 1992, pp. 745-75

[2] Barro, Robert J., *Rare Disasters and Asset Markets in the Twentieth Century*, The Quarterly Review of Economics, vol. 121, no. 3, August 2006, pp. 823-866.

[3] Barro, Robert J., *Rare Disasters, Asset Prices, and Welfare Costs*, American Economic Review, vol. 99, no. 1, March 2009, pp. 243-264.

[4] Bansal, Ravi; Lundblad, Christian, *Market Efficiency, Asset Returns, and the Size of the Risk Premium in Global Equity Markets*, Journal of Econometrics, vol. 109, August 2002, pp. 195-237.

[5] Bansal, Ravi; Yaron, Amir, *Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles*, Journal of Finance, vol. 59, no. 4, August 2004, pp. 1481-1509

[6] Baxter, Marianne; Crucini, Mario J., *Business Cycles and the Asset Structure of Foreign Trade*, International Economic Review, vol. 36, no. 4, November 1995, pp. 821-54

[7] Baxter, Marianne; Jermann, Urban J., *The International Diversification Puzzle Is Worse Than You Think*, American Economic Review, vol. 87, no. 1, March 1997, pp. 170-80

[8] Brandt, Michael W; Cochrane, John H; Santa-Clara, Pedro. *International Risk Sharing Is Better Than You Think, or Exchange Rates Are Too Smooth,* Journal of Monetary Economics, vol. 53. no. 4, May 2006, pp.671-698.

[9] Campbell, John Y., *Intertemporal Asset Pricing without Consumption Data*, American Economic Review, vol. 83, no. 3, June 1993, pp. 487-512.

[10] Campbell, John Y., *Consumption-Based Asset Pricing*, Handbook of the Economics of Finance, vol. 1B, George Constantinides, Milton Harris, and Rene Stulz eds., North Holland, Amsterdam, 2003.

[11] Campbell, John Y; Cochrane, John H., *By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior*, Journal of Political Economy, vol. 107, no. 2, April 1999, pp. 205-51

[12] Campbell, John Y; Shiller, Robert J., *The dividend-price ratio and expectations of future dividends and discount factors*, Review of Financial Studies, vol. 1, 1988, pp. 195-228.

[13] Coeurdacier, Nicolas; Rey, Helene; *Home Bias in Open Economy Financial Macroeconomics*, Journal of Economic Literature, forthcoming.

[14] Colacito, Riccardo; Croce, Mariano M. *The Short and Long Run Benefits of Financial Integration,*American Economic Review, vol. 100, no. 2, May 2010, 527-531.

[15] Colacito, Riccardo; Croce, Mariano M. *Risks for the Long Run and the Real Exchange Rate*, Journal of Political Economy, vol. 119, no. 1, February 2011, 153-181.

[16] Colacito, Riccardo; Croce, Mariano M. *International Asset Pricing with Recursive Preferences*, North Carolina Working Paper, September 2012.

[17] Cole, Harold L; Obstfeld, Maurice, *Commodity Trade and International Risk Sharing: How Much Do Financial Markets Matter?*, Journal of Monetary Economics, vol. 28, no. 1, August 1991, pp. 3-24

[18] Dumas, Bernard; Harvey, Campbell R; Ruiz, Pierre. *Are Correlations of Stock Returns Justified by Subsequent Changes in National Outputs?* Journal of International Money and Finance, vol. 22. no. 6, Nov 2003, pp. 777-811.

[19] Epstein, Larry G; Zin, Stanley E., *Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework*, Econometrica, vol. 57, no. 4, July 1989, pp. 937-69.

[20] Farhi, Emmanuel; Fraiberger, Samuel; Gabaix, Xavier; Ranciere, Romain; Verdelhan, Adrien. *Crash Risk in Currency Markets*, NBER Working Paper 15062, 2009.

[21] Fitzgerald, Doireann. *Trade Costs, Asset Market Frictions and Risk Sharing* American Economic Review, forthcoming.

[22] Gallant, A Ronald; Tauchen, George.*The Relative Efficiency of Method of Moments Estimators* Journal of Econometrics vol. 92, no.1, Sep 1999, pp. 149-172.

[23] Jonathan Heathcote; Perri, Fabrizio, *The international diversification puzzle is not as bad as you think*, University of Minnesota Working Paper, April 2008.

[24] Lewis, Karen K.,*Trying to Explain Home Bias in Equities and Consumption*, Journal of Economic Literature, vol. 37, no. 2, June 1999, pp. 571-608.

[25] Lewis, Karen K., *Why Do Stocks and Consumption Imply Such Different Gains from International Risk Sharing?*, Journal of International Economics, vol. 52, October 2000, pp. 1-35

[26] Lewis, Karen K., *Global Asset Pricing*, Annual Review of Financial Economics, vol. 3, 2011, pp. 435-466.

[27] Lewis, Karen K.; Liu, Edith X., *International Consumption Risk is Shared After All: An Asset Return View*, NBER Working Paper No. 17872, Feburary 2012

[28] Lucas, Robert E., Jr.; Stokey, Nancy L. *Recursive Methods in Economic Dynamics*, Harvard University Press, 1989.

[29] Mehra, Rajnish; Prescott, Edward C., *The Equity Premium: A Puzzle*, Journal of Monetary Economics, vol. 15, no. 2,March 1985, pp. 145-61

[30] Nakamura, Emi; Sergeyev, Dmitriy; Steinsson, Jon. *Growth-Rate and Uncertainty Shocks in Consumption: Cross-Country Evidence* NBER Working Paper w18128, June 2012.

[31] Nakamura, Emi; Steinsson, Jon; Barro, Robert; Ursua, Jose'.*Crises and Recoveries in an Empirical Model of Consumption Disasters* NBER Working Paper w15920, April 2010.

[32] Obstfeld, Maurice, *Evaluating Risky Consumption Paths: The Role of Intertemporal Substitutability*, European Economic Review, vol. 38, no. 7, August 1994a, pp. 1471-86

[33] Obstfeld, Maurice, *Risk-Taking, Global Diversification, and Growth*, American Economic Review, vol. 84, no. 5, December 1994b, pp. 1310-29

[34] Rietz, Thomas A., *The Equity Risk Premium: A Solution*, Journal of Monetary Economics, vol. 22, no. 1, July 1988, pp. 117-131

[35] Stathopoulos, Andreas, *Asset Prices and Risk Sharing in Open Economies*, Working Paper, University of Southern California, January 2012

[36] Stockman, Alan C; Tesar, Linda L., *Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements*, American Economic Review, vol. 85, no. 1, March 1995, pp. 168-85

[37] Tesar, Linda L., *Evaluating the Gains from International Risksharing*, Carnegie-Rochester Conference Series on Public Policy, vol. 42, June 1995, pp. 95-143

[38] Tesar, Linda L; Werner, Ingrid M., *Home Bias and High Turnover*, Journal of International Money and Finance, vol. 14, no. 4, August 1995, pp. 467-92

[39] van Wincoop, Eric, *Welfare Gains from International Risksharing*, Journal of Monetary Economics, vol. 34, no. 2, October 1994, pp. 175-200

[40] van Wincoop, Eric, *How Big are Potential Welfare Gains from International Risksharing?* Journal of International Economics, Vol. 47, Feburary 1999, pp. 109-135.

[41] Varian, Hal R., *Microeconomic Analysis*, W.W. Norton and Company, 1978.

[42] Verdelhan, Adrien, *A Habit-Based Explanation of the Exchange Rate Risk Premium* Journal of Finance 65: 1 February 2010, pp. 123-146.

[43] Weil, Philippe, *The Equity Premium Puzzle and the Risk-Free Rate Puzzle*, Journal of Monetary Economics, vol. 24, no. 3, 1989, pp. 401-21

[44] Weil, Philippe, *Nonexpected Utility in Macroeconomics*, Quarterly Journal of Economics, vol. 105, no. 1, February 1990, pp. 29-42

WEB APPENDICES: FOR REFEREE REFERENCE, NOT FOR PUBLICATION

A Appendix: Country Consumption Weights

In this appendix we show that the country weights in aggregate consumption are determined by the solution to a planner's problem. We first assume identically sized economies and then extend these results to differing population weights. Finally, we characterize the set of Pareto efficient allocations.

A.1 The Consumption Allocation with Identically Sized Countries

Proposition 1: *Let a^j be the planner weights on utility of country j, Q^j_τ be the state-price for country j at time τ, and $U^j(C^j_t, U^j_{t+1})$ be given by:*

$$U^j(C^j_t, U^j_{t+1}) = \left\{ {C^j_t}^{\frac{1-\gamma}{\theta}} + \beta E_t \left[\left(U^j_{t+1} \right)^{1-\gamma} \right]^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}}. \tag{18}$$

Then the solution to the planner's problem:

$$\underset{\substack{\{C^j_t\} \\ \forall t}}{MaxS} = \sum_{j=1}^{J} a^j U^j(C^j_0, U^j_1) \tag{19}$$

$$\begin{aligned} \text{s.t.} \sum_{j=1}^{J} C^j_t &= \sum_{j=1}^{J} C^{jB}_t, \forall t & (20) \\ E_0 \sum_{\tau=0}^{\infty} Q^j_\tau C^j_\tau &= E_0 \sum_{\tau=0}^{\infty} Q^j_\tau C^{jB}_\tau, \forall j & (21) \\ U^j(C^j_0, U^j_1) &\in R, < \infty, \forall j & (22) \end{aligned}$$

is given by:

$$C^j_t = \varpi^j C^w_t, \forall t,$$

where

$$\varpi^j = \frac{C^{jB}_0 + P^{j*}_0}{C^w_0 + P^{w*}_0} \tag{23}$$

for $C_t^w \equiv \sum_{j=1}^{J} C_t^{jB}$*, the world consumption in each period and for* $P_0^{j*} = E_0 \sum_{\tau=1}^{\infty} Q_\tau^* C_\tau^{jB}$ *and* $P_0^{w*} = E_0 \sum_{\tau=1}^{\infty} Q_\tau^* C_\tau^w$*, the present value of country j's benchmark consumption and the world consumption, respectively, at the world stochastic discount factor,* Q_τ^**.*

Discussion: Note that the planner maximizes utility across agents in each country given three constraints. The first constraint given in equation (20) is the resource constraint that total benchmark consumption levels from each country equals total world consumption in each period. The second constraint given in equation (21) is the lifetime budget constraint for each country. This constraint says that the expected lifetime value of consumption for each country equals the expected lifetime value of its output. The budget constraint holds in expectations both because of uncertainty (Lucas and Stokey (1989), pp. 487-490) and because current utility depends upon expected future utility. Finally, the third constraint in equation (22) requires utility to be bounded and rational along the equilibrium path.

Proof: The planner's problem can be simplified by solving for the value function of each country given the budget constraint (21). For this purpose, first note that since the state-price at time 0 is one (i.e., $Q_0^j = 1$), the lifetime budget constraint can be rewritten as:

$$C_0^j + E_0 \sum_{\tau=1}^{\infty} Q_\tau^j C_\tau^j = C_0^{jB} + E_0 \sum_{\tau=1}^{\infty} Q_\tau^j C_\tau^{jB}, \forall j$$

Or as:

$$C_0^j + P_0^{jc} = W_0^j \equiv C_0^{jB} + P_0^{jB}, \forall j \tag{24}$$

where P_t^{jB} is the price of country j benchmark consumption at state prices Q_τ^j and P_t^{jc} is the value of country j consumption. Thus, the budget constraint simply states that current consumption plus the future expected value of consumption equals current country benchmark consumption plus the expected value its future stream.

To solve for the value function of each country, we then solve for the Bellman equation:

$$V(C_0^j, W_0^j) = \underset{\substack{\{C_t^j\} \\ \forall t}}{Max} \left\{ C_t^{j\,\frac{1-\gamma}{\theta}} + \beta E_t \left[\left(U_{t+1}^j \right)^{1-\gamma} \right]^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \tag{25}$$

$$s.t.(24) \text{ holds}$$

This problem has the solution (Campbell (1993), Obstfeld (1994a)):

$$V(C_t^j, W_t^j) = \left(C_t^j \right)^{-\frac{1/\psi}{1-(1/\psi)}} \left(W_t^j \right)^{\frac{1}{1-(1/\psi)}} = C_t^j \left(\frac{W_t^j}{C_t^j} \right)^{\frac{1}{1-(1/\psi)}} \tag{26}$$

Then the value function can be determined given the solution to equilibrium wealth. This solution in turn depends upon the equilibrium price of benchmark consumption, P_t^{jB}. But this price can be determined using the Euler equation for the return on the asset paying out benchmark consumption (Epstein and Zin (1989)):

$$E_t \left\{ \beta^{\theta} (C_{t+1}^j / C_t^j)^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^{jc})^{(\theta-1)} R_{t+1}^{jB} \right\} = 1 \tag{27}$$

where $R_{t+1}^{jc} \equiv (C_{t+1}^j + P_{t+1}^{jc})/P_t^{jc}$and $R_{t+1}^{jB} \equiv (C_{t+1}^{jB} + P_{t+1}^{jB})/P_t^{jB}$. Similarly, the equilibrium price of the consumption asset can be determined using the Euler equation for its return:

$$E_t \left\{ \beta^{\theta} (C_{t+1}^j / C_t^j)^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^{jc})^{\theta} \right\} = 1 \tag{28}$$

So these solutions to the value functions give us J value functions in terms of J sets of country state prices, Q_τ^j. However, in a Pareto competitive equilibrium with heterogeneous agents but identical preferences, these state prices must be equal across agents. (See for example, Varian (1978), p. 152.) Using our notation above, these equilibrium state prices correspond to the common Q_τ^*. Therefore, all agents share the same Euler equations (27) and (28). As a consequence, consumption growth rates are equated by the planner:

$$(C_{t+1}^j / C_t^j) = (C_{t+1}^i / C_t^i), \forall i, j, t$$

Thus, in the equilibrium, per capita consumption levels are proportional to aggregate consumption. Defining this proportion for country j as ϖ^{jw},

$$C_t^j = \varpi^{jw} C_t^w, \forall t. \tag{29}$$

In this case, the lifetime expected consumption in the budget constraint (24) becomes:

$$E_0 \sum_{\tau=0}^{\infty} Q_\tau^* C_\tau^j = \varpi^{jw} C_0^w + E_0 \sum_{\tau=1}^{\infty} Q_\tau^* \varpi^{jw} C_\tau^w = \varpi^{jw} \left(C_0^j + P_0^{w*} \right)$$

We now substitute the value function for each country (26) into the planner problem (19) to rewrite the problem as:

$$\underset{\substack{\{C_t^j\} \\ \forall t}}{Max} S = \sum_{j=1}^{J} a^j V(C_0^j, W_0^j)$$

$$\begin{aligned} \text{s.t.} \sum_{j=1}^{J} C_t^j &= \sum_{j=1}^{J} C_t^{jB}, \forall t \\ \varpi^{jw} (C_0^w + P_0^{w*}) &= W_0^j \equiv C_0^{jB} + P_0^{*j}, \forall j \end{aligned}$$

Clearly then $\varpi^{jw} = \frac{C_0^{jB}+P_0^{*j}}{C_0^w+P_0^{w*}}$ as in equation (23), verifying the proposition above.

Using the definition of wealth, note also that:

$$\frac{W_t^j}{C_t^j} = \frac{\varpi^{jw}\left(C_t^w + P_t^{w*}\right)}{\varpi^{jw} C_t^w} = \frac{C_t^w + P_t^{w*}}{C_t^w}$$

Therefore, the wealth-consumption ratio is equal for all countries in equilibrium.

Moreover, the planner weights are equalized across countries. To see why, note that the first-order condition for period 0 is:

$$\frac{\partial S}{\partial C_0^j} = \frac{a^j}{1-\psi}\left(\frac{W_t^j}{C_t^j}\right)^{\frac{1}{1-(1/\psi)}} - \lambda = 0 \tag{30}$$

Rearranging equation (30) and using the fact that $\frac{W_t^j}{C_t^j} = \frac{W_t^i}{C_t^i}, \forall i, j$ implies that $a^i = a^j$ as required for a utilitarian planner (Varian (1978), pp. 152-154.), thus verifying the proposition.

A.2 The Consumption Allocation with Differing Population Sizes

The consumption allocations above are derived assuming all countries have the same number of agents or, alternatively, the planner cares about countries equally regardless of size. Here we recalculate the planner allocations assuming that in each country j there are N^j people and the planner cares about maximizing over all individual utilities.

Each person in each country is endowed with the claim to the stream of one unit of *per capita* benchmark consumption in his home country, C_t^{jB}, $\forall t$. Defining the number of people in country j as N^j, total benchmark consumption in country j is $N^j C^{jB}$. Thus, there are now N^j claims to benchmark consumption of country j available. At time 0, each person in country j sells his share and purchases shares in the world consumption process. Thus, the budget constraint for country j as a whole implies

$$N^j\left(C_0^{jB} + P_0^{j*}\right) = N^w\left(C_0^w + P_0^{w*}\right)\varpi_0^{jw}$$

where total population is $N^w = \Sigma_{j=1}^J N^j$. Solving for the share of country j agents in world markets then implies: $\varpi^{jw} = n^j\left(C_0^{jB} + P_0^{j*}\right)/\left(C_0^w + P_0^{w*}\right)$ where $n^j \equiv N^j/N^w$. That is, the share of country j in the world market is equal to its share in the world wealth, $\frac{C_0^j+P_0^{j*}}{C_0^w+P_0^{w*}}$, multiplied by its share in world population, n^j.

Proposition 2: *Let a^{ji} be the planner weights on utility of resident i in country j, Q_τ^j be the state-price for country j at time τ, and consumption and utility of agent i in country j at time t*

be C_t^{ji} and U_t^{ji}, respectively, where U_t^{ji} is the Epstein-Zin utility given in equation (18). Then the solution to the planner's problem:

$$\underset{\underset{\forall t}{\{C_t^j\}}}{Max} S = \sum_{j=1}^{J}\sum_{i=1}^{N^j} a^{ji} U^j(C_0^{ji}, U_1^{ji}) \tag{31}$$

$$\text{s.t.} \sum_{j=1}^{J}\sum_{i=1}^{N^j} C_t^{ji} = \sum_{j=1}^{J}\sum_{i=1}^{N^j} C_t^{jB}, \forall t \tag{32}$$

$$E_0 \sum_{\tau=0}^{\infty} Q_\tau^j C_\tau^{ji} = E_0 \sum_{\tau=0}^{\infty} Q_\tau^j C_\tau^{jiB}, \forall i, j \tag{33}$$

is given by:

$$C_t^j = \varpi^{jw} N^w \widetilde{C}_t^w, \forall t,$$

where

$$\varpi^{jw} = n^j \frac{C_0^{jB} + P_0^{j*}}{\widetilde{C}_0^w + \widetilde{P}_0^{w*}} \tag{34}$$

for $C_t^j \equiv N^j C_t^{ji}$, the consumption in each country j; $N^w \equiv \Sigma_{j=1}^J N^j$, the world population; $\widetilde{Y}_t^w = Y_t^w / N^w$, the world per capital output; $n^j \equiv \left(N^j/N^w\right)$, the country j population share; $\widetilde{P}_0^{w*} = E_0 \sum\limits_{\tau=1}^{\infty} Q_\tau^* C_\tau^w$, the price of world per capita output. and as before, $P_0^{j*} = E_0 \sum\limits_{\tau=1}^{\infty} Q_\tau^* C_\tau^{jB}$, the price of country j per capital benchmark consumption at world prices.

Proof: The population-weighted planner problem can be solved as a straightforward extension to the identical sized country version above. First note that as above the identical preferences implies that consumption growth rates are equalized or that:

$$(C_{t+1}^{j\ell}/C_t^{j\ell}) = (C_{t+1}^{iq}/C_t^{iq}), \forall i, j, \ell, q, t$$

Therefore, consumption across individuals differ only by a proportional initial condition. Moreover, since agents in each country are identical, in equilibrium $C_t^{ji} = C_t^{j\ell}\ \forall i, \ell$ so each agent holds identical shares in world output of ϖ_0^{jw}/N^j. As a result, individual consumption can be rewritten:

$$C_t^{ji} = \left(\varpi_0^{jw}/n^j\right) \widetilde{C}_t^w, \forall t,$$

where we have used the fact that aggregate world output can be written as world per capita output times world population or $C_t^w = \widetilde{C}_t^w \Sigma_{\ell=1}^J N^\ell$. Using this solution in the individual lifetime budget constraint in equation (33) and solving for ϖ^{jw} verifies the consumption allocations in equation (34). Moreover, by the competitive equilibrium, $Q_\tau^j = Q_\tau^*$ as before. Thus, substituting the solutions for the prices P_0^{j*} and $\widetilde{P}_0^{w*}$ into the individual value function in (26) and then solving for the initial period first order condition to the planner problem (31), verifies that $a^{ji} = a^{\ell q}, \forall j, i, \ell, q$ corresponding to utilitarian planner weights, thereby proving the proposition.

Discussion: Thus, equation (34) implies the shares in world consumption are the same as the equal population case in Proposition 1 except for two differences. First, the shares are weighted by population shares, n^j. As such larger countries have higher shares in world output. Second, the price of world consumption is now a population weighted average of country benchmark consumption.

A.3 The Set of Pareto Efficient Allocations

The solution to the planner's problem does not always correspond to a steady state equilibrium. This tendency becomes more pronounced when the consumption parameters differ significantly. For these cases, we characterize the set of efficient allocations so that risk-sharing generates gains for some countries without making others worse off. These allocations provide the boundaries for the efficient set.

Proposition 3: *Let $V(C_t^i, W_t^i)$ be the value functions given by the individual Bellman equation (25) and let $V(C_0^{iB}, W_0^{iB}) = C_t^{iB}\left[1 + \frac{P_t^{iB}}{C_t^{iB}}\right]^{\left(\frac{1}{1-(\frac{1}{\psi})}\right)}$ be the value function in the benchmark economy. Then the initial consumption allocations that maximize country j utility without making all other countries i worse off solves the problem:*

$$\underset{C_\tau^\ell, \forall \tau, \ell}{Max}\, V(C_0^j, W_0^j) \tag{35}$$

$$\begin{aligned} \text{s.t. } V(C_0^i, W_0^i) &\geq V(C_0^{iB}, W_0^{iB}), \forall i \neq j \\ \text{s.t. } \sum_{j=1}^{J} C_t^j &= C_t^w \equiv \sum_{j=1}^{J} C_t^{jB}, \forall t \end{aligned}$$

and is given by the set of $\left\{\widehat{C}_0^{j*}, \widehat{C}_0^{i*}\ \forall i \neq j\right\}$ determined by giving the reservation initial consumption levels $\widehat{C}_0^{i*}$ to all $i \neq j$ countries (equation (15)):

$$1 = \left(\frac{\widehat{C}_0^{i*}}{C_0^{iB}}\right)\left\{\frac{W_0^*/C_0^*}{W_0^{iB}/C_0^{iB}}\right\}^{\frac{1}{1-\frac{1}{\psi}}}$$

and giving the residual consumption from the resource constraint to country j (equation (16)):

$$(1 + \Delta^j) = \left(\frac{C_0^w - \sum\limits_{i \neq j} \widehat{C}_0^{i*}}{C_0^j}\right)\left\{\frac{W_0^{j*}/C_0^{j*}}{W_0^{jB}/C_0^{jB}}\right\}^{\frac{1}{1-\frac{1}{\psi}}}$$

Proof: Since the efficient Pareto allocation implies common state prices, the consumption growth rates across countries are shared as above and consumption levels are a constant share of world consumption. Thus, as before, the wealth-to-consumption ratios are equalized across countries and the problem to determine the boundary of the efficient set in equation (35) can be rewritten:

$$\begin{aligned} \underset{C_\tau^\ell,\forall\tau,\ell}{Max}\, C_0^j V\left(\frac{W_0^j}{C_0^j}\right) \text{ s.t. } C_0^i V\left(\frac{W_0^*}{C_0^*}\right) &= C_0^i V\left(\frac{W_0^{iB}}{C_0^{iB}}\right), \forall i \neq j \\ \text{s.t. } \sum_{j=1}^{J} C_0^j &= C_0^w \end{aligned}$$

Using the fact that the gains can be written as:

$$1+\Delta^i = \left(\frac{C_0^{i*}}{C_0^i}\right)\left\{\frac{W_0^*/C_0^*}{W_0^{iB}/C_0^{iB}}\right\}^{\frac{1}{1-\frac{1}{\psi}}}$$

the constraints clearly imply that for all but country j, the allocations are determined by setting $\Delta^i = 0$ so that equation (15) holds. Maximizing the utility to country j then means allocating all remaining consumption to country j so that $\widehat{C}_0^{j*}$ is determined by equation (16) verifying the proposition.

B Appendix: Model Solutions and Analysis

In this appendix, we describe the solutions to the risk sharing gains as well as asset returns for the model. To calculate the gains from risk sharing, we require solutions to the value function under the benchmark economy and the full risk-sharing economy. As noted above, the general solution to this value function is[31]:

$$V(C_t^j, W_t^j) = C_t^j\left(\frac{W_t^j}{C_t^j}\right)^{\frac{1}{1-(1/\psi)}}$$

for $W_t^j = C_t^j + P_t^{jc}$ where P_t^{jc} is the time t expected value of lifetime consumption for investor j. All prices are determined by the Euler equation (4) in the text:

$$E_t\left\{\beta^\theta (C_{t+1}^j/C_t^j)^{\left(-\frac{\theta}{\psi}\right)}(R_{t+1}^P)^{(\theta-1)}R_{t+1}^\ell\right\} = 1$$

where $R_{t+1}^P = (C_{t+1}^j + P_{t+1}^{jc})/P_t^{jc}$ is the return on the asset that pays out consumption and R_{t+1}^ℓ is the return on any asset. Then clearly the Euler equation for the consumption asset can be written

[31] We also checked our solution against the solution implied by the guess-and-verify approach substituting consumption growth in the utility function as in Lewis (2000).

as:

$$E_t \left\{ \beta^{\theta} (C_{t+1}^j / C_t^j)^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^P)^{\theta} \right\} = 1 \tag{36}$$

We next describe the solution for P_t^{jc} and the value function $V(C_t^j, W_t^j)$.

When consumption includes persistent risk, consumption growth is given by equation (8), reproduced here:

$$\begin{aligned} g_{y,t+1}^j &= \mu^j + x_t^j + \eta_{t+1}^j \\ x_{t+1}^j &= \rho x_t^j + e_{t+1}^j \end{aligned}$$

We now substitute the process for $\exp(g_{c,t+1}^j) = \left(C_{t+1}^j / C_t^j\right)$ into the Euler equation (36). With persistent risk, it is not possible to solve the value function in closed form. Therefore, we follow Bansal and Yaron (2004) in assuming returns can be approximated using the Campbell-Shiller approximation[32]:

$$R_{t+1}^j = k_0^j + k_1^j z_{t+1}^j - z_t^j + g_{t+1}^j \tag{37}$$

where $z_t^j = \ln(P_t^j / D_t^j)$, the log of the price-to-payout ratio for the asset, and where k_0^j and k_1^j are approximating constants[33]. For the return on the consumption asset, for example, $z_t^j \equiv \ln(P_t^{cj} / C_t^j)$, the log of the price-to-consumption ratio while for the asset paying country j benchmark consumption, $z_t^j \equiv \ln(P_t^{*j} / C_t^{jB})$. We use these relationships below to determine the welfare in the benchmark and the risk sharing economy.

B.1 Benchmark Economy Welfare

Since the value function and the return process depends upon the price-to-payout ratio, it is necessary to solve for this ratio, $P_t^{cj} / C_t^j = \exp(z_t^j)$. In the benchmark economy, consumption is just given by the benchmark level so that $\exp(z_t^j) \equiv P_t^{cj} / C_t^{jB}$. Following Bansal and Yaron (2004), we conjecture that the log price-to-consumption ratio is linear in the persistent risk. Thus,

$$z_t^j = A_0^j + A_1^j x_t^j. \tag{38}$$

[32] However, these approximations can lead to misleading conclusions. As pointed out by Hansen (forthcoming), the true returns from recursive preferences depend upon a nontrivial factorization.

[33] The constants are $k_1^j = \frac{exp(\bar{z}^j)}{1+exp(\bar{z}^j)}$ and $k_0^j = log(1 + exp(\bar{z}^j)) - k_1^j \bar{z}_j$, where $\bar{z}^j$ is the steady state log price to consumption ratio.

Substituting equation (38) into the consumption asset Euler equation above and taking expectations implies:

$$A_1^j = \frac{1-\frac{1}{\psi}}{1-k_1^j\rho} \tag{39}$$

$$A_0^j = \ln(\beta) + \left(1-\frac{1}{\psi}\right)\left[\widetilde{\mu}^j - \frac{1}{2}\gamma\sigma^2\left(1+\frac{\varphi_e^2\ k_1^{j2}}{(1-k_1\rho)^2}\right)\right] + \frac{k_0^j}{1-k_1^j} \tag{40}$$

where $k_0^j = log(1+exp(\bar{z}^j)) - k_1^j\bar{z}_j$ and $k_1^j = exp(\bar{z}^j)/\left(1+exp(\bar{z}^j)\right)$. Note that the approximating constants k_0^j, k_1^j depend upon the solution to the long run value of z_t^j so our solution solves for the fixed point between the z_t^j equation (38) and the constant A_0^j in equation (40).

Defining $Z_t^j \equiv \exp(z_t^j)$, the value function can be found by substituting the solution for the price-to-consumption ratio into the wealth equation giving:

$$V(C_t^{jB}, W_t^{jB}) = C_t^{jB}\left(1+\frac{P_t^j}{C_t^{jB}}\right)^{\frac{1}{1-(1/\psi)}} = C_t^{jB}\left(1+Z_t^j\right)^{\frac{1}{1-(1/\psi)}}$$

B.2 Risk Sharing Economy Welfare

In the full risk sharing economy, we follow the same steps to find the consumption for country j as a weighted share of world consumption: $C_t^j = \varpi^j C_t^w$ where $\varpi^j = \left(C_0^{jB} + P_0^{j*}\right)/(C_0^w + P_0^{w*})$.

We begin with the price of world consumption, P_0^{w*}. In this case, the common growth rate across countries is the weighted sum of the country growth rates:

$$\begin{aligned} g_{c,t+1}^* &= \mu^* + x_t^* + \eta_{t+1}^* \\ x_{t+1}^* &= \rho x_t^* + e_{t+1}^* \end{aligned}$$

where $\mu^* \equiv \frac{1}{J}\Sigma_{j=1}^J\ \mu^j$, $x_t^* \equiv \frac{1}{J}\Sigma_{j=1}^J\ x_t^j$, $\eta_t^* \equiv \frac{1}{J}\Sigma_{j=1}^J\ \eta_t^j$ and $e_t^* \equiv \frac{1}{J}\Sigma_{j=1}^J e_t^j$ so that $\sigma^{*2} = \left(\frac{1}{J}\right)^2 \iota'\ \Sigma\ \iota$ and $\sigma_e^{*2} = \left(\frac{1}{J}\right)^2 \iota'\ \Sigma_e\ \iota$ for Σ and Σ_e, the variance-covariance matrix of transitory and persistent shocks, respectively, and ι, a J-dimensional unit vector. Note that this specification assumes the autocorrelation in persistent shocks ρ are common across countries. We also solved the model relaxing this assumption, though it significantly complicated the analysis without altering the results much.

With this process for world consumption, the log price-to-consumption process can be rewritten:

$$z_t^w = A_0^w + \sum_{i=1}^J A_j^w x_t^i \tag{41}$$

Substituting equation (41) and the world process $g_{c,t+1}^*$ into the consumption asset Euler equation (36) above and taking expectations implies:

$$A_1^w = \frac{1-\frac{1}{\psi}}{1-k_1^*\rho}$$

$$A_0^w = \ln(\beta) + \left(1-\frac{1}{\psi}\right)\left[\widetilde{\mu}^* - \frac{1}{2}\gamma\sigma^{*2}\left(1+\frac{\varphi_e^{*2}\ k_1^{*2}}{(1-k_1^*\rho)^2}\right)\right] + \frac{k_0^*}{1-k_1^*}$$

where the approximating constants k_0^*, k_1^* are the same as before but correspond to the world price-to-consumption ratio. The solution to this fixed point problem determines $Z_t^w \equiv \exp(z_t^w)$.

Next, we require the price of country j benchmark consumption in world markets. For this purpose, we solve for the price-to-payout ratio given by:

$$z_t^{*j} = A_0^{*j} + \sum_{i=1}^{J} A_{i,1}^{*j} x_t^i$$

Substituting this price-to-payout equation into the Euler equation and taking expectations yields:

$$A_{i,1}^{*j} = \frac{(\theta-1-\frac{\theta}{\psi}) + (\theta-1)(k_1^w\rho-1)A_j^w}{1-k_1^{*i}\rho},$$

$$A_0^{*i} = \frac{[\theta\ln\beta + (\theta-1-\frac{\theta}{\psi})\sum_j w_j\mu^j + (\theta-1)(k_0^w - A_0^w(1-k_1^w)) + k_0^{*i} + \mu^i] + \frac{1}{2}\sigma^{*2} + \frac{1}{2}\sigma_e^{*2}}{1-k_1^{*i}}.$$

Solving the fixed point for the new approximating constants, k_0^{*i} and k_1^{*i} determine the equilibrium $Z_t^{*j} = \exp(z_t^{*j})$. Then the country share is given by:

$$\varpi^j = \frac{\left(C_0^{jB} + P_0^{j*}\right)}{\left(C_0^w + P_0^{w*}\right)} = \frac{C_0^{jB}\left(1+Z_0^{*j}\right)}{C_0^w\left(1+Z_0^w\right)} \tag{42}$$

When we allow for differing population sizes, we amend the process for the sum of growth rates to include population weights. Using these population-weighted shares, the world parameters become $\mu^* \equiv \Sigma_{j=1}^J n^j\ \mu^j$ for the mean growth rate and $\sigma^{*2} = \left(\frac{1}{J}\right)^2 n'\ \Sigma\ n$ and $\sigma_e^{*2} = \left(\frac{1}{J}\right)^2 n'\ \Sigma_e\ n$ for the transitory and persistent variance, respectively, where n is the j dimensioned vector of population shares and Σ and Σ_e are the variance-covariance matrices of transitory and persistent shocks, respectively.

B.3 Welfare gains

We can now calculate the welfare gains as before using the Z solutions. The general form for the welfare gains is given by Δ^j in:

$$V_0((1+\Delta^j)C_0^{jB}, (1+\Delta^j)W_0^{jB}) = V_0(C_0^{j*}, W_0^{j*})$$

$$(1+\Delta^j) = \frac{\varpi^j C_0^w}{C_0^{jB}} \left[\frac{1+\frac{P_0^{w*}}{C_0^w}}{1+\frac{P_0^j}{C_0^{jB}}}\right]^{\left(\frac{1}{1-\frac{1}{\psi}}\right)} = \frac{\varpi^j C_0^w}{C_0^{jB}} \left[\frac{1+Z_0^w}{1+Z_0^j}\right]^{\left(\frac{1}{1-\frac{1}{\psi}}\right)}$$

where the share ϖ^j is given above in equation (42).

B.4 Implied Returns

Given the price-to-consumption solutions for benchmark economy, Z_t^j, and the risk sharing economy, Z_t^w, we then calculate the returns using the Campbell-Shiller equation (37) as well as the risk-free rate by solving the Euler equation for:

$$E_t\left\{\beta^\theta (C_{t+1}^j/C_t^j)^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^P)^{(\theta-1)}\right\} R_t^{Rfree} = 1$$

C Appendix: Empirical Methods

In this appendix we describe the empirical methods used in our analysis.

C.1 Data Description

Our analysis requires data for consumption, asset returns, and dividends. Moreover, our framework considers risk from variations in a common good. Therefore, we must adjust all consumption, returns, and dividends to insure they are valued in units of this common good. For consumption, we use per capita consumption from the Penn World Tables National Accounts measured with a common Purchasing Power Parity (PPP) price deflator from 1950 to 2009. As such, real exchange rate variations appear as purchasing power deviations that add to the variability in our consumption data.

For dividend and equity return data, we use quarterly data from the Total Market Indices in Datastream-Thomson Financial from 1970 to 2009. For the risk-free rates we update the series in Campbell (2003) using the IMF's International Financial Statistics. To be consistent with the annual consumption data, we first aggregate the quarterly data to annual. We then use the common good deflator from Penn World Tables to form real annual equity returns, risk free rates, and dividend growth rates. Therefore, as with our consumption measures, the real value of these asset returns incorporate real exchange rate risk through PPP deviations.

C.2 Solutions and Simulated Method of Moments

We solve for the consumption process parameters in our model by fitting target moments from a reduced Simulated Method of Moments (SMM). We conduct this analysis for both versions of our model: (a) equity as the "consumption asset"; and (b) equity as the "dividend asset".

To generate the parameter values, we first calibrate the monthly growth rates μ and μ_d to the annual means of consumption growth and dividend growth. For this purpose, we calculate the mean annual growth rates from the data and divide by 12. In trial runs of the SMM procedure described below, we find that this change makes little difference in the estimation of the remaining parameters and greatly decreases the computation time.

We then use the reduced SMM to fit the remaining parameters for each country: $\left[\sigma^j, \sigma_e^j, \rho^j\right]$ for the "consumption asset" case, and $\left[\sigma^j, \sigma_e^j, \sigma_d^j, \rho^j\right]$ for the "dividend asset" case. Implementing the SMM procedure involves the following steps. For every set of parameter values, we first solve the model using the analytical solutions for returns in the benchmark economy. We choose a set of targeted moments to best represent both consumption and asset pricing data[34]. We then compute a weighted difference between a targeted set of model-generated moments and the data moments using a weighting matrix. To treat all targets equally, we report the estimates using the identity matrix.[35] The set of parameter values that minimizes this difference is the SMM estimate.

For the "consumption asset" model, we choose the following set of target data moments for each country: the standard deviation of log consumption growth (σ_{gc}), the first order auto-correlation of log consumption growth (ρ_{gc}), the mean equity premium ($E(R^p - R^{rf})$), the mean risk free rate ($E(R^{rf})$), the standard deviation of the market return ($\sigma(R^p)$), and the standard deviation of the risk free rate ($\sigma(R^{rf})$). Using these six moments per country, we estimate the three parameters capturing the transitory risk, σ^j, persistent risk, σ_e^j, and degree of persistence, ρ^j. As a practical matter, we find that fitted values of ρ^j are quite similar across countries so we equate them in the analysis reported in the paper.

For the "dividend asset" model, we first follow Bansal and Yaron (2004) in setting the sensitivity of dividends to persistent risk; i.e., $\phi^j = 3$. For this version of the model, we augment the number of targets in the six moment "consumption asset" set to include the standard deviation of dividend growth (σ_{gd}), and the first order auto-correlation of log consumption growth (ρ_{gd}). Using these eight moments per country, we fit the same three consumption parameters along with the standard

[34]See Gallant and Tauchen (1999) for a discussion on efficient method of moments and problems with moment selection.

[35]We also implemented the reduced SMM procedure using a diagonal matrix with typical components equal to the sample variance. This procedure gave qualitatively similar results.

deviation of monthly dividend growth σ_d^j. Once again, the ρ^j values obtained are similar across countries so that we set them equal in the reported results.

Our approach requires a set of preference parameters. For this purpose, we use parameter estimates that have been found to fit asset returns best in the US. We therefore take the parameters from Bansal and Yaron (2004) of IES = 1.5, $\gamma = 10$, and monthly $\beta = .998$ or annualized $\beta = .985$. As is required from our model, these parameters are the same across all countries.

The model is estimated at the monthly level and therefore the simulated data from the model must be time-aggregated to match the annual data moments. To time-aggregate, we compute the growth between the levels at t and $t + 12$, given the realizations of 12 monthly growth rates.[36] To match our annual consumption, dividend growth and asset return moments, we then time-aggregate the model-generated data from monthly to annual frequency. Parameter estimates and simulated model moments are the averages of 500 simulations, each with 840 time-aggregated monthly observations.

C.3 Monte Carlo Experiments

As noted in the Appendix B, the solution for the world equilibrium approximates the aggregate world consumption growth rate as the weighted sum of the individual country growth rates. This approximation treats the log growth rate of the sum of outputs as the sum of the log growth rates of output. For example, in the equally-weighted model, the world consumption growth rate is assumed to follow: $g_{c,t}^w \equiv \frac{1}{J}\Sigma_{j=1}^J \; g_{c,t}^j$ Since each of the processes are conditionally log normal and the solution to the Euler equation assumes log normality, this approximation may render the solution approach invalid.

To evaluate this approximation, we conducted a Monte Carlo experiment First we used the processes for the individual log-normally distributed growth rates $g_{c,t}^j$ to generate 1000 draws. We then constructed the resulting world growth rate process $g_{c,t}^w$. On this simulated series, we calculated the skewness and kurtosis moments. We found that these moments matched closely the normal distribution, suggesting that the approximated world growth rate is close to being log-normally distributed.

[36]By comparison, we multiply monthly rates times 12 when we annualize as opposed to time aggregate.

D Appendix: Persistent Consumption Risk Correlation

In this appendix, we detail the identification of correlation in the shock to persistent risk, e_t^i. Here we outline the three ways to identifying persistent risk. First, we identify the persistent risk with equity returns, priced as a consumption asset, combined with consumption growth. Second, we show that a similar relationship holds when we specify equity returns as a dividend asset. And third, we identify the persistent risk directly through the risk free asset return.

D.1 Consumption Asset

The consumption process with persistent risk is given by:

$$\begin{aligned} g_{y,t+1}^j &= \mu^j + x_t^j + \eta_{t+1}^j \\ x_{t+1}^j &= \rho x_t^j + e_{t+1}^j \end{aligned}$$

where $\eta_{t+1}^j \backsim N(0, \sigma^j)$ and $e_{t+1}^j \backsim N(0, \sigma_e^j)$.

Then clearly the covariance of consumption across any two countries i and j is given by equation (9) in the text:

$$Cov(g_c^i, g_c^j) = \sigma^i \sigma^j Corr(\eta^i, \eta^j) + \frac{\sigma_e^i \sigma_e^j}{1-\rho^2} Corr(e^i, e^j)$$

In order to identify the correlations from η separately from e, we require an independent observation of these correlations. Recall that the Campbell-Shiller approximation of returns in equation (37) states:

$$R_{t+1}^j = k_0^j + k_1^j z_{t+1}^j - z_t^j + g_{t+1}^j$$

where here $z_t^j \equiv \ln(P_t^{c,j}/C_t^j)$, since equity is assumed to payout consumption. Moreover, we have solved above in Appendix B for the the log price-to-consumption ratio z_t^j as

$$z_t^j = A_0^j + A_1^j x_t^j$$

where A_0^j and A_1^j are given by equations (40) and (39), respectively. Substituting these solutions into the equation for z_t^j and the result into the Campbell-Shiller approximation in (37) and rearranging implies that equity returns for country i can be written in the form:

$$R_{t+1}^i = a_0^i + a_1^i x_t^i + a_2^i e_{t+1}^i + \eta_{t+1}^i$$

where a_0^i, a_1^i, a_2^i are given by:

$$\begin{aligned} a_0^i &= k_0^i + k_1^i A_0^i - A_0^i + \mu^i \\ a_1^i &= k_1^i A_1^i \rho - A_1^i + 1 \\ a_2^i &= k_1^i A_1^i \end{aligned}$$

Calculating the covariance of equity returns between any two countries i and j using this solution yields:

$$Cov(R_{t+1}^i, R_{t+1}^j) = \sigma^i \sigma^j Corr(\eta_{t+1}^i, \eta_{t+1}^j) + \left[\frac{a_1^i a_1^j}{1-\rho^2} + a_2^i a_2^j\right] \sigma_e^i \sigma_e^j Corr(e_{t+1}^i, e_{t+1}^j)$$

given as equation (11) in the text.

Combining the consumption covariances in equation (9) with the equity covariance in equation (11), we solve for the correlation in the persistent shock as:

$$Corr(e^i, e^j) = D_o \frac{\sigma_R^i \sigma_R^j}{\sigma_e^i \sigma_e^j} \left[Corr(R^i, R^j) - \frac{\sigma_c^i \sigma_c^j}{\sigma_R^i \sigma_R^j} Corr(g_c^i, g_c^j)\right]$$

where $D_o \equiv \left[\frac{a_1^i a_1^j - 1}{1-\rho^2} + a_2^i a_2^j\right]^{-1}$. Substituting the solutions for A_1^i for the $a_1^i, a_1^j, a_2^i, a_2^j$ parameters and using the fact that $\psi > 1$ and k_1^i and k_1^j in our analysis verifies that $D_o > 0$. Since the data implies $\sigma_R^i \sigma_R^j \gg \sigma_c^i \sigma_c^j > \sigma_e^i \sigma_e^j$ and since $Corr(R^i, R^j) > Corr(g_c^i, g_c^j)$, the correlation on the persistent risk, $Corr(e^i, e^j)$, must be high. Since the standard deviations of e and η are fitted to the data, the implied correlations of the components can in principle exceed 1. In such instance, we restrict the correlations to equal 1.

D.2 Dividend Asset

The consumption process with persistent risk in dividends is given by:

$$\begin{aligned} g_{c,t+1}^j &= \mu^j + x_t^j + \eta_{t+1}^j \\ x_{t+1}^j &= \rho^j x_t^j + e_{t+1}^j \\ g_{d,t+1}^j &= \mu_d^j + \phi^j x_t^j + u_{t+1}^j \end{aligned}$$

where $\eta_{t+1}^j \sim N(0, \sigma^j)$, $e_{t+1}^j \sim N(0, \sigma_e^j)$, $u_{t+1}^j \sim N(0, \sigma_u^j)$, $u_{t+1}^j \perp \eta_{t+1}^j \perp e_{t+1}^j$ and μ_d^j is the growth rate of dividends.

The covariance of the consumption process across countries is the same as before as given by equation (9). However, now equity pays out dividends so we must solve for the price-dividend ratio. Defining the log price-to-dividend ratio as $z^j_{mt} \equiv \ln(P^{;j}_t / D^j_t)$, we conjecture the form of the process:

$$z^j_{mt} = A^j_{0,m} + A^j_{1,m} x^j_t \tag{43}$$

Substituting the return process into the Euler equation and solving for the constants implies[37]:

$$A^j_{1,m} = \frac{\phi - \frac{1}{\psi}}{1 - k^j_{1,m}\rho}$$

where k^j_{1m} is the approximating constant for the dividend paying asset. Substituting the solutions for $A^j_{1,m}$ into equation (43) and the resut into the Campbell-Shiller equation (37) generates equity returns of the form:

$$R^i_{t+1} = b^i_0 + b^i_1 x^i_t + b^i_2 e^i_{t+1} + u^i_{t+1}$$

where b^i_0, b^i_1, b^i_2 are given by:

$$\begin{aligned} b^i_0 &= k^i_{0,m} + k^i_{1,m} A^i_{0,m} - A^i_{0,m} + \mu^i_d \\ b^i_1 &= k^i_{1,m} A^i_{1,m}\rho - A^i_{1,m} + \phi^i \\ b^i_2 &= k^i_{1,m} A^i_{1,m} \end{aligned}$$

where k^j_{1m} is the approximating constant counterpart to k^j_1 for the dividend paying asset. Calculating the covariance of equity returns between i and j implies:

$$Cov(R^i, R^j) = \sigma^i_u \sigma^j_u Corr(u^i, u^j) + \left[\frac{b^i_1 b^j_1}{1-\rho^2} + b^i_2 b^j_2\right] \sigma^i_e \sigma^j_e Corr(e^i, e^j) \tag{44}$$

Comparing the covariance in equation (44) with the implied covariances when equity pays out consumption in equation (11) shows that the relationships are similar except that the correlation and volatility in transitory dividend shocks (u) replace their counterparts for transitory consumption shocks (η).

Therefore, to identify the effects of dividend shocks, we require the covariance of dividends across countries. Using the expression for dividend growth in equation (13), the covariance between dividend growth in country i and j can be written:

$$Cov(g^i_d, g^j_d) = \sigma^i_u \sigma^j_u Corr(u^i, u^j) + \phi^i \phi^j \frac{\sigma^i_e \sigma^j_e}{1-\rho^2} Corr(e^i, e^j) \tag{45}$$

[37]The correlation does not depend upon A^j_{0m} so its solution is ommitted to save space.

Note that the dividend covariance in equation (45) has the same form as the covariance of consumption growth in equation (9) with two important changes. First, the covariance in transitory consumption shocks is replaced by the covariance in transitory dividend shocks. Second, "leverage" parameters $\phi^i\phi^j$ now appear in the second term, reflecting the covariance of persistent consumption risk.

Given the covariance in equity returns (equation (44)) and the covariance in dividends (equation (45)), we can now solve for the correlation in persistent consumption risk, $Corr(e^i, e^j)$, in terms of the equity return and dividend growth cross-country correlations:

$$Corr(e^i, e^j) = B_o \frac{\sigma_R^i \sigma_R^j}{\sigma_e^i \sigma_e^j} \left[Corr(R^i, R^j) - \frac{\sigma_d^i \sigma_d^j}{\sigma_R^i \sigma_R^j} Corr(g_d^i, g_d^j) \right]$$

where $B_o \equiv \left[\frac{b_1^i b_1^j - \phi^i \phi^j}{1-\rho^2} + b_2^i b_2^j \right]^{-1}$. Given our parametrization, $B_o > 0$ when $\phi^i \phi^j > 1$, a condition that is satisfied by the BY assumptions that $\phi = 3$. As with the consumption asset case, the data relationships imply high correlations in persistent risk, e^i. As the correlation in equity returns, $Corr(R^i, R^j)$, increases relative to the correlation in dividends, $Corr(g_d^i, g_d^j)$, the implied correlation of persistent shocks rises, an effect reinforced when the variability in dividends, σ_d^i, is less than that of equity returns, σ_R^i. That is, empirically we find $\sigma_R^i \sigma_R^j > \sigma_d^i \sigma_d^j > \sigma_e^i \sigma_e^j$. Moreover, $Corr(R^i, R^j) > Corr(g_d^i, g_d^j)$. As a result, $Corr(e^i, e^j)$ is high and near one.

D.3 Using the Risk Free Asset

Using Euler condition in Equation 4, we can solve for the return of the risk free rate.

$$r_{f,t} = -\theta ln\delta + \frac{\theta}{\psi} E_t(g_{t+1}^j) + (1-\theta) E_t(r_{a,t+1}^j) - 0.5 * Var_t(-\frac{\theta}{\psi} g_{t+1}^j + (\theta - 1) r_{a,t+1}^j) \quad (46)$$

Following the previous two sections, we assume that the log price-to-consumption ratio is linear in the persistent risk and the Campbell-Shiller approximation of returns. The return on the consumption asset in the stochastic discount factor can be represented as:

$$r_{a,t+1}^j = k_0^j + k_1^j z_{t+1}^j - z_t^j + g_{c,t+1}^j$$

where z_{t+1}^j is the price-to-consumption ratio. Substituting the above equation and the specification of consumption growth from Equation 8, we can easily express the covariance of the risk free rate for country j and country i can as a function of the long run correlation, $corr[e_t^i, e_t^j]$:

$$cov(r^i_{f,t}, r^j_{f,t}) = (\frac{\theta}{\psi} + (\theta-1)(A^i_1 - 1 - k^i_1 A^i_1 \rho))(\frac{\theta}{\psi} + (\theta-1)(A^j_1 - 1 - k^j_1 A^j_1 \rho))(\frac{(\sigma^i \varphi^i_e)(\sigma^j \varphi^j_e)}{1-\rho^2})corr[e^i_t, e^j_t] \tag{47}$$

However, the standard deviation of the risk free rate of any country j is simply:

$$\sigma(r^j_{f,t}) = (\frac{\theta}{\psi} + (\theta-1)(A^j_1 - 1 - k^j_1 A^j_1 \rho))\frac{(\sigma^j \varphi^j_e)}{\sqrt{1-\rho^2}} \tag{48}$$

Substituting the above standard deviation for country i and country j, in to the left side of the covariance of the risk free rate in Equation 47, we can easily see that the correlation on the risk free rate, $corr[r^i_{f,t}, r^j_{f,t}]$, is exactly equal to the correlation on the persistent risk, $corr[e^i_t, e^j_t]$.

Table 1: Consumption, Welfare Gains and Persistent Risk

Panel A: Consumption Growth Statistics				Correlation		
	Mean	S.D.	AC	U.S.	U.K.	Can
United States	2.08	1.76	0.27	1.00	0.49	0.63
United Kingdom	1.99	1.72	0.40	0.49	1.00	0.32
Canada	1.96	1.73	0.38	0.63	0.32	1.00

Panel B: Welfare Gains and Persistent Risk Correlation

Cross-Country Correlation		**Corr**$(\mathbf{e}^j, \mathbf{e}^w) =$				
	$\sigma_e = \mathbf{0}$	**0.0**	**0.2**	**0.5**	**0.8**	**1.0**
US	10.2	70.0	54.2	34.0	17.4	7.8
Portfolio Share	(28.4)	(29.7)	(30.0)	(37.4)	(30.7)	(30.9)
UK	12.6	86.0	65.5	40.3	20.2	9.0
Portfolio Share	(36.9)	(33.0)	(32.6)	(32.2)	(31.9)	(31.7)
Canada	8.4	75.7	58.1	35.9	17.8	7.6
Portfolio Share	(34.7)	(37.3)	(37.4)	(30.4)	(37.4)	(37.4)

Notes: All variables in percent. For each country, first line gives total % gains in consumption implied by Table 2 parameters. Second line in parenthesis reports percentage shares in world output, ϖ^j.

Table 2: Parameters and Targeted Moments			
Country	United States	United Kingdom	Canada
Panel A: Monthly Parameters			
Mean (μ)	.173	.166	.164
Transitory Std Dev (σ)	.920	.630	.660
Persistence Std Dev (σ_e)	.027	.030	.026
Cons Std Dev (σ_{gc})	.929	.648	.673
Panel B: Targeted Moments			
Equity Premium-Mean	4.3	4.5	6.5
Equity Return-Std Dev	17.6	23.5	17.6
Risk-free Rate - Mean	1.5	3.9	2.5
Risk-free Rate - Std Dev	2.2	2.8	6.0
Consn Growth - Std Dev	1.8	1.7	1.7
Consn Growth - Autocorrelation	0.3	0.4	0.4
Panel C: Simulated Moments			
Equity Premium-Mean	1.6	1.2	1.1
Equity Return-Std Dev	3.6	2.8	2.7
Risk-free Rate - Mean	1.8	2.2	2.2
Risk-free Rate - Std Dev	0.5	0.5	0.4
Consn Growth - Std Dev	2.9	2.3	2.2
Consn Growth - Autocorrelation	0.3	0.5	0.4

Notes: All variables in percent. Model assumes common mean $\mu^* = .168$. All reported simulations based upon $\rho = 0.979$, $\gamma = 10$,$\psi = 1.5$, and annual $\beta = 0.985$

Table 3: Equity Correlations and Gains			
Equity as Consumption Asset			
Country	United States	United Kingdom	Canada
A. Equity Return Correlation:			
United States	1.00	0.75	0.72
United Kingdom	0.75	1.00	0.59
Canada	0.72	0.59	1.00
B. Implied Correlations[a]			
Corr($\mathbf{e}^i, \mathbf{e}^w$):	1.00	1.00	1.00
Corr(η^i, η^j):			
United States	1.00	0.48	0.62
United Kingdom	0.48	1.00	0.29
Canada	0.62	0.29	1.00
Corr(η^i, η^w):	0.70	0.59	0.64
C: Implied World	σ^*	σ_e^*	σ_{gc}^*
Standard Deviations	.599	.028	.614
D: Welfare Gains	United States	United Kingdom	Canada
Total Gain	7.9	9.4	7.8
Portfolio Share	(30.9)	(31.7)	(37.4)
Gain from W^j/C^j	17	15	-4
Gain from C^{j*}/C^{jB}	-7	-5	12
E. Std Dev (Δ Exchange rate)[b]			
Real (PPP against US)	NA	3.05	1.24

[a]"Implied Correlations" determined from cross-country equity and consumption correlations (Table 1A). [b]Data standard deviation of annual changes in real exchange rate measured as purchasing power of GDP basket in column country against the U.S.

Table 4: Dividend Model Parameters and Gains			
Country	United States	United Kingdom	Canada
Panel A: Monthly Parameters			
Transitory Std Dev (σ)	.604	.469	.454
Persistence Std Dev (σ_e)	.044	.040	.044
Cons Std Dev (σ_{gc})	.641	.509	.499
Dividend Mean (μ_d)	.186	.339	.201
Dividend SD (σ_d)	3.03	3.74	3.63
Panel B: Targeted Moments[a]			
Dividend - Std Dev	7.1	6.8	13.0
Dividend - Autocorrelation	0.1	0.3	0.3
Panel C: Simulated Moments			
Equity Premium-Mean	5.0	5.7	6.5
Equity Return-Std Dev	15.2	18.5	18.3
Risk-free Rate - Mean	2.0	2.0	1.9
Risk-free Rate - Std Dev	0.7	0.7	0.8
Consn Growth - Std Dev	2.6	2.4	2.6
Consn Growth - Autocorrelation	0.6	0.6	0.7
Dividend - Std Dev	9.6	12.1	12.2
Dividend - Autocorrelation	0.4	0.4	0.4
Panel D: Dividend Correlation:			
United States	1.00	0.35	0.37
United Kingdom	0.35	1.00	0.12
Canada	0.37	0.12	1.00
Panel E: Implied Correlations[b]			
$\mathbf{Corr}(\mathbf{e}^i, \mathbf{e}^w)$:	0.996	0.996	1.000
Panel F: Welfare Gains	2.7	4.2	2.7
Portfolio Share	(30.2)	(38.3)	(31.4)
Gain from W^j/C^j	13	-9	9
Gain from C^{j*}/C^{jB}	-9	15	-6

Notes: All variables in percent. Model assumes common $\mu^* = .168$, $\mu_d^* = .201$. All reported simulations set $\rho = 0.979$, $\gamma = 10$, $\psi = 1.5$, and annual $\beta = 0.985$. [a] Additional to those in Table 2B. [b] Implied Correlations based on dividend and equity correlations.

Table 5: Differing Means, Sizes and Gains

Country	United States	United Kingdom	Canada
A: Differing Means and Gains			
1. Annual Means	2.08	1.99	1.96
2. Equity paying Consumption			
Welfare Gains	8.3	9.5	7.5
Portfolio Share	(32.6)	(31.4)	(36.0)
Gain from W^j/C^j	10.8	16.3	-0.6
Gain from C^{j*}/C^{jA}	-2.2	-5.9	8.1
3. Equity paying Dividends			
Welfare Gains	2.3	4.5	2.4
Portfolio Share	(31.3)	(38.1)	(30.6)
Gain from W^j/C^j	8.9	-8.5	11.4
Gain from C^{j*}/C^{jA}	-6.1	14.2	-8.1
B: Differing Sizes and Gains			
1. Population Weights	.70	.23	.06
2. Equity paying Consumption			
Gain from W^j/C^j	8	6	-11
Maximum Gains[a]	8.8	26.3	78.6
Portfolio Share	(71)	(27)	(13)
Gain from C^{j*}/C^{jB}	1	19	101
Minimum Portfolio Share[b]	(65)	(22)	(7)
3. Equity paying Dividends			
Gain from W^j/C^j	8	-14	4
Maximum Gains[a]	3.0	7.2	31.0
Portfolio Share	(67)	(29)	(8)
Gain from C^{j*}/C^{jB}	-5	24	26
Minimum Portfolio Share[b]	(65)	(27)	(6)

Notes: All variables in percent. Panel A reports gains for the consumption asset case with differing means. Panel B gives the gains for dividend asset case. Panel C reports gains for the consumption asset case with differing population sizes. Panel D gives the gains for dividend asset case. [a] Results give bounds for efficient allocations, where $\Delta^j = 0$. [b] Shares that imply $\Delta^\ell = 0$ for column country ℓ.

Table 6: Many Countries and Gains

A. Parameters	Consumption Mean and SD				Dividend			
	Mean (μ)	Trans (σ)	Persist (σ_e)	Total (σ_{gc})	Mean (μ_d)	SD (σ_d)		
Australia	0.170	0.620	0.051	0.668	0.280	4.32		
France	0.212	0.672	0.049	0.714	0.267	5.25		
Germany	0.157	0.562	0.044	0.602	0.398	4.50		
Japan	0.322	1.092	0.036	1.111	0.233	5.15		
Implied World	0.195	0.403	0.042	0.454	NA	NA		
B: Target Moments	Equity Prem	Equity S.D.	Rfree Mean	Rfree. S.D	Con S.D	Con A.C.	Div S.D	Div A.C.
Australia	7.1	22.1	1.6	6.3	2.2	0.03	11.8	0.48
France	7.6	25.6	1.8	5.9	1.8	0.52	14.0	0.19
Germany	6.4	23.1	4.2	4.5	1.6	0.61	12.6	0.43
Japan	2.2	25.0	2.6	5.2	3.2	0.68	10.2	0.61
C: Implied Moments								
Australia	7.6	20.4	1.6	0.8	3.0	0.62	13.7	0.4
France	7.9	23.4	1.9	0.9	3.1	0.60	16.4	0.4
Germany	6.6	21.2	1.8	0.8	2.7	0.62	14.3	0.4
Japan	3.6	20.9	2.4	0.6	3.6	0.39	15.0	0.3
D: Welfare Gains and Correlations	Div Corr with World	Implied Corr		Efficient Set Range				
				Max[a]				Min[b]
		(e^i, e^w)	(η^i, η^w)	Gains	Share	W/C	C^*/C^B	Share
United States	0.44	0.96	0.42	127	28	16	96	12
United Kingdom	0.33	1.0	0.28	100	31	-9	119	16
Canada	0.54	0.91	0.37	122	29	11	100	13
Australia	0.50	0.88	0.06	192	24	75	67	08
France	0.47	1.0	0.35	180	24	64	70	09
Germany	0.51	0.92	0.32	125	28	14	97	13
Japan	0.48	0.83	0.38	111	30	1	108	14

Notes: All reported simulations based upon $\gamma = 10$, $\psi = 1.5$,and annual $\beta = 0.985$

[a]Results give bounds for efficient allocations where $\Delta^j = 0$ for all countries but row country.

[b]Shares that imply $\Delta^\ell = 0$ for row country ℓ.

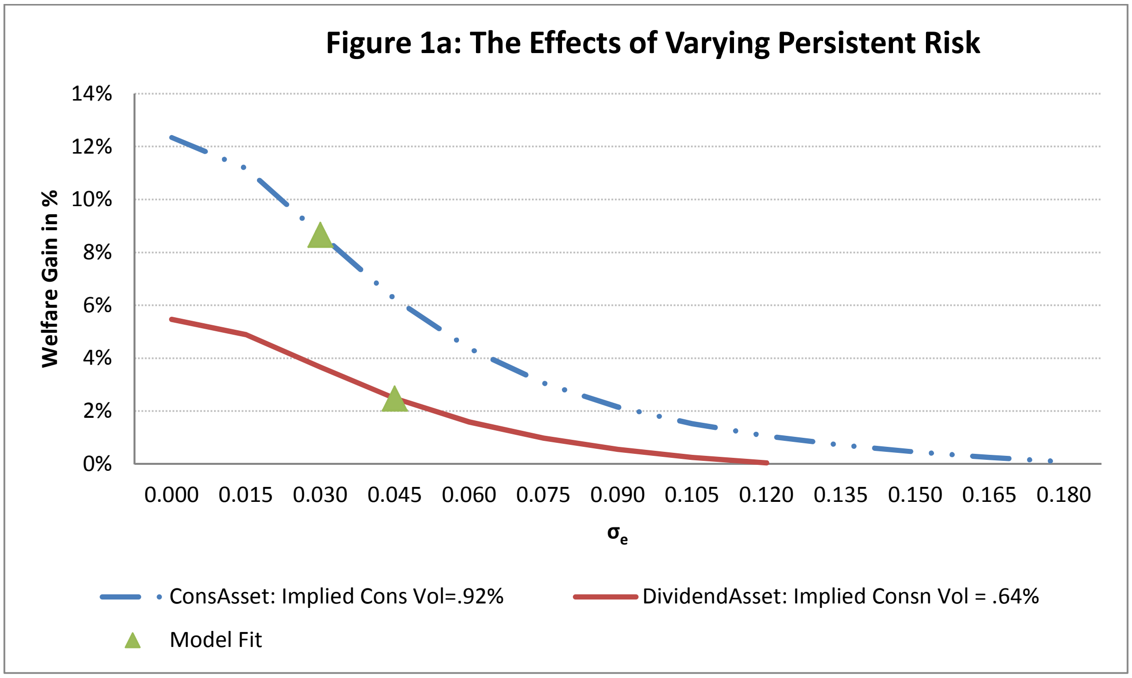

Figure 1a: The Effects of Varying Persistent Risk

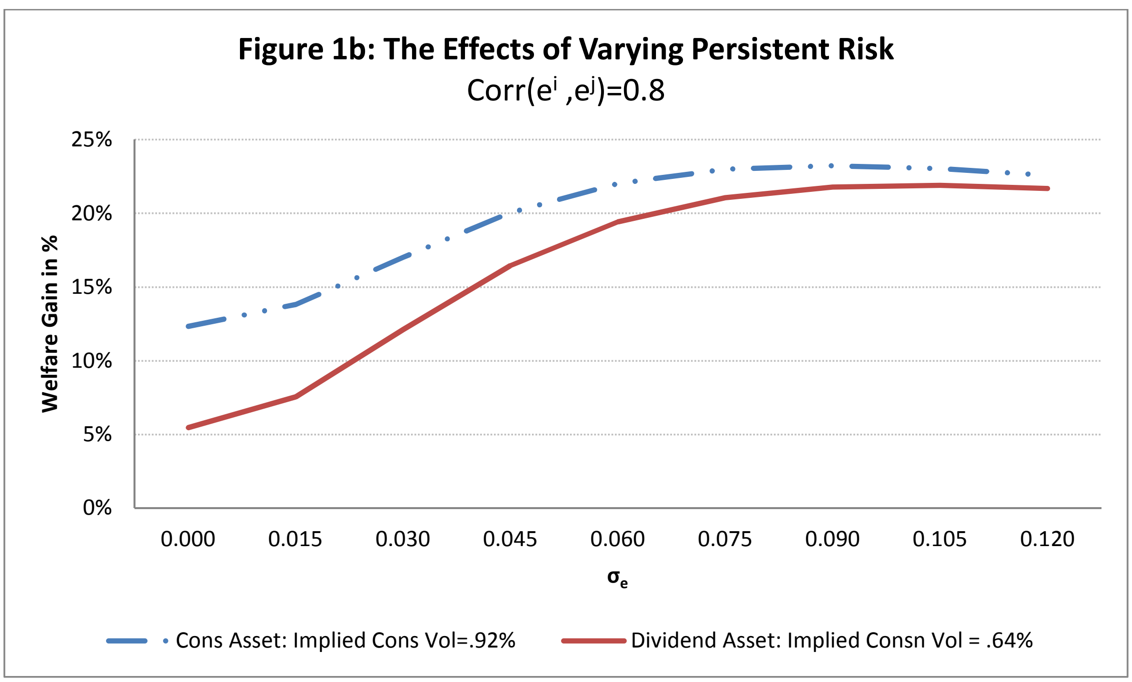

Figure 1b: The Effects of Varying Persistent Risk

Corr(e^i, e^j)=0.8